7.3. Prove that Maxwell's equations in the physical form of Exercise 2.16(a) form a Hamiltonian system with Poisson bracket

$$\{\mathscr{F}, \mathscr{H}\} = \int \left(\frac{\delta \mathscr{F}}{\delta E} \cdot \nabla \times \frac{\delta \mathscr{H}}{\delta B} - \frac{\delta \mathscr{H}}{\delta E} \cdot \nabla \times \frac{\delta \mathscr{F}}{\delta B} \right) dx.$$

Discuss symmetries and conservation laws. (See also Exercises 4.6 and 5.25.) (Born and Infeld, [1], Marsden, [1].)

7.4. Derive the conservation laws $\mathscr{P}_\alpha$, $\mathscr{P}_\beta$ for the two-dimensional Euler equations found in Example 7.17 directly from the conservation law of energy using Proposition 5.64. (Ibragimov, [1; p. 357].)

*7.5. Prove that the three-dimensional Euler equations for incompressible fluid flow, when replaced by the corresponding vorticity equations for $\boldsymbol{\omega} = \nabla \times \mathbf{u}$, form a Hamiltonian system relative to the operator $\mathscr{D}$, where

$$\mathscr{D}P = \boldsymbol{\omega} \cdot \nabla P - (\nabla \boldsymbol{\omega}) \nabla \times P$$

(∇ denoting total gradient, curl or divergence). Find the conservation laws corresponding to known symmetry groups. Prove that the only nontrivial distinguished functional is the "total helicity" $\mathscr{C} = \int (\mathbf{u} \cdot \boldsymbol{\omega})\, d\mathbf{x}$. (Olver, [5]; see Serre, [1], and Khesin and Chekanov, [1], for the n-dimensional case.)

7.6. Let $\mathscr{L}[u]$ be a variational problem with Euler–Lagrange equations $\delta \mathscr{L} = 0$. Suppose $\mathbf{v}_Q$ generates a variational symmetry group with conservation law $\operatorname{Div} P = 0$. Prove that the corresponding dynamical Hamiltonian equations $u_t = \mathscr{D} \cdot \delta \mathscr{L}$ have a corresponding conservation law if and only if $\mathbf{v}_Q = \hat{\mathbf{v}}_{\mathscr{P}}$ is Hamiltonian with respect to the given Poisson bracket.

7.7. The dynamical equations of elasticity take the form

$$\frac{\partial^2 u^\alpha}{\partial t^2} = \sum_{i=1}^{p} D_i \left(\frac{\partial W}{\partial u_i^\alpha} \right), \qquad \alpha = 1, \ldots, q,$$

where $W(x, \nabla u)$ is the stored energy function, cf. Example 4.32. Prove that these can be put into Hamiltonian form using the total energy

$$\mathscr{H} = \int [\tfrac{1}{2} |u_t|^2 + W(x, \nabla u)]\, dx$$

as the Hamiltonian and u, $v = u_t$ as canonical variables. Discuss the conservation laws of this system in light of Exercise 7.6 and Example 4.32. (D. C. Fletcher, [1], Marsden and Hughes, [1; § 5.5].)

7.8. (a) Let $\mathscr{D} = \mathscr{A}^q \to \mathscr{A}^q$ be a differential operator. Prove that if $\mathscr{C}[u]$ is any functional satisfying $\mathscr{D}^* \cdot \delta \mathscr{C} = 0$, then $\mathscr{C}$ is a conservation law for any evolutionary system of the form $u_t = \mathscr{D} Q$ for $Q \in \mathscr{A}^q$.

(b) Prove that any evolution equation of the form $u_t = D_x^m Q$, with $x, u \in \mathbb{R}$, always conserves the first $m + 1$ moments $\mathscr{M}_j = \int x^j u\, dx$, $j = 0, 1, \ldots, m$, of any solution.

7.9. Prove that the operators

$$\mathscr{D} = D_x, \qquad \mathscr{E} = D_x^3 + \tfrac{2}{3} D_x \cdot u D_x^{-1} \cdot u D_x,$$

form a Hamiltonian pair making the modified Korteweg–de Vries equation $u_t = u_{xxx} + u^2 u_x$ into a bi-Hamiltonian system. Find the recursion operator and the first few symmetries. How do these relate to the Korteweg–de Vries equation under the Miura transformation of Exercise 5.11? (Magri, [2])

7.10. The Harry Dym equation is $u_t = D_x^3(u^{-1/2})$. Prove that this is a bi-Hamiltonian system with $\mathscr{D} = 2uD_x + u_x$, $\mathscr{E} = D_x^3$. Discuss distinguished functionals, symmetries and conservation laws for this equation. The change of variables $v = u^{-1/2}$ changes this equation to $v_t = -\frac{1}{2}v^3 v_{xxx}$. Discuss its effects on the bi-Hamiltonian structure. (Magri, [1], Leo, Leo, Soliani, Solombrino and Mancarella, [1]; Ibragimov, [1; p. 300], shows how this equation can be transformed into the Korteweg–de Vries equation.)

**7.11. The system of equations

$$u_t = uu_x + v_x - \tfrac{1}{2}u_{xx}, \qquad v_t = (uv)_x + \tfrac{1}{2}v_{xx},$$

is equivalent, under a change of variables, to a system of Boussinesq equations modelling the bi-directional propagation of long waves in shallow water, first found by Whitham, [1] and Broer, [1]. Prove that this system is *tri-Hamiltonian*, meaning that it can be written as a Hamiltonian system using any one of the three Hamiltonian operators

$$\mathscr{D}_0 = \begin{pmatrix} 0 & D_x \\ D_x & 0 \end{pmatrix}, \qquad \mathscr{D}_1 = \begin{pmatrix} 2D_x & D_x \cdot u - D_x^2 \\ uD_x + D_x^2 & 2vD_x + v_x \end{pmatrix},$$

$$\mathscr{D}_2 = \begin{pmatrix} 4uD_x + 2u_x & 4vD_x + 2v_x + D_x(D_x - u)^2 \\ 4vD_x + 2v_x + (D_x + u)^2 D_x & (D_x + u)(2vD_x + v_x) - (2vD_x + v_x)(D_x - u) \end{pmatrix},$$

and any two of these operators form a Hamiltonian pair. Discuss symmetries and conservation laws of the system. (Kupershmidt, [2].)

7.12. (a) Prove that if $\mathscr{D}$ is a self-adjoint (respectively, skew-adjoint) matrix differential operator, and $\mathbf{v}_Q$ is any evolutionary vector field, then the Lie derivative $\operatorname{pr} \mathbf{v}_Q(\mathscr{D})$ is self-adjoint (skew-adjoint).
(b) Prove directly that (7.11) is an alternating, trilinear function of P, Q, R.

7.13. Prove that if $\mathscr{D}\colon \mathscr{A} \to \mathscr{A}$ and $\mathscr{E}\colon \mathscr{A} \to \mathscr{A}$ are nonzero scalar differential operators, then $\mathscr{E} \cdot \mathscr{D}\colon \mathscr{A} \to \mathscr{A}$ is a nonzero differential operator. Deduce that any scalar differential operator is nondegenerate in the sense of Definition 7.23.

*7.14. Let $\mathscr{D}\colon \mathscr{A}^r \to \mathscr{A}^s$ be a differential operator, and let $\mathscr{K}^* = \{Q \in \mathscr{A}^s\colon \mathscr{D}^*Q = 0\}$ be the kernel of its adjoint. Prove that if $\mathscr{K}^*$ is a finite-dimensional vector space over $\mathbb{R}$, then $\mathscr{D}$ is nondegenerate in the sense of Definition 7.23. How many distinguished functionals does a nondegenerate Hamiltonian operator have?

*7.15. The equations of polytropic gas dynamics have the form

$$u_t + uu_x + v^\sigma v_x = 0, \qquad v_t + (uv)_x = 0,$$

in which u represents the velocity, v the density and $\sigma = \gamma - 2$, where γ is the physical ratio of specific heats appearing in the pressure-density relation. Show that this system can be written in Hamiltonian form in three distinct ways, using the Hamiltonian operators

$$\mathscr{D}_1 = \begin{pmatrix} 0 & D_x \\ D_x & 0 \end{pmatrix},$$

$$\mathscr{D}_2 = \begin{pmatrix} 2v^\sigma D_x + (v^\sigma)_x & (\sigma+1)uD_x + u_x \\ (\sigma+1)uD_x + \sigma u_x & 2vD_x + v_x \end{pmatrix},$$

$$\mathscr{D}_3 = \begin{bmatrix} 2uv^\sigma D_x + (uv^\sigma)_x & \left[\tfrac{1}{2}(\sigma+1)u^2 + \dfrac{2}{\sigma+1}v^\sigma\right]D_x + uu_x + v^\sigma v_x \\ \left[\tfrac{1}{2}(\sigma+1)u^2 + \dfrac{2}{\sigma+1}v^\sigma\right]D_x + \sigma uu_x + v^\sigma v_x & 2uvD_x + (uv)_x \end{bmatrix}.$$

Prove that each operator is Hamiltonian. Which pairs are compatible? Discuss the consequential recursion operators, symmetries and conservation laws. (Whitham, [2], Nutku, [1], Olver and Nutku, [1].)

7.16. Prove that the Toda lattice equations of Exercise 6.11 are a bi-Hamiltonian system for the two Poisson brackets with structure matrices

$$\begin{bmatrix} 0 & 0 & 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & -1 & 1 & 0 \\ 0 & 0 & 0 & 0 & -1 & 1 \\ -1 & 1 & 0 & 0 & 0 & 0 \\ 0 & -1 & 1 & 0 & 0 & 0 \\ 1 & 0 & -1 & 0 & 0 & 0 \end{bmatrix}, \qquad \begin{bmatrix} 0 & -e^{q^1} & e^{q^3} & p^1 & 0 & -p^1 \\ e^{q^1} & 0 & -e^{q^2} & -p^2 & p^2 & 0 \\ -e^{q^3} & e^{q^2} & 0 & 0 & -p^3 & p^3 \\ -p^1 & p^2 & 0 & 0 & 1 & -1 \\ 0 & -p^2 & p^3 & -1 & 0 & 1 \\ p^1 & 0 & -p^3 & 1 & -1 & 0 \end{bmatrix},$$

relative to the coordinates $(p^1, p^2, p^3, q^1, q^2, q^3)$. Are these two Poisson brackets compatible? (Arnol'd and Novikov, [1; p. 58], Leo, Leo, Soliani, Solombrino and Mancarella, [2].)

*7.17. Show that the structure matrices

$$J_1 = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \end{bmatrix}, \qquad J_2 = \begin{bmatrix} 0 & 0 & e^{p_1} & -p_2 e^{p_1} \\ 0 & 0 & 0 & -e^{p_1} \\ -e^{p_1} & 0 & 0 & 0 \\ p_2 e^{p_1} & e^{p_1} & 0 & 0 \end{bmatrix},$$

using coordinates (p_1, p_2, q_1, q_2), are Hamiltonian, but do not form a Hamiltonian pair. Discuss the integrability of any associated "bi-Hamiltonian systems." (Olver, [15].)

7.18. Let $u_t = \mathscr{D}\delta\mathscr{H}$ be a scalar Hamiltonian evolution equation. Prove that the pseudo-differential operator $\mathscr{D}^{-1}$ is a formal conservation law of rank ∞.

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