Assignment # 09

Date

Q3

$$U \times V = \begin{vmatrix} i & j & k \\ 1 & 2 & -3 \\ -4 & 1 & 2 \end{vmatrix}$$

$$= (4-(-3))i - j(2-12) + (1-(-8))k$$

$$\boxed{U \times V = 7i + 10j + 9k}$$

if two vectors a & b are orthogonal then: $a \cdot b = 0$

$$U \cdot (U \times V) = 0$$

$$(1i + 2j - 3k) \cdot (7i + 10j + 9k) = 0$$

$$(1 \times 7) + (2 \times 10) + (-3 \times 9) = 0$$

$$\boxed{0 = 0}$$

$$V \cdot (U \times V) = 0$$

$$(-4i + j + 2k) \cdot (7i + 10j + 9k) = 0$$

$$(-4 \times 7) + (1 \times 10) + (2 \times 9) = 0$$

$$\boxed{0 = 0}$$

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Q5

let C be U×V

$$C = \begin{vmatrix} i & j & k \\ 0 & 1 & -2 \\ 3 & 0 & -4 \end{vmatrix}$$

~~$C = (-4+0)i - (0+$~~

$C = (-4-0)i - (0-(-6))j + (0-3)k$

$\boxed{C = -4i - 6j - 3k}$

orthogonal condition

$$\vec{a} \cdot \vec{b} = 0$$

$U \cdot C = (0\hat{i} + 1\hat{j} + -2\hat{k}) \cdot (-4\hat{i} - 6\hat{j} - 3\hat{k})$

$0 = (0 \times -4) + (1 \times -6) + (-2 \times -3)$

$\boxed{0 = 0}$

$V \cdot C = 0$

$(3\hat{i} + 0\hat{j} + -4\hat{k})(-4i - 6j - 3k) = 0$

$(3 \times -4) + (0 \times -6) + (-4 \times -3) = 0$

$\boxed{0 = 0}$

Q7

a) $u \times (v \times w)$

$$v \times w = \begin{vmatrix} i & j & k \\ 0 & 1 & 7 \\ 1 & 4 & 5 \end{vmatrix}$$

$$= (5 - 28)i - (0 - 7)j + (0 - 1)k$$

$$= -23i + 7j - k$$

$$u \times (v \times w) = \begin{vmatrix} i & j & k \\ 2 & -1 & 3 \\ -23 & 7 & -1 \end{vmatrix}$$

$$= (1 - 21)i - (-2 + 69)j + (14 + 23)k$$

$$u \times (v \times w) = -20i - 67j + 37k$$

b) $(u \times v) \times w$

$$u \times v = \begin{vmatrix} i & j & k \\ 2 & -1 & 3 \\ 0 & 1 & 7 \end{vmatrix}$$

$$= (-7 - 3)i - (14 - 0)j + (2 - 0)k$$

$$u \times v = -10i - 14j + 2k$$

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$$(u \times v) \times w = \begin{vmatrix} i & j & k \\ -10 & -14 & 2 \\ 1 & 4 & 5 \end{vmatrix}$$

$$= (-70 - 8)i - (-50 - 2)j + (-40 + 14)k$$

$$\boxed{(u \times v) \times w = -78i + 52j - 26k}$$

c) $(u \times v) \times (v \times w)$

$$u \times v = \begin{vmatrix} i & j & k \\ 2 & -1 & 3 \\ 0 & 1 & 7 \end{vmatrix}$$

$$= (-7 - 3)i - (14 - 0)j + (2 - 0)k$$

$$u \times v = -10i - 14j + 2k$$

$$v \times w = \begin{vmatrix} i & j & k \\ 0 & 1 & 7 \\ 1 & 4 & 5 \end{vmatrix}$$

$$= (5 - 28)i - (0 - 7)j + (0 - 1)k$$

$$v \times w = -23i + 7j - k$$

$$(u \times v) \times (v \times w) = \begin{vmatrix} i & j & k \\ -10 & -14 & 2 \\ -23 & 7 & -1 \end{vmatrix}$$

$$= (14 - 14)i + (10 + 46)j + (-70 - 322)k$$

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$(u \times v) \times (v \times w) = -56j - 392k$

d) $(v \times w) \times (u \times v)$

$$v \times w = \begin{vmatrix} i & j & k \\ 0 & 1 & 7 \\ 1 & 4 & 5 \end{vmatrix}$$

$$= (5 - 28)i - (0 - 7)j + (0 - 1)k$$

$$v \times w = -23j + 7j - k$$

$$u \times v = \begin{vmatrix} i & j & k \\ 2 & -1 & 3 \\ 0 & 1 & 7 \end{vmatrix}$$

$$= (-7 - 3)i - (14 - 0)j + (2 - 0)k$$

$$u \times v = -10i - 14j + 2k$$

$$(v \times w) \times (u \times v) = \begin{vmatrix} i & j & k \\ -23 & 7 & -1 \\ -10 & -14 & 2 \end{vmatrix}$$

$$= (14 - 14)i - (-46 - 10)j + (322 + 70)k$$

$$= 56j + 392k$$

Q11

A(0, -2, 1) B(1, -1, -2) C(-1, 1, 0)

Let c be the normal vector to the plane.

$AB = \langle 1, 1, -3 \rangle$

$AC = \langle -1, 3, -1 \rangle$

$$c = \begin{vmatrix} i & j & k \\ 1 & 1 & -3 \\ -1 & 3 & -1 \end{vmatrix}$$

$$c = 8i + 4j + 4k$$

$$\hat{c} = \frac{8i + 4j + 4k}{\sqrt{(8)^2 + (4)^2 + (4)^2}}$$

$$= \pm \frac{1}{4\sqrt{6}} (8i + 4j + 4k)$$

$$\hat{c} = \pm \left(\frac{2}{\sqrt{6}} i + \frac{1}{\sqrt{6}} j + \frac{1}{\sqrt{6}} k \right)$$

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$$\hat{c} = \pm \frac{1}{\sqrt{6}} (2i + j + k)$$

Q16 Q17

$u = i - j + 2k \qquad v = 3j + k$

Area of parallelogram = $\|u \times v\|$

$$u \times v = \begin{vmatrix} i & j & k \\ 1 & -1 & 2 \\ 0 & 3 & 1 \end{vmatrix}$$

$$u \times v = -7i - j + 3\hat{k}$$

$$\|u \times v\| = \sqrt{(7)^2 + (1)^2 + (3)^2}$$

area of parallelogram = $\sqrt{59}$

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Q19

$P(1,5,-2), Q(0,0,0), R(3,5,1)$

$\overrightarrow{PQ} = \langle -1, -5, 2 \rangle$

$\overrightarrow{PR} = \langle 2, 0, 3 \rangle$

$$\overrightarrow{PQ} \times \overrightarrow{PR} = \begin{vmatrix} i & j & k \\ -1 & -5 & 2 \\ 2 & 0 & 3 \end{vmatrix}$$

Let n be the normal vector

$$n = -15i + 7j + 10k$$

$$\|n\| = \sqrt{(-15)^2 + (7)^2 + (10)^2}$$

$$\|n\| = \sqrt{374}$$

$$\text{area of triangle} = \frac{\|n\|}{2} = \frac{\sqrt{374}}{2}$$

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Q21

$$v \times w = \begin{vmatrix} i & j & k \\ 4 & 1 & -3 \\ 0 & 1 & 5 \end{vmatrix}$$

$$v \times w = 8i - 20j + 4k$$

$$u \cdot (v \times w) = (2i - 3j + k) \cdot (8i - 20j + 4k)$$

$$= (2 \times 8) + (-3 \times -20) + (1 \times 4)$$

$$\boxed{u \cdot (v \times w) = 80}$$

Q25

volume of parallelopipped $= |u \cdot (v \times w)|$

$$v \times w = \begin{vmatrix} i & j & k \\ 0 & 4 & -2 \\ 2 & 2 & -4 \end{vmatrix}$$

$$= -12i + 4j - 8k$$

$$|u \cdot (v \times w)| = |(2i - 6j + 2k) \cdot (-12i - 4j - 8k)|$$

$$= |(2 \times -12) + (-6 \times -4) + (2 \times -8)|$$

$$= |-16|$$

$$\boxed{|u \cdot (v \times w)| = 16}$$

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Q27

a)

$$v \times w = \begin{vmatrix} i & j & k \\ 3 & 0 & -2 \\ 5 & -4 & 0 \end{vmatrix}$$

$$v \times w = -8i - 10j - 12k$$

$$u \cdot (v \times w) = (i - 2j + k) \cdot (-8i - 10j - 12k)$$

$$= (1 \times -8) + (-2 \times -10) + (1 \times -12)$$

$$\boxed{u \cdot (v \times w) = 0}$$

yes, vectors lie on same plane.

b)

$$v \times w = \begin{vmatrix} i & j & k \\ 4 & -1 & 1 \\ 1 & -1 & 0 \end{vmatrix}$$

$$= i + j - 3k$$

$$u \cdot (v \times w) = (5i - 2j + k) \cdot (i + j - 3k)$$

$$= (5 \times 1) + (-2 \times 1) + (1 \times -3)$$

$$\boxed{u \cdot (v \times w) = 0}$$

yes, vectors lie on same plane

c)

$$V \times W = \begin{vmatrix} i & j & k \\ 2 & 1 & -2 \\ 3 & -4 & 12 \end{vmatrix}$$

$$V \times W = 4i - 30j - 11k$$

$$U \cdot (V \times W) = (4i - 8j + k) \cdot (4i - 30j - 11k)$$

$$= (4 \times 4) + (-8 \times -30) + (1 \times -11)$$

$$U \cdot (V \times W) = 245$$

No, vectors does not lie on same plane

Q31

a)

$$d = \frac{\| \overrightarrow{AP} \times \overrightarrow{AB} \|}{\| AB \|}$$

$$AP = -4i + 2k$$

$$AB = -3i + 2j - 4k$$

$$\| AB \| = \sqrt{3^2 + 2^2 + 4^2} = \sqrt{29}$$

$$AP \times AB = \begin{vmatrix} i & j & k \\ -4 & 0 & 2 \\ -3 & 2 & -4 \end{vmatrix}$$

$$AP \times AB = -4i - 22j - 8k$$

$$\| AP \times AB \| = \sqrt{4^2 + 22^2 + 8^2}$$

$$\| AP \times AB \| = 2\sqrt{141}$$

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$$d = \frac{2\sqrt{141}}{\sqrt{29}} \text{ units}$$

b)

$$d = \frac{\|AP \times AB\|}{\|AB\|}$$

$$AP = 2i + 2j$$

$$AB = -2i + j$$

$$AP \times AB = \begin{vmatrix} i & j & k \\ 2 & 2 & 0 \\ -2 & 1 & 0 \end{vmatrix}$$

$$AP \times AB = 6k$$

$$\|AP \times AB\| = 6$$

$$\|AB\| = \sqrt{5}$$

$$d = \frac{6}{\sqrt{5}} \text{ units}$$