TASK 9: Introduction to Statistics and Probability Distribution:

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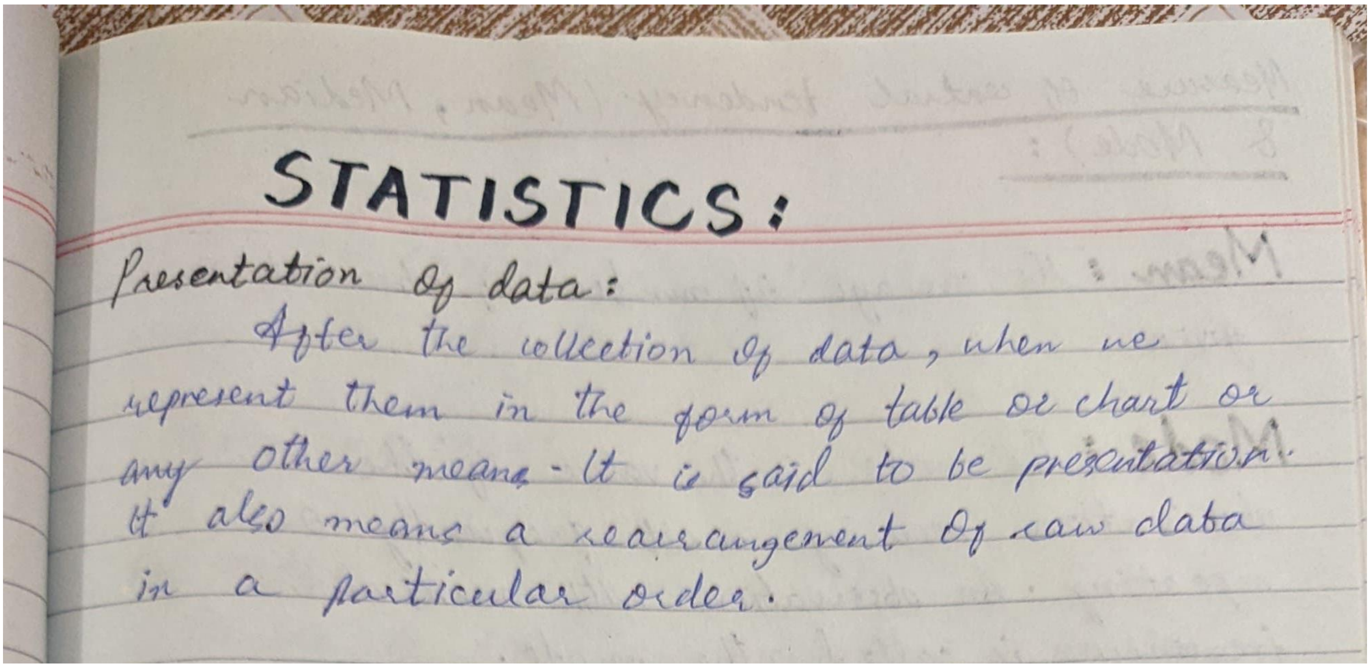
Statistics deals with the collection, presentation and analysis of the data:

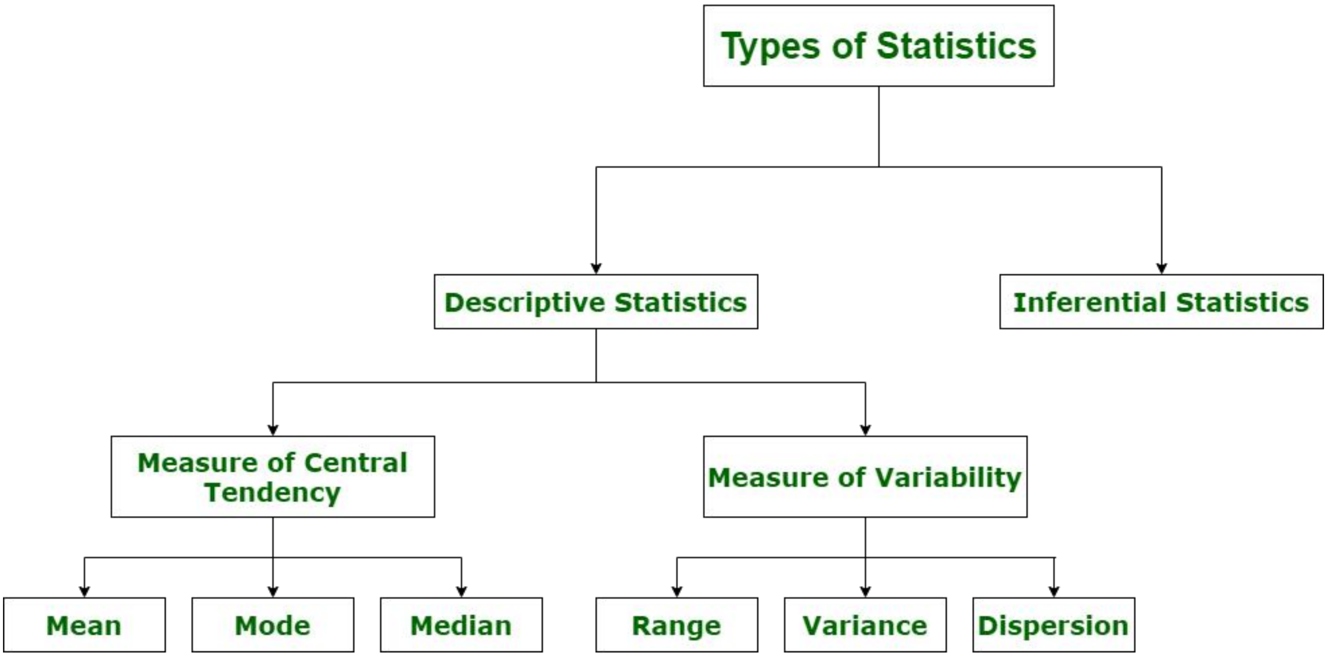


STATISTICS:

Presentation of data:

After the collection of data, when we represent them in the form of table or chart or any other means. It is said to be presentation. It also means a rearrangement of raw data in a particular order.

Types of Statistics:



Some basic statistics questions:

Q NO: 1

Q: If marks obtained by students in a class test is given as per below:

55 36 95 73 60 42 25 78 75 62

Arrange the marks from lowest to highest.

Solution:

We need to arrange the marks obtained by each student in ascending order.

25, 36, 42, 55, 60, 62, 73, 75, 78, 95.

Q NO: 2

. Check the following frequency distribution table, consisting of weights of 38 students of a class:

Weights (in kg)	Number of students
31 – 35	9
36 – 40	5
41 – 45	14
46 – 50	3
51 – 55	1
56 – 60	2
61 – 65	2
66 – 70	1
71 – 75	1

(i) What is class-interval for classes 31 – 35?

(ii) How many students are there in the range of 41-45 kgs?

Solution :

i) ∵ Class interval = upper class limit − lower class limit

= 35 − 31

class interval = 4

ii) For the 41-45 range, there are 14 students.

Measure of Central Tendency:

Mean : The average of number of observations given.

Mode : The mode is the value of the observation occuring most frequently or repeating. An observation with maximum frequency is called the mode.

Median : The median which divides the given observation into exactly two parts.

If n is Even :

- Median = $[(n/2)^{th}$ term $+ (n/2+1)^{th}$ term$]/2$

If n is odd :

- Median = $(n+1)/2$

Q NO: 3

. Consider a small unit of a factory where there are 5 employees : a supervisor and four labourers. The workers earn a salary of Rs. 5,000 per month each while the supervisor gets Rs. 15,000 per month. Calculate the mean, median and mode of the salaries.

Solution :

For Mean :

Mean = (5000 + 5000 + 5000 + 5000 + 15000) / 5

= 35000 / 5

= 7000

So, the mean salary is Rs. 7000 per month.

For Median :

To obtain the median, let us arrange the salaries in ascending order

5000, 5000, 5000, 5000, 15000

Median = (n+1)/2 = (5+1)/2 = 6/2

= 3rd observation

Median = Rs. 5000/-

For Mode :

Number of times an observation is repeated.

= 5000/-

Measure of Variability:

The measure of variability is also known as the measure of dispersion and is used to describe the variability in a sample or population.

1. Range of Data: how to spread value in a sample set or data set.

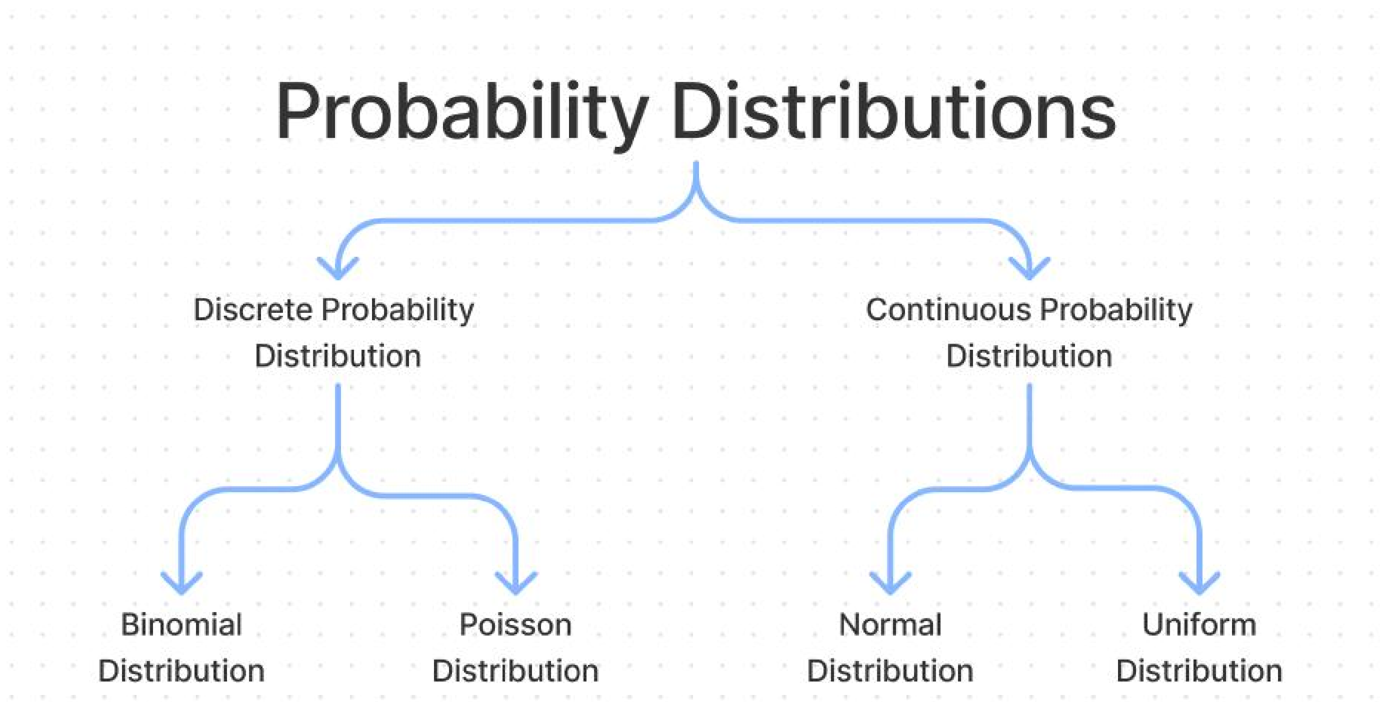
Range = Maximum value − Minimum value

2. Variance:

It is calculated by averaging the squared deviations from the mean. It is represented by σ^2.

There are two types of variance.

- Population Variance: represented as σ^2.
- Sample variance: represented as S^2.



Normal Distribution:

Q NO: 4

Q: What is the Empirical Rule for a normal distibution?

The Empirical Rule, also known as the 68-95-99.7 rule, describes how data in a normal distibution are distibuted.

- About 68% of the data fall within one standard deviation (σ) of the mean (μ).
- About 95% of the data fall within two standard deviations of the mean.
- About 99.7% of the data fall within three standard deviations of the mean.

Q NO: 5

A company's employee salaries are normally distributed with a mean of \$50,000 and a standard deviation of \$5,000. What is the probability that a randomly selected employee earns between \$45,000 and \$55,000?

Solutions:

To find probability, we use the properties of the normal distibution:

- $\mu = 50,000$
- $\sigma = 5,000$

We convert the values to z-scores.

$$\therefore z = \frac{X - \mu}{\sigma}$$

For 45,000:

$$z = \frac{45,000 - 50,000}{5000}$$

$$z = -1$$

For 55,000:

$$z = \frac{55,000 - 50,000}{5000}$$

$Z = 1$

Using the standard normal distribution table:

$$P(-1 < Z < 1) \approx 0.6826$$

So, the probability of randomly selected employee earns between 45,000 and 55,000 is approximately 68.26%.

Binomial Distribution:

Q NO: 6

A coin is flipped 10 times. What is the probability of getting exactly 6 heads?

Solution:

This is a binomial distribution problem where

$n = 10$ (no. of trials)

$p = 0.5$ (probability of success on each trial)

$k = 6$ (number of successes)

$\because P(X = k) = \binom{n}{k} p^k (1-p)^{n-k}$

$k = 6, n = 10, p = 0.5$:

$$P(X = 6) = \binom{10}{6} (0.5)^6 (0.5)^4$$

Using binomial distribution coefficient

$$\binom{10}{6} = \frac{10!}{6!(10-6)!} = 210$$

$$P(X = 6) = 210 \times (0.5)^{10}$$

$$= 210 \times 1/1024$$

$$\boxed{P(X = 6) = 0.2051}$$ Ans.

Q NO: 7

If the probability of a product being defective is 0.02, what is the probability that in a shipment of 50 products, at most 1 product is defective?

Solution:

This is a binomial dist. problem with,

$n = 50$

$p = 0.02$

We need $P(X \leq 1)$ where X is the no. of defective products.

$$P(X \leq 1) = P(X=0) + P(X=1)$$

For $P(X=0)$:

$$P(X=0) = \binom{50}{0}(0.02)^0 (0.98)^{50}$$

$$= 0.3642$$

For $P(X=1)$:

$$P(X=1) = \binom{50}{1}(0.02)^1 (0.98)^{40}$$

$$= 50 \times 0.02 \times (0.98)^{40}$$

$$= 0.3718$$

Therefore,

$$P(X \leq 1) = 0.3642 + 0.3718$$

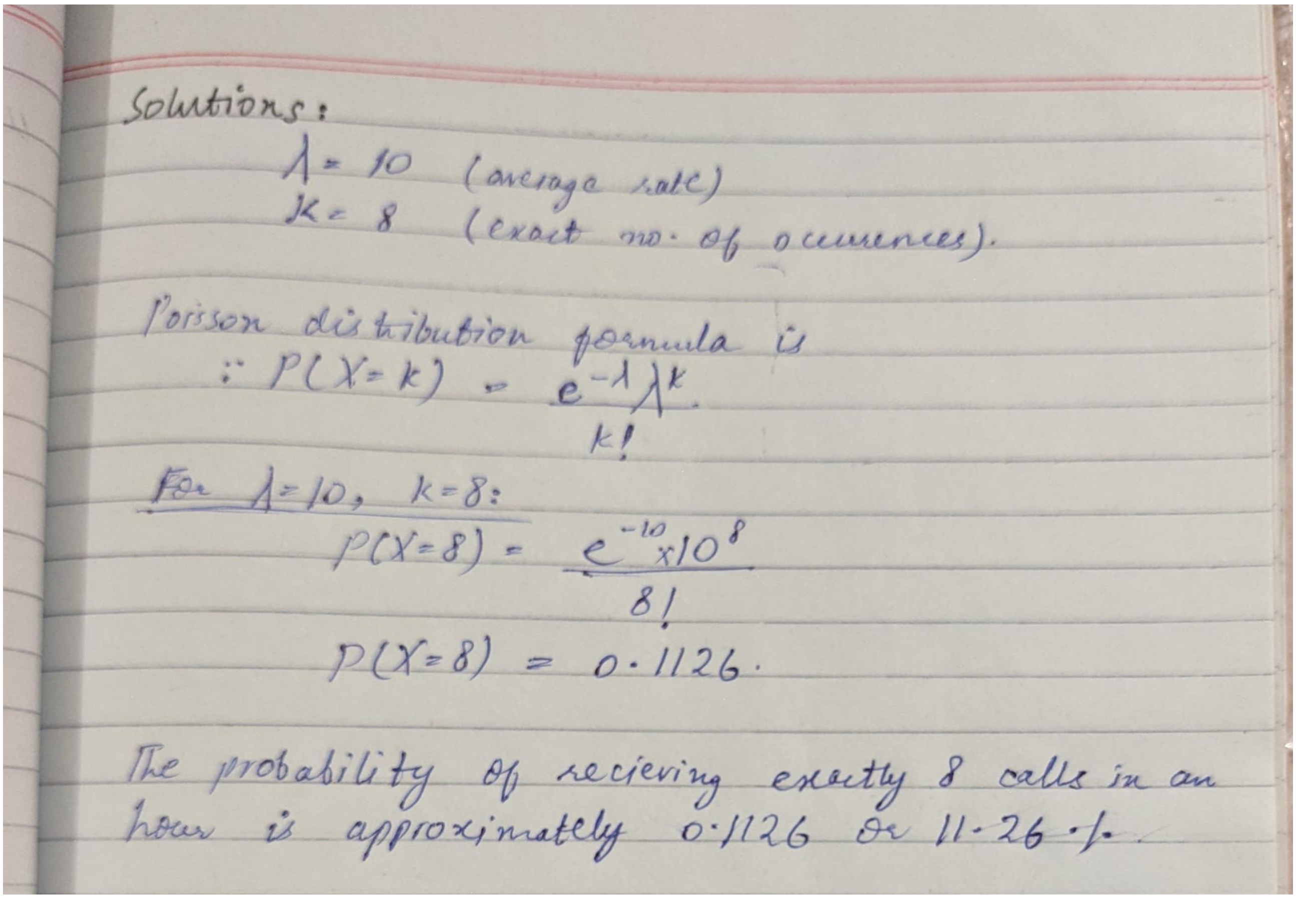
$$P(X \leq 1) = 0.7360$$

So, the probability that at most 1 product is defective is approximately 0.7360 or 73.60%.

Poisson Distribution:

Q NO: 8

A call center receives an average of 10 calls per hour. What is the probability that they receive exactly 8 calls in a given hour?



Solutions:

$\lambda = 10$ (average rate)

$k = 8$ (exact no. of occurences).

Poisson distribution formula is

$\therefore P(X=k) = \frac{e^{-\lambda}\lambda^{k}}{k!}$

For $\lambda = 10$, $k = 8$:

$P(X=8) = \frac{e^{-10} \times 10^{8}}{8!}$

$P(X=8) = 0.1126$.

The probability of recieving exactly 8 calls in an hour is approximately 0.1126 or 11.26 %.

Uniform Distribution:

Q NO: 9

If the waiting time for a bus at a certain stop is uniformly distributed between 0 and 20 minutes, what is the probability that a person has to wait more than 15 minutes?

Solution:

Uniform distibution $U(a, b)$

$a = 0$

$b = 20$

The probability density function is:

$$f(x) = \frac{1}{20-0} = \frac{1}{20}$$

To find $P(X > 15)$:

$$P(X > 15) = \int_{15}^{20} \frac{1}{20}\, dx$$

$$= \frac{1}{20}(20-15) \Rightarrow \frac{5}{20}$$

$$= 1/4.$$

Probability ~~that~~ that a person has to wait more than 15 minutes is $\frac{1}{4}$ or 25%..