Representasi sistem dalam state space

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Representasi sistem dalam state space

- Definisi state space
- Kegunaan state space
- State space persamaan diferensial orde 1
- State space persamaan diferensial orde 2
- Konversi Laplace ke State Space
- State space dengan matlab
- State space dengan python

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Definisi state space

“state space” atau “Ruang keadaan” mengacu pada representasi matematis dari keadaan internal sistem dinamis. Dalam sistem kendali, sistem sering direpresentasikan sebagai model matematika yang terdiri dari variabel keadaan yang menggambarkan keadaan internal sistem dan persamaan yang menggambarkan bagaimana keadaan tersebut berubah seiring waktu.

State equation / persamaan keadaan	$\dot{x}(t) = Ax(t) + Bu(t)$
output equation / persamaan output	$y(t) = Cx(t) + Du(t)$

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Definisi state space

State equation / persamaan keadaan

Kombinasi linear

$$\dot{x}(t) = Ax(t) + Bu(t)$$

Perubahan vektor

state vector

Kombinasi linear input

output equation / persamaan output

$$y(t) = \underbrace{Cx(t) + Du(t)}_{\text{Kombinasi linear dari state dan input}}$$

output

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Kegunaan state space

- Kalman Filter
- Linear Quadratic Regulator (LQR)
- Robust Control
- Model Predictive Control

State space persamaan diferensial orde 1

$Persamaan\ differensial$

$$\tau_p \frac{dy}{dt} = -y + k_p u$$

$: \tau_p$

$$\frac{dy}{dt} = -\frac{y}{\tau_p} + \frac{k_p}{\tau_p} u$$

y=x

$$\frac{dx}{dt} = -\frac{x}{\tau_p} + \frac{k_p}{\tau_p} u$$

$\frac{dx}{dt} = \dot{x}$

$$\dot{x} = \frac{-1}{\tau_p} x + \frac{k_p}{\tau_p} u$$

$$y = [1]x + [0]u$$

A = $\frac{-1}{\tau_p}$; B = $\frac{k_p}{\tau_p}$; C = 1; D = 0

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State space persamaan diferensial orde 2

$Persamaan\ differensial$

$$\tau_{p1}\frac{dx_1}{dt} = -x_1 + \ k_p u$$

$$\tau_{p2}\frac{dx_2}{dt} = -x_2 + x_1$$

$$y = x_2$$

$$\tau_{p1}\frac{dx_1}{dt} = -x_1 + \ k_p u$$

$$\frac{dx_1}{dt} = \frac{-1}{\tau_{p1}}x_1 + \frac{K_{p1}}{\tau_{p1}}u$$

$$\dot{x}_1 = \frac{-1}{\tau_{p1}}x_1 + \frac{K_{p1}}{\tau_{p1}}u$$

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State space persamaan diferensial orde 2

$$Persamaan\ differensial \qquad \tau_{p1}\frac{dx_1}{dt} = -x_1 + \ k_p u$$

$$\tau_{p2}\frac{dx_2}{dt} = -x_2 + x_1$$

$$y = x_2$$

$$\tau_{p2}\frac{dx_2}{dt} = -x_2 + x_1$$

$$\frac{dx_2}{dt} = \frac{-1}{\tau_{p2}}x_2 + \frac{1}{\tau_{p2}}x_1$$

$$\dot{x}_2 = \frac{-1}{\tau_{p2}}x_2 + \frac{1}{\tau_{p2}}x_1$$

State space persamaan diferensial orde 2

Persamaan differensial

$$\dot{x}_1 = \frac{-1}{\tau_{p1}} x_1 + \frac{K_{p1}}{\tau_{p1}} u$$

$$\dot{x}_2 = \frac{-1}{\tau_{p2}} x_2 + \frac{1}{\tau_{p2}} x_1$$

$$y = x_2$$

---

bentuk state space

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} \frac{-1}{\tau_{p1}} & 0 \\ \frac{1}{\tau_{p2}} & \frac{1}{\tau_{p2}} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} \frac{K_{p1}}{\tau_{p1}} \\ 0 \end{bmatrix} u$$

$$y = \begin{bmatrix} 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + [0]u$$

$$A = \begin{bmatrix} \frac{-1}{\tau_{p1}} & 0 \\ \frac{1}{\tau_{p2}} & \frac{1}{\tau_{p2}} \end{bmatrix}; B = \begin{bmatrix} \frac{K_{p1}}{\tau_{p1}} \\ 0 \end{bmatrix}; C = \begin{bmatrix} 0 & 1 \end{bmatrix}; D = [0]$$

Konversi Laplace ke State Space

$$G(s) = \frac{y(s)}{u(s)} = \frac{b_m s^m + b_{m-1} s^{n-1} + \cdots + b_1 s + b_0}{s^n + a_{n-1} s^{n-1} + \cdots + a_1 s + a_0}$$

$contoh\ kasus$

$$G(s) = \frac{b_2 s^2 + b_1 s + b_0}{s^3 + a_2 s^2 + a_1 s + a_0}$$

$$G(s) = \frac{b_2 s^2}{s^3+a_2 s^2+a_1 s+a_0} + \frac{b_1 s}{s^3+a_2 s^2+a_1 s+a_0} + \frac{b_0}{s^3+a_2 s^2+a_1 s+a_0}$$

$$G_c\ (s) = \frac{1}{s^3 + a_2 s^2 + a_1 s + a_0}$$

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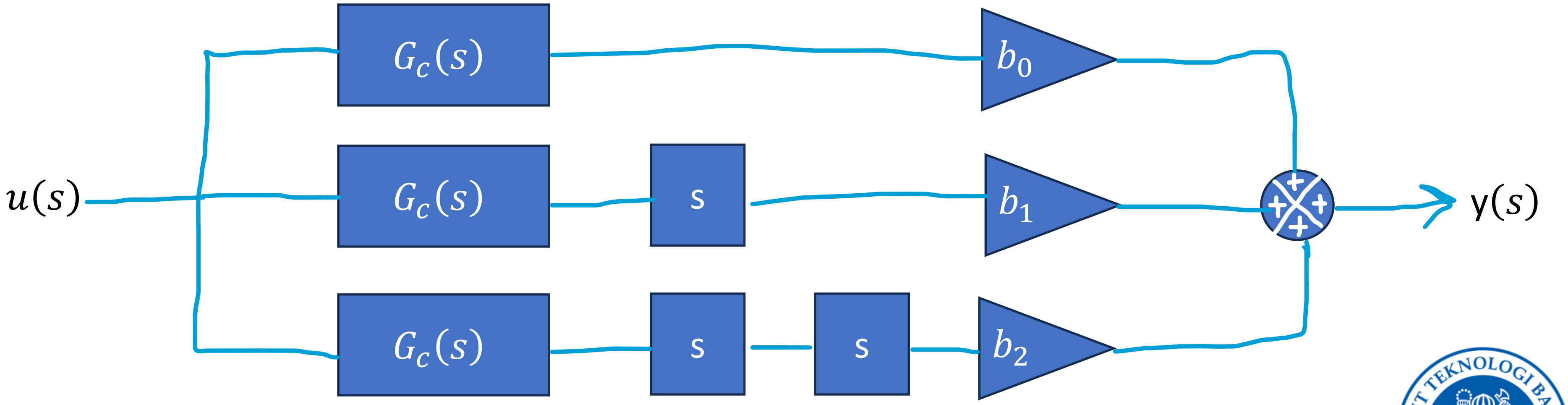
Konversi Laplace ke State Space

$$G(s) = \frac{y(s)}{u(s)} = b_2 s^2 * G_c(s) + b_1 s * G_c(s) + b_0 * G_c(s)$$

$$y(s) = b_2 s^2 * G_c(s).u(s) + b_1 s * G_c(s).u(s) + b_0 * G_c(s).u(s)$$



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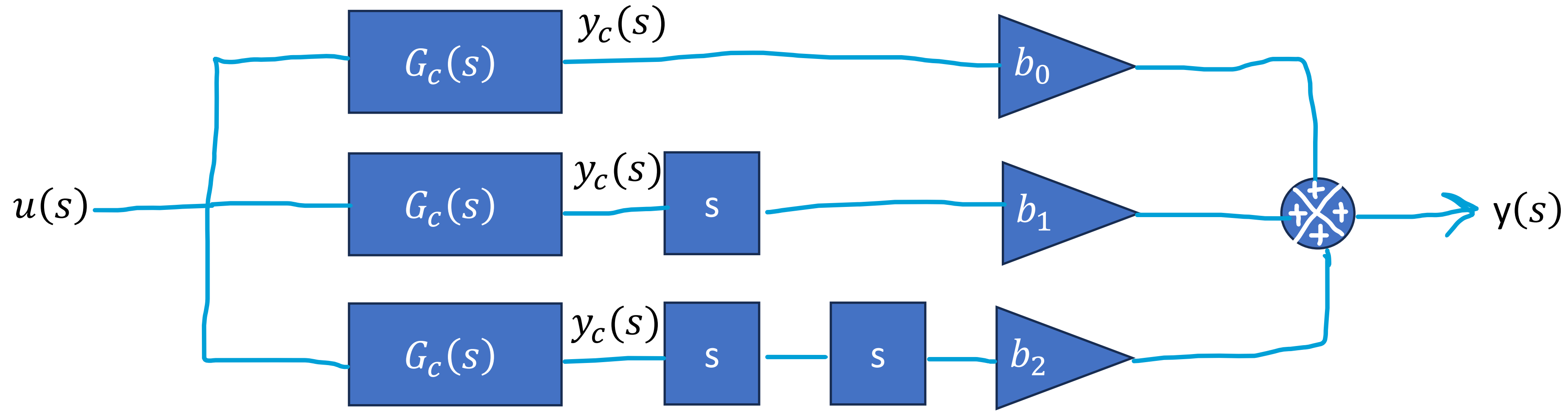
Konversi Laplace ke State Space



$$y_c(s) = u(s) * G_c(s)$$

$$y_c(s) = u(s) * \frac{1}{s^3 + a_2 s^2 + a_1 s + a_0}$$

$$u(s) = y_c(s)(s^3 + a_2 s^2 + a_1 s + a_0)$$

$$u(s) = y_c(s).s^3 + y_c(s).a_2 s^2 + y_c(s).a_1 s + y(s).a_0$$

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Konversi Laplace ke State Space

$$u(s) = y_c(s).s^3 + y_c(s).a_2 s^2 + y_c(s).a_1 s + y(s).a_0$$

$$\mathcal{L}^{-1}[u(s)] = [y_c(s).s^3 + y_c(s).a_2 s^2 + y_c(s).a_1 s + y(s).a_0]$$

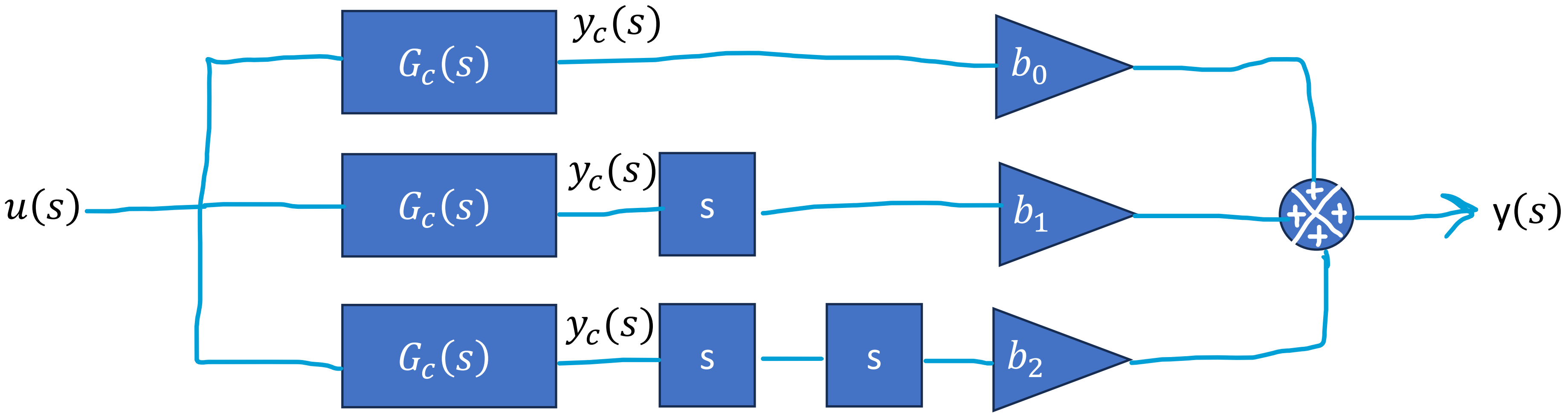
$$u(t) = \dddot{y}_c(t) + a_2 \ddot{y}_c(t) + a_1 \dot{y}_c(t) + a_0 y(t)$$

$$\dddot{y}_c(t) = -a_2 \ddot{y}_c(t) - a_1 \dot{y}_c(t) - a_0 y(t) + u(t)$$

$$\bar{x}(t) = \begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \end{bmatrix} = \begin{bmatrix} y_c(t) \\ \dot{y}_c(t) \\ \ddot{y}_c(t) \end{bmatrix}$$

$$\dot{\bar{x}}\,(t) = \begin{bmatrix} \dot{y}_c(t) \\ \ddot{y}_c(t) \\ \dddot{y}_c(t) \end{bmatrix} = \begin{bmatrix} \dot{y}_c(t) \\ \ddot{y}_c(t) \\ -a_2 \ddot{y}_c(t) - a_1 \dot{y}_c(t) - a_0 y(t) + u(t) \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -a_0 & -a_1 & -a_2 \end{bmatrix} \begin{bmatrix} y_c(t) \\ \dot{y}_c(t) \\ \ddot{y}_c(t) \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} u(t)$$

Konversi Laplace ke State Space



$$y(t) = b_0 y_c(t) + b_1 \dot{y}_c(t) + b_2 \ddot{y}_c(t)$$

$$y(t) = \begin{bmatrix} b_0 & b_1 & b_2 \end{bmatrix} \begin{bmatrix} y_c(t) \\ \dot{y}_c(t) \\ \ddot{y}_c(t) \end{bmatrix} + [0]\mathrm{u}$$

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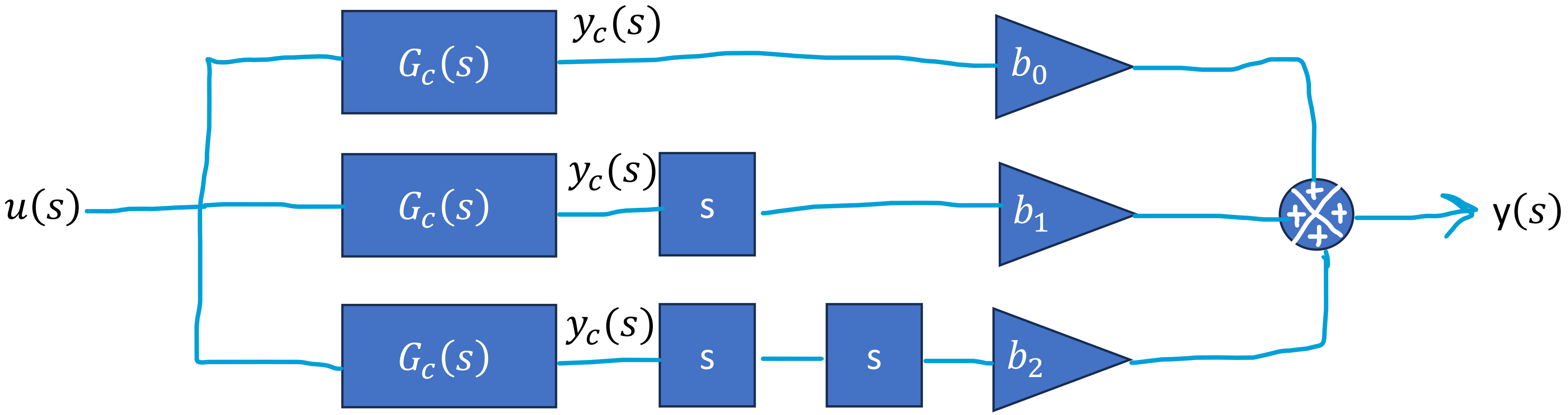
Konversi Laplace ke State Space



Persamaan state space nya adalah :

$$\dot{x}\,(t) = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -a_0 & -a_1 & -a_2 \end{bmatrix} \begin{bmatrix} y_c(t) \\ \dot{y}_c(t) \\ \ddot{y}_c(t) \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} u(t)$$

$$y(t) = \begin{bmatrix} b_0 & b_1 & b_2 \end{bmatrix} \begin{bmatrix} y_c(t) \\ \dot{y}_c(t) \\ \ddot{y}_c(t) \end{bmatrix} + [0]\mathrm{u(t)}$$

Contoh Kasus $\dfrac{5s^2+10s+1}{s^3+6s^2+2s+7}$

Urutan matriks yang digunakan oleh matlab dan python

Urutan matriks yang digunakan oleh textbook

$$\dot{x}(t) = \begin{bmatrix} \dot{y}_c(t) \\ \ddot{y}_c(t) \\ \dddot{y}_c(t) \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -a_0 & -a_1 & -a_2 \end{bmatrix} \begin{bmatrix} y_c(t) \\ \dot{y}_c(t) \\ \ddot{y}_c(t) \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} u(t)$$

$$\dot{x}(t) = \begin{bmatrix} \dddot{y}_c(t) \\ \ddot{y}_c(t) \\ \dot{y}_c(t) \end{bmatrix} = \begin{bmatrix} -a_2 & -a_1 & -a_0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} \ddot{y}_c(t) \\ \dot{y}_c(t) \\ y_c(t) \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} u(t)$$

$$y(t) = \begin{bmatrix} b_0 & b_1 & b_2 \end{bmatrix} \begin{bmatrix} y_c(t) \\ \dot{y}_c(t) \\ \ddot{y}_c(t) \end{bmatrix} + [0]\mathrm{u(t)}$$

$$y(t) = \begin{bmatrix} b_2 & b_1 & b_0 \end{bmatrix} \begin{bmatrix} \ddot{y}_c(t) \\ \dot{y}_c(t) \\ y_c(t) \end{bmatrix} + [0]\mathrm{u(t)}$$

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Contoh Kasus $\dfrac{5s^2+10s+1}{s^3+6s^2+2s+7}$

$$\begin{array}{ll} b & 5s^2 + 10s + 1 \\ \hline a & s^3 + 6s^2 + 2s + 7 \end{array}$$

3 2 1 0

$$\dot{x}\,(t) = \begin{bmatrix} \dddot{y}_c(t) \\ \ddot{y}_c(t) \\ \dot{y}_c(t) \end{bmatrix} = \begin{bmatrix} -a_2 & -a_1 & -a_0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} \ddot{y}_c(t) \\ \dot{y}_c(t) \\ y_c(t) \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} u(t)$$

$$y(t) = \begin{bmatrix} b_2 & b_1 & b_0 \end{bmatrix} \begin{bmatrix} \ddot{y}_c(t) \\ \dot{y}_c(t) \\ y_c(t) \end{bmatrix} + [0]\mathrm{u(t)}$$

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Contoh Kasus $\frac{5s^2+10s+1}{s^3+6s^2+2s+7}$

$$\dot{x}\,(t) = \begin{bmatrix} \dddot{y}_c(t) \\ \ddot{y}_c(t) \\ \dot{y}_c(t) \end{bmatrix} = \begin{bmatrix} -6 & -2 & -7 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} \ddot{y}_c(t) \\ \dot{y}_c(t) \\ y_c(t) \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} u(t)$$

$$y(t) = \begin{bmatrix} 5 & 10 & 1 \end{bmatrix} \begin{bmatrix} \ddot{y}_c(t) \\ \dot{y}_c(t) \\ y_c(t) \end{bmatrix} + [0]\mathrm{u(t)}$$

State Space dengan Matlab

```
% Deklarasi variabel
b = [5, 10, 1]; % Koefisien numerator
a = [1, 6, 2, 7]; % Koefisien denumerator

% Mendapatkan bentuk state space dalam bentuk kontrolabel canonical form
[A, B, C, D] = tf2ss(b, a);

% Menampilkan matriks A, B, C, dan D
disp('A matrix:');
disp(A)
disp('B matrix:');
disp(B)
disp('C matrix:');
disp(C);
disp('D matrix:');
disp(D);
#menampilkan hasil perhitungan konversi state space
print(Gss)
```

```
>> tf_ke_ss
A matrix:
    -6    -2    -7
     1     0     0
     0     1     0

B matrix:
     1
     0
     0

C matrix:
     5    10     1

D matrix:
     0
```

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State Space dengan python

```
#import library
import control
import numpy as np

#deklarasi variabel
b = [5, 10, 1]
a = [1, 6, 2, 7]
print(control.tf(b, a))

#memasukkan persamaan laplace b numerator dan a denumerator
Gtf = control.tf(b, a)

#mengkonversi laplace ke state space
Gss = control.ss(Gtf)

#menampilkan hasil perhitungan konversi state space
print("A matrix:")
print(Gss.A)
print("B matrix:")
print(Gss.B)
print("C matrix:")
print(Gss.C)
print("D matrix:")
print(Gss.D)
```

```
>>> %Run laplace_statespace.py

  5 s^2 + 10 s + 1
---------------------
s^3 + 6 s^2 + 2 s + 7

A matrix:
[[-6. -2. -7.]
 [ 1.  0.  0.]
 [ 0.  1.  0.]]
B matrix:
[[1.]
 [0.]
 [0.]]
C matrix:
[[ 5. 10.  1.]]
D matrix:
[[0.]]
```

Kesimpulan

$$\dot{\bar{x}}(t) = \begin{bmatrix} \dddot{y}_c(t) \\ \ddot{y}_c(t) \\ \dot{y}_c(t) \end{bmatrix} = \begin{bmatrix} -6 & -2 & -7 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} \ddot{y}_c(t) \\ \dot{y}_c(t) \\ y_c(t) \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} u(t)$$

$$y(t) = \begin{bmatrix} 5 & 10 & 1 \end{bmatrix} \begin{bmatrix} \ddot{y}_c(t) \\ \dot{y}_c(t) \\ y_c(t) \end{bmatrix} + [0]\mathrm{u}(\mathrm{t})$$

```
>> tf_ke_ss
A matrix:
    -6    -2    -7
     1     0     0
     0     1     0

B matrix:
     1
     0
     0

C matrix:
     5    10     1

D matrix:
     0
```

```
>>> %Run laplace_statespace.py

  5 s^2 + 10 s + 1
---------------------
s^3 + 6 s^2 + 2 s + 7

A matrix:
[[-6. -2. -7.]
 [ 1.  0.  0.]
 [ 0.  1.  0.]]
B matrix:
[[1.]
 [0.]
 [0.]]
C matrix:
[[ 5. 10.  1.]]
D matrix:
[[0.]]
```

Perhitungan Manual

Perhitungan Matlab

Perhitungan Python

Perhitungan Manual sama dengan perhitungan dengan matlab dan python akan tetapi ada sedikit perbedaan pada urutan matriks di textbook dengan di Bahasa pemrograman python dan matlab

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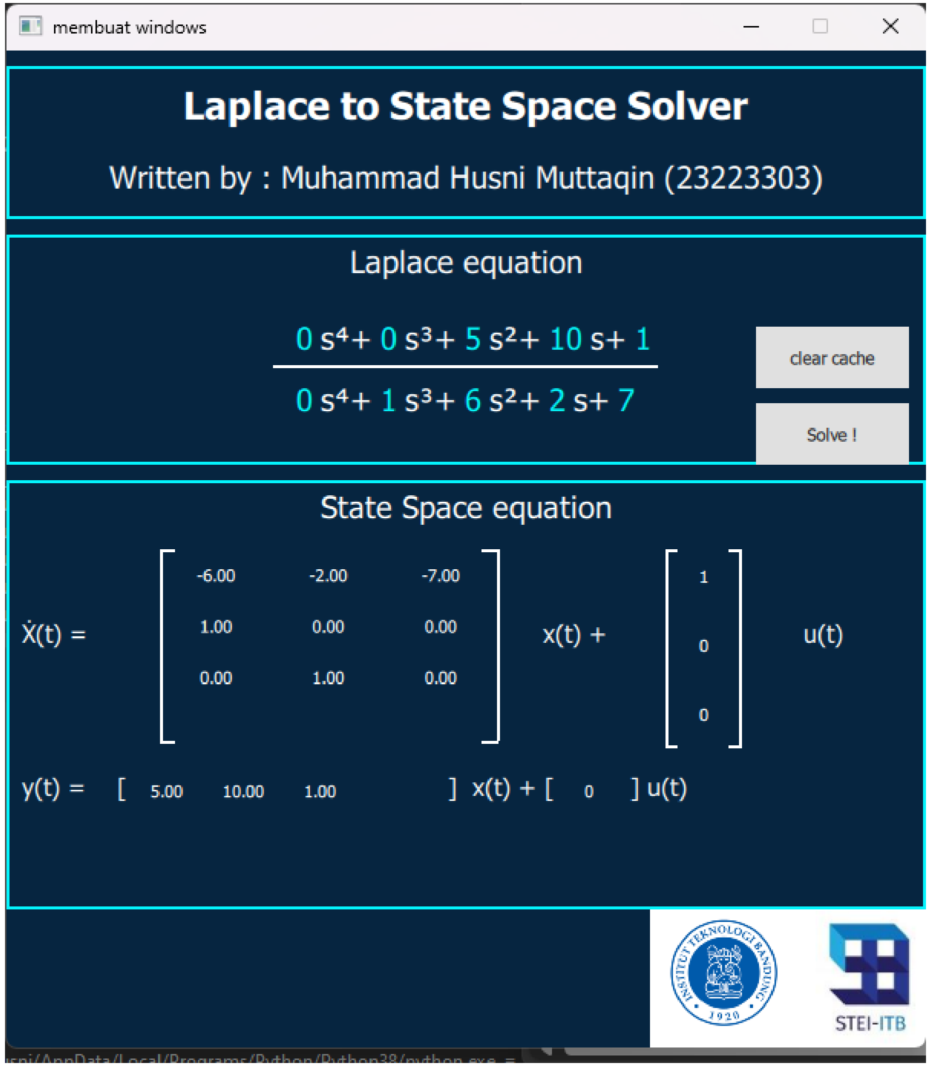
Pengembangan

- Pembuatan aplikasi state space solver dengan python dan qml



https://github.com/muhammadhusni777/laplace-to-state-space-solver