Assignment 1 (Remaining 3 Questions)

Q1

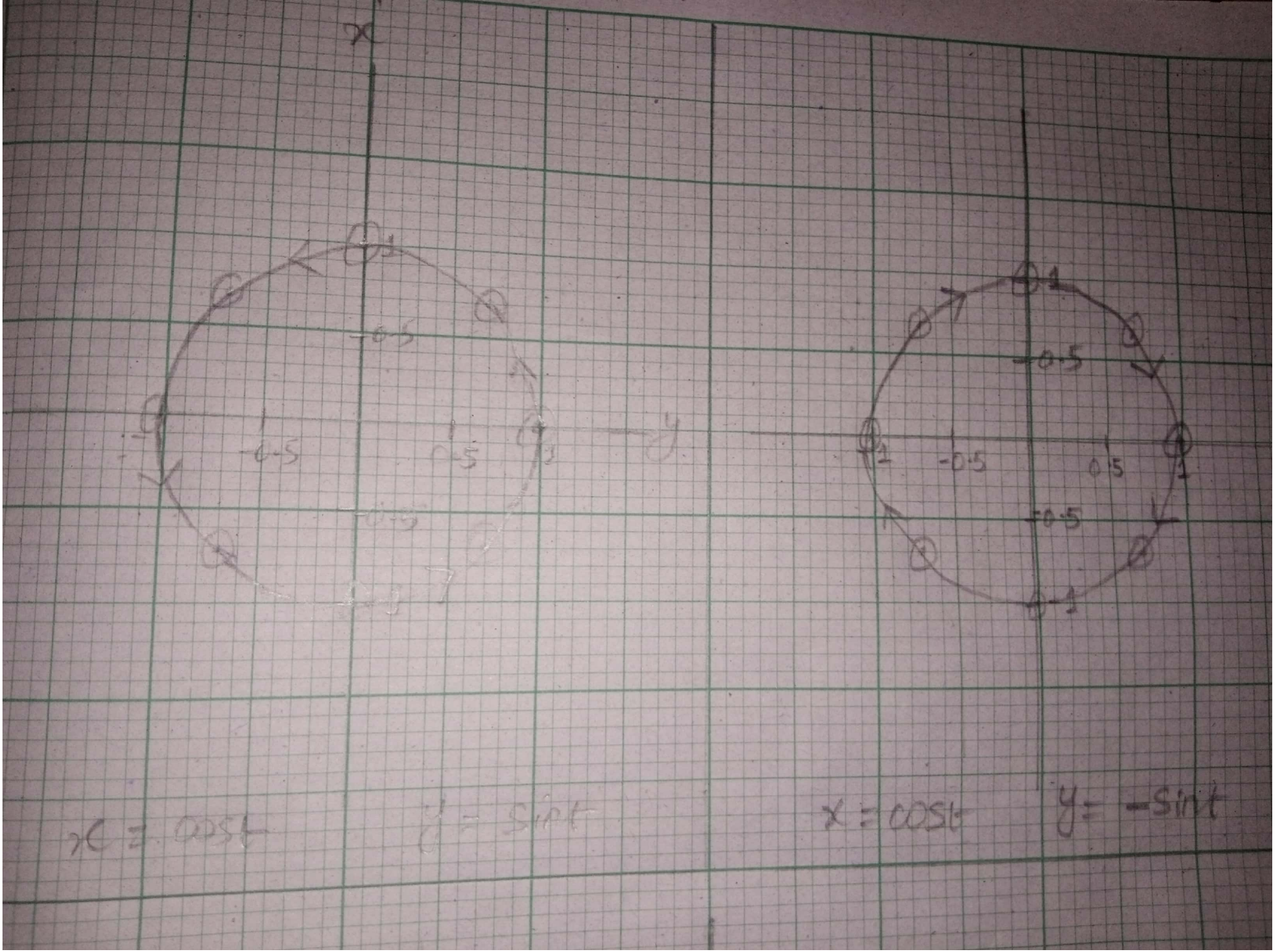
$x = \cos t$ $\quad y = \sin t$

t	$x = \cos t$	$y = \sin t$
0	1	0
$\pi/4$	0.7	0.7
$\pi/2$	0	1
$3\pi/4$	-0.7	0.7
π	-1	0
$5\pi/4$	-0.7	-0.7
$3\pi/2$	0	-1
$7\pi/4$	0.7	-0.7
2π	1	0

Q2

$x = \cos t$ $\quad y = -\sin t$.

t	x	y
0	1	0
$\pi/4$	0.7	-0.7
$\pi/2$	0	-1
$3\pi/4$	-0.7	-0.7
π	-1	0
$5\pi/4$	-0.7	0.7
$3\pi/2$	0	1
$7\pi/4$	0.7	0.7
2π	1	0

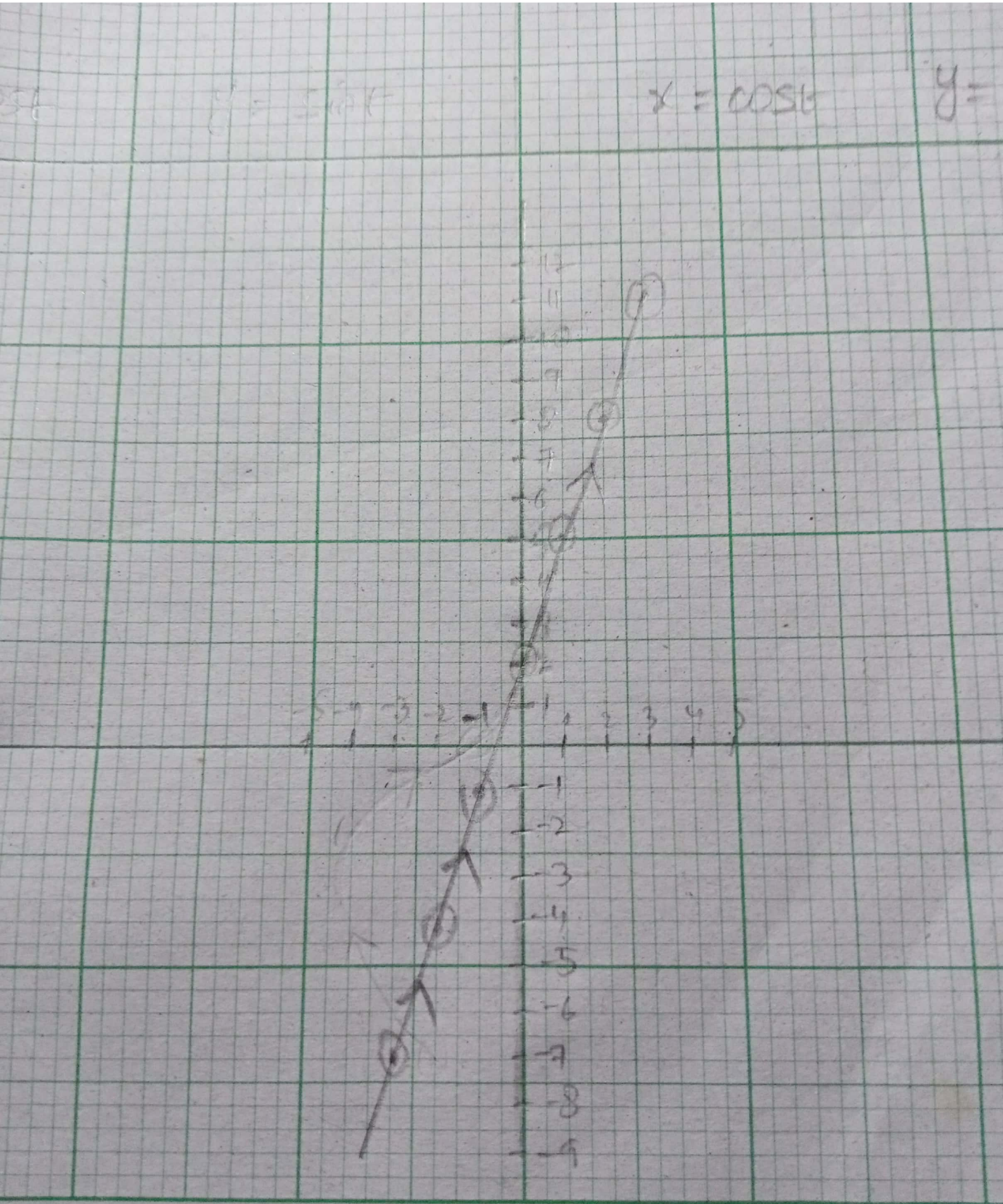


Q3)

$x = 2t - 3$ $\quad y = 6t - 7$

t	x	y
0	-3	-7
0.5	-2	-4
1	-1	-1
1.5	0	2
2	1	5
2.5	2	8
3	3	11

$x = \cos t$ $y =$



$x = 2t - 3$ $y = 6t - 7$

Assignment #01

Question # 03

$$x = 3t - 4 \qquad y = 6t + 2$$

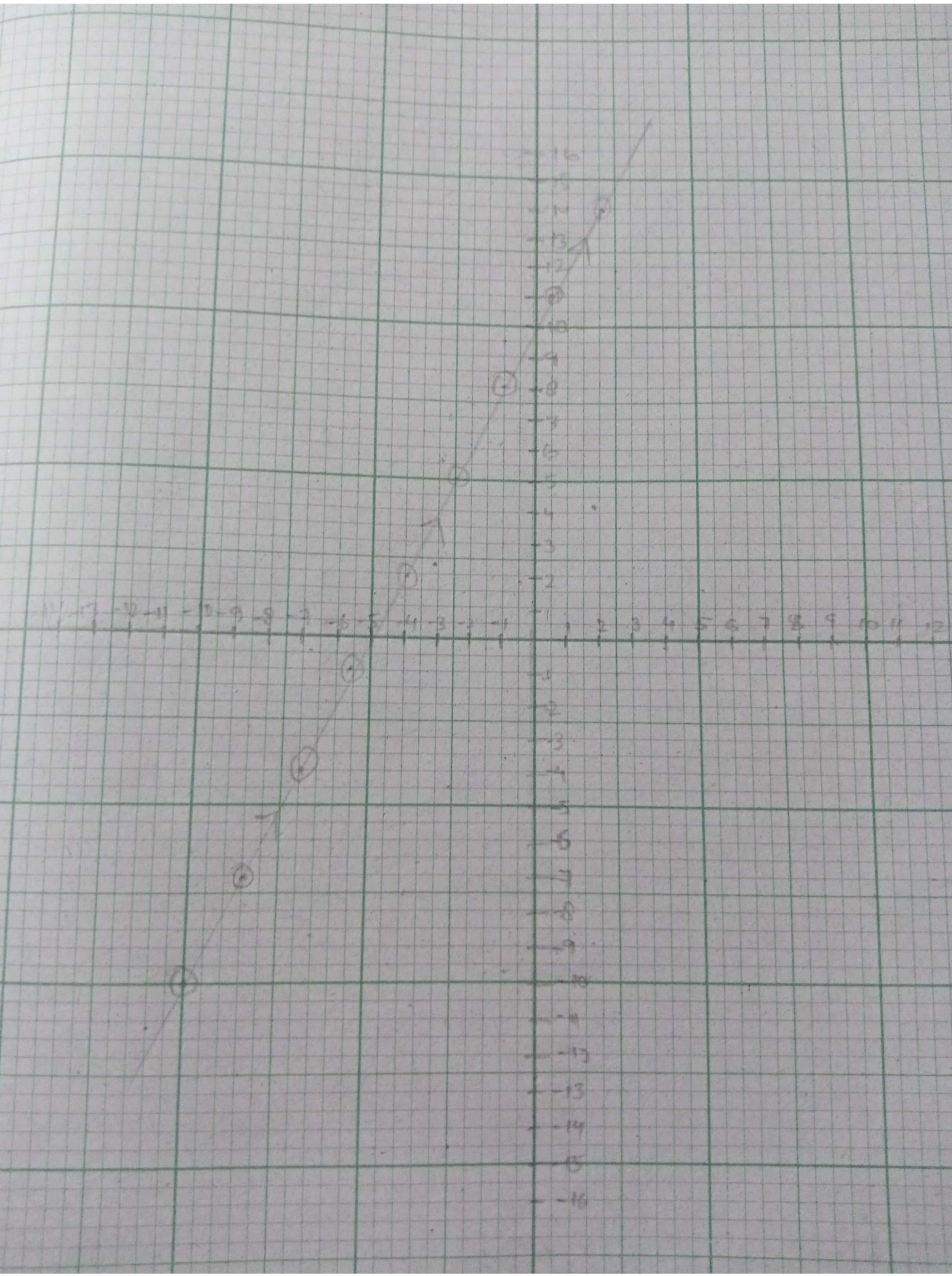
$$t = \frac{x+4}{3} \qquad y = 6t + 2$$

$$y = 6\left(\frac{x+4}{3}\right) + 2$$

$$y = 2x + 8 + 2$$

$$\boxed{y = 2x + 10}$$

t	x	y
-2	-10	-10
-1.5	-8.5	-7
-1	-7	-4
-0.5	-5.5	-1
0	-4	2
0.5	-2.5	5
1	-1	8
1.5	0.5	11
2	2	14



Question # 5

$x = 2\cos t$ $\quad (0 \leq t \leq 2\pi)$

$y = 5\sin t$

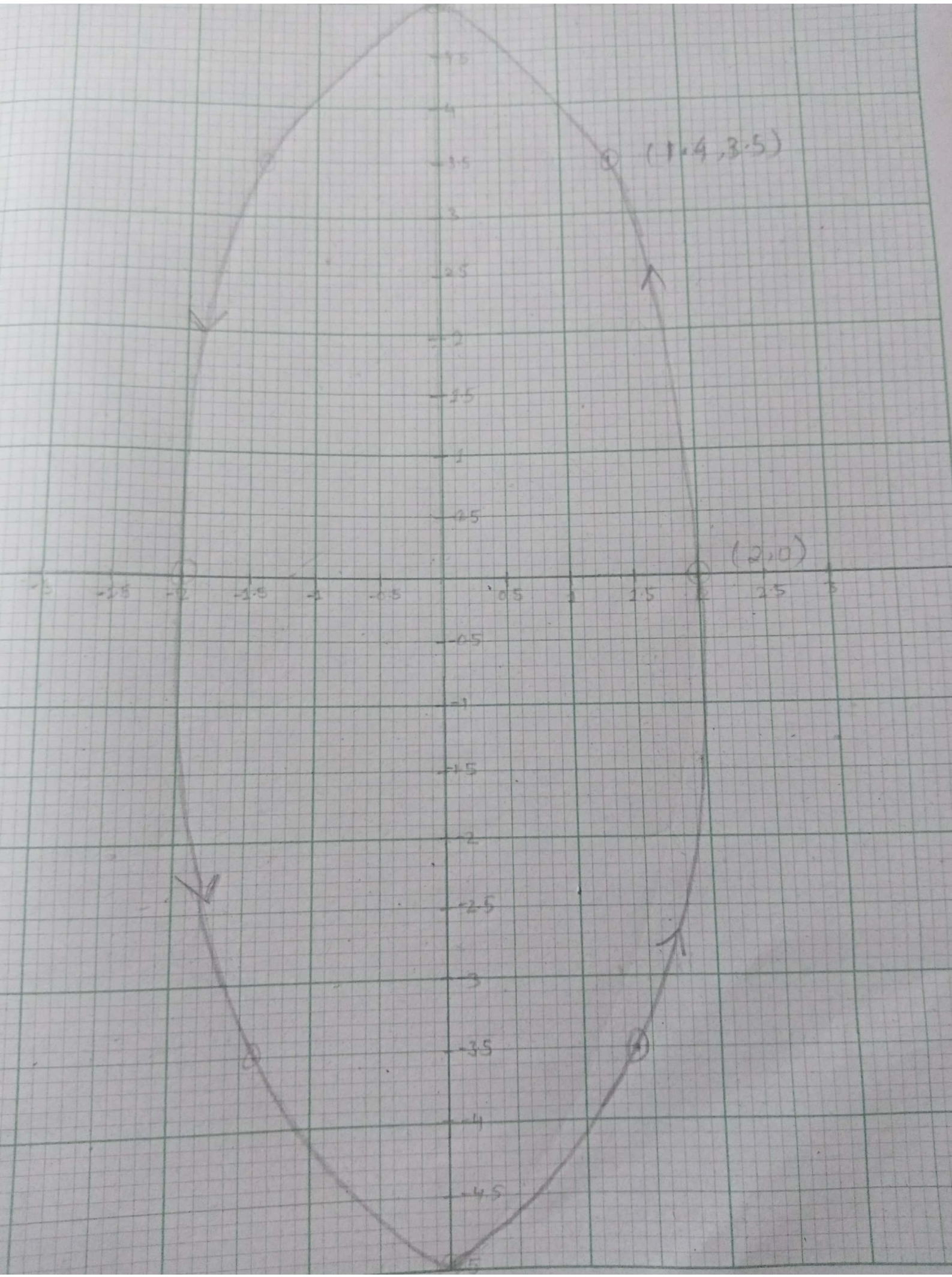
$\cos t = \frac{x}{2}$ $\qquad \sin t = \frac{y}{5}$

$1 = \frac{x^2}{2^2} + \frac{y^2}{5^2}$

$1 = \frac{5x + 2y}{10}$ $\qquad \frac{x^2}{4} + \frac{y^2}{25} = 1$

$5x + 2y = 10$

t	x	y
0	2	0
$\frac{\pi}{4}$	1.4	3.5
$\frac{\pi}{2}$	0	5
$3\pi/4$	-1.4	3.5
π	-2	0
$5\pi/4$	-1.4	-3.5
$3\pi/2$	0	-5
$7\pi/4$	1.4	-3.5
2π	2	0



Question # 09

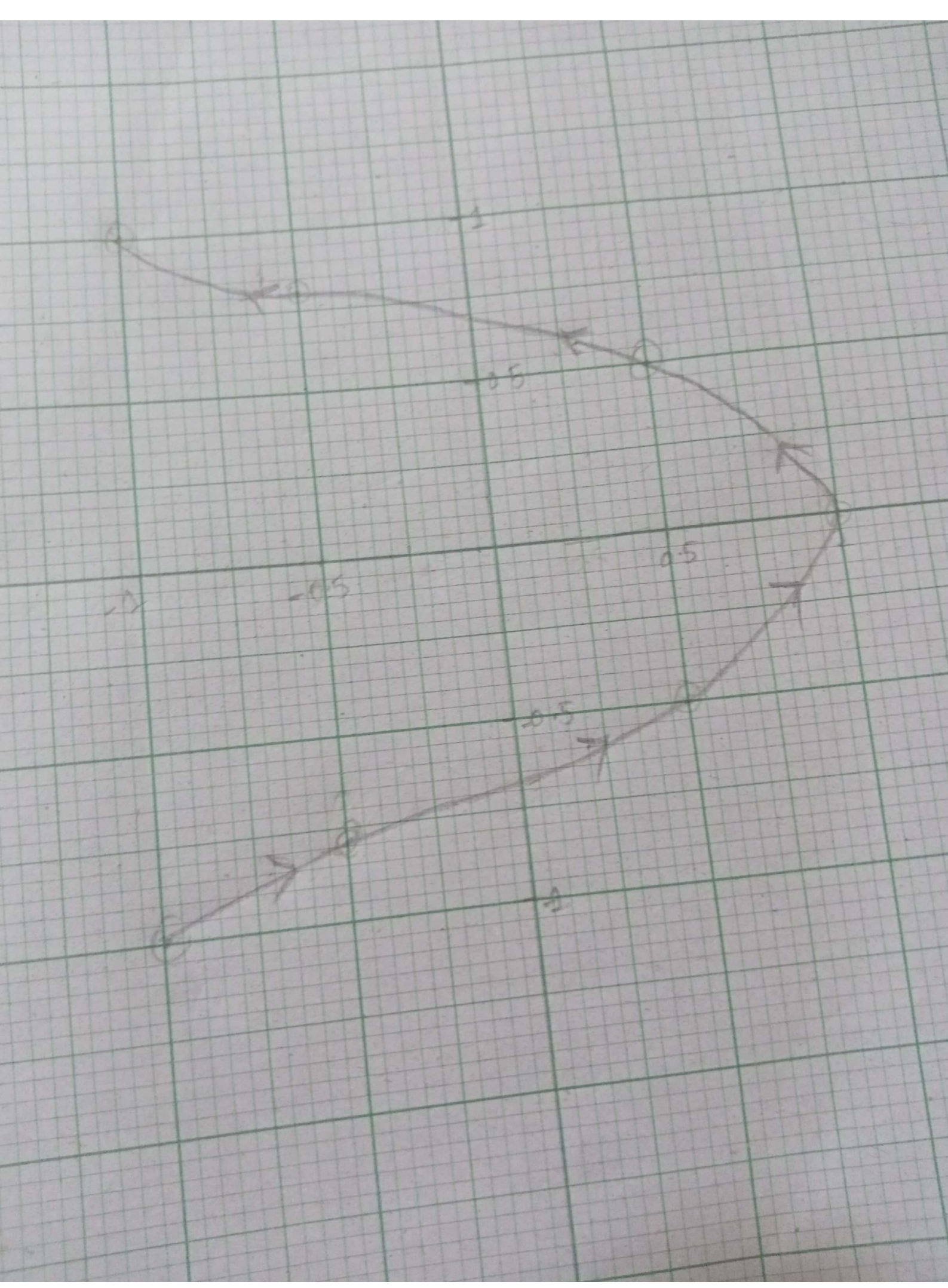
$x = \cos 2t \qquad y = \sin t \qquad \left(-\frac{\pi}{2} \leq t \leq \frac{\pi}{2}\right)$

$\cos 2t = 1 - 2\sin^2 t$

$x = 1 - 2\sin^2 t$

$\boxed{x = 1 - 2y^2}$

t	x	y
$-\frac{\pi}{2}$	-1	-1
$-\pi/3$	-0.5	-0.8
$-\pi/6$	0.5	-0.5
0	1	0
$\pi/6$	0.5	0.5
$\pi/3$	-0.5	0.866
$\pi/2$	-1	1



Question # 26

Q) Find parametric equation.

Two points given

$P(2,-1) = (x_1, y_1)$

$Q(3,1) = (x_2, y_2)$

First we find normal equation using two point form

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$$

$$y - (-1) = \frac{1 - (-1)}{3 - 2}(x - 2)$$

$$y + 1 = 2x - 4$$

~~$2x - y - 5 = 0$~~

~~y~~ $= 2x - 5$

$$\boxed{\begin{aligned} x &= t \\ y &= 2t - 5 \end{aligned}}$$

(a) Midpoint between P & Q :-

$$M(x,y) = \frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}$$

$$M(x,y) = \left(\frac{5}{2}, 0\right)$$

b) the point is one fourth of the way from P to

ratio = one fourth

$$m_1 : m_2 = 1 : 3$$

$$(x_1, y_1) = (2, -1)$$

$$(x_2, y_2) = (3, 1)$$

$$M(x,y) = \frac{m_1x_2 + m_2x_1}{m_1+m_2}, \frac{m_1y_2 + m_2y_1}{m_1+m_2}$$

$$= \frac{3+6}{4}, \frac{1+(-3)}{4}$$

$$M(x,y) = \left(\frac{9}{4}, \frac{-2}{2}\right)$$

c) the point is three fourth of the way from P

ratio = three fourths

$m_1 : m_2 = 3 : 1$

$x_1, y_1 = 2, -1$

$x_2, y_2 = 3, 1$

$$M(x,y) = \frac{m_1x_2 + m_2x_1}{m_1 + m_2}, \frac{m_1y_2 + m_2y_1}{m_1 + m_2}$$

$$\frac{9 + 2}{4}, \frac{3 + (-1)}{4}$$

$$M(x,y) = \left(\frac{11}{4}, \frac{1}{2}\right)$$

Question 45

x_2 $\sqrt{t}$

Question 45

$x = \sqrt{t}$, $y = 2t + 4$

$$\frac{dx}{dt} = \frac{1}{2\sqrt{t}} \qquad \frac{dy}{dt} = 2$$

$$\frac{dy}{dx} = \frac{2}{1/2\sqrt{t}}$$

$$= 2 \div \frac{1}{2\sqrt{t}}$$

$$= 2 \times 2\sqrt{t}$$

$$\frac{dy}{dx} = 4\sqrt{t}$$

$$\frac{d^2y}{dx^2} = \frac{\frac{d}{dt}\left(\frac{dy}{dx}\right)}{dx/dt} = \frac{\frac{d}{dt}(4\sqrt{t})}{1/(2\sqrt{t})}$$

$$= \frac{4\left(1/(2\sqrt{t})\right)}{1/(2\sqrt{t})}$$

$$\frac{d^2y}{dx^2} = 4$$

at $t = 1$

$$\boxed{\frac{d^2y}{dx^2} = 4}$$

Question # 49

$$x = \theta + \cos\theta \qquad y = 1 + \sin\theta$$

$$\frac{dx}{d\theta} = 1 - \sin\theta \qquad \frac{dy}{d\theta} = \cos\theta$$

$$\frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta} = \frac{\cos\theta}{1-\sin\theta}$$

$$\frac{d^2y}{dx^2} = \frac{\frac{d}{d\theta}\left(\frac{dy}{dx}\right)}{dx/d\theta}$$

$$= \frac{\frac{d}{d\theta}\left(\frac{\cos\theta}{1-\sin\theta}\right)}{1-\sin\theta}$$

$$= \left(\frac{(1-\sin\theta)(-\sin\theta) - \cos\theta(-\cos\theta)}{(1-\sin\theta)^2}\right) \div 1-\sin\theta$$

$$= \frac{-\sin\theta + \sin^2\theta + \cos^2\theta}{1 - 2\sin\theta + \sin^2\theta} \times \frac{1}{1-\sin\theta}$$

$$= \frac{1 - \sin\theta}{1 - 2\sin\theta + \sin^2\theta} \times \frac{1}{1-\sin\theta}$$

$$\frac{d^2y}{dx^2} = \frac{1}{(1-\sin\theta)^2}$$

here $\theta = \frac{\pi}{6}$

$$\frac{d^2y}{dx^2} = 4$$

Question # 51

$x = e^t$, $y = e^{-t}$

$\frac{dx}{dt} = e^t$ $\quad$ $\frac{dy}{dt} = -e^{-t}$

$\frac{dy}{dx} = \frac{-e^{-t}}{e^t}$

$= -e^{-t} e^{-t}$

$\frac{dy}{dx} = -e^{-2t}$ $\quad$ $t = 1$

$m = -e^{-2}$

$x = e^1$ $\quad$ $y = e^{-t}$

$x = e$ $\quad$ $y = e^{-1}$

$(x_1, y_1) = (e, e^{-1})$ $\quad$ $m = -e^{-2}$

point slope form

$y - y_1 = m(x - x_1)$

$y - e^{-1} = -e^{-2}(x - e)$

$y = \frac{-x}{e^2} + \frac{e}{e^2} + e^{-1}$

$y = -\frac{1}{e^2}x + 2e^{-1}$

2

Question 53

$x = 2\sin t$ $\quad y = 4\cos t$ $\quad 0 \leq t \leq 2\pi$

$\frac{dx}{dt} = 2\cos t$ $\quad \frac{dy}{dt} = -4\sin t$

$\frac{dy}{dx} = \frac{-4\sin t}{2\cos t} = \frac{-2\sin t}{\cos t}$

For horizontal tangent lines

$-2\sin t = 0$

$-\sin 0 = 0$

$0 \leq 0 \leq 2\pi$ $\quad\quad$ $\cos 0 \neq 0$

$-\sin \pi = 0$

$0 \leq \pi \leq 2\pi$ $\quad\quad$ $\cos \pi \neq 0$

$-\sin 2\pi = 0$

$0 \leq 2\pi \leq 2\pi$ $\quad\quad$ $\cos 2\pi \neq 0$

For H. Tangent $\boxed{0, \pi, 2\pi}$

For Vertical Tangent lines

$\cos t = 0$

$\cos \frac{\pi}{2} = 0$

$0 \leq \frac{\pi}{2} \leq 2\pi$ $\quad\quad$ $-2\sin \frac{\pi}{2} \neq 0$

$\cos \frac{3\pi}{2} = 0$

$0 \leq \frac{3\pi}{2} \leq 2\pi$ $\quad\quad$ $-2\sin \frac{3\pi}{2} \neq 0$

$$\boxed{\frac{\pi}{2}, \frac{3\pi}{2}}$$

Question # 65

$x = t^2$, $y = \frac{1}{3}t^3$ $\quad 0 \le t \le 1$

$\frac{dx}{dt} = 2t \qquad \frac{dy}{dt} = t^2$

$$L = \int_b^a \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt$$

$$\int_0^1 \sqrt{(2t)^2 + (t^2)^2}\, dt$$

$$2\int_0^1 t\sqrt{1+t^2}\, dt$$

let $x = 1 + t^2$

$\frac{dx}{dt} = 2t$

$dt = \frac{dx}{2t}$

$\frac{dx}{2} = t\,dt$

$$\frac{2}{2}\int_2^1 \sqrt{x}\, dx$$

$$\left[\frac{2x^{3/2}}{3}\right]_0^1$$

$$\boxed{L = \frac{2}{3}}$$

Question # 67

$$x = \cos 3t \qquad y = \sin 3t$$

$$\frac{dx}{dt} = -3\sin 3t \qquad \frac{dy}{dt} = 3\cos 3t$$

$$0 \leq t \leq \pi$$

$$L = \int_b^a \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt$$

$$\int_0^{\pi} \sqrt{(-3\sin 3t)^2 + (3\cos 3t)^2}$$

$$3\int_0^{\pi} dt$$

$$\boxed{3\pi}$$