Ampere’s Law II

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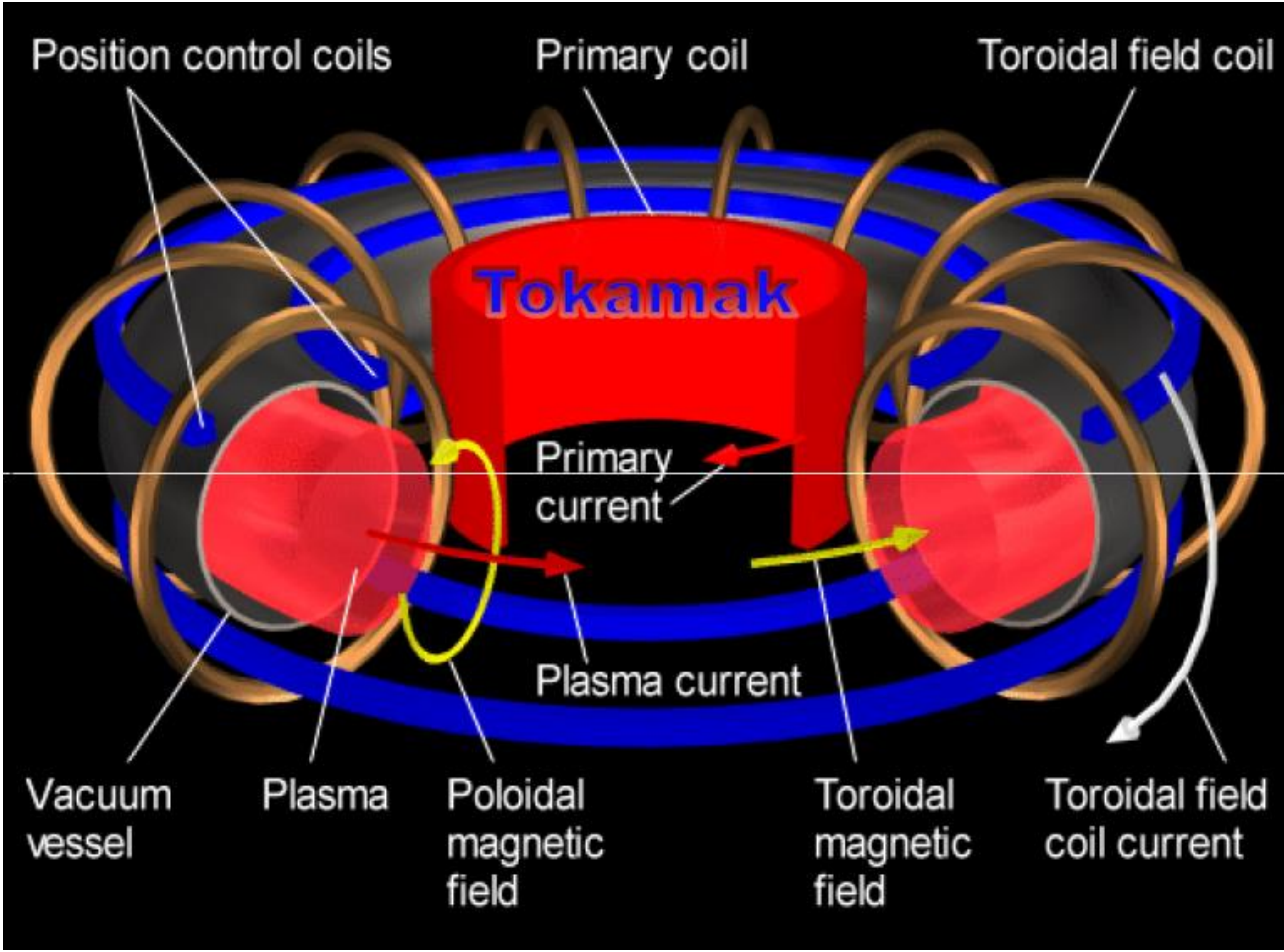

Position control coils
Primary coil
Toroidal field coil
Tokamak
Primary
current
Plasma current
Vacuum
vessel
Plasma
Poloidal
magnetic
field
Toroidal
magnetic
field
Toroidal field
coil current

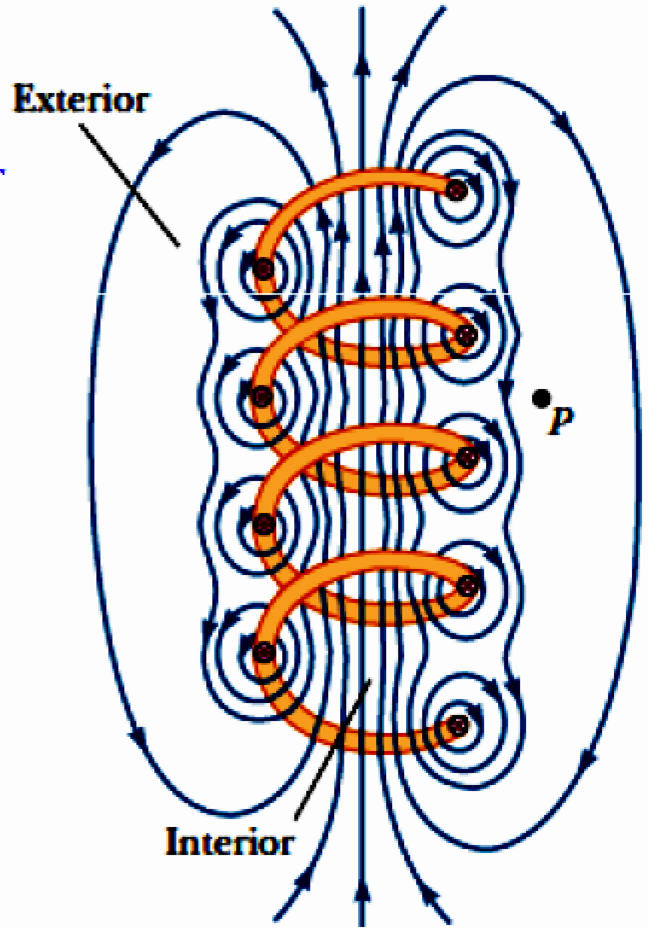
B Due To a Current Carrying Solenoid

Let's consider a long, tightly wound helical coil of wire, which is called a *solenoid*.

With this configuration, a reasonably uniform magnetic field can be produced in the *interior of the solenoid—when the solenoid* carries a current.

When the turns are closely spaced, each can be approximated as a circular loop, and the net magnetic field is the vector sum of the fields resulting from all the turns.

The field lines in the interior are nearly parallel, are uniformly distributed, and are close together, indicating that the field in this space is uniform and strong.

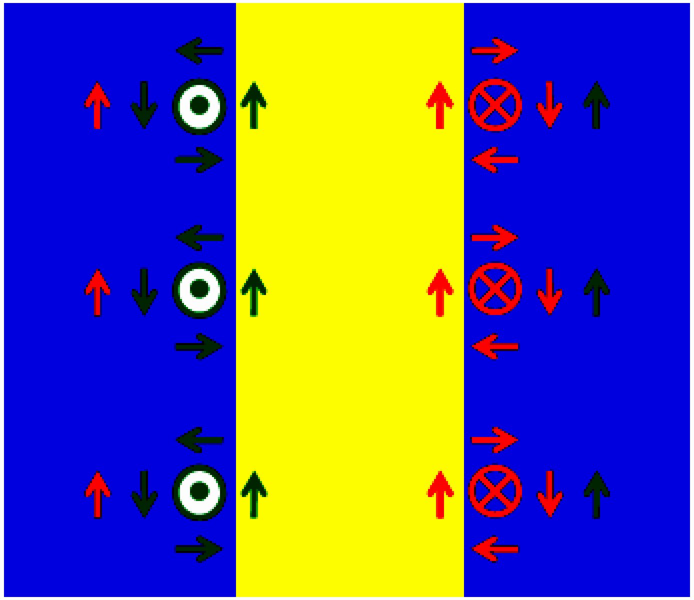


The field lines between current elements on two adjacent turns tend to cancel each other because the field vectors from the two elements are in opposite directions.

The field at exterior points such as P is weak because the field due to current elements on the right-hand portion of a turn tends to cancel the field due to current elements on the left-hand portion.

The figure to the near right clearly illustrates how the external fields cancel each other while the internal filed add. The green arrows represent fields produced from left part of the turns while the red arrows represents fields produced from the right part of the turns.

In the limiting case of the ideal solenoid, the external field is zero.



B inside the Solenoid

Consider a rectangular Amperian loop of height h and width w.

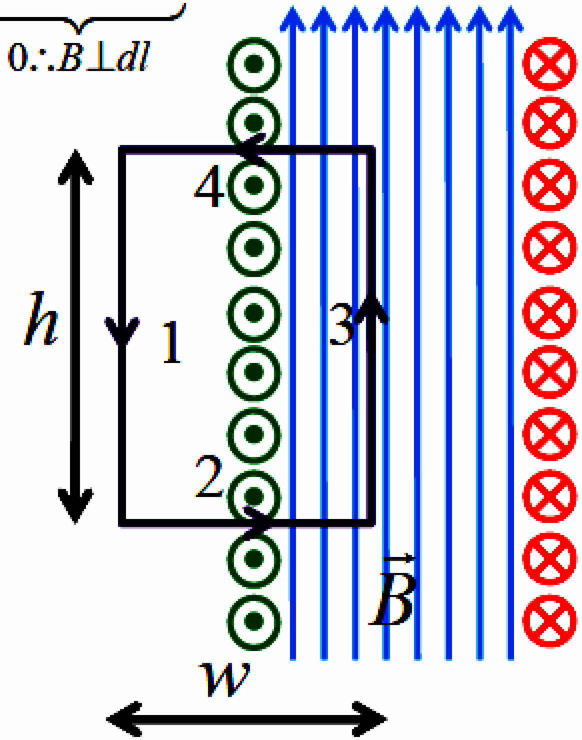
$$\oint \vec{B} \bullet d\vec{l} = \mu_0 i_{encl} \qquad (1)$$

$$\oint \vec{B} \bullet d\vec{l} = \underbrace{\int_1 \vec{B} \bullet d\vec{l}}_{0 \because B=0} + \underbrace{\int_2 \vec{B} \bullet d\vec{l}}_{0 \because B \perp dl} + \underbrace{\int_3 \vec{B} \bullet d\vec{l}}_{Bh \because B \,||\, dl} + \underbrace{\int_4 \vec{B} \bullet d\vec{l}}_{0 \because B \perp dl}$$

$$\oint \vec{B} \bullet d\vec{l} = Bh$$

If solenoid has n turns per unit length and each turns carry current i then total enclosed current is

$$i_{encl} = nih$$



From (1) $\quad Bh = \mu_\circ nih \Rightarrow \boxed{B = \mu_\circ ni}$

A solenoid 95.6cm long has a radius of 1.9cm, a winding of 1230 turns, and carries a current of 3.58A. Calculate the strength of magnetic field inside the solenoid

As the magnetic field inside the solenoid is

$$B = \mu_{\circ} n i$$

Where n is the number of turns per unit length, that is $n = N / l$

$$B = \mu_{\circ} i \frac{N}{l}$$

$$B = 4\pi \times 10^{-7} N / A^2 (3.58A) \frac{1230}{0.956}$$

$$= 5.8 \times 10^{-3} T$$

A solenoid 1.33 m long and 2.6cm in diameter carries a current of 17.8A. The strength of magnetic field inside the solenoid is 22.4mT. Find the length of the wire forming the solenoid.

As the magnetic field inside the solenoid is

$$B = \mu_{\circ} n i$$

$$n = \frac{B}{\mu_{\circ} i} = 1\times10^{3}/m$$

Where n is the number of turns per unit length, that is $n = N/l$

$$N = nl = 1\times10^{3}/m(1.33m) = 1330$$

since each turn has a length of one circumference, then the total length of the wire which makes up the solenoid is

$$L = N(2\pi r) = N(\pi D) = 109m$$

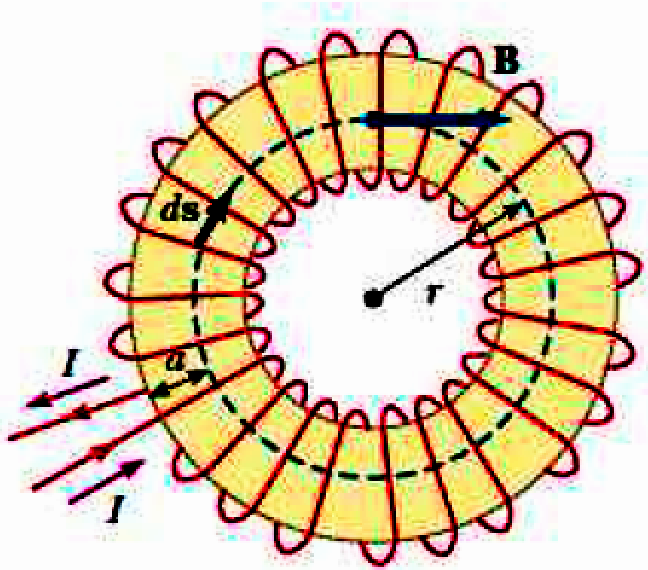
B Due To a Current Carrying Toroid

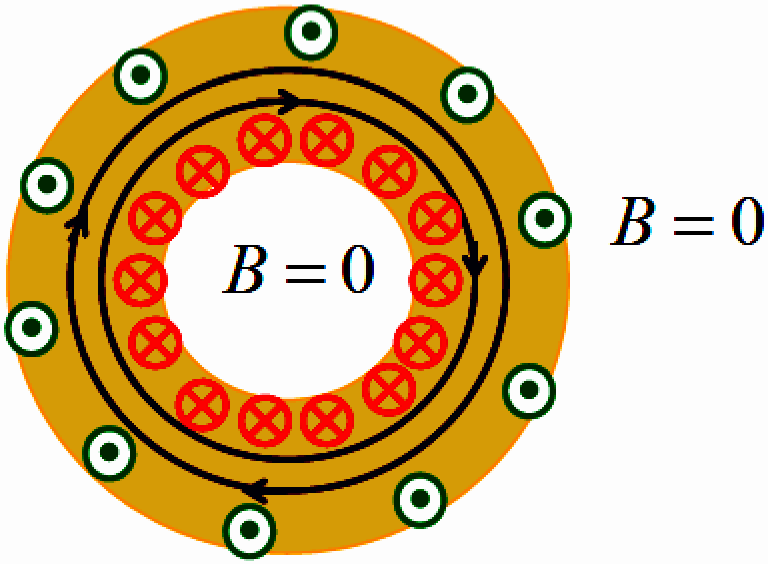
A device called a *toroid* consists of a conducting wire wrapped around a ring (a torus) made of a nonconducting material.

Toroid is often used to create magnetic field in some enclosed area.

Toroid is a solenoid bent in the shape of a doughnut.

For ideal toroid, B=0 for point outside and in the central cavity.





Consider a toroid having N closely spaced turns of wire carrying current i as shown in figure. For calculating the magnetic field in the region occupied by the torus, a distance r from the center, consider a circular Amperian loop of radius r. By symmetry, we see that the magnitude of the field is constant on this Amperian loop and tangent to it, so

$$\oint \vec{B} \bullet d\vec{l} = \mu_0 i_{encl}$$

$$B(2\pi r) = \mu_0 N i \qquad\qquad i_{encl} = Ni$$

$$\boxed{B = \frac{\mu_0 N i}{2\pi r}}$$

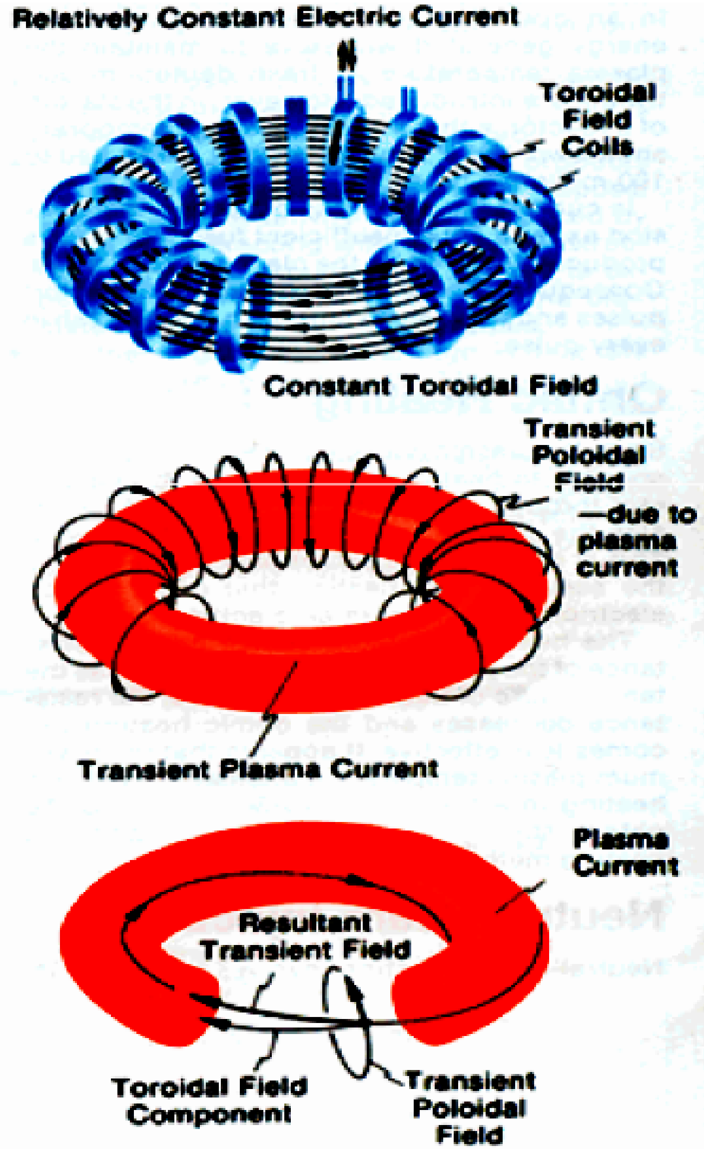
B varies as 1/r and hence is non uniform in the region occupied by the torus. However, if r is very large compared with the cross-sectional radius of the torus, then the field is approximately uniform inside the torus.

TOKOMAK: A Way Towards Fusion

A **tokamak** is a type of machine that uses a magnetic field to confine a plasma in the shape of a torus (donut).

The tokamak is one of several types of magnetic confinement devices, and is one of the most-researched candidates for producing **controlled thermonuclear fusion.**

Achieving a stable plasma equilibrium requires magnetic field lines that move around the torus in a **helical shape**.



ITER-TOKOMAK

Such a helical field can be generated by adding a toroidal field (traveling around the torus in circles) and a poloidal field (traveling in circles orthogonal to the toroidal field).

In a tokamak, the toroidal field is produced by electromagnets that surround the torus, and the poloidal field is the result of a toroidal electric current that flows inside the plasma.

This current is induced inside the plasma with a second set of electromagnets.