MACT 4212: Stochastic Processes
Assignment 2

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Question 1

Consider the following Markov chain with state space$\{1, 2, 3\}$ and transition matrix and Find:

$$P = \begin{pmatrix} 0.2 & 0.8 & 0 \\ 0.5 & 0 & 0.5 \\ 0.25 & 0.25 & 0.5 \end{pmatrix}$$

1. $P[X_2 = 2|X_0 = 1]$.
2. $P[X_3 = 2|X_0 = 1, X_2 = 1]$.
3. $P[X_1 = 2|X_2 = 1]$.
4. $P[X_2 = 3]$ given that $\pi^T = [0.2 \quad 0.3 \quad 0.5]$

Question 2

Three white and three black balls are distributed equally into two urns. We say that the system is in state $i, i = 0, 1, 2, 3$, if the first urn contains 3 white balls. At each step, we draw one ball from each urn and exchange them. Let X_n denote the state of the system after the n-th step. Find the transition probability matrix.

Question 3: R application

Open the R code attached, modify and run the code to answer the following questions

1. Draw the transition diagram in question 1 using the R code provided. Label the states as 1, 2 and 3.
2. Find $P(X_4 = 2|X_0 = 1)$ using the R code.