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Model Predictive Controller Using Laguerre Functions for Dynamic Positioning System

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Abstract: This paper develops an improved model predictive controller (MPC) for the dynamic positioning system of vessels by using the Laguerre functions. The set of Laguerre networks is introduced to describe the future control increment trajectory, which has fewer parameter to adjust. The improved model predictive controller have the advantages of the traditional model predictive control, which has the good control performance to deal with the multivariable and constraints. Besides, the number of parameters required in the optimization algorithm is fewer than the one required by the conventional MPC and the computation load online was also reduced. It is shown that the proposed MPC makes the closed-loop system asymptotically stable in the presence of input constraints. Simulation studies with comparisons on a supply vessel are carried out, and the results illustrate the effectiveness of the proposed control scheme.

Key Words: Dynamic Positioning System, Model Predictive Control, Laguerre Functions, Constraints

1 Introduction

Dynamic Positioning (DP) system means the complete installation necessary for a dynamically positioning vessel including: power system, thruster system, and DP-control system[1]. In the offshore industry, it has been widely used in shuttle tankers, pipelay vessel, diving support vessels and platform supply vessels etc. The first DP systems were designed using conventional PID controllers in cascade with low-pass and/or notch filters to suppress the wave-induced motion components. From the middle of the 1970s, more advanced control techniques based on optimal control and Kalman-filter theory proposed by Balchen[2]. Recently, the third generation of DP system is researched by using the advance control algorithms, such as neural networks control[3], fuzzy control[4], adaptive control[5] etc.

Model predictive control (MPC) scheme effectively handles the constraints on the control input and state by numerically optimizing a cost index of the feasible state space[6].It has been applied in dynamic positioning system and a certain amount of research achievements has been received. Amin[7] designed a nonlinear MPC for the coordinated dynamic positioning of a Mobile Offshore Base based on a combined approach of feedback linearization and MPC. Wang Yuanhui[8] studied the application of model predictive control to restriction control of marine dynamic positioning system. Fannemel[9] studied the application of the unscented Kalman filter and nonlinear model predictive control in DP system. Sui Yufeng[10] explored the application of nonlinear model predictive control to dynamic positioning control system of surface vessels, seeking to overcome constraints between thrust and torque. Liang Haizhi[11] adopted Model predictive control to integrate the virtual control force calculation and thrust allocation, which fully considered the propellers' physical performance. The key technique in the design of MPC is based on optimizing the future control trajectory, that is the difference of the control signal, $\Delta u(k)$. As a consequence, to obtain the best close loop performance, satisfactory approximation of the control signal requires a very large number of forward shift operators, and leads to poorly numerically conditioned solutions and heavy computational load when implemented on-line. The paper proposed an improved Model Predictive control algorithm for dynamic positioning system, which is based on the Laguerre functions.

The rest of the paper is organized as follows. The problem formulation and preliminaries are described in Section 2. Section 3 presents the model predictive controller design procedures for the DP system of vessel using the Laguerre functions with stability analysis. In Section 4, simulation studies with comparisons on a supply vessel are provided to illustrate the effectiveness of the control scheme. Section 5 is the conclusions part.

2 Problem Description and Preliminaries

2.1 Mathematic Model

Due to the environmental forces caused by waves, wind and ocean currents, ship motion model can be divided into two components: low-frequency (LF) motion model and high-frequency (HF) motion model. Generally, Dynamic Positioning control is only concerned with the LF motion model of vessel. Therefore, two reference coordinate frames of the ship motion are established in Figure 1, which are north-east coordinate system and horizontal vessel fixed coordinate system. A three-DOF LF kinematics model of the vessel is obtained only considering the vessel's surge, sway and yaw[2]:

$$\begin{cases}\dot{\eta} = J(\psi)\nu \\ M\dot{\nu} + D\nu = \tau + \tau_{Env}\end{cases} \tag{1}$$

Where $\eta = [x, y, \psi]^{\mathrm{T}} \in \mathbb{R}^3$ expresses the vessel's position

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and heading, and $\nu=[u,v,r]^{\mathrm{T}}\in\mathbb{R}^3$ indicates the vessel's velocity and angle velocity; the rotation matrix $J(\psi)$ is denoted as:

$$J(\psi)=\begin{bmatrix}\cos\psi & -\sin\psi & 0\\ \sin\psi & \cos\psi & 0\\ 0 & 0 & 1\end{bmatrix} \tag{2}$$

The control forces and moments τ are provided by the propulsion system, M donates the inertia matrix including add mass, D represents damping matrix.

Considering the slowly varying heading and low frequency motion, the linear state-space model is usually expressed in vessel parallel coordinates:

$$\begin{aligned}\dot{x}_m &= \boldsymbol{A}_c x_m + \boldsymbol{B}_c u + \boldsymbol{E}_c\omega\\ y &= \boldsymbol{C}_c x_m\end{aligned} \tag{3}$$

Where, $x_m=[\eta_p^{\mathrm{T}},\nu^{\mathrm{T}}]^{\mathrm{T}}$ is the present state of vessel, y is the present position and heading of vessel, u represents the control force and moments τ, ω indicates the marine environment disturbance. And

$$\begin{aligned}&\boldsymbol{A}_c=\begin{bmatrix}0 & I\\ 0 & -\boldsymbol{M}^{-1}\boldsymbol{D}\end{bmatrix},\ \boldsymbol{B}_c=\left[0,\boldsymbol{M}^{-1}\right]^{\mathrm{T}}\\ &\boldsymbol{E}_c=\left[0,\boldsymbol{M}^{-1}\right]^{\mathrm{T}},\ \boldsymbol{C}_c=\left[\boldsymbol{I}\quad 0\right]\end{aligned} \tag{4}$$

The control objective in the paper is to design the improved model predictive controller using the Laguerre functions to force the vessel to reach and maintain the desired position $\eta_d=[x_d,y_d,\psi_d]^{\mathrm{T}}$.

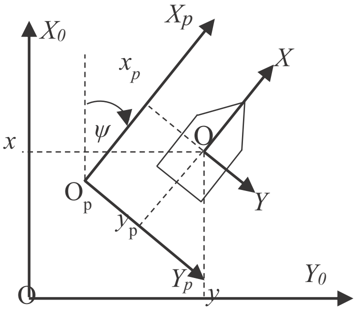


Fig. 1: Coordinate Frames of Ship Motion

2.2 Preliminaries

The application of Laguerre networks so far is mainly in the area of system identification, due to its orthogonality. The discrete-time Laguerre network is described in Figure 2[12]

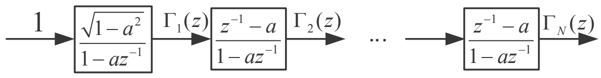


Fig. 2: Discrete-time Laguerre Network

$$\Gamma_k(z)=\Gamma_{k-1}(z)\frac{z^{-1}-a}{1-az^{-1}},\ \Gamma_1(z)=\frac{\sqrt{1-a^2}}{1-az^{-1}} \tag{5}$$

Where a is the pole of the discrete-time Laguerre network, and $0\leqslant a<1$ for stability of the network.

From the inverse z-transform of $\Gamma_1(z),\Gamma_2(z),\cdots,\Gamma_N(z)$, the discrete-time Laguerre functions in a vector form can be :

$$L(k)=\left[l_1(k)\quad l_2(k)\quad l_3(k)\quad\cdots\quad l_N(k)\right]^T \tag{6}$$

Laguerre functions satisfies the following property:

$$L(k+1)=A_l L(k) \tag{7}$$

Where matrix A_l is a function of parameters a and $\beta=1-a^2$, the initial condition is given by

$$L(0)=\sqrt{\beta}\left[1\quad -a\quad\cdots\quad(-a)^{N-1}\right]^T \tag{8}$$

$$A_l=\begin{bmatrix}a & 0 & \cdots & 0\\ \beta & a & \cdots & 0\\ \vdots & \vdots & \ddots & \vdots\\ (-a)^{N-2}\beta & (-a)^{N-3}\beta & \cdots & a\end{bmatrix} \tag{9}$$

Besides the other important property is its orthogonality:

$$\begin{cases}\sum_{k=0}^{\infty} l_i(k)l_j(k)=0, i\neq j\\ \sum_{k=0}^{\infty} l_i(k)l_j(k)=1, i=j\end{cases} \tag{10}$$

Based on the above properties, the impulse response of a stable system $H(k)$ can be represented by the set of discrete time Laguerre network:

$$H(k)=L(k)^T\eta=L(k)^T\left[c_1\quad c_2\quad c_3\quad\cdots\quad c_N\right]^T \tag{11}$$

Note that $c_1,c_2,c_3,\cdots,c_N$ are the coefficients to be determined from the system datas.

3 Model Predictive Control design for Dynamic Positioning system

3.1 Controller Design

In this section, by combining the model predictive control algorithm with Laguerre functions, an improved model predictive controller for the DP system of vessels is designed to achieve the control objective stated in Section 2.1. The design process consists of the following three steps.

Step 1: Prediction

The discrete state-space model is obtained from the continue state-space model(3):

$$\begin{cases}x_m(k+1)=A_d x_m(k)+B_d u(k)+E_d\omega(k)\\ y(k)=C_d x_m(k)\end{cases} \tag{12}$$

Where $x_m(k)$ indicates the state variable vector of the plant at the sample time k, $u(k)$ is the control signal, $y(k)$ is the system output, ω is the marine environment disturbance due to wind, wave and current. A_d, B_d, C_d and E_d are the constant matrixes.

Define $x(k)=\left[\Delta x_m(k)\quad y(k)\right]^T$ as a new state variable vector, then an augment state-space model with an integrator is established:

$$\begin{cases}x(k+1)=Ax(k)+B\Delta u(k)\\ y(k)=Cx(k)\end{cases} \tag{13}$$

Where A, B and C are system matrices:

$$A=\begin{bmatrix} A_d & 0_{6\times3} \\ C_d A_d & I_{3\times3} \end{bmatrix},\ B=\begin{bmatrix} B_d \\ C_d B_d \end{bmatrix} \tag{14}$$
$$C=\begin{bmatrix} 0_{3\times6} & I_{3\times3} \end{bmatrix}$$

Let N_c and N_p donate the control horizon and predictive horizon respectively. So the future output Y can be predicted at the instant k :

$$Y = Fx(k) + \Phi\Delta U \tag{15}$$

Where the system matrix F and Φ can be formulated from the augment state space model(13).

$$\Delta U=\begin{bmatrix} \Delta u(k) \\ \Delta u(k+1) \\ \vdots \\ \Delta u(k+N_c-1) \end{bmatrix} Y=\begin{bmatrix} y(k+1\mid k) \\ y(k+2\mid k) \\ \vdots \\ y(k+N_p\mid k) \end{bmatrix}$$

$$F=\begin{bmatrix} CA \\ CA^2 \\ \vdots \\ CA^{N_p} \end{bmatrix} \Phi=\begin{bmatrix} CB & 0_{3\times3} & \cdots & 0_{3\times3} \\ CAB & CB & \cdots & 0_{3\times3} \\ \vdots & \vdots & \ddots & \vdots \\ CA^{N_p-1}B & CA^{N_p-2}B & \cdots & CA^{N_p-N_c}B \end{bmatrix} \tag{16}$$

Now the set of Laguerre networks is introduced to feature the future increment control trajectory at the future instant m:

$$\Delta u(k+m) = L(m)^T \eta_k \tag{17}$$

Where

$$L(m)=\begin{bmatrix} L_1(m)^T & 0_{1\times N} & 0_{1\times N} \\ 0_{1\times N} & L_2(m)^T & 0_{1\times N} \\ 0_{1\times N} & 0_{1\times N} & L_3(m)^T \end{bmatrix} \eta_k=\begin{bmatrix} \eta_{k_1} \\ \eta_{k_2} \\ \eta_{k_3} \end{bmatrix} \tag{18}$$

Then the future state variable and output become:

$$x(k+m\mid k) = A^m x(k) + \phi(m)^T \eta \tag{19}$$

$$y(k+m\mid k) = CA^m x(k) + C\phi(m)^T \eta \tag{20}$$

Where $\phi(m)^T = \sum_{i=0}^{m-1} A^{m-i-1} BL(i)^T$.

Step 2: Optimization

The term receding horizon control (RHC) has occasionally been used and MPC is the conventional name of this technique, which solves the finite domain optimization problems online. The cost function J is defined as

$$J=\sum_{m=1}^{N_p}\left(\eta_d - y\left(k+m\middle|k\right)\right)^T\left(\eta_d - y\left(k+m\middle|k\right)\right) + \sum_{m=0}^{N_p-1} \Delta u(k+m)^T r_w \Delta u(k+m) \tag{21}$$

Where r_w is the control input weight matrix Δu . Suppose

$$\Delta u(k{+}m) = \begin{bmatrix} 0 & 0 & 0 \end{bmatrix}^T \quad (N_c \le m \le N_p - 1) \tag{22}$$

Define $x_f(k) = \begin{bmatrix} \Delta x_m(k) & e(k) \end{bmatrix}^T$ and substitute (20) into (21), then the cost function J becomes:

$$J{=}\sum_{m=1}^{N_P}\left(x_f(k)^T (A^T)^m QA^m x_f(k)\right) + 2\eta^T \Psi x_f(k) + \eta^T \Omega \eta \tag{23}$$

Where

$$\Omega = \sum_{m=1}^{N_P} \phi(m) Q \phi(m)^T + R_L \tag{24}$$

$$\Psi = \sum_{m=1}^{N_P}\left(\phi(m) QA^m\right) \tag{25}$$

Without constraints, the optimal control forces and moments increments produced by thrusters are found:

$$\Delta u(k) = -K_{mpc} x(k) = -L(0)^T \Omega^{-1} \Psi x_f(k) \tag{26}$$

Step 3: Constraints

One of the key features of model predictive control is the ability to handle hard constraints in the design. Besides, the control forces and moments of thruster and the operating areas are constrained.

The constrained control forces and moments at the future time m, $m = 1,2,\cdots,N_c$ can be expressed as:

$$U^{\min} \le \sum_{i=0}^{m-1} L(i)^T \eta_k + u(k-1) \le U^{\max} \tag{27}$$

The operating areas constraints at the future time m, m=1, 2,⋯,N_p is represented as

$$y^{\min} \le CA^m x(k_i) + \sum_{i=0}^{m-1} CA^{m-i-1} BL(i)^T \eta_k \le y^{\max} \tag{28}$$

The optimal value of η_k can be solved by quadratic programming. Finally, the control forces and moment is calculated by

$$u(k) = u(k-1) + \Delta u(k) {=} u(k-1) + L(0)^T \eta_k \tag{29}$$

3.2 Stability Analysis

This section shows the stability of the DP system with the novel MPC, which is a nonlinear control problem when the thruster power and the operating areas are constrained.

Theorem 1. Assume that

(1) The terminal state of the receding horizon optimization problem is constrained: $x(k+N_p\mid k)=0$, which is resulting from the control sequence $\Delta u(k+m) = L(m)\eta_k$, $m = 0,1,2,\cdots,N_p$; and

(2) For each sampling instant k, there exists a solution such that the cost function J is minimized subject to the inequality constraints and the terminal state constraint.

Subject to the above assumptions, the closed-loop DP system is asymptotically stable.

Proof. Choose the cost function J as the Lyapunov function $V(x(k),k)$:

$$V(x(k),k) = \sum_{m=1}^{N_P} x_f(k+m\mid k)^T Q x_f(k+m\mid k) + \sum_{n=0}^{N_p-1} \Delta u(k+n)^T r_w \Delta u(k+n) \tag{30}$$

Obviously, $V(x(k),k)$ is positive definite. Similarly, the Lyapunov function $V(x(k+1),k+1)$ at time k+1 becomes:

$$V(x(k+1),k+1)=\sum_{m=1}^{N_P} x_f(k+1+m\,|\,k+1)^T Q * x_f(k+1+m\,|\,k+1) + \sum_{n=0}^{N_p-1} \Delta u(k+1+n)^T r_w \Delta u(k+1+n) \quad (31)$$

From the second assumption, the optimal control sequence $\Delta U(k)$ and $\Delta U(k+1)$ at time k and k+1 respectively are

$$\Delta U(k)=\left[L(0)^T\eta_k, L(1)^T\eta_k,\cdots,L(N_p-1)^T\eta_k\right]^T \quad (32)$$

$$\Delta U(k+1)=\left[L(0)^T\eta_{k+1}, L(1)^T\eta_{k+1},\cdots,L(N_p-1)^T\eta_{k+1}\right]^T \quad (33)$$

Notes that η_k and η_{k+1} is the optimal solution for time k and k+1 respectively. According to the state space model(13), the feasible control sequence $\Delta\bar{U}(k+1)$ at time k+1 can be got by shift $\Delta U(k)$ one step forward and the last element was replace by zero:

$$\Delta U(k+1)=\left[L(1)^T\eta_k, L(2)^T\eta_k,\cdots,L(N_p-1)^T\eta_k,0\right]^T \quad (34)$$

Where η_k is not optimal solution for the time k. The corresponding Lyapunov function $\bar{V}(x(k+1),k+1)$ at time k+1 is got, and

$$V(x(k+1),k+1)\le\bar{V}(x(k+1),k+1) \quad (35)$$

The difference between $V(x(k+1),k+1)$ and $V(x(k),k)$ bounded by

$$V(x(k+1),k+1)-V(x(k),k)\le \bar{V}(x(k+1),k+1)-V(x(k),k) \quad (36)$$

From the first assumption, the right part of inequality (36) becomes

$$\begin{aligned}&\bar{V}(x(k+1),k+1)-V(x(k),k)\\ &=\left(x(k+1+N_p\,|\,k+1)\right)^T Q\left(x(k+1+N_p\,|\,k+1)\right)\\ &\quad -x(k+1\,|\,k)^T Qx(k+1\,|\,k)\text{-}(L(0)^T\eta_k)^T r_w(L(0)^T\eta_k)\\ &=-\left(x(k+1\,|\,k)^T Qx(k+1\,|\,k)\text{-}(L(0)^T\eta_k)^T r_w(L(0)^T\eta_k)\right)\\ &<0\end{aligned} \quad (37)$$

Therefore, the difference between $V(x(k+1),k+1)$ and $V(x(k),k)$ is smaller than 0.

$$V(x(k+1),k+1)-V(x(k),k)<0 \quad (38)$$

It can be seen that the Lyapunov function (30) is positive definite, while the difference (38) of Lyapunov function is negative definite. Hence, the closed-loop DP system with the novel MPC is asymptotically stable.

4 Simulation and Comparison Studies

In order to verify the effectiveness of the above algorithm, a simulation is carried out based on a support vessel. Table 1 show the vessel's main parameters and the condition of the simulation. The dynamic parameters in the motion model (1) of the supply vessel are as follows[13]::

$$\boldsymbol{M}''=\begin{bmatrix}1.1274 & 0.0000 & 0.0000\\ 0.0000 & 1.8902 & -0.0744\\ 0.0000 & -0.0744 & 0.0308\end{bmatrix}$$

$$\boldsymbol{D}''=\begin{bmatrix}0.0358 & 0.0000 & 0.0000\\ 0.0000 & 0.1183 & -0.0124\\ 0.0000 & -0.0041 & 0.0308\end{bmatrix}$$

In order to illustrate the control effect of the model predictive controller clearly, this paper only takes the input constraints of the controller into account, including the X-direction force, Y-direction force, and yaw moment input constraints.

The simulation result is shown in the figure 3, figure 4 and table 2. Experimental analysis have been done:

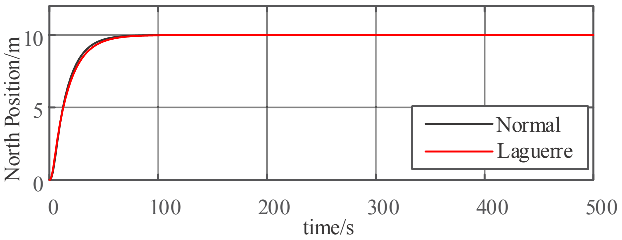
1) Position and heading: It is observed that both normal MPC and Laguerre MPC can force the vessel to arrive at the desired position (10 m, 10 m, 10°) in each of two case. As can be seen from figure 3(a) and (b),and figure 4(a) and (b) the curves of Laguerre MPC is similar to normal, which indicates the former owns the same control performance with the latter. However, from figure 4(c), Laguerre MPC is better control performance than normal MPC, especially the heading. The former has the smaller overshoot than the latter.

2) Forces and moment: As a whole, both normal MPC and Laguerre MPC can overcome the environment disturbance and converge to opposite values of the environment disturbance. As can be seen from figure 3(d) and (e), the X-direction force and Y-direction force reach the constraint boundary in the beginning and after a few seconds become small, but the curve of Laguerre MPC is smoother than the normal MPC, which help to reduce mechanical wear of thrusters.

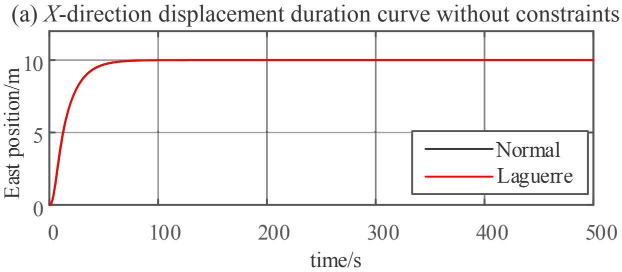
3) Calculate time: As can be seen from table 2, the computation load of the Laguerre MPC is reduced comparing to the normal MPC. For the detail, the mean calculate time of Laguerre MPC is about 87% less than the normal MPC without constraints, while the mean calculate time of Laguerre MPC is about 74% less than the normal MPC with constraints.

Tab. 1 Parameters in the Simulation

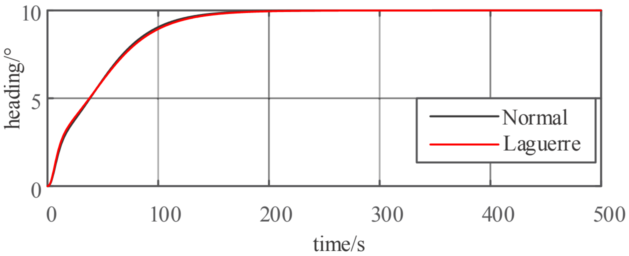
Parameters	Value
Initial position and heading	$[0\text{m}\ \ 0\text{m}\ \ 0°]^T$
Desired position and heading	$[10\text{m}\ \ 10\text{m}\ \ 10°]^T$
X-direction force constraint	-1000 kN ~ 1000 kN
Y-direction force constraint	-300 kN ~300 kN
Moment constraint	-7620 kN·m ~ 7620 kN·m
Environment disturbances	[-11.5kN 115kN 762kN·m]
Control horizon	10
Prediction horizon	150
Sampling time	0.5 s
Simulation time	500 s
a	0.2
N	5



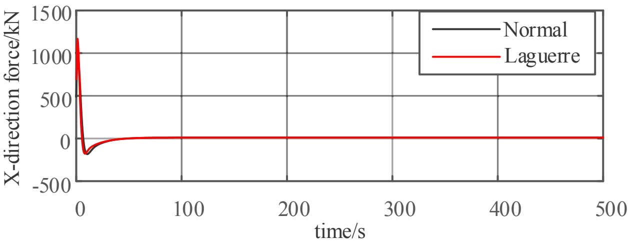
(a) *X*-direction displacement duration curve without constraints



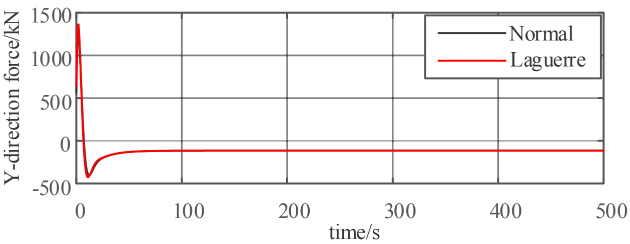
(b) *Y*-direction displacement duration curve without constraints



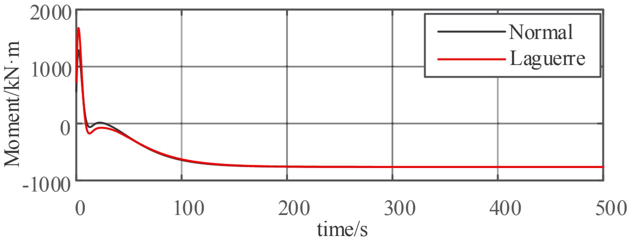
(c) Heading variation duration without constraints



(d) X-direction force input duration curve without constraints

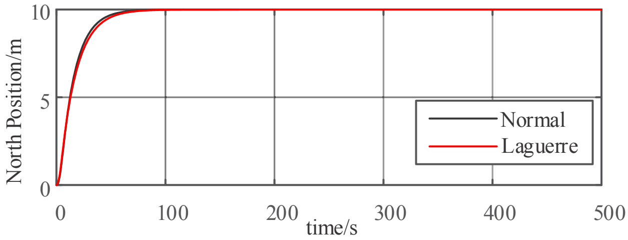


(e) Y-direction force input duration curve without constraints

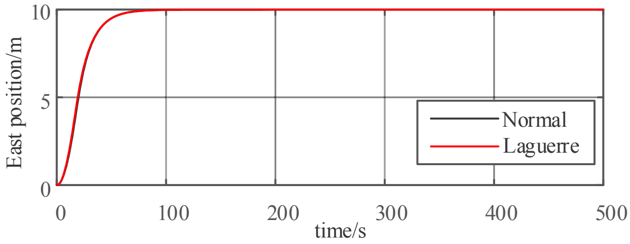


(f) Yaw moment input duration curve without constraints

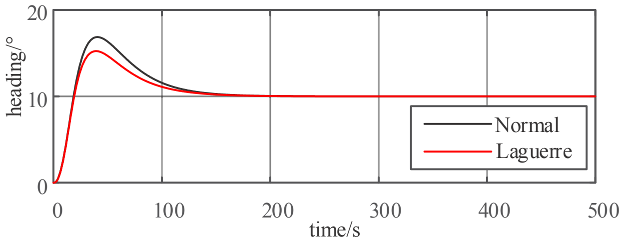
Fig. 3: Result of simulation without constraints



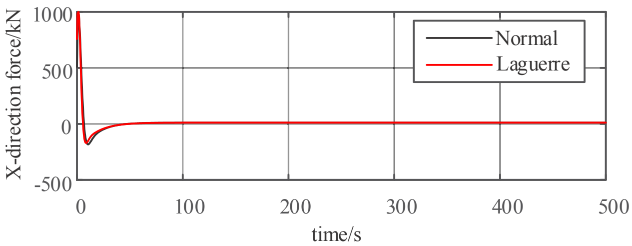
(a) *X*-direction displacement duration curve with constraints



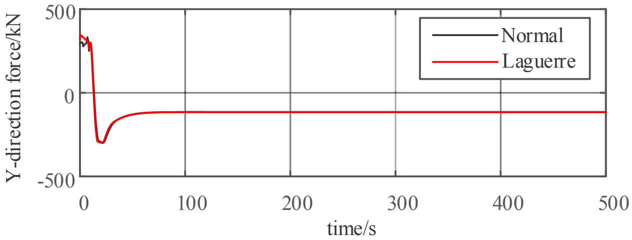
(b) *Y*-direction displacement duration curve with constraints



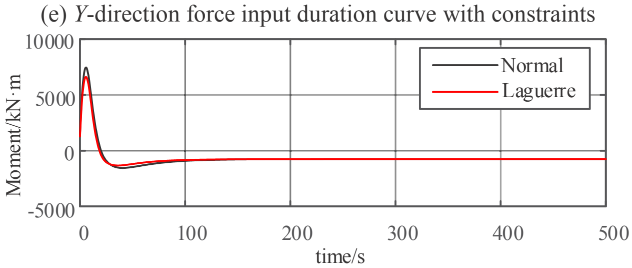
(c) Heading variation duration with constraints



(d) *X*-direction force input duration curve with constraints



(e) *Y*-direction force input duration curve with constraints



(f) Yaw moment input duration curve with constraints

Fig. 4: Result of simulation with constraints

Table 2 Comparison of mean computation time

	Normal MPC	Laguerre MPC
Without constraints	0.23ms	0.03ms
With constraints	1.25ms	0.32ms

5 Conclusion

In this paper, an improved model predictive controller was designed for the DP system of vessel by using the Laguerre functions. MPC is an advanced control algorithm that has the good control performance to deal with the multivariable and constraints. By using the Laguerre functions into the MPC, the number of parameters required in the optimization algorithm was fewer than the one required by the conventional MPC, and computation load online was also reduced. Additionally, stability of the closed-loop DP system is guaranteed in the presence of constraints. Simulation results and simulation comparisons on a supply vessel have confirmed the effectiveness of the proposed control scheme.

In the next step research work, the nonlinear model predictive control algorithm for the DP system of vessel will be studied.

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