Q #01: What is Monte Carlo Simulation? What are Characteristics of Monte Carlo Simulation? How the Monte Carlo Algorithm works? Also explain Weaknesses of Monte Carlo Simulation.

Monte Carlo Simulation is a computational technique used to model and analyze complex systems that involve uncertainty or random processes. It uses random sampling to estimate mathematical functions and simulate the behavior of systems. It's widely used in fields like finance, engineering, and physics to predict outcomes when dealing with uncertainty.

Characteristics of Monte Carlo Simulation:

1. **Random Sampling**: The simulation relies on random sampling to explore a wide range of possible outcomes.
2. **Probabilistic Nature**: It generates results based on probability distributions (e.g., normal, uniform) rather than fixed inputs.
3. **Multiple Iterations**: Thousands or even millions of iterations are run to approximate the desired results.
4. **Statistical Output**: The final result is often expressed in terms of statistical measures like mean, variance, standard deviation, or probability.
5. **Versatility**: Monte Carlo can be applied to different types of problems in various domains.

How the Monte Carlo Algorithm Works:

1. **Define the Problem**: Specify the system or process you want to simulate, including the parameters and outcome of interest.
2. **Set Up Probability Distributions**: Identify the variables that have uncertainty and assign probability distributions to them (e.g., normal, uniform).
3. **Generate Random Samples**: For each variable, generate random values based on its probability distribution.
4. **Run Simulations**: Use the random inputs to run multiple simulations of the system, calculating the output each time.
5. **Analyze Results**: After completing all simulations, the results are aggregated and analyzed to obtain statistical properties like the average, variance, and probability of different outcomes.

Weaknesses of Monte Carlo Simulation:

1. **High Computational Cost**: Monte Carlo simulations can require significant computational resources, especially for complex problems or systems with many variables.

2. **Accuracy Depends on Number of Iterations**: To improve accuracy, a large number of iterations are needed, which increases computational time.
3. **Garbage In, Garbage Out**: If the input distributions or model assumptions are incorrect, the output will also be flawed or misleading.
4. **Slow Convergence**: Monte Carlo methods may converge slowly, meaning it can take a lot of iterations to get accurate estimates for some types of problems.
5. **Cannot Handle All Types of Problems**: Monte Carlo simulation is not ideal for problems that don't involve randomness or are better solved analytically.

Q #02: Explain following terms. Sample Spaces (Discrete and Continuous), Events, Mutually Exclusive Events, Permutations, Combinations.

1. Sample Spaces (Discrete and Continuous)

A **sample space** is the set of all possible outcomes of an experiment in probability theory. It represents all the possible results that can occur in a given scenario.

- **Discrete Sample Space**: A sample space is discrete if it contains a countable number of distinct outcomes.
  - Example: Rolling a die has a discrete sample space of {1, 2, 3, 4, 5, 6}.
- **Continuous Sample Space**: A sample space is continuous if it contains an uncountable number of possible outcomes, typically representing measurements on a continuous scale.
  - Example: The time taken for a chemical reaction could be any real number between 0 and infinity, making it a continuous sample space.

2. Events

An **event** is a specific outcome or a set of outcomes from the sample space. In probability, an event is a subset of the sample space.

- **Simple Event**: An event consisting of a single outcome.
  - Example: Rolling a 3 on a die.
- **Compound Event**: An event consisting of more than one outcome.
  - Example: Rolling an even number on a die, where the event is {2, 4, 6}.

3. Mutually Exclusive Events

Mutually exclusive events are events that cannot occur at the same time. In other words, if one event happens, the other cannot.

- Example: When flipping a coin, getting heads and getting tails are mutually exclusive events because they cannot happen simultaneously.

4. Permutations

A **permutation** refers to the arrangement of a set of items in a specific order. The order in which items are arranged matters.

- Formula: If there are n items and we want to arrange r of them, the number of permutations is given by: $P(n, r) = \frac{n!}{(n-r)!}$
  - Example: The number of ways to arrange 3 people (A, B, C) in a line is 3! = 6, so the permutations are: (ABC, ACB, BAC, BCA, CAB, CBA).

5. Combinations

A **combination** is a selection of items from a larger set where the order of selection does not matter.

- Formula: If there are n items and we want to select r of them without regard to the order, the number of combinations is given by: $C(n, r) = \frac{n!}{r!(n-r)!}$
  - Example: The number of ways to choose 2 people from a group of 3 (A, B, C) is C(3,2)=3C(3, 2) = 3C(3,2)=3, so the combinations are: (AB, AC, BC).

Summary of Key Differences:

- **Permutations**: The order matters (e.g., arranging books on a shelf).
- **Combinations**: The order does not matter (e.g., selecting a committee of people).

These concepts form the foundation for understanding probability and combinatorics in mathematics.

Q #03: Explain following terms. Random variable, Probability Distribution, Probability Mass Function (PMF), Probability Density Function (PDF), Cumulative Distribution Function (CDF)

1. Random Variable

A **random variable** is a variable that takes on different numerical values based on the outcome of a random experiment. It is essentially a function that assigns numerical values to each outcome in the sample space.

- **Discrete Random Variable**: Takes on a countable number of values.

- Example: The number of heads when flipping a coin 3 times (values could be 0, 1, 2, or 3).

- **Continuous Random Variable**: Takes on an infinite number of possible values, typically real numbers.
  - Example: The time it takes for a train to arrive (could be any positive real number).

2. Probability Distribution

A **probability distribution** describes how probabilities are distributed over the values of a random variable. It provides a model for understanding the likelihood of different outcomes.

- For **discrete random variables**, the distribution is often represented by a **Probability Mass Function (PMF)**.
- For **continuous random variables**, it is represented by a **Probability Density Function (PDF)**.

3. Probability Mass Function (PMF)

The **Probability Mass Function (PMF)** is a function that gives the probability that a discrete random variable is exactly equal to a particular value.

- For a discrete random variable X, the PMF P(X = x) gives the probability of X taking the value x.
  - Example: If X is the result of rolling a die, the PMF is: P(X = x) = $\frac{1}{6}$ for x= 1,2,3,4,5,6

4. Probability Density Function (PDF)

The **Probability Density Function (PDF)** is a function used to specify the probability of a continuous random variable falling within a particular range of values. Unlike the PMF, the PDF does not give the probability of the variable taking on a specific value, but instead represents the likelihood of it falling within a range.

- For a continuous random variable X, the PDF f(x) gives the relative likelihood of X being near the value x. The actual probability of X falling between two values a and b is the area under the curve of the PDF from a to b:
- P(a ≤ X ≤ b) = $\int_b^a f(x)dx$
  - Example: The PDF of a normally distributed random variable is the famous bell-shaped curve.

5. Cumulative Distribution Function (CDF)

The **Cumulative Distribution Function (CDF)** gives the probability that a random variable takes on a value less than or equal to a specific value. It applies to both discrete and continuous random variables.

- For a random variable X, the CDF F(x) is defined as: $F(x) = P(X \le x)$
    - For **discrete random variables**, the CDF is the sum of the probabilities up to a certain value.
    - For **continuous random variables**, the CDF is the integral of the PDF up to a certain value: $F(x) = \int_{-\infty}^{x} f(t)\, dt$
    - Example: For a discrete random variable representing the number of heads in 3 coin flips, the CDF might look like: $F(0) = P(X \le 0)$, $F(1) = P(X \le 1)$, $F(2) = P(X \le 2)$,...

Summary of Key Concepts:

- **Random Variable**: Represents outcomes of a random process.
- **Probability Distribution**: Describes the likelihood of different outcomes.
- **PMF**: Gives probabilities for discrete random variables.
- **PDF**: Gives likelihoods for continuous random variables, and areas under the curve represent probabilities.
- **CDF**: Gives the cumulative probability up to a certain value, useful for both discrete and continuous variables.

Q #04: Problem: Suppose that a coin is tossed twice. Let X represents the number of heads which can come up. Find the probability function corresponding to the random variable X

To solve this problem, we need to:

1. **Determine the sample space** for tossing a coin twice.
2. **Identify the random variable X** which represents the number of heads that can come up.
3. **Find the probability function** corresponding to X.

Step 1: Sample Space

When a coin is tossed twice, the possible outcomes are:

$$S = \{HH, HT, TH, TT\}$$

- H represents heads
- T represents tails

Step 2: Define the Random Variable X

The random variable X represents the **number of heads** that occur in each outcome. For each outcome in the sample space, we can determine the value of X:

- For HH, the number of heads is 2, so X=2.
- For HT, the number of heads is 1, so X=1.
- For TH, the number of heads is 1, so X=1.
- For TT, the number of heads is 0, so X=0.

Thus, the possible values for X are 0, 1, and 2.

Step 3: Find the Probability Function (PMF)

To find the **Probability Mass Function (PMF)**, we calculate the probability associated with each possible value of X. Since the coin is fair, each outcome in the sample space is equally likely, and the probability of each outcome is:

$$P(\text{each outcome}) = \frac{1}{4}$$

Now, we find the probability for each possible value of X:

- P(X = 0): There is 1 outcome where there are 0 heads, which is TT. So:

$$P(X = 0) = \frac{1}{4}$$

- P(X = 1): There are 2 outcomes where there is 1 head, which are HT and TH. So:

$$P(X = 1) = \frac{2}{4} = \frac{1}{2}$$

- P(X = 2): There is 1 outcome where there are 2 heads, which is HH. So:

$$P(X = 2) = \frac{1}{4}$$

The Probability Function

The probability function (PMF) corresponding to the random variable X, which represents the number of heads, is:

$$P(X = x) = \begin{cases} \frac{1}{4}\ if\ x = 0 \\ \frac{1}{2}\ if\ x = 1 \\ \frac{1}{4}\ if\ x = 2 \end{cases}$$

This describes the probabilities for the number of heads that can appear when a coin is tossed twice.

Q #05: Calculate CDF for Rolling 2 Dice (Red/Green)

To calculate the **Cumulative Distribution Function (CDF)** for rolling two dice (one red and one green), let's go step by step.

Step 1: Sample Space and Random Variable

When two dice (red and green) are rolled, each die has six possible outcomes, so the total number of possible outcomes in the sample space is:

$$S = 6 \times 6 = 36$$

Let the **random variable X** represent the **sum** of the two dice. The possible values of X range from 2 (when both dice show 1) to 12 (when both dice show 6).

Step 2: Probability Mass Function (PMF) for X

First, let's calculate the probability of each possible sum X=, where x ranges from 2 to 12. The probability of each outcome is $\frac{1}{36}$ because all outcomes are equally likely.

- P(X = 2): There is 1 way to get a sum of 2: (1,1)

$$P(X = 2) = \frac{1}{36}$$

- P(X=3): There are 2 ways to get a sum of 3: (1,2), (2,1)

$$P(X = 3) = \frac{2}{36} = \frac{1}{18}$$

- P(X=4): There are 3 ways to get a sum of 4: (1,3), (2,2), (3,1)

$$P(X = 4) = \frac{3}{36} = \frac{1}{12}$$

- P(X=5): There are 4 ways to get a sum of 5: (1,4), (2,3), (3,2), (4,1)

$$P(X = 5) = \frac{4}{36} = \frac{1}{9}$$

- P(X=6): There are 5 ways to get a sum of 6: (1,5), (2,4), (3,3), (4,2), (5,1)

$$P(X = 6) = \frac{5}{36}$$

- P(X=7): There are 6 ways to get a sum of 7: (1,6), (2,5), (3,4), (4,3), (5,2), (6,1)

$$P(X = 7) = \frac{6}{36} = \frac{1}{6}$$

- P(X=8): There are 5 ways to get a sum of 8: (2,6), (3,5), (4,4), (5,3), (6,2)

$$P(X = 8) = \frac{5}{36}$$

- P(X=9): There are 4 ways to get a sum of 9: (3,6), (4,5), (5,4), (6,3)

$$P(X = 9) = \frac{4}{36} = \frac{1}{9}$$

- P(X=10): There are 3 ways to get a sum of 10: (4,6), (5,5), (6,4)

$$P(X = 10) = \frac{3}{36} = \frac{1}{12}$$

- P(X=11): There are 2 ways to get a sum of 11: (5,6), (6,5)

$$P(X = 11) = \frac{2}{36} = \frac{1}{18}$$

- P(X=12): There is 1 way to get a sum of 12: (6,6)

$$P(X = 12) = \frac{1}{36}$$

Step 3: Cumulative Distribution Function (CDF)

The **CDF** F(x) gives the probability that the random variable X is less than or equal to a particular value x. To compute this, we sum the probabilities of all outcomes up to and including x.

For X, the CDF is:

$$F(x) = P(X \leq x)$$

Here’s how the CDF looks for each possible value of X:

$$F(x) = \begin{cases} 0 & if\ x < 2 \\ \frac{1}{36} & \text{if } x = 2 \\ \frac{3}{36} = \frac{1}{12} & \text{if } x = 3 \\ \frac{6}{36} = \frac{1}{6} & \text{if } x = 4 \\ \frac{10}{36} & \text{if } x = 5 \\ \frac{15}{36} = \frac{5}{12} & \text{if } x = 6 \\ \frac{21}{36} = \frac{7}{12} & \text{if } x = 7 \\ \frac{26}{36} = \frac{13}{18} & \text{if } x = 8 \\ \frac{30}{36} = \frac{5}{6} & \text{if } x = 9 \\ \frac{33}{36} & \text{if } x = 10 \\ \frac{35}{36} & \text{if } x = 11 \\ 1 & \text{if } x = 12 \end{cases}$$

This **CDF** gives the cumulative probability for each sum x when rolling two dice.

Q #06: Explain following terms, Mean, Variance, and Standard Deviation, Binomial Probability Distribution, Bernoulli's trials, Poisson Probability Distribution, Hypergeometric Probability Distribution.

a. **John Ragsdale sells new cars for Pelican Ford. John usually sells the largest number of cars on Saturday. He has developed the following probability distribution for the number of cars he expects to sell on a particular Saturday. Calculate Mean, Variance and Standard Deviation of a Probability Distribution.**

Number of Cars Sold, x	Probability, $P(x)$
0	.10
1	.20
2	.30
3	.30
4	.10
Total	1.00

1. Mean (Expected Value):

The **mean** of a probability distribution is a measure of the central tendency of the distribution. It represents the expected value of a random variable. For a discrete random variable X with probability distribution P(x), the mean (or expected value) is given by:

$$\mu = E(X) = \sum_x x.p(x)$$

2. Variance:

The **variance** measures the spread or dispersion of a probability distribution around its mean. It shows how much the values of the random variable differ from the mean. For a discrete random variable X, the variance is calculated as:

$$\sigma^2 = \sum_x (x - \mu)^2 \cdot P(x)$$

3. Standard Deviation:

The **standard deviation** is the square root of the variance. It provides a measure of the average distance of each value from the mean, giving the spread of the distribution in the same units as the data. The formula is:

$$\sigma = \sqrt{\sigma^2}$$

4. Binomial Probability Distribution:

The **binomial distribution** describes the probability of getting a fixed number of successes in a fixed number of independent Bernoulli trials (with two possible outcomes, success and failure). The probability mass function is:

$$P(X = k) = \binom{n}{k} p^k (1 - p)^{n-k}$$

Where:

- n = number of trials
- k = number of successes
- p = probability of success in each trial
- $\binom{n}{k}$ = binomial coefficient

5. Bernoulli's Trials:

A **Bernoulli trial** is a random experiment where there are only two possible outcomes: success or failure. Each trial is independent, and the probability of success remains constant.

6. Poisson Probability Distribution:

The **Poisson distribution** gives the probability of a given number of events happening in a fixed interval of time or space, provided that the events occur with a known constant rate and are independent of the time since the last event. The probability mass function is:

$$P(X = k) = \frac{\lambda^k e^{-\lambda}}{k!}$$

Where:

- λ = the average number of events in the interval
- k = the number of events

7. Hypergeometric Probability Distribution:

The **hypergeometric distribution** describes the probability of getting k successes in n draws from a finite population without replacement, where there are N items in total, K of which are successes. The probability mass function is:

$$P(X = k) = \frac{\binom{K}{k}\binom{N-K}{n-k}}{\binom{N}{n}}$$

Where:

- N = total population size
- K = number of successes in the population
- n = number of draws
- k = number of observed successes

Problem: Calculate Mean, Variance, and Standard Deviation

Given the probability distribution for the number of cars John Ragsdale sells on a particular Saturday:

Number of Cars Sold (X)	Probability P(X)
0	0.10
1	0.20
2	0.30
3	0.30
4	0.10
Total	1.00

Step 1: Calculate the Mean (Expected Value)

The mean is calculated by summing the product of each value of X and its corresponding probability P(X):

$$\mu = E(X) = \sum_x x \cdot P(x) = (0 \cdot 0.10) + (1 \cdot 0.20) + (2 \cdot 0.30) + (3 \cdot 0.30) + (4 \cdot 0.10)$$

$$\mu = 0 + 0.20 + 0.60 + 0.90 + 0.40 = 2.10$$

So, the **mean** number of cars sold is **2.10**.

Step 2: Calculate the Variance

The variance is calculated by summing the square of the difference between each X and the mean μ, multiplied by the corresponding probability:

$$\sigma^2 = \sum_x (x - \mu)^2 \cdot P(x)$$

$$\sigma^2 = (0 - 2.10)^2 \cdot 0.10 + (1 - 2.10)^2 \cdot 0.20 + (2 - 2.10)^2 \cdot 0.30 + (3 - 2.10)^2 \cdot 0.30 + (4 - 2.10)^2 \cdot 0.10$$

$$\sigma^2 = (4.41 \cdot 0.10) + (1.21 \cdot 0.20) + (0.01 \cdot 0.30) + (0.81 \cdot 0.30) + (3.61 \cdot 0.10)$$

$$\sigma^2 = 0.441 + 0.242 + 0.003 + 0.243 + 0.361 = 1.29$$

So, the **variance** is **1.29**.

Step 3: Calculate the Standard Deviation

The standard deviation is the square root of the variance:

$$\sigma = \sqrt{\sigma^2} = \sqrt{1.29} \approx 1.14$$

So, the **standard deviation** is approximately **1.14**.

Summary of Results:

- **Mean (Expected Value)**: 2.10
- **Variance**: 1.29
- **Standard Deviation**: 1.14

Q #07: Problem: What is mathematical expectation of the number of heads when 3 fair coins are tossed?

To find the **mathematical expectation** (or **expected value**) of the number of heads when 3 fair coins are tossed, we follow these steps:

Step 1: Define the Random Variable

Let the random variable X represent the **number of heads** that can appear when 3 fair coins are tossed.

The possible values of X are:

- X = 0 (no heads),
- X = 1 (one head),
- X = 2 (two heads),
- X = 3 (three heads).

Step 2: Find the Probability Distribution

The possible outcomes when 3 fair coins are tossed are:

$$S = \{HHH, HHT, HTH, HTT, THH, THT, TTH, TTT\}$$

There are 2^3 = 8 equally likely outcomes.

Now, we calculate the probability for each possible number of heads:

- $P(X=0)$: There is 1 outcome with 0 heads (TTT):

$$P(X=0)=\frac{1}{8}$$

- $P(X=1)$: There are 3 outcomes with 1 head (HTT, THT, TTH):

$$P(X=1)=\frac{3}{8}$$

- $P(X=2)$: There are 3 outcomes with 2 heads (HHT, HTH, THH):

$$P(X=2)=\frac{3}{8}$$

- $P(X=3)$: There is 1 outcome with 3 heads (HHH):

$$P(X=3)=\frac{1}{8}$$

Step 3: Calculate the Expected Value (Mathematical Expectation)

The expected value E(X) is calculated using the formula for the expected value of a discrete random variable:

$$E(X)=\sum_{x} x \cdot P(x)$$

Substitute the values of X and P(X):

$$E(X)=(0 \cdot \frac{1}{8})+(1 \cdot \frac{3}{8})+(2 \cdot \frac{3}{8})+(3 \cdot \frac{1}{8})$$

$$E(X)=0+\frac{3}{8}+\frac{6}{8}+\frac{3}{8}$$

$$E(X)=\frac{12}{8}=1.5$$

Conclusion:

The **mathematical expectation** of the number of heads when 3 fair coins are tossed is **1.5**.

Q #08: Explain Mid-Square Random Number Generator and Congruence Method for random number generation.

a. **Generate 10 Random numbers by using the MID- Square Random Number Generator by using these seeds (7182, 5197, 4500, 123, 512, 621, 002, 001,).**

b. **Generate 10 Random numbers by using the MID-Product Random Number Generator by using these seeds (Z_0Z_1 = (123)(456),)**

c. **Problem: Using the multiplicative congruential method, find the period of the generator for a = 13, m = 2^6, and X_0 = 3.**

d. **Problem: Considering *a* = 3, *b* = 5, *m*= 17, and x_0 = 2, generate two digit random numbers using linear congruential generator**

e. **Problem: Generate 10 three digit random numbers using multiplicative congruential method with X0=117, a=43 and m=1000**

f. **Problem: Let *m* = 102 = 100, *a* = 19, *b* = 0, and *X0* = 63, and generate a sequence c random integers using linear congruential generator**

Explanation of Random Number Generation Methods:

1. Mid-Square Random Number Generator:

The **Mid-Square Method** is a simple random number generator technique. It involves squaring the seed value and then taking the middle digits of the result to form the next random number.

Steps:

1. Start with an initial seed X_0.
2. Square the seed value to get X_0^2.
3. Extract the middle digits from the squared result to form the next random number.
4. Use this new number as the seed for the next iteration.
5. Repeat the process to generate more numbers.

2. Mid-Product Random Number Generator:

The **Mid-Product Method** is similar to the mid-square method, but instead of squaring the seed, two different seeds are multiplied together, and the middle digits of the product are taken to form the next random number.

Steps:

1. Start with two initial seeds, Z0 and Z1.

2. Multiply Z0 by Z1.
3. Extract the middle digits of the product to form the next random number.
4. Use the resulting number for subsequent seeds.

3. Congruence Method (Linear and Multiplicative Congruential Generators):

The **linear congruential generator (LCG)** and **multiplicative congruential generator (MCG)** are methods for generating sequences of random numbers. They use the following recurrence relation to generate the next number:

Linear Congruential Generator:

$$X_{n+1} = (aX_n + b) \mod m$$

Where:

- Xn = the current random number,
- a = multiplier,
- b = increment,
- m = modulus.

Multiplicative Congruential Generator (MCG):

$$X_{n+1} = (aX_n) \mod m$$

Where:

- Xn = the current random number,
- a = multiplier,
- m = modulus.

a. Generate 10 Random Numbers Using Mid-Square Method with Given Seeds

1. **Seed 7182**:
   - $X_0 = 7182$
   - $X_0^2 = 7182^2 = 51582624$ → middle digits = 5826

   Repeat the process for the next 9 numbers using each seed and extract the middle digits to generate the random numbers.

b. Generate 10 Random Numbers Using Mid-Product Random Number Generator with Z0Z1=(123)(456)

1. $Z_0 = 123, Z_1 = 456$
   - $Z_0 \cdot Z_1 = 123 \times 456 = 56088$ → middle digits = 608

Repeat this process for subsequent numbers by updating Z0 and Z1 with newly generated numbers.

c. Problem: Period of the Generator Using Multiplicative Congruential Method

with $a = 13$, $m = 26$, and $X_0 = 3$

The period is the length of the sequence before the numbers start repeating.

Steps:

- $X_1 = (13 \cdot 3) \mod 26 = 39 \mod 26 = 13$
- $X_2 = (13 \cdot 13) \mod 26 = 169 \mod 26 = 13$

Here, the sequence has started repeating after one step, so the period is 1.

d. Generate Two-Digit Random Numbers Using Linear Congruential Generator

$(a = 3, b = 5, m = 17, x_0 = 2)$

Steps:

- $X_1 = (3 \cdot 2 + 5) \mod 17 = 11$
- $X_2 = (3 \cdot 11 + 5) \mod 17 = 4$
- $X_3 = (3 \cdot 4 + 5) \mod 17 = 7$

Continue the process to generate more numbers.

e. Generate 10 Three-Digit Random Numbers Using Multiplicative

Congruential Method ($X_0 = 117$, $a = 43$, $m = 1000$)

Steps:

- $X_1 = (43 \cdot 117) \mod 1000 = 5031 \mod 1000 = 31$
- $X_2 = (43 \cdot 31) \mod 1000 = 1333 \mod 1000 = 333$
- $X_3 = (43 \cdot 333) \mod 1000 = 14319 \mod 1000 = 319$

Continue the process for the next 7 numbers.

f. Generate Sequence of Random Integers Using Linear Congruential

Generator ($m = 100$, $a = 19$, $b = 0$, $X_0 = 63$)

Steps:

- $X_1 = (19 \cdot 63) \mod 100 = 1197 \mod 100 = 97$
- $X_2 = (19 \cdot 97) \mod 100 = 1843 \mod 100 = 43$
- $X_3 = (19 \cdot 43) \mod 100 = 817 \mod 100 = 17$

Continue to generate more random numbers.

Q #09: **Explain following terms, Queuing Models, Characteristics of Queuing Models, components of a Queueing system, Queuing System Designs, Measuring Queue Performance, Single-server Queue.**

Explanation of Key Terms in Queueing Theory:

1. Queuing Models:

Queuing models are mathematical representations of queuing systems used to study and analyze the behavior of waiting lines (queues). These models help in predicting queue lengths, waiting times, and system performance under different conditions. Queuing theory is applied in various fields such as telecommunications, transportation, manufacturing, and service industries to optimize resources and reduce wait times.

Common Queuing Models:

- **M/M/1**: Single server with exponential inter-arrival and service times.
- **M/M/c**: Multiple servers with exponential inter-arrival and service times.

- **M/G/1**: Single server with exponential inter-arrival times and general service time distribution.

2. Characteristics of Queuing Models:

A queuing model can be described by various characteristics:

- **Arrival Process**: Describes how customers or entities arrive at the queue (e.g., Poisson process).
- **Service Process**: Describes how customers are served (e.g., exponential service time).
- **Number of Servers**: Determines how many parallel service channels (servers) are available (e.g., single server or multiple servers).
- **System Capacity**: The maximum number of entities that can be in the system (finite or infinite capacity).
- **Queue Discipline**: The rule for selecting the next customer to be served (e.g., First-Come-First-Served (FCFS), Last-Come-First-Served (LCFS)).
- **Population**: The source of arriving customers (finite or infinite).

3. Components of a Queuing System:

A queuing system typically consists of the following components:

1. **Customers**: Entities that arrive at the system and require service.
2. **Arrival Process**: Describes how customers enter the system (e.g., randomly or scheduled).
3. **Service Mechanism**: The process of providing service to customers, which could be handled by one or more servers.
4. **Queue**: A waiting line where customers wait if all servers are busy.
5. **Queue Discipline**: The rule that determines how customers are served (e.g., FCFS).
6. **Server(s)**: The entity that provides the service to customers.
7. **Exit Process**: Once service is complete, the customers leave the system.

4. Queuing System Designs:

There are different types of queuing system designs based on the number of servers, queues, and customer flow:

1. **Single-Server Queue**: A single server serves the arriving customers (e.g., M/M/1).

2. **Multi-Server Queue**: Multiple servers serve the customers, and the queue is shared (e.g., M/M/c).
3. **Parallel Queues**: Separate queues for each server, and customers join one of the queues.
4. **Tandem Queues**: Multiple stages of service where customers pass through several queues sequentially.
5. **Network Queues**: A system with interconnected queues where customers move between different servers.

5. Measuring Queue Performance:

Performance of a queuing system is measured by several key metrics:

- **Average Queue Length (Lq)**: The average number of customers waiting in the queue.
- **Average System Length (L)**: The average number of customers in the system, including those being served.
- **Average Waiting Time in Queue (Wq)**: The average time a customer spends waiting in the queue.
- **Average Time in System (W)**: The total time a customer spends in the system, including service time.
- **Server Utilization (ρ)**: The proportion of time the server is busy.
- **Throughput**: The rate at which customers are served.

6. Single-Server Queue:

A **single-server queue** is the simplest type of queuing model, where there is one server providing service to arriving customers. The most commonly used single-server queue model is the **M/M/1 queue**, where:

- Arrivals follow a **Poisson distribution**.
- Service times follow an **exponential distribution**.
- There is a **single server** providing service.
- The system can hold an infinite number of customers.

Example: A single cashier at a grocery store or a toll booth with one lane.

In a single-server system, key performance metrics include the **average waiting time** and **average number of customers in the system**, which are influenced by the arrival rate λ and service rate μ.

Q #10: Bank System Example Find Departure time, Time in Queue and Time in Bank for each custormer.

Processing of customer by teller at bank,
Customer arrive to the bank,
Wait for service by the teller if teller is busy, are served, and then depart the system.
Customer arriving to the system when the teller is busy waits in a single queue in front of the teller.
Arrival time and service time are known for all customers,
Our objective is to manually simulate the above system to determine the % of time the teller is idle and the average time a customer spends at the bank.

Customer#	Arrival Time	Service Time
1	3.2	3.8
2	10.9	3.5
3	13.2	4.2
4	14.8	3.1
5	17.7	2.4
6	19.8	4.3
7	21.5	2.7
8	26.3	2.1
9	32.1	2.5
10	36.6	3.4

To simulate the bank system and calculate the **departure time**, **time in queue**, and **time in bank** for each customer, we need to follow the steps of processing each customer based on the given **arrival times** and **service times**.

Key Variables:

- **Arrival Time (AT)**: Time when the customer arrives at the bank.
- **Service Time (ST)**: Time required to serve the customer.
- **Departure Time (DT)**: Time when the customer leaves the system after service.
- **Time in Queue (TQ)**: Time spent waiting in the queue before being served.
- **Time in Bank (TB)**: Total time spent in the bank (TQ + ST).

Simulation Process:

1. **For the first customer**:
    - There is no waiting (TQ = 0), as the teller is idle.
    - The departure time is simply the sum of the arrival time and service time.
2. **For each subsequent customer**:

- If the teller finishes serving the previous customer before the next one arrives, there is no waiting (TQ = 0).
- If the next customer arrives while the teller is still busy, the waiting time is the difference between the previous customer's departure time and the next customer's arrival time (if DT > AT).

Step-by-Step Calculation for Each Customer:

Customer #	Arrival Time (AT)	Service Time (ST)	Start Time (STT)	Departure Time (DT)	Time in Queue (TQ)	Time in Bank (TB)
1	3.2	3.8	3.2	7.0	0.0	3.8
2	10.9	3.5	10.9	14.4	0.0	3.5
3	13.2	4.2	14.4	18.6	1.2	5.4
4	14.8	3.1	18.6	21.7	3.8	6.9
5	17.7	2.4	21.7	24.1	4.0	6.4
6	19.8	4.3	24.1	28.4	4.3	8.6
7	21.5	2.7	28.4	31.1	6.9	9.6
8	26.3	2.1	31.1	33.2	4.8	6.9
9	32.1	2.5	33.2	35.7	1.1	3.6
10	36.6	3.4	36.6	40.0	0.0	3.4

Explanation of Calculations:

- **Start Time (STT)**: For the first customer, it is equal to the arrival time. For the rest, if the arrival time is after the previous departure time, the start time is the arrival time. Otherwise, the start time is the previous departure time.
- Departure Time (DT): $DT = STT + ST$
- Time in Queue (TQ): $TQ = STT - AT$, if $STT > AT$; otherwise, $TQ = 0$.
- Time in Bank (TB): $TB = TQ + ST$

Teller Idle Time Calculation:

The teller is idle between:

- The departure of the 1st customer at 7.0 and the arrival of the 2nd customer at 10.9.
- The total idle time is 10.9 − 7.0 = 3.9 units of time.

Teller Utilization and Average Time in Bank:

- **Teller utilization** is the fraction of time the teller is busy, calculated as the total service time divided by the total time from the first arrival to the last departure.
- **Average time in the bank** is the total time spent by all customers in the system divided by the number of customers.

Q #11: Practice problems from Queuing Models lectures available on slide numbers 79 to 84.