

- 7.3. Prove that Maxwell's equations in the physical form of Exercise 2.16(a) form a Hamiltonian system with Poisson bracket

$$\{\mathcal{F}, \mathcal{H}\} = \int \left(\frac{\delta \mathcal{F}}{\delta E} \cdot \nabla \times \frac{\delta \mathcal{H}}{\delta B} - \frac{\delta \mathcal{H}}{\delta E} \cdot \nabla \times \frac{\delta \mathcal{F}}{\delta B} \right) dx.$$

Discuss symmetries and conservation laws. (See also Exercises 4.6 and 5.25.) (Born and Infeld, [1], Marsden, [1].)

- 7.4. Derive the conservation laws $\mathcal{P}_\alpha, \mathcal{P}_\beta$ for the two-dimensional Euler equations found in Example 7.17 directly from the conservation law of energy using Proposition 5.64. (Ibragimov, [1; p. 357].)
- *7.5. Prove that the three-dimensional Euler equations for incompressible fluid flow, when replaced by the corresponding vorticity equations for $\omega = \nabla \times u$, form a Hamiltonian system relative to the operator \mathcal{D} , where

$$\mathcal{D}P = \omega \cdot \nabla P - (\nabla \omega) \nabla \times P$$

(∇ denoting total gradient, curl or divergence). Find the conservation laws corresponding to known symmetry groups. Prove that the only nontrivial distinguished functional is the “total helicity” $\mathcal{C} = \int (u \cdot \omega) dx$. (Olver, [5]; see Serre, [1], and Khesin and Chekanov, [1], for the n -dimensional case.)

- 7.6. Let $\mathcal{L}[u]$ be a variational problem with Euler–Lagrange equations $\delta \mathcal{L} = 0$. Suppose v_Q generates a variational symmetry group with conservation law $\text{Div } P = 0$. Prove that the corresponding dynamical Hamiltonian equations $u_i = \mathcal{D} \cdot \delta \mathcal{L}$ have a corresponding conservation law if and only if $v_Q = \mathcal{V}_\mathcal{P}$ is Hamiltonian with respect to the given Poisson bracket.
- 7.7. The dynamical equations of elasticity take the form

$$\frac{\partial^2 u^\alpha}{\partial t^2} = \sum_{i=1}^p D_i \left(\frac{\partial W}{\partial u_i^\alpha} \right), \quad \alpha = 1, \dots, q,$$

where $W(x, \nabla u)$ is the stored energy function, cf. Example 4.32. Prove that these can be put into Hamiltonian form using the total energy

$$\mathcal{H} = \int [\frac{1}{2} |u_i|^2 + W(x, \nabla u)] dx$$

as the Hamiltonian and $u, v = u_i$ as canonical variables. Discuss the conservation laws of this system in light of Exercise 7.6 and Example 4.32. (D. C. Fletcher, [1], Marsden and Hughes, [1; § 5.5].)

- 7.8. (a) Let $\mathcal{D} : \mathcal{A}^q \rightarrow \mathcal{A}^q$ be a differential operator. Prove that if $\mathcal{C}[u]$ is any functional satisfying $\mathcal{D}^* \cdot \delta \mathcal{C} = 0$, then \mathcal{C} is a conservation law for any evolutionary system of the form $u_i = \mathcal{D}Q$ for $Q \in \mathcal{A}^q$.
- (b) Prove that any evolution equation of the form $u_i = D_x^m Q$, with $x, u \in \mathbb{R}$, always conserves the first $m+1$ moments $\mathcal{M}_j = \int x^j u dx, j = 0, 1, \dots, m$, of any solution.

- 7.9. Prove that the operators

$$\mathcal{D} = D_x, \quad \mathcal{E} = D_x^3 + \frac{2}{3} D_x \cdot u D_x^{-1} \cdot u D_x,$$

form a Hamiltonian pair making the modified Korteweg–de Vries equation $u_t = u_{xxx} + u^2 u_x$ into a bi-Hamiltonian system. Find the recursion operator and the first few symmetries. How do these relate to the Korteweg–de Vries equation under the Miura transformation of Exercise 5.11? (Magri, [2])

- 7.10. The Harry Dym equation is $u_t = D_x^3(u^{-1/2})$. Prove that this is a bi-Hamiltonian system with $\mathcal{D} = 2uD_x + u_x$, $\mathcal{E} = D_x^3$. Discuss distinguished functionals, symmetries and conservation laws for this equation. The change of variables $v = u^{-1/2}$ changes this equation to $v_t = -\frac{1}{2}v^3 v_{xxx}$. Discuss its effects on the bi-Hamiltonian structure. (Magri, [1], Leo, Leo, Soliani, Solombrino and Mancarella, [1]; Ibragimov, [1; p. 300], shows how this equation can be transformed into the Korteweg–de Vries equation.)

- **7.11. The system of equations

$$u_t = uu_x + v_x - \frac{1}{2}u_{xx}, \quad v_t = (uv)_x + \frac{1}{2}v_{xx},$$

is equivalent, under a change of variables, to a system of Boussinesq equations modelling the bi-directional propagation of long waves in shallow water, first found by Whitham, [1] and Broer, [1]. Prove that this system is *tri-Hamiltonian*, meaning that it can be written as a Hamiltonian system using any one of the three Hamiltonian operators

$$\begin{aligned} \mathcal{D}_0 &= \begin{pmatrix} 0 & D_x \\ D_x & 0 \end{pmatrix}, & \mathcal{D}_1 &= \begin{pmatrix} 2D_x & D_x \cdot u - D_x^2 \\ uD_x + D_x^2 & 2vD_x + v_x \end{pmatrix}, \\ \mathcal{D}_2 &= \begin{pmatrix} 4uD_x + 2u_x & 4vD_x + 2v_x + D_x(D_x - u)^2 \\ 4vD_x + 2v_x + (D_x + u)^2 D_x & (D_x + u)(2vD_x + v_x) - (2vD_x + v_x)(D_x - u) \end{pmatrix}, \end{aligned}$$

and any two of these operators form a Hamiltonian pair. Discuss symmetries and conservation laws of the system. (Kupershmidt, [2]).

- 7.12. (a) Prove that if \mathcal{D} is a self-adjoint (respectively, skew-adjoint) matrix differential operator, and v_Q is any evolutionary vector field, then the Lie derivative pr $v_Q(\mathcal{D})$ is self-adjoint (skew-adjoint).
 (b) Prove directly that (7.11) is an alternating, trilinear function of P, Q, R .
- 7.13. Prove that if $\mathcal{D}: \mathcal{A} \rightarrow \mathcal{A}$ and $\mathcal{E}: \mathcal{A} \rightarrow \mathcal{A}$ are nonzero scalar differential operators, then $\mathcal{E} \cdot \mathcal{D}: \mathcal{A} \rightarrow \mathcal{A}$ is a nonzero differential operator. Deduce that any scalar differential operator is nondegenerate in the sense of Definition 7.23.
- *7.14. Let $\mathcal{D}: \mathcal{A}' \rightarrow \mathcal{A}^*$ be a differential operator, and let $\mathcal{K}^* = \{Q \in \mathcal{A}^*: \mathcal{D}^*Q = 0\}$ be the kernel of its adjoint. Prove that if \mathcal{K}^* is a finite-dimensional vector space over \mathbb{R} , then \mathcal{D} is nondegenerate in the sense of Definition 7.23. How many distinguished functionals does a nondegenerate Hamiltonian operator have?

- *7.15. The equations of polytropic gas dynamics have the form

$$u_t + uu_x + v^\sigma v_x = 0, \quad v_t + (uv)_x = 0,$$

in which u represents the velocity, v the density and $\sigma = \gamma - 2$, where γ is the physical ratio of specific heats appearing in the pressure-density relation. Show that this system can be written in Hamiltonian form in three distinct ways, using the Hamiltonian operators

$$\mathcal{D}_1 = \begin{pmatrix} 0 & D_x \\ D_x & 0 \end{pmatrix},$$

$$\mathcal{D}_2 = \begin{pmatrix} 2v^\sigma D_x + (v^\sigma)_x & (\sigma + 1)uD_x + u_x \\ (\sigma + 1)uD_x + \sigma u_x & 2vD_x + v_x \end{pmatrix},$$

$$\mathcal{D}_3 = \begin{pmatrix} 2uv^\sigma D_x + (uv^\sigma)_x & \left[\frac{1}{2}(\sigma + 1)u^2 + \frac{2}{\sigma + 1}v^\sigma \right] D_x \\ \left[\frac{1}{2}(\sigma + 1)u^2 + \frac{2}{\sigma + 1}v^\sigma \right] D_x & uu_x + v^\sigma v_x \\ + \sigma uu_x + v^\sigma v_x & 2uvD_x + (uv)_x \end{pmatrix}.$$

Prove that each operator is Hamiltonian. Which pairs are compatible? Discuss the consequential recursion operators, symmetries and conservation laws. (Whitham, [2], Nutku, [1], Olver and Nutku, [1].)

- 7.16. Prove that the Toda lattice equations of Exercise 6.11 are a bi-Hamiltonian system for the two Poisson brackets with structure matrices

$$\begin{pmatrix} 0 & 0 & 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & -1 & 1 & 0 \\ 0 & 0 & 0 & 0 & -1 & 1 \\ -1 & 1 & 0 & 0 & 0 & 0 \\ 0 & -1 & 1 & 0 & 0 & 0 \\ 1 & 0 & -1 & 0 & 0 & 0 \end{pmatrix}, \quad \begin{pmatrix} 0 & -e^{q^1} & e^{q^3} & p^1 & 0 & -p^1 \\ e^{q^1} & 0 & -e^{q^2} & -p^2 & p^2 & 0 \\ -e^{q^3} & e^{q^2} & 0 & 0 & -p^3 & p^3 \\ -p^1 & p^2 & 0 & 0 & 1 & -1 \\ 0 & -p^2 & p^3 & -1 & 0 & 1 \\ p^1 & 0 & -p^3 & 1 & -1 & 0 \end{pmatrix},$$

relative to the coordinates $(p^1, p^2, p^3, q^1, q^2, q^3)$. Are these two Poisson brackets compatible? (Arnol'd and Novikov, [1]; p. 58], Leo, Leo, Soliani, Solombrino and Mancarella, [2].)

- *7.17. Show that the structure matrices

$$J_1 = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \end{pmatrix}, \quad J_2 = \begin{pmatrix} 0 & 0 & e^{p_1} & -p_2 e^{p_1} \\ 0 & 0 & 0 & -e^{p_1} \\ -e^{p_1} & 0 & 0 & 0 \\ p_2 e^{p_1} & e^{p_1} & 0 & 0 \end{pmatrix},$$

using coordinates (p_1, p_2, q_1, q_2) , are Hamiltonian, but do not form a Hamiltonian pair. Discuss the integrability of any associated “bi-Hamiltonian systems.” (Olver, [15].)

- 7.18. Let $u_t = \mathcal{D}\delta\mathcal{H}$ be a scalar Hamiltonian evolution equation. Prove that the pseudo-differential operator \mathcal{D}^{-1} is a formal conservation law of rank ∞ .

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