Assignment # 11

Date

Q3

Point $\Rightarrow$ $P(2,6,1)$

$n = \langle 1, 4, 2 \rangle$

Equation of plane

$$a(x-x_0) + b(y-y_0) + c(z-z_0) = 0$$

$$1(x-2) + 4(y-6) + 2(z-1) = 0$$

$$x - 2 + 4y - 24 + 2z - 2 = 0$$

$$x + 4y + 2z - 28 = 0$$

$$\boxed{x + 4y + 2z = 28}$$

Q7

$P_1(0,0,1)$

$P_2(1,0,0)$

$P_3(0$

Q7

$P_1(0,0,0)$

$P_2(0,0,1)$

$P_3(1,1,0)$

$P_1P_2 = \langle 0, 0, 1 \rangle$

$P_1P_3 = \langle 1, 1, 0 \rangle$

$$P_1P_2 \times P_1P_3 = \begin{vmatrix} i & j & k \\ 0 & 0 & 1 \\ 1 & 1 & 0 \end{vmatrix}$$

$n = -i + j$

$-1(x-0) + 1(y-0) + 0(z-0) = 0$

$-x + y = 0$

$-(x-y) = 0$

$\boxed{x - y = 0}$

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Q 11

$P_1(-2, 1, 1)$
$P_2(0, 2, 3)$
$P_3(1, 0, -1)$

$P_1P_2 = \langle 2, 1, 2 \rangle$
$P_1P_3 = \langle 3, -1, -2 \rangle$

$$n = \begin{vmatrix} i & j & k \\ 2 & 1 & 2 \\ 3 & -1 & -2 \end{vmatrix}$$

$$= 0i + 10j - 5k$$

$n = 10j - 5k$
$n = 2j - k$

$a(x - x_0) + b(y - y_0) + c(z - z_0) = 0$
$0(x - \cancel{(+2)})) + 2(y - 1) - 1(z - 1) = 0$
$2y - 2 - z + 1 = 0$
$\boxed{2y - z = 1}$

Q13

a)

$n_1 = \langle 2, -8, -6 \rangle$

$n_2 = \langle -1, 4, 3 \rangle$

since ~~vec~~ normal vectors of plane are parallel, so the planes are also parallel.

b)

$n_1 = \langle 3, -2, 1 \rangle$

$n_2 = \langle 4, 5, -2 \rangle$

planes are not parallel

$n_1 \cdot n_2 = 0$

$(3 \times 4) + (-2 \times 5) + (1 \times -2) = 0$

$12 - 10 - 2 = 0$

$\boxed{0 = 0}$

since normal vectors are perpendicular so the planes are also perpendicular

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c)

$n_1 = \langle 1, -1, 3 \rangle$
$n_2 = \langle 2, 0, 1 \rangle$

planes are not parallel

$n_1 \cdot n_2 = 0$
$(1 \times 2) + (-1 \times 0) + (3 \times 1) = 0$
$2 + 3 = 0$
$5 \neq 0$

planes are not perpendicular

$\boxed{\text{neither}}$

Q15

a)

$\cancel{n_1 =}$ $V = \langle 2, -1, -4 \rangle$
$n = \langle 3, 2, 1 \rangle$

plane & line is not perpendicular

$n \cdot V = 0$
$(3 \cdot 2) + (2 \cdot -1) + (1 \cdot -4) = 0$

$6 - 6 = 0$

$\boxed{0 = 0}$

plane and line is parallel.

b)

$V = \langle 1, 2, 3 \rangle$

$n = \langle 1, -1, 2 \rangle$

not perpendicular

$n \cdot V = 0$

$(1.1) + (-1.2) + (2.3) = 0$

$1 - 2 + 6 = 0$

$5 \neq 0$

not parallel

$\boxed{\text{neither}}$

c)

$V = \langle 2, 1, -1 \rangle$

$n = \langle 4, 2, -2 \rangle$

plane and line is perpendicular,

Q17

a)

$v = \langle 1, 1, 1 \rangle$

$n = \langle 3, -2, 1 \rangle$

$n \cdot v = 0$

$(1 \cdot 3) + (1 \cdot -2) + (1 \cdot 1) = 0$

$3 - 2 + 1 = 0$

$2 \neq 0$

Yes, it is intersect.

Let (x_0, y_0, z_0) be the point of intersection

satisfy the line :

$x_0 = t$

$y_0 = t$

$z_0 = t$

satisfy the plane :

$3x_0 - 2y_0 + z_0 - 5 = 0$

$3t - 2t + t - 5 = 0$

$t = \frac{5}{2}$

$$\boxed{P(x_0, y_0, z_0) = \left(\frac{5}{2}, \frac{5}{2}, \frac{5}{2}\right)}$$

b)

$v = \langle -1, 1, 1 \rangle$

$n = \langle 2, 1, 1 \rangle$

$n.v = 0$

$(2 \cdot -1) + (1 \cdot 1) + (1 \cdot 1) = 0$

$-2 + 2 = 0$

$\boxed{0 = 0}$

$\boxed{\text{no point of intersection}}$

Q19

$n_1 = \langle 1, 0, 0 \rangle$

$n_2 = \langle 2, -1, 1 \rangle$

$$\cos\theta = \frac{n_1 \cdot n_2}{\|n_1\| \cdot \|n_2\|}$$

$$= \frac{(1, 0, 0)(2, -1, 1)}{\sqrt{1^2+0^2+0^2}\sqrt{2^2+1^2+1^2}}$$

$$\cos\theta = \frac{2}{\sqrt{6}}$$

$$\theta = \cos^{-1}\left(\frac{2}{\sqrt{6}}\right)$$

$\theta \approx 35°$

Date

Q25

Point $\Rightarrow P(0,0,0)$

normal vector of parallel plane $= n' = \langle 4,-2,7\rangle$

$n = n'$

$n = \langle 4,-2,7\rangle$

$a(x-x_0)+b(y-y_0)+c(z-z_0)=0$

$4(x-0)+-2(y-0)+\cancel{7}(z-0)=0$

$\boxed{4x-2y+7z=0}$

Q29

~~put t=0 in one plane~~

~~A~~ .

Point $\Rightarrow P(1,2,-1)$

$n_1 = \langle 2,1,1\rangle$

$n_2 = \langle 1,2,1\rangle$

$n = n_1 \times n_2$

$$= \begin{vmatrix} i & j & k \\ 2 & 1 & 1 \\ 1 & 2 & 1 \end{vmatrix}$$

$n = -i - j + 3k$

$$a(x - x_0) + b(y - y_0) + c(z - z_0) = 0$$

$$-1(x - 1) - 1(y - 2) + 3(z + 1) = 0$$

$$-x + 1 - y + 2 + 3z + 3 = 0$$

$$x + y - 3z - 6 = 0$$

$$\boxed{x + y - 3z = 6}$$

Q31

Point $\Rightarrow$ $P(-1, 2, -5)$

$n_1 = \langle 2, -1, 1 \rangle$

$n_2 = \langle 1, 1, -2 \rangle$

$n = n_1 \times n_2$

$$n = \begin{vmatrix} i & j & k \\ 2 & -1 & 1 \\ 1 & 1 & -2 \end{vmatrix}$$

$$n = \cancel{-2}\; i + 5j + 3k$$

$$a(x - x_0) + b(y - y_0) + c(z - z_0)$$

$$1(x + 1) + 5(y - 2) + 3(z + 5)$$

$$x + 1 + 5y - 10 + 3z + 15 = 0$$

$$x + 5y + 3z + 6 = 0$$

$$\boxed{x + 5y + 3z = -6}$$

Date

Q33

$P_1(2,-1,1)$
$P_2(3,1,5)$

$$P(x,y,z) = \left(\frac{2+3}{2}, \frac{-1+1}{2}, \frac{1+5}{2}\right)$$

$$= \frac{5}{2}, 0 \ \frac{6}{2}$$

$$P(x,y,z) = \left(\frac{5}{2}, 0, 3\right)$$

$n = P_1P_2$
$n = \langle 1, 2, 4 \rangle$

$$a(x-x_0) + b(y-y_0) + c(z-z_0) = 0$$
$$1\left(x-\frac{5}{2}\right) + 2(y-0) + 4(z-3) = 0$$

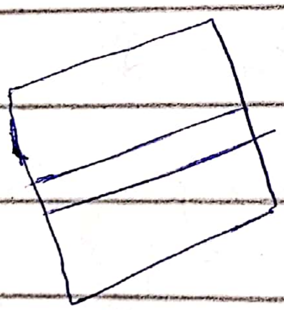
$$x - \frac{5}{2} + 2y + 4z - 12 = 0$$

$$\boxed{x + 2y + 4z = \frac{29}{2}}$$

Q37

$V_1 = \langle 1, 2, -1 \rangle$
$V_2 = \langle -1, -2, 1 \rangle$

Lines are parallel because their vectors are multiple of each and other



put $t=0$ in L_1 & L_2

$A(x,y,z) = (-2, 3, 4)$
$B(x,y,z) = (3, 4, 0)$
$\vec{AB} = \langle 5, 1, -4 \rangle$
$n = V_2 \times AB$

$$= \begin{vmatrix} i & j & k \\ -1 & -2 & 1 \\ 5 & 1 & \end{vmatrix}$$

$n = v_1 \times AB$

$$= \begin{vmatrix} i & j & k \\ 1 & 2 & -1 \\ 5 & 1 & -4 \end{vmatrix}$$

$n = -7i - j - 9k$

$P(-2, 3, 4)$

$a(x - x_0) + b(y - y_0) + c(z - z_0) = 0$

$+7(x+2) + 1(y-3) + 9(z-4) = 0$

$7x + 14 + y - 3 + 9z - 36 = 0$

$$\boxed{7x + y + 9z = 25}$$

Q41

$n_1 = \langle -2, 3, 7 \rangle$

$n_2 = \langle 1, 2, -3 \rangle$

$v = n_1 \times n_2$

$$v = \begin{vmatrix} i & j & k \\ -2 & 3 & 7 \\ 1 & 2 & -3 \end{vmatrix}$$

$v = -23i + j - 7k$

put $\cancel{z} = 0$ in P_1 & P_2

$-2x + 3y + z(0) + 2 = 0$

$x + 2y - 3(0) + 5 = 0$

$-2x + 3y = -2$

$x + 2y = -5$

$x = \frac{-19}{7}$ $\quad y = \frac{-12}{7}$ $\quad z = 0$

$P\left(\frac{-19}{7}, \frac{-12}{7}, 0\right)$

$v = \langle -23, 1, -7 \rangle$

$$\boxed{\begin{aligned} x &= \frac{-19}{7} - 23t \\ y &= \frac{-12}{7} + t \\ z &= -7t \end{aligned}}$$

Q43

$$d = \frac{|ax_0 + by_0 + cz_0 + d|}{\sqrt{a^2 + b^2 + c^2}}$$

$P(x_0, y_0, z_0) = P(1, -2, 3)$

$2x - 2y + z = 4$

$a = 2, \; b = -2, \; c = 1 \quad d = -4$

$$d = \frac{|2(1) - 2(-2) + 1(3) - 4|}{\sqrt{2^2 + 2^2 + 1^2}}$$

$$\boxed{d = \frac{5}{3} \text{ units}}$$

Q45

$-2x + y + z = 0 \rightarrow (1)$

$6x - 3y - 3z - 5 = 0$

put $y = 0$, $z = 0$ in (1)

$x = 0$

$P(x_0, y_0, z_0) = P(0, 0, 0)$

$a = 6 \quad b = -3 \quad c = -3 \quad d = -5$

$$d = \frac{|ax_0 + by_0 + cz_0 + d|}{\sqrt{a^2 + b^2 + c^2}}$$

$$d = \frac{|6(0) + -3(0) + -3(0) - 5|}{\sqrt{6^2 + 3^2 + 3^2}}$$

$$\boxed{d = \frac{5}{\sqrt{54}} \text{ units}}$$

$L_1 \Rightarrow x = 1 + 7t, y = 3 + t, z = 5 - 3t$

$L_2 \Rightarrow x = 4 - t, y = 6, z = 7 + 2t$

Put $t = 0$ in L_1 & L_2

$L_1 \Rightarrow$ ~~Pt~~ $x = 1, y = 3, z = 5$

$P(x_0, y_0, z_0) = P(1, 3, 5)$

$L_2 \Rightarrow x = 4, y = 6, z = 7$

$A(x', y', z') = A(4, 6, 7)$

$n' = V_1 \times V_2$

$$= \begin{vmatrix} i & j & k \\ 7 & 1 & -3 \\ -1 & 0 & 2 \end{vmatrix}$$

$n' = 2i - 11j + k$

$L_2 \Rightarrow$ Plane $\Rightarrow 2(x-4) - 11(y-6) + 1(z-7) = 0$

$2x - 8 - 11y + 66 + z - 7 = 0$

$2x - 11y + z + 51 = 0$

$a = 2 \quad b = -11 \quad c = 1 \quad d = +51$

$P(x_0, y_0, z_0) = P(1, 3, 5)$

$$d = \frac{|ax_0 + by_0 + cz_0 + d|}{\sqrt{a^2 + b^2 + c^2}} = \frac{|2(1) - 11(3) + 1(5) + 51|}{\sqrt{2^2 + 11^2 + 1^2}}$$

$$\boxed{d = \frac{25}{3\sqrt{14}} \text{ units}}$$