

- 7.3. Prove that Maxwell's equations in the physical form of Exercise 2.16(a) form a Hamiltonian system with Poisson bracket

$$\{\mathcal{F}, \mathcal{H}\} = \int \left(\frac{\delta \mathcal{F}}{\delta E} \cdot \nabla \times \frac{\delta \mathcal{H}}{\delta B} - \frac{\delta \mathcal{H}}{\delta E} \cdot \nabla \times \frac{\delta \mathcal{F}}{\delta B} \right) dx.$$

Discuss symmetries and conservation laws. (See also Exercises 4.6 and 5.25.) (Born and Infeld, [1], Marsden, [1].)

- 7.4. Derive the conservation laws \mathcal{P}_α , \mathcal{P}_β for the two-dimensional Euler equations found in Example 7.17 directly from the conservation law of energy using Proposition 5.64. (Ibragimov, [1; p. 357].)
- *7.5. Prove that the three-dimensional Euler equations for incompressible fluid flow, when replaced by the corresponding vorticity equations for $\omega = \nabla \times u$, form a Hamiltonian system relative to the operator \mathcal{D} , where

$$\mathcal{D}P = \omega \cdot \nabla P - (\nabla \omega) \nabla \times P$$

(∇ denoting total gradient, curl or divergence). Find the conservation laws corresponding to known symmetry groups. Prove that the only nontrivial distinguished functional is the "total helicity" $\mathcal{H} = \int (u \cdot \omega) dx$. (Olver, [5]; see Serre, [1], and Khesin and Chekanov, [1], for the n -dimensional case.)

- 7.6. Let $\mathcal{L}[u]$ be a variational problem with Euler–Lagrange equations $\delta \mathcal{L} = 0$. Suppose v_Q generates a variational symmetry group with conservation law $\text{Div } P = 0$. Prove that the corresponding dynamical Hamiltonian equations $u_t = \mathcal{D} \cdot \delta \mathcal{L}$ have a corresponding conservation law if and only if $v_Q = \mathfrak{V}_{\mathcal{D}}$ is Hamiltonian with respect to the given Poisson bracket.

- 7.7. The dynamical equations of elasticity take the form

$$\frac{\partial^2 u^\alpha}{\partial t^2} = \sum_{i=1}^p D_i \left(\frac{\partial W}{\partial u_i^\alpha} \right), \quad \alpha = 1, \dots, q,$$

where $W(x, \nabla u)$ is the stored energy function, cf. Example 4.32. Prove that these can be put into Hamiltonian form using the total energy

$$\mathcal{H} = \int \left[\frac{1}{2} |u_t|^2 + W(x, \nabla u) \right] dx$$

as the Hamiltonian and u , $v = u_t$ as canonical variables. Discuss the conservation laws of this system in light of Exercise 7.6 and Example 4.32. (D. C. Fletcher, [1], Marsden and Hughes, [1; § 5.5].)

- 7.8. (a) Let $\mathcal{D} = \mathcal{A}^q \rightarrow \mathcal{A}^q$ be a differential operator. Prove that if $\mathcal{C}[u]$ is any functional satisfying $\mathcal{D}^* \cdot \delta \mathcal{C} = 0$, then \mathcal{C} is a conservation law for any evolutionary system of the form $u_t = \mathcal{D}Q$ for $Q \in \mathcal{A}^q$.
- (b) Prove that any evolution equation of the form $u_t = D_x^m Q$, with $x, u \in \mathbb{R}$, always conserves the first $m + 1$ moments $\mathcal{M}_j = \int x^j u dx$, $j = 0, 1, \dots, m$, of any solution.

- 7.9. Prove that the operators

$$\mathcal{D} = D_x, \quad \mathcal{E} = D_x^3 + \frac{2}{3} D_x \cdot u D_x^{-1} \cdot u D_x,$$

form a Hamiltonian pair making the modified Korteweg–de Vries equation $u_t = u_{xxx} + u^2 u_x$ into a bi-Hamiltonian system. Find the recursion operator and the first few symmetries. How do these relate to the Korteweg–de Vries equation under the Miura transformation of Exercise 5.11? (Magri, [2])

- 7.10. The Harry Dym equation is $u_t = D_x^3(u^{-1/2})$. Prove that this is a bi-Hamiltonian system with $\mathcal{D} = 2uD_x + u_x$, $\mathcal{E} = D_x^3$. Discuss distinguished functionals, symmetries and conservation laws for this equation. The change of variables $v = u^{-1/2}$ changes this equation to $v_t = -\frac{1}{2}v^3 v_{xxx}$. Discuss its effects on the bi-Hamiltonian structure. (Magri, [1], Leo, Leo, Soliani, Solombrino and Mancarella, [1]; Ibragimov, [1; p. 300], shows how this equation can be transformed into the Korteweg–de Vries equation.)

- **7.11. The system of equations

$$u_t = uu_x + v_x - \frac{1}{2}u_{xx}, \quad v_t = (uv)_x + \frac{1}{2}v_{xx},$$

is equivalent, under a change of variables, to a system of Boussinesq equations modelling the bi-directional propagation of long waves in shallow water, first found by Whitham, [1] and Broer, [1]. Prove that this system is *tri-Hamiltonian*, meaning that it can be written as a Hamiltonian system using any one of the three Hamiltonian operators

$$\begin{aligned} \mathcal{D}_0 &= \begin{pmatrix} 0 & D_x \\ D_x & 0 \end{pmatrix}, & \mathcal{D}_1 &= \begin{pmatrix} 2D_x & D_x \cdot u - D_x^2 \\ uD_x + D_x^2 & 2vD_x + v_x \end{pmatrix}, \\ \mathcal{D}_2 &= \begin{pmatrix} 4uD_x + 2u_x & 4vD_x + 2v_x + D_x(D_x - u)^2 \\ 4vD_x + 2v_x + (D_x + u)^2 D_x & (D_x + u)(2vD_x + v_x) - (2vD_x + v_x)(D_x - u) \end{pmatrix}, \end{aligned}$$

and any two of these operators form a Hamiltonian pair. Discuss symmetries and conservation laws of the system. (Kupershmidt, [2].)

- 7.12. (a) Prove that if \mathcal{D} is a self-adjoint (respectively, skew-adjoint) matrix differential operator, and \mathbf{v}_Q is any evolutionary vector field, then the Lie derivative $\text{pr } \mathbf{v}_Q(\mathcal{D})$ is self-adjoint (skew-adjoint).
 (b) Prove directly that (7.11) is an alternating, trilinear function of P, Q, R .
- 7.13. Prove that if $\mathcal{D}: \mathcal{A} \rightarrow \mathcal{A}$ and $\mathcal{E}: \mathcal{A} \rightarrow \mathcal{A}$ are nonzero scalar differential operators, then $\mathcal{E} \cdot \mathcal{D}: \mathcal{A} \rightarrow \mathcal{A}$ is a nonzero differential operator. Deduce that any scalar differential operator is nondegenerate in the sense of Definition 7.23.
- *7.14. Let $\mathcal{D}: \mathcal{A}^* \rightarrow \mathcal{A}^*$ be a differential operator, and let $\mathcal{K}^* = \{Q \in \mathcal{A}^*: \mathcal{D}^*Q = 0\}$ be the kernel of its adjoint. Prove that if \mathcal{K}^* is a finite-dimensional vector space over \mathbb{R} , then \mathcal{D} is nondegenerate in the sense of Definition 7.23. How many distinguished functionals does a nondegenerate Hamiltonian operator have?
- *7.15. The equations of polytropic gas dynamics have the form

$$u_t + uu_x + v^\sigma v_x = 0, \quad v_t + (uv)_x = 0,$$

in which u represents the velocity, v the density and $\sigma = \gamma - 2$, where γ is the physical ratio of specific heats appearing in the pressure-density relation. Show that this system can be written in Hamiltonian form in three distinct ways, using the Hamiltonian operators

$$\mathcal{D}_1 = \begin{pmatrix} 0 & D_x \\ D_x & 0 \end{pmatrix},$$

$$\mathcal{D}_2 = \begin{pmatrix} 2v^\sigma D_x + (v^\sigma)_x & (\sigma + 1)uD_x + u_x \\ (\sigma + 1)uD_x + \sigma u_x & 2vD_x + v_x \end{pmatrix},$$

$$\mathcal{D}_3 = \begin{pmatrix} 2uv^\sigma D_x + (uv^\sigma)_x & \left[\frac{1}{2}(\sigma + 1)u^2 + \frac{2}{\sigma + 1}v^\sigma \right] D_x + uu_x + v^\sigma v_x \\ \left[\frac{1}{2}(\sigma + 1)u^2 + \frac{2}{\sigma + 1}v^\sigma \right] D_x + \sigma uu_x + v^\sigma v_x & 2uvD_x + (uv)_x \end{pmatrix}.$$

Prove that each operator is Hamiltonian. Which pairs are compatible? Discuss the consequential recursion operators, symmetries and conservation laws. (Whitham, [2], Nutku, [1], Olver and Nutku, [1].)

- 7.16. Prove that the Toda lattice equations of Exercise 6.11 are a bi-Hamiltonian system for the two Poisson brackets with structure matrices

$$\begin{pmatrix} 0 & 0 & 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & -1 & 1 & 0 \\ 0 & 0 & 0 & 0 & -1 & 1 \\ -1 & 1 & 0 & 0 & 0 & 0 \\ 0 & -1 & 1 & 0 & 0 & 0 \\ 1 & 0 & -1 & 0 & 0 & 0 \end{pmatrix}, \quad \begin{pmatrix} 0 & -e^{q^1} & e^{q^3} & p^1 & 0 & -p^1 \\ e^{q^1} & 0 & -e^{q^2} & -p^2 & p^2 & 0 \\ -e^{q^3} & e^{q^2} & 0 & 0 & -p^3 & p^3 \\ -p^1 & p^2 & 0 & 0 & 1 & -1 \\ 0 & -p^2 & p^3 & -1 & 0 & 1 \\ p^1 & 0 & -p^3 & 1 & -1 & 0 \end{pmatrix},$$

relative to the coordinates $(p^1, p^2, p^3, q^1, q^2, q^3)$. Are these two Poisson brackets compatible? (Arnol'd and Novikov, [1; p. 58], Leo, Leo, Soliani, Solombrino and Mancarella, [2].)

- *7.17. Show that the structure matrices

$$J_1 = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \end{pmatrix}, \quad J_2 = \begin{pmatrix} 0 & 0 & e^{p_1} & -p_2 e^{p_1} \\ 0 & 0 & 0 & -e^{p_1} \\ -e^{p_1} & 0 & 0 & 0 \\ p_2 e^{p_1} & e^{p_1} & 0 & 0 \end{pmatrix},$$

using coordinates (p_1, p_2, q_1, q_2) , are Hamiltonian, but do not form a Hamiltonian pair. Discuss the integrability of any associated "bi-Hamiltonian systems." (Olver, [15].)

- 7.18. Let $u_t = \mathcal{D}\delta\mathcal{H}$ be a scalar Hamiltonian evolution equation. Prove that the pseudo-differential operator \mathcal{D}^{-1} is a formal conservation law of rank ∞ .

References

- Abellanas, L. and Galindo, A.
1. Conserved densities for linear evolution systems, *Commun. Math. Phys.* **79** (1981), 341–351.
- Ablowitz, M. J. and Kodama, Y.
1. Note on asymptotic solutions of the Korteweg–de Vries equation with solitons, *Stud. Appl. Math.* **66** (1982), 159–170.
- Ablowitz, M. J., Ramani, A. and Segur, H.
1. Nonlinear evolution equations and ordinary differential equations of Painlevé type, *Lett. Nuovo Cim.* **23** (1978), 333–338.
 2. A connection between nonlinear evolution equations and ordinary differential equations of P -type. I, *J. Math. Phys.* **21** (1980), 715–721.
- Abraham, R. and Marsden, J. E.
1. *Foundations of Mechanics*, 2nd ed., Benjamin-Cummings, Reading, Mass., 1978.
- Abramowitz, M., and Stegun, I.
1. *Handbook of Mathematical Functions*, National Bureau of Standards Appl. Math. Series, No. 55, U.S. Govt. Printing Office, Washington, D.C., 1970.
- Adams, M., Ratiu, T. and Schmidt, R.
1. A Lie group structure for pseudodifferential operators, *Math. Ann.* **273** (1986), 529–551.
 2. A Lie group structure for Fourier integral operators, *Math. Ann.* **276** (1986), 19–41.
- Adler, M.
1. On a trace functional for formal pseudo-differential operators and the symplectic structure of the Korteweg–de Vries type equations, *Invent. Math.* **50** (1979), 219–248.
- Ado, I. D.
1. The representation of Lie algebras by matrices, *Bull. Soc. Phys.-Math. Kazan* **7** (1935), 3–43; also *Usp. Mat. Nauk.* **2** (1947), 159–173, and Amer. Math. Soc. Transl., No. 2, 1949.

- Akhmatov, I. S., Gazizov, R. K. and Ibragimov, N. H.
 1. Nonlocal symmetries. Heuristic approach, *J. Sov. Math.* **55** (1991), 1401–1450.
- Aldersley, S. J.
 1. Higher Euler operators and some of their applications, *J. Math. Phys.* **20** (1979), 522–531.
- Alonso, L. M.
 1. On the Noether map, *Lett. Math. Phys.* **3** (1979), 419–424.
- Ames, W. F.
 1. *Nonlinear Partial Differential Equations in Engineering*, Academic Press, New York, 1965, 1972.
- Anderson, I. M.
 1. Introduction to the variational bicomplex, *Comtemp. Math.* **132** (1992), 51–73.
- Anderson, I. M. and Duchamp, T. E.
 1. On the existence of global variational principles, *Amer. J. Math.* **102** (1980), 781–868.
 2. Variational principles for second-order quasi-linear scalar equations, *J. Diff. Eq.* **51** (1984), 1–47.
- Anderson, I. M., Kamran, N. and Olver, P. J.
 1. Internal, external and generalized symmetries, *Adv. Math.* **100** (1993), 53–100.
- Anderson, I. and Thompson, G.
 1. The inverse problem of the calculus of variations for ordinary differential equations, *Mem. Amer. Math. Soc.* **473** (1992), 1–110.
- Anderson, R. L. and Ibragimov, N. H.
 1. *Lie-Bäcklund Transformations in Applications*, SIAM Studies in Appl. Math., No. 1, Philadelphia, 1979.
- Anderson, R. L., Kumei, S. and Wulfman, C. E.
 1. Generalizations of the concept of invariance of differential equations. Results of applications to some Schrödinger equations, *Phys. Rev. Lett.* **28** (1972), 988–991.
- Appell, P.
 1. Sur l'équation $\partial^2 z / \partial x^2 - \partial z / \partial y = 0$ et la théorie de la chaleur, *J. de Math. Pures et Appl.* **8**(4), (1892), 187–216.
- Arnol'd, V. I.
 1. Sur la géométrie différentielle des groupes de Lie de dimension infinie et ses applications à l'hydrodynamique des fluides parfaits, *Ann. Inst. Fourier Grenoble* **16** (1966), 319–361.
 2. The Hamiltonian nature of the Euler equations in the dynamics of a rigid body and an ideal fluid, *Usp. Mat. Nauk*, **24** (1969), 225–226 (in Russian).
 3. *Mathematical Methods of Classical Mechanics*, Springer-Verlag, New York, 1978.
- Arnol'd, V. I. and Novikov, S. P. (eds.)
 1. *Dynamical Systems IV*, Encyclopaedia of Mathematical Sciences, vol. 4, Springer-Verlag, New York, 1990.
- Astashov, A. M. and Vinogradov, A. M.
 1. On the structure of Hamiltonian operators in field theory, *J. Geom. Phys.* **3** (1986), 263–287.

Atherton, R. W. and Homsy, G. M.

1. On the existence and formulation of variational principles for nonlinear differential equations, *Stud. Appl. Math.* **54** (1975), 31–60.

Bäcklund, A. V.

1. Ueber Flächentransformationen, *Math. Ann.* **9** (1876), 297–320.

Baikov, V. A., Gazizov, R. K. and Ibragimov, N. H.

1. Approximate symmetries, *Math. USSR Sbornik* **64** (1989), 427–441.

Baker, J. W. and Tavel, M. A.

1. The applications of Noether's theorem to optical systems, *Amer. J. Physics* **42** (1974), 857–861.

Bakirov, I. M.

1. On the symmetries of some system of evolution equations, preprint, 1991.

Barenblatt, G. I.

1. *Similarity, Self-similarity, and Intermediate Asymptotics*, Consultants Bureau, New York, 1979.

Bargmann, V.

1. Irreducible unitary representations of the Lorentz group, *Ann. Math.* **48** (1947), 568–640.

Bateman, H.

1. The conformal transformations of a space of four dimensions and their applications to geometrical optics, *Proc. London Math. Soc.* **7** (1909), 70–89.
2. On dissipative systems and related variables, *Phys. Rev.* **38** (1931), 815–819.

Benjamin, T. B.

1. The stability of solitary waves, *Proc. Roy. Soc. London A* **328** (1972), 153–183.

Benjamin, T. B. and Olver, P. J.

1. Hamiltonian structure, symmetries and conservation laws for water waves, *J. Fluid Mech.* **125** (1982), 137–185.

Berezin, F. A.

1. Some remarks about the associated envelope of a Lie algebra, *Func. Anal. Appl.* **1** (1967), 91–102.

Berker, R.

1. Intégration des équations du mouvement d'un fluide visqueux incompressible, in *Handbuch der Physik*, VIII/2, Springer-Verlag, Berlin, 1963, pp. 1–384.

Bessel-Hagen, E.

1. Über die Erhaltungssätze der Elektrodynamik, *Math. Ann.* **84** (1921), 258–276.

Beyer, W. A.

1. Lie-group theory for symbolic integration of first order ordinary differential equations, in *Proceedings of the 1979 Macsyma Users Conference*, V. E. Lewis, ed., MIT Laboratory for Computer Science, Cambridge, Mass., 1979, pp. 362–384.

Bianchi, L.

1. *Lezioni sulla teoria dei Gruppi Continui Finiti di Trasformazioni*, Enrico Spoerri, Pisa, 1918.

Bilby, B. A., Miller, K. J. and Willis, J. R.

1. *Fundamentals of Deformation and Fracture*, Cambridge University Press, Cambridge, 1985.

Birkhoff, G.

1. Lie groups isomorphic with no linear group, *Bull. Amer. Math. Soc.* **42** (1936), 883–888.
2. *Hydrodynamics—A Study in Logic, Fact and Similitude*, 1st ed., Princeton University Press, Princeton, 1950.

Bluman, G. W. and Cole, J. D.

1. The general similarity solution of the heat equation, *J. Math. Mech.* **18** (1969), 1025–1042.
2. *Similarity Methods for Differential Equations*, Appl. Math. Sci., No. 13, Springer-Verlag, New York, 1974.

Bluman, G. W. and Kumei, S.

1. On the remarkable nonlinear diffusion equation $(\partial/\partial x)[a(u+b)^{-2}(\partial u/\partial x)] - (\partial u/\partial t) = 0$, *J. Math. Phys.* **21** (1980), 1019–1023.
2. *Symmetries and Differential Equations*, Springer-Verlag, New York, 1989.

Boltzmann, L.

1. Zur integration der Diffusionsgleichung bei variabeln Diffusionscoefficienten, *Ann. der Physik und Chemie* **53** (1894), 959–964.

Boothby, W. M.

1. *An Introduction to Differentiable Manifolds and Riemannian Geometry*, Academic Press, New York, 1975.

Born, M. and Infeld, L.

1. On the quantization of the new field theory. II, *Proc. Roy. Soc. London* **150A** (1935), 141–166.

Bott, R. and Tu, L. W.

1. *Differential Forms in Algebraic Topology*, Springer-Verlag, New York, 1982.

Bourlet, M. C.

1. Sur les équations aux dérivées partielles simultanées, *Ann. Sci. École Norm. Sup.* **8**(3) (1891), Suppl. S.3–S.63.

Boyer, C. P., Kalnins, E. G. and Miller, W., Jr.

1. Symmetry and separation of variables for the Helmholtz and Laplace equations, *Nagoya Math. J.* **60** (1976), 35–80.

Broer, L. J. F.

1. Approximate equations for long water waves, *Appl. Sci. Res.* **31** (1975), 377–395.

Brown, A. B.

1. Functional dependence, *Trans. Amer. Math. Soc.* **38** (1935), 379–394.

Buchnev, A. A.

1. The Lie group admitted by the motion of an ideal incompressible fluid, *Dinamika Splosh. Sredi* **7** (1971), 212–214 (in Russian).

Byrd, P. F. and Friedman, M. D.

1. *Handbook of Elliptic Integrals for Engineers and Scientists*, Springer-Verlag, New York, 1971.

Carathéodory, C.

1. *Calculus of Variations and Partial Differential Equations of the First Order*, Vol. 1, Holden-Day, New York, 1965.

Carmichael, R. D.

1. Transformations leaving invariant certain partial differential equations of physics, *Amer. J. Math.* **49** (1927), 97–116.

Cartan, E.

1. *Leçons sur les Invariants Intégraux*, Hermann, Paris, 1922.
2. *La Théorie des Groupes Finis et Continus et l'Analysis Situs*, Mém. Sci. Math. No. 42, Gauthier-Villars, Paris, 1930; also, *Oeuvres Complètes*, vol. 1, Gauthier-Villars, Paris, 1952, pp. 1165–1225.
3. *La Topologie des Groupes de Lie*, Exp. de Géométrie, vol. 8, Hermann, Paris, 1936; also, *Oeuvres Complètes*, vol. 1, Gauthier-Villars, Paris, 1952, pp. 1307–1330.

Cartan, E. and Einstein, A.

1. *Letters on Absolute Parallelism 1929–1932*, Princeton Univ. Press, Princeton, N.J., 1979.

Champagne, B., Hereman, W. and Winternitz, P.

1. The computer calculation of Lie point symmetries of large systems of differential equations, *Comp. Phys. Commun.* **66** (1991), 319–340.

Chen, H. H., Lee, Y. C. and Lin, J.-E.

1. On a new hierarchy of symmetries for the integrable nonlinear evolution equations, in *Advances in Nonlinear Waves*, Vol. 2, L. Debnath, ed., Research Notes in Math., Vol. 111, Pitman Publ. Inc., Marshfield, Mass., 1985, pp. 233–239.

Chevalley, C. C.

1. *Theory of Lie Groups*, vol. 1, Princeton University Press, Princeton, N.J., 1946.

Clarkson, P. and Kruskal, M.

1. New similarity reductions of the Boussinesq equation, *J. Math. Phys.* **30** (1989), 2201–2213.

Cohen, A.

1. *An Introduction to the Lie Theory of One-Parameter Groups, with Applications to the Solution of Differential Equations*, D. C. Heath & Co., New York, 1911.

Conn, J. F.

1. Normal forms for analytic Poisson structures, *Ann. Math.* **119** (1984), 577–601.

Cooke, D. B.

1. Classification results and the Darboux theorem for low order Hamiltonian operators, *J. Math. Phys.* **32** (1991), 109–119.

Copson, E. T.

1. *Partial Differential Equations*, Cambridge University Press, Cambridge, 1975.

Courant, R. and Hilbert, D.

1. *Methods of Mathematical Physics*, Interscience, New York, 1953.

Crampin, M.

1. A note on non-Noether constants of motion, *Phys. Lett.* **95A** (1983), 209–212.

Cunningham, E.

1. The principle of relativity in electrodynamics and an extension thereof, *Proc. London Math. Soc.* **8** (1909), 77–98.

Dedecker, P.

1. Calcul des variations et topologie algébrique, *Mém. Soc. Roy. Sci. de Liège* **29** (1957), 7–216.

Delassus, E.

1. Extension du théorème de Cauchy aux systèmes les plus généraux d'équations aux dérivées partielles, *Ann. Sci. École Norm. Sup.* **13**(3) (1896), 421–467.

Delong, R. P., Jr.

1. Killing tensors and the Hamilton–Jacobi equation, Ph.D. thesis, Univ. of Minnesota, 1982.

Dickey, L.

1. *Soliton Equations and Hamiltonian Systems*, World Scientific, Singapore, 1991.

DiPerna, R. J.

1. Decay of solutions of hyperbolic systems of conservation laws with a convex extension, *Arch. Rat. Mech. Anal.* **64** (1977), 1–46.
2. Uniqueness of solutions to hyperbolic conservation laws, *Indiana Univ. Math. J.* **28** (1979), 137–188.

Dirac, P. A. M.

1. Generalized Hamiltonian dynamics, *Canad. J. Math.* **2** (1950), 129–148.

Dorfman, I. Y.

1. On differential operators that generate Hamiltonian structures, *Phys. Lett. A* **140** (1989), 378–382.

Dorodnitsyn, V. A.

1. Transformation groups in net spaces, *J. Sov. Math.* **55** (1991), 1490–1517.

Douglas, J.

1. Solution of the inverse problem of the calculus of variations, *Trans. Amer. Math. Soc.* **50** (1941), 71–128.

Doyle, P.

1. *Differential Geometric Poisson Bivectors and Quasilinear Systems in One Space Variable*, Ph.D. Thesis, University of Minnesota, 1992.

Dresner, L.

1. *Similarity Solutions of Nonlinear Partial Differential Equations*, Research Notes in Math., No. 88, Pitman, Boston, 1983.

Drinfel'd, V. G.

1. Quantum groups, in *Proc. Int. Cong. Math., Berkeley*, Vol. 1, 1986, pp. 798–820.

Dubrovin, B. A. and Novikov, S. A.

1. Hamiltonian formalism of one-dimensional systems of hydrodynamic type and the Bogolyubov–Whitham averaging method, *Sov. Math. Dokl.* **27** (1983), 665–669.
2. On Poisson brackets of hydrodynamic type, *Sov. Math. Dokl.* **30** (1984), 651–654.
3. Hydrodynamics of weakly deformed soliton lattices. Differential geometry and Hamiltonian theory, *Russian Math. Surveys* **44**: 6 (1989), 35–124.

Edelen, D. G. B.

1. *Isovector Methods for Equations of Balance*, Sijthoff and Noordhoff, Germantown, Md., 1980.

Ehresmann, C.

1. Les prolongements d'une variété différentiable, *C.R. Acad. Sci. Paris* **233** (1951), 598–600, 777–779, 1081–1083; **234** (1952), 1028–1030, 1424–1425.
2. Introduction à la théorie des structures infinitésimales et des pseudo-groupes de Lie, in *Géométrie Différentielle*, Colloq. Inter. du Centre Nat. de la Recherche Scientifique, Strasbourg, 1953, 97–110.

Eisenhart, L. P.

1. *Riemannian Geometry*, Princeton University Press, Princeton, 1926.
2. *Continuous Groups of Transformations*, Princeton University Press, Princeton, 1933.

Elkana, Y.

1. *The Discovery of the Conservation of Energy*, Hutchinson Educational Ltd., London, 1974.

Engel, F.

1. Über die zehn allgemeinen Integrale der klassischen Mechanik, *Nachr. König. Gesell. Wissen. Göttingen, Math.-Phys. Kl.* (1916), 270–275.

Ericksen, J. L.

1. On the formulation of St.-Venant's problem, in *Nonlinear Analysis and Mechanics: Heriot-Watt Symposium*, R. J. Knops, ed., Research Notes in Math., No. 17, Pitman, San Francisco, 1977, pp. 158–186.

Eshelby, J. D.

1. The continuum theory of lattice defects, in *Solid State Physics*, Vol. 3, F. Seitz and D. Turnbull, eds., Academic Press, New York, 1956, pp. 79–144.

Finzi, A.

1. Sur les systèmes d'équations aux dérivées partielles qui, comme les systèmes normaux, comportent autant d'équations que de fonctions inconnues, *Proc. Kon. Neder. Akad. v. Wetenschappen* **50** (1947), 136–142, 143–150, 288–297, 351–356.

Fletcher, D. C.

1. Conservation laws in linear elastodynamics, *Arch. Rat. Mech. Anal.* **60** (1976), 329–353.

Fletcher, J. G.

1. Local conservation laws in generally covariant theories, *Rev. Mod. Phys.* **32** (1960), 65–87.

Fokas, A. S.

1. Group theoretical aspects of constants of motion and separable solutions in classical mechanics, *J. Math. Anal. Appl.* **68** (1979), 347–370.
2. Generalized symmetries and constants of motion of evolution equations, *Lett. Math. Phys.* **3** (1979), 467–473.
3. A symmetry approach to exactly solvable evolution equations, *J. Math. Phys.* **21** (1980), 1318–1325.
4. Symmetries and integrability, *Stud. Appl. Math.* **77** (1987), 253–299.

Fokas, A. S. and Fuchssteiner, B.

1. The hierarchy of the Benjamin-Ono equation, *Phys. Lett.* **86A** (1981), 341–345.

Fokas, A. S. and Santini, P.

1. Recursion operators and bi-Hamiltonian structures in multi-dimensions. II, *Commun. Math. Phys.* **116** (1988), 449–474.

Fokas, A. S. and Yortsos, Y. C.

1. On the exactly solvable equation $S_t = [(\beta S + \gamma)^{-2} S_x]_x + \alpha(\beta S + \gamma)^{-2} S_x$ occurring in two-phase flow in porous media, *SIAM J. Appl. Math.* **42** (1982), 318–332.

Forsyth, A. R.

1. *The Theory of Differential Equations*, Cambridge University Press, Cambridge, 1890, 1900, 1902, 1906.

Frobenius, G.

1. Über das Pfaffsche Probleme, *J. Reine Angew. Math.* **82** (1877), 230–315.