20.02.2019

GTU Computer Science and Engineering
CSE 222/505 – Spring 2019
Homework #01
Due Date: 28.02.2019 17:00

1. Given any two functions f(n) and g(n), show that f(n) + g(n) = Θ(max{f(n),g(n)}) (10P).

2. Show that f(n) =n^2+ 2n+ 1 is Θ(n^2) using induction. (If you use l'Hopital, you will lose points.)(5P).

3. Prove the functions below (40P).
   a) If f(n) = 10 log(n) + 5 $(\log(n))^3$ + 7n + 3n^2 + 6n^3, then f(n) = O(n^3) (5P)
   b) 1 = O(n) (5P)
   c) n = O(n^2) (5P)
   d) log(n) = O(n), 2 n + 1 = O(n) (5P)
   e) n = Ω(1) (5P)
   f) n^2 = Ω(n) (5P)
   g) n^2 = Ω(n log(n)) (5P)
   h) 2 n + 1 = Θ(n) (5P)

4. Sort the following functions from fastest to slowest with respect to their growth rate. Do not use l'Hopital! Prove all of them using induction (20P).

   n!, n^{k+n} , n, logn, n(logn), e^7, 2019, -7^{n+m}, n^4, 100*n

   *k and m are constants.

5. Explain the time complexity of the code snippets below (10P).

```
a-)System.out.println = SOP
void method4(int [] arr) {
   for(int i = 0; i < arr.length; i++) {
        for(int k = arr.length - 1; k > 0; k = k / 3 ) {
                  SOP(arr[i]);
        }
   }
}

b-)
void method3(int [] arr)
{
     for(int i = 0; i < arr.length; i++)
     {
          method1(arr);
          method2(arr);
     }
```

```
}
void method2(int [] arr)
{
      for(int i = 0; i <arr.length; i++)
      {
            for(int k = 0; k <arr.length; ++k)
            {
                  SOP (Math.log(arr[i]));
            }
      }
}
```

```
void method1(int [] arr)
{
     int n = arr.length;

     for(int i = n - 1 ; i >= 0; i = i - 3)
     {
          SOP (arr[i]);
     }
}
```

6. Calculate the time complexity of the following recurrence functions (Use the master theorem)(10P)
   - $T(n) = T(n/7) + n^4$
   - $T(n) = T(n/99) + n^{75}$
   - $T(n) = 2^3T(n/12) + 6$

7. Write mergesort with pseudo-code and analyze the algorithm’s worst case, best case and average case using asymptotic notations(15P).

- **PS:**
- If you have any questions about the hw, please send an email to ogoksu@gtu.edu.tr
- Your submission should be handwritten. Do not send any files as .pdf, .pptx etc.
- You should hand over your submission to (Office No: 109) Özgü Göksu, before the due date.

Good Luck!