Assignment 1 (Remaining 3 question)

Q1

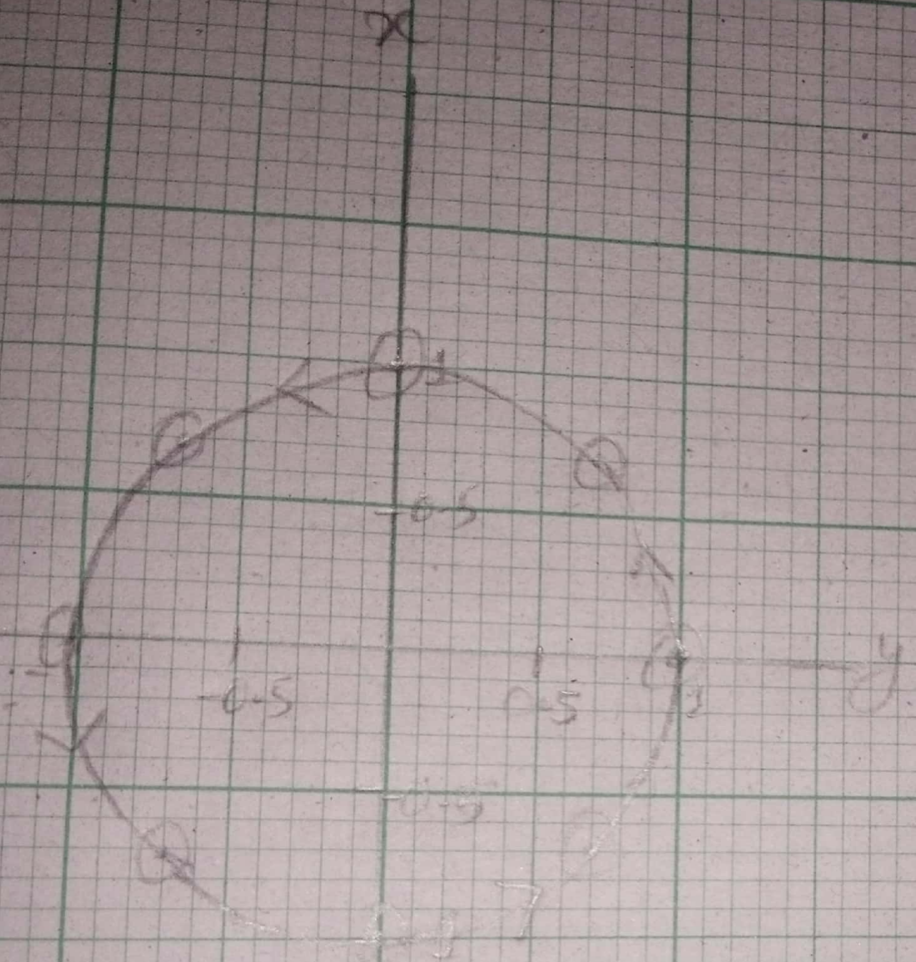
$x = \cos t$ $y = \sin t$

t	$x = \cos t$	$y = \sin t$
0	1	0
$\pi/4$	0.7	0.7
$\pi/2$	0	1
$3\pi/4$	-0.7	0.7
π	-1	0
$5\pi/4$	-0.7	-0.7
$3\pi/2$	0	-1
$7\pi/4$	0.7	-0.7
2π	1	0

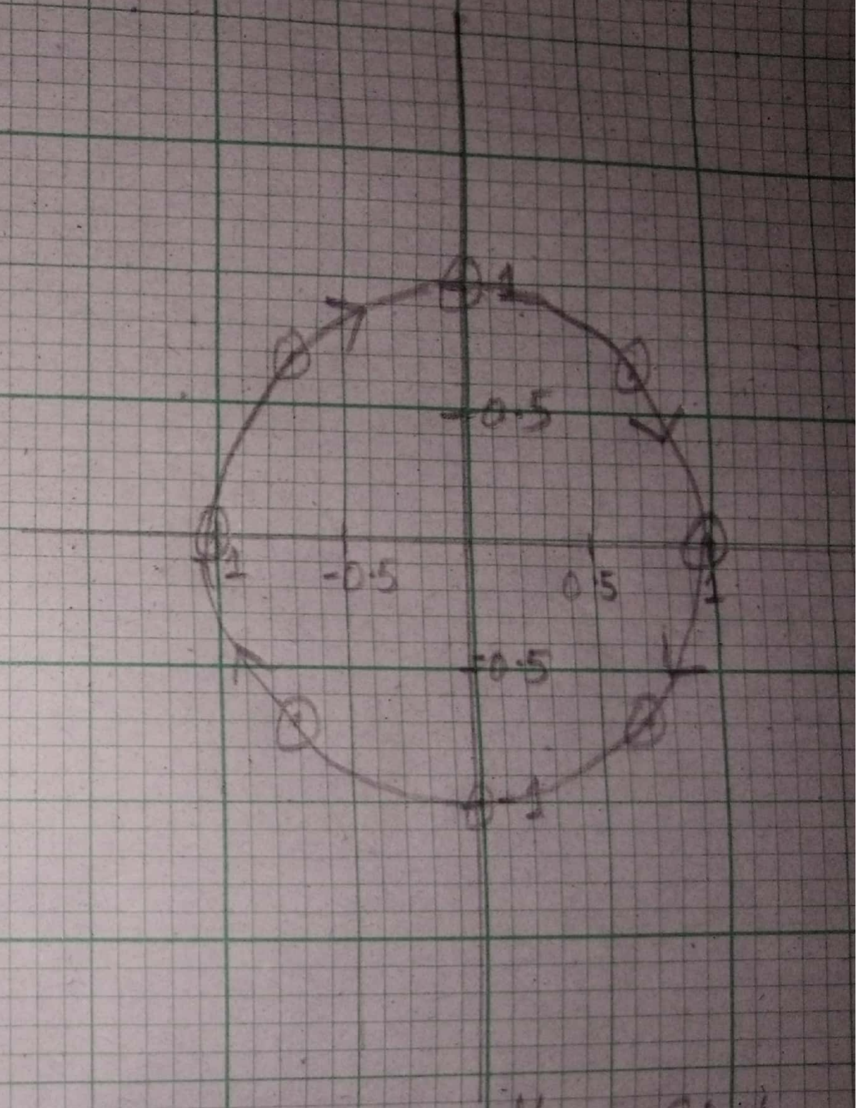
Q2

$x = \cos t$ $y = -\sin t$.

t	x	y
0	1	0
$\pi/4$	0.7	-0.7
$\pi/2$	0	-1
$3\pi/4$	-0.7	-0.7
π	-1	0
$5\pi/4$	-0.7	0.7
$3\pi/2$	0	1
$7\pi/4$	0.7	0.7
2π	1	0



$x = \cos t$ $y = \sin t$

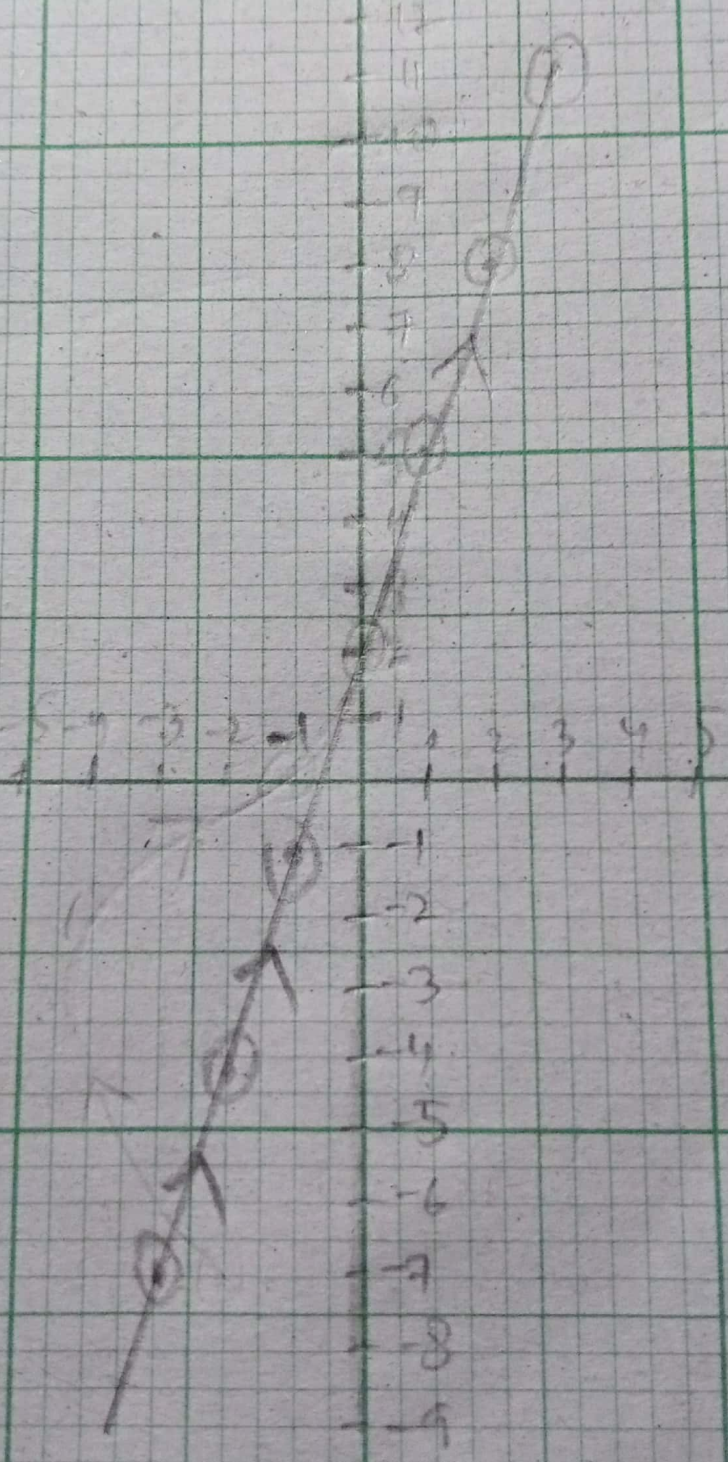


$x = \cos t$ $y = -\sin t$

Q3)

$x = 2t - 3$ $\qquad$ $y = 6t - 7$

t	x	y
0	-3	-7
0.5	-2	-4
1	-1	-1
1.5	0	2
2	1	5
2.5	2	8
3	3	11



$x = 2t - 3$ $y = 6t - 7$

Assignment # 01

Question # 03

$x = 3t - 4 \qquad y = 6t + 2$

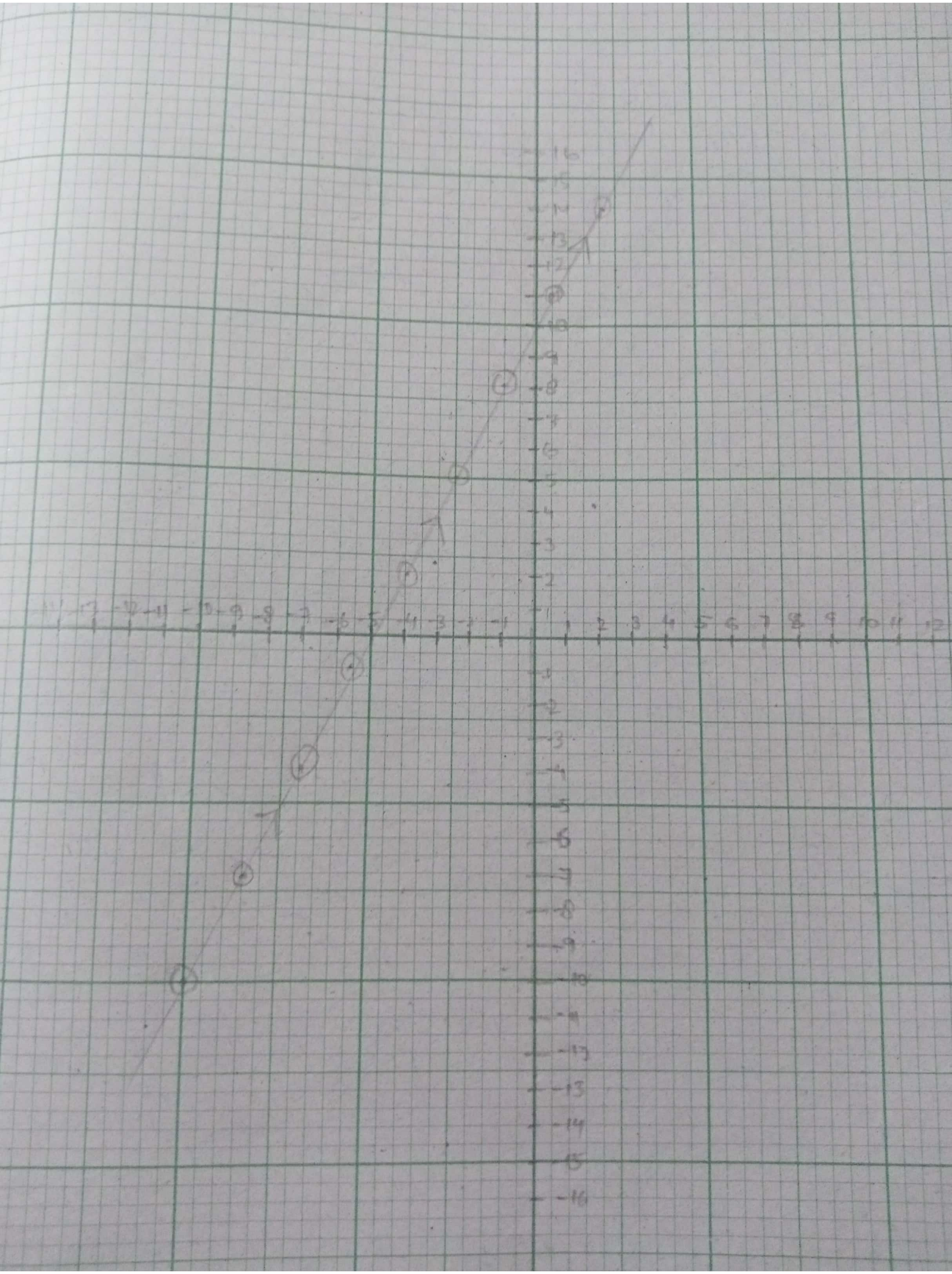
$t = \dfrac{x+4}{3} \qquad y = 6t + 2$

$y = \cancel{6}^{2}\left(\dfrac{x+4}{\cancel{3}}\right) + 2$

$y = 2x + 8 + 2$

$\boxed{y = 2x + 10}$

t	x	y
-2	-10	-10
-1.5	-8.5	-7
-1	-7	-4
-0.5	-5.5	-1
0	-4	2
0.5	-2.5	5
1	-1	8
1.5	0.5	11
2	2	14



Question # 5

$x = 2\cos t$ $\quad (0 \leq t \leq 2\pi)$

$y = 5\sin t$

$\cos t = \dfrac{x}{2}$ $\quad \sin t = \dfrac{y}{5}$

$$1 = \frac{x^2}{2^2} + \frac{y^2}{5^2}$$

$$\cancel{1 = \frac{5x + 2y}{10}} \qquad \frac{x^2}{4} + \frac{y^2}{25} = 1$$

$$\cancel{5x + 2y = 10}$$

t	x	y
0	$\cancel{0}$ 2	0
$\frac{\pi}{4}$	1.4	3.5
$\frac{\pi}{2}$	0	5
$3\pi/4$	-1.4	3.5
π	-2	0
$5\pi/4$	-1.4	-3.5
$3\pi/2$	0	-5
$7\pi/4$	1.4	-3.5
2π	2	0

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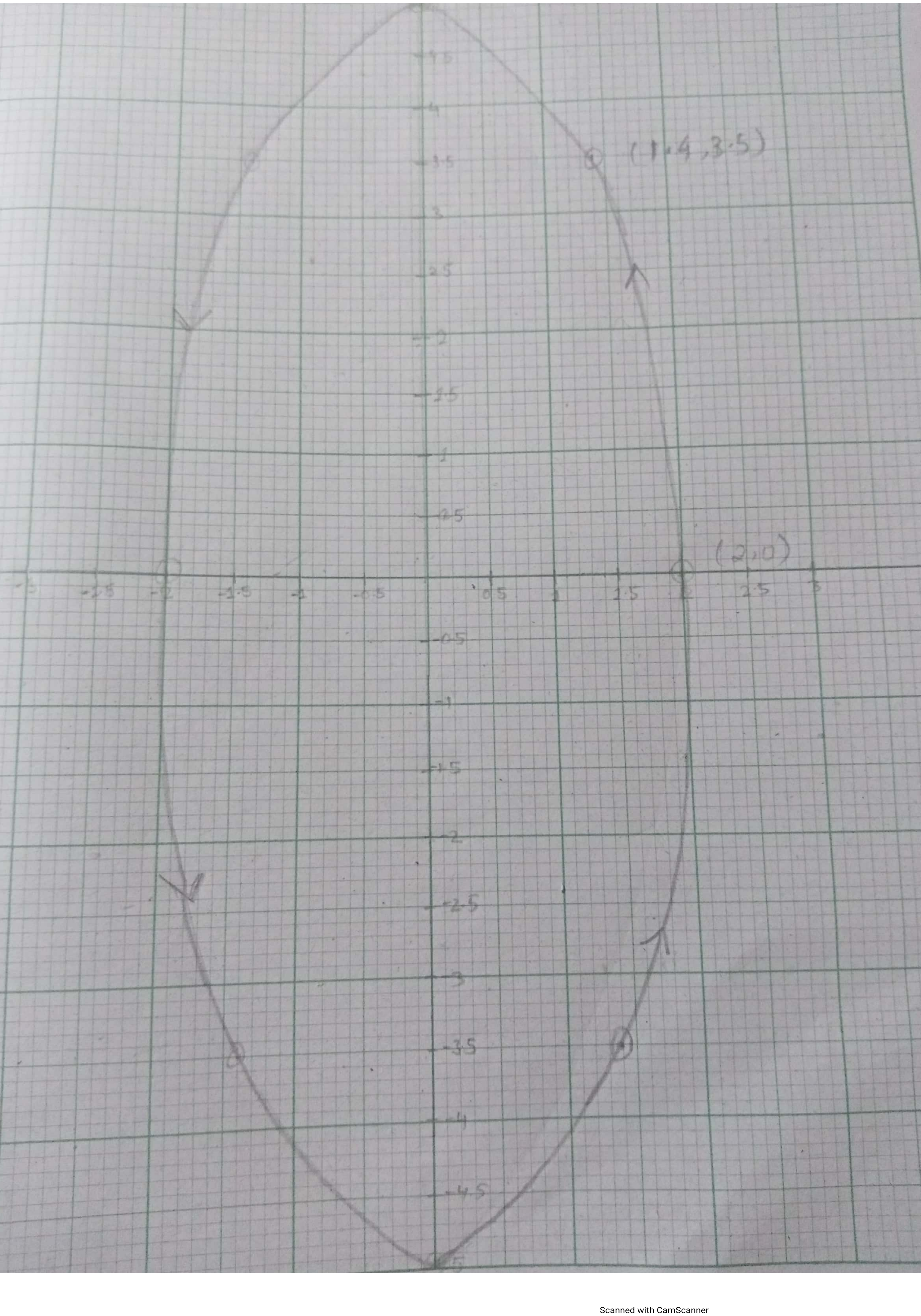

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Question # 09

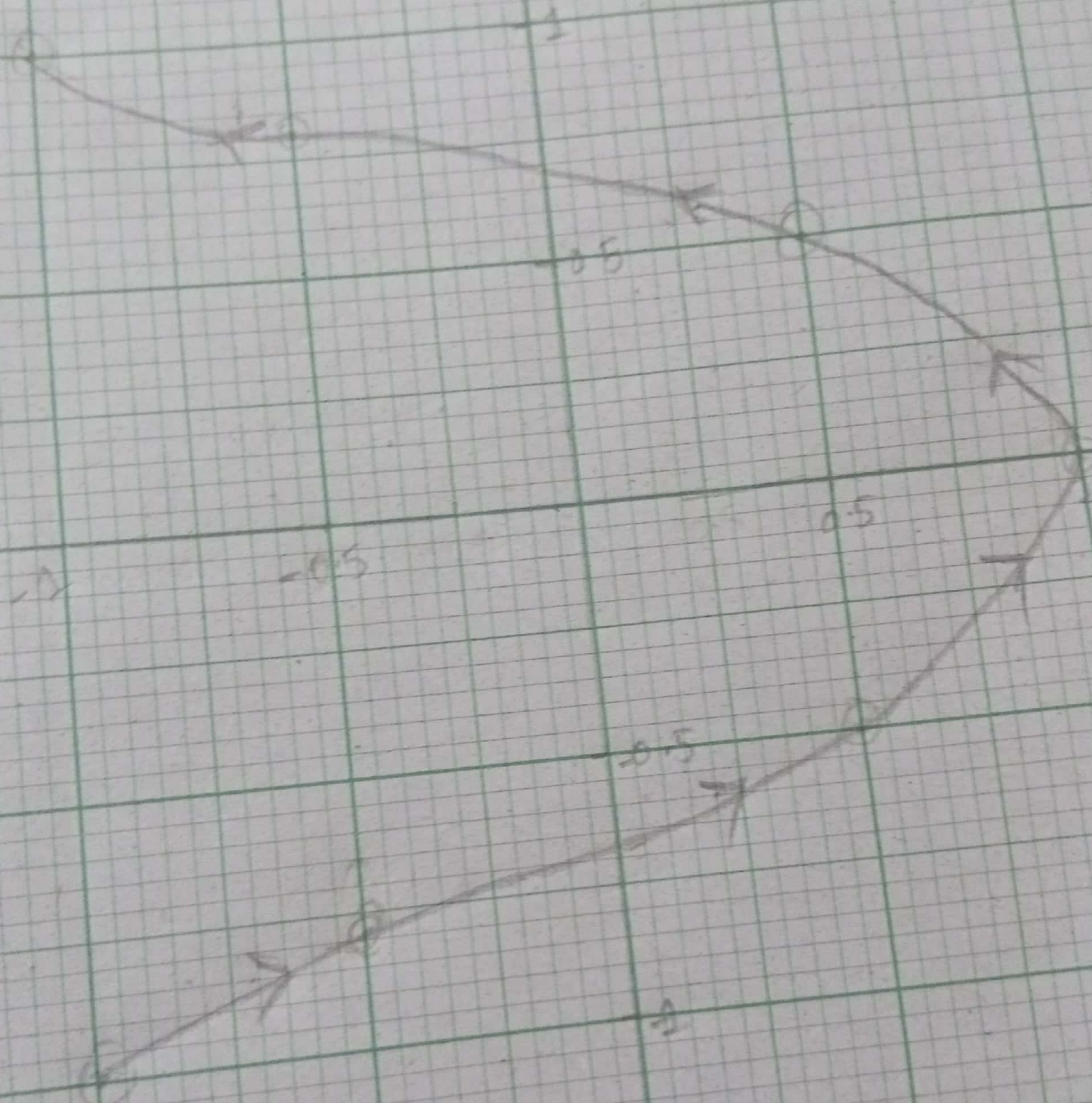
$x = \cos 2t \qquad y = \sin t \qquad \left(-\frac{\pi}{2} \le t \le \frac{\pi}{2}\right)$

$\cancel{x} \quad \cos 2t = 1 - 2\sin^2 t$

$x = 1 - 2\sin^2 t$

$\boxed{x = 1 - 2y^2}$

t	x	y
$-\frac{\pi}{2}$	-1	-1
$-\pi/3$	-0.5	-0.8
$-\pi/6$	0.5	-0.5
0	1	0
$\pi/6$	0.5	0.5
$\pi/3$	-0.5	0.866
$\pi/2$	-1	1


1
0.5
-1
-0.5
0.5
-0.5
-1

Question # 26

Q) Find parametric equation.

Two points given

$P(2,-1) = (x_1, y_1)$

$Q(3,1) = (x_2, y_2)$

First we find normal equation using two point form

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$$

$$y - (-1) = \frac{1-(-1)}{3-2}(x-2)$$

$$y + 1 = 2x - 4$$

$$\cancel{2x - y - 5 = 0}$$

$$y = 2x - 5$$

$$\boxed{\begin{aligned} x &= t \\ y &= 2t - 5 \end{aligned}}$$

a) Midpoint between P & Q :-

$$M(x,y) = \frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}$$

$$M(x,y) = \left(\frac{5}{2}, 0\right)$$

b) the point is one fourth of the way from P to

ratio = one fourth

$m_1 : m_2 = 1 : 3$

$(x_1, y_1) = (2, -1)$

$(x_2, y_2) = (3, 1)$

$$M(x,y) = \frac{m_1 x_2 + m_2 x_1}{m_1 + m_2}, \frac{m_1 y_2 + m_2 y_1}{m_1 + m_2}$$

$$= \frac{3+6}{4}, \frac{1+(-3)}{4}$$

$$M(x,y) = \left(\frac{9}{4}, \frac{-1}{2}\right)$$

c) the point is three fourth of the way from P to

ratio = three fourths

$m_1 : m_2 = 3 : 1$

$x_1, y_1 = 2, -1$

$x_2, y_2 = 3, 1$

$$M(x,y) = \frac{m_1 x_2 + m_2 x_1}{m_1 + m_2}, \frac{m_1 y_2 + m_2 y_1}{m_1 + m_2}$$

$$\frac{9+2}{4}, \frac{3+(-1)}{4}$$

$$M(x,y) = \left(\frac{11}{4}, \frac{1}{2}\right)$$

Question 45

$x = \sqrt{t}$

Question 45

$$x = \sqrt{t} \quad , \quad y = 2t + 4$$

$$\frac{dx}{dt} = \frac{1}{2\sqrt{t}} \qquad \frac{dy}{dt} = 2 .$$

$$\frac{dy}{dx} = \frac{2}{1/2\sqrt{t}}$$

$$= 2 \div \frac{1}{2\sqrt{t}}$$

$$= 2 \times 2\sqrt{t}$$

$$\frac{dy}{dx} = 4\sqrt{t}$$

$$\frac{d^2y}{dx^2} = \frac{\frac{d}{dt}\left(\frac{dy}{dx}\right)}{dx/dt} = \frac{\frac{d}{dt}(4\sqrt{t})}{1/(2\sqrt{t})}$$

$$= \frac{4\left(\cancel{1/(2\sqrt{t})}\right)}{\cancel{1/(2\sqrt{t})}}$$

$$\frac{d^2y}{dx^2} = 4$$

at $t = 1$

$$\boxed{\frac{d^2y}{dx^2} = 4}$$

Question # 49

$$x = \theta + \cos\theta \qquad y = 1 + \sin\theta$$

$$\frac{dx}{d\theta} = 1 - \sin\theta \qquad \frac{dy}{d\theta} = \cos\theta$$

$$\frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta} = \frac{\cos\theta}{1-\sin\theta}$$

$$\frac{d^2y}{dx^2} = \frac{\frac{d}{d\theta}\left(\frac{dy}{dx}\right)}{dx/d\theta}$$

$$= \frac{\frac{d}{d\theta}\left(\frac{\cos\theta}{1-\sin\theta}\right)}{1-\sin\theta}$$

$$= \left(\frac{(1-\sin\theta)(-\sin\theta) - \cos\theta(-\cos\theta)}{(1-\sin\theta)^2}\right) \div 1-\sin\theta$$

$$= \frac{-\sin\theta + \sin^2\theta + \cos^2\theta}{1 - 2\sin\theta + \sin^2\theta} \times \frac{1}{1-\sin\theta}$$

$$= \frac{1-\cancel{\sin\theta}}{1-2\sin\theta+\sin^2\theta} \times \frac{1}{1-\cancel{\sin\theta}}$$

$$\frac{dy^2}{dx^2} = \frac{1}{(1-\sin\theta)^2}$$

here $\theta = \frac{\pi}{6}$

$$\frac{d^2y}{dx^2} = 4$$

Question # 51

$x = e^{t}$, $y = e^{-t}$.

$\frac{dx}{dt} = e^{t}$ $\frac{dy}{dt} = -e^{-t}$

$\frac{dy}{dx} = \frac{-e^{-t}}{e^{t}}$

$= -e^{-t} e^{-t}$

$\frac{dy}{dx} = -e^{-2t}$ $t = 1$

$m = -e^{-2}$

$x = e^{1}$ $y = e^{-t}$

$x = e$ $y = e^{-1}$

$(x_1, y_1) = (e, e^{-1})$ $m = -e^{-2}$

point slope form

$y - y_1 = m(x - x_1)$

$y - e^{-1} = -e^{-2} \cdot (x - e)$

$y = \frac{-x}{e^2} + \frac{e}{e^2} + e^{-1}$

$$\boxed{y = -\frac{1}{e^2} x + 2e^{-1}}$$

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Question 53

$x = 2\sin t \qquad y = 4\cos t \qquad 0 \le t \le 2\pi$

$\frac{dx}{dt} = 2\cos t \qquad \frac{dy}{dt} = -4\sin t$

$\frac{dy}{dx} = \frac{-4\sin t}{2\cos t} = \frac{-2\sin t}{\cos t}$

For horizontal tangen lines

$-2\sin t = 0$

$-\sin 0 = 0$

$0 \leq 0 \leq 2\pi$ $\qquad \cos 0 \neq 0$

$-\sin \pi = 0$

$0 \leq \pi \leq 2\pi$ $\qquad \cos \pi \neq 0$

$-\sin 2\pi = 0$

$0 \leq 2\pi \leq 2\pi$ $\qquad \cos 2\pi \neq 0$

For H. Tangen $\boxed{0, \pi, 2\pi}$

For Vertical Tangent lines

$\cos t = 0$

$\cos \frac{\pi}{2} = 0$

$0 \leq \frac{\pi}{2} \leq 2\pi$ $\qquad -2\sin \frac{\pi}{2} \neq 0$

$\cos \frac{3\pi}{2} = 0$

$0 \leq \frac{3\pi}{2} \leq 2\pi$ $\qquad -2\sin \frac{3\pi}{2} \neq 0$

$$\boxed{\frac{\pi}{2}, \frac{3\pi}{2}}$$

Question # 65

$x = t^2$, $y = \frac{1}{3}t^3$ $\quad 0 \le t \le 1$

$\frac{dx}{dt} = 2t$ $\quad \frac{dy}{dt} = t^2$

$$L = \int_b^a \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt$$

$$\int_0^1 \sqrt{(2t)^2 + (t^2)^2}\, dt$$

$$2\int_0^1 t\sqrt{1+t^2}\, dt$$

$x = 1 + t^2$

$\frac{dx}{dt} = 2t$

$\frac{dx}{2} = t\,dt$

$$\frac{2}{2}\int_0^1 \sqrt{x}\, dx$$

$$\left[\frac{2x^{3/2}}{3}\right]_0^1$$

$$\boxed{L = \frac{2}{3}}$$

Question # 67

$x = \cos 3t \qquad y = \sin 3t$

$\frac{dx}{dt} = -3\sin 3t \qquad \frac{dy}{dt} = 3\cos 3t$

$0 \leq t \leq \pi$

$$L = \int_b^a \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt$$

$$\int_0^\pi \sqrt{(-3\sin 3t)^2 + (3\cos 3t)^2}$$

$$3\int_0^\pi dt$$

$$\boxed{3\pi}$$

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Assignment # 02

Q9

a) $r = 2$

$\because r = \sqrt{x^2+y^2}$

$\sqrt{x^2+y^2} = 2$

$\boxed{x^2+y^2 = 4}$

b) $r\sin\theta = 4$

$\because y = r\sin\theta$

$y = 4$

c) $r = 3\cos\theta$

multiply r on bs

$r^2 = 3r\cos\theta$

$\because r^2 = x^2+y^2$

$r\cos\theta = x$

$\boxed{x^2+y^2 = 3x}$

d) $r = \dfrac{6}{3\cos\theta + 2\sin\theta}$

$3r\cos\theta + 2r\sin\theta = 6$

$\because x = r\cos\theta$

$\because y = r\sin\theta$

$\boxed{3x + 2y = 6}$

~~Q12~~ Q11

a) ~~$y = 3$~~ $x = 3$

$\because x = r\cos\theta$

$$\boxed{r\cos\theta = 3}$$

b) $x^2+y^2=7$

$\because r^2 = x^2+y^2$

$r^2 = 7$

$$\boxed{r = \sqrt{7}}$$

c) $x^2+y^2+6y=0$

$\because r^2 = x^2+y^2$

$\because y = r\sin\theta$

$r^2+6r\sin\theta=0$

$$\boxed{r = -6\sin\theta}$$

d) $9xy = 4$

$\because x = r\cos\theta$

$y = r\sin\theta$

$9r^2\cos\theta\sin\theta = 4$

$$\boxed{r^2 = \sqrt{\frac{4}{9\cos\theta\sin\theta}}}$$

Q19

a) $r = 3\sin 2\theta$

) $r = 3 + 2\sin\theta$

) $r^2 = 3\cos 2\theta$

Q25

$r = 6\sin\theta$

θ	r
0	0
$\pi/6$	3
$\pi/3$	5.2
$\pi/2$	6
$2\pi/3$	5.2
$5\pi/6$	3
π	0

Q29

Q) $r = 4-4\cos\theta$

Symmetric Test:

- For x-Axis

$r = 4-4\cos(-\theta)$

$r = 4-4\cos\theta$

TRUE

- For y-axis

$r = 4-4\cos(\pi-\theta)$

$= 4-4[\cos\overset{-1}{\pi}\cos\theta + \cancel{\sin\pi\sin\theta}^{0}]$

$r = 4+4\cos\theta$

False

θ	r
0	0
$\pi/6$	0.5
$\pi/3$	2
$\pi/2$	4
$2\pi/3$	6
$5\pi/6$	7.4
π	8

Q33

$r = 3 - \sin\theta$

Symmetric Test :-

For y-axis

$r = 3 - \sin(\pi - \theta)$

$= 3 - [\sin\pi \cos\theta - \sin\theta \cos\pi]$

$r = 3 - \sin\theta$

TRUE

For x-axis

False because sine is odd function

θ	r
$\pi/2$	2
$2\pi/3$	2.1
$5\pi/6$	2.5
π	3
$7\pi/6$	3.5
$4\pi/3$	3.8
$3\pi/2$	4

Q37

$r = -3 - 4\sin\theta$

Symmetric Test :-

For x-axis :-

$r = -3 - 4\sin(-\theta)$

$r = -3 + 4\sin\theta$

False

For y-axis :-

$r = -3 - 4\sin(\pi - \theta)$

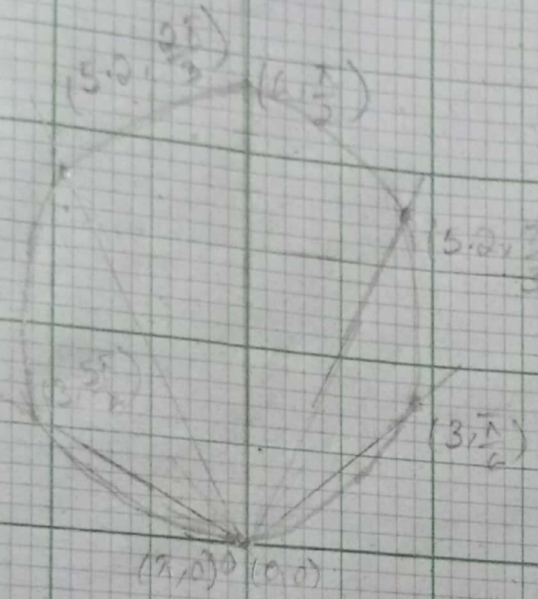
$= -3 - 4[\overset{0}{\sin\pi}\cos\theta - \overset{-1}{\cos\pi}\sin\theta]$

$r = -3 - 4\sin\theta$

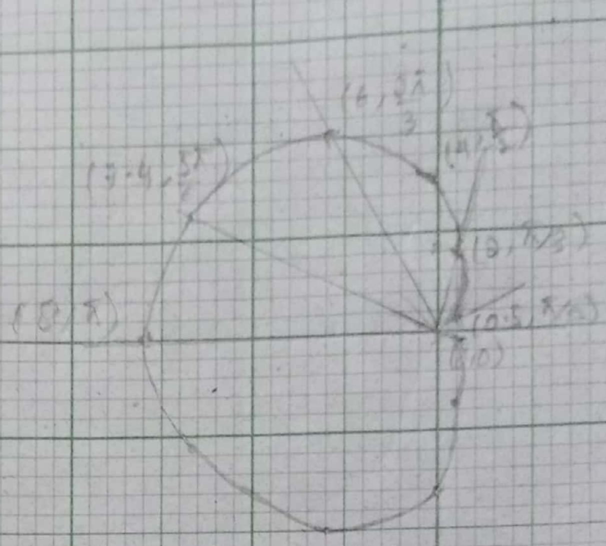
True

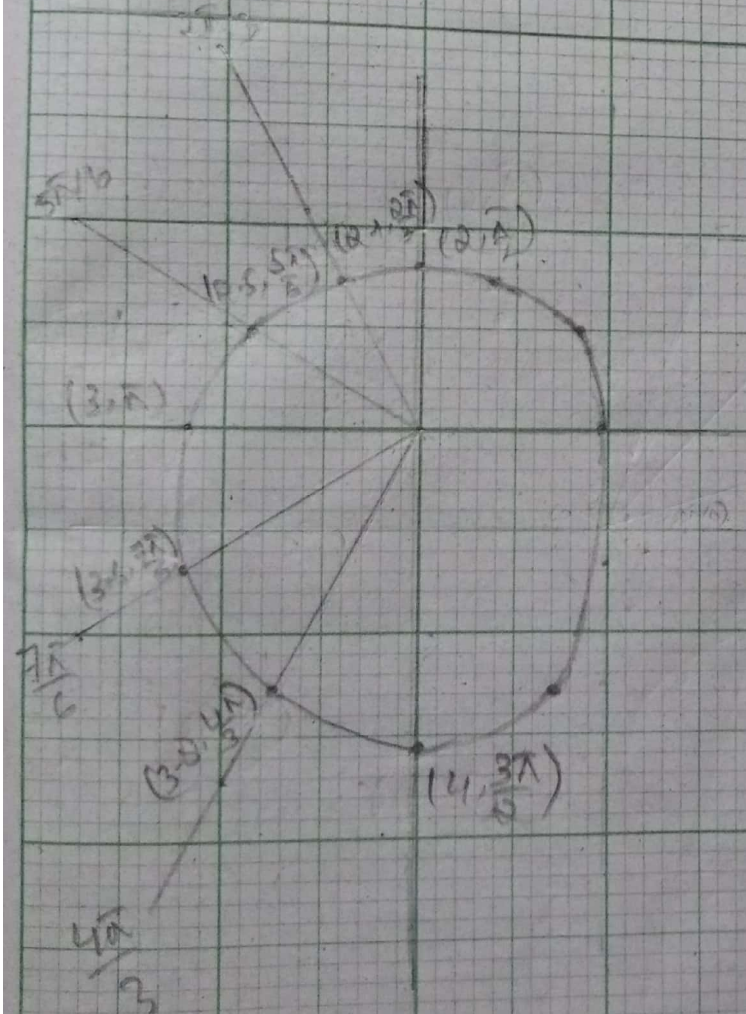
r	θ	r	θ
-7	$\pi/2$	0.4	$5\pi/3$
-6.4	$2\pi/3$		
-5	$5\pi/6$	-1	$11\pi/6$
-3	π	-3	2π
-1	$7\pi/6$	-3	0
0.4	$4\pi/3$	-6.4	$\pi/3$
1	$3\pi/2$	-5	$\pi/6$

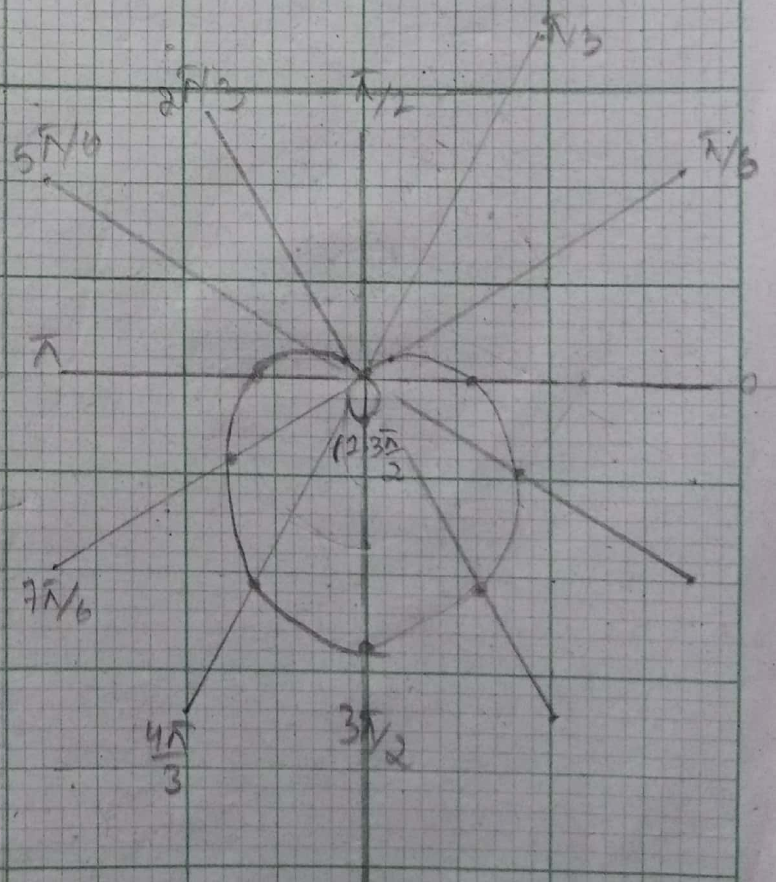
$r = 6\sin\theta$



$r = 4 - 4\cos\theta$







$r = 3 - \sin\theta$

Q39

$r^2 = 16\sin 2\theta$

Symmetric Test

For x-axis

$r^2 = 16\sin 2(-\theta)$

$r^2 = -16\sin 2\theta$

False

For Pole

$r^2 = 16\sin 2(\theta+\pi)$

$= 16[\sin 2\theta \cos 2\pi - \cos 2\theta \sin 2\pi]$

$r^2 = 16\sin 2\theta$

TRUE

For y-axis:-

$r^2 = 16\sin 2(\pi-\theta)$

$= 16\sin(2\pi - 2\theta)$

$= 16[\cancel{\sin 2\pi \cos 2\theta} - \cos 2\pi \sin 2\theta]$

$r^2 = 16\sin 2\theta$

Yes, TRUE.

r	θ
0	0
3.7	$\pi/6$
4	$\pi/4$
3.7	$\pi/3$
0	$\pi/2$
2.8	$\pi/12$
2.8	$5\pi/12$

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Q41

$r = 4\theta$

θ	r
0	0
$\pi/4$	0.7
$\pi/2$	1.5
$3\pi/4$	2.3
π	3.1
$5\pi/4$	3.9
$3\pi/2$	4.7
$7\pi/4$	5.4
2π	6.2
$9\pi/4$	7.0
$5\pi/2$	7.8
$11\pi/4$	8.6
3π	9.42

Q45

$r = 9 \sin 4\theta$

Symmetric Test

For x-axis

$-r = 9\sin(4\pi - 4\theta)$

$-r = 9[\sin 4\pi \cos 4\theta - \cos 4\pi \sin 4\theta]$

$r = 9 \sin 4\theta$

TRUE

For y-axis

$-r = 9 \sin 4(-\theta)$

$r = 9 \sin 4\theta$

TRUE

θ	r
0	0
$\pi/12$	7.7
$\pi/6$	7.7
$\pi/4$	0
$\pi/3$	-7.7
$5\pi/12$	-7.7
$\pi/2$	0

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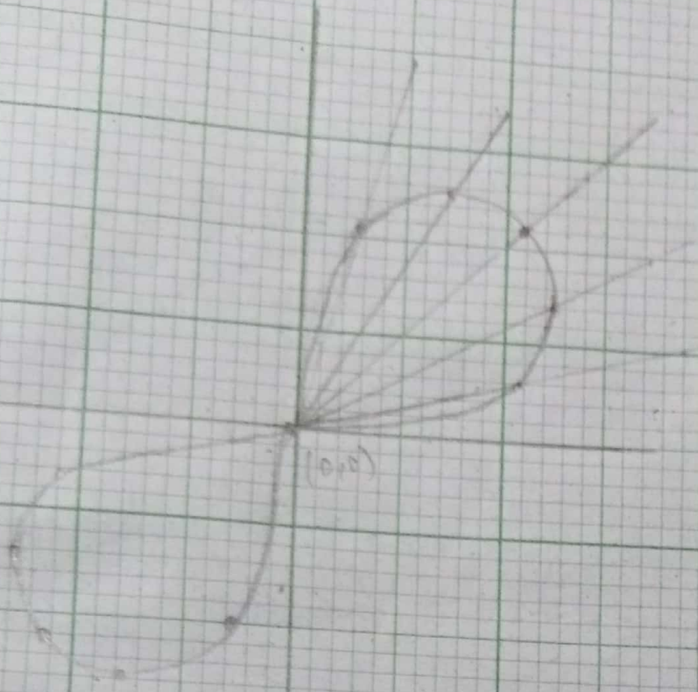
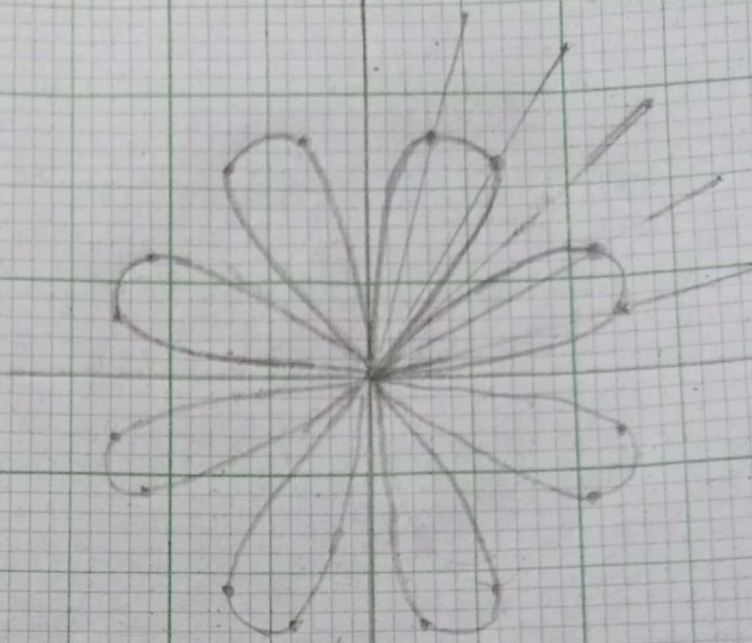
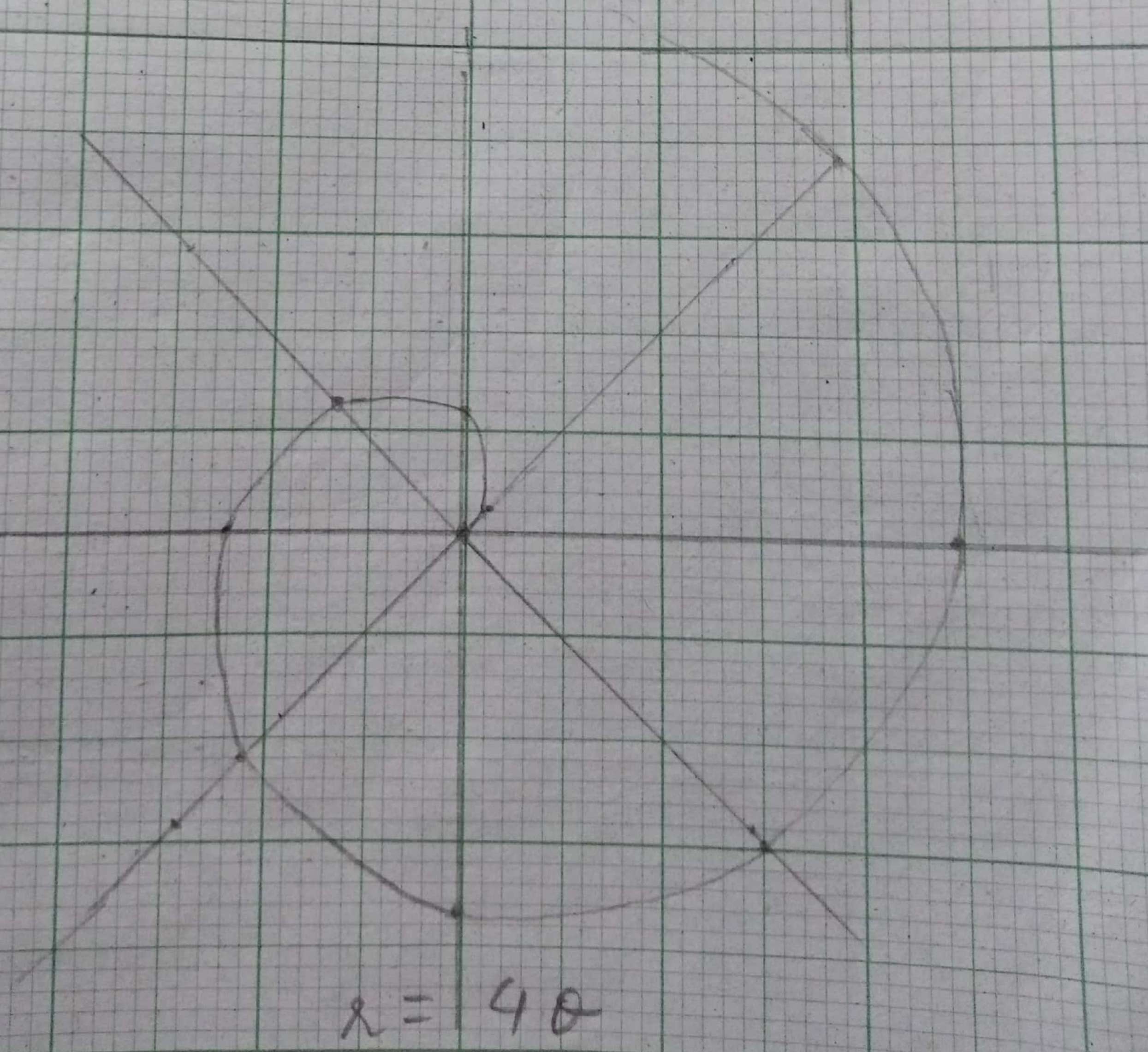


$r^2 = 16 \sin 2\theta$



$r = 9 \sin 4\theta$



$r = 4\theta$

Assignment 03

Q5

$r = \sin 3\theta \qquad \theta = \pi/4$

$x = r\cos\theta \qquad y = r\sin\theta$

$\frac{dx}{d\theta} = -r\sin\theta + \frac{dr}{d\theta}\cos\theta \qquad \frac{dy}{d\theta} = r\cos\theta + \frac{dr}{d\theta}\sin\theta$

$$\frac{dy}{dx} = \frac{r\cos\theta + \frac{dr}{d\theta}\sin\theta}{-r\sin\theta + \frac{dr}{d\theta}\cos\theta}$$

~~here $\theta = \frac{\pi}{4}$~~ here $r = \sin 3\theta$

$$\cancel{\frac{dy}{dx} = \sin 3\theta \cos\left(\frac{\pi}{4}\right) + \frac{dr}{d\theta}}$$

$$\frac{dy}{dx} = \frac{\sin 3\theta \cos\theta + 3\cos 3\theta \sin\theta}{-\sin 3\theta \sin\theta + 3\cos 3\theta \cos\theta}$$

here $\theta = \frac{\pi}{4}$

$$\frac{dy}{dx} = \frac{\frac{1}{2} + \left(-\frac{3}{2}\right)}{-\frac{1}{2} + \left(-\frac{3}{2}\right)}$$

$$\left.\frac{dy}{dx}\right|_{\theta = \frac{\pi}{4}} = \frac{1}{2}$$

Q13

$r = 2\cos 3\theta$

Symmetric Test

For x-axis

$r = 2\cos 3(-\theta)$

$r = 2\cos 3\theta$

TRUE

For y-axis

$r = 2\cos(3\pi - 3\theta)$

$r = 2[\cos 3\pi \cos 3\theta + \sin 3\pi \sin 3\theta]$

$r = -2\cos 3\theta$

False

θ	r
0	2
$\pi/6$	0
$\pi/3$	-2
$\pi/2$	0
$2\pi/3$	2
$5\pi/6$	0
π	-2

$\theta = \frac{\pi}{6}$:

$r = 0$

$\frac{dr}{d\theta} = 6$

$\underline{\theta = \pi/2 :}$

$r = 0$

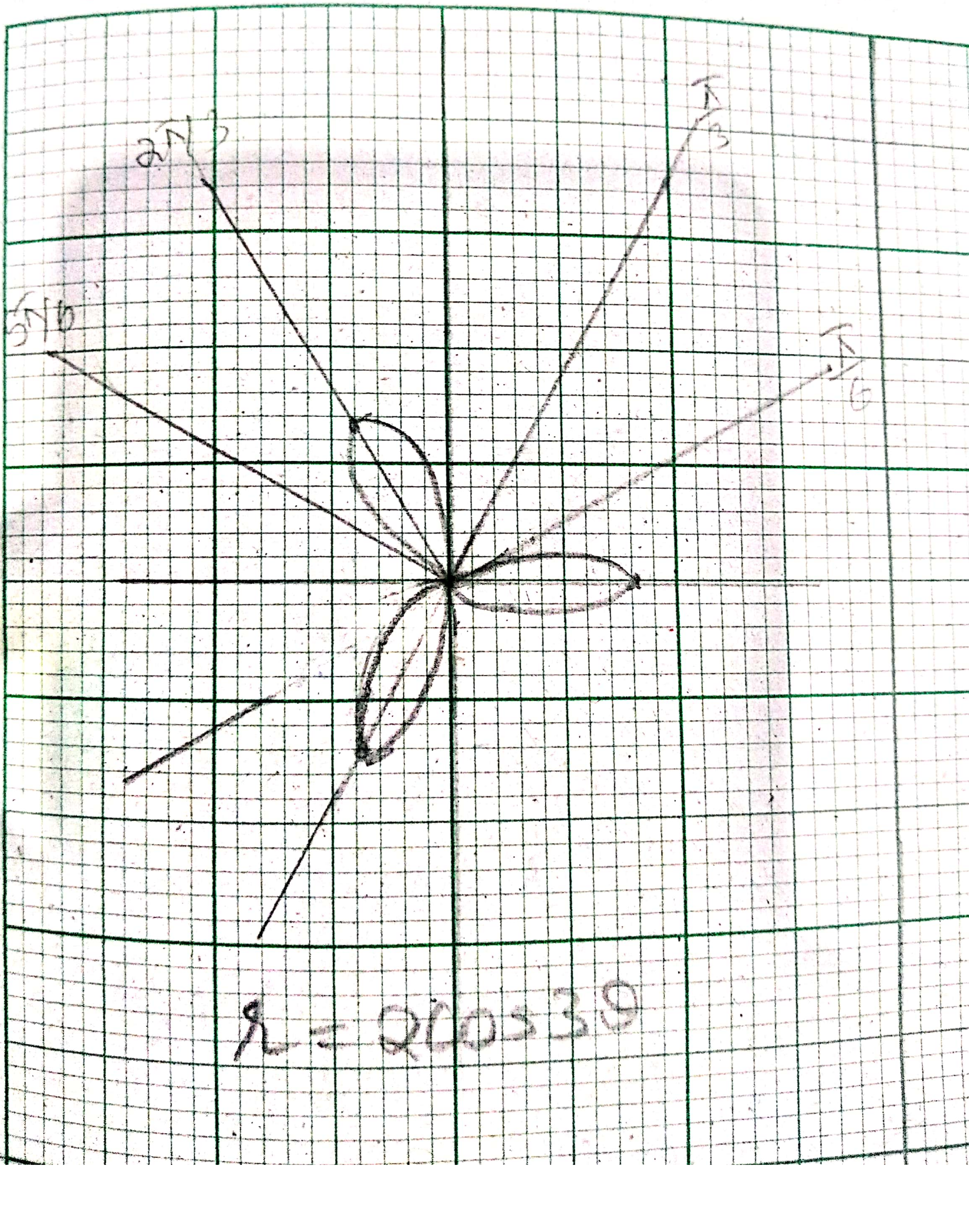
$$\frac{dr}{d\theta} = 6$$

$\underline{\theta = 5\pi/6}$

$r = 0$

$$\frac{dr}{d\theta} = -6$$

Q19


2π/3
π/3
5π/6
π/6
r = 2cos3θ

$$\frac{dx}{d\theta} = a\cos\theta \sin\ldots$$

$$= a\cos\theta\sin\theta - \sin\theta + \sin\ldots$$

Q19

$r = a$

$$\frac{dr}{d\theta} = 0$$

$$L = \int_{0}^{2\pi} \sqrt{a^2 + \left(\frac{dr}{d\theta}\right)^2}\, d\theta$$

$$L = a\int_{0}^{2\pi} d\theta$$

$$L = a[\theta]_0^{2\pi}$$

$$\boxed{L = 2\pi a}$$

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Q21

$r = a(1-\cos\theta)$

$\frac{dr}{d\theta} = a\sin\theta$

$$L = \int_{\alpha}^{\beta} \sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2}\, d\theta$$

$$L = \int_{0}^{2\pi} \sqrt{a^2(1-\cos\theta)^2 + a^2\sin^2\theta}\, d\theta$$

$$L = a\int_{0}^{2\pi} \sqrt{1-2\cos\theta + \cos^2\theta + \sin^2\theta}\, d\theta$$

$$L = a\int_{0}^{2\pi} \sqrt{2(1-\cos\theta)}\, d\theta$$

$$L = \sqrt{2}a\int_{0}^{2\pi} \sqrt{1-\cos\theta}\, d\theta$$

$$L = \sqrt{2}a\int_{0}^{2\pi} \sqrt{2}\sin\frac{\theta}{2}\, d\theta$$

Let

$x = \frac{\theta}{2}$

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$$\frac{dx}{d\theta} = \frac{1}{2}$$

$$d\theta = 2dx$$

$$L = \sqrt{2}a \int_0^{2\pi} 2\sqrt{2} \sin x \, dx$$

$$L = 4a \int_0^{2\pi} \sin x \, dx$$

$$\cancel{L = 4a \left[-\cos x\right]_0^{2\pi}}$$

$$=$$

$$\cancel{= 4a \left[-(1-1)\right]}$$

$$= 4a \left[-\cos x\right]_0^{2\pi}$$

$$L = 4a \left[-\cos\frac{\theta}{2}\right]_0^{2\pi}$$

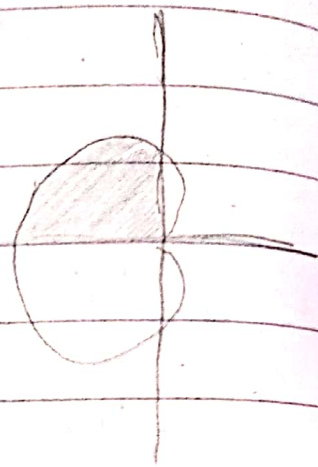
$$L = 4a(+1-(-1))$$

$$\boxed{L = 8a}$$

Q25

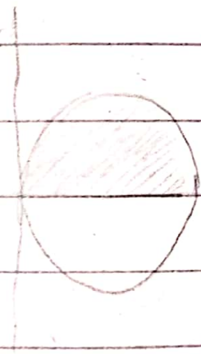
a) $r = 1 - \cos\theta$

$$A = \int_{\pi/2}^{\pi} \frac{1}{2}(1-\cos\theta)^2 \, d\theta$$



b) $r = 2\cos\theta$

$$A = \int_{0}^{\frac{\pi}{2}} \frac{1}{2}(2\cos\theta)^2 \, d\theta$$

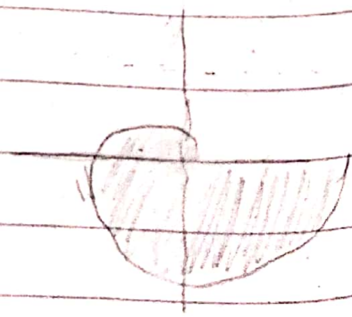


c) $r = \sin 2\theta$

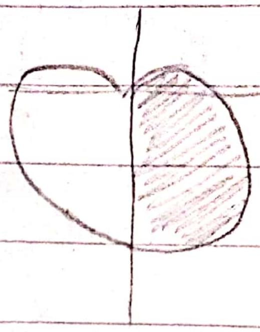
$$A = \int_{0}^{\frac{\pi}{2}} \frac{1}{2}(\sin 2\theta)^2 \, d\theta$$

d) $r = \theta$

$$A = \int_{0}^{2\pi} \frac{1}{2}\theta^2 \, d\theta$$

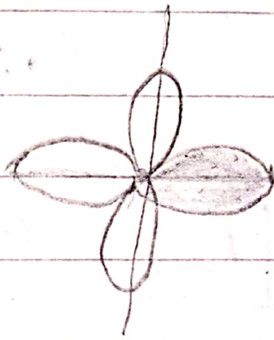


e) $\gamma = 1 - \sin\theta$



$$A = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{1}{2} (1 - \sin\theta)^2 \, d\theta$$

f) $r = \cos 2\theta$



$$A = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{1}{2} (\cos 2\theta)^2 \, d\theta$$

Q29

$r = 2 + 2\sin\theta$

$$A = \int_{0}^{2\pi} \frac{1}{2} (2 + 2\sin\theta)^2 \, d\theta$$

$$= \frac{1}{2} \int_{0}^{2\pi} 4 + 8\sin\theta + 4\sin^2\theta \, d\theta$$

$$= \frac{1}{2} \int_{0}^{2\pi} 4 + 8\sin\theta + 2(1 - \cos 2\theta) \, d\theta$$

$$A = \frac{1}{2} \int_{0}^{2\pi} 4 + 8\sin\theta + 2 - 2\cos 2\theta \, d\theta$$

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$$A = \frac{1}{2}\left[4\theta - 8\cos\theta + 2\theta - \sin 2\theta\right]_0^{2\pi}$$

$$= \frac{1}{2}\left[(12\pi - 8) + 8\right]$$

$$= \frac{12\pi - 0}{2}$$

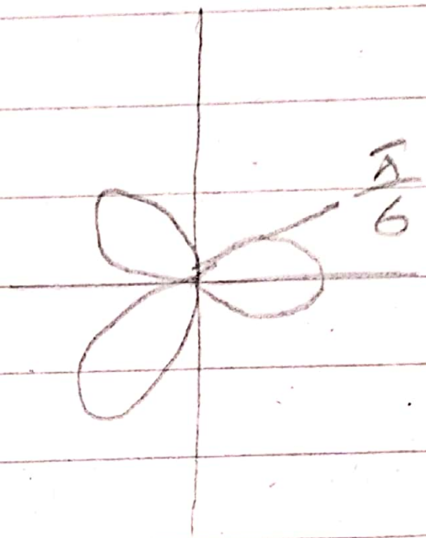
$$\boxed{A = 6\pi}$$

Q31

$$r = 4\cos 3\theta$$

Q31

$r = 4\cos 3\theta$



$$A = 3\int_{-\frac{\pi}{6}}^{\frac{\pi}{6}} \frac{1}{2}(4\cos 3\theta)^2$$

$$= 24\int_{-\pi/6}^{\pi/6} \cos^2 3\theta$$

$$= 24\int_{\pi/6}^{\pi/6} \frac{1+\cos 6\theta}{2}$$

$$= 12\left[\theta + \frac{\sin 6\theta}{6}\right]_{-\pi/6}^{\pi/6}$$

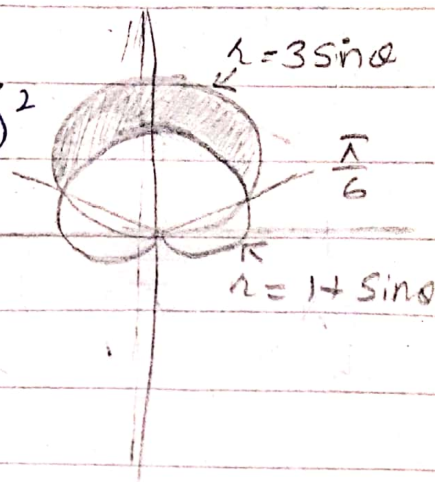
$$= 12\left[\frac{\pi}{6} + \frac{\pi}{6}\right]$$

$$= {}^{4}\cancel{12} \times \frac{\pi}{\cancel{3}}$$

$$\boxed{A = 4\pi}$$

Q39

$$A = \cancel{\int_0^{\pi} \frac{1}{2}(3\sin\theta)^2} - \cancel{\int_0^{\pi} \frac{1}{2}(1+\sin\theta)^2}$$



$3\sin\theta = 1 + \sin\theta$

$2\sin\theta = 1$

$\theta = \sin^{-1}\left(\frac{1}{2}\right)$

$\theta = \frac{\pi}{6}$

$$A = 2\left[\int_{\pi/6}^{\pi/2} \frac{1}{2}(3\sin\theta)^2 d\theta - \int_{\pi/6}^{\frac{\pi}{2}} \frac{1}{2}(1+\sin\theta)^2 d\theta\right]$$

$$= 2\int_{\pi/6}^{\pi/2} \frac{1}{2}\left[(3\sin\theta)^2 - (1+\sin\theta)^2\right] d\theta$$

$$= 2\int_{\pi/6}^{\pi/2} \frac{1}{2}\left(9\sin^2\theta - 1 - 2\sin\theta - \sin^2\theta\right) d\theta$$

$$= \int_{\pi/6}^{\pi/2} \overset{4}{\cancel{8}}\left(\frac{1-\cos 2\theta}{\cancel{2}}\right) - 1 - 2\sin\theta \, d\theta$$

$$A = \int_{\pi/6}^{\pi/2} 3 - 4\cos 2\theta - 2\sin\theta \, d\theta$$

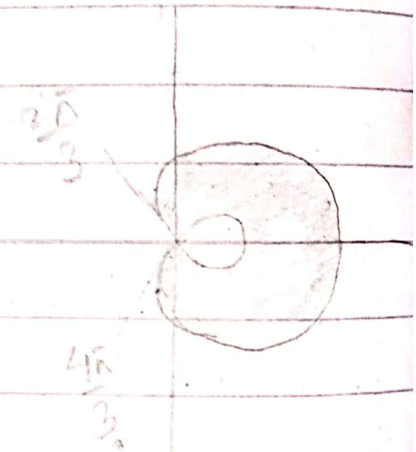
$$= \left[3\theta \cancel{+} - 2\sin 2\theta + 2\cos\theta\right]_{\pi/6}^{\pi/2}$$

$$= \frac{3\pi}{2} - \left(\frac{\pi}{2} - \cancel{\sqrt{3}} + \cancel{\sqrt{3}}\right)$$

$$\boxed{A = \pi}$$

Q43

$$r = \frac{1}{2} + \cos\theta$$



$$A = \cancel{A}\, 2\left[\int_0^{2\pi/3} \frac{1}{2}\left(\frac{1}{2} + \cos\theta\right)^2 - \int_{\pi}^{4\pi/3} \frac{1}{2}\left(\frac{1}{2} + \cos\theta\right)^2\right].$$

$$= \int_0^{\frac{2\pi}{3}} \frac{1}{4} + \cos\theta + \frac{1+\cos 2\theta}{2} - \int_{\pi}^{\frac{4\pi}{3}} \frac{1}{4} + \cos\theta + \frac{1+\cos 2\theta}{2}.$$

$$= \left[\frac{1}{4}\theta + \sin\theta + \frac{1}{2}\theta + \frac{\sin 2\theta}{4}\right]_0^{\frac{2\pi}{3}} - \left[\frac{1}{4}\theta + \sin\theta + \frac{1}{2}\theta + \frac{\sin 2\theta}{4}\right]_{\pi}^{\frac{4\pi}{3}}$$

$$= \frac{\pi}{6} + \frac{\sqrt{3}}{2} + \frac{\pi}{3} - \frac{\sqrt{3}}{8} - \left(\frac{\pi}{3} - \frac{\sqrt{3}}{2} + \frac{2\pi}{3} + \frac{\sqrt{3}}{8} - \frac{\pi}{4} - \frac{\pi}{2}\right).$$

$$= \frac{\pi}{2} + \frac{3\sqrt{3}}{8} - \left(\frac{\pi}{4} - \frac{3\sqrt{3}}{8}\right)$$

$$A = \frac{\pi}{4} + \frac{3\sqrt{3}}{4} \Rightarrow \boxed{A = \frac{\pi + 3\sqrt{3}}{4}}$$

Assignment #04

Q3

a) $y^2 = 4x$

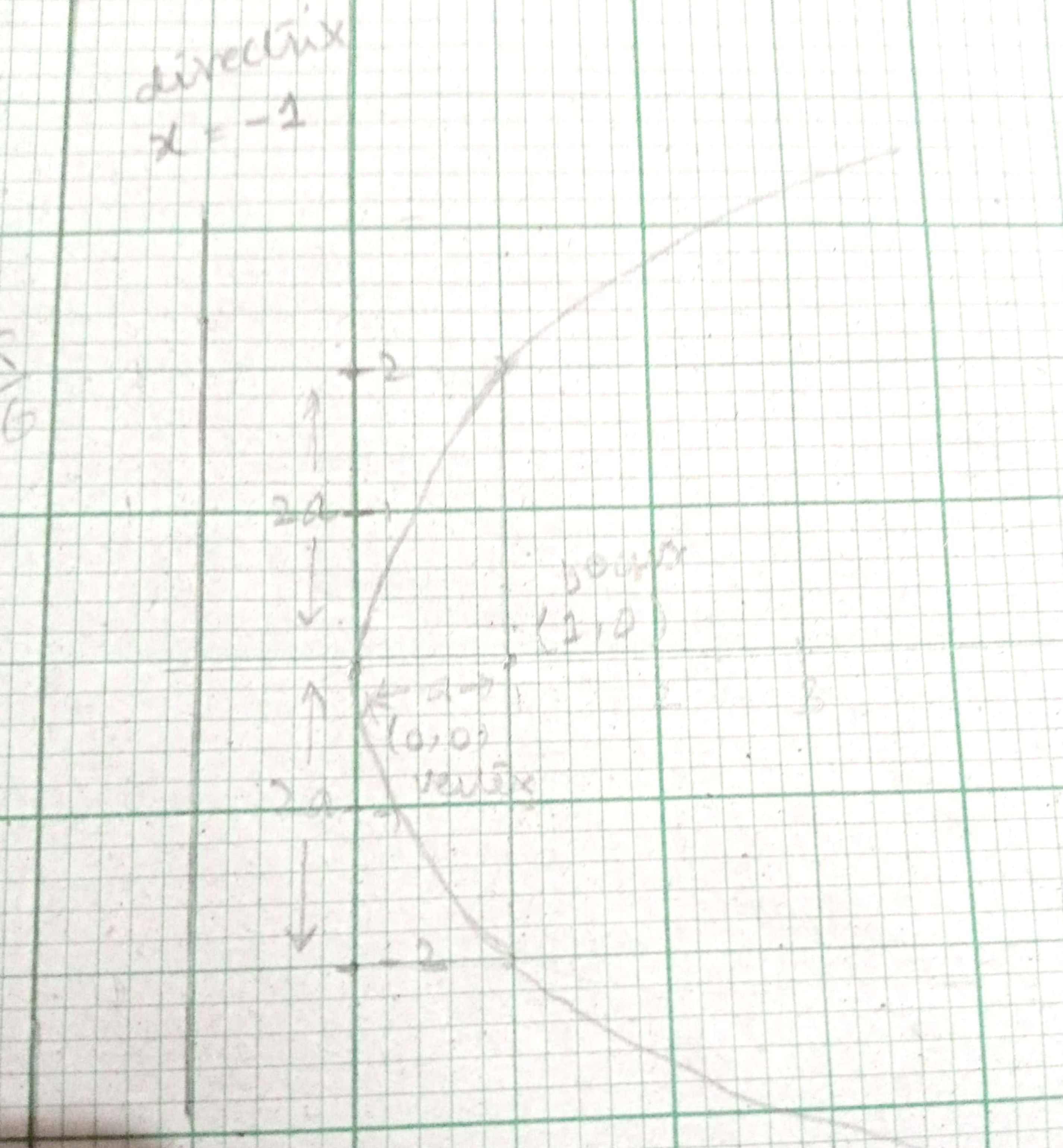
vertex = $(0,0)$

symmetric axis = x-axis

$a = 1$ $\because y^2 = 4 \times 1 x$

focus = $(1,0)$

directrix $\rightarrow$ $x = -1$



$y^2 = 4x$

Q3

b) $x^2 = -8y$

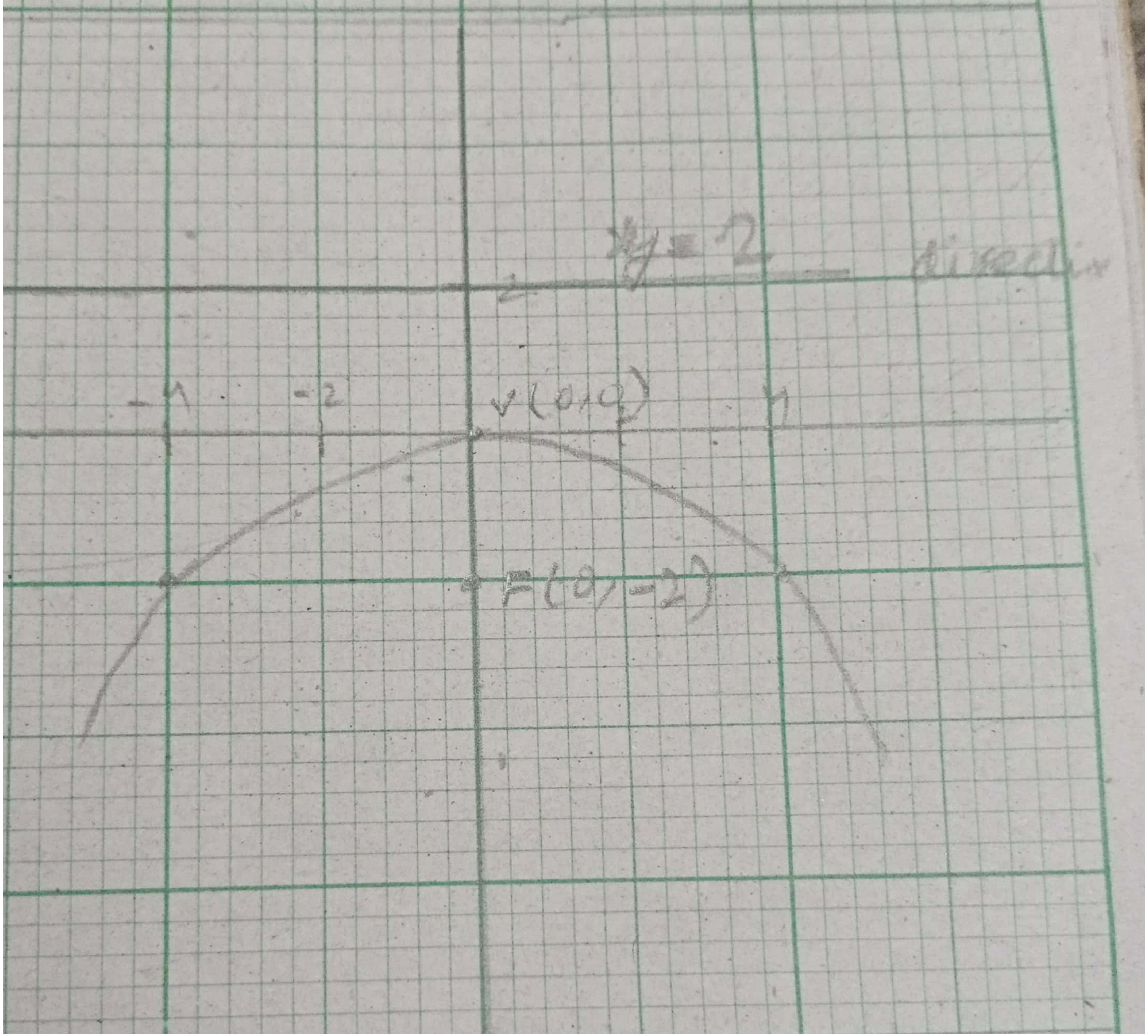
$4a = -8$

$a = -2$

vertex. $(0, 0)$

Focus $\rightarrow (0, -2)$

directrix $\rightarrow y = 2$


y = 2
directrix
-4
-2
V(0,0)
4
F(0,-2)

Q5

a) $(y-1)^2 = -12(x+4)$

vertex $= (-4, 1)$

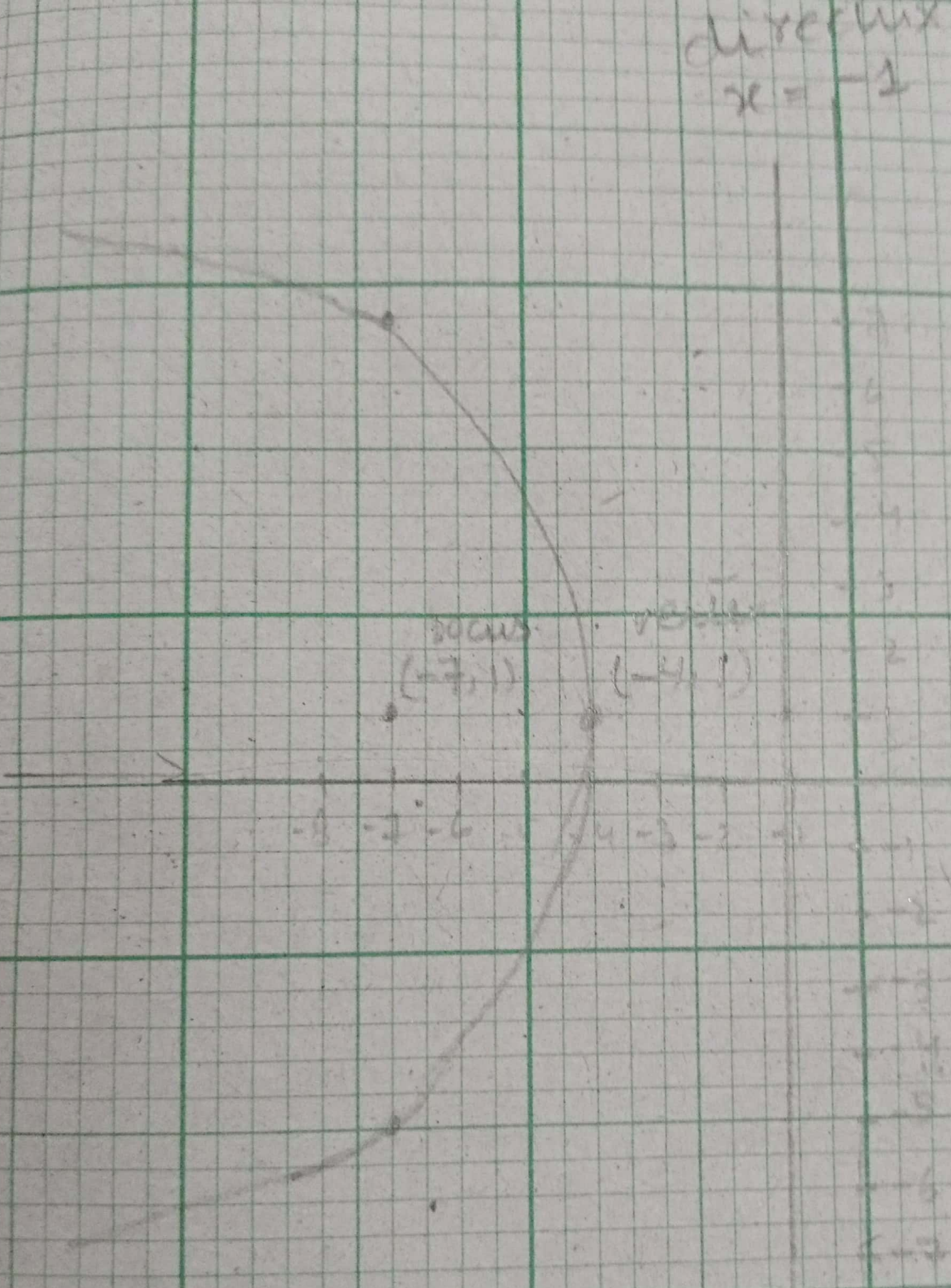
$4a = 12$

$a = 3$

symmetric axis : x-axis

focus $= (-7, 1)$

directrix $\Rightarrow x = -1$



$$(y-1)^2 = -12(x+4)$$

Q5

b) ~~Foci $(\pm 3, 0)$ asymptotes. $y = \pm 2x$~~

$(x-1)^2 = 2\left(y - \frac{1}{2}\right)$

Vertex $= (h, k) = \left(1, \frac{1}{2}\right)$

$4a = 2$

$a = \frac{1}{2}$

Focus $\rightarrow (1, 1)$

directrix $\rightarrow y = 0$

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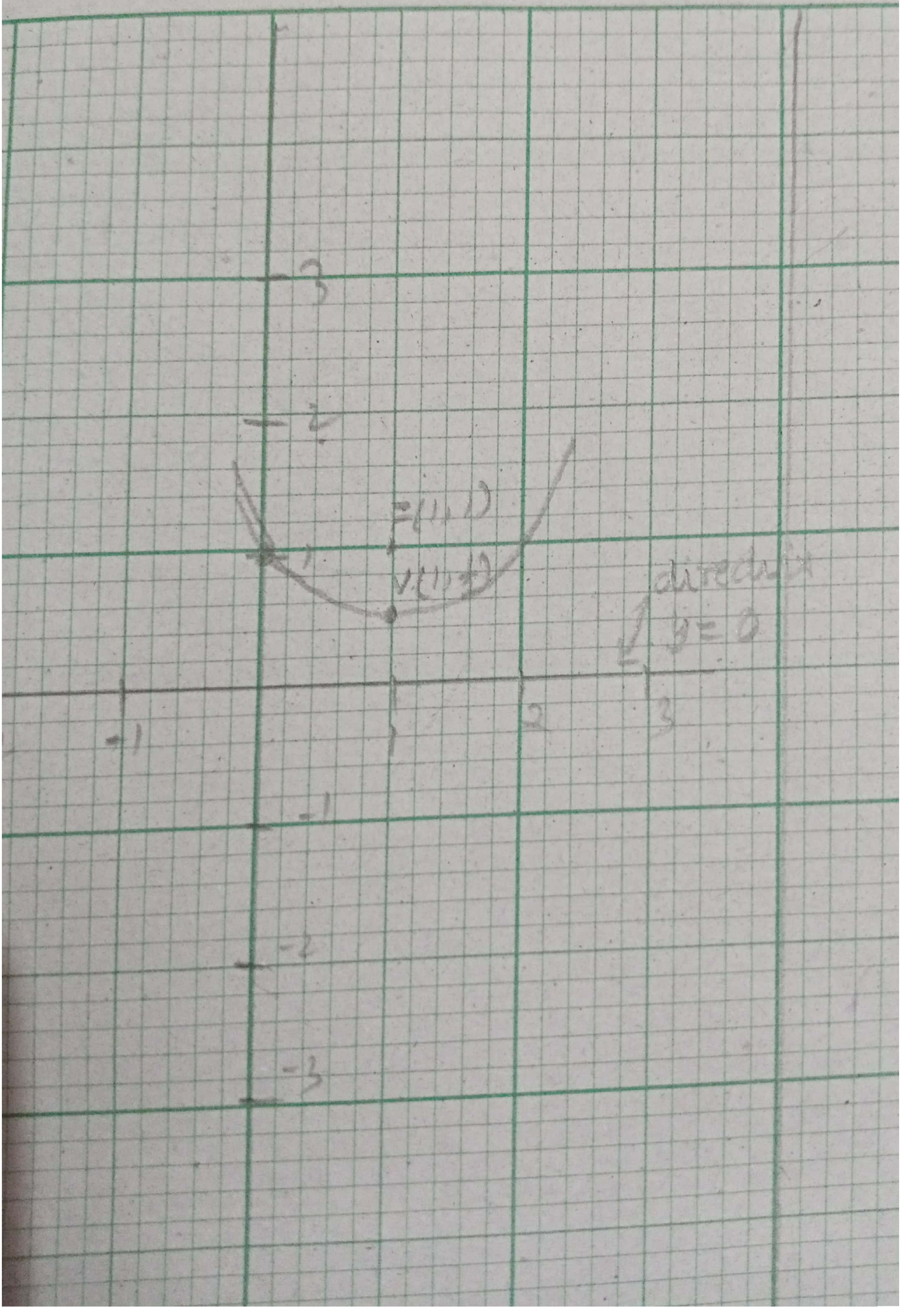

3
2
F(1,1)
1
V(1,½)
directrix
y = 0
-1
1
2
3
-1
-2
-3

Q7

a) $\frac{x^2}{16} + \frac{y^2}{9} = 1$

major axis $\rightarrow$ x-axis

$a = 4$ $b = 3$

center ~~vertex~~ $\rightarrow (0,0)$

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~~foci~~ $\Rightarrow$ $c = \sqrt{a^2 - b^2}$

$c = \sqrt{16 - 9}$

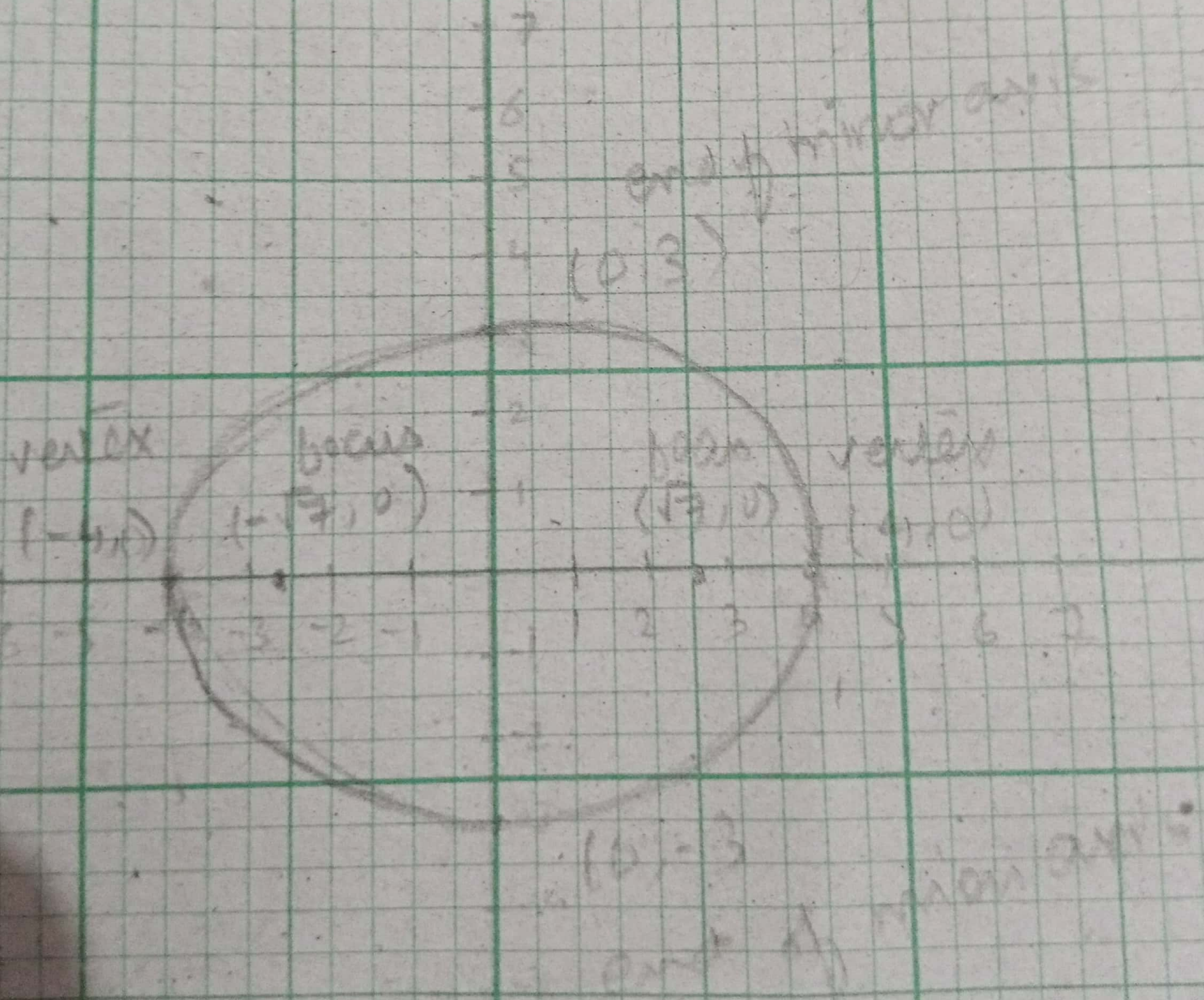
$c = \pm\sqrt{7}$

vertices $\Rightarrow$ $(-4, 0)$ $(4, 0)$

foci $\Rightarrow$ $(-\sqrt{7}, 0), (\sqrt{7}, 0)$

Ends of Minor axis $\Rightarrow$ $(0, 3), (0, -3)$

Scanned with CamScanner


vertex
(-4,0)
focus
(-√7,0)
focus
(√7,0)
vertex
(4,0)
end of minor axis
(0 3)
(0,-3)
end of minor axis

Q7

b) $9x^2 + y^2 = 9$

$\div$ by 9

$$\frac{x^2}{1} + \frac{y^2}{9} = 1$$

major axis y-axis

Vertex (0,0)

$a = 3$ $b = 1$

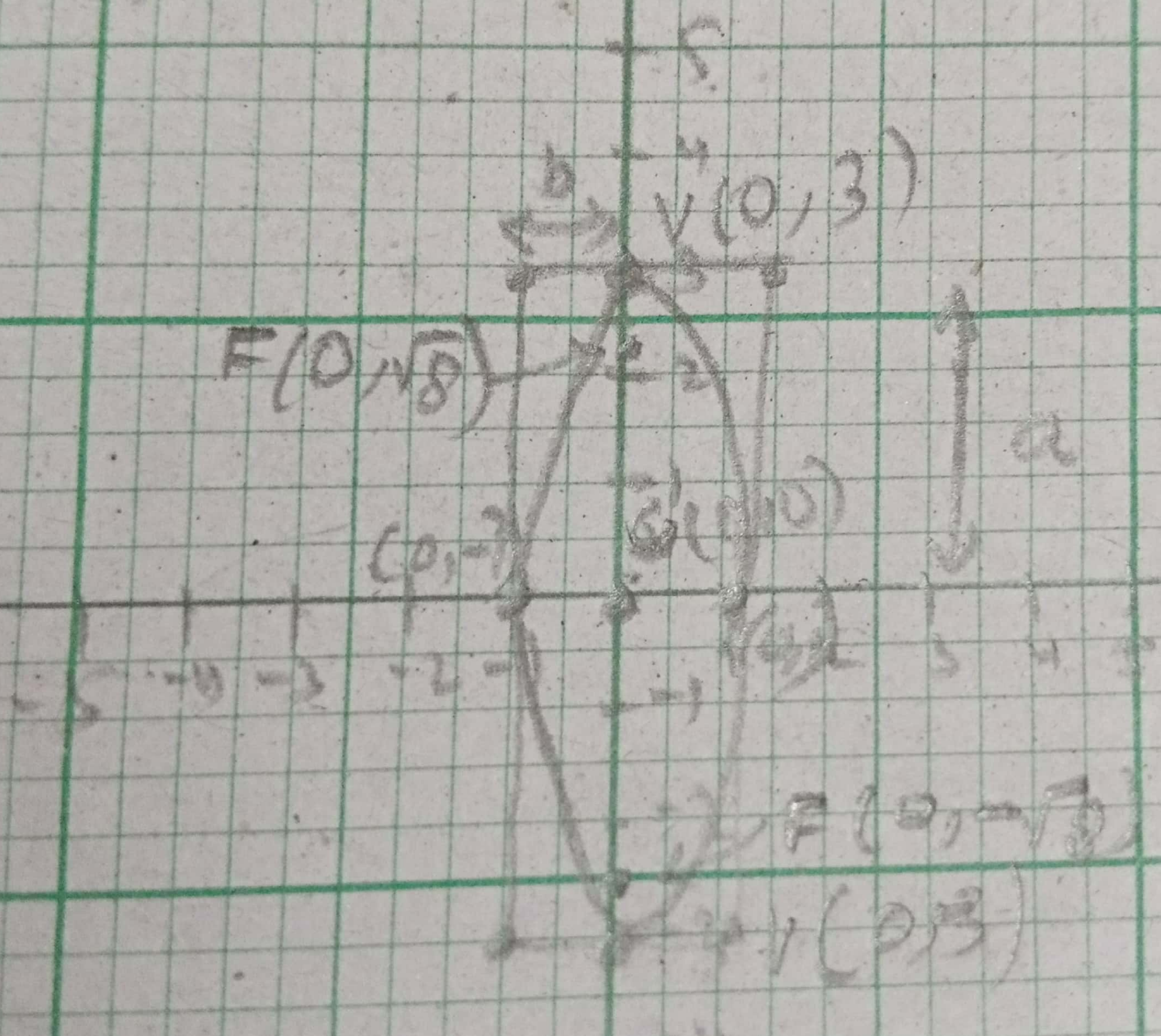
~~Foci~~ Vertices $\rightarrow (0, \pm 3)$

$c = \sqrt{a^2 - b^2}$

$c = \sqrt{8}$

Foci $\rightarrow$ $(0, \pm\sqrt{8})$

ends of minor axis $\rightarrow$ $(0, \pm 1)$



b
V(0,3)
F(0,√8)
a
(0,-1)
(0,0)
(1,0)
F(0,-√8)
V(0,-3)

Q9

a) $(x+3)^2 + 4(y-5)^2 = 16$

$\div$ by 16

$$\frac{(x+3)^2}{16} + \frac{(y-5)^2}{4} = 1$$

major axis $\rightarrow$ x-axis

$a = 4 \qquad b = 2$

center ~~vertex~~ $\Rightarrow (-3, 5)$

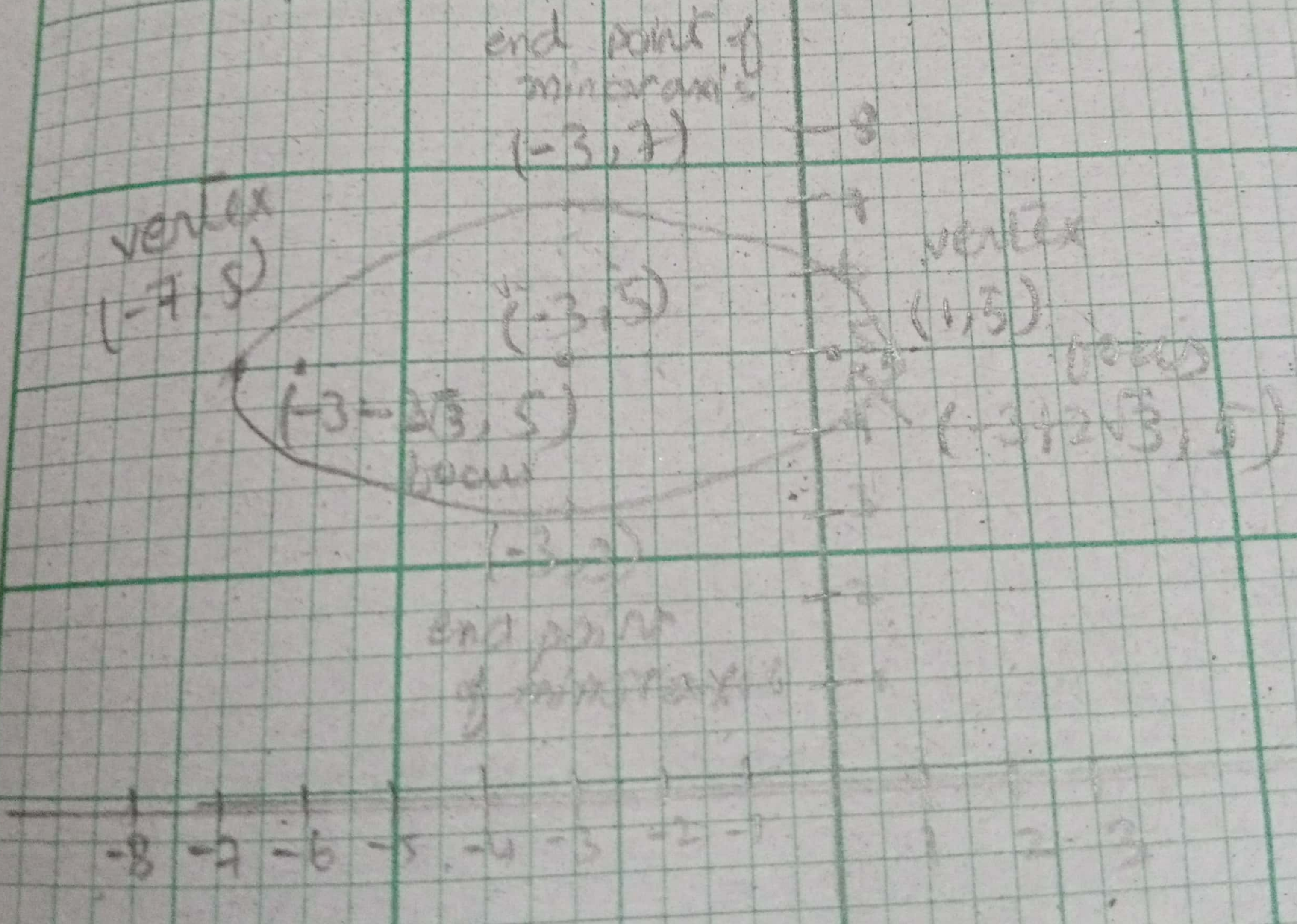
$c = \sqrt{a^2 - b^2}$

$c = \pm\sqrt{12}$

foci $\rightarrow (-3-\sqrt{12}, 5), (-3+\sqrt{12}, 5)$

vertices $\rightarrow (-7, 5), (1, 5)$

endpoints of minor axis $\rightarrow (-3, 3), (-3, 7)$



end point of minor axis
(-3, 7)
vertex
(-7, 5)
(-3, 5)
vertex
(1, 5)
(-3-2√3, 5)
focus
focus
(-3+2√3, 5)
(-3, 3)
end point of minor axis
8
7
-8
-7
-6
-5
-4
-3
-2
-1
1
2
3

Q91

b) $\frac{1}{4}x^2 + \frac{1}{9}(y+2)^2 - 1 = 0$

$\frac{1}{4}x^2 + \frac{1}{9}\overset{(y+2)^2}{\cancel{(y^2+4y+4)}} = 1 \cancel{-0}$

$\frac{x^2}{4} + \frac{(y+2)^2}{9} = 1$

major axis is y-axis

Centre $(0, -2)$

$a = 3$ $\qquad b = 2$

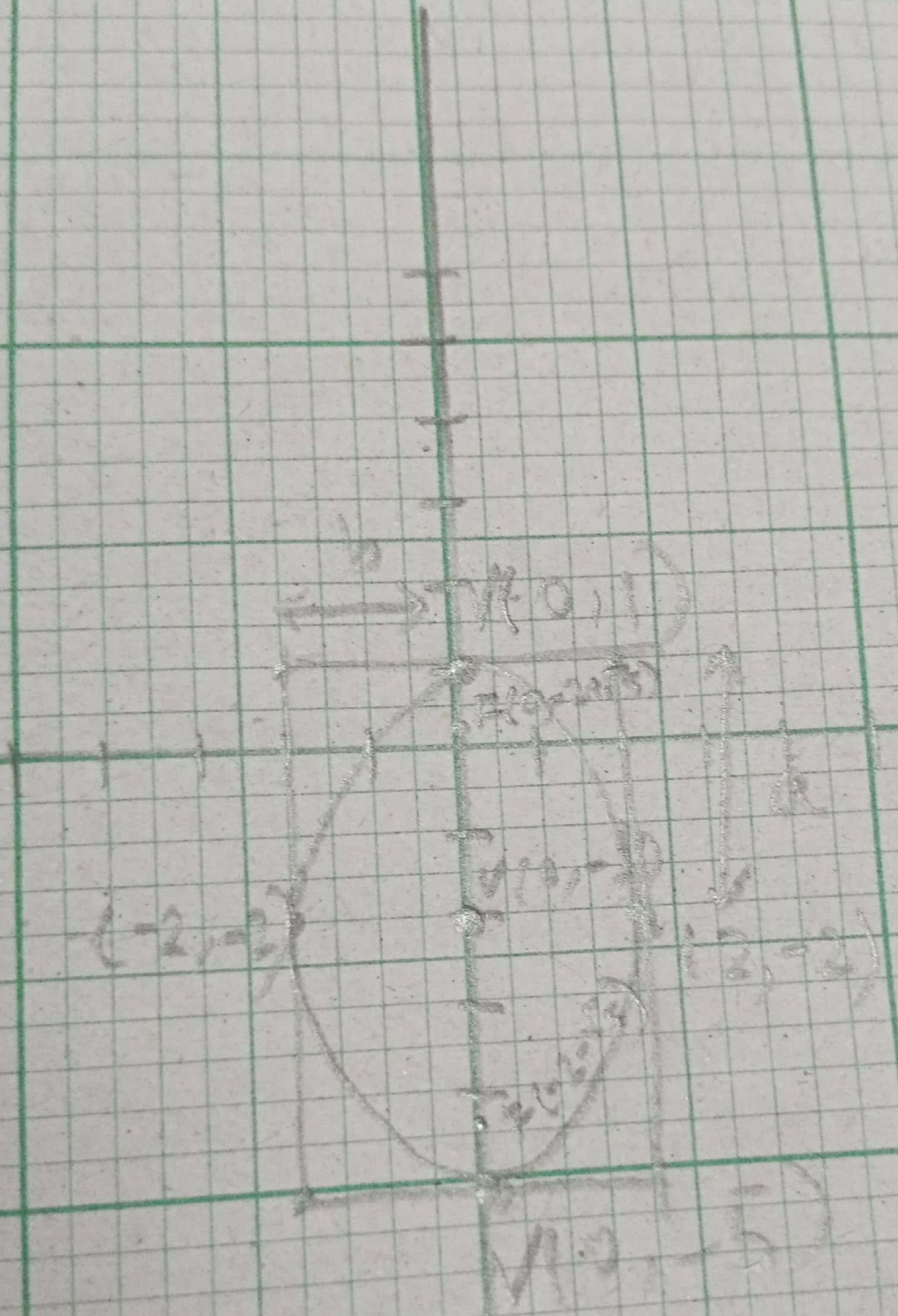
$c = \sqrt{a^2 - b^2}$

$c = \sqrt{5}$

Foci $\rightarrow (0, -2 \pm \sqrt{5})$

Vertices $\rightarrow (0, +1), (0, -5)$

ends of minor axis $\rightarrow (\pm 2, -2)$


V(0,1)
(-2,-2)
(2,-2)
V(0,-5)

Q11

a) $\frac{x^2}{16} - \frac{y^2}{9} = 1$

focal axis $\rightarrow$ x-axis

$a = 4 \quad b = 3$

center $(0, 0)$

$c = \sqrt{a^2 + b^2}$

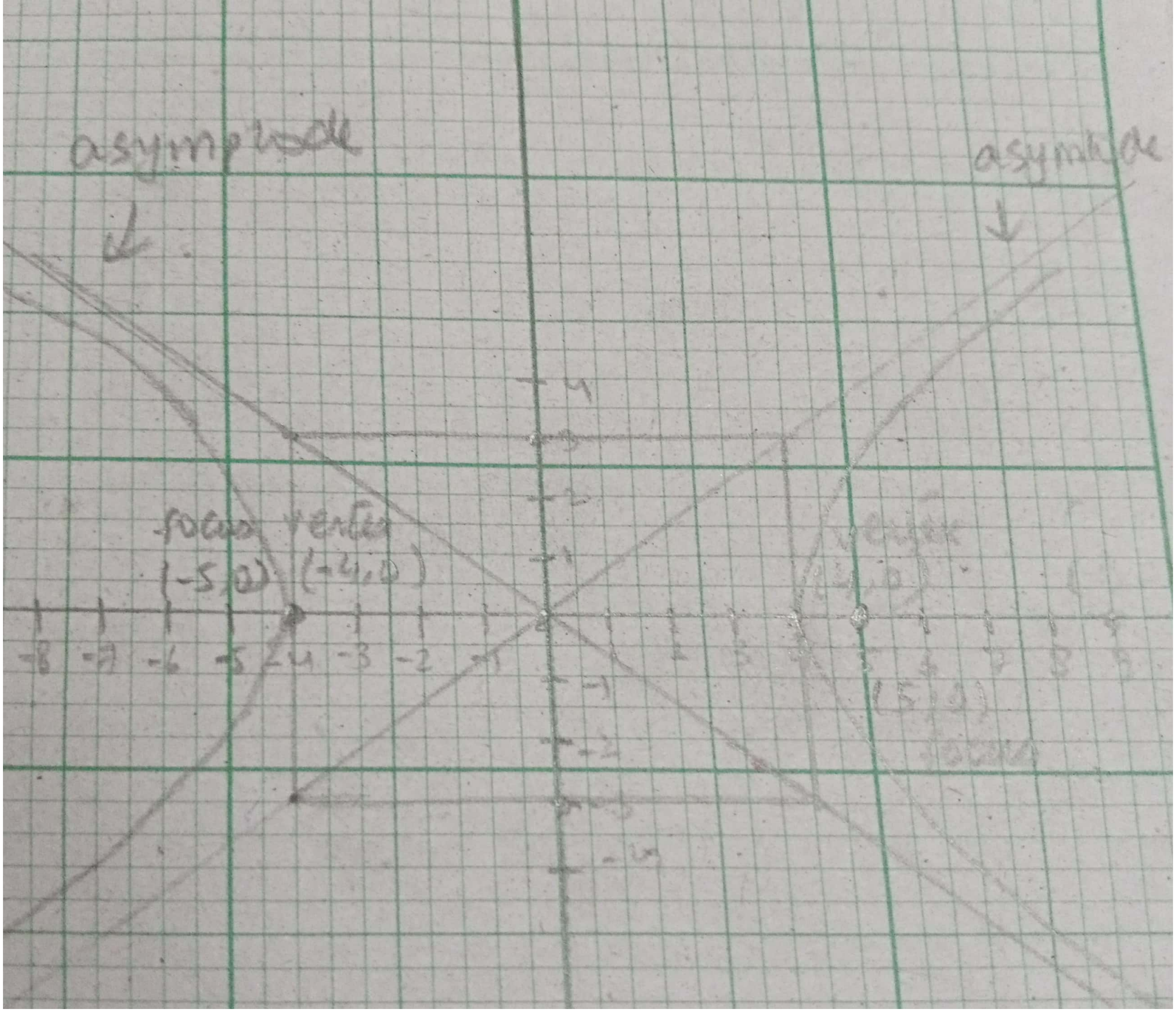
$c = \pm 5$

vertices $\rightarrow (-4,0), (4,0)$

foci $\rightarrow (-5,0), (5,0)$

Eq of asymptote

$y = \pm \frac{3}{4} x$


asymptote
asymptote
focus vertex
(-5,0) (-4,0)
vertex
(4,0)
(5,0)
focus

Q11

b) $9y^2 - x^2 = 36$

$\div$ by 36

$$\frac{y^2}{4} - \frac{x^2}{36} = 1$$

major axis $\rightarrow$ y-axis

$a = 2$, $b = 6$

vertices $\rightarrow$ $(0, \pm 2)$

$c = \sqrt{a^2 + b^2}$

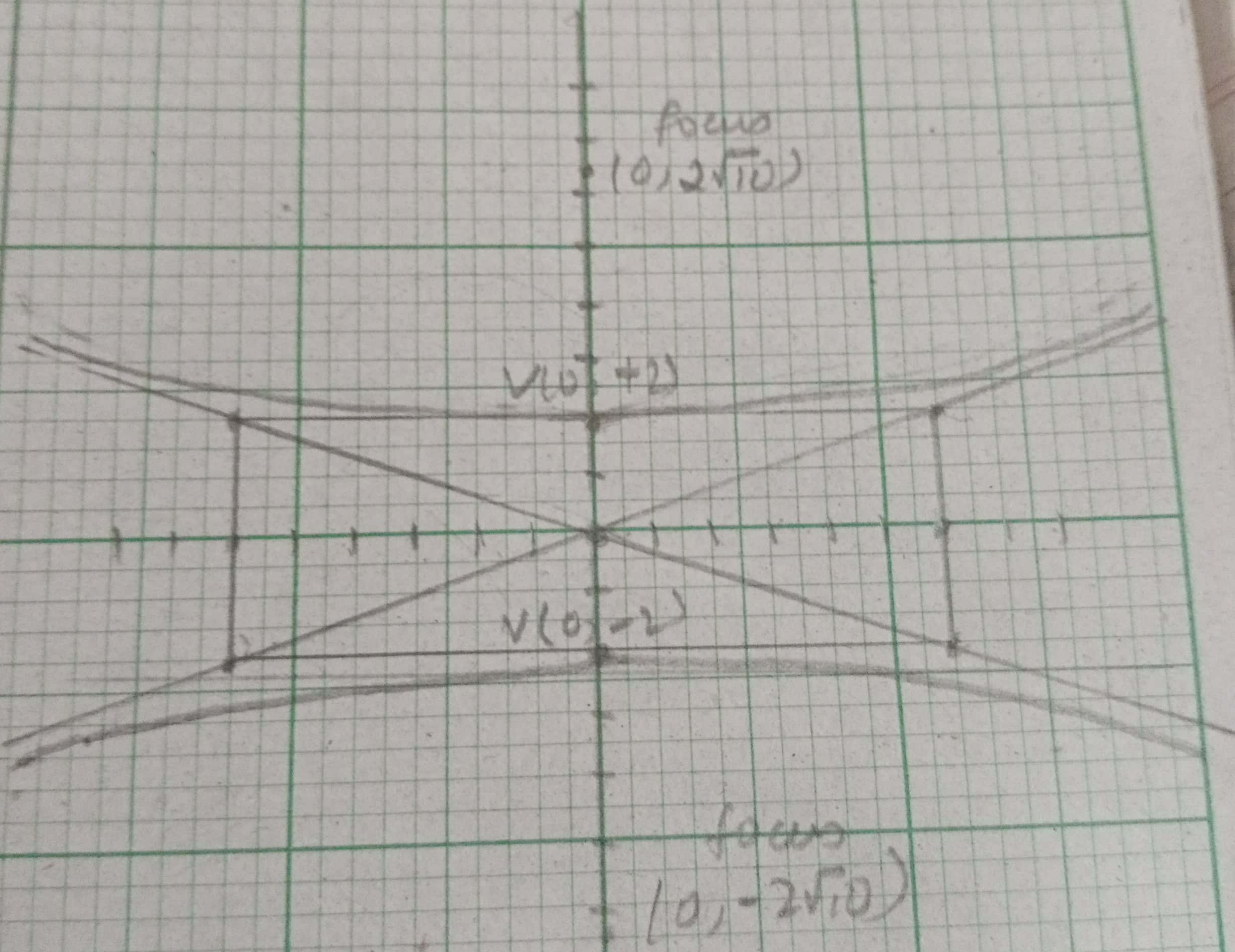
$c = 2\sqrt{10}$

Foci $\Rightarrow$ $(0, \pm 2\sqrt{10})$

asymptotes :

$$y = k \pm \frac{a}{b}(x - h)$$

$$\boxed{y = \pm \frac{1}{3} x}$$



focus
(0, 2√10)
V(0, +2)
V(0, -2)
focus
(0, -2√10)

Q13

a) $$\frac{(y+4)^2}{3} - \frac{(x-2)^2}{5} = 1$$

focal axis $\rightarrow$ y-axis

$a = \sqrt{3}$, $b = \sqrt{5}$

Center $(2, -4)$

$c = \sqrt{a^2 + b^2}$

$c = \pm\sqrt{8}$

vertices $(2, -4+\sqrt{3})$, $(2, -4-\sqrt{3})$

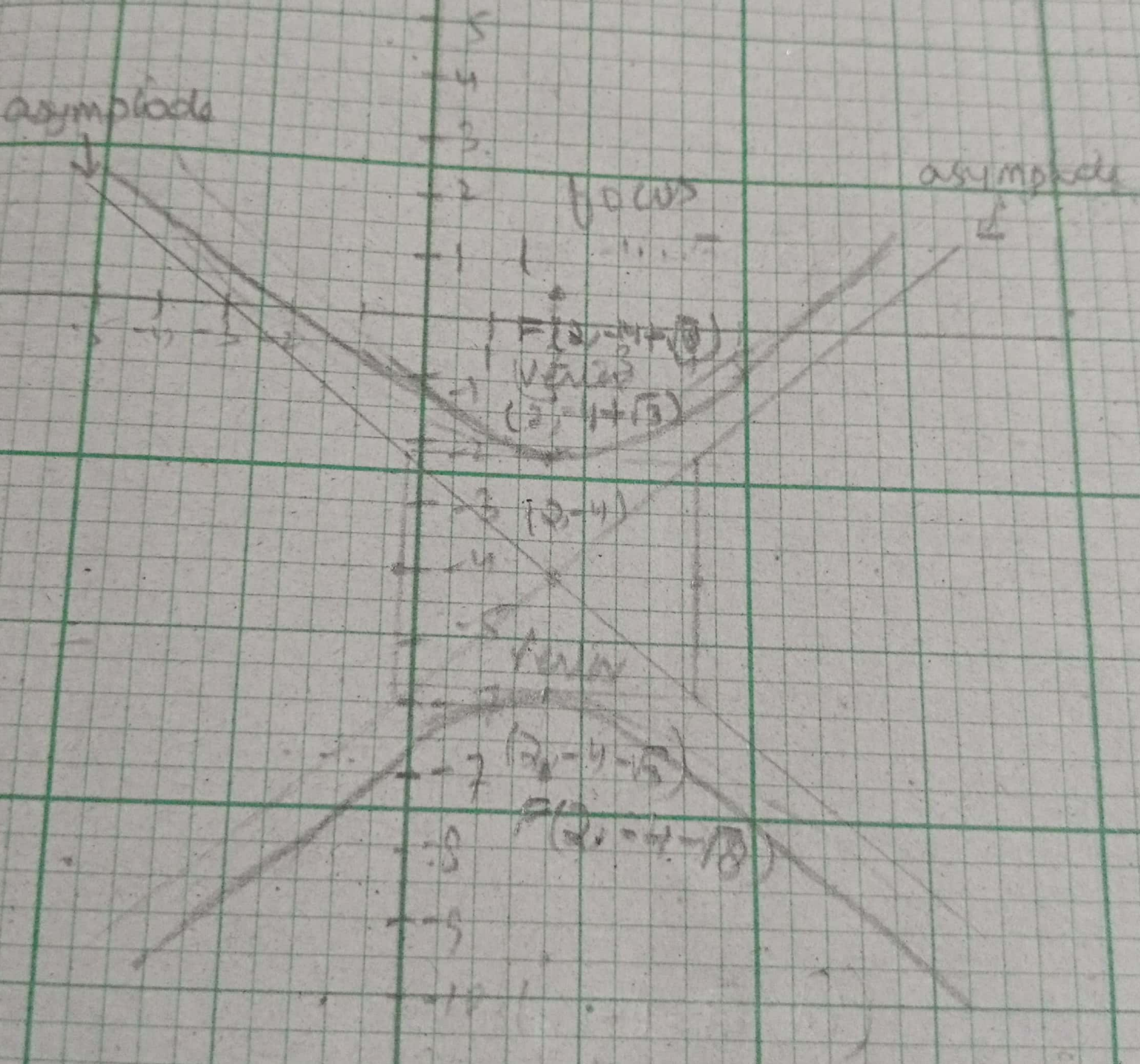
~~foci $\rightarrow$ $(2, -4+\sqrt{3}+\sqrt{8})$, $(2, -4-\sqrt{3}-\sqrt{8})$~~

foci $\rightarrow$ $(2, -4 \pm \sqrt{8})$

Eq of asymptotes

$$y = -4 \pm \frac{\sqrt{3}}{\sqrt{5}}(x-2)$$

$$\frac{(y+4)^2}{3} - \frac{(x-2)^2}{5} = 1$$



Q13

b) $16(x+1)^2 - 8(y-3)^2 = 16$

$\div$ by 16

$$\frac{(x+1)^2}{1} - \frac{(y-3)^2}{2} = 1$$

focal axis $\rightarrow$ x-axis

$a = 1$ $\quad$ $b = \sqrt{2}$

Center $(-1, 3)$

vertices $\rightarrow$ $(0,3)$ $(-2,3)$

$c = \sqrt{a^2 + b^2}$

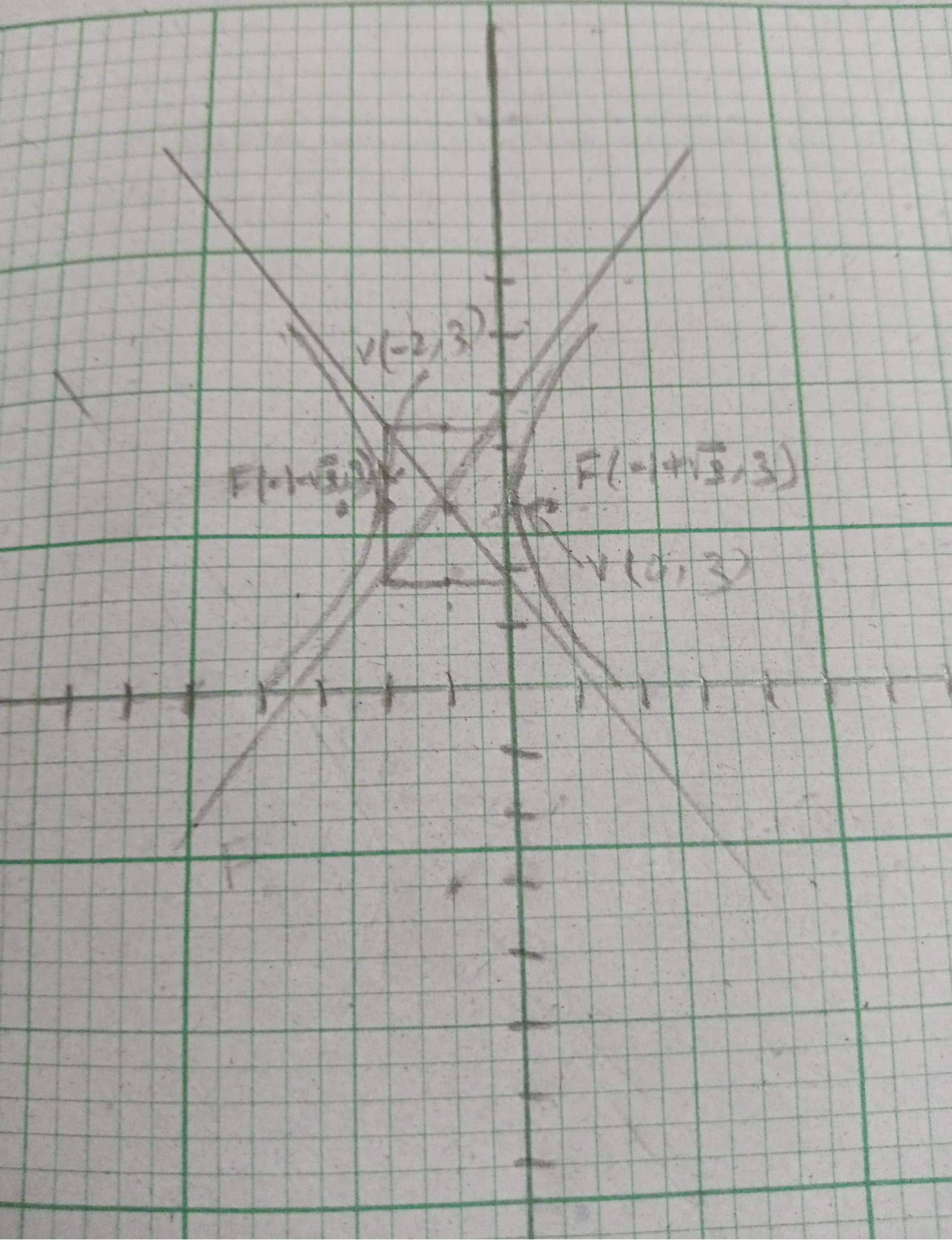
$c = \sqrt{3}$

foci $\rightarrow$ $(-1 \pm \sqrt{3}, 3)$

Eq of asymptotes:

$$y = k \pm \frac{b}{a}(x-h)$$

$$\boxed{y = 3 \pm \sqrt{2}(x+1)}$$



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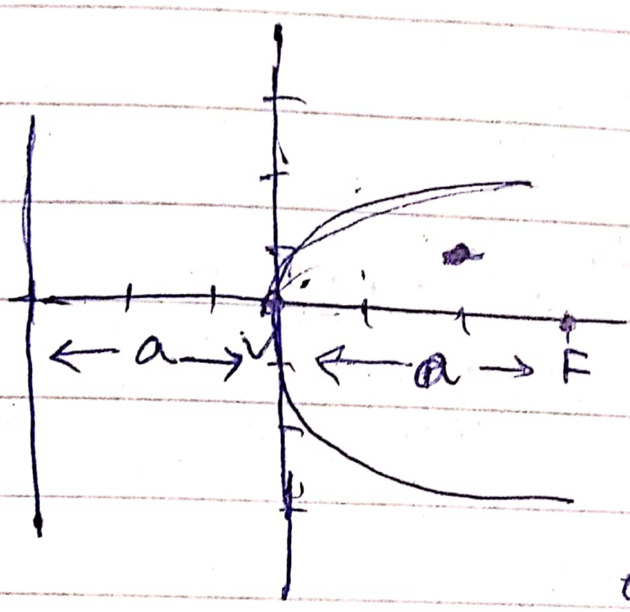
Q15

a) vertex $(0,0)$ focus $(3,0)$



$\cancel{a = \sqrt{\ }}$

$a = VF$

$a = \sqrt{(3-0)^2 + (0-0)^2}$

$\boxed{a = 3}$

$(x-h)^2 = 4a(y-k)$

$(x-0)^2 = 4a(y-0)$

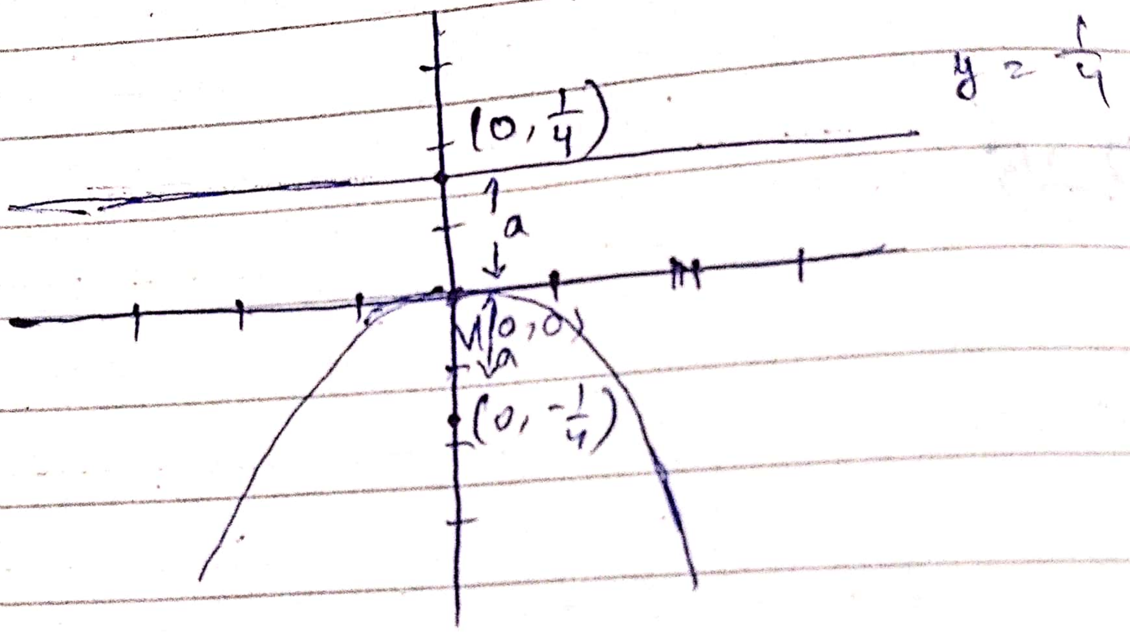
$x^2 = 4ay$

$x^2 = 4(3)y$

$x^2 = 12y.$

b) Vertex $(0,0)$ directrix $y = \frac{1}{4}$

First plot the given equations.



$a = \frac{1}{4}$

$V(h,k) = (0,0)$

$(x-h)^2 = -4a(y-k)$

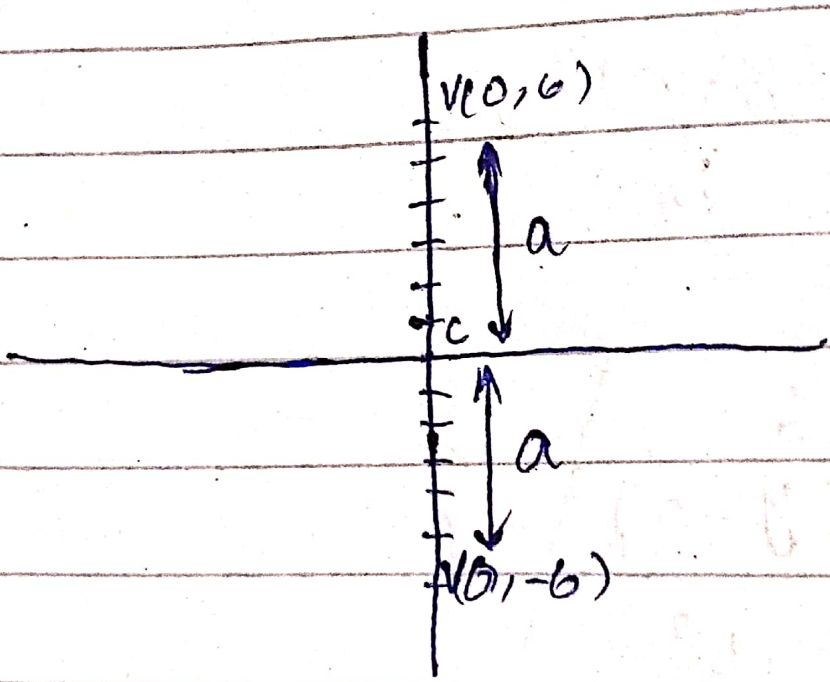
$x^2 = -4 \times \frac{1}{4} y$

$x^2 = -y$

Q21

a) End points of major axis : $(0, \pm 6)$
passes through $(-3, 2)$

Vertices → $(0, \pm 6)$



vertices is on vertical axis

$$\frac{(x-h)^2}{b^2} + \frac{(y-k)^2}{a^2} = 1 \longrightarrow (A)$$

$C(0,0)$

$a = CV = 6$

put in equation (A)

$$\frac{(x-0)^2}{b^2} + \frac{(y-0)^2}{(6)^2} = 1$$

$$\frac{x^2}{b^2} + \frac{y^2}{36} = 1 \longrightarrow (B)$$

$(-3,2)$ most satisfy this equation.

$$\frac{(-3)^2}{b^2} + \frac{(2)^2}{36} = 1$$

$$\frac{9}{b^2} = 1 - \frac{1}{9}$$

$$\frac{9}{b^2} = \frac{8}{9}$$

$$b^2 = \frac{9 \times 9}{8}$$

$$b^2 = \frac{81}{8}$$

put in equation (B)

$$\boxed{\frac{x^2}{81/8} + \frac{y^2}{36} = 1}$$

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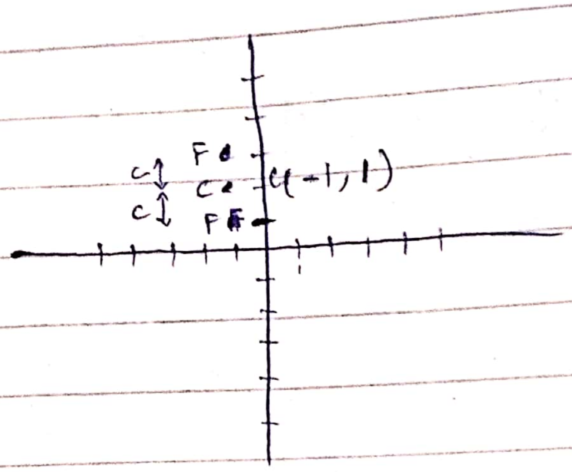
b) Foci $(-1,1)$ and $(-1,3)$
minor axis of length 4

plot the information



$\boxed{c = 1}$ foci is on vertical axis

$2b = 4$ so major axis y-axis

$\boxed{b = 2}$

$c^2 = a^2 - b^2$

$1 = a^2 - 4$

$a^2 = 5$

center $(-1,1) = C(h,k)$

$$\frac{(x-h)^2}{b^2} + \frac{(y-k)^2}{a^2} = 1$$

$$\frac{(x-(-1))^2}{(2)^2} + \frac{(y-1)^2}{5} = 1$$

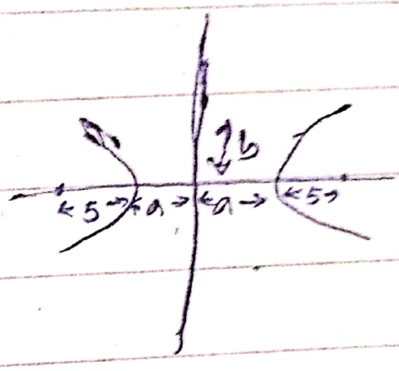
$$\boxed{\frac{(x+1)^2}{4} + \frac{(y-1)^2}{5} = 1}$$

Q25

a) asymptodes $y = \pm \frac{3}{4} x$ $\quad c = 5$

If focal axis is x-axis :-

$$y = \pm \frac{b}{a} x = \pm \frac{3}{4} x$$



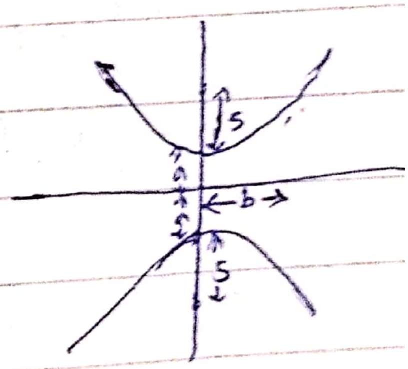
$a = 4 \qquad b = 3$

the equation is :

$$\boxed{\frac{x^2}{16} - \frac{y^2}{9} = 1}$$

If focal axis is y-axis

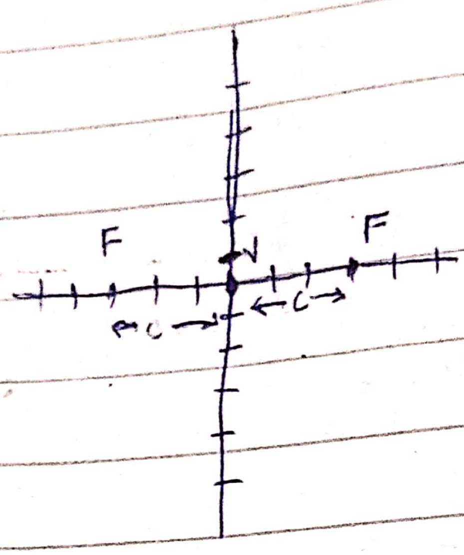
$$y = \pm \frac{a}{b} x = \pm \frac{3}{4} x$$



$a = 3 \qquad b = 4$

$$\boxed{\frac{y^2}{16} - \frac{x^2}{9} = 1}$$

b) Asymptotes $y = \pm 2x$
Foci $(\pm 3, 0)$



center $(0,0)$

$c = 3$

Compare $y = \pm 2x$ with general equation of asymptotes

$$y = k \pm \frac{b}{a}(x-h)$$

$$2 = \frac{b}{a}$$

$$b = 2a$$

$$c^2 = a^2 + b^2$$

$$9 = a^2 + 4a^2$$

$$9 = 5a^2$$

$$\boxed{a^2 = \frac{9}{5}}$$

put it back

$$c^2 = a^2 + b^2$$

$$9 = \frac{9}{5} + b^2$$

$$\boxed{b^2 = \frac{36}{5}}$$

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

$$\boxed{\frac{x^2}{9/5} - \frac{y^2}{36/5} = 1}$$

Q59

a) $x^2 - 5y^2 - 2x - 10y - 9 = 0$

$x^2 - 2x - 5y^2 - 10y = 9$

$x^2 - 2x - 5(y^2 + 2y) = 9$

For x: $\left(\frac{b}{2a}\right)^2 = \left(\frac{+2}{2(1)}\right)^2 = 1$

For y: $\left(\frac{2}{2(1)}\right)^2 = 1$

$x^2 - 2x + 1 - 5(y^2 + 2y + 1) = 9 + 1 - 5$

$(x-1)^2 - 5(y+1)^2 = 5$

$\frac{(x-1)^2}{5} - \frac{(y+1)^2}{1} = 1$

~~Ellipse~~ Hyperbola

b) ~~$x^2 - 3y^2 - 6y - 3 = 0$~~

$4x^2 + 8y^2 + 16x + 16y + 20 = 0$

$4(x^2 + 4x) + 8(y^2 + 2y) = -20$

$4(x^2 + 4x + 4) + 8(y^2 + 2y + 1) = -20 + 16 + 8$

$4(x+2)^2 + 8(y+1)^2 = 4$

$\div$ by

$\frac{(x+2)^2}{1} + \frac{(y+1)^2}{\frac{1}{2}} = 1$

Elipse

Scanned with CamScanner

$x^2+8x+2y+14=0$

$x^2+8x+16+2(y+7)=0+16$

$(x+4)^2 = -2y-14+16$

$(x+4)^2 = -2y+2$

$(x+4)^2 = -2(y-1)$

parabola

1) $5x^2+40x+2y+94=0$

$5(x^2+8x)+2y+94=0$

$5(x^2+8x+16) = 80-94-2y$

$(x+4)^2 = \frac{-14-2y}{5}$

$(x+4)^2 = \frac{-14}{5} - \frac{2y}{5}$

$(x+4)^2 = \frac{-2}{5}(y+7)$

parabola

Assignment #05

Q3

$xy = -9$

First we find angle of rotational axis

$\cot 2\theta = \frac{A-C}{B}$ $\because Ax^2 + Cy^2 + Bxy$

$\cot 2\theta = \frac{0-0}{1}$

$\cot 2\theta = 0$

$\frac{1}{\cot 2\theta} = \frac{1}{0}$

$\tan 2\theta = \infty$

$2\theta = \tan^{-1}(\infty)$

$2\theta = 90$

$\theta = 45^\circ$

Find x & y co-ordinate in rotational axis

$x = x'\cos\theta - y'\sin\theta$ $\rightarrow ①$

$y = x'\sin\theta + y'\cos\theta$ $\rightarrow ②$

For x :

$x = x'\cos(45) - y'\sin(45)$

$x = \frac{x'}{\sqrt{2}} - \frac{y'}{\sqrt{2}}$

For y :

$$y = x'\sin 45 + y'\cos 45$$

$$y = \frac{x'}{\sqrt{2}} + \frac{y'}{\sqrt{2}}$$

put in given equation

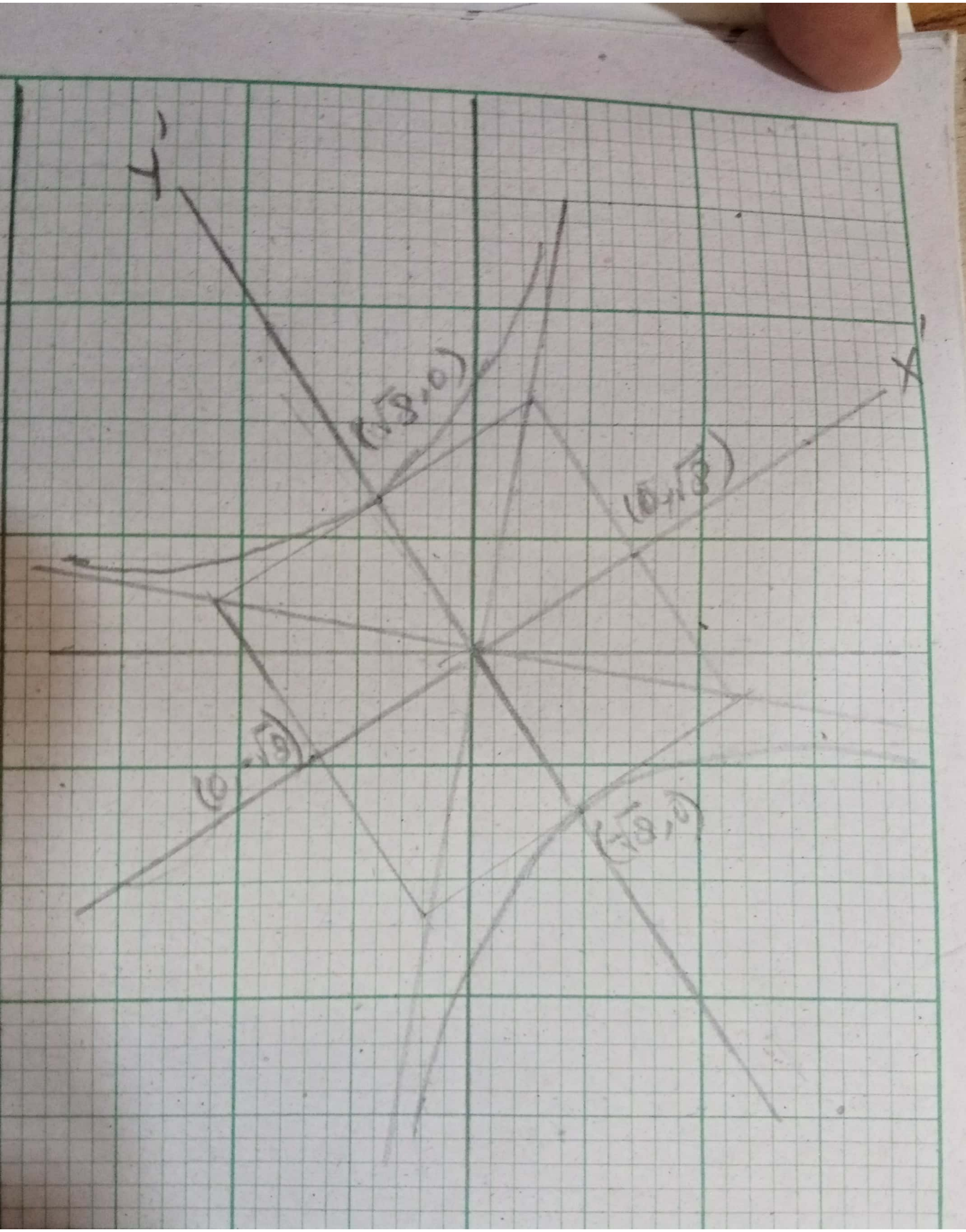
$$\left(\frac{x'}{\sqrt{2}} - \frac{y'}{\sqrt{2}}\right)\left(\frac{x'}{\sqrt{2}} + \frac{y'}{\sqrt{2}}\right) = -9$$

$$\frac{x'}{2} - \frac{y^2}{2} = -9$$

multiply whole equation by $-\frac{1}{9}$

$$\frac{y'^2}{18} - \frac{x'^2}{18} = 1$$

Type of Conic : Hyperbola


Y'
X'
(√8,0)
(0,√8)
(0,-√8)
(-√8,0)

Q7

$$x^2 + 2\sqrt{3}xy + 3y^2 + 2\sqrt{3}x - 2y = 0$$

$$\cancel{x^2 + 2\sqrt{3}x + 3\left(y^2 - \frac{2}{3}y\right) + 2\sqrt{3}xy = 0}$$

$$\cancel{x^2 + 2\sqrt{3}x + 3 + 3\left(y^2 - \frac{2}{3}y + \frac{1}{9}\right) + 2\sqrt{3}xy = 3 + \frac{1}{3}}$$

$$\cancel{(x + \sqrt{3})^2 + 3\left(y + \frac{1}{3}\right)^2 + 2\sqrt{3}xy = \frac{10}{3}}$$

Find angle

$$\cot 2\theta = \frac{A - C}{B}$$

$$= \frac{1 - 3}{2\sqrt{3}}$$

$$\cot 2\theta = -\frac{1}{\sqrt{3}} \Rightarrow \tan 2\theta = -\sqrt{3}$$

$$2\theta = \tan^{-1}(-\sqrt{3})$$

$$\theta = 60$$

$$\boxed{\begin{aligned} x &= x'\cos\theta - y'\sin\theta \\ y &= x'\sin\theta + y'\cos\theta \end{aligned}}$$

For x

$$x = x'\cos(60) - y'\sin(60)$$

$$x = \frac{x'}{2} - \frac{\sqrt{3}y'}{2}$$

For y :

$$y = x'\sin(60) + y'\cos 60$$

$$y = \frac{\sqrt{3}x}{2} + \frac{y'}{2}$$

put in given equation

$$x^2 + 2\sqrt{3}xy + 3y^2 + 2\sqrt{3}x - 2y = 0 \quad \left(\frac{\sqrt{3}}{2}x' + \frac{y'}{2}\right)$$

$$\left(\frac{x'}{2} - \frac{\sqrt{3}}{2}y'\right)^2 + 2\sqrt{3}\left(\frac{x'}{2} - \frac{\sqrt{3}}{2}y'\right) + 3\left(\frac{\sqrt{3}}{2}x' + \frac{y'}{2}\right)^2$$

$$+ 2\sqrt{3}\left(\frac{x'}{2} - \frac{\sqrt{3}}{2}y'\right) - 2\left(\frac{\sqrt{3}}{2}x' + \frac{y'}{2}\right) = 0$$

$$\frac{x'^2}{4} - 2\left(\frac{x'}{2}\right)\left(\frac{\sqrt{3}}{2}y\right) + \frac{3y'^2}{4} + 2\sqrt{3}\left(\frac{\sqrt{3}}{4}x'^2 + \frac{x'y'}{4} - \frac{3}{4}x'y'\right.$$

$$\left. - \frac{\sqrt{3}}{4}y'^2\right) + \sqrt{3}x' - 3y' - \sqrt{3}x' - y' + 3\left(\frac{3}{4}x'^2 + 2\left(\frac{\sqrt{3}}{2}\right)\left(\frac{1}{2}\right)x'y'\right.$$

$$\frac{x'^2}{4} - \cancel{\frac{\sqrt{3}}{2}x'y'} + \frac{3y'^2}{4} + \frac{3x'^2}{2} + \cancel{\frac{\sqrt{3}}{2}x'y'} - \quad \left. + \frac{y'^2}{4}\right) = 0$$

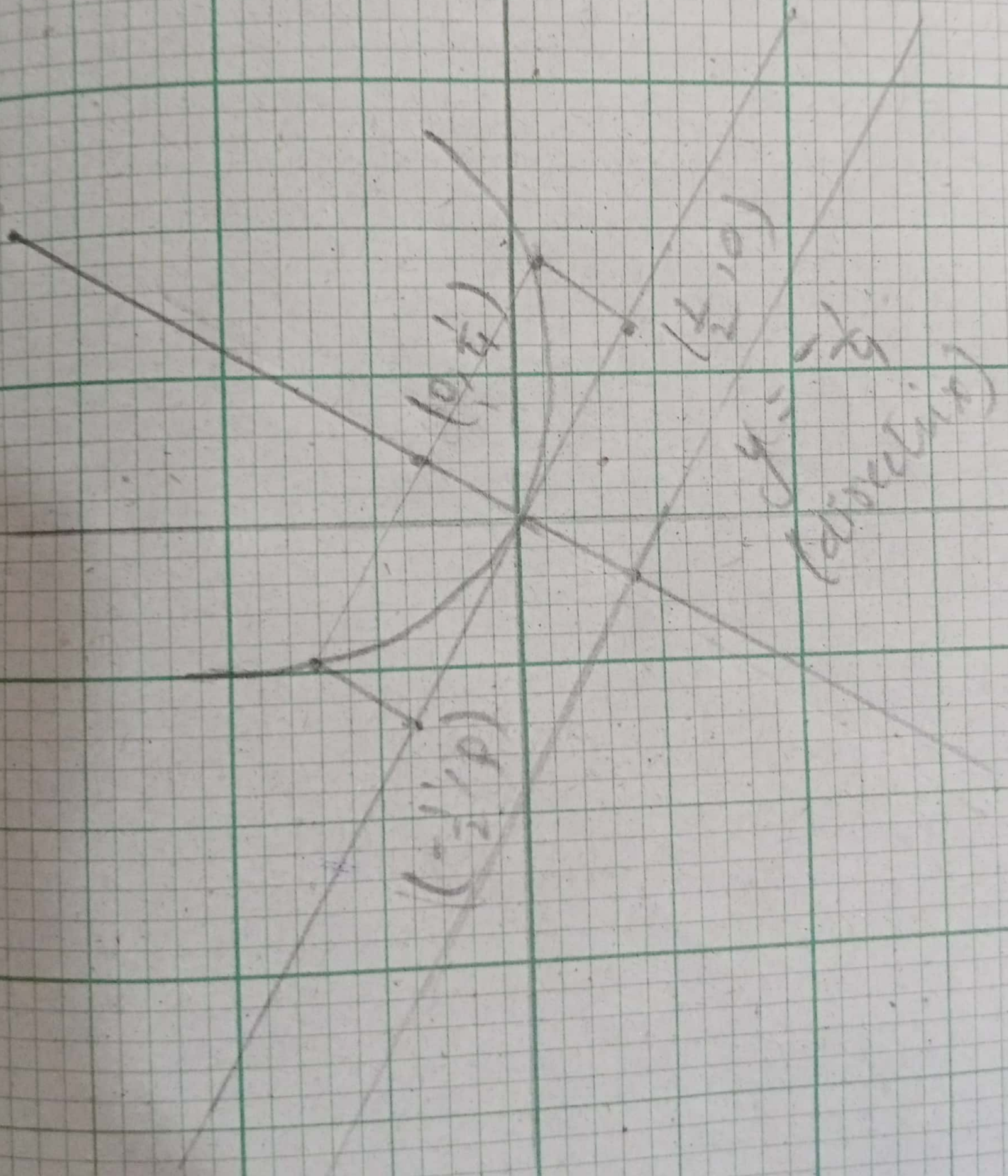
$$\cancel{\frac{3\sqrt{3}}{2}x'y'} - \frac{3}{2}y'^2 + \cancel{\sqrt{3}x'} - 3y' - \cancel{\sqrt{3}x'} - y' + \frac{9}{4}x'^2 \cancel{+ \frac{3\sqrt{3}}{2}x'y'}$$

$$+ \frac{3y'^2}{4} = 0$$

$$4x'^2 - 4y' = 0$$

$$\boxed{x'^2 = y'}$$

Type of conic : Parabola.



Q11

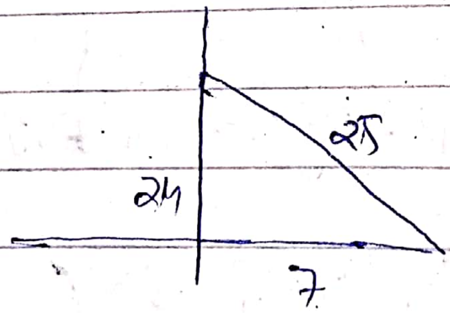
$$52x^2 - 72xy + 73y^2 + 40x + 30y - 75 = 0$$

$$x = x'\cos\theta - y'\sin\theta$$

$$y = x'\sin\theta + y'\cos\theta$$

$$\cot 2\theta = \frac{A-C}{B} = \frac{52-73}{-72}$$

$$\cot 2\theta = \frac{7}{24} = \frac{B}{P}$$



$$\cos 2\theta = \frac{7}{25}$$

$$\cos\theta = \sqrt{\frac{1+\cos 2\theta}{2}} = \sqrt{\frac{1+7/25}{2}}$$

$$\cos\theta = 4/5$$

$$\sin\theta = \sqrt{\frac{1-\cos 2\theta}{2}} = \sqrt{\frac{1-7/25}{2}}$$

$$\sin\theta = \frac{3}{5}$$

$$x = \frac{4x'}{5} - \frac{3y'}{5}$$

$$y = \frac{3}{5}x' + \frac{4}{5}y'$$

put in given equations

$$= 52\left(\frac{4x'}{5} - \frac{3y'}{5}\right)^2 - 72\left(\frac{4x'}{5} - \frac{3y'}{5}\right)\left(\frac{3}{5}x' + \frac{4}{5}y'\right) + 73\left(\frac{3}{5}x' + \frac{4}{5}y'\right)^2 + 40\left(\frac{4x'}{5} - \frac{3y'}{5}\right) + 30\left(\frac{3}{5}x' + \frac{4}{5}y'\right) - 75 = 0$$

$$\Rightarrow 52\left(\frac{16x'^2}{25} - \frac{24}{25}x'y' + \frac{9y'^2}{25}\right) - 72\left(\frac{12x'^2}{25} + \frac{16x'y'}{25} - \frac{9}{25}x'y' - \frac{12}{25}y'^2\right) + 73\left(\frac{9}{25}x'^2 + \frac{24x'y'}{25} + \frac{16}{25}y'^2\right)$$
$$+ \frac{40\times 4x'}{5} - \frac{40\times 3}{5}y' + \frac{30\times 3}{5}x' + \frac{4\times 30}{5}y' - 75 = 0$$

$$\Rightarrow \frac{832}{25}x'^2 - \frac{1248}{25}x'y' + \frac{468}{25}y'^2 - \frac{864x'^2}{25} - \frac{1152}{25}x'y' + \frac{648}{25}x'y' + \frac{864}{25}y'^2 + \frac{657}{25}x'^2 + \frac{1752}{25}x'y' + \frac{1168}{25}y'^2$$
$$+ \frac{40\times 4x'}{5} - \frac{40\times 3}{5}y' + \frac{30\times 3x'}{5} + \frac{4\times 30y'}{5} - 75 = 0$$

$$25x'^2 + 100y'^2 + 50x' = 75$$

$\div$ by 25

$$x'^2 + 4y'^2 + 2x' = 3$$

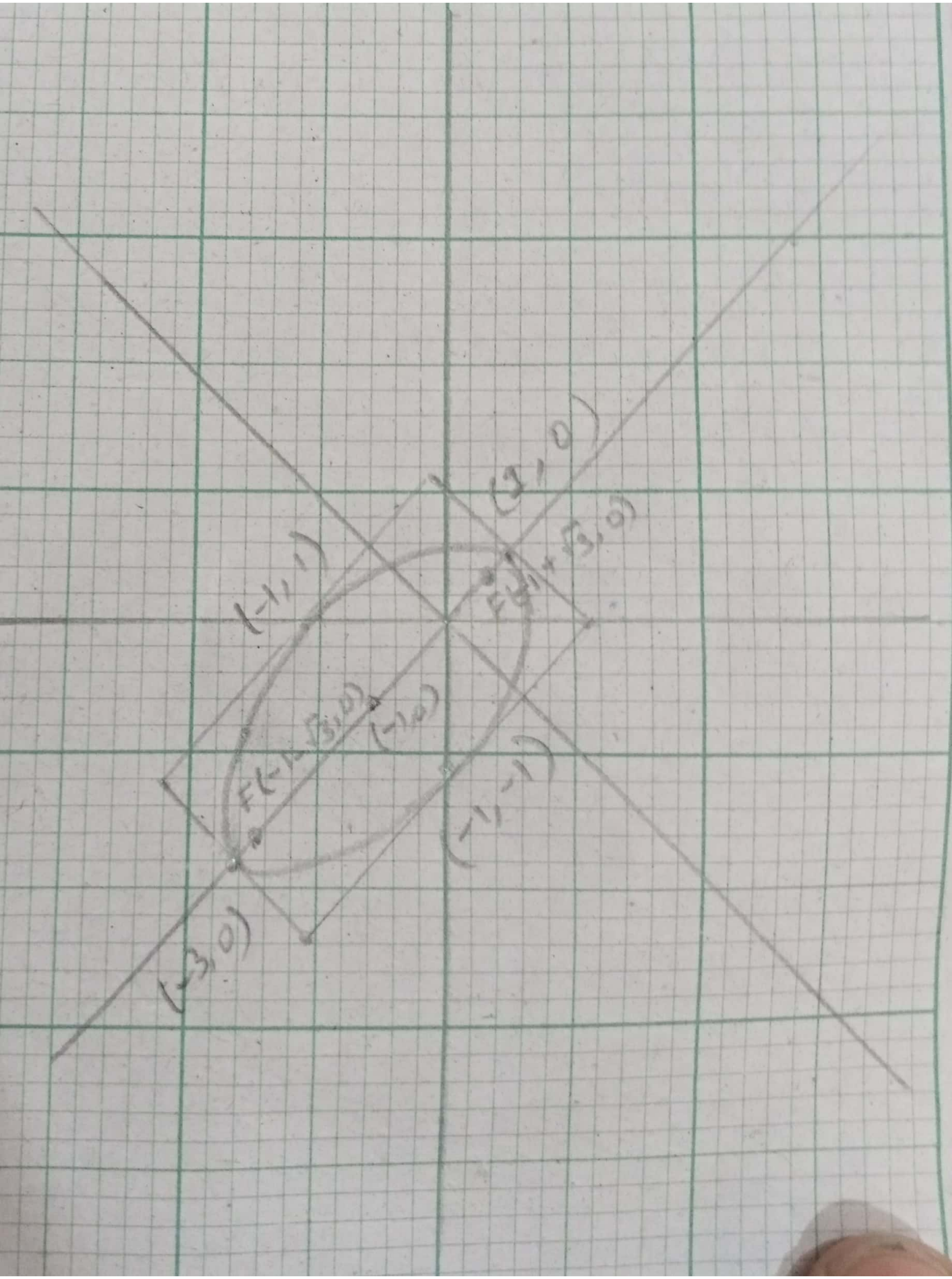
$$\frac{x'^2 + 2x' +}{3} + \frac{4y'^2}{3} = 1$$

$$\frac{(x'+1)^2}{3} + \frac{y'^2}{\frac{3}{4}} = 1$$

$$(x'+1)^2 + 4y'^2 = 4$$

$$\boxed{\frac{(x'+1)^2}{4} + y'^2 = 1}$$

Type of Conic : Elipse



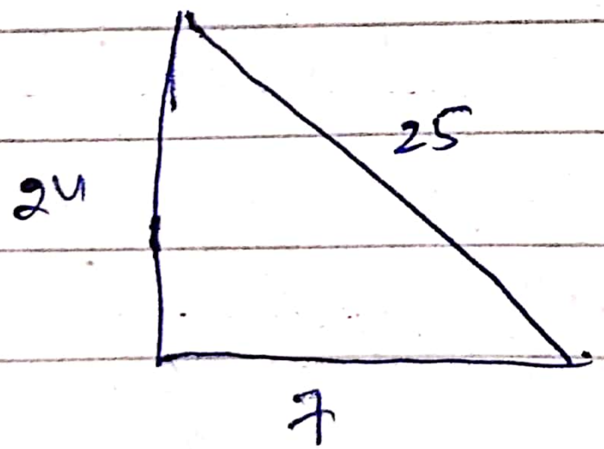
(1, 0)
(-1, 1)
F(-1+√3, 0)
F(-1-√3, 0)
(-1, 0)
(-1, -1)
(-3, 0)

Q21

$$9x^2 - 24xy + 16y^2 - 80x - 60y + 100 = 0$$

$$\cot 2\theta = \frac{A - C}{B} = \frac{9 - 16}{-24}$$

$$\cot 2\theta = \frac{7}{24} = \frac{B}{P}$$



$$\cos 2\theta = \frac{7}{25}$$

$$\cos\theta = \sqrt{\frac{1+\cos 2\theta}{2}} = \sqrt{\frac{1+(7/25)}{2}}$$

$$\cos\theta = \frac{4}{5}$$

$$\sin\theta = \sqrt{\frac{1-\cos 2\theta}{2}} = \sqrt{\frac{1-(7/25)}{2}}$$

$$\sin\theta = \frac{3}{5}$$

$$x = x'\cos\theta - y'\sin\theta$$

$$x = \frac{4x'}{5} - \frac{3}{5}y'$$

$$y = x'\sin\theta + y'\cos\theta$$

$$y = \frac{3}{5}x' + \frac{4}{5}y'$$

put in given equation

$$\Rightarrow 9\left(\frac{4}{5}x' - \frac{3}{5}y'\right)^2 - 24\left(\frac{4}{5}x' - \frac{3}{5}y'\right)\left(\frac{3}{5}x' + \frac{4}{5}y'\right) +$$

$$16\left(\frac{3}{5}x' + \frac{4}{5}y'\right)^2 - 80\left(\frac{4}{5}x' - \frac{3}{5}y'\right) - 60\left(\frac{3}{5}x'\right.$$

$$\left. + \frac{4}{5}y'\right) + 100 = 0$$

$$\Rightarrow 9\left(\frac{16}{25}x'^2 + \frac{24}{25}x'y' + \frac{9}{25}y'^2\right) - 24\left(\frac{12}{25}x'^2 + \frac{16}{25}x'y' - \frac{9}{25}x'y' - \frac{12}{25}y'^2\right) + 16\left(\frac{9}{25}x'^2 + \frac{24}{25}x'y' + \frac{16}{25}y'^2\right)$$

$$-64x' + \cancel{48}y' - 36x' - \cancel{48}y' + 100 = 0$$

$$\Rightarrow \frac{144}{25}x'^2 + \frac{216}{25}x'y' + \frac{81}{25}y'^2 - \frac{288}{25}x'^2 - \frac{168}{25}x'y'$$

$$+ \frac{288}{25}y'^2 + \frac{144}{25}x'^2 + \frac{384}{25}x'y' + \frac{256}{25}y'^2 - 100x' + 100$$

$$25y'^2 - 100x' = -100$$

$$y'^2 - 4x' = -4$$

$$y'^2 = 4(x'-1)$$

Type of conics : Parabola

Vertex $\rightarrow$ $(1, 0)$

Focus $\rightarrow$ $(2, 0)$

Directrix $\rightarrow$ $x' = 0$

For vertex in original co-ordinates

$$x = (1)(4/5) - (0)(3/5) = 4/5$$

$$y = (1)(3/5) - (0)(4/5) = 3/5$$

$$\boxed{\text{Vertex}(x, y) \Rightarrow (4/5, 3/5)}$$

For Focus :-

$x = (2)(4/5) - (0)(3/5) = 8/5$

$y = (2)(3/5) + (0)(4/5) = 6/5$

Focus $(x, y) \Rightarrow (8/5, 6/5)$

For directrix

$\therefore x' = x\cos\theta + y\sin\theta$

$x\cos\theta + y\sin\theta = 0$

$x\left(\frac{4}{5}\right) + y\left(\frac{3}{5}\right) = 0$

$\frac{1}{5}(4x + 3y) = 0$

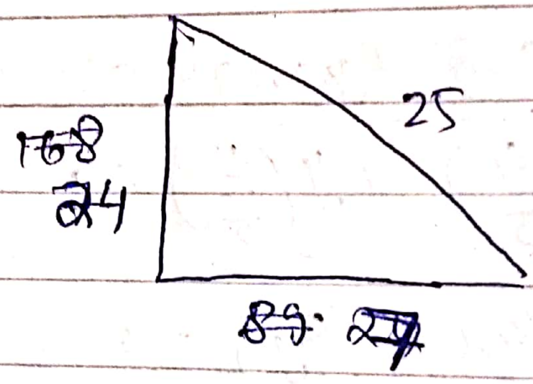
$\boxed{4x + 3y = 0}$

Q 23

$$288x^2 - 168xy + 337y^2 - 3600 = 0$$

$$\cot 2\theta = \frac{A-C}{B} = \frac{288-337}{-168}$$

$$\cot 2\theta = \frac{\cancel{89}\,7}{\cancel{168}\,24} = \frac{B}{P}$$



$$\cos 2\theta = \frac{7}{25}$$

$$\cos\theta = \sqrt{\frac{1+\cos 2\theta}{2}} = \sqrt{\frac{1+(7/25)}{2}}$$

$$\cos\theta = \frac{4}{5}$$

$$\sin\theta = \sqrt{\frac{1-\cos 2\theta}{2}} = \sqrt{\frac{1-(7/25)}{2}}$$

$$\sin\theta = \frac{3}{5}$$

$$x = \frac{4}{5}x' - \frac{3}{5}y'$$

$$y = \frac{3}{5}x' + \frac{4}{5}y'$$

put in given equation

$$\Rightarrow 288\left(\frac{4}{5}x' - \frac{3}{5}y'\right)^2 - 168\left(\frac{4}{5}x' - \frac{3}{5}y'\right)\left(\frac{3}{5}x' + \frac{4}{5}y'\right) + 337\left(\frac{3}{5}x' + \frac{4}{5}y'\right)^2 - 3600 = 0$$

$$\Rightarrow 288\left(\frac{16}{25}x'^2 - \frac{24}{25}x'y' + \frac{9}{25}y'^2\right) - 168\left(\frac{12}{25}x'^2 + \frac{16}{25}x'y' - \frac{9}{25}x'y' - \frac{12}{25}y'^2\right) + 337\left(\frac{9}{25}x'^2 + \frac{24}{25}x'y' + \frac{16}{25}y'^2\right) - 3600 = 0$$

$$\Rightarrow \frac{4608}{25}x'^2 - \frac{6912}{25}x'y' + \frac{2592}{25}y'^2 - \frac{2016}{25}x'^2 - \frac{2688}{25}x'y' + \frac{1512}{25}x'y' + \frac{2016}{25}y'^2 + \frac{3033}{25}x'^2 + \frac{8088}{25}x'y' + \frac{5392}{25}y'^2 - 3600 = 0$$

$$225x'^2 + 400y'^2 = 3600$$

$\div$ by 3600

$$\frac{x'^2}{16} + \frac{y'^2}{9} = 1$$

~~Vertices $\Rightarrow$ $(\pm 4, 0)$~~

~~Focus $\Rightarrow$ $(0, \pm\sqrt{7})$~~

~~Directrix $\Rightarrow$ (~~

In $x'y'$-coordinate :

Vertices $\Rightarrow (\pm 4, 0)$

Foci $\Rightarrow (\pm\sqrt{7}, 0)$

End points of minor axis $= (0, \pm 3)$

For vertices

$$x = (4)\frac{4}{5} - 10)\left(\frac{3}{5}\right) = \frac{16}{5}$$

$$y = 4\left(\frac{3}{5}\right) + 10)\left(\frac{4}{5}\right) = \frac{12}{5}$$

$$\left(\frac{16}{5}, \frac{12}{5}\right)$$

$$x = (-4)\left(\frac{4}{5}\right) - (0)\left(\frac{3}{5}\right) = \frac{-16}{5}$$

$$y = (-4)\left(\frac{3}{5}\right) + 10)\left(\frac{4}{5}\right) = \frac{-12}{5}$$

$$(-16/5, -12/5)$$

$$\boxed{\text{vertices} \Rightarrow (16/5, 12/5)\ (-16/5, -12/5)}$$

For Foci

when $(x', y') = (\sqrt{7}, 0)$

$$x = \sqrt{7}\left(\frac{4}{5}\right) - 10)\left(\frac{3}{5}\right) = \frac{4\sqrt{7}}{5}$$

$$y = \sqrt{7}\left(\frac{3}{5}\right) - (0)\left(\frac{4}{5}\right) = \frac{3\sqrt{7}}{5}$$

$$(x, y) \Rightarrow (4\sqrt{7}/5, 3\sqrt{7}/5)$$

when $(x', y') = (-\sqrt{7}, 0)$

$x = (-\sqrt{7})\left(\frac{4}{5}\right) - (0)\left(\frac{3}{5}\right) = \frac{-4\sqrt{7}}{5}$

$y = (-\sqrt{7})\left(\frac{3}{5}\right) - 0\left(\frac{4}{5}\right) = -\frac{3\sqrt{7}}{5}$

$(x, y) \Rightarrow \left(-\frac{4\sqrt{7}}{5}, -\frac{3\sqrt{7}}{5}\right)$

$\boxed{\text{Foci} \Rightarrow \left(\frac{4\sqrt{7}}{5}, \frac{3\sqrt{7}}{5}\right), \left(\frac{-4\sqrt{7}}{5}, \frac{-3\sqrt{7}}{5}\right)}$

For end points of minor axis :-

when $(x', y') = (0, 3)$

$x = 0(4/5) - 3(3/5) = -9/5$

$y = 0(3/5) + 3(4/5) = 12/5$

$(x, y) \Rightarrow (-9/5, 12/5)$

when $(x', y') = (0, -3)$

$x = (0)(4/5) - (-3)(3/5) = 9/5$

$y = (0)(3/5) + (-3)(4/5) = -12/5$

$(x, y) \Rightarrow (9/5, -12/5)$

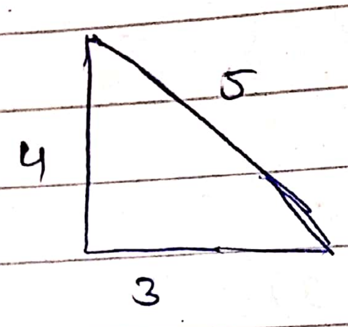
$\boxed{\text{End points of minor axis} \Rightarrow \left(-\frac{9}{5}, \frac{12}{5}\right), \left(\frac{9}{5}, -\frac{12}{5}\right)}$

Q29

$$32y^2 - 52xy\ 7x^2 + 72\sqrt{5}x - 144\sqrt{5}y + 900 = 0$$

$$\cot 2\theta = \frac{A-C}{B} = \frac{-7+32}{-52}$$

$$\cot 2\theta = \frac{3}{4} = \frac{B}{P}$$



$$\cos 2\theta = \frac{3}{5}$$

$$\cos\theta = \sqrt{\frac{1+\cos 2\theta}{2}} = \frac{2\sqrt{5}}{5} = \frac{2}{\sqrt{5}}$$

$$\sin\theta = \sqrt{\frac{1-\cos 2\theta}{2}} = \frac{\sqrt{5}}{5} = \frac{1}{\sqrt{5}}$$

$$X = \frac{2\sqrt{5}}{5}x' - \frac{\sqrt{5}}{5}y'$$

$$Y = \frac{\sqrt{5}}{5}x' + \frac{2\sqrt{5}}{5}y'$$

$$\Rightarrow 32\left(\frac{\sqrt{5}}{5}x' + \frac{2\sqrt{5}}{5}y'\right)^2 - 52\left(\frac{2\sqrt{5}}{5}x' - \frac{\sqrt{5}}{5}y'\right)\left(\frac{\sqrt{5}}{5}x' + \frac{2\sqrt{5}}{5}y'\right)$$

$$-7\left(\frac{2\sqrt{5}}{5}x' - \frac{\sqrt{5}}{5}y'\right)^2 + 72\sqrt{5}\left(\frac{2\sqrt{5}}{5}x' - \frac{\sqrt{5}}{5}y'\right) -$$

$$144\sqrt{5}\left(\frac{\sqrt{5}}{5}x' + \frac{2\sqrt{5}}{5}y'\right) + 900 = 0$$

$$\Rightarrow 32\left(\frac{\cancel{5}}{\cancel{2}5}x'^2 + \frac{4}{5}x'y' + \frac{4}{5}y'^2\right) - 52\left(\frac{2}{5}x'^2 + \frac{4}{5}x'y' -\right.$$

$$\left.\frac{1}{5}x'y' - \frac{2}{5}y'^2\right) \cancel{+72\sqrt{5}} - 7\left(\frac{4}{5}x'^2 - \frac{4}{5}x'y' + \frac{1}{5}y'^2\right)$$

$$+ \cancel{72\sqrt{5}} \; 144\cancel{x'} - 72y' - 4\cancel{4}x' - 288y' + 900 = 0$$

$$\Rightarrow \frac{32}{5}x'^2 + \frac{128}{5}x'y' + \frac{128}{5}y'^2 - \frac{104}{5}x'^2 - \frac{208}{5}x'y' +$$

$$\frac{52}{5}x'y' + \frac{104}{5}y'^2 - \frac{28}{5}x'^2 + \frac{28}{5}x'y' - \frac{7}{5}y'^2 - 360y'$$

$$= -900$$

$$\Rightarrow -20x'^2 + 45y'^2 - 360y' = -900$$

$$-20x'^2 + 45(y'^2 - 8y') = -900$$

$$-20x'^2 + 45(y'^2 - 8y' + 16) = -900 + 720$$

$$-20x'^2 + 45(y' - 4)^2 = -180$$

$$-4x'^2 + 9(y'^2 - 4)^2 = -36$$

$$\cancel{4x^2 = 9(y'}$$

$$\frac{x'^2}{9} - \frac{(y'-4)^2}{4} = 1$$

Vertices $\Rightarrow (\pm 3, 4)$

Foci $\Rightarrow (\pm\sqrt{13}, 4)$

Eq of asymptotes $y' = 4 \pm \frac{2}{3}x'$

<u>For Vertices</u> :-

when $(x', y') = (3, 4)$

$$x = 3\left(\frac{2\sqrt{5}}{5}\right) - 4\left(\frac{\sqrt{5}}{5}\right) = \frac{2\sqrt{5}}{5}$$

$$y = 3\left(\frac{\sqrt{5}}{5}\right) + 4\left(\frac{2\sqrt{5}}{5}\right) = \frac{11\sqrt{5}}{5}$$

$$\left(\frac{2\sqrt{5}}{5}, \frac{11\sqrt{5}}{5}\right)$$

when $(x, y) = (-3, 4)$

$$x = -3\left(\frac{2\sqrt{5}}{5}\right) - 4\left(\frac{\sqrt{5}}{5}\right) = -2\sqrt{5}$$

$$y = -3\left(\frac{\sqrt{5}}{5}\right) + 4\left(\frac{2\sqrt{5}}{5}\right) = \sqrt{5}$$

$$(-2\sqrt{5}, \sqrt{5})$$

vertices $\Rightarrow \left(\frac{2\sqrt{5}}{5}, \frac{11\sqrt{5}}{5}\right), (-2\sqrt{5}, \sqrt{5})$

For Foci:-

when $(x', y') = (\sqrt{13}, 4)$

$$x = \sqrt{13}\left(\frac{2\sqrt{5}}{5}\right) - 4\left(\frac{\sqrt{5}}{5}\right) = \frac{2\sqrt{65} - 4\sqrt{5}}{5}$$

$$y = \sqrt{13}\left(\frac{\sqrt{5}}{5}\right) + 4\left(\frac{2\sqrt{5}}{5}\right) = \frac{\sqrt{65} + 8\sqrt{5}}{5}$$

$$\left(\frac{2\sqrt{65} - 4\sqrt{5}}{5}, \frac{\sqrt{65} - 8\sqrt{5}}{5}\right)$$

when $(x', y') = (-\sqrt{13}, 4)$

$$x = (-\sqrt{13})\left(\frac{2\sqrt{5}}{5}\right) - 4\left(\frac{\sqrt{5}}{5}\right) = \frac{-2\sqrt{65} - 4\sqrt{5}}{5}$$

$$y = (-\sqrt{13})\left(\frac{\sqrt{5}}{5}\right) + 4\left(\frac{2\sqrt{5}}{5}\right) = \frac{-\sqrt{65} + 8\sqrt{5}}{5}$$

$$\text{Foci} \Rightarrow \left(\frac{2\sqrt{65} - 4\sqrt{5}}{5}, \frac{\sqrt{65} - 8\sqrt{5}}{5}\right), \left(\frac{-2\sqrt{65} - 4\sqrt{5}}{5}, \frac{-\sqrt{65} + 8\sqrt{5}}{5}\right)$$

For asymptotes :-

First case:-

$$y' = 4 + \frac{2}{3}x'$$

$$-x\sin\theta + y\cos\theta = 4 + \frac{2}{3}\left(x\cos\theta + y\sin\theta\right)$$

$$\cancel{-\frac{2x}{\sqrt{5}} + \frac{2}{\sqrt{5}}y}$$

$$-\frac{1}{\sqrt{5}}x + \frac{2}{\sqrt{5}}y = 4 + \frac{2}{3}\left(\frac{2}{\sqrt{5}}x + \frac{1}{\sqrt{5}}y\right)$$

$$-\frac{1}{\sqrt{5}}x + \frac{2}{\sqrt{5}}y = 4 + \frac{4}{3\sqrt{5}}x + \frac{2}{3\sqrt{5}}y$$

$$\frac{4}{3\sqrt{5}}x + \frac{1}{\sqrt{5}}x + \frac{2}{3\sqrt{5}}y - \frac{2}{\sqrt{5}}y = -4$$

$$\frac{7}{3\sqrt{5}}x - \frac{4}{3\sqrt{5}}y = -4$$

$$\frac{7}{3\sqrt{5}}x + 4 = \frac{4}{3\sqrt{5}}y$$

$$\boxed{\frac{7}{4}x + 3\sqrt{5} = y}$$

<u>case 2 :-</u>

$$y' = 4 - \frac{2}{3}x'$$

$$-x\sin\theta + y\cos\theta = 4 - \frac{2}{3}(x\cos\theta + y\sin\theta)$$

$$-\frac{1}{\sqrt{5}}x + \frac{2}{\sqrt{5}}y = 4 - \frac{2}{3}\left(\frac{2}{\sqrt{5}}x + \frac{1}{\sqrt{5}}y\right)$$

$$-\frac{1}{\sqrt{5}}x + \frac{2}{\sqrt{5}}y = 4 - \frac{4}{3\sqrt{5}}x - \frac{2}{3\sqrt{5}}y$$

$$\frac{2}{\sqrt{5}}y + \frac{2}{3\sqrt{5}}y = 4 - \frac{4}{3\sqrt{5}}x + \frac{1}{\sqrt{5}}x$$

$$\frac{8}{3\sqrt{5}}y = 4 - \frac{1}{3\sqrt{5}}x$$

$$\boxed{y = \frac{3\sqrt{5}}{2} - \frac{1}{8}x}$$

" Assignment # 06 "

Q23) $x^2 + y^2 + z^2 + 10x + 4y + 2z - 19 = 0$

$x^2 + 10x + y^2 + 4y + z^2 + 2z = 19$

$x^2 + 10x + 25 + y^2 + 4y + 4 + z^2 + 2z + 1 = 19 + 25 + 4 + 1$

$(x+5)^2 + (y+2)^2 + (z+1)^2 = 49$

It is the equation of sphere whose general equation is

$(x - x_0)^2 + (y - y_0)^2 + (z - z_0)^2 = r^2$

$(x_0, y_0, z_0) = (-5, -2, -1)$ & $r = 7$

Q25) $2x^2 + 2y^2 + 2z^2 - 2x - 3y + 5z - 2 = 0$

$\div$ by 2

$x^2 + y^2 + z^2 - x - \frac{3}{2}y + \frac{5}{2}z - 1 = 0$

$x^2 - x + y^2 - \frac{3}{2}y + z^2 + \frac{5}{2}z = 1$

$x^2 - x + \frac{1}{4} + y^2 - \frac{3}{2}y + \frac{9}{16} + z^2 + \frac{5}{2}z + \frac{25}{16} = \frac{27}{8}$

$\left(x - \frac{1}{2}\right)^2 + \left(y - \frac{3}{4}\right)^2 + \left(z + \frac{5}{4}\right)^2 = +\frac{27}{8}$

It is the equation of sphere whose general equation is $(x - x_0)^2 + (y - y_0)^2 + (z - z_0)^2 = r^2$

$(x_0, y_0, z_0) = \left(\frac{1}{2}, \frac{3}{4}, -\frac{5}{4}\right)$

$r = \frac{3\sqrt{6}}{4}$

Q27) $x^2 + y^2 + z^2 - 3x + 4y - 8z + 25 = 0$

$x^2 \cancel{+ y^2} - 3x + y^2 + 4y + z^2 - 8z + 25 = 0$

$x^2 - 3x + \frac{9}{4} + y^2 + 4y + 4 + z^2 - 8z + 16 = -25 + \frac{89}{4}$

$\left(x - \frac{3}{2}\right)^2 + \left(y + 2\right)^2 + \left(z - 4\right)^2 = -\frac{11}{4}$

no graph

~~This is the equation of sphere whose general equation is~~

$\cancel{(x - x_0)^2 + (y - y_0)^2 + (z - z_0)^2 = r^2}$

$\cancel{(x_0, y_0, z_0) = \left(\frac{3}{2}, -2, 4\right)}$ $\cancel{r =}$

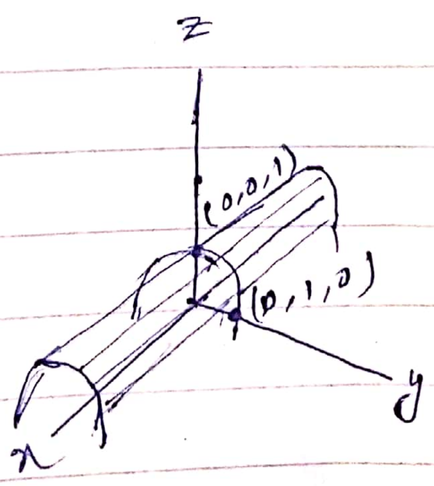
Q37) $z = 1 - y^2$

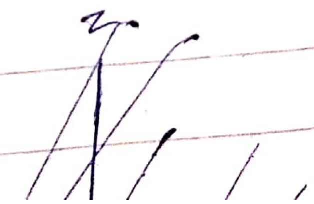
put $y = 0$

$z = 1$

put $z = 0$

$y = 1$





41) $4x^2 + 9z^2 = 36$

$$\cancel{(2x)^2 + (3z)^2 =}$$

put $x = 0$

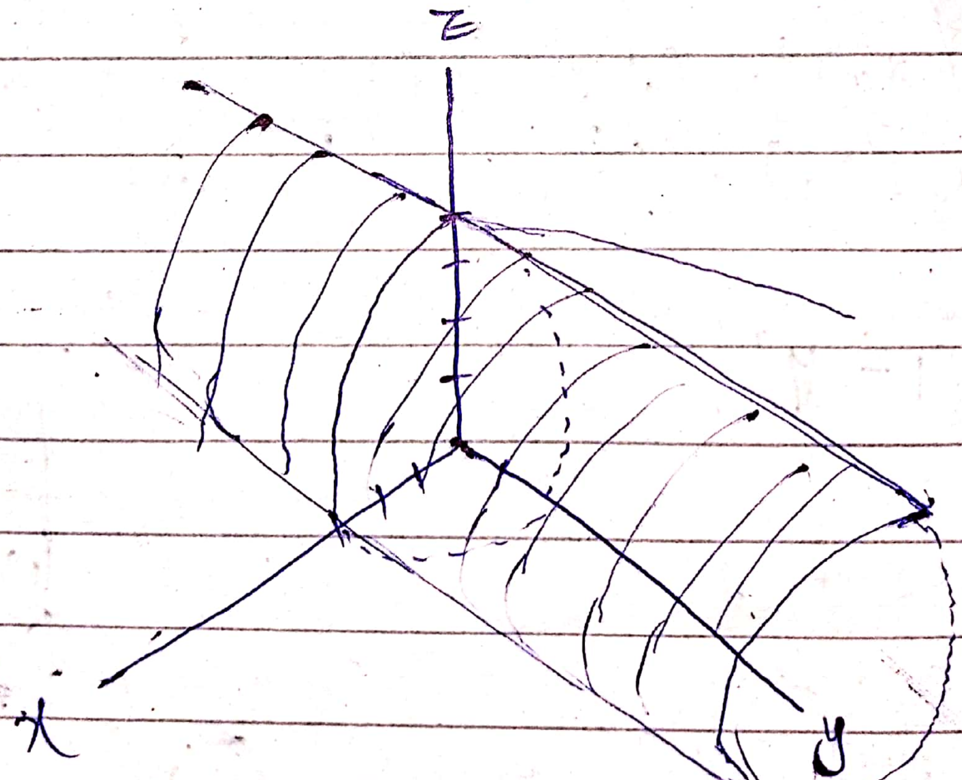
$$z^2 = 4$$

$$\boxed{z = \pm 2}$$

put $z = 0$

$$x^2 = 9$$

$$x = \pm 3$$



Assignment 07

Q7

a) $P_1(3,5)$ $P_2(2,8)$

$P_1(x_1, y_1) = P_1(3,5)$

$P_2(x_2, y_2) = P_2(2,8)$

$\overrightarrow{P_1P_2} = \langle x_2 - x_1 , y_2 - y_1 \rangle$

$= \langle 2-3 , 8-5 \rangle$

$\overrightarrow{P_1P_2} = \langle -1 , 3 \rangle$

b) $P_1(7,-2)$ $P_2(0,0)$

$P_1(x_1, y_1) = P_1(7,-2)$

$P_2(x_2, y_2) = P_2(0,0)$

$\overrightarrow{P_1P_2} = \langle x_2 - x_1 , y_2 - y_1 \rangle$

$\overrightarrow{P_1P_2} = \langle -7 , 2 \rangle$

c) $P_1(5,-2,1)$, $P_2(2,4,2)$

$P_1(x_1, y_1, z_1) = P_1(5,-2,1)$

$P_2(x_2, y_2, z_2) = P_2(2,4,2)$

$\overrightarrow{P_1P_2} = \langle x_2 - x_1 , y_2 - y_1 , z_2 - z_1 \rangle$

$\overrightarrow{P_1P_2} = \langle -3 , 6 , 1 \rangle$

~~A8~~ Q9

a) Find the terminal point of $V = 3i - 2j$ if the initial point is $(1, -2)$

$P_1(x_1, y_1) = P_1(1, -2)$

$P_2(x_2, y_2) = ?$

$\overrightarrow{P_1P_2} = \langle x_2 - x_1, y_2 - y_1 \rangle$

$\langle 3, -2 \rangle = \langle x_2 - 1, y_2 + 2 \rangle$

By compairing

$x_2 - 1 = 3$ $\quad$ $y_2 + 2 = -2$

$x_2 = 4$ $\quad$ $y_2 = -4$

$P_2(x_2, y_2) = \langle 4, -4 \rangle$

b) Find the initial point of $v = (-3, 1, 2)$ if the terminal point is $(5, 0, -1)$

$P_1(x_1, y_1, z_1) = \cancel{(5, 0, -1)}\,)\ ?$

$P_2(x_2, y_2, z_2) = P_2(5, 0, -1)$

$\overrightarrow{P_1P_2} = V = \langle -3, 1, 2 \rangle$

$\overrightarrow{P_1P_2} = \langle x_2 - x_1, y_2 - y_1, z_2 - z_1 \rangle$

$\langle -3, 1, 2 \rangle = \langle 5 - x_1, -y_1, -1 - z_1 \rangle$

By compairing

$-3 = 5 - x_1$ $\quad 1 = -y_1$ $\quad 2 = -1 - z_1$

$x_1 = 8$ $\quad y_1 = -1$ $\quad z_1 = -3$

Initial point $P_1(8, -1, -3)$

Q15

$U = i - 3j + 2k$

$V = i + j$

$w = 2i + 2j - 4k$

a) $\|U + V\|$

$U + V = 2i - 2j + 2k$

$\|P\| = \sqrt{x^2 + y^2 + z^2}$

$\|U + V\| = \sqrt{(2)^2 + (-2)^2 + (2)^2}$

$\|U + V\| = 2\sqrt{3}$

b) $\|U\| + \|V\|$

$\|U\| = \sqrt{1^2 + (-3)^2 + (2)^2} = \sqrt{14}$

$\|V\| = \sqrt{1^2 + 1^2 + 0^2} = \sqrt{2}$

$\|U\| + \|V\| = \sqrt{14} + \sqrt{2}$

c) $\|-2u\| + 2\|v\|$

$-2u = -2i + 6j - 4k$

$\|-2u\| = \sqrt{(-2)^2 + (6)^2 + (-4)^2}$

$\|-2u\| = 8$

$\|v\| = \sqrt{1^2 + 1^2 + 0^2}$

$\|v\| = \sqrt{2}$

$\|-2u\| + 2\|v\| = 8 + 2\sqrt{2}$

$2\|u\| + 2\|v\|$

$\|u\| = \sqrt{1^2 + (-3)^2 + 2^2} = \sqrt{14}$

$\|v\| = \sqrt{1^2 + 1^2 + 0^2} = \sqrt{2}$

$2\|u\| + 2\|v\| = 2\sqrt{14} + 2\sqrt{2}$

d) $\|3u - 5v + w\|$

$3u = 3i - 9j + 6k$

$5v = 5i + 5j$

$3u - 5v + w = -12j + 2k$

$\|3u - 5v + w\| = \sqrt{(-12)^2 + (2)^2}$

$$\boxed{\|3u - 5v + w\| = 2\sqrt{37}}$$

f) $\left\| \frac{1}{\|w\|} w \right\|$

$\|w\| = \sqrt{2^2 + 2^2 + (-4)^2}$

$\|w\| = 2\sqrt{6}$

$\frac{1}{\|w\|} w = \frac{1}{\sqrt{6}} i + \frac{1}{\sqrt{6}} j - \frac{2}{\sqrt{6}} k$

$\left\| \frac{1}{\|w\|} w \right\| = \sqrt{\left(\frac{1}{\sqrt{6}}\right)^2 + \left(\frac{1}{\sqrt{6}}\right)^2 + \left(\frac{-2}{\sqrt{6}}\right)^2}$

$$\boxed{\left\| \frac{1}{\|w\|} w \right\| = 1}$$

e) $\frac{1}{\|w\|} w$

$\|w\| = \sqrt{2^2 + 2^2 + 4^2}$

$\|w\| = 2\sqrt{6}$

$$\boxed{\frac{1}{\|w\|} w = \frac{1}{\sqrt{6}} i + \frac{1}{\sqrt{6}} j - \frac{2}{\sqrt{6}} k}$$

Q21

a) same direction as $-i+4j$

$$\vec{U} = -i + 4j$$

$$\|U\| = \sqrt{(-1)^2 + (4)^2} = \sqrt{17}$$

$$\hat{U} = \frac{-i + 4j}{\|U\|}$$

$$\boxed{\hat{U} = \frac{-1}{\sqrt{17}} i + \frac{4}{\sqrt{17}} j}$$

b) Opposite directed to $6i - 4j + 2k$

$$\vec{U} = 6i - 4j + 2\hat{k}$$

$$\|U\| = \sqrt{(6)^2 + (-4)^2 + (2)^2} = \cancel{8}\, 2\sqrt{14}$$

$$\hat{U} = \frac{\vec{U}}{\|U\|} = \frac{6i - 4\hat{j} + 2\hat{k}}{2\sqrt{14}}$$

$$\hat{U} = \frac{3}{\sqrt{14}} i - \frac{\cancel{2}}{\cancel{2}\sqrt{14}} j + \frac{1}{\sqrt{14}} \hat{k}$$

$$\boxed{-\hat{V} = \frac{-3}{\sqrt{14}} \hat{i} + \frac{2}{\sqrt{14}} j - \frac{1}{\sqrt{14}} \hat{k}}$$

c) same direction as the vector from the point $A(-1,0,2)$ to the point $B(3,1,1)$

let $\vec{U}$ be the vector

$\vec{U} = \vec{AB}$

$= \langle 3+1, 1-0, 1-2 \rangle$

$\vec{U} = 4i + j - \hat{k}$

$\|U\| = \sqrt{4^2 + 1^2 + (-1)^2} = 3\sqrt{2}$

$\hat{U} = \dfrac{\vec{U}}{\|U\|} = \dfrac{4i + j - \hat{k}}{3\sqrt{2}}$

$\hat{U} = \dfrac{4}{3\sqrt{2}}\hat{i} + \dfrac{1}{3\sqrt{2}}\hat{j} - \dfrac{1}{3\sqrt{2}}\hat{k}$

Q23

a) oppositely directed to $v = (3, -4)$ and half the length of v

let $\vec{x}$ be the vector

$\vec{x} = \dfrac{-v}{2}$

$$\boxed{\vec{x} = \dfrac{-3i}{2} + 2\hat{j}}$$

b) Length $\sqrt{17}$ and same direction as $v = (7, 0, -6)$

Let $\vec{x}$ be the vector

$$\vec{x} = \sqrt{17}\,\hat{v}$$

$$\vec{x} = \sqrt{17}\,\frac{v}{\|v\|} \rightarrow (1)$$

$$\|v\| = \sqrt{7^2 + 0^2 + (-6)^2} = \sqrt{85}$$

$$\vec{x} = \frac{\sqrt{17}}{\sqrt{85}}(7i - 6k)$$

$$= \frac{1}{\sqrt{5}}(7i - 6k)$$

$$\boxed{\vec{x} = \frac{7}{\sqrt{5}}i - \frac{6}{\sqrt{5}}k}$$

Q31

$$2u - v + x = 7x + w$$

$$2(1,3) - (2,1) + x = 7x + (4,-1)$$

$$(2,6) - (2,1) - (4,-1) = 6x$$

$$6x = (-4, 6)$$

$$\boxed{x = \left(-\frac{2}{3}, 1\right)}$$

$$\boxed{x = -\frac{2}{3}i + j}$$

Q57

a)

$$w = c_1 v_1 + c_2 v_2$$
$$4j = c_1(2i - j) + c_2(4i + 2j)$$
$$4j = (2c_1 + 4c_2)\,i + (-c_1 + 2c_2)\,j$$

$$2c_1 + 4c_2 = 0 \qquad -c_1 + 2c_2 = 4$$
$$c_1 = -2c_2 \qquad 2c_2 + 2c_2 = 4$$
$$c_1 = -2(1) \qquad \boxed{c_2 = 1}$$
$$\boxed{c_1 = -2}$$

b)

$$w = c_1 v_1 + c_2 v_2$$
$$\cancel{2i + j - k}\ (3,5) = c_1(1,-3) + c_2(-2,6)$$
$$(3,5) = (c_1 - 2c_2,\ -3c_1 + 6c_2)$$

$$c_1 - 2c_2 = 3 \qquad -3c_1 + 6c_2 = 5$$
$$c_1 - 2c_2 = -\frac{5}{3}$$

The scalar equation has same LHS but different RHS, so the vector "w" cannot be as linear combination of vectors v_1 & v_2

Assignment # 08

Date

Q3

a) $U = 7i + 3j + 5k$, $V = -8i + 4j + 2k$

$$\cos\theta = \frac{U \cdot V}{\|U\| \|V\|}$$

$\|U\| = \sqrt{7^2 + 3^2 + 5^2} = \sqrt{83}$

$\|V\| = \sqrt{8^2 + 4^2 + 2^2} = 2\sqrt{21}$

$U \cdot V = (7 \times -8) + (3 \times 4) + (5 \times 2) = -34$

$$\cos\theta = \frac{-34}{(\sqrt{83})(2\sqrt{21})}$$

$\theta = 114.02^\circ$

Angle is obtuse.

b) $U = 6i + j + 3k$, $V = 4i - 6k$

$\|U\| = \sqrt{6^2 + 1^2 + 3^2} = \sqrt{46}$

$\|V\| = \sqrt{4^2 + 0^2 + 6^2} = 2\sqrt{13}$

$U \cdot V = (6 \times 4) + (1 \times 0) + (3 \times -6) = 6$

$$\cos\theta = \frac{6}{(\sqrt{46})(2\sqrt{13})}$$

$\theta = 82.95^\circ$

angle is acute.

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c) $U = (1,1,1)$ $V = (-1,0,0)$

$\|U\| = \sqrt{1^2+1^2+1^2} = \sqrt{3}$

$\|V\| = \sqrt{1^2} = 1$

$U \cdot V = (1\times-1) + (1\times0) + (1\times0) = -1$

$\cos\theta = \dfrac{-1}{\sqrt{3}}$

$\theta = 125.26°$

angle is obtuse.

d) $U = (4,1,6)$, $V = (-3,0,2)$

$\|U\| = \sqrt{4^2+1^2+6^2} = \sqrt{53}$

$\|V\| = \sqrt{(-3)^2+0^2+2^2} = \sqrt{13}$

$U \cdot V = (4\times-3) + (1\times0) + (6\times2) = 0$

$\cos\theta = \dfrac{0}{(\sqrt{53})(\sqrt{13})}$

$\theta = 90$

angle is orthogonal.

Date

Q13

let U be the AB vector and V be the AP vector.

when angle is orthogonal.

$\boxed{U \cdot V = 0}$

$U = B - A = (3-1)i + (0-(-1))j + (5-3)k$

$U = 2i + j + 2k$

$V = P - A = (r-1)i + (r-(-1))j + (r-3)k$

$V = (r-1)i + (r+1)j + (r-3)k$

$U \cdot V = 0$

$(2 \times (r-1)) + (1 \times (r+1)) + (2 \times (r-3)) = 0$

$2r - 2 + r + 1 + 2r - 6 = 0$

$\boxed{r = \frac{7}{5}}$

Q15

a) $v = i + j - k$

$\|v\| = \sqrt{3}$, $v_1 = 1$, $v_2 = 1$, $v_3 = -1$

$\boxed{\cos\alpha = \frac{1}{\sqrt{3}}}$

$\boxed{\cos\beta = \frac{1}{\sqrt{3}}}$

$$\boxed{\cos\gamma = \frac{-1}{\sqrt{3}}}$$

$$\boxed{\begin{array}{l}\alpha = \beta \approx 55^\circ \\ \gamma \approx 125^\circ\end{array}}$$

$$\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$$

$$\left(\frac{1}{\sqrt{3}}\right)^2 + \left(\frac{1}{\sqrt{3}}\right)^2 + \left(\frac{-1}{\sqrt{3}}\right)^2 = 1$$

$$\frac{1}{3} + \frac{1}{3} + \frac{1}{3} = 1$$

$$\boxed{1 = 1}$$

b) $v = 2i - 2j + k$

$\cos\alpha = \frac{v_1}{\|v\|} = \frac{2}{3}$ $\qquad \alpha \approx 48^\circ$

$\cos\beta = \frac{v_2}{\|v\|} = \frac{-2}{3}$ $\qquad \beta \approx 132^\circ$

$\cos\gamma = \frac{v_3}{\|v\|} = \frac{1}{3}$ $\qquad \gamma \approx 71^\circ$

$$\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$$

$$(2/3)^2 + (-2/3)^2 + (1/3)^2 = 1$$

$$\boxed{1 = 1}$$

Date

Q39

$$W = \vec{F} \cdot d(\hat{v})$$

$$\hat{v} = \frac{V}{\|V\|} = \frac{i+j+k}{\sqrt{3}} = \frac{1}{3}\hat{i} + \frac{1}{3}\hat{j} + \frac{1}{3}\hat{k}$$

$$d\hat{v} = \frac{15}{\cancel{\sqrt{3}}}\left(\frac{1}{\sqrt{3}}\hat{i} + \frac{1}{\sqrt{3}}\hat{j} + \frac{1}{\sqrt{3}}\hat{k}\right)$$

$$d\hat{v} = 5\sqrt{3}\,\hat{i} + 5\sqrt{3}\,j + 5\sqrt{3}\,\hat{k}$$

$$W = F \cdot d\hat{v}$$

$$W = (4i - 6j + k) \cdot (5\sqrt{3}\,\hat{i} + 5\sqrt{3}\,\hat{j} + 5\sqrt{3}\,\hat{k})$$

$$W = (4 \times 5\sqrt{3}) + (-6 \times 5\sqrt{3}) + (1 \times 5\sqrt{3})$$

$$\boxed{W = -5\sqrt{3}\ J}$$

Assignment # 09

Q3

$$U \times V = \begin{vmatrix} i & j & k \\ 1 & 2 & -3 \\ -4 & 1 & 2 \end{vmatrix}$$

$$= (4-(-3))i - j(2-12) + (1-(-8))k$$

$$\boxed{U \times V = 7i + 10j + 9\hat{k}}$$

if two vectors a & b are orthogonal then: $a \cdot b = 0$

$$U \cdot (U \times V) = 0$$

$$(1i + 2j - 3k) \cdot (7i + 10j + 9\hat{k}) = 0$$

$$(1 \times 7) + (2 \times 10) + (-3 \times 9) = 0$$

$$\boxed{0 = 0}$$

$$V \cdot (U \times V) = 0$$

$$(-4i + j + 2k) \cdot (7i + 10j + 9\hat{k}) = 0$$

$$(-4 \times 7) + (1 \times 10) + (2 \times 9) = 0$$

$$\boxed{0 = 0}$$

Date

Q5

let C be U×V

$$C = \begin{vmatrix} i & j & k \\ 0 & 1 & -2 \\ 3 & 0 & -4 \end{vmatrix}$$

~~C = (-4+0)i - (0+~~

$C = (-4-0)i - (0-(-6))j + (0-3)k$

$\boxed{C = -4i - 6j - 3k}$

orthogonal condition

$\vec{a} \cdot \vec{b} = 0$

$U \cdot C = (0\hat{i} + 1\hat{j} + -2\hat{k}) \cdot (-4\hat{i} - 6\hat{j} - 3\hat{k})$

$0 = (0 \times -4) + (1 \times -6) + (-2 \times -3)$

$\boxed{0 = 0}$

$V \cdot C = 0$

$(3\hat{i} + 0\hat{j} + -4\hat{k})(-4i - 6j - 3\hat{k}) = 0$

$(3 \times -4) + (0 \times -6) + (-4 \times -3) = 0$

$\boxed{0 = 0}$

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Q7

a) $U \times (V \times W)$

$$V \times W = \begin{vmatrix} i & j & k \\ 0 & 1 & 7 \\ 1 & 4 & 5 \end{vmatrix}$$

$$= (5-28)i - (0-7)j + (0-1)k$$

$$= -23i + 7j - k$$

$$U \times (V \times W) = \begin{vmatrix} i & j & k \\ 2 & -1 & 3 \\ -23 & 7 & -1 \end{vmatrix}$$

$$= (1-21)i - (-2+69)j + (14 + 23)k$$

$$\boxed{U \times (V \times W) = -20i - 67j + 37k}$$

b) $(U \times V) \times W$

$$U \times V = \begin{vmatrix} i & j & k \\ 2 & -1 & 3 \\ 0 & 1 & 7 \end{vmatrix}$$

$$= (-7-3)i - (14-0)j + (2-0)k$$

$$U \times V = -10i - 14j + 2k$$

$$(U\times V)\times W = \begin{vmatrix} i & j & k \\ -10 & -14 & 2 \\ 1 & 4 & 5 \end{vmatrix}$$

$$= (-70-8)i - (-50-2)j + (-40+14)k$$

$$\boxed{(U\times V)\times W = -78i + 52j - 26k}$$

c) $(U\times V)\times(V\times W)$

$$U\times V = \begin{vmatrix} i & j & k \\ 2 & -1 & 3 \\ 0 & 1 & 7 \end{vmatrix}$$

$$= (-7-3)i - (14-0)j + (2-0)k$$

$$U\times V = -10i - 14j + 2k$$

$$V\times W = \begin{vmatrix} i & j & k \\ 0 & 1 & 7 \\ 1 & 4 & 5 \end{vmatrix}$$

$$= (5-28)i - (0-7)j + (0-1)k$$

$$V\times W = -23i + 7j - k$$

$$(U\times V)\times(V\times W) = \begin{vmatrix} i & j & k \\ -10 & -14 & 2 \\ -23 & 7 & -1 \end{vmatrix}$$

$$= (14-14)i + (10+46)j + (-70-322)k$$

Date

$$\boxed{(u \times v) \times (v \times w) = -56j - 392k}$$

d) $(v \times w) \times (u \times v)$

$$v \times w = \begin{vmatrix} i & j & k \\ 0 & 1 & 7 \\ 1 & 4 & 5 \end{vmatrix}$$

$$= (5 - 28)i - (0 - 7)j + (0 - 1)k$$

$$v \times w = -23i + 7j - k$$

$$u \times v = \begin{vmatrix} i & j & k \\ 2 & -1 & 3 \\ 0 & 1 & 7 \end{vmatrix}$$

$$= (-7 - 3)i - (14 - 0)j + (2 - 0)k$$

$$u \times v = -10i - 14j + 2k$$

$$(v \times w) \times (u \times v) = \begin{vmatrix} i & j & k \\ -23 & 7 & -1 \\ -10 & -14 & 2 \end{vmatrix}$$

$$= (14 - 14)i - (-46 - 10)j + (322 + 70)k$$

$$= 56j + 392k$$

RC

Q11

$A(0,-2,1)$ $B(1,-1,-2)$ $C(-1,1,0)$

Let c be the normal vector to the plane.

$AB = \langle 1, 1, -3 \rangle$

$AC = \langle -1, 3, -1 \rangle$

$$c = \begin{vmatrix} \hat{i} & \hat{j} & k \\ 1 & 1 & -3 \\ -1 & 3 & -1 \end{vmatrix}$$

$c = 8i + 4j + 4k$

$$\hat{c} = \frac{8i + 4j + 4k}{\sqrt{(8)^2 + (4)^2 + (4)^2}}$$

$$= \pm \frac{1}{4\sqrt{6}} (8i + 4j + 4k)$$

$$\hat{c} = \pm \left(\frac{2}{\sqrt{6}} i + \frac{1}{\sqrt{6}} j + \frac{1}{\sqrt{6}} k \right)$$

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$$\boxed{\hat{c} = \pm\frac{1}{\sqrt{6}}(2i + j + k)}$$

~~Q16~~ Q17

$u = i - j + 2k \qquad v = 3j + k$

Area of parallelogram = $\|u \times v\|$

$$u \times v = \begin{vmatrix} i & j & k \\ 1 & -1 & 2 \\ 0 & 3 & 1 \end{vmatrix}$$

$u \times v = -7i \; \cancel{+} - \hat{j} + 3\hat{k}$

$\|u \times v\| = \sqrt{(7)^2 + (1)^2 + (3)^2}$

$\boxed{\text{area of parallelogram} = \sqrt{59}}$

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Q19

$P(1, 5, -2), Q(0, 0, 0), R(3, 5, 1)$

$PQ = \langle -1, -5, 2 \rangle$

$PR = \langle 2, 0, 3 \rangle$

$$PQ \times PR = \begin{vmatrix} i & j & k \\ -1 & -5 & 2 \\ 2 & 0 & 3 \end{vmatrix}$$

let n be the normal vector

$n = -15i + 7j + 10k$

$\|n\| = \sqrt{(-15)^2 + (7)^2 + (10)^2}$

$\|n\| = \sqrt{374}$

$\boxed{\text{area of triangle} = \frac{\|n\|}{2} = \frac{\sqrt{374}}{2}}$

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Q21

$$v \times w = \begin{vmatrix} i & j & k \\ 4 & 1 & -3 \\ 0 & 1 & 5 \end{vmatrix}$$

$$v \times w = 8i - 20j + 4k$$

$$u \cdot (v \times w) = (2i - 3j + k) \cdot (8i - 20j + 4k)$$

$$= (2 \times 8) + (-3 \times -20) + (1 \times 4)$$

$$\boxed{u \cdot (v \times w) = 80}$$

Q25

volume of parallelopipped $= |u \cdot (v \times w)|$

$$v \times w = \begin{vmatrix} i & j & k \\ 0 & 4 & -2 \\ 2 & 2 & -4 \end{vmatrix}$$

$$= -12i - 4j - 8k$$

$$|u \cdot (v \times w)| = |(2i - 6j + 2k) \cdot (-12i - 4j - 8k)|$$

$$= |(2 \times -12) + (-6 \times -4) + (2 \times -8)|$$

$$= |-16|$$

$$\boxed{|u \cdot (v \times w)| = 16}$$

No.

Q27

a)

$$v \times w = \begin{vmatrix} i & j & k \\ 3 & 0 & -2 \\ 5 & -4 & 0 \end{vmatrix}$$

$v \times w = -8i - 10j - 12k$

$u \cdot (v \times w) = (i - 2j + k) \cdot (-8i - 10j - 12k)$

$= (1 \times -8) + (-2 \times -10) + (1 \times -12)$

$\boxed{u \cdot (v \times w) = 0}$

yes, vectors lie on same plane

b)

$$v \times w = \begin{vmatrix} i & j & k \\ 4 & -1 & 1 \\ 1 & -1 & 0 \end{vmatrix}$$

$= i + j - 3k$

$u \cdot (v \times w) = (5i - 2j + k) \cdot (i + j - 3k)$

$= (5 \times 1) + (-2 \times 1) + (1 \times -3)$

$\boxed{u \cdot (v \times w) = 0}$

yes, vectors lie on same plane

c)

$$V \times W = \begin{vmatrix} i & j & k \\ 2 & 1 & -2 \\ 3 & -4 & 12 \end{vmatrix}$$

$$V \times W = 4i - 30j - 11k$$

$$U \cdot (V \times W) = (4i - 8j + k) \cdot (4i - 30j - 11k)$$

$$= (4 \times 4) + (-8 \times -30) + (1 \times -11)$$

$$U \cdot (V \times W) = 245$$

No, vectors does not lie on same plane

Q31

a)

$$d = \frac{\| \vec{AP} \times \vec{AB} \|}{\| AB \|}$$

$$AP = -4i + 2k$$

$$AB = -3i + 2j - 4k$$

$$\| AB \| = \sqrt{3^2 + 2^2 + 4^2} = \sqrt{29}$$

$$AP \times AB = \begin{vmatrix} i & j & k \\ -4 & 0 & 2 \\ -3 & 2 & -4 \end{vmatrix}$$

$$AP \times AB = -4i - 22j - 8k$$

$$\| AP \times AB \| = \sqrt{4^2 + 22^2 + 8^2}$$

$$\| AP \times AB \| = 2\sqrt{141}$$

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$$\boxed{d = \frac{2\sqrt{141}}{\sqrt{29}} \text{ units}}$$

b)

$$d = \frac{\| AP \times AB \|}{\| AB \|}$$

$AP = 2i + 2j$

$AB = -2i + j$

$$AP \times AB = \begin{vmatrix} i & j & k \\ 2 & 2 & 0 \\ -2 & 1 & 0 \end{vmatrix}$$

$AP \times AB = 6k$

$\| AP \times AB \| = 6$

$\| AB \| = \sqrt{5}$

$$\boxed{d = \frac{6}{\sqrt{5}} \text{ units}}$$

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Assignment # 10

Q3

a) $\vec{P_1P_2} = \langle 2, 3 \rangle$

$P_1(3, -2)$ $\quad$ $P_2(5, 1)$

For line:

$$\boxed{\begin{aligned} x &= 3 + 2t \\ y &= -2 + 3t \end{aligned}}$$

For line segment:

Put P_1 in equation

$3 = 3 + 2t$

$\boxed{t = 0}$

Put P_2 in equation

$5 = 3 + 2t$

$\boxed{t = 1}$

$$\boxed{\begin{aligned} x &= 3 + 2t \qquad 0 \le t \le 1 \\ y &= -2 + 3t \end{aligned}}$$

b) $P_1(5, -2, 1)$ $\quad$ $P_2(2, 4, 2)$

$\vec{P_1P_2} = \langle -3, 6, 1 \rangle$

For line:

$$\boxed{\begin{aligned} x &= 5 - 3t \\ y &= -2 + 6t \\ z &= 1 + t \end{aligned}}$$

Date

For line segment :-

put P_1 in equation

$5 = 5 - 3t$

$t = 0$

put P_2 in equation

$2 = 5 - 3t$

$t = 1$

$$\boxed{\begin{aligned} x &= 5 - 3t \\ y &= -2 + 6t \qquad 0 \leq t \leq 1 \\ z &= 1 + t \end{aligned}}$$

Q7

a) $xi + yj = (2i - j) + t(4i - j)$

$x = 2 + 4t$

$y = -1 - t$

$$\boxed{\begin{aligned} &\text{Point} = P(2, -1) \\ &\text{vector} = V = \cancel{4}\ 4i - j \end{aligned}}$$

b) $(x, y, z) = (-1, 2, 4) + t(5, 7, 8)$

$x = -1 + 5t$
$y = 2 + 7t$
$z = 4 + 8t$

$$\boxed{\begin{array}{l} \text{Point} \Rightarrow P(-1, 2, 4) \\ \text{vector} \Rightarrow 5i + 7j + 8k \end{array}}$$

Q15

Point $\Rightarrow P(-5, 2)$
Vector $\Rightarrow 2i - 3j$

$$\boxed{\begin{array}{l} x = -5 + 2t \\ y = 2 - 3t \end{array}}$$

Q17

$x^2 + y^2 = 25$ → equation of circle
$P_1(x, y) = (3, -4)$ → point on line & circle

The center point of circle is $O(0, 0)$
$\vec{OP} = \langle 3, -4 \rangle$
Let v is the vector parallel to the line & ⊥ to OP

Date

$$V \cdot OP = 0$$

$$(ai + bj) \cdot (3i - 4j) = 0$$

$$3a - 4b = 0$$

$$3a = 4b$$

$$\frac{a}{b} = \frac{4}{3}$$

$$a = 4 \quad b = 3$$

so the parallel vector $v = 4i + 3j$

Point $\Rightarrow P(3, -4)$

$$\boxed{\begin{aligned} x &= 3 + 4t \\ y &= -4 + 3t \end{aligned}}$$

Q19

$P(-1, 2, 4)$

$V = 3i - 4j + k$

$$\boxed{\begin{aligned} x &= -1 + 3t \\ y &= 2 - 4t \\ z &= 4 + t \end{aligned}}$$

Q 21

Point $\Rightarrow P(-2, 0, 5)$

Parallel line $\Rightarrow$ $x = 1 + 2t$

$y = 4 - t$

$z = 6 + 2t$

Parallel vector $\Rightarrow$ $v = \langle 2, -1, 2 \rangle$

Equation of line $\Rightarrow$

$$\boxed{\begin{aligned} x &= -2 + 2t \\ y &= -t \\ z &= 5 + 2t \end{aligned}}$$

Q23

a) the x-axis

Put $y = 0$

$0 = 2 - t$

$t = 2$

$x = 1 + 3(2)$

$x = 7$

$\boxed{P(7, 0)}$

Date

b) the y-axis

put $x = 0$

$0 = 1 + 3t$

$t = -\frac{1}{3}$

$y = 2 - \left(-\frac{1}{3}\right)$

$y = \frac{7}{3}$

$\boxed{P(0, 7/3)}$

c) the parabola $y = x^2$

c) the parabola $y = x^2$

put line in parabola

$$2-t = (1+3t)^2$$

$$2-t = 1+6t+9t^2$$

$$9t^2+7t-1=0$$

$\cancel{t}$

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$$t = \frac{-7+\sqrt{85}}{18}$$

$$x = 1+3\left(\frac{-7+\sqrt{85}}{18}\right)$$

$$\boxed{x = \frac{-1+\sqrt{85}}{6}}$$

$$y = 2-\left(\frac{-7+\sqrt{85}}{18}\right)$$

$$\boxed{y = \frac{43-\sqrt{85}}{18}}$$

$$t = \frac{-7-\sqrt{85}}{18}$$

$$x = 1+3\left(\frac{-7-\sqrt{85}}{18}\right)$$

$$\boxed{x = \frac{-1-\sqrt{85}}{6}}$$

$$y = 2-\left(\frac{-7-\sqrt{85}}{18}\right)$$

$$\boxed{y = \frac{43+\sqrt{85}}{18}}$$

$$\boxed{x = \frac{-1\pm\sqrt{85}}{6} \qquad y = \frac{43\pm\sqrt{85}}{18}}$$

Q25

the xy plane :-

$z = -3+t = 0$

$t = 3$

$\boxed{x = -2}$

$y = 4+2(3)$

$\boxed{y = 10}$

$\boxed{P(-2, 10, 0)}$

the xz plane :-

$y = 4+2t$

$0 = 4+2t$

$t = -2$

$x = -2$

$z = -3+(-2) = -5$

$\boxed{P(-2, 0, -5)}$

the yz plane :-

$x = -2$

there is no parameter bond on this equation so x never be zero:
the line does not intersect yz plane

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Q27

equation of line

$x = 1+t , \quad y = 3-t , \quad z = 2t$

put in cylinder equation

$x^2 + y^2 = 16$

$(1+t)^2 + (3-t)^2 = 16$

$1 + 2t + t^2 + 9 - 6t + t^2 = 16$

$2t^2 - 4t - 6 = 0$

$t = 3$

$x = 1 + 3 = 4$

$y = 3 - 3 = 0$

$z = 2(3) = 6$

$\boxed{P(4,0,6)}$

$t = -1$

$x = 1 + (-1) = 0$

$y = 3 - (-1) = 4$

$z = 2(-1) = -2$

$\boxed{P(0,4,-2)}$

Q28

Q29

Let (x_0, y_0, z_0) be the point of intersection

$L_1 \Rightarrow x_0 = 2 + t_1,\ y_0 = 2 + 3t_1,\ z_0 = 3 + t_1$

$L_2 \Rightarrow x_0 = 2 + t_2,\ y_0 = 3 + 4t_2,\ z_0 = 4 + 2t_2$

By equaling

$2 + t_1 = 2 + t_2 \longrightarrow (1)$

$2 + 3t_1 = 3 + 4t_2 \longrightarrow (2)$

$3 + t_1 = 4 + 2t_2 \longrightarrow (3)$

No.

~~Simultaneously solving eq ② & ③~~

~~② ⇒ t = −1~~

Simultaneously solving eq ① & ②

① ⇒ $t_1 = t_2$

put in eq ②

$2 + 3t_1 = 3 + 4t_1$

$\boxed{t_1 = -1}$

$\boxed{t_2 = -1}$

put in eq ③

$3 + (-1) = 4 + 2(-1)$

$3 - 1 = 4 - 2$

$\boxed{2 = 2}$

$\boxed{\text{Yes, these lines are intersecting}}$

$x_0 = 2 + (-1) = 2$

$y_0 = 2 + 3(-1) = -1$

$z_0 = 3 + (-1) = 2$

$\boxed{P(x_0, y_0, z_0) = P(2, -1, 2)}$

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Q31

we have to proof :-

a) Lines are not parallel

b) Lines are not intersect

a)

$V_1 = <7, 1, -3>$

$V_2 = <-1, 0, 2>$

vectors are not multiple of one & another:

$\boxed{\text{Lines are not parallel}}$

b)

Let (x_0, y_0, z_0) be the point of intersection

L_1	L_2
$x_0 = 1+7t_1$	$x_0 = 4-t_2$
$y_0 = 3+t_1$	$y_0 = 6$
$z_0 = 5-3t_1$	$z_0 = 7+2t_2$

$1+7t_1 = 4-t_2$

$t_2 = 7t_1-3$

$5-3t_1 = 7+2t_2$

$5-3t_1 = 7+2(7t_1-3)$

$t_1 = \frac{4}{17}$

No. ______

$3 + t_1 = 6$

$3 + \frac{4}{17} = 6$

$\frac{55}{17} \neq 6$

$\boxed{\text{lines are not intersecting}}$

Yes it is skew lines

Q33

$V_1 = \langle -2, 1, -1 \rangle$
$V_2 = \langle -4, 2, -2 \rangle$

V_2 is multiple of V_1 so:
$\boxed{\text{lines are parallel}}$

Q35

$P_1P_2 = \langle 3, -7, -7 \rangle$
~~$P_3P_2 = \langle +9, +14, -10 \rangle$~~
$P_2P_3 = \langle -9, -7, -3 \rangle$

Vectors are not multiple of each & other:
~~lines~~ points does not lie on the same line

Q37

~~$v_1 = \langle -1, 2 \rangle$~~

~~$v_2 = \langle 3, -6 \rangle$~~

~~$x = 3 - t$~~ ~~$y = 1 + 2t$~~

~~$v_1 = \langle -1, 2 \rangle$~~

From L_1

$y = 1 + 2(3 - x)$

$y = 1 + 6 - 2x$

$\boxed{y = 7 - 2x}$

From L_2

$y = 9 - 6\left(\frac{x+1}{3}\right)$

$y = 9 - 2x - 2$

$\boxed{y = 7 - 2x}$

lines are same : prooved

43

$\langle x, y \rangle = \langle 1, 0 \rangle + t\langle -2, 3 \rangle \quad 0 \leq t \leq 2$

$x = 1 - 2t$

$y = 3t$

put $t = 0$

$x = 1 \qquad y = 0$

Put $t = 2$

$x = -3, \quad y = 6$

the line segment joining the points
(1, 0) and (−3, 6)

Q47

P(-2, 1, 1)

put t = 0 in line

A(x, y, z) = (3, 0, 1)

put t = 1 in line

B(x, y, z) = (2, 1, 3)

AP = < -5, 1, 0 >

AB = < -1, 1, 2 >

$$AP \times AB = \begin{vmatrix} i & j & k \\ -5 & 1 & 0 \\ -1 & 1 & 2 \end{vmatrix}$$

$$AP \times AB = 2i + 10j - 4k$$

$$\| AP \times AB \| = \sqrt{2^2 + 10^2 + 42}$$

$$\| AP \times AB \| = 2\sqrt{30}$$

$$\| AB \| = \sqrt{1^2 + 1^2 + 2^2} = \sqrt{6}$$

$$d = \frac{\| AP \times AB \|}{\| AB \|}$$

$$= \frac{2\sqrt{30}}{\sqrt{6}}$$

$$\boxed{d = 2\sqrt{5}} \text{ units}$$

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Q49

put $t=0$ in L_1

$P(x,y,z) = (2,0,1)$

put $t=0$ in L_2

$A(x,y,z) = (1,3,5)$

put $t=1$ in L_2

$B(x,y,z) = (3,-1,3)$

$$\boxed{d = \frac{\|AP \times AB\|}{\|AB\|}}$$

$$AP \times AB = \begin{vmatrix} i & j & k \\ 1 & -3 & -4 \\ 2 & -4 & -2 \end{vmatrix}$$

$AP \times AB = -10i - 6j + 2k$

$\|AP \times AB\| = \sqrt{10^2 + 6^2 + 2^2}$

$\|AP \times AB\| = 2\sqrt{35}$

$\|AB\| = \sqrt{2^2 + 4^2 + 2^2} = 2\sqrt{6}$

$$d = \frac{\cancel{2}\sqrt{35}}{\cancel{2}\sqrt{6}}$$

$$\boxed{d = \sqrt{\frac{35}{6}} \text{ unit}}$$

No.

Q55

a)

put $t = 3$ in L_1

$x = 1 + 2(3) = 7$

$y = 2 - 3 = -1$

$z = 4 - 2(3) = -2$

$\boxed{P(7, -1, -2)}$

put $t = -2$ in L_2

$x = 9 + 1(-2) = 7$

$y = 5 + 3(-2) = -1$

$z = -4 - (-2) = -2$

$\boxed{P(7, -1, -2)}$

b)

parallel vectors are

$v_1 = \langle 2, -1, -2 \rangle$

$v_2 = \langle 1, 3, -1 \rangle$

$$\cos\theta = \frac{v_1 \cdot v_2}{\|v_1\| \, \|v_2\|} = \frac{(2, -1, -2) \cdot (1, 3, -1)}{\sqrt{2^2 + 1^2 + 2^2} \, \sqrt{1^2 + 3^2 + 1^2}} = \frac{\sqrt{11}}{33}$$

$$\theta = \cos^{-1}\left(\frac{\sqrt{11}}{33}\right)$$

$\boxed{\theta \approx 84^\circ}$

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c)

parallel vectors are

$v_1 = \langle 2, -1, -2 \rangle$

$v_2 = \langle 1, 3, -1 \rangle$

let c is the vector perpendicular to v_1 & v_2

$$c = v_1 \times v_2$$

$$= \begin{vmatrix} i & j & k \\ 2 & -1 & -2 \\ 1 & 3 & -1 \end{vmatrix}$$

$$c = 7i + 7k$$

$$\hat{c} = i + k$$

$P(7, -1, -2)$

$$\boxed{\begin{aligned} x &= 7 + t \\ y &= -1 \\ z &= -2 + t \end{aligned}}$$

Assignment # 11

Date

Q3

Point $\Rightarrow P(2, 6, 1)$

$n = \langle 1, 4, 2 \rangle$

Equation of plane

$$a(x - x_0) + b\cdot(y - y_0) + c(z - z_0) = 0$$

$$1(x - 2) + 4(y - 6) + 2(z - 1) = 0$$

$$x - 2 + 4y - 24 + 2z - 2 = 0$$

$$x + 4y + 2z - 28 = 0$$

$$\boxed{x + 4y + 2z = 28}$$

Q7

$P_1(0, 0, 1)$

$P_2(1, 0, 0)$

$P_3($ [illegible] $)$

Q7

$P_1(0,0,0)$
$P_2(0,0,1)$
$P_3(1,1,0)$

$P_1P_2 = \langle 0, 0, 1\rangle$
$P_1P_3 = \langle 1, 1, 0\rangle$

$$P_1P_2 \times P_1P_3 = \begin{vmatrix} i & j & k \\ 0 & 0 & 1 \\ 1 & 1 & 0 \end{vmatrix}$$

$$n = -i + j$$

$$-1(x-0) + 1(y-0) + 0(z-0) = 0$$

$$-x + y = 0$$

$$-(x-y) = 0$$

$$\boxed{x - y = 0}$$

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Q 11

$P_1(-2, 1, 1)$
$P_2(0, 2, 3)$
$P_3(1, 0, -1)$

$P_1P_2 = \langle 2, 1, 2 \rangle$
$P_1P_3 = \langle 3, -1, -2 \rangle$

$$n = \begin{vmatrix} i & j & k \\ 2 & 1 & 2 \\ 3 & -1 & -2 \end{vmatrix}$$

$$= 0i + 10j - 5k$$

$n = 10j - 5k$
$n = 2j - k$

$a(x - x_0) + b(y - y_0) + c(z - z_0) = 0$
$0(x - \cancel{(+2)}) + 2(y - 1) - 1(z - 1) = 0$
$2y - 2 - z + 1 = 0$
$\boxed{2y - z = 1}$

Q13

a)

$n_1 = \langle 2, -8, -6 \rangle$

$n_2 = \langle -1, 4, 3 \rangle$

since ~~vec~~ normal vectors of plane are parallel, so the planes are also parallel.

b)

$n_1 = \langle 3, -2, 1 \rangle$

$n_2 = \langle 4, 5, -2 \rangle$

planes are not parallel

$n_1 \cdot n_2 = 0$

$(3 \times 4) + (-2 \times 5) + (1 \times -2) = 0$

$12 - 10 - 2 = 0$

$\boxed{0 = 0}$

since normal vectors are perpendicular so the planes are also perpendicular c)

$n_1 = \langle 1, -1, 3 \rangle$

$n_2 = \langle 2, 0, 1 \rangle$

planes are not parallel

$n_1 \cdot n_2 = 0$

$(1 \times 2) + (-1 \times 0) + (3 \times 1) = 0$

$2 + 3 = 0$

$5 \neq 0$

planes are not perpendicular

$\boxed{\text{neither}}$

Q15

a)

$V = \langle 2, -1, -4 \rangle$

$n = \langle 3, 2, 1 \rangle$

plane & line is not perpendicular

$n \cdot V = 0$

$(3 \cdot 2) + (2 \cdot -1) + (1 \cdot -4) = 0$

No.

$6-6=0$

$\boxed{0=0}$

plane and line is parallel.

b)

$v = \langle 1, 2, 3 \rangle$

$n = \langle 1, -1, 2 \rangle$

not perpendicular

$n \cdot v = 0$

$(1.1) + (-1.2) + (2.3) = 0$

$1 - 2 + 6 = 0$

$5 \neq 0$

not parallel

neither

c)

$v = \langle 2, 1, -1 \rangle$

$n = \langle 4, 2, -2 \rangle$

plane and line is perpendicular,

Q17

a)

$v = \langle 1, 1, 1 \rangle$

$n = \langle 3, -2, 1 \rangle$

$n \cdot v = 0$

$(1 \cdot 3) + (1 \cdot -2) + (1 \cdot 1) = 0$

$3 - 2 + 1 = 0$

$2 \neq 0$

Yes, it is intersect.

Let (x_0, y_0, z_0) be the point of intersection

satisfy the line:

$x_0 = t$

$y_0 = t$

$z_0 = t$

satisfy the plane:

$3x_0 - 2y_0 + z_0 - 5 = 0$

$3t - 2t + t - 5 = 0$

$t = \frac{5}{2}$

$$\boxed{P(x_0, y_0, z_0) = \left(\frac{5}{2}, \frac{5}{2}, \frac{5}{2}\right)}$$

b)

$v = \langle -1, 1, 1 \rangle$

$n = \langle 2, 1, 1 \rangle$

$n \cdot v = 0$

$(2 \cdot -1) + (1 \cdot 1) + (1 \cdot 1) = 0$

$-2 + 2 = 0$

$\boxed{0 = 0}$

$\boxed{\text{no point of intersection}}$

Q19

$n_1 = \langle 1, 0, 0 \rangle$

$n_2 = \langle 2, -1, 1 \rangle$

$$\cos\theta = \frac{n_1 \cdot n_2}{\|n_1\| \cdot \|n_2\|}$$

$$= \frac{(1,0,0)(2,-1,1)}{\sqrt{1^2+0^2+0^2}\sqrt{2^2+1^2+1^2}}$$

$$\cos\theta = \frac{2}{\sqrt{6}}$$

$$\theta = \cos^{-1}\left(\frac{2}{\sqrt{6}}\right)$$

$$\theta \approx 350$$

Date

Q25

Point $\Rightarrow P(0,0,0)$

normal vector of parallel plane $= n' = \langle 4,-2,7\rangle$

$n = n'$

$n = \langle 4,-2,7\rangle$

$a(x-x_0) + b(y-y_0) + c(z-z_0) = 0$

$4(x-0) + -2(y-0) + 7(z-0) = 0$

$\boxed{4x - 2y + 7z = 0}$

Q29

~~put t=0 in one plane~~

~~At~~.

Point $\Rightarrow P(1,2,-1)$

$n_1 = \langle 2,1,1\rangle$

$n_2 = \langle 1,2,1\rangle$

$n = n_1 \times n_2$

$= \begin{vmatrix} i & j & k \\ 2 & 1 & 1 \\ 1 & 2 & 1 \end{vmatrix}$

$n = -i - j + 3k$

$a(x-x_0)+b(y-y_0)+c(z-z_0)=0$

$-1(x-1)-1(y-2)+3(z+1)=0$

$-x+1-y+2+3z+3=0$

$x+y-3z-6=0$

$\boxed{x+y-3z=6}$

Q31

Point $\Rightarrow P(-1, 2, -5)$

$n_1 = \langle 2, -1, 1\rangle$

$n_2 = \langle 1, 1, -2\rangle$

$n = n_1 \times n_2$

$$n = \begin{vmatrix} i & j & k \\ 2 & -1 & 1 \\ 1 & 1 & -2 \end{vmatrix}$$

$n = \cancel{-2} \; i + 5j + 3k$

$a(x-x_0)+b(y-y_0)+c(z-z_0)$

$1(x+1)+5(y-2)+3(z+5)$

$x+1+5y-10+3z+15=0$

$x+5y+3z+6=0$

$\boxed{x+5y+3z=-6}$

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Date

Q33

$P_1(2,-1,1)$
$P_2(3,1,5)$

$$P(x,y,z) = \left(\frac{2+3}{2}, \frac{-1+1}{2}, \frac{1+5}{2}\right)$$

$$= \frac{5}{2}, 0 \quad \frac{6}{2}$$

$$P(x,y,z) = \left(\frac{5}{2}, 0, 3\right)$$

$n = P_1P_2$
$n = \langle 1, 2, 4 \rangle$

$$a(x-x_0) + b(y-y_0) + c(z-z_0) = 0$$

$$1\left(x-\frac{5}{2}\right) + 2(y-0) + 4(z-3) = 0$$

$$x - \frac{5}{2} + 2y + 4z - 12 = 0$$

$$\boxed{x + 2y + 4z = \frac{29}{2}}$$

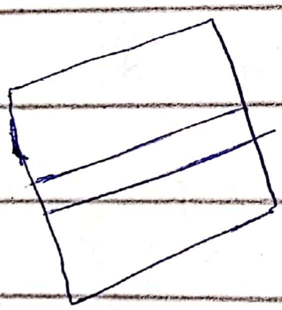
Date

Q37

$V_1 = \langle 1, 2, -1 \rangle$

$V_2 = \langle -1, -2, 1 \rangle$

lines are parallel because their vectors are multiple of each and other



put $t=0$ in L_1 & L_2

$A(x, y, z) = (-2, 3, 4)$

$B(x, y, z) = (3, 4, 0)$

$\overrightarrow{AB} = \langle 5, 1, -4 \rangle$

$n = V_2 \times AB$

$$= \begin{vmatrix} i & j & k \\ -1 & -2 & 1 \\ 5 & 1 & \end{vmatrix}$$

$n = V_1 \times AB$

$$= \begin{vmatrix} i & j & k \\ 1 & 2 & -1 \\ 5 & 1 & -4 \end{vmatrix}$$

$n = -7i - j - 9k$

$P(-2, 3, 4)$

$a(x - x_0) + b(y - y_0) + c(z - z_0) = 0$

$+7(x+2) + 1(y-3) + 9(z-4) = 0$

$7x + 14 + y - 3 + 9z - 36 = 0$

$$\boxed{7x + y + 9z = 25}$$

Q41

$n_1 = \langle -2, 3, 7 \rangle$

$n_2 = \langle 1, 2, -3 \rangle$

$v = n_1 \times n_2$

$$v = \begin{vmatrix} i & j & k \\ -2 & 3 & 7 \\ 1 & 2 & -3 \end{vmatrix}$$

$v = -23i + j - 7k$

put $z = 0$ in P_1 & P_2

$-2x + 3y + z(0) + 2 = 0$

$x + 2y - 3(0) + 5 = 0$

x

$-2x + 3y = -2$

$x + 2y = -5$

$x = \frac{-19}{7}$ $\quad y = \frac{-12}{7}$ $\quad z = 0$

$P\left(\frac{-19}{7}, \frac{-12}{7}, 0\right)$

$V = \langle -23, 1, -7 \rangle$

$$\boxed{\begin{aligned} x &= \frac{-19}{7} - 23t \\ y &= \frac{-12}{7} + t \\ z &= -7t \end{aligned}}$$

Q43

$$d = \frac{|ax_0 + by_0 + cz_0 + d|}{\sqrt{a^2 + b^2 + c^2}}$$

$P(x_0, y_0, z_0) = P(1, -2, 3)$

$2x - 2y + z = 4$

$a = 2, b = -2, c = 1 \quad d = -4$

Date

$$d = \frac{|2(1) - 2(-2) + 1(3) - 4|}{\sqrt{2^2 + 2^2 + 1^2}}$$

$$\boxed{d = \frac{5}{3} \text{ units}}$$

Q45

$-2x + y + z = 0 \quad \rightarrow (1)$

$6x - 3y - 3z - 5 = 0$

put $y = 0$, $z = 0$ in (1)

$x = 0$

$P(x_0, y_0, z_0) = P(0, 0, 0)$

$a = 6 \quad b = -3 \quad c = -3 \quad d = -5$

$$d = \frac{|ax_0 + by_0 + cz_0 + d|}{\sqrt{a^2 + b^2 + c^2}}$$

$$d = \frac{|6(0) + -3(0) + -3(0) - 5|}{\sqrt{6^2 + 3^2 + 3^2}}$$

$$\boxed{d = \frac{5}{\sqrt{54}} \text{ units}}$$

47

$L_1 \Rightarrow x = 1+7t,\ y = 3+t,\ z = 5-3t$

$L_2 \Rightarrow x = 4-t,\ y = 6,\ z = 7+2t$

Put $t=0$ in L_1 & L_2

$L_1 \Rightarrow$ Pt $x = 1,\ y = 3,\ z = 5$

$P(x_0, y_0, z_0) = P(1, 3, 5)$

$L_2 \Rightarrow x = 4,\ y = 6,\ z = 7$

$A(x', y', z') = A(4, 6, 7)$

$n' = v_1 \times v_2$

$$= \begin{vmatrix} i & j & k \\ 7 & 1 & -3 \\ -1 & 0 & 2 \end{vmatrix}$$

$n' = 2i - 11j + k$

$L_2 \Rightarrow$ Plane $\Rightarrow 2(x-4) - 11(y-6) + 1(z-7) = 0$

$2x - 8 - 11y + 66 + z - 7 = 0$

$2x - 11y + z + 51 = 0$

$a = 2 \quad b = -11 \quad c = 1 \quad d = +51$

$P(x_0, y_0, z_0) = P(1, 3, 5)$

$$d = \frac{|ax_0 + by_0 + cz_0 + d|}{\sqrt{a^2+b^2+c^2}} = \frac{|2(1) - 11(3) + 1(5) + 51|}{\sqrt{2^2+11^2+1^2}}$$

$$\boxed{d = \frac{25}{3\sqrt{14}} \text{ units}}$$

RC

Ex # 11.8

Assignment 12

Date

Q1

a) $(4\sqrt{3}, 4, -4)$

$x = 4\sqrt{3}$ $y = 4$ $z = -4$

$r = \sqrt{x^2+y^2} = \sqrt{(4\sqrt{3})^2 + (4)^2}$

$\boxed{r = 8}$

$\theta = \tan^{-1}\left(\frac{4}{+4\sqrt{3}}\right)$

$\theta = \tan^{-1}\left(\frac{+1}{\sqrt{3}}\right)$

$\boxed{\theta = +\frac{\pi}{6}}$

$\boxed{z = -4}$

$\boxed{\left(8, \frac{\pi}{6}, -4\right)}$

b). $(-5, 5, 6)$

$x = -5$ $y = 5$ $z = 6$

$r = \sqrt{x^2+y^2} = \sqrt{(-5)^2+(5)^2}$

$r = \sqrt{50}$

$\boxed{r = 5\sqrt{2}}$

$\theta = \tan^{-1}\left(\frac{y}{x}\right) = \tan^{-1}\left(\frac{5}{-5}\right)$

$= \tan^{-1}(-1)$

$\boxed{\theta = -\frac{\pi}{4}}$

$\boxed{z = 6}$

$7\pi/4$

$\left(5\sqrt{2}, \cancel{+\frac{3\pi}{4}}, 6\right)$

No.

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c) $(0, 2, 0)$

$x = 0 \quad y = 2 \quad z = 0$

$r = \sqrt{x^2 + y^2} = \sqrt{0^2 + 2^2}$

$\boxed{r = 2}$

$\tan\theta = \frac{y}{x} = \frac{2}{0} = \infty$

$\theta = \tan^{-1}(\infty)$

$\boxed{\theta = \frac{\pi}{2}}$

$\boxed{z = 0}$

$\boxed{2, \frac{\pi}{2}, 0}$

d) $(4, -4\sqrt{3}, 6)$

$x = 4, \quad y = -4\sqrt{3}, \quad z = 6$

$r = \sqrt{x^2 + y^2} = \sqrt{(4)^2 + (-4\sqrt{3})^2}$

$\boxed{r = 8}$

$\tan\theta = \frac{-4\sqrt{3}}{4} = -\sqrt{3}$

$\theta = \tan^{-1}(-\sqrt{3})$

$\boxed{\theta = 5\pi/3}$

$\boxed{z = 6}$

$\boxed{(8, \frac{2\pi}{3}, 6)}$

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Q3

a) $(4, \frac{\pi}{6}, 3)$.

$r = 4$, $\theta = \pi/6$ $z = 3$

$x = r\cos\theta = 4\cos(\pi/6) = 2\sqrt{3}$

$y = r\sin\theta = 4\sin(\pi/6) = 2$

$z = 3$

$(2\sqrt{3}, 2, 3)$

b) $(8, \frac{3\pi}{4}, -2)$

$r = 8$, $\theta = \frac{3\pi}{4}$ $z = 3$

$x = r\cos\theta = 8\cos(3\pi/4) = -4\sqrt{2}$

$y = r\sin\theta = 8\sin(3\pi/4) = 4\sqrt{2}$

$z = -2$

$(-4\sqrt{2}, 4\sqrt{2}, -2)$

c) $(5, 0, 4)$

$r = 5$, $\theta = 0$, $z = 4$

$x = r\cos\theta = 5\cos 0 = 5$

$y = r\sin\theta = 5\sin(0) = 0$

$z = 4$

$(5, 0, 4)$

d) $(7, \pi, -9)$

$x = r\cos\theta = 7\cos\pi = -7$

$y = r\sin\theta = 7\sin\pi = 0$

$z = -9$

$(-7, 0, -9)$

Q5

a) $(1, \sqrt{3}, -2)$

$x = 1, \quad y = \sqrt{3}, \quad z = -2$

$\rho = \sqrt{x^2 + y^2 + z^2}$

$\boxed{\rho = 2\sqrt{2}}$

$\tan\theta = \dfrac{y}{x}$

$\theta = \tan^{-1}\left(\dfrac{\sqrt{3}}{1}\right)$

$\boxed{\theta = \dfrac{\pi}{3}}$

$\cos\phi = \dfrac{z}{\sqrt{x^2 + y^2 + z^2}}$

$= \dfrac{-\cancel{2}}{\cancel{2}\sqrt{2}}$

$\phi = \cos^{-1}\left(\dfrac{-1}{\sqrt{2}}\right)$

$\boxed{\phi = \dfrac{3\pi}{4}}$ $\left(2\sqrt{2}, \dfrac{\pi}{3}, \dfrac{3\pi}{4}\right)$

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b) $(1, -1, \sqrt{2})$

$x = 1$, $y = -1$, $z = \sqrt{2}$

$\rho = \sqrt{x^2+y^2+z^2}$

$\boxed{\rho = 2}$

$\tan\theta = \frac{y}{x}$

$\theta = \tan^{-1}\left(\frac{-1}{1}\right)$

$\boxed{\theta = \frac{7\pi}{4}}$

$\cos\phi = \frac{z}{\sqrt{x^2+y^2+z^2}}$

$\phi = \cos^{-1}\left(\frac{\sqrt{2}}{2}\right)$

$\boxed{\phi = \frac{\pi}{4}}$

$\boxed{\left(2, \frac{7\pi}{4}, \frac{\pi}{4}\right)}$

c) $(0, 3\sqrt{3}, 3)$

$x = 0$ $y = 3\sqrt{3}$ $z = 3$

$\rho = \sqrt{x^2+y^2+z^2}$

$\rho = 6$

$\tan\theta = \frac{y}{x}$

$\theta = \tan^{-1}\left(\frac{3\sqrt{3}}{0}\right)$

$\theta = \frac{\pi}{2}$

$\cos\phi = \frac{z}{\sqrt{x^2+y^2+z^2}}$

$\phi = \cos^{-1}\left(\frac{3}{6}\right)$

$\phi = \frac{\pi}{3}$

$\boxed{\left(6, \frac{\pi}{2}, \frac{\pi}{3}\right)}$

d) $(-5\sqrt{3}, 5, 0)$

$x = -5\sqrt{3}$ $y = 5$ $z = 0$

$\rho = \sqrt{x^2+y^2+z^2}$

$\rho = 10$

$\tan\theta = \frac{y}{x}$

$\theta = \tan^{-1}\left(\frac{5}{-5\sqrt{3}}\right)$

$\theta = -\frac{\pi}{6}$

$\cos\phi = \frac{z}{\sqrt{x^2+y^2+z^2}}$

Date

$\phi = \cos^{-1}\left(\frac{0}{10}\right)$

$\boxed{\phi = \frac{\pi}{2}}$

$\left(10, -\frac{\pi}{6}, \frac{\pi}{2}\right)$

Q7

a) $\left(5, \frac{\pi}{6}, \frac{\pi}{4}\right)$

$\rho = 5 \quad \theta = \frac{\pi}{6} \quad \phi = \frac{\pi}{4}$

$x = \rho \sin\phi \cos\theta$

$x = 5\sin\left(\frac{\pi}{4}\right)\cos\left(\frac{\pi}{6}\right)$

$x = \frac{5\sqrt{6}}{4}$

$y = \rho \sin\phi \sin\theta$

$= 5\sin(\pi/4)\sin(\pi/6)$

$y = \frac{5\sqrt{2}}{4}$

$z = \rho\cos\phi$

$= 5\cos\left(\frac{\pi}{4}\right)$

$z = \frac{5\sqrt{2}}{2}$

$\boxed{\left(\frac{5\sqrt{6}}{4}, \frac{5\sqrt{2}}{4}, \frac{5\sqrt{2}}{2}\right)}$

b) $(7, 0, \pi/2)$

$\rho = 7 \quad \theta = 0 \quad \phi = \pi/2$

$x = \rho \sin\phi \cos\theta$

$x = 7 \sin\frac{\pi}{2} \cos 0$

$x = 7$

$y = \rho \sin\phi \sin\theta$

$y = 0$

$z = \rho \cos\phi$

$= 7 \cos \pi/2$

$z = 0$

$(7, 0, 0)$

c) $(1, \pi, 0)$

$\rho = 1, \quad \theta = \pi \quad \phi = 0$

$x = \rho \sin\phi \cos\theta \qquad x = 0$

$y = \rho \sin\phi \sin\theta \qquad y = 0$

$z = \rho \cos\phi$

$= 1 \cos 0$

$z = 1$

$(0, 0, 1)$

Date

d) $(2, 3\pi/2, \pi/2)$

$\rho = 2 \quad \theta = 3\pi/2 \quad \phi = \pi/2$

$x = \rho \sin\phi \cos\theta$

$= 2\sin(\frac{\pi}{2})\cos(\frac{3\pi}{2})$

$x = 0$

$y = \rho \sin\phi \sin\theta$

$= 2\sin(\pi/2)\sin(3\pi/2)$

$y = -2$

$z = \rho\cos\phi$

$z = 2\cos\frac{\pi}{2}$

$z = 0$

$(0, -2, 0)$

Q9

a) $(\sqrt{3}, \frac{\pi}{6}, 3)$

$r = \sqrt{3}, \quad \theta = \pi/6, \quad z = 3$

$\rho = \sqrt{r^2 + z^2}$

$\rho = 2\sqrt{3}$

$\phi = \tan^{-1}\left(\frac{r}{z}\right)$

$\phi = \tan^{-1}\left(\frac{\sqrt{3}}{3}\right)$

$\phi = \frac{\pi}{6} \qquad \theta = \frac{\pi}{6}$

$\boxed{(2\sqrt{3}, \frac{\pi}{6}, \frac{\pi}{6})}$

No.

b) $(1, \pi/4, -1)$

$r = 1 \quad \theta = \pi/4 \quad z = -1$

$\rho = \sqrt{r^2 + z^2}$

$\rho = \sqrt{(1)^2 + (-1)^2}$

$\rho = \sqrt{2}$

$\phi = \tan^{-1}\left(\frac{r}{z}\right)$

$\phi = \tan^{-1}\left(\frac{1}{-1}\right)$

$\phi = 7\pi/4$

$\theta = \pi/4$

$(\sqrt{2}, \pi/4, 7\pi/4)$

c) $(2, 3\pi/4, 0)$

$r = 2, \quad \theta = 3\pi/4 \quad z = 0$

$\rho = \sqrt{r^2 + z^2}$

$\rho = 2$

$\phi = \tan^{-1}\left(\frac{r}{z}\right)$

$\phi = \pi/2$

$\theta = 3\pi/4$

$(2, \frac{3\pi}{4}, \pi/2)$

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Date

d) $(6, 1, -2\sqrt{3})$

$r = 6, \quad \theta = 1 \qquad z = -2\sqrt{3}$

$\rho = \sqrt{r^2 + z^2}$

$\rho = 4\sqrt{3}$

$\phi = \tan^{-1}\left(\frac{r}{z}\right)$

$\phi = 5\pi/3$

$\theta = 1$

$(4\sqrt{3}, 1, 5\pi/3)$

Q11

a) $(5, \pi/4, 2\pi/3)$

$\rho = 5 \qquad \theta = \pi/4 \qquad \phi = 2\pi/3$

$r = \rho \sin\phi$

$r = 5\sin(2\pi/3)$

$r = \frac{5\sqrt{3}}{2}$

$\theta = \pi/4$

$z = \rho\cos\phi$

$= 5\cos(2\pi/3)$

$z = -\frac{5}{2}$

$\left(\frac{5\sqrt{3}}{2}, \frac{\pi}{4}, \frac{-5}{2}\right)$

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b) $\left(1, \frac{7\pi}{6}, \pi\right)$

$\rho = 1$, $\theta = 7\pi/6$ $\quad \phi = \pi$

$r = \rho \sin \phi$

$= 1 \sin(\pi)$

$r = 0$

$\theta = \frac{7\pi}{6}$

$\phi\ z = \rho \cos \phi$

$= 1 \cos(\pi)$

$z = -1$

$\left(0, \frac{7\pi}{6}, -1\right)$

c) $(3, 0, 0)$

$\rho = 3$, $\theta = 0$, $\phi = 0$

$r = \rho \sin \phi$

$= 3 \sin 0$

$r = 0$

$\theta = 0$

$z = \rho \cos \phi$

$= 3 \cos 0$

$z = 3$

$(0, 0, 3)$

Date

d) $(4, \pi/6, \pi/2)$

$\rho = 4, \quad \theta = \pi/6 \quad \phi = \pi/2$

$r = \rho \sin\phi$

$= 4\sin(\pi/2)$

$r = 4$

$\theta = \pi/6$

$z = \rho \cos\phi$

$= 4\cos(\pi/2)$

$z = 0$

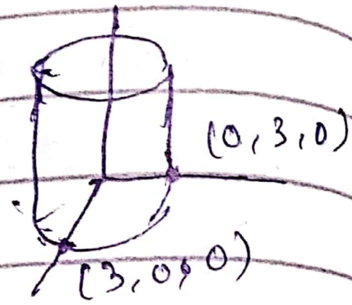
$(4, \pi/6, 0)$

Q19

$$r = 3$$

$$\sqrt{x^2+y^2} = 3$$

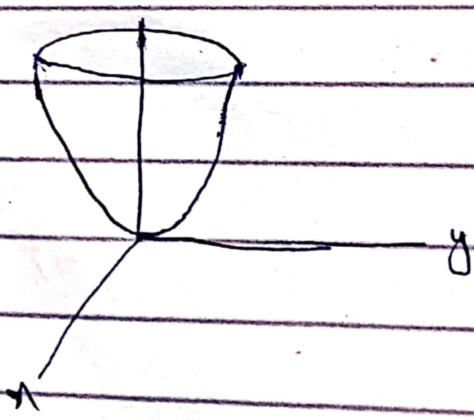
$$x^2+y^2 = 9$$



Q21

$$z = r^2$$

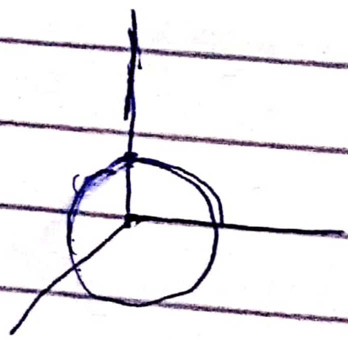
$$z = x^2+y^2$$



Q25

$$r^2+z^2 = 1$$

$$x^2+y^2+z^2 = 1$$



Date

Q29

$$\phi = \pi/4$$

$$\cos\phi = \frac{z}{\sqrt{x^2+y^2+z^2}}$$

$$\cos\left(\frac{\pi}{4}\right) = \frac{z}{\sqrt{x^2+y^2+z^2}}$$

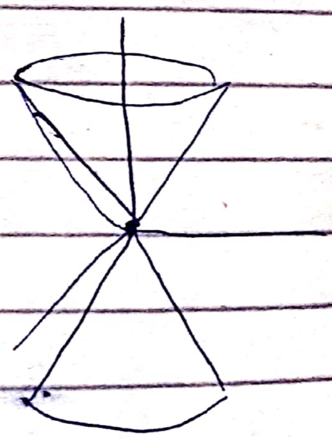
$$\frac{\sqrt{2}}{2} = \frac{z}{\sqrt{x^2+y^2+z^2}}$$

$$\frac{2}{4} = \frac{z^2}{x^2+y^2+z^2}$$

$$2z^2 = x^2+y^2+z^2$$

$$x^2+y^2-z^2=0$$

$$z = \sqrt{x^2+y^2}$$



No.

Q33

$\rho \sin\phi = 2\cos\theta$

$\cancel{\rho \sin\phi \cos\theta = 2\cos^2\theta}$

$\rho^2 \sin^2\phi = 2\rho \sin\phi \cos\theta$ $\quad \because \rho \sin\phi \sin\theta = y$

$\rho^2 \sin^2\phi = 2x$ $\quad \because \rho \sin\phi \cos\theta = x$

$\rho^2 \sin^2\phi \, (\sin^2\theta + \cos^2\theta) = 2x$

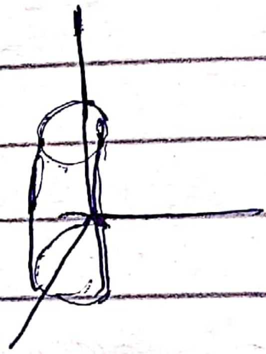
$\rho^2 \sin^2\phi \sin^2\theta + \rho^2 \sin^2\phi \cos^2\theta = 2x$

$y^2 + x^2 = 2x$

$x^2 - 2x + y^2 = 0$

$x^2 - 2x + 1 + y^2 = 1$

$\boxed{(x-1)^2 + y^2 = 1}$



Q41

$x^2 + y^2 + z^2 = 9$

a) cylinderical Co-ordinates.

$\cancel{r^2 = 9}$ $\Rightarrow \boxed{r^2 + z^2 = 9}$

$\boxed{r = 3}$

b) Spherical Co-ordinates

$\rho^2 = 9$

$\boxed{\rho = 3}$

Date

Q 45

$x^2 = 16 - z^2$

a) Cylinderical Co-ordinates:

$r^2\cos^2\theta = 16 - z^2$

b) Spherical Coordinates:

$\cancel{\rho^2\sin^2\phi\cos^2\theta = 16 - \rho^2\cos^2\phi}$

$\cancel{\rho^2\sin^2\phi\cos^2\theta + \rho^2\cos^2\theta = 16}$

$\cancel{\rho^2(\sin^2\phi\cos^2\theta + \cos^2\theta) = 16}$

$x^2 + z^2 = 16$

$x^2 + z^2 + y^2 = 16 + y^2$

$\rho^2 = 16 + \rho^2\sin^2\phi\sin^2\theta$

$\rho^2 - \rho^2\sin^2\phi\sin^2\theta = 16$

$$\boxed{\rho^2(1 - \sin^2\phi\sin^2\theta) = 16}$$

Exercise 12.1

Assignment 16

Date

Q1

Domain:-

$r(t) = \cos t\, i - 3t\, j$

$\cos t \qquad (-\infty, +\infty)$

$-3t \qquad (-\infty, +\infty)$

Intersection of above domains

$(-\infty, +\infty)$

value of $r(t_0)$:

$t_0 = \pi$

$r(t_0) = (\cos\pi)i - 3(\pi)j$

$r(t_0) = -i - 3\pi j$

Q2

Domain:

$r(t) = \cos\pi t\, i - \ln t\, j + \sqrt{t-2}\,k$

$\cos\pi t \qquad (-\infty, +\infty)$

$\ln t \qquad (0, +\infty)$

$\sqrt{t-2} \qquad t-2 \geq 0$

$t \geq 2 \qquad [2, +\infty)$

Date:

intersection of above domains

$[2, +\infty)$

value of $r(t_0)$

$t_0 = 3$

$r(t_0) = \cos 3\pi\, i - \ln(3)\, j + \sqrt{1}\, k$

$r(t_0) = -i - \ln 3\, j + k$

Q5

$x = 3\cos t\,, \quad y = t + \sin t$

$x(t) = 3\cos t \quad y(t) = t + \sin t$

$r = x(t)\, i + y(t)\, j$

$$\boxed{r = 3\cos t\, i + (t + \sin t)\, j}$$

Q7

$r = 3t^2 i - 2j$

$r = x(t) i - y(t) j$

$x = 3t^2 \quad y = -2$

Date

Q9

$r = (3-2t)i + 5tj$

$x = 3-2t$

$y = 0 + 5t.$

A line in 2 space L is passing through point (3,0) and parallel to the vector $-2i + 5j$

Q11

$r = 2ti - 3j + (1+3t)k$

$x = 0 + 2t$

$y = -3 + 0t$

$z = 1 + 3t$

A line in 3-space passing through (0,-3,1) and parallel to the vector $2i + 3k$

Exercise 12.2

Assignment 17

Date

Q1

$$\lim_{t\to\infty}\left\langle \frac{t^2+1}{3t^2+2}, \frac{1}{t}\right\rangle$$

$$\left\langle \lim_{t\to\infty}\frac{t^2+1}{3t^2+2}, \lim_{t\to\infty}\frac{1}{t}\right\rangle$$

$$\left\langle \lim_{t\to\infty}\frac{\cancel{t^2}(1+\frac{1}{t^2})}{\cancel{t^2}(3+\frac{2}{t^2})}, \lim_{t\to\infty}\frac{1}{t}\right\rangle$$

Apply limit

$$\left\langle \frac{1}{3}, 0\right\rangle$$

Q2

$$\lim_{t\to0}\left(\sqrt{t}\,i+\frac{\sin t}{t}\,j\right)$$

Apply first hopital rule

$$\lim_{t\to0}\left(\sqrt{t}\,i+\frac{\cos t}{1}\,j\right)$$

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Applying Limit

(0,1)

Q4

$$\lim_{t \to 1} \left\langle \frac{3}{t^2}, \frac{\ln t}{t^2-1}, \sin 2t \right\rangle$$

Apply First Hopital Rule

$$\lim_{t \to 1} \left\langle \frac{3}{t^2}, \frac{1}{2t^2}, \sin 2t \right\rangle$$

Apply limit

$$\left(\frac{3}{1}, \frac{1}{2}, \sin 2 \right)$$

Date

Q5

a) $r(t) = (3\sin t)i - (2t)j$

$t = 0$

1) $r(t)$ must be defined:

$r(0) = \langle 3\sin(0), -2(0) \rangle$

$\langle 0, 0 \rangle \rightarrow (1)$

2) $\lim_{t \to 0} r(t)$ must exist:

$\lim_{t \to 0} 3\sin t - 2t$

$0 \rightarrow (2)$

3) $\lim_{t \to 0} r(t) \stackrel{?}{=} r(t)$

$(1) = (2)$

The function is continuous

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b) $r(t) = t^2 i + \frac{1}{t} j + tk$

$t = 0$

i) $r(0)$ must be defined

$= (0)^2 i + \frac{1}{0} j + 0k$

$r(0) =$ UNDEFINED

The function is not continuous.

Q9

$r(t) = 4i - \cos t j$

$r'(t) = 0i - (\sin t) j$

$\boxed{r'(t) = \sin t \, j}$

Q11

$r(t) = \langle t, t^2 \rangle \quad t_0 = 2$

$r'(t) = \langle 1, 2t \rangle$

$r'(t_0) = \langle 1, 2(2) \rangle$

$r'(t_0) = \langle 1, 4 \rangle$

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Q13

$r(t) = \sec t\, i + \tan t\, j - \sqrt{t}\, k$

$r'(t) = \sec t \tan t\, i + \sec^2 t\, j - \frac{1}{2\sqrt{t}} k$ $\quad -\frac{1}{2}(t)^{-\frac{1}{2}} k$

$t_0 = 0$

$r'(t_0) = 0i + 1j - 0k$

$\boxed{r'(t_0) = j}$

Q15

$r(t) = 2\sin t\, i + j + 2\cos t\, k$

$r'(t) = 2\cos t\, i + 0j - 2\sin t\, k$

$t_0 = \pi/2$

$r'(t_0) = 0 + 0 - 2k$

$r'(t_0) = -2k$

Q19

$r_0 = r(t_0) = t_0^2 i + (2 - \ln t_0) j$

$v_0 = r'(t_0) = 2t_0 i - \frac{1}{t_0} j$

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$\therefore r = r_0 + t v_0$

$= \left(t_0^2 i + (2 - \ln t_0) j\right) + \left(2t_0 i - \frac{1}{t_0} j\right)t$

$= \left(t_0^2 i + (2 - \ln t_0) j\right) + \left(2 t t_0 i - \frac{t}{t_0} j\right)$

$\langle x, y \rangle = \left(t_0^2 + 2t\, t_0\right) i + \left(2 - \ln t_0 - \frac{t}{t_0}\right) j$

$x = t_0^2 + 2t\, t_0$

$y = 2 - \ln t_0 - \frac{t}{t_0}$

$t_0 = 1$

$x = 1 + 2t$

$y = 2 - t$

Q21

$r_0 = r(t_0) = 2\cos \pi t_0\, i + 2 \sin \pi t_0\, j + 3 t_0 k$

$v_0 = r'(t_0) = -2\pi \sin \pi t_0\, i + 2\pi \cos \pi t_0\, j + 3k$

$r = r_0 + t v_0$

$\langle x, y, z \rangle = 2\cos \pi t_0\, i + 2 \sin \pi t_0\, j + 3t_0 k - 2t\pi \sin \pi t_0\, i + 2t\pi \cos \pi t_0\, j + 3tk$

$x = 2\cos \pi t_0 - 2t\pi \sin \pi t_0$

$y = 2 \sin \pi t_0 + 2t\pi \cos \pi t_0$

$z = 3t_0 + 3t$

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$$t_0 = \frac{1}{3}$$

$x = 1 - \sqrt{3}\pi t$

$y = \sqrt{3} + t\pi$

$z = 1 + 3t$

Q23

$r(t) = (2t-1)i + \sqrt{3t+4}\,j \qquad P_0(-1,2)$

$2t - 1 = -1$

$\sqrt{3t+4} = 2$

solve for t.

$\boxed{t=0}$

$$v_0 = r'(t) = 2i + \frac{3}{2\sqrt{3t+4}}j$$

at $t = 0$

$v_0 = 2i + \frac{3}{4}j$

$r_0 = -i + 2j$

$r = r_0 + t v_0$

$$\boxed{r = (-i + 2j) + (2i + \frac{3}{4}j)t}$$

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Q25

$$r(t) = t^2 i - \frac{1}{t+1} j + (4-t^2) k \qquad P_0(4,1,0)$$

$$t^2 = 4$$

$$-\frac{1}{t+1} = 1$$

$$4 - t^2 = 0$$

~~S~~solve for t

$$t = -2$$

$$r'(t) = 2t i + \frac{1}{(t+1)^2} j - 2t k$$

at $t = -2$

$$r_0 = r(-2) = 4i + j$$

$$V_0 = r'(-2) = -4i + j + 4k$$

$$r = r_0 + v_0 t$$

$$r = (4i + j) + (-4i + j + 4k) t$$

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Q27

$r(t) = \cos t\, i + \sin t\, j + k$

$r'(t) = -\sin t\, i + \cos t\, j$

a) $\cancel{r'(t)} =$

$\lim_{t \to 0} \left[(\cos t\, i + \sin t\, j + k) - (-\sin t\, i + \cos t\, j + 0k) \right]$

$\lim_{t \to 0} \left[(\cos t + \sin t) i + (\sin t + \cos t) j + k \right]$

$\lim_{t \to 0} (\cos t + \sin t) i + \lim_{t \to 0} (\sin t + \cos t) j + \lim_{t \to 0} (k)$

$= i + j + k$

b) $\lim (r(t) \times r'(t))$

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b) $\lim_{t \to 0} [r(t) \times r'(t)]$

$r(t) \times r'(t) = \begin{vmatrix} i & j & k \\ \cos t & \sin t & 1 \\ -\sin t & \cos t & 0 \end{vmatrix}$

$= -\cos t\, i - \sin t\, j + k$

$\lim_{t \to 0} -\cos t\, i - \lim_{t \to 0} \sin t\, j + \lim_{t \to 0} k$

$\boxed{-i + k}$

c) $\lim_{t \to 0} [r(t) \cdot r'(t)]$

$r(t) \cdot r'(t) = (\cos t\, i + \sin t\, j + k) \cdot (-\sin t\, i + \cos t\, j + 0k)$

$= -\cos t \sin t\, i + \sin t \cos t\, j + 0k$

$\lim_{t \to 0} -\cos t \sin t\, i + \lim_{t \to 0} \sin t \cos t\, j$

$\boxed{0}$

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Q29

$r_1(t) = 2ti + 3t^2 j + t^3 k$

$r_2(t) = t^4 k$

$\frac{d}{dt}[r_1(t) \cdot r_2(t)] = r_1(t) \cdot \frac{dr_2}{dt} + \frac{dr_1}{dt} \cdot r_2(t)$

$= (2ti + 3t^2 j + t^3 k) \cdot 4t^3 k + (2i + 6tj + 3t^2 k)(t^4 k)$

$= 4t^6 + 3t^6$

$= \boxed{7t^6}$

$\frac{d}{dt}[r_1(t) \times r_2(t)] = r_1(t) \times \frac{dr_2}{dt} + \frac{dr_1}{dt} \times r_2(t)$

$= (2ti + 3t^2 j + t^3 k) \times 4t^3 k + (2i + 6tj + 3t^2 k) \times t^4 k$

$\begin{vmatrix} i & \hat{j} & k \\ 2t & 3t^2 & t^3 \\ 0 & 0 & 4t^3 \end{vmatrix} = 12t^5 i + 8t^4 j$

$\begin{vmatrix} i & \hat{j} & k \\ 2t & 6t & 3t^2 \\ 0 & 0 & t^4 \end{vmatrix} = \langle 6t^5 i + 2t^4 j \rangle$

$= 12t^5 i + 8t^4 j + 6t^5 i + 2t^4 j$

$= 18t^5 i + 10t^4 j$

$$\int u \cdot v \, dx = u \int v \, dx - \int \left(u' \int v \, dx\right) dx$$

$$\int u \cdot v = u \int v \, dx - \int \left(u' \int v \, dx\right) dx$$

Q31

$$\int (3i + 4tj) \, dt$$

$$3ti + 2t^2 j + C$$

Q33

$$\int \langle te^t, \ln t \rangle \, dt$$

$$\left\langle \int \overset{u}{t} \overset{v}{e^t}, \int \overset{u}{\ln t} \times \overset{v}{1} \right\rangle$$

$$\left\langle t \int e^t - \int \frac{d(t)}{dt} \int e^t, \ \ln t \int 1 - \int \frac{1}{t} \int 1 \right\rangle$$

$$\langle te^t - e^t, \ t\ln t - t \rangle$$

$$\langle e^t(t-1), \ t(\ln t - 1) \rangle$$

Q35

$$\int_0^{\frac{\pi}{2}} \langle \cos 2t, \sin 2t \rangle \, dt$$

$$\left\langle \frac{\sin 2t}{2}, -\frac{\cos 2t}{2} \right\rangle \Bigg|_0^{\frac{\pi}{2}}$$

$$-\left(\frac{\cos \pi}{2} - \frac{\cos 0}{2}\right)$$

$$-\left(-\frac{1}{2} - \frac{1}{2}\right)$$

$$-(-1) = 1$$

$$\langle 0, 1 \rangle$$

No. ______

Q 37

$$\int_0^2 \| ti + t^2 j \| \, dt$$

$$\int_0^2 \sqrt{t^2 + t^4} \, dt$$

$$\int_0^2 t\sqrt{1+t^2} \, dt$$

let

$u = 1 + t^2$

$\frac{du}{dt} = 2t$

$\frac{du}{2} = t\,dt$

$$\int_0^2 \sqrt{u} \, \frac{du}{2}$$

$$\frac{1}{2} \frac{u^{3/2}}{3/2} \Big|_0^2$$

$$= \frac{u^{3/2}}{3} \Big|_0^2$$

$$= \frac{(1+t^2)^{3/2}}{3} \Big|_0^2$$

$$\frac{1}{3}\left[(1+2^2)^{\frac{3}{2}} - (1+0)^{3/2} \right]$$

$$\frac{1}{3}\left(\sqrt{5^3} - 1 \right)$$

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$$\boxed{\frac{5\sqrt{5}-1}{3}}$$

Q39

$$\int_1^9 \left(t^{\frac{1}{2}} i + t^{-\frac{1}{2}} j\right) dt$$

$$\frac{2t^{\frac{3}{2}}}{3} i + 2t^{\frac{1}{2}} j \Big|_1^9$$

$$\cancel{2\left[\left(\frac{(9)^{\frac{3}{2}}}{3} i + 2(9)^{\frac{1}{2}} j\right) - \left(2(1)^{\frac{3}{2}} i + 2(1)^{\frac{1}{2}} j\right)\right]}$$

$$\cancel{2(15-4)}$$

$$\cancel{\boxed{22}}$$

$$(18i + 6j) - \left(\frac{2}{3} i + 2j\right)$$

$$\boxed{\frac{52}{3} i + 4j}$$

Assignment # 13

Exercise 13.1

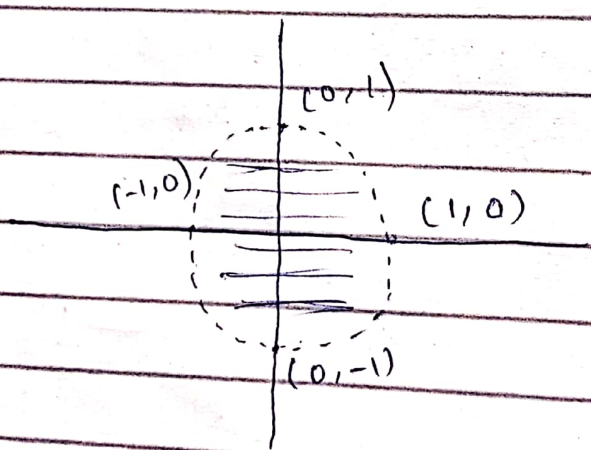
Q23

$f(x,y) = \ln(1-x^2-y^2)$

$1-x^2-y^2 \geq 1$ $\qquad$ $1-x^2-y^2 > 0$

$0 \geq x^2+y^2$ $\qquad$ $x^2+y < 1$

$x^2+y^2 \leq 0$



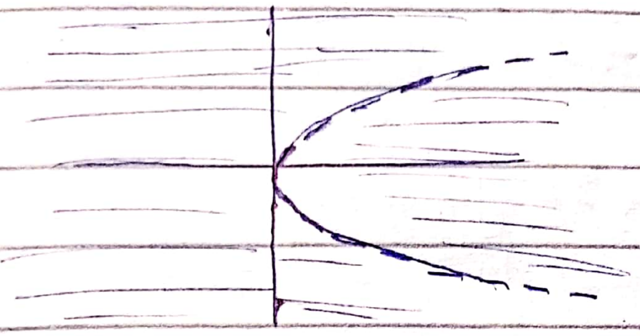
Q25

$$f(x,y) = \frac{1}{x-y^2}$$

$x-y^2 \neq 0$

$x \neq y^2$

Date:



Q27

a) $f(x,y) = xe^{-\sqrt{y+2}}$

$y+2 \geq 0$

$y \geq -2$

All the points above or on the line

$y = -2$

b) $f(x,y,z) = \sqrt{25 - x^2 - y^2 - z^2}$

$25 - x^2 - y^2 - z^2 \geq 0$

$x^2 + y^2 + z^2 \leq 25$

All the points inside or on the sphere $x^2+y^2+z^2 = 25$

c) $f(x,y,z) = e^{xyz}$

All points in three space.

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Exercise 13.3

Q5

~~$z = \sin(5x^3y + 7xy^2)$~~

~~$\frac{\partial z}{\partial x} = \cos(5x^3y + 7xy^2)\frac{\partial}{\partial x}(5x^3y +$~~

$$z = (x^2 + 5x - 2y)^8$$

$$\frac{\partial z}{\partial x} = 8(x^2+5x-2y)^7 \frac{\partial}{\partial x}(x^2+5x-2y)$$

$$= 8(x^2+5x-2y)^7(2x+5)$$

$$\boxed{\frac{\partial z}{\partial x} = (16x+40)(x^2+5x-2y)^7}$$

$$\frac{\partial z}{\partial y} = 8(x^2+5x-2y)^7 \frac{\partial}{\partial y}(x^2+5x-2y)$$

$$\boxed{\frac{\partial z}{\partial y} = -16(x^2+5x-2y)^7}$$

Q9

$$z = \sin(5x^3y + 7xy^2)$$

$$\frac{\partial z}{\partial x} = \cos(5x^3y + 7xy^2)\frac{\partial}{\partial x}(5x^3y + 7xy^2)$$

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$$\boxed{\frac{\partial z}{\partial x} = (15x^2y + 7y^2)\left(\cos(5x^3y + 7xy^2)\right)}$$

$$\frac{\partial z}{\partial y} = \cos(5x^3y + 7xy^2)\frac{\partial}{\partial y}(5x^3y + 7xy^2)$$

$$= (5x^3 + 14xy)\cos(5x^3y + 7xy^2)$$

Q 13

$$z = \sin(y^2 - 4x)$$

a)

$$\frac{\partial z}{\partial x} = \cos(y^2 - 4x)\frac{\partial}{\partial x}(y^2 - 4x)$$

$$\frac{\partial z}{\partial x} = \cos(y^2 - 4x) \times -4$$

point $(2, 1)$

$$= -4\cos(1^2 - 4(2))$$

$$\boxed{\frac{\partial z}{\partial x} = -4\cos 7}$$

b)

$$\frac{\partial z}{\partial y} = \cos(y^2 - 4x)\frac{\partial}{\partial y}(y^2 - 4x)$$

No.

$$\frac{\partial z}{\partial y} = 2y\cos(y^2-4x)$$

point $(2,1)$

$$\boxed{\frac{\partial z}{\partial y} = 2\cos 7}$$

Q27

$$z = x^3 \ln(1+xy^{-3/5})$$

$$\frac{\partial z}{\partial x} = x^3 \times \frac{1}{1+xy^{-3/5}} \frac{\partial}{\partial x}(1+xy^{-3/5}) + 3x^2\ln(1+xy^{-\frac{3}{5}})$$

$$= \frac{x^3y^{-3/5}}{1+xy^{-3/5}} + 3x^2\ln(1+xy^{-3/5})$$

$$\boxed{\frac{\partial z}{\partial x} = \frac{x^3}{y^{3/5}+x} + 3x^2\ln(1+xy^{-3/5})}$$

$$\frac{\partial z}{\partial y} = \frac{x^3}{1+xy^{-3/5}} \frac{\partial}{\partial y}(1+xy^{-3/5})$$

$$= \frac{x^3}{1+xy^{-\frac{3}{5}}} \times -\frac{3}{5}xy^{-8/5}$$

$$\boxed{\frac{\partial z}{\partial y} = \frac{-(3/5)x^2}{y^{8/5}+xy}}$$

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Q29

$$z = \frac{xy}{x^2+y^2}$$

$$\frac{\partial z}{\partial x} = \frac{(x^2+y^2)\frac{\partial}{\partial x}(xy) - xy\frac{\partial}{\partial x}(x^2+y^2)}{(x^2+y^2)^2}$$

$$= \frac{x^2y + y^3 - 2x^2y}{\cancel{x^4+2x^2y^2+y^4}\ (x^2+y^2)^2}$$

$$= \frac{y^3 - x^2y}{(x^2+y^2)^2}$$

$$= \frac{y(y^2-x^2)}{(x^2+y^2)^2}$$

$$\boxed{\frac{\partial z}{\partial x} = \frac{-y(x^2-y^2)}{(x^2+y^2)^2}}$$

$$\frac{\partial z}{\partial y} = \frac{x^2+y^2\frac{\partial}{\partial y}(xy) - xy\frac{\partial}{\partial y}(x^2+y^2)}{(x^2+y^2)^2}$$

$$= \frac{x^3 + xy^2 - 2xy^2}{(x^2+y^2)^2}$$

$$= \frac{x^3 - xy^2}{(x^2+y^2)^2}$$

$$\boxed{\frac{\partial z}{\partial y} = \frac{x(x^2-y^2)}{(x^2+y^2)^2}}$$

Q31

$$f(x,y) = \sqrt{3x^5y - 7x^3y}$$

$$f_x(x,y) = \frac{\frac{\partial}{\partial x}(3x^5y - 7x^3y)}{2\sqrt{3x^5y - 7x^3y}}$$

$$= \frac{15x^4y - 21x^2y}{2\sqrt{3x^5y - 7x^3y}}$$

$$\boxed{f_x(x,y) = \left(\frac{3}{2}\right)x^2y\,(5x^2-7)(3x^5y - 7x^3y)^{-\frac{1}{2}}}$$

$$f_y(x,y) = \frac{1}{2}(3x^5y - 7x^3y)^{-\frac{1}{2}} \frac{\partial}{\partial y}(3x^5y - 7x^3y)$$

$$= \frac{1}{2}(3x^5y - 7x^3y)^{-\frac{1}{2}}(3x^5 - 7x^3)$$

$$\boxed{f_y(x,y) = \frac{1}{2}x^3(3x^2-7)(3x^5y - 7x^3y)^{-\frac{1}{2}}}$$

Q33

$$f(x,y) = y^{-\frac{3}{2}} \tan^{-1}(x/y)$$

$$f_x(x,y) = y^{-\frac{3}{2}} \frac{1}{1+\left(\frac{x}{y}\right)^2} \times \frac{1}{y}$$

$$= \frac{y^2 \times y^{-\frac{3}{2}} \times y^{-1}}{y^2+x^2}$$

Date

$$f_x(x,y) = \frac{y^{-\frac{1}{2}}}{y^2+x^2}$$

Q39

$$z = \sqrt{x^2+4y^2}$$

$$\frac{\partial z}{\partial x} = \frac{1}{2}(x^2+4y^2)^{-1/2}\frac{\partial}{\partial x}(x^2+4y^2)$$

$$\frac{\partial z}{\partial x} = \frac{x}{\sqrt{x^2+4y^2}}$$

$$\cancel{\frac{\partial z}{\partial x}\left(\frac{1}{2}\right)} \quad \frac{\partial z}{\partial x}(1,2) = \frac{1}{\sqrt{1^2+4(2)^2}}$$

$$\boxed{\frac{\partial z}{\partial x}(1,2) = \frac{1}{\sqrt{17}}}$$

$$\frac{\partial z}{\partial y} = \frac{1}{2}(x^2+4y^2)^{-\frac{1}{2}} \times 8y$$

$$\frac{\partial z}{\partial y} = \frac{4y}{\sqrt{x^2+4y^2}}$$

$$\frac{\partial z}{\partial y}(1,2) = \frac{4(2)}{\sqrt{1^2+4(2)^2}}$$

$$\boxed{\frac{\partial z}{\partial y}(1,2) = \frac{8}{\sqrt{17}}}$$

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Q41

$$f(x,y,z) = x^2y^4z^3 + xy + z^2 + 1$$

a) $f_x(x,y,z)$

$$f_x(x,y,z) = 2xy^4z^3 + y$$

b) $f_y(x,y,z) = 4x^2y^3z^3 + x$

c) $f_z(x,y,z) = 3x^2y^4z^2 + 2z$

d) $f_x(1,y,z) = 2y^4z^3 + y$

e) $f_y(1,2,z) = 4(1)^2(2)^3z^3 + 1$

$= 32z^3 + 1$

f) $f_z(1,2,3) = 3(1)^2(2)^4(3)^2 + 2(3)$

$= 438$

Q43

$$f(x,y,z) = z\ln(x^2y\cos z)$$

$$f_x = \frac{z}{x^2y\cos z} \times \frac{\partial}{\partial x}(\cos z\, x^2y)$$

$$= \frac{2\cancel{x}yz\cos z}{x^2y\cos z}$$

x^{-1}

$-1\, x^{-2}$

$-\frac{1}{x^2}$

Date

$$\boxed{f_x = \frac{\partial z}{x}}$$

$$f_y = \frac{z}{x^2 y \cos z} \frac{\partial}{\partial y}(x^2 y \cos z)$$

$$= \frac{\cancel{x^2} z \cancel{\cos} z}{\cancel{x^2} y \cancel{\cos} z}$$

$$\boxed{f_y = \frac{z}{y}}$$

$$f_z = z \times \frac{1}{x^2 y \cos z} \frac{\partial}{\partial z}(x^2 y \cos z) + \ln(x^2 y \cos z)$$

$$= \frac{-\cancel{x^2}\cancel{y}\, z \sin z}{\cancel{x^2}\cancel{y} \cos z} + \ln(x^2 y \cos z)$$

$$\boxed{f_z = \ln(x^2 y \cos z) - z \tan z}$$

Q45

$$f(x,y,z) = \tan^{-1}\left(\frac{1}{xy^2z^3}\right)$$

$$f_x = \frac{1}{1+\left(\frac{1}{xy^2z^3}\right)^2}\,\frac{\partial}{\partial x}\left(\frac{1}{xy^2z^3}\right)$$

$$= \frac{x^2y^4z^6}{x^2y^4z^6+1} \times \frac{-1}{x^2y^2z^3}$$

$$\boxed{f_x = \frac{-y^2z^3}{1+x^2y^4z^6}}$$

$$f_y = \frac{1}{1+\left(\frac{1}{xy^2z^3}\right)^2} \times \frac{\partial}{\partial y}\left(\frac{1}{xy^2z^3}\right)$$

$$= \frac{x^2y^4z^6}{x^2y^4z^6+1} \times \frac{-2}{xy^3z^3}$$

$$\boxed{f_y = \frac{-2xyz^3}{x^2y^4z^6+1}}$$

$$f_z = \frac{1}{1+\left(\frac{1}{xy^2z^3}\right)^2} \times \frac{\partial}{\partial z}\left(\frac{1}{xy^2z^3}\right)$$

$$= \frac{x^2y^4z^6}{x^2y^4z^6+1} \times \frac{-3}{xy^2z^4}$$

Date

$$\boxed{f_z = \frac{-3xy^2z^2}{x^2y^4z^6+1}}$$

Q47

$$w = ye^z \sin(xz)$$

$$\frac{\partial w}{\partial x} = ye^z \cos(xz) \frac{\partial}{\partial x}(xz)$$

$$\boxed{\frac{\partial w}{\partial x} = yze^z \cos(xz)}$$

$$\boxed{\frac{\partial w}{\partial y} = e^z \sin(xz)}$$

$$\frac{\partial w}{\partial z} = y\left[e^z \cos(xz) \cdot x + \sin(xz)\, e^z\right]$$

$$= xye^z \cos(xz) + e^z y \sin(xz)$$

$$\boxed{\frac{\partial w}{\partial z} = ye^z\left(x\cos(xz) + \sin(xz)\right)}$$

Q51

$$f(x,y,z) = y^2 e^{xz}$$

$$f_x = zy^2e^{xz}$$

$$\boxed{f_x(1,1,1) = e}$$

$$f_y = 2ye^{xz}$$

$$\boxed{f_y(1,1,1) = 2e}$$

No.

$$\cancel{f}z = xy^2 e^{xz}$$

$$\boxed{f_z(1,1,1) = e}$$

Q69

$$(x^2+y^2+z^2)^{3/2} = 1$$

$$\frac{\partial}{\partial x}\left((x^2+y^2+z^2)^{3/2}\right) = \frac{\partial}{\partial x}(1)$$

$$\frac{3}{2}(x^2+y^2+z^2)^{\frac{1}{2}}\frac{\partial}{\partial x}(x^2+y^2+z^2) = 0$$

$$\frac{3}{2}(x^2+y^2+z^2)\left(2x + 2z\frac{\partial z}{\partial x}\right) = 0$$

$$\cancel{\frac{3}{2}x^2(2x) + \frac{3}{2}x^2(2z)} =$$

$$\cancel{2x + 2z\frac{\partial z}{\partial x} = \frac{2}{3(x^2+y^2+z^2)}}$$

$$2x + 2z\frac{\partial z}{\partial x} = 0$$

$$\frac{\partial z}{\partial x} = \frac{-\cancel{2}x}{+\cancel{2}z}$$

$$\boxed{\frac{\partial z}{\partial x} = \frac{-x}{z}}$$

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$$\frac{\partial}{\partial y}\left[(x^2+y^2+z^2)^{3/2}\right] = \frac{\partial}{\partial y}(1)$$

$$\frac{3}{2}(x^2+y^2+z^2)^{\frac{1}{2}}\frac{\partial}{\partial y}(x^2+y^2+z^2) = 0$$

$$2y + 2z\frac{\partial z}{\partial y} = 0$$

$$\boxed{\frac{\partial z}{\partial y} = \frac{-y}{z}}$$

Q75

$$w^2 + w\sin xyz = 1$$

$$\frac{\partial}{\partial x}(w^2 + w\sin xyz) = \frac{\partial}{\partial x}(1)$$

$$2w\frac{\partial w}{\partial x} + w\cos(xyz)\cdot yz + \frac{\partial w}{\partial x}\sin xyz = 0$$

$$\frac{\partial w}{\partial x}(2w + \sin xyz) = -wyz\cos(xyz)$$

$$\boxed{\frac{\partial w}{\partial x} = \cancel{-wxyz}\ \frac{-wyz\cos(xyz)}{2w + \sin(xyz)}}$$

$$\frac{\partial}{\partial y}(w^2 + w\sin(xyz)) = \frac{\partial}{\partial y}(1)$$

$$2w\frac{\partial w}{\partial y} + w\cos(xyz)\cdot xz + \frac{\partial w}{\partial y}\sin xyz = 0$$

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$$\frac{\partial w}{\partial y}(2w+\sin xyz) = -wxz\cos(xyz)$$

$$\boxed{\frac{\partial w}{\partial y} = \frac{-wxz\cos(xyz)}{2w+\sin xyz}}$$

$$\frac{\partial}{\partial z}(w^2+w\sin xyz) = \frac{\partial}{\partial z}(1)$$

$$2w\frac{\partial w}{\partial z} + \frac{\partial w}{\partial z}\sin(xyz) + w\cos(xyz)\cdot xy = 0$$

$$\frac{\partial w}{\partial z}(2w+\sin xyz) = -wxy\cos xyz$$

$$\boxed{\frac{\partial w}{\partial z} = \frac{-wxy\cos xyz}{2w+\sin xyz}}$$

Q81

$$z = \sqrt{x}\cos y$$

a)

$$\cancel{\frac{\partial z}{\partial x} = \frac{\cos y}{2\sqrt{x}}}$$

$$\cancel{\frac{\partial^2 z}{\partial x^2} = \cos y \cdot \sqrt{x}}$$

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a)

$$\frac{\partial z}{\partial x} = \frac{1}{2}(x)^{-\frac{1}{2}} \cos y$$

$$\frac{\partial^2 z}{\partial x^2} = -\frac{1}{4}(x)^{-\frac{3}{2}} \cos y$$

$$\boxed{\frac{\partial^2 z}{\partial x^2} = -\frac{1}{4x^{3/2}} \cos y}$$

b)

$$\frac{\partial z}{\partial y} = -\sin y \cdot \sqrt{x}$$

$$\boxed{\frac{\partial^2 z}{\partial y^2} = -\cos y \sqrt{x}}$$

c)

$$\frac{\partial z}{\partial y} = -\sqrt{x} \sin y$$

$$\boxed{\frac{\partial^2 z}{\partial x \partial y} = \frac{-\sin y}{2\sqrt{x}}}$$

d)

$$\frac{\partial z}{\partial x} = \frac{\cos y}{2\sqrt{x}}$$

$$\boxed{\frac{\partial^2 z}{\partial y \partial x} = \frac{-\sin y}{2\sqrt{x}}}$$

Date

Q83

$$f(x, y) = \sin(3x^2 + 6y^2)$$

a)

$$f_x = 6x\cos(3x^2+6y^2)$$

$$f_{xx} = 6\left[-x\sin(3x^2+6y^2)6x + \cos(3x^2+6y^2)\right]$$

$$\boxed{f_{xx} = -36x\sin(3x^2+6y^2) + 6\cos(3x^2+6y^2)}$$

b)

$$f_y = 12y\cos(3x^2+6y^2).$$

$$f_{yy} = 12\left[-y\sin(3x^2+6y^2) * 12y + \cos(3x^2+6y^2)\right]$$

$$\boxed{f_{yy} = +12\cos(3x^2+6y^2) - 144y^2\sin(3x^2+6y^2)}$$

c)

$$f_x = 6x\cos(3x^2+6y^2)$$

$$\boxed{f_{xy} = -72xy\sin(3x^2+6y^2)}$$

d)

$$f_y = 12y\cos(3x^2+6y^2)$$

$$f_{yx} = -72xy\sin(3x^2+6y^2).$$

Date.

Q87

$f(x,y) = e^x \cos y$

$f_x = e^x \cos y$

$\boxed{f_{xy} = -\sin y \, e^x}$

$f_y = -\sin y \, e^x$

$\boxed{f_{yx} = -\sin y \, e^x}$

$f_{xy} = f_{yx}$

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Q91

$$f(x,y) = \frac{(x-y)}{x+y}$$

$$f_x = \frac{(x+y)\frac{\partial}{\partial x}(x-y) - (x-y)\frac{\partial}{\partial x}(x+y)}{(x+y)^2}$$

$$= \frac{\cancel{x}+y-\cancel{x}+y}{(x+y)^2}$$

$$f_x = \frac{2y}{(x+y)^2}$$

$$f_{xy} = \frac{(x+y)^2\frac{\partial}{\partial y}2y - 2y\frac{\partial}{\partial y}(x+y)^2}{(x+y)^4}$$

$$= \frac{2(x+y)^2 - 4y(x+y)}{(x+y)^4}$$

$$= \frac{\cancel{(x+y)}[2(x+y)-4y]}{(x+y)^{\cancel{4}}}$$

$$= \frac{2x+2y-4y}{(x+y)^3}$$

$$\boxed{f_{xy} = \frac{2(x-y)}{(x+y)^3}}$$

$$f_y = \frac{(x+y)\frac{\partial}{\partial y}(x-y) - (x-y)\frac{\partial}{\partial y}(x+y)}{(x+y)^2}$$

Date

$$= \frac{\cancel{x+y-x} \quad -x-\cancel{y}-x+\cancel{y}}{(x+y)^2}$$

$$f_y = \frac{-2x}{(x+y)^2}$$

$$f_{yx} = \frac{(x+y)^2 \frac{\partial}{\partial x}(-2x) - (-2x)\frac{\partial}{\partial x}(x+y)^2}{(x+y)^4}$$

$$= \frac{-2(x+y)^2 + 4x(x+y)}{(x+y)^4}$$

$$= \frac{\cancel{(x+y)}[-2(x+y)+4x]}{(x+y)^{\cancel{4}}}$$

$$= \frac{-2x-2y+4x}{(x+y)^3}$$

$$= \frac{2x-2y}{(x+y)^3}$$

$$\boxed{f_{yx} = \frac{2(x-y)}{(x+3)^3}}$$

$$f_{xy} = f_{yx} = \frac{2(x-y)}{(x+3)^3}$$

Date

95

$$f(x,y) = x^3y^5 - 2x^2y + x$$

a) f_{xxy}

$f_x = 3x^2y^5 - 4xy$

$f_{xx} = 6xy^5 - 4y$

$f_{xxy} = 30xy^4 - 4$

b) f_{yxy}

$f_y = 5x^3y^4 - 2x^2$

$f_{yx} = 15x^2y^4 - 4x$

$f_{yxy} = 60x^2y^3$

c) f_{yyy}

$f_y = 5x^3y^4 - 2x^2$

$f_{yy} = 20x^3y^3$

$f_{yyy} = 60x^3y^2$

Date

Q99

$$f(x,y,z) = x^3y^5z^7 + xy^2 + y^3z$$

a) f_{xy}

$f_x = 3x^2y^5z^7 + y^2$

$f_{xy} = 15x^2y^4z^7 + 2y$

b) f_{yz}

$f_y = 5x^3y^4z^7 + 2xy + 3y^2z$

$f_{yz} = 35x^3y^4z^6 + 3y^2$

c) f_{xz}

$f_x = 3x^2y^5z^7 + y^2$

$f_{xz} = 21x^2y^5z^6$

d) f_{zz}

$f_z = 7x^3y^5z^6 + y^3$

$f_{zz} = 42x^3y^5z^5$

e) f_{zyy}

$f_z = 7x^3y^5z^6 + y^3$

$f_{zy} = 35x^3y^4z^6 + 3y^2$

$f_{zyy} = 140x^3y^3z^6 + 6y$

Date

f) f_{xxy}

$f_x = 3x^2y^5z^7 + y^2$

$f_{xx} = 6xy^5z^7$

$f_{xxy} = 30xy^4z^7$

g) f_{zyx}

$f_z = 7x^3y^5z^6 + y^3$

$f_{zy} = 35x^3y^4z^6 + 3y^2$

$f_{zyx} = 105x^2y^4z^6$

h) f_{xxyz}

$f_x = 3x^2y^5z^7 + y^2$

$f_{xx} = 6xy^5z^7$

$f_{xxy} = 30xy^4z^7$

$f_{xxyz} = 210xy^4z^6$

Q 101

a) $z = x^2 - y^2 + 2xy$

$$\frac{\partial z}{\partial x} = 2x + 2y$$

$$\frac{\partial^2 z}{\partial x^2} = 2$$

Date

$$\frac{\partial z}{\partial y} = -2y + 2x$$

$$\frac{\partial^2 z}{\partial y^2} = -2$$

put in Laplace equation

$$\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2} = 0$$

$$2 - 2 = 0$$

$$\boxed{0 = 0}$$

b) $z = e^x \sin y + e^y \cos x$

$$\frac{\partial z}{\partial x} = e^x \sin y + -\sin x \, e^y$$

$$\frac{\partial^2 z}{\partial x^2} = e^x \sin y - \cos x \cdot e^y$$

$$\cancel{\frac{\partial z}{\partial y} = e^x \cos y - \sin x \cdot e^y}$$ $$\frac{\partial z}{\partial y} = \cos y \cdot e^x + \cos x \, e^y$$

$$\cancel{\frac{\partial^2 z}{\partial y^2} = -e^x \sin y - \cos x \, e}$$ $$\frac{\partial^2 z}{\partial y^2} = -\sin y \, e^x + \cos x \, e^y$$

put in laplace

$$(e^x \sin y - e^y \cos x) + (\cos x e^y - e^x \sin y) = 0$$

$$\boxed{0 = 0}$$

No.

c) $z = \ln(x^2+y^2) + 2\tan^{-1}(y/x)$

$$\frac{\partial z}{\partial x} = \frac{2x}{x^2+y^2} + \frac{2}{1+\left(\frac{y}{x}\right)^2} \frac{-y}{x^2}$$

$$= \frac{2x}{x^2+y^2} + \frac{2\cancel{x^2}}{x^2+y^2} \times \frac{-y}{\cancel{x^2}}$$

$$\frac{\partial z}{\partial x} = \frac{2x-2y}{x^2+y^2}$$

$$\frac{\partial^2 z}{\partial x^2} = \frac{(x^2+y^2)\frac{\partial}{\partial x}(2x-2y) - (2x-2y)\frac{\partial}{\partial x}(x^2+y^2)}{(x^2+y^2)^2}$$

$$= \frac{2(x^2+y^2) - 4x(x-y)}{(x^2+y^2)^2}$$

$$= \frac{2x^2+2y^2-4x^2+4xy}{(x^2+y^2)^2}$$

$$\frac{\partial^2 z}{\partial x^2} = \frac{2y^2-2x^2+4xy}{(x^2+y^2)^2}$$

$$\boxed{\frac{\partial^2 z}{\partial x^2} = \frac{2(y^2-x^2+2xy)}{(x^2+y^2)^2}}$$

$$\frac{\partial z}{\partial y} = \frac{2y}{x^2+y^2} + \frac{2}{1+\left(\frac{y}{x}\right)^2} \times \frac{1}{x}$$

$$= \frac{2y}{x^2+y^2} + \frac{2x^2}{x^2+y^2} \times \frac{1}{x}$$

$$\frac{\partial z}{\partial y} = \frac{2x+2y}{x^2+y^2}$$

$$\frac{\partial^2 z}{\partial y^2} = \frac{(x^2+y^2)\frac{\partial}{\partial y}(2x+2y) - (2x+2y)\frac{\partial}{\partial y}(x^2+y^2)}{(x^2+y^2)^2}$$

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$$\frac{\partial^2 z}{\partial y^2} = \frac{2(x^2+y^2) - 2y(2x+2y)}{(x^2+y^2)^2}$$

$$= \frac{2x^2 + 2y^2 - 4xy - 4y^2}{(x^2+y^2)^2}$$

$$\boxed{\frac{\partial^2 z}{\partial y^2} = \frac{2x^2 - 2y^2 - 4xy}{(x^2+y^2)^2}}$$

$$\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2} = 0$$

$$\frac{2y^2 - 2x^2 - 4xy}{(x^2+y^2)^2} + \frac{2x^2 - 2y^2 - 4xy}{(x^2+y^2)^2} = 0$$

$$\frac{\cancel{2y^2} - \cancel{2x^2} + \cancel{4xy} + \cancel{2x^2} - \cancel{2y^2} - \cancel{4xy}}{(x^2+y^2)^2} = 0$$

$$\frac{0}{(x^2+y^2)^2} = 0$$

$$\boxed{0 = 0}$$

Q109

$$f(v_1, v_2, v_3, v_4) = \frac{v_1^2 - v_2^2}{v_3^2 + v_4^2}$$

$$\boxed{\frac{\partial f}{\partial v_1} = \frac{2v_1}{v_3^2 + v_4^2}}$$

$$\boxed{\frac{\partial f}{\partial v_2} = \frac{-2v_2}{v_3^2 + v_4^2}}$$

$$\cancel{\frac{\partial f}{\partial v_3} = \frac{v_1^2 - v_2^2}{2v_3}}$$

$$\cancel{\frac{\partial f}{\partial v_4} = \frac{v_1^2 - v_2^2}{2v_4}}$$

$$\frac{\partial f}{\partial v_3} = \frac{(v_3^2 + v_4^2)\frac{\partial}{\partial v_3}(v_1^2 - v_2^2) - (v_1^2 - v_2^2)\frac{\partial}{\partial v_3}(v_3^2 + v_4^2)}{(v_3^2 + v_4^2)^2}$$

$$\boxed{\frac{\partial f}{\partial v_3} = \frac{-2v_3(v_1^2 - v_2^2)}{(v_3^2 + v_4^2)^2}}$$

$$\frac{\partial f}{\partial v_4} = \frac{(v_3^2 + v_4^2)\frac{\partial}{\partial v_4}(v_1^2 - v_2^2) - (v_1^2 - v_2^2)\frac{\partial}{\partial v_4}(v_3^2 + v_4^2)}{(v_3^2 + v_4^2)^2}$$

$$\boxed{\frac{\partial f}{\partial v_4} = \frac{-2v_4(v_1^2 - v_2^2)}{(v_3^2 + v_4^2)}}$$

Date

Q111

$$U(w, x, y, z) = xe^{yw} \sin^2 z$$

$$\cancel{\frac{\partial U}{\partial x} = e^{yw} \sin^2 z}$$

$$\cancel{\frac{\partial U}{\partial x}(0, 0, 1, \pi) = e^0 \sin^2(\pi)}$$

$$\frac{\partial U}{\partial x} = e^{yw} \sin^2 z$$

$$\frac{\partial U}{\partial x}(0, 0, 1, \pi) = e^{1 \times 0} \sin^2(\pi)$$

$$\boxed{\frac{\partial U}{\partial x}(0, 0, 1, \pi) = 0}$$

$$\frac{\partial U}{\partial y} = wx\, e^{yw} \sin^2 z$$

$$\boxed{\frac{\partial U}{\partial y}(0, 0, 1, \pi) = 0}$$

$$\frac{\partial U}{\partial w} = xy\, e^{yw} \sin^2 z$$

$$\boxed{\frac{\partial U}{\partial w}(0, 0, 1, \pi) = 0}$$

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$$\frac{\partial u}{\partial z} = xe^{yw}\, 2\sin z \cos z$$

$$\boxed{\frac{\partial u}{\partial z}(0,0,1,\pi) = 0}$$

$$\frac{\partial u}{\partial z} = xe^{yw}\, 2\sin z \cos z$$

$$\frac{\partial^2 u}{\partial w \partial z} = \cancel{xwe} \quad xy\, e^{yw}\, 2\sin z \cos z$$

$$\frac{\partial^3 u}{\partial y \partial w \partial z} = x\, 2\sin z \cos z\, (wy\, e^{yw} + e^{yw})$$

$$\frac{\partial^3 u}{\partial y \partial w \partial z} = 2x(1+wy)\, e^{yw} \sin z \cos z$$

$$\boxed{\frac{\partial^4 u}{\partial x \partial y \partial w \partial z} = 2(1+wy)\, e^{yw} \sin z \cos z}$$

$$\frac{\partial u}{\partial y} = wx\, e^{yw} \sin^2 z$$

$$\frac{\partial^2 u}{\partial y^2} = w^2 x e^{yw} \sin^2 z$$

$$\frac{\partial^3 u}{\partial z \partial y^2} = w^2 x e^{yw}\, 2\sin z \cos z$$

$$\frac{\partial^4 u}{\partial w \partial z \partial y^2} = \cancel{2x} \cdot (w^2 y e^{yw} + 2we^{yw})\, 2x \sin z \cos z$$

$$\boxed{\frac{\partial^4 u}{\partial w \partial z \partial y^2} = 2xw(2+yw)\, e^{yw} \sin z \cos z}$$

No.

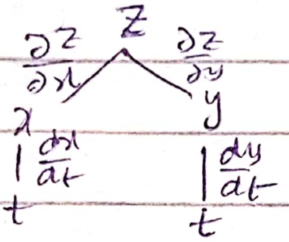
Exercise # 13.5

Assignment # 14

Date

Q3

$z = 3\cos x - \sin xy \; ; \; x = 1/t \; , \; y = 3t$



$$\frac{dz}{dt} = \frac{\partial z}{\partial x} \cdot \frac{dx}{dt} + \frac{\partial z}{\partial y} \cdot \frac{dy}{dt}$$

$$\frac{dx}{dt} = -\frac{1}{t^2} \qquad \frac{\partial z}{\partial x} = -y\cos xy - 3\sin x$$

$$\frac{dy}{dt} = 3 \qquad \frac{\partial z}{\partial y} = -x\cos xy$$

$$\frac{dz}{dt} = \frac{-(-y\cos xy - 3\sin x)}{t^2} + -3x\cos xy.$$

$$\frac{dz}{dt} = \frac{3\sin x + y\cos xy}{t^2} - 3x\cos xy.$$

$$= \frac{3\sin x + y\cos xy - 3xt^2\cos xy}{t^2}$$

$$\cancel{3(\sin x - xt^2) + \cos xy(y}$$

$$\frac{3\sin x + \cos xy(y - 3xt^2)}{t^2}$$

$$\frac{dz}{dt} = \frac{3\sin\left(\frac{1}{t}\right) + 3t\cos 3}{t^2} - \frac{3\cos 3}{t}$$

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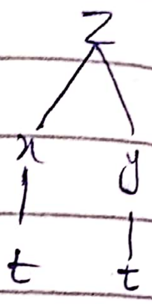
$$= \frac{3t\sin\left(\frac{1}{t}\right) + \cancel{3t^2\cos 3} - \cancel{3t^2\cos 3}}{t^3}$$

$$= \frac{3\cancel{t}\sin\left(\frac{1}{t}\right)}{t^{\cancel{3}}}$$

$$\frac{dz}{dt} = 3t^{-2}\sin\left(\frac{1}{t}\right)$$

Q5

$\cancel{z = \sqrt{1+x} = 2}$



$$z = e^{1-xy}; \quad x = t^{1/3}, \quad y = t^3$$

$$\frac{dz}{dt} = \frac{\partial z}{\partial x}\frac{dx}{dt} + \frac{\partial z}{\partial y}\frac{dy}{dt}$$

$$\frac{\partial z}{\partial x} = -ye^{1-xy} \qquad \frac{dx}{dt} = \frac{1}{3}t^{-2/3}$$

$$\frac{\partial z}{\partial y} = -xe^{1-xy} \qquad \frac{dy}{dt} = 3t^2$$

$$\frac{dz}{dt} = \frac{-yt^{\frac{4}{3}}e^{1-xy}}{3} + 3t^2(-xe^{1-xy})$$

$$= \frac{-t^{\frac{7}{3}}e^{1-t^{10/3}}}{3} - 3t^{7/3}e^{1-t^{10/3}}$$

$$= \frac{-t^{\frac{7}{3}}e^{1-t^{10/3}} - 9t^{7/3}e^{1-t^{10/3}}}{3}$$

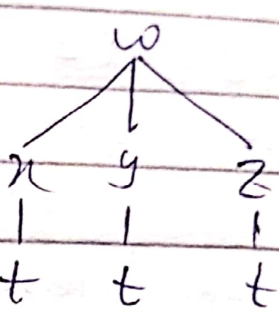
$$\frac{dz}{dt} = -\frac{10}{3}t^{7/3}e^{1-t^{10/3}}$$

Date

Q9

$w = 5\cos xy - \sin xz$; $x = \frac{1}{t}$, $y = t$, $z = t^3$

$$\frac{dw}{dt} = \frac{\partial w}{\partial x}\cdot\frac{dx}{dt} + \frac{\partial w}{\partial y}\frac{dy}{dt} + \frac{\partial w}{\partial z}\frac{dz}{dt}$$



$\frac{\partial w}{\partial x} = -5y\sin xy - z\cos xz$; $\frac{dx}{dt} = -\frac{1}{t^2}$

$\frac{\partial w}{\partial y} = -5x\sin xy$ $\quad \frac{dy}{dt} = 1$

$\frac{\partial w}{\partial z} = -x\cos xz$ $\quad \frac{dz}{dt} = 3t^2$

$$\frac{dw}{dt} = \frac{5y\sin xy + z\cos xz}{t^2} - 5x\sin xy - 3xt^2\cos xz$$

$$= \frac{5y\sin xy + z\cos xz - 5xt^2\sin xy - 3xt^4\cos xz}{t^2}$$

$$= \frac{5t\cancel{\sin(1)} + t^3\cos t^2 - 5t\cancel{\sin(1)} - 3t^3\cos t^2}{t^2}$$

$$= \frac{-2t^{\cancel{3}}\cos t^2}{\cancel{t^2}}$$

$$\boxed{\frac{dw}{dt} = -2t\cos t^2}$$

No.

Date

Q17

$z = 8x^2y - 2x + 3y \,;\; x = uv,\; y = u - v$

$$\boxed{\frac{\partial z}{\partial u} = \frac{\partial z}{\partial x}\frac{\partial x}{\partial u} + \frac{\partial z}{\partial y}\frac{\partial y}{\partial u}}$$

$$\frac{\partial z}{\partial x} = 16xy - 2 \qquad \frac{\partial x}{\partial u} = v$$

$$\frac{\partial z}{\partial y} = 8x^2 + 3 \qquad \frac{\partial y}{\partial u} = 1$$

$$\frac{\partial z}{\partial u} = v(16xy - 2) + 8x^2 + 3$$

$$= 16uv(u-v)v - 2v + 8u^2v^2 + 3$$

$$= 16u^2v^2 - 16uv^3 - 2v + 8u^2v^2 + 3$$

$$\boxed{\frac{\partial z}{\partial u} = 24u^2v^2 - 16uv^3 - 2v + 3}$$

$$\boxed{\frac{\partial z}{\partial v} = \frac{\partial z}{\partial x}\cdot\frac{\partial x}{\partial v} + \frac{\partial z}{\partial y}\cdot\frac{\partial y}{\partial v}}$$

$$\cancel{\partial z}\ \frac{\partial x}{\partial v} = u \qquad \frac{\partial y}{\partial v} = -1$$

Date

$$\frac{\partial z}{\partial v} = 16uxy - 2u - 8x^2 - 3$$

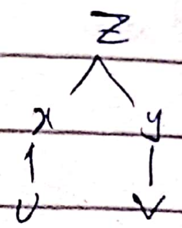
$$= 16u^2v(u-v) - 2u - 8u^2v^2 - 3$$

$$= 16u^3v - 16u^2v^2 - 2u - 8u^2v^2 - 3$$

$$\boxed{\frac{\partial z}{\partial v} = 16u^3v - 24u^2v^2 - 2u - 3}$$

Q19

$$z = x/y \qquad x = 2\cos u \qquad y = 3\sin v$$



$$\boxed{\frac{\partial z}{\partial u} = \frac{\partial z}{\partial x} \cdot \frac{dx}{du}}$$

$$\frac{\partial z}{\partial x} = \frac{1}{y} \quad ; \quad \frac{dx}{du} = -2\sin u$$

$$\frac{\partial z}{\partial u} = \frac{-2\sin u}{y}$$

$$\boxed{\frac{\partial z}{\partial u} = \frac{-2\sin u}{3\sin v}}$$

$$\boxed{\frac{\partial z}{\partial v} = \frac{\partial z}{\partial y} \cdot \frac{dy}{dv}}$$

$$\frac{\partial z}{\partial y} = \frac{-x}{y^2} \qquad \frac{dy}{dv} = 3\cos v$$

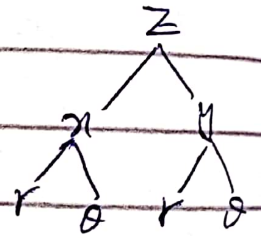
$$\frac{\partial z}{\partial v} = \frac{-3x\cos v}{y^2}$$

$$= \frac{-6\cos u\cos v}{9\sin^2 v}$$

$$\boxed{\frac{\partial z}{\partial v} = \frac{-2\cos u\cos v}{3\sin^2 v}}$$

Q23

$T = x^2y - xy^3 + 2$; $x = r\cos\theta$, $y = r\sin\theta$



$$\boxed{\frac{\partial T}{\partial r} = \frac{\partial z}{\partial x}\cdot\frac{\partial x}{\partial r} + \frac{\partial z}{\partial y}\cdot\frac{\partial y}{\partial r}}$$

$$\frac{\partial z}{\partial x} = 2xy - y^3 \qquad \frac{\partial x}{\partial r} = \cos\theta$$

$$\frac{\partial z}{\partial y} = x^2 - 3xy^2 \qquad \frac{\partial y}{\partial r} = \sin\theta$$

$$\frac{\partial T}{\partial r} = 2xy\cos\theta - y^3\cos\theta + x^2\sin\theta - 3xy^2\sin\theta$$

$$\cancel{x^2\sin\theta - 3xy^2\sin\theta}$$

$$\frac{\partial T}{\partial r} = 2r^2\cos^2\theta\sin\theta - r^3\sin^3\theta\cos\theta + r^2\cos^2\theta\sin\theta - 3r^3\sin^3\theta\cos\theta$$

$$\boxed{\frac{\partial T}{\partial r} = 3r^2\cos^2\theta\sin\theta - 4r^3\sin^3\theta\cos\theta}$$

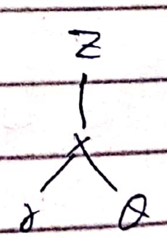
$$\boxed{\frac{\partial T}{\partial \theta} = \frac{\partial z}{\partial x}\cdot\frac{\partial x}{\partial \theta} + \frac{\partial z}{\partial y}\frac{\partial y}{\partial \theta}}$$

$$\frac{\partial x}{\partial \theta} = -r\sin\theta \quad , \quad \frac{\partial y}{\partial \theta} = r\cos\theta$$

$$\frac{\partial T}{\partial \theta} = -2rxy\sin\theta + ry^3\sin\theta + rx^2\cos\theta - 3rxy^2\cos\theta$$

$$\boxed{\frac{\partial T}{\partial \theta} = -2r^3\sin^2\theta\cos\theta + r^4\sin^2\theta + r^3\cos^3\theta - 3r^4\cos^2\theta\sin^2\theta}$$

Q27



$z = \ln(x^2+1)$; $x = r\cos\theta$

$$\boxed{\frac{\partial z}{\partial r} = \frac{dz}{dx}\cdot\frac{\partial x}{\partial r}}$$

$$\frac{dz}{dx} = \frac{2x}{x^2+1} \qquad \frac{\partial x}{\partial r} = \cos\theta$$

Date

$$\frac{\partial z}{\partial r} = \frac{2x\cos\theta}{x^2+1}$$

$$\boxed{\frac{\partial z}{\partial r} = \frac{2r\cos^2\theta}{r^2\cos^2\theta+1}}$$

$$\boxed{\frac{\partial z}{\partial \theta} = \frac{dz}{dx}\,\frac{\partial x}{\partial \theta}}$$

$$\frac{\partial x}{\partial \theta} = -r\sin\theta$$

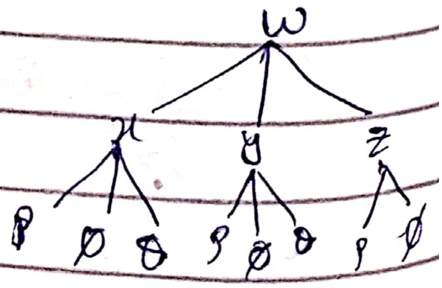
$$\frac{\partial z}{\partial \theta} = \frac{-2xr\sin\theta}{x^2+1}$$

$$\boxed{\frac{\partial z}{\partial \theta} = \frac{-2r^2\cos\theta\sin\theta}{r^2\cos^2\theta+1}}$$

Q 29

$w = 4x^2+4y^2+z^2$, $x = \rho\sin\phi\cos\theta$, $y = \rho\sin\phi\sin\theta$
$z = \rho\cos\phi$

$$\boxed{\frac{\partial w}{\partial \rho} = \frac{\partial w}{\partial x}\cdot\frac{\partial x}{\partial \rho} + \frac{\partial w}{\partial y}\,\frac{\partial y}{\partial \rho} + \frac{\partial w}{\partial z}\,\frac{\partial z}{\partial \rho}}$$



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$\frac{\partial w}{\partial x} = 8x$, $\frac{\partial x}{\partial \rho} = \sin\phi\cos\theta$

$\frac{\partial w}{\partial y} = 8y$, $\frac{\partial y}{\partial \rho} = \sin\phi\sin\theta$

$\frac{\partial w}{\partial z} = 2z$, $\frac{\partial z}{\partial \rho} = \cos\phi$

$$\frac{\partial w}{\partial \rho} = 8x\sin\phi\cos\theta + 8y\sin\phi\sin\theta + 2z\cos\phi$$

$$= 8\rho\sin^2\phi\cos^2\theta + 8\rho\sin^2\phi\sin^2\theta + 2\rho\cos^2\phi$$

$$= 2\rho\left(4\sin^2\phi\cos^2\theta + 4\sin^2\phi\sin^2\theta + \cos^2\phi\right)$$

$$= 2\rho\left(4\sin^2\phi\left(\cos^2\theta + \sin^2\theta\right) + \cos^2\phi\right)$$

$$\boxed{\frac{\partial w}{\partial \rho} = 2\rho\left(4\sin^2\phi + \cos^2\phi\right)}$$

$$\boxed{\frac{\partial w}{\partial \phi} = \frac{\partial w}{\partial x}\cdot\frac{\partial x}{\partial \phi} + \frac{\partial w}{\partial y}\frac{\partial y}{\partial \phi} + \frac{\partial w}{\partial z}\frac{\partial z}{\partial \phi}}$$

$\frac{\partial x}{\partial \phi} = \rho\cos\phi\cos\theta$, $\frac{\partial y}{\partial \phi} = \rho\cos\phi\sin\theta$, $\frac{\partial z}{\partial \phi} = -\rho\sin\phi$

$$\frac{\partial w}{\partial \phi} = 8x\rho\cos\phi\cos\theta + 8y\rho\cos\phi\sin\theta - 2z\rho\sin\phi$$

$$= 8\rho^2\sin\phi\cos\phi\cos^2\theta + 8\rho^2\sin\phi\cos\phi\sin^2\theta - 2\rho^2\sin\phi\cos\phi$$

$$= 2\rho^2\sin\phi\cos\phi\left(4\cos^2\theta + 4\sin^2\theta - 1\right)$$

$$\boxed{\frac{\partial w}{\partial \phi} = 6\rho^2\sin\phi\cos\phi}$$

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$$\boxed{\frac{\partial w}{\partial \theta} = \frac{\partial w}{\partial x}\frac{\partial x}{\partial \theta} + \frac{\partial w}{\partial y}\frac{\partial y}{\partial \theta}}$$

$$\frac{\partial x}{\partial \theta} = -\rho \sin\phi \sin\theta$$

$$\frac{\partial y}{\partial \theta} = \rho \sin\phi \cos\theta$$

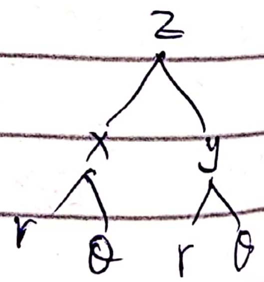
$$\frac{\partial w}{\partial \theta} = -8x\rho \sin\phi \sin\theta + 8y\rho \sin\phi \cos\theta$$

$$= \cancel{-8\rho^2 \sin^2\phi \sin\theta \cos\theta} + \cancel{8\rho^2 \sin^2\phi \sin\theta \cos\theta}$$

$$\boxed{\frac{\partial w}{\partial \theta} = 0}$$

Q33

$$z = xye^{\frac{x}{y}} \; ; \; x = r\cos\theta, \; y = r\sin\theta$$



$$\boxed{\frac{\partial z}{\partial r} = \frac{\partial z}{\partial x}\frac{\partial x}{\partial r} + \frac{\partial z}{\partial y}\frac{\partial y}{\partial r}}$$

$$\frac{\partial z}{\partial x} = y\left(\frac{x}{y}e^{\frac{x}{y}} + e^{\frac{x}{y}}\right)$$

$$\frac{\partial z}{\partial x} = xe^{\frac{x}{y}} + ye^{\frac{x}{y}}$$

Date

$$\frac{\partial x}{\partial r} = \cos\theta$$

$$\frac{\partial z}{\partial y} = x\left(-\frac{x}{y^2}e^{\frac{x}{y}} + e^{\frac{x}{y}}\right)$$

$$\frac{\partial z}{\partial y} = xe^{\frac{x}{y}} - \frac{x^2}{y^2}e^{\frac{x}{y}}$$

$$\frac{\partial y}{\partial r} = \sin\theta$$

$$\frac{\partial z}{\partial r} = xe^{\frac{x}{y}}\cos\theta + ye^{\frac{x}{y}}\cos\theta + xe^{\frac{x}{y}}\sin\theta - \frac{x^2}{y^2}e^{\frac{x}{y}}\sin\theta$$

$$= e^{\frac{x}{y}}\left(x\cos\theta + y\cos\theta + x\sin\theta - \frac{x^2}{y^2}\sin\theta\right)$$

$$= e^{\frac{\cos\theta}{\sin\theta}}\left(r\cos^2\theta + r\sin\theta\cos\theta + r\cos\theta\sin\theta - \frac{\cos^2\theta}{\sin\theta}\right)$$

$$\frac{\partial z}{\partial r} = e^{\frac{\cos\theta}{\sin\theta}}\left(r\cos^2\theta + 2r\sin\theta\cos\theta - \frac{\cos^2\theta}{\sin\theta}\right)$$

$$\left.\frac{\partial z}{\partial r}\right|_{r=2,\theta=\pi/6} = e^{\sqrt{3}}\left(\cancel{\frac{3}{2}} + \sqrt{3} - \cancel{\frac{3}{2}}\right)$$

$$\boxed{\left.\frac{\partial z}{\partial r}\right|_{r=2,\theta=\frac{\pi}{6}} = \sqrt{3}\,e^{\sqrt{3}}}$$

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$$\boxed{\frac{\partial z}{\partial \theta} = \frac{\partial z}{\partial x}\frac{\partial x}{\partial \theta} + \frac{\partial z}{\partial y}\frac{\partial y}{\partial \theta}}$$

$$\frac{\partial x}{\partial \theta} = -r\sin\theta$$

$$\frac{\partial y}{\partial \theta} = r\cos\theta$$

$$\frac{\partial z}{\partial \theta} = -rxe^{\frac{x}{y}}\sin\theta - rye^{\frac{x}{y}}\sin\theta + rxe^{\frac{x}{y}}\cos\theta - \frac{rx^2e^{\frac{x}{y}}\cos\theta}{y^2}$$

$$= -r^2e^{\frac{\cos\theta}{\sin\theta}}\cos\theta\sin\theta - r^2e^{\frac{\cos\theta}{\sin\theta}}\sin^2\theta + r^2e^{\frac{\cos\theta}{\sin\theta}}\cos^2\theta - \frac{r\cos^3\theta}{\sin^2\theta}\cdot e^{\frac{\cos\theta}{\sin\theta}}$$

$$\cancel{= re^{\frac{\cos\theta}{\sin\theta}}(r\cos}$$

$$= -re^{\frac{\cos\theta}{\sin\theta}}\left(r\cos\theta\sin\theta + r\sin^2\theta - r\cos^2\theta + \frac{\cos^3\theta}{\sin^2\theta}\right)$$

$$\left.\frac{\partial z}{\partial \theta}\right|_{r=2,\theta=\frac{\pi}{6}} = -2e^{\sqrt{3}}\left(\frac{\sqrt{3}}{2} + \frac{1}{2} - \frac{3}{2} + \frac{3\sqrt{3}}{2}\right)$$

$$= -2e^{\sqrt{3}}(-1 + 2\sqrt{3})$$

$$\boxed{\left.\frac{\partial z}{\partial \theta}\right|_{r=2,\theta=\frac{\pi}{6}} = e^{\sqrt{3}}(2 - 4\sqrt{3})}$$

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Date

Q41

$$x^2y^3 + \cos y = 0$$

$$f(x,y) = x^2y^3 + \cos y$$

$$\frac{dy}{dx} = -\frac{\partial f/\partial x}{\partial f/\partial y}$$

$$\frac{\partial f}{\partial x} = 2xy^3$$

$$\frac{\partial f}{\partial y} = 3x^2y^2 - \sin y$$

$$\boxed{\frac{dy}{dx} = \frac{-2xy^3}{3x^2y^2 - \sin y}}$$

Q43

$$f(x,y) = e^{xy} + ye^y - 1$$

$$\frac{\partial f}{\partial x} = ye^{xy}$$

$$\frac{\partial f}{\partial y} = xe^{xy} + ye^y + e^y$$

$$\frac{dy}{dx} = -\frac{\partial f/\partial x}{\partial f/\partial y}$$

$$\boxed{\frac{dy}{dx} = \frac{-ye^{xy}}{xe^{xy} + ye^y + e^y}}$$

No.

Q47

$$ye^x - 5\sin 3z = 3z$$

$$f = ye^x - 5\sin 3z - 3z$$

$$\frac{\partial z}{\partial x} = -\frac{\partial f/\partial x}{\partial f/\partial z}$$

$$\frac{\partial z}{\partial x} = -\frac{ye^x}{-15\cos 3z - 3}$$

$$\boxed{\frac{\partial z}{\partial x} = \frac{ye^x}{3(\cos 3z + 1)}}$$

$$\frac{\partial z}{\partial y} = \frac{\partial f/\partial y}{\partial f/\partial z}$$

$$= -\frac{e^x}{-15\cos 3z - 3}$$

$$\boxed{\frac{\partial z}{\partial y} = \frac{e^x}{15\cos 3z - 3}}$$

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Date

Q51

$$z = f(x+2y)$$

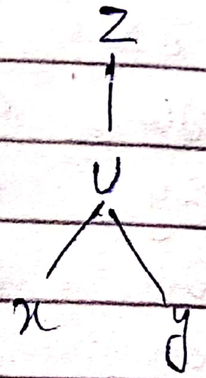
~~let $v = x+2y$~~

~~$z = f(v)$~~

let $u = x+2y$ then $z = f(u)$

$$\frac{\partial z}{\partial x} = \frac{dz}{du}\frac{\partial u}{\partial x}$$

$$\frac{\partial z}{\partial x} = \frac{dz}{du}$$



$$\frac{\partial z}{\partial y} = \frac{dz}{du}\frac{\partial u}{\partial y}$$

$$\frac{\partial z}{\partial y} = 2\frac{dz}{du}$$

put in given equation

$$2\frac{\partial z}{\partial x} - \frac{\partial z}{\partial y} = 0$$

$$2\frac{\partial z}{\partial u} - 2\frac{\partial z}{\partial u} = 0$$

$$\boxed{0 = 0}$$

showed.

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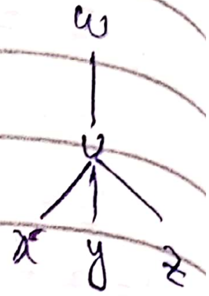
Q53



$$w = f(u)$$

$$u = x + 2y + 3z$$

$$\frac{\partial w}{\partial x} = \frac{dw}{du}\frac{\partial u}{\partial x}$$

$$\frac{\partial w}{\partial x} = \frac{dw}{du} \qquad \because \frac{\partial u}{\partial x} = 1$$

$$\frac{\partial w}{\partial y} = \frac{dw}{du}\frac{\partial u}{\partial y}$$

$$\frac{\partial w}{\partial y} = \frac{2\,dw}{du} \qquad \because \frac{\partial u}{\partial y} = 2$$

$$\frac{\partial w}{\partial z} = \frac{dw}{du}\frac{\partial u}{\partial z}$$

$$\frac{\partial w}{\partial z} = \frac{3\,dw}{\partial u} \qquad \because \frac{\partial u}{\partial z} = 3$$

$$\frac{\partial w}{\partial x} + \frac{\partial w}{\partial y} + \frac{\partial w}{\partial z} = \frac{6\,dw}{du}$$

$$\frac{dw}{du} + \frac{2\,dw}{du} + \frac{3\,dw}{du} = \frac{6\,dw}{du}$$

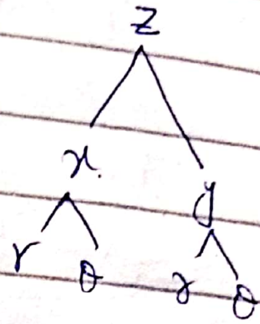
$$\boxed{\frac{6\,dw}{du} = \frac{6\,dw}{du}}$$ Showed!

Q57

a) $x = r\cos\theta$ — (1)

$\cancel{\frac{\partial}{\partial x}(x = r\cos\theta)}$

$\cancel{1 = \frac{\partial r}{\partial x}}$



$y = r\sin\theta$ — (2)

Squaring & adding

$x^2+y^2 = r^2(\cos^2\theta + \sin^2\theta)$

$r = \sqrt{x^2+y^2} \rightarrow$ (3)

$$\frac{\partial r}{\partial x} = \frac{\cancel{2}x}{\cancel{2}\sqrt{x^2+y^2}}$$

$\because x = r\cos\theta$ $\quad\because r = \sqrt{x^2+y^2}$

$$\frac{\partial r}{\partial x} = \frac{\cancel{r}\cos\theta}{\cancel{r}}$$

$$\boxed{\frac{\partial r}{\partial x} = \cos\theta}$$

Dividing eq (2) by eq (1)

$$\frac{y}{x} = \frac{\cancel{r}\sin\theta}{\cancel{r}\cos\theta}$$

$$\tan\theta = \frac{y}{x}$$

Date

$$\theta = \tan^{-1}\left(\frac{y}{x}\right) \rightarrow (4)$$

$$\frac{\partial \theta}{\partial x} = \frac{1}{1+\left(\frac{y}{x}\right)^2} \cdot \frac{-y}{x^2}$$

$$\frac{\partial \theta}{\partial x} = \frac{-y}{x^2+y^2}$$

$$\frac{\partial \theta}{\partial x} = \frac{-r\sin\theta}{r^2}$$

$$\boxed{\frac{\partial \theta}{\partial x} = -\frac{\sin\theta}{r}}$$

b) from eq (3)

$$r = \sqrt{x^2+y^2}$$

$$\frac{\partial r}{\partial y} = \frac{\cancel{2}y}{2\sqrt{x^2+y^2}}$$

$$\frac{\partial r}{\partial y} = \frac{\cancel{r}\sin\theta}{\cancel{r}}$$

$$\boxed{\frac{\partial r}{\partial y} = \sin\theta}$$

c)

(A)

(B)

Date

θ. From eq (4)

$$\theta = \tan^{-1} \frac{y}{x}$$

$$\frac{\partial \theta}{\partial y} = \frac{1}{1+\left(\frac{y}{x}\right)^2} \times \frac{1}{x}$$

$$= \frac{x}{x^2+y^2}$$

$$= \frac{\cancel{r}\cos\theta}{r^{\cancel{2}}}$$

$$\boxed{\frac{\partial \theta}{\partial y} = \frac{\cos\theta}{r}}$$

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Exercise 13.8

Assignment 15

Date

Q9

$f(x,y) = y^2 + xy + 3y + 2x + 3$

$f_x = y + 2$

$f_y = 2y + x + 3$

For critical points

$y + 2 = 0 \qquad 2y + x + 3 = 0$

$y = -2 \qquad 2(-2) + x + 3 = 0$

$x = 1$

$(x_0, y_0) = (1, -2)$

$D = f_{xx}(x_0, y_0)\, f_{yy}(x_0, y_0) - f^2_{xy}(x_0, y_0)$

$f_{xx}(1,-2) = 0$

$f_{yy}(1,-2) = 2$

$f^2_{xy} = 1$

$D = (0)(2) - 1$

$D = -1$

$D < 0$, so ~~f has~~ $(1,-2)$ is a saddle point of function.

Q11

$f(x,y) = x^2 + xy + y^2 - 3x$

$f_x = 2x + y - 3$

$f_y = 2y + x$

For critical point :-

$2x + y = 3$

$x + 2y = 0$

$x = -2y$

$2(-2y) + y = 3$

$-4y + y = 3$

$-3y = 3$

$y = -1$

$x = -2(-1)$

$x = 2$

$(x_0, y_0) = (2, -1)$

$D = f_{xx}(x_0, y_0)\, f_{yy}(x_0, y_0) - f_{xy}^2(x_0, y_0)$

$f_{xx} = 2$

$f_{yy} = 2$

$f^2 xy = 1$

$D = (2)(2) - 1$

$D = 3$

$D > 0 \qquad f_{xx} > 0$

function has relative minima at $(2,-1)$

Q 13

$f(x,y) = x^2 + y^2 + \frac{2}{xy}$

$f_x = 2x - \frac{2}{x^2 y}$

$f_y = 2y - \frac{2}{xy^2}$

For critical point

$2x - \frac{2}{x^2 y} = 0$

$\cancel{2} x^3 y = \cancel{2}$

$y = \frac{1}{x^3}$

$2y = \frac{2}{xy^2} = 0$

$\cancel{2} x y^3 = \cancel{2}$

$x \left(\frac{1}{x^3} \right)^3 = 1$

$\frac{1}{x^8} = 1$

$x^8 = 1 \Rightarrow x = \pm 1$

$y = 1 \quad , \quad y = -1$

Date

Critical points are

$(1,1), (-1,-1)$

$$f_{xx} = 2 + \frac{2}{x^3 y}$$

$$f_{yy} = 2 + \frac{2}{x y^3}$$

$$f_{xy} = \frac{2}{x^2 y^2}$$

$D = f_{xx} f_{yy} - f_{xy}^2$

x	y	f_{xx}	f_{xy}	f_{xy}	D	
1	1	4	4	2	12	Minima
-1	-1	4	4	2	12	Minima

function has relative minimum at $(1,1)$ and $(-1,-1)$.

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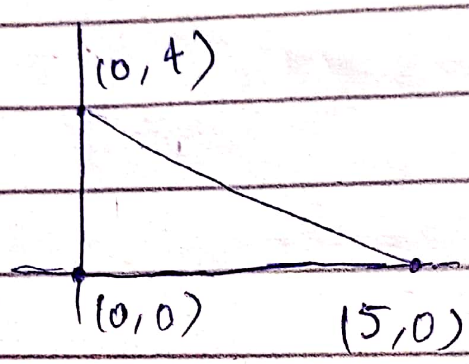
Q31

$f(x,y) = xy - x - 3y$



Critical points

$f_x = y - 1$

$f_y = x - 3$

$f_x = 0 \quad f_y = 0$

$(x_0, y_0) = (3,1)$

Boundry Points

a) line segment joining $(0,0)$ & $(5,0)$

Eq $\rightarrow$ $y = 0$

$U(x) = f(x,0) = -x \qquad 0 \leq x \leq 5$

$U'(x) = -1$

$(0,0)$, $(5,0)$

Date

b) line segment joining $(0,0)$ & $(0,4)$

Eq $\rightarrow x = 0$

$f(0,y) = -3y = v(x)$ $\qquad 0 \leq y \leq 4$

$v'(x) = -3$

$(0,0), (0,4)$

c) line segment joining $(0,4)$ & $(5,0)$

$0 \le x \le 5$

$$Eq \rightarrow \quad y = -\frac{4}{5}x + 4$$

$$f(x,y) = x\left(-\frac{4}{5}x+4\right) - x - 3\left(-\frac{4}{5}x + 4\right)$$

$$= -\frac{4}{5}x^2 + 4x - x + \frac{12}{5}x - 12$$

$$f(x,y) = -\frac{4}{5}x^2 + \frac{27}{5}x - 12 = w(x)$$

$$w'(x) = -\frac{8}{5}x + \frac{27}{5}$$

$$-\frac{8}{5}x + \frac{27}{5} = 0$$

$$x = \frac{27}{8}$$

$$y = \frac{13}{10} \qquad \left(\frac{27}{8}, \frac{13}{10}\right)$$

Put critical points in function

critical points are: $(3,1)$, $(0,0)$, $(5,0)$, $(0,4)$, $(27/8, 13/10)$

x	3	0	5	0	27/8
y	1	0	0	4	13/10
$f(x,y)$	-3	0	-5	-12	-2.88

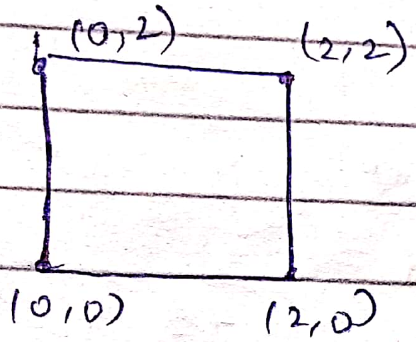
Date

Absolute minimum -12

Absolute maximum 0

Q33

$$f(x,y) = x^2 - 3y^2 - 2x + 6y$$



Critical point

$f_x = 2x - 2$

$f_y = -6y + 6$

$f_x = 0 \qquad f_y = 0$

$(x_0, y_0) = (1, +1)$

Boundary Point.

a) Line segment joining (0,0) & (0,2)

$Eq \rightarrow x = 0$

$t(x) = f(x,y) = -3y^2 + 6y$

$t'(x) = 6y - 6$

$6y - 6 = 0$

$x = 0 \qquad y = 1 \qquad (0,1)$

No.

b) line segment joining $(2,0)$ & $(2,2)$

Eq $\rightarrow$ $x = 2$

$u(y) = f(2,y) = -3y^2 + 6y$

$u'(y) = -6y + 6$

$-6y + 6 = 0$

$y = +1$

$(2, +1)$

c) line segment joining $(0,0)$ & $(2,0)$

$y = 0$

$v(x) = f(x,0) = x^2 - 2x$

$v'(x) = 2x - 2$

$v'(x) = 0$

$2x - 2 = 0$

$x = 1$

$(1, 0)$

d) line segment joining $(0,2)$ & $(2,2)$

$y = 2$

$w(x) = f(x,2) = x^2 - 2x$

$w'(x) = 2x - 2$

$2x - 2 = 0$

$x = 1$

$(1, 2)$

put critical point in $f(x,y)$

critical points $\Rightarrow$ $(1,-1)$ & $(1,2)$, $(2,-1)$, $(1,0)$

x	1	0	2	1	1
y	+1	1	+1	0	2
$f(x,y)$	~~-10~~ 2	~~1~~ 3	+3	-1	-1

Absolute minimum $\rightarrow$ -1

Absolute maximum $\rightarrow$ $+3$

Q35

$$f(x,y) = x^2 + 2y^2 - x$$

critical point :-

$$f_x = 2x - 1$$

$$f_y = 4y$$

$$f_x = 0 \qquad f_y = 0$$

$$(x_0, y_0) = \left(\tfrac{1}{2}, 0\right)$$

critical Point along Region :-

$$x^2 + y^2 = 4$$

$$y = \sqrt{4 - x^2}$$

$$f(x,y) = x^2 + 2(4 - x^2) - x$$

$$u(x) \Rightarrow f(x,y) = 8 - x^2 - x$$

$$u'(x) = -2x - 1$$

Date

$-2x-1=0$

$x = \frac{-1}{2}$, $y = \pm\frac{\sqrt{15}}{2}$

$\left(-\frac{1}{2}, \frac{\sqrt{15}}{2}\right)$, $\left(-\frac{1}{2}, \frac{-\sqrt{15}}{2}\right)$

Find absolute maximum & minimums

critical points are ;

$\left(\frac{1}{2}, 0\right)$, $\left(-\frac{1}{2}, \frac{\sqrt{15}}{2}\right)$, $\left(-\frac{1}{2}, \frac{-\sqrt{15}}{2}\right)$

x	1/2	$-\frac{1}{2}$	$-\frac{1}{2}$
y	0	$\frac{\sqrt{15}}{2}$	$\frac{-\sqrt{15}}{2}$
f(x, y)	$-\frac{1}{4} = -0.25$	$33/4 = 8.25$	$33/4$

Absolute minimum → $-1/4$

Absolute maximum → $33/4$

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Exercise 14.1

Assignment 1B

Date

Q5

$$\int_0^{\ln 3}\int_0^{\ln 2} e^{x+y}\, dy\, dx$$

$$\int_0^{\ln 3}\int_0^{\ln 2} e^{x}\cdot e^{y}\, dy\, dx$$

$$\int_0^{\ln 3} e^{x}\left[e^{y}\right]_0^{\ln 2}\, dx$$

$$\int_0^{\ln 3} e^{x}\, dx$$

$$\left[e^{x}\right]_0^{\ln 3}$$

$$\boxed{2}$$

Q7

$$\int_{-1}^{0}\int_{2}^{5} dx\, dy$$

$$\int_{-1}^{0}\left[x\right]_2^5\, dy$$

$$3\int_{-1}^{0} dy$$

$$3\left[y\right]_{-1}^{0}$$

$$\boxed{3}$$

Date

Q9

$$\int_0^1\int_0^1 \frac{x}{(xy+1)^2}\,dy\,dx$$

$$\int_0^1\int_0^1 x(xy+1)^{-2}\,dy\,dx$$

$$-\int_0^1 \left[\frac{1}{xy+1}\right]_0^1 dx$$

$$-\int_0^1 \frac{1}{x(1)+1} - \frac{1}{x(0)+1}\,dx$$

$$-\int_0^1 \frac{1}{x+1} - 1 \quad dx$$

$$-\left[\int_0^1 \frac{1}{x+1}\,dx - \int_0^1 dx\right]$$

$$\cancel{-\left[\int_0^1 (x+1)^{-1}dx - \int_0^1 dx\right]}$$

$$-\left[\ln(x+1) - x\right]_0^1$$

$$1 - \ln 2$$

No.

Date

Q15

$$\iint_R x\sqrt{1-x^2}\, dA \qquad R=\{(x,y): 0\le x\le 1,\ 2\le y\le 3\}$$

$$\int_2^3\int_0^1 x\sqrt{1-x^2}\, dx\,dy$$

$$-\frac{1}{2}\int_2^3\int_0^1 -2x\sqrt{1-x^2}\, dx\, dy$$

$$-\frac{1}{2}\int_2^3\left[\frac{2(1-x^2)^{3/2}}{3}\right]_0^1 dy$$

$$-\frac{1}{2}\int_2^3\left(\frac{2(0)^{3/2}}{3}-\frac{2(1)^{3/2}}{3}\right) dy$$

$$+\frac{1}{2}\int_2^3 \frac{2}{3}\, dy$$

$$+\frac{1}{3}\left[y\right]_2^3$$

$$\boxed{+\frac{1}{3}}$$

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Date

Q 29

$$\int_1^2 \int_3^5 2x+y \, dx \, dy$$

$$\int_1^2 \left(\left[\frac{2x^2}{2} \right]_3^5 + \left[xy \right]_3^5 \right) dy$$

$$\int_1^2 (16 + 2y) \, dy$$

$$\left[16y \right]_1^2 + \left[\frac{2y^2}{2} \right]_1^2$$

$$16 + 3$$

$$\boxed{19}$$

Q 31

$$\int_0^3 \int_0^2 x^2 \, dx \, dy$$

$$\int_0^3 \left[\frac{x^3}{3} \right]_0^2 dy$$

$$\int_0^3 \frac{8}{3} \, dy$$

Date

$$\frac{8}{3}\left[y\right]_0^3$$

$$\boxed{8}$$

Q35

$A(R) = (8-0)(6-0) = 48$

$$\frac{1}{48}\int_0^6\int_0^8 xy^2 \, dx\, dy$$

$$\frac{1}{48}\int_0^6 \frac{y^2}{2}\left[x^2 \cancel{y^2}\right]_0^8 dy$$

$$\frac{1}{48} 32 \int_0^6 y^2 \, dy$$

$$\frac{1}{48} \times \frac{32}{3}\left[y^3\right]_0^6$$

$$\frac{2304}{48}$$

$$\boxed{48}$$

Exercise 14.2

Assignment 17

Date.

Q3

$$= \int_0^3 \int_0^{\sqrt{9-y^2}} y \, dx \, dy \, \cancel{dx}$$

$$= \int_0^3 \cancel{x} y \, |x|_0^{\sqrt{9-y^2}} \, dy$$

$$= \int_0^3 y\sqrt{9-y^2} \, dy$$

$$\int f(x)^n f'(x) \, dx = \frac{f(x)^{n+1}}{n+1}$$

$$= -\frac{1}{2} \int_0^3 -2y \sqrt{9-y^2}$$

$$= -\frac{1}{2} \times \frac{(9-y^2)^{\frac{1}{2}+1}}{\frac{1}{2}+1} \Big|_0^3$$

$$= -\frac{1}{\cancel{2}} \times \frac{\cancel{2}(9-y^2)^{3/2}}{3}$$

$$= -\frac{1}{3} |(9-y^2)^{3/2}|_0^3$$

$$= -\frac{1}{3} \left\{ (9-(3)^2)^{3/2} - (9-0^2)^{3/2} \right\}$$

$$= -\frac{1}{3} \times -27$$

$$= 9$$

No.

Date

Q5

$$= \int_{\sqrt{\pi}}^{\sqrt{2\pi}} \int_{0}^{x^3} \sin \frac{y}{x} \, dy \, dx$$

$$= \int_{\sqrt{\pi}}^{\sqrt{2\pi}} \left(-x \cos \frac{y}{x} \Big|_{0}^{x^3} \right) dx$$

$$= - \int_{\sqrt{\pi}}^{\sqrt{2\pi}} x \left(\cos \frac{x^3}{x} - \cos \frac{0}{x} \right)$$

$$= - \int_{\sqrt{\pi}}^{\sqrt{2\pi}} (x \cos x^2 - x) \, dx$$

$$= - \frac{1}{2} \int_{\sqrt{\pi}}^{\sqrt{2\pi}} 2x \cos x^2 - \int_{\sqrt{\pi}}^{\sqrt{2\pi}} x$$

$$= - \left(\frac{\sin x^2}{2} - \frac{x^2}{2} \right) \Big|_{\sqrt{\pi}}^{\sqrt{2\pi}}$$

$$= -\frac{1}{2} \left(\sin x^2 - x^2 \right)_{\sqrt{\pi}}^{\sqrt{2\pi}}$$

$$= -\frac{1}{2} \left[\cancel{\sin 4\pi} \; (\sin 2\pi - 2\pi) - (\sin \pi - \pi) \right]$$

$$= + \frac{\pi}{2}$$

No.

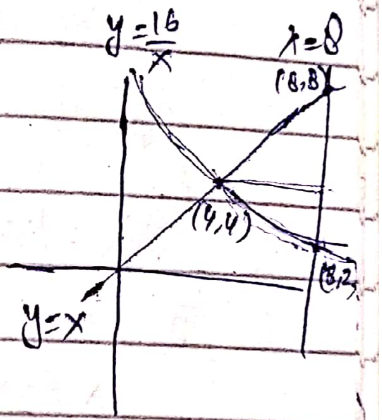
Date

Q15

$$\iint_R x^2 \, dA$$

$y = \frac{16}{x}$ $\quad y = x \quad x = 8$

Type II :-



$$= \int_4^8 \int_{\frac{16}{x}}^{x} x^2 \, dy \, dx$$

$$= \int_4^8 x^2 y \Big|_{\frac{16}{x}}^{x} dx$$

$$= \int_4^8 x^2 \left(x - \frac{16}{x}\right) dx$$

$$= \int_4^8 x(x^2 - 16) \, dx$$

$$= \frac{1}{2} \int_4^8 2x \, (x^2 - 16) \, dx$$

$$= \frac{1}{2} \times \frac{(x^2-16)^2}{2} \Big|_4^8$$

$$= \frac{1}{4} \left[(8^2 - 16)^2 - (4^2 - 16)^2 \right]$$

$y = x$

$y = \frac{16}{x}$

$x = \frac{16}{x}$

$x^2 = 16$

$x = \pm 4$

$$= \frac{2304}{4}$$

$$= 576$$

Type II

$$\int_{\frac{16}{y}}^{y} x^2 \, dx \, dy$$

4 8

8

Type I

$$= \int_{2}^{4}\int_{16/y}^{8} x^2 \, dx\,dy + \int_{4}^{8}\int_{y}^{8} x^2 \, dx\,dy$$

$$= \int_{2}^{4} \frac{x^3}{3}\Big|_{16/y}^{8} dy + \int_{4}^{8} \frac{x^3}{3}\Big|_{y}^{8}$$

$$= \int_{2}^{4} \frac{8^3 - (16/y)^3}{3} dy + \int_{4}^{8} \frac{8^3 - y^3}{3} dy.$$

$$= \int_{2}^{4} \frac{8^3}{3} - \frac{1}{3}\int_{2}^{4} \frac{16^3}{y^3} + \int_{4}^{8} \frac{8^3}{3} - \int_{4}^{8} \frac{y^3}{3}$$

$$= \frac{8^3}{3} y\Big|_{2}^{4} - \frac{16^3}{3(-2)y^2}\Big|_{2}^{4} + \frac{8^3}{3} y\Big|_{4}^{8} - \frac{1}{3(4)} y^4\Big|_{4}^{8}$$

$$\cancel{\frac{8^3}{3}(2) + \frac{16^3}{\cancel{6}\,3(2)}(\cancel{2}) + \frac{8^3}{3}(4) - \frac{1}{\cancel{12}\,3}(4)}$$

$$= \frac{1024}{3} - 128 + \frac{2048}{3} - 320$$

$$= 576$$

Q17

$$= \iint_R (3x - 2y)\, dA$$

$$= \quad x^2 + y^2 = 1$$

Type I

$$= \int_{-1}^{1}\int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} 3x - 2y \quad dy\, dx$$

$$= \int_{-1}^{1} 3xy - \frac{\cancel{2}y^2}{\cancel{2}} \Bigg|_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} dx$$

$$= \int_{-1}^{1} \left(3x(\sqrt{1-x^2}) - \frac{\cancel{2}(1-x^2)}{\cancel{2}} \right) - \left(3x(-\sqrt{1-x^2}) - (1-x^2) \right)$$

$$= \int_{-1}^{1} 3x\sqrt{1-x^2} \cancel{-1} + \cancel{x^2} + 3x\sqrt{1-x^2} \cancel{+1} \cancel{-x^2}$$

$$= \int_{-1}^{1} 6x\sqrt{1-x^2}$$

$$= -\frac{6}{2}\int_{-1}^{1} -2x\sqrt{1-x^2}$$

$$= -\frac{6}{2} \times \frac{(1-x^2)^{3/2}}{3/2} \Bigg|_{-1}^{1}$$

Date

$$= \frac{-6}{2}\left(\frac{(1-1^2)^{3/2}}{3/2} - \frac{(1-(-1)^2)^{3/2}}{3/2}\right)$$

$$= 0$$

Type II

$$\int_{-1}^{1}\int_{-\sqrt{1-y^2}}^{\sqrt{1-y^2}} 3x-2y \; dx\,dy$$

$$\int_{-1}^{1} \frac{3x^2}{2} - 2xy \Big|_{-\sqrt{1-y^2}}^{\sqrt{1-y^2}} dy.$$

$$\int_{-1}^{1}\left(\left[\frac{3}{2}\cancel{x^2}(1-y^2) - 2(\sqrt{1-y^2})y\right] - \left[\frac{3}{2}(1-y^2) + 2(\sqrt{1-y^2})y\right]\right)$$

$$\int_{-1}^{1}\left(\frac{3}{2} - \frac{3}{2}y^2 - 2y\sqrt{1-y^2} - \frac{3}{2} + \frac{3}{2}y^2 - 2y\sqrt{1-y^2}\right)$$

$$\frac{3}{2}y - \frac{3}{2}\frac{y^3}{3} + \frac{(1-y^2)^{3/2}}{3/2} - \frac{3}{2}y + \frac{3}{2}\frac{y^2}{2} + \frac{(1-y^2)^{3/2}}{3/2}\Bigg|_{-1}^{1}$$

$$\left(\frac{3}{2}(1) - \frac{3}{2}\frac{(1)^3}{3} + \frac{(1-1^2)^{3/2}}{3/2} - \frac{3}{2}(1) + \frac{3}{2}\frac{(1)^2}{2} + \frac{(1-1^2)^{3/2}}{3/2}\right) -$$

$$\left(\frac{3}{2}(-1) - \frac{3}{2}\frac{(-1)^3}{3} + \frac{(1-1^2)^{3/2}}{3/2} - \frac{3}{2}(-1) + \frac{3}{2}\frac{(-1)^2}{2} + \frac{(1-)^{2})^{3/2}}{3/2}\right)$$

No.

$$= \left(\frac{3}{2} - \frac{1}{2} + 0 - \frac{3}{2} + \frac{3}{4} + 0\right) - \left(-\frac{3}{2} + \frac{1}{2} + 0 + \frac{3}{2} - \frac{3}{4} + 0\right)$$

$$= \frac{1}{4} - \frac{1}{4}$$

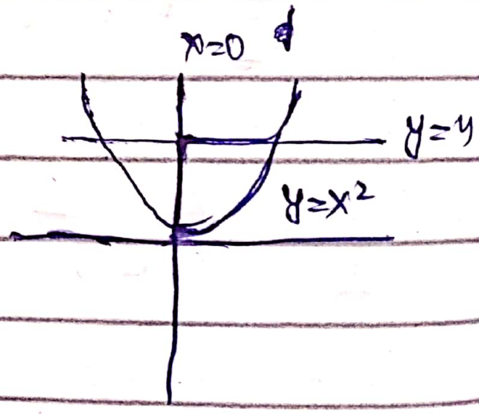
$$= 0$$

Q19

$$\iint_R x(1+y^2)^{-\frac{1}{2}}\, dA$$

$y = x^2$, $y = 4$, and $x = 0$

$$\int_0^4 \int_0^{\sqrt{y}} x(1+y^2)^{-\frac{1}{2}}\, dx\,dy$$



$$\int_0^4 \frac{x^2}{2}(1+y^2)^{-\frac{1}{2}} \Big|_0^{\sqrt{y}} dy$$

$$\int_0^4 \frac{y}{2}(1+y^2)^{-\frac{1}{2}}\, dy$$

$$\frac{1}{4}\int_0^4 2y(1+y^2)^{-\frac{1}{2}}\, dy$$

$$= \frac{1}{4} \times \frac{(1+y^2)^{\frac{1}{2}}}{\frac{1}{2}} \Bigg|_0^4$$

$$= \frac{1}{4}\left(2(1+(4)^2)^{\frac{1}{2}} - 2(1+0^2)^{1/2}\right)$$

$$= \frac{1}{4}\left(2\sqrt{17} - 2\right)$$

$$= \frac{\sqrt{17}}{2} - \frac{1}{2}$$

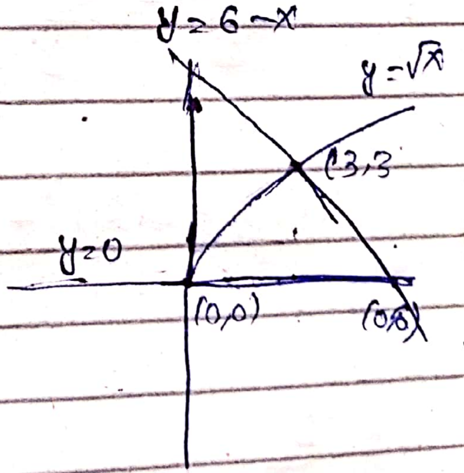
$$= \frac{\sqrt{17}-1}{2}$$

Q21

$$\iint_R xy \, dA$$

$y = \sqrt{x}$, $y = 6-x$, $y = 0$

$$\int_0^2 \int_{y^2}^{6-y} xy \; dx\,dy$$



R.W

$6 - x = \sqrt{x}$

$36 - 12x + x^2 = x$

$x^2 - 13x + 36 = 0$

$x^2 - 4x - 9x + 36 = 0$

$x(x-4) - 9(x-4) = 0$

$x = 4$, $x = 9$

$\cancel{6 - x = \sqrt{x}}$

$\cancel{36 - 12x + x^2 = x}$

$\cancel{12x = 36}$ $\cancel{x^2 - 13x + 36}$

$\cancel{x = 3}$

$\cancel{x(x-13) = -36}$

$$= \int_0^2 \frac{x^2 y}{2}\Big|_{y^2}^{6-y} dy$$

$$= \int_0^2 \left[\frac{(6-y)^2 y}{2} - \frac{(y^2)^2 y}{2}\right] dy$$

$$= \int_0^2 \left(\frac{y(36-12y+y^2)}{2} - \frac{y^5}{2}\right) dy$$

$$= \int_0^2 \left(\frac{\overset{18}{\cancel{36}}\,y}{\cancel{2}} - \frac{\overset{6}{\cancel{12}}\,y^2}{\cancel{2}} + \frac{y^3}{2} - \frac{y^5}{2}\right) dy$$

$$= \int_0^2 \left(18y - 6y^2 + \frac{y^3}{2} - \frac{y^5}{2}\right) dy$$

$$= \frac{\overset{9}{\cancel{18}}\,y^2}{\cancel{2}} - \frac{\overset{2}{\cancel{6}}\,y^3}{\cancel{3}} + \frac{y^4}{2(4)} - \frac{y^6}{2(6)}\Big|_0^2$$

$$= 9(2)^2 - 2(2)^3 + \frac{(2)^4}{8} - \frac{(2)^6}{12}$$

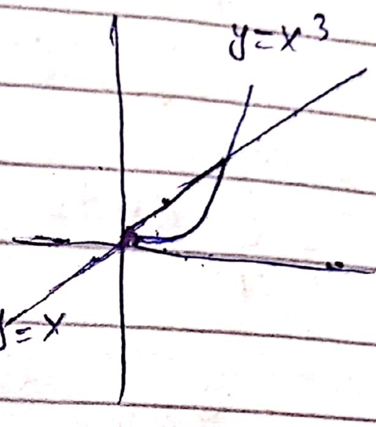
$$= \frac{50}{3}$$

Date

Q23

$$\iint_R (x-1)\,dA$$

$y = x \qquad y = x^3$



$$\int_0^1\int_y^{y^{\frac{1}{3}}} (x-1)\,dx\,dy$$

$$\int_0^1 \frac{x^2}{2} - x \Big|_y^{y^{\frac{1}{3}}} dy$$

$$\int_0^1 \left(\frac{y^{2/3}}{2} - y^{\frac{1}{3}}\right) - \left(\frac{y^2}{2} - y\right) dy$$

$x^3 = x$

$x^3 - x = 0$

$x(x^2 - 1) = 0$

$x = 0 \qquad x^2 - 1 = 0$

$x = 0 \qquad x = \pm 1$

$$\int_0^1 \frac{y^{\frac{2}{3}}}{2} - y^{\frac{1}{3}} - \frac{y^2}{2} + y\,dy$$

$$\frac{y^{\frac{5}{3}}}{2(\frac{5}{3})} - \frac{y^{\frac{4}{3}}}{4/3} - \frac{y^3}{2(3)} + \frac{y^2}{2}\Big|_0^1$$

$$\frac{3y^{10/9}}{10} - \frac{3y^{4/3}}{4} - \frac{y^3}{6} + \frac{y^2}{2}\Big|_0^1$$

$$= \frac{3(1)^{10/9}}{10} - \frac{3(1)^{4/3}}{4} - \frac{(1)^3}{6} + \frac{(1)^2}{2}$$

$$= \frac{-7}{60}$$

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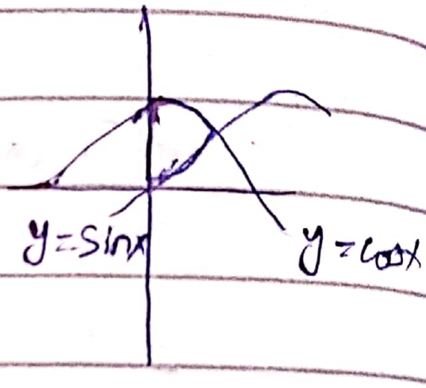
Q29

$$\iint_R dA$$

$y = \sin x \qquad y = \cos x \qquad 0 \le x \le \pi/4$

$$= \int_0^{\frac{\pi}{4}} \int_{\sin x}^{\cos x} dy\, dx$$



$$= \int_0^{\frac{\pi}{4}} y \Big|_{\sin x}^{\cos x} dx$$

$$= \int_0^{\frac{\pi}{4}} \cos x - \sin x \; dx$$

$$= (\sin x + \cos x) \Big|_0^{\frac{\pi}{4}}$$

$$= \left(\sin\frac{\pi}{4} + \cos\frac{\pi}{4}\right) - \left(\sin 0 + \cos(0)\right)$$

$$= \sqrt{2} - 1$$

$$= \cancel{\sqrt{2}}$$

$$= \boxed{\sqrt{2} - 1}$$

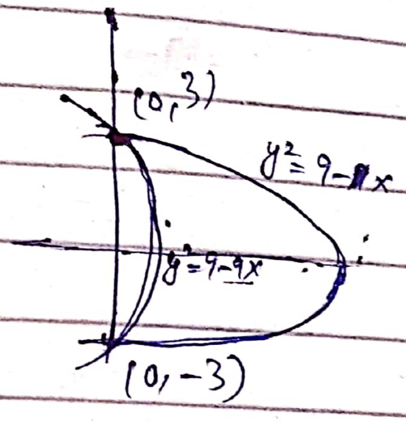
Date

Q31

$$\iint_R dA$$

$y^2 = 9 - x \qquad y^2 = 9 - 9x$

$$\int_{-3}^{3}\int_{\frac{9-y^2}{9}}^{9-y^2} dx\,dy$$



$\cancel{9} - \cancel{x} = 9 - 9x$

$8x = 0$

$x = 0$

$$\int_{-3}^{3} x^2 \Big|_{\frac{9-y^2}{9}}^{9-y^2} dy$$

$$\int_{-3}^{3}\left(9-y^2 - \frac{9-y^2}{9}\right)dy$$

$$\int_{-3}^{3}\left(9-y^2-1+\frac{y^2}{9}\right)dy$$

$$8y - \frac{8y^3}{9(3)}\Big|_{-3}^{3}$$

$$\left(8(3)-\frac{8}{9}\frac{(3)^3}{3}\right)-\left(8(-3)-\frac{8}{9}\frac{(-3)^3}{3}\right)$$

$= 16 - (-16)$

$= 32$

Date

Q39

$\iint_R dA$

$x^2+y^2=9$

$z=0, \ z=3-x$

$$= \int_{-3}^{3}\int_{-\sqrt{9-x^2}}^{\sqrt{9-x^2}} 3-x \ dydx$$

$$= (3-x)y \Big|_{-\sqrt{9-x^2}}^{\sqrt{9-x^2}}$$

$$= \int_{-3}^{3} (3-x)(\sqrt{9-x^2}) + (3-x)(\sqrt{9-x^2}) \ dx$$

$$= \int_{-3}^{3} 3\sqrt{9-x^2} - x\sqrt{9-x^2} + 3\sqrt{9-x^2} - x\sqrt{9-x^2}$$

$$= \int_{-3}^{3} 6\sqrt{9-x^2} - 2x\sqrt{9-x^2} \ dx$$

$$= 6\int_{-3}^{3} \sqrt{9-x^2} + \int_{-3}^{3} -2x\sqrt{9-x^2}$$

$\because \sqrt{a^2-x^2} = \frac{1}{2}x\sqrt{a^2-x^2} + \frac{1}{2}a^2\sin^{-1}\left(\frac{x}{a}\right)+c$

~~$u = 9-x^2$~~

~~$\frac{du}{dx} = -2x$~~

$$= 6\left(\frac{1}{2}x\sqrt{9-x^2} + \frac{1}{2}9\sin^{-1}\left(\frac{x}{3}\right)\right)\Bigg|_{-3}^{3} + \frac{(9-x^2)^{3/2}}{3/2}\Bigg|_{-3}^{3}$$

Date

$$= \frac{6}{\cdot}\left[\left(\frac{1}{2}(3)\sqrt{9-(3)^2}+\frac{1}{2}9\sin^{-1}\left(\frac{3}{3}\right)\right)-\left(\frac{1}{2}(-3)\sqrt{9-(-3)^2}+\frac{1}{2}9\sin^{-1}\left(\frac{-3}{3}\right)\right)\right]+\left[\frac{(9-3^2)^{3/2}}{3/2}-\frac{(9-(-3)^2)^{3/2}}{3/2}\right]$$

$$= 6\left(\frac{9\pi}{2}\right)+\frac{9\pi}{4}$$

$$\boxed{= 27\pi}$$

Q43

$$\iint_R y+3 \; dA$$

$$4x^2+y^2=9$$

$$\cancel{\int_{-\frac{\sqrt{9-y^2}}{2}}^{\frac{\sqrt{9-y^2}}{2}}\int y+3 \; dx\,dy}$$

$$\int_{-\frac{3}{2}}^{\frac{3}{2}}\int_{-\sqrt{9-4x^2}}^{+\sqrt{9-4x^2}} y+3 \; dy\,dx$$

$$\int_{-\frac{3}{2}}^{\frac{3}{2}} \left.\frac{y^2}{2}+3y\right|_{\sqrt{9-4x^2}}^{\sqrt{9-4x^2}} dx$$

$$\int_{-3/2}^{3/2}\left(\frac{9-4x^2}{2}+3\sqrt{9-4x^2}\right)-\left(\frac{9-4x^2}{2}-3\sqrt{9-4x^2}\right)dx$$

No.

$$\int_{-\frac{3}{2}}^{\frac{3}{2}} 6\sqrt{9-4x^2}\;dx$$

$$6\int_{-3/2}^{3/2} 2\sqrt{\frac{9}{4}-x^2}$$

$$12\cancel{\int_{-3/2}^{3/2}} \quad \because \sqrt{a^2-x^2} = \frac{1}{2}x\sqrt{a^2-x^2} + \frac{a^2}{2}\sin^{-1}\left(\frac{x}{a}\right)$$

$$12\left[\left(\frac{1}{2}x\sqrt{\frac{9}{4}-x^2} + \frac{9}{4(2)}\sin^{-1}\left(\frac{x}{3/2}\right)\right)\right]_{-3/2}^{3/2}$$

$$12\left[\left[\frac{1}{2}\left(\frac{3}{2}\right)\cancel{\sqrt{\frac{9}{4}-\left(\frac{3}{2}\right)^2}} + \frac{9}{4(2)}\sin^{-1}\left(\frac{3/2}{3/2}\right)\right] -\right.$$

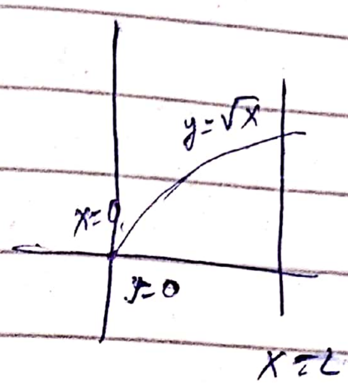
$$\left.\left[\cancel{\frac{1}{2}\left(\frac{-3}{2}\right)\sqrt{\frac{9}{4}-\left(\frac{-3}{2}\right)^2}} + \frac{9}{4(2)} \times \sin^{-1}\left(\frac{-3/2}{3/2}\right)\right]\right]$$

$$12\left(+\frac{9}{16}\pi - \left(-\frac{9}{16}\pi\right)\right)$$

$$12\left(+\frac{9\pi}{8}\right) = +\frac{27}{2}\pi$$

Q47

$$\int_0^2 \int_0^{\sqrt{x}} f(x,y)\, dy\, dx$$



for dx

$\boxed{x = 2}$

$y = \sqrt{x}$

$y^2 = x$

$\boxed{x = y^2}$

for dy

$\boxed{y = 0}$

$y = \sqrt{x}$

$\cancel{\sqrt{x} = 0}$ $\boxed{y = \sqrt{2}}$

$\cancel{x = 0}$

$$\int_0^{\sqrt{2}} \int_{y^2}^{2} f(x,y)\, dx\, dy$$

Q53

$$\int_0^1 \int_{4x}^{4} e^{-y^2}\, dy\, dx$$

for dx

$\boxed{x = 0}$

$y = 4x$

$\boxed{x = \frac{4}{y}}$

for dy

$\boxed{y = 4}$

$y = 4(x)$

$y = 4(0)$

$\boxed{y = 0}$

$$\int_0^4\int_0^{y/4} e^{-y^2}\,dx\,dy$$

$$\int_0^4 x e^{-y^2}\Big|_0^{y/4} dy$$

$$\int_0^4 \frac{y}{4} e^{-y^2} dy$$

$$\frac{-2}{-2}\int_0^4 \frac{y}{4} e^{-y^2} dy$$

$$-\frac{1}{8}\int_0^4 -2y\, e^{-y^2} dy$$

$$= -\frac{1}{8} e^{-y^2}\Big|_0^4$$

$$= -\frac{1}{8}\left[e^{-4^2} - e^{-0^2}\right]$$

$$= -\frac{1}{8}\left(e^{-16} - e^{0}\right)$$

$$= \frac{1 - e^{-16}}{8}$$

Exercise 14.3

Assignment 28

Date

Q5

$$= \int_0^{\pi}\int_0^{1-\sin\theta} r^2 \cos\theta \, dr\, d\theta$$

$$= \int_0^{\pi} \frac{r^3}{3}\cos\theta \Big|_0^{1-\sin\theta} d\theta$$

$$= \int_0^{\pi} \frac{(1-\sin\theta)^3}{3}\cos\theta \, d\theta$$

$$= -\frac{1}{3}\int_0^{\pi} -\cos\theta\,(1-\sin\theta)^3 \, d\theta$$

$$= -\frac{1}{3} \times \frac{(1-\sin\theta)^4}{4}\Big|_0^{\pi}$$

$$= -\frac{1}{3}\left(\frac{1-\sin\pi}{4} - \frac{1-\sin 0}{4}\right)$$

$$= +\frac{1}{3}(0)$$

$$= 0$$

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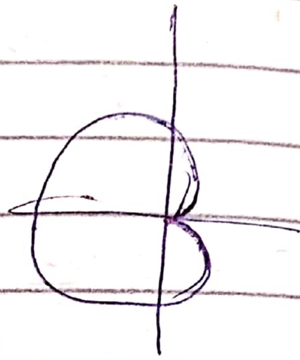
Date

Q7

$$= \iint_R dA$$



$$dA = r\,dr\,d\theta$$

$$\int_0^{2\pi}\int_0^{1-\cos\theta} r\,dr\,d\theta$$

$$\int_0^{2\pi} \frac{r^2}{2}\Big|_0^{1-\cos\theta} d\theta$$

$$\int_0^{2\pi} \frac{(1-\cos\theta)^2}{2}\, d\theta$$

$$\int_0^{2\pi} \frac{1-2\cos\theta+\cos^2\theta}{2}\, d\theta$$

$$\because \cos^2\theta = \frac{1+\cos 2\theta}{2}$$

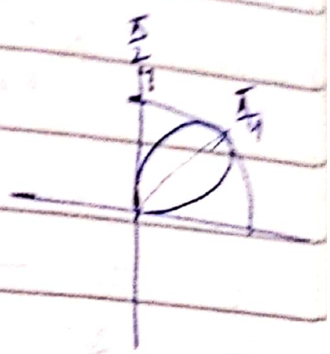
$$\int_0^{2\pi} \frac{1}{2}\, d\theta - \int_0^{2\pi} \cos\theta + \frac{1}{2}\int_0^{2\pi} \frac{1+\cos 2\theta}{2}$$

$$\frac{\theta}{2}\Big|_0^{2\pi} - \sin\theta\Big|_0^{2\pi} + \frac{1}{4}\theta\Big|_0^{2\pi} + \frac{\sin 2\theta}{4\times 2}\Big|_0^{2\pi}$$

$$= \frac{2\pi}{2} - 0 + \frac{2\pi}{4} - 0$$

$$= \frac{3\pi}{2}$$

Q9



$$A = \iint_R r\,dr\,d\theta$$

$$\frac{\pi}{4} \le \theta \le \frac{\pi}{2}$$

$$\sin 2\theta \le r \le 1$$

$$\int_{\pi/4}^{\pi/2}\int_{\sin 2\theta}^{1} r\,dr\,d\theta$$

$$\int_{\pi/4}^{\pi/2} \frac{r^2}{2}\Big|_{\sin 2\theta}^{1} d\theta$$

$$\int_{\pi/4}^{\pi/2} \frac{1}{2} - \frac{\sin^2 2\theta}{2}\,d\theta$$

$$\frac{1}{2}\int_{\pi/4}^{\pi/2} d\theta - \frac{1}{2}\int_{\pi/4}^{\pi/2} \sin^2 2\theta\,d\theta$$

$$\because \sin\frac{\theta}{2} = \sqrt{\frac{1-\cos\theta}{2}}$$

$$= \frac{1}{2}\theta\Big|_{\pi/4}^{\pi/2} - \frac{1}{2}\int_{\pi/4}^{\pi/2} \frac{1-\cos 4\theta}{2}\, d\theta$$

$$= \frac{1}{2}\theta\Big|_{\pi/4}^{\pi/2} - \frac{1}{4}\int_{\pi/4}^{\pi/2} d\theta + \frac{1}{4}\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \cos 4\theta\, d\theta$$

$$= \frac{\pi}{8} - \frac{\pi}{16} + \frac{\sin 4\theta}{16}\Big|_{\pi/4}^{\pi/2}$$

$$= \frac{\pi}{16}$$

Q29

$$\int_0^2 \int_0^{\sqrt{2x-x^2}} \sqrt{x^2+y^2}\, dy\, dx$$

$$r = \sqrt{x^2+y^2}$$

$$y = \sqrt{2x-x^2}$$

$\because x = r\cos\theta$ $\quad\because y = r\sin\theta$

$$r\sin\theta = \sqrt{2r\cos\theta - r^2\cos^2\theta}$$

$$r^2\sin^2\theta = 2r\cos\theta - r^2\cos^2\theta$$

$$r^2\sin^2\theta + r^2\cos^2\theta = 2r\cos\theta$$

Date

$r^2 = 2r\cos\theta$

$r = 2\cos\theta$

$y = 0$

$r\sin\theta = 0$

$r = \theta$

$x = 0$

$r\cos\theta = 0$

$\cos\theta = 0$

$\cos^{-1}(0) = \theta$

$\theta =$

$$\int_0^{\frac{\pi}{2}}\int_0^{2\cos\theta} r \cdot r\,dr\,d\theta$$

$$\int_0^{\frac{\pi}{2}} \frac{r^3}{3}\Big|_0^{2\cos\theta} d\theta$$

$$\int_0^{\frac{\pi}{2}} \frac{8\cos^3\theta}{3}\,d\theta$$

$$\frac{8}{3}\int_0^{\frac{\pi}{2}} \cos\theta \cdot \cos^2\theta \;\; d\theta$$

$$= \frac{8}{3}\int_0^{\frac{\pi}{2}} \cos(1-\sin^2\theta)\, d\theta$$

$$= \frac{8}{3}\int_0^{\frac{\pi}{2}} \cos\theta - \cos\sin^2\theta \, d\theta$$

$$= \frac{8}{3}\left[\sin\theta \Big|_0^{\frac{\pi}{2}} - \frac{\sin^3\theta}{3}\Big|_0^{\frac{\pi}{2}} \right]$$

$$= \frac{8}{3}\left[(1-0) - \left(\frac{1}{3} - 0\right) \right]$$

$$= \frac{8}{3} \times \frac{2}{3}$$

$$= \frac{16}{9}$$

Q31

$$\int_0^a \int_0^{\sqrt{a^2-x^2}} \frac{dy\,dx}{(1+x^2+y^2)^{3/2}} \quad (a>0)$$

$1+x^2+y^2 = 1+r^2$

$y = \sqrt{a^2-x^2}$

$r\sin\theta = \sqrt{a^2 - r^2\cos^2\theta}$

$r^2\sin^2\theta + r^2\cos^2\theta = a^2$

Date

$r = a$

$$\int_0^{\frac{\pi}{2}}\int_0^{a} \frac{1}{(1+r^2)^{3/2}} \, r\,dr\,d\theta$$

$$\frac{1}{2}\int_0^{\frac{\pi}{2}}\int_0^{a} 2r(1+r^2)^{-3/2}\,dr\,d\theta$$

$$\frac{1}{2}\int_0^{\frac{\pi}{2}} \left.\frac{(1+r^2)^{-1/2}}{-1/2}\right|_0^{a} d\theta$$

$$\cancel{\frac{1}{2} - \int_0^{\frac{\pi}{2}} (1+a^2)^{-1/2}} \; \cancel{\theta} = 1$$

$$\frac{1}{2}\int_0^{\frac{\pi}{2}} \left(-2(1+a^2)^{-1/2} + 2(1)^{-1/2}\right) d\theta$$

$$\int_0^{\frac{\pi}{2}} -(1+a^2)^{-1/2} d\theta + \int_0^{\frac{\pi}{2}} d\theta$$

$$-\theta(1+a^2)^{-1/2}\Big|_0^{\frac{\pi}{2}} + \theta\Big|_0^{\pi/2}$$

$$-\frac{\pi}{2}(1+a^2)^{-1/2} \cancel{\pi} \; \cancel{+0} + \frac{\pi}{2}$$

$$\boxed{\frac{\pi}{2}\left[1-(1+a^2)^{-1/2}\right]}$$

No.

Exercise 14.5

Assignment 29

Date

Q7

$$\int_0^2 \int_0^{\sqrt{4-x^2}} \int_{-5+x^2+y^2}^{3-x^2-y^2} x \, dz\,dy\,dx$$

$$\int_0^2 \int_0^{\sqrt{4-x^2}} xz \Big|_{-5+x^2+y^2}^{3-x^2-y^2} dy\,dx$$

$$= \int_0^2 \int_0^{\sqrt{4-x^2}} x(3-x^2-y^2) - x(-5+x^2+y^2)\, dy\,dx$$

$$= \int_0^2 \int_0^{\sqrt{4-x^2}} 3x - x^3 - xy^2 + 5x - x^3 - xy^2 \, dy\,dx$$

$$= \int_0^2 3xy \Big|_0^{\sqrt{4-x^2}} - x^3 y \Big|_0^{\sqrt{4-x^2}} - \frac{xy^3}{3} \Big|_0^{\sqrt{4-x^2}} + 5xy \Big|_0^{\sqrt{4-x^2}} - x^3 y \Big|_0^{\sqrt{4-}}$$
$$- \frac{xy^3}{3} \Big|_0^{\sqrt{4-x^2}}$$

$$= \int_0^2 3x\sqrt{4-x^2} - x^3\sqrt{4-x^3} - \frac{x(\sqrt{4-x^2})^3}{3} + 5x\sqrt{4-x^2}$$
$$- x^3\sqrt{4-x^2} - \frac{x(\sqrt{4-x^2})^3}{3} \, dx$$

$$= \int_0^2 8x\sqrt{4-x^2} - 2x^3\sqrt{4-x^2} - \frac{2x}{3}(\sqrt{4-x^2})^3 \, dx$$

Date

$$-4\int_0^2 -2x\sqrt{4-x^2}\,dx - \int_0^2 2x^3\sqrt{4-x^2}\,dx - \int_0^2 \frac{2x}{3}\left(\sqrt{4-x^2}\right)^3 dx$$

Appy substitution for middle part

$$\text{let } 4-x^2 = t^2$$

$$2x\,dx = 2t\,dt$$

$$dx = \frac{t}{x}\,dt$$

$$dx = \frac{t}{\sqrt{4-t^2}} \qquad \because x = \sqrt{4-t^2}$$

$$= \int_0^2 2(4-t^2)^{3/2}\frac{t^2}{(4-t^2)^{1/2}}$$

$$= \int_0^2 2(4-t^2)t^2$$

$$= \int_0^2 (8t^2 - 2t^4)\,dt$$

$$= \left.\frac{8t^3}{3} - \frac{2t^5}{5}\right|_0^2$$

$$= \left.\frac{8\left(\sqrt{4-x^2}\right)^3}{3} - \frac{2\left(\sqrt{4-x^2}\right)^5}{5}\right|_0^2$$

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$$= \frac{\cancel{8}(4-x^2)^{3/2}}{3/2}\Big|_0^2 - \left(\frac{8(\sqrt{4-x^2})^3}{3} - \frac{2(\sqrt{4-x^2})^5}{5}\right)\Big|_0^2$$

$$- \frac{1}{3}\frac{(4-x^2)^{5/2}}{5/2} \cancel{dx}\Big|_0^2$$

$$= -4\left(\cancel{0}0 - \frac{\cancel{8}(4)^{3/2}}{3/2}\right) - \left[0 - \left(\frac{8(4)^{3/2}}{3} - \frac{2(4)^{5/2}}{5}\right)\right]$$

$$- \left[0 - \frac{1}{3}\frac{(4)^{5/2}}{5/2}\right]$$

$$= \overset{64/3}{\cancel{-\frac{128}{3}}} + \frac{64}{3} - \frac{64}{5} + \frac{64}{15}$$

$$= \frac{128}{15}$$

Date

Q11

$= \iiint_G xyz \, dv$

Z limit

$\boxed{Z = 0}$ $\boxed{Z = 2\cancel{x} - x^2}$

Y - limit.

$\boxed{y = 0}$ $\boxed{y = x}$

X - limit

$\boxed{X = 0}$ $\because y = x \;\&\; y = 0$.

$z = 2 - x^2$

$0 = 2 - x^2$

$\boxed{X = 2}$

$= \int_0^2 \int_0^x \int_0^{2-x^2} xyz \, dz \, dy \, dx$

$= \int_0^2 \int_0^x \frac{xyz^2}{2} \Big|_0^{2-x^2} dy \, dx$

No.

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$$= \int_0^2 \int_0^x \frac{xy(2-x^2)^2}{2}\, dy\, dz$$

$$= \int_0^2 \frac{xy^2(2-x^2)^2}{4} \Big|_0^x$$

$$= \int_0^2 \frac{x^3(2-x^2)^2}{4}\, dx$$

let $u^2 = 2-x^2$

$du\, 2u = -2x\, dx$

$dx = -\frac{u}{x}\, du$

$dx = \frac{-u}{\sqrt{2-u^2}}\, du$

$$= \frac{1}{4}\int_0^2 (2-u^2)^{3/2}\, u^4 \left(\frac{-u}{(2-u^2)^{1/2}}\right) du$$

$$= \frac{-1}{4}\int_0^2 (2-u^2)\, u^5$$

$$= -\frac{1}{4}\int_0^2 2u^5 - u^7\, du$$

Date

$$= -\frac{1}{4}\left[\frac{u^6}{3} - \frac{u^8}{8}\right]$$

$$= -\frac{1}{4}\left[\frac{(2 \cancel{x} - x^2)^3}{3} - \frac{(2-x^2)^4}{8}\right]_0^2$$

$$= -\frac{1}{4}\left[0 - \left(\frac{2^3}{3} - \frac{2^4}{8}\right)\right]$$

$$= -\frac{1}{\cancel{4}_2}\left(-\frac{2}{3}\right)$$

$$= \boxed{\frac{1}{6}}$$

Q15

$$V = \iiint_G dz\,dy\,dx$$

$x = 0$, $y = 0$, $z = 0$

$3x + 6y + 4z = 12$

z - limits

$z = 0$

$z = \frac{12 - 3x - 6y}{4}$

y - limits

$y = 0$

$y = \frac{12 - 3x}{6} \Rightarrow y = \frac{4 - x}{2}$

x - limits

$x = 0$

$x = 4$

$$= \int_0^4 \int_0^{\frac{4-x}{2}} \int_0^{\frac{12-3x-6y}{4}} dz\,dy\,dx$$

Date

$$= \int_0^4 \int_0^{\frac{4-x}{2}} z \Big|_0^{\frac{12-3x-6y}{4}} dy\,dz$$

$$= \int_0^4 \int_0^{\frac{4-x}{2}} \frac{12-3x-6y}{4}\, dy\,dz$$

$$= \int_0^4 \frac{12}{4}y - \frac{3}{4}xy - \frac{6y^2}{8} \Big|_0^{\frac{4-x}{2}} dx$$

$$= \int_0^4 3y - \frac{3}{4}xy - \frac{3}{4}y^2 \Big|_0^{\frac{4-x}{2}} dx$$

$$= \int_0^4 \left[3\left(\frac{4-x}{2}\right) - \frac{3}{4}x\left(\frac{4-x}{2}\right) - \frac{3}{4}\left(\frac{4-x}{2}\right)^2\right] dx$$

$$\int_0^4 \frac{12-3x}{2} - \frac{3}{4}\left(\frac{4x-x^2}{2}\right) - \cancel{\left(\frac{12-3x}{8}\right)}$$

$$-\frac{3}{4}\left(\frac{4^2-8x+x^2}{4}\right)$$

$$\int_0^4 6 - \frac{3}{2}x - \cancel{\frac{3}{2}x} + \frac{3x^2}{8} - \frac{3}{2}$$

$$+\cancel{\frac{3}{2}x} - \frac{3}{16}x^2$$

$$\int_0^4 3 - \frac{3}{2}x + \frac{3x^2}{16}\,dx$$

Date

$$= 3x - \frac{3}{4}x^2 + \frac{3x^3}{16\times 3}\Big|_0^4$$

$$= \frac{3}{4}(4) - \frac{3}{4}(4)^2 + \frac{3(4)^3}{16\times 3}$$

$= -4$

$= 4$

Q17

$y = x^2$

$y + z = 4$

$z = 0$

<u>z limits</u>

$z = 0 \quad z = 4 - y$

<u>y - limit</u>

$y = x^2 \quad y = 4$

<u>x - limits</u>

$x = \sqrt{y}$

$x = \sqrt{4}$

$x = \pm 2$

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$$\int_{-2}^{2}\int_{x^2}^{4}\int_{0}^{4-y} dz\,dy\,dx$$

$$\int_{-2}^{2}\int_{x^2}^{4} z\Big|_0^{4-y} \; dy\,dx$$

$$\int_{-2}^{2}\int_{x^2}^{4} 4-y \quad dy\,dx$$

$$\int_{-2}^{2} 4y-\frac{y^2}{2}\Big|_{x^2}^{4} \quad dx$$

$$\int_{-2}^{2}\left(16-\frac{16}{2}\right)-\left(4x^2-\frac{x^4}{2}\right) dx$$

$$\int_{-2}^{2} 8-4x^2+\frac{x^4}{2} \quad dx$$

$$8x-\frac{4x^3}{3}+\frac{x^5}{10}\Big|_{-2}^{2}$$

$$\left[8(2)-\frac{4(2)^3}{3}+\frac{(2)^5}{10}\right]-\left[8(-2)-\frac{4(-2)^3}{3}+\frac{(-2)^5}{10}\right]$$

$$=\frac{128}{15}-\left(\frac{-128}{15}\right)$$

$$=\frac{256}{15}$$

No.