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Section:- 3E
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DISCRETE ASSIGNMENT #3

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Q1) Sol:-

a) $\{x \mid x$ is a real number such that $x^2 = 1\} = \{-1, 1\}$

b) $\{x \mid x$ is a positive integer less than $12\} = \{1,2,3,4,5,6,7,8,9,1$

c) $\{x \mid x$ is the square of an integer and $x < 100\} = \{0, 1,4,9,16,25,36,$

d) $\{x \mid x$ is an integer such that $x^2 = 2\} = \emptyset$

2) Sol:-

a) The set of nonstop airline flights from New York to New Delh
a subset of airline flights from New York to New Delhi.

) Neither is a subset of the other.

) The set of flying squirrels is a subset of the set of living c
at can fly.

). Sol:-

The second is the subset of the first.

The second is the subset of the first.

Neither is a subset of the other.

Sol:-

The two sets are equal as both contain common elements 1,

The two sets are not equal as the cardinality of sets is
$|\{\{1\}\}| = 1$ and $|\{1, \{1\}\}| = 2$

he two sets are not equal as the cardinality of sets
$|\emptyset| = 0$ and $|\{\emptyset\}| = 1$

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Q5) Sol:-

B is a subset of A, while C is a subset of both A and D.

Q6) Sol:-

ⓐ $\{2,3,4,5,\dots\}$, Yes, 2 is an element of this set.

ⓑ $\{1,4,9,16,\dots\}$, NO, 2 is not an element of this set.

ⓒ $\{2,\{2\}\}$, Yes, 2 is an element of this set.

ⓓ $\{2\} \neq 2$, $\{\{2\}\} \neq 2$, So NO, 2 is not an element of this set.

ⓔ $\{2\} \neq 2$, $\{2,\{2\}\} \neq 2$, So NO, 2 is not an element of this set.

ⓕ $\{\{\{2\}\}\} \neq 2$, So NO, 2 is not an element of this set.

Q7) Sol:-

ⓐ $0 \in \emptyset$, False

- Empty set doesn't contain any elements.

ⓑ $\emptyset \in \{0\}$, False

- Empty set can not be an element of a set that contains only zero

ⓒ $\{0\} \subset \emptyset$, False

- Set that contains zero, is not a subset of empty set.

ⓓ $\emptyset \subset \{0\}$, True

- Set zero contains empty set, so empty set is subset of set zero.

ⓔ $\{0\} \in \{0\}$, False

- Zero element belongs to the set of zero not zero set $\{0\}$.

ⓕ $\{0\} \subset \{0\}$, False

- As the sets are completely identical, so they are not subsets of each other.

ⓖ $\{\emptyset\} \subseteq \{\emptyset\}$, True

- A set is subset of itself.

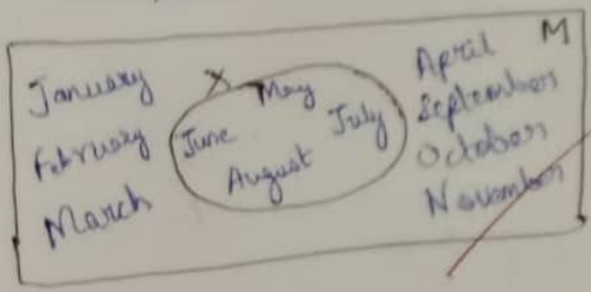
Q10)

Sol:- Set of all months of the year whose names do not contain letter

$X = \{May, June, July, August\}$

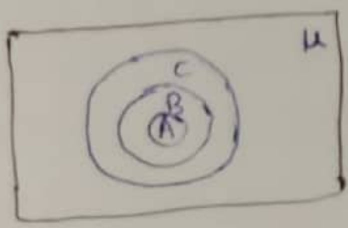
Universal set = set containing all months

$M = \{$January, February, March, April, May, June, July, August, September, October, November, December$\}$



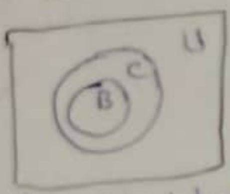
Q11) Sol:-

$A \subset B$ and $B \subset C$

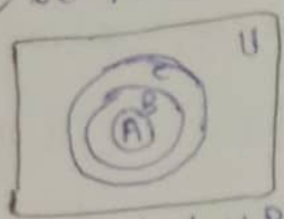


Q12) Sol:-

$A \subseteq B$ and $B \subseteq C$, so $A \subseteq C$



B is subset to C



A is subset of B
B is subset of C
A is subset of C

Q13) Sol:-

(a) $\{a\}$, Cardinality = 1

(b) $\{\{a\}\}$, Cardinality = 1

(c) $\{a, \{a\}\}$, Cardinality = 2

(d) $\{a, \{a\}, \{a, \{a\}\}\}$, Cardinality = 3

Q14) Sol:-

(a) $\emptyset$, Cardinality = 0

(b) $\{\emptyset\}$, Cardinality = 1

(c) $\{\emptyset, \{\emptyset\}\}$, Cardinality = 2

(d) $\{\emptyset, \{\emptyset\}, \{\emptyset, \{\emptyset\}\}\}$, Cardinality = 3

Q18) Sol:-

(a) $A \times B = \{(a,y),(a,z),(b,y),(b,z),(c,y),(c,z),(d,y),(d,z)\}$

(b) $B \times A = \{(y,a),(z,a),(y,b),(z,b),(y,c),(z,c),(y,d),(z,d)\}$

Q19) Sol:-

The Cartesian product of two sets, A and set B is the set of all ordered pairs (a,b) such that $a \in A$ and $b \in B$ which is denoted as $A \times B$. The Cartesian product $A \times B$ consists of all ordered pairs of the form(a,b) where a is a course offered by the mathematics department at University and b is a mathematics professor at this university. One way to use the set $A \times B$ is to represent all possible courses offered by the mathematics department taught by the mathematics professors at the university.

Q20) Sol:-

The Cartesian product $A \times B \times C$ consists of all ordered triples of the form (a,b,c) where a is an airline and both b and c are cities in the United States.

One way to use the set $A \times B \times C$ represents all airlines that fly from the city B to city C of the United States and $A \times B \times C$ would give us information on the connectivity of the cities using the airline.

Eg; Let $A = \{a,b\}$, $B = \{x,y\}$ and $C = \{1\}$

$A \times B \times C = a,b \times [x,y \times 1]$

$= a,b \times [x,1,y,1]$

$= (a,x,1),(a,y,1),(b,x,1),(b,y,1)$

$A \cup B = \{0,1,2,3,4,5,6\}$

$A \cap B = \{3\}$

(c) $A - B = \{1,2,4,5\}$

(d) $B - A = \{0,6\}$

Q28) Sol:-

(a) $A \cup B = \{a,b,c,d,e,f,g,h\}$

(b) $A \cap B = \{a,b,c,d,e\}$

(c) $A - B = \{\}$

(d) $B - A = \{f,g,h\}$

Q29) Sol:-

(a) $A \cup B$

26 alphabets = 26 bits

	a	b	c	d	e	f	g	h	i	j	k	l	m	n	o	p	q	r	s	t	u	v	w	x
$A=\{a,b,c,d,e\}$	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
$B=\{b,c,d,g,p,t,v\}$	0	1	1	1	0	0	1	0	0	0	0	0	0	0	0	1	0	0	0	1	0	1	0	0
$C=\{c,e,i,o,u,x,y,z\}$	0	0	1	0	1	0	0	0	1	0	0	0	0	0	1	0	0	0	0	0	1	0	0	1
$D=\{d,e,h,i,n,o,t,u,x,y\}$	0	0	0	1	1	0	0	1	1	0	0	0	0	1	1	0	0	0	0	1	1	0	0	1

(a) $A \cup B = \cancel{A \times B}\ A \vee B$

$\boxed{A \cup B = 11111010000000001000101000 0}$

(b) $A \cap B = A \wedge B$

$\boxed{A \cap B = 0111000000000000000000000 0}$

(c) $(A \cup D) \cap (B \cup C)$

$A \cup D = 11111001100001100001100110$

$B \cup C = 01111010100000110001110111$

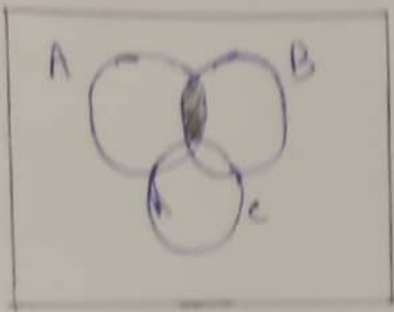
$\boxed{(A \cup D) \cap (B \cup C) = 01111000100000100001100110}$

(d) $A \cup B \cup C \cup D = A \vee B \vee C \vee D$

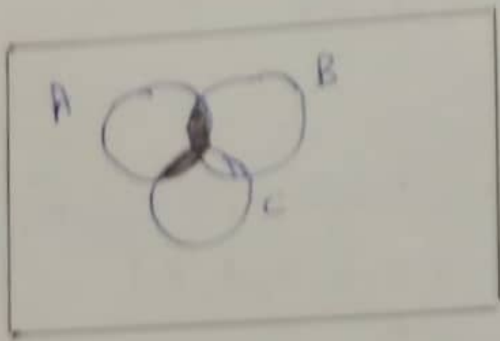
$\boxed{A \cup B \cup C \cup D = 1\,1\,1\,1\,1\,0\,1\,1\,1\,0000\,1\,1\,1\,000\,1\,1\,1\,0\,1\,1\,1}$

(Q30) Sol:

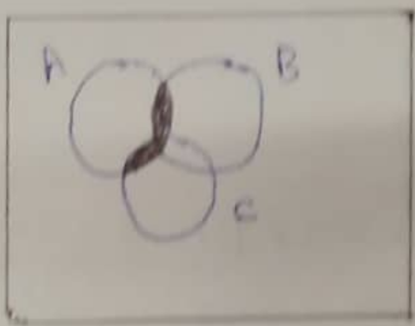
(a) $A \cap (B - C)$:



(b) $(A \cap B) \cup (A \cap C)$



(c) $(A \cap B) \cup (A \cap C)$



Sol:-
$A\times B\times C = \{(a,x,0), (a,x,1), (a,y,0), (a,y,1), (b,x,0), (b,x,1), (b,y,0), (b,y,1), (c,x,0), (c,x,1), (c,y,0), (c,y,1)\}$

(b) $C\times B\times A = \{(0,x,a), (0,x,b), (0,x,c), (0,y,a), (0,y,b), (0,y,c), (1,x,a), (1,x,b), (1,x,c), (1,y,a), (1,y,b), (1,y,c)\}$

(c) $C\times A\times B = \{(0,a,x), (0,a,y), (0,b,x), (0,b,y), (0,c,x), (0,c,y), (1,a,x), (1,a,y), (1,b,x), (1,b,y), (1,c,x), (1,c,y)\}$

(d) $B\times B\times B = \{(x,x,x), (x,x,y), (x,y,x), (x,y,y), (y,x,x), (y,x,y), (y,y,x), (y,y,y)\}$

Q22) Sol:-

(a) $A = \{0,1,3\}$, $A^2 = A\times A$

$A^2 = \{(0,0), (0,1), (0,3), (1,0), (1,1), (1,3), (3,0), (3,1), (3,3)\}$

(b) $A = \{1,2,a,b\}$, $A^2 = A\times A$

$A^2 = \{(1,1), (1,2), (1,a), (1,b), (2,1), (2,2), (2,a), (2,b), (a,1), (a,2), (a,a), (a,b), (b,1), (b,2), (b,a), (b,b)\}$

Q23) Sol:-

(a) $A = \{a\}$, $A^3 = A\times A\times A$

$A^3 = \{(a,a,a)\}$

(b) $A = \{0,a\}$, $A^3 = A\times A\times A$

$A^3 = \{(0,0,0), (0,0,a), (0,a,0), (0,a,a), (a,0,0), (a,0,a), (a,a,0), (a,a,a)\}$

Q24) Sol:-

(a) $\forall x\in R\,(x^2=-1)$

- For all real values of x, $x^2=-1$.
- Truth value = False

(b) $\exists x\in Z\,(x^2=2)$

- There exists an integer value x such that $x^2=2$
- Truth value = False

Sol:-

$P(\{a\}) = \{\emptyset, \{a\}\}$

$P(\{a,b\}) = \{\emptyset, \{a\}, \{b\}, \{a,b\}\}$

$P(\{\emptyset, \{\emptyset\}\}) = \{\emptyset, \{\emptyset\}, \{\{\emptyset\}\}, \{\emptyset, \{\emptyset\}\}\}$

Q16) Sol:-

ⓐ $P(\{a, b, \{a,b\}\}) = 2^3 = 2\times 2\times 2$

$\boxed{= 8 \text{ elements}}$

ⓑ $P(\{\emptyset, a, \{a\}, \{\{a\}\}\}) = 2^4 = 2\times 2\times 2\times 2$

$\boxed{= 16 \text{ elements}}$

ⓒ $P(P(\emptyset)) = 2^1 = 2$

$\boxed{= 2 \text{ elements}}$

Q17) Sol:-

$P(A) \subseteq P(B) \longleftrightarrow A \subseteq B$

- Proof($\rightarrow$):- Suppose that A and B are sets and $P(A) \subseteq P(B)$. By definition of power se[t] $A \in P(A)$. Since $A \in P(A)$ and $P(A) \subseteq P(B)$, we know that $A \in P(B)$ (definition of subset). So, by definition of power set, $A \subseteq B$.

- Proof($\leftarrow$):- Suppose that A and B are sets and $A \subseteq B$. Suppose that x is a[n] element of $P(A)$. By definition of power set, x must be a subs[et] of A. Since $x \subseteq A$ and $A \subseteq B$, so $x \subseteq B$. Since $x \subseteq B$, t[he] definition of power set implies that $x \in P(B)$. Since we have s[hown] that any element of $P(A)$ is also an element of $P(B)$, we have $P(A) \subseteq P(B)$.

ⓒ $\forall x \in Z (x^2 > 0)$

- For all integer values of x, the square of x is positive
- Truth value = False

ⓓ $\exists x \in R (x^2 = x)$

- There exist an x in Real values of x, such that the square of x is itself.
- Truth value = True

Q25) Sol:-

ⓐ $P(x) : x^2 < 3$

- The truth set of $P(x)$ is the set $\{-1, 0, 1\}$

ⓑ $Q(x) : x^2 > x$

- The truth set of $Q(x)$ is the set $\{-\infty, \ldots, -1, 2, \ldots, \infty\}$

ⓒ $R(x) : 2x + 1 = 0$

- The truth set of $R(x)$ is the set $\{\emptyset\}$ or $\{\}$

Q26) Sol:-

ⓐ $P(x) : x^3 \geq 1$

The truth set of $P(x)$ is the set $\{1, 2, 3, 4, \ldots\}$

ⓑ $Q(x) : x^2 = 2$

The truth set of $Q(x)$ is the set $\{\emptyset\}$ or $\{\}$

ⓒ $R(x) : x < x^2$

The truth set of $R(x)$ is the set $\{-\infty, \ldots, -1, 2, 3, \ldots \infty\}$

Sol:-

$\emptyset \in \{\emptyset\}$, True

Empty set contains empty, so empty string belongs to empty set.

b) $\emptyset \in \{\emptyset, \{\emptyset\}\}$, True

- Empty set belongs to set & subset of empty.

c) $\{\emptyset\} \in \{\emptyset\}$, False

- The set containing elements, not containing sets.

d) $\{\emptyset\} \in \{\{\emptyset\}\}$, True

- The empty set is an element of set.

e) $\{\emptyset\} \subset \{\emptyset, \{\emptyset\}\}$, True

- The empty string is subset of given set.

f) $\{\{\emptyset\}\} \subset \{\emptyset, \{\emptyset\}\}$, True

- The empty string is subset of given set.

g) $\{\{\emptyset\}\} \subset \{\{\emptyset\}, \{\emptyset\}\}$, False

- Proper set not equal sets.

Q9) Sol:-

a) $x \in \{x\}$, True

- x is an element of set $\{x\}$.

b) $\{x\} \subseteq \{x\}$, True

- The set $\{x\}$ is a subset of itself.

c) $\{x\} \in \{x\}$, False

- The set $\{x\}$ is not an element of $\{x\}$.

d) $\{x\} \in \{\{x\}\}$, True

- The set $\{x\}$ is an element of set $\{\{x\}\}$.

e) ~~$\{\emptyset\} \subseteq \{\emptyset$~~ $\emptyset \subseteq \{x\}$, True

g) $\emptyset \in \{x\}$, True

- Empty set belongs to set $\{x\}$.

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