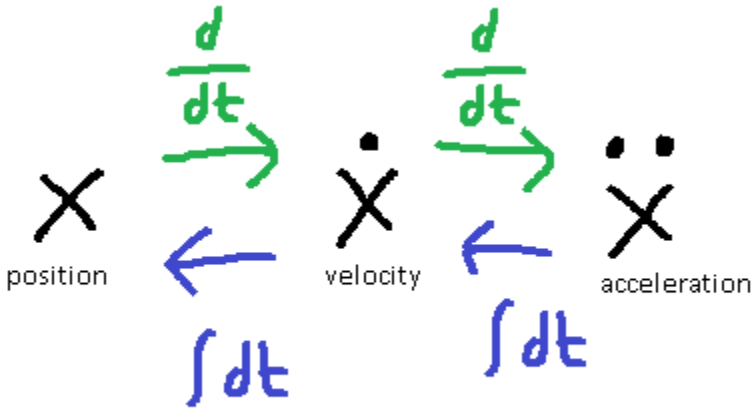
Implementing differential math using printer encoder



Written by :

Muhammad Husni

Goal of this experiment :

- Understanding what is position, speed, and acceleration
- Understanding how to implement that equation and make coding using Arduino IDE.

Theory behind that

- Position, speed, and acceleration

  Position is also known location. In cartesian graph (x,y) position is how many value x and y.

  The velocity of an object is the rate of change of its position with respect to a frame of reference, and is a function of time. Remember this equation :

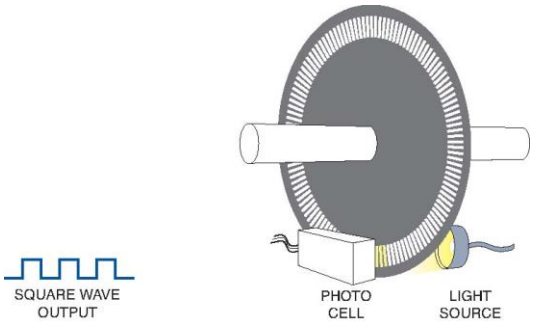
$$v = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt}.$$

https://en.wikipedia.org/wiki/Velocity

  Acceleration is the rate of change of the velocity of an object with respect to time. Accelerations are vector quantities (in that they have magnitude and direction).

$$\mathbf{a} = \frac{d\mathbf{v}}{dt} = \frac{d^2\mathbf{x}}{dt^2}$$

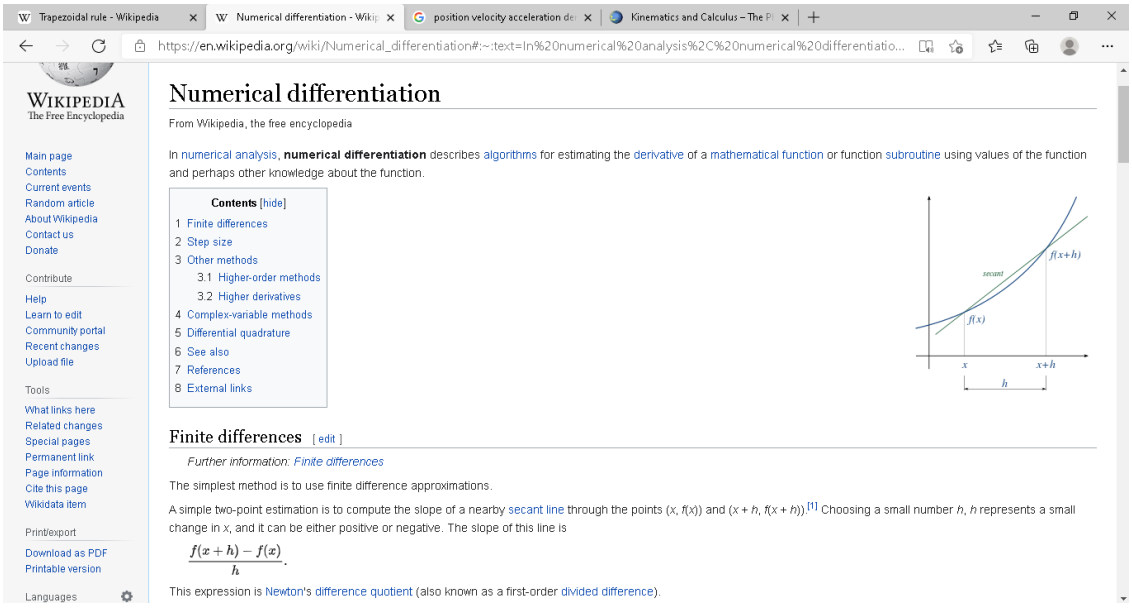
- How Encoder work



Encoder is a device that convert rotary motion to a square wave electronics signal. More speed more square wave. To read the position of encoder we need a microcontroller. We use Arduino uno microcontroller to make easier.

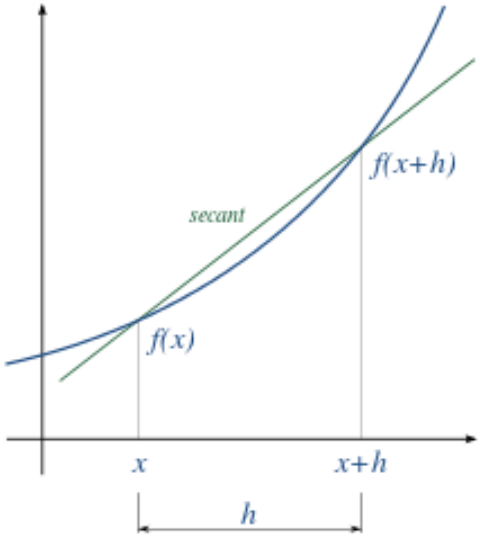
Little bit math

- Numerical derivation



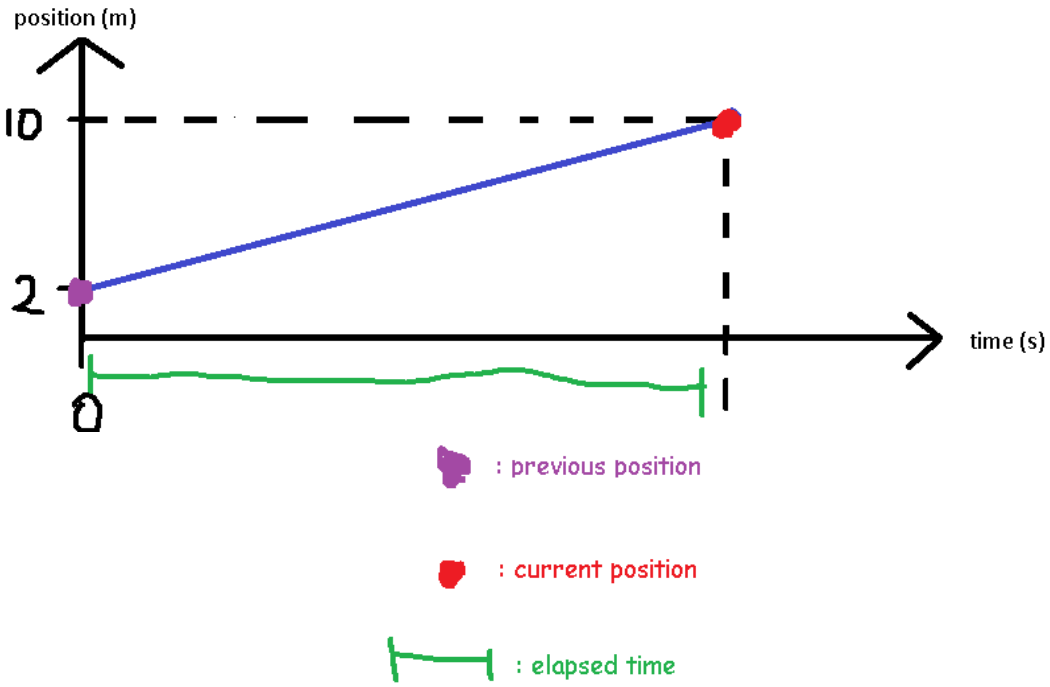
https://en.wikipedia.org/wiki/Numerical_differentiation#:~:text=In%20numerical%20analysis%2C%20numerical%20differentiation,other%20knowledge%20about%20the%20function.

Are you still confused ? me too 😊 hahaha nonono just kidding. On Wikipedia there are lot of equation that you must confused which one you will use. I will pick and modify the equation so you can use easily 😊



$$\frac{f(x+h)-f(x)}{h}.$$

I guess you have no idea what this is. Okay remember speed equation that you got from position differentiation. I will make simpler I promise haha



This graph have so many similarity. Let's replace every variable on that equation. So the equation will looks like this

$$\frac{f(x+h)-f(x)}{h} = \frac{\text{current position} - \text{previous position}}{\text{elapsed time}}$$

Let's insert that value. The result will surprise you

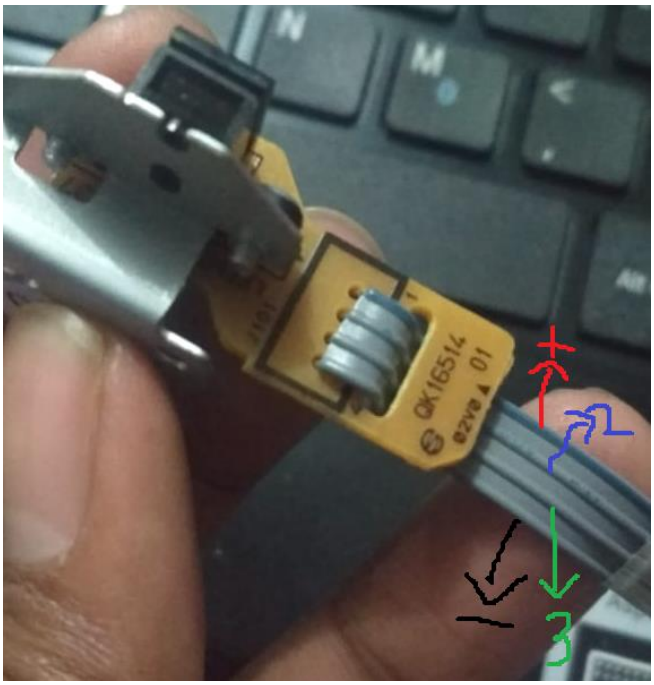
$$\frac{f(x+h)-f(x)}{h} = \frac{\text{current position} - \text{previous position}}{\text{elapsed time}}$$

$$= \frac{10\text{ m} - 2\text{m}}{1\text{ s}} = 8\,\text{m/s}$$

…… VOILAAA you got velocity from position derivation 😊

So you can calculate the acceleration by replacing position with velocity on that equation. So let's try by yourself

Electronics circuit



Red = +5v
Blue = D2
Green = D3
Black = GND

Connect to Arduino uno

Code explained line by line

```
//////////////////////////////////////////////////////////
////Implementing differential math using printer encoder/////
////written by : Muhammad Husni                          //////
//////////////////////////////////////////////////////////


#define encoder0PinA 2
#define encoder0PinB 3
#define encoder0Btn 4
int encoder0Pos = 0;

float Time;
float elapsedTime;
float timePrev;

void setup() {
  Serial.begin(9600);
  pinMode(encoder0PinA, INPUT_PULLUP);
  pinMode(encoder0PinB, INPUT_PULLUP);
  attachInterrupt(0, doEncoder, CHANGE);
}
int x,last_x;
int x_dot, last_x_dot;
int x_dot_dot;
void loop() {
```

```
//direction
if(x>last_x)
{
Serial.print("  CW");
}

if(x<last_x)  {
Serial.print("  CCW");
}

//got position value from interrupt
//pulse
Serial.print(" position : ");
Serial.print(x);

//got velocity/speed value from position using numerical derivation
//pulse per second
x_dot = (x-last_x)/elapsedTime;
Serial.print(" speed : ");
Serial.print(x_dot);
last_x = x;

//got acceleration value from velocity using numerical derivation
//pulse per second square
x_dot_dot = (x_dot-last_x_dot)/elapsedTime;
```

```
  Serial.print(" acceleration : ");
  Serial.print(x_dot_dot);
  last_x_dot = x_dot;
  Serial.println(" ");
  delay(250);

  //TIMER
  timePrev = Time;                        // the previous time is stored before the actual time read
  Time = millis();                        // actual time read
  elapsedTime = (Time - timePrev) / 1000;
}

//position value interrupt
void doEncoder()
{
  if (digitalRead(encoder0PinA) == digitalRead(encoder0PinB))
  {
  encoder0Pos++;
  }
  else
  {
  encoder0Pos--;
  }
  x = encoder0Pos/2.5;
}
```

Serial monitor result

serial monitor result						manual calculation		
pulse	pulse/s	pulse/s^2				pulse	pulse/s	pulse/s^2
0	0	0				0	0	0
4	16	64				4	16	64
28	95	314				28	96	316
47	75	-79				47	76	-80
92	180	420				92	180	420
168	302	486				168	304	488
210	167	-537				210	168	-540
226	63	-414				226	64	-416
245	76	52				245	76	52
242	-12	-352				242	-12	-352
243	3	59				243	4	60
382	556	2212				382	556	2212
460	310	-980				460	312	-984
598	549	952				598	552	956
702	414	-537				702	416	-540
733	123	-1159				733	124	-1164

Here is the comparation between Arduino calculation and excel calculation. Not to far because of sampling rate and Arduino performance. But all and all you can use Arduino for differential math to calculate speed and acceleration.