Thermodynamics I

Lecture 16

Energy Analysis of Closed Systems (Ch-4)
Moving Boundary Work

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Objectives

- Examine the moving boundary work or *P dV* work commonly encountered in reciprocating devices such as automotive engines and compressors.
- Identify the first law of thermodynamics as simply a statement of the conservation of energy principle for closed (fixed mass) systems.
- Develop the general energy balance applied to closed systems.
- Define the specific heat at constant volume and the specific heat at constant pressure.
- Relate the specific heats to the calculation of the changes in internal energy and enthalpy of ideal gases.
- Describe incompressible substances and determine the changes in their internal energy and enthalpy.
- Solve energy balance problems for closed (fixed mass) systems that involve heat and work interactions for general pure substances, ideal gases, and incompressible substances.

MOVING BOUNDARY WORK

Moving boundary work (*P dV* work): The expansion and compression work in a piston-cylinder device.

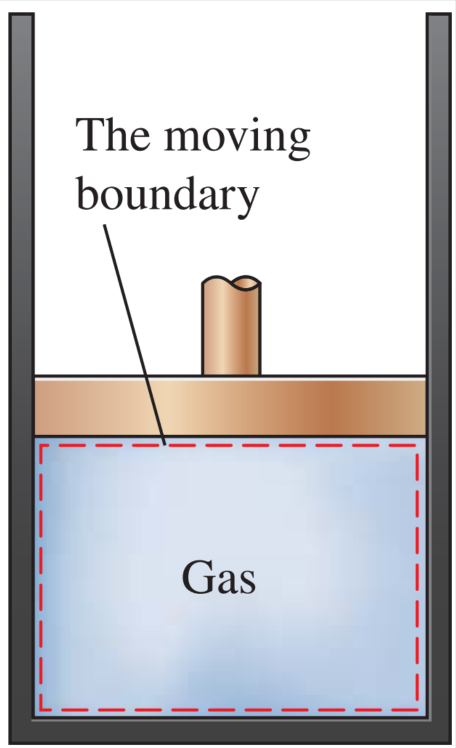
$$\delta W_b = F\,ds = PA\,ds = P\,dV$$

$$W_b = \int_1^2 P\,dV \qquad \text{(kJ)}$$

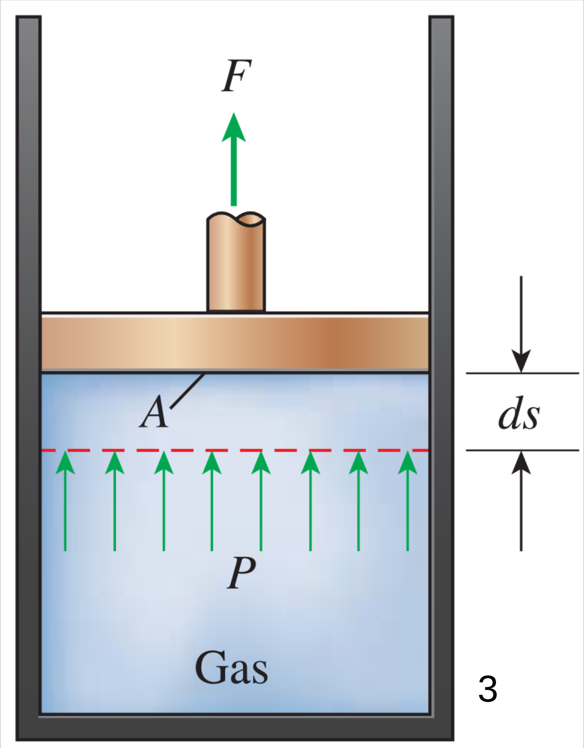
Quasi-equilibrium process: A process during which the system remains nearly in equilibrium at all times.

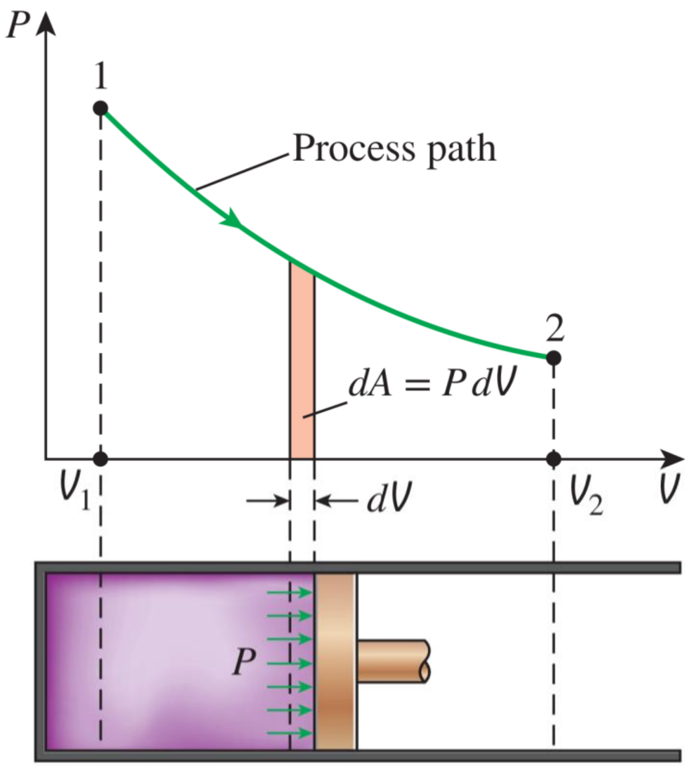
W_b is positive $\rightarrow$ for expansion
W_b is negative $\rightarrow$ for compression

The work associated with a moving boundary is called boundary work.



A gas does a differential amount of work δW_b as it forces the piston to move by a differential amount *ds*.

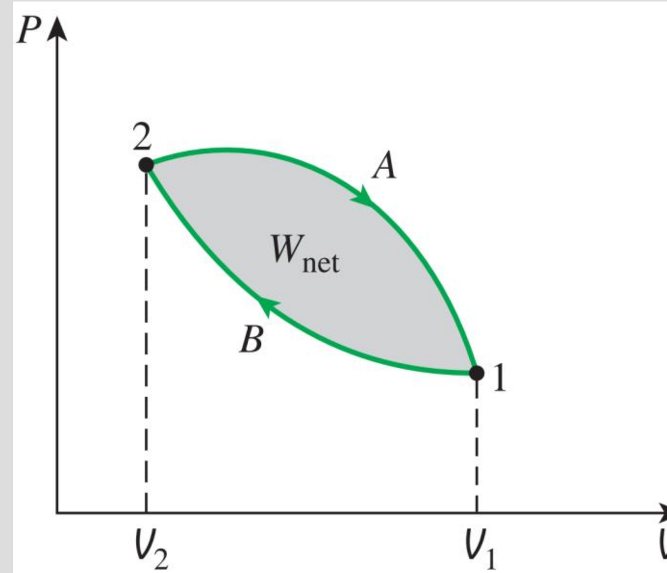




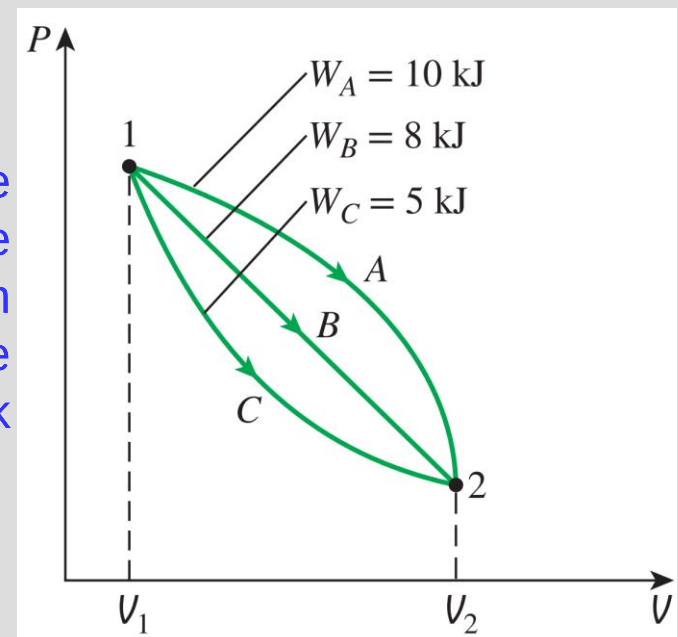
The area under the process curve on a *P-V* diagram represents the boundary work.

$$\text{Area} = A = \int_1^2 dA = \int_1^2 P\,dV$$

The boundary work done during a process depends on the path followed as well as the end states.

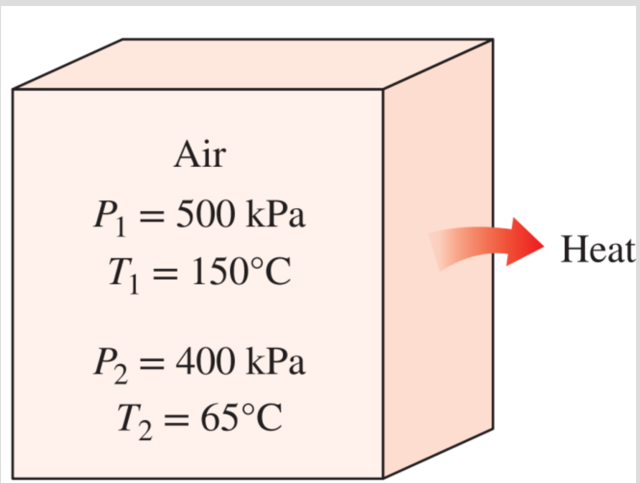


The net work done during a cycle is the difference between the work done by the system and the work done on the system.

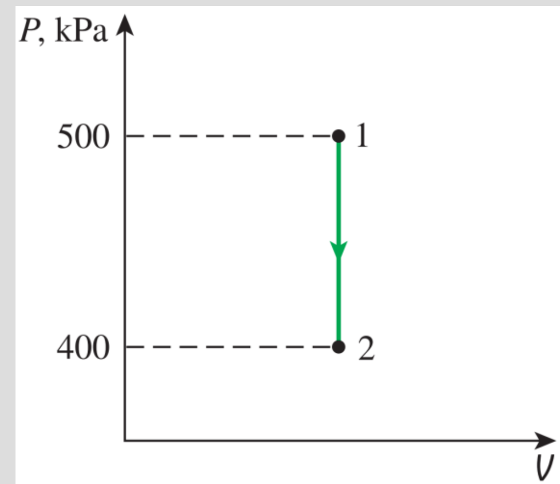


Example 4-1

A rigid tank contains air at 500 kPa and 150°C. As a result of heat transfer to the surroundings, the temperature and pressure inside the tank drop to 65°C and 400 kPa, respectively. Determine the boundary work done during this process.



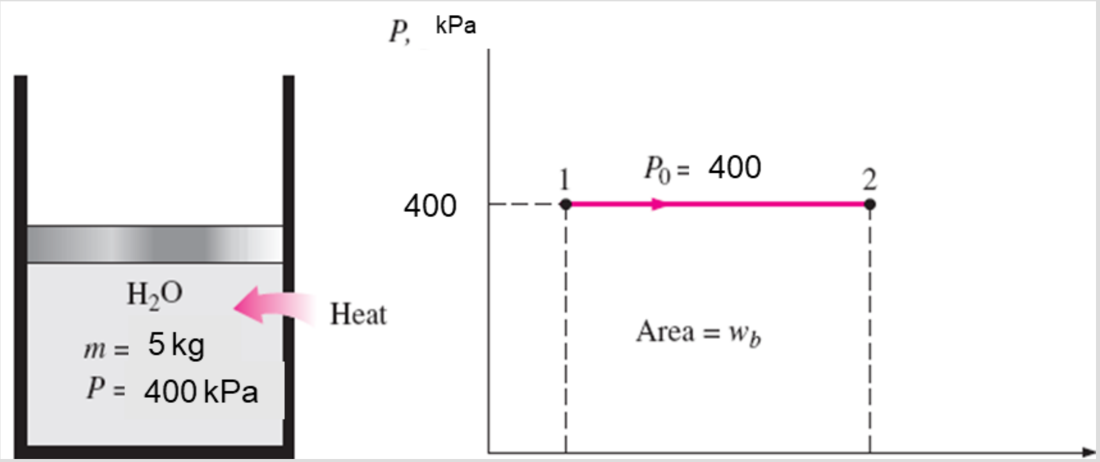
$$W_b = \int_1^2 P\,dV^{\nearrow 0} = \mathbf{0}$$



This is expected since a rigid tank has a constant volume and dV = 0 in this equation. Therefore, there is no boundary work done during this process. That is, the boundary work done during a constant-volume process is always zero. This is also evident from the P-V diagram of the process (the area under the process curve is zero).

Example 4-2 Boundary Work for a Constant-Pressure Process

A frictionless piston-cylinder device contains 5 kg of steam at 400 kPa and 200 ˚C. Heat is now transferred to the steam until the temperature reaches 250 ˚C. If the piston is not attached to a shaft and its mass is constant, determine the work done by the steam during this process



$$W_b = \int_1^2 P\,dV = P_0 \int_1^2 dV = P_0(V_2 - V_1)$$

$$W_b = mP_0(v_2 - v_1) \quad \text{since } V = mv.$$

Example 4-2 Boundary Work for a Constant-Pressure Process

From superheated vapor table (**Table A-6**), the specific volumes are

$\mathbf{v_1}$ = 0.53434 m^3/kg at state 1 (400 kPa, 200 ˚C)

$\mathbf{v_2}$ = 0.59520 m^3/kg at state 2 (400 kPa, 250 ˚C)

$$\boxed{W_b = mP_0(v_2 - v_1)}$$

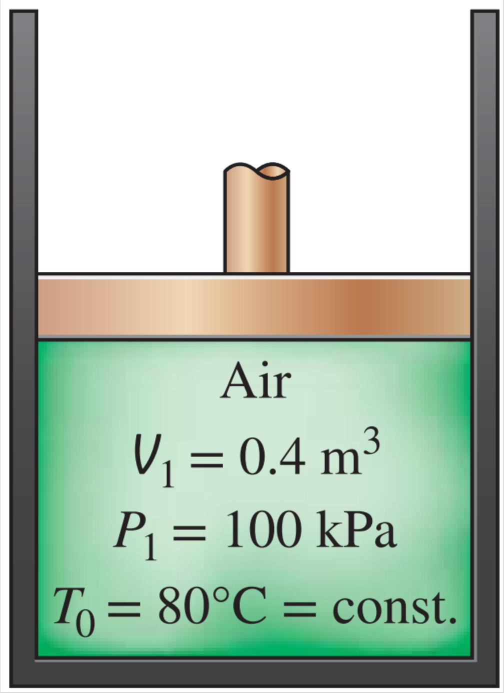
$\mathbf{W_b}$ = (5 kg)(400 kPa)[0.59520 – 0.53434) [m^3/kg](1kJ/ 1kPa.m^3)

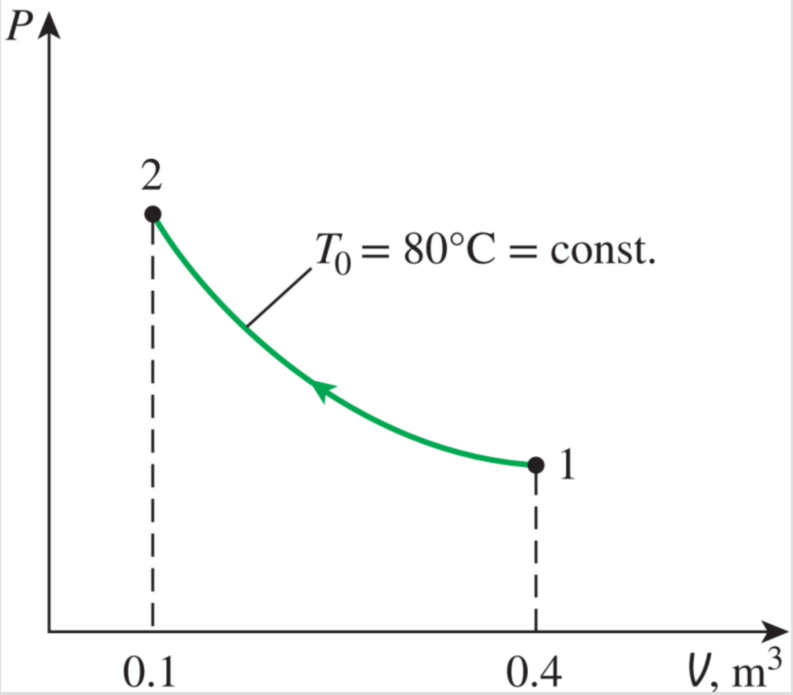
$\mathbf{W_b}$= 121.7 kJ

The positive sign indicates that the work is done by the system.

Example 4-3

A piston–cylinder device initially contains 0.4 m^3 of air at 100 kPa and 80°C. The air is now compressed to 0.1 m^3 in such a way that the temperature inside the cylinder remains constant. Determine the work done during this process.





Example 4-3

For an ideal gas at constant temperature T_0,

$$PV = mRT_0 = C \quad \text{or} \quad P = \frac{C}{V}$$

where C is a constant. Substituting this into movable work equation, we have:

$$W_b = \int_1^2 P\,dV = \int_1^2 \frac{C}{V}\,dV$$

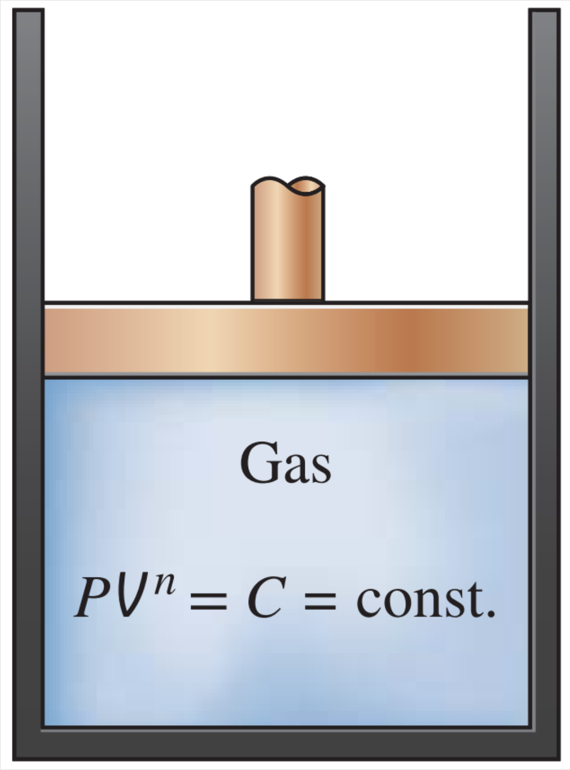
$$= C\int_1^2 \frac{dV}{V} = C\ln\frac{V_2}{V_1} = P_1V_1\ln\frac{V_2}{V_1}$$

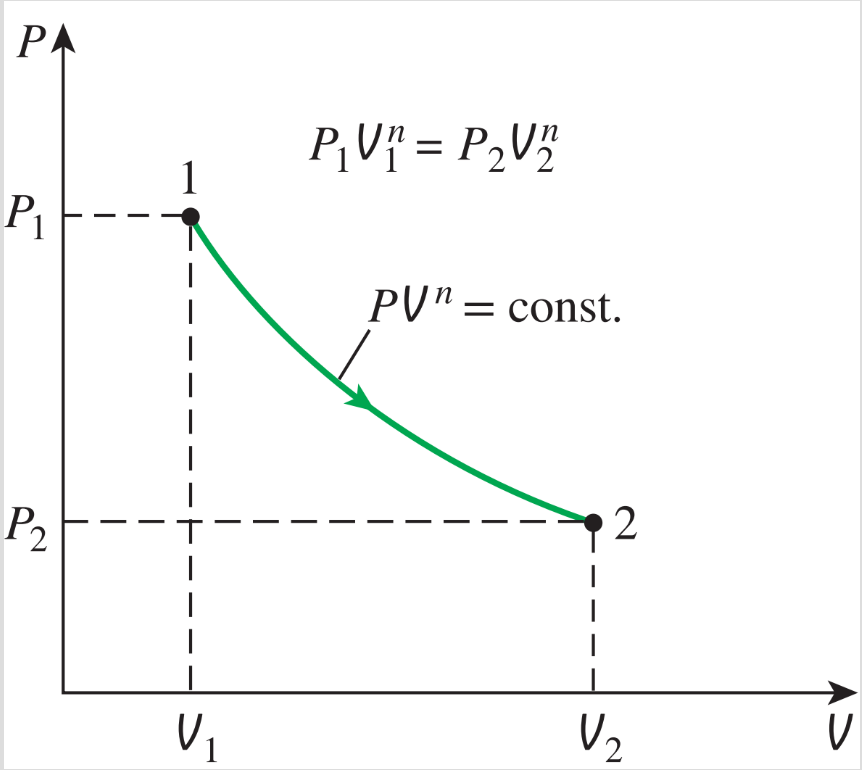
$$W_b = (100\text{ kPa})(0.4\text{ m}^3)\left(\ln\frac{0.1}{0.4}\right)\left(\frac{1\text{ kJ}}{1\text{ kPa}\cdot\text{m}^3}\right) = \mathbf{-55.5\text{ kJ}}$$

The negative sign indicates that this work is done on the system (a work input), which is always the case for compression processes.

POLYTROPIC PROCESS

During expansion and compression processes of gases, pressure and volume are often related by **$PV^n = C$**, where n (polytropic exponent) and C are constants. A process of this kind is called a **polytropic process.**





Schematic and *P-V* diagram for a polytropic process.

Polytropic, Isothermal, and Isobaric processes

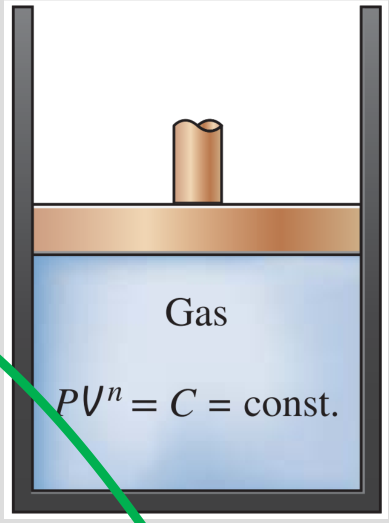
$$P = CV^{-n}$$ Polytropic process: *C*, *n* (polytropic exponent) constants

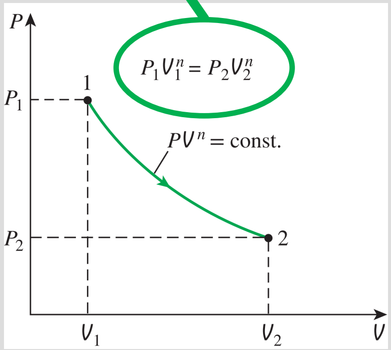
$$W_b = \int_1^2 P\,dV = \int_1^2 CV^{-n}\,dV$$

$$= C\frac{V_2^{-n+1} - V_1^{-n+1}}{-n+1} = \frac{P_2V_2 - P_1V_1}{1-n}$$

For an ideal gas ($PV = mRT$)

$$W_b = \frac{mR(T_2 - T_1)}{1-n} \qquad n \neq 1$$





When n = 1 (isothermal process)

$$W_b = \int_1^2 P\,dV = \int_1^2 \frac{C}{V}dV = C\int_1^2 \frac{dV}{V} = C\ln\frac{V_2}{V_1} = P_1V_1\ln\frac{V_2}{V_1}$$

Constant pressure process

$$W_b = \int_1^2 P\,dV = P_0\int_1^2 dV = P_0(V_2 - V_1)$$

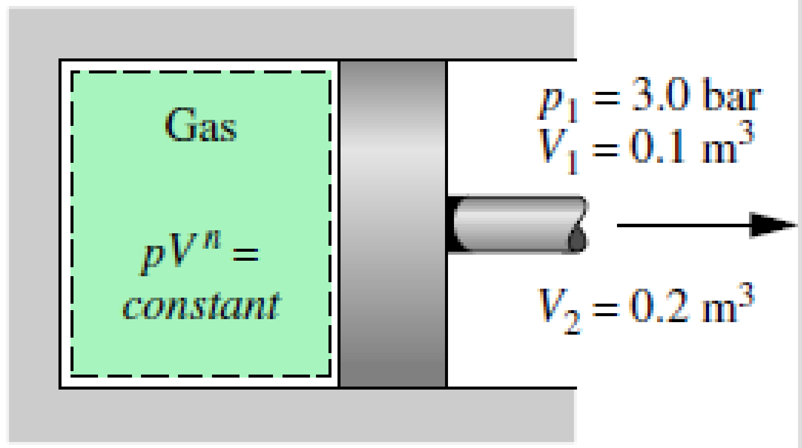
What is the boundary work for a constant-volume process?

Example: Evaluating Expansion Work

A gas in a piston–cylinder assembly undergoes an expansion process for which the relationship between pressure and volume is given by

$$pV^n = constant$$

The initial pressure is 3 bar, the initial volume is 0.1 m^3, and the final volume is 0.2 m^3. Determine the work for the process, in kJ, if (a) $n = 1.5$, (b) $n = 1.0$, and (c) $n = 0$.



Assumptions:

1. The gas is a closed system.
2. The moving boundary is the only work mode.
3. The expansion is a polytropic process.

Known: A gas in a piston–cylinder assembly undergoes an expansion for which $pV^n = constant$.

Find: Evaluate the work if (a) $n = 1.5$, (b) $n = 1.0$, (c) $n = 0$.

Example: Evaluating Expansion Work

For n = 1.5

$$W_b = \int_1^2 P\,dV = \int_1^2 CV^{-n}\,dV = C\,\frac{V_2^{-n+1} - V_1^{-n+1}}{-n+1} = \frac{P_2V_2 - P_1V_1}{1-n}$$

P_2 can be found by using

$$C = P_1V_1^n = P_2V_2^n$$

$$p_2 = p_1\left(\frac{V_1}{V_2}\right)^n = (3\text{ bar})\left(\frac{0.1}{0.2}\right)^{1.5} = 1.06\text{ bar}$$

$$W = \left(\frac{(1.06\text{ bar})(0.2\text{ m}^3) - (3)(0.1)}{1 - 1.5}\right)\left|\frac{10^5\text{ N/m}^2}{1\text{ bar}}\right|\left|\frac{1\text{ kJ}}{10^3\text{ N}\cdot\text{m}}\right|$$

$$= +17.6\text{ kJ}$$

Example: Evaluating Expansion Work

For n = 1.0

$$W_b = \int_1^2 P\,dV = \int_1^2 \frac{C}{V}dV = C\int_1^2 \frac{dV}{V} = C\ln\frac{V_2}{V_1} = P_1V_1\ln\frac{V_2}{V_1}$$

$$W = (3\text{ bar})(0.1\text{ m}^3)\left|\frac{10^5\text{ N/m}^2}{1\text{ bar}}\right|\left|\frac{1\text{ kJ}}{10^3\text{ N}\cdot\text{m}}\right|\ln\left(\frac{0.2}{0.1}\right) = +20.79\text{ kJ}$$

For n = 0 the relation becomes p = constant

$$W_b = \int_1^2 P\,dV = P_0\int_1^2 dV = P_0(V_2 - V_1)$$

$$W = +30\text{ kJ}$$