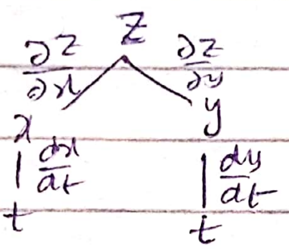
Exercise # 13.5

Assignment # 14

Date

Q 3

$z = 3\cos x - \sin xy \; ; \; x = 1/t \; , \; y = 3t$



$$\frac{dz}{dt} = \frac{\partial z}{\partial x} \cdot \frac{dx}{dt} + \frac{\partial z}{\partial y} \cdot \frac{dy}{dt}$$

$$\frac{dx}{dt} = \frac{-1}{t^2} \qquad \frac{\partial z}{\partial x} = -y\cos xy - 3\sin x$$

$$\frac{dy}{dt} = 3 \qquad \frac{\partial z}{\partial y} = -x\cos xy$$

$$\frac{dz}{dt} = \frac{-(-y\cos xy - 3\sin x)}{t^2} + -3x\cos xy.$$

$$\frac{dz}{dt} = \frac{3\sin x + y\cos xy}{t^2} - 3x\cos xy.$$

$$= \frac{3\sin x + y\cos xy - 3xt^2\cos xy}{t^2}$$

$$\cancel{3(\sin x - xt^2) + \cos xy(y}$$

$$\frac{3\sin x + \cos xy(y - 3xt^2)}{t^2}$$

$$\frac{dz}{dt} = \frac{3\sin(\frac{1}{t}) + 3t\cos 3}{t^2} - \frac{3\cos 3}{t}$$

No.

$$= \frac{3t\sin\left(\frac{1}{t}\right) + \cancel{3t^2\cos 3} - \cancel{3t^2\cos 3}}{t^3}$$

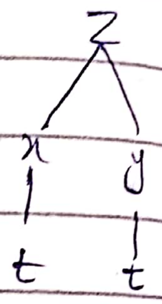
$$= \frac{3\cancel{t}\sin\left(\frac{1}{t}\right)}{t^{\cancel{3}}}$$

$$\frac{dz}{dt} = 3t^{-2}\sin\left(\frac{1}{t}\right)$$

Q5

$\cancel{z = \sqrt{1+x} = 2}$

$$z = e^{1-xy};\quad x = t^{1/3},\quad y = t^3$$



$$\frac{dz}{dt} = \frac{\partial z}{\partial x}\frac{dx}{dt} + \frac{\partial z}{\partial y}\frac{dy}{dt}$$

$$\frac{\partial z}{\partial x} = -ye^{1-xy} \qquad \frac{dx}{dt} = \frac{1}{3}t^{-2/3}$$

$$\frac{\partial z}{\partial y} = -xe^{1-xy} \qquad \frac{dy}{dt} = 3t^2$$

$$\frac{dz}{dt} = \frac{-yt^{\frac{4}{3}}e^{1-xy}}{3} + 3t^2(-xe^{1-xy})$$

$$= \frac{-t^{\frac{7}{3}}e^{1-t^{10/3}}}{3} - 3t^{7/3}e^{1-t^{10/3}}$$

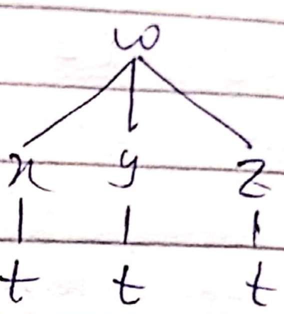
$$= \frac{-t^{\frac{7}{3}}e^{1-t^{10/3}} - 9t^{7/3}e^{1-t^{10/3}}}{3}$$

$$\frac{dz}{dt} = -\frac{10}{3}t^{7/3}e^{1-t^{10/3}}$$

Q9

$w = 5\cos xy - \sin xz$; $x = \frac{1}{t}$, $y = t$, $z = t^3$

$$\frac{dw}{dt} = \frac{\partial w}{\partial x}\cdot\frac{dx}{dt} + \frac{\partial w}{\partial y}\frac{dy}{dt} + \frac{\partial w}{\partial z}\frac{dz}{dt}$$



$\frac{\partial w}{\partial x} = -5y\sin xy - z\cos xz$; $\frac{dx}{dt} = -\frac{1}{t^2}$

$\frac{\partial w}{\partial y} = -5x\sin xy$ $\quad \frac{dy}{dt} = 1$

$\frac{\partial w}{\partial z} = -x\cos xz$ $\quad \frac{dz}{dt} = 3t^2$

$$\frac{dw}{dt} = \frac{5y\sin xy + z\cos xz}{t^2} - 5x\sin xy - 3xt^2\cos xz$$

$$= \frac{5y\sin xy + z\cos xz - 5xt^2\sin xy - 3xt^4\cos xz}{t^2}$$

$$= \frac{5t\cancel{\sin(1)} + t^3\cos t^2 - 5t\cancel{\sin(1)} - 3t^3\cos t^2}{t^2}$$

$$= \frac{-2t^{\cancel{3}}\cos t^2}{\cancel{t}}$$

$$\boxed{\frac{dw}{dt} = -2t\cos t^2}$$

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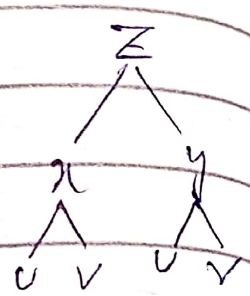
Q17

$z = 8x^2y - 2x + 3y$; $x = uv$, $y = u - v$

$$\boxed{\frac{\partial z}{\partial u} = \frac{\partial z}{\partial x}\frac{\partial x}{\partial u} + \frac{\partial z}{\partial y}\frac{\partial y}{\partial u}}$$



$\frac{\partial z}{\partial x} = 16xy - 2$ $\qquad$ $\frac{\partial x}{\partial u} = v$

$\frac{\partial z}{\partial y} = 8x^2 + 3$ $\qquad$ $\frac{\partial y}{\partial u} = 1$

$$\frac{\partial z}{\partial u} = v(16xy - 2) + 8x^2 + 3$$

$$= 16uv(u-v)v - 2v + 8u^2v^2 + 3$$

$$= 16u^2v^2 - 16uv^3 - 2v + 8u^2v^2 + 3$$

$$\boxed{\frac{\partial z}{\partial u} = 24u^2v^2 - 16uv^3 - 2v + 3}$$

$$\boxed{\frac{\partial z}{\partial v} = \frac{\partial z}{\partial x}\cdot\frac{\partial x}{\partial v} + \frac{\partial z}{\partial y}\cdot\frac{\partial y}{\partial v}}$$

$\cancel{\partial z}$ $\frac{\partial x}{\partial v} = u$ $\qquad$ $\frac{\partial y}{\partial v} = -1$

$$\frac{\partial z}{\partial v} = 16uxy - 2u - 8x^2 - 3$$

$$= 16u^2v(u-v) - 2u - 8u^2v^2 - 3$$

$$= 16u^3v - 16u^2v^2 - 2u - 8u^2v^2 - 3$$

$$\boxed{\frac{\partial z}{\partial v} = 16u^3v - 24u^2v^2 - 2u - 3}$$

Q19

$$z = x/y \qquad x = 2\cos u \qquad y = 3\sin v$$

$$\begin{tikzcd} & z \arrow[dl] \arrow[dr] & \\ x \arrow[d] & & y \arrow[d] \\ u & & v \end{tikzcd}$$

$$\boxed{\frac{\partial z}{\partial u} = \frac{\partial z}{\partial x} \cdot \frac{dx}{du}}$$

$$\frac{\partial z}{\partial x} = \frac{1}{y} \quad ; \quad \frac{dx}{du} = -2\sin u$$

$$\frac{\partial z}{\partial u} = \frac{-2\sin u}{y}$$

$$\boxed{\frac{\partial z}{\partial u} = \frac{-2\sin u}{3\sin v}}$$

$$\boxed{\frac{\partial z}{\partial v} = \frac{\partial z}{\partial y} \cdot \frac{dy}{dv}}$$

$$\frac{\partial z}{\partial y} = \frac{-x}{y^2} \qquad \frac{dy}{dv} = 3\cos v$$

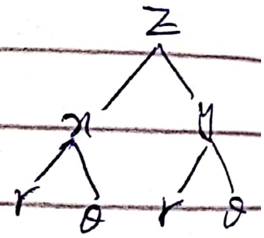
$$\frac{\partial z}{\partial v} = \frac{-3x\cos v}{y^2}$$

$$= \frac{-6\cos u\cos v}{9\sin^2 v}$$

$$\boxed{\frac{\partial z}{\partial v} = \frac{-2\cos u\cos v}{3\sin^2 v}}$$

Q23

$T = x^2y - xy^3 + 2 \; ; \; x = r\cos\theta, \; y = r\sin\theta$



$$\boxed{\frac{\partial T}{\partial r} = \frac{\partial z}{\partial x}\cdot\frac{\partial x}{\partial r} + \frac{\partial z}{\partial y}\cdot\frac{\partial y}{\partial r}}$$

$$\frac{\partial z}{\partial x} = 2xy - y^3 \qquad \frac{\partial x}{\partial r} = \cos\theta$$

$$\frac{\partial z}{\partial y} = x^2 - 3xy^2 \qquad \frac{\partial y}{\partial r} = \sin\theta$$

$$\frac{\partial T}{\partial r} = 2xy\cos\theta - y^3\cos\theta + x^2\sin\theta - 3xy^2\sin\theta$$

$$\cancel{x^2\sin\theta - 3xy^2\sin\theta}$$

$$\frac{\partial T}{\partial r} = 2r^2\cos^2\theta \sin\theta - r^3\sin^3\theta\cos\theta + r^2\cos^2\theta \sin\theta - 3r^3\sin^3\theta\cos\theta$$

$$\boxed{\frac{\partial T}{\partial r} = 3r^2\cos^2\theta \sin\theta - 4r^3\sin^3\theta\cos\theta}$$

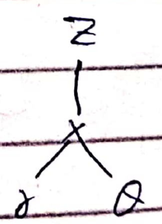
$$\boxed{\frac{\partial T}{\partial \theta} = \frac{\partial z}{\partial x}\cdot\frac{\partial x}{\partial \theta} + \frac{\partial z}{\partial y}\frac{\partial y}{\partial \theta}}$$

$$\frac{\partial x}{\partial \theta} = -r\sin\theta \quad , \quad \frac{\partial y}{\partial \theta} = r\cos\theta$$

$$\frac{\partial T}{\partial \theta} = -2rxy\sin\theta + ry^3\sin\theta + rx^2\cos\theta - 3rxy^2\cos\theta$$

$$\boxed{\frac{\partial T}{\partial \theta} = -2r^3\sin^2\theta\cos\theta + r^4\sin^2\theta + r^3\cos^3\theta - 3r^4\cos^2\theta\sin^2\theta}$$

Q27



$$z = \ln(x^2+1) \quad ; \quad x = r\cos\theta$$

$$\boxed{\frac{\partial z}{\partial r} = \frac{dz}{dx}\cdot\frac{\partial x}{\partial r}}$$

$$\frac{dz}{dx} = \frac{2x}{x^2+1} \qquad \frac{\partial x}{\partial r} = \cos\theta$$

$$\frac{\partial z}{\partial r} = \frac{2x\cos\theta}{x^2+1}$$

$$\boxed{\frac{\partial z}{\partial r} = \frac{2r\cos^2\theta}{r^2\cos^2\theta+1}}$$

$$\boxed{\frac{\partial z}{\partial \theta} = \frac{dz}{dx}\,\frac{\partial x}{\partial \theta}}$$

$$\frac{\partial x}{\partial \theta} = -r\sin\theta$$

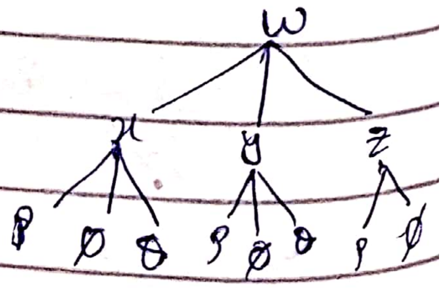
$$\frac{\partial z}{\partial \theta} = \frac{-2xr\sin\theta}{x^2+1}$$

$$\boxed{\frac{\partial z}{\partial \theta} = \frac{-2r^2\cos\theta\sin\theta}{r^2\cos^2\theta+1}}$$

Q29

$w = 4x^2+4y^2+z^2$, $x = \rho\sin\phi\cos\theta$, $y = \rho\sin\phi\sin\theta$
$z = \rho\cos\phi$

$$\boxed{\frac{\partial w}{\partial \rho} = \frac{\partial w}{\partial x}\cdot\frac{\partial x}{\partial \rho} + \frac{\partial w}{\partial y}\frac{\partial y}{\partial \rho} + \frac{\partial w}{\partial z}\frac{\partial z}{\partial \rho}}$$



$$\frac{\partial w}{\partial x} = 8x, \quad \frac{\partial x}{\partial \rho} = \sin\phi\cos\theta$$

$$\frac{\partial w}{\partial y} = 8y, \quad \frac{\partial y}{\partial \rho} = \sin\phi\sin\theta$$

$$\frac{\partial w}{\partial z} = 2z, \quad \frac{\partial z}{\partial \rho} = \cos\phi$$

$$\frac{\partial w}{\partial \rho} = 8x\sin\phi\cos\theta + 8y\sin\phi\sin\theta + 2z\cos\phi$$

$$= 8\rho\sin^2\phi\cos^2\theta + 8\rho\sin^2\phi\sin^2\theta + 2\rho\cos^2\phi$$

$$= 2\rho\left(4\sin^2\phi\cos^2\theta + 4\sin^2\phi\sin^2\theta + \cos^2\phi\right)$$

$$= 2\rho\left(4\sin^2\phi\left(\cos^2\theta + \sin^2\theta\right) + \cos^2\phi\right)$$

$$\boxed{\frac{\partial w}{\partial \rho} = 2\rho\left(4\sin^2\phi + \cos^2\phi\right)}$$

$$\boxed{\frac{\partial w}{\partial \phi} = \frac{\partial w}{\partial x}\cdot\frac{\partial x}{\partial \phi} + \frac{\partial w}{\partial y}\frac{\partial y}{\partial \phi} + \frac{\partial w}{\partial z}\frac{\partial z}{\partial \phi}}$$

$$\frac{\partial x}{\partial \phi} = \rho\cos\phi\cos\theta, \quad \frac{\partial y}{\partial \phi} = \rho\cos\phi\sin\theta, \quad \frac{\partial z}{\partial \phi} = -\rho\sin\phi$$

$$\frac{\partial w}{\partial \phi} = 8x\rho\cos\phi\cos\theta + 8y\rho\cos\phi\sin\theta - 2z\rho\sin\phi$$

$$= 8\rho^2\sin\phi\cos\phi\cos^2\theta + 8\rho^2\sin\phi\cos\phi\sin^2\theta - 2\rho^2\sin\phi\cos\phi$$

$$= 2\rho^2\sin\phi\cos\phi\left(4\cos^2\theta + 4\sin^2\theta - 1\right)$$

$$\boxed{\frac{\partial w}{\partial \phi} = 6\rho^2\sin\phi\cos\phi}$$

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$$\boxed{\frac{\partial w}{\partial \theta} = \frac{\partial w}{\partial x}\frac{\partial x}{\partial \theta} + \frac{\partial w}{\partial y}\frac{\partial y}{\partial \theta}}$$

$$\frac{\partial x}{\partial \theta} = -\rho \sin\phi \sin\theta$$

$$\frac{\partial y}{\partial \theta} = \rho \sin\phi \cos\theta$$

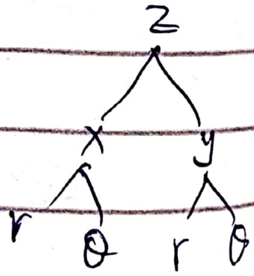
$$\frac{\partial w}{\partial \theta} = -8x\rho \sin\phi \sin\theta + 8y\rho \sin\phi \cos\theta$$

$$= -8\rho^2 \sin^2\phi \cancel{\sin\theta \cos\theta} + 8\rho^2 \cancel{\sin^2\phi} \sin\theta \cos\theta$$

$$\boxed{\frac{\partial w}{\partial \theta} = 0}$$

Q33

$$z = xye^{\frac{x}{y}} \; ; \; x = r\cos\theta, \; y = r\sin\theta$$



$$\boxed{\frac{\partial z}{\partial r} = \frac{\partial z}{\partial x}\frac{\partial x}{\partial r} + \frac{\partial z}{\partial y}\frac{\partial y}{\partial r}}$$

$$\frac{\partial z}{\partial x} = y\left(\frac{x}{y}e^{\frac{x}{y}} + e^{\frac{x}{y}}\right)$$

$$\frac{\partial z}{\partial x} = xe^{\frac{x}{y}} + ye^{\frac{x}{y}}$$

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$$\frac{\partial x}{\partial r} = \cos\theta$$

$$\frac{\partial z}{\partial y} = x\left(-\frac{x}{y^2}e^{\frac{x}{y}} + e^{\frac{x}{y}}\right)$$

$$\frac{\partial z}{\partial y} = xe^{\frac{x}{y}} - \frac{x^2}{y^2}e^{\frac{x}{y}}$$

$$\frac{\partial y}{\partial r} = \sin\theta$$

$$\frac{\partial z}{\partial r} = xe^{\frac{x}{y}}\cos\theta + ye^{\frac{x}{y}}\cos\theta + xe^{\frac{x}{y}}\sin\theta - \frac{x^2}{y^2}e^{\frac{x}{y}}\sin\theta$$

$$= e^{\frac{x}{y}}\left(x\cos\theta + y\cos\theta + x\sin\theta - \frac{x^2}{y^2}\sin\theta\right)$$

$$= e^{\frac{\cos\theta}{\sin\theta}}\left(r\cos^2\theta + r\sin\theta\cos\theta + r\cos\theta\sin\theta - \frac{\cos^2\theta}{\sin\theta}\right)$$

$$\frac{\partial z}{\partial r} = e^{\frac{\cos\theta}{\sin\theta}}\left(r\cos^2\theta + 2r\sin\theta\cos\theta - \frac{\cos^2\theta}{\sin\theta}\right)$$

$$\left.\frac{\partial z}{\partial r}\right|_{r=2,\theta=\pi/6} = e^{\sqrt{3}}\left(\frac{\cancel{3}}{2} + \sqrt{3} - \frac{\cancel{3}}{2}\right)$$

$$\boxed{\left.\frac{\partial z}{\partial r}\right|_{r=2,\theta=\frac{\pi}{6}} = \sqrt{3}\,e^{\sqrt{3}}}$$

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$$\boxed{\frac{\partial z}{\partial \theta} = \frac{\partial z}{\partial x}\frac{\partial x}{\partial \theta} + \frac{\partial z}{\partial y}\frac{\partial y}{\partial \theta}}$$

$$\frac{\partial x}{\partial \theta} = -r\sin\theta$$

$$\frac{\partial y}{\partial \theta} = r\cos\theta$$

$$\frac{\partial z}{\partial \theta} = -rxe^{\frac{x}{y}}\sin\theta - rye^{\frac{x}{y}}\sin\theta + rxe^{\frac{x}{y}}\cos\theta - \frac{rx^2e^{\frac{x}{y}}\cos\theta}{y^2}$$

$$= -r^2e^{\frac{\cos\theta}{\sin\theta}}\cos\theta\sin\theta - r^2e^{\frac{\cos\theta}{\sin\theta}}\sin^2\theta + r^2e^{\frac{\cos\theta}{\sin\theta}}\cos^2\theta - \frac{r\cos^3\theta}{\sin^2\theta}\cdot e^{\frac{\cos\theta}{\sin\theta}}$$

$$\cancel{= re^{\frac{\cos\theta}{\sin\theta}}(r\cos}$$

$$= -re^{\frac{\cos\theta}{\sin\theta}}\left(r\cos\theta\sin\theta + r\sin^2\theta - r\cos^2\theta + \frac{\cos^3\theta}{\sin^2\theta}\right)$$

$$\left.\frac{\partial z}{\partial \theta}\right|_{r=2,\theta=\frac{\pi}{6}} = -2e^{\sqrt{3}}\left(\frac{\sqrt{3}}{2} + \frac{1}{2} - \frac{3}{2} + \frac{3\sqrt{3}}{2}\right)$$

$$= -2e^{\sqrt{3}}(-1 + 2\sqrt{3})$$

$$\boxed{\left.\frac{\partial z}{\partial \theta}\right|_{r=2,\theta=\frac{\pi}{6}} = e^{\sqrt{3}}(2 - 4\sqrt{3})}$$

Date

Q41

$$x^2y^3 + \cos y = 0$$

$$f(x,y) = x^2y^3 + \cos y$$

$$\frac{dy}{dx} = -\frac{\partial f/\partial x}{\partial f/\partial y}$$

$$\frac{\partial f}{\partial x} = 2xy^3$$

$$\frac{\partial f}{\partial y} = 3x^2y^2 - \sin y$$

$$\boxed{\frac{dy}{dx} = \frac{-2xy^3}{3x^2y^2 - \sin y}}$$

Q43

$$f(x,y) = e^{xy} + ye^y - 1$$

$$\frac{\partial f}{\partial x} = ye^{xy}$$

$$\frac{\partial f}{\partial y} = xe^{xy} + ye^y + e^y$$

$$\frac{dy}{dx} = -\frac{\partial f/\partial x}{\partial f/\partial y}$$

$$\boxed{\frac{dy}{dx} = \frac{-ye^{xy}}{xe^{xy} + ye^y + e^y}}$$

No.

Q47

$$ye^x - 5\sin 3z = 3z$$

$$f = ye^x - 5\sin 3z - 3z$$

$$\frac{\partial z}{\partial x} = -\frac{\partial f/\partial x}{\partial f/\partial z}$$

$$\frac{\partial z}{\partial x} = -\frac{ye^x}{-15\cos 3z - 3}$$

$$\boxed{\frac{\partial z}{\partial x} = \frac{ye^x}{3(\cos 3z + 1)}}$$

$$\frac{\partial z}{\partial y} = \frac{\partial f/\partial y}{\partial f/\partial z}$$

$$= -\frac{e^x}{-15\cos 3z - 3}$$

$$\boxed{\frac{\partial z}{\partial y} = \frac{e^x}{15\cos 3z - 3}}$$

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Q51

$$z = f(x+2y)$$

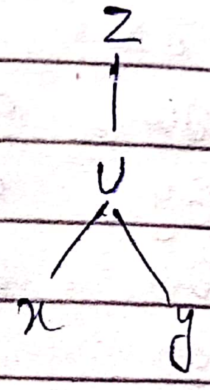
~~let v = x+2y~~

~~z = f(v)~~

let $u = x+2y$ then $z = f(u)$

$$\frac{\partial z}{\partial x} = \frac{dz}{du}\frac{\partial u}{\partial x}$$

$$\frac{\partial z}{\partial x} = \frac{dz}{du}$$



$$\frac{\partial z}{\partial y} = \frac{dz}{du}\frac{\partial u}{\partial y}$$

$$\frac{\partial z}{\partial y} = 2\frac{dz}{du}$$

put in given equation

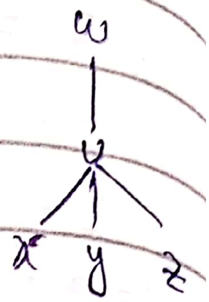
$$2\frac{\partial z}{\partial x} - \frac{\partial z}{\partial y} = 0$$

$$2\frac{\partial z}{\partial u} - 2\frac{\partial z}{\partial u} = 0$$

$$\boxed{0 = 0}$$

showed.

Q53



$$w = f(u)$$

$$u = x + 2y + 3z$$

$$\frac{\partial w}{\partial x} = \frac{dw}{du}\frac{\partial u}{\partial x}$$

$$\frac{\partial w}{\partial x} = \frac{dw}{du} \qquad \because \frac{\partial u}{\partial x} = 1$$

$$\frac{\partial w}{\partial y} = \frac{dw}{du}\frac{\partial u}{\partial y}$$

$$\frac{\partial w}{\partial y} = \frac{2dw}{du} \qquad \because \frac{\partial u}{\partial y} = 2$$

$$\frac{\partial w}{\partial z} = \frac{dw}{du}\frac{\partial u}{\partial z}$$

$$\frac{\partial w}{\partial z} = 3\frac{dw}{du} \qquad \because \frac{\partial u}{\partial z} = 3$$

$$\frac{\partial w}{\partial x} + \frac{\partial w}{\partial y} + \frac{\partial w}{\partial z} = 6\frac{dw}{du}$$

$$\frac{dw}{du} + 2\frac{dw}{du} + 3\frac{dw}{du} = 6\frac{dw}{du}$$

$$\boxed{6\frac{dw}{du} = 6\frac{dw}{du}}$$

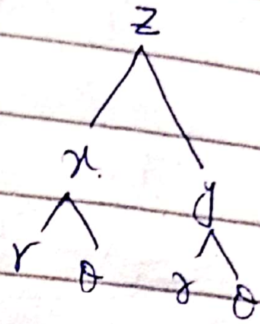
Showed!

Q57

a) $x = r\cos\theta$ — (1)

$\cancel{\frac{\partial}{\partial x}(x = r\cos\theta)}$

$\cancel{1 = \frac{\partial r}{\partial x}}$



$y = r\sin\theta$ — (2)

Squaring & adding

$$x^2 + y^2 = r^2(\cos^2\theta + \sin^2\theta)$$

$$r = \sqrt{x^2 + y^2} \longrightarrow (3)$$

$$\frac{\partial r}{\partial x} = \frac{\cancel{2}x}{\cancel{2}\sqrt{x^2+y^2}}$$

$\because x = r\cos\theta$ $\because r = \sqrt{x^2+y^2}$

$$\frac{\partial r}{\partial x} = \frac{\cancel{r}\cos\theta}{\cancel{r}}$$

$$\boxed{\frac{\partial r}{\partial x} = \cos\theta}$$

Dividing eq (2) by eq (1)

$$\frac{y}{x} = \frac{\cancel{r}\sin\theta}{\cancel{r}\cos\theta}$$

$$\tan\theta = \frac{y}{x}$$

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$$\theta = \tan^{-1}\left(\frac{y}{x}\right) \rightarrow (4)$$

$$\frac{\partial\theta}{\partial x} = \frac{1}{1+\left(\frac{y}{x}\right)^2} \cdot \frac{-y}{x^2}$$

$$\frac{\partial\theta}{\partial x} = \frac{-y}{x^2+y^2}$$

$$\frac{\partial\theta}{\partial x} = \frac{-\cancel{r}\sin\theta}{r^{\cancel{2}}}$$

$$\boxed{\frac{\partial\theta}{\partial x} = -\frac{\sin\theta}{r}}$$

b) from eq (3)

$$r = \sqrt{x^2+y^2}$$

$$\frac{\partial r}{\partial y} = \frac{\cancel{2}y}{2\sqrt{x^2+y^2}}$$

$$\frac{\partial r}{\partial y} = \frac{\cancel{r}\sin\theta}{\cancel{r}}$$

$$\boxed{\frac{\partial r}{\partial y} = \sin\theta}$$

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θ. From eq (4)

$$\theta = \tan^{-1}\frac{y}{x}$$

$$\frac{\partial \theta}{\partial y} = \frac{1}{1+\left(\frac{y}{x}\right)^2} \times \frac{1}{x}$$

$$= \frac{x}{x^2+y^2}$$

$$= \frac{\cancel{r}\cos\theta}{r^{\cancel{2}}}$$

$$\boxed{\frac{\partial \theta}{\partial y} = \frac{\cos\theta}{r}}$$

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