The American University in Cairo

Computer Science and Engineering Department

Fall 2022

CSCE 363/3611 – Digital Signal Processing

Assignment #2

(Due on: November 6, 2022)

(This assignment is individual – Submit on Blackboard as one .zip file)

Problem 1

Determine the z-transforms of the following signals including the Region of Convergence (ROC) and sketch the corresponding pole-zero plot. In the solution, you can use the z-transform pairs and properties tables:

a) $x(n)=\{-1,-2,\underset{\uparrow}{0},1,1,1,0,0\}$

b) $x(n)=\{\underset{\uparrow}{10},3,-1,5\}$

c) $x(n)=\left(\frac{1}{5}\right)^n u(n-5)$

d) $x(n)=\begin{cases}\left(\frac{1}{2}\right)^n, & 3\le n\le 10\\ 0, & Otherwise\end{cases}$

e) $x(n)=n^2 u(n-5)$

Solution:

a) $X(z)=\sum_{n=-\infty}^{\infty} x[n]z^{-n}=-z^2-2z+z^{-1}+z^{-2}+z^{-3}=\frac{-z^5-2z^4+z^3+z+1}{z^3}$

ROC: Entire z-plane except at $z = 0$ and $z = \infty$

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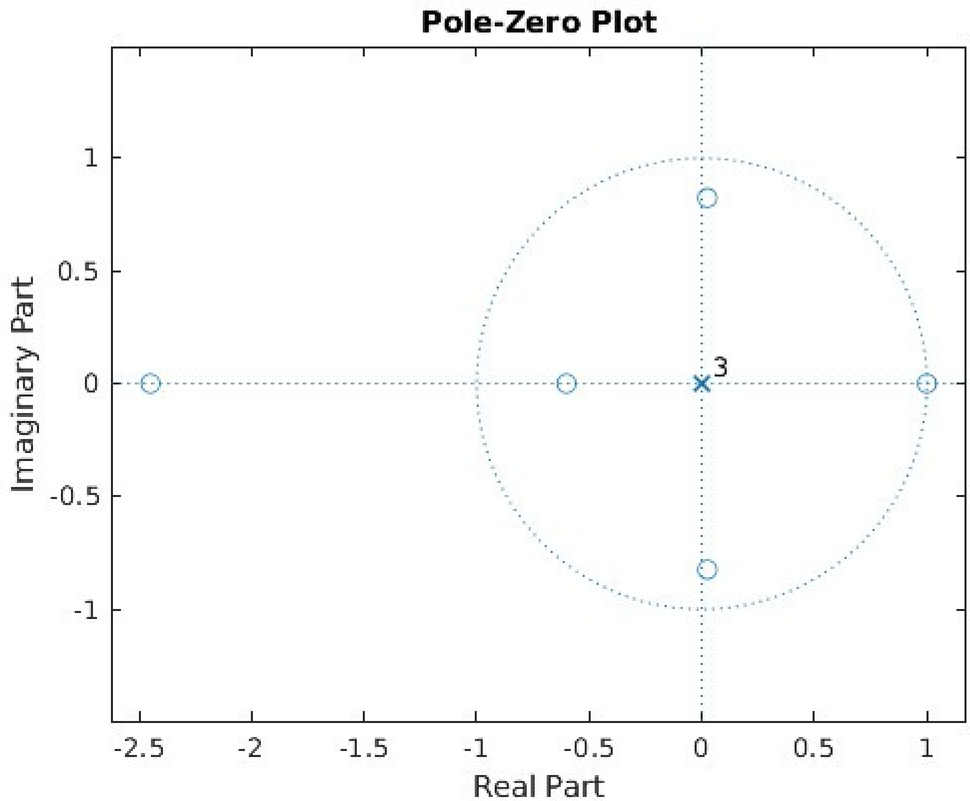

b) $X(z)=\sum_{n=-\infty}^{\infty} x[n]z^{-n}=10+3z^{-1}-z^{-2}+5z^{-3}=\frac{10z^3+3z^2-z+5}{z^3}$

ROC: Entire z-plane except at z = 0

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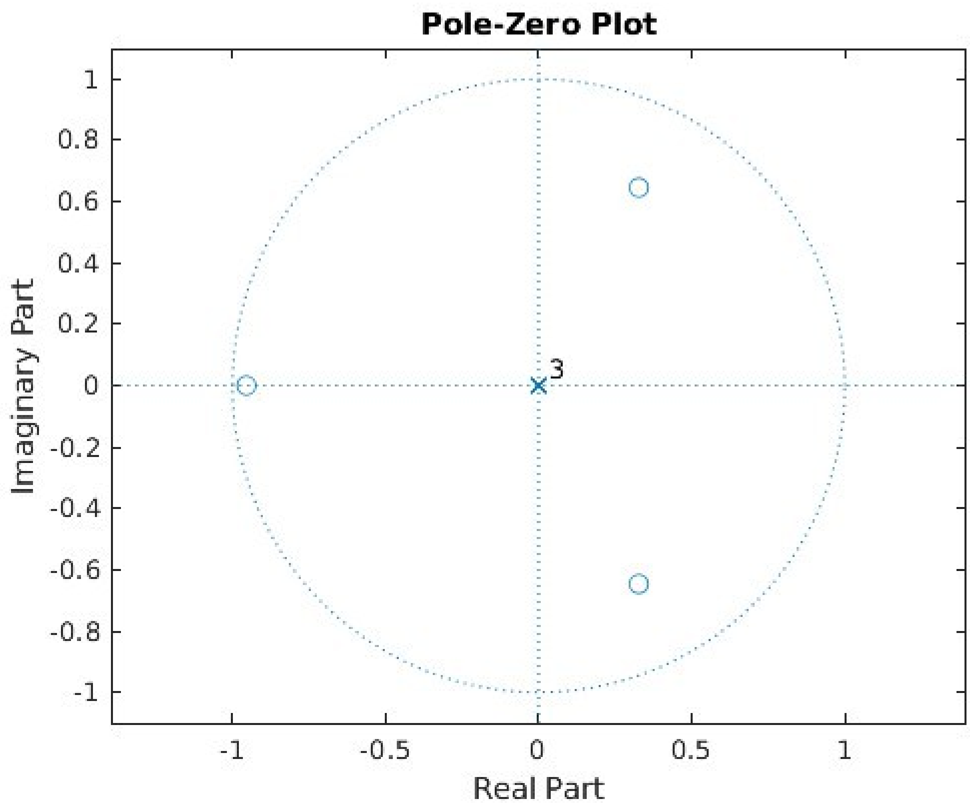

c)

$$X(z)=\sum_{n=-\infty}^{\infty} x[n]z^{-n}=\sum_{n=-\infty}^{\infty}\left(\frac{1}{5}\right)^{n}u(n-5)z^{-n}=\sum_{n=5}^{\infty}\left(\frac{1}{5}\right)^{n}z^{-n}=\sum_{n=5}^{\infty}\left(\frac{1}{5z}\right)^{n}=\frac{\left(\frac{1}{5z}\right)^{5}}{1-\frac{1}{5z}}=\frac{1}{625z^{4}(5z-1)}$$

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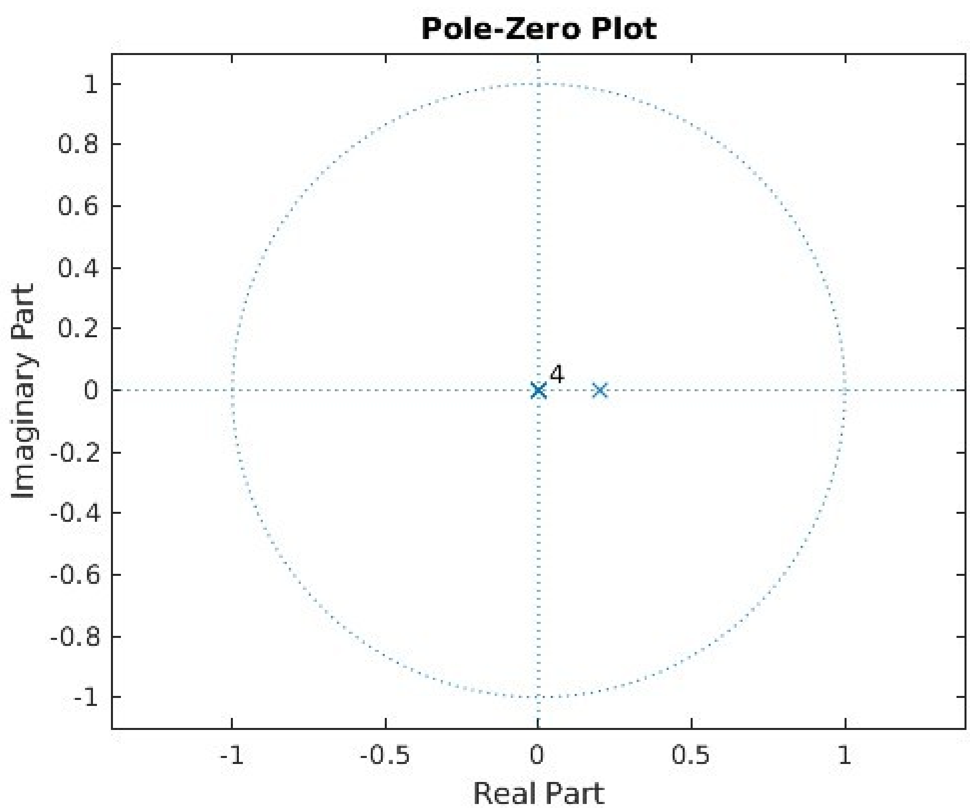
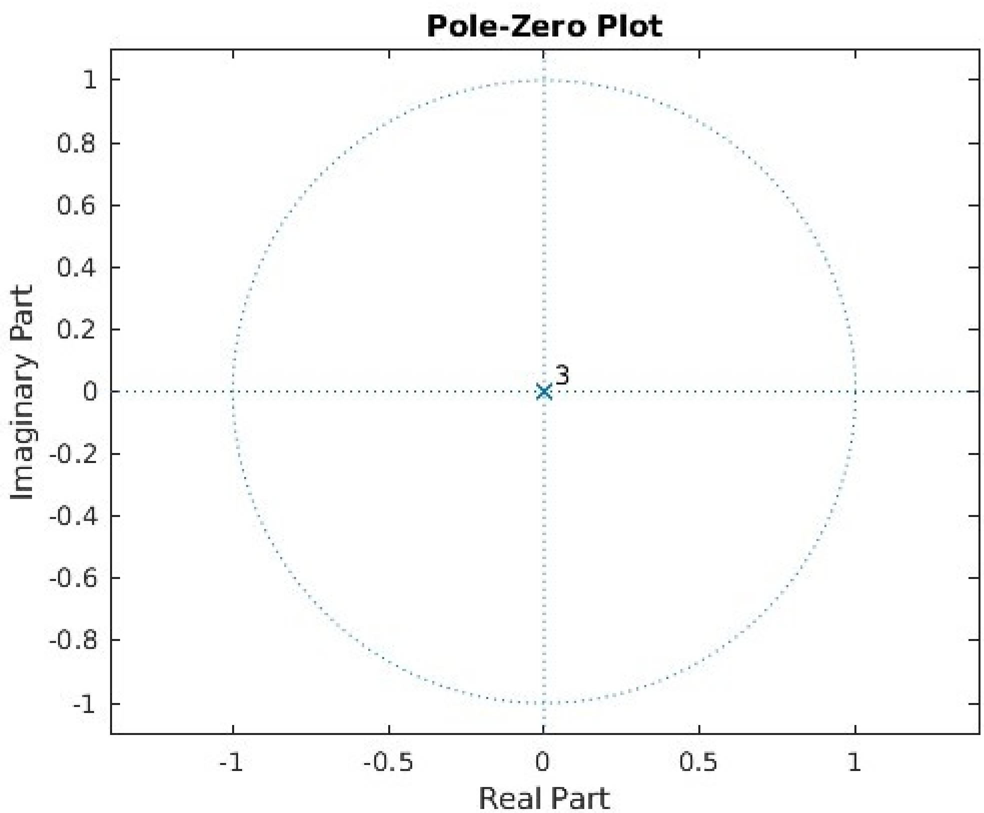
ROC: Entire z-plane except at $z = 0$ and $z = 1/5$



d) $$X(z)=\sum_{n=-\infty}^{\infty} x[n]z^{-n}=\sum_{n=3}^{10}\left(\frac{1}{2}\right)^{n} z^{-n}=\sum_{n=3}^{10}\left(\frac{1}{2z}\right)^{n}=\frac{\left(\frac{1}{2z}\right)^{3}\left(1-\left(\frac{1}{2z}\right)^{8}\right)}{1-\frac{1}{2z}}=\left(\frac{1}{2z}\right)^{3}=\frac{1}{8z^{3}}$$

ROC: Entire z-plane



e) $X(z)=\sum_{n=-\infty}^{\infty} x[n]z^{-n}=\sum_{n=-\infty}^{\infty} n^2 u(n-5)\,z^{-n}=\sum_{n=5}^{\infty} n^2 z^{-n}$

The z-transform does not exist.

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Problem 2

Determine the causal system that corresponds to the following z-transforms:

a) $X(z)=\frac{2}{1+1.5z^{-1}-0.5z^{-2}}$

b) $X(z)=\frac{1-z^{-1}+z^{-7}-0.5z^{-8}}{1-1.5z^{-1}+0.5z^{-2}}$

c) $X(z)=\frac{z^{-4}+z^{-5}}{1-0.5z^{-1}}$

Solution:

a) **Inverse Z-Transform: Distinct Poles**

First, we eliminate the negative powers, by multiplying both numerator and denominator by z^2

$$X(z)=\frac{2z^2}{z^2+1.5z-0.5}$$

$$\frac{X(z)}{z}=\frac{2z}{z^2+1.5z-0.5}$$

$$\frac{X(z)}{z}=\frac{2z}{z^2+1.5z-0.5}$$

To find the roots of the denominator, we use the formula

$$p_{1,2}=\frac{-b\pm\sqrt{b^2-4ac}}{2a}=\frac{-3+\sqrt{17}}{4},\frac{-3-\sqrt{17}}{4}$$

Therefore,

$$\frac{X(z)}{z}=\frac{2z}{\left(z+\frac{3-\sqrt{17}}{4}\right)\left(z+\frac{3+\sqrt{17}}{4}\right)} \qquad (1)$$

We now need to express this in the form

$$\frac{X(z)}{z}=\frac{A_1}{\left(z+\frac{3-\sqrt{17}}{4}\right)}+\frac{A_2}{\left(z+\frac{3+\sqrt{17}}{4}\right)} \qquad (2)$$

To find A_1 and A_2, we express equation (2) as (1) again and compare the coefficients

$$\frac{X(z)}{z}=\frac{A_1\left(z+\frac{3+\sqrt{17}}{4}\right)+A_2\left(z+\frac{3-\sqrt{17}}{4}\right)}{\left(z+\frac{3-\sqrt{17}}{4}\right)\left(z+\frac{3+\sqrt{17}}{4}\right)} \qquad (3)$$

Comparing the coefficients of the numerator of (3) to (1), we get that

$$A_1+A_2=2 \;\; and \;\; A_1\left(\frac{3+\sqrt{17}}{4}\right)+A_2\left(\frac{3-\sqrt{17}}{4}\right)=0$$

Solving the last 2 equations together, we get

$$A_1=\frac{17-3\sqrt{17}}{17}, A_2=\frac{17+3\sqrt{17}}{17}$$

Therefore, $X(z)/z$ can be written as

$$\frac{X(z)}{z}=\frac{\frac{17-3\sqrt{17}}{17}}{\left(z+\frac{3-\sqrt{17}}{4}\right)}+\frac{\frac{17+3\sqrt{17}}{17}}{\left(z+\frac{3+\sqrt{17}}{4}\right)}$$

$$X(z)=\frac{\frac{17-3\sqrt{17}}{17}z}{\left(z+\frac{3-\sqrt{17}}{4}\right)}+\frac{\frac{17+3\sqrt{17}}{17}z}{\left(z+\frac{3+\sqrt{17}}{4}\right)}$$

$$X(z)=\frac{\frac{17-3\sqrt{17}}{17}}{\left(1+\frac{3-\sqrt{17}}{4}z^{-1}\right)}+\frac{\frac{17+3\sqrt{17}}{17}}{\left(1+\frac{3+\sqrt{17}}{4}z^{-1}\right)}$$

By using the z-transform pairs table, and given that the signal is causal we get

$$x(n)=\frac{17-3\sqrt{17}}{17}\left(\frac{-3+\sqrt{17}}{4}\right)^n u(n)+\frac{17+3\sqrt{17}}{17}\left(\frac{-3-\sqrt{17}}{4}z^{-1}\right)^n u(n)$$

b) **Inverse Z-Transform: Multiple-order Poles**

First, we eliminate the negative powers, by multiplying both numerator and denominator by z^9

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$$X(z)=\frac{z^9-z^8+z^2-0.5z}{z^9-1.5z^8+0.5z^7}$$

$$\frac{X(z)}{z}=\frac{z^8-z^7+z-0.5}{z^9-1.5z^8+0.5z^7}$$

$$\frac{X(z)}{z}=\frac{z^8-z^7+z-0.5}{(z^2-1.5z+0.5)z^7}$$

To find the roots of the denominator, we use the formula

$$p_{1,2}=\frac{-b\pm\sqrt{b^2-4ac}}{2a}=1,0.5$$

Therefore,

$$\frac{X(z)}{z}=\frac{z^8-z^7+z-0.5}{(z-1)(z-0.5)z^7} \qquad (1)$$

We now need to express this in the form

$$\frac{X(z)}{z}=\frac{A_1}{z-1}+\frac{A_2}{z-0.5}+\frac{A_3}{z}+\frac{A_4}{z^2}+\frac{A_5}{z^3}+\frac{A_6}{z^4}+\frac{A_7}{z^5}+\frac{A_8}{z^6}+\frac{A_9}{z^7} \qquad (2)$$

To find A_1, A_2, A_3, A_4, A_5, A_6, A_7, A_8, and A_9, we express equation (2) as (1) again and compare the coefficients

(3)

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Comparing the coefficients of the numerator of (3) to (1), we get that

$A_1+A_2+A_3=1, -0.5A_1-A_2-1.5A_3+A_4, 0.5A_3-1.5A_4+A_5, 0.5A_4-1.5A_5+A_6=0,$

$0.5A_5-1.5A_6+A_7=0, 0.5A_6-1.5A_7+A_8=0, 0.5A_7-1.5A_8+A_9=0,$

$0.5A_8-1.5A_9=1$ *and* $0.5A_9=-0.5$

Solving the 9 equations together, we get

$A_1=1, A_2=1, A_3=-1, A_4=-1, A_5=-1, A_6=-1, A_7=-1, A_8=-1, A_9=-1$

Therefore,

$$\frac{X(z)}{z}=\frac{1}{z-1}+\frac{1}{z-0.5}+\frac{-1}{z}+\frac{-1}{z^2}+\frac{-1}{z^3}+\frac{-1}{z^4}+\frac{-1}{z^5}+\frac{-1}{z^6}+\frac{-1}{z^7}$$

$$X(z)=\frac{z}{z-1}+\frac{z}{z-0.5}-\frac{z}{z}-\frac{z}{z^2}-\frac{z}{z^3}-\frac{z}{z^4}-\frac{z}{z^5}-\frac{z}{z^6}-\frac{z}{z^7}$$

$$X(z)=\frac{z}{z-1}+\frac{z}{z-0.5}-1-\frac{1}{z}-\frac{1}{z^2}-\frac{1}{z^3}-\frac{1}{z^4}-\frac{1}{z^5}-\frac{1}{z^6}$$

$$X(z)=\frac{1}{1-z^{-1}}+\frac{1}{1-0.5z^{-1}}-1-\frac{1}{z}-\frac{1}{z^2}-\frac{1}{z^3}-\frac{1}{z^4}-\frac{1}{z^5}-\frac{1}{z^6}$$

By using the z-transform pairs table, we get

$$x(n)=u(n)+(0.5)^n u(n)-\delta(n)-\delta(n-1)-\delta(n-2)-\delta(n-3)-\delta(n-4)-\delta(n-5)-\delta(n-6)$$

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c) **Inverse Z-Transform: Multiple-order Poles**

First, we eliminate the negative powers by multiplying both the numerator and denominator by z^6

$$X(z)=\frac{z^2+z}{z^6-0.5z^5}$$

$$\frac{X(z)}{z}=\frac{z+1}{z^6-0.5z^5}$$

Therefore,

$$\frac{X(z)}{z}=\frac{z+1}{z^5(z-0.5)} \qquad (1)$$

We now need to express this in the form

$$\frac{X(z)}{z}=\frac{A_1}{(z-0.5)}+\frac{A_2}{z}+\frac{A_3}{z^2}+\frac{A_4}{z^3}+\frac{A_5}{z^4}+\frac{A_6}{z^5} \qquad (2)$$

To find A_1, A_2, A_3, A_4, A_5, and A_6 we express equation (2) as (1) again and compare the coefficients

$$A_1+A_2=0,-0.5A_2+A_3=0,-0.5A_3+A_4=0,-0.5A_4+A_5=0,-0.5A_5+A_6=1 \text{ and } -0.5A_6=1$$

Solving the 6 equations together, we get

$$A_1=48\,,A_2=-48\,,A_3=-24\,,A_4=-12\,,A_5=-6\,,A_6=-2$$

Therefore,

$$\frac{X(z)}{z}=\frac{48}{(z-0.5)}-\frac{48}{z}-\frac{24}{z^2}-\frac{12}{z^3}-\frac{6}{z^4}-\frac{2}{z^5}$$

$$\frac{X(z)}{z}=\frac{48z}{(z-0.5)}-48-\frac{24}{z}-\frac{12}{z^2}-\frac{6}{z^3}-\frac{2}{z^4}$$

By using the z-transform pairs table, we get

$$x(n)=48(0.5)^n u(n)-48\,\delta(n)-24\,\delta(n-1)-12\,\delta(n-2)-6\,\delta(n-3)-2\,\delta(n-4)$$

Problem 3

For each of the systems defined by their impulse response function below, find the output of the system in response to the given input:

a) $h(n)=\{\underset{\uparrow}{1},-1\}$, $x(n)=\{1,\underset{\uparrow}{0},1,0\}$

b) $h(n)=\left(\frac{1}{2}\right)^n u(n)$, $x(n)=\{1,\underset{\uparrow}{0},1,0\}$

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c) $h(n)=\left(\frac{1}{5}\right)u(n),$ $\qquad x(n)=\left(\frac{1}{2}\right)^{n}u(n)$

Solution:

a) First, we need to get the z-transform of the system equation

$$H(z)=1-z^{-1}=1-\frac{1}{z}=\frac{z-1}{z}$$

$$X(z)=z+z^{-1}$$

Therefore,

$$Y(z)=H(z)X(z)=(1-z^{-1})(z+z^{-1})=z-1+z^{-1}-z^{-2}=z-1+\frac{1}{z}-\frac{1}{z^{2}}$$

By using the z-transform pairs table, and given that the signal is causal we get

$$y(n)=\delta(n+1)-\delta(n)+\delta(n-1)-\delta(n-2)$$

b) First, we need to get the z-transform of the system equation

$$H(z)=\frac{1}{1-0.5z^{-1}}$$

$$X(z)=z+z^{-1}=z+\frac{1}{z}=\frac{z^{2}+1}{z}$$

Therefore a pole-zero cancellation occurs,

$$Y(z)=H(z)X(z)=\left(\frac{1}{1-0.5z^{-1}}\right)(z+z^{-1})=\frac{z+z^{-1}}{1-0.5z^{-1}}=\frac{z^2+1}{z-0.5}$$

Inverse Z-Transform: Improper Rational

Long Division

Since the order of the remainder polynomial is less than the polynomial we are dividing by, we stop

$$Y(z)=z+0.5+\frac{1.25}{z-0.5}=z+0.5+\frac{1.25z^{-1}}{1-0.5z^{-1}}$$

The inverse z-transform can be then written as

$$y(n)=\delta(n+1)+0.5\,\delta(n)+Z^{-1}\left\{\frac{1.25z^{-1}}{1-0.5z^{-1}}\right\}$$

$$Let\,M(z)=\frac{1.25z^{-1}}{1-0.5z^{-1}}$$

First, we eliminate the negative powers by multiplying both the numerator and denominator by z^2

$$M(z)=\frac{1.25z}{z^2-0.5z}$$

$$\frac{M(z)}{z}=\frac{1.25}{z^2-0.5z}$$

$$\frac{M(z)}{z}=\frac{1.25}{z(z-0.5)}$$

Then

$$\frac{M(z)}{z}=\frac{1.25}{z^2-0.5z}=\frac{1.25}{z(z-0.5)} \qquad (1)$$

We now need to express this in the form

$$\frac{M(z)}{z}=\frac{A_1}{z}+\frac{A_2}{z-0.5} \qquad (2)$$

To find A_1, A_2, and A_3, we express equation (2) as (1) again and compare the coefficients

$$\frac{M(z)}{z}=\frac{A_1(z-0.5)+A_2 z}{z(z-0.5)} \qquad (3)$$

Comparing the coefficients of the numerator of (3) to (1), we get

$A_1+A_2=0$ and $-0.5A_1=1.25$

Solving the 2 equations together, we get

$A_1=-2.5, A_2=2.5$

Therefore, $M(z)/z$ can be written as

$$\frac{M(z)}{z}=\frac{-2.5}{z}+\frac{2.5}{z-0.5}$$

$$M(z)=-2.5+\frac{2.5z}{z-0.5}$$

$$M(z)=-2.5+\frac{2.5}{1-0.5z^{-1}}$$

By using the z-transform pairs table, and given that the signal is causal we get

$$y(n)=\delta(n+1)-2\delta(n)+2.5(0.5)^n u(n)$$

c) First, we get the z-Transform of the system equation

$$H(z)=\frac{0.5}{1-z^{-1}}=\frac{0.2z}{z-1}$$

$$X(z)=\frac{1}{1-0.5z^{-1}}=\frac{z}{z-0.5}$$

Therefore,

$$Y(z)=H(z)X(z)=\left(\frac{0.2z}{z-1}\right)\left(\frac{z}{z-0.5}\right)=\frac{0.2z^2}{z^2-1.5z+0.5}$$

Inverse Z-Transform: Improper Rational

Long Division

Since the order of the remainder polynomial is less than the polynomial we are dividing by, we stop

$$Y(z)=0.2+\frac{1.5z-0.1}{z^2-1.5z+0.5}$$

The inverse z-transform can be then written as

$$y(n)=0.2\,\delta(n)+Z^{-1}\left\{\frac{1.5z-0.1}{z^2-1.5z+0.5}\right\}$$

$$Let\ M(z)=\frac{1.5z-0.1}{z^2-1.5z+0.5}$$

$$M(z)=\frac{1.5z^2-0.1z}{z^3-1.5z^2+0.5z}$$

$$\therefore \frac{M(z)}{z}=\frac{1.5z-0.1}{z^3-1.5z^2+0.5z}$$

$$\frac{M(z)}{z}=\frac{1.5z-0.1}{z(z-1)(z-0.5)}$$

Then

$$\frac{M(z)}{z}=\frac{1.5z-0.1}{z(z-1)(z-0.5)}=\frac{1.5z-0.1}{z(z-1)(z-0.5)} \qquad (1)$$

We now need to express this in the form

$$\frac{M(z)}{z}=\frac{A_1}{z}+\frac{A_2}{z-1}+\frac{A_3}{z-0.5} \tag{2}$$

To find A_1, A_2, and A_3, we express equation (2) as (1) again and compare the coefficients

$$\frac{M(z)}{z}=\frac{A_1(z-1)(z-0.5)+A_2 z(z-0.5)+A_3 z(z-1)}{z(z-1)(z-0.5)} \tag{3}$$

Comparing the coefficients of the numerator of (3) to (1), we get

$$A_1+A_2+A_3=0, -1.5A_1-0.5A_2-A_3=1.5 \text{ and } 0.5A_1=-0.1$$

Solving the last 3 equations together, we get

$$A_1=-0.2, A_2=2.8, A_3=-2.6$$

Therefore, $M(z)/z$ can be written as

$$\frac{M(z)}{z}=\frac{-0.2}{z}+\frac{2.8}{z-1}-\frac{2.6}{z-0.5}$$

$$M(z)=-0.2+\frac{2.8z}{z-1}-\frac{2.6z}{z-0.5}$$

$$M(z)=-0.2+\frac{2.8}{1-z^{-1}}-\frac{2.6}{1-0.5z^{-1}}$$

By using the z-transform pairs table, and given that the signal is causal we get

$$y(n)=2.8u(n)-2.6(0.5)^n u(n)$$

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Problem 4

Implement the following function using MATLAB or Python:

ExponentialMovingAverageFilter: A function that takes as input the file name of an input audio file, the moving average window size *M*, the parameter *a*, and the file name of the output filtered audio file.

This function should convolve the exponential moving average filter with the input signal. The impulse response of the exponential moving average filter takes the form

$$h(n)=\frac{(1-a)^{|n|}}{\sum_{i=-L}^{L}(1-a)^{|i|}}, \qquad -L\leq n\leq L$$

where $M = 2L + 1$.

To adjust for the attenuation that occurs after filtering, scale the output obtained after filtering to have the same maximum value as the input audio.

Deliverables:

i- Your code (either MATLAB .m files or Python .py files or Jupyter notebook files).

ii- Apply the implemented function to the audio file provided "NoisyTone.wav" using $a = 0$, $a = 0.25$, $a = 0.9$. In all cases, use M = 201. Name the output audio files "Filtered_a_0.wav", "Filtered_a_0_25.wav", "Filtered_a_0_9.wav", respectively. The sampling rate of the original audio is 10 KHz.

iii- The file "CleanTone.wav" includes the same audio without noise. Plot the first 1000 samples of the clean tone, the noisy tone and each of the 3 filtered signals obtained in (ii). Comment on the quality of the output obtained in (ii) relative to the clean tone based on the figure.

iv- Apply the implemented function to the audio provided "NoisyTone.wav" using $M = 3$, $M = 30$, and $M = 50$. In all cases, use $a = 0.25$. Name the output audio files "Filtered_M_3.wav", "Filtered_M_30.wav", and "Filtered_M_50.wav", respectively.

v- Comment on the impact of increasing M on the quality of the filtered output obtained in (iv).

Solution:

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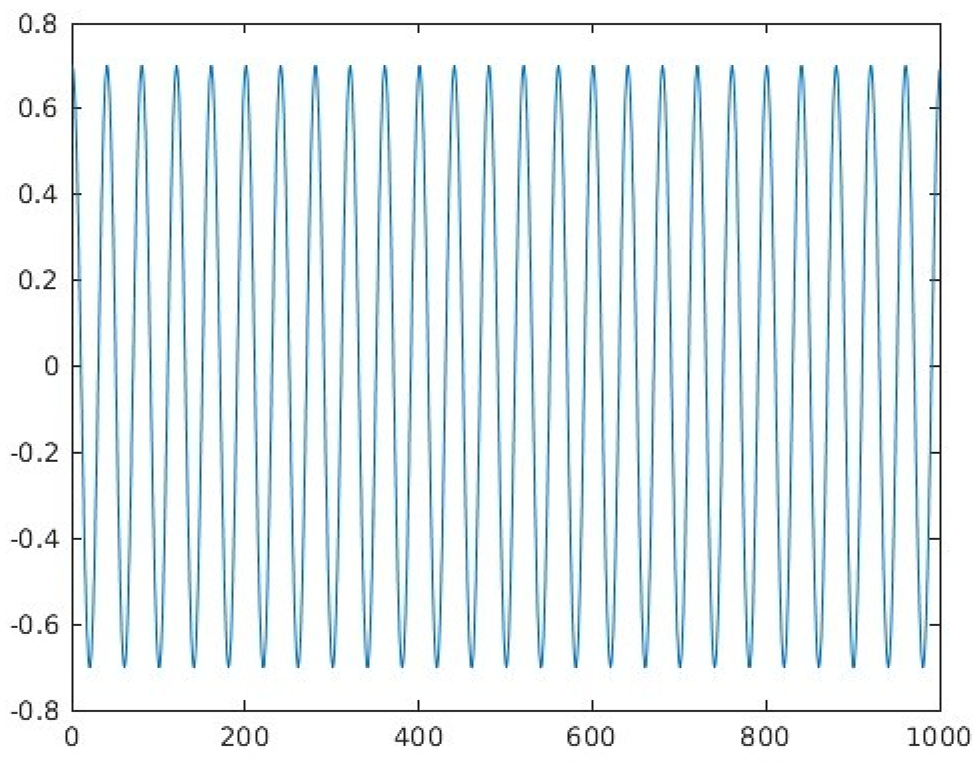

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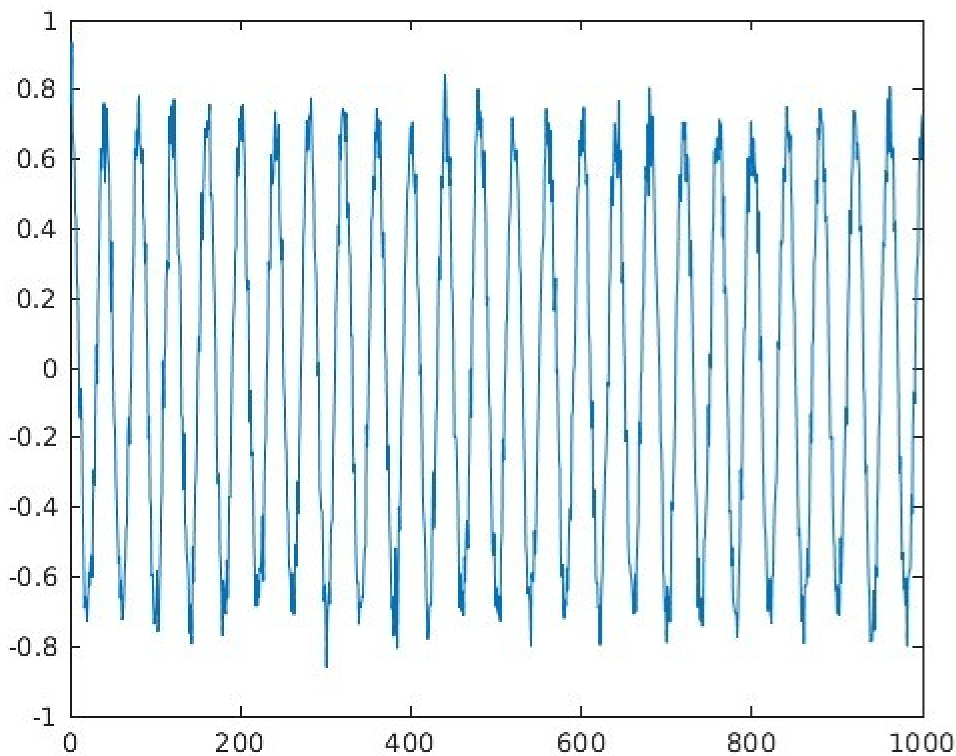

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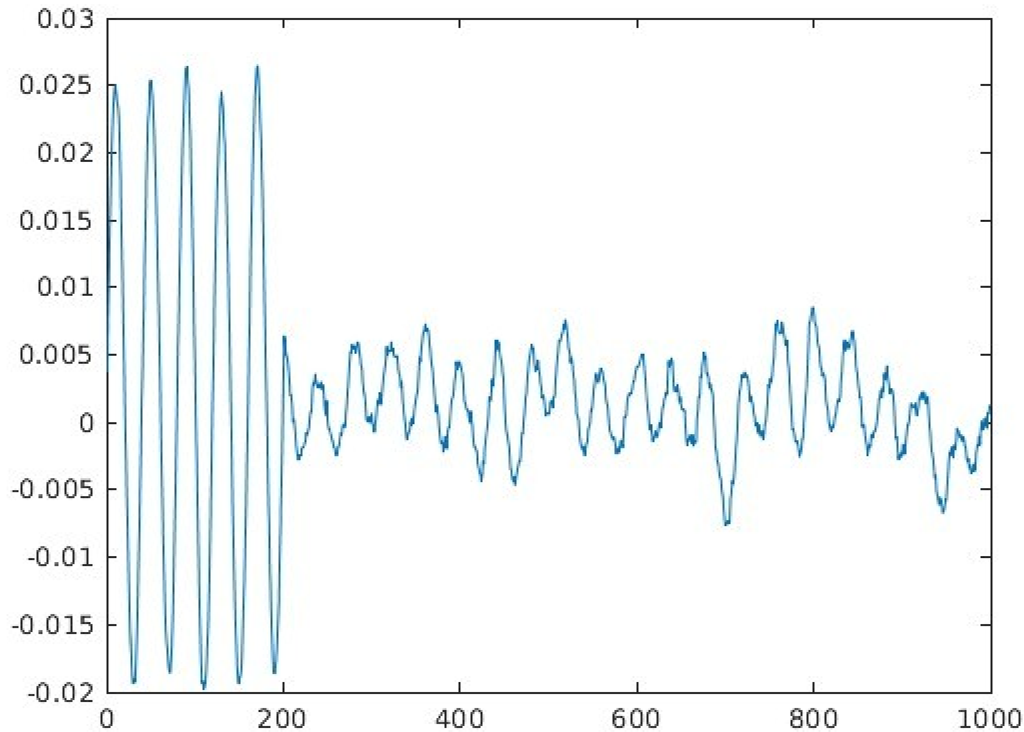

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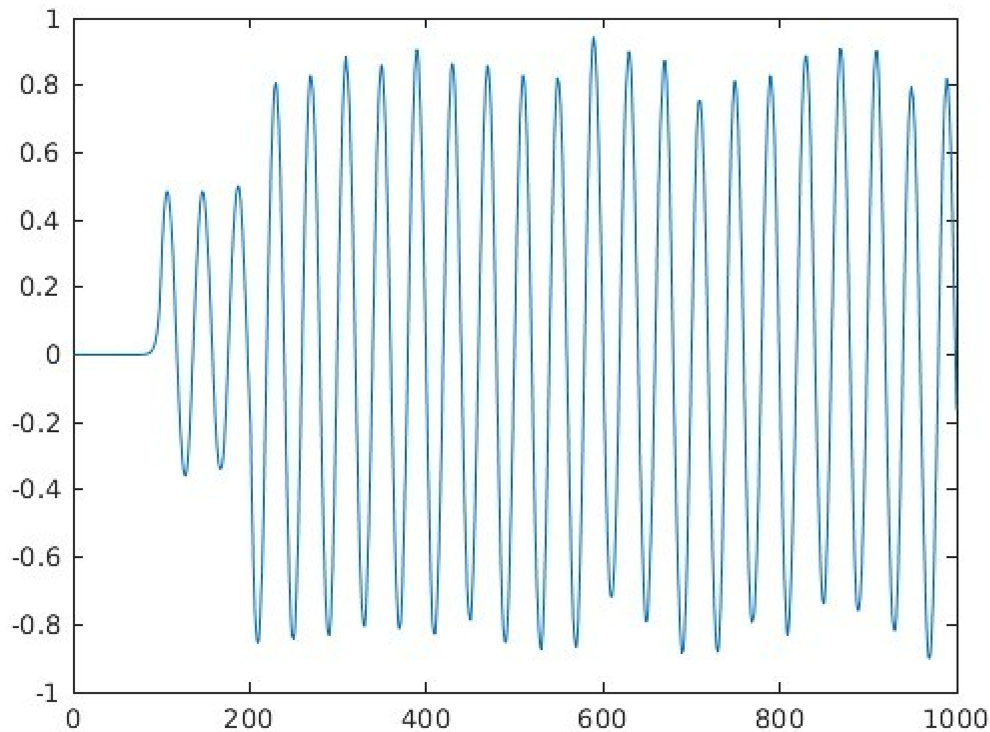

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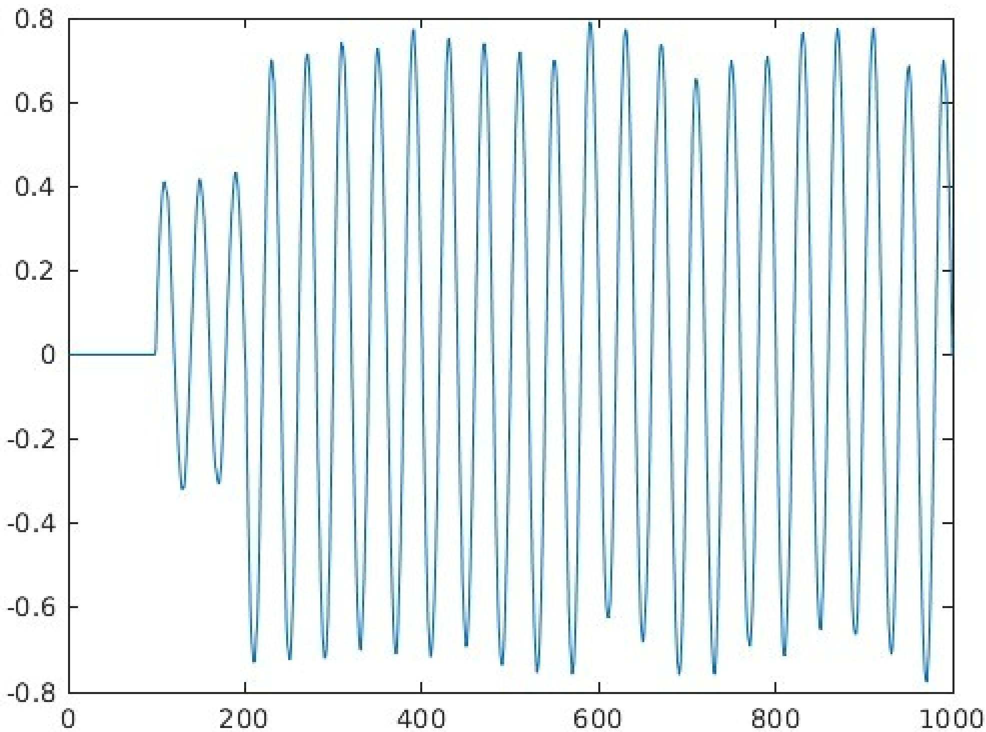

Based on the figure, the quality of the output obtained in (ii) is lower than the quality of the clean tone. However, the quality of the output obtained in (ii) when a = 0.25 was closest to the quality of the clean tone.

The impact of increasing M is an increase in the quality of the filtered output obtained in (iii).

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Important Notes:

- All deliverables should be included in one .zip file.
- For Problems 1, 2 and 3, scanned version of handwritten solutions is acceptable.
- For Problem 4, you can use the convolution function available in MATLAB or Python to apply the filter to the input audio. Don't use ready made functions that you might find available in MATLAB or Python that directly implement the Exponential Moving Average Filter.
- This is an individual assignment.