Exercise 13.3

Q5

$$\cancel{z = \sin(5x^3y + 7xy^2)}$$

$$\cancel{\frac{\partial z}{\partial x} = \cos(5x^3y + 7xy^2)\frac{\partial}{\partial x}(5x^3y +}$$

$$z = (x^2 + 5x - 2y)^8$$

$$\frac{\partial z}{\partial x} = 8(x^2+5x-2y)^7 \frac{\partial}{\partial x}(x^2+5x-2y)$$

$$= 8(x^2+5x-2y)^7(2x+5)$$

$$\boxed{\frac{\partial z}{\partial x} = (16x+40)(x^2+5x-2y)^7}$$

$$\frac{\partial z}{\partial y} = 8(x^2+5x-2y)^7 \frac{\partial}{\partial y}(x^2+5x-2y)$$

$$\boxed{\frac{\partial z}{\partial y} = -16(x^2+5x-2y)^7}$$

Q9

$$z = \sin(5x^3y + 7xy^2)$$

$$\frac{\partial z}{\partial x} = \cos(5x^3y + 7xy^2)\frac{\partial}{\partial x}(5x^3y + 7xy^2)$$

Date

$$\boxed{\frac{\partial z}{\partial x} = (15x^2y + 7y^2)(\cos(5x^3y + 7xy^2))}$$

$$\frac{\partial z}{\partial y} = \cos(5x^3y + 7xy^2)\frac{\partial}{\partial y}(5x^3y + 7xy^2)$$

$$= (5x^3 + 14xy)\cos(5x^3y + 7xy^2)$$

Q 13

$$z = \sin(y^2 - 4x)$$

a)

$$\frac{\partial z}{\partial x} = \cos(y^2 - 4x)\frac{\partial}{\partial x}(y^2 - 4x)$$

$$\frac{\partial z}{\partial x} = \cos(y^2 - 4x) \times -4$$

point $(2, 1)$

$$= -4\cos(1^2 - 4(2))$$

$$\boxed{\frac{\partial z}{\partial x} = -4\cos 7}$$

b)

$$\frac{\partial z}{\partial y} = \cos(y^2 - 4x)\frac{\partial}{\partial y}(y^2 - 4x)$$

No.

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$$\frac{\partial z}{\partial y} = 2y\cos(y^2-4x)$$

point $(2,1)$

$$\boxed{\frac{\partial z}{\partial y} = 2\cos 7}$$

Q27

$$z = x^3 \ln(1+xy^{-3/5})$$

$$\frac{\partial z}{\partial x} = x^3 \times \frac{1}{1+xy^{-3/5}} \frac{\partial}{\partial x}(1+xy^{-3/5}) + 3x^2\ln(1+xy^{-\frac{3}{5}})$$

$$= \frac{x^3y^{-3/5}}{1+xy^{-3/5}} + 3x^2\ln(1+xy^{-3/5})$$

$$\boxed{\frac{\partial z}{\partial x} = \frac{x^3}{y^{3/5}+x} + 3x^2\ln(1+xy^{-3/5})}$$

$$\frac{\partial z}{\partial y} = \frac{x^3}{1+xy^{-3/5}} \frac{\partial}{\partial y}(1+xy^{-3/5})$$

$$= \frac{x^3}{1+xy^{-\frac{3}{5}}} \times -\frac{3}{5}xy^{-8/5}$$

$$\boxed{\frac{\partial z}{\partial y} = \frac{-(3/5)x^2}{y^{8/5}+xy}}$$

Date

Q29

$$z = \frac{xy}{x^2+y^2}$$

$$\frac{\partial z}{\partial x} = \frac{(x^2+y^2)\frac{\partial}{\partial x}(xy) - xy\frac{\partial}{\partial x}(x^2+y^2)}{(x^2+y^2)^2}$$

$$= \frac{x^2y + y^3 - 2x^2y}{\cancel{x^4+2x^2y^2+y^4}\ (x^2+y^2)^2}$$

$$= \frac{y^3 - x^2y}{(x^2+y^2)^2}$$

$$= \frac{y(y^2-x^2)}{(x^2+y^2)^2}$$

$$\boxed{\frac{\partial z}{\partial x} = \frac{-y(x^2-y^2)}{(x^2+y^2)^2}}$$

$$\frac{\partial z}{\partial y} = \frac{x^2+y^2\frac{\partial}{\partial y}(xy) - xy\frac{\partial}{\partial y}(x^2+y^2)}{(x^2+y^2)^2}$$

$$= \frac{x^3 + xy^2 - 2xy^2}{(x^2+y^2)^2}$$

$$= \frac{x^3 - xy^2}{(x^2+y^2)^2}$$

$$\boxed{\frac{\partial z}{\partial y} = \frac{x(x^2-y^2)}{(x^2+y^2)^2}}$$

No.

Q31

$$f(x,y) = \sqrt{3x^5y - 7x^3y}$$

$$f_x(x,y) = \frac{\frac{\partial}{\partial x}(3x^5y - 7x^3y)}{2\sqrt{3x^5y - 7x^3y}}$$

$$= \frac{15x^4y - 21x^2y}{2\sqrt{3x^5y - 7x^3y}}$$

$$\boxed{f_x(x,y) = \left(\frac{3}{2}\right)x^2y\,(5x^2 - 7)\,(3x^5y - 7x^3y)^{-\frac{1}{2}}}$$

$$f_y(x,y) = \frac{1}{2}(3x^5y - 7x^3y)^{-\frac{1}{2}}\,\frac{\partial}{\partial y}(3x^5y - 7x^3y)$$

$$= \frac{1}{2}(3x^5y - 7x^3y)^{-\frac{1}{2}}(3x^5 - 7x^3)$$

$$\boxed{f_y(x,y) = \frac{1}{2}x^3\,(3x^2 - 7)\,(3x^5y - 7x^3y)^{-\frac{1}{2}}}$$

Q33

$$f(x,y) = y^{-\frac{3}{2}}\tan^{-1}(x/y)$$

$$f_x(x,y) = y^{-\frac{3}{2}}\,\frac{1}{1 + \left(\frac{x}{y}\right)^2} \times \frac{1}{y}$$

$$= \frac{y^2 \times y^{-\frac{3}{2}} \times y^{-1}}{y^2 + x^2}$$

Date

$$f_x(x,y) = \frac{y^{-\frac{1}{2}}}{y^2+x^2}$$

Q39

$$z = \sqrt{x^2+4y^2}$$

$$\frac{\partial z}{\partial x} = \frac{1}{2}(x^2+4y^2)^{-1/2} \frac{\partial}{\partial x}(x^2+4y^2)$$

$$\frac{\partial z}{\partial x} = \frac{x}{\sqrt{x^2+4y^2}}$$

$$\cancel{\frac{\partial z}{\partial x}\left(\frac{1}{2}\right)} \quad \frac{\partial z}{\partial x}(1,2) = \frac{1}{\sqrt{1^2+4(2)^2}}$$

$$\boxed{\frac{\partial z}{\partial x}(1,2) = \frac{1}{\sqrt{17}}}$$

$$\frac{\partial z}{\partial y} = \frac{1}{2}(x^2+4y^2)^{-\frac{1}{2}} \times 8y$$

$$\frac{\partial z}{\partial y} = \frac{4y}{\sqrt{x^2+4y^2}}$$

$$\frac{\partial z}{\partial y}(1,2) = \frac{4(2)}{\sqrt{1^2+4(2)^2}}$$

$$\boxed{\frac{\partial z}{\partial y}(1,2) = \frac{8}{\sqrt{17}}}$$

Date

Q 41

$$f(x,y,z) = x^2y^4z^3 + xy + z^2 + 1$$

a) $f_x(x,y,z)$

$$f_x(x,y,z) = 2xy^4z^3 + y$$

b) $f_y(x,y,z) = 4x^2y^3z^3 + x$

c) $f_z(x,y,z) = 3x^2y^4z^2 + 2z$

d) $f_x(1,y,z) = 2y^4z^3 + y$

e) $f_y(1,2,z) = 4(1)^2(2)^3z^3 + 1$

$= 32z^3 + 1$

f) $f_z(1,2,3) = 3(1)^2(2)^4(3)^2 + 2(3)$

$= 438$

Q 43

$$f(x,y,z) = z\ln(x^2y\cos z)$$

$$f_x = \frac{z}{x^2y\cos z} \times \frac{\partial}{\partial x}(\cos z\, x^2 y)$$

$$= \frac{2\cancel{x}yz\cos z}{x^2y\cos z}$$

x^{-1}

$-1\, x^{-2}$

$-\frac{1}{x^2}$

Date

$$\boxed{f_x = \frac{\partial z}{x}}$$

$$f_y = \frac{z}{x^2 y \cos z}\frac{\partial}{\partial y}(x^2 y \cos z)$$

$$= \frac{\cancel{x^2}\, z\, \cancel{\cos} z}{\cancel{x^2} y\, \cancel{\cos} z}$$

$$\boxed{f_y = \frac{z}{y}}$$

$$f_z = z \times \frac{1}{x^2 y \cos z}\frac{\partial}{\partial z}(x^2 y \cos z) + \ln(x^2 y \cos z)$$

$$= -\frac{\cancel{x^2}\cancel{y}\, z \sin z}{\cancel{x^2}\cancel{y} \cos z} + \ln(x^2 y \cos z)$$

$$\boxed{f_z = \ln(x^2 y \cos z) - z \tan z}$$

Q45

$$f(x,y,z) = \tan^{-1}\left(\frac{1}{xy^2z^3}\right)$$

$$f_x = \frac{1}{1+\left(\frac{1}{xy^2z^3}\right)^2}\,\frac{\partial}{\partial x}\left(\frac{1}{xy^2z^3}\right)$$

$$= \frac{x^2y^4z^6}{x^2y^4z^6+1} \times \frac{-1}{x^2y^2z^3}$$

$$\boxed{f_x = \frac{-y^2z^3}{1+x^2y^4z^6}}$$

$$f_y = \frac{1}{1+\left(\frac{1}{xy^2z^3}\right)^2} \times \frac{\partial}{\partial y}\left(\frac{1}{xy^2z^3}\right)$$

$$= \frac{x^2y^4z^6}{x^2y^4z^6+1} \times \frac{-2}{xy^3z^3}$$

$$\boxed{f_y = \frac{-2xyz^3}{x^2y^4z^6+1}}$$

$$f_z = \frac{1}{1+\left(\frac{1}{xy^2z^3}\right)^2} \times \frac{\partial}{\partial z}\left(\frac{1}{xy^2z^3}\right)$$

$$= \frac{x^2y^4z^6}{x^2y^4z^6+1} \times \frac{-3}{xy^2z^4}$$

Date

$$\boxed{f_z = \frac{-3xy^2z^2}{x^2y^4z^6+1}}$$

Q47

$$w = ye^z\sin(xz)$$

$$\frac{\partial w}{\partial x} = ye^z\cos(xz)\frac{\partial}{\partial x}(xz)$$

$$\boxed{\frac{\partial w}{\partial x} = yze^z\cos(xz)}$$

$$\boxed{\frac{\partial w}{\partial y} = e^z\sin(xz)}$$

$$\frac{\partial w}{\partial z} = y\left[e^z\cos(xz).x + \sin(xz)e^z\right]$$

$$= xye^z\cos(xz) + e^z_y\sin(xz)$$

$$\boxed{\frac{\partial w}{\partial z} = ye^z\left(x\cos(xz) + \sin(xz)\right)}$$

Q51

$$f(x,y,z) = y^2e^{xz}$$

$$f_x = zy^2e^{xz}$$

$$\boxed{f_x(1,1,1) = e}$$

$$f_y = 2ye^{xz}$$

$$\boxed{f_y(1,1,1) = 2e}$$

No.

$\cancel{f}z = xy^2e^{xz}$

$\boxed{fz(1,1,1) = e}$

Q69

$(x^2+y^2+z^2)^{3/2} = 1$

$\frac{\partial}{\partial x}\left((x^2+y^2+z^2)^{3/2}\right) = \frac{\partial}{\partial x}(1)$

$\frac{3}{2}(x^2+y^2+z^2)^{\frac{1}{2}}\frac{\partial}{\partial x}(x^2+y^2+z^2) = 0$

$\frac{3}{2}(x^2+y^2+z^2)\left(2x + 2z\frac{\partial z}{\partial x}\right) = 0$

$\cancel{\frac{3}{2}x^2(2x) + \frac{3}{2}x^2(2z)} =$

$\cancel{2x + 2z\frac{\partial z}{\partial x} = \frac{2}{3(x^2+y^2+z^2)}}$

$2x + 2z\frac{\partial z}{\partial x} = 0$

$\frac{\partial z}{\partial x} = \frac{-\cancel{2}x}{+\cancel{2}z}$

$$\boxed{\frac{\partial z}{\partial x} = \frac{-x}{z}}$$

Date

$$\frac{\partial}{\partial y}\left[(x^2+y^2+z^2)^{3/2}\right] = \frac{\partial}{\partial y}(1)$$

$$\frac{3}{2}(x^2+y^2+z^2)^{\frac{1}{2}} \frac{\partial}{\partial y}(x^2+y^2+z^2) = 0$$

$$2y + 2z\frac{\partial z}{\partial y} = 0$$

$$\boxed{\frac{\partial z}{\partial y} = \frac{-y}{z}}$$

Q75

$$w^2 + w\sin xyz = 1$$

$$\frac{\partial}{\partial x}(w^2 + w\sin xyz) = \frac{\partial}{\partial x}(1)$$

$$2w\frac{\partial w}{\partial x} + w\cos(xyz)\cdot yz + \frac{\partial w}{\partial x}\sin xyz = 0$$

$$\frac{\partial w}{\partial x}(2w + \sin xyz) = -wyz\cos(xyz)$$

$$\boxed{\frac{\partial w}{\partial x} = \cancel{wxyz}\ \frac{-wyz\cos(xyz)}{2w + \sin(xyz)}}$$

$$\frac{\partial}{\partial y}(w^2 + w\sin(xyz)) = \frac{\partial}{\partial y}(1)$$

$$2w\frac{\partial w}{\partial y} + w\cos(xyz)\cdot xz + \frac{\partial w}{\partial y}\sin xyz = 0$$

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Date

$$\frac{\partial w}{\partial y}(2w + \sin xyz) = -wxz\cos(xyz)$$

$$\boxed{\frac{\partial w}{\partial y} = \frac{-wxz\cos(xyz)}{2w + \sin xyz}}$$

$$\frac{\partial}{\partial z}(w^2 + w\sin xyz) = \frac{\partial}{\partial z}(1)$$

$$2w\frac{\partial w}{\partial z} + \frac{\partial w}{\partial z}\sin(xyz) + w\cos(xyz)\cdot xy = 0$$

$$\frac{\partial w}{\partial z}(2w + \sin xyz) = -wxy\cos xyz$$

$$\boxed{\frac{\partial w}{\partial z} = \frac{-wxy\cos xyz}{2w + \sin xyz}}$$

Q81

$$z = \sqrt{x}\cos y$$

a)

$$\cancel{\frac{\partial z}{\partial x} = \frac{\cos y}{2\sqrt{x}}}$$

$$\cancel{\frac{\partial^2 z}{\partial x^2} = \cos y \cdot \sqrt{x}}$$

a)

$$\frac{\partial z}{\partial x} = \frac{1}{2}(x)^{-\frac{1}{2}} \cos y$$

$$\frac{\partial^2 z}{\partial x^2} = -\frac{1}{4}(x)^{-\frac{3}{2}} \cos y$$

$$\boxed{\frac{\partial^2 z}{\partial x^2} = -\frac{1}{4x^{3/2}} \cos y}$$

b)

$$\frac{\partial z}{\partial y} = -\sin y \cdot \sqrt{x}$$

$$\boxed{\frac{\partial^2 z}{\partial y^2} = -\cos y \sqrt{x}}$$

c)

$$\frac{\partial z}{\partial y} = -\sqrt{x} \sin y$$

$$\boxed{\frac{\partial^2 z}{\partial x \partial y} = \frac{-\sin y}{2\sqrt{x}}}$$

d)

$$\frac{\partial z}{\partial x} = \frac{\cos y}{2\sqrt{x}}$$

$$\boxed{\frac{\partial^2 z}{\partial y \partial x} = \frac{-\sin y}{2\sqrt{x}}}$$

Date

Q83

$$f(x,y) = \sin(3x^2+6y^2)$$

a)

$f_x = 6x\cos(3x^2+6y^2)$

$f_{xx} = 6\left[-x\sin(3x^2+6y^2)6x + \cos(3x^2+6y^2)\right]$

$$\boxed{f_{xx} = -36x\sin(3x^2+6y^2) + 6\cos(3x^2+6y^2)}$$

b)

$f_y = 12y\cos(3x^2+6y^2)$.

$f_{yy} = 12\left[-y\sin(3x^2+6y^2) * 12y + \cos(3x^2+6y^2)\right]$

$$\boxed{f_{yy} = +12\cos(3x^2+6y^2) - 144y^2\sin(3x^2+6y^2)}$$

c)

$f_x = 6x\cos(3x^2+6y^2)$

$$\boxed{f_{xy} = -72xy\sin(3x^2+6y^2)}$$

d)

$f_y = 12y\cos(3x^2+6y^2)$

$f_{yx} = -72xy\sin(3x^2+6y^2)$

Date.

Q87

$f(x,y) = e^x \cos y$

$f_x = e^x \cos y$

$\boxed{f_{xy} = -\sin y \, e^x}$

$f_y = -\sin y \, e^x$

$\boxed{f_{yx} = -\sin y \, e^x}$

$f_{xy} = f_{yx}$

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Q91

$$f(x,y) = \frac{(x-y)}{x+y}$$

$$f_x = \frac{(x+y)\frac{\partial}{\partial x}(x-y) - (x-y)\frac{\partial}{\partial x}(x+y)}{(x+y)^2}$$

$$= \frac{\cancel{x}+y-\cancel{x}+y}{(x+y)^2}$$

$$f_x = \frac{2y}{(x+y)^2}$$

$$f_{xy} = \frac{(x+y)^2\frac{\partial}{\partial y}2y - 2y\frac{\partial}{\partial y}(x+y)^2}{(x+y)^4}$$

$$= \frac{2(x+y)^2 - 4y(x+y)}{(x+y)^4}$$

$$= \frac{\cancel{(x+y)}[2(x+y)-4y]}{(x+y)^{\cancel{4}}}$$

$$= \frac{2x+2y-4y}{(x+y)^3}$$

$$\boxed{f_{xy} = \frac{2(x-y)}{(x+y)^3}}$$

$$f_y = \frac{(x+y)\frac{\partial}{\partial y}(x-y) - (x-y)\frac{\partial}{\partial y}(x+y)}{(x+y)^2}$$

Date

$$= \frac{\cancel{x+y-x} \quad -x-\cancel{y}-x+\cancel{y}}{(x+y)^2}$$

$$f_y = \frac{-2x}{(x+y)^2}$$

$$f_{yx} = \frac{(x+y)^2 \frac{\partial}{\partial x}(-2x) - (-2x)\frac{\partial}{\partial x}(x+y)^2}{(x+y)^4}$$

$$= \frac{-2(x+y)^2 + 4x(x+y)}{(x+y)^4}$$

$$= \frac{\cancel{(x+y)^2}\left[-2(x+y)+4x\right]}{(x+y)^{\cancel{4}}}$$

$$= \frac{-2x-2y+4x}{(x+y)^3}$$

$$= \frac{2x-2y}{(x+y)^3}$$

$$\boxed{f_{yx} = \frac{2(x-y)}{(x+3)^3}}$$

$$f_{xy} = f_{yx} = \frac{2(x-y)}{(x+3)^3}$$

Date

95

$$f(x,y) = x^3y^5 - 2x^2y + x$$

a) f_{xxy}

$f_x = 3x^2y^5 - 4xy$

$f_{xx} = 6xy^5 - 4y$

$f_{xxy} = 30xy^4 - 4$

b) f_{yxy}

$f_y = 5x^3y^4 - 2x^2$

$f_{yx} = 15x^2y^4 - 4x$

$f_{yxy} = 60x^2y^3$

c) f_{yyy}

$f_y = 5x^3y^4 - 2x^2$

$f_{yy} = 20x^3y^3$

$f_{yyy} = 60x^3y^2$

Date

Q99

$$f(x,y,z) = x^3y^5z^7 + xy^2 + y^3z$$

a) f_{xy}

$f_x = 3x^2y^5z^7 + y^2$

$f_{xy} = 15x^2y^4z^7 + 2y$

b) f_{yz}

$f_y = 5x^3y^4z^7 + 2xy + 3y^2z$

$f_{yz} = 35x^3y^4z^6 + 3y^2$

c) f_{xz}

$f_x = 3x^2y^5z^7 + y^2$

$f_{xz} = 21x^2y^5z^6$

d) f_{zz}

$f_z = 7x^3y^5z^6 + y^3$

$f_{zz} = 42x^3y^5z^5$

e) f_{zyy}

$f_z = 7x^3y^5z^6 + y^3$

$f_{zy} = 35x^3y^4z^6 + 3y^2$

$f_{zyy} = 140x^3y^3z^6 + 6y$

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f) f_{xxy}

$f_x = 3x^2y^5z^7 + y^2$

$f_{xx} = 6xy^5z^7$

$f_{xxy} = 30xy^4z^7$

g) f_{zyx}

$f_z = 7x^3y^5z^6 + y^3$

$f_{zy} = 35x^3y^4z^6 + 3y^2$

$f_{zyx} = 105x^2y^4z^6$

h) f_{xxyz}

$f_x = 3x^2y^5z^7 + y^2$

$f_{xx} = 6xy^5z^7$

$f_{xxy} = 30xy^4z^7$

$f_{xxyz} = 210xy^4z^6$

Q 101

a) $z = x^2 - y^2 + 2xy$

$$\frac{\partial z}{\partial x} = 2x + 2y$$

$$\frac{\partial^2 z}{\partial x^2} = 2$$

Date

$$\frac{\partial z}{\partial y} = -2y + 2x$$

$$\frac{\partial^2 z}{\partial y^2} = -2$$

put in Laplace equation

$$\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2} = 0$$

$$2 - 2 = 0$$

$$\boxed{0 = 0}$$

b) $z = e^x \sin y + e^y \cos x$

$$\frac{\partial z}{\partial x} = e^x \sin y + -\sin x \, e^y$$

$$\frac{\partial^2 z}{\partial x^2} = e^x \sin y - \cos x \cdot e^y$$

$$\cancel{\frac{\partial z}{\partial y} = e^x \cos y - \sin x \cdot e^y} \qquad \frac{\partial z}{\partial y} = \cos y \cdot e^x + \cos x \, e^y$$

$$\cancel{\frac{\partial^2 z}{\partial y^2} = -e^x \sin y - \cos x \, e} \qquad \frac{\partial^2 z}{\partial y^2} = -\sin y \, e^x + \cos x \, e^y$$

put in laplace

$$(e^x \sin y - e^y \cos x) + (\cos x \, e^y - e^x \sin y = 0$$

$$\boxed{0 = 0}$$

No. ____

c) $z = \ln(x^2+y^2) + 2\tan^{-1}(y/x)$

$$\frac{\partial z}{\partial x} = \frac{2x}{x^2+y^2} + \frac{2}{1+\left(\frac{y}{x}\right)^2} \frac{-y}{x^2}$$

$$= \frac{2x}{x^2+y^2} + \frac{2\cancel{x^2}}{x^2+y^2} \times \frac{-y}{\cancel{x^2}}$$

$$\frac{\partial z}{\partial x} = \frac{2x-2y}{x^2+y^2}$$

$$\frac{\partial^2 z}{\partial x^2} = \frac{(x^2+y^2)\frac{\partial}{\partial x}(2x-2y) - (2x-2y)\frac{\partial}{\partial x}(x^2+y^2)}{(x^2+y^2)^2}$$

$$= \frac{2(x^2+y^2) - 4x(x-y)}{(x^2+y^2)^2}$$

$$= \frac{2x^2+2y^2-4x^2+4xy}{(x^2+y^2)^2}$$

$$\frac{\partial^2 z}{\partial x^2} = \frac{2y^2-2x^2+4xy}{(x^2+y^2)^2}$$

$$\boxed{\frac{\partial^2 z}{\partial x^2} = \frac{2(y^2-x^2+2xy)}{(x^2+y^2)^2}}$$

$$\frac{\partial z}{\partial y} = \frac{2y}{x^2+y^2} + \frac{2}{1+\left(\frac{y}{x}\right)^2} \times \frac{1}{x}$$

$$= \frac{2y}{x^2+y^2} + \frac{2x^{\cancel{2}}}{x^2+y^2} \times \frac{1}{\cancel{x}}$$

$$\frac{\partial z}{\partial y} = \frac{2x+2y}{x^2+y^2}$$

$$\frac{\partial^2 z}{\partial y^2} = \frac{(x^2+y^2)\frac{\partial}{\partial y}(2x+2y) - (2x+2y)\frac{\partial}{\partial y}(x^2+y^2)}{(x^2+y^2)^2}$$

Date

$$\frac{\partial^2 z}{\partial y^2} = \frac{2(x^2+y^2) - 2y(2x+2y)}{(x^2+y^2)^2}$$

$$= \frac{2x^2 + 2y^2 - 4xy - 4y^2}{(x^2+y^2)^2}$$

$$\boxed{\frac{\partial^2 z}{\partial y^2} = \frac{2x^2 - 2y^2 - 4xy}{(x^2+y^2)^2}}$$

$$\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2} = 0$$

$$\frac{2y^2 - 2x^2 - 4xy}{(x^2+y^2)^2} + \frac{2x^2 - 2y^2 - 4xy}{(x^2+y^2)^2} = 0$$

$$\frac{\cancel{2y^2} - \cancel{2x^2} + \cancel{4xy} + \cancel{2x^2} - \cancel{2y^2} - \cancel{4xy}}{(x^2+y^2)^2} = 0$$

$$\frac{0}{(x^2+y^2)^2} = 0$$

$$\boxed{0 = 0}$$

Q109

$$f(v_1, v_2, v_3, v_4) = \frac{v_1^2 - v_2^2}{v_3^2 + v_4^2}$$

$$\boxed{\frac{\partial f}{\partial v_1} = \frac{2v_1}{v_3^2 + v_4^2}}$$

$$\boxed{\frac{\partial f}{\partial v_2} = \frac{-2v_2}{v_3^2 + v_4^2}}$$

$$\cancel{\frac{\partial f}{\partial v_3} = \frac{v_1^2 - v_2^2}{2v_3}}$$

$$\cancel{\frac{\partial f}{\partial v_4} = \frac{v_1^2 - v_2^2}{2v_4}}$$

$$\frac{\partial f}{\partial v_3} = \frac{(v_3^2 + v_4^2)\frac{\partial}{\partial v_3}(v_1^2 - v_2^2) - (v_1^2 - v_2^2)\frac{\partial}{\partial v_3}(v_3^2 + v_4^2)}{(v_3^2 + v_4^2)^2}$$

$$\boxed{\frac{\partial f}{\partial v_3} = \frac{-2v_3(v_1^2 - v_2^2)}{(v_3^2 + v_4^2)^2}}$$

$$\frac{\partial f}{\partial v_4} = \frac{(v_3^2 + v_4^2)\frac{\partial}{\partial v_4}(v_1^2 - v_2^2) - (v_1^2 - v_2^2)\frac{\partial}{\partial v_4}(v_3^2 + v_4^2)}{(v_3^2 + v_4^2)^2}$$

$$\boxed{\frac{\partial f}{\partial v_4} = \frac{-2v_4(v_1^2 - v_2^2)}{(v_3^2 + v_4^2)}}$$

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$$u(w, x, y, z) = x e^{yw} \sin^2 z$$

$$\cancel{\frac{\partial u}{\partial x} = e^{yw} \sin^2 z}$$

$$\cancel{\frac{\partial u}{\partial x}(0, 0, 1, \pi) = e^{0} \sin^2(\pi)}$$

$$\frac{\partial u}{\partial x} = e^{yw} \sin^2 z$$

$$\frac{\partial u}{\partial x}(0, 0, 1, \pi) = e^{1 \times 0} \sin^2(\pi)$$

$$\boxed{\frac{\partial u}{\partial x}(0, 0, 1, \pi) = 0}$$

$$\frac{\partial u}{\partial y} = w x e^{yw} \sin^2 z$$

$$\boxed{\frac{\partial u}{\partial y}(0, 0, 1, \pi) = 0}$$

$$\frac{\partial u}{\partial w} = x y e^{yw} \sin^2 z$$

$$\boxed{\frac{\partial u}{\partial w}(0, 0, 1, \pi) = 0}$$

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$$\frac{\partial u}{\partial z} = x e^{yw}\, 2\sin z \cos z$$

$$\boxed{\frac{\partial u}{\partial z}(0,0,1,\pi) = 0}$$

$$\frac{\partial u}{\partial z} = x e^{yw}\, 2\sin z \cos z$$

$$\frac{\partial^2 u}{\partial w \partial z} = \cancel{xwe} \quad xy\, e^{yw}\, 2\sin z \cos z$$

$$\frac{\partial^3 u}{\partial y \partial w \partial z} = x\, 2\sin z \cos z\,(wy\, e^{yw} + e^{yw})$$

$$\frac{\partial^3 u}{\partial y \partial w \partial z} = 2x(1+wy)\, e^{yw} \sin z \cos z$$

$$\boxed{\frac{\partial^4 u}{\partial x \partial y \partial w \partial z} = 2(1+wy)\, e^{yw} \sin z \cos z}$$

$$\frac{\partial u}{\partial y} = wx\, e^{yw} \sin^2 z$$

$$\frac{\partial^2 u}{\partial y^2} = w^2 x\, e^{yw} \sin^2 z$$

$$\frac{\partial^3 u}{\partial z \partial y^2} = w^2 x\, e^{yw}\, 2\sin z \cos z$$

$$\frac{\partial^4 u}{\partial w \partial z \partial y^2} = \cancel{2xw} \cdot (w^2 y\, e^{yw} + 2w\, e^{yw})\, 2x \sin z \cos z$$

$$\boxed{\frac{\partial^4 u}{\partial w \partial z \partial y^2} = 2xw(2+yw)\, e^{yw} \sin z \cos z}$$

No.