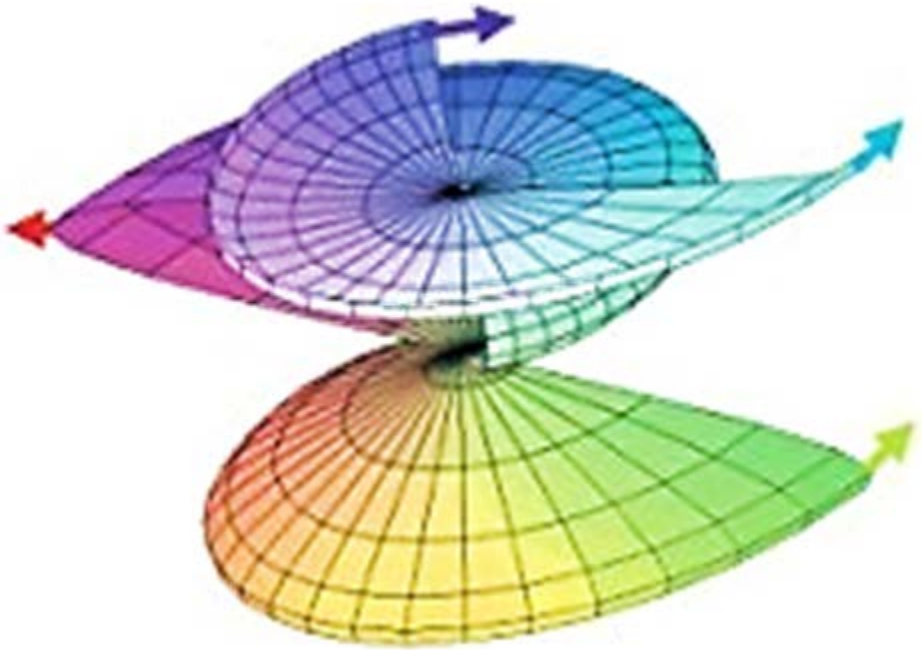
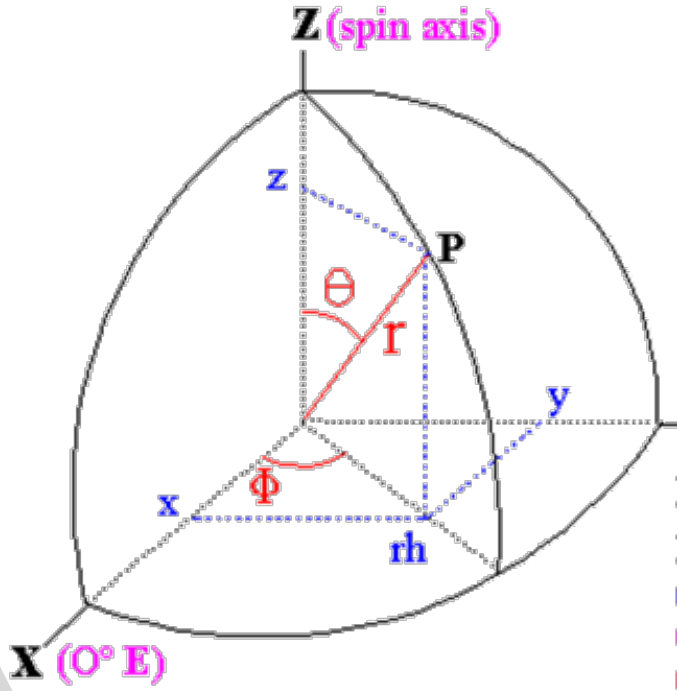
Vector Calculus

MATH- 243





$rh = r \sin(\Theta)$
$x = r \sin(\Theta) \cos(\Phi)$
$y = r \sin(\Theta) \sin(\Phi)$
$z = r \cos(\Theta)$
$dv = dx\ dy\ dz$
$dv = r^2 \sin(\Theta)\ dr\ d(\Theta)d(\Phi)$

In various coordinates, point P is at:

(x, y, z) - cartesian
(λ,Φ) - latitude, longitude
(r, Θ, Φ) - spherical

Instructor: Dr. Naila Amir

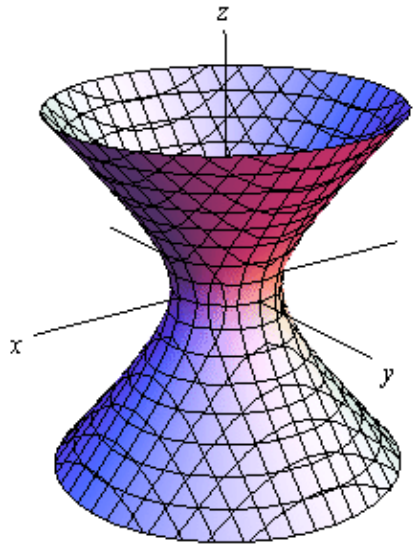
What is Vector Calculus??

And the Answer is……

- Multivariable calculus (also known as multivariate calculus) is the extension of calculus in one variable to calculus in more than one variables: the differentiation and integration of functions involving multiple variables, rather than just one.

- Vector calculus is a field of mathematics within multivariable calculus. It is concerned with multivariate real analysis of vectors in an inner product space of two or more dimensions. It deals with scalar fields, which associate a scalar to every point in space, and vector fields, which associate a vector to every point in space.

- Vector Calculus is used extensively in physics and engineering, especially in the description of electromagnetic fields, gravitational fields and fluid flow.



Vector Calculus

- The course of vector calculus is a course that is very much related to our daily-life experiences and observations. A solid object carries a ***representation*** of a multivariable function. Motions of these objects under the influence of gravitational, electromagnetic, wind and pressure fields are primary examples of vector calculus.

- It would be crucial to have clear and better understanding of the concepts in vector calculus before one could apply them to ***set-up an engineering design*** which is a combination of 3D objects. Needless to say, research at advanced level also *rely* on the understanding of basic concepts in vector calculus.

Vector Calculus

- In the subsequent slide a list of advanced courses (elective and non-elective) is given where vector calculus is a pre-requisite. Although vector calculus is ***only*** a pre-requisite of ***Complex Variables and Transform*** (CVT) at SEECS but at MIT and elsewhere it is a pre-requisite of many advanced courses. In short vector calculus is a pre-requisite of all courses for which CVT is a pre-requisite.

Vector Calculus

Complex Variables and Transform

Non-Electives

- Electromagnetic Field Theory → Microwave Engineering; Transmission Lines & Waveguides
- Communication Systems → Communication Systems II; Mobile Communication Systems
- Electrical Network Analysis → Electrical Machines

Electives

Main Topics

Analytical Geometry in 3-space	Quadratic Surfaces	Cylindrical and Spherical coordinates	Parametric representation of curves, Arc length Curvature & Torsion
Gradient of a Scalar Field and directional derivatives	Divergence of a Vector Field.	Curl of a Vector Field.	Line integral, integration around closed curves.
Application of double integrals, Green's theorem.	Surface Integrals.	Triple integrals, Divergence theorem of Gauss.	Stokes's theorem.
Partial differential equations solvable as ODEs (separation of variables)	Modeling a Vibrating String, Derivation of Wave Equation	Solution by the Method of Separation of Variables using Fourier Series.	Heat Equation; its Solution by Fourier Series.

Course Objectives:

- The objective is to develop understanding of vector valued functions, partial differential equations and multiple integrals. The applications will be covered from several engineering problems.
- The other objective is to learn basic vector differential operators, gradient, divergence and curl along with their applications to calculate surface integrals, flows and flux across surfaces.
- The understanding of partial differential equations is developed which is a strong tool for various mathematical models. Objective is to learn solution techniques of partial differential equations.

Course Learning Outcomes (CLOs)

Course Learning Outcomes (CLOs):	PLO	BT Level*
[CLO - 1] Interpret the consequences of del (nabla) operator on scalar and vector fields.	2	C-6
[CLO - 2] Solve line- and surface integrals directly or by using known integrals theorems.	2	C-3
[CLO - 3] Develop analytical solutions of partial differential equations.	3	C-5
* BT= Bloom's Taxonomy, C=Cognitive domain, P=Psychomotor domain, A= Affective domain		

Assessments

- Quizzes (FTF)
- Assignments (DL/FTF)
- One Hour Tests (FTF)
- ESE (FTF)
- Class Participation (DL/FTF)
- Communication/Presentation Skills (DL/FTF)

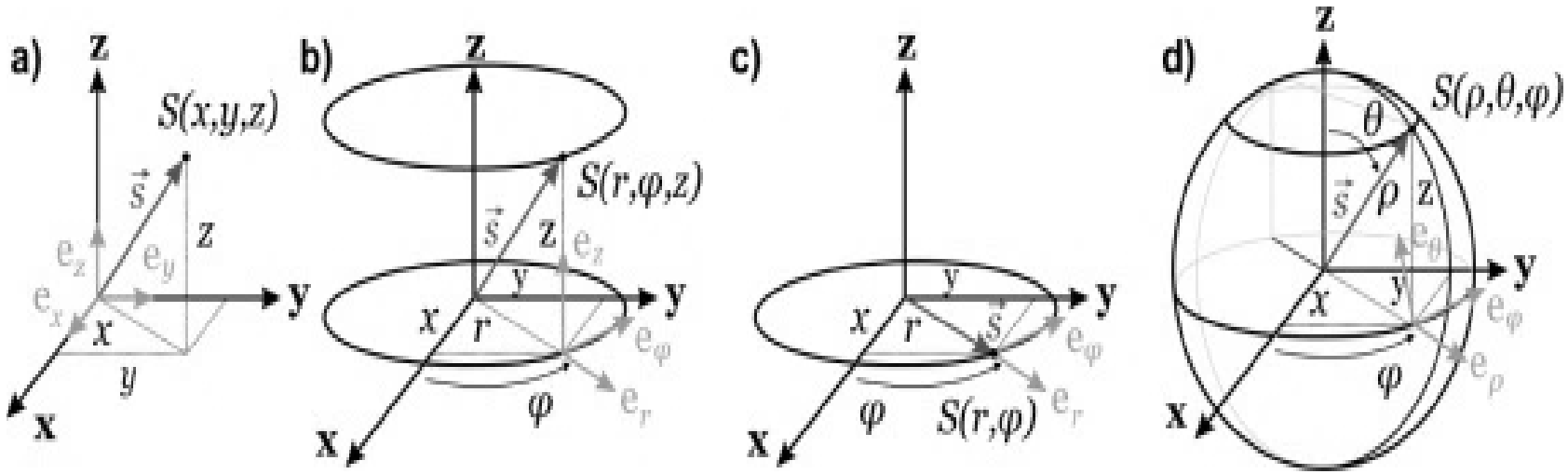
Text Books

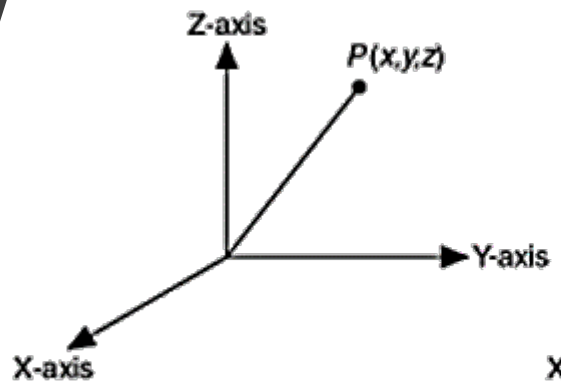
- Thomas's Calculus (11th Edition) George B. Thomas, Jr.
- Calculus (6th Edition) James Stewert.
- Advanced Engineering Mathematics (9th Edition) Ervin Kreyszig
- Calculus (6th Edition) Swokowski, Olinick and Pence

Reference Books

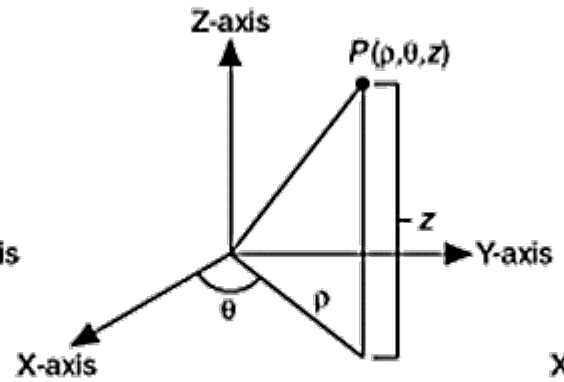
- Borisenko & Taranov, Vector and Tensor Analysis with Applications.

Coordinate Systems

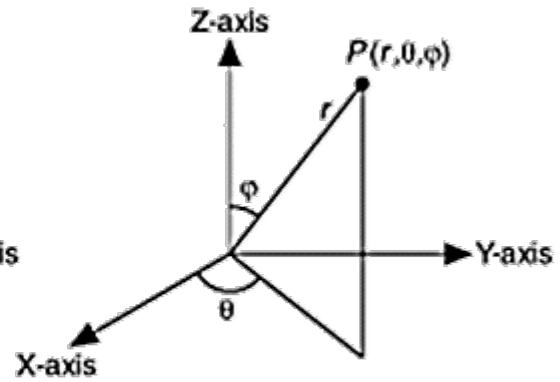




Cartesian Coordinate



Cylindrical Coordinate



Spherical Coordinate

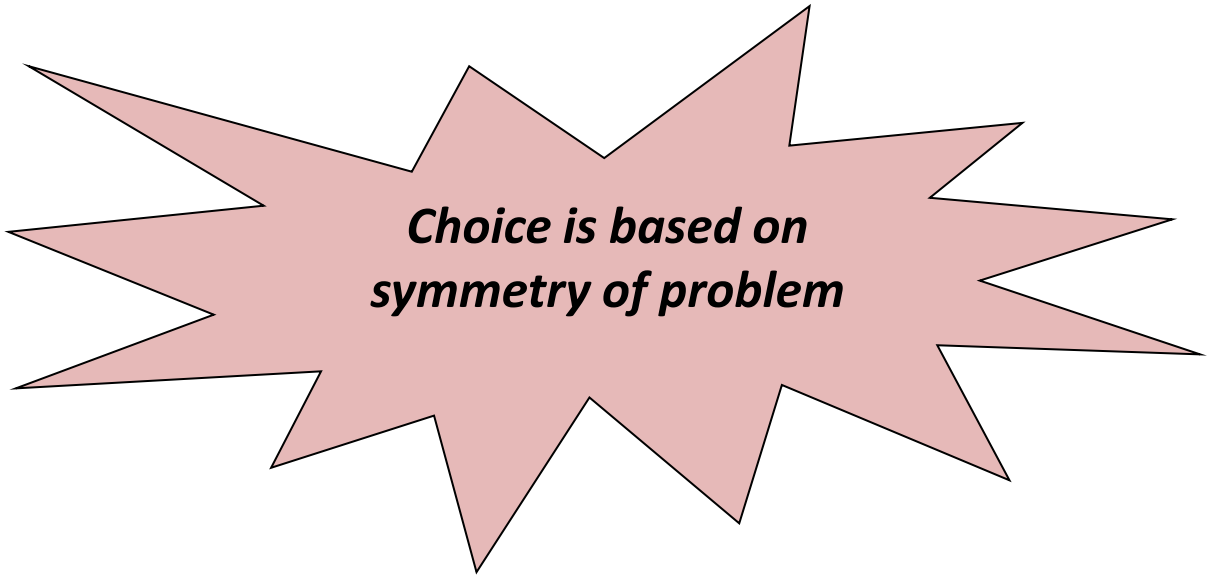
Coordinate Systems

- To understand the Electromagnetics, we must know basic vector algebra and coordinate systems.

- A good understanding of coordinate systems can be very helpful in solving problems related to Maxwell's Equations. So let us start with the discussion coordinate systems.

Coordinate Systems

- ***Rectangular or Cartesian***
- ***Cylindrical***
- ***Spherical***



<u>Examples:</u>

- ***Sheets - Rectangular***
- ***Wires/Cables - Cylindrical***
- ***Spheres - Spherical***

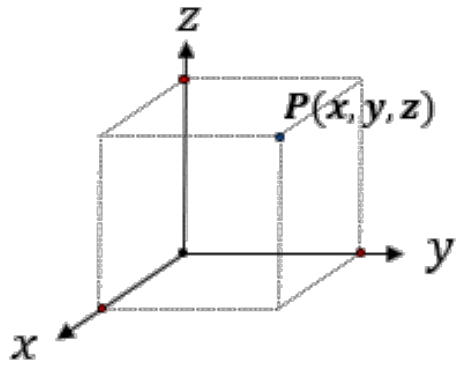
Orthogonal Coordinate Systems:

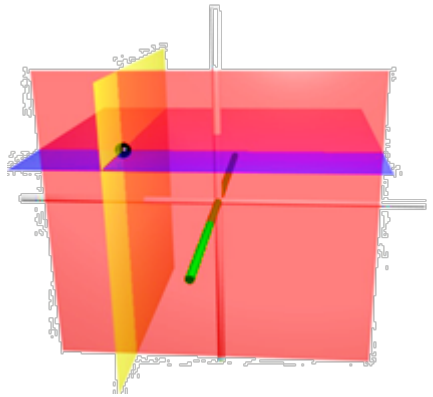
1. Cartesian Coordinates

Or

Rectangular Coordinates

$$P\ (x, y, z)$$

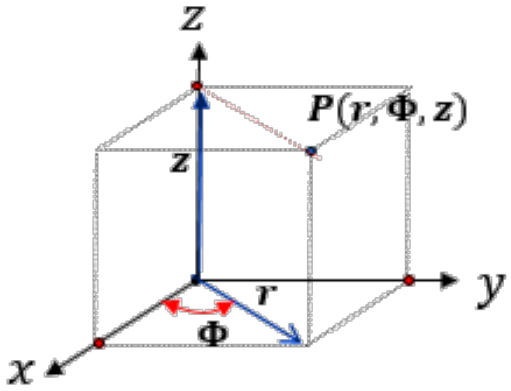


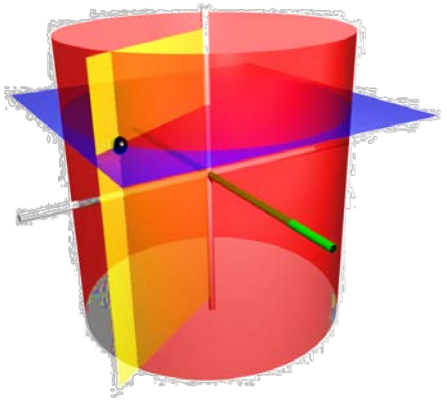


2. Cylindrical Coordinates

$$P\ (r, \Phi, z)$$

$$x = r\cos\Phi,$$
$$y = r\sin\Phi,$$
$$z = z.$$

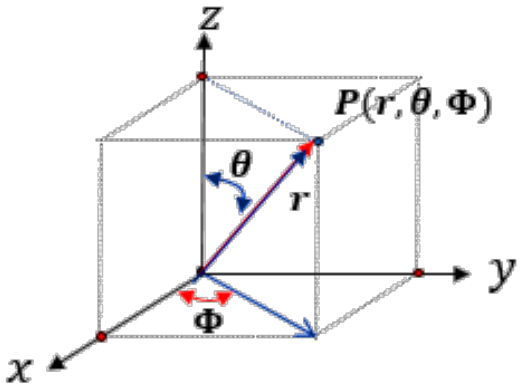


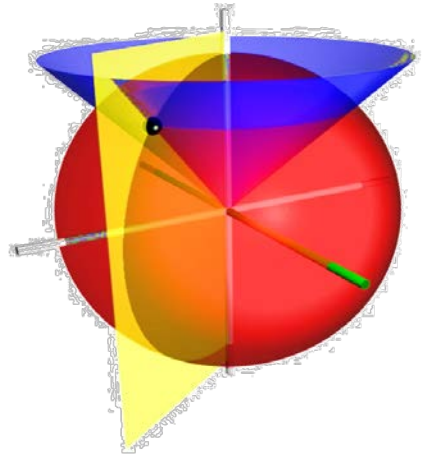


3. Spherical Coordinates

$$P\ (r, \theta, \Phi)$$

$$x = r\sin\theta\cos\Phi\,,$$
$$y = r\sin\theta\sin\Phi,$$
$$z = z\cos\theta\,.$$





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VECTORS AND THE GEOMETRY OF SPACE

Book: Thomas' Calculus Early Transcendentals (14th Edition) By George B. Thomas, Jr., Joel Hass, Christopher Heil, Maurice D. Weir.

Book: Calculus Early Transcendentals (6th Edition) By James Stewart.

Vectors And The Geometry Of Space

12.1

Three-Dimensional Coordinate Systems

In this section, we will learn about:

aspects of three-dimensional coordinate systems.