Assignment #05

Q3

$xy = -9$

First we find angle of rotational axis $\because Ax^2 + Cy^2 + Bxy$

$$\cot 2\theta = \frac{A-C}{B}$$

$$\cot 2\theta = \frac{0-0}{1}$$

$$\cot 2\theta = 0$$

$$\frac{1}{\cot 2\theta} = \frac{1}{0}$$

$$\tan 2\theta = \infty$$

$$2\theta = \tan^{-1}(\infty)$$

$$2\theta = 90$$

$$\theta = 45^\circ$$

Find x & y co-ordinate in rotational axis

$$x = x'\cos\theta - y'\sin\theta \quad (a)$$

$$y = x'\sin\theta + y'\cos\theta \quad (b)$$

For x :

$$x = x'\cos(45) - y'\sin(45)$$

$$x = \frac{x'}{\sqrt{2}} - \frac{y'}{\sqrt{2}}$$

For y :

$y = x'\sin 45 + y'\cos 45$

$$y = \frac{x'}{\sqrt{2}} + \frac{y'}{\sqrt{2}}$$

put in given equation

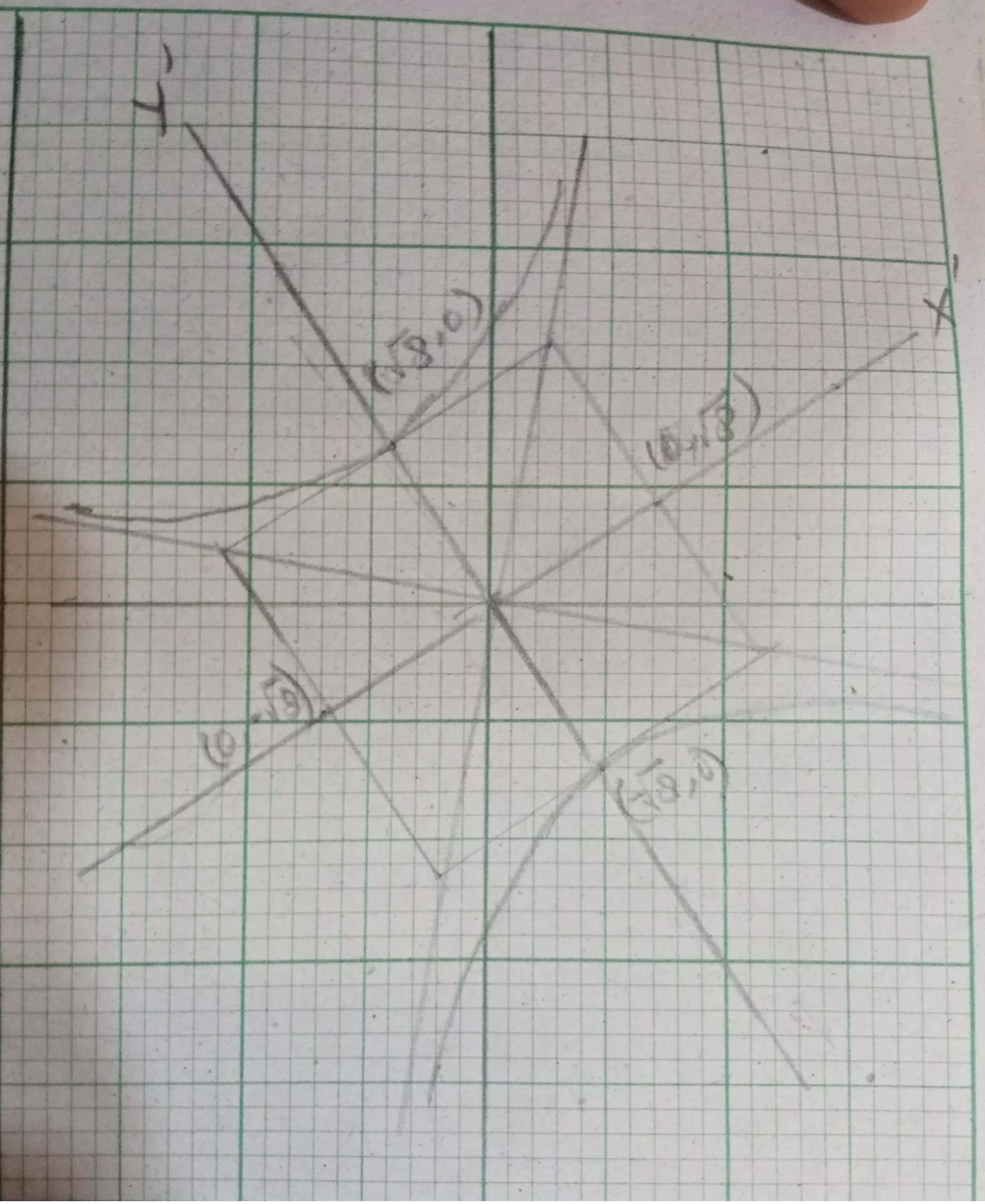
$$\left(\frac{x'}{\sqrt{2}} - \frac{y'}{\sqrt{2}}\right)\left(\frac{x'}{\sqrt{2}} + \frac{y'}{\sqrt{2}}\right) = -9$$

$$\frac{x'}{2} - \frac{y^2}{2} = -9$$

multiply whole equation by $-\frac{1}{9}$

$$\frac{y'^2}{18} - \frac{x'^2}{18} = 1$$

Type of Conic : Hyperbola


Y'
X'
(√8,0)
(0,√8)
(0,-√8)
(-√8,0)

Q7

$x^2 + 2\sqrt{3}xy + 3y^2 + 2\sqrt{3}x - 2y = 0$

$x^2 + 2\sqrt{3}x + 3\left(y^2 - \frac{2}{3}y\right) + 2\sqrt{3}xy = 0$

$x^2 + 2\sqrt{3}x + 3 + 3\left(y^2 - \frac{2}{3}y + \frac{1}{9}\right) + 2\sqrt{3}xy = 3 + \frac{1}{3}$

$(x+\sqrt{3})^2 + 3\left(y + \frac{1}{3}\right)^2 + 2\sqrt{3}xy = \frac{10}{3}$

Find angle

$$\cot 2\theta = \frac{A - C}{B}$$

$$= \frac{1-3}{2\sqrt{3}}$$

$$\cot 2\theta = -\frac{1}{\sqrt{3}} \Rightarrow \tan 2\theta = -\sqrt{3}$$

$$2\theta = \tan^{-1}(-\sqrt{3})$$

$$\theta = 30$$

$$\boxed{\begin{aligned} x &= x'\cos\theta - y'\sin\theta \\ y &= x'\sin\theta + y'\cos\theta \end{aligned}}$$

For x

$x = x'\cos(30) - y'\sin(30)$

$x = \frac{x'}{2} - \frac{\sqrt{3}y'}{2}$

For y

$$y = x'\sin(60) + y'\cos 60$$

$$y = \frac{\sqrt{3}x}{2} + \frac{y'}{2}$$

put in given equation

$$x^2 + 2\sqrt{3}xy + 3y^2 + 2\sqrt{3}x - 2y = 0 \qquad \left(\frac{\sqrt{3}}{2}x' + \frac{y'}{2}\right)$$

$$\left(\frac{x'}{2} - \frac{\sqrt{3}}{2}y'\right)^2 + 2\sqrt{3}\left(\frac{x'}{2} - \frac{\sqrt{3}y'}{2}\right) + 3\left(\frac{\sqrt{3}}{2}x' + \frac{y'}{2}\right)^2$$

$$+ 2\sqrt{3}\left(\frac{x'}{2} - \frac{\sqrt{3}}{2}y'\right) - 2\left(\frac{\sqrt{3}}{2}x' + \frac{y'}{2}\right) = 0$$

$$\frac{x'^2}{4} - 2\left(\frac{x'}{2}\right)\left(\frac{\sqrt{3}}{2}y\right) + \frac{3y'^2}{4} + 2\sqrt{3}\left(\frac{\sqrt{3}}{4}x'^2 + \frac{x'y'}{4} - \frac{3}{4}x'y'\right.$$

$$\left.-\frac{\sqrt{3}}{4}y'^2\right) + \sqrt{3}x' - 3y' - \sqrt{3}x' - y' + 3\left(\frac{3}{4}x'^2 + 2\left(\frac{\sqrt{3}}{2}\right)\left(\frac{1}{2}\right)x'y'\right.$$

$$\frac{x'^2}{4} - \cancel{\frac{\sqrt{3}}{2}x'y'} + \frac{3y'^2}{4} + \frac{3x'^2}{2} + \cancel{\frac{\sqrt{3}x'y'}{2}} - \qquad \left.+\frac{y'^2}{4}\right) = 0$$

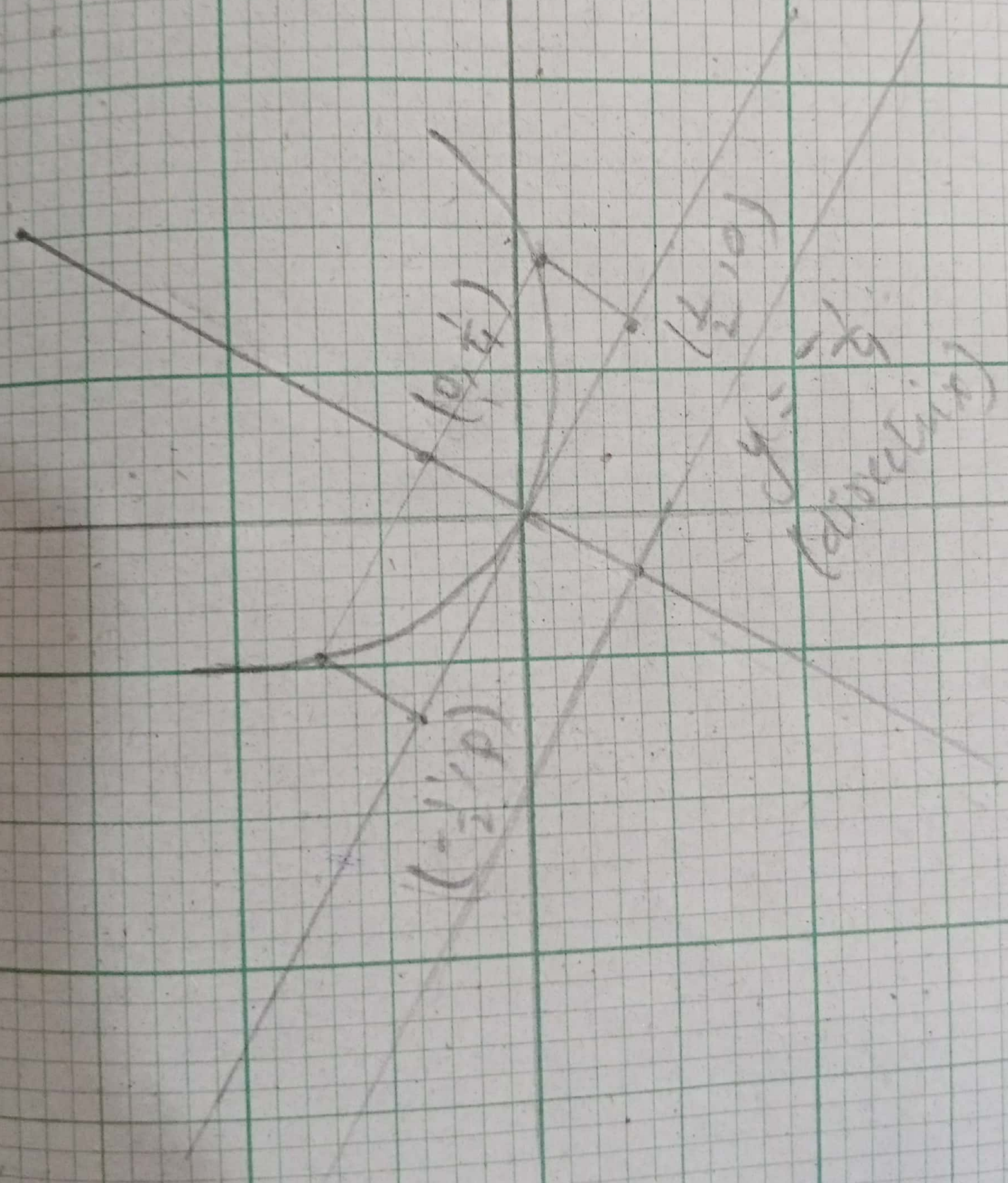
$$\cancel{\frac{3\sqrt{3}}{2}x'y'} - \frac{3}{2}y'^2 + \cancel{\sqrt{3}x'} - 3y' - \cancel{\sqrt{3}x'} - y' + \frac{9}{4}x'^2 + \cancel{\frac{3\sqrt{3}}{2}x'y'}$$

$$+ \frac{3y'^2}{4} = 0$$

$$4x'^2 - 4y' = 0$$

$$\boxed{x'^2 = y'}$$

Type of conic : Parabola.


(0, 1/4)
(directrix)

Q11

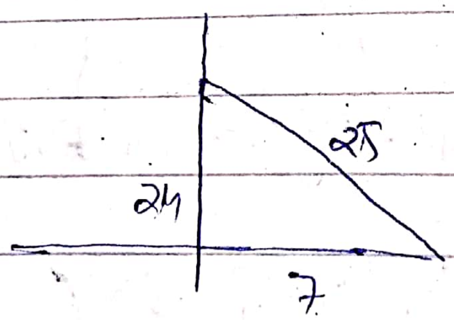
$$52x^2 - 72xy + 73y^2 + 40x + 30y - 75 = 0$$

$$x = x'\cos\theta - y'\sin\theta$$

$$y = x'\sin\theta + y'\cos\theta$$

$$\cot 2\theta = \frac{A-C}{B} = \frac{52-73}{-72}$$

$$\cot 2\theta = \frac{7}{24} = \frac{B}{P}$$



$$\cos 2\theta = \frac{7}{25}$$

$$\cos\theta = \sqrt{\frac{1+\cos 2\theta}{2}} = \sqrt{\frac{1+7/25}{2}}$$

$$\cos\theta = 4/5$$

$$\sin\theta = \sqrt{\frac{1-\cos 2\theta}{2}} = \sqrt{\frac{1-7/25}{2}}$$

$$\sin\theta = \frac{3}{5}$$

$x = \frac{4x'}{5} - \frac{3y'}{5}$

$y = \frac{3}{5}x' + \frac{4}{5}y'$

put in given equations

$= 52\left(\frac{4x'}{5} - \frac{3y'}{5}\right)^2 - 72\left(\frac{4x'}{5} - \frac{3y'}{5}\right)\left(\frac{3}{5}x' + \frac{4}{5}y'\right) +$

$73\left(\frac{3}{5}x' + \frac{4}{5}y'\right)^2 + 40\left(\frac{4x'}{5} - \frac{3y'}{5}\right) + 30\left(\frac{3}{5}x' + \frac{4}{5}y'\right)$

$-75 = 0$

$\Rightarrow 52\left(\frac{16x'^2}{25} - \frac{24}{25}x'y' + \frac{9y'^2}{25}\right) - 72\left(\frac{12x'^2}{25} + \frac{16x'y'}{25}\right.$

$\left.-\frac{9}{25}x'y' - \frac{12}{25}y'^2\right) + 73\left(\frac{9}{25}x'^2 + \frac{24x'y'}{25} + \frac{16}{25}y'^2\right)$

$+ \frac{40\times4x'}{5} - \frac{40\times3}{5}y' + \frac{30\times3}{5}x' + \frac{4\times30}{5}y' - 75 = 0$

$\Rightarrow \frac{832}{25}x'^2 - \frac{1248}{25}x'y' + \frac{468}{25}y'^2 - \frac{864}{25}x'^2 - \frac{1152}{25}x'y'$

$+ \frac{648}{25}x'y' + \frac{864}{25}y'^2 + \frac{657}{25}x'^2 + \frac{1752}{25}x'y' + \frac{1168}{25}y'^2$

$+ \frac{40\times4x'}{5} - \frac{40\times3}{5}y' + \frac{30\times3}{5}x' + \frac{4\times30}{5}y' - 75 = 0$

$$25x'^2 + 100y'^2 + 50x' = 75$$

$\div$ by 25

$$x'^2 + 4y'^2 + 2x' = 3$$

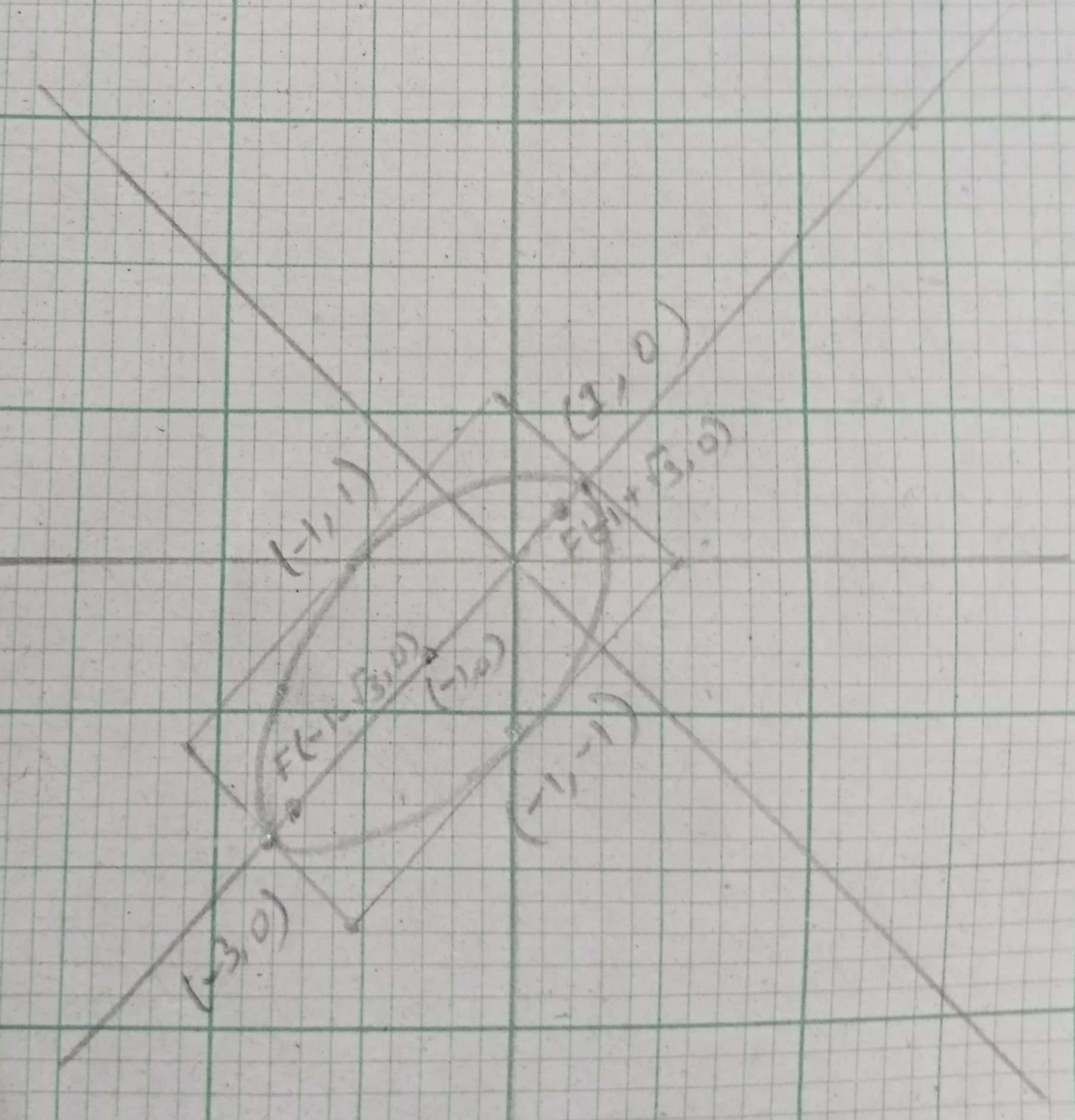
$$\frac{x'^2 + 2x' +}{3} + \frac{4y'^2}{3} = 1$$

$$\frac{(x'+1)^2}{3} + \frac{y'^2}{\frac{3}{4}} = 1$$

$$(x'+1)^2 + 4y'^2 = 4$$

$$\boxed{\frac{(x'+1)^2}{4} + y'^2 = 1}$$

Type of Conic : Elipse

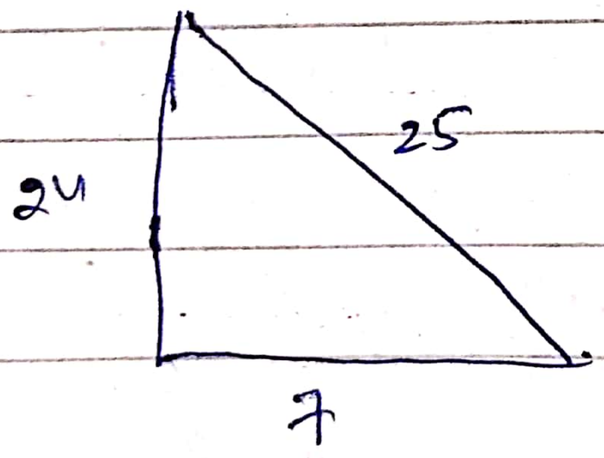

(1, 0)
F(-1+√3, 0)
(-1, 1)
(-1, 0)
F(-1-√3, 0)
(-1, -1)
(-3, 0)

Q21

$$9x^2 - 24xy + 16y^2 - 80x - 60y + 100 = 0$$

$$\cot 2\theta = \frac{A-C}{B} = \frac{9-16}{-24}$$

$$\cot 2\theta = \frac{7}{24} = \frac{B}{P}$$



$$\cos 2\theta = \frac{7}{25}$$

$$\cos\theta = \sqrt{\frac{1+\cos 2\theta}{2}} = \sqrt{\frac{1+(7/25)}{2}}$$

$$\cos\theta = \frac{\cancel{3}\,4}{5}$$

$$\sin\theta = \sqrt{\frac{1-\cos 2\theta}{2}} = \sqrt{\frac{1-(7/25)}{2}}$$

$$\sin\theta = \frac{3}{5}$$

$$x = x'\cos\theta - y'\sin\theta$$

$$x = \frac{4x'}{5} - \frac{3}{5}y'$$

$$y = x'\sin\theta + y'\cos\theta$$

$$y = \frac{3}{5}x' + \frac{4}{5}y'$$

put in given equation

$$\Rightarrow 9\left(\frac{4}{5}x' - \frac{3}{5}y'\right)^2 - 24\left(\frac{4}{5}x' - \frac{3}{5}y'\right)\left(\frac{3}{5}x' + \frac{4}{5}y'\right) +$$

$$16\left(\frac{3}{5}x' + \frac{4}{5}y'\right)^2 - 80\left(\frac{4}{5}x' - \frac{3}{5}y'\right) - 60\left(\frac{3}{5}x'\right.$$

$$\left. + \frac{4}{5}y'\right) + 100 = 0$$

$$\Rightarrow 9\left(\frac{16}{25}x'^2 + \frac{24}{25}x'y' + \frac{9}{25}y'^2\right) - 24\left(\frac{12}{25}x'^2 + \frac{16}{25}x'y' - \frac{9}{25}x'y' - \frac{12}{25}y'^2\right) + 16\left(\frac{9}{25}x'^2 + \frac{24}{25}x'y' + \frac{16}{25}y'^2\right)$$

$$-64x' + \cancel{48y'} - 36x' - \cancel{48y'} + 100 = 0$$

$$\Rightarrow \frac{144}{25}x'^2 + \frac{216}{25}x'y' + \frac{81}{25}y'^2 - \frac{288}{25}x'^2 - \frac{168}{25}x'y' + \frac{288}{25}y'^2 + \frac{144}{25}x'^2 + \frac{384}{25}x'y' + \frac{256}{25}y'^2 - 100x' + 100$$

$$25y'^2 - 100x' = -100$$

$$y'^2 - 4x' = -4$$

$$y'^2 = 4(x'-1)$$

Type of conics : Parabola

Vertex $\rightarrow$ $(1, 0)$

Focus $\rightarrow$ $(2, 0)$

Directrix $\rightarrow$ $x' = 0$

For vertex in original co-ordinates

$$x = (1)(4/5) - (0)(3/5) = 4/5$$

$$y = (1)(3/5) - (0)(4/5) = 3/5$$

$$\boxed{\text{Vertex}(x, y) \Rightarrow (4/5, 3/5)}$$

For Focus :-

$x = (2)(4/5) - (0)(3/5) = 8/5$

$y = (2)(3/5) + (0)(4/5) = 6/5$

Focus $(x, y) \Rightarrow (8/5, 6/5)$

For directrix

$\therefore x' = x\cos\theta + y\sin\theta$

$x\cos\theta + y\sin\theta = 0$

$x\left(\frac{4}{5}\right) + y\left(\frac{3}{5}\right) = 0$

$\frac{1}{5}(4x + 3y) = 0$

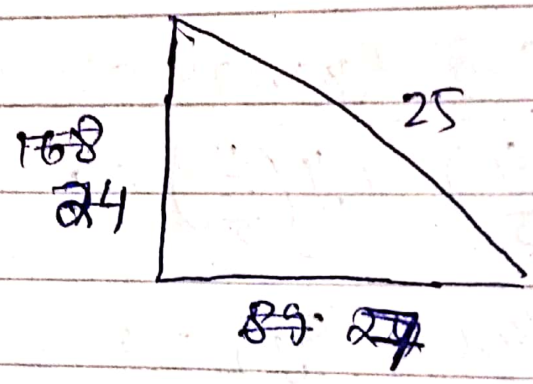
$\boxed{4x + 3y = 0}$

Q23

$$288x^2 - 168xy + 337y^2 - 3600 = 0$$

$$\cot 2\theta = \frac{A-C}{B} = \frac{288-337}{-168}$$

$$\cot 2\theta = \frac{\cancel{89}\,7}{\cancel{168}\,24} = \frac{B}{P}$$



$$\cos 2\theta = \frac{7}{25}$$

$$\cos\theta = \sqrt{\frac{1+\cos 2\theta}{2}} = \sqrt{\frac{1+(7/25)}{2}}$$

$$\cos\theta = \frac{4}{5}$$

$$\sin\theta = \sqrt{\frac{1-\cos 2\theta}{2}} = \sqrt{\frac{1-(7/25)}{2}}$$

$$\sin\theta = \frac{3}{5}$$

$$x = \frac{4}{5}x' - \frac{3}{5}y'$$

$$y = \frac{3}{5}x' + \frac{4}{5}y'$$

put in given equation

$$\Rightarrow 288\left(\frac{4}{5}x' - \frac{3}{5}y'\right)^2 - 168\left(\frac{4}{5}x' - \frac{3}{5}y'\right)\left(\frac{3}{5}x' + \frac{4}{5}y'\right) + 337\left(\frac{3}{5}x' + \frac{4}{5}y'\right)^2 - 3600 = 0$$

$$\Rightarrow 288\left(\frac{16}{25}x'^2 - \frac{24}{25}x'y' + \frac{9}{25}y'^2\right) - 168\left(\frac{12}{25}x'^2 + \frac{16}{25}x'y' - \frac{9}{25}x'y' - \frac{12}{25}y'^2\right) + 337\left(\frac{9}{25}x'^2 + \frac{24}{25}x'y' + \frac{16}{25}y'^2\right) - 3600 = 0$$

$$\Rightarrow \frac{4608}{25}x'^2 - \frac{6912}{25}x'y' + \frac{2592}{25}y'^2 - \frac{2016}{25}x'^2 - \frac{2688}{25}x'y' + \frac{1512}{25}x'y' + \frac{2016}{25}y'^2 + \frac{3033}{25}x'^2 + \frac{8088}{25}x'y' + \frac{5392}{25}y'^2 - 3600 = 0$$

$$225x'^2 + 400y'^2 = 3600$$

$\div$ by 3600

$$\frac{x'^2}{16} + \frac{y'^2}{9} = 1$$

~~Vertices $\Rightarrow$ $(\pm 4, 0)$~~

~~Focus $\Rightarrow$ $(0, \pm\sqrt{7})$~~

~~Directrix $\Rightarrow$ (~~

In $x'y'$-coordinate :

Vertices $\Rightarrow (\pm 4, 0)$

Foci $\Rightarrow (\pm\sqrt{7}, 0)$

End points of minor axis $= (0, \pm 3)$

For vertices

$$x = (4)\frac{4}{5} - (0)\left(\frac{3}{5}\right) = \frac{16}{5}$$

$$y = 4\left(\frac{3}{5}\right) + (0)\left(\frac{4}{5}\right) = \frac{12}{5}$$

$$\left(\frac{16}{5}, \frac{12}{5}\right)$$

$$x = (-4)\left(\frac{4}{5}\right) - (0)\left(\frac{3}{5}\right) = \frac{-16}{5}$$

$$y = (-4)\left(\frac{3}{5}\right) + (0)\left(\frac{4}{5}\right) = \frac{-12}{5}$$

$$(-16/5, -12/5)$$

$$\boxed{\text{vertices} \Rightarrow (16/5, 12/5)\ (-16/5, -12/5)}$$

For Foci

when $(x', y') = (\sqrt{7}, 0)$

$$x = \sqrt{7}\left(\frac{4}{5}\right) - (0)\left(\frac{3}{5}\right) = \frac{4\sqrt{7}}{5}$$

$$y = \sqrt{7}\left(\frac{3}{5}\right) - (0)\left(\frac{4}{5}\right) = \frac{3\sqrt{7}}{5}$$

$$(x, y) \Rightarrow (4\sqrt{7}/5, 3\sqrt{7}/5)$$

when $(x', y') = (-\sqrt{7}, 0)$

$$x = (-\sqrt{7})\left(\frac{4}{5}\right) - (0)\left(\frac{3}{5}\right) = \frac{-4\sqrt{7}}{5}$$

$$y = (-\sqrt{7})\left(\frac{3}{5}\right) - 0\left(\frac{4}{5}\right) = -\frac{3\sqrt{7}}{5}$$

$$(x, y) \Rightarrow \left(-\frac{4\sqrt{7}}{5}, -\frac{3\sqrt{7}}{5}\right)$$

$$\boxed{\text{Foci} \Rightarrow \left(\frac{4\sqrt{7}}{5}, \frac{3\sqrt{7}}{5}\right), \left(\frac{-4\sqrt{7}}{5}, \frac{-3\sqrt{7}}{5}\right)}$$

For end points of minor axis :-

when $(x', y') = (0, 3)$

$$x = 0(4/5) - 3(3/5) = -9/5$$

$$y = 0(3/5) + 3(4/5) = 12/5$$

$$(x, y) \Rightarrow (-9/5, 12/5)$$

when $(x', y') = (0, -3)$

$$x = (0)(4/5) - (-3)(3/5) = 9/5$$

$$y = (0)(3/5) + (-3)(4/5) = -12/5$$

$$(x, y) \Rightarrow (9/5, -12/5)$$

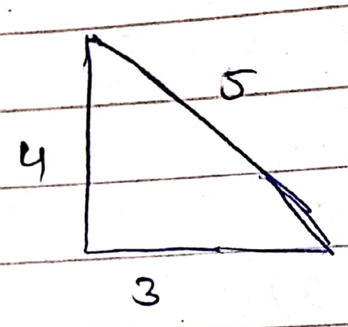
$$\boxed{\text{End points of minor axis} \Rightarrow \left(-\frac{9}{5}, \frac{12}{5}\right), \left(\frac{9}{5}, -\frac{12}{5}\right)}$$

Q29

$$32y^2 - 52xy\ 7x^2 + 72\sqrt{5}x - 144\sqrt{5}y + 900 = 0$$

$$\cot 2\theta = \frac{A-C}{B} = \frac{\cancel{-7}+32}{-52}$$

$$\cot 2\theta = \frac{3}{4} = \frac{B}{P}$$



$$\cos 2\theta = \frac{3}{5}$$

$$\cos\theta = \sqrt{\frac{1+\cos 2\theta}{2}} = \frac{2\sqrt{5}}{5} = \frac{2}{\sqrt{5}}$$

$$\sin\theta = \sqrt{\frac{1-\cos 2\theta}{2}} = \frac{\sqrt{5}}{5} = \frac{1}{\sqrt{5}}$$

$$X = \frac{2\sqrt{5}}{5}x' - \frac{\sqrt{5}}{5}y'$$

$$Y = \frac{\sqrt{5}}{5}x' + \frac{2\sqrt{5}}{5}y'$$

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$$\Rightarrow 32\left(\frac{\sqrt{5}}{5}x' + \frac{2\sqrt{5}}{5}y'\right)^2 - 52\left(\frac{2\sqrt{5}}{5}x' - \frac{\sqrt{5}}{5}y'\right)\left(\frac{\sqrt{5}}{5}x' + \frac{2\sqrt{5}}{5}y'\right)$$

$$-7\left(\frac{2\sqrt{5}}{5}x' - \frac{\sqrt{5}}{5}y'\right)^2 + 72\sqrt{5}\left(\frac{2\sqrt{5}}{5}x' - \frac{\sqrt{5}}{5}y'\right) -$$

$$144\sqrt{5}\left(\frac{\sqrt{5}}{5}x' + \frac{2\sqrt{5}}{5}y'\right) + 900 = 0$$

$$\Rightarrow 32\left(\frac{1}{5}x'^2 + \frac{4}{5}x'y' + \frac{4}{5}y'^2\right) - 52\left(\frac{2}{5}x'^2 + \frac{4}{5}x'y' -\right.$$

$$\left.\frac{1}{5}x'y' - \frac{2}{5}y'^2\right) \cancel{+72\sqrt{5}} - 7\left(\frac{4}{5}x'^2 - \frac{4}{5}x'y' + \frac{1}{5}y'^2\right)$$

$$+ \cancel{72\sqrt{5}}\ 144\cancel{x'} - 72y' - 4\cancel{4}x' - 288y' + 900 = 0$$

$$\Rightarrow \frac{32}{5}x'^2 + \frac{128}{5}x'y' + \frac{128}{5}y'^2 - \frac{104}{5}x'^2 - \frac{208}{5}x'y' +$$

$$\frac{52}{5}x'y' + \frac{104}{5}y'^2 - \frac{28}{5}x'^2 + \frac{28}{5}x'y' - \frac{7}{5}y'^2 - 360y'$$

$$= -900$$

$$\Rightarrow -20x'^2 + 45y'^2 - 360y' = -900$$

$$-20x'^2 + 45(y'^2 - 8y') = -900$$

$$-20x'^2 + 45(y'^2 - 8y' + 16) = -900 + 720$$

$$-20x'^2 + 45(y' - 4)^2 = -180$$

$$-4x'^2 + 9(y'^2 - 4)^2 = -36$$

$$\cancel{4x^2 = 9(y'}$$

$$\frac{x'^2}{9} - \frac{(y'-4)^2}{4} = 1$$

Vertices $\Rightarrow$ $(\pm 3, 4)$

Foci $\Rightarrow$ $(\pm\sqrt{13}, 4)$

Eq of asymptotes $y' = 4 \pm \frac{2}{3}x'$

For Vertices :-

when $(x', y') = (3, 4)$

$$x = 3\left(\frac{2\sqrt{5}}{5}\right) - 4\left(\frac{\sqrt{5}}{5}\right) = \frac{2\sqrt{5}}{5}$$

$$y = 3\left(\frac{\sqrt{5}}{5}\right) + 4\left(\frac{2\sqrt{5}}{5}\right) = \frac{11\sqrt{5}}{5}$$

$$\left(\frac{2\sqrt{5}}{5}, \frac{11\sqrt{5}}{5}\right)$$

when $(x, y) = (-3, 4)$

$$x = -3\left(\frac{2\sqrt{5}}{5}\right) - 4\left(\frac{\sqrt{5}}{5}\right) = -2\sqrt{5}$$

$$y = -3\left(\frac{\sqrt{5}}{5}\right) + 4\left(\frac{2\sqrt{5}}{5}\right) = \sqrt{5}$$

$$(-2\sqrt{5}, \sqrt{5})$$

vertices $\Rightarrow$ $\left(\frac{2\sqrt{5}}{5}, \frac{11\sqrt{5}}{5}\right), (-2\sqrt{5}, \sqrt{5})$

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For Foci:-

when $(x', y') = (\sqrt{13}, 4)$

$$x = \sqrt{13}\left(\frac{2\sqrt{5}}{5}\right) - 4\left(\frac{\sqrt{5}}{5}\right) = \frac{2\sqrt{65}-4\sqrt{5}}{5}$$

$$y = \sqrt{13}\left(\frac{\sqrt{5}}{5}\right) + 4\left(\frac{2\sqrt{5}}{5}\right) = \frac{\sqrt{65}+8\sqrt{5}}{5}$$

$$\left(\frac{2\sqrt{65}-4\sqrt{5}}{5}, \frac{\sqrt{65}-8\sqrt{5}}{5}\right)$$

when $(x', y') = (-\sqrt{13}, 4)$

$$x = (-\sqrt{13})\left(\frac{2\sqrt{5}}{5}\right) - 4\left(\frac{\sqrt{5}}{5}\right) = \frac{-2\sqrt{65}-4\sqrt{5}}{5}$$

$$y = (-\sqrt{13})\left(\frac{\sqrt{5}}{5}\right) + 4\left(\frac{2\sqrt{5}}{5}\right) = \frac{-\sqrt{65}+8\sqrt{5}}{5}$$

Foci $\Rightarrow \left(\frac{2\sqrt{65}-4\sqrt{5}}{5}, \frac{\sqrt{65}-8\sqrt{5}}{5}\right), \left(\frac{-2\sqrt{65}-4\sqrt{5}}{5}, \frac{-\sqrt{65}+8\sqrt{5}}{5}\right)$

__For asymptotes :-__

__First case :-__

$$y' = 4 + \frac{2}{3}x'$$

$$-x\sin\theta + y\cos\theta = 4 + \frac{2}{3}\left(x\cos\theta + y\sin\theta\right)$$

$$\cancel{-\frac{2x}{\sqrt{5}} + \frac{2}{\sqrt{5}}y}$$

$$-\frac{1}{\sqrt{5}}x + \frac{2}{\sqrt{5}}y = 4 + \frac{2}{3}\left(\frac{2}{\sqrt{5}}x + \frac{1}{\sqrt{5}}y\right)$$

$$-\frac{1}{\sqrt{5}}x + \frac{2}{\sqrt{5}}y = 4 + \frac{4}{3\sqrt{5}}x + \frac{2}{3\sqrt{5}}y$$

$$\frac{4}{3\sqrt{5}}x + \frac{1}{\sqrt{5}}x + \frac{2}{3\sqrt{5}}y - \frac{2}{\sqrt{5}}y = -4$$

$$\frac{7}{3\sqrt{5}}x - \frac{4}{3\sqrt{5}}y = -4$$

$$\frac{7}{3\sqrt{5}}x + 4 = \frac{4}{3\sqrt{5}}y$$

$$\boxed{\frac{7}{4}x + 3\sqrt{5} = y}$$

case 2 :-

$$y' = 4 - \frac{2}{3}x'$$

$$-x\sin\theta + y\cos\theta = 4 - \frac{2}{3}(x\cos\theta + y\sin\theta)$$

$$-\frac{1}{\sqrt{5}}x + \frac{2}{\sqrt{5}}y = 4 - \frac{2}{3}\left(\frac{2}{\sqrt{5}}x + \frac{1}{\sqrt{5}}y\right)$$

$$-\frac{1}{\sqrt{5}}x + \frac{2}{\sqrt{5}}y = 4 - \frac{4}{3\sqrt{5}}x - \frac{2}{3\sqrt{5}}y$$

$$\frac{2}{\sqrt{5}}y + \frac{2}{3\sqrt{5}}y = 4 - \frac{4}{3\sqrt{5}}x + \frac{1}{\sqrt{5}}x$$

$$\frac{8}{3\sqrt{5}}y = 4 - \frac{1}{3\sqrt{5}}x$$

$$\boxed{y = \frac{3\sqrt{5}}{2} - \frac{1}{8}x}$$