Chapter3: Gate-Level Minimization

Lecture3- Function Simplification using Quine McCluskey Minimization Algorithm

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Objectives

- Functions Simplification in Sum-of-Products (SOP) form using Quine McCluskey Minimization Algorithm

Function Simplification using Quine McCluskey Method

- The Quine-McCluskey method is an exact algorithm which finds minimum cost sum-of-products implementation of a Boolean function.
- There are four steps in the Quine-McCluskey algorithm:
  - ❑ Generate Prime Implicants
  - ❑ Construct Prime Implicant Table
  - ❑ Reduce the Prime Implicant Table by
    - – Removing Essential Prime Implicants
    - – Column Dominance
    - – Row Dominance
  - ❑ Solve Prime Implicant Table by (i) Petrick’s Method (ii) Branching Method

Example

$f(a,b,c,d,e)=\sum_m(1,3,4,5,6,7,10,11,12,13,14,15,$
$18,19,20,21,22,23,26,27)$

Example Cont…

❑ minterms sorted into groups according to number of 1's in each term.

Group	minterms	Variables a b c d e
0	1 4	0 0 0 0 1 0 0 1 0 0
1	3 5 6 10 12 18 20	0 0 0 1 1 0 0 1 0 1 0 0 1 1 0 0 1 0 1 0 0 1 1 0 0 1 0 0 1 0 1 0 1 0 0
2	7 11 13 14 19 21 22 26	0 0 1 1 1 0 1 0 1 1 0 1 1 0 1 0 1 1 1 0 1 0 0 1 1 1 0 1 0 1 1 0 1 1 0 1 1 0 1 0
3	15 23 27	0 1 1 1 1 1 0 1 1 1 1 1 0 1 1

Column 1

Group	minterms	a b c d e
0	1 4	0 0 0 0 1✓ 0 0 1 0 0✓
1	3 5 6 10 12 18 20	0 0 0 1 1✓ 0 0 1 0 1✓ 0 0 1 1 0✓ 0 1 0 1 0✓ 0 1 1 0 0✓ 1 0 0 1 0✓ 1 0 1 0 0✓
2	7 11 13 14 19 21 22 26	0 0 1 1 1✓ 0 1 0 1 1✓ 0 1 1 0 1✓ 0 1 1 1 0✓ 1 0 0 1 1✓ 1 0 1 0 1✓ 1 0 1 1 0✓ 1 1 0 1 0✓
3	15 23 27	0 1 1 1 1✓ 1 0 1 1 1✓ 1 1 0 1 1✓

Column 2

Group	minterms	a b c d e
0	1,3 1,5 4,5 4,6 4,12 4,20	0 0 0 - 1✓ 0 0 - 0 1✓ 0 0 1 0 -✓ 0 0 1 - 0✓ 0 - 1 0 0✓ - 0 1 0 0✓
1	3,7 3,11 3,19 5,7 5,13 5,21 6,7 6,14 6,22 10,11 10,14 10,26 12,13 12,14 18,19 18,22 18,26 20,21 20,22	0 0 - 1 1✓ 0 - 0 1 1✓ - 0 0 1 1✓ 0 0 1 - 1✓ 0 - 1 0 1✓ - 0 1 0 1✓ 0 0 1 1 -✓ 0 - 1 1 0✓ - 0 1 1 0✓ 0 1 0 1 -✓ 0 1 - 1 0✓ - 1 0 1 0✓ 0 1 1 0 -✓ 0 1 1 - 0✓ 1 0 0 1 -✓ 1 0 - 1 0✓ 1 - 0 1 0✓ 1 0 1 0 -✓ 1 0 1 - 0✓
2	7,15 7,23 11,15 11,27 13,15 14,15 19,23 19,27 21,23 22,23 26,27	0 - 1 1 1✓ - 0 1 1 1✓ 0 1 - 1 1✓ - 1 0 1 1✓ 0 1 1 - 1✓ 0 1 1 1 -✓ 1 0 - 1 1✓ 1 - 0 1 1✓ 1 0 1 - 1✓ 1 0 1 1 -✓ 1 1 0 1 -✓

Column3

Gp	minterms	a b c d e
0	1,3,5,7	0 0 - - 1 ×
	4,5,6,7	0 0 1 - - ✓
	4,5,12,13	0 - 1 0 - ✓
	4,5,20,21	- 0 1 0 - ✓
	4,6,12,14	0 - 1 - 0 ✓
	4,6,20,22	- 0 1 - 0 ✓
1	3,7,11,15	0 - - 1 1 ×
	3,7,19,23	- 0 - 1 1 ×
	3,11,19,27	- - 0 1 1 ×
	5,7,13,15	0 - 1 - 1 ✓
	5,7,21,23	- 0 1 - 1 ✓
	6,7,14,15	0 - 1 1 - ✓
	6,7,22,23	- 0 1 1 - ✓
	10,11,14,15	0 1 - 1 - ×
	10,11,26,27	- 1 0 1 - ×
	12,13,14,15	0 1 1 - - ✓
	18,19,22,23	1 0 - 1 - ×
	18,19,26,27	1 - 0 1 - ×
	20,21,22,23	1 0 1 - - ✓

Column4

Gp	minterms	a b c d e
0	4,5,6,7,12,13,14,15	0 - 1 - - ×
	4,5,6,7,20,21,22,23	- 0 1 - - ×
	~~4,5,12,13,6,7,14,15~~	~~0 - 1 - -~~
	~~4,5,20,21,6,7,22,23~~	~~- 0 1 - -~~
	~~4,6,12,14,5,7,13,15~~	~~0 - 1 - -~~
	~~4,6,20,22,5,7,21,23~~	~~- 0 1 - -~~

The Prime implicants generated:-

a′b′e ∑(1,3,5,7)

a′d e ∑(3,7,11,15)

b′d e ∑(3,7,19,23)

c′d e ∑(3,11,19,27)

a′bd ∑(10,11,14,15)

bc′d ∑(10,11,26,27)

ab′d ∑(18,19,22,23)

ac′d ∑(18,19,26,27)

a′c ∑(4,5,6,7,12,13,14,15)

b′c ∑(4,5,6,7,20,21,22,23)

Prime Implicant Table

PI	minterms	1	3	4	5	6	7	10	11	12	13	14	15	18	19	20	21	22	23	26	27
a'b'e*	1,3,5,7	⊗	x		x		x														
a'de	3,7,11,15		x				x		x				x								
b'de	3,7,19,23		x				x								x				x		
c'de	3,11,19,27		x						x						x						x
a'bd	10,11,14,15							x	x			x	x								
bc'd	10,11,26,27							x	x											x	x
ab'd	18,19,22,23													x	x			x	x		
ac'd	18,19,26,27													x	x					x	x
a'c*	4,5,6,7,12,13,14,15			x	x	x	x			⊗	⊗	x	x								
b'c*	4,5,6,7,20,21,22,23			x	x	x	x									⊗	⊗	x	x		

Reduced Prime Implicant Table

❖ The Prime Implicant Table was constructed in the previous slide.

❖ Essential Prime Implicants (EPIs) identified and eliminated from the table and corresponding minterms also struck.

❖ The Essential Prime Implicants (EPIs) obtained in this way are:-

_ a’b’e ∑(1,3,5,7)

_ a’c ∑(4,5,6,7,12,13,14,15)

_ b’c ∑(4,5,6,7,20,21,22,23)

❖ Now we can construct Reduced Prime Implicant Table and apply column dominance to reduce it further.

❖ Eliminate dominating column.

PIs	10	11	18	19	26	27
a’de (3,7,11,15)		×				
b’de (3,7,19,23)				×		
c’de (3,11,19,27)		×		×		×
a’bd (10,11,14,15)	×	×				
bc’d (10,11,26,27)	×	×			×	×
ab’d (18,19,22,23)			×	×		
ac’d (18,19,26,27)			×	×	×	×

Further Reduced Prime Implicant Table

- ❖ The table is further reduced by applying Column Dominance.
- ❖ We can now apply Row Dominance and eliminate dominated rows.
- ❖ Rows bc'd and ac'd dominate a'bd and ab'd. Hence dominated rows a'bd and ab'd can be eliminated. The secondary EPIs bc'd and ac'd cover all minterms and are selected for minimal solution.
- ❖ We can also apply row dominance first and then column dominance.

PIs	10	18	26
~~a'bd (10,11,14,15)~~	×		
bc'd (10,11,26,27)	⊗		×
~~ab'd (18,19,22,23)~~		×	
ac'd (18,19,26,27)		⊗	×

f(a,b,c,d,e)=a'b'e+a'c+b'c+bc'd+ac'd

Petrick Method

- In Petrick's method, a Boolean expression P is formed which describes all possible solutions of the table.
- The prime implicants in the table are numbered in order, from 1 to 6. For each prime implicant pi, a Boolean variable Pi is used which is true whenever prime implicant pi is included in the solution.
- Remember that pi is prime implicant whereas Pi is corresponding Boolean proposition(true/false statement) which is true(1) or false(0) value.
- Using these Pi variables, a larger Boolean expression P can be formed, which captures the precise conditions for every row in the table to be covered.

PIs	10	11	18	19	26	27
p1 a'de (3,7,11,15)		×				
p2 b'de (3,7,19,23)				×		
p3 c'de (3,11,19,27)		×		×		×
p4 a'bd (10,11,14,15)	×	×				
p5 bc'd (10,11,26,27)	×	×			×	×
p6 ab'd (18,19,22,23)			×	×		
p7 ac'd (18,19,26,27)			×	×	×	×

All Possible Solutions using Petrick's Method

P=(P4+P5)(P1+P3+P4+P5)(P6+P7)(P2+P3+P6+P7)(P5+P7)
(P3+P5+P7)

P=(P4+P5)(P6+P7)(P5+P7) Absorption theorem

P=(P4+P5)(P7+P5P6) + dist over .

P=P4P7 +P4P5P6+P5P7+P5P6

Each product term in the above Boolean expression describes a solution for the table.

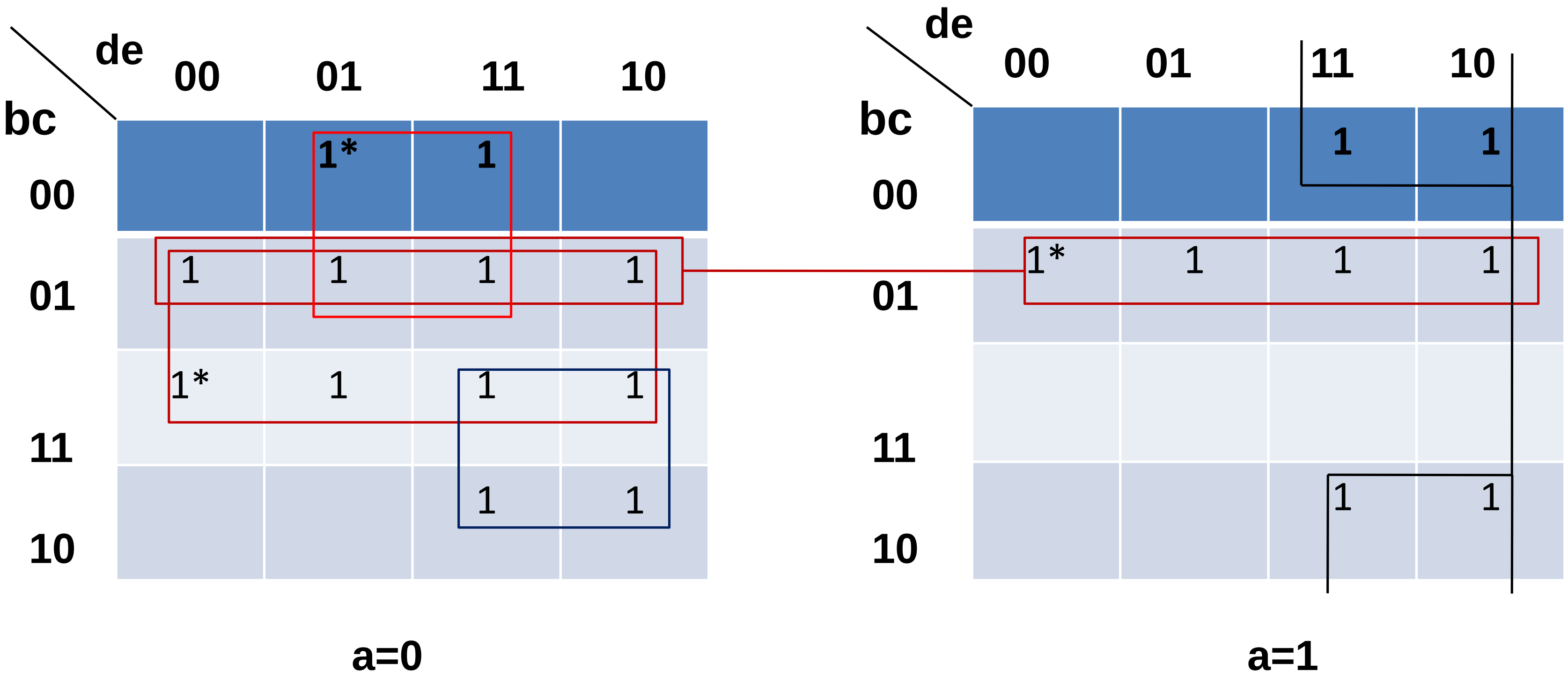
- All possible solutions are

f1(a,b,c,d,e)=a'b'e+a'c+b'c+a'bd+ac'd

f1(a,b,c,d,e)=a'b'e+a'c+b'c+bc'd+ac'd

f1(a,b,c,d,e)=a'b'e+a'c+b'c+ab'd+bc'd

Map Simplification of the Same Function



f(a,b,c,d,e)=a′c+b′c+a′b′e+(a′bd+ac′d or bc′d+ab′d or bc′d+ac′d)

The End