

Ex # 11-8

Assignment 12

Date \_\_\_\_\_

G1

a)  $(4\sqrt{3}, 4, -4)$

$$x = 4\sqrt{3} \quad y = 4 \quad z = -4$$

$$r = \sqrt{x^2 + y^2} = \sqrt{(4\sqrt{3})^2 + (4)^2}$$

$$\boxed{r = 8}$$

$$\theta = \tan^{-1}\left(\frac{4}{4\sqrt{3}}\right)$$

$$\theta = \tan^{-1}(+1)$$

$$\boxed{\theta = +\frac{\pi}{6}}$$

$$\boxed{z = -4}$$

$$\boxed{(8, \frac{\pi}{6}, -4)}$$

b).  $(-5, 5, 6)$

$$x = -5 \quad y = 5 \quad z = 6$$

$$r = \sqrt{x^2 + y^2} = \sqrt{(-5)^2 + (5)^2}$$

$$r = \sqrt{50}$$

$$\boxed{r = 5\sqrt{2}}$$

$$\theta = \tan^{-1}\left(\frac{y}{x}\right) = \tan^{-1}\left(\frac{5}{-5}\right)$$

$$= \tan^{-1}(-1)$$

$$\boxed{\theta = -\frac{\pi}{4}}$$

$$\boxed{z = 6}$$

$$\frac{7\pi}{4}$$

$$(5\sqrt{2}, \frac{7\pi}{4}, 6)$$

RC

No. \_\_\_\_\_

c)  $(0, 2, 0)$

$$x=0 \quad y=2 \quad z=0$$

$$r = \sqrt{x^2 + y^2} = \sqrt{0^2 + 2^2}$$

$$\boxed{r=2}$$

$$\tan \theta = \frac{y}{x} = \frac{2}{0} = \infty$$

$$\theta = \tan^{-1}(\infty)$$

$$\boxed{\theta = \frac{\pi}{2}}$$

$$\boxed{z=0}$$

$$\boxed{(2, \frac{\pi}{2}, 0)}$$

d)  $(4, -4\sqrt{3}, 6)$

$$x=4, y=-4\sqrt{3}, z=6$$

$$r = \sqrt{x^2 + y^2} = \sqrt{(4)^2 + (-4\sqrt{3})^2}$$

$$\boxed{r=8}$$

$$\tan \theta = \frac{-4\sqrt{3}}{4} = -\sqrt{3}$$

$$\theta = \tan^{-1}(-\sqrt{3})$$

$$\boxed{\theta = 5\pi/3}$$

$$\boxed{z=6}$$

$$\boxed{(8, \frac{5\pi}{3}, 6)}$$

Q3

a)  $(4, \frac{\pi}{6}, 3)$

$$r = 4, \theta = \frac{\pi}{6}, z = 3$$

$$x = r \cos \theta = 4 \cos(\frac{\pi}{6}) = 2\sqrt{3}$$

$$y = r \sin \theta = 4 \sin(\frac{\pi}{6}) = 2$$

$$z = 3$$

$$(2\sqrt{3}, 2, 3)$$

b)  $(8, \frac{3\pi}{4}, -2)$

$$r = 8, \theta = \frac{3\pi}{4}, z = -2$$

$$x = r \cos \theta = 8 \cos(\frac{3\pi}{4}) = -4\sqrt{2}$$

$$y = r \sin \theta = 8 \sin(\frac{3\pi}{4}) = 4\sqrt{2}$$

$$z = -2$$

$$(-4\sqrt{2}, 4\sqrt{2}, -2)$$

c)  $(5, 0, 4)$

$$r = 5, \theta = 0, z = 4$$

$$x = r \cos \theta = 5 \cos 0 = 5$$

$$y = r \sin \theta = 5 \sin(0) = 0$$

$$z = 4$$

$$(5, 0, 4)$$

$$d) (7, \pi, -9)$$

$$x = r \cos \theta = 7 \cos \pi = -7$$

$$y = r \sin \theta = 7 \sin \pi = 0$$

$$z = -9$$

$$(-7, 0, -9)$$

Q5

$$a) (1, \sqrt{3}, -2)$$

$$x = 1, y = \sqrt{3}, z = -2$$

$$r = \sqrt{x^2 + y^2 + z^2}$$

$$(r = 2\sqrt{2})$$

$$\tan \theta = \frac{y}{x}$$

$$\theta = \tan^{-1} \left( \frac{\sqrt{3}}{1} \right)$$

$$\boxed{\theta = \frac{\pi}{3}}$$

$$\cos \phi = \frac{z}{r}$$

$$\begin{aligned} &= \frac{-2}{2\sqrt{2}} \\ &= -\frac{1}{\sqrt{2}} \end{aligned}$$

$$\phi = \cos^{-1} \left( -\frac{1}{\sqrt{2}} \right)$$

$$\boxed{\phi = \frac{3\pi}{4}}$$

$$\left( 2\sqrt{2}, \frac{\pi}{3}, \frac{3\pi}{4} \right)$$

BC

b)  $(1, -1, \sqrt{2})$

$$x = 1, y = -1, z = \sqrt{2}$$

$$r = \sqrt{x^2 + y^2 + z^2}$$

$$r = 2$$

$$\tan \theta = \frac{y}{x}$$

$$\theta = \tan^{-1} \left( \frac{-1}{1} \right)$$

$$\theta = \frac{7\pi}{4}$$

$$\cos \phi = \frac{z}{\sqrt{x^2 + y^2 + z^2}}$$

$$\phi = \cos^{-1} \left( \frac{\sqrt{2}}{2} \right)$$

$$\phi = \frac{\pi}{4}$$

$$\left( 2, \frac{7\pi}{4}, \frac{\pi}{4} \right)$$

c)  $(0, 3\sqrt{3}, 3)$

$$x = 0, y = 3\sqrt{3}, z = 3$$

$$r = \sqrt{x^2 + y^2 + z^2}$$

$$r = 6$$

$$\tan \theta = \frac{y}{x}$$

$$\theta = \tan^{-1} \left( \frac{3\sqrt{3}}{6} \right)$$

$$\theta = \frac{\pi}{2}$$

$$\cos \phi = \frac{z}{\sqrt{x^2 + y^2 + z^2}}$$

$$\phi = \cos^{-1} \left( \frac{3}{6} \right)$$

$$\phi = \frac{\pi}{3}$$

$$\left( 6, \frac{\pi}{2}, \frac{\pi}{3} \right)$$

d)  $(-5\sqrt{3}, 5, 0)$

$$x = -5\sqrt{3} \quad y = 5 \quad z = 0$$

$$r = \sqrt{x^2 + y^2 + z^2}$$

$$r = 10$$

$$\tan \theta = \frac{y}{x}$$

$$\theta = \tan^{-1} \left( \frac{5}{-5\sqrt{3}} \right)$$

$$\theta = -\frac{\pi}{6}$$

$$\cos \phi = \frac{z}{\sqrt{x^2 + y^2 + z^2}}$$

$$\sqrt{x^2 + y^2 + z^2}$$

$$\phi = \cos^{-1} \left( \frac{10}{10} \right)$$

$$\boxed{\phi = \frac{\pi}{2}}$$

$$(10, -\frac{\pi}{6}, \frac{\pi}{2})$$

**Q7**

a)  $(5, \frac{\pi}{6}, \frac{\pi}{4})$

$$r = 5 \quad \theta = \frac{\pi}{6} \quad \phi = \frac{\pi}{4}$$

$$x = r \sin \theta \cos \phi$$

$$x = 5 \sin \left( \frac{\pi}{4} \right) \cos \left( \frac{\pi}{6} \right)$$

$$x = \frac{5\sqrt{6}}{4}$$

$$y = r \sin \phi \sin \theta$$

$$= 5 \sin \left( \frac{\pi}{4} \right) \sin \left( \frac{\pi}{6} \right)$$

$$y = \frac{5\sqrt{2}}{4}$$

$$4$$

$$z = r \cos \phi$$

$$= 5 \cos \left( \frac{\pi}{4} \right)$$

$$z = \frac{5\sqrt{2}}{2}$$

$$\boxed{\left( \frac{5\sqrt{6}}{4}, \frac{5\sqrt{2}}{4}, \frac{5\sqrt{2}}{2} \right)}$$

$$b) (7, 0, \pi/2)$$

$$\rho = 7 \quad \theta = 0 \quad \phi = \pi/2$$

$$x = \rho \sin \theta \cos \phi$$

$$x = 7 \sin 0 \cos \frac{\pi}{2}$$

$$x = 0$$

$$y = \rho \sin \theta \sin \phi$$

$$y = 0$$

$$z = \rho \cos \theta$$

$$= 7 \cos \frac{\pi}{2}$$

$$z = 0$$

$$(7, 0, 0)$$

$$c) (1, \pi, 0)$$

$$\rho = 1, \theta = \pi, \phi = 0$$

$$x = \rho \sin \theta \cos \phi \quad x = 0$$

$$y = \rho \sin \theta \sin \phi \quad y = 0$$

$$z = \rho \cos \theta$$

$$= 1 \cos 0$$

$$z = 1$$

$$(0, 0, 1)$$

d)  $(2, \frac{3\pi}{2}, \frac{\pi}{2})$

$$r = 2 \quad \theta = \frac{3\pi}{2} \quad \phi = \frac{\pi}{2}$$

$$x = r \sin \theta \cos \phi$$

$$= 2 \sin\left(\frac{3\pi}{2}\right) \cos\left(\frac{3\pi}{2}\right)$$

$$x = 0$$

$$y = r \sin \theta \sin \phi$$

$$= 2 \sin\left(\frac{\pi}{2}\right) \sin\left(\frac{3\pi}{2}\right)$$

$$y = -2$$

$$z = r \cos \theta$$

$$z = 2 \cos\left(\frac{\pi}{2}\right)$$

$$z = 0$$

$$(0, -2, 0)$$

**Q9**

a)  $(\sqrt{3}, \frac{\pi}{6}, 3)$

$$r = \sqrt{3}, \quad \theta = \frac{\pi}{6}, \quad z = 3$$

$$r = \sqrt{x^2 + z^2}$$

$$r = 2\sqrt{3}$$

$$\phi = \tan^{-1}\left(\frac{y}{z}\right)$$

$$\theta = \tan^{-1}\left(\frac{\sqrt{3}}{3}\right)$$

$$\phi = \frac{\pi}{6}$$

$$\theta = \frac{\pi}{6}$$

$$\boxed{(2\sqrt{3}, \frac{\pi}{6}, \frac{\pi}{6})}$$

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$$6) (1, \pi/4, -1)$$

$$r = 1 \quad \theta = \pi/4 \quad z = -1$$

$$P = \sqrt{r^2 + z^2}$$

$$P = \sqrt{(1)^2 + (-1)^2}$$

$$P = \sqrt{2}$$

$$\phi = \tan^{-1} \left( \frac{y}{z} \right)$$

$$\phi = \tan^{-1} \left( \frac{1}{-1} \right)$$

$$\phi = 7\pi/4$$

$$\theta = \pi/4$$

$$(\sqrt{2}, \pi/4, 7\pi/4)$$

$$c) (2, 3\pi/4, 0)$$

$$r = 2, \theta = 3\pi/4, \phi = 0$$

$$P = \sqrt{r^2 + z^2}$$

$$P = 2$$

$$\phi = \tan^{-1} \left( \frac{y}{z} \right)$$

$$\phi = \pi/2$$

$$\theta = 3\pi/4$$

$$(2, 3\pi/4, \pi/2)$$

$$d) (6, 1, -2\sqrt{3})$$

$$r = 6, \quad \theta = 1 \quad z = -2\sqrt{3}$$

$$r = \sqrt{x^2 + z^2}$$

$$r = 4\sqrt{3}$$

$$\phi = \tan^{-1}\left(\frac{z}{x}\right)$$

$$\phi = 5\pi/3$$

$$\theta = 1$$

$$(4\sqrt{3}, 1, 5\pi/3)$$

**Q11**

$$a) (5, \pi/4, 2\pi/3)$$

$$r = 5, \quad \theta = \pi/4, \quad \phi = 2\pi/3$$

$$x = r \sin \phi$$

$$x = 5 \sin(2\pi/3)$$

$$x = \frac{5\sqrt{3}}{2}$$

$$\theta = \pi/4$$

$$z = r \cos \phi$$

$$z = 5 \cos(2\pi/3)$$

$$z = -\frac{5}{2}$$

$$\left(\frac{5\sqrt{3}}{2}, \frac{\pi}{4}, -\frac{5}{2}\right)$$

$$b) \left(1, \frac{7\pi}{6}, \pi\right)$$

$$\rho = 1 \rightarrow \theta = \frac{7\pi}{6} \quad \phi = \pi$$

$$x = \rho \sin \theta \\ = 1 \sin(\pi)$$

$$y = 0$$

$$\theta = \frac{\frac{7\pi}{6}}{6}$$

$$\phi z = \rho \cos \phi$$

$$= 1 \cos(\pi)$$

$$z = -1$$

$$\left(0, \frac{7\pi}{6}, -1\right)$$

$$c) (3, 0, 0)$$

$$\rho = 3, \theta = 0, \phi = 0$$

$$x = \rho \sin \theta$$

$$= 3 \sin 0$$

$$y = 0$$

$$\theta = 0$$

$$z = \rho \cos \phi$$

$$= 3 \cos 0$$

$$z = 3$$

$$(0, 0, 3)$$

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d)  $(4, \pi/6, \pi/2)$

$$r = 4, \theta = \pi/6, \phi = \pi/2$$

$$\begin{aligned}x &= r \sin \theta \\&= 4 \sin(\pi/2)\end{aligned}$$

$$x = 4$$

$$\theta = \pi/6$$

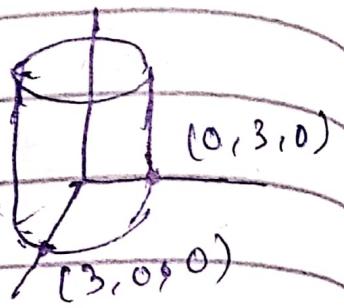
$$\begin{aligned}y &= r \cos \theta \\&= 4 \cos(\pi/2)\end{aligned}$$

$$y = 0$$

$$(4, \pi/6, 0)$$

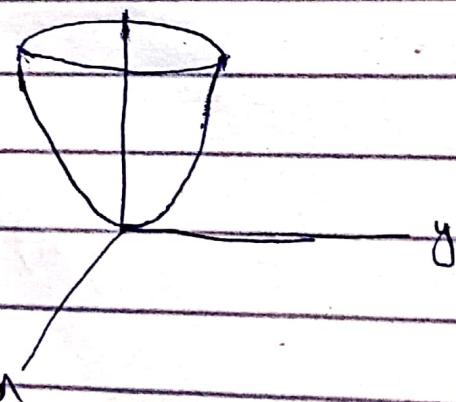
Q19

$$\begin{aligned}r &= 3 \\ \sqrt{x^2 + y^2} &= 3 \\ x^2 + y^2 &= 9\end{aligned}$$



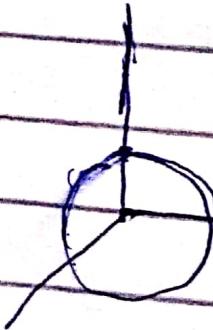
Q21

$$\begin{aligned}z &= r^2 \\ z &= x^2 + y^2\end{aligned}$$



Q25

$$\begin{aligned}r^2 + z^2 &= 1 \\ x^2 + y^2 + z^2 &= 1\end{aligned}$$



Q29

$$\phi = \frac{\pi}{4}$$

$$\cos \phi = \frac{z}{\sqrt{x^2 + y^2 + z^2}}$$

$$\cos\left(\frac{\pi}{4}\right) = \frac{z}{\sqrt{x^2 + y^2 + z^2}}$$

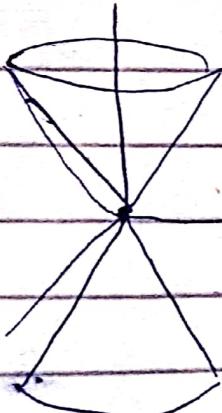
$$\frac{\sqrt{2}}{2} = \frac{z}{\sqrt{x^2 + y^2 + z^2}}$$

$$\frac{z}{\sqrt{x^2 + y^2 + z^2}} = \frac{z^2}{x^2 + y^2 + z^2}$$

$$z^2 = x^2 + y^2 + z^2$$

$$x^2 + y^2 - z^2 = 0$$

$$z = \sqrt{x^2 + y^2}$$



Q33

$$p \sin \theta = 2 \cos \varphi$$

$$p \sin \theta \cos \varphi = 2 \cos^2 \varphi$$

$$p^2 \sin^2 \theta = 2 p \sin \theta \cos \varphi \quad \because p \sin \theta \sin \varphi = y$$

$$p^2 \sin^2 \theta = 2x \quad \therefore p \sin \theta \cos \varphi = x$$

$$p^2 \sin^2 \theta (\sin^2 \varphi + \cos^2 \varphi) = 2x$$

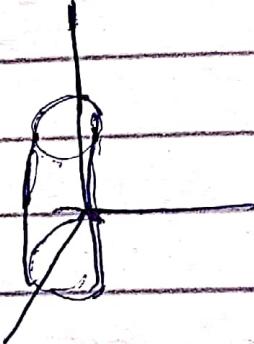
$$p^2 \sin^2 \theta \sin^2 \varphi + p^2 \sin^2 \theta \cos^2 \varphi = 2x$$

$$y^2 + x^2 = 2x$$

$$x^2 - 2x + y^2 = 0$$

$$x^2 - 2x + 1 + y^2 = 1$$

$$(x-1)^2 + y^2 = 1$$



Q41

$$x^2 + y^2 + z^2 = 9$$

a) Cylindrical Co-ordinates.

$$x^2 + z^2 = 9 \Rightarrow [y^2 + z^2 = 9]$$

$$[x = \sqrt{3}]$$

b) Spherical Co-ordinates

$$p^2 = 9$$

$$[p = 3]$$

RC

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## Q 45

$$x^2 = 16 - z^2$$

a) Cylindrical Co-ordinates :

$$r^2 \cos^2 \theta = 16 - z^2$$

b) Spherical Co-ordinates :

$$\rho^2 \sin^2 \phi \cos^2 \theta = 16 - \rho^2 \cos^2 \phi$$

$$\rho^2 \sin^2 \phi \cos^2 \theta + \rho^2 \cos^2 \phi = 16$$

$$\rho^2 (\sin^2 \phi \cos^2 \theta + \cos^2 \phi) = 16$$

$$x^2 + z^2 = 16$$

$$x^2 + z^2 + y^2 = 16 + y^2$$

$$\rho^2 = 16 + \rho^2 \sin^2 \phi \sin^2 \theta$$

$$\rho^2 - \rho^2 \sin^2 \phi \sin^2 \theta = 16$$

$$\boxed{\rho^2 (1 - \sin^2 \phi \sin^2 \theta) = 16}$$