Artificial Intelligence

Lecture 9 – Knowledge-Based Agents and Logic

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Outline

- Knowledge-based agents
- Wumpus world
- Logic in general - models and entailment
- Propositional (Boolean) logic
- Equivalence, validity, satisfiability
- Inference rules and theorem proving
  - Forward chaining
  - Backward chaining
  - Resolution.

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Knowledge Base

Inference engine	← domain-independent algorithms
Knowledge base	← domain-specific content

- Knowledge base: Set of sentences in a formal language
- Declarative approach for building an agent
  - TELL it what it needs to know
  - Then it can ASK itself what to do - answers should follow from the KB
- Agents can be viewed at the knowledge level
  - What they know, regardless of implementation
- Or at the implementation level
  - Data structures in KB and algorithms that manipulate them.

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A Knowledge-based Agent

- The agent must be able to:
  - Represent states, actions, etc.
  - Incorporate new percepts
  - Update internal representations of the world
  - Deduce hidden properties of the world
  - Deduce appropriate actions.

```
function KB-Agent(percept) returns an action
  static: KB, a knowledge base
          t, a counter, initially 0, indicating time
  Tell(KB, Make-Percept-Sentence(percept, t))
  action ← Ask(KB, Make-Action-Query(t))
  Tell(KB, Make-Action-Sentence(action, t))
  t ← t + 1
  return action
```

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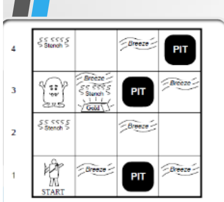
Wumpus World



Performance measure:
- Gold +1000, Death -1000, -1 per step, -10 for using the arrow

Environment:
- Squares adjacent to wumpus are smelly
- Squares adjacent to pit are breezy
- Glitter iff gold is in the same square
- Shooting kills wumpus if you are facing it
- Shooting uses up the only arrow
- Grabbing picks up gold if in same square
- Releasing drops the gold in same square

Actuators:
- Left turn, Right turn, Forward, Grab, Release, Shoot

Sensors:
- Breeze, Glitter, Smell.

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Wumpus World Characterization

- **Observable**? No - only local perception
- **Deterministic**? Yes -outcomes exactly specified
- **Episodic**? No - sequential at the level of actions
- **Static**? Yes -Wumpus and Pits do not move
- **Discrete**? Yes
- **Single-agent**? Yes

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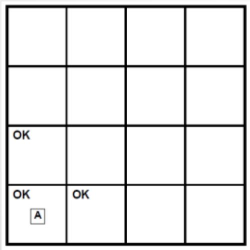
Exploring the Wumpus World



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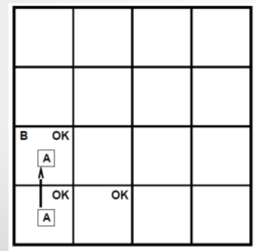
Exploring the Wumpus World



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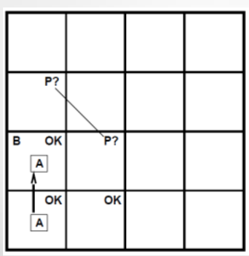
Exploring the Wumpus World



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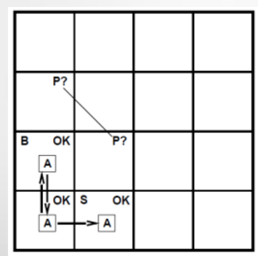
Exploring the Wumpus World



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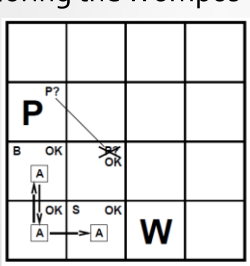
Exploring the Wumpus World



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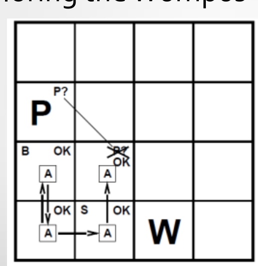
Exploring the Wumpus World



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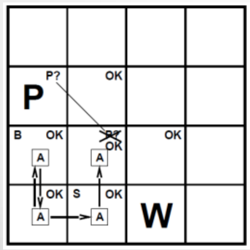
Exploring the Wumpus World



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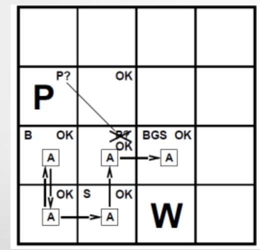
Exploring the Wumpus World



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Logic In General

- Logics are formal languages for representing information, such that conclusions can be drawn
- Syntax defines the sentences in the language
- Semantics define the "meaning" of sentences, i.e., define truth of a sentence in a world
- E.g., the language of arithmetic
- x + 2 ≥ y is a sentence; x2 + y > is not a sentence
- x + 2 ≥ y is true iff the number x + 2 is no less than the number y
- x + 2 ≥ y is true in a world where x=7; y=1
- x + 2 ≥ y is false in a world where x=0; y =6

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Entailment

Entailment means that one thing **follows** from another

$KB \models \alpha$

Knowledge base KB entails sentence B iff B is true in all worlds where KB is true

E.g., the KB containing "the Giants won" and "the Reds won" entails "Either the Giants won or the Reds won"

E.g., x + y =4 entails 4=x + y

Entailment is a relationship between sentences (i.e., syntax) that is based on semantics.

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Models

- Logicians typically think in terms of **models**, which are formally structured worlds with respect to which truth can be evaluated
- We say *m* is a model of a sentence B if B is true in *m*
  - M(B) is the set of all models of B
- Then KB |= B if and only if M(KB) ⊆ M(B)
  - E.g. KB = Giants won and Reds won
  - B = Giants won

M(B) could be also true for worlds that are different than the worlds of KB

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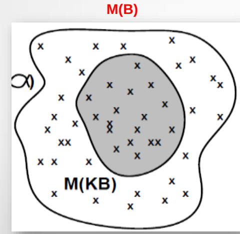
Models

M(B)



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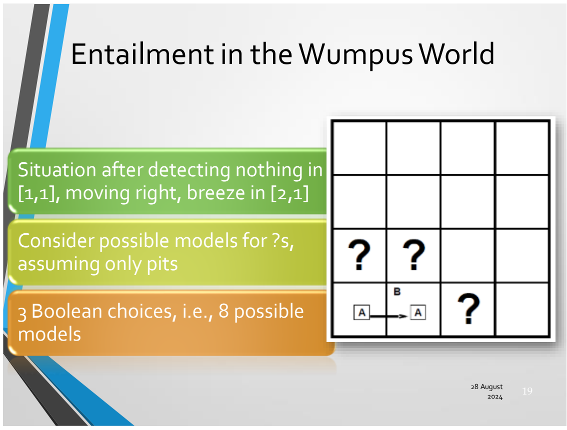

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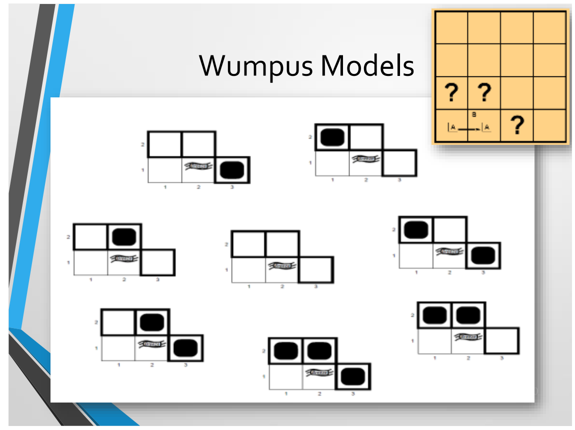

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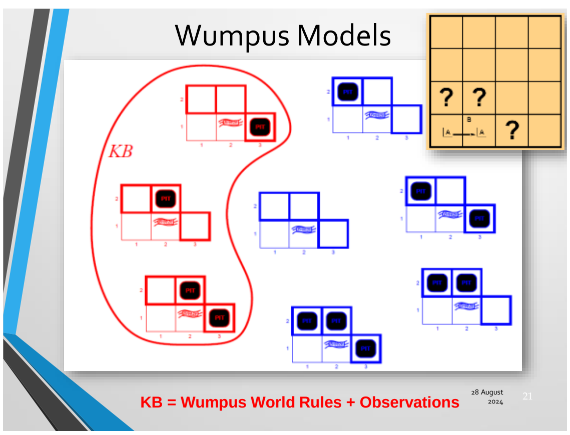

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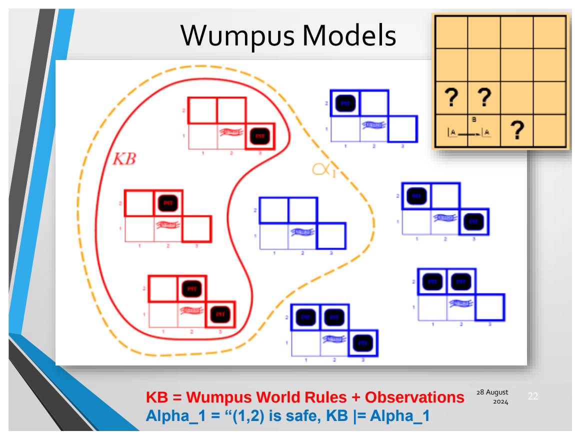

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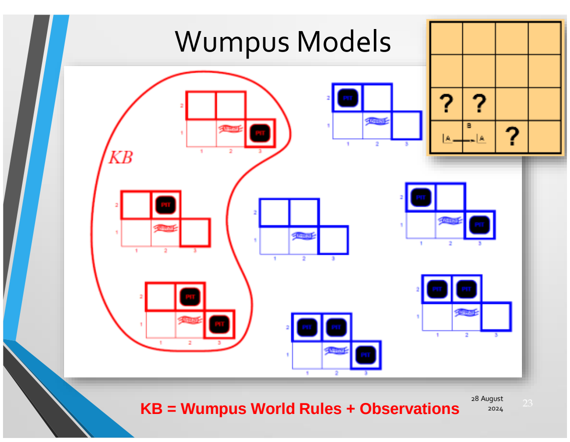

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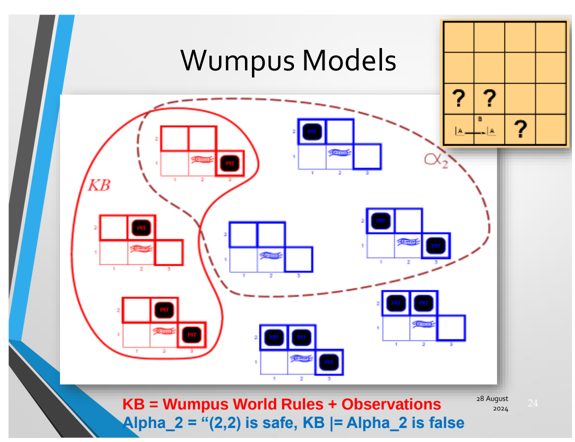

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Inference

$KB \vdash_i \alpha$ = sentence α can be derived from KB by procedure i

Consequences of KB are a haystack; α is a needle.
Entailment = needle in haystack; inference = finding it

Soundness: i is sound if
whenever $KB \vdash_i \alpha$, it is also true that $KB \models \alpha$

Completeness: i is complete if
whenever $KB \models \alpha$, it is also true that $KB \vdash_i \alpha$

- First Order Logic (FOL): Allows complete and sound inference procedures
  - Will answer any question whose answer follows from what is known by the KB.

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Propositional Logic

Propositional logic is the simplest logic—illustrates basic ideas

The proposition symbols P_1, P_2 etc are sentences

If S is a sentence, $\neg S$ is a sentence (negation)

If S_1 and S_2 are sentences, $S_1 \wedge S_2$ is a sentence (conjunction)

If S_1 and S_2 are sentences, $S_1 \vee S_2$ is a sentence (disjunction)

If S_1 and S_2 are sentences, $S_1 \Rightarrow S_2$ is a sentence (implication)

If S_1 and S_2 are sentences, $S_1 \Leftrightarrow S_2$ is a sentence (biconditional)

Order of Precedence

If and Only If

If S1 then S2

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Propositional Logic: Semantics

Each model specifies true/false for each proposition symbol

E.g. $P_{1,2}$ $P_{2,2}$ $P_{3,1}$
$true$ $true$ $false$

If S1 is true, then I am claiming that S2 is true

(With these symbols, 8 possible models, can be enumerated automatically.)

Rules for evaluating truth with respect to a model m:

$\neg S$	is true iff	S	is false		
$S_1 \wedge S_2$	is true iff	S_1	is true **and**	S_2	is true
$S_1 \vee S_2$	is true iff	S_1	is true **or**	S_2	is true
$S_1 \Rightarrow S_2$	is true iff	S_1	is false **or**	S_2	is true
i.e.,	is false iff	S_1	is true **and**	S_2	is false
$S_1 \Leftrightarrow S_2$	is true iff	$S_1 \Rightarrow S_2$	is true **and**	$S_2 \Rightarrow S_1$	is true

Simple recursive process evaluates an arbitrary sentence, e.g.,
$\neg P_{1,2} \wedge (P_{2,2} \vee P_{3,1}) = true \wedge (false \vee true) = true \wedge true = true$

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Truth Table for Connectives

P	Q	$\neg P$	$P \wedge Q$	$P \vee Q$	$P \Rightarrow Q$	$P \Leftrightarrow Q$
$false$	$false$	$true$	$false$	$false$	$true$	$true$
$false$	$true$	$true$	$false$	$true$	$true$	$false$
$true$	$false$	$false$	$false$	$true$	$false$	$false$
$true$	$true$	$false$	$true$	$true$	$true$	$true$

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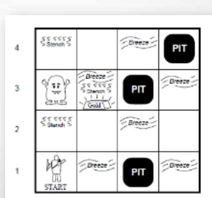
Wumpus World Sentences

Let $P_{i,j}$ be true if there is a pit in $[i,j]$.
Let $B_{i,j}$ be true if there is a breeze in $[i,j]$.

$\neg P_{1,1}$
$\neg B_{1,1}$
$B_{2,1}$

"Pits cause breezes in adjacent squares"



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Wumpus World Sentences

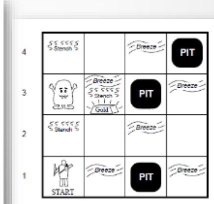
Let $P_{i,j}$ be true if there is a pit in $[i,j]$.
Let $B_{i,j}$ be true if there is a breeze in $[i,j]$.

$\neg P_{1,1}$ R1
$\neg B_{1,1}$ R4
$B_{2,1}$ R5

"Pits cause breezes in adjacent squares"

$B_{1,1} \Leftrightarrow (P_{1,2} \vee P_{2,1})$ R2
$B_{2,1} \Leftrightarrow (P_{1,1} \vee P_{2,2} \vee P_{3,1})$ R3

"A square is breezy **if and only if** there is an adjacent pit"



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Wumpus World Truth Table

$B_{1,1}$	$B_{2,1}$	$P_{1,1}$	$P_{1,2}$	$P_{2,1}$	$P_{2,2}$	$P_{3,1}$	R_1	R_2	R_3	R_4	R_5	KB
false	*false*	*false*	*false*	*false*	*false*	*false*	*true*	*true*	*true*	*true*	*false*	*false*
false	*false*	*false*	*false*	*false*	*false*	*true*	*true*	*true*	*false*	*true*	*false*	*false*
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
false	*true*	*false*	*false*	*false*	*false*	*false*	*true*	*true*	*false*	*true*	*true*	*false*
false	*true*	*false*	*false*	*false*	*false*	*true*	*true*	*true*	*true*	*true*	*true*	*true*
false	*true*	*false*	*false*	*false*	*true*	*false*	*true*	*true*	*true*	*true*	*true*	*true*
false	*true*	*false*	*false*	*false*	*true*	*true*	*true*	*true*	*true*	*true*	*true*	*true*
false	*true*	*false*	*false*	*true*	*false*	*false*	*true*	*false*	*false*	*true*	*true*	*false*
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
true	*true*	*true*	*true*	*true*	*true*	*true*	*false*	*true*	*true*	*false*	*true*	*false*

Enumerate rows (different assignments to symbols),
if KB is true in row, check that α is too

128 possible rows, only 3 make the KB true

$P_{1,2}$ is false for all 3; hence, no pit in $P_{1,2}$

Evaluate the entailed sentences in these rows

$P_{2,2}$ is both false and true; hence, no conclusion can be drawn about the pit being in $P_{2,2}$

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Inference by Enumeration

Depth-first enumeration of all models is sound and complete

function TT-ENTAILS?(KB, α) **returns** *true* or *false*
  inputs: KB, the knowledge base, a sentence in propositional logic
    α, the query, a sentence in propositional logic
  symbols ← a list of the proposition symbols in KB and α
  return TT-CHECK-ALL(KB, α, *symbols*, [ ])

function TT-CHECK-ALL(KB, α, *symbols*, *model*) **returns** *true* or *false*
  if EMPTY?(*symbols*) **then**
    if PL-TRUE?(KB, *model*) **then return** PL-TRUE?(α, *model*)
    else return *true*
  else do
    P ← FIRST(*symbols*); *rest* ← REST(*symbols*)
    return TT-CHECK-ALL(KB, α, *rest*, EXTEND(P, *true*, *model*)) **and**
      TT-CHECK-ALL(KB, α, *rest*, EXTEND(P, *false*, *model*))

$O(2^n)$ for n symbols; problem is **co-NP-complete**

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Logical Equivalence

Two sentences are logically equivalent iff true in same models:
$\alpha \equiv \beta$ if and only if $\alpha \models \beta$ and $\beta \models \alpha$

$$
\begin{aligned}
(\alpha \land \beta) &\equiv (\beta \land \alpha) \quad \text{commutativity of } \land \\
(\alpha \lor \beta) &\equiv (\beta \lor \alpha) \quad \text{commutativity of } \lor \\
((\alpha \land \beta) \land \gamma) &\equiv (\alpha \land (\beta \land \gamma)) \quad \text{associativity of } \land \\
((\alpha \lor \beta) \lor \gamma) &\equiv (\alpha \lor (\beta \lor \gamma)) \quad \text{associativity of } \lor \\
\neg(\neg\alpha) &\equiv \alpha \quad \text{double-negation elimination} \\
(\alpha \Rightarrow \beta) &\equiv (\neg\beta \Rightarrow \neg\alpha) \quad \text{contraposition} \\
(\alpha \Rightarrow \beta) &\equiv (\neg\alpha \lor \beta) \quad \text{implication elimination} \\
(\alpha \Leftrightarrow \beta) &\equiv ((\alpha \Rightarrow \beta) \land (\beta \Rightarrow \alpha)) \quad \text{biconditional elimination} \\
\neg(\alpha \land \beta) &\equiv (\neg\alpha \lor \neg\beta) \quad \text{De Morgan} \\
\neg(\alpha \lor \beta) &\equiv (\neg\alpha \land \neg\beta) \quad \text{De Morgan} \\
(\alpha \land (\beta \lor \gamma)) &\equiv ((\alpha \land \beta) \lor (\alpha \land \gamma)) \quad \text{distributivity of } \land \text{ over } \lor \\
(\alpha \lor (\beta \land \gamma)) &\equiv ((\alpha \lor \beta) \land (\alpha \lor \gamma)) \quad \text{distributivity of } \lor \text{ over } \land
\end{aligned}
$$

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Validity and Satisfiability

A sentence is valid if it is true in all models,
  e.g., $True$, $A \lor \neg A$, $A \Rightarrow A$, $(A \land (A \Rightarrow B)) \Rightarrow B$

Validity is connected to inference via the Deduction Theorem:
  $KB \models \alpha$ if and only if $(KB \Rightarrow \alpha)$ is valid

If KB is true, alpha is always true. Hence, I can say that alpha follows from KB

A sentence is satisfiable if it is true in some model
  e.g., $A \lor B$, C

A sentence is unsatisfiable if it is true in no models
  e.g., $A \land \neg A$

Satisfiability is connected to inference via the following:
  $KB \models \alpha$ if and only if $(KB \land \neg\alpha)$ is unsatisfiable
i.e., prove α by *reductio ad absurdum*

Suppose that KB=true. Then, this will become unsatisfiable only when alpha is true. Hence, I can say that alpha follows from KB

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Two Famous Inference Rules

- Modus Ponens $\dfrac{P \Rightarrow Q \;,\; P}{Q}$

Given that P implies Q, and I know that P is true, then I can infer Q

- And Rule $P \land Q$

Given that P AND Q is true, I can infer that P is true, and I can also infer that Q is true.

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Resolution

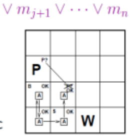
Conjunctive Normal Form (CNF—universal)
  conjunction of $\underbrace{\textbf{disjunctions} \text{ of } \textbf{literals}}_{\textbf{clauses}}$
  E.g., $(A \lor \neg B) \land (B \lor \neg C \lor \neg D)$

Resolution inference rule (for CNF): complete for propositional logic

$$\frac{\ell_1 \lor \cdots \lor \ell_k, \qquad m_1 \lor \cdots \lor m_n}{\ell_1 \lor \cdots \lor \ell_{i-1} \lor \ell_{i+1} \lor \cdots \lor \ell_k \lor m_1 \lor \cdots \lor m_{j-1} \lor m_{j+1} \lor \cdots \lor m_n}$$

where ℓ_i and m_j are complementary literals. E.g.,

$$\frac{P_{1,3} \lor P_{2,2}, \qquad \neg P_{2,2}}{P_{1,3}}$$



Resolution is sound and complete for propositional logic

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Conversion to CNF

$B_{1,1} \Leftrightarrow (P_{1,2} \vee P_{2,1})$

1. Eliminate $\Leftrightarrow$, replacing $\alpha \Leftrightarrow \beta$ with $(\alpha \Rightarrow \beta) \wedge (\beta \Rightarrow \alpha)$.

   $(B_{1,1} \Rightarrow (P_{1,2} \vee P_{2,1})) \wedge ((P_{1,2} \vee P_{2,1}) \Rightarrow B_{1,1})$

2. Eliminate $\Rightarrow$, replacing $\alpha \Rightarrow \beta$ with $\neg\alpha \vee \beta$.

   $(\neg B_{1,1} \vee P_{1,2} \vee P_{2,1}) \wedge (\neg(P_{1,2} \vee P_{2,1}) \vee B_{1,1})$

3. Move $\neg$ inwards using de Morgan's rules and double-negation:

   $(\neg B_{1,1} \vee P_{1,2} \vee P_{2,1}) \wedge ((\neg P_{1,2} \wedge \neg P_{2,1}) \vee B_{1,1})$

4. Apply distributivity law ($\vee$ over $\wedge$) and flatten:

   $(\neg B_{1,1} \vee P_{1,2} \vee P_{2,1}) \wedge (\neg P_{1,2} \vee B_{1,1}) \wedge (\neg P_{2,1} \vee B_{1,1})$

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Resolution Algorithm

- Resolution rule:

$$\frac{\alpha \vee \beta \qquad \neg\beta \vee \gamma}{\alpha \vee \gamma}$$

- Resolution refutation:
  - Convert all sentences to CNF
  - Negate the desired conclusion (converted to CNF)
  - Apply resolution rule until either
    - Derive false (a contradiction)
    - Can't apply any more
- Resolution refutation is sound and complete
  - If we derive a contradiction, then the conclusion follows from the axioms
  - If we can't apply any more, then the conclusion cannot be proved from the axioms.

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Example

Propositional Resolution Example

Prove R

1	P v Q
2	P → R
3	Q → R

false v R

¬ R v false

false v false

Step	Formula	Derivation
1	P v Q	Given
2	¬ P v R	Given
3	¬ Q v R	Given
4	¬ R	Negated conclusion
5	Q v R	1,2
6	¬ P	2,4
7	¬ Q	3,4
8	R	5,7
9	•	4,8

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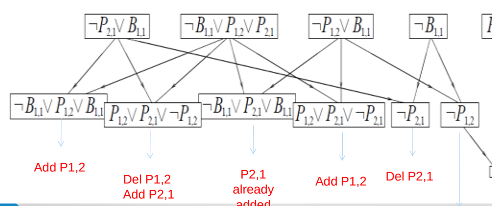
Resolution Example

$(\neg B_{1,1} \vee P_{1,2} \vee P_{2,1}) \wedge (\neg P_{1,2} \vee B_{1,1}) \wedge (\neg P_{2,1} \vee B_{1,1})$

$KB = (B_{1,1} \Leftrightarrow (P_{1,2} \vee P_{2,1})) \wedge \neg B_{1,1}$ $\alpha = \neg P_{1,2}$



This returns an empty clause. So, alpha is true.

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Questions

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