Exercise 14.2

Assignment 17

Date

Q3

$$= \int_0^3 \int_0^{\sqrt{9-y^2}} y \, dx \, dy \, \cancel{dx}$$

$$= \int_0^3 \cancel{x} y \, |x|_0^{\sqrt{9-y^2}} \, dy$$

$$= \int_0^3 y\sqrt{9-y^2} \, dy$$

$$\int f(x)^n f'(x) \, dx = \frac{f(x)^{n+1}}{n+1}$$

$$= -\frac{1}{2} \int_0^3 -2y \sqrt{9-y^2}$$

$$= -\frac{1}{2} \times \frac{(9-y^2)^{\frac{1}{2}+1}}{\frac{1}{2}+1} \Big|_0^3$$

$$= -\frac{1}{\cancel{2}} \times \frac{\cancel{2}(9-y^2)^{3/2}}{3}$$

$$= -\frac{1}{3} \left|(9-y^2)^{3/2}\right|_0^3$$

$$= -\frac{1}{3} \left\{ (9-(3)^2)^{3/2} - (9-0^2)^{3/2} \right\}$$

$$= -\frac{1}{3} \times -27$$

$$= 9$$

No.

Date

Q5

$$= \int_{\sqrt{\pi}}^{\sqrt{2\pi}} \int_{0}^{x^3} \sin \frac{y}{x} \, dy \, dx$$

$$= \int_{\sqrt{\pi}}^{\sqrt{2\pi}} \left(-x \cos \frac{y}{x} \Big|_0^{x^3} \right) dx$$

$$= - \int_{\sqrt{\pi}}^{\sqrt{2\pi}} x \left(\cos \frac{x^3}{x^2} - \cos \frac{0}{x} \right)$$

$$= - \int_{\sqrt{\pi}}^{\sqrt{2\pi}} (x \cos x^2 - x) \, dx$$

$$= - \frac{1}{2} \int_{\sqrt{\pi}}^{\sqrt{2\pi}} 2x \cos x^2 - \int_{\sqrt{\pi}}^{\sqrt{2\pi}} x$$

$$= - \left(\frac{\sin x^2}{2} - \frac{x^2}{2} \right) \Big|_{\sqrt{\pi}}^{\sqrt{2\pi}}$$

$$= - \frac{1}{2} \left(\sin x^2 - x^2 \right)_{\sqrt{\pi}}^{\sqrt{2\pi}}$$

$$= - \frac{1}{2} \left[\cancel{\sin 4\pi} \; (\sin 2\pi - 2\pi) - (\sin \pi - \pi) \right]$$

$$= + \frac{\pi}{2}$$

No.

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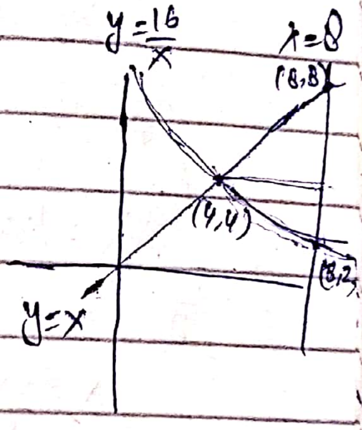
Q15

$$\iint_R x^2 \, dA$$

$y = \frac{16}{x}$ $y = x$ $x = 8$



Type I :-

$$= \int_4^8 \int_{\frac{16}{x}}^{x} x^2 \, dy\,dx$$

$$= \int_4^8 x^2 y \Big|_{\frac{16}{x}}^{x} dx$$

$$= \int_4^8 x^2 \left(x - \frac{16}{x}\right) dx$$

$$= \int_4^8 x(x^2 - 16)\, dx$$

$$= \frac{1}{2} \int_4^8 2x\,(x^2 - 16)\, dx$$

$$= \frac{1}{2} \times \frac{(x^2-16)^2}{2} \Big|_4^8$$

$$= \frac{1}{4}\left[(8^2-16)^2 - (4^2-16)^2\right]$$

$y = x$

$y = \frac{16}{x}$

$x = \frac{16}{x}$

$x^2 = 16$

$x = \pm 4$

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$$= \frac{2304}{4}$$

$$= 576$$

Type II

$$\int_{\frac{16}{y}}^{y} x^2 \, dx \, dy$$

$4 \quad 8$

8

Type I

$$= \int_2^4 \int_{16/y}^8 x^2 \, dx\,dy + \int_4^8 \int_y^8 x^2 \, dx\,dy$$

$$= \int_2^4 \frac{x^3}{3}\Big|_{16/y}^{8} dy + \int_4^8 \frac{x^3}{3}\Big|_y^8$$

$$= \int_2^4 \frac{8^3 - (16/y)^3}{3} dy + \int_4^8 \frac{8^3 - y^3}{3} dy.$$

$$= \int_2^4 \frac{8^3}{3} - \frac{1}{3}\int_2^4 \frac{16^3}{y^3} + \int_4^8 \frac{8^3}{3} - \int_4^8 \frac{y^3}{3}$$

$$= \frac{8^3}{3} y\Big|_2^4 - \frac{16^3}{3(-2)y^2}\Big|_2^4 + \frac{8^3}{3} y\Big|_4^8 - \frac{1}{3(4)} y^4\Big|_4^8$$

$$\cancel{\frac{8^3}{3}(2) + \frac{16^3(2)}{6\,3(2)} + \frac{8^3}{3}(4) - \frac{1}{12\,3}(4)}$$

$$= \frac{1024}{3} - 128 + \frac{2048}{3} - 320$$

$$= 576$$

No. ______

Q17

$$= \iint_R (3x - 2y)\, dA$$

$$= x^2 + y^2 = 1$$

Type I

$$= \int_{-1}^{1}\int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} 3x - 2y \; dy\, dx$$

$$= \int_{-1}^{1} 3xy - \frac{\cancel{2}y^2}{\cancel{2}} \Big|_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} dx$$

$$= \int_{-1}^{1} \left(3x(\sqrt{1-x^2}) - \frac{\cancel{2}(1-x^2)}{\cancel{2}}\right) - \left(3x(-\sqrt{1-x^2}) - (1-x^2)\right)$$

$$= \int_{-1}^{1} 3x\sqrt{1-x^2} \cancel{-1} + \cancel{x^2} + 3x\sqrt{1-x^2} \cancel{+1} \cancel{-x^2}$$

$$= \int_{-1}^{1} 6x\sqrt{1-x^2}$$

$$= -\frac{6}{2}\int_{-1}^{1} -2x\sqrt{1-x^2}$$

$$= -\frac{6}{2} \times \frac{(1-x^2)^{3/2}}{3/2}\Big|_{-1}^{1}$$

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$$= \frac{-6}{2}\left(\frac{(1-1^2)^{3/2}}{3/2} - \frac{(1-(-1)^2)^{3/2}}{3/2}\right)$$

$$= 0$$

Type II

$$\int_{-1}^{1}\int_{-\sqrt{1-y^2}}^{\sqrt{1-y^2}} 3x-2y \; dx\,dy$$

$$\int_{-1}^{1} \frac{3x^2}{2} - 2xy \Big|_{-\sqrt{1-y^2}}^{\sqrt{1-y^2}} dy.$$

$$\int_{-1}^{1}\left(\left[\frac{3}{2}\cancel{x^2}(1-y^2) - 2(\sqrt{1-y^2})y\right] - \left[\frac{3}{2}(1-y^2)+2(\sqrt{1-y^2})y\right]\right)$$

$$\int_{-1}^{1}\left(\frac{3}{2} - \frac{3}{2}y^2 - 2y\sqrt{1-y^2} - \frac{3}{2} + \frac{3}{2}y^2 - 2y\sqrt{1-y^2}\right)$$

$$\frac{3}{2}y - \frac{3}{2}\frac{y^3}{3} + \frac{(1-y^2)^{3/2}}{3/2} - \frac{3}{2}y + \frac{3}{2}\frac{y^2}{2} + \frac{(1-y^2)^{3/2}}{3/2}\Bigg|_{-1}^{1}$$

$$\left(\frac{3}{2}(1) - \frac{3}{2}\frac{(1)^3}{3} + \frac{(1-1^2)^{3/2}}{3/2} - \frac{3}{2}(1) + \frac{3}{2}\frac{(1)^2}{2} + \frac{(1-1^2)^{3/2}}{3/2}\right) -$$

$$\left(\frac{3}{2}(-1) - \frac{3}{2}\frac{(-1)^3}{3} + \frac{(1-1^2)^{3/2}}{3/2} - \frac{3}{2}(-1) + \frac{3}{2}\frac{(-1)^2}{2} + \frac{(1-)^{2})^{3/2}}{3/2}\right)$$

No.

$$= \left(\frac{3}{2} - \frac{1}{2} + 0 - \frac{3}{2} + \frac{3}{4} + 0\right) - \left(-\frac{3}{2} + \frac{1}{2} + 0 + \frac{3}{2} - \frac{3}{4} + 0\right)$$

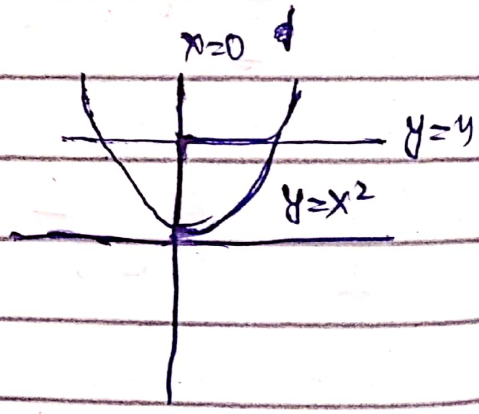
$$= \frac{1}{4} - \frac{1}{4}$$

$$= 0$$

Q19

$$\iint_R x(1+y^2)^{-\frac{1}{2}}\, dA$$

$y^{\cancel{2}} = x^2$, $y = 4$, and $x = 0$



$$\int_0^4 \int_0^{\sqrt{y}} x(1+y^2)^{-\frac{1}{2}}\, dx\, dy$$

$$\int_0^4 \left. \frac{x^2}{2}(1+y^2)^{-\frac{1}{2}} \right|_0^{\sqrt{y}} dy$$

$$\int_0^4 \frac{y}{2}(1+y^2)^{-\frac{1}{2}}\, dy$$

$$\frac{1}{4}\int_0^4 2y\,(1+y^2)^{-\frac{1}{2}}\, dy$$

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$$= \frac{1}{4} \times \frac{(1+y^2)^{\frac{1}{2}}}{\frac{1}{2}} \Big|_0^4$$

$$= \frac{1}{4}\left(2(1+(4)^2)^{\frac{1}{2}} - 2(1+0^2)^{1/2}\right)$$

$$= \frac{1}{4}(2\sqrt{17} - 2)$$

$$= \frac{\sqrt{17}}{2} - \frac{1}{2}$$

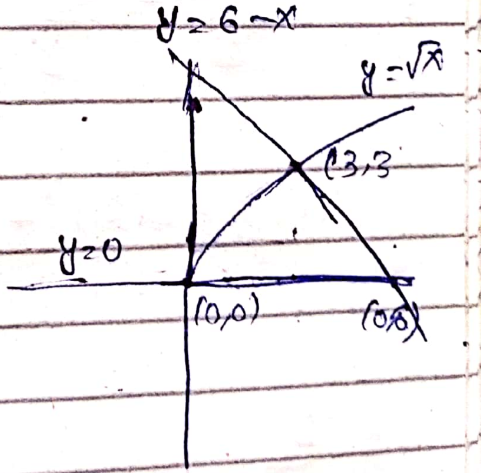
$$= \frac{\sqrt{17}-1}{2}$$

Q21

$$\iint_R xy \, dA$$

$y = \sqrt{x}$, $y = 6-x$, $y = 0$

$$\int_0^{2}\int_{y^2}^{6-y} xy \; dx\,dy$$



R.W

$6 - x = \sqrt{x}$

$36 - 12x + x^2 = x$

$x^2 - 13x + 36 = 0$

$x^2 - 4x - 9x + 36 = 0$

$x(x-4) - 9(x-4) = 0$

$x = 4$, $x = 9$

$\cancel{6 - x = \sqrt{x}}$

$\cancel{36 - 12x + x^2 = x}$

$12x = 36$ $\quad \cancel{x^2 + 13x + 36}$

$x = 3$

$\cancel{x(x-13) = 36}$

No.

$$= \int_0^2 \frac{x^2 y}{2} \Big|_{y^2}^{6-y} dy$$

$$= \int_0^2 \left[\frac{(6-y)^2 y}{2} - \frac{(y^2)^2 y}{2}\right] dy$$

$$= \int_0^2 \left(\frac{y(36-12y+y^2)}{2} - \frac{y^5}{2}\right) dy$$

$$= \int_0^2 \left(\frac{\overset{18}{\cancel{36}}y}{\cancel{2}} - \frac{\overset{6}{\cancel{12}}y^2}{\cancel{2}} + \frac{y^3}{2} - \frac{y^5}{2}\right) dy$$

$$= \int_0^2 \left(18y - 6y^2 + \frac{y^3}{2} - \frac{y^5}{2}\right) dy$$

$$= \frac{{}^{9}\cancel{18}y^2}{\cancel{2}} - \frac{{}^{2}\cancel{6}y^3}{\cancel{3}} + \frac{y^4}{2(4)} - \frac{y^6}{2(6)} \Bigg|_0^2$$

$$= 9(2)^2 - 2(2)^3 + \frac{(2)^4}{8} - \frac{(2)^6}{12}$$

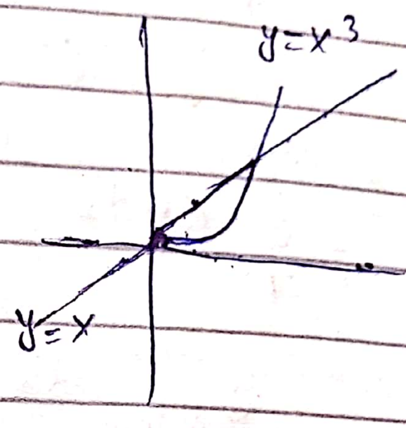
$$= \frac{50}{3}$$

Date

Q23

$$\iint_R (x-1)\,dA$$

$y = x \qquad y = x^3$



$$\int_0^1 \int_y^{y^{1/3}} (x-\cancel{1})\,dx\,dy$$

$$\int_0^1 \frac{x^2}{2} - x \Big|_y^{y^{1/3}} dy$$

$x^3 = x$

$x^3 - x = 0$

$x(x^2-\cancel{1}) = 0$

$x = 0 \quad x^2 - 1 = 0$

$x = 0 \quad x = \pm 1$

$$\int_0^1 \left(\frac{y^{\cancel{2/3}}}{2} - y^{\frac{1}{3}}\right) - \left(\frac{y^2}{2} - y\right) dy$$

$$\int_0^1 \frac{y^{\frac{2}{3}}}{2} - y^{\frac{1}{3}} - \frac{y^2}{2} + y \; dy$$

$$\frac{y^{\frac{\cancel{5}}{3}}}{2(\frac{\cancel{5}}{3})} - \frac{y^{\frac{4}{3}}}{4/3} - \frac{y^3}{2(3)} + \frac{y^2}{2}\Big|_0^1$$

$$\frac{\cancel{3}y^{10/9}}{10} - \frac{3y^{4/3}}{4} - \frac{y^3}{6} + \frac{y^2}{2}\Big|_0^1$$

$$= \frac{3(1)^{10/9}}{10} - \frac{3(1)^{4/3}}{4} - \frac{(1)^3}{6} + \frac{(1)^2}{2}$$

$$= \frac{-7}{60}$$

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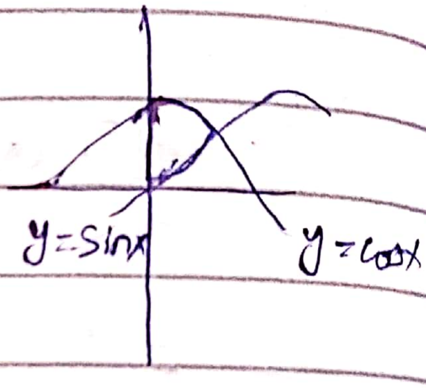
Q29

$$\iint_R dA$$

$y = \sin x \quad y = \cos x \quad 0 \le x \le \pi/4$

$$= \int_0^{\frac{\pi}{4}} \int_{\sin x}^{\cos x} dy\,dx$$



$$= \int_0^{\frac{\pi}{4}} y \Big|_{\sin x}^{\cos x} dx$$

$$= \int_0^{\frac{\pi}{4}} \cos x - \sin x \; dx$$

$$= (\sin x + \cos x) \Big|_0^{\frac{\pi}{4}}$$

$$= \left(\sin \frac{\pi}{4} + \cos \frac{\pi}{4}\right) - \left(\sin 0 + \cos(0)\right)$$

$$= \sqrt{2}\ \cancel{(\ )} - 1$$

$$= \cancel{\sqrt{2}}$$

$$= \boxed{\sqrt{2} - 1}$$

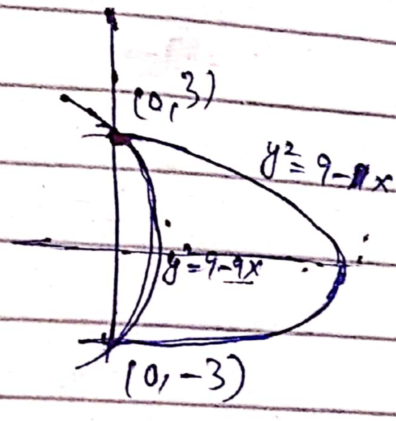
Date

Q31

$$\iint_R dA$$

$y^2 = 9 - x$ $\quad$ $y^2 = 9 - 9x$

$$\int_{-3}^{3}\int_{\frac{9-y^2}{9}}^{9-y^2} dx\,dy$$



$\cancel{9} - \cancel{x} = 9 - 9x$

$8x = 0$

$x = 0$

$$\int_{-3}^{3} x \Big|_{\frac{9-y^2}{9}}^{9-y^2} dy$$

$$\int_{-3}^{3}\left(9-y^2 - \frac{9-y^2}{9}\right)dy$$

$$\int_{-3}^{3}\left(9-y^2 - 1 + \frac{y^2}{9}\right)dy$$

$$8y - \frac{8y^3}{9(3)}\Big|_{-3}^{3}$$

$$\left(8(3) - \frac{8}{9}\frac{(3)^3}{3}\right) - \left(8(-3) - \frac{8}{9}\frac{(-3)^3}{3}\right)$$

$= 16 - (-16)$

$= 32$

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Date

Q39

$\iint_R dA$

$x^2+y^2=9$

$z=0,\ z=3-x$

$$= \int_{-3}^{3}\int_{-\sqrt{9-x^2}}^{\sqrt{9-x^2}} 3-x \ dy dx$$

$$= (3-x)y \Big|_{-\sqrt{9-x^2}}^{\sqrt{9-x^2}}$$

$$= \int_{-3}^{3} (3-x)(\sqrt{9-x^2}) + (3-x)(\sqrt{9-x^2}) \ dx$$

$$= \int_{-3}^{3} 3\sqrt{9-x^2} - x\sqrt{9-x^2} + 3\sqrt{9-x^2} - x\sqrt{9-x^2}$$

$$= \int_{-3}^{3} 6\sqrt{9-x^2} - 2x\sqrt{9-x^2} \ dx$$

$$= 6\int_{-3}^{3} \sqrt{9-x^2} + \int_{-3}^{3} -2x\sqrt{9-x^2}$$

$\because \sqrt{a^2-x^2} = \frac{1}{2}x\sqrt{a^2-x^2} + \frac{1}{2}a^2\sin^{-1}\left(\frac{x}{a}\right)+c$

~~$u = 9-x^2$~~

~~$\frac{du}{dx}$~~ $= -2x$

$$= 6\left(\frac{1}{2}x\sqrt{9-x^2} + \frac{1}{2}9\sin^{-1}\left(\frac{x}{3}\right)\right)\Big|_{-3}^{3} + \frac{(9-x^2)^{3/2}}{3/2}\Big|_{-3}^{3}$$

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$$= 6\left[\left(\frac{1}{2}(3)\sqrt{9-(3)^2} + \frac{1}{2}9\sin^{-1}\left(\frac{3}{3}\right)\right) - \left(\frac{1}{2}(-3)\sqrt{9-(-3)^2} + \frac{1}{2}9\sin^{-1}\left(\frac{-3}{3}\right)\right)\right] + \left[\frac{(9-3^2)^{3/2}}{3/2} - \frac{(9-(-3)^2)^{3/2}}{3/2}\right]$$

$$= 6\left(\frac{9\pi}{4}\right) + \frac{9\pi}{4}$$

$$\boxed{= 27\pi}$$

Q43

$$\iint_R y+3 \; dA$$

$$4x^2+y^2=9$$

$$\cancel{\int_{-\frac{\sqrt{9-y^2}}{2}}^{\frac{\sqrt{9-y^2}}{2}} y+3 \; dx\,dy}$$

$$\int_{-\frac{3}{2}}^{\frac{3}{2}}\int_{-\sqrt{9-4x^2}}^{+\sqrt{9-4x^2}} y+3 \; dy\,dx$$

$$\int_{-\frac{3}{2}}^{\frac{3}{2}} \left[\frac{y^2}{2} + 3y\right]_{\sqrt{9-4x^2}}^{\sqrt{9-4x^2}} dx$$

$$\int_{-3/2}^{3/2}\left(\frac{9-4x^2}{2} + 3\sqrt{9-4x^2}\right) - \left(\frac{9-4x^2}{2} - 3\sqrt{9-4x^2}\right) dx$$

No.

$$\int_{-\frac{3}{2}}^{\frac{3}{2}} 6\sqrt{9-4x^2}\,dx$$

$$6\int_{-3/2}^{3/2} 2\sqrt{\frac{9}{4}-x^2}$$

$$\cancel{12\int_{-3/2}^{3/2}} \quad \because \sqrt{a^2-x^2} = \frac{1}{2}x\sqrt{a^2-x^2} + \frac{a^2}{2}\sin^{-1}\left(\frac{x}{a}\right)$$

$$12\left[\left(\frac{1}{2}x\sqrt{\frac{9}{4}-x^2} + \frac{9}{4(2)}\sin^{-1}\left(\frac{x}{3/2}\right)\right)\right]_{-3/2}^{3/2}$$

$$12\left[\left[\frac{1}{2}\left(\frac{3}{2}\right)\cancel{\sqrt{\frac{9}{4}-\left(\frac{3}{2}\right)^2}} + \frac{9}{4(2)}\sin^{-1}\left(\frac{3/2}{3/2}\right)\right] - \right.$$

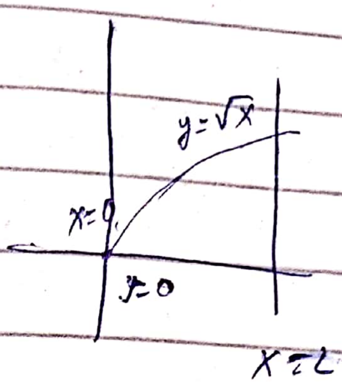
$$\left.\left[\frac{1}{2}\cancel{\left(\frac{-3}{2}\right)}\sqrt{\frac{9}{4}-\left(\frac{-3}{2}\right)^2} + \frac{9}{4(2)} \times \sin^{-1}\left(\frac{-3/2}{3/2}\right)\right]\right]$$

$$12\left(+\frac{9}{16}\pi - \left(-\frac{9}{16}\pi\right)\right)$$

$$12\left(+\frac{9\pi}{8}\right) = +\frac{27}{2}\pi$$

Q47

$$\int_0^2\int_0^{\sqrt{x}} f(x,y)\,dy\,dx$$



for dx

$\boxed{x=2}$

$y=\sqrt{x}$

$y^2=x$

$\boxed{x=y^2}$

for dy

$\boxed{y=0}$

$y=\sqrt{x}$

$\cancel{\sqrt{x}=0}$ $\boxed{y=\sqrt{2}}$

$\cancel{x=0}$

$$\int_0^{\sqrt{2}}\int_{y^2}^{2} f(x,y)\,dx\,dy$$

Q53

$$\int_0^1\int_{4x}^{4} e^{-y^2}\,dy\,dx$$

for dx

$\boxed{x=0}$

$y=4x$

$\boxed{x=\frac{4}{y}}$

for dy

$\boxed{y=4}$

$y=4(x)$

$y=4(0)$

$\boxed{y=0}$

$$\int_0^4 \int_0^{y/4} e^{-y^2}\, dx\, dy$$

$$\int_0^4 x e^{-y^2} \Big|_0^{y/4} dy$$

$$\int_0^4 \frac{y}{4} e^{-y^2} dy$$

$$\frac{-2}{-2} \int_0^4 \frac{y}{4} e^{-y^2} dy$$

$$-\frac{1}{8} \int_0^4 -2y e^{-y^2} dy$$

$$= -\frac{1}{8} e^{-y^2} \Big|_0^4$$

$$= -\frac{1}{8} \left[e^{-4^2} - e^{-0^2} \right]$$

$$= -\frac{1}{8} \left(e^{-16} - e^{0} \right)$$

$$= \frac{1 - e^{-16}}{8}$$