

EPM500 İLERİ MÜHENDİSLİK MATEMATİĞİ

ÖDEV -2 SORU ve ÇÖZÜMLERİ

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İleri Mühendislik Matematiği - Ödev 2

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Soru-1

$$2x + 3y - z = 5$$

$$3x + 2y + z = 10$$

$$x - 5y + 3z = 0$$

Yukarıda verilen denklem sistemini Cramer kuralı ile çözün.

Cözüm-1

x, y ve z 'yi bulmak için Cramer kuralını kullanırız.

$$\frac{x}{D_x} = \frac{-y}{D_y} = \frac{z}{D_z} = \frac{-1}{D}$$

$$D_x = \begin{vmatrix} 3 & -1 & -5 \\ 2 & 1 & -10 \\ -5 & 3 & 0 \end{vmatrix} = 3 \times \begin{vmatrix} 1 & -10 \\ 3 & 0 \end{vmatrix} + 1 \times \begin{vmatrix} 2 & -10 \\ -5 & 0 \end{vmatrix} - 5 \times \begin{vmatrix} 2 & 1 \\ -5 & 0 \end{vmatrix}$$

$$= 3 \cdot (1 \cdot 0 - (-10) \cdot 3) + 1 \cdot (2 \cdot 0 - (-10) \cdot (-5)) - 5 \cdot (2 \cdot 3 - 1 \cdot (-5))$$

$$= 3 \cdot (0 + 30) + 1 \cdot (0 - 50) - 5 \cdot (6 + 5)$$

$$= 3 \cdot 30 + 1 \cdot (-50) - 5 \cdot 11$$

$$= 90 - 50 - 55$$

$$= -15$$

Görem - 1

$$\begin{aligned}D_y &= \begin{vmatrix} 2 & -1 & -5 \\ 3 & 1 & -10 \\ 1 & 3 & 0 \end{vmatrix} = 2 \cdot \begin{vmatrix} 1 & -10 \\ 3 & 0 \end{vmatrix} + 1 \cdot \begin{vmatrix} 3 & -10 \\ 1 & 0 \end{vmatrix} - 5 \cdot \begin{vmatrix} 3 & 1 \\ 1 & 3 \end{vmatrix} \\&= 2 \cdot (1 \cdot 0 - (-10) \cdot 3) + 1 \cdot (3 \cdot 0 - (-10) \cdot 1) - 5 \cdot (3 \cdot 3 - 1 \cdot 1) \\&= 2 \cdot (0 + 30) + 1 \cdot (0 + 10) - 5 \cdot (9 - 1) \\&= 2 \cdot (30) + 1 \cdot (10) - 5 \cdot (8) \\&= 60 + 10 - 40 \\&= 30\end{aligned}$$

$$\begin{aligned}D_2 &= \begin{vmatrix} 2 & 3 & -5 \\ 3 & 2 & -10 \\ 1 & -5 & 0 \end{vmatrix} = 2 \cdot \begin{vmatrix} 2 & -10 \\ -5 & 0 \end{vmatrix} - 3 \cdot \begin{vmatrix} 3 & -10 \\ 1 & 0 \end{vmatrix} - 5 \cdot \begin{vmatrix} 3 & 2 \\ 1 & -5 \end{vmatrix} \\&= 2 \cdot (2 \cdot 0 - (-10) \cdot (-5)) - 3 \cdot (3 \cdot 0 - (-10) \cdot 1) - 5 \cdot (3 \cdot (-5) - 2 \cdot 1) \\&= 2 \cdot (0 - 50) - 3 \cdot (0 + 10) - 5 \cdot (-15 - 2) \\&= 2 \cdot (-50) - 3 \cdot (10) - 5 \cdot (-17) \\&= -100 - 30 + 85 \\&= -45\end{aligned}$$

$$D = \begin{vmatrix} 2 & 3 & -1 \\ 3 & 2 & 1 \\ 1 & -5 & 3 \end{vmatrix} = 2 \begin{vmatrix} 2 & 1 \\ -5 & 3 \end{vmatrix} - 3 \cdot \begin{vmatrix} 3 & 1 \\ 1 & 3 \end{vmatrix} - 1 \cdot \begin{vmatrix} 3 & 2 \\ 1 & -5 \end{vmatrix}$$

Götz - 1

$$\begin{aligned} &= 2 \cdot (2, 3-1, (-5)) - 3 \cdot (3, 3-1, 1) - 1 \cdot (3, (-5)-2, 1) \\ &= 2(6+5) - 3 \cdot (9-1) - 1 \cdot (-15-2) \\ &= 2 \cdot (11) - 3 \cdot (8) - 1 \cdot (-17) \\ &= 22 - 24 + 17 \\ &= 15 \end{aligned}$$

$$\frac{x}{D_x} = \frac{-y}{D_y} = \frac{z}{D_z} = \frac{-1}{D}$$

$$\frac{x}{-15} = \frac{-y}{30} = \frac{z}{-45} = \frac{-1}{15}$$

$$\frac{x}{-15} = \frac{-1}{15} \quad y \cdot x = \frac{15}{15} = 1$$

$$-\frac{y}{30} = -\frac{1}{15} \quad y = \frac{30}{15} = 2$$

$$\frac{z}{-45} = \frac{-1}{15} \rightarrow z = \frac{45}{15} = 3$$

$$x = 1, y = 2, z = 3$$

Soru - 2

$y'' - 5y' + 4y = 0$, $y(0) = 5$, $y'(0) = -4$ başlangıç değer problemini çözünüz.

G 520M-2

$$y^{\text{genel}}_g = y_h \rightarrow \text{homogen} \\ \Rightarrow r^2 - 5r + 4 = 0 \quad (r-4), (r-1) = 0 \\ \begin{matrix} r & -4 \\ r & -1 \end{matrix} \quad r = 4, r = 1$$

$$y = c_1 \cdot e^{4x} + c_2 \cdot e^x \rightarrow \begin{cases} c_1 + c_2 = 5 \\ 4c_1 + c_2 = -4 \end{cases} \rightarrow \begin{cases} c_1 = -3 \\ c_2 = 8 \end{cases}$$

$$y = -3 \cdot e^{6x} + 8e^x$$

Soru-3

$$x + 4y + 5z = 0$$

$$2x + 2y + z = 5$$

$$x - 5y + 4z = 7$$

Yukarıda verilen denklem sisteminin Gauss elimesi ile çözün.
Gözlemler-3

Genişletilmiş matrisi indirgenmiş eşeler matrise çevirnej

$$\left(\begin{array}{ccc|c} 1 & 4 & 5 & 0 \\ 2 & 2 & 1 & 5 \\ 1 & -5 & 4 & 7 \end{array} \right) \xrightarrow{\cdot(-2)} R_2 - 2 \cdot R_1 \rightarrow R_2$$

$$\left(\begin{array}{ccc|c} 1 & 4 & 5 & 0 \\ 0 & -6 & -9 & 5 \\ 1 & -5 & 4 & 7 \end{array} \right) \xrightarrow{\cdot(-1)} R_3 - R_1 \rightarrow R_3$$

$$\left(\begin{array}{ccc|c} 1 & 4 & 5 & 0 \\ 0 & -6 & -9 & 5 \\ 0 & -9 & -1 & 7 \end{array} \right) \xrightarrow{\cdot\left(\frac{-1}{3}\right)} R_3 - \left(\frac{3}{2}\right) \cdot R_2 \rightarrow R_3$$

$$\left(\begin{array}{ccc|c} 1 & 4 & 5 & 0 \\ 0 & -6 & -9 & 5 \\ 0 & 0 & \frac{25}{2} & -\frac{1}{2} \end{array} \right)$$

Gözüm-3

$$x + 4y + 5z = 0$$

$$-6y - 9z = 5$$

$$\frac{25}{2} z = -\frac{1}{2}$$

z deðiðkenini bulmak icin;

$$\frac{25}{2} z = -\frac{1}{2}$$

$$z = -\frac{1}{25}$$

y deðiðkenini bulmak icin;

$$-6y = 5 + 9z = 5 + 9 \cdot \left(-\frac{1}{25}\right) = \frac{116}{25} \Rightarrow y = -\frac{58}{75}$$

x deðiðkenini bulmak icin;

$$x = -4y - 5z = -4 \cdot \left(-\frac{58}{75}\right) - 5 \cdot \left(-\frac{1}{25}\right) = \frac{247}{75}$$

$$x = \frac{247}{75}, \quad y = -\frac{58}{75}, \quad z = -\frac{1}{25}$$