



Discrete Random Variable

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Random Variable

A random variable is a variable that assumes numerical values associated with the random outcome of an experiment, where one (and only one) numerical value is assigned to each sample point.

Discrete Random Variables

A discrete random variable can assume a countable number of values.

Examples

- Number of heads
- Number of calls
- Numbers of Errors
- Students in class
- Mistakes per page

Probability Distributions for Discrete Random Variables

The probability distribution of a discrete random variable is a graph, table or formula that specifies the probability associated with each possible outcome the random variable can assume.

- $p(x) \geq 0$ for all values of x
- $\sum p(x) = 1$

Probability mass function

$$p(x) = p_x(x) = P(X = x)$$

Discreate Probability Distribution

Example: Three Coins

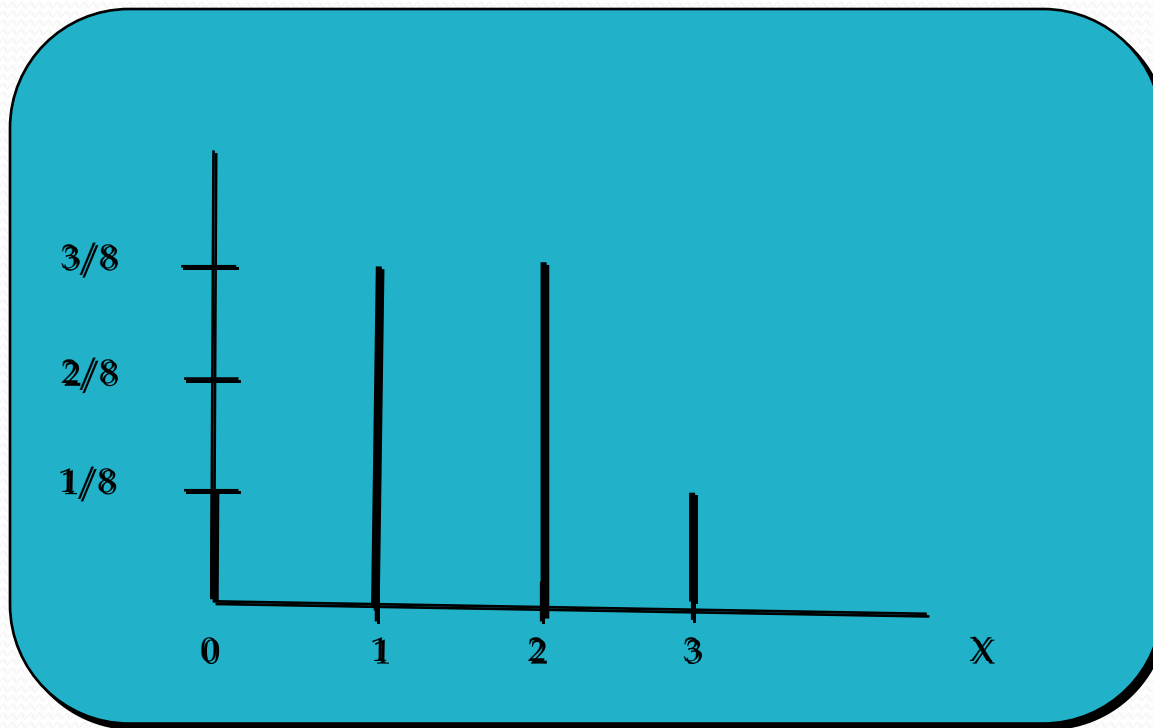
X =No of Heads

X	Outcomes	$P(X=x)$
0	TTT	$\frac{1}{8}$
1	TTH, THT, HTT	$\frac{3}{8}$
2	HHT, THH, HTH	$\frac{3}{8}$
3	HHH	$\frac{1}{8}$
	Total sample Points=8	$\sum p=1$

Graph of Discrete probability distribution

X-Axis \longrightarrow X=No of Heads

Y-Axis \longrightarrow Probabilities



Probability Mass Function

$$P(X=x) = \frac{\binom{3}{x}}{8} \quad x=0, 1, 2, \text{ and } 3$$

X	Outcomes	P(X=x)
0	TTT	$\frac{\binom{3}{0}}{8} = \frac{1}{8}$
1	TTH, THT, HTT	$\frac{\binom{3}{1}}{8} = \frac{3}{8}$
2	HHT, THH, HTH	$\frac{\binom{3}{2}}{8} = \frac{3}{8}$
3	HHH	$\frac{\binom{3}{3}}{8} = \frac{1}{8}$
	Total sample Points=8	$\sum p=1$

Distribution Function or Cumulative Distribution Function

The Distribution function of a random variable X , denoted by $F(x)$, is defined by $F(x) = P(X \leq x)$. The function $F(x)$ gives the probability of the event takes a value less than or equal to a specified value x . The Distribution function is abbreviated to d.f and is also called the cumulative distribution function(cdf) as it is the cumulative probability function of the X from the smallest up to specific value of x .

Discrete Example: Toss of Three coins

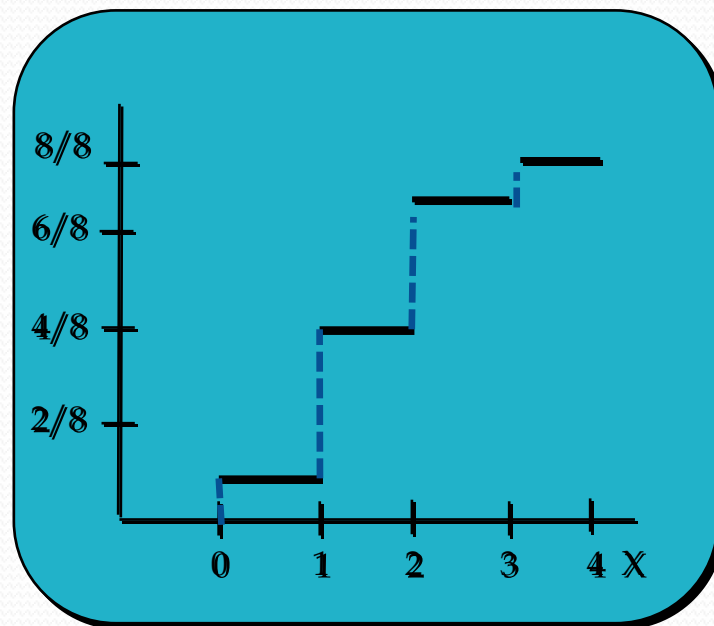
Cumulative probability distribution

X	$P(X=X)$	$F(X)=P(X\leq x)$
0	$\frac{1}{8}$	$\frac{1}{8}$
1	$\frac{3}{8}$	$\frac{4}{8}$
2	$\frac{3}{8}$	$\frac{7}{8}$
3	$\frac{1}{8}$	$\frac{8}{8}$

Graph of cumulative probability distribution

X-Axis \longrightarrow X=No of Heads

Y-Axis \longrightarrow Cumulative Probabilities

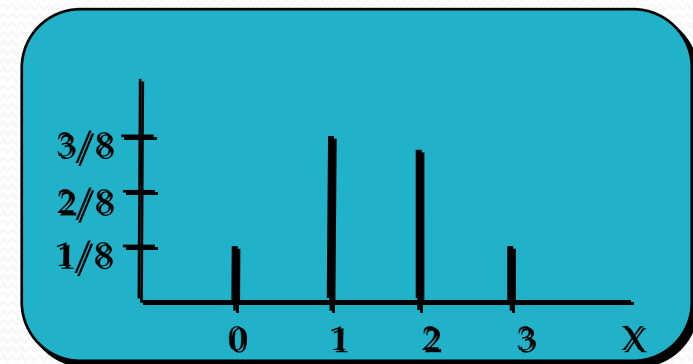


Summary: Toss of Three Coins

Discrete probability distribution

X	$p(x)$
0	$1/8$
1	$3/8$
2	$3/8$
3	$1/8$
$\sum p(x)$	1

Graph of Discrete probability distribution



Probability mass function

$$P(X=x) = \frac{\binom{3}{x}}{8} \quad x=0, 1, 2, 3$$

Cumulative probability distribution

X	F(X)
0	$\frac{1}{8}$
1	$\frac{4}{8}$
2	$\frac{7}{8}$
3	$\frac{8}{8}$

Question 1

A weighted coin is tossed 4 times. If the variable X denotes the numbers of Heads and the probability of head is equal to $1/3$. Find

- Discrete probability distribution
- Probability mass function
- Cumulative probability distribution
- Graph of discrete probability distribution
- Graph of cumulative probability distribution

Question 2

Find the probability distribution and probability mass function of the sum of the dots when two fair dice is thrown.

Question 3

A bag contains 4 red and 6 black balls. A sample of 4 balls is selected from the bag without replacement. Let X be the number of black balls. Find the

- probability distribution of X .
- probability mass function.