Assignment # 10

Date

Q3

a) $P_1P_2 = \langle 2, 3 \rangle$

$P_1(3, -2)$ $\quad P_2(5, 1)$

For line :

$$\boxed{\begin{aligned} x &= 3 + 2t \\ y &= -2 + 3t \end{aligned}}$$

For line segment :

Put P_1 in equation

$3 = 3 + 2t$

$\boxed{t = 0}$

Put P_2 in equation

$5 = 3 + 2t$

$\boxed{t = 1}$

$$\boxed{\begin{aligned} x &= 3 + 2t \qquad 0 \le t \le 1 \\ y &= -2 + 3t \end{aligned}}$$

b) $P_1(5, -2, 1)$ $\quad P_2(2, 4, 2)$

$P_1P_2 = \langle -3, 6, 1 \rangle$

For line :

$$\boxed{\begin{aligned} x &= 5 - 3t \\ y &= -2 + 6t \\ z &= 1 + t \end{aligned}}$$

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For line segment :-

put P_1 in equation

$5 = 5 - 3t$

$t = 0$

put P_2 in equation

$2 = 5 - 3t$

$t = 1$

$$\boxed{\begin{aligned} x &= 5 - 3t \\ y &= -2 + 6t \qquad 0 \leq t \leq 1 \\ z &= 1 + t \end{aligned}}$$

Q7

a) $xi + yj = (2i - j) + t(4i - j)$

$x = 2 + 4t$

$y = -1 - t$

$$\boxed{\begin{aligned} &\text{Point} = P(2, -1) \\ &\text{vector} = V = \cancel{4}\ 4i - j \end{aligned}}$$

b) $(x, y, z) = (-1, 2, 4) + t(5, 7, 8)$

$x = -1 + 5t$
$y = 2 + 7t$
$z = 4 + 8t$

$\boxed{\begin{array}{l} \text{Point} \Rightarrow P(-1, 2, 4) \\ \text{vector} \Rightarrow 5i + 7j + 8k \end{array}}$

Q15

Point $\Rightarrow P(-5, 2)$
Vector $\Rightarrow 2i - 3j$

$\boxed{\begin{array}{l} x = -5 + 2t \\ y = 2 - 3t \end{array}}$

Q17

$x^2 + y^2 = 25$ → equation of circle
$P_1(x, y) = (3, -4)$ → point on line & circle

The center point of circle is $O(0, 0)$
$\vec{OP} = \langle 3, -4 \rangle$
Let v is the vector parallel to the line & $\perp$ to OP

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$$V \cdot OP = 0$$

$$(ai + bj) \cdot (3i - 4j) = 0$$

$$3a \cancel{+} - 4b = 0$$

$$3a = 4b$$

$$\frac{a}{b} = \frac{4}{3}$$

$$a = 4 \quad b = 3$$

So the parallel vector $V = 4i + 3j$

Point $\Rightarrow P(3, -4)$

$$\boxed{\begin{aligned} x &= 3 + 4t \\ y &= -4 + 3t \end{aligned}}$$

Q19

$P(-1, 2, 4)$

$V = 3i - 4j + k$

$$\boxed{\begin{aligned} x &= -1 + 3t \\ y &= 2 - 4t \\ z &= 4 + t \end{aligned}}$$

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Q 21

Point $\Rightarrow$ $P(-2, 0, 5)$

Parallel line $\Rightarrow$ $x = 1 + 2t$

$y = 4 - t$

$z = 6 + 2t$

Parallel vector $\Rightarrow$ $v = \langle 2, -1, 2 \rangle$

Equation of line $\Rightarrow$

$$\boxed{\begin{aligned} x &= -2 + 2t \\ y &= -t \\ z &= 5 + 2t \end{aligned}}$$

Q23

a) the x-axis

put. $y = 0$

$0 = 2 - t$

$t = 2$

$x = 1 + 3(2)$

$x = 7$

$\boxed{P(7, 0)}$

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b) the y-axis

put $x = 0$

$0 = 1 + 3t$

$t = -\frac{1}{3}$

$y = 2 - \left(-\frac{1}{3}\right)$

$y = \frac{7}{3}$

$\boxed{P(0, 7/3)}$

c) the parabola $y = x^2$

c) the parabola $y = x^2$

put line in parabola

$$2 - t = (1 + 3t)^2$$

$$2 - t = 1 + 6t + 9t^2$$

$$9t^2 + 7t - 1 = 0$$

$\cancel{t}$

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$$t = \frac{-7 + \sqrt{85}}{18}$$

$$x = 1 + 3\left(\frac{-7 + \sqrt{85}}{18}\right)$$

$$\boxed{x = \frac{-1 + \sqrt{85}}{6}}$$

$$y = 2 - \left(\frac{-7 + \sqrt{85}}{18}\right)$$

$$\boxed{y = \frac{43 - \sqrt{85}}{18}}$$

$$t = \frac{-7 - \sqrt{85}}{18}$$

$$x = 1 + 3\left(\frac{-7 - \sqrt{85}}{18}\right)$$

$$\boxed{x = \frac{-1 - \sqrt{85}}{6}}$$

$$y = 2 - \left(\frac{-7 - \sqrt{85}}{18}\right)$$

$$\boxed{y = \frac{43 + \sqrt{85}}{18}}$$

$$\boxed{x = \frac{-1 \pm \sqrt{85}}{6} \qquad y = \frac{43 \pm \sqrt{85}}{18}}$$

Q25

the xy plane :-

$z = -3 + t = 0$

$t = 3$

$\boxed{x = -2}$

$y = 4 + 2(3)$

$\boxed{y = 10}$

$\boxed{P(-2, 10, 0)}$

the xz plane :-

$y = 4 + 2t$

$0 = 4 + 2t$

$t = -2$

$x = -2$

$z = -3 + (-2) = -5$

$\boxed{P(-2, 0, -5)}$

the yz plane :-

$x = -2$

there is no parameter bond on this equation so x never be zero: the line does not intersect yz plane

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Q27

equation of line

$x = 1+t$, $y = 3-t$, $z = 2t$

put in cylinder equation

$x^2+y^2 = 16$

$(1+t)^2+(3-t)^2 = 16$

$1+2t+t^2+9-6t+t^2 = 16$

$2t^2-4t-6 = 0$

$t = 3$

$x = 1+3 = 4$

$y = 3-3 = 0$

$z = 2(3) = 6$

$\boxed{P(4,0,6)}$

$t = -1$

$x = 1+(-1) = 0$

$y = 3-(-1) = 4$

$z = 2(-1) = -2$

$\boxed{P(0,4,-2)}$

Q23

Q29

Let (x_0, y_0, z_0) be the point of intersection

$L_1 \Rightarrow x_0 = 2+t_1,\ y_0 = 2+3t_1,\ z_0 = 3+t_1$

$L_2 \Rightarrow x_0 = 2+t_2,\ y_0 = 3+4t_2,\ z_0 = 4+2t_2$

By equaling

$2+t_1 = 2+t_2 \longrightarrow (1)$

$2+3t_1 = 3+4t_2 \longrightarrow (2)$

$3+t_1 = 4+2t_2 \longrightarrow (3)$

No. ____

~~Simultaneously solving eq ② & ③~~

~~② ⇒ t = -1~~

Simultaneously solving eq ① & ②

① ⇒ $t_1 = t_2$

put in eq ②

$2 + 3t_1 = 3 + 4t_1$

$\boxed{t_1 = -1}$

$\boxed{t_2 = -1}$

put in eq ③

$3 + (-1) = 4 + 2(-1)$

$3 - 1 = 4 - 2$

$\boxed{2 = 2}$

$\boxed{\text{Yes, these lines are intersecting}}$

$x_0 = 2 + (-1) = 2$

$y_0 = 2 + 3(-1) = -1$

$z_0 = 3 + (-1) = 2$

$\boxed{P(x_0, y_0, z_0) = P(2, -1, 2)}$

Q31

we have to proof :-

a) Lines are not parallel

b) Lines are not intersect

a)

$V_1 = <7, 1, -3>$

$V_2 = <-1, 0, 2>$

vectors are not multiple of one & another:

$\boxed{\text{Lines are not parallel}}$

b)

Let (x_0, y_0, z_0) be the point of intersection

L_1	L_2
$x_0 = 1 + 7t_1$	$x_0 = 4 - t_2$
$y_0 = 3 + t_1$	$y_0 = 6$
$z_0 = 5 - 3t_1$	$z_0 = 7 + 2t_2$

$$1 + 7t_1 = 4 - t_2$$
$$t_2 = 7t_1 - 3$$

$$5 - 3t_1 = 7 + 2t_2$$
$$5 - 3t_1 = 7 + 2(7t_1 - 3)$$
$$t_1 = \frac{4}{17}$$

$3 + t_1 = 6$

$3 + \frac{4}{17} = 6$

$\frac{55}{17} \neq 6$

$\boxed{\text{lines are not intersecting}}$

Yes it is skew lines

Q33

$V_1 = \langle -2, 1, -1 \rangle$
$V_2 = \langle -4, 2, -2 \rangle$

V_2 is multiple of V_1 so:

$\boxed{\text{lines are parallel}}$

Q35

$P_1P_2 = \langle 3, -7, -7 \rangle$
~~$P_3P_2 = \langle +9, +14, -10 \rangle$~~
$P_2P_3 = \langle -9, -7, -3 \rangle$

Vectors are not multiple of each other:
~~lines~~ points does not lie on the same line

Q37

$\cancel{v_1 = \langle -1, 2 \rangle}$

$\cancel{v_2 = \langle 3, -6 \rangle}$

$\cancel{x = 3 - t \qquad y = 1 + 2t}$

$\cancel{v_1 = \langle -1, 2 \rangle}$

From L_1

$y = 1 + 2(3 - x)$

$y = 1 + 6 - 2x$

$\boxed{y = 7 - 2x}$

From L_2

$y = 9 - 6\left(\frac{x+1}{3}\right)$

$y = 9 - 2x - 2$

$\boxed{y = 7 - 2x}$

lines are same ; prooved

43

$\langle x, y \rangle = \langle 1, 0 \rangle + t\langle -2, 3 \rangle \quad 0 \le t \le 2$

$x = 1 - 2t$

$y = 3t$

put $t = 0$ Put $t = 2$

$x = 1 \quad y = 0 \qquad x = -3 \, , \quad y = 6$

the line segment joining the points
(1, 0) and (−3, 6)

Q47

P(-2, 1, 1)

put t = 0 in line

A(x, y, z) = (3, 0, 1)

put t = 1 in line

B(x, y, z) = (2, 1, 3)

AP = <-5, 1, 0>

AB = <-1, 1, 2>

$$AP \times AB = \begin{vmatrix} i & j & k \\ -5 & 1 & 0 \\ -1 & 1 & 2 \end{vmatrix}$$

$AP \times AB = 2i + 10j - 4k$

$\|AP \times AB\| = \sqrt{2^2 + 10^2 + 4 2}$

$\|AP \times AB\| = 2\sqrt{30}$

$\|AB\| = \sqrt{1^2 + 1^2 + 2^2} = \sqrt{6}$

$$d = \frac{\|AP \times AB\|}{\|AB\|}$$

$$= \frac{2\sqrt{30}}{\sqrt{6}}$$

$\boxed{d = 2\sqrt{5}}$ units

RC

Q49

put $t=0$ in L_1

$P(x,y,z) = (2,0,1)$

put $t=0$ in L_2

$A(x,y,z) = (1,3,5)$

put $t=1$ in L_2

$B(x,y,z) = (3,-1,3)$

$$\boxed{d = \frac{\| AP \times AB \|}{\| AB \|}}$$

$$AP \times AB = \begin{vmatrix} i & j & k \\ 1 & -3 & -4 \\ 2 & -4 & -2 \end{vmatrix}$$

$AP \times AB = -10i - 6j + 2k$

$\| AP \times AB \| = \sqrt{10^2 + 6^2 + 2^2}$

$\| AP \times AB \| = 2\sqrt{35}$

$\| AB \| = \sqrt{2^2 + 4^2 + 2^2} = 2\sqrt{6}$

$$d = \frac{\cancel{2}\sqrt{35}}{\cancel{2}\sqrt{6}}$$

$$\boxed{d = \sqrt{\frac{35}{6}} \text{ unit}}$$

Scanned with CamScanner

Q55

a)

put $t = 3$ in L_1

$x = 1 + 2(3) = 7$

$y = 2 - 3 = -1$

$z = 4 - 2(3) = -2$

$\boxed{P(7, -1, -2)}$

put $t = -2$ in L_2

$x = 9 + 1(-2) = 7$

$y = 5 + 3(-2) = -1$

$z = -4 - (-2) = -2$

$\boxed{P(7, -1, -2)}$

b)

parallel vectors are

$v_1 = \langle 2, -1, -2 \rangle$

$v_2 = \langle 1, 3, -1 \rangle$

$$\cos\theta = \frac{v_1 \cdot v_2}{\|v_1\| \|v_2\|} = \frac{(2, -1, -2) \cdot (1, 3, -1)}{\sqrt{2^2 + 1^2 + 2^2}\sqrt{1^2 + 3^2 + 1^2}} = \frac{\sqrt{11}}{33}$$

$$\theta = \cos^{-1}\left(\frac{\sqrt{11}}{33}\right)$$

$\boxed{\theta \approx 84^\circ}$

Date

c)

parallel vectors are

$v_1 = \langle 2, -1, -2 \rangle$

$v_2 = \langle 1, 3, -1 \rangle$

Let c is the vector perpendicular to v_1 & v_2

$$c = v_1 \times v_2$$

$$= \begin{vmatrix} i & j & k \\ 2 & -1 & -2 \\ 1 & 3 & -1 \end{vmatrix}$$

$$c = 7i + 7k$$

$$\hat{c} = i + k$$

$P(7, -1, -2)$

$$\boxed{\begin{aligned} x &= 7 + t \\ y &= -1 \\ z &= -2 + t \end{aligned}}$$