**Assignment 3: Advanced computational fluid dynamics (ME670)-2024**

Department of Mechanical Engineering, Indian Institute of Technology Guwahati

- Provide: (i) the grid detail, (ii) the discretized equations detail, (iii) the boundary condition implementation detail, (iv) a well-documented code, (v) the required output (plots/any other such means).
- Items (i), (ii), and (iii) above should be written out/typed on a separate sheet and attached before items (iv) and (v).

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**1**. Consider 2D conduction problem governed by equation

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} = 0$$

On a unit square domain with homogamous Dirichlet boundary conditions i.e. $T = 0$ at the left, right and bottom boundaries. The top wall (non-dimensional) temperature is 1 unit. The analytical solution of the problem is

$$T(x, y) = \frac{2}{\pi} \sum_{n=1}^{\infty} \frac{(-1)^{n+1} + 1}{n} \sin(n\pi x) \frac{\sinh(n\pi y)}{\sinh(n\pi)}$$

Discretize the governing equation using finite volume method. The FV mesh is uniform in both the directions with $N_i = N_j = 128$ finite volumes in $x$- and $y$-directions, respectively. The FV cells are numbered in lexicographic ordering by lines of constant $i$. Show the final expression of the discrete set of equations in the form $Ax = b$.

Solve the system of equations using conjugate gradient method, preconditioned conjugate gradient method with Jacobi, ILU and SIP preconditioners. Use residual 2-norm falling below $10^{-6}$ as convergence criteria.

(a) Compare the iterations vs residual 2-norm plot for all four iterative methods.
(b) Compare the temperature contours of the analytical and numerical solutions obtained from iterative methods.
(c) Compare the temperature variation with *y* along mid-vertical plane *x* = 0.5 obtained from the analytical and numerical solutions.
(d) Compare the temperature variation with *x* along mid-vertical plane *y* = 0.5 obtained from the analytical and numerical solutions.
(e) For $N_i = N_j = 4$, tabulate the values of diagonals of $L$ and $U$ matrices coming from ILU and SIP factorization.

| $n$ | $L_W^n$ | $L_S^n$ | $L_P^n$ | $U_N^n$ | $U_E^n$ |
|---|---|---|---|---|---|
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Coding guidelines/tips:

- Refer to assignment 1.