Exercise 12.2

Assignment 17

Date

Q1

$$\lim_{t\to\infty} \left\langle \frac{t^2+1}{3t^2+2}, \frac{1}{t} \right\rangle$$

$$\left\langle \lim_{t\to\infty} \frac{t^2+1}{3t^2+2}, \lim_{t\to\infty} \frac{1}{t} \right\rangle$$

$$\left\langle \lim_{t\to\infty} \frac{\cancel{t^2}(1+\frac{1}{t^2})}{\cancel{t^2}(3+\frac{2}{t^2})}, \lim_{t\to\infty} \frac{1}{t} \right\rangle$$

Apply limit

$$\left\langle \frac{1}{3}, 0 \right\rangle$$

Q2

$$\lim_{t\to 0} \left(\sqrt{t}\, i + \frac{\sin t}{t} j \right)$$

Apply first hopital rule

$$\lim_{t\to 0} \left(\sqrt{t}\, i + \frac{\cos t}{1} j \right)$$

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Applying Limit

(0,1)

Q4

$$\lim_{t \to 1} \left\langle \frac{3}{t^2}, \frac{\ln t}{t^2-1}, \sin 2t \right\rangle$$

Apply First Hopital Rule

$$\lim_{t \to 1} \left\langle \frac{3}{t^2}, \frac{1}{2t^2}, \sin 2t \right\rangle$$

Apply limit

$$\left(\frac{3}{\cancel{0}}, \frac{1}{2}, \sin 2 \right)$$

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Q5

a) $r(t) = (3\sin t)i - (2t)j$

$t = 0$

1) $r(t)$ must be defined:

$r(0) = \langle 3\sin(0), -2(0) \rangle$

$\langle 0, 0 \rangle \rightarrow (1)$

2) $\lim_{t \to 0} r(t)$ must exist:

$\lim_{t \to 0} 3\sin t - 2t$

$0 \rightarrow (2)$

3) $\lim_{t \to 0} r(t) \stackrel{?}{=} r(t)$

$(1) = (2)$

The function is continuous

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b) $r(t) = t^2 i + \frac{1}{t} j + tk$

$t = 0$

i) $r(0)$ must be defined

$= (0)^2 i + \frac{1}{0} j + 0k$

$r(0) =$ UNDEFINED

The function is not continuous.

Q9

$r(t) = 4i - \cos t j$

$r'(t) = 0i - (\sin t) j$

$\boxed{r'(t) = \sin t \, j}$

Q11

$r(t) = \langle t, t^2 \rangle \qquad t_0 = 2$

$r'(t) = \langle 1, 2t \rangle$

$r'(t_0) = \langle 1, 2(2) \rangle$

$r'(t_0) = \langle 1, 4 \rangle$

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Q13

$r(t) = \sec t\, i + \tan t\, j - \sqrt{t}\, k$

$r'(t) = \sec t \tan t\, i + \sec^2 t\, j - \frac{1}{2\sqrt{t}} k$ $-\frac{1}{2}(t)^{-\frac{1}{2}} k$

$t_0 = 0$

$r'(t_0) = 0i + 1j - 0k$

$\boxed{r'(t_0) = j}$

Q15

$r(t) = 2\sin t\, i + j + 2\cos t\, k$

$r'(t) = 2\cos t\, i + 0j - 2\sin t\, k$

$t_0 = \pi/2$

$r'(t_0) = 0 + 0 - 2k$

$r'(t_0) = -2k$

Q19

$r_0 = r(t_0) = t_0^2 i + (2 - \ln t_0) j$

$v_0 = r'(t_0) = 2t_0 i - \frac{1}{t_0} j$

Date

$\therefore r = r_0 + t v_0$

$= (t_0^2 i + (2 - \ln t_0) j) + (2 t_0 i - \frac{1}{t_0} j) t$

$= (t_0^2 i + (2 - \ln t_0) j) + \left(2 t t_0 i - \frac{t}{t_0} j\right)$

$\langle x, y \rangle = (t_0^2 + 2t t_0) i + \left(2 - \ln t_0 - \frac{t}{t_0}\right) j$

$x = t_0^2 + 2t t_0$

$y = 2 - \ln t_0 - \frac{t}{t_0}$

$t_0 = 1$

$x = 1 + 2t$

$y = 2 - t$

Q21

$r_0 = r(t_0) = 2\cos \pi t_0\, i + 2 \sin \pi t_0\, j + 3 t_0 k$

$v_0 = r'(t_0) = -2\pi \sin \pi t_0\, i + 2\pi \cos \pi t_0\, j + 3k$

$r = r_0 + t v_0$

$\langle x, y, z \rangle = 2 \cos \pi t_0\, i + 2 \sin \pi t_0\, j + 3 t_0 k - 2t\pi \sin \pi t_0\, i +$
$2t\pi \cos \pi t_0\, j + 3t k$

$x = 2 \cos \pi t_0 - 2t\pi \sin \pi t_0$

$y = 2 \sin \pi t_0 + 2t\pi \cos \pi t_0$

$z = 3t_0 + 3t$

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Date

$$t_0 = \frac{1}{3}$$

$x = 1 - \sqrt{3}\pi t$
$y = \sqrt{3} + t\pi$
$z = 1 + 3t$

Q23

$r(t) = (2t-1)i + \sqrt{3t+4}\,j \qquad P_0(-1,2)$

$2t - 1 = -1$
$\sqrt{3t+4} = 2$
solve for t.
$\boxed{t=0}$

$$v_0 = r'(t) = 2i + \frac{3}{2\sqrt{3t+4}}j$$

at $t = 0$

$$v_0 = 2i + \frac{3}{4}j$$

$r_0 = -i + 2j$

$r = r_0 + t v_0$

$$\boxed{r = (-i+2j) + \left(2i + \frac{3}{4}j\right)t}$$

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Q25

$$r(t) = t^2 i - \frac{1}{t+1} j + (4 - t^2) k \qquad P_0(4,1,0)$$

$$t^2 = 4$$

$$-\frac{1}{t+1} = 1$$

$$4 - t^2 = 0$$

Solve for t

$$t = -2$$

$$r'(t) = 2t i + \frac{1}{(t+1)^2} j - 2t k$$

at $t = -2$

$$r_0 = r(-2) = 4i + j$$

$$V_0 = r'(-2) = -4i + j + 4k$$

$$r = r_0 + v_0 t$$

$$r = (4i + j) + (-4i + j + 4k)t$$

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Q27

$r(t) = \cos t\, i + \sin t\, j + k$

$r'(t) = -\sin t\, i + \cos t\, j$

a) $\cancel{r'(t)} =$

$\lim_{t \to 0} \left[(\cos t\, i + \sin t\, j + k) - (-\sin t\, i + \cos t\, j + 0k) \right]$

$\lim_{t \to 0} \left[(\cos t + \sin t) i + (\sin t + \cos t) j + k \right]$

$\lim_{t \to 0} (\cos t + \sin t) i + \lim_{t \to 0} (\sin t + \cos t) j + \lim_{t \to 0} (k)$

$= i + j + k$

b) $\lim (r(t) \times r'(t))$

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b) $\lim_{t\to 0} [r(t) \times r'(t)]$

$$r(t) \times r'(t) = \begin{vmatrix} i & j & k \\ \cos t & \sin t & 1 \\ -\sin t & \cos t & 0 \end{vmatrix}$$

$= -\cos t\, i - \sin t\, j + k$

$\lim_{t\to 0} -\cos t\, i - \lim_{t\to 0} \sin t\, j + \lim_{t\to 0} k$

$\boxed{-i + k}$

c) $\lim_{t\to 0} [r(t) \cdot r'(t)]$

$r(t) \cdot r'(t) = (\cos t\, i + \sin t\, j + k) \cdot (-\sin t\, i + \cos t\, j + 0k)$

$= -\cos t \sin t\, i + \sin t \cos t\, j + 0k$

$\lim_{t\to 0} -\cos t \sin t\, i + \lim_{t\to 0} \sin t \cos t\, j$

$\boxed{0}$

No.

Date

Q29

$r_1(t) = 2ti + 3t^2 j + t^3 k$

$r_2(t) = t^4 k$

$$\frac{d}{dt}[r_1(t) \cdot r_2(t)] = r_1(t) \cdot \frac{dr_2}{dt} + \frac{dr_1}{dt} \cdot r_2(t)$$

$= (2ti + 3t^2 j + t^3 k) \cdot 4t^3 k + (2i + 6tj + 3t^2 k)(t^4 k)$

$= 4t^6 + 3t^6$

$= \boxed{7t^6}$

$$\frac{d}{dt}[r_1(t) \times r_2(t)] = r_1(t) \times \frac{dr_2}{dt} + \frac{dr_1}{dt} \times r_2(t)$$

$= (2ti + 3t^2 j + t^3 k) \times 4t^3 k + (2i + 6tj + 3t^2 k) \times t^4 k$

$$\begin{vmatrix} i & \hat{j} & k \\ 2t & 3t^2 & t^3 \\ 0 & 0 & 4t^3 \end{vmatrix} = 12t^5 i + 8t^4 j$$

$$\begin{vmatrix} i & \hat{j} & k \\ 2t & 6t & 3t^2 \\ 0 & 0 & t^4 \end{vmatrix} = \langle 6t^5 i + 2t^4 \rangle$$

$= 12t^5 i + 8t^4 j + 6t^5 i + 2t^4 j$

$= 18t^5 j + 10t^4 j$

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$$\int u\cdot v\,dx = u\int v\,dx - \int\left(u'\int v\,dx\right)dx$$

$$\int u\cdot v = u\int v\,dx \; \int\left(u'\int v\,dx\right)dx$$

Q31

$$\int (3i + 4tj)\,dt$$

$$3t\,i + 2t^2 j + C$$

Q33

$$\int \langle te^t, \ln t \rangle\, dt$$

$$\left\langle \int t e^t , \int \ln t \times 1 \right\rangle$$

$$\left\langle t\int e^t - \int \frac{d(t)}{dt}\int e^t , \ln t \int 1 - \int \frac{1}{t}\int 1 \right\rangle$$

$$\langle te^t - e^t , t\ln t - t \rangle$$

$$\langle e^t(t-1) , t(\ln t - 1) \rangle$$

Q35

$$\int_0^{\frac{\pi}{2}} \langle \cos 2t , \sin 2t \rangle\, dt$$

$$\left\langle \frac{\sin 2t}{2} , -\frac{\cos 2t}{2} \right\rangle \Bigg|_0^{\frac{\pi}{2}}$$

$$-\left(\frac{\cos\pi}{2} - \frac{\cos 0}{2}\right)$$

$$-\left(\frac{-1}{2} - \frac{1}{2}\right)$$

$$-(-1) = 1$$

$$\langle 0 , 1 \rangle$$

No.

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Q 37

$$\int_0^2 \|ti + t^2 j\| \, dt$$

$$\int_0^2 \sqrt{t^2 + t^4} \, dt$$

$$\int_0^2 t\sqrt{1+t^2} \, dt$$

let

$U = 1 + t^2$

$\frac{du}{dt} = 2t$

$\frac{du}{2} = t \, dt$

$$\int_0^2 \sqrt{U} \, \frac{du}{2}$$

$$\frac{1}{2} \frac{U^{3/2}}{3/2} \Big|_0^2$$

$$= \frac{U^{3/2}}{3} \Big|_0^2$$

$$= \frac{(1+t^2)^{3/2}}{3} \Big|_0^2$$

$$\frac{1}{3}\left[(1+2^2)^{\frac{3}{2}} - (1+0)^{3/2} \right]$$

$$\frac{1}{3}\left(\sqrt{5^3} - 1 \right)$$

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$$\boxed{\frac{5\sqrt{5}-1}{3}}$$

Q39

$$\int_1^9 \left(t^{\frac{1}{2}} i + t^{-\frac{1}{2}} j\right) dt$$

$$\frac{2t^{\frac{3}{2}}}{3} i + 2t^{\frac{1}{2}} j \Big|_1^9$$

$$\cancel{2\left[\left(\frac{(9)^{\frac{3}{2}}}{3} i + 2(9)^{\frac{1}{2}} j\right) - \left(\frac{2}{3}(1)^{\frac{3}{2}} i + 2(1)^{\frac{1}{2}} j\right)\right]}$$

$$\cancel{2(15-4)}$$

$$\cancel{\boxed{22}}$$

$$(18i + 6j) - \left(\frac{2}{3} i + 2j\right)$$

$$\boxed{\frac{52}{3} i + 4j}$$