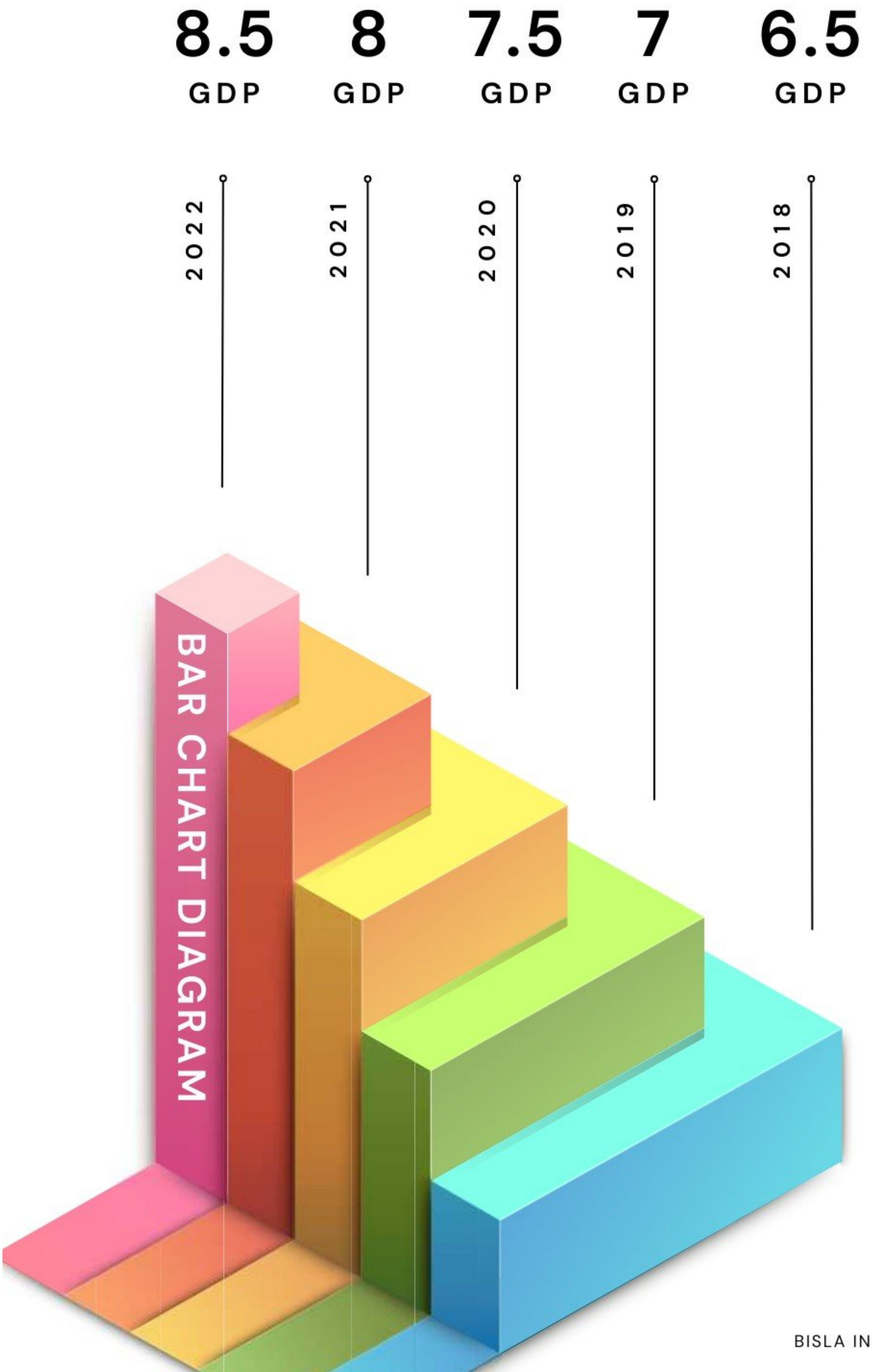
STATISTICS

MEAN, MEDIAN, MODE



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MEAN

There are several kinds of mean in mathematics, especially in statistics. Each mean serves to summarize a given group of data, often to better understand the overall value (magnitude and sign) of a given data set.

TYPES OF MEAN

There are three 03 types of mean:

- Arithmetic Mean
- Geometric Mean
- Harmonic Mean

ARITHMETIC MEAN

The arithmetic mean (or simply *mean*) of a list of numbers, is the sum of all of the numbers divided by the number of numbers. Similarly, the mean of a sample $x_1, x_2, \ldots, x_n$, usually denoted by $\bar{x}$, is the sum of the sampled values divided by the number of items in the sample.

$$\bar{x} = \frac{1}{n}\left(\sum_{i=1}^{n} x_i\right) = \frac{x_1 + x_2 + \cdots + x_n}{n}$$

For example, the arithmetic mean of five values: 4, 36, 45, 50, 75 is:

$$\frac{4 + 36 + 45 + 50 + 75}{5} = \frac{210}{5} = 42.$$

GEOMETRIC MEAN

The geometric mean is an average that is useful for sets of positive numbers, that are interpreted according to their product (as is the case with rates of growth) and not their sum (as is the case with the arithmetic mean):

$$\bar{x} = \left(\prod_{i=1}^{n} x_i\right)^{\frac{1}{n}} = (x_1 x_2 \cdots x_n)^{\frac{1}{n}}$$ [2]

For example, the geometric mean of five values: 4, 36, 45, 50, 75 is:

$$(4 \times 36 \times 45 \times 50 \times 75)^{\frac{1}{5}} = \sqrt[5]{24\,300\,000}$$

MARMONIC MEAN

The harmonic mean is an average which is useful for sets of numbers which are defined in relation to some unit, as in the case of speed (i.e., distance per unit of time):

$$\bar{x} = n\left(\sum_{i=1}^{n} \frac{1}{x_i}\right)^{-1}$$

For example, the harmonic mean of the five values: 4, 36, 45, 50, 75 is

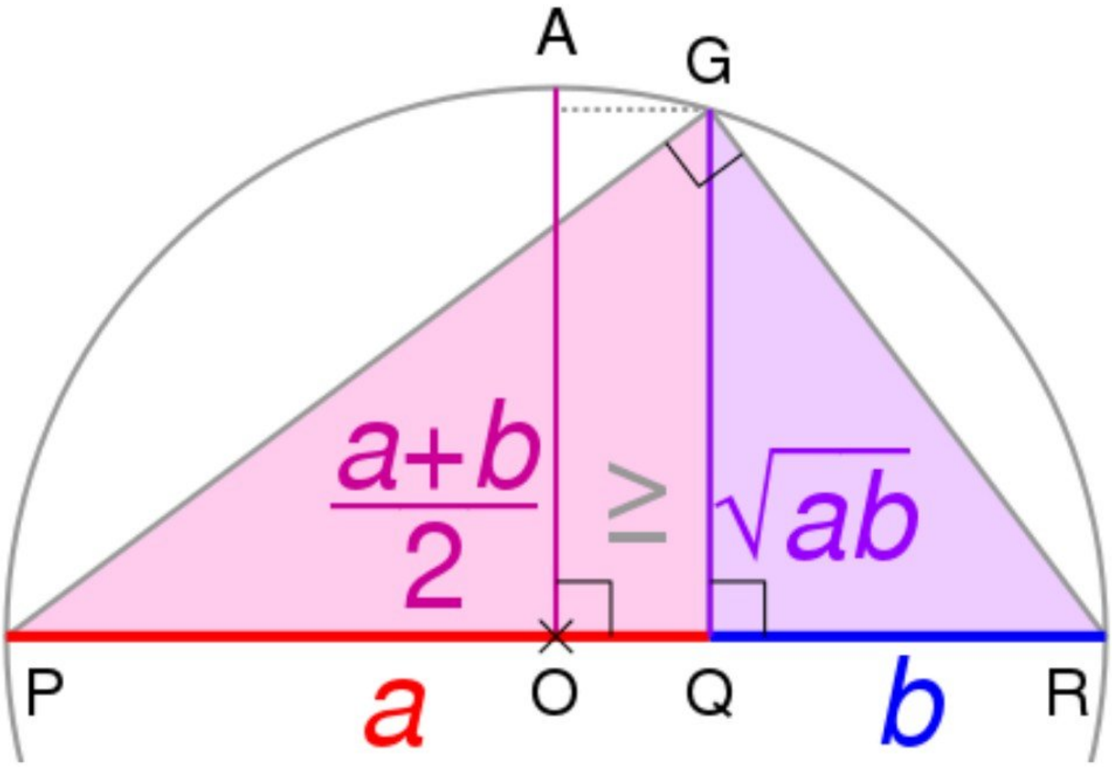
$$\frac{5}{\frac{1}{4} + \frac{1}{36} + \frac{1}{45} + \frac{1}{50} + \frac{1}{75}} = \frac{5}{\frac{1}{3}} = 15.$$

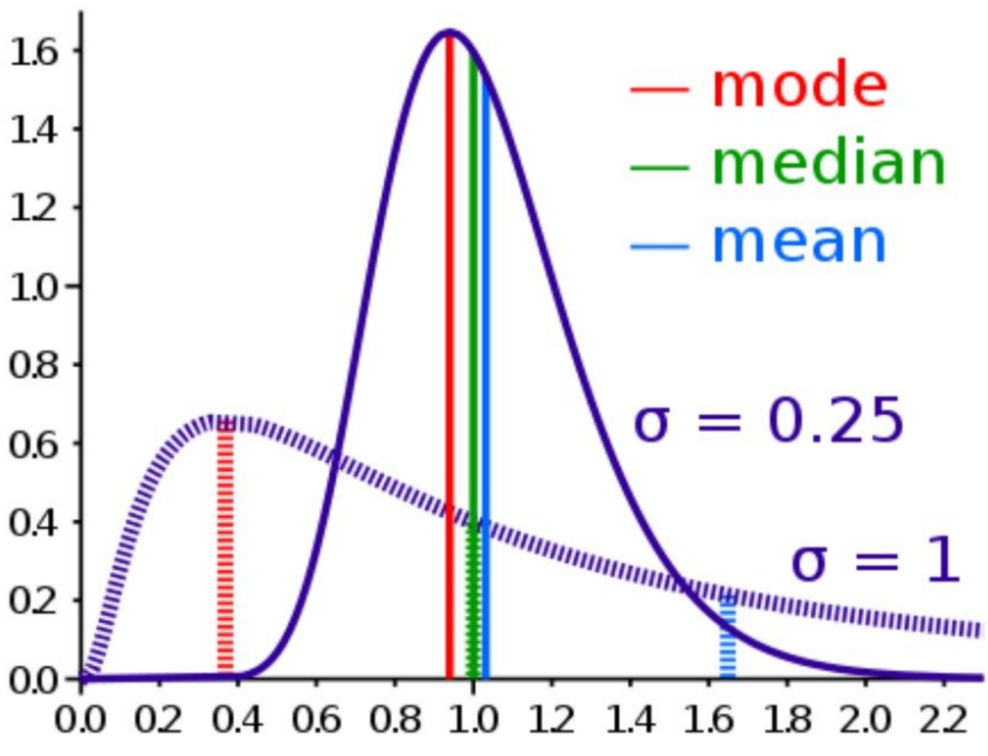
RELATIONSHIP BETWEEN MEANS

AM, GM, and HM satisfy these inequalities:

$$\mathrm{AM} \geq \mathrm{GM} \geq \mathrm{HM}$$

Equality holds if all the elements of the given sample are equal.





MEDIAN

In statistics and probability theory, the median is the value separating the higher half from the lower half of a data sample, a population, or a probability distribution.

For a data set, it may be thought of as "the middle" value. The basic feature of the median in describing data compared to the mean (often simply described as the "average") is that it is not skewed by a small proportion of extremely large or small values, and therefore provides a better representation of the center.

1, 3, 3, **6**, 7, 8, 9

Median = 6

1, 2, 3, **4**, **5**, 6, 8, 9

Median = (4 + 5) ÷ 2

= 4.5

Finding the median in sets of data with an odd and even number of values

In general, with this convention, the median can be defined as follows: For a data set x of n elements, ordered from smallest to greatest,

if n is odd, $\operatorname{median}(x) = x_{(n+1)/2}$

if n is even,

$$\operatorname{median}(x) = \frac{x_{(n/2)} + x_{((n/2)+1)}}{2}$$

Type	Description	Example	Result
Midrange	Midway point between the minimum and the maximum of a data set	**1**, 2, 2, 3, 4, 7, **9**	**5**
Arithmetic mean	Sum of values of a data set divided by number of values: $\bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i$	(1 + 2 + 2 + 3 + 4 + 7 + 9) / 7	**4**
Median	Middle value separating the greater and lesser halves of a data set	1, 2, 2, **3**, 4, 7, 9	**3**
Mode	Most frequent value in a data set	1, **2**, **2**, 3, 4, 7, 9	**2**

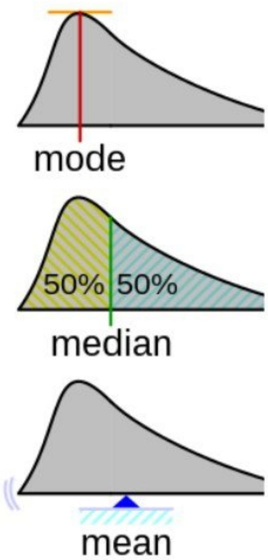
MODE

The mode is the value that appears most often in a set of data values. If X is a discrete random variable, the mode is the value x at which the probability mass function takes its maximum value.

The mode of a sample is the element that occurs most often in the collection. For example, the mode of the sample [1, 3, 6, 6, 6, 6, 7, 7, 12, 12, 17] is 6. Given the list of data [1, 1, 2, 4, 4] its mode is not unique. A dataset, in such a case, is said to be bimodal, while a set with more than two modes may be described as multimodal.

Comparison of common averages of values { 1, 2, 2, 3, 4, 7, 9 }

Type	Description	Example	Result
Arithmetic mean	*Sum of values of a data set divided by number of values*	(1+2+2+3+4+7+9) / 7	**4**
Median	*Middle value separating the greater and lesser halves of a data set*	1, 2, 2, **3**, 4, 7, 9	**3**
Mode	*Most frequent value in a data set*	1, **2**, **2**, 3, 4, 7, 9	**2**



$$\begin{aligned} \text{mean} &= e^{\mu+\sigma^2/2} && = e^{0+1^2/2} && \approx 1.649 \\ \text{mode} &= e^{\mu-\sigma^2} && = e^{0-1^2} && \approx 0.368 \\ \text{median} &= e^{\mu} && = e^{0} && = 1 \end{aligned}$$

$$\begin{aligned} \text{mean} &= e^{\mu+\sigma^2/2} && = e^{0+0.25^2/2} && \approx 1.032 \\ \text{mode} &= e^{\mu-\sigma^2} && = e^{0-0.25^2} && \approx 0.939 \\ \text{median} &= e^{\mu} && = e^{0} && = 1 \end{aligned}$$

THANK
YOU