Assignment #04

Q3

a) $y^2 = 4x$

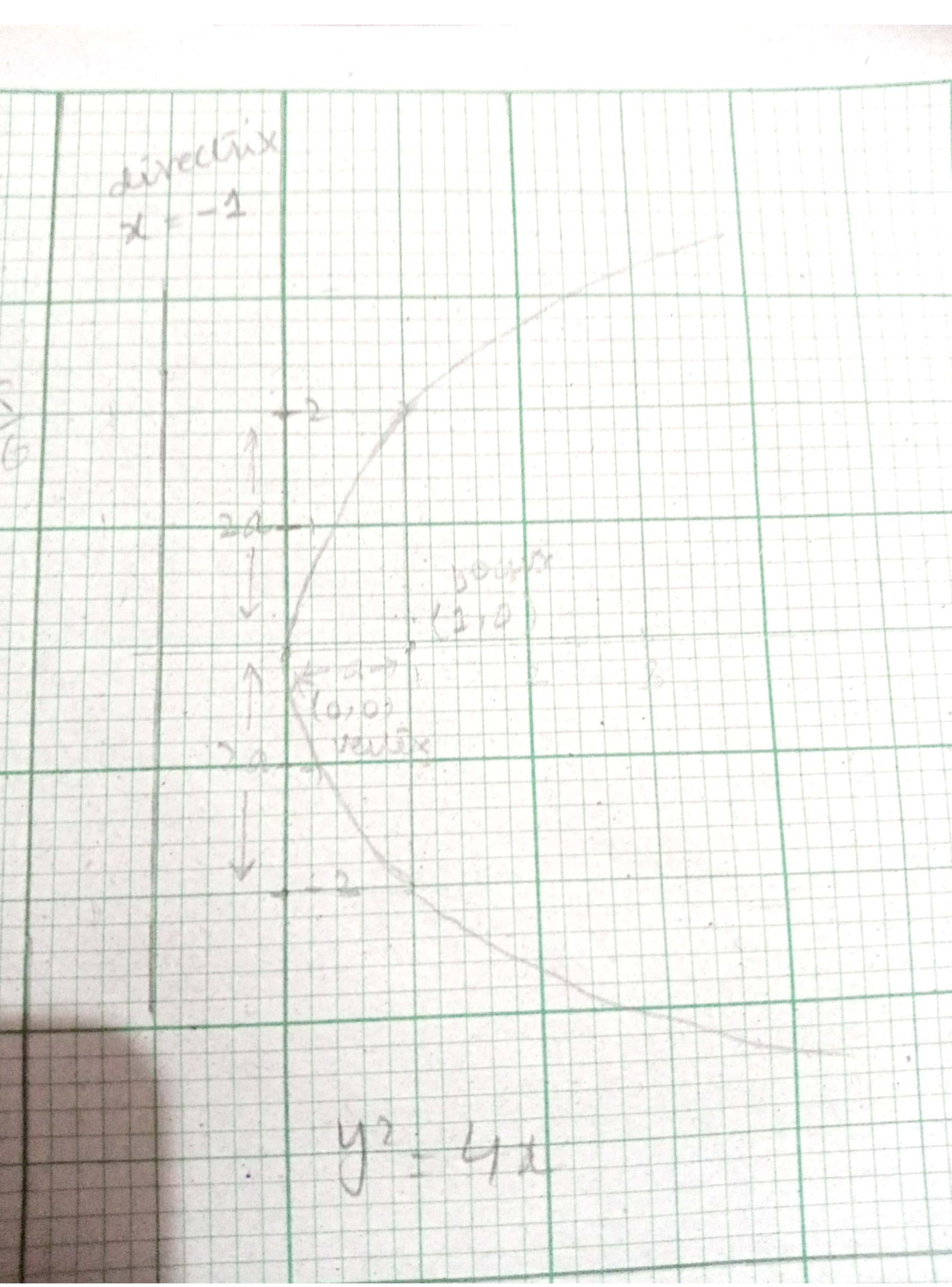
Vertex = (0, 0)

Symmetric axis = x-axis

$a = 1$ $\quad \therefore y^2 = 4 \times 1 x$

focus = (1, 0)

directrix → $x = -1$



Q3

b) $x^2 = -8y$

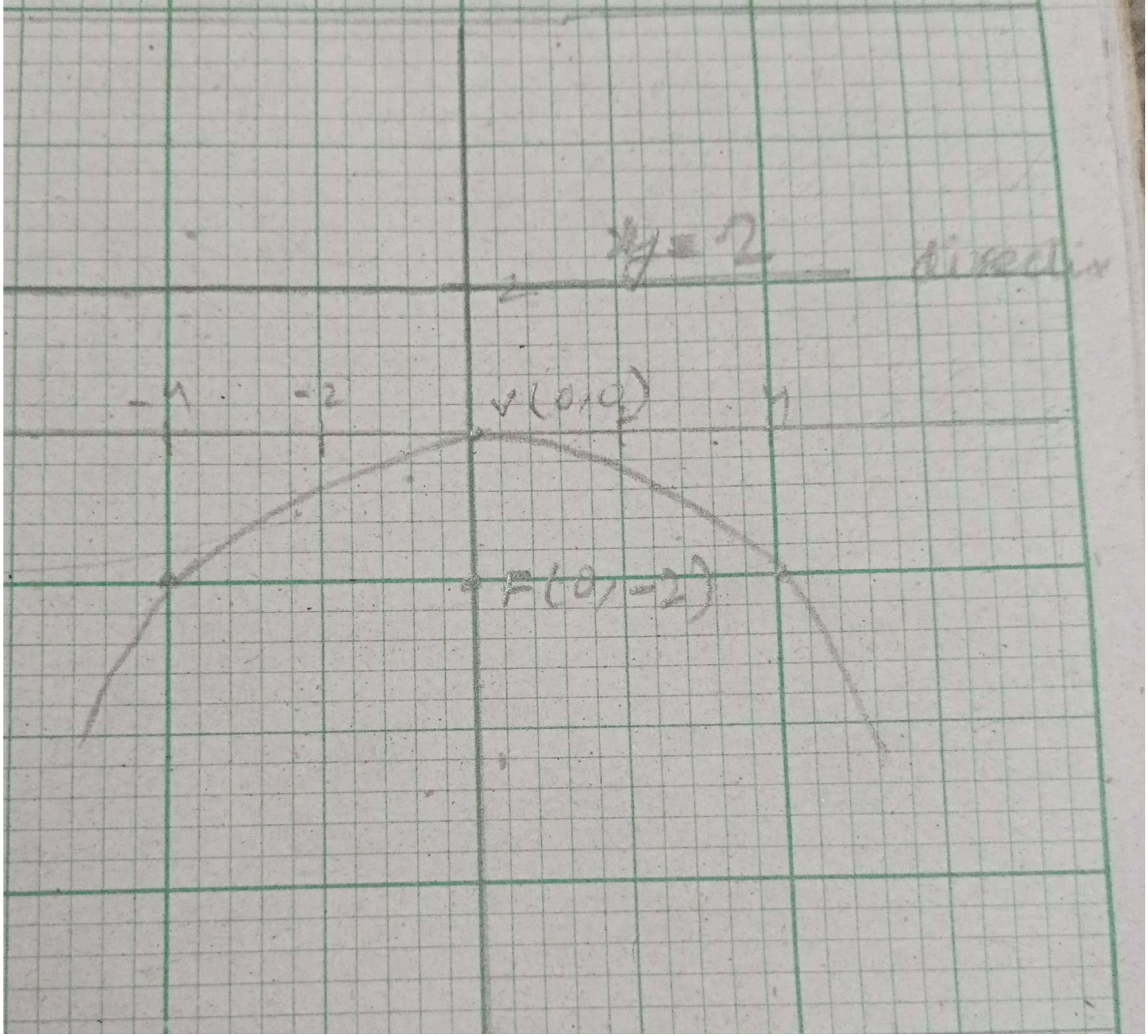
$4a = -8$

$a = -2$

vertex $(0, 0)$

Focus $\rightarrow (0, -2)$

directrix $\rightarrow$ $y = 2$



Q5

a) $(y-1)^2 = -12(x+4)$

vertex $= (-4, 1)$

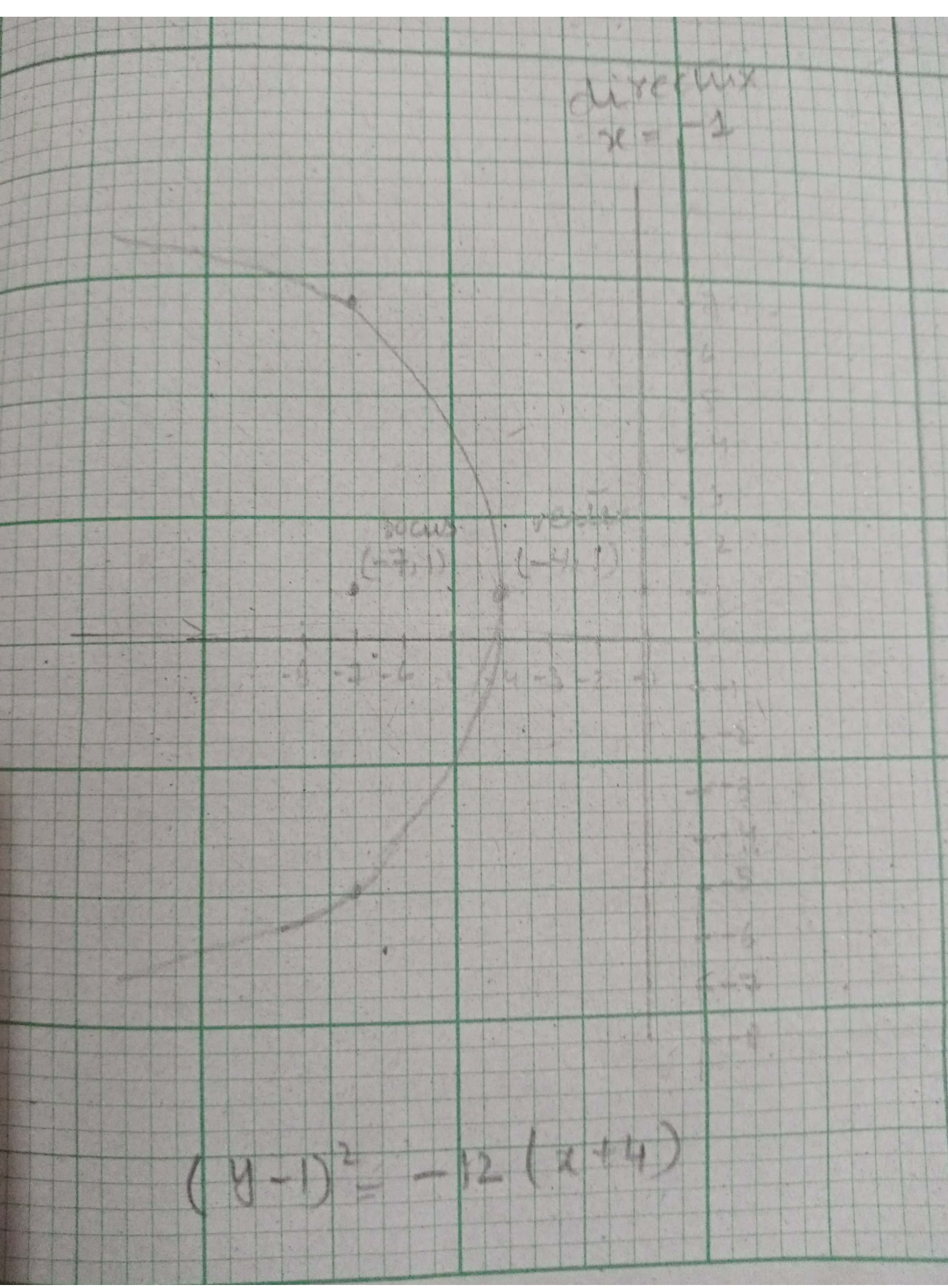
$4a = -12$

$a = -3$

Symmetric axis : x-axis

focus $= (-7, 1)$

directrix $\Rightarrow x = -1$



$$(y-1)^2 = -12(x+4)$$

Q5

b) ~~Foci $(\pm 3, 0)$ asymptotes $y = \pm 2x$~~

$$(x-1)^2 = 2\left(y - \frac{1}{2}\right)$$

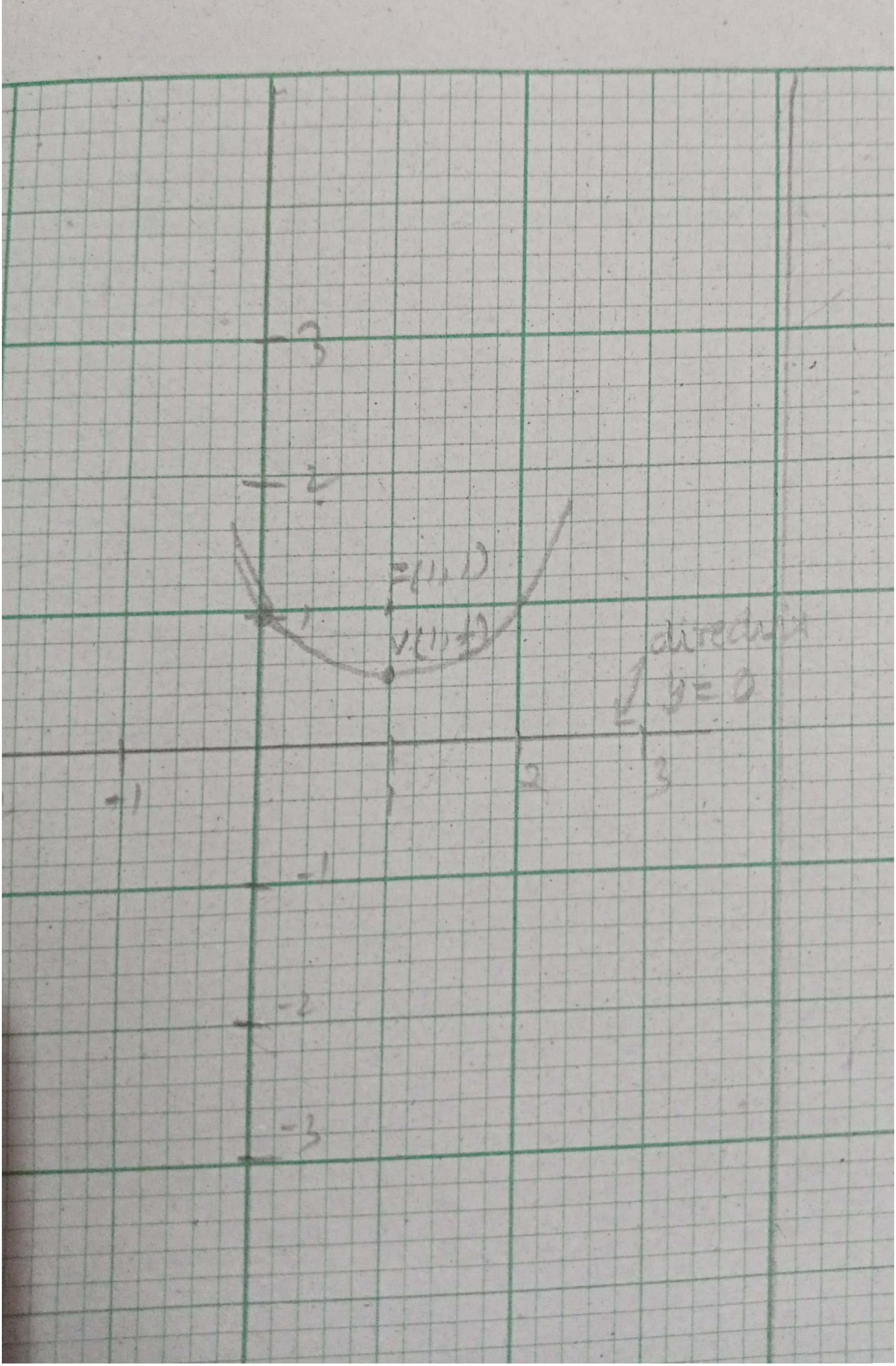
Vertex $= (h, k) = \left(1, \frac{1}{2}\right)$

$$4a = 2$$

$$a = \frac{1}{2}$$

Focus $\rightarrow (1, 1)$

directrix $\rightarrow y = 0$



Q7

a) $\frac{x^2}{16} + \frac{y^2}{9} = 1$

major axis → x-axis

$a = 4$ $\quad b = 3$

center ~~vertex~~ ⇒ (0, 0)

$c = \sqrt{a^2 - b^2}$

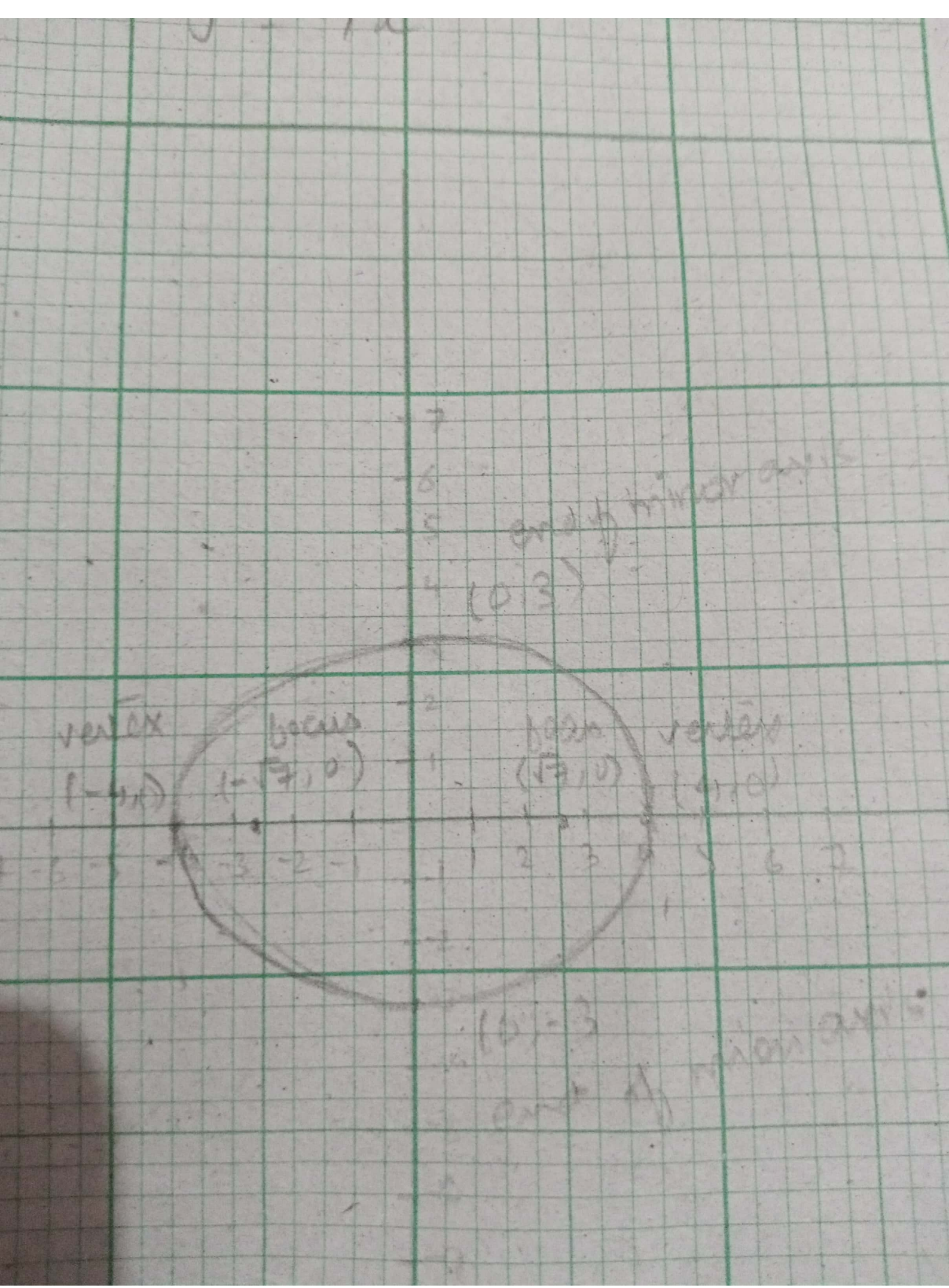
$c = \sqrt{16 - 9}$

$c = \pm\sqrt{7}$

vertices ⇒ $(-4, 0)$ $(4, 0)$

foci ⇒ $(-\sqrt{7}, 0)$, $(\sqrt{7}, 0)$

ends of Minor axis ⇒ $(0, 3)$, $(0, -3)$



q7

b) $9x^2 + y^2 = 9$

$\div$ by. 9

$$\frac{x^2}{1} + \frac{y^2}{9} = 1$$

major axis y-axis

Vertex (0, 0)

$a = 3 \quad b = 1$

~~Foci~~ → $(0, \pm 3)$

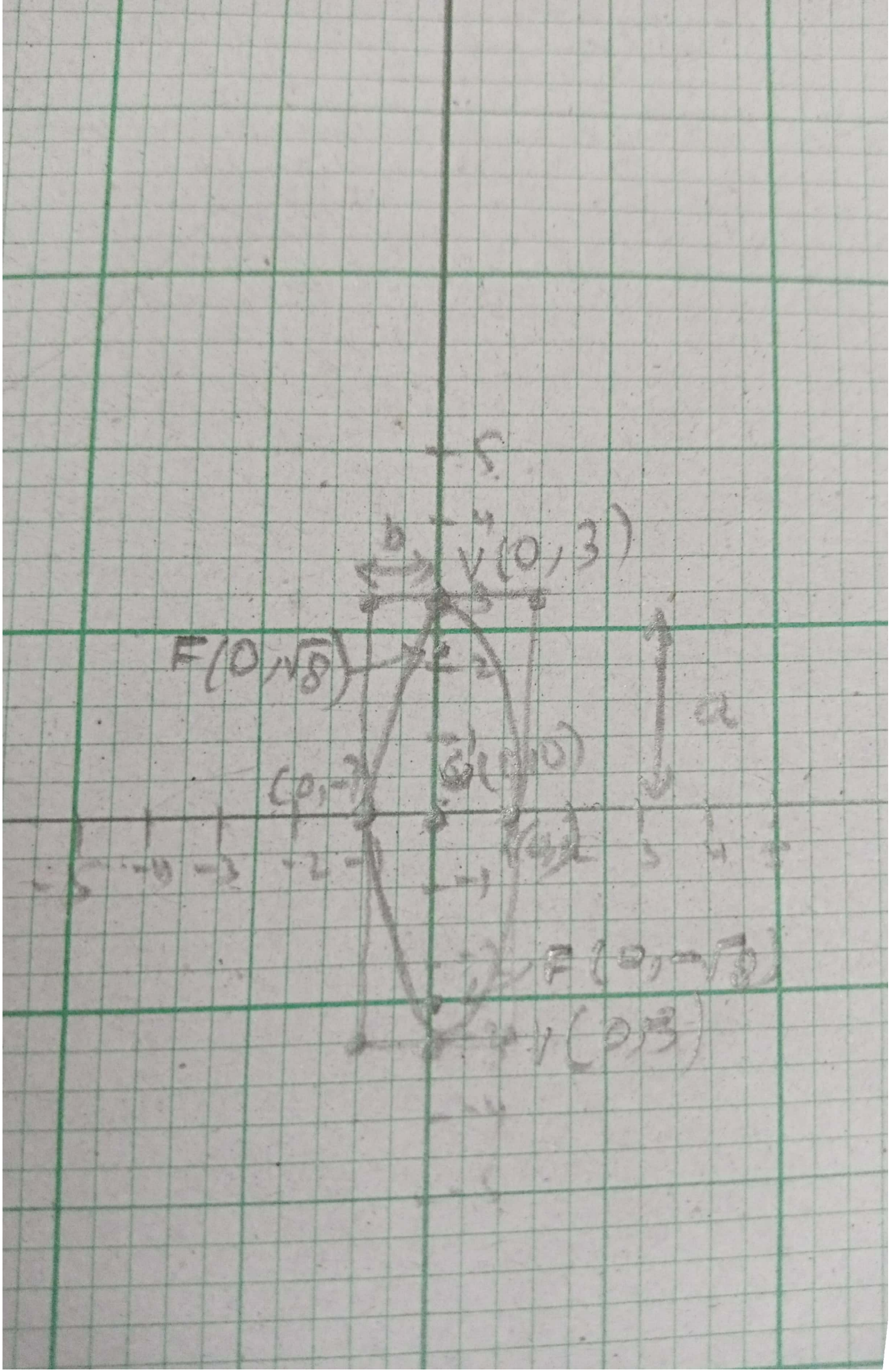
Vertices

$$c = \sqrt{a^2 - b^2}$$

$$c = \sqrt{8}$$

Foci $\rightarrow$ $(0, \pm\sqrt{8})$

ends of minor axis $\rightarrow$ $(0, \pm 1)$



99

a) $(x+3)^2 + 4(y-5)^2 = 16$

÷ by 16

$$\frac{(x+3)^2}{16} + \frac{(y-5)^2}{4} = 1$$

major axis → x-axis

$a = 4 \qquad b = 2$

center ~~vertex~~ ⇒ $(-3, 5)$

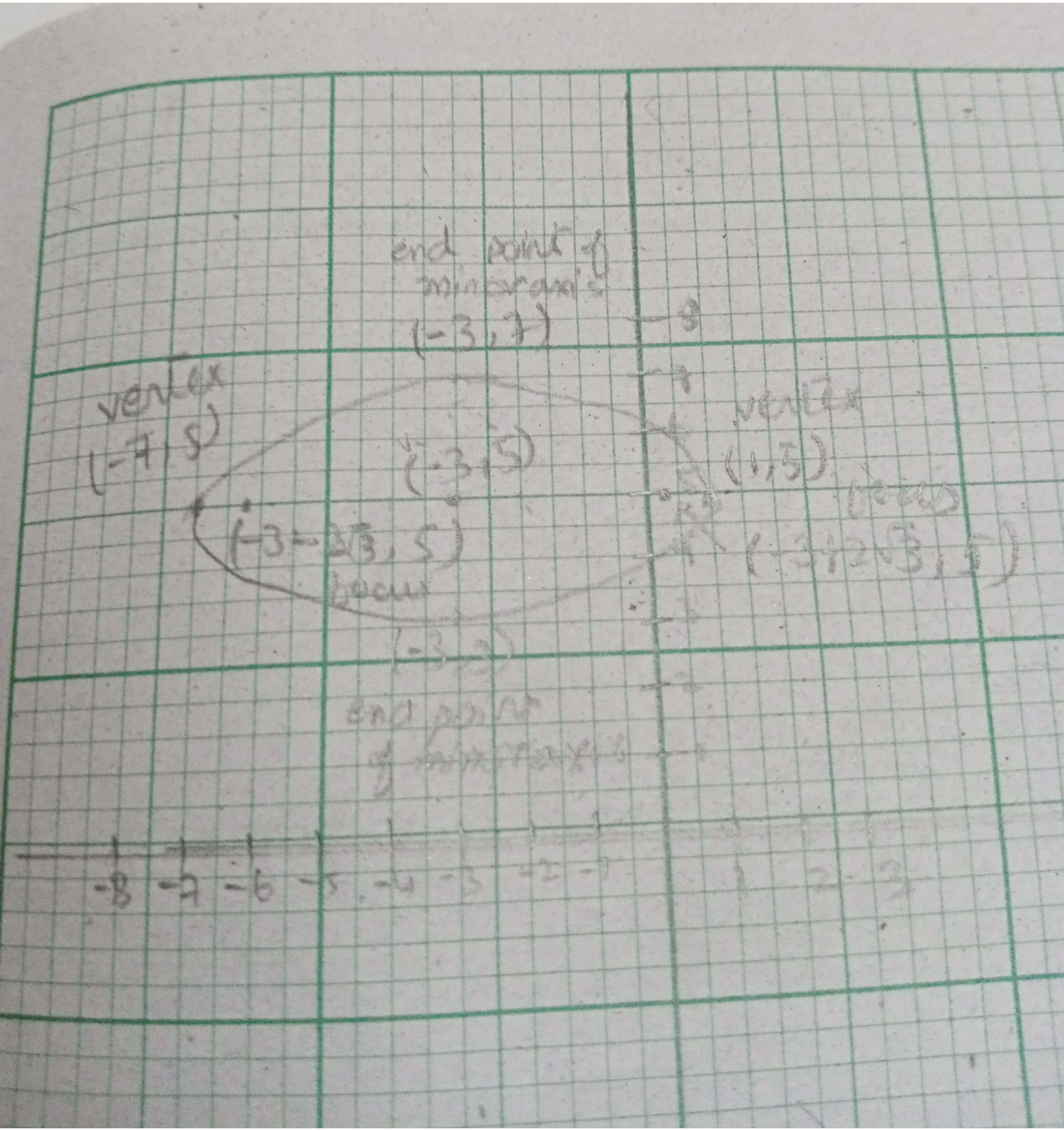
$c = \sqrt{a^2 - b^2}$

$c = \pm\sqrt{12}$

foci → $(-3-\sqrt{12}, 5)$, $(-3+\sqrt{12}, 5)$

vertices → $(-7, 5)$, $(1, 5)$

endpoints of minor axis → $(-3, 3)$, $(-3, 7)$



Q9

b) $\frac{1}{4}x^2 + \frac{1}{9}(y+2)^2 - 1 = 0$

$$\frac{1}{4}x^2 + \frac{1}{9}(y^2+4y+4) = 1$$

$$\frac{x^2}{4} + \frac{(y+2)^2}{9} = 1$$

major axis is y-axis

Centre $(0, -2)$

$a = 3 \qquad b = 2$

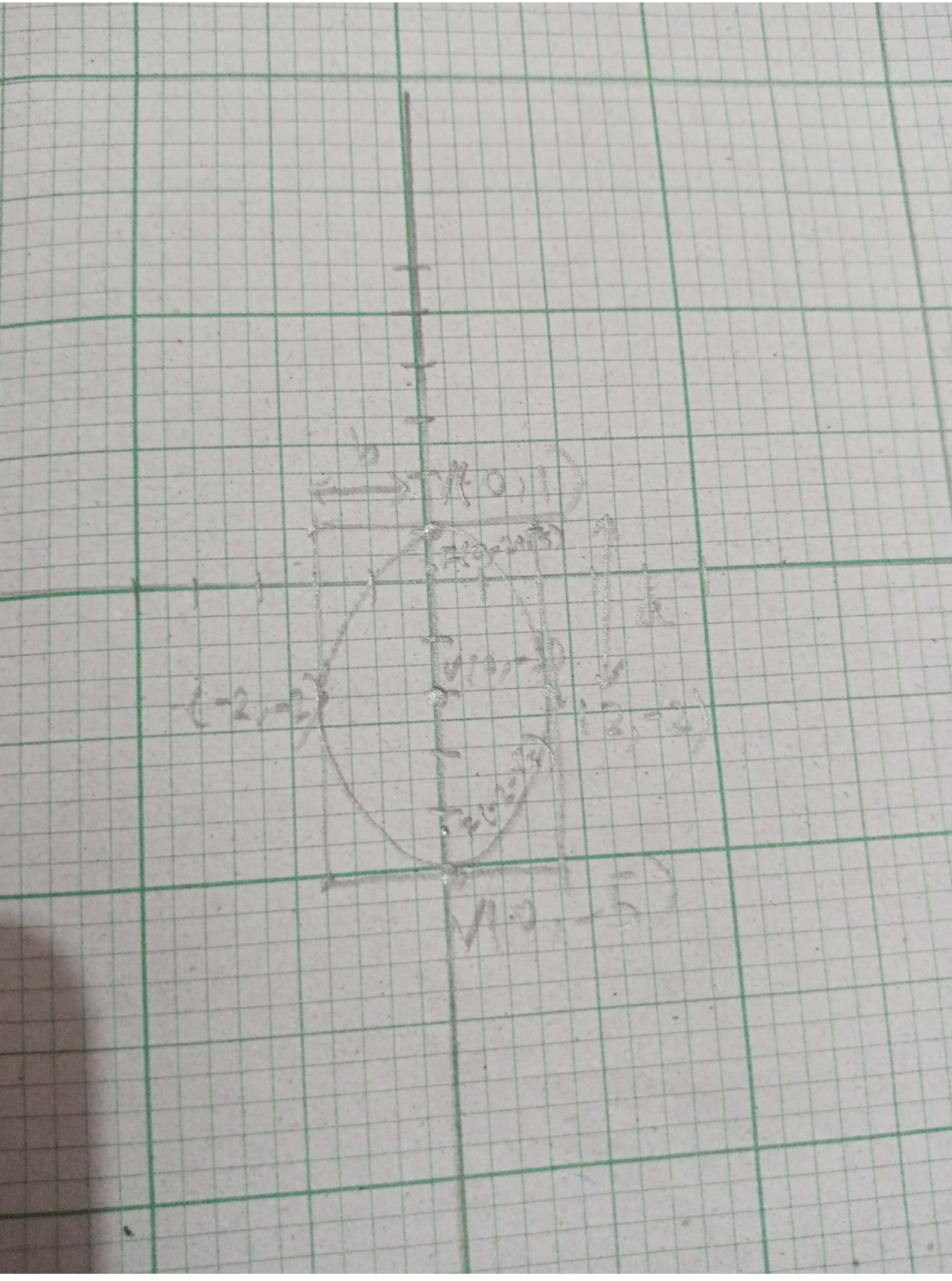
$c = \sqrt{a^2 - b^2}$

$c = \sqrt{5}$

Foci $\rightarrow (0, -2 \pm \sqrt{5})$

Vertices $\rightarrow (0, +1), (0, -5)$

ends of minor axis $\rightarrow (\pm 2, -2)$



Q11

a) $\frac{x^2}{16} - \frac{y^2}{9} = 1$

focal axis → x-axis

$a = 4 \quad b = 3$

center $(0, 0)$

$c = \sqrt{a^2 + b^2}$

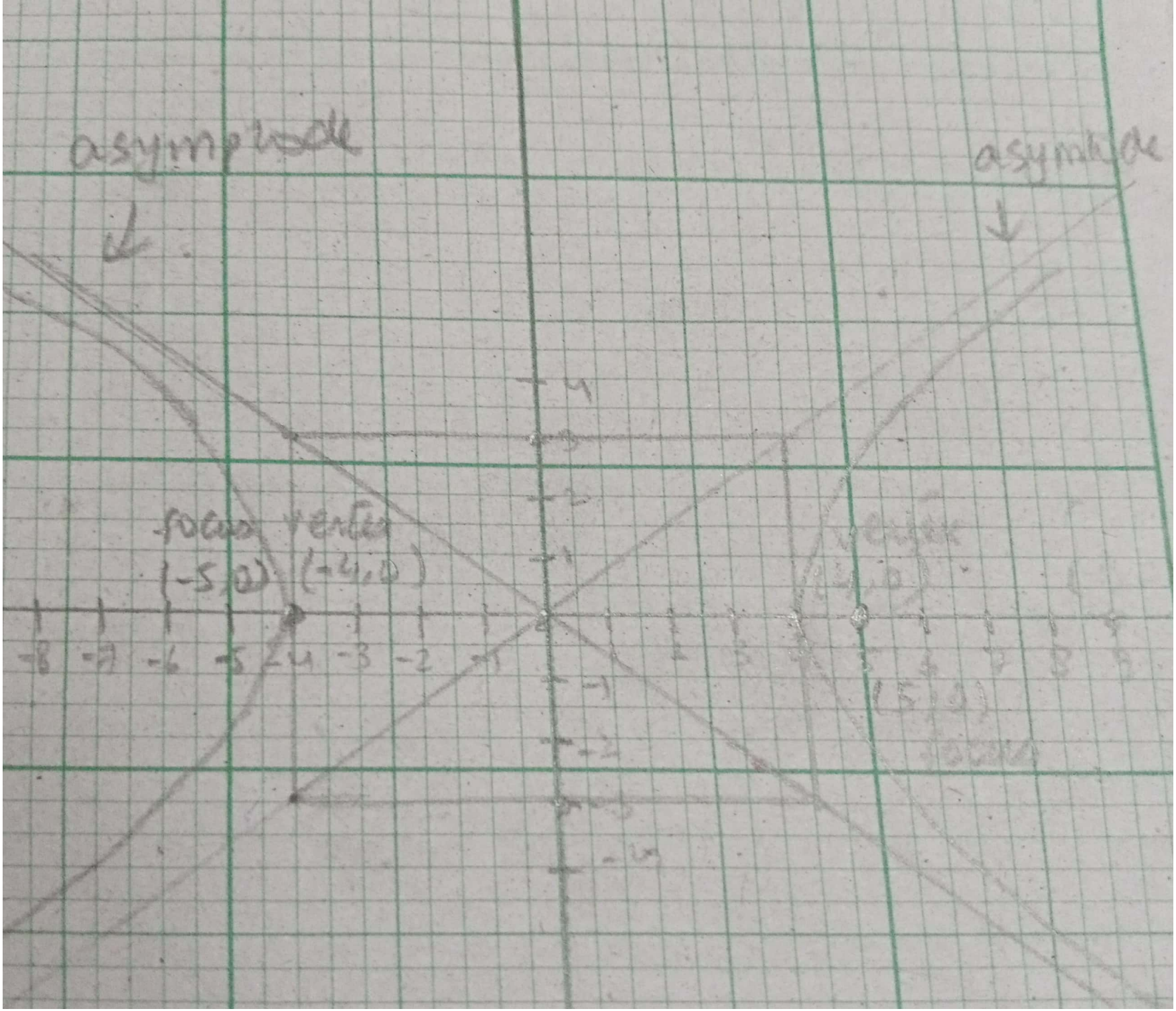
$c = \pm 5$

vertices → $(-4, 0), (4, 0)$

foci → $(-\sqrt{5}, 0), (\sqrt{5}, 0)$

Eq of asymptote

$y = \pm \frac{3}{4} x$



q11

b) $9y^2 - x^2 = 36$

÷ by 36

$$\frac{y^2}{4} - \frac{x^2}{36} = 1$$

major axis → y-axis

$a = 2$, $b = 6$

vertices → $(0, \pm 2)$

$c = \sqrt{a^2 + b^2}$

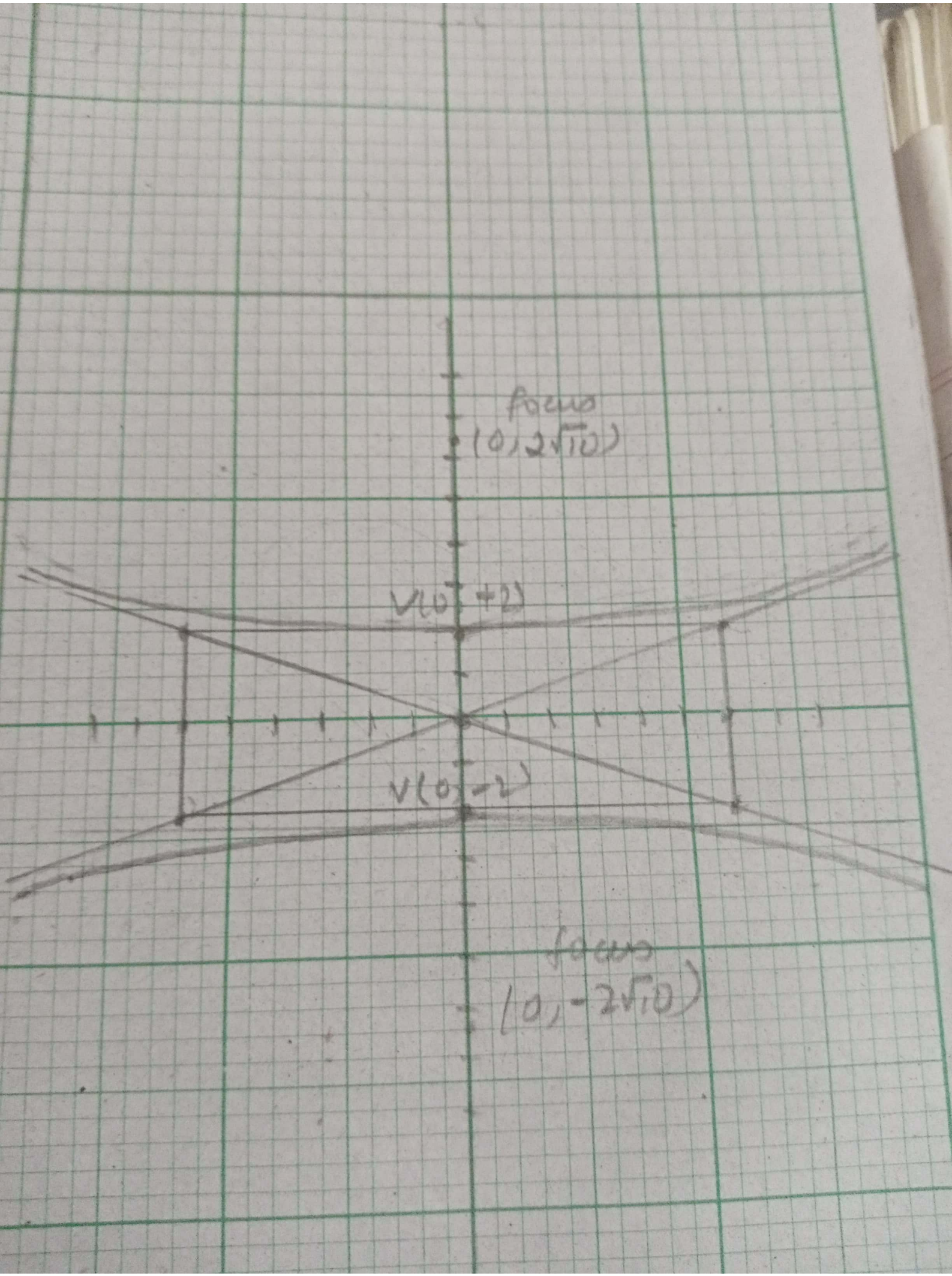
$c = 2\sqrt{10}$

Foci → $(0, \pm 2\sqrt{10})$

asymptotes :-

$$y = K \pm \frac{a}{b}(x - h)$$

$$\boxed{y = \pm \frac{1}{3}x}$$



Q13

a) $\frac{(y+4)^2}{3} - \frac{(x-2)^2}{5} = 1$

focal axis → y-axis

$a = \sqrt{3}$, $b = \sqrt{5}$

Center $(2, -4)$

$c = \sqrt{a^2 + b^2}$

$c = \pm\sqrt{8}$

vertices $(2, -4+\sqrt{3})$, $(2, -4-\sqrt{3})$

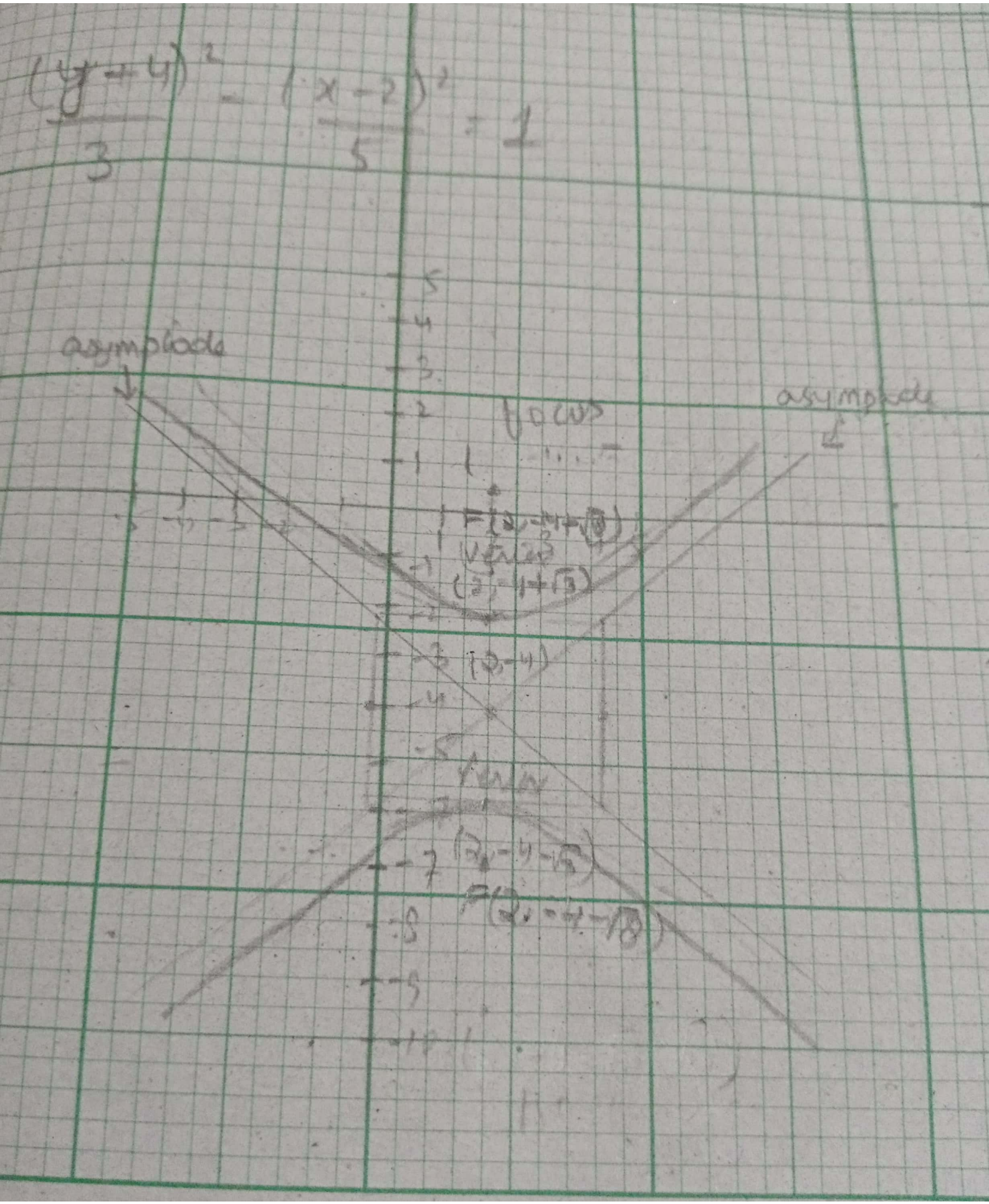
~~foci → $(2, -4+\sqrt{3}+\sqrt{8})$, $(2, -4-\sqrt{3}-\sqrt{8})$~~

foci → $(2, -4 \pm \sqrt{8})$

Eq of asymptotes

$y = -4 \pm \frac{\sqrt{3}}{\sqrt{5}}(x-2)$

$$\frac{(y-4)^2}{3} - \frac{(x-2)^2}{5} = 1$$



Q13

b) $16(x+1)^2 - 8(y-3)^2 = 16$

$\div$ by 16

$$\frac{(x+1)^2}{1} - \frac{(y-3)^2}{2} = 1$$

focal axis → x-axis

$a = 1 \qquad b = \sqrt{2}$

Centre $(-1, 3)$

vertices → $(0,3)\ (-2,3)$

$c = \sqrt{a^2 + b^2}$

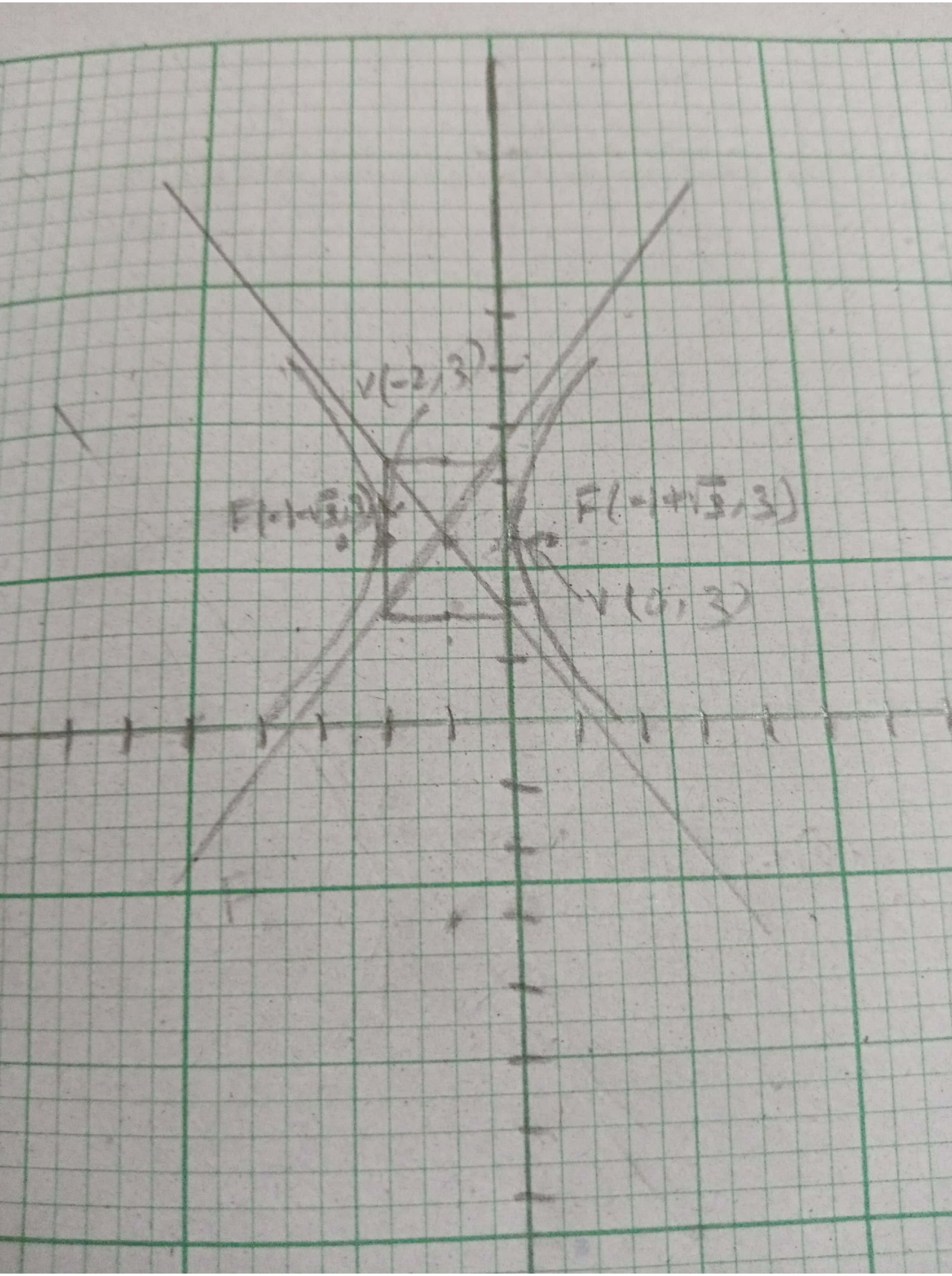
$c = \sqrt{3}$

foci → $(-1 \pm \sqrt{3}, 3)$

Eq of asymptotes is

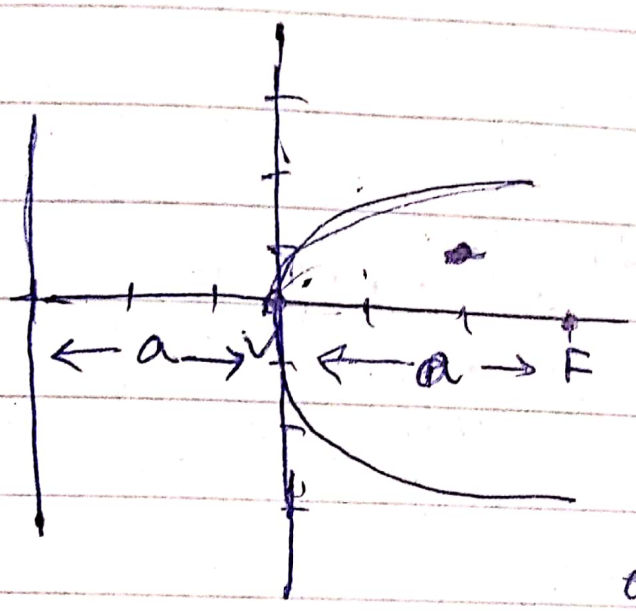
$$y = k \pm \frac{b}{a}(x-h)$$

$$\boxed{y = 3 \pm \sqrt{2}(x+1)}$$



Q15

a) Vertex (0, 0) focus (3, 0)



~~a = √~~

$a = VF$

$a = \sqrt{(3-0)^2 + (0-0)^2}$

$(x-h)^2 = 4a(y-k)$

$(x-0)^2 = 4a(y-0)$

$\boxed{a = 3}$

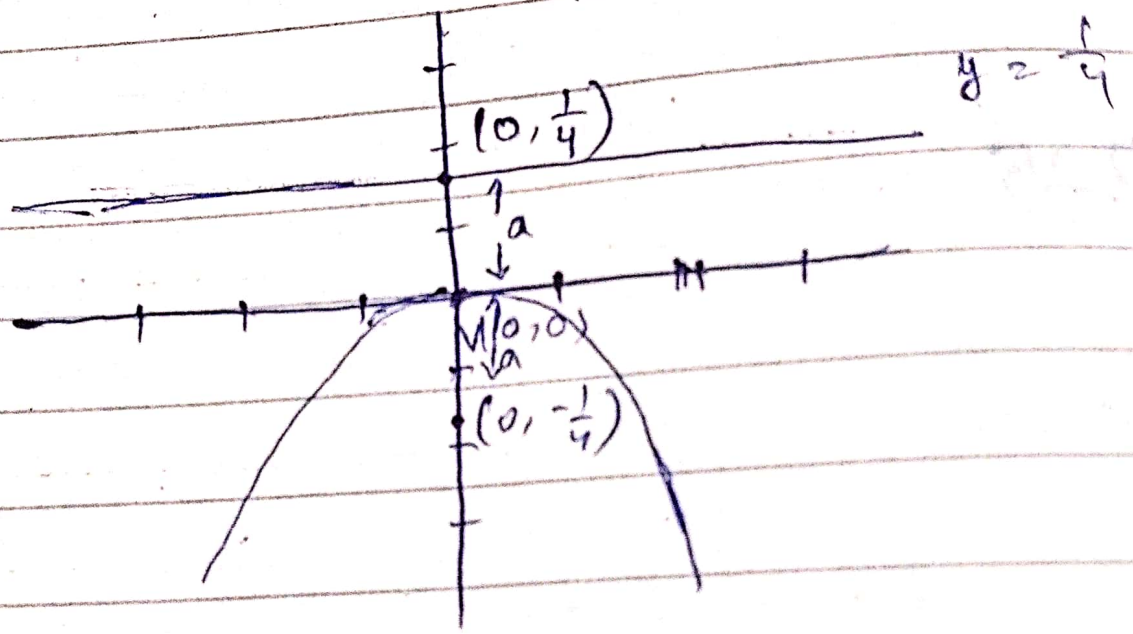
$x^2 = 4ay$

$x^2 = 4(3)y$

$x^2 = 12y$.

b) vertex $(0,0)$ directrix $y = \frac{1}{4}$

First plot the given equations



$$a = \frac{1}{4}$$

$$V(h,k) = (0,0)$$

$$(x-h)^2 = -4a(y-k)$$

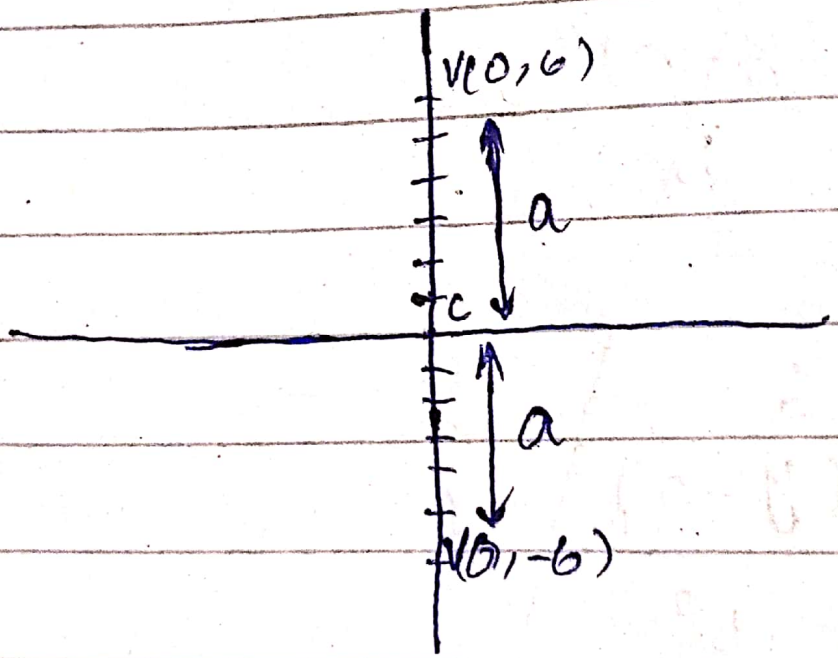
$$x^2 = -4 \times \frac{1}{4} y$$

$$x^2 = -y$$

Q21

a) End points of major axis : $(0, \pm 6)$
passes through $(-3, 2)$

Vertices $\rightarrow (0, \pm 6)$



vertices is on vertical axis

$$\frac{(x-h)^2}{b^2} + \frac{(y-k)^2}{a^2} = 1 \longrightarrow (A)$$

$C(0,0)$

$a = CV = 6$

put in equation (A)

$$\frac{(x-0)^2}{b^2} + \frac{(y-0)^2}{(6)^2} = 1$$

$$\frac{x^2}{b^2} + \frac{y^2}{36} = 1 \longrightarrow (B)$$

$(-3, 2)$ must satisfy this equation.

$$\frac{(-3)^2}{b^2} + \frac{(2)^2}{36} = 1$$

$$\frac{9}{b^2} = 1 - \frac{1}{9}$$

$$\frac{9}{b^2} = \frac{8}{9}$$

$$b^2 = \frac{9 \times 9}{8}$$

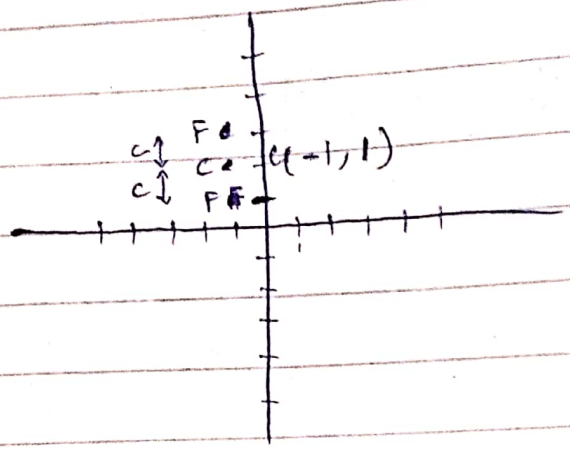
$$b^2 = \frac{81}{8}$$

put in equation (B)

$$\boxed{\frac{x^2}{81/8} + \frac{y^2}{36} = 1}$$

b) Foci $(-1, 1)$ and $(-1, 3)$
minor axis of length 4

plot the information



$\boxed{c = 1}$

foci is on vertical axis

$2b = 4$

so major axis y-axis

$\boxed{b = 2}$

$c^2 = a^2 * - b^2$

$1 = a^2 - 4$

$a^2 = 5$

center $(-1, 1) = C(h, k)$

$$\frac{(x-h)^2}{b^2} + \frac{(y-k)^2}{a^2} = 1$$

$$\frac{(x-(-1))^2}{(2)^2} + \frac{(y-1)^2}{5} = 1$$

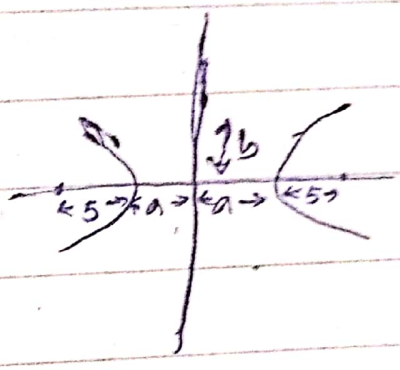
$$\boxed{\frac{(x+1)^2}{4} + \frac{(y-1)^2}{5} = 1}$$

Q25

i) asymptotes $y = \pm \frac{3}{4} x$ $\quad c = 5$

If focal axis is x-axis :-

$$y = \pm \frac{b}{a} x = \pm \frac{3}{4} x$$



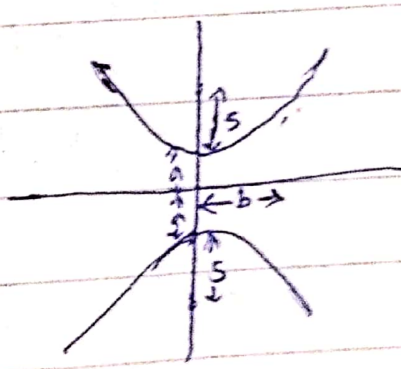
$a = 4 \qquad b = 3$

the equation is

$$\boxed{\frac{x^2}{16} - \frac{y^2}{9} = 1}$$

If focal axis is y-axis

$$y = \pm \frac{a}{b} x = \pm \frac{3}{4} x$$

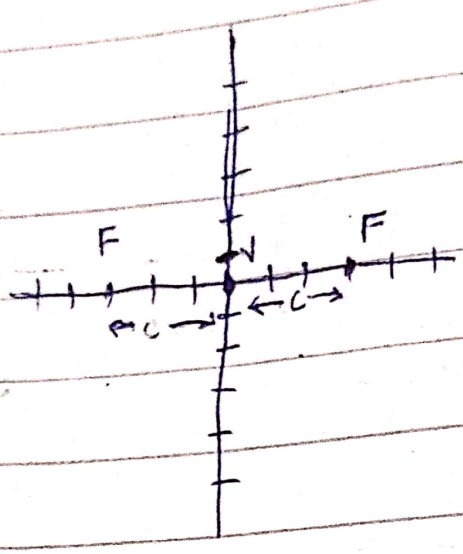


$a = 3 \qquad b = 4$

$$\boxed{\frac{y^2}{16} - \frac{x^2}{9} = 1}$$

b) Asymptotes $y = \pm 2x$

Foci $(\pm 3, 0)$



center $(0, 0)$

$c = 3$

Compare $y = \pm 2x$ with general equation of asymptotes

$$y = k \pm \frac{b}{a}(x - h)$$

$$\frac{b}{a} = 2$$

$$b = 2a$$

$$c^2 = a^2 + b^2$$

$$9 = a^2 + 4a^2$$

$$9 = 5a^2$$

$$\boxed{a^2 = \frac{9}{5}}$$

put it back

$$c^2 = a^2 + b^2$$

$$9 = \frac{9}{5} + b^2$$

$$\boxed{b^2 = \frac{36}{5}}$$

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

$$\boxed{\frac{x^2}{9/5} - \frac{y^2}{36/5} = 1}$$

Q59

a) $x^2 - 5y^2 - 2x - 10y - 9 = 0$

$x^2 - 2x - 5y^2 - 10y = 9$

$x^2 - 2x - 5(y^2 + 2y) = 9$

For x: $\left(\frac{b}{2a}\right)^2 = \left(\frac{-2}{2(1)}\right)^2 = 1$

For y: $\left(\frac{2}{2(1)}\right)^2 = 1$

$x^2 - 2x + 1 - 5(y^2 + 2y + 1) = 9 + 1 - 5$

$(x-1)^2 - 5(y+1)^2 = 5$

$\frac{(x-1)^2}{5} - \frac{(y+1)^2}{1} = \frac{1}{}$

Ellipse Hyperbola

b) ~~$x^2 - 3y^2 - 6y - 3 = 0$~~

$4x^2 + 8y^2 + 16x + 16y + 20 = 0$

$4(x^2 + 4x) + 8(y^2 + 2y) = -20$

$4(x^2 + 4x + 4) + 8(y^2 + 2y + 1) = -20 + 16 + 8$

$4(x+2)^2 + 8(y+1)^2 = 4$

÷ by

$\frac{(x+2)^2}{1} + \frac{(y+1)^2}{\frac{1}{2}} = 1$

Ellipse

$x^2 + 8x + 2y + 14 = 0$

$x^2 + 8x + 16 + 2(y + 7) = 0 + 16$

$(x + 4)^2 = -2y - 14 + 16$

$(x+4)^2 = -2y + 2$

$(x+4)^2 = -2(y-1)$

parabola

1) $5x^2 + 40x + 2y + 94 = 0$

$5(x^2 + 8x) + 2y + 94 = 0$

$5(x^2 + 8x + 16) = 80 - 94 - 2y$

$(x + 4)^2 = \frac{-14 - 2y}{5}$

$(x + 4)^2 = \frac{-14}{5} - \frac{2y}{5}$

$(x + 4)^2 = \frac{-2}{5}(y + 7)$

parabola.