Exercise 13.8

Assignment 15

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$$f(x,y) = y^2 + xy + 3y + 2x + 3$$

$$f_x = y + 2$$

$$f_y = 2y + x + 3$$

For critical point

$y + 2 = 0$ $\quad$ $2y + x + 3 = 0$

$y = -2$ $\quad$ $2(-2) + x + 3 = 0$

$x = 1$

$(x_0, y_0) = (1, -2)$

$$D = f_{xx}(x_0, y_0)\, f_{yy}(x_0, y_0) - f^2_{xy}(x_0, y_0)$$

$f_{xx}(1,-2) = 0$

$f_{yy}(1,-2) = 2$

$f^2_{xy} = 1$

$D = (0)(2) - 1$

$D = -1$

$D < 0$, so $(1,-2)$ is a saddle point of function.

Q11

$$f(x,y) = x^2 + xy + y^2 - 3x$$

$$f_x = 2x + y - 3$$
$$f_y = 2y + x$$

For critical point :-

$$2x + y = 3$$
$$x + 2y = 0$$

$$x = -2y$$

$$2(-2y) + y = 3$$
$$-4y + y = 3$$
$$-3y = 3$$
$$y = -1$$

$$x = -2(-1)$$

$$x = 2$$

$$(x_0, y_0) = (2, -1)$$

$$D = f_{xx}(x_0, y_0)\, f_{yy}(x_0, y_0) - f^2_{xy}(x_0, y_0)$$

$$f_{xx} = 2$$

$$f_{yy} = 2$$

$f^2xy = 1$

$D = (2)(2) - 1$

$D = 3$

$D > 0 \quad . \quad fxx > 0$

function has relative minima at $(2, -1)$

Q 13

$$f(x,y) = x^2 + y^2 + \frac{2}{xy}$$

$$fx = 2x - \frac{2}{x^2y}$$

$$fy = 2y - \frac{2}{xy^2}$$

For critical point

$$2x - \frac{2}{x^2y} = 0$$

$$2x^3y = 2$$

$$y = \frac{1}{x^3}$$

$y = 1$, $y = -1$

$$2y = \frac{2}{xy^2} = 0$$

$$2xy^3 = 2$$

$$x\left(\frac{1}{x^3}\right)^3 = 1$$

$$\frac{1}{x^8} = 1$$

$x^8 = 1 \Rightarrow x = \pm 1$

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Critical points are

$(1,1), (-1,-1)$

$$f_{xx} = 2 + \frac{2}{x^3 y}$$

$$f_{yy} = 2 + \frac{2}{x y^3}$$

$$f_{xy} = \frac{2}{x^2 y^2}$$

$$D = f_{xx} f_{yy} - f_{xy}^2$$

x	y	f_{xx}	f_{xy}	f_{xy}	D	
1	1	4	4	2	12	Minima
-1	-1	4	4	2	12	Minima

function has relative minimum at $(1,1)$ and $(-1,-1)$.

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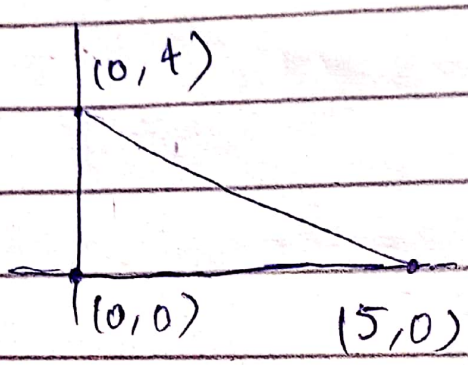
Q31

$$f(x,y) = xy - x - 3y$$



Critical points

$f_x = y - 1$

$f_y = x - 3$

$f_x = 0 \quad f_y = 0$

$(x_0, y_0) = (3, 1)$

Boundry Points

a) line segment joining (0, 0) & (5, 0)

Eq $\rightarrow y = 0$

$U(x) = f(x, 0) = -x \qquad 0 \leq x \leq 5$

$U'(x) = -1$

(0,0), (5,0)

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b) line segment joining (0,0) & (0,4)

Eq → $x = 0$

$f(0,y) = -3y = v(x)$ $\quad 0 \le y \le 4$

$v'(x) = -3$

(0,0), (0,4)

c) Line segment joining (0,4) & (5,0)

Eq → $y = -\frac{4}{5}x + 4$ $\quad 0 \le x \le 5$

$$f(x,y) = x\left(-\frac{4}{5}x+4\right) = x - 3\left(-\frac{4}{5}x + 4\right)$$

$$= -\frac{4}{5}x^2 + 4x - x + \frac{12}{5}x - 12$$

$$f(x,y) = -\frac{4}{5}x^2 + \frac{27}{5}x - 12 = w(x)$$

$$w'(x) = -\frac{8}{5}x + \frac{27}{5}$$

$$-\frac{8}{5}x + \frac{27}{5} = 0$$

$$x = \frac{27}{8}$$

$$y = \frac{13}{10} \qquad \left(\frac{27}{8}, \frac{13}{10}\right)$$

Put critical points in function

critical points are: (3,1), (0,0), (5,0), (0,4), (27/8, 13/10)

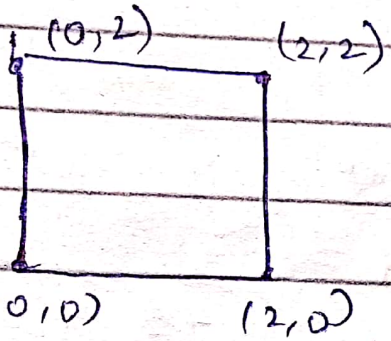
x	3	0	5	0	27/8
y	1	0	0	4	13/10
f(x,y)	-3	0	-5	-12	-2.88

Absolute minimum -12

Absolute maximum 0

Q33

$$f(x,y) = x^2 - 3y^2 - 2x + 6y$$



Critical point

$$fx = 2x - 2$$

$$fy = -6y + 6$$

$fx = 0 \qquad fy = 0$

$(x_0, y_0) = (1, +1)$

Boundary point:

a) Line segment joining $(0,0)$ & $(0,2)$

Eq $\rightarrow x = 0$

$t(x) \Rightarrow f(0,y) = -3y^2 + 6y$

$t'(x) = 6y - 6$

$6y - 6 = 0$

$x = 0 \qquad y = 1 \qquad (0,1)$

b) line segment joining (2,0) & (2,2)

Eq → $x = 2$

$v(y) = f(2,y) = -3y^2 + 6y$

$v'(y) = -6y + 6$

$-6y + 6 = 0$

$y = +1$

$(2, +1)$

c) line segment joining (0,0) & (2,0)

$y = 0$

$v(x) = f(x,0) = x^2 - 2x$

$v'(x) = 2x - 2$

$v'(x) = 0$

$2x - 2 = 0$

$x = 1$

$(1, 0)$

d) line segment joining (0,2) & (2,2)

$y = 2$

$w(x) = f(x,2) = x^2 - 2x$

$w'(x) = 2x - 2$

$2x - 2 = 0$

$x = 1$

$(1, 2)$

put critical point in f(x,y)

critical points ⇒ (1,-1) & (1,2), (2,-1), (1,0)

x	1	0	2	1	1
y	+1	1	+1	0	2
f(x,y)	2	3	+3	-1	-1

Absolute minimum → -1

Absolute maximum → +3

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$f(x,y) = x^2 + 2y^2 - x$

Critical point :-

$f_x = 2x - 1$

$f_y = 4y$

$f_x = 0 \quad f_y = 0$

$(x_0, y_0) = (\frac{1}{2}, 0)$

critical Point along Region :-

$x^2 + y^2 = 4$

$y = \sqrt{4 - x^2}$

$f(x,y) = x^2 + 2(4 - x^2) - x$

$u(x) = f(x,y) = 8 - x^2 - x$

$u'(x) = -2x - 1$

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$-2x - 1 = 0$

$x = -\frac{1}{2}$, $y = \pm\frac{\sqrt{15}}{2}$

$\left(-\frac{1}{2}, \frac{\sqrt{15}}{2}\right)$, $\left(-\frac{1}{2}, -\frac{\sqrt{15}}{2}\right)$

Find absolute maximum & minimum

Critical points are;

$\left(\frac{1}{2}, 0\right)$, $\left(-\frac{1}{2}, \frac{\sqrt{15}}{2}\right)$, $\left(-\frac{1}{2}, -\frac{\sqrt{15}}{2}\right)$

x	1/2	$-\frac{1}{2}$	$-\frac{1}{2}$
y	0	$\frac{\sqrt{15}}{2}$	$\frac{-\sqrt{15}}{2}$
f(x, y)	$-\frac{1}{4} = -0.25$	$33/4 = 8.25$	$33/4$

Absolute minimum → $-1/4$

Absolute maximum → $33/4$