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Exercise 1.1

Assignment No 1

(1)

Transpose of Matrix :-

The transpose of a matrix is obtained by changing the rows into columns and columns into rows for a given matrix. Using transpose of matrix, we can view data in different aspects. It can be used to find the in a line closer to a given position outside the line in the plane.

Trace of Matrix :-

The trace of a matrix is the sum of the principle diagonal elements of the matrix. Trace of a 2x2 matrix is used to classify Mobius transformations. Trace of matrix is the sum of eigenvalues, It has a lot of nice properties such as linearity, invariance. It also used for generalization.

Exercise 1.2

Assignment # 02

①

Q1

a) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

Both

b) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

Both

c) $\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$

Both

d) $\begin{bmatrix} 1 & 0 & 3 & 1 \\ 0 & 1 & 2 & 4 \end{bmatrix}$

Both

e) $\begin{bmatrix} 1 & 2 & 0 & 3 & 0 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$

Both.

f) $\begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$

Both

g) $\begin{bmatrix} 1 & -7 & 5 & 5 \\ 0 & 1 & 3 & 2 \end{bmatrix}$

Row echelon form

Date

Q3

a)
$$\begin{bmatrix} 1 & -3 & 4 & 7 \\ 0 & 1 & 2 & 2 \\ 0 & 0 & 1 & 5 \end{bmatrix}$$

$$x_1 - 3x_2 + 4x_3 = 7$$
$$x_2 + 2x_3 = 2$$
$$x_3 = 5$$

Apply back substitution

$$x_3 = 5$$
$$x_2 = -8$$
$$x_1 - 3(-8) + 4(5) = 7$$
$$x_1 = -37$$
$$(x_1, x_2, x_3) = (-37, -8, 5)$$

b)
$$\begin{bmatrix} 1 & 0 & 8 & -3 & 6 \\ 0 & 1 & 4 & -9 & 3 \\ 0 & 0 & 1 & 1 & 2 \end{bmatrix}$$

$$x_1 + 8x_3 - 3x_4 = 6$$
$$x_2 + 4x_3 - 9x_4 = 3$$
$$x_3 + x_4 = 2$$

Date

$x_3 = 2 - x_4$

Let $\boxed{x_4 = r}$

$\boxed{x_3 = 2 - r}$

$x_2 = 9x_4 - 4x_3 + 3$

$x_2 = 9r - 8 + 4r + 3$

$\boxed{x_2 = 13r - 5}$

$x_1 + 8(2-r) - 3r = 6$

$x_1 + 16 - 8r - 3r = 6$

$x_1 = 5r - 10$

c) $\begin{bmatrix} 1 & 7 & -2 & 0 & -8 & -3 \\ 0 & 0 & 1 & 1 & 6 & 5 \\ 0 & 0 & 0 & 1 & 3 & 9 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$

$x_1 + 7x_2 - 2x_3 \quad - 8x_5 = -3$

$x_3 + x_4 + 6x_5 = 5$

$x_4 + 3x_5 = 9$

$0 = 0$

Date

$$x_4 = 9 - 3x_5$$

let

$$\boxed{x_5 = r}$$

$$\boxed{x_4 = 9 - 3r}$$

$$x_3 + 9 - 3r + 6r = 5$$

$$\boxed{x_3 = -3r - 4}$$

$$x_1 + 7x_2 - 2(-3r - 4) - 8(r) = -3$$

$$x_1 + 7x_2 + 6r + 8 - 8r = -3$$

$$x_1 + 7x_2 = -3 - 8 + 2r$$

$$x_1 + 7x_2 = -11 + 2r$$

let

$$\boxed{x_2 = s}$$

$$\boxed{x_1 = -11 + 2r - 7a}$$

d) $\begin{bmatrix} 1 & -3 & 7 & 1 \\ 0 & 1 & 4 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

$$0x_1 + 0x_2 + 0x_3 = \cancel{0}\,1$$

System is inconsistent.

Q5

Q5)

$$x_1 + x_2 + 2x_3 = 8$$
$$-x_1 + 2x_2 + 3x_3 = 1$$
$$3x_1 - 7x_2 + 4x_3 = 10$$

$$\begin{array}{c} R_1 \\ R_2 \\ R_3 \end{array} \begin{bmatrix} 1 & 1 & 2 & 8 \\ -1 & -2 & 3 & 1 \\ 3 & -7 & 4 & 10 \end{bmatrix}$$

- Multiply R_1 with 1 and add in R_2
- Multiply R_1 with -3 and add in R_3.

$$\begin{array}{c} R_1 \\ R_2 \\ R_3 \end{array} \begin{bmatrix} 1 & 1 & 2 & 8 \\ 0 & -1 & 5 & 9 \\ 0 & -10 & -2 & -14 \end{bmatrix}$$

Divide by R_3 by -1

$$\begin{array}{c} R_1 \\ R_2 \\ R_3 \end{array} \begin{bmatrix} 1 & 1 & 2 & 8 \\ 0 & 1 & -5 & -9 \\ 0 & -10 & -2 & -14 \end{bmatrix}$$

- Multipy R_2 with -1 and add in R_1
- Multiply R_2 with 10 and add in R_3

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$$\begin{array}{c} R_1 \\ R_2 \\ R_3 \end{array} \begin{bmatrix} 1 & 0 & 7 & 17 \\ 0 & 1 & -5 & -9 \\ 0 & 0 & -52 & -104 \end{bmatrix}$$

Divide R_3 by -52

$$\begin{array}{c} R_1 \\ R_2 \\ R_3 \end{array} \begin{bmatrix} 1 & 0 & 7 & 17 \\ 0 & 1 & -5 & -9 \\ 0 & 0 & 1 & +2 \end{bmatrix}$$

Multiply R_3 by 5 and add in R_2

Multiply R_3 by -7 and add in R_1

$$\begin{bmatrix} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 2 \end{bmatrix}$$

$x_1 = 3$

$x_2 = 1$

$x_3 = 2$

$(x_1, x_2, x_3) = (3, 1, 2)$

Scanned with CamScanner

Date

Q7

7)
$$x - y + 2z - w = -1$$
$$2x + y - 2z - 2w = -2$$
$$-x + 2y - 4z + w = 1$$
$$3x - 3w = -3$$

$$\begin{array}{c} R_1 \\ R_2 \\ R_3 \\ R_4 \end{array} \begin{bmatrix} 1 & -1 & 2 & -1 & -1 \\ 2 & 1 & -2 & -2 & -2 \\ -1 & 2 & -4 & 1 & 1 \\ 3 & 0 & 0 & -3 & -3 \end{bmatrix}$$

- Multiply R_1 with -2 and add in R_2
- Multiply R_1 with 1 and add in R_3
- Multiply R_1 with -3 and add in R_4

$$\begin{array}{c} R_1 \\ R_2 \\ R_3 \\ R_4 \end{array} \begin{bmatrix} 1 & -1 & 2 & -1 & -1 \\ 0 & 3 & -6 & 0 & 0 \\ 0 & 1 & -2 & 0 & 0 \\ 0 & 3 & -6 & 0 & 0 \end{bmatrix}$$

Divide by R_3 with 3

$$\begin{array}{c} R_1 \\ R_2 \\ R_3 \\ R_4 \end{array} \begin{bmatrix} 1 & -1 & 2 & -1 & -1 \\ 0 & 1 & -2 & 0 & 0 \\ 0 & 1 & -2 & 0 & 0 \\ 0 & 3 & -6 & 0 & 0 \end{bmatrix}$$

Date

Multiply R_2 with 1 and add in R_1
Multiply R_2 with -1 and add in R_3
Multiply R_2 with -3 and add in R_4

$$\begin{bmatrix} 1 & 0 & 0 & -1 & -1 \\ 0 & 1 & -2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$x - w = -1$$
$$y - 2z = 0$$

Let ~~w = r~~

$w = a$
$z = b$

then $x = -1 + a$
$y = 2b$

Date

(10)

Q9

$$x_1 + x_2 + 2x_3 = 8$$
$$-x_1 - 2x_2 + 3x_3 = 1$$
$$3x_1 - 7x_2 + 4x_3 = 10$$

$$\begin{array}{c} R_1 \\ R_2 \\ R_3 \end{array} \begin{bmatrix} 1 & 1 & 2 & 8 \\ -1 & -2 & 3 & 1 \\ 3 & -7 & 4 & 10 \end{bmatrix}$$

Multiply 1 with R_1 and add in R_2

Multiply -3 with R_1 and add in R_3

$$\begin{array}{c} R_1 \\ R_2 \\ R_3 \end{array} \begin{bmatrix} 1 & 1 & 2 & 8 \\ 0 & -1 & 5 & 9 \\ 0 & -10 & -2 & -14 \end{bmatrix}$$

Divide R_2 by -1

Multiply R_2 with -1 and add in R_1

Multiply R_2 with 10 and add in R_3

$$\begin{array}{c} R_1 \\ R_2 \\ R_3 \end{array} \begin{bmatrix} 1 & 0 & 7 & 17 \\ 0 & 1 & -5 & -9 \\ 0 & 0 & -52 & +104 \end{bmatrix}$$

Divide R_3 by -52
Multiply R_3 with -17 and add in R_1
Multiply R_3 with 5 and add in R_2

$$\begin{bmatrix} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 2 \end{bmatrix}$$

$x_1 = 3$

$x_2 = 1$

$x_3 = 2$

Example 5

$$\begin{aligned} x_1 + 3x_2 - 2x_3 \quad\quad\quad + 2x_5 \quad\quad &= 0 \\ 2x_1 + 6x_2 - 5x_3 - 2x_4 + 4x_5 - 3x_6 &= -1 \\ 5x_3 + 10x_4 \quad\quad + 15x_6 &= 5 \\ 2x_1 + 6x_2 \quad\quad + 8x_4 + 4x_5 + 18x_6 &= 6 \end{aligned}$$

$$\begin{array}{c} R_1 \\ R_2 \\ R_3 \\ R_4 \end{array} \begin{bmatrix} 1 & 3 & -2 & 0 & 2 & 0 & 0 \\ 2 & 6 & -5 & -2 & 4 & -3 & -1 \\ 0 & 0 & 5 & 10 & 0 & 15 & 5 \\ 2 & 6 & 0 & 8 & 4 & 18 & 6 \end{bmatrix}$$

Multiply R_1 with -2 & add in R_2, R_4

$$\begin{array}{c} R_1 \\ R_2 \\ R_3 \\ R_4 \end{array} \begin{bmatrix} 1 & 3 & -2 & 0 & 2 & 0 & 0 \\ 0 & 0 & -1 & -2 & 0 & -3 & -1 \\ 0 & 0 & 5 & 10 & 0 & 15 & 5 \\ 0 & 0 & 4 & 8 & 0 & 18 & 6 \end{bmatrix}$$

Divide R_2 by -1

Multiply R_2 with 2 and add in R_1.

Multiply R_2 with -5 and add in R_3

Multiply R_2 with -4 and add in R_4

$$\begin{array}{c} R_1 \\ R_2 \\ R_3 \\ R_4 \end{array} \begin{bmatrix} 1 & 3 & 0 & 4 & 2 & 6 & 2 \\ 0 & 0 & 1 & 2 & 0 & 3 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 6 & 2 \end{bmatrix}$$

Interchange R_3 & R_4

$$\begin{array}{c} R_1 \\ R_2 \\ R_3 \\ R_4 \end{array} \begin{bmatrix} 1 & 3 & 0 & 4 & 2 & 6 & 2 \\ 0 & 0 & 1 & 2 & 0 & 3 & 1 \\ 0 & 0 & 0 & 0 & 0 & 6 & 2 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

(13)

Date

Multiply R_3 with $\frac{1}{6}$
Multiply R_3 with -6 and add in R_1
Multiply R_3 with -3 and add in R_2

$$\begin{bmatrix} 1 & 3 & 0 & 4 & 2 & 0 & 0 \\ 0 & 0 & 1 & 2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1/3 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\begin{aligned} x_1 + 3x_2 + 4x_4 + 2x_5 &= 0 \\ x_3 + 2x_4 &= 0 \\ x_6 &= 1/3 \end{aligned}$$

Solving for Leading Variables

$x_1 = -3x_2 - 4x_4 - 2x_5$
$x_3 = -2x_4$
$x_6 = \frac{1}{3}$

Let

$x_2 = r$
$x_4 = s$
$x_5 = t$

then $x_1 = -3r - 4s - 2t$
$x_3 = -2s$
$x_6 = 1/3$

Date

Example 7

$$\begin{bmatrix} 1 & 3 & -2 & 0 & 2 & 0 & 0 \\ 0 & 0 & 1 & 2 & 0 & 3 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 & \frac{1}{3} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$x_1 + 3x_2 - 2x_3 + 2x_5 = 0$$
$$x_3 + 2x_4 + 3x_6 = 1$$
$$x_6 = \frac{1}{3}$$

Solve for Leading variable

$x_1 = -3x_2 + 2x_3 - 2x_5 \rightarrow (1)$

$x_3 = 1 - 2x_4 - 3x_6 \rightarrow (2)$

$\boxed{x_6 = 1/3} \rightarrow (3)$

put (3) in (1) & (2)

$x_1 = -3x_2 + 2x_3 - 2x_5$

$x_3 = 1 - 2x_4 - 1$

$x_3 = -2x_4 \rightarrow (4)$

put (4) in (1)

$x_1 = -3x_2 - 4x_4 - 2x_5 \rightarrow (5)$

let

$x_2 = r$

$x_4 = s$

$x_5 = t$

then

eq(4) $\rightarrow x_3 = -2s$

eq(5) $\rightarrow x_1 = -3r - 4s - 2t$

$\boxed{(x_1, x_2, x_3, x_4, x_5, x_6) = (-3r - 4s - 2t, r, -2s, s, t, \frac{1}{3})}$

Exercise 1.2

Assignment # 03

Q(13) **Q13**

$$2x_1 - 3x_2 + 4x_3 - x_4 = 0$$
$$7x_1 + x_2 - 8x_3 + 9x_4 = 0$$
$$2x_1 + 8x_2 + x_3 - x_4 = 0$$

This system has non trivial solution

Q14

$$x_1 + 3x_2 - x_3 = 0$$
$$\cancel{7x_1} \; x_2 - 8x_3 = 0$$
$$4x_3 = 0$$

This system has trivial solution.

Q25

$$x + 2y - 3z = 4$$
$$3x - y + 5z = 2$$
$$4x + y + (a^2 - 14)z = a + 2$$

$$\left[\begin{array}{ccc|c} 1 & 2 & -3 & 4 \\ 3 & -1 & 5 & 2 \\ 4 & 1 & a^2 - 14 & a + 2 \end{array}\right]$$

$$\begin{bmatrix} 1 & 2 & -3 & 4 \\ 0 & -7 & 14 & -10 \\ 0 & -7 & a^2-2 & a-14 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 1 & 8/7 \\ 0 & 1 & -2 & 10/7 \\ 0 & 0 & a^2-16 & a-4 \end{bmatrix}$$

For no solution

$a^2-16=0$

$a = \pm 4$

For $a-4 \neq 0$ a must be -4.

$\boxed{a=-4}$

the last row becomes

$[0 \quad 0 \quad 0 \quad \cancel{a}-8]$

which means the system is inconsistent

For exactly one solution

a^2-16 should be not equal zero.

$a^2-16 \neq 0$

means

$\boxed{a \neq \pm 4}$

For infinitely many solutions:-

$a^2-16=0 \qquad a-4=0$

solve for a,

$\boxed{a=4}$

Q 26

$x+2y+z=2$

$2x-2y+3z=1$

$x+2y-(a^2-3)z=a$

$$\left[\begin{array}{ccc|c} 1 & 2 & 1 & 2 \\ 2 & -2 & 3 & 1 \\ 1 & 2 & -(a^2-3) & a \end{array}\right]$$

$$\begin{bmatrix} 1 & 2 & 1 & 2 \\ 0 & -6 & 1 & -3 \\ 0 & 0 & -(a^2-4) & a-2 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 2 & 1 & 2 \\ 0 & 1 & -1/6 & 1/2 \\ 0 & 0 & -(a^2-4) & a-2 \end{bmatrix}$$

For no solution :-

$a^2 - \cancel{4} = 0$

$\boxed{\cancel{a = \pm 2}}$ $\boxed{a = \sqrt{2}}$

For exactly one solution :-

$\cancel{a^2 - 4 \neq 0}$ $a^2 - 2 \neq 0$

$\boxed{\cancel{a \neq \pm 2}}$ $a \neq \sqrt{2}$

For infinitely many solutions

$a^2 - \cancel{2} = 0$ $a - 2 = 0$

$\boxed{\cancel{a = 2}}$ no solution for a.

Q27

$$x + 3y - z = a$$
$$x + y + 2z = b$$
$$2y - 3z = c$$

$$\begin{bmatrix} 1 & 3 & -1 & a \\ 1 & 1 & 2 & b \\ 0 & 2 & -3 & c \end{bmatrix}$$

$$\begin{bmatrix} 1 & 3 & -1 & a \\ 0 & -2 & 3 & b-a \\ 0 & 2 & -3 & c \end{bmatrix}$$

$$\begin{bmatrix} 1 & 3 & -1 & a \\ 0 & 1 & -3/2 & (b-a)/-2 \\ 0 & 0 & 0 & b-a+c \end{bmatrix}$$

$b-a+c$ must ~~should not~~ be equal to zero

$b - a + c \neq 0$

Q28

$$x + 3y + z = a$$
$$-x - 2y + z = b$$
$$3x + 7y - z = c$$

$$\begin{bmatrix} 1 & 3 & 1 & a \\ -1 & -2 & 1 & b \\ 3 & 7 & -1 & c \end{bmatrix}$$

$$\begin{bmatrix} 1 & 3 & 1 & a \\ 0 & 1 & 2 & a+b \\ 0 & -2 & -4 & -3a+c \end{bmatrix}$$

$$\left[\begin{array}{cccc} 1 & 3 & 1 & a \\ 0 & 1 & 2 & a+b \\ 0 & 0 & 0 & -a+2b+c \end{array}\right]$$

$-a+2b+c \neq 0$

$-a+2b+c$ most be equals to zero.

Exercise 1.3

Assignment # 04

Date

Homwork

prove

$$\begin{bmatrix} 1 & -9 & -3 \end{bmatrix} \begin{bmatrix} -1 & 3 & 2 \\ 1 & 2 & -3 \\ 2 & 1 & -2 \end{bmatrix} = \begin{bmatrix} -16 & -18 & 35 \end{bmatrix}$$

$1(-1) + -9(1) + 3(2) = -16$

$1(3) - 9(2) - 3(1) = -18$

$1(2) - 9(-3) - 3(-2) = 35$

$$\begin{bmatrix} -16 & -18 & 35 \end{bmatrix}$$

Q 7

a) the first row of AB.

$$\begin{bmatrix} 3 & -2 & 7 \end{bmatrix} \begin{bmatrix} 6 & -2 & 4 \\ 0 & 1 & 3 \\ 7 & 7 & 5 \end{bmatrix}$$

$$\begin{bmatrix} 3(6)-2(0)+7(7) & 3(-2)-2(1)+7(7) & 3(4)-2(3)+7(5) \end{bmatrix}$$

$$\begin{bmatrix} 67 & 41 & 41 \end{bmatrix}$$

b) the third row of AB

$$\begin{bmatrix} 0 & 4 & 9 \end{bmatrix} \begin{bmatrix} 6 & -2 & 4 \\ 0 & 1 & 3 \\ 7 & 7 & 5 \end{bmatrix}$$

$$\begin{bmatrix} 0(6)+4(0)+9(7) & 0(-2)+4(1)+9(7) & 0(4)+4(3)+9(5) \end{bmatrix}$$

$$\begin{bmatrix} 63 & 67 & 57 \end{bmatrix}$$

c) ~~Row +~~ second column of AB

$$\begin{bmatrix} 3 & -2 & 7 \\ 6 & 5 & 4 \\ 0 & 4 & 9 \end{bmatrix} \begin{bmatrix} -2 \\ 1 \\ 7 \end{bmatrix}$$

$$\begin{bmatrix} 3(-2)-2(1)+7(7) \\ 6(-2)+5(1)+4(7) \\ 0(-2)+4(1)+9(7) \end{bmatrix}$$

$$\begin{bmatrix} 41 \\ 21 \\ 67 \end{bmatrix}$$

d) the first column of BA

$$\begin{bmatrix} 6 & -2 & 4 \\ 0 & 1 & 3 \\ 7 & 7 & 5 \end{bmatrix} \begin{bmatrix} 3 \\ 6 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 6(3)-2(6)+4(0) \\ 0(3)+1(6)+3(0) \\ 7(3)+7(6)+5(0) \end{bmatrix}$$

$$\begin{bmatrix} 6 \\ 6 \\ 63 \end{bmatrix}$$

e) the third row of AA

$$\begin{bmatrix} 0 & 4 & 9 \end{bmatrix} \begin{bmatrix} 3 & -2 & 7 \\ 6 & 5 & 4 \\ 0 & 4 & 9 \end{bmatrix}$$

$$\begin{bmatrix} 0(3)+4(6)+9(0) & 0(-2)+4(5)+9(4) & 0(7)+4(4)+9(9) \end{bmatrix}$$

$$\begin{bmatrix} 24 & 56 & 97 \end{bmatrix}$$

f) the third column of AA

$$\begin{bmatrix} 3 & -2 & 7 \\ 6 & 5 & 4 \\ 0 & 4 & 9 \end{bmatrix} \begin{bmatrix} 7 \\ 4 \\ 9 \end{bmatrix}$$

$$\begin{bmatrix} 3(7) - 2(4) + 7(9) \\ 6(7) + 5(4) + 4(9) \\ 0(7) + 4(4) + 9(9) \end{bmatrix}$$

$$\begin{bmatrix} 76 \\ 98 \\ 97 \end{bmatrix}$$

Q8

a) the first column of AB

$$\begin{bmatrix} 3 & -2 & 7 \\ 6 & 5 & 4 \\ 0 & 4 & 9 \end{bmatrix} \begin{bmatrix} 6 \\ 0 \\ 7 \end{bmatrix}$$

$$\begin{bmatrix} 3(6) - 2(0) + 7(7) \\ 6(6) + 5(0) + 4(7) \\ 0(6) + 4(0) + 9(7) \end{bmatrix} \Rightarrow \begin{bmatrix} 67 \\ 64 \\ 63 \end{bmatrix}$$

b) the third column of BB

$$\begin{bmatrix} 6 & -2 & 4 \\ 0 & 1 & 3 \\ 7 & 7 & 5 \end{bmatrix} \begin{bmatrix} 4 \\ 3 \\ 5 \end{bmatrix}$$

$$\begin{bmatrix} 6(4) - 2(3) + 4(5) \\ 0(4) + 1(3) + 3(5) \\ 7(4) + 7(3) + 5(5) \end{bmatrix} \Rightarrow \begin{bmatrix} 63 \\ 18 \\ 74 \end{bmatrix}$$

c) the second row of BB

$$\begin{bmatrix} 0 & 1 & 3 \end{bmatrix} \begin{bmatrix} 6 & -2 & 4 \\ 0 & 1 & 3 \\ 7 & 7 & 5 \end{bmatrix}$$

$$\begin{bmatrix} 0(6) + 1(0) + 3(7) & 0(-2) + 1(1) + 3(7) & 0(4) + 1(3) + 3(5) \end{bmatrix}$$

$$\begin{bmatrix} 21 & 22 & 18 \end{bmatrix}$$

d) the first column of AA

$$\begin{bmatrix} 3 & -2 & 7 \\ 6 & 5 & 4 \\ 0 & 4 & 9 \end{bmatrix} \begin{bmatrix} 3 \\ 6 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 3(3)-2(6)+7(0) \\ 6(3)+5(6)+4(0) \\ 0(3)+4(6)+9(0) \end{bmatrix} \Rightarrow \begin{bmatrix} -3 \\ 48 \\ 24 \end{bmatrix}$$

e) the third column of AB

$$\begin{bmatrix} 3 & -2 & 7 \\ 6 & 5 & 4 \\ 0 & 4 & 9 \end{bmatrix} \begin{bmatrix} 4 \\ 3 \\ 5 \end{bmatrix}$$

$$\begin{bmatrix} 3(4)-2(3)+7(5) \\ 6(4)+5(3)+4(5) \\ 0(4)+4(3)+9(5) \end{bmatrix} \Rightarrow \begin{bmatrix} 41 \\ 59 \\ 57 \end{bmatrix}$$

f) the first row of BA

$$\begin{bmatrix} 6 & -2 & 4 \end{bmatrix} \begin{bmatrix} 3 & -2 & 7 \\ 6 & 5 & 4 \\ 0 & 4 & 9 \end{bmatrix}$$

$$\begin{bmatrix} 6(3)-2(6)+4(0) & 6(-2)-2(5)+4(4) & 6(7)-2(4)+4(9) \end{bmatrix}$$

$$\begin{bmatrix} 6 & -6 & 54 \end{bmatrix}$$

Exercise 1.6

Assignment # 05

27

Date

Q1

$$x_1 + x_2 = 2$$
$$5x_1 + 6x_2 = 9$$

$$\begin{bmatrix} x_1 + x_2 \\ 5x_1 + 6x_2 \end{bmatrix} = \begin{bmatrix} 2 \\ 9 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 \\ 5 & 6 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 2 \\ 9 \end{bmatrix}$$

$$AX = B$$
$$X = A^{-1}B$$

Finding A^{-1}:

$$\left[\begin{array}{cc|cc} 1 & 1 & 1 & 0 \\ 5 & 6 & 0 & 1 \end{array}\right]$$

$$\left[\begin{array}{cc|cc} 1 & 1 & 1 & 0 \\ 0 & 1 & -5 & 1 \end{array}\right]$$

$$\left[\begin{array}{cc|cc} 1 & 0 & 6 & -1 \\ 0 & 1 & -5 & 1 \end{array}\right]$$

$$A^{-1} = \begin{bmatrix} 6 & -1 \\ -5 & 1 \end{bmatrix}$$

$$X = A^{-1}B$$

$$X = \begin{bmatrix} 6 & -1 \\ -5 & 1 \end{bmatrix} \begin{bmatrix} 2 \\ 9 \end{bmatrix}$$

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = 2 \begin{bmatrix} 6 \\ -5 \end{bmatrix} + 9 \begin{bmatrix} -1 \\ 1 \end{bmatrix} = \begin{bmatrix} +3 \\ -1 \end{bmatrix}$$

$x_1 = 3$, $x_2 = -1$

Q2

$4x_1 - 3x_2 = -3$

$2x_1 - 5x_2 = 9$

$$\begin{bmatrix} 4 & -3 \\ 2 & -5 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} -3 \\ 9 \end{bmatrix}$$

$AX = B$

$X = A^{-1}B$

Finding A^{-1}

$$\left[\begin{array}{cc|cc} 4 & -3 & 1 & 0 \\ 2 & -5 & 0 & 1 \end{array}\right]$$

$$\left[\begin{array}{cc|cc} 1 & -3/4 & 1/4 & 0 \\ 2 & -5 & 0 & 1 \end{array}\right]$$

$$\left[\begin{array}{cc|cc} 1 & -3/4 & 1/4 & 0 \\ 0 & -7/2 & -1/2 & 1 \end{array}\right]$$

$$\left[\begin{array}{cc|cc} 1 & 0 & -1/8 & 3/4 \\ 0 & 1 & -1/2 & 1 \end{array}\right]$$

$$\left[\begin{array}{cc|cc} 1 & -3/4 & 1/4 & 0 \\ 0 & 1 & 1/7 & -2/7 \end{array}\right]$$

$$\left[\begin{array}{cc|cc} 1 & 0 & 5/14 & -3/14 \\ 0 & 1 & 1/7 & -2/7 \end{array}\right]$$

$$A^{-1} = \begin{bmatrix} 5/14 & -3/14 \\ 1/7 & -2/7 \end{bmatrix}$$

$$X = A^{-1}B$$

$$X = \begin{bmatrix} 5/14 & -3/14 \\ 1/7 & -2/7 \end{bmatrix} \begin{bmatrix} -3 \\ 9 \end{bmatrix}$$

$$X = -3 \begin{bmatrix} 5/14 \\ 1/7 \end{bmatrix} + 9 \begin{bmatrix} -3/14 \\ -2/7 \end{bmatrix}$$

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} -3 \\ -3 \end{bmatrix}$$

$x_1 = -3$ $x_2 = -3$

Q3

$x_1 + 3x_2 + x_3 = 4$

$2x_1 + 2x_2 + x_3 = -1$

$2x_1 + 3x_2 + x_3 = 3$

$$\begin{bmatrix} 1 & 3 & 1 \\ 2 & 2 & 1 \\ 2 & 3 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 4 \\ -1 \\ 3 \end{bmatrix}$$

$AX = B$

$X = A^{-1}B$

Finding A^{-1}

$$\left[\begin{array}{ccc|ccc} 1 & 3 & 1 & 1 & 0 & 0 \\ 2 & 2 & 1 & 0 & 1 & 0 \\ 2 & 3 & 1 & 0 & 0 & 1 \end{array}\right]$$

$$\left[\begin{array}{ccc|ccc} 1 & 3 & 1 & 1 & 0 & 0 \\ 0 & -4 & -1 & -2 & 1 & 0 \\ 0 & -3 & -1 & -2 & 0 & 1 \end{array}\right]$$

$$\left[\begin{array}{ccc|ccc} 1 & 3 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1/4 & 1/2 & -1/4 & 0 \\ 0 & -3 & -1 & -2 & 0 & 1 \end{array}\right]$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 1/4 & -\frac{1}{2} & -3/4 & 0 \\ 0 & 1 & 1/4 & 1/2 & -1/4 & 0 \\ 0 & 0 & -1/4 & -1/2 & -3/4 & 1 \end{array}\right]$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -1 & 0 & 1 \\ 0 & 1 & 0 & 0 & -1 & +1 \\ 0 & 0 & 1 & +2 & +3 & -4 \end{array}\right]$$

$X = A^{-1}B$

$$x = \begin{bmatrix} -1 & 0 & 1 \\ 0 & -1 & 1 \\ 2 & 3 & -4 \end{bmatrix} \begin{bmatrix} 4 \\ -1 \\ 3 \end{bmatrix}$$

$$X = 4\begin{bmatrix} -1 \\ 0 \\ 2 \end{bmatrix} - 1\begin{bmatrix} 0 \\ -1 \\ 3 \end{bmatrix} + 3\begin{bmatrix} 1 \\ 1 \\ -4 \end{bmatrix}$$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -1 \\ 4 \\ -7 \end{bmatrix}$$

$x_1 = -1, \quad x_2 = 4, \quad x_3 = -7$

Q4

$$5x_1 + 3x_2 + 2x_3 = 4$$
$$3x_1 + 3x_2 + 2x_3 = 2$$
$$x_2 + x_3 = 5$$

$$\begin{bmatrix} 5 & 3 & 2 \\ 3 & 3 & 2 \\ 0 & 1 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 4 \\ 2 \\ 5 \end{bmatrix}$$

$$AX = B$$
$$X = A^{-1}B.$$

Finding A^{-1}

$$\left[\begin{array}{ccc|ccc} 5 & 3 & 2 & 1 & 0 & 0 \\ 3 & 3 & 2 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 \end{array}\right]$$

$$\left[\begin{array}{cccccc} 1 & 3/5 & 2/5 & 1/5 & 0 & 0 \\ 0 & 6/5 & 4/5 & -3/5 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 \end{array}\right]$$

$$\left[\begin{array}{cccccc} 1 & 0 & 0 & +\frac{1}{2} & -\frac{1}{2} & 0 \\ 0 & 1 & 2/3 & -\frac{1}{2} & \frac{5}{6} & 0 \\ 0 & 0 & 1/3 & 1/2 & -5/6 & 1 \end{array}\right]$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & +\frac{1}{2} & -\frac{1}{2} & 0 \\ 0 & 1 & 0 & -3/2 & 5/2 & -2 \\ 0 & 0 & 1 & 3/2 & -5/2 & 3 \end{array}\right]$$

$$X = A^{-1}B$$
$$= \begin{bmatrix} 1/2 & -1/2 & 0 \\ -3/2 & 5/2 & -2 \\ 3/2 & -5/2 & 3 \end{bmatrix} \begin{bmatrix} 4 \\ 2 \\ 5 \end{bmatrix}$$

$$X = 4\begin{bmatrix} 1/2 \\ -3/2 \\ 3/2 \end{bmatrix} + 2\begin{bmatrix} -1/2 \\ 5/2 \\ -5/2 \end{bmatrix} + 5\begin{bmatrix} 0 \\ -2 \\ 3 \end{bmatrix}$$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 \\ -11 \\ 16 \end{bmatrix}$$

$x_1 = 1$, $x_2 = -11$, $x_3 = 16$

Q5

$x + y + z = 5$
$x + y - 4z = 10$
$-4x + y + z = 0$

$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & -4 \\ -4 & 1 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 5 \\ 10 \\ 0 \end{bmatrix}$$

$AX = B$

$X = A^{-1}B$

Finding A^{-1} :-

$$\left[\begin{array}{ccc|ccc} 1 & 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & -4 & 0 & 1 & 0 \\ -4 & 1 & 1 & 0 & 0 & 1 \end{array}\right]$$

$$\left[\begin{array}{ccc|ccc} 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & -5 & -1 & 1 & 0 \\ 0 & 5 & 5 & 4 & 0 & 1 \end{array}\right]$$

$$\left[\begin{array}{ccc|ccc} 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 5 & 5 & 4 & 0 & 1 \\ 0 & 0 & -5 & -1 & 1 & 0 \end{array}\right]$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1/5 & 0 & -1/5 \\ 0 & 1 & 1 & 4/5 & 0 & 1/5 \\ 0 & 0 & -5 & -1 & 1 & 0 \end{array}\right]$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1/5 & 0 & -1/5 \\ 0 & 1 & 0 & 3/5 & 1/5 & 1/5 \\ 0 & 0 & 1 & 1/5 & -1/5 & 0 \end{array}\right]$$

$X = A^{-1}B$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1/5 & 0 & -1/5 \\ 3/5 & 1/5 & 1/5 \\ 1/5 & -1/5 & 0 \end{bmatrix}\begin{bmatrix} 5 \\ 10 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = 5\begin{bmatrix} 1/5 \\ 3/5 \\ 1/5 \end{bmatrix} + 10\begin{bmatrix} 0 \\ 1/5 \\ -1/5 \end{bmatrix} + 0$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 5 \\ -1 \end{bmatrix}$$

$x = 1,\ y = 5,\ z = -1$

Q6

$$-x - 2y - 3z = 0$$
$$w + x + 4y + 4z = \cancel{0}\,7$$
$$w + 3x + 7y + 9z = 4$$
$$-w - 2x - 4y - 6z = 6$$

$$\begin{bmatrix} 0 & -1 & -2 & -3 \\ 1 & 1 & 4 & 4 \\ 1 & 3 & 7 & 9 \\ -1 & -2 & -4 & -6 \end{bmatrix} \begin{bmatrix} w \\ x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 7 \\ 4 \\ 6 \end{bmatrix}$$

$$AX = B$$
$$X = A^{-1}B$$

Finding A^{-1}:

$$\left[\begin{array}{cccc|cccc} 0 & -1 & -2 & -3 & 1 & 0 & 0 & 0 \\ 1 & 1 & 4 & 4 & 0 & 1 & 0 & 0 \\ 1 & 3 & 7 & 9 & 0 & 0 & 1 & 0 \\ -1 & -2 & -4 & -6 & 0 & 0 & 0 & 1 \end{array}\right]$$

$$\left[\begin{array}{cccc|cccc} 1 & 1 & +4 & 4 & 0 & 1 & 0 & 0 \\ 0 & -1 & -2 & -3 & 1 & 0 & 0 & 0 \\ 1 & 3 & 7 & 9 & 0 & 0 & 1 & 0 \\ -1 & -2 & -4 & -6 & 0 & 0 & 0 & 1 \end{array}\right]$$

$$\left[\begin{array}{cccc|cccc} 1 & 1 & 4 & 4 & 0 & 1 & 0 & 0 \\ 0 & -1 & -2 & -3 & 1 & 0 & 0 & 0 \\ 0 & 2 & 3 & 5 & 0 & -1 & 1 & 0 \\ 0 & -1 & 0 & -2 & 0 & 1 & 0 & +1 \end{array}\right]$$

$$\left[\begin{array}{cccc|cccc} 1 & 0 & 2 & 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 2 & 3 & -1 & 0 & 0 & 0 \\ 0 & 0 & -1 & -1 & 2 & -1 & 1 & 0 \\ 0 & 0 & 2 & 1 & -1 & 1 & 0 & 1 \end{array}\right]$$

$$\left[\begin{array}{cccc|cccc} 1 & 0 & 0 & -3 & -3 & -1 & -2 & 0 \\ 0 & 1 & 0 & 5 & -5 & -2 & -2 & 0 \\ 0 & 0 & 1 & \cancel{+1}\,-1 & -2 & 1 & +1 & 0 \\ 0 & 0 & 0 & +3 & -5 & -1 & -2 & 1 \end{array}\right]$$

$$\left[\begin{array}{cccc|cccc} 1 & 0 & 0 & 0 & 2 & 0 & 0 & -1 \\ 0 & 1 & 0 & 0 & 10/3 & -1/3 & 4/3 & -5/3 \\ 0 & 0 & 1 & 0 & -11/3 & 2/3 & 1/3 & 1/3 \\ 0 & 0 & 0 & 1 & -5/3 & -1/3 & -2/3 & 1/3 \end{array}\right]$$

$$X = A^{-1}B$$

$$\begin{bmatrix} w \\ x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 & 0 & 0 & -1 \\ 10/3 & -1/3 & 4/3 & -5/3 \\ -11/3 & 2/3 & 1/3 & 1/3 \\ -5/3 & -1/3 & -2/3 & 1/3 \end{bmatrix} \begin{bmatrix} 0 \\ 7 \\ 4 \\ 6 \end{bmatrix}$$

$$\begin{bmatrix} w \\ x \\ y \\ z \end{bmatrix} = 0 + 7\begin{bmatrix} 0 \\ -1/3 \\ 2/3 \\ -1/3 \end{bmatrix} + 4\begin{bmatrix} [illegible] \\ 4/3 \\ 1/3 \\ -2/3 \end{bmatrix} + 6\begin{bmatrix} -1 \\ -5/3 \\ 1/3 \\ 1/3 \end{bmatrix}$$

$$\begin{bmatrix} w \\ x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -6 \\ -202/9 \\ 8 \\ -3 \end{bmatrix}$$

$w = -6 \quad x = -202/9 \quad y = 8 \quad z = -3$

Q7

$$3x_1 + 5x_2 = b_1$$
$$x_1 + 2x = b_2$$

$$\begin{bmatrix} 3 & 5 \\ 1 & 2 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \end{bmatrix}$$

$AX = B$

$X = A^{-1}B$

Finding A^{-1}

$$\cancel{\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}} = \left[\begin{array}{cc|cc} 3 & 5 & 1 & 0 \\ 1 & 2 & 0 & 1 \end{array}\right]$$

$$\left[\begin{array}{cc|cc} 1 & 5/3 & 1/3 & 0 \\ 0 & 1/3 & -1/3 & 1 \end{array}\right]$$

$$\left[\begin{array}{cc|cc} 1 & 0 & 2 & -5 \\ 0 & 1 & -1 & 3 \end{array}\right]$$

$X = A^{-1}B$

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 2 & -5 \\ -1 & 3 \end{bmatrix}\begin{bmatrix} b_1 \\ b_2 \end{bmatrix}$$

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 2b_1 - 5b_2 \\ 3b_2 - b_1 \end{bmatrix}$$

$x_1 = 2b_1 - 5b_2 \qquad x_2 = 3b_2 - b_1$

Q8

$$x_1 + 2x_2 + 3x_3 = b_1$$
$$2x_1 + 5x_2 + 5x_3 = b_2$$
$$3x_1 + 5x_2 + 8x_3 = b_3$$

$$\begin{bmatrix} 1 & 2 & 3 \\ 2 & 5 & 5 \\ 3 & 5 & 8 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$$

$AX = B$

$$X = A^{-1} B$$

Finding A^{-1}

$$\left[\begin{array}{ccc|ccc} 1 & 2 & 3 & 1 & 0 & 0 \\ 2 & 5 & 5 & 0 & 1 & 0 \\ 3 & 5 & 8 & 0 & 0 & 1 \end{array}\right]$$

$$\begin{bmatrix} 1 & 2 & 3 & 1 & 0 & 0 \\ 0 & 1 & -1 & -2 & 1 & 0 \\ 0 & -4 & -1 & -3 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 5 & 5 & -2 & 0 \\ 0 & 1 & -1 & -2 & 1 & 0 \\ 0 & 0 & -2 & -5 & 4 & 1 \end{bmatrix}$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -15/2 & 1/2 & 5/2 \\ 0 & 1 & 0 & 1/2 & 1/2 & -1/2 \\ 0 & 0 & 1 & 5/2 & -1/2 & -1/2 \end{array}\right]$$

$$X = A^{-1} B$$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -15/2 & 1/2 & 5/2 \\ 1/2 & 1/2 & -1/2 \\ 5/2 & -1/2 & -1/2 \end{bmatrix} \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$$

$$x_1 = \frac{1}{2}(-15b_1 + b_2 + 5b_3), \quad x_2 = \frac{1}{2}(b_1 + b_2 - b_3), \quad x_3 = \frac{1}{2}(5b_1 - b_2 - b_3)$$

Exercise 1.7

Assignment # 06

41

Date

Q1

a) $\begin{bmatrix} 2 & 1 \\ 0 & 3 \end{bmatrix}$

upper triangular & invertible

b) $\begin{bmatrix} 0 & 0 \\ 4 & 0 \end{bmatrix}$

lower triangular & not invertible

c) $\begin{bmatrix} -1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1/5 \end{bmatrix}$

diagonal, upper triangular, lower triangular and invertible

d) $\begin{bmatrix} 3 & -2 & 7 \\ 0 & 0 & 3 \\ 0 & 0 & 8 \end{bmatrix}$

upper triangular & non invertible

Q7

$$A = \begin{bmatrix} 1 & 0 \\ 0 & -2 \end{bmatrix}$$

$$A^2 = \begin{bmatrix} (1)^2 & 0^2 \\ 0^2 & (-2)^2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 4 \end{bmatrix}$$

$$A^{-2} = \begin{bmatrix} (1)^{-2} & (0)^{-2} \\ (0)^{-2} & (-2)^{-2} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & \frac{1}{4} \end{bmatrix}$$

$$A^{-k} = \begin{bmatrix} (1)^{-k} & (0)^{-k} \\ (0)^{-k} & (-2)^{-k} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & \frac{1}{(-2)^k} \end{bmatrix}$$

Q8

$$A = \begin{bmatrix} -6 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 5 \end{bmatrix}$$

$$A^2 = \begin{bmatrix} (-6)^2 & 0^2 & 0^2 \\ 0^2 & (3)^2 & 0^2 \\ 0^2 & 0^2 & (5)^2 \end{bmatrix} = \begin{bmatrix} 36 & 0 & 0 \\ 0 & 9 & 0 \\ 0 & 0 & 25 \end{bmatrix}$$

$$A^{-2} = \begin{bmatrix} (-6)^{-2} & (0)^{-2} & (0)^{-2} \\ (0)^{-2} & (3)^{-2} & (0)^{-2} \\ (0)^{-2} & (0)^{-2} & (5)^{-2} \end{bmatrix} = \begin{bmatrix} 1/36 & 0 & 0 \\ 0 & 1/9 & 0 \\ 0 & 0 & 1/25 \end{bmatrix}$$

$$A^{-k} = \begin{bmatrix} (-6)^{-k} & (0)^{-k} & (0)^{-k} \\ (0)^{-k} & (3)^{-k} & (0)^{-k} \\ (0)^{-k} & (0)^{-k} & (5)^{-k} \end{bmatrix} \Rightarrow \begin{bmatrix} \frac{1}{(-6)^k} & 0 & 0 \\ 0 & \frac{1}{(3)^k} & 0 \\ 0 & 0 & \frac{1}{(5)^k} \end{bmatrix}$$

Q9

$$A = \begin{bmatrix} \frac{1}{2} & 0 & 0 \\ 0 & \frac{1}{3} & 0 \\ 0 & 0 & \frac{1}{4} \end{bmatrix}$$

$$A^2 = \begin{bmatrix} (\frac{1}{2})^2 & (0)^2 & (0)^2 \\ (0)^2 & (\frac{1}{3})^2 & (0)^2 \\ (0)^2 & (0)^2 & (\frac{1}{4})^2 \end{bmatrix} \Rightarrow \begin{bmatrix} \frac{1}{4} & 0 & 0 \\ 0 & \frac{1}{9} & 0 \\ 0 & 0 & \frac{1}{16} \end{bmatrix}$$

$$A^{-2} = \begin{bmatrix} (\frac{1}{2})^{-2} & (0)^{-2} & (0)^{-2} \\ (0)^{-2} & (\frac{1}{3})^{-2} & (0)^{-2} \\ (0)^{-2} & (0)^{-2} & (\frac{1}{4})^{-2} \end{bmatrix} \Rightarrow \begin{bmatrix} 4 & 0 & 0 \\ 0 & 9 & 0 \\ 0 & 0 & 16 \end{bmatrix}$$

$$A^{-k} = \begin{bmatrix} (\frac{1}{2})^{-k} & (0)^{-k} & (0)^{-k} \\ (0)^{-k} & (\frac{1}{3})^{-k} & (0)^{-k} \\ (0)^{-k} & (0)^{-k} & (\frac{1}{4})^{-k} \end{bmatrix} \Rightarrow \begin{bmatrix} (2)^k & 0 & 0 \\ 0 & (3)^k & 0 \\ 0 & 0 & (4)^k \end{bmatrix}$$

Q10

$$A = \begin{bmatrix} -2 & 0 & 0 & 0 \\ 0 & -4 & 0 & 0 \\ 0 & 0 & -3 & 0 \\ 0 & 0 & 0 & 2 \end{bmatrix}$$

$$A^2 = \begin{bmatrix} (-2)^2 & 0^2 & 0^2 & 0^2 \\ 0^2 & (-4)^2 & 0^2 & 0^2 \\ 0^2 & 0^2 & (-3)^2 & 0^2 \\ 0^2 & 0^2 & 0^2 & (2)^2 \end{bmatrix} = \begin{bmatrix} 4 & 0 & 0 & 0 \\ 0 & 16 & 0 & 0 \\ 0 & 0 & 9 & 0 \\ 0 & 0 & 0 & 4 \end{bmatrix}$$

$$A^{-2} = \begin{bmatrix} (-2)^{-2} & 0^{-2} & 0^{-2} & 0^{-2} \\ 0^{-2} & (-4)^{-2} & 0^{-2} & 0^{-2} \\ 0^{-2} & 0^{-2} & (-3)^{-2} & 0^{-2} \\ 0^{-2} & 0^{-2} & 0^{-2} & (2)^{-2} \end{bmatrix} \Rightarrow \begin{bmatrix} \frac{1}{4} & 0 & 0 & 0 \\ 0 & \frac{1}{16} & 0 & 0 \\ 0 & 0 & \frac{1}{9} & 0 \\ 0 & 0 & 0 & \frac{1}{4} \end{bmatrix}$$

$$A^{-k} = \begin{bmatrix} (-2)^{-k} & 0^{-k} & 0^{-k} & 0^{-k} \\ 0^{-k} & (-4)^{-k} & 0^{-k} & 0^{-k} \\ 0^{-k} & 0^{-k} & (-3)^{-k} & 0^{-k} \\ 0^{-k} & 0^{-k} & 0^{-k} & (2)^{-k} \end{bmatrix} \Rightarrow \begin{bmatrix} \frac{1}{(-2)^k} & 0 & 0 & 0 \\ 0 & \frac{1}{(-4)^k} & 0 & 0 \\ 0 & 0 & \frac{1}{(-3)^k} & 0 \\ 0 & 0 & 0 & \frac{1}{(2)^k} \end{bmatrix}$$

Ex 2.1 – 2.3

Assignment # 07

45

EXERCISE 2.1

Q1

$$A = \begin{bmatrix} 1 & -2 & 3 \\ 6 & 7 & -1 \\ -3 & 1 & 4 \end{bmatrix}$$

For a_{11}

$$M_{11} = \begin{vmatrix} 7 & -1 \\ 1 & 4 \end{vmatrix}$$

$$\boxed{M_{11} = 29}$$

$$C_{11} = (-1)^{1+1} M_{11}$$

$$\boxed{C_{11} = 29}$$

For a_{12}

$$M_{12} = \begin{vmatrix} 6 & -1 \\ -3 & 4 \end{vmatrix}$$

$$\boxed{M_{12} = 21}$$

$$C_{12} = (-1)^{1+2} M_{12}$$

$$\boxed{C_{12} = -21}$$

For a_{13} :-

$$M_{13} = \cancel{(-1)^{1+3}} \begin{vmatrix} 6 & -3 \\ 7 & 1 \end{vmatrix}$$

$\boxed{M_{13} = 27}$

$C_{13} = (-1)^{1+3} M_{13}$

$\boxed{C_{13} = 27}$

For a_{21} :-

$$M_{21} = \begin{vmatrix} -2 & 3 \\ 1 & 4 \end{vmatrix}$$

$\boxed{M_{21} = -11}$

$C_{21} = (-1)^{2+1} M_{21}$

$\boxed{C_{21} = +11}$

For a_{22} :-

$$M_{22} = \begin{vmatrix} 1 & 3 \\ -3 & 4 \end{vmatrix}$$

$M_{22} = 13$

$C_{22} = (-1)^{2+2} M_{22}$

$\boxed{C_{22} = 13}$

For a_{23} :-

$$M_{23} = \begin{vmatrix} 1 & -2 \\ -3 & 1 \end{vmatrix}$$

$\boxed{M_{23} = -5}$

$C_{23} = (-1)^{2+3} M_{23}$

$\boxed{C_{23} = 5}$

For a_{31} :-

$$M_{31} = \begin{vmatrix} -2 & 3 \\ 7 & -1 \end{vmatrix}$$

$\boxed{M_{31} = -19}$

$C_{31} = (-1)^{3+1} M_{31}$

$\boxed{C_{31} = -19}$

For a_{32} :-

$$M_{32} = \begin{vmatrix} 1 & 3 \\ 6 & -1 \end{vmatrix}$$

$M_{32} = -19$

$C_{32} = (-1)^{3+2} M_{32}$

$\boxed{C_{32} = 19}$

For a_{33} :-

$$M_{33} = \begin{vmatrix} 1 & -2 \\ 6 & 7 \end{vmatrix}$$

$\boxed{M_{33} = 19}$

$C_{33} = (-1)^{3+3} M_{33}$

$\boxed{C_{33} = 19}$

Q5

→ let $A = \begin{bmatrix} 3 & 5 \\ -2 & 4 \end{bmatrix}$

$\det(A) = \begin{vmatrix} 3 & 5 \\ -2 & 4 \end{vmatrix}$

$\boxed{\det(A) = 22}$

$A^{-1} = \dfrac{1}{\det(A)} \begin{vmatrix} 4 & -5 \\ 2 & 3 \end{vmatrix}$

$= \dfrac{1}{22} \begin{vmatrix} 4 & -5 \\ 2 & 3 \end{vmatrix}$

$\boxed{A^{-1} = \begin{vmatrix} 4/22 & -5/22 \\ 1/11 & -3/22 \end{vmatrix}}$

Q10

Let $A = \begin{vmatrix} -2 & 7 & 6 \\ 5 & 1 & -2 \\ 3 & 8 & 4 \end{vmatrix}$

$A = \left| \begin{array}{ccc|cc} -2 & 7 & 6 & -2 & 7 \\ 5 & 1 & -2 & 5 & 1 \\ 3 & 8 & 4 & 3 & 8 \end{array} \right|$

$\det(A) = [(-2)(1)(4) + (7)(-2)(+3) + (6)(5)(8)] -$
$[(6)(1)(3) + (-2)(-2)(8) + (7)(5)(4)]$

$\det(A) = 0$

Q11

Let $A = \begin{vmatrix} -2 & 1 & 4 \\ 3 & 5 & -7 \\ 1 & 6 & 2 \end{vmatrix}$

$A = \left| \begin{array}{ccc|cc} -2 & 1 & 4 & -2 & 1 \\ 3 & 5 & -7 & 3 & 5 \\ 1 & 6 & 2 & 1 & 6 \end{array} \right|$

$\det(A) = [(-2)(5)(2) + (1)(-7)(1) + (4)(3)(6)] - [(4)(5)(1)$
$+ (-2)(-7)(6) + (1)(3)(2)]$

$d = 45 - 110$

$d = -65$

Q13

let $A = \begin{vmatrix} 3 & 0 & 0 \\ 2 & -1 & 5 \\ 1 & 9 & -4 \end{vmatrix}$

$$A = \left|\begin{array}{ccc|cc} 3 & 0 & 0 & 3 & 0 \\ 2 & -1 & 5 & 2 & -1 \\ 1 & 9 & -4 & 1 & 9 \end{array}\right|$$

$= [(3)(-1)(-4) + (0)(5)(1) + (0)(2)(9)] - [0(-1)(1) + (3)(5)(9) + 0]$

$d = -123$

Q15

let $A = \begin{bmatrix} \lambda-2 & 1 \\ -5 & \lambda+4 \end{bmatrix}$

$\det(A) = \begin{vmatrix} \lambda-2 & 1 \\ -5 & \lambda+4 \end{vmatrix} = 0$

$(\lambda-2)(\lambda+4) + 5 = 0$

$\lambda^2 + 4\lambda - 2\lambda - 8 + 5 = 0$

$\lambda^2 + 2\lambda - 3 = 0$

$\boxed{\lambda = 1}$ $\boxed{\lambda = -3}$

Q19

a) by First row

$d = a_{11}C_{11} + a_{12}C_{12} + a_{13}C_{13}$

$= 3C_{11} + 0C_{12} + 0C_{13}$

$d = 3C_{11}$

$d = 3 \times (-1)^{1+1} \begin{vmatrix} -1 & 5 \\ 9 & -4 \end{vmatrix}$

$\boxed{d = -123}$

b) the first column

$d = a_{11}C_{11} + a_{21}C_{21} + a_{31}C_{31}$

$= 3(-1)^{1+1}\begin{vmatrix} -1 & 5 \\ 9 & -4 \end{vmatrix} + 2(-1)^{2+1}\begin{vmatrix} 0 & 0 \\ 9 & -4 \end{vmatrix} + 1(-1)^{3+1}\begin{vmatrix} 0 & 0 \\ -1 & 5 \end{vmatrix}$

$\boxed{d = -123}$

c) the second row

$d = a_{21}C_{21} + a_{22}C_{22} + a_{23}C_{23}$

$d = 2(-1)^{2+1}\begin{vmatrix} 0 & 0 \\ 9 & -4 \end{vmatrix} + (-1)(-1)^{2+2}\begin{vmatrix} 3 & 0 \\ 1 & -4 \end{vmatrix} + 5(-1)^{2+3}\begin{vmatrix} 3 & 0 \\ 1 & 9 \end{vmatrix}$

$d = -2(0) - 1(-12) - 5(27)$

$\boxed{d = -123}$

Date

d) the second column

$$d = a_{12}C_{12} + a_{22}C_{22} + a_{32}C_{32}$$

$$= 0C_{12} + (-1)(-1)^{2+2}\begin{vmatrix} 3 & 0 \\ 1 & -4 \end{vmatrix} + 9(-1)^{3+2}\begin{vmatrix} 3 & 0 \\ 2 & 5 \end{vmatrix}$$

$d = 12 - 135$

$\boxed{d = -123}$

e) the third row

$$d = a_{31}C_{31} + a_{32}C_{32} + a_{33}C_{33}$$

$$= 1(-1)^{3+1}\begin{vmatrix} 0 & 0 \\ -1 & 5 \end{vmatrix} + 9(-1)^{3+2}\begin{vmatrix} 3 & 0 \\ 2 & 5 \end{vmatrix} + (-4)(-1)^{3+3}\begin{vmatrix} 3 & 0 \\ 2 & -1 \end{vmatrix}$$

$d = 0 - 135 + 12$

$\boxed{d = -123}$

f) the third column.

$$d = a_{13}C_{13} + a_{23}C_{23} + a_{33}C_{33}$$

$$= 0C_{13} + 5(-1)^{2+3}\begin{vmatrix} 3 & 0 \\ 1 & 9 \end{vmatrix} - 4(-1)^{3+3}\begin{vmatrix} 3 & 0 \\ 2 & -1 \end{vmatrix}$$

$= -135 + 12$

$\boxed{d = -123}$

Date

Exercise 2.2

Q 11

$$A = \begin{bmatrix} 2 & 1 & 3 & 1 \\ 1 & 0 & 1 & 1 \\ 0 & 2 & 1 & 0 \\ 0 & 1 & 2 & 3 \end{bmatrix}$$

$$A = (-1)\begin{vmatrix} 1 & 0 & 1 & 1 \\ 2 & 1 & 3 & 1 \\ 0 & 2 & 1 & 0 \\ 0 & 1 & 2 & 3 \end{vmatrix}$$

$R_2 = R_2 - 2R_1$

$$A = (-1)\begin{vmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & -1 \\ 0 & 2 & 1 & 0 \\ 0 & 1 & 2 & 3 \end{vmatrix}$$

$R_3 = R_3 - 2R_2$ $\qquad$ $R_4 = R_4 - R_2$

$$A = (-1)\begin{vmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & -1 \\ 0 & 0 & -1 & 2 \\ 0 & 0 & 1 & 4 \end{vmatrix}$$

$R_3 = R_3$

$$A = \begin{vmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & -1 \\ 0 & 0 & 1 & 4 \\ 0 & 0 & -1 & 2 \end{vmatrix}$$

(-1)(-1)

$R_4 = R_4 + R_3$

$$A = \begin{vmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & -1 \\ 0 & 0 & 1 & 4 \\ 0 & 0 & 0 & 6 \end{vmatrix}$$

$\det(A) = (1)(1)(1)(6)$

$\boxed{\det(A) = 6}$

Q13

let $A = \begin{bmatrix} 1 & 3 & 1 & 5 & 3 \\ -2 & -7 & 0 & -4 & 2 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 2 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 \end{bmatrix}$

$R_2 = R_2 + 2R_1$

$$A = \begin{bmatrix} 1 & 3 & 1 & 5 & 3 \\ 0 & -1 & 2 & 6 & 8 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 2 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 \end{bmatrix}$$

~~$R_5 = R_5 -$~~ $R_4 = R_4 - 2R_3$

$$A = \begin{bmatrix} 1 & +3 & 1 & 5 & 3 \\ 0 & -1 & 2 & 6 & 8 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 1 & 1 \end{bmatrix}$$

$R_5 = R_5 - R_4$

$$A = \begin{bmatrix} 1 & 3 & 1 & 5 & 3 \\ 0 & -1 & 2 & 6 & 8 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 & 2 \end{bmatrix}$$

$\det(A) = (1)(-1)(1)(1)(2)$

$\boxed{\det(A) = -2}$

Date

Q15

$$\det(A) = \begin{vmatrix} d & e & f \\ g & h & i \\ a & b & c \end{vmatrix}$$

$$\det(A) = (-1)\begin{vmatrix} d & e & f \\ a & b & c \\ g & h & i \end{vmatrix}$$

$$= (-1)(-1)\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix}$$

$$\det(A) = (-1)(-1)(6)$$

$$\det(A) = -6$$

Q17

$$= (3)(4)(-1)\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix}$$

$$= (3)(4)(-1)(-6)$$

$$\boxed{d = 72}$$

Exercise 2.3

Q7

$$A = \begin{bmatrix} 2 & 5 & 5 \\ -1 & -1 & 0 \\ 2 & 4 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 5 & 5 \\ -1 & -1 & 0 \\ 2 & 4 & 3 \end{bmatrix} \begin{matrix} 2 & 5 \\ -1 & -1 \\ 2 & 4 \end{matrix}$$

$\det(A) = [-6 + 0 - 20] - [-10 + 0 - 15]$

$\det(A) = -1$

Matrix is invertible

~~Q8~~ Q9

$$A = \begin{bmatrix} 2 & -3 & 5 \\ 0 & 1 & -3 \\ 0 & 0 & 2 \end{bmatrix}$$

$$A = \begin{bmatrix} 2 & -3 & 5 \\ 0 & 1 & -3 \\ 0 & 0 & 2 \end{bmatrix} \begin{matrix} 2 & -3 \\ 0 & 1 \\ 0 & 0 \end{matrix}$$

$\det(A) = 4 - 0$

$\boxed{\det(A) = 4}$

Matrix is invertible

Q11

$$A = \begin{bmatrix} 4 & 2 & 8 \\ -2 & 1 & -4 \\ 3 & 1 & 6 \end{bmatrix}$$

$$\begin{bmatrix} 4 & 2 & 8 \\ -2 & 1 & -4 \\ 3 & 1 & 6 \end{bmatrix} \begin{matrix} 4 & 2 \\ -2 & 1 \\ 3 & 1 \end{matrix}$$

$\det(A) = [(4)(1)(6) + (2)(-4)(3) + (8)(-2)(1)] - [8(1)(3) + 4(-4)(1) + 2(-2)(6)]$

$= [-16] - [-16]$

$\det(A) = 0$

Matrix is non-invertible

Q19

$$A = \begin{bmatrix} 2 & 5 & 5 \\ -1 & -1 & 0 \\ 2 & 4 & 3 \end{bmatrix}$$

$$\boxed{A^{-1} = \frac{1}{\det(A)} \operatorname{Adj}(A)}$$

For $\det(A)$:

$$\det(A) = \begin{bmatrix} 2 & 5 & 5 \\ -1 & -1 & 0 \\ 2 & 4 & 3 \end{bmatrix} \begin{matrix} 2 & 5 \\ -1 & -1 \\ 2 & 4 \end{matrix}$$

$\det(A) = [-6 + 0 - 20] - [-10 + 0 - 15]$

$\det(A) = -1$

For $\operatorname{Adj}(A)$

$$\operatorname{Adj}(A) = \begin{bmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{bmatrix}$$

$C_{11} = (-1)^{1+1} \begin{vmatrix} -1 & 0 \\ 4 & 3 \end{vmatrix}$

$\boxed{C_{11} = -3}$

$C_{12} = (-1)^{1+2} \begin{vmatrix} -1 & 0 \\ 2 & 3 \end{vmatrix}$

$\boxed{C_{12} = 3}$

$C_{13} = (-1)^{1+3} \begin{vmatrix} -1 & -1 \\ 2 & 4 \end{vmatrix}$

$\boxed{C_{13} = -2}$

$C_{21} = (-1)^{2+1} \begin{vmatrix} 5 & 5 \\ 4 & 3 \end{vmatrix}$

$\boxed{C_{21} = 5}$

$C_{22} = (-1)^{2+2} \begin{vmatrix} 2 & 5 \\ 2 & 3 \end{vmatrix}$

$\boxed{C_{22} = -4}$

$C_{23} = (-1)^{2+3} \begin{vmatrix} 2 & 5 \\ 2 & 4 \end{vmatrix}$

$\boxed{C_{23} = 2}$

$C_{31} = (-1)^{3+1} \begin{vmatrix} 5 & 5 \\ -1 & 0 \end{vmatrix}$

$= (-1)^4 (5)$

$\boxed{C_{31} = 5}$

$C_{32} = (-1)^{3+2} \begin{vmatrix} 2 & 5 \\ -1 & 0 \end{vmatrix}$

$\boxed{C_{32} = -5}$

$C_{33} = (-1)^{3+3} \begin{vmatrix} 2 & 5 \\ -1 & -1 \end{vmatrix}$

$\boxed{C_{33} = 3}$

$\cancel{A^{-1} = \frac{1}{-1} \begin{bmatrix} -3 & 5 & -2 \\ 5 & -4 & 2 \\ 5 & -5 & 3 \end{bmatrix}}$ $A^{-1} = \frac{1}{-1} \begin{bmatrix} -3 & 5 & 5 \\ 3 & -4 & -5 \\ -2 & 2 & 3 \end{bmatrix}$

$\boxed{A^{-1} = \begin{bmatrix} 3 & -3 & 2 \\ -5 & 4 & -2 \\ -5 & 5 & -3 \end{bmatrix}}$ $\boxed{A^{-1} = \begin{bmatrix} 3 & -5 & -5 \\ -3 & 4 & 5 \\ 2 & -2 & -3 \end{bmatrix}}$

Q21

$A = \begin{bmatrix} 2 & -3 & 5 \\ 0 & 1 & -3 \\ 0 & 0 & 2 \end{bmatrix}$ f.

For det (A):-

$\det(A) \begin{bmatrix} 2 & -3 & 5 \\ 0 & 1 & -3 \\ 0 & 0 & 2 \end{bmatrix} = 2 \begin{vmatrix} 1 & -3 \\ 0 & 2 \end{vmatrix} - 0 + 0$

$\det(A) = 4$

For Adj(A) :-

$Adj(A) = \begin{bmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{bmatrix}$

$$C_{11} = (-1)^{1+1} \begin{vmatrix} \cancel{0}1 & -3 \\ 0 & 2 \end{vmatrix}$$

$$\boxed{C_{11} = 2}$$

$$C_{22} = (-1)^{1+2} \begin{vmatrix} 0 & -3 \\ 0 & 2 \end{vmatrix}$$

$$\boxed{C_{12} = 0}$$

$$C_{13} = (-1)^{1+3} \begin{vmatrix} 0 & 1 \\ 0 & 0 \end{vmatrix}$$

$$\boxed{C_{13} = 0}$$

$$C_{21} = (-1)^{2+1} \begin{vmatrix} -3 & 5 \\ 0 & 2 \end{vmatrix}$$

$$\boxed{C_{21} = 6}$$

$$C_{22} = (-1)^{2+2} \begin{vmatrix} 2 & 5 \\ 0 & 2 \end{vmatrix}$$

$$\boxed{C_{22} = 4}$$

$$C_{23} = (-1)^{2+3} \begin{vmatrix} 2 & -3 \\ 0 & 0 \end{vmatrix}$$

$$\boxed{C_{23} = 0}$$

$$C_{31} = (-1)^{3+1} \begin{vmatrix} -3 & 5 \\ 1 & -3 \end{vmatrix}$$

$$\boxed{C_{31} = 4}$$

$$C_{32} = (-1)^{3+2} \begin{vmatrix} 2 & 5 \\ 0 & -3 \end{vmatrix}$$

$$\boxed{C_{32} = 6}$$

$$C_{33} = (-1)^{3+3} \begin{vmatrix} 2 & -3 \\ 0 & 1 \end{vmatrix}$$

$$\boxed{C_{33} = 2}$$

$$\text{Adj } A = \begin{bmatrix} 2 & 0 & 0 \\ 6 & 4 & 0 \\ 4 & 6 & 2 \end{bmatrix}^t = \begin{bmatrix} 2 & 6 & 4 \\ 0 & 4 & 6 \\ 0 & 0 & 2 \end{bmatrix}$$

$$A^{-1} = \frac{1}{\det(A)} \text{Adj}(A)$$

$$A^{-1} = \frac{1}{4} \begin{bmatrix} 2 & 6 & 4 \\ 0 & 4 & 6 \\ 0 & 0 & 2 \end{bmatrix}$$

$$\boxed{A^{-1} = \begin{bmatrix} \frac{1}{2} & 3/2 & 1 \\ 0 & 1 & 3/2 \\ 0 & 0 & 1/2 \end{bmatrix}}$$

Q27

$$x - 3x_2 + x_3 = 4$$
$$2x_1 - x_2 = -2$$
$$4x_1 - 3x_3 = 0$$

$$D = \begin{vmatrix} 1 & -3 & 1 \\ 2 & -1 & 0 \\ 4 & 0 & -3 \end{vmatrix}$$

$$= \begin{vmatrix} 2 & -1 \\ 4 & 0 \end{vmatrix} \cancel{70} + (-3) \begin{vmatrix} 1 & -3 \\ 2 & -1 \end{vmatrix}$$

$$= 4 - 15$$

$$\boxed{D = -11}$$

$$Dx = \begin{vmatrix} 4 & -3 & 1 \\ -2 & -1 & 0 \\ 0 & 0 & -3 \end{vmatrix}$$

$$Dx = -3 \begin{vmatrix} 4 & -3 \\ -2 & -1 \end{vmatrix}$$

$$\boxed{Dx_1 = 30}$$

Q29

$$3x_1 - x_2 + x_3 = 4$$
$$-x_1 + 7x_2 - 2x_3 = 1$$
$$2x_1 + 6x_2 - x_3 = 5$$

$$D = \begin{vmatrix} 3 & -1 & 1 \\ -1 & 7 & -2 \\ 2 & 6 & -1 \end{vmatrix}$$

$$= 3 \begin{vmatrix} 7 & -2 \\ 6 & -1 \end{vmatrix} + 1 \begin{vmatrix} -1 & -2 \\ 2 & -1 \end{vmatrix} + 1 \begin{vmatrix} -1 & 7 \\ 2 & 6 \end{vmatrix}$$

$$\boxed{D = 0}$$

∴ cramer's rule can't be apply ∴ solution does not exist.

$$Dx_2 = \begin{vmatrix} 1 & 4 & 1 \\ 2 & -2 & 0 \\ 4 & 0 & -3 \end{vmatrix}$$

$$= \begin{vmatrix} 2 & -2 \\ 4 & 0 \end{vmatrix} - 0 + (-3) \begin{vmatrix} 1 & 4 \\ 2 & -2 \end{vmatrix}$$

$$\boxed{Dx_2 = 38}$$

$$Dx_3 = \begin{vmatrix} 1 & -3 & 4 \\ 2 & -1 & -2 \\ 4 & 0 & 0 \end{vmatrix}$$

$$Dx_3 = 4 \begin{vmatrix} -3 & 4 \\ -1 & -2 \end{vmatrix} - 0 + 0$$

$$Dx_3 = 40$$

$$x_1 = \frac{Dx_1}{D} \qquad x_2 = \frac{Dx_2}{D} \qquad x_3 = \frac{Dx_3}{D}$$

$$x = -\frac{30}{11} \qquad x_2 = -\frac{38}{11} \qquad x_3 = \frac{-40}{11}$$

Assignment # 08

Exercise 3.1

Date

Q1

a)

$A = (1, 5)$

$B = (4, 1)$

$V = \overrightarrow{AB}$

$V = B - A$

$V = (4-1, 1-5)$

$V = (3, -4)$

b)

$A = (2, 3, 0)$

$B = (0, 0, 4)$

$V = B - A$

$= (0-2, 0-3, 4-0)$

$V = (-2, -3, 4)$

Q2

a)

$A = (2, 3)$

$B = (-3, 3)$

$V = B - A$

$V = (-3-2, 3-3)$

$V = (-5, 0)$

b)

$A = (3,0,4)$

$B = (0,4,4)$

$V = B - A$

$= (0-3, 4-0, 4-4)$

$V = (-3, 4, 0)$

Q3

a)

$P_1 = (3,5)$

$P_2 = (2,8)$

$\overrightarrow{P_1P_2} = P_2 - P_1$

$\overrightarrow{P_1P_2} = (2-3, 8-5)$

$\overrightarrow{P_1P_2} = (-1, 3)$

b)

$P_1(5,-2,1)$

$P_2(2,4,2)$

$\overrightarrow{P_1P_2} = P_2 - P_1$

$= (2-5, 4+2, 2-1)$

$\overrightarrow{P_1P_2} = (-3, 6, -1)$

Q4

a)

$P_1 = (-6,2)$

$P_2 = (-4,-1)$

$\overrightarrow{P_1P_2} = (-4+6, -1-2)$

$\overrightarrow{P_1P_2} = (2, -3)$

b)

$P_1 = (0,0,0)$

$P_2 = (-1,6,1)$

$\overrightarrow{P_1P_2} = (-1-0, 6-0, 1-0)$

$\overrightarrow{P_1P_2} = (-1, 6, 1)$

Q5

a)

$A(1,1)$

$B(x,y)$

$V = (1,2)$

$V = B - A$

$(1,2) = (x-1, y-1)$

$x-1 = 1 \Rightarrow x = 2$

$y-1 = 2 \Rightarrow y = 3$

$B(x,y) = B(2,3)$

b)

$U = (1, 1, 3)$

$A = (x, y, z)$

$B = (-1, -1, 2)$

$U = B - A$

$(1, 1, 3) = (-1-x, -1-y, 2-z)$

$-1-x = 1 \quad -1-y = 1 \quad 2-z = 3$

$x = -2 \quad y = -2 \quad z = -1$

$A(x, y, z) = (-2, 2, -1)$

Q6

a)

$U = (1, 2)$

$A = (x, y)$

$B = (2, 0)$

$(1, 2) = (2-x, 0-y)$

$2-x = 1 \quad 0-y = 2$

$x = 1 \quad y = -2$

$A(x, y) = (1, -2)$

b)

$A = (0, 2, 0)$

$B = (x, y, z)$

$U = (1, 1, 3)$

$U = B - A$

$(1, 1, 3) = (x-0, y-2, z-0)$

$x = 1 \quad y-2 = 1 \quad z = 3$

$x = 1 \quad y = 3 \quad z = 3$

$B(x, y, z) = (1, 3, 3)$

Q9

$U = (4, -1) \quad V = (0, 5) \quad W = (-3, -3)$

a) $U + W$

$U + W = (4 + (-3), -1 + (-3))$

$U + W = (1, -4)$

b) $V - 3U$

$3U = (12, -3)$

$V - 3U = (0-12, 5-(-3))$

$V - 3U = (-12, 8)$

c) $2(U - 5W)$

$-5W = (+15, +15)$

$U - 5W = (19, 14)$

$2(U - 5W) = (38, 28)$

d) $3v - 2(u+2w)$

$3v = (0, 15)$
$2w = (-6, -6)$
$u + 2w = (-2, -7)$
$2(u+2w) = (-4, -14)$
$3v - 2(u+2w) = (+4, 29)$

Q16

$u = (4, -1)$

a) $8t, -2$

$2(4t, -1)$ parallel to $(4, -1)$

$4t = 4$
$\boxed{t = 1}$

b) $(8t, 2t)$

$2t(4, 1)$ is not parallel to $(4, -1)$

c) $(1, t^2)$

square of any number is positive
$(1, t^2)$ not parallel to $(4, -1)$

Q17

$u = (2, 1, 0, 1, -1)$
$v = (-2, 3, 1, 0, 2)$
$au + bv = (-8, 8, 3, -1, 7)$

$a(2, 1, 0, 1, -1) + b(-2, 3, 1, 0, 2) = (-8, 8, 3, -1, 7)$
$2a, a, 0,$

$u = (1, -1, 3, 5)$
$v = (2, 1, 0, -3)$
$au + bv = (1, -4, 9, 18)$
$(a, -a, 3a, 5a) + (2b, b, 0, -3b) = (1, -4, 9, 18)$
$(a+2b, -a+b, 3a, 5a-3b) = (1, -4, 9, 18)$

$3a = 9$
$a = 3$

$a + 2b = 1$
$\cancel{2b = \frac{1}{a}}$ $b = \frac{1-a}{2}$

$b = \frac{1-3}{2}$

$\boxed{a = 3}$ $\boxed{b = -1}$

Q26

a)
$U = \cancel{(5,5) - (0,0) = (5,5)}$
$V = \cancel{(-5,-3) - (5,5) = (-10,-8)}$
$W = \cancel{(-3,5) - (-5,-3) = (2,8)}$

$\cancel{V + W = (-10+2, -8+8)}$
$\cancel{V + W = (-8, 0)}$
$\cancel{U - V + W = (5,5) - (-8,0)}$
$\cancel{U - V + W = (13, 5)}$

a) $U = (5,-5) - (0,0) = \langle 5, -5 \rangle$
$V = (-5,-3) - (5,-5) = \langle -10, 2 \rangle$
$W = (-2,5) - (-5,-3) = \langle 3, 8 \rangle$
$U - V = \langle 5, -5 \rangle - \langle -10, 2 \rangle = \langle 15, -7 \rangle$
$U - V + W = \langle 15, -7 \rangle + \langle 3, 8 \rangle = \langle 18, 1 \rangle$

b) $U = (5,-3) - (-5,4) = \langle 10, -7 \rangle$
$V = (2,5) - (5,-3) = \langle -3, 8 \rangle$
$W = (-2,-4) - (2,5) = \langle -4, -9 \rangle$
$U - V = \langle 10, -7 \rangle - \langle -3, 8 \rangle = \langle 13, -15 \rangle$
$U - V + W = \langle 13, -15 \rangle + \langle -4, -9 \rangle$
$= \langle 9, -24 \rangle$

Assignment # 09

Exercise 3-2

73

Date

Q1

a) $V = (2, 2, 2)$

$\|V\| = \sqrt{2^2 + 2^2 + 2^2}$

$\boxed{\|V\| = \sqrt{12}}$

$\hat{V} = \frac{\sqrt{12}}{6}(i + j + k)$

$\boxed{\hat{V} = -\frac{\sqrt{12}}{6}(i + j + k)}$

b) $V = (1, 0, 2, 1, 3)$

$\|V\| = \sqrt{1^2 + 0^2 + 2^2 + 1^2 + 3^2}$

$\|V\| = \sqrt{15}$

$\hat{N} = \frac{1}{\sqrt{15}} i + \frac{2}{\sqrt{15}} K + \frac{1}{\sqrt{15}} l + \frac{3}{\sqrt{15}} m$

$\boxed{\hat{V} = -\frac{1}{\sqrt{15}} i - \frac{2}{\sqrt{15}} K \cancel{+} - \frac{1}{\sqrt{15}} l - \frac{\sqrt{15}}{5} m}$

Q3

$U = (2,-2,3)$
$V = (1,-3,4)$
$w = (3,6,-4)$

a) $\|U+V\|$

$U+V = (3, -5, 7)$
$\|U+V\| = \sqrt{3^2+(5)^2+7^2}$
$\|U+V\| = \sqrt{83}$

b) $\|U\| + \|V\|$

$\|U\| = \sqrt{2^2+2^2+3^2} = \sqrt{17}$
$\|V\| = \sqrt{1^2+3^2+4^2} = \sqrt{26}$
$\|U\|+\|V\| = \sqrt{17}+\sqrt{26}$

c) $\|-2U+2V\|$

$-2U = (-4, 4, -6)$
$2V = (2, -6, 8)$
$-2U+2V = (-2, -2, 2)$
$\|-2U+2V\| = \sqrt{2^2+2^2+2^2}$
$\|-2U+2V\| = \sqrt{12}$

d) $\|3U-5V+w\|$

$3U = (6, -6, 9)$
$5V = (5, -15, 20)$
$3U-5V = (1, 9, -11)$
$3U-5V+w = (4, 15, -15)$
$\|3U-5V+w\| = \sqrt{4^2+15^2+15^2}$
$\|3U-5V+w\| = \sqrt{466}$

Q5

$U = (-2,1,4,5)$ $V = (3,1,-5,7)$, $w = (-6,2,1,1)$

a) $\|3U-5V+w\|$

$3U = (-6, -3, 12, 15)$
$5V = (15, 5, -25, 35)$
$3U-5V = (-21, -8, 37, -20)$
$3U-5V+w = (-27, -6, 38, -19)$
$\|3U-5V+w\| = \sqrt{27^2+6^2+38^2+19^2}$
$\|3U-5V+w\| = \sqrt{2570}$

b) $\|3U\| - 5\|V\| + \|w\|$

$3U = (-6, -3, 12, 15)$
$\|3U\| = \sqrt{6^2+3^2+12^2+15^2} = 3\sqrt{46}$
$\|V\| = \sqrt{3^2+1^2+5^2+7^2} = 2\sqrt{21}$

$\|w\| = \sqrt{6^2+2^2+1^2+1^2}$

$\|w\| = \sqrt{42}$

$\|3u\| - 5\|v\| + \|w\| = 3\sqrt{46} - 5(2\sqrt{21}) + \sqrt{42}$

$= 3\sqrt{46} - 10\sqrt{21} + \sqrt{42}$

Q7

$v = (-2, 3, 0, 6)$

$kv = (-2k, 3k, 0, 6k)$

$\|kv\| = \sqrt{4k^2+9k^2+36k^2}$

$\sqrt{4k^2+9k^2+36k^2} = 5$

$\pm 7k = 5$

$\boxed{k = \pm\frac{5}{7}}$

Q9

a) $u = (3,1,4) \quad v = (2,2,-4)$

$u.v = 3(2)+1(2)+4(-4) = -8$

$u.u = 3(3)+1(1)+4(4) = 26$

$v.v \quad 2(2)+2(2)+(-4)(-4) = 24$

b) $u = (1,1,4,6) \quad v = (2,-2,3,-2)$

$u.v = 1(2)+1(-2)+4(3)+6(-2) = 0$

$u.u = 1(1)+1(1)+4(4)+6(6) = 54$

$v.v = 2(2)+(-2)(-2)+3(3)+(-2)(-2) = 21$

Q15

a) Does not make sense, v.w is scalar and dot product is only defined between vectors.

b) Makes sense

c) ~~Does not make sense~~ ↳ u.v is scalar and norm is defined only for vectors only

d) Makes sense

ASSIGNMENT 109

Exercise 3.3

Q1

a)

$u = (6, 1, 4) \qquad v = (2, 0, -3)$

$u \cdot v = (6, 1, 4) \cdot (2, 0, -3)$

$= (6 \times 2) + (1 \times 0) + (4 \times -3)$

$u \cdot v = 0$

u & v are orthogonal vectors

b) $u = (0, 0, -1) \qquad v = (1, 1, 1)$

$u \cdot v = (0, 0, -1) \cdot (1, 1, 1)$

$= (0 \times 1) + (0 \times 1) + (-1 \times 1)$

$u \cdot v = -1$

u & v are not orthogonal vectors

c) $u = (3, -2, 1, 3) \qquad v = (-4, 1, -3, 7)$

$u \cdot v = (3, -2, 1, 3) \cdot (4, 1, -3, 7)$

$= (3 \times 4) + (-2 \times 1) + (1 \times -3) + (3 \times 7)$

$u \cdot v = 28$

u & v are not orthogonal vectors

d) $U = (5,-4,0,3)$ $V = (-4,1,-3,7)$

$U \cdot V = (5,-4,0,3) \cdot (-4,1,-3,7)$

$= (5\times -4) + (-4\times 1) + (0\times -3) + (3\times 7)$

$U \cdot V = -12 - 5$

U & V are not orthognal vectors.

Q2

a) $U = (2,3)$ $V = (5,-7)$

$U \cdot V = (2,3)\cdot(5,-7)$

$= (2\times 5) + (3\times -7)$

$U \cdot V = -11$

Not ORThognal

b) $U = (1,1,1)$ $V = (0,0,0)$

$U \cdot V = (1,1,1)\cdot(0,0,0)$

$= (1\times 0) + (1\times 0) + (1\times 0)$

$U \cdot V = 0$

orthogonal.

c) $U = (1,-5,4)$ $V = (3,3,3)$

$U \cdot V = (1,-5,4)\cdot(3,3,3)$

$= (1\times 3) + (-5\times 3) + (4\times 3)$

$U \cdot V = 0$

orthogonal.

d) $U = (4,1,-2,5)$ $V = (-1,5,3,1)$

$U \cdot V = (4,1,-2,5)\cdot(-1,5,3,1)$

$= (4\times -1) + (1\times 5) + (-2\times 3) + (5\times 1)$

$U \cdot V = 0$

orthogonal.

Q3

$n = (a,b,c) = (-2,1,-1)$

$P = (x_0, y_0, z_0) = (-1,3,-2)$

$a(x-x_0) + b(y-y_0) + c(z-z_0) = 0$

$-2(x+1) + (y-3) - (z+2) = 0$

Q4

$P(x_0, y_0, z_0) = (1,1,4)$

$n = (a,b,c) = (1,9,8)$

$a(x-x_0) + b(y-y_0) + c(z-z_0) = 0$

$(x-1) + 9(y-1) + 8(z-4) = 0$

Q5

$P(x_0, y_0, z_0) = (2,0,0)$ $n = (a,b,c) = (0,0,2)$

$a(x-x_0) + b(y-y_0) + c(z-z_0) = 0$

$2z = 0$

Scanned with CamScanner

Q6

$P(x_0, y_0, z_0) = (0,0,0)$ $\quad n = (a,b,c) = (1,2,3)$

$a(x-x_0) + b(y-y_0) + c(z-z_0) = 0$

$x + 2b + 3z = 0$

Q7

Plane 1 → $4x - \cancel{2}y + 2z = 5$

normal vector $= n_1 = \langle 4, -1, 2 \rangle$

Plane 2 → $7x - 3y + 4z = 8$

normal vector $= n_2 = \langle 7, -3, 4 \rangle$.

n_1 & n_2 are not scalar multiple of each other;

planes are not parallel.

Q8

Plane 1 → $x - 4y - 3z - 2 = 0$

normal vector $= n_1 = \langle 1, -4, -3 \rangle$

plane 2 → $3x - 12y - 9z - 7 = 0$

normal vector $= n_2 = \langle 3, -12, -9 \rangle$

Planes are parallel because n_1 & n_2 are scalar multiple of each other.

Q9

Plane 1 → $8x - 2y - 4z + 5 = 0$

normal vector $= n_1 = \langle 8, -2, -4 \rangle$

Plane 2 → $x - \frac{1}{2}z - \frac{1}{4}y = 0$

normal vector $= n_2 = \langle 1, -\frac{1}{4}, -\frac{1}{2} \rangle$

n_1 & n_2 are scalar multiples of each other

planes are parallel.

Q10

Plane 1 → $-4x + y + 2z = 0$

normal vector $= n_1 = \langle -4, 1, 2 \rangle$.

Plane 2 → $8x - 2y - 4z = 0$

normal vector $= n_2 = \langle 8, -2, -4 \rangle$

n_1 & n_2 are scalar multiples of each other

planes are parallel.

Q11

Plane 1 → $3x - y + z - 4 = 0$

normal vector $= n_1 = \langle 3, -1, 1 \rangle$

Plane 2 → $x + 2z = -1$

normal vector $= n_2 = \langle 1, 0, 2 \rangle$

$n_1 . n_2 = (3,-1,1) \cdot (1,0,2)$
$= (3\times1) + (-1\times0) + (1\times2)$
$n_1.n_2 = 5$
$n_1.n_2 \neq 0$, so the planes are not perpendicular

Q12

Plane 1 → $x - 2y + 3z = 4$
$n_1 \rightarrow \langle 1,-2,3 \rangle$
Plane 2 → $-2x + 5y + 4z = -1$
$n_2 \rightarrow \langle -2,5,4 \rangle$
$n_1 . n_2 = (1,-2,3)(-2,5,4)$
$= (1\times-2) + (-2\times5) + (3\times4)$
$n_1 . n_2 = 0$
planes are perpendicular

Q13

a) $u = (1,-2) \quad a = (-4,-3)$
$\|proj_a u\| = \frac{|u.a|}{\|a\|}$
$u.a = (1,-2).(-4,-3)$
$= (1\times-4) + (-2\times-3)$
$u.a = 2$

$\|a\| = \sqrt{(-4)^2+(-3)^2}$
$\|a\| = 5$
$\boxed{\|proj_a u\| = \frac{2}{5}}$

b) $u = (3,0,4) \quad a = (2,3,3)$
$\|proj_a u\| = \frac{|u.a|}{\|a\|}$
$u.a = (3\times2)+(0\times3)+(4\times3)$
$u.a = 18$
$\|a\| = \sqrt{2^2+3^2+3^2} = \sqrt{22}$
$\boxed{\|proj_a u\| = \frac{18}{\sqrt{22}}}$

Q14

a) $u = (5,6) \quad a = (2,-1)$
$u.a = (5,6).(2,-1)$
$= (5\times2) + (6\times-1)$
$u.a = 4$
$\|a\| = \sqrt{2^2+(-1)^2} = \sqrt{5}$
$\|proj_a u\| = \frac{4}{\sqrt{5}}$

b) $v = (3,-2,6)$, $a = (1,2,-7)$

$v \cdot a = (3\times1) + (-2\times2) + (6\times-7)$

$v \cdot a = -43$

$\|a\| = \sqrt{1^2 + 2^2 + (-7)^2} = \sqrt{54}$.

$\|proj_a v\| = \cancel{\sqrt{54}} \quad \frac{|v\cdot a|}{\|a\|} = \frac{43}{\sqrt{54}}$

Q15

$v = (6,2)$ $\quad a = (3,-9)$

vector component of v along a:

$proj_a v = \frac{v \cdot a}{\|a\|^2} a$

$v \cdot a = (6\times3) + (2\times-9)$

$v \cdot a = 0$

$\boxed{proj_a v = \text{vector component of } v \text{ along } a = (0,0)}$

$v - proj_a v = $ vector component of v orthogonal to a $= v - (0,0)$

$= v$

$\boxed{v - proj_a v = (6,2)}$

Q16

$v = (-1,-2)$ $\quad a = (-2,3)$

$proj_a v = \frac{v \cdot a}{\|a\|^2} a$

$v \cdot a = (-1\times-2) + (-2\times3)$

$v \cdot a = -4$

$\|a\| = (\sqrt{(-2)^2 + (3)^2})^2 = \sqrt{13}$

$proj_a v = \frac{-4}{(\sqrt{13})^2}(-2,3)$

$\boxed{\text{vector comp. of } v \text{ along } a = \left(\frac{8}{13}, \frac{-12}{13}\right)}$

vector comp. of v orthogonal to a $= v - proj_a v$.

$= (-1,-2) - \left(\frac{8}{13}, \frac{-12}{13}\right)$

$\boxed{\text{vector component of } v \text{ orthogonal to } a = \cancel{\frac{-13-8\sqrt{13}}{13}}, \cancel{\frac{-26+12\sqrt{13}}{13}}}$

$-\frac{21}{13} \qquad -\frac{14}{13}$

Q17

$u = (3, 1, -7) \quad a = (1, 0, 5)$

$\text{proj}_a u = \dfrac{u \cdot v}{\|a\|^2} a$

$u \cdot v = (3 \times 1) + (1 \times 0) + (-7 \times 5)$

$u \cdot v = -32$

$\|a\|^2 = \left(\sqrt{(1)^2 + (0)^2 + (5)^2}\right)^2 = 26$

$\text{proj}_a u = \dfrac{-32}{26}(1, 0, 5)$

$\boxed{\text{vector component of } u \text{ along } a = \left(\dfrac{-16}{13}, 0, \dfrac{-80}{13}\right)}$

$u - \text{proj}_a u = (3, 1, -7) - \left(\dfrac{-16}{13}, 0, \dfrac{-80}{13}\right)$

$u - \text{proj}_a u = \left(\dfrac{55}{13}, 1, \dfrac{-11}{13}\right)$

$\boxed{\text{vector component of } u \text{ orthogonal to } a = \left(\dfrac{55}{13}, 1, \dfrac{-11}{13}\right)}$

Q18

$u = (2, 0, 1) \quad a = (1, 2, 3)$

$u \cdot a = (2 \times 1) + 0 + (1 \times 3)$

$u \cdot a = 5$

$\|a\|^2 = \left(\sqrt{1^2 + 2^2 + 3^2}\right)^2 = 14$

$\text{proj}_a u = \dfrac{u \cdot a}{\|a\|^2} a$

$= \dfrac{5}{14}(1, 2, 3)$

$\boxed{\text{vector component of } u \text{ along } a = \dfrac{5}{14}, \dfrac{5}{07}, \dfrac{15}{14}}$

$u - \text{proj}_a u = (2, 0, 1) - \left(\dfrac{5}{14}, \dfrac{5}{7}, \dfrac{15}{14}\right)$

$\boxed{\text{vector component of } u \text{ orthogonal to } a = \left(\dfrac{23}{14}, \dfrac{-5}{7}, -\dfrac{1}{14}\right)}$

Q19

$u = (2, 1, 1, 2) \quad a = (4, -4, 2, -2)$

$u \cdot a = (2 \times 4) + (1 \times -4) + (1 \times 2) + (2 \times -2)$

$u \cdot a = 2$

$\|a\|^2 = \left(\sqrt{(4)^2 + (-4)^2 + (2)^2 + (-2)^2}\right)^2$

$\|a\|^2 = \sqrt{40}$

$proj_a U = \frac{2}{40}(4, -4, 2, -2)$

vector comp. of U along a = $\left(\frac{1}{5}, \frac{-1}{5}, \frac{1}{10}, \frac{-1}{10}\right)$

$U - proj_a U = (2, 1, 1, 2) - \left(\frac{1}{5}, \frac{-1}{5}, \frac{1}{10}, \frac{-1}{10}\right)$

vector component of U orthogonal to a = $\left(\frac{9}{5}, \frac{6}{5}, \frac{9}{10}, \frac{-9}{10}\right)$

Q20

$U = (5, 0, -3, 7)$ $\quad a = (2, 1, -1, -1)$

$U \cdot a = (5 \times 2) + (-3 \times -1) + (7 \times -1)$

$U \cdot a = 6$

$\|a\|^2 = \left(\sqrt{2^2 + 1^2 + (-1)^2 + (-1)^2}\right)^2$

$\|a\|^2 = 7$

$proj_a U = \frac{6}{7}(2, 1, -1, -1)$

vector comp of U along a = $\left(\frac{12}{7}, \frac{6}{7}, \frac{-6}{7}, \frac{-6}{7}\right)$

$U - proj_a U = (5, 0, -3, 7) - \left(\frac{12}{7}, \frac{6}{7}, \frac{-6}{7}, \frac{-6}{7}\right)$

Vector comp. of U orthogonal to a = $\frac{23}{7}, \frac{-6}{7}, \frac{-15}{7}, \frac{55}{7}$

Q21

Point → $P(-3, 1)$

Line → $4x + 3y + 4 = 0$

$D = \frac{|ax_0 + by_0 + c|}{\sqrt{a^2 + b^2}}$

$D = \frac{|4(-3) + 3(1) + 4|}{\sqrt{4^2 + 3^2}}$

$D = 1$

Q22

Point → $(-1, 4)$

Line → $x - 3y + 2 = 0$

$D = \frac{|ax_0 + by_0 + c|}{\sqrt{a^2 + b^2}}$

$= \frac{|1(-1) + (-3)(4) + 2|}{\sqrt{1^2 + (-3)^2}}$

$D = \frac{11}{\sqrt{10}}$

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Q23

Point $\rightarrow$ $P(2,-5)$

Line $\rightarrow$ $y = -4x + 2$

$$D = \frac{|ax_0 + by_0 + c|}{\sqrt{a^2+b^2}}$$

$$= \frac{|4(2) + (1)(-5) - 2|}{\sqrt{(4)^2 + (1)^2}}$$

$$D = \frac{1}{\sqrt{17}}$$

Q24

Point $\rightarrow$ $P(1,8)$

Line $\rightarrow$ $3x + y = 5$

$$D = \frac{|ax_0 + by_0 + c|}{\sqrt{a^2+b^2}}$$

$$= \frac{|3(1) + 1(8) - 5|}{\sqrt{3^2+1^2}}$$

$$D = \frac{6}{\sqrt{10}}$$

Q25

Point $\rightarrow$ $P(3,1,-2)$

Plane $\rightarrow$ $x + 2y - 2z - 4 = 0$

$$D = \frac{|ax_0 + by_0 + cz_0 + d|}{\sqrt{a^2+b^2+c^2}}$$

$$D = \frac{|1(3) + 2(1) - 2(-2) - 4|}{\sqrt{1^2 + 2^2 + (-2)^2}}$$

$$D = \frac{5}{3}$$

Q26

Point $\rightarrow$ $(-1,-1,2)$

Plane $\rightarrow$ $2x + 5y - 6z = 4$

$$D = \frac{|ax_0 + by_0 + cz_0 + d|}{\sqrt{a^2+b^2+c^2}}$$

$$= \frac{|2(-1) + 5(-1) - 6(2) - 4|}{\sqrt{2^2 + 5^2 + 6^2}}$$

$$D = \frac{23}{\sqrt{65}}$$

Assignment 11

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Ex 3.5

Exercise 3.5

Date

Q11

$P_1(1,2)$, $P_2(4,4)$, $P_3(7,5)$, $P_4(4,3)$

$\vec{U} = P_1P_2 = (3,2)$

$\vec{V} = P_1P_3 = (6,3)$

$\cancel{\vec{U} \times \vec{V} = \begin{vmatrix} i & j & k \\ 3 & 2 & \\ 6 & 3 & \end{vmatrix}}$

$\cancel{= 6i}$

Area of parallelogram $= \det \begin{bmatrix} u_1 & u_2 \\ v_1 & v_2 \end{bmatrix}$

$= \begin{vmatrix} 3 & 2 \\ 6 & 3 \end{vmatrix}$

$\boxed{A = 3}$

Q12

$P_1(3,2)$, $P_2(5,4)$, $P_3(9,4)$; $P_4(7,2)$

$\vec{U} = P_1P_2 = (2,2)$

$\vec{V} = P_1P_3 = (6,2)$

Area $= \det \begin{bmatrix} u_1 & u_2 \\ v_1 & v_2 \end{bmatrix} = \det \begin{bmatrix} 2 & 2 \\ 6 & 2 \end{bmatrix}$

$\boxed{\text{Area} = 8.}$

Q13

A(2,0) B(3,4) C(-1,2)

$\vec{u}$ = AB = (1, 4)

$\vec{v}$ = AC = (-3, 2)

$$\text{Area} = \frac{1}{2} \det \begin{bmatrix} u_1 & u_2 \\ v_1 & v_2 \end{bmatrix}$$

$$= \frac{1}{2} \det \begin{bmatrix} 1 & 4 \\ -3 & 2 \end{bmatrix}$$

$\boxed{\text{Area} = 5}$

Q14

A(1,1) , B(2,2) C(3,-3)

$\vec{u}$ = AB = (1, 1)

$\vec{v}$ = AC = (2, -4)

$$\text{Area} = \frac{1}{2} \det \begin{bmatrix} u_1 & u_2 \\ v_1 & v_2 \end{bmatrix}$$

$$= \frac{1}{2} \det \begin{bmatrix} 1 & 1 \\ 2 & -4 \end{bmatrix}$$

$\boxed{\text{Area} = 3}$

Q19

u = (-1, -2, 1) v = (3, 0, -2) w = (5, -4, 0)

$$u \cdot (v \times w) = \begin{vmatrix} u_1 & u_2 & u_3 \\ v_1 & v_2 & v_3 \\ w_1 & w_2 & w_3 \end{vmatrix}$$

$$= \begin{vmatrix} -1 & -2 & 1 \\ 3 & 0 & -2 \\ 5 & -4 & 0 \end{vmatrix}$$

$= 8 + 20 - 12$

$= -24$

$u \cdot (v \times w) \neq 0$

Does not lie in same plane.

Q20

u = (5, -2, 1) v = (4, -1, 1) w = (1, -1, 0)

$$u \cdot (v \times w) = \begin{vmatrix} u_1 & u_2 & u_3 \\ v_1 & v_2 & v_3 \\ w_1 & w_2 & w_3 \end{vmatrix}$$

$$= \begin{vmatrix} 5 & -2 & 1 \\ 4 & -1 & 1 \\ 1 & -1 & 0 \end{vmatrix}$$

$u \cdot (v \times w) = +5 + 2 - 3 \neq 0$

$u\cdot(v\times w)=0$

Lie on the same plane.

Q 27

a) $A(1,0,1)$ $B(0,2,3)$ $C(2,1,0)$

$\vec{u}=AB=(-1,2,2)$

$\vec{v}=AC=(1,1,-1)$

$$u\times v=\begin{vmatrix} i & j & k\\ -1 & 2 & 2\\ 1 & 1 & -1\end{vmatrix}$$

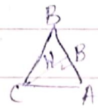
$u\times v=-4i+j-3k$

$\|u\times v\|=\sqrt{(1)^2+(3)^2+(4)^2}$

$\|u\times v\|=\sqrt{26}$

Area of triangle $=\frac{1}{2}\|u\times v\|=\frac{1}{2}\sqrt{26}$

$\boxed{\text{Area of triangle}=\frac{\sqrt{26}}{2}}$



b)

$B=\sqrt{(3-1)^2+(2-0)^2+(0-1)^2}=3$

$2\Delta=B\times H$

$H=\frac{2\Delta}{B}=\frac{2\left(\frac{\sqrt{26}}{2}\right)}{3}$

$\boxed{H=\frac{\sqrt{26}}{3}}$

Q1

a) $v\times w$

$$v\times w=\begin{vmatrix} i & j & k\\ 0 & 2 & -3\\ 2 & 6 & 7\end{vmatrix}$$

$=(14+18)i-(0+6)j+(0-4)k$

$v\times w=32\hat{i}-6\hat{j}-4\hat{k}$

b) $w\times v$

$$w\times v=\begin{vmatrix} i & j & k\\ 2 & 6 & 7\\ 0 & 2 & -3\end{vmatrix}$$

$=(-18-14)i-(-6-0)j+(4-0)k$

$w\times v=-32i+6j+4k$

c) $(u+v)\times w$

$u+v=(3,2,-1)+(0,2,-3)$

$u+v=(3,4,-4)$

$$(u+v)\times w=\begin{vmatrix} i & j & k\\ 3 & 4 & -4\\ 2 & 6 & 7\end{vmatrix}$$

$=(28+24)i-(21+8)j+(18-8)k$

$(u+v)\times w=52i-29j+10k$

d) $v\cdot(v\times w)$

$$v\times w = \begin{vmatrix} i & j & k \\ 0 & 2 & -3 \\ 2 & 6 & 7 \end{vmatrix}$$

a

$$= (14+18)i-(0+6)j+(0-4)k$$

$$v\times w = 32i-6j-4k$$

$$v\cdot(v\times w) = (0,2,-3)\cdot(32,-6,-4)$$

$$= 0 - 12 + 12$$

$$\boxed{v\cdot(v\times w) = 0}$$

e) $v\times v$

$$v\times v = \begin{vmatrix} i & j & k \\ 0 & 2 & -3 \\ 0 & 2 & -3 \end{vmatrix}$$

$$v\times v = 0i+0j+0k$$

$$v\times v = (0,0,0)$$

b

f) $(u-3w)\times(u-3w)$

$3w=(6,18,21)$, $u-3w=(-3,-16,-22)$

$$(u-3w)\times(u-3w) = \begin{vmatrix} i & j & k \\ -3 & -16 & -22 \\ -3 & -16 & -22 \end{vmatrix}$$

$$= (0,0,0)$$

Q2

a) $u\times v$

$$u\times v = \begin{vmatrix} i & j & k \\ 3 & 2 & -1 \\ 0 & 2 & -3 \end{vmatrix}$$

$$u\times v = -4i+9j+6k$$

b) $-(u\times v)$

$$u\times v = -4i+9j+6k$$

$$-(u\times v) = 4i-9j-6k$$

c) $u\times(v+w)$

$$v+w = (0,2,-3)+(2,6,7)$$

$$v+w = (2,8,4)$$

$$u\times(v+w) = \begin{vmatrix} i & j & k \\ 3 & 2 & -1 \\ 2 & 8 & 4 \end{vmatrix}$$

$$u\times(v+w) = 16i-14j+20k$$

d) $w\cdot(w\times v)$

$$w\times v = \begin{vmatrix} i & j & k \\ 2 & 6 & 7 \\ 0 & 2 & -3 \end{vmatrix}$$

$$w\times v = -32i+6j+4k$$

Date

$w\cdot(w\times v) = (2,6,7)\cdot(-32,6,4)$

$= -64+36+28$

$w\cdot(w\times v) = 0$

e) $w \times w$

$$w\times w = \begin{vmatrix} i & j & k \\ 2 & 6 & 7 \\ 2 & 6 & 7 \end{vmatrix}$$

$= 0i + 0j + 0k$

$w\times w = (0,0,0)$

f) $(7v-3u)\times(7v-3u)$

$3u = (9, 6, -3)$

$7v = (0, 14, -21)$

$7v-3u = (-9, 8, -18)$

$$(7v-3u)\times(7v-3u) = \begin{vmatrix} i & j & k \\ -9 & 8 & -18 \\ -9 & 8 & -18 \end{vmatrix}$$

$= 0i + 0j + 0k$

$(7v-3u)\times(7v-3u) = (0,0,0)$

Q3

$$\|U \times W\|^2 = \|U\|^2 \|W\|^2 - (U \cdot W)^2$$

e) Taking L.H.S

$$U \times W = \begin{vmatrix} i & j & k \\ 3 & 2 & -1 \\ 2 & 6 & 7 \end{vmatrix}$$

$$U \times W = 20i - 23j + 14k$$

$$\|U \times W\|^2 = \left(\sqrt{(20)^2 + (-23)^2 + (14)^2}\right)^2$$

$$\|U \times W\|^2 = 1125$$

f

Taking R.H.S

$$\|U\|^2 = \left(\sqrt{3^2 + 2^2 + (-1)^2}\right)^2 = 14$$

$$\|W\|^2 = \left(\sqrt{2^2 + 6^2 + 7^2}\right)^2 = 89$$

$$U \cdot V = (3, 2, -1) \cdot (2, 6, 7)$$

$$U \cdot V = 11$$

$$(U \cdot V)^2 = 121$$

$$= 14 \times 89 - 121$$

$$= 1125$$

L.H.S = R.H.S

Q4

$$\|V \times U\|^2 = \|V\|^2 \|U\|^2 - (U \cdot V)^2$$

Taking L.H.S

$$V \times U = \begin{vmatrix} i & j & k \\ 0 & 2 & -3 \\ 3 & 2 & -1 \end{vmatrix}$$

$$V \times U = 4i + 9j - 6k$$

$$\|V \times U\|^2 = \left(\sqrt{(4)^2 + (9)^2 + (-6)^2}\right)^2$$

$$\|V \times U\|^2 = 133$$

Taking R.H.S.

$$\|V\|^2 = \left(\sqrt{0^2 + 2^2 + 3^2}\right)^2 = 13$$

$$\|U\|^2 = \left(\sqrt{3^2 + 2^2 + 1^2}\right)^2 = 14$$

$$V \cdot U = (0, 2, 3) \cdot (3, 2, 1)$$

$$V \cdot U = 7$$

$$(V \cdot U)^2 = 49.$$

$$= 13 \times 14 - 49$$

$$= 13$$

L.H.S = R.H.S

Date

Q7

Let $\vec{c}$ is the orthogonal to u & v

$$\vec{c} = \vec{u} \times \vec{v}$$

$$= \begin{vmatrix} i & j & k \\ -6 & 4 & 2 \\ 3 & 1 & 5 \end{vmatrix}$$

$$\vec{c} = 18i + 36j - 18k$$

Q8

Let $\vec{w}$ is the orthogonal to u & v

$$\vec{w} = \vec{u} \times \vec{v}$$

$$= \begin{vmatrix} i & j & k \\ 1 & 1 & -2 \\ 2 & -1 & 2 \end{vmatrix}$$

$$\vec{w} = 0i - \cancel{6}i - 3k$$

$$\vec{w} = -6i - 3k$$

Q9

$$u \times v = \begin{vmatrix} i & j & k \\ 1 & -1 & 2 \\ 0 & 3 & 1 \end{vmatrix}$$

$$u \times v = -7i \cancel{j} - j + 3k$$

$$\|u \times v\| = \sqrt{59}$$

Date

Q10

$$u \times v = \begin{vmatrix} i & j & k \\ 3 & -1 & 4 \\ 6 & -2 & 8 \end{vmatrix}$$

$$u \times v = 0i + 0j + 0k$$

$$\|u \times v\| = 0$$

vector are parallel

Q17

$$v \times w = \begin{vmatrix} i & j & k \\ 0 & 4 & -2 \\ 2 & 2 & -4 \end{vmatrix}$$

$$v \times w = -12i - 4j - 8k$$

$$u \cdot (v \times w) = (2, -6, 2) \cdot (-12, -4, -8)$$

$$= -24 + 24 - 16$$

$$u \cdot (v \times w) = -16$$

$$|u \cdot (v \times w)| = 16$$

Q18

$$v \times w = \begin{vmatrix} i & j & k \\ 4 & 5 & 1 \\ 1 & 2 & 4 \end{vmatrix}$$

$v \times w = 18i - 15j + 3k$

$u \cdot (v \times w) = (3, 1, 2) \cdot (18, -15, 3)$

$u \cdot (v \times w) = 45$

Q28

$\| u \times v \| = \| u \| \| v \| \sin\theta$

$$\sin\theta = \frac{\| u \times v \|}{\| u \| \| v \|}$$

$$u \times v = \begin{vmatrix} i & j & k \\ 2 & 3 & -6 \\ 2 & 3 & 6 \end{vmatrix}$$

$u \times v = 36i - 24j$

$\| u \times v \| = \sqrt{36^2 + 24^2}$

$\| u \times v \| = 12\sqrt{13}$

$\| u \| = \sqrt{2^2 + 3^2 + (-6)^2} = 7$

$\| v \| = \sqrt{2^2 + 3^2 + 6^2} = 7$

$$\boxed{\sin\theta = \frac{12\sqrt{13}}{49}}$$

Assignment 12

Exercise 4.2

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Date

Q7

a)

Let $\vec{w} = (2,2,2)$

$\vec{w}$ is a linea combination of vector u & v

if

$$\vec{w} = k_1\vec{u} + k_2\vec{v}$$

$$(2,2,2) = K_1(0,-2,2) + K_2(1,3,-1)$$

$$(2,2,2) = (K_2, -2K_1+3K_2, 2K-K_2)$$

$$K_2 = 2$$

$$-2K_1+3K_2 = 2$$

$$2K_1-K_2 = 2$$

$$\boxed{K_2 = 2}$$

$$-2K_1 + 3(2) = 2$$

$$\boxed{K_1 = 2}$$

$$w = K_1U + K_2V$$

$$(2,2,2) = 2(0,-2,2) + 2(1,3,-1)$$

$$\boxed{(2,2,2) = (2,2,2)}$$

Yes, w is the linear combination of u and v.

b) $(0,4,5)$

Let $\vec{w} = (0,4,5)$

$\vec{w}$ is the linear combination of u & v if

$w = K_1 u + K_2 V$

$(0,4,5) = K_1(0,-2,2) + K_2(1,3,-1)$

$(0,4,5) = (K_2, -2K_1 + 3K_2, 2K_1 - K_2)$

$K_2 = 0 \rightarrow (1)$

$-2K_1 + 3K_2 = 4 \rightarrow (2)$

$2K_1 - K_2 = 5 \rightarrow (3)$

$-2(0) + 3K_2 = 4$

$K_2 = \frac{4}{3}$

put in eq (3)

$-2(0) - \frac{4}{3} = 5$

$-\frac{4}{3} \neq 5$

value of constant does not exist so w is not linear combination of u & v.

c) $(0,0,0)$

Let $\vec{w} = (0,0,0)$

$\vec{w}$ is linear combination of u & v, if

$\vec{w} = K_1 U + K_2 V$

$(0,0,0) = K_1(0,-2,2) + K_2(1,3,-1)$

$(0,0,0) = (K_2, -2K_1 + 3K_2, 2K_1 - K_2)$

$K_2 = 0 \rightarrow (1)$

$-2K_1 + 3K_2 = 0 \rightarrow (2)$

$2K_1 - K_2 = 0 \rightarrow (3)$

put (1) in (2)

$-2K_1 + 3(0) = 0$

$K_1 = 0$

put in (3)

$2(0) - 0 = 0$

$0 = 0$

$w = K_1 U + K_2 V$

$(0,0,0) = 0(0,-2,2) + 0(1,3,-1)$

$(0,0,0) = (0,0,0)$

w is linear combination of u & v.

Date

Q8

a) let $\vec{z} = (-9, -7, -15)$

z is the linear combination of u, v and w if:-

$$z = k_1 u + k_2 v + k_3 w$$

$$(-9,-7,-15) = k_1(2,1,4) + k_2(1,-1,3) + k_3(3,2,5)$$

$$(-9,-7,-15) = 2k_1 + k_2 + 3k_3,\ k_1 - k_2 + 2k_3,\ 4k_1 + 3k_2 + 5k_3$$

$$2k_1 + k_2 + 3k_3 = -9$$

$$k_1 - k_2 + 2k_3 = -7$$

$$4k_1 + 3k_2 + 5k_3 = -15$$

Solving above system of eq

$$\begin{bmatrix} 2 & 1 & 3 & -9 \\ 1 & -1 & 2 & -7 \\ 4 & 3 & 5 & -15 \end{bmatrix}$$

Interchange R_1 & R_2

$$\begin{bmatrix} 1 & -1 & 2 & -7 \\ 2 & 1 & 3 & -9 \\ 4 & 3 & 5 & -15 \end{bmatrix}$$

$R_2 = R_2 - 2R_1$

$R_3 = R_3 - 4R_1$

Date

$$\begin{bmatrix} 1 & -1 & 2 & -7 \\ 0 & 3 & -1 & 5 \\ 0 & 7 & -3 & 13 \end{bmatrix}$$

$R_2 = R_2 \div 3$

$$\begin{bmatrix} 1 & -1 & 2 & -7 \\ 0 & 1 & -1/3 & 5/3 \\ 0 & 7 & -3 & 13 \end{bmatrix}$$

$R_1 = R_1 + R_2$

$R_3 = R_3 - 7R_2$

$$\begin{bmatrix} 1 & 0 & 5/3 & -\frac{16}{3} \\ 0 & 1 & -\frac{1}{3} & \frac{5}{3} \\ 0 & 0 & -2/3 & 4/3 \end{bmatrix}$$

$R_3 = R_3 \div -\frac{2}{3}$

$$\begin{bmatrix} 1 & 0 & \frac{5}{3} & -\frac{16}{3} \\ 0 & 1 & -\frac{1}{3} & \frac{5}{3} \\ 0 & 0 & 1 & -2 \end{bmatrix}$$

$R_1 = R_1 - \frac{5}{3}R_3$

$R_2 = R_2 + \frac{1}{3}R_3$

$$\begin{bmatrix} 1 & 0 & 0 & -2 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & -2 \end{bmatrix}$$

$k_1 = -2 \quad k_2 = 1 \quad k_3 = -2$

$$\boxed{z = -2u + v - 2w}$$

b)

let $Z = (6, 11, 6)$

$\therefore$ Z is the linear combination of u, v and w if

$$Z = k_1 u + k_2 v + k_3 w$$

$$(6, 11, 6) = k_1(2,1,4) + k_2(1,-1,3) + k_3(3,2,5)$$

$$(6, 11, 6) = (2k_1 + k_2 + 3k_3,\ k_1 - k_2 + 2k_3,\ 4k_1 + 3k_2 + 5k_3)$$

$$2k_1 + k_2 + 3k_3 = 6$$

$$k_1 - k_2 + 2k_3 = 11$$

$$4k_1 + 3k_2 + 5k_3 = 6$$

$$\begin{bmatrix} 2 & 1 & 3 & 6 \\ 1 & -1 & 2 & 11 \\ 4 & 3 & 5 & 6 \end{bmatrix}$$

Interchang R_1 & R_2

$$\begin{bmatrix} 1 & -1 & 2 & 11 \\ 2 & 1 & 3 & 6 \\ 4 & 3 & 5 & 6 \end{bmatrix}$$

$R_2 = R_2 - 2R_1$

$R_3 = R_3 - 4R_1$

$$\begin{bmatrix} 1 & -1 & 2 & 11 \\ 0 & 3 & -1 & -16 \\ 0 & 7 & -3 & -38 \end{bmatrix}$$

$R_2 = R_2 \div 3$

$$\begin{bmatrix} 1 & -1 & 2 & 11 \\ 0 & 1 & -\frac{1}{3} & -\frac{16}{3} \\ 0 & 7 & -3 & -38 \end{bmatrix}$$

$R_1 = R_1 + R_2$

$R_3 = R_3 - 7R_2$

$$\begin{bmatrix} 1 & 0 & 5/3 & 17/3 \\ 0 & 1 & -\frac{1}{3} & -\frac{16}{3} \\ 0 & 0 & -\frac{2}{3} & -\frac{2}{3} \end{bmatrix}$$

$R_3 = R_3 \times -\frac{3}{2}$

$$\begin{bmatrix} 1 & 0 & 5/3 & 17/3 \\ 0 & 1 & -1/3 & -\frac{16}{3} \\ 0 & 0 & 1 & \text{[illegible]} \end{bmatrix}$$

$R_1 = R_1 - \frac{5}{3}R_3$

$R_2 = R_2 + \frac{1}{3}R_3$

$$\begin{bmatrix} 1 & 0 & 0 & 4 \\ 0 & 1 & 0 & -5 \\ 0 & 0 & 1 & 1 \end{bmatrix}$$

$k_1 = 4 \quad k_2 = -5 \quad k_3 = 1$

$$\boxed{Z = 4u - 5v + w}$$

C)

let $z = (0,0,0)$

z is linear combination of u, v & w if.

$$z = k_1u + k_2v + k_3w$$

$$(0,0,0) = (2k_1 - k_2 + 3k_3,\ k_1 - k_2 + 2k_3,\ 4k_1 + 3k_2 + 5k_3)$$

$$2k_1 - k_2 + 3k_3 = 0$$
$$k_1 - k_2 + 2k_3 = 0$$
$$4k_1 + 3k_2 + 5k_3 = 0$$

$k_1 = 0 \qquad k_2 = 0 \qquad k_3 = 0$

$$\boxed{z = 0u + 0v + 0w}$$

Q11

a)

vectors span R_3 if thier determinant is not zero.

$$d = \begin{vmatrix} 2 & 2 & 2 \\ 0 & 0 & 3 \\ 0 & 1 & 1 \end{vmatrix}$$

$d = -6 - 0 + 0$

$\boxed{d = -6}$

vectors not span

b)

vectors span R_3 if thier determinant is not zero.

$$d = \begin{vmatrix} 2 & -1 & 3 \\ 4 & 1 & 2 \\ 8 & -1 & 8 \end{vmatrix}$$

$d = 2(10) + 16 + 3(-12)$

$d = 0$

vectors span R_3

Q12

a) $(2, 3, -7, 3)$

The above vector span $\{v_1, v_2, v_3\}$ if and only

$$(2, 3, -7, 3) = k_1v_1 + k_2v_2 + k_3v_3$$

$$(2,3,-7,3) = 2k_1 + 3k_2 - k_3,\ k_1 - k_2,\ 5k_2 + 2k_3,\ 3k_1 + 2k_2 + k$$

$2k_1 + 3k_2 - k_3 = 2 \rightarrow (1)$

$k_1 - k_2 = 3 \rightarrow (2)$

$5k_2 + 2k_3 = -7 \rightarrow (3)$

$3k_1 + 2k_2 + k_3 = 3 \rightarrow (4)$

C

$$\begin{bmatrix} 2 & 3 & -1 & 2 \\ 1 & -1 & 0 & 3 \\ 0 & 5 & 2 & -7 \\ 3 & 2 & 1 & 3 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -1 & 0 & 3 \\ 0 & 5 & -1 & -4 \\ 0 & 5 & 2 & -7 \\ 0 & 5 & 1 & -6 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & -\frac{1}{5} & \frac{11}{5} \\ 0 & 1 & -\frac{1}{5} & -\frac{4}{5} \\ 0 & 0 & 3 & -3 \\ 0 & 0 & 2 & -2 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$K_1 = 2 \quad K_2 = -1 \quad K_3 = -1$

$\boxed{(2, 3, -7, 3) = 2V_1 - V_2 - V_3}$

B)

$(0,0,0,0)$ is span $\{V_1, V_2, V_3\}$ if and only

$(0,0,0,0) = K_1V_1 + K_2V_2 + K_3V_3$

$(0,0,0,0) = 2K_1+3K_2-K_3,\ K_1-K_2,\ 5K_2+2K_3,\ 3K_1+2K_2+K_3$

$2K_1+3K_2-K_3 = 0$

$K_1 - K_2 = 0$

$5K_2+2K_3 = 0$

$3K_1+2K_2+K_3 = 0$

one solution is always exist in homogeneous

$K_1 = 0 \quad K_2 = 0 \quad K_3 = 0$

$\boxed{(0,0,0,0) = 0V_1 + 0V_2 + 0V_3}$

c)

$(1,1,1,1)$ is span $\{V_1, V_2, V_3\}$ if and only

$(1,1,1,1) = K_1V_1 + K_2V_2 + K_3V_3$

$(1,1,1,1) = 2K_1+3K_2-K_3,\ K_1-K_2,\ 5K_2+2K_3,\ 3K_1+2K_2+K_3$

$2K_1+3K_2-K_3 = 1$ (1)

$K_1 - K_2 = 1$ (2)

$5K_2+2K_3 = 1$ (3)

$3K_1+2K_2+K_3 = 1$ (4)

(2) $\Rightarrow K_1 = 1+K_2$ (5)

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(3) => $k_3 = \frac{1-5k_2}{2}$ (6)

put (5) & (6) in (1)

$2(1+k_2)+3k_2-\frac{(1-5k_2)}{2} = 1$

$10k_2 - 1 + 5k_2 = -2$

$\boxed{k_2 = -1/15}$

put in (5)

$\boxed{k_1 = 14/15}$

put in (6)

$\boxed{k_3 = 2/3}$

$$\boxed{(1,1,1,1) = \frac{14}{15}v_1 + \left(\frac{-1}{15}\right)v_2 + \frac{2}{3}v_3}$$

d)

$(-4,6,-13,4)$ span $\{v_1, v_2, v_3\}$ if & only

$(-4,6,-13,4) = k_1v_1 + k_2v_2 + k_3v_3$

$2k_1+3k_2-k_3=-4$ (1)

$k_1-k_2=6$ (2)

$5k_2+2k_3=-13$ (3)

$3k_1+2k_2+\cancel{3}k_3=4$ (4)

(2) => $k_1=6+k_2$ (5)

(3) => $k_3=(-13-5k_2)/2$ (6)

put (5) & (6) in (1)

$2(6+k_2)+3k_2-(-13-5k_2)/2=-4$

$10k_2+5k_2=-45$

$k_2=-3$

put in (5)

$k_1 = 3$

put in (6)

$k_3 = 1$

$\boxed{(-4,6,-13,4) = 3v_1 - 3v_2 + v_3}$

Exercise 4.3

Date

Q1

a) They have the same angle. ; $U_2 = -5U_1$

b) $r > n$

c) $P_2 = 2P_1$

d) $A = (-1)B$

Q3

a)

$(3,8,7,-3), (1,5,3,-1), (2,-1,2,6); (4,2,6,4)$

$r_1V_1 + r_2V_2 + r_3V_3 + r_4V_4 = 0$

$r_1(3,8,7,-3) + r_2(1,5,3,-1) + r_3(2,-1,2,6) + r_4(4,2,6,4) = 0$

$$\begin{aligned}
3r_1 + r_2 + 2r_3 + 4r_4 &= 0 \\
8r_1 + 5r_2 - r_3 + 2r_4 &= 0 \\
7r_1 + 3r_2 + 2r_3 + 6r_4 &= 0 \\
-3r_1 - r_2 + 6r_3 + 4r_4 &= 0
\end{aligned}$$

$$\left[\begin{array}{cccc|c}
3 & 1 & 2 & 4 & 0 \\
8 & 5 & -1 & 2 & 0 \\
7 & 3 & 2 & 6 & 0 \\
-3 & -1 & 6 & 4 & 0
\end{array}\right]$$

$$\left[\begin{array}{ccccc} 1 & \frac{1}{3} & \frac{2}{3} & \frac{4}{3} & 0 \\ 8 & 5 & -1 & 2 & 0 \\ 7 & 3 & 2 & 6 & 0 \\ -3 & -1 & 6 & 4 & 0 \end{array}\right].$$

$R_2 = R_2 - 8R_1$

$R_3 = R_3 - 7R_1$

$R_4 = R_4 + 3R_1$

$$\left[\begin{array}{ccccc} 1 & \frac{1}{3} & \frac{2}{3} & \frac{4}{3} & 0 \\ 0 & \frac{7}{3} & -\frac{19}{3} & -\frac{26}{3} & 0 \\ 0 & \frac{2}{3} & -\frac{8}{3} & -\frac{10}{3} & 0 \\ 0 & 0 & 8 & 8 & 0 \end{array}\right]$$

d)

$$\left[\begin{array}{ccccc} 1 & \frac{1}{3} & \frac{2}{3} & \frac{4}{3} & 0 \\ 0 & 1 & -\frac{19}{7} & -\frac{26}{7} & 0 \\ 0 & \frac{2}{3} & -\frac{8}{3} & -\frac{10}{3} & 0. \\ 0 & 0 & 8 & 8 & 0 \end{array}\right]$$

$R_1 = R_1 - (\frac{1}{3})R_2$

$R_3 = R_3 - (\frac{2}{3})R_2$

$$\left[\begin{array}{ccccc} 1 & 0 & \frac{11}{7} & \frac{18}{7} & 0 \\ 0 & 1 & -\frac{19}{7} & -\frac{26}{7} & 0 \\ 0 & 0 & -\frac{6}{7} & -\frac{6}{7} & 6 \\ 0 & 0 & 8 & 8 & 0 \end{array}\right]$$

$$\left[\begin{array}{ccccc} 1 & 0 & \frac{11}{7} & \frac{18}{7} & 0 \\ 0 & 1 & -\frac{19}{7} & -\frac{26}{7} & 0 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 8 & 8 & 0 \end{array}\right]$$

$R_1 = R_1 - \frac{11}{7}R_3$

$R_2 = R_2 + \frac{19}{7}R_3$

$R_4 = R_4 - 8R_3$

$$\left[\begin{array}{ccccc} 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & -1 & 0 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

$\lambda_1 + \lambda_4 = 0$

$\lambda_2 - \lambda_4 = 0$

$\lambda_3 + \lambda_4 = 0$

Let

$\lambda_4 = t$

$\lambda_1 = -t$

$\lambda_2 = t$

$\lambda_3 = -t$

Vectors are linearly dependent.

b)

Vectors are:

(-2,0,1), (3,2,5), (6,-1,1), (7,0,-2)

$$-2r_1 + 3r_2 + 6r_3 + 7r_4 = 0$$
$$+2r_2 - r_3 = 0$$
$$r_1 + 5r_2 + r_3 - 2r_4 = 0$$

$$3r_1 + 0r_2 + 0r_3 - 2r_4 = 0$$
$$0r_1 + 2r_2 - 2r_3 + r_4 = 0$$
$$-3r_1 + 3r_2 - 2r_3 + 2r_4 = 0$$
$$6r_1 + 1r_2 + 0r_3 + r_4 = 0$$

$$\begin{bmatrix} 3 & 0 & 0 & -2 & 0 \\ 0 & 2 & -2 & 1 & 0 \\ -3 & 3 & -2 & 2 & 0 \\ 6 & 1 & 0 & 1 & 0 \end{bmatrix}$$

$R_1 = R_1/3$

$$\begin{bmatrix} 1 & 0 & 0 & -2/3 & 0 \\ 0 & 2 & -2 & 1 & 0 \\ -3 & 3 & -2 & 2 & 0 \\ 6 & 1 & 0 & 1 & 0 \end{bmatrix}$$

$R_3 = R_3 + 3R_1$

$R_4 = R_4 - 6R_1$

$$\begin{bmatrix} 1 & 0 & 0 & -\frac{2}{3} & 0 \\ 0 & 2 & -2 & 1 & 0 \\ 0 & 3 & -2 & 0 & 0 \\ 0 & 1 & 0 & +5 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 & -2/3 & 0 \\ 0 & 2 & -2 & 1 & 0 \\ 0 & 1 & -2/3 & 0 & 0 \\ 0 & 1 & 0 & 5 & 0 \end{bmatrix}$$

$R_2 = R_2 - 2R_3$ $\qquad$ $R_4 = R_4 - R_3$

$$\begin{bmatrix} 1 & 0 \\ 0 & 0 \\ 0 & \\ 0 & \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 & -\frac{2}{3} & 0 \\ 0 & 1 & -1 & \frac{1}{2} & 0 \\ 0 & 3 & -2 & 0 & 0 \\ 0 & 1 & 0 & 5 & 0 \end{bmatrix}$$

$R_3 = R_3 - 3R_2$

$R_4 = R_4 - R_2$

$$\begin{bmatrix} 1 & 0 & 0 & -2/3 & 0 \\ 0 & 1 & -1 & 1/2 & 0 \\ 0 & 0 & 1 & -3/2 & 0 \\ 0 & 0 & 1 & 9/2 & 0 \end{bmatrix}$$

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$R_2 = R_2 + R_3 \qquad R_4 = R_4 - R_3$

b:

$$\begin{bmatrix} 1 & 0 & 0 & -2/3 & 0 \\ 0 & 1 & 0 & -1 & 0 \\ 0 & 0 & 1 & -3/2 & 0 \\ 0 & 0 & 0 & 6 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 & -2/3 & 0 \\ 0 & 1 & 0 & -1 & 0 \\ 0 & 0 & 1 & -3/2 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

$R_1 = R_1 + \frac{2}{3} R_4 \qquad R_2 = R_2 + R_4$

$R_3 = R_3 + \frac{3}{2} R_4$

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

vectors are linearly independent

Q5

a)

$$\lambda_1 \begin{bmatrix} 1 & 0 \\ 1 & 2 \end{bmatrix} + \lambda_2 \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix} + \lambda_3 \begin{bmatrix} 0 & 1 \\ 2 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$\lambda_1 + \lambda_2 = 0$

$+ 2\lambda_2 + \lambda_3 = 0$

$\lambda_1 + 2\lambda_2 + 2\lambda_3 = 0$

$2\lambda_1 + \lambda_2 + \lambda_3 = 0$

$$\begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 2 & 1 & 0 \\ 1 & 2 & 2 & 0 \\ 2 & 1 & 1 & 0 \end{bmatrix}$$

$R_3 = R_3 - R_1 \qquad R_4 = R_4 - 2R_1$

$$\begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 2 & 1 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & -1 & 1 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 1 & 1/2 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & -1 & 1 & 0 \end{bmatrix}$$

$R_1 = R_1 - R_2$
$R_3 = R_3 - R_2$
$R_4 = R_4 + R_2$

$$\begin{bmatrix} 1 & 0 & -1/2 & 0 \\ 0 & 1 & 1/2 & 0 \\ 0 & 0 & 3/2 & 0 \\ 0 & 0 & 3/2 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & -1/2 & 0 \\ 0 & 1 & 1/2 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 3/2 & 0 \end{bmatrix}$$

$R_1 = R_1 + \frac{1}{2}R_3$
$R_2 = R_2 - \frac{1}{2}R_3$
$R_4 = R_4 - \frac{3}{2}R_3$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

vector are linearly independent

b)

$$k_1 \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} + k_2 \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} + k_3 \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

vectors are linearly independent.

Q7

a)

$v_1 = (2, -2, 0)$ $v_2 = (6, 1, 4)$, $v_3 = (2, 0, -4)$

$$\left[\begin{array}{ccc|c} 2 & 6 & 2 & 0 \\ -2 & 1 & 0 & 0 \\ 0 & 4 & -4 & 0 \end{array}\right]$$

$$\begin{bmatrix} 1 & 3 & 1 & 0 \\ -2 & 1 & 0 & 0 \\ 0 & 4 & -4 & 0 \end{bmatrix}$$

$R_2 = R_2 + 2R_1$

$$\begin{bmatrix} 1 & 3 & 1 & 0 \\ 0 & 7 & 2 & 0 \\ 0 & 4 & -4 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 3 & 1 & 0 \\ 0 & 1 & 2/7 & 0 \\ 0 & 4 & -4 & 0 \end{bmatrix}$$

$R_1 = R_1 - 3R_2 \quad R_3 = R_3 - 4R_2$

$$\begin{bmatrix} 1 & 0 & 1/7 & 0 \\ 0 & 1 & 2/7 & 0 \\ 0 & 0 & -\frac{36}{7} & 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 1/7 & 0 \\ 0 & 1 & 2/7 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

$R_1 = R_1 - \frac{1}{7}R_3 \quad R_2 = R_2 - \frac{2}{7}R_3$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

∴ vectors are linearly independent so lie in the same plane.

b)

$$\begin{bmatrix} -6 & 3 & 4 & 0 \\ 7 & 2 & -1 & 0 \\ 2 & 4 & 2 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -1/2 & -2/3 & 0 \\ 7 & 2 & -1 & 0 \\ 2 & 4 & 2 & 0 \end{bmatrix}$$

$R_2 = R_2 - 7R_1 \quad R_3 = R_3 - 2R_1$

$$\begin{bmatrix} 1 & -1/2 & -2/3 & 0 \\ 0 & \frac{11}{2} & 11/3 & 0 \\ 0 & 5 & 10/3 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -\frac{1}{2} & -\frac{2}{3} & 0 \\ 0 & 1 & 2/3 & 0 \\ 0 & 5 & 10/3 & 0 \end{bmatrix}$$

$R_1 = R_1 + \frac{1}{2}R_2 \quad R_3 = R_3 - 5R_2$

$$\begin{bmatrix} 1 & 0 & -1/3 & 0 \\ 0 & 1 & 2/3 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

vectors are linearly dependent so does not lie in the same plane.

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Q9

$$\begin{bmatrix} 0 & 6 & 4 & 0 \\ 3 & 0 & -7 & 0 \\ 1 & 5 & 1 & 0 \\ -1 & 1 & 3 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 5 & 1 & 0 \\ 3 & 0 & -7 & 0 \\ 0 & 6 & 4 & 0 \\ -1 & 1 & 3 & 0 \end{bmatrix}$$

$R_2 = R_2 - 3R_1 \qquad R_4 = R_4 + R_1$

$$\begin{bmatrix} 1 & 5 & 1 & 0 \\ 0 & -15 & -10 & 0 \\ 0 & 6 & 4 & 0 \\ 0 & 6 & 4 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 5 & 1 & 0 \\ 0 & 1 & 2/3 & 0 \\ 0 & 6 & 4 & 0 \\ 0 & 6 & 4 & 0 \end{bmatrix}$$

$R_1 = R_1 - 5R_2 \quad R_3 = R_3 - 6R_2 \quad R_4 = R_4 - 6R_2$

$$\begin{bmatrix} 1 & 6 & -7/3 & 0 \\ 0 & 1 & 2/3 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Date

vectors are linearly dependent

b)

$V_1 = K_1V_2 + K_2V_3$

$$\begin{bmatrix} \cancel{0} & 6 & 4 & 0 \\ \cancel{3} & 0 & -7 & 3 \\ \cancel{1} & 5 & 1 & 1 \\ \cancel{-1} & 1 & 3 & -1 \end{bmatrix}$$

Interchanging R_1 & R_4 ; $R_3 = R_3 - 5R_1$; $R_4 = R_4 - 6R_1$

$$\begin{bmatrix} 1 & 3 & -1 \\ 0 & -7 & 3 \\ 0 & -14 & 6 \\ 0 & -14 & 6 \end{bmatrix}$$

$R_2 = R_2 / -7$; $R_1 = R_1 - 3R_2$; $R_3 = R_3 + 14R_2$; $R_4 = R_4 + 14R$

$$\begin{bmatrix} 1 & 0 & +2/7 \\ 0 & 1 & -3/7 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\boxed{V_1 = \frac{2}{7}V_2 - \frac{3}{7}V_3}$$

$v_2 = K_1 V_1 + K_2 V_3$

$$\begin{bmatrix} 0 & 4 & 6 \\ 3 & -7 & 0 \\ 1 & 1 & 5 \\ -1 & 3 & 1 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 1 & 5 \\ 3 & -7 & 0 \\ 0 & 4 & 6 \\ -1 & 3 & 1 \end{bmatrix}$$

$R_2 = R_2 - 3R_1 \qquad R_4 = R_4 + R_1$

$$\begin{bmatrix} 1 & 1 & 5 \\ 0 & -10 & -15 \\ 0 & 4 & 6 \\ 0 & 4 & 6 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 1 & 5 \\ 0 & 1 & 3/2 \\ 0 & 4 & 6 \\ 0 & 4 & 6 \end{bmatrix}$$

$R_1 = R_1 - R_2 \qquad R_3 = R_3 - 4R_2 \qquad R_4 = R_4 - 4R_2$

$$\begin{bmatrix} 1 & 0 & 7/2 \\ 0 & 1 & 3/2 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\boxed{V_2 = \frac{7}{2} V_1 + \frac{3}{2} V_3}$$

$V_3 = K_1 V_1 + K_2 V_2$ (ii)

$$\begin{bmatrix} 0 & 6 & 4 \\ 3 & 0 & -7 \\ 1 & 5 & 1 \\ -1 & 1 & 3 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 5 & 1 \\ 3 & 0 & -7 \\ 0 & 6 & 4 \\ -1 & 1 & 3 \end{bmatrix}$$

$R_2 = R_2 - 3R_1 \qquad R_4 = R_4 + R_1$

$$\begin{bmatrix} 1 & 5 & 1 \\ 0 & -15 & -10 \\ 0 & 6 & 4 \\ 0 & 6 & 4 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 5 & 1 \\ 0 & 1 & 2/3 \\ 0 & 6 & 4 \\ 0 & 6 & 4 \end{bmatrix}$$

$R_1 = R_1 - 5R_2 \qquad R_3 = R_3 - 6R_1 \qquad R_4 = R_4 - 6R_1$

$$\begin{bmatrix} 1 & 0 & -7/3 \\ 0 & 1 & 2/3 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\boxed{V_3 = -\frac{7}{3} V_1 + \frac{2}{3} V_2}$$

Date

Q11

If v_1, v_2 & v_3 are linearly dependent thats its determinant is zero.

$$\begin{array}{ccc} v_1 & v_2 & v_3 \end{array}$$

$$\begin{vmatrix} \lambda & -\frac{1}{2} & -\frac{1}{2} \\ -\frac{1}{2} & \lambda & -\frac{1}{2} \\ -\frac{1}{2} & -\frac{1}{2} & \lambda \end{vmatrix} = 0$$

$$\lambda \begin{vmatrix} \lambda & -\frac{1}{2} \\ -\frac{1}{2} & \lambda \end{vmatrix} + \frac{1}{2} \begin{vmatrix} -\frac{1}{2} & -\frac{1}{2} \\ -\frac{1}{2} & \lambda \end{vmatrix} - \frac{1}{2} \begin{vmatrix} -\frac{1}{2} & \lambda \\ -\frac{1}{2} & -\frac{1}{2} \end{vmatrix} = 0$$

$$\lambda\left(\lambda^2 - \frac{1}{4}\right) + \frac{1}{2}\left(-\frac{\lambda}{2} - \frac{1}{4}\right) - \frac{1}{2}\left(\frac{1}{4} + \frac{\lambda}{2}\right) = 0$$

$$\lambda^3 - \frac{\lambda}{4} - \frac{\lambda}{4} - \frac{1}{8} - \frac{1}{8} - \frac{\lambda}{4} = 0$$

$$\lambda^3 - \frac{3\lambda}{4} - \frac{1}{4} = 0$$

$$4\lambda^3 - 3\lambda - 1 = 0$$

$$4\lambda^3 - 4\lambda^2 + 4\lambda^2 - 4\lambda + \lambda - 1 = 0$$

$$4\lambda^2(\lambda - 1) + 4\lambda(\lambda - 1) + 1(\lambda - 1) = 0$$

$$(\lambda - 1)(4\lambda^2 + 4\lambda + 1) = 0$$

$$(\lambda - 1)(2\lambda + 1)^2 = 0$$

$\lambda - 1 = 0$ $\qquad$ $(2\lambda + 1)^2 = 0$

$\boxed{\lambda = 1}$ $\qquad$ $\boxed{\lambda = -\frac{1}{2}}$

Exercise 4.4

Date

Q1

Given set of vectors forms a basis for R_2 if the determinant of its coefficients not equal to zero.

$$D = \begin{vmatrix} 2 & 3 \\ 1 & 0 \end{vmatrix}$$

of $ad - bc$

$D = 2(0) - 3(1)$

$D = -3$

$D \neq 0$

Q5

we have to prove matrices

For linear independency :-

$c_1M_1 + c_2M_2 + c_3M_3 + c_4M_4 = 0$

$c_1 = c_2 = c_3 = c_4 = 0$

For span ;

$c_1M_1 + c_2M_2 + c_3M_3 + c_4M_4 = b$

where $b = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$.

$3c_1 + 0c_2 + 0c_3 + c_4 = a$

$6c_1 - c_2 - 8c_3 + 0c_4 = b$

$3c_1 - c_2 - 12c_3 - c_4 = 0$

$-6c_1 + 0c_2 - 4c_3 + 2c_4 = 0$

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$$\begin{bmatrix} 3 & 0 & 0 & 1 & 0 \\ 6 & -1 & -8 & 0 & 0 \\ 3 & -1 & -12 & -1 & 0 \\ -6 & 0 & -4 & 2 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 & 1/3 & 0 \\ 6 & -1 & -8 & 0 & 0 \\ 3 & -1 & -12 & -1 & 0 \\ -6 & 0 & -4 & 2 & 0 \end{bmatrix}$$

$R_2 = R_2 - 6R_1$ $R_3 = R_3 - 3R_1$ $R_4 = R_4 + 6R_1$

$$\begin{bmatrix} 1 & 0 & 0 & 1/3 & 0 \\ 0 & -1 & -8 & -2 & 0 \\ 0 & -1 & -12 & -2 & 0 \\ 0 & 0 & -4 & 4 & 0 \end{bmatrix}$$

$R_2 = R_2/-1$ then $R_3 = R_3 + R_2$

$$\begin{bmatrix} 1 & 0 & 0 & 1/3 & 0 \\ 0 & 1 & 8 & 2 & 0 \\ 0 & 0 & -4 & 0 & 0 \\ 0 & 0 & -4 & 4 & 0 \end{bmatrix}$$

$R_3 = R_3/-4$ then $R_2 = R_2 - 8R_3$, $R_4 = R_4 + 4R_3$

$$\begin{bmatrix} 1 & 0 & 0 & 1/3 & 0 \\ 0 & 1 & 0 & 2 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 4 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

vectors are linearly independent.

Q7

a) $c_1V_1 + c_2V_2 + c_3V_3 = 0$

$c_1(2,-3,1) + c_2(4,1,1) + c_3(0,-7,1) = (0,0,0)$

$2c_1 + 4c_2 + 0c_3 = 0$

$-3c_1 + c_2 - 7c_3 = 0$

$c_1 + c_2 + c_3 = 0$

$$d = \begin{vmatrix} 2 & 4 & 0 \\ -3 & 1 & -7 \\ 1 & 1 & 1 \end{vmatrix}$$

$$2\begin{vmatrix} 1 & -7 \\ 1 & 1 \end{vmatrix} - 4\begin{vmatrix} -3 & -7 \\ 1 & 1 \end{vmatrix} + 0$$

$2(8) - 4(4)$

$16 - 16$

$d = 0$

determinant is zero so the vectors is not basis of R_3

Date

b)

$$c_1 v_1 + c_2 v_2 + c_3 v_3 = 0$$

$$c_1(1,6,4) + c_2(2,4,-1) + c_3(-1,2,5)$$

~~$(c_1 + 6c_1 + 4c_1)$~~ $+$

$$c_1 + 2c_2 - c_3 = 0$$

$$6c_1 + 4c_2 + 2c_3 = 0$$

$$4c_1 - c_2 + 5c_3 = 0$$

$$\begin{vmatrix} 1 & 2 & -1 \\ 6 & 4 & 2 \\ 4 & -1 & 5 \end{vmatrix}$$

$$1\begin{vmatrix} 4 & 2 \\ -1 & 5 \end{vmatrix} - 2\begin{vmatrix} 6 & 2 \\ 4 & 5 \end{vmatrix} - 1\begin{vmatrix} 6 & 4 \\ 4 & -1 \end{vmatrix}$$

$$1(22) - 2(22) - 1(-22)$$

$$22 - 44 + 22$$

$$d = 0$$

determinant is zero so the vectors is not basis of R_3.

Exercise 4.5

Date

Q1

$$\begin{bmatrix} 1 & 1 & -1 & 0 \\ -2 & -1 & 2 & 0 \\ -1 & 0 & 1 & 0 \end{bmatrix}$$

$R_2 = R_2 + 2R_1$, $R_3 = R_3 + R_1$

$$\begin{bmatrix} 1 & 1 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}$$

$R_1 = R_1 - R_2$ $\quad$ $R_3 = R_3 - R_2$

$$\begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$x_1 - x_3 = 0$ $\quad$ $x_2 = 0$

$x_1 = x_3$

Let

$x_3 = t$

$x_1 = t$, $x_2 = 0$, $x_3 = t$

$(x_1, x_2, x_3) = (t, 0, t)$

$(x_1, x_2, x_3) = t(1, 0, 1)$

Basis $= \{(1, 0, 1)\}$

dimension $= 1$

b) **Q3**

$$\left[\begin{array}{ccc|c} 2 & 1 & 3 & 0 \\ 1 & 0 & 5 & 0 \\ 0 & 1 & 1 & 0 \end{array}\right]$$

$R_1 = R_1/2$

$$\begin{bmatrix} 1 & 1/2 & 3/2 & 0 \\ 1 & 0 & 5 & 0 \\ 0 & 1 & 1 & 0 \end{bmatrix}$$

$R_2 = R_2 - R_1$

$$\begin{bmatrix} 1 & 1/2 & 3/2 & 0 \\ 0 & -1/2 & 7/2 & 0 \\ 0 & 1 & 1 & 0 \end{bmatrix}$$

$R_2 = R_2 \times -2$

$$\begin{bmatrix} 1 & 1/2 & 3/2 & 0 \\ 0 & 1 & -7 & 0 \\ 0 & 1 & 1 & 0 \end{bmatrix}$$

$R_1 = R_1 - \frac{1}{2}R_2 \qquad R_3 = R_3 - R_2$

$$\begin{bmatrix} 1 & 0 & 5 & 0 \\ 0 & 1 & -7 & 0 \\ 0 & 0 & 8 & 0 \end{bmatrix}$$

$R_3 = R_3/8$ then $R_1 = R_1 - 5R_3$, $R_2 = R_2 + 7R_3$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

$x_1 = 0, \quad x_2 = 0, \quad x_3 = 0$

No Basis

Dimension $= 0$

Q5

$$\begin{bmatrix} 1 & -3 & 1 & 0 \\ 2 & -6 & 2 & 0 \\ 3 & -9 & 3 & 0 \end{bmatrix}$$

$R_2 = R_2 - 2R_1 \qquad R_3 = R_3 - 3R_1$

$$\begin{bmatrix} 1 & -3 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$x_1 - 3x_2 + x_3 = 0$

let $x_2 = t, \quad x_3 = s$

$x_1 = +3t - s$

$x_2 = t$

$x_3 = s$

$(x_1, x_2, x_3) = (3t - s, t, s)$

$(x_1, x_2, x_3) = t(3,1,0) + s(-1,0,1)$

Basis $= \{(3,1,0), (-1,0,1)\}$

Dimension $= 2$

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Q17

$av_1 + bv_2 + cv_3 + dv_4 = 0$

$$\begin{bmatrix} 1 & 1 & 2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & -1 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 & 2 & 0 & 0 \\ 0 & 1 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$a + c + d = 0 \Rightarrow a = -c - d$

$b + c - d = 0 \Rightarrow b = d - c$

c, d are free variables then any of the vectors v_3, v_4 can be expressed as a linear combination of the vectors v_1 & v_2

Basis $\Rightarrow \{(1, 0, 0), (1, 0, 1)\}$

Date

Q18

$a_1v_1 + bv_2 + cv_3 + dv_4 = 0$

$$\begin{bmatrix} 1 & 2 & 0 & 3 & 0 \\ 1 & 2 & 0 & 3 & 0 \\ 1 & 2 & 0 & 3 & 0 \\ 1 & 0 & 3 & 4 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 2 & 0 & 3 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & -2 & 3 & 1 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 2 & 0 & 3 & 0 \\ 0 & 1 & -3/2 & -1/2 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 & 4 & 0 \\ 0 & 1 & -3/2 & -1/2 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$a = -4d$

$b = \frac{3}{2}c + \frac{1}{2}d$

c, d are free vectors, 8 1 D

Basis = $\{ \cancel{\ } v_1 , v_2 \}$

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TRUE / FALSE

Date

EXERCISE 1.1

a) True
b) False
c) True
d) False
e) False
f) False
g) True
h) False

EXERCISE 1.2

a) True
b) False
c) False
d) True
e) True
f) False
g) True
h) False
i) False.

EXERCISE 1.3

a) True
b) False
c) False
d) False
e) True
f) False
g) False
h) True
i) True.
j) True
K True
l) False
m) True
n) True
o) False

EXERCISE 1.6

a) True
b) True
c) True
d) True
e) True
f) True
g) True.

c, d
Basr

EXERCISE 1.7

a) True
b) False
c) False
d) True
e) True
f) False
g) False
h) True
i) True
j) False
k) False
l) False
m) True

EXERCISE 2.1

a) False
b) False
c) True
d) True
e) True
f) True
g) False
h) False
i) False
j) True

EXERCISE 2.2

a) True
b) True
c) False
d) False
e) True
f) True

EXERCISE 2.3

a) False
b) False
c) True
d) False
e) True
f) True
g) True
h) True
i) True
j) True
k) True
l) False

EXERCISE 3.1

a] False
b] False
c] False
d] True
e] True
f] False
g] False
h) True
i) False
j) True
k) False

EXERCISE ~~2.1~~ 3.2

a) True
b) True
c) False
d) True
e) True
f) False
g) False
h) False
i) True
j) True
~~k)~~

EXERCISE 3.3

a) True
b) True
c) True
d) True
e) True
f) False
g) False

EXERCISE 3.5

a) True
b) True
c) False
d) True
e) False
f False

Date

Exercise 4.2

a) True
b) True
c) False
d) False
e) False
f) True
g) True
h) False
i) False
j) True
k) False

Ex 4.3

a) False
b) True
c) False
d) True
e) True
f) False
g) True
h) False

Exercise 4.4

a) False
b) False
c) True
d) True
e) False

Exercise 4.5

a) True
b) True
c) False
~~d) True~~
e) True
f) True
g) True
h) True
i) True
j) False
k) False