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Geometrical Formulation of Renormalization-Group Method as an Asymptotic Analysis

With Applications to Derivation of Causal Fluid Dynamics

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Preface

The purpose of this monograph is twofold: The one is to present a comprehensive account of the so-called Renormalization-Group (RG) method and its extension, which we named the doublet scheme, in a geometrical point of view with various examples. The second is the application of the doublet scheme in the RG method to the derivation of the so-called second-order (causal) fluid dynamics from the relativistic as well as non-relativistic Boltzmann equations with quantum statistics. All the contents of the monograph are virtually based on the authors original work except for some review parts.

The RG method is a global and asymptotic analysis of differential equations, which was developed by an Illinois group and others some thirty years ago. We introduce the method in a purely mathematical way on the basis of the classical theory of envelopes, a notion in elementary differential geometry.

Then a focus is put on the fact that the RG method provides us with a powerful and transparent method for the reduction of dynamics, which includes an elementary method of a construction of the invariant/attractive manifolds and reduced dynamical equations written in terms of the variables that constitute the coordinates of the manifolds. They are the key concepts in the reduction theory of dynamical systems, and thereby naturally lead to a foundation to existing theories in the specific physical systems such as Krylov-Bogoliubov-Mitropolsky theory for non-linear oscillators and Kuramoto's reduction theory for evolution equations. Examples treated include stochastic equations like Langevin and Fokker-Planck equations.

Although the RG method is applicable to discrete systems and thereby provides us with an optimized discretization scheme of differential equations, we have omitted the once prepared part on the discrete systems partly because of its irrelevance to the second part and partly for the sake of making the already lengthy monograph too voluminous. We hope that there will be a chance to publish the omitted part somewhere else in the future.

The usual reduction theory including the RG method based on the perturbation theory utilizes the zero modes of the linear operator in the unperturbed equation. However, in the derivation of the so-called causal second-order fluid dynamics, which has a nature of the mesoscopic dynamics of the given system, from the Boltzmann equation, one needs to extend the invariant manifold so as to incorporate appropriate excited modes as the additional coordinate variables. In this monograph, a general reduction theory is presented for constructing the mesoscopic dynamics with inclusion of appropriate excited modes from the given microscopic equation, which is formulated as an extension of the RG method and called the doublet scheme.

In the second part, we work out for the derivation of the second-order or causal fluid dynamics on the basis of the doublet scheme: Thus we obtain not only the fluid dynamic equations that are uniquely those in the energy frame but also the microscopic formulae of the transport coefficients and the relaxation times in Kubo-like formulae that admit natural physical interpretations. It is shown that the resultant fluid dynamics is not only causal but also stable dissipative (non-)relativistic fluid dynamics. The derivation of the mesoscopic dynamics beyond the fluid dynamics in the non-relativistic regime is also one of the hot topics in physics of cold atoms. The present monograph includes numerical analyses on these interesting physical systems. We also provide an accurate and efficient numerical method for computing the transport equations and relaxation times using the microscopic expressions. The numerical method utilizes the double exponential formulae for integrations and the direct matrix inversion method. The numerical calculations are fully worked out for typical model systems composed of classical particles, a fermion system with a Yukawa interaction, and boson system described by a chiral Lagrangian. The numerical calculations are also presented for the non-relativistic system and a critical comparison is made with the relaxation-time approximation, which is commonly used in the current literature.

The presentation of the monograph, at least in the first part, is intended to be as pedagogical as possible so that not only researchers who are not familiar with the RG theory in physics but also motivated undergraduate students with mathematical backgrounds such as introductory calculus including linear differential equations and linear algebra may appreciate and understand the method.

Conversely, the monograph is not intended to be a systematic review of the current status of the studies of the RG method and causal fluid dynamics either with respect to its foundations or applications. Since the literature on these subjects is huge now that making a systematic review on this subject is not the intention of the authors and beyond the authors' ability. Therefore we apologize to those whose important articles are not cited in this monograph in advance.

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He is indebted to Shoji Yotsutani for his question made to him, which prompted him to realize the significance of the 'initial' condition in the formulation of the RG method in a purely mathematical manner. He is also deeply indebted to late Professor Masaya Yamaguti for his interest in his work on the envelope formulation of the RG method and encouragement.

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