Calculus

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* Natural Numbers: 1, 2, ---
Whole Numbers: 0, 1, 2, ---
Integers: '+' and '−' and 0
Rational Number:
i) P/q form where $q \neq 0$ ii) Te
iii) recurrence means same n
⇒ 0.333, 0.141414.
Irrational Numbers: Which can
P/q from e.g: $\sqrt{3}$, $\sqrt{5}$, $1+\sqrt{2}$, cos 1, are denoted
by Q'
Real Numbers: All numbers that consists of all above numbers so rational and irrational number. $R = Q \cup Q'$
→ Natural, whole no. and integers can be converted to rational numbers easily.

Division by Zero
eg:
$$y = \frac{P}{0 \rightarrow \text{zero}} \quad \text{if } P \neq 0$$
if P = 0
$$y = \frac{0}{0} \Rightarrow \text{und}$$

Complex Number:-
$x^2 = -1$ ⇒ sq root on b.
$x = \sqrt{-1}$ ⇒ → These are complex no.
$x = i$ → iota
mostly come in quadratic equation.
$$ax^2 + bx + c = 0$$
$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{}$$

Calculus & Analytical Geometry:-
(NS − 1008)
NS → Natural sciences
* calculus → change
* Analytical Geometry → shapes
* Differentiation (17th century)
* Integration (250BC)
* ○ of Area = πr^2
* Reimann Integral (Technique hoti ha) un curves keliye calculus & analytical use kita ha.

Binary Operation (+, ×)
(+) ⇒ zero is additive identity
(*) ⇒ one is multiplicative "
addition handles sub / multi. handles div.
Properties of Real No.s:
1- Closure Property:-
if a & $b \in \mathbb{R}$, then $c \in \mathbb{R}$
$a * b = c$
$a = 2$, $b = -1$

w.r.t (+)
$a + b = c$
$2 + (-1) = c$
$\boxed{c = 1}$

w.r.t (×)
~~a*b = b*a~~
$2 \times -1 = c$
$\boxed{c = -2}$

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D here is a discriminant:
$D^2 = b^2 - 4ac$

if $D > 0$
so, x = real number
→ if $D \leq 0$ → as this will mak
so, x = imaginary no./ complex no

Interval

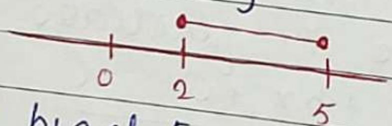
Set: $A = \{1, 2, 3, 4, 5\}$
set of 1st five natural n

Inseit builder notation:-
$\{ x : x \in$ →belong $\mathbb{N}$ $\wedge$ →And $x \leqslant$
natural no.

→ Interval is sets of real nu
→ Geometrically interval
$a < b$, where b is larger

1) Closed Interval:
We use square bracket eg
Now how to write interval
$\{x : a \leq x \leq b\}$
↳ means x lies b/w a to b so greater than a & less than b where a and b included as well.

↳ so all no. b/w a & b
↳ including a & b
eg:- $[2, 5] = \{x \mid 2 \leq x \leq 5\}$
On no. line:-



2) Open Intervals: Round brackets used
(a, b) means excluding a & b.

2) Commutative Property:-
$a * b = b * a$
w.r.t ($\times$)
$1 \times 2 = 2 \times 1$
$2 = 2$
w.r.t (+)
$1 + 2 = 2 + 1$
$3 = 3$

3) Associative Property:-
$a * (b * c) = (a * b) * c$
$a = 2, b = -1, c = 1/2$
w.r.t (+)
$2 + (-1 + 1/2) = (2 - 1) + 1/2$
$2 + (-1/2) = 1 + 1/2$
$3/2 = 3/2$

4) Identity: $a * e = a$
w.r.t (+)
$e = 0$
$2 + 0 = 2$
$-1/7 + 0 = -1/7$
w.r.t ($\times$)
$e = 1$
$2 \times 1 = 2$
$-1/7 \times 1 = -1/7$

5) Inverse: $a * a' = e$
w.r.t (+)
→ $2 + (-2) = 0$
→ $-1/7 + 1/7 = 0$
$2 \times 1/2 = 1$
$-1/7 \times -7 = 1$

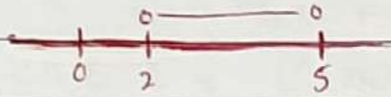
*Practice Questions on Notepad.-

↳ can be written as $\{x : a < x < b\}$ → means x is greater than a & less than b 'a' & 'b' no. here are not included 'b' all no. greater than a & less than b are included.

↳ So all no. b/w a & b but not a & b.

example: $(2,5) = \{x \mid 2 < x < 5\}$

Number line:

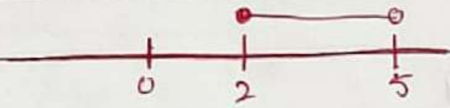


3) Semi closed (or Half open)

- $[a, b)$ → includes a, but not b
- $(a, b]$ → include b, but not a

example: $[2,5)$ means

- All numbers ≥ 2 and < 5
- Solid dot at 2, hollow circle at 5



Tip = Solid dot = Included = [ ]
= Hollow dot = Excluded = ( )

4) Infinited intervals : (go on forever)

- (a, ∞) → greater than a
- $[a, \infty)$ → greater than or equal to a
- $(-\infty, b)$ → less than b
- $(-\infty, b]$ → less than or equal to b.
- $(-\infty, +\infty)$ → Both open & closed

5) Whole real line: $(-\infty, +\infty)$

- All real no.

$(-\infty, 0) \cup (0, \infty)$

↳ This means all number except

Inequality

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$<, \leq, >, \geq$

For eg:- $3 + 7x \leq 2x - 9$

linear :- $3 + 7x \leq 2x - 9$

$7x - 2x \leq -9 - 3$

$5x \leq -12$

$x \leq \frac{-12}{5}$

$\left(-\infty, \frac{-12}{5}\right]$ → but this is included so all no. less than & included -12/5 will in be solution set.

This tells us that $-\infty$ is not included or defined.

Linear:

eg 2: $7 \leq 2 - 5x < 9$

$7 - 2 \leq -5x < 9 - 2$

$5 \leq -5x < 7$

$-\frac{5}{5} \leq x > -\frac{7}{5}$

$-1 \geq x > -\frac{7}{5}$

$-\frac{7}{5} < x \leq -1$

$\left(\frac{-7}{5}, -1\right]$

All no.s greater than $-\frac{7}{5}$ are included but $-\frac{7}{5}$ is not included so all no.s b/w $-\frac{7}{5}$ to -1 and -1 also included.

Cases to solve Inequalities:

1) Linear Eqn: → (x ki power 1)

$3 + 7x \leq 2x - 9$

2) Quadratic Eqn:

$x^2 - 5x + 6 < 0$

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3) Rational Inequalities:

$$\frac{2x+5}{x-2} < 1$$

4) Absolute Functions

$$|5x+6| \geq 5$$

case 1 # linear.

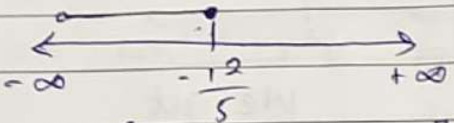
1) $3+7x \leq 2x-9 \rightarrow$ linear as x ki power 1
no mod
~~no fraction~~

Solve:-

$7x - 2x \leq -3-9$

$5x \leq -12$

$x \leq \frac{-12}{5} \rightarrow$ x is less than equals to $-\frac{12}{5}$



S.S = $[-\infty, -12/5]$

2) $7 \leq 2-5x < 9$

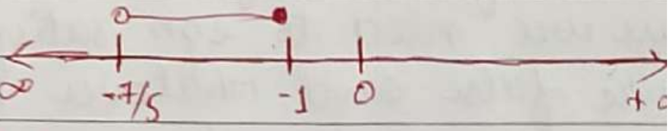
$7 \leq 2-5x$ | $2-5x < 9$
$7-2 \leq -5x$ | $-5x < 9-2$
$5 \leq -5x$ | $-5x < 7$
$-\frac{\cancel{5}}{\cancel{5}} \leq x$ | $x < -7/5$

$-1 \leq x$

$-1 \leq x < -7/5 \rightarrow$ can be -1.4

inequality sign reverse:

$-7/5 > x \geq -1$



$\frac{-7}{5} > x \geq -1 \rightarrow -1$ included as $\geq / \leq$ means closed interval

$\downarrow$ ~~included nhi as >/< means open interval~~

$\left(\frac{-7}{5}, -1\right)$

(solution set mai hamesha)
(a , b)
$\downarrow$ 1st no. yahan se shuru hua
$\downarrow$ kahan khatam hua

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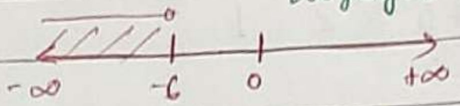
3) $3x + 5 < x - 7$

$5 + 7 < x - 3x$

$\cancel{12} < -\cancel{2}x$

$-6 > x$

+ ∞ ke sth open interval lagega hamesha.



$S.S = (-\infty, -6)$

↳ as not included so open interval

Case # 2 Quadratic → isme test kena pata ha values ko.

① $x^2 - 3x > 10$

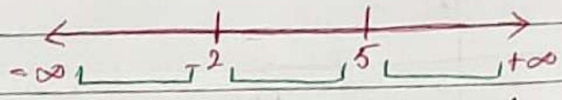
$x^2 - 3x - 10 > 0$

$x^2 - 5x + 2x - 10 > 0$

$x(x-5) + 2(x-5) > 0$

$(x+2)(x-5) > 0$ → no equal to sign so -2 would be open interval & ∞ always open interval

$x = -2$, $x = -5$



Region / Test interval	Test point	We get
1) $(-\infty, -2)$	-3	$18 > 10$
2) $(-2, 5)$	0	$0 > 10$
3) $(5, +\infty)$	6	$18 > 10$

Result:

1) True → as 18 is greater than 10
2) False → as 0 is not greater than 10
3) True → as 18 is greater than 10

1) 1st we need to write intervals
2) Then for 1st region -∞ to -2 in b/w these whatever no. are we need to eqn satisfied then result = True otherwise false and whatever interval satisfies would be the part of solution set.

For Region b: now take any no b/w -2 & 5 and then put in eqn replace it the ans would be written with in equality sign given in question.
Same for region c.

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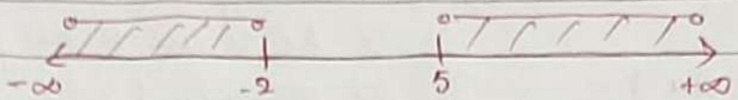
Now Solution set : $(-\infty, -2) \cup (5, \infty)$ * open interval so hollow circle

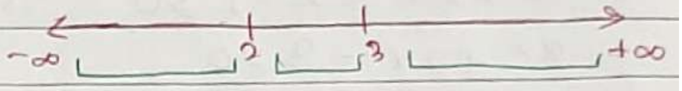


② $x^2 - 5x + 6 < 0$ → no equal to sign, so open interval

$x^2 - 3x - 2x + 6 < 0$

$x(x-3) - 2(x-3) < 0$

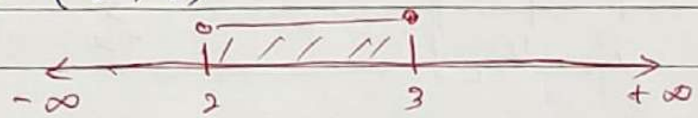
$(x-3)(x-3) < 0$



$x = 3$ or $x = 2 < 0$

Interval	Test point	We get	Result
$(-\infty, 2)$	1	$2 < 0$	False
$(2, 3)$	2.5	$0.25 < 0$	True
$(3, \infty)$	4	$2 < 0$	False

Solution set $(2, 3)$



* Case # 2 Rational Inequalities:

① $\dfrac{2x-5}{x-2} < 1$ ↳ x linear hai par fraction form mai so rational inequalities In case ma yaha x ki ~~ott~~ values nikalenge ~~jese nikalte hain or doos~~ denominator se.

$2x - 5 < 1(x-2)$

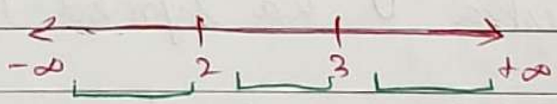
$2x - 5 < x - 2$

$2x - x < -2 + 5$

$x < 3$

$x - 2 = 0$

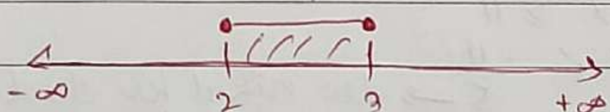
$x = 2$



→ eqn me ye values x ki jaga hogi.

Intervals	Test point	We get	Result
$(-\infty, 2)$	1	$3 < 1$	False
$(2, 3)$	2.5	$0 < 1$	True
$(3, \infty)$	4	$1.5 < 1$	False

Solution Set $(2, 3)$



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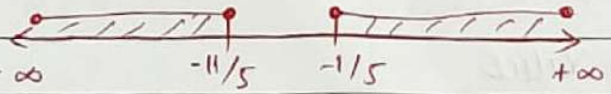
* Absolute Functions recognised by ||

equals to sign here $\leq / \geq$ so not open interval with no.

Intervals	Test point	We get	Result
$(-\infty, -11/5)$	-3	$9 \geq 5$	True
$[-11/5, -1/5]$	-1	$1 \geq 5$	False
$[-1/5, \infty)$	1	$11 \geq 5$	True

Solution set :- $(-\infty, -11/5] \cup [-1/5, \infty)$



yaha we get k
ans -9 ath-a tho
so incase 4 mod he
hai so will becom
+9

② $\left| 5 - \frac{2}{x} \right| < 1$

Absolute function mix with rational

$+\left(5 - \frac{2}{x}\right) < 1$ $\qquad\qquad -\left(5 - \frac{2}{x} < 1\right)$

so third value of x as well in denominator

$+\left(\frac{5x}{x} - \frac{2}{x}\right) < 1$ $\qquad\qquad -\left(5 - \frac{2}{x}\right) < 1$

LCM

$\frac{5x-2}{x} < 1$ $\qquad\qquad \frac{-5x+2}{x} < 1$

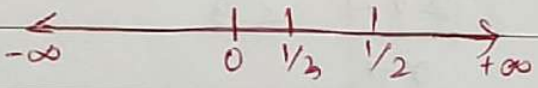
$5x - 2 < 1x$ $\qquad\qquad -5x + 2 < x$

$5x - 1x < 2$ $\qquad\qquad 2 < x + 5x$

$\cancel{4}^{2}x < \cancel{2}$ $\qquad\qquad \cancel{2}^{1} < \cancel{6}^{3}x$

$x < 1/2 \Rightarrow x = 1/2$ $\qquad\qquad x < 1/3 \Rightarrow x = 1/3$

For this value we used to have like this $x - 2 = 0 \Rightarrow x$
but over here $x = 0$



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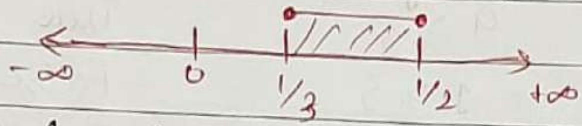
Interval	Testpoint	We get	Result
$(-\infty, 0)$	-1	$7 < 1$	False
$(0, 1/3)$	0.2	$+5 < 1$	False
$(1/3, 1/2)$	0.4	$0 < 1$	True
$(1/2, \infty)$	1	$3 < 1$	False

Solution set: $(1/3, 1/2)$ x - 5 aya tha but due to mod +5.



Methods of Boundary Values:

$$\frac{2x+1}{x-1} < 3$$

For free boundary value

$$x - 1 = 0$$

$$x = 1$$

For b.v:-

$$\frac{2x+1}{x-3} = 3 \rightarrow \text{associated eqn.}$$

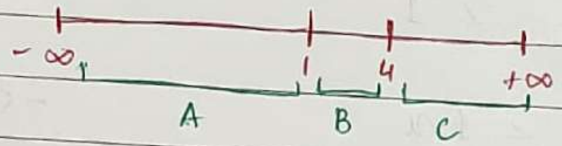
$$(2x+1) = 3(x-1)$$

$$2x+1 = 3x-3$$

$$2x - 3x = -3 - 1$$

$$-x = -4$$

$$\boxed{x = 4}$$



Region Test:-

A: $-\infty, 1$

let $x = 0$

$$\frac{2x+1}{x-1} < 3$$

$$\frac{2(0)+1}{0-1} < 3$$

$$\frac{1}{-1} < 3$$

$-1 < 3$ ✓

True

B: $1, 4$

let $x = 2$

$$\frac{2x+1}{x-1} < 3$$

$$\frac{2x+1}{x-1} < 3$$

$$\frac{2(2)+1}{2-1} < 3$$

$5 < 3$ ✗

Fal...

C: $4, +\infty$

let $x = 6$

$$\frac{2(6)+1}{6-1} < 2$$

$13/5 < 2$

$2.6 < 3$ ✓

True

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Conclusion: $(-\infty, 1) \cup (4, +\infty)$

(2) $\dfrac{2x}{x+2} \geq \dfrac{x}{x-2}$

For Free b.V:-

$x+2=0$, $x-2=0$

$x=-2$, $x=2$

For boundary value

$\dfrac{2x}{x+2} = \dfrac{x}{x-2}$

$2x(x-2) = x(x+2)$

$2x^2 - 4x = x^2 + 2x$

$2x^2 - x^2 = 2x + 4x$

$x^2 = 6x$

$x^2 - 6x - 0 = 0$

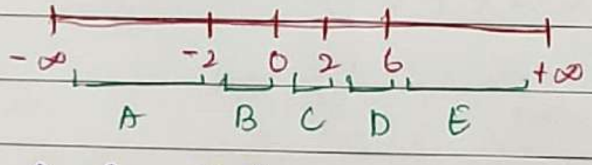
$\Rightarrow \dfrac{-b \pm \sqrt{b^2-4ac}}{2a}$

$\Rightarrow \dfrac{-(-6) \pm \sqrt{(6)^2 - 4(1)(0)}}{2(1)}$

$\Rightarrow \dfrac{6+6}{2}$, $\dfrac{6-\cancel{6}\,0}{\cancel{2}}$

$\Rightarrow 6$, 0

$\Rightarrow \boxed{x=6}$, $\boxed{x=0}$



Region Test:-

A: $-\infty, -2$

let $x=-4$

$\dfrac{2(-4)}{-4+2} \geq \dfrac{-4}{-4-2}$

$-\cancel{8}^{4} / \cancel{2} \geq -\cancel{4}^{2} / \cancel{6}\,3$

True

B: $-2, 0$ let $x=-1$

$\Rightarrow \dfrac{2(-1)}{-1+2} \geq \dfrac{-1}{-1-2}$

$\Rightarrow \dfrac{-2}{1} \geq \dfrac{1}{3}$

$\Rightarrow -2 \geq 1/3$ ×

False

C: $0, 2$ let $x=1$

$\Rightarrow \dfrac{2(1)}{1+2} \geq \dfrac{1}{1-2}$

$\Rightarrow 2/3 \geq 1/-1$ True

D: $2, 6$ let $x=4$

$\Rightarrow \dfrac{2(4)}{4+2} \geq \dfrac{4}{4-2}$

$\Rightarrow {}^{4}\cancel{8} / \cancel{6}\,3 \geq {}^{2}\cancel{4} / \cancel{2}\,1$

$\Rightarrow 1.33 \geq 2$ × False

E: $6, +\infty$ let $x=8$

$= \dfrac{2(8)}{8+2} \geq \dfrac{8}{8-2}$

$\Rightarrow \dfrac{\cancel{16}\,8}{\cancel{10}\,5} \geq \dfrac{\cancel{8}\,4}{\cancel{6}\,3}$

$\Rightarrow 1.6 > 1.33$

True

Conclusion:

$(-\infty, -2) \cup [0, 2) \cup [6, +\infty$

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(1) $\frac{x-1}{2} - \frac{1}{x} > \frac{4}{x} + 5$

Sol^n $\Rightarrow \frac{x-1}{2} - \frac{1}{x} > \frac{4}{x} + 5$

$\Rightarrow \frac{x^2-x-2}{2x} > \frac{4+5x}{x}$

For F.b.v:-

$2x = 0$; $x = 0$

$x = 0$

For b.v:-

$\Rightarrow \frac{x^2-x-2}{2x} = \frac{4+5x}{x}$

$\Rightarrow x(x^2-x-2) = 2x(4+5x)$

$\Rightarrow x^3 - x^2 - 2x = 8x + 10x^2$

$\Rightarrow x^3 - 11x^2 - 10x = 0$

$\Rightarrow x(x^2 - 11x - 10) = 0$

$\Rightarrow x^2 - 11x - 10 = 0$

$\therefore x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$

$x = \frac{-(-11) \pm \sqrt{(-11)^2 - 4(1)(-10)}}{2(+1)}$

$x = \frac{11 \pm \sqrt{121+40}}{+2}$ $\Rightarrow$

$x = \frac{11 \pm \sqrt{161}}{2}$

$x = \frac{11+\sqrt{161}}{2}$, $\frac{11-\sqrt{161}}{2}$

$x = 11.84$, $x = -0.84$

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① $x^4 - 5x^3 - 4x^2 + 20x \leq 0$

For b.v:-

$\Rightarrow x^4 - 5x^3 - 4x^2 + 20x \leq 0$

$\Rightarrow x(x^3 - 5x^2 - 4x + 20) \leq 0$

$\Rightarrow x^3 - 5x^2 - 4x + 20 = 0$

check Polynomial:-

Put $x = 2$

$\Rightarrow (2)^3 - 5(2)^2 - 4(2) + 20 = 0$

$\Rightarrow 8 - 20 - 8 + 20 = 0$

$\Rightarrow 0$, so, $x = 2$

Now, applying synthetic Div:

2	1	-5	-4	20
		2	-6	-20
	1	-3	-10	$\boxed{0}$

Factors: $(x-2)(x^2 - 3x - 10)$

$\Rightarrow x^2 - 3x - 10 = 0$

$\Rightarrow x^2 + 2x - 5x - 10 = 0$

$\Rightarrow x(x+2) - 5(x+2) = 0$

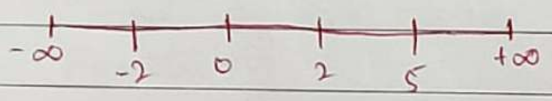
$\Rightarrow (x-5)(x+2) = 0$

$\Rightarrow x + 2 = 0$; $x - 5 = 0$

$x = -2$; $x = 5$

complete Fact:-

$x = 0, x = 2, x = 5, x = -2$



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(8) $12x^2 - 25x + 12 \geq 0$

For b.o.v :-

$\Rightarrow 12x^2 - 25x + 12 = 0$

$\Rightarrow 12x^2 - 16x - 9x + 12 = 0$

$\Rightarrow 4x(3x-4) - 3(3x-4) = 0$

$\Rightarrow (4x-3)(3x-4) = 0$

$\Rightarrow 4x - 3 = 0 \ , \ 3x - 4 = 0$

$\Rightarrow x = 3/4 \ , \ x = 4/3$

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Functions:-

→ every input values should map.

→ Output unique → Just like students & their marks

- Domain = (Input)
- codomain = (Possible output)
- Range = (Real output)

Types of Functions -

① Injective : (One - One function

⇒ domain unique:-

A 1
B 2
C 3
 4

A → 1
B → 2 → Injective bcz unique domain
C → 4
D → 3
 5

② surjective (onto Function):-

⇒ codomain = Range

A → 1
B → 2
C → 3
D → 3

C.D = {1, 2, 3}

R = {1, 2, 3}

③ Bijective (Inj + Sur):-

(Inverse exist)

A → 1
B → 3
C → 2
D → 4

Q. Check whether given relations are functions or not. If possible identify its type otherwise justify your comments

i) A → 2
B → 2
C → 3
 1, 4
D → 5

Function

iv) 1 → 2
2 → 4
3 → 6
4 → 8
5 → 10

Bijective

ii) 1 → 2
2 → 2
3 → 2
4 → 2
5 → 3
 1

Function

vi) A → 4
B → 4
C → 6

Function surjective

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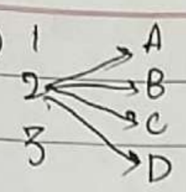
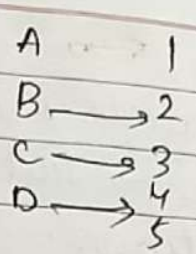
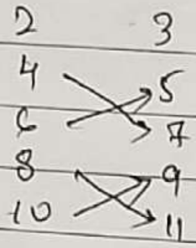
v) 

Not Function bcz their is not mapping on values & no unique output.

iii) 

Not Function bcz their is not mapping on values and no unique output.

vii) 

Not Function, bcz not mapping in input value.

Injective Function :-

① $f(x) = f(y)$, $f(x) = 3 - e^x$

then $x = y$,

$f(y) = 3 - e^y$

$f(x) = f(y)$

$3 - e^x = 3 - e^y$

$-e^x = \cancel{3} - \cancel{3} e^y$

$\cancel{-} e^x = \cancel{-} e^y$

Appling ln on b.s

$\ln e^x = \ln e^y$

$\boxed{x = y}$

② $f(x) = 2 + \sin x$

$f(x) = f(y)$

then $x = y$

$f(y) = 2 + \sin y$

$\cancel{2} + \sin y = \cancel{2} + \sin x$

$y = \sin^{-1}(\cancel{\sin} x)$

$\boxed{y = x}$

③ $f(x) = x + \ln x$

$f(x) = f(y)$

then, $x = y$

$f(y) = y + \ln y$

$f(x) = f(y)$

$x + \ln x \ne = y + \ln y$

$e^{x + \ln x} = e^{y + \ln y}$

$e^x \cdot e^{\ln x} = e^y \cdot e^{\ln y}$

$e^x \cdot x = e^y \cdot y$

Not injective bcz $x \neq y$

Surjective Function:

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convert x in terms of y. Then, $f(x) = y \rightarrow$ codomain

① $f(x) = 3 - e^x \rightarrow$ ①

$$y = 3 - e^x$$

$$e^x = 3 - y$$

$$\ln e^x = \ln(3-y)$$

$$x = \ln(3-y)$$

Put $x = \ln(3-y)$ in eqn ①

$$f(x) = 3 - e^{\ln(3-y)}$$

$$f(x) = 3 - (3-y)$$

$$f(x) = \cancel{3} - \cancel{3} + y$$

$$f(x) = y$$

Surjective.

② $f(x) = 2 + \sin x \rightarrow$ (i)

$$y = 2 + \sin x$$

$$y - 2 = \sin x$$

Applying $\sin^{-1}$ on b/s

$$\sin^{-1}(y-2) = x$$

Put value of x in (i)

$$f(x) = 2 + \cancel{\sin}\{\cancel{\sin^{-1}}(y-2)\}$$

$$f(x) = \cancel{2} + y - \cancel{2}$$

$$f(x) = y$$

③ $f(x) = x + \ln x$

$$y = x + \ln x$$

In this function we are not separate "x" so that's why we are not to represent y as x so this is not a surjective function.

" Function is not in term of y ".

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Q: Check wheather given function is Injective, surjective or both.

① $f(x) = 2x + 3$

$f(x) = f(y)$

Then, $x = y$

$f(y) = 2y + 3$

$f(x) = f(y)$

$2x + \cancel{3} = 2y + \cancel{3}$

$\cancel{2}x = \cancel{2}y$

$\boxed{x = y}$

For surjective

$y = 2x + 3$

$\frac{y - 3}{2} = x$

Put value of x in ①

$f(x) = \cancel{2}\left(\frac{y - \cancel{3}}{\cancel{2}}\right) + 3$

$\boxed{f(x) = y}$

The given function is injective & surjective both.

so, the given function is also a bijective.

Domain, Range, Co-domain

① $f(x) = 2x + 3$ Domain $= \mathbb{R}$

② $f(x) = \frac{1}{x}$ Domain $= (-\infty, 0) \cup (0, +\infty)$

$\mathbb{R} - [0]$

③ $f(x) = \frac{x+3}{2}$ Domain $= \mathbb{R}$

④ $f(x) = \sqrt{x}$ Domain $= [0, +\infty]$, $\mathbb{R} - (-\infty, 0)$

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① $f(x) = \sqrt{1-x^2}$

$1 - x^2 \geq 0$

using m.o.b.n

$1 - x^2 = 0$

$1 = x^2$

$\pm\sqrt{1} = \sqrt{x^2}$

$\pm 1 = x$

$-\infty \quad -1 \quad +1 \quad +\infty$

Region Test:-

A :- $-\infty, -1$

$x = -2$

$1-(-2)^2 \geq 0$

$1 - 4 \geq 0$

$-3 \geq 0$

False

B : $-1, 1$

$x = 0$

$1 - 0^2 \geq 0$

$1 \geq 0$ True

C = $-1, +\infty$

$x = 2$

$1 - 2^2 \geq 0$

$-3 \geq 0$

False

Domain of $f(x) = \sqrt{1-x^2}$ is from $[-1,1]$

find domain of $f(x) = \dfrac{\sqrt{x+1}}{x-1} + \dfrac{1}{x}$

Break:

$\underbrace{\sqrt{x+1}}_{g} \cdot \underbrace{\dfrac{1}{x-1}}_{h} + \underbrace{\dfrac{1}{x}}_{i}$

(i) $1/x$ = undefined

$x = 0$

Interval of i = $-(\infty, 0) \cup (0, +\infty)$

(h) $\dfrac{1}{x-1}$ = undefined

$x - 1 = 0$

$x = 1$

Interval of h = $(-\infty, 1) \cup (1, -\infty$

(g) $\sqrt{x+1}$

$x + 1 \geq 0$

$x + 1 = 0$

$x = -1$

$-\infty \quad -1 \quad +\infty$

A B

Region test:

A : $-\infty, -1$

$-2 + 1 \geq 0$

$-1 \geq 0$

False

B : $-1, +\infty$

$1 + 1 \geq 0$

$2 \geq 0$

True

Domain of $\dfrac{\sqrt{x+1}}{x-1} + \dfrac{1}{x}$ is form

Q. Find domain of $f(x) = \dfrac{1}{(x-1)(x-2)} + \dfrac{2x+3}{\sqrt{x}}$

Break :-

$g = \dfrac{1}{(x-1)(x-2)}$; $h = 2x+3$; $i = \dfrac{1}{\sqrt{x}}$

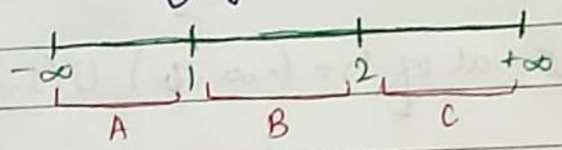
$h : \quad 2x+3 = R$

$g : \quad \dfrac{1}{(x-1)(x-2)}$

$x-1 = 0$; $x-2=0$

$x = 1$; $x = 2$

Interval of g : $(-\infty, 1) \cup (1,2) \cup (2,+\infty)$



$i = \dfrac{1}{\sqrt{x}}$

$\sqrt{x} = 0$, $x = 0$

Interval : $(-\infty, 0) \cup (0, +\infty)$

Operations And Functions:

i) $f(x) + g(x) = g(x) + f(x)$

ii) $f(x) - g(x) \neq g(x) - f(x)$

iii) $f(x) \times g(x) = g(x) \times f(x)$

iv) $f(x) / g(x) = g(x) / f(x)$

v) $f \circ g = f(g(x))$
$g \circ f = g(f(x))$ < composition of function.

vi) $f(-x) = f(x) \longrightarrow$ even
$f(-x) = -f(x) \longrightarrow$ odd

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Q. $f(x) = 2x+1$, $g(x) = \sin x - 1$

•) $f(x) + g(x) = 2x + \cancel{1} + \sin x - \cancel{1} = 2x + \sin x$

$g(x) + f(x) = \sin x - 1 + 2x + 1 = \sin x + 2x$

•) $f(x) - g(x) = (2x+1) - (\sin x - 1)$

$= 2x + 1 - \sin x + 1$

$= 2x + 2 - \sin x$

•) $g(x) - f(x) = (\sin x - 1) - (2x+1)$

$= \sin x - 1 - 2x - 1$

$= \sin x - 2 - 2x$

•) $f \circ g = f(gx) \Rightarrow f(x) = 2x+1$

$f(g(x)) = 2g(x) + 1$

$\Rightarrow f(g(x)) = 2(\sin - 1) + 1$

$f(g(x)) = 2\sin x - 2 + 1$

$f(g(x)) = 2\sin x - 1$

Composition Example:

•2) $f(x) = x^3 + x$

$f(-x) = (-x)^3 + (-x)$

$= -(x^3 + x)$

$= -f(x)$ (odd function)

$f(x) = x^2 + 1$

$= (-x^2) + 1$

$= x^2 + 1$

$= f(x)$ (even function)

$f(x) = \ln x$, $g(x) = \sqrt{x} + 2$

$h(x) = \sin x + e^x$

(a) find $h \circ g$

(b) check $h(x)$ is even or odd or neither even nor odd

(c) find $g * h$ and h/f

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Limits:

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Operations and Functions:-

1. $f(x) + g(x) = g(x) + f(x)$
$f(x) - g(x) \neq g(x) - f(x)$
$f(x) \cdot g(x) = g(x) \cdot f(x)$
$f(x) / g(x) \neq g(x) / f(x)$

$f(x) = 2x - 1,\ g(x) = \sin x - 1$
$f(x) - g(x) = 2x + \sin x + \cancel{1} - \cancel{1}$
$= 2x + \sin x$
$g(x) - f(x) = \sin x - \cancel{1} + 2x + \cancel{1}$
$= \sin x + 2x$

∴ Additive follows commutative law:
$A + B = B + A$

Even Functions:

A function said to be a if $f(-x) = f(x)$
eg: $y = \cos x$ and $y = x^2$

Odd Function:

A function is said to be an odd function if $(-x) = -f(x)$
$y = \sin x$ & $y = x^3$

Note:
- sum, diff and quotient of even (odd) functions are even (odd)
- Product of an even and odd fx is odd.
- $f(x) = 0$ is the only function which is even and odd.

Right hand limit:

Right Hand limit of $f(x)$ as x tend to 'a' exists and is equal to l_1 if as x approaches 'a' through values greater than 'a' the values of $f(x)$ approach a definit unique real num l_1 and we write $\lim_{x \to a} f(x)$.
or $f(a+0) = l_2$

Left hand limit:
$f(x)$ as x tend to 'a' excel and is equal to l_2 if as x approaches 'a' through values less than 'a' the value of $f(x)$ approach to a definite unique real no. l_2 and write
limit $f(x) = l_2$ or $f(a-0) = l_2$

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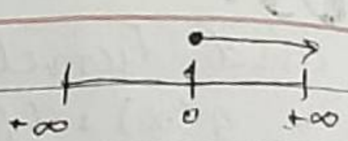
$$\lim_{x \to 0} \frac{1}{x^2}$$



$x \to 0^+$

x	0.1	0.01	0.001
$f(x^2)$	100	10,000	1,000,000

$\therefore f(x^2)$ $x \to 0^+$, $x \to \infty^+$ (diverge)

$x \to 0^-$

x	0.1	-0.01	-0.001
$f(x^2)$	100	10,000	1,000,000

$\therefore f(x^2)$ $x \to 0^+$, $x \to \infty^+$

* The limits of the fn exists b/c limit x^+ = limit x^- so it's the fn which exist.

$$\lim_{x \to 2} \frac{x^2 - 4}{x - 2}$$

$x \to 2^-$	1.9	1.99	1.999
$f(x^2-4/x-2)$	3.9	3.99	3.999

$x \to 2^+$

x	2.1	2.01	2.001
$f(x^2-4/x-2)$	4.1	4.01	4.001

limit 2^- = limit 2^+

if it's giving the specific point it's existing and is also converge.

Types of limits:-

Easy limit: In which point is ah defined.

0/0 form: Which is solved or can be solved by factorization / Rationalization.

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Not 0/0 form: the form in which you have to break into L. hand limit and R. hand limit.

Trignometric form:

for finding the limits of trignometric ln we use trigonometric trans formation and quite useful.

$$\lim_{x \to 0} \frac{\sin x}{x} = 1$$

Peice wise function:-

0/0 form

$$\lim_{x \to 2} \frac{x^2 - 4}{x - 2}$$

$$\lim_{x \to 2} \frac{\cancel{(x-2)}(x+2)}{\cancel{(x-2)}}$$

$$\lim_{x \to 2} x + 2 = 2 + 2 = 4$$

$$\lim_{x \to 9} \frac{\sqrt{x} - 3}{x - 9}$$

$$\lim_{x \to 9} \frac{\sqrt{x} - 3}{x - 9} \times \frac{\sqrt{x} + 3}{\sqrt{x} + 3}$$

$$\lim_{x \to 9} \frac{\cancel{(x - 9)}}{\cancel{(x-9)}(\sqrt{x} + 3)}$$

$$\lim_{x \to 9} \frac{1}{\sqrt{9} + 3} \Rightarrow \frac{1}{3 + 3} \Rightarrow \frac{1}{6}$$

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Limits on Trigonometric function:

i) $\lim_{x \to 0} \frac{\sin x}{x} = 1$

=> $x \to 0^+$

x	0.1	0.001	0.0001
$\sin x / x$	0.9	0.999	0.9999

as $x \to 0^+$, $f(x) \to \cancel{0}\ 1$

=> $x \to 0^-$

x	-0.1	-0.001	-0.0001
$\sin x / x$	0.9	0.999	0.9999

as $x \to 0^-$, $f(x) \to 1$

(ii) $\lim_{x \to 0} \frac{1 - \cos x}{x}$

$1 - \cos^2 x = \sin^2 x$

$\lim_{x \to 0} \frac{1 - \cos x}{x} \times \frac{1 + \cos x}{1 + \cos x}$

$\lim_{x \to 0} \frac{1 - \cos^2 x}{x(1 + \cos x)}$

$\lim_{x \to 0} \frac{\sin^2 x}{x(1 + \cos x)}$

$\lim_{x \to 0} \left(\frac{\sin x}{x} \cdot \sin x \cdot \frac{1}{(1 + \cos x)} \right)$

$\lim_{x \to 0} 1 \cdot 0 \cdot \frac{1}{2}$

=> 0 Ans

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$f(x) - g(x) = 2x + 1 \ (\sin 1)$

$2x + 2 - \sin x$

$g(x) - f(x) = \sin x - 1 - (2x + 1)$

$\sin x - 1 - 2x - 1$

$\sin x - 3x - 2$

* The commutative law is not followed by subtraction so $f(x) - g(x) \neq g(x) - f(x)$

$fog \Rightarrow f = 2x + 1 \quad g = \sin x - 1$

$fog = 2(\sin x - 1) + 1$

$= 2\sin x - 2 + 1$

$= 2\sin x - 1$

function is odd / Even

$f(x) = x^3 + x$

$f(-x) = (-x)^3 + (-x)$

$= -x^3 + -x$

$= -(x^3 + x)$

$f(x) = -f(x)$ (odd function)

$f(x) = x^2 + 1$

$= (x)^2 + 1$

$f(x) = x^2 + 1$

$f(x) = f(x)$ (Even function)

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Limits on absolute

(i) $\lim_{x \to 0} |x|$

$= 0$ → $\lim_{x \to 0^+} |x| = 0$

→ $\lim_{x \to 0^-} |x| = 0$

x	0.1	0.001	0.00001
$\lvert x \rvert$	0.1	0.001	0.00001

as $x \to 0^+$, $f(x) \to 0$

x	-0.1	-0.001	-0.0001
$\lvert x \rvert$	0.1	0.001	0.0001

→ variable terms se mode hatae tw ± sign lagate h.

→ number ke term se mode hatae tw jo no. he wahi aye ga liken (−) gi tw (+) hogi value

(ii) $\lim_{x \to 0} \frac{|x|}{x}$ → $\lim_{x \to 0^+} = 1$

→ $\lim_{x \to 0^-} = -1$

⇒ $x \to 0^+$

x	0.1	0.001	0.0001
$\lvert x \rvert / x$	1	1	1

as $x \to 0^+$, $f(x) \Rightarrow 1$

⇒ $x \to 0^-$

x	-0.1	-0.001	-0.0001
$\lvert x \rvert / x$	-1	-1	-1

as $x \to 0^-$, $f(x) \to -1$

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$\Rightarrow \quad x \to 0^+$

$\rightarrow \lim_{x \to 0} \frac{\csc x - \cot x}{x}$

$\lim_{x \to 0} \frac{\csc x - \cot x}{x} \times \frac{\csc + \cot x}{\csc + \cot x}$

$\lim_{x \to 0} \frac{\csc^2 x - \cot^2 x}{x(\csc + \cot x)}$

$\frac{1/\sin^2 x - \cos^2 x/\sin^2 x}{x\ (}$

$\lim_{x \to 0} \frac{1/\sin x - \cos x/\sin x}{x}$

$\frac{1 - \cos x}{(\sin x)(x)} \left(\frac{1 + \cos x}{1 + \cos x}\right)$

$\frac{1 - \cos^2 x}{(\sin x)(x)(1 + \cos x)}$

$\frac{\sin^{\cancel{2}} x}{(1 + \cos x)(\cancel{\sin x})(x)}$

$\frac{\sin x}{(1 + \cos x)(x)} \Rightarrow \frac{\sin x}{x}\left(\frac{1}{1 + \cos x}\right) \Rightarrow 1\ ($

Limits on Piecewise Function :-

is $f(x) = \begin{cases} x+3 & x<0 \quad (-\infty, 0) \\ x^2-1 & 0 \le x < 2 \quad [0, 2) \\ x-6 & x \geq 2 \quad [2, +\infty) \end{cases}$

$\lim_{x \to 0} \rightarrow \lim_{x \to 0^+} x^2 - 1 = -1$

$\rightarrow \lim_{x \to 0^-} x+3 = 3$

$\lim_{x \to 0}$ doesn't exist.

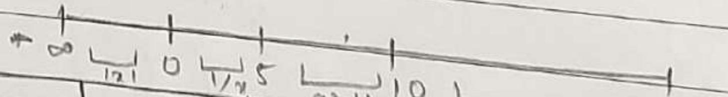
$\lim_{x \to 2} \rightarrow \lim_{x \to 2^+} (x-6) = -4$

$\rightarrow \lim_{x \to 2^-} (x^2-1) = 3$

$\lim_{x \to 2}$ doesn't exist

Q: $f(x) \begin{cases} |x| & x \le 0 \quad (-\infty, 0] \\ 1/x & 0 < x < 5 \quad [0, 5) \\ x^2+1 & 5 \le x < 10 \quad [5, 10) \\ 101 & x \geq 10 \quad [10, +\infty) \end{cases}$

$\lim_{x \to 0^-} |x| = \boxed{0}$



$\lim_{x \to 0^+} 1/x \; \boxed{= +\infty}$

x	0.1	0.001	0.0001
$1/x$	10	1000	10,000

limits doesn't exist

$\lim_{x \to 5} \rightarrow \lim_{x \to 5^+} x^2+1 = 26$

$\rightarrow \lim_{x \to 5^-} 1/x = 1/5$

limits doesn't exist

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$\lim_{x \to 10^+} + 101 = 101$

- → $-\infty$ or $+\infty$ = diverge
- → In number → converge

$\lim_{x \to 10^-} x^2 + 1 = 101$

Limits exist.

Continuous Function:

A function $f(x)$ is continuo

i - $f(a)$ is defined.

ii - $\lim_{x \to a} f(x)$ exist

iii - $\lim_{x \to a} f(x) = f(a)$

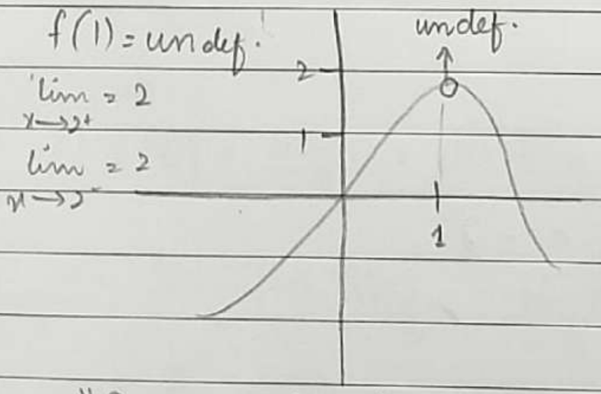
Types of Discontinuity:-

ii: Infinite (f is undefined + lim diverge)

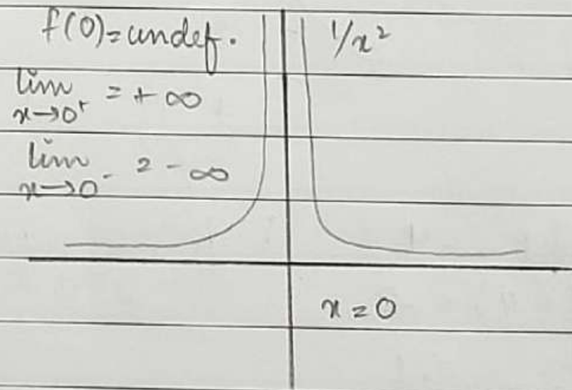
i: Removeable (if it is undefined + lim coverge)

iii: Jump (f is defined + lim doesn't exist)

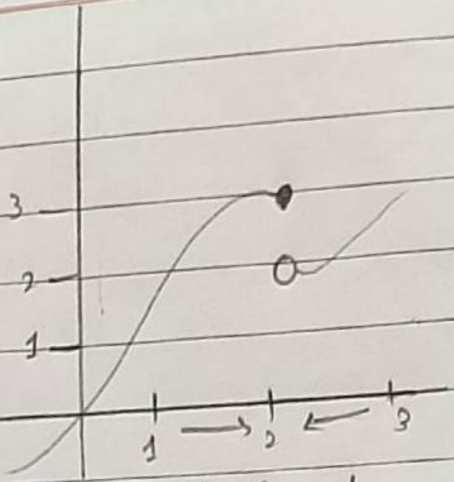
iv) Normal ($f(a) \neq \lim_{x \to a} f(x)$)



"Removable"



"Infinite"

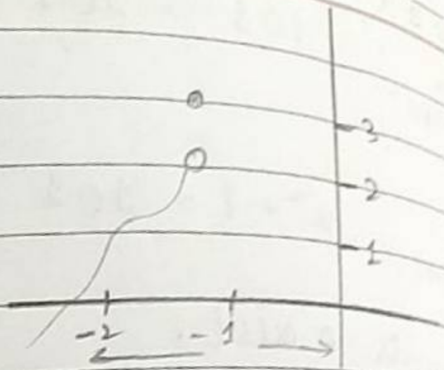


$f(2) = 3 \rightarrow$ defined

$\lim_{x \to 2^+} = 2$; $\lim_{x \to 2^-} = 3$

limit doesn't exist

"Jump"

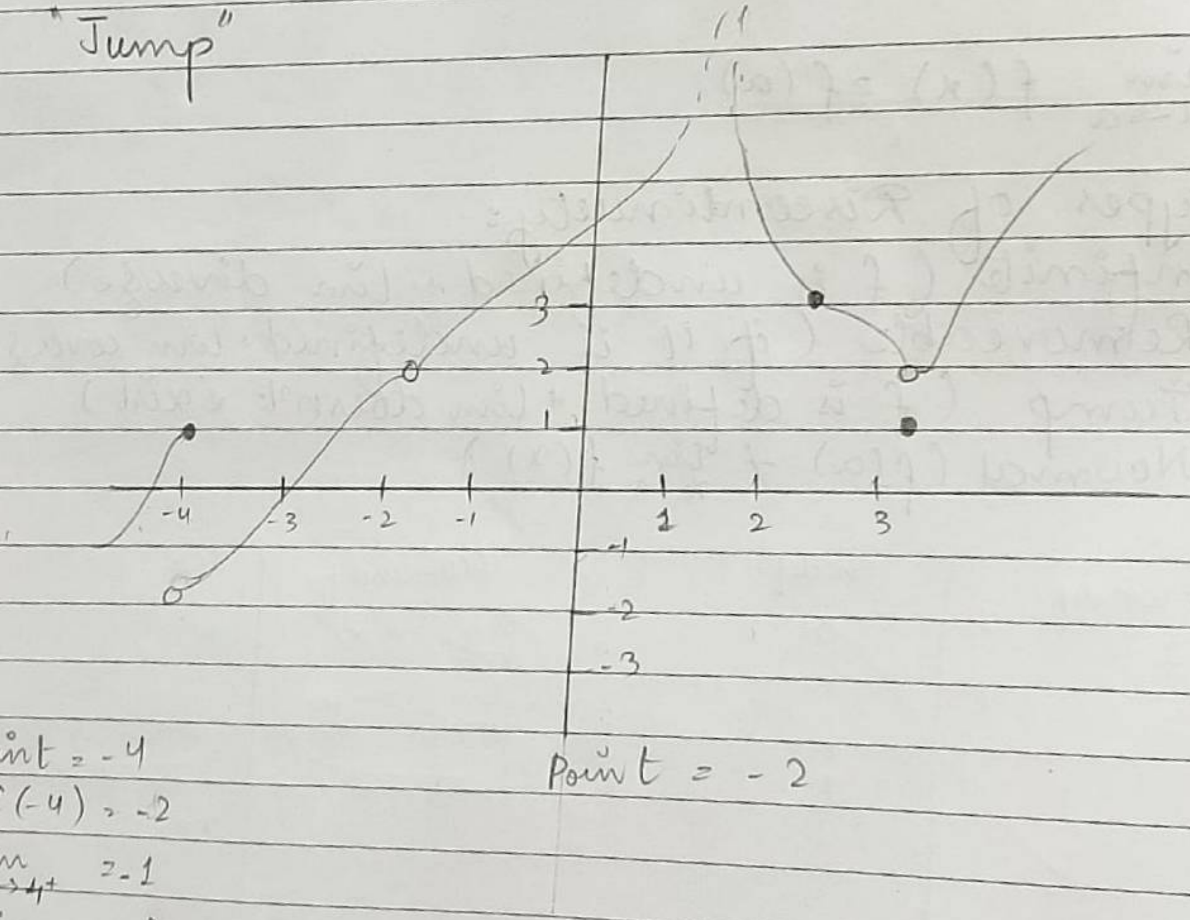


$f(-1) = 3$ defined

$\lim_{x \to -1^+} = 2$; $\lim_{x \to -1^-} = 2$

limit is exist.

"Normal"



Point $= -4$

$f(-4) = -2$

$\lim_{x \to -4^+} = -1$

$\lim_{x \to -4^-} = 2$

"limit doesn't exist

Jumpal function

Point $= -2$

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Q. $f(x) = \begin{cases} |x| & x \leq 0 \quad (-\infty, 0] \\ \sin x / x & 0 < x \leq 6 \quad (0, 6] \\ \dfrac{x^2 - 36}{x - 6} & 6 < x \quad (6, +\infty) \end{cases}$

$x = 0$

i) $f(0) = |x|$

ii) $\lim_{x \to 0} f(x) = \begin{cases} \lim_{x \to 0^+} \sin x / x \Rightarrow 1 \\ \lim_{x \to 0^-} |x| = 0 \end{cases}$

$\lim_{x \to 0^+} \neq \lim_{x \to 0^-}$ (Thus limit doesn't exist)

$f(x)$ is discontinous at $x = 0$ "Jump".

$x = 6$

i) $f(6) = \dfrac{\sin x}{x} = \dfrac{\sin 6}{6} \Rightarrow -0.046$

ii) $\lim_{x \to 6} \begin{cases} \lim_{x \to 6^-} \dfrac{\sin x}{x} \Rightarrow -0.046 \\ \lim_{x \to 6^+} \dfrac{x^2 - 36}{x - 6} \Rightarrow 12 \end{cases}$

limit doesn't exist

Jump

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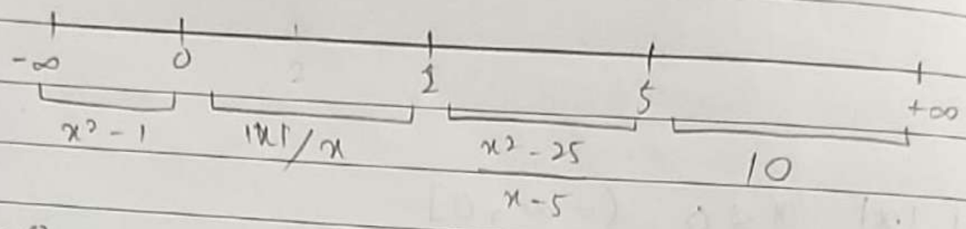
$$f(x) \begin{cases} x^2 - 1 & x \leq 0 & (-\infty, 0] \\ |x|/x & 0 < x < 2 & (0, 2) \\ \dfrac{x^2-25}{x-5} & 2 < x < +5 & (2, +5) \\ 10 & x \geq +5 & [5, +\infty) \end{cases}$$



i: for $x = 0$

$f(0) = -1$

$\lim_{x \to 0^+} \frac{|x|}{x} = 1$

$\lim_{x \to 0^-} x^2 - 1 = -1$

$\lim_{x \to 0}$ doesn't exist

ii: for $2 \rightarrow f(2)$ = undefined

$\rightarrow \lim_{x \to 2^+} \frac{x^2 - 25}{x - 5} = 7$

$\lim_{x \to 2^-} \frac{|x|}{x} = 1$

removable, Infinite, Jump

iii: for 5

$f(5) = 10$

$\lim_{x \to 5^+} 10 = 10$

$\lim_{x \to 5^-} \frac{x^2 - 25}{x - 5} = 10$

Continuous at $x = 5$