CSE 321 - HW 1 151044084

Muhammed ~~...~~ ÖZKAN

1

1) a) $\lim_{n\to\infty} \frac{\log_2 n^2+1}{n} = \lim_{n\to\infty} \frac{2\log_2 n+1}{n} = \lim_{n\to\infty} \frac{\frac{2}{n\ln 2}}{1} = 0$ ✓

it's true

$\log_2 n^2+1 \in O(n)$ ✓

b) $\lim_{n\to\infty} \frac{\sqrt{n(n+1)}}{n} = \lim_{n\to\infty} \frac{\cancel{n}\sqrt{1+\frac{1}{n}}}{\cancel{n}} = \sqrt{1} = 1$ ✓

$\sqrt{n(n+1)} \in O(n)$ and $\sqrt{n(n+1)} \in \Omega(n)$ ✓ it's true

c) $\lim_{n\to\infty} \frac{n^{n-1}}{n^n} = \lim_{n\to\infty} \frac{n^n \cdot n^{-1}}{n^n} = \lim_{n\to\infty} \frac{1}{n} = 0$ ✗

it's not true

$n^{n-1} \in O(n^n)$ ✓ not Θ(theta)

d) $\lim_{n\to\infty} \frac{2^n+n^3}{4^n} = \lim_{n\to\infty} \left(\left(\frac{2}{4}\right)^n + \frac{n^3}{4^n}\right) = 0 + \lim_{n\to\infty} \frac{n^3}{4^n} = \lim_{n\to\infty} \frac{3n^2}{4^n \ln 4}$

$= \lim_{n\to\infty} \frac{6n}{4^n \ln^2 4} = \lim_{n\to\infty} \frac{6}{4^n \ln^3 4} = \frac{1}{\infty} = 0$ ✓

$O(2^n+n^3) \subset O(4^n)$ ✓ it's true

e) $\lim_{n\to\infty} \frac{2\log_3 \sqrt[3]{n}}{3\log_2 n^2} = \lim_{n\to\infty} \frac{\frac{2}{3}\log_3 n}{6\log_2 n} = \frac{1}{9} \lim_{n\to\infty} \frac{\frac{\log n}{\log 3}}{\frac{\log n}{\log 2}} = \frac{1}{9} \frac{\log 2}{\log 3}$ ✗

$O(2\log_3 \sqrt[3]{n}) \subset O(3\log_2 n^2)$ it's not true because constant

f) $\lim_{n\to\infty} \frac{\log_2 (n^{\frac{1}{2}})}{(\log_2 n)^2} = \lim_{n\to\infty} \frac{\frac{1}{2}}{\log_2 n} = \frac{1}{\infty} = 0$ ✗ it's not true

$\log_2 \sqrt{n} \in O((\log_2 n)^2)$ ✓ not Θ(theta)

2

2)

$$-\lim_{n\to\infty}\frac{2^n}{10^n}=\lim_{n\to\infty}\left(\frac{2}{10}\right)^n=0\Rightarrow 2^n<10^n$$

$$-\lim_{n\to\infty}\frac{n^3}{2^n}=\lim_{n\to\infty}\frac{3n^2}{2^n\ln 2}=\lim_{n\to\infty}\frac{6n}{2^n\ln^2 2}=\lim_{n\to\infty}\frac{6}{2^n\ln^3 2}=0$$

$$\Rightarrow n^3<2^n$$

$\boxed{8^{\log n}=n^3}$

$$-\lim_{n\to\infty}\frac{8^{\log n}}{n^3}=\lim_{n\to\infty}\frac{n^3}{n^3}=1\Rightarrow 8^{\log n}=n^3$$

$$-\lim_{n\to\infty}\frac{n^2\log n}{8^{\log n}}=\lim_{n\to\infty}\frac{n^2\log n}{n^3}=\lim_{n\to\infty}\frac{\log n}{n}=\lim_{n\to\infty}\frac{\frac{1}{n\ln 2}}{1}=0$$

$$\Rightarrow n^2\log n<8^{\log n}$$

$$-\lim_{n\to\infty}\frac{n^2}{n^2\log n}=\lim_{n\to\infty}\frac{1}{\log n}=0\Rightarrow n^2<n^2\log n$$

$$-\lim_{n\to\infty}\frac{\sqrt{n}}{n^2}=\lim_{n\to\infty}\frac{1}{n^{3/2}}=0\Rightarrow \sqrt{n}<n^2$$

$$-\lim_{n\to\infty}\frac{\log n}{\sqrt{n}}=\lim_{n\to\infty}\frac{\frac{1}{n\ln 2}}{\frac{1}{2\sqrt{n}}}=\lim_{n\to\infty}\frac{2\sqrt{n}}{n\ln 2}=\frac{2}{\ln 2}\lim_{n\to\infty}\frac{1}{\sqrt{n}}=0$$

$$\Rightarrow \log n<\sqrt{n}$$

result

$$\boxed{10^n>2^n>n^3=8^{\log n}>n^2\log n>n^2>\sqrt{n}>\log n}$$

3)

a) when the function is examined, we see that the variable that the "for" loop depends on is i, which depends on the size of the array.
the complexity of this function is $\Theta(n)$, since the loop will run the size of array in all probability. The first element variable has the largest element of the element of the array and second element variable has the second largest element.

```
for (int i = 0; i < sizeofArray; i++)
{
    :
}
```

$$\text{sizeofArray} \downarrow n \qquad \boxed{\Theta(n)}$$

b) Time complexity of a loop is considered $O(\log\log n)$ if the loop variables is reduced/increased exponentially by a constant amount. The for loop can be modified as the following and the both loop is exactly the same with n together

```
for (int i=2; i<=n; i=(i*i)+i)   // i^2+i
    count++;
```

in this statement; the cursor(i) increases exponential as it increases i^2+i on each step. So time complexity of the code is $O(\log\log n)$

$$2,\ 2^k,\ (2^k)^k = 2^{k^2},\ (2^{k^2})^k = 2^{k^3},\ \ldots\ 2^{k^{\log_k(\log(n))}}$$

$$2^{k^{\log_k(\log(n))}} = 2^{\log(n)} = n \ \ldots \quad \log_k(\log(n)) \quad \boxed{O(\log\log n)}$$

4) a) $\sum_{i=1}^{n} i^2 \log i = f(n)$ $\quad i^2 \log i = g(i)$ is nondecreasing func.

apply $\int_0^n g(x)\,dx \le f(n) \le \int_1^{n+1} g(x)\,dx$

$\int_0^n x^2 \log x \,dx = ?$ $\quad \log x = u \quad x^2 = v'$ $\quad u.v - \int v.du$

$\frac{1}{x\ln 2}dx = du \quad \frac{x^3}{3} = v$

$$\int x^2 \log x\,dx = \log x \cdot \frac{x^3}{3} - \int \frac{x^3}{3} \cdot \frac{1}{x\ln 2} \cdot dx$$

$$= \log x \cdot \frac{x^3}{3} - \frac{1}{3\ln 2}\int x^2 dx$$

$$= \log x \cdot \frac{x^3}{3} - \frac{1}{3\ln 2} \cdot \frac{x^3}{3} + c$$

$$\int_0^n x^2 \log x\,dx = \log x \frac{x^3}{3} - \frac{1}{3\ln 2} \cdot \frac{x^3}{3} \Big|_0^n = \frac{n^3 \log n}{3} - \frac{n^3}{9\ln 2}$$

$$\int_1^{n+1} x^2 \log x\,dx = \log x \frac{x^3}{3} - \frac{1}{3\ln 2} \cdot \frac{x^3}{3} \Big|_1^{n+1}$$

$$= \frac{(n+1)^3 \cdot \log(n+1)}{3} - \frac{(n+1)^3}{9\ln 2} - 0 + \frac{1}{9\ln 2}$$

$$\frac{(n^3 \log n) \cdot 3\ln 2 - n^3}{9\ln 2} \le f(x) \le \frac{(n+1)^3 \log(n+1) \cdot 3\ln 2 - (n+1)^3 + 1}{9\ln 2}$$

$$\Rightarrow f(n) \in \Theta(n^3 \log n)$$

5

4) b) $\sum_{i=1}^{n} i^3 = f(n)$ $\quad i^3 = g(i)$ is non decreasing func.

apply $\int_0^n g(x)\,dx \leq f(n) \leq \int_1^{n+1} g(x)\,dx$

$$\int_0^n x^3\,dx = ? = \frac{x^4}{4}\Big|_0^n = \frac{n^4}{4} - 0$$

$$\int_0^{n+1} x^3\,dx = ? \ \frac{x^4}{4}\Big|_1^{n+1} = \frac{(n+1)^4}{4} - \frac{1}{4}$$

$$\frac{n^4}{4} \leq \frac{(n+1)^4 - 1}{4} \Rightarrow f(n) \in \Theta(n^4)$$

c) $\sum_{i=1}^{n} \frac{1}{2\sqrt{i}} = f(n)$ $\quad \frac{1}{2\sqrt{i}} = g(i)$ is non increasing func

apply $\int_1^{n+1} g(x)\,dx \leq f(n) \leq \int_0^n g(x)\,dx$

$$\left.\begin{aligned} &\int_1^{n+1} \frac{1}{2\sqrt{x}}\,dx = ? = \sqrt{x}\Big|_1^{n+1} = \sqrt{n+1} - 1 \\ &\int_0^n \frac{1}{2\sqrt{x}}\,dx = ? = \sqrt{x}\Big|_0^n = \sqrt{n} \end{aligned}\right\} \quad \begin{aligned} &\sqrt{n+1} - 1 \leq f(n) \leq \sqrt{n} \\ &\Rightarrow f(n) \in \Theta(\sqrt{n}) \end{aligned}$$

d) $\sum_{i=1}^{n} \frac{1}{i} = f(n)$ $\quad \frac{1}{i} = g(i)$ is non increasing func.

apply $\int_{1}^{n+1} g(x)\,dx \leq f(n) \leq \int_{0}^{n} g(x)\,dx$

$\int_{1}^{n+1} \frac{1}{x}\,dx = ? = \ln x \Big|_{1}^{n+1} = \ln(n+1) - \ln 1 \rightarrow f(n) \in \Omega(\log n)$ is lower bound

$\int_{0}^{n} \frac{1}{x}\,dx = ? = \ln x \Big|_{0}^{n} = \ln(n) - \underbrace{\ln(0)}_{-\infty} \longrightarrow$ There is no upper bound

different way

$f(n) = \sum_{n=1}^{i} \frac{1}{i} = 1 + \sum_{i=2}^{n} \frac{1}{i} \Rightarrow f(n) \leq 1 + \int_{1}^{n} g(x)\,dx$

$f(n) \leq 1 + \int_{1}^{n} \frac{1}{x}\,dx = 1 + \ln x \Big|_{1}^{n}$

$f(n) \leq 1 + \ln(n) \Rightarrow f(n) \in O(\log n)$ is upper bound

$f(n) \in \Omega(\log n)$ is lower bound
$f(n) \in O(\log n)$ is upper bound

also

$f(n) \in \Theta(\log n)$

5)

pseudocode

```
function linearSearch ( L[1:n], x )    a list with element / searched element
    for i=1 to n do
        if (L[i]=x) then
            return(i)
        end if
    end for
    return 0
end
```

Best case: if $x = L[1]$, then best case occures

$$B(n) = 1 \in O(1)$$

Worst case: if $x = L[n]$, then worst case occures

$$W(n) = n \in O(n)$$

A comparison based searching algorithm that is based on making comparisions. Involving list elements and then making decisions based on the comparisons. whether the elements in the list are repeated or not does not affect the complexity. Because the linear search algorithm terminates the search where it first finds the element it looking for. Even if, it does not, this does not affect the best and worst complexity.