Problem 1

Sketch the spectrum of the following periodic signals:

a) $x(n)=3\sin\left(\frac{2\pi}{5}n\right)$

b) $x(n)=3\sin\left(\frac{4\pi}{5}n\right)+\cos\left(\frac{2\pi}{3}n\right)$

c) $x(n)=\{\ldots,1,0,1,\underset{\uparrow}{0},1,0,\ldots\}$

Solution:

a) For this signal $2\pi f=\frac{2\pi}{5}$

$$\therefore\ f=\frac{1}{5}$$

Since f is rational, then the signal is periodic with a fundamental period $N = 5$

Then the signal can be described as

$$x(n)=\sum_{k=0}^{4} c_k e^{j2\pi kn/5}$$

where $c_k=\frac{1}{5}\sum_{n=0}^{4} x(n)e^{-j2\pi kn/5}$

We can either compute c_0, c_1, …, c_4 or given that we can describe $x(n)$ as

$$x(n)=\sin\left(\frac{2\pi n}{5}\right)$$

$$=\sin\left(\frac{2\pi n}{5}\right)=\frac{3}{2j}\left(e^{j\frac{2\pi n}{5}}-e^{-j\frac{2\pi n}{5}}\right)$$

This is the same form as the Fourier series

$$x(n)=\sum_{k=0}^{N-1} c_k e^{j2\pi kn/N}$$

So, we can conclude that from the first exponential, c_1 = -1.5*j*

The second exponential would correspond to c_{-1} = 1.5*j*. However, we cannot have *k* = -1 since *k* = 0, 1, …, 4. Given the periodicity property of the coefficients, we can conclude that $c_{-1} = c_{-1+5} = c_4$ = 1.5*j*

All other coefficients c_0, c_2, c_3 = 0

Each coefficient can be described as magnitude and phase as follows

$$|c_0|=0, \measuredangle c_0=undefined$$

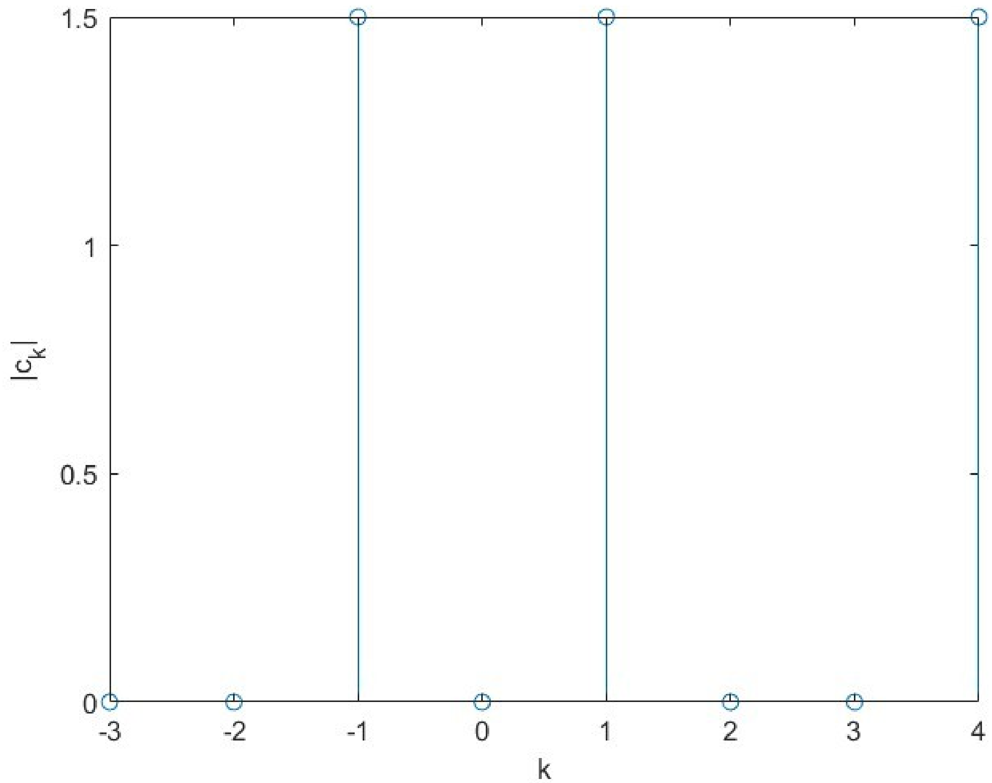
$$|c_1|=\frac{3}{2}, \measuredangle c_1=\frac{-\pi}{2}$$

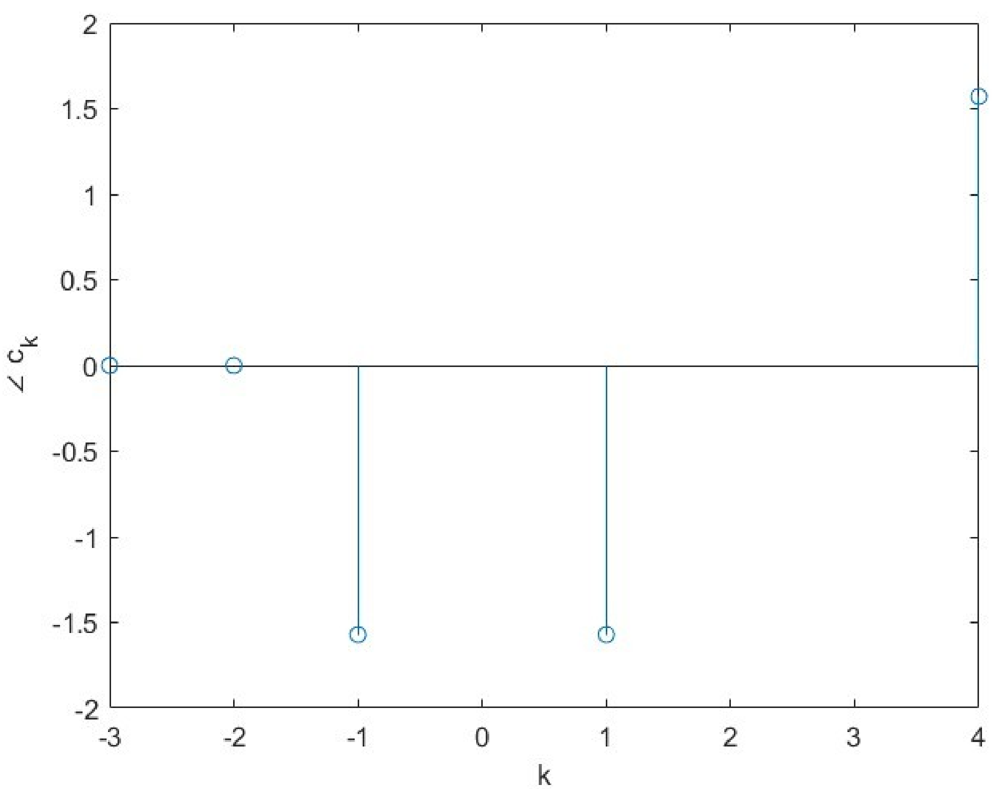
$$|c_2|=0, \measuredangle c_2=undefined$$

$$|c_3|=0, \measuredangle c_3=undefined$$

$$|c_4|=\frac{3}{2}, \measuredangle c_4=\frac{\pi}{2}$$

The spectrum can be described as follows


1.5
1
0.5
0
$|c_k|$
-3
-2
-1
0
1
2
3
4
k


2
1.5
1
0.5
0
-0.5
-1
-1.5
-2
$\angle c_k$
-3
-2
-1
0
1
2
3
4
k

b) For this signal $2\pi f = \frac{4\pi}{5}$

$$\therefore\ f = \frac{2}{5}$$

$$2\pi f = \frac{2\pi}{3}$$

$$f = \frac{1}{3}$$

Since f is rational, then the signal is periodic with a fundamental period $N = 15$

Then the signal can be described as

$$x(n) = \sum_{k=0}^{14} c_k e^{j2\pi kn/15}$$

where $c_k = \frac{1}{15}\sum_{n=0}^{14} x(n)e^{-j2\pi kn/15}$

We can either compute $c_0, c_1, \ldots, c_{14}$ or given that we can describe $x(n)$ as

$$x(n) = 3\sin\left(\frac{4\pi}{5}n\right) + \cos\left(\frac{2\pi}{3}n\right)$$

$$= \frac{3}{2j}\left(e^{j\frac{4\pi n}{5}} - e^{-j\frac{4\pi n}{5}}\right) + \frac{1}{2}\left(e^{j\frac{2\pi n}{3}} + e^{-j\frac{2\pi n}{3}}\right)$$

This is the same form as the Fourier series

$$x(n) = \sum_{k=0}^{N-1} c_k e^{j2\pi kn/N}$$

So, we can conclude that from the first exponential, $c_5 = 0.5$

The second exponential would correspond to $c_6 = -1.5j$

The third exponential would correspond to $c_9 = 1.5j$

The last exponential would correspond to $c_{10} = 0.5$

All other coefficients $c_0, c_1, c_2, c_3, c_4, c_7, c_8, c_{11}, c_{12}, c_{13}, c_{14} = 0$

Each coefficient can be described as magnitude and phase as follows

$$|c_0|=0, \measuredangle c_0=undefined$$

$$|c_1|=0, \measuredangle c_1=undefined$$

$$|c_2|=0, \measuredangle c_2=undefined$$

$$|c_3|=0, \measuredangle c_3=undefined$$

$$|c_4|=0, \measuredangle c_4=undefined$$

$$|c_5|=\frac{1}{2}, \measuredangle c_5=0$$

$$|c_6|=\frac{3}{2}, \measuredangle c_6=\frac{-\pi}{2}$$

$$|c_7|=0, \measuredangle c_7=undefined$$

$$|c_8|=0, \measuredangle c_8=undefined$$

$$|c_9|=\frac{3}{2}, \measuredangle c_9=\frac{\pi}{2}$$

$$|c_{10}|=\frac{1}{2}, \measuredangle c_{10}=0$$

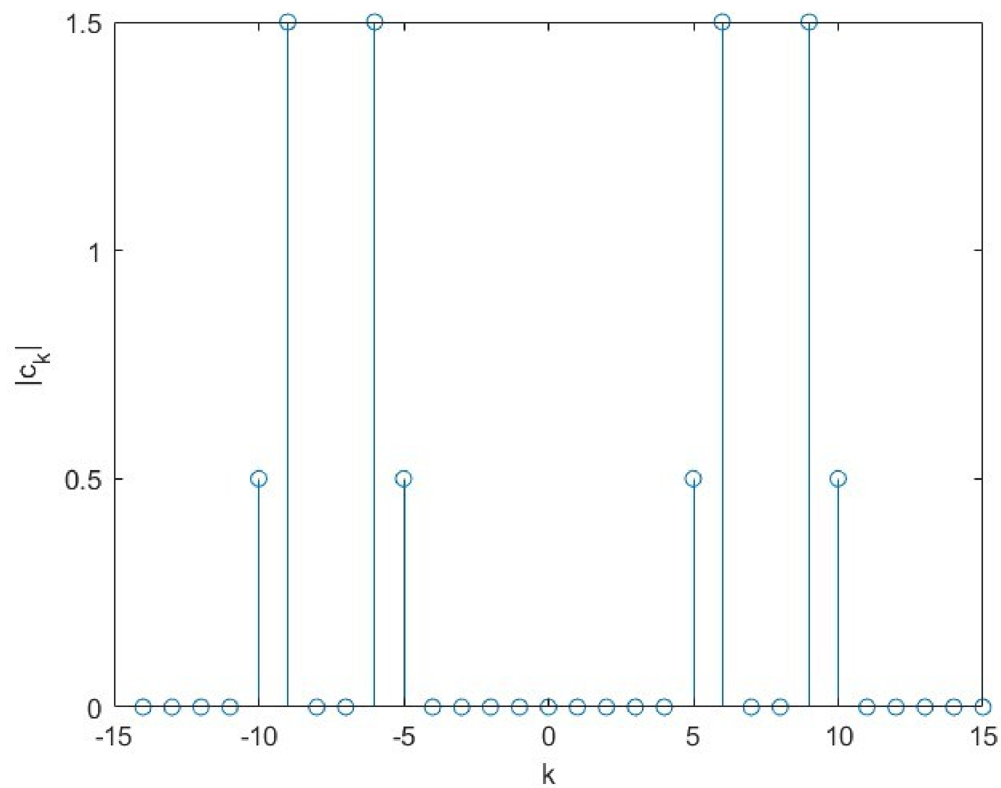
$$|c_{11}|=0, \measuredangle c_{11}=undefined$$

$$|c_{12}|=0, \measuredangle c_{12}=undefined$$

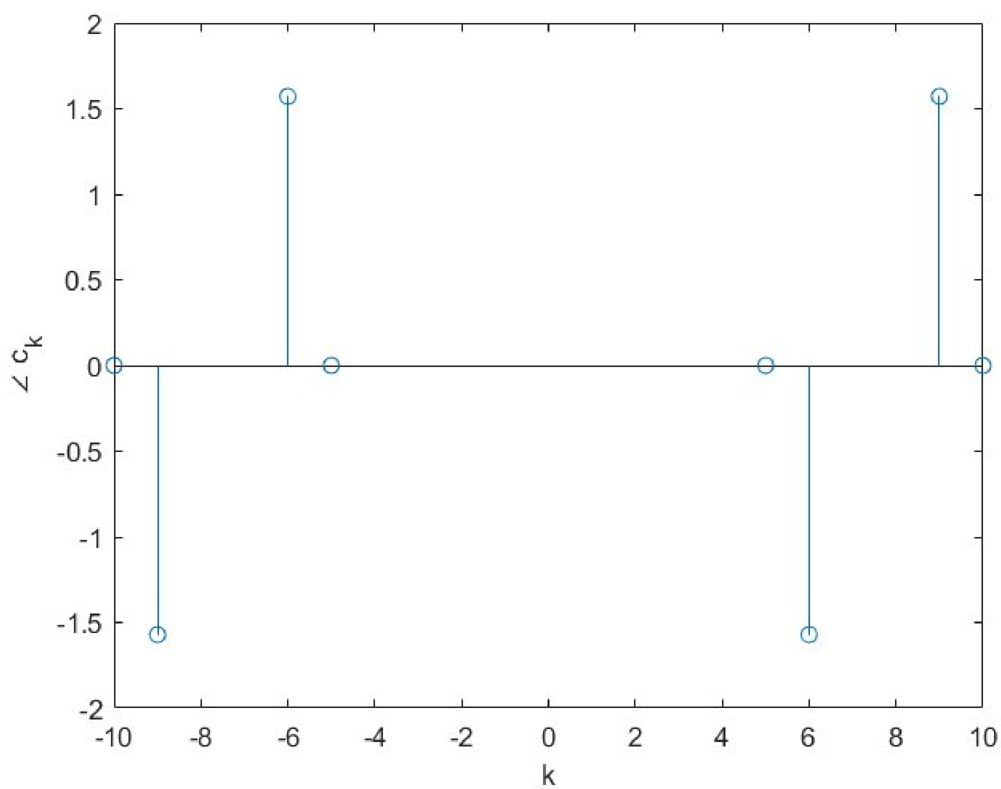
$$|c_{13}|=0, \measuredangle c_{13}=undefined$$

$$|c_{14}|=0, \measuredangle c_{14}=undefined$$

The spectrum can be described as follows



1.5
1
0.5
0
-15
-10
-5
0
5
10
15
k
$|c_k|$



2
1.5
1
0.5
0
-0.5
-1
-1.5
-2
-10
-8
-6
-4
-2
0
2
4
6
8
10
k
$\angle c_k$

c) The signal can be described as:

$$x(n)=\sum_{k=0}^{1} c_k e^{j2\pi kn/2}$$

where $c_k=\frac{1}{2}\sum_{n=0}^{1} x(n)e^{-j2\pi kn/2}$

Given $x(n)$, then

$$c_k=\frac{1}{2}e^{-j\pi k}$$

Therefore,

$$c_0=\frac{1}{2}, c_1=\frac{1}{2}e^{-j\pi}$$

The coefficients can be further reduced using $e^{j\theta}=\cos(\theta)+j\sin(\theta)$

$$c_0=\frac{1}{2}$$

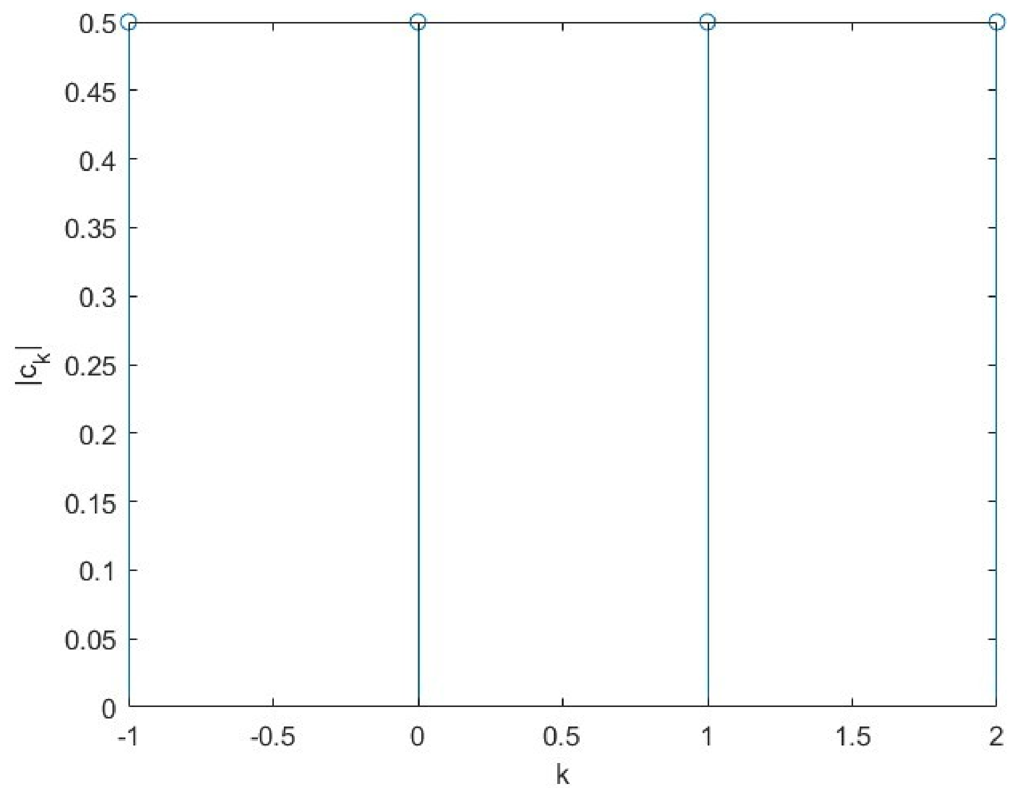
$$c_1=\frac{-1}{2}$$

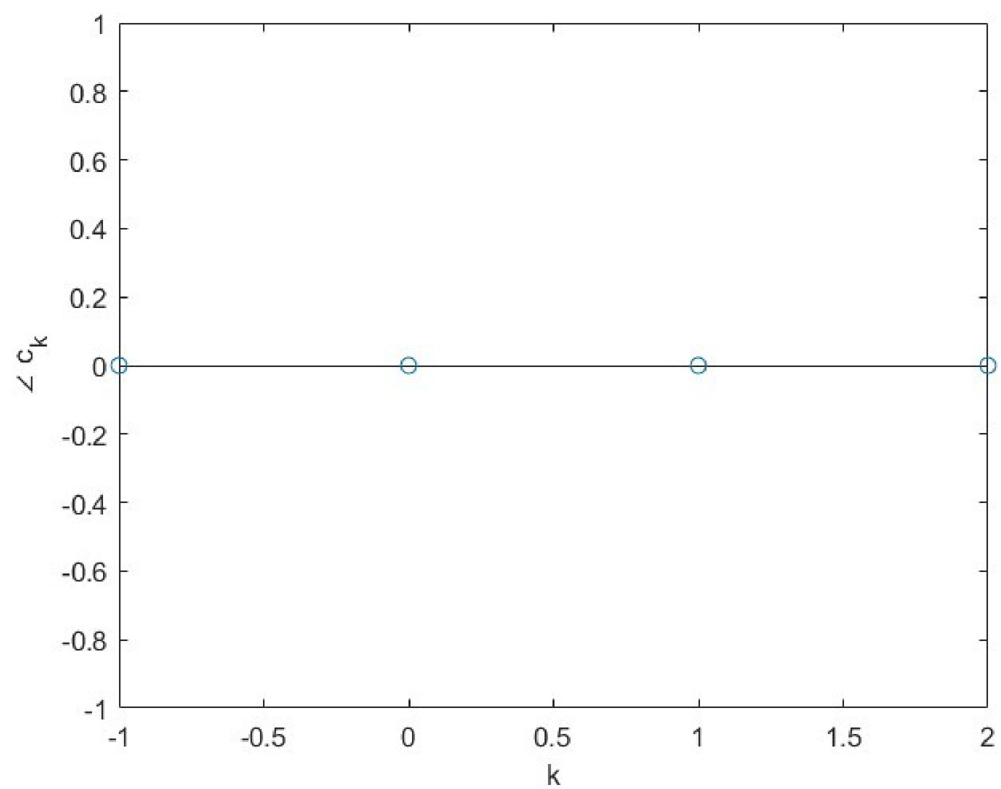
Each coefficient can be described as magnitude and phase as follows

$$|c_0|=\frac{1}{2}, \measuredangle c_0=0$$

$$|c_1|=\frac{1}{2}, \measuredangle c_1=0$$

The spectrum can be described as follows


0.5
0.45
0.4
0.35
0.3
0.25
0.2
0.15
0.1
0.05
0
$|c_k|$
-1
-0.5
0
0.5
1
1.5
2
k


1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-1
$\angle c_k$
-1
-0.5
0
0.5
1
1.5
2
k

Problem 2

Find the Fourier Transform of the following signals:

a) $x(n)=\{\underset{\uparrow}{0},1,5,6\}$

b) $x(n)=4^{-n}u(n-5)$

c) $x(n)=\begin{cases}\frac{1}{2}\sin\left(\frac{2\pi n}{5}\right), & 0\le n\le 6\\ 0, & \textit{Otherwise}\end{cases}$

Solution:

a) $X(\omega)=\sum_{n=-\infty}^{\infty}x(n)e^{-j\omega n}$

$$=e^{-j\omega}+5e^{-j2\omega}+6e^{-j3\omega}$$

$$=\cos(\omega)-j\sin(\omega)+5\cos(2\omega)-5j\sin(2\omega)+6\cos(3\omega)-6j\sin(3\omega)$$

b) $x(n)=\left(\frac{1}{4}\right)^{5}\left(\frac{1}{4}\right)^{n-5}u(n-5)$

$$X(\omega)=\left(\frac{1}{4}\right)^{5}F\left\{\left(\frac{1}{4}\right)^{n-5}u(n-5)\right\}$$

$$=\left(\frac{1}{4}\right)^{5}F\left\{\left(\frac{1}{4}\right)^{n}u(n)\right\}e^{-j5\omega}$$

$$=\frac{1}{1024-256e^{-j\omega}}e^{-j5\omega}$$

c) $X(\omega)=\sum_{n=-\infty}^{\infty}x(n)e^{-j\omega n}$

$$=\sum_{n=0}^{6}\frac{1}{2}\sin\left(\frac{2\pi n}{5}\right)e^{-j\omega n}$$

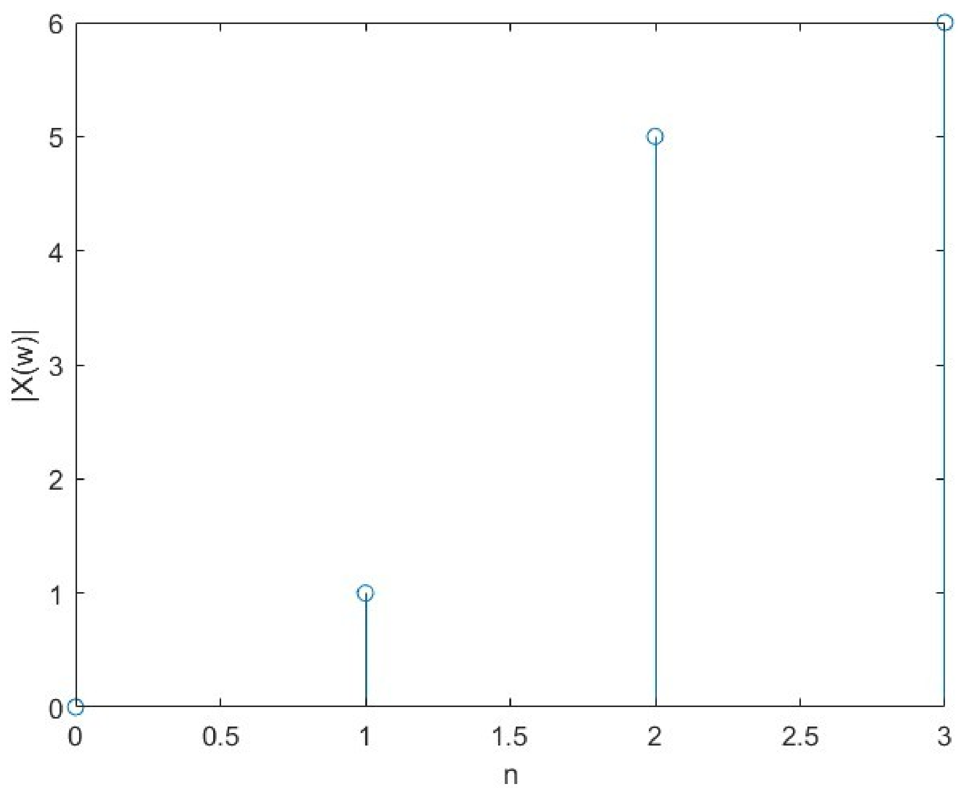
$$=\frac{1}{2}\sum_{n=0}^{6}\sin\left(\frac{2\pi n}{5}\right)e^{-j\omega n}$$

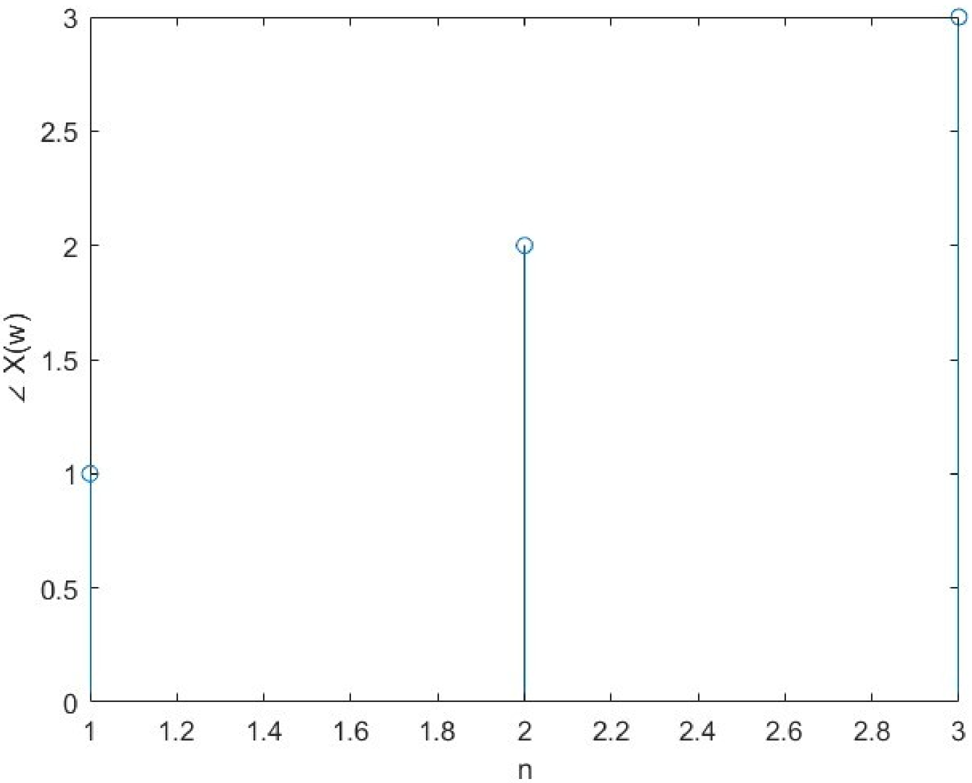
$$=\frac{1}{2}\left(\sin\left(\frac{2\pi}{5}\right)e^{-j\omega}+\sin\left(\frac{4\pi}{5}\right)e^{-j2\omega}+\sin\left(\frac{8\pi}{5}\right)e^{-j3\omega}+\sin\left(\frac{16\pi}{5}\right)e^{-j4\omega}+\sin\left(\frac{32\pi}{5}\right)e^{-j5\omega}+\sin\left(\frac{64\pi}{5}\right)e^{-j6\omega}\right)$$

Problem 3

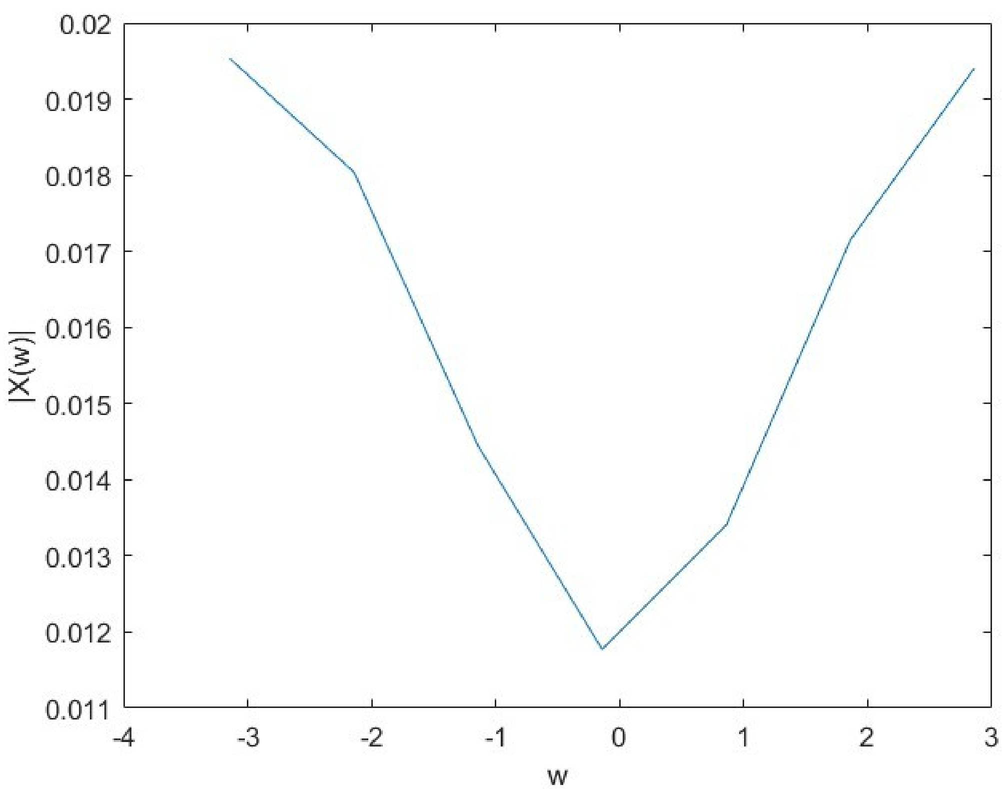
Using MATLAB or Python, if the Fourier Transform is complex, plot the magnitude and phase of the Fourier Transform of the signals given in Problem 2. If the Fourier Transform is real, plot the Fourier transform. In this problem, you do not need to use the Fourier Transform function. You are supposed to plot the expression you get in Problem 2.

Solution:

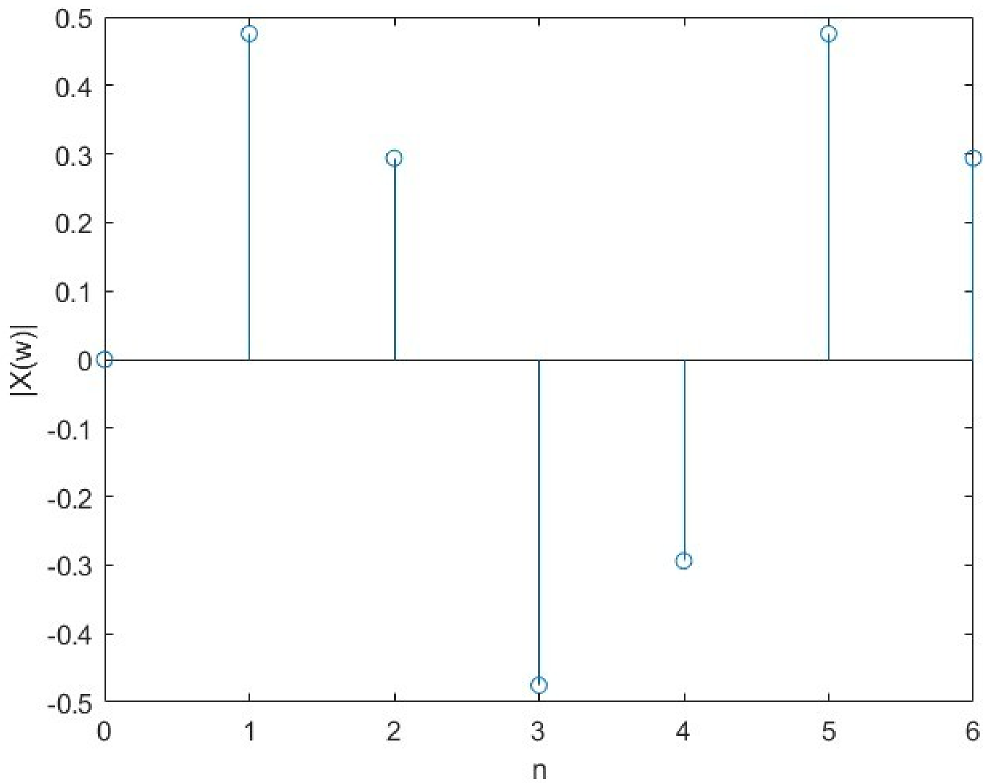


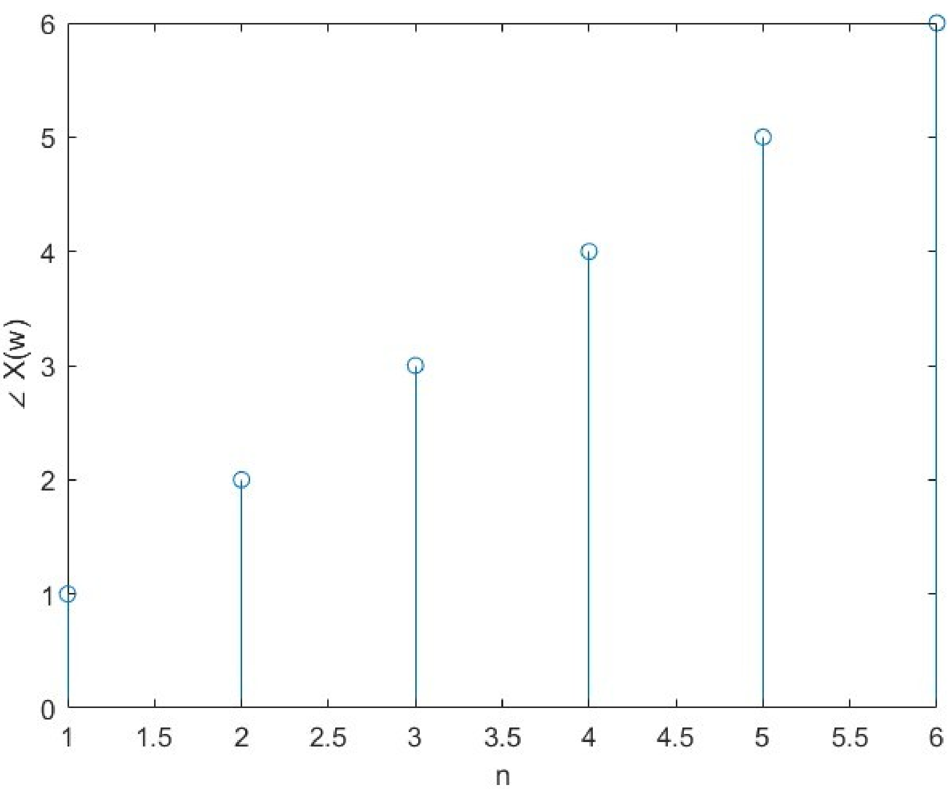


3
2.5
2
1.5
1
0.5
0
∠ X(w)
1
1.2
1.4
1.6
1.8
2
2.2
2.4
2.6
2.8
3
n



0.02
0.019
0.018
0.017
0.016
0.015
0.014
0.013
0.012
0.011
|X(w)|
-4
-3
-2
-1
0
1
2
3
w


0.5
0.4
0.3
0.2
0.1
0
-0.1
-0.2
-0.3
-0.4
-0.5
|X(w)|
0
1
2
3
4
5
6
n


6
5
4
3
2
1
0
∠ X(w)
1
1.5
2
2.5
3
3.5
4
4.5
5
5.5
6
n

Problem 4

Implement the following functions using MATLAB or Python:

- ApplyDFT: A function that takes as input the file name of an input audio file, the length *N* of DFT, the length *M* of Inverse DFT, and the file name of the output audio file. This function should compute the *N*-point DFT of the input audio and then the M-point Inverse DFT of the obtained DFT. If the output of the Inverse DFT is complex, replace it with its magnitude.
- DropFrequencyRange: A function that takes as input the file name of an input audio file, the length *N* of DFT, the range of frequencies to drop (given as samples of the output DFT), and the file name of the output filtered audio file. This function should compute the *N*-point DFT of the input audio, replace the coefficients of the dropped frequencies by 0, then compute the N-point inverse DFT and save the output file. The function can use the function above ApplyDFT.

Deliverables:

i- Your code (either MATLAB .m files or Python .py or Jupyter notebook files).

ii- Apply the function ApplyDFT to the audio file provided “Audio.wav” (N = 441000, M = 441000), (N = 220500, M = 220500), (N = 441000, M = 220500), and (N = 220500, M = 441000). Name the output audio files “Inverse_X_Y.wav”, where X = N and Y = M.

iii- For each case in (ii), plot the original signal and the signal obtained after applying the Inverse DFT on the same plot. Name the output images as “Inverse_X_Y.jpg”, where X = N and Y = M.

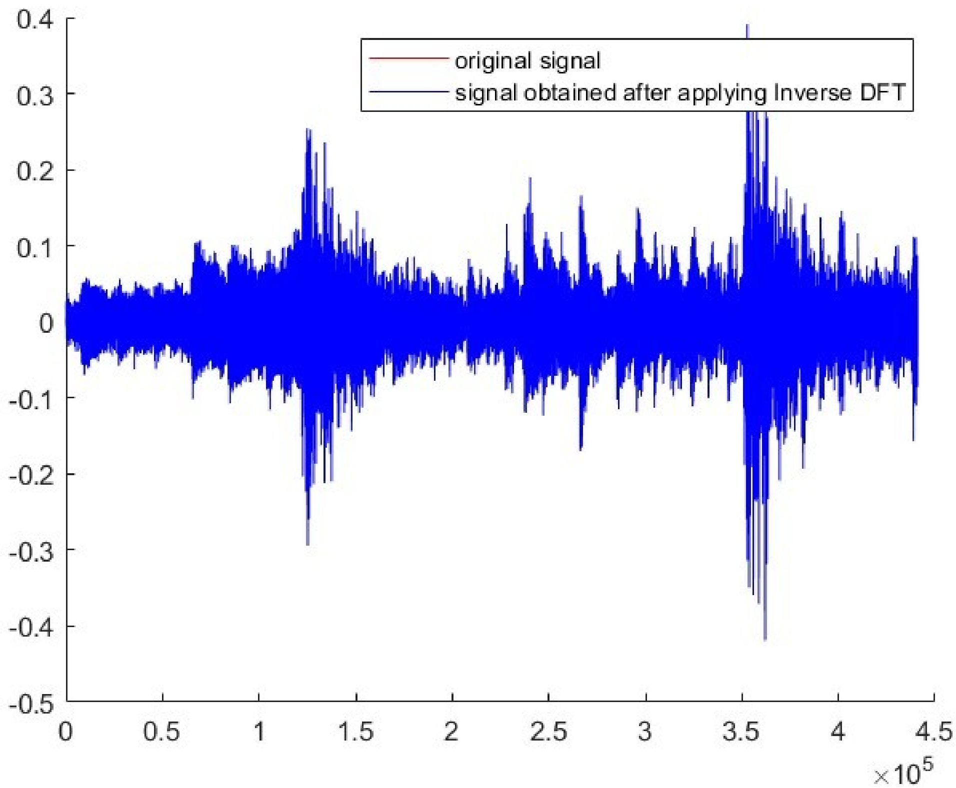
iv- Based on the outcomes in (ii) and (iii), comment on the output when $M = N$ and less than the length of the original signal, when $M < N$, and when $M > N$.

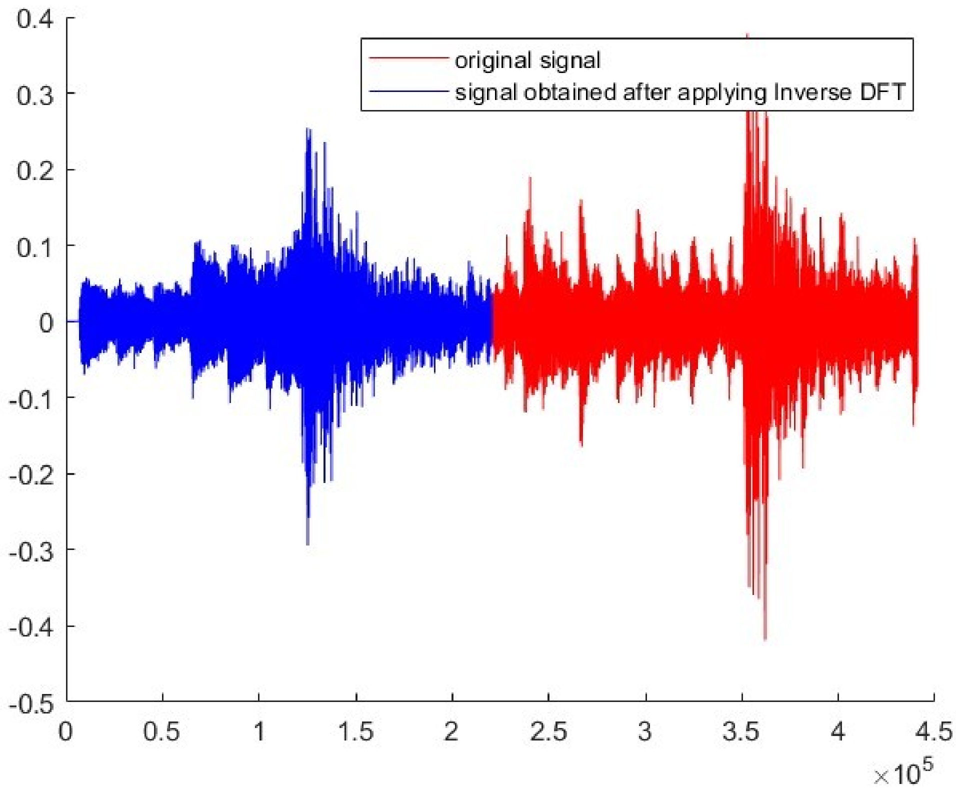
v- Apply the function DropFrequencyRange to the audio file provided “Audio.wav” with N = 441000 and the following range of frequencies given as samples of the DFT as follows (1000, 30000), (100, 30000), and (30000, 40000). Name the output audio files

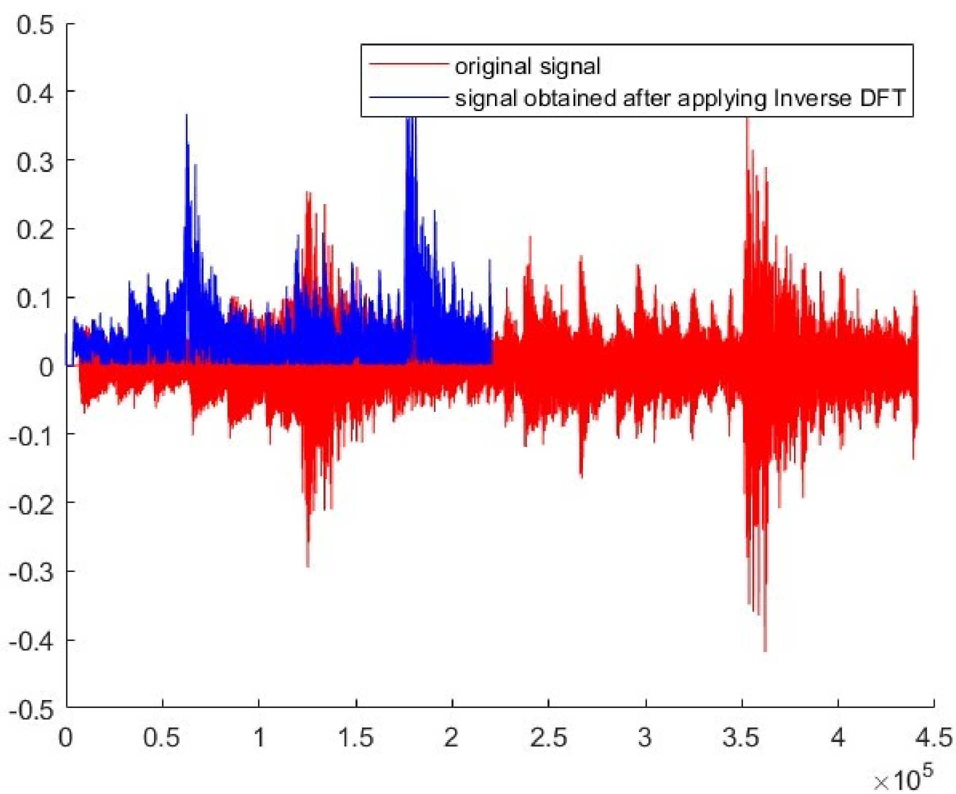
"Drop_1000_30000.wav", "Drop_100_30000.wav", and "Drop_30000_40000.wav", respectively.

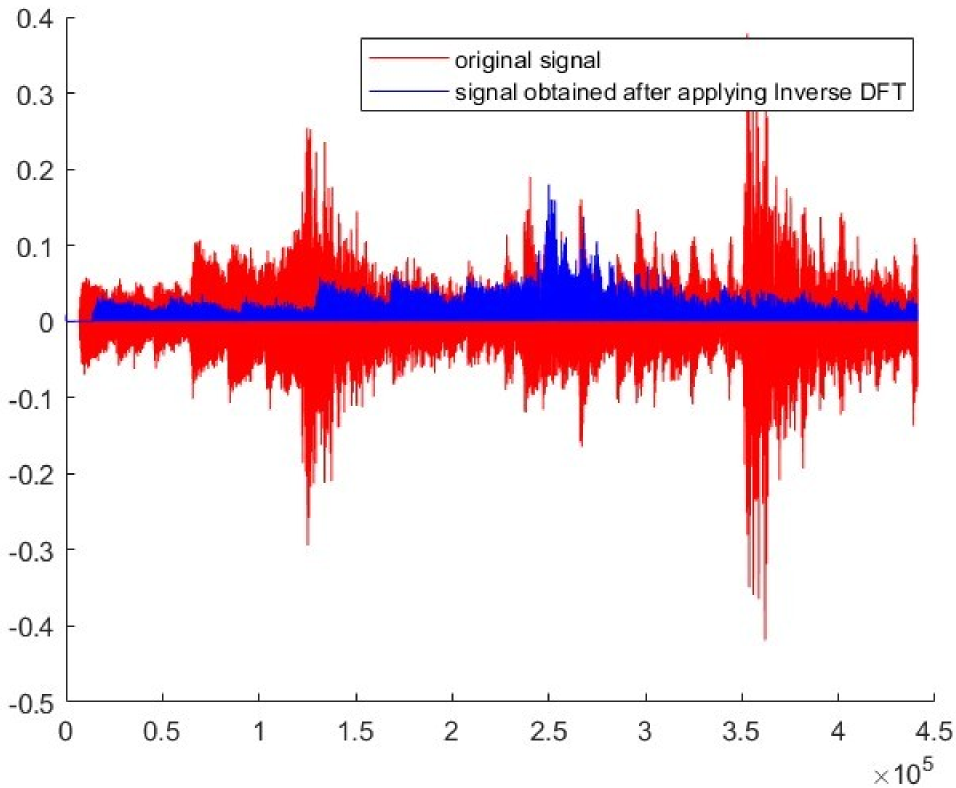
vi- For each case in (v), plot the original signal and the signal obtained after applying the function on the same plot. Name the output images as "Drop_1000_30000.jpg", "Drop_100_30000.jpg", and "Drop_30000_40000.jpg".

vii- Based on the outcomes in (v) and (vi), comment on the output based on the change in the range.

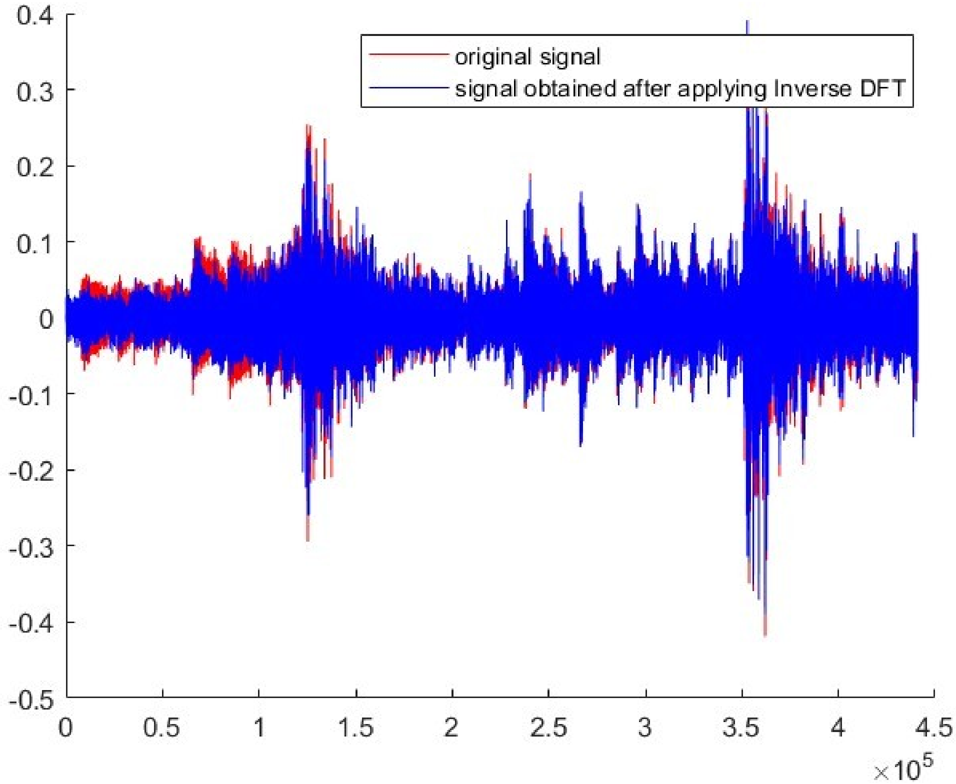


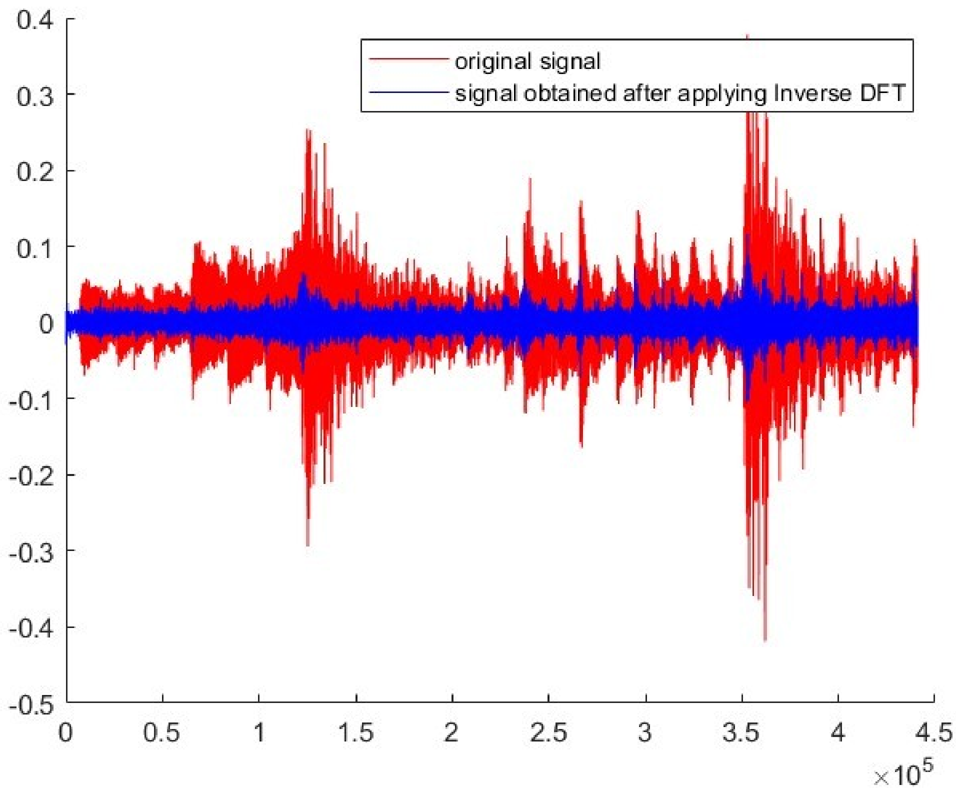

original signal
signal obtained after applying Inverse DFT
0.4
0.3
0.2
0.1
0
-0.1
-0.2
-0.3
-0.4
-0.5
0
0.5
1
1.5
2
2.5
3
3.5
4
4.5
$\times 10^5$

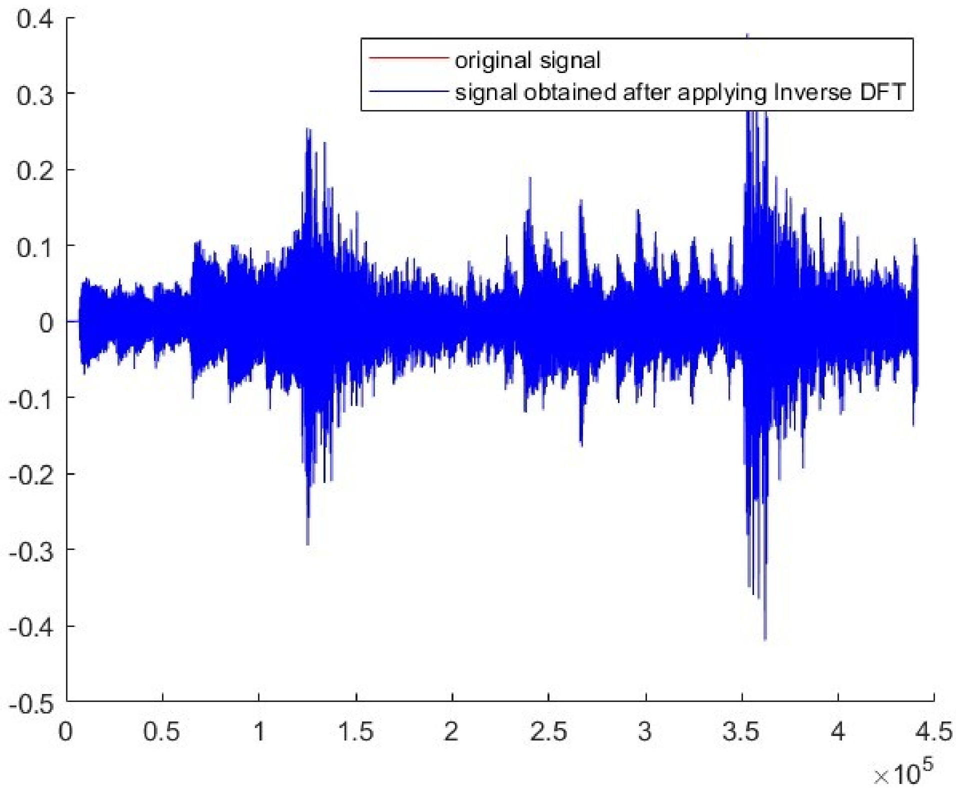

original signal
signal obtained after applying Inverse DFT
0.5
0.4
0.3
0.2
0.1
0
-0.1
-0.2
-0.3
-0.4
-0.5
0
0.5
1
1.5
2
2.5
3
3.5
4
4.5
$\times 10^5$



The output when $N = M$ and less than the length of the original signal is better than the output when $N > M$ and the output when $N < M$. The output when $N < M$ has the same length as that of the original signal. The output when $N > M$ has half the length as that of the original signal.


original signal
signal obtained after applying Inverse DFT
0.4
0.3
0.2
0.1
0
-0.1
-0.2
-0.3
-0.4
-0.5
0
0.5
1
1.5
2
2.5
3
3.5
4
4.5
$\times 10^5$


original signal
signal obtained after applying Inverse DFT
0.4
0.3
0.2
0.1
0
-0.1
-0.2
-0.3
-0.4
-0.5
0
0.5
1
1.5
2
2.5
3
3.5
4
4.5
$\times 10^5$



When the range of frequencies was (1000, 30000), the output was similar to the original signal. When the range of frequencies was (100, 30000), the output was very different from the original signal. When the range of frequencies was (30000, 40000), the output was nearly identical to the original signal.

Important Notes:

- All deliverables should be included in one .zip file.
- For Problems 1 and 2, a scanned version of handwritten solutions is acceptable.
- For Problem 3, include the plots in a report.
- For Problem 4, you can use DFT, Inverse DFT, and any necessary functions available in MATLAB or Python.
- Include the plots of Problem 4 in the report along with the comments required in parts (iv) and (vii).
- This is an individual assignment.