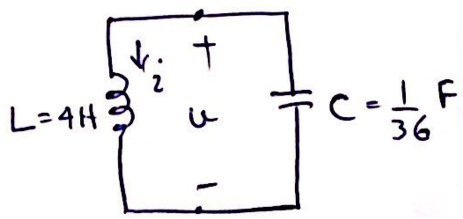
9.7 The Lossless LC Circuit ($\alpha < \omega_0$)

(pp 357 7th Ed HKD)

— when the value of the resistance in a parallel RLC circuit becomes infinite, or zero in a series RLC circuit, it results in a simple LC loop in which an oscillatory response can be maintained for ever.

— Consider



$i(0) = -\frac{1}{6} A$

and $v(0) = 0\ V$

— Now $\alpha = \frac{1}{2RC}$ parallel ——— R infinite $\alpha \Rightarrow 0$

or $\alpha = \frac{R}{2L}$ Series ——— R zero $\alpha \Rightarrow 0$

— However, $\omega_0 = \frac{1}{\sqrt{LC}}$ for both (resonant frequency)

— So in lossless circuit $\alpha = 0$ (undamped response)

and $\omega_0 = \frac{1}{\sqrt{4 \times \frac{1}{36}}} = \sqrt{9}$ (undamped natural frequency)

— The damped natural resonant frequency

$\omega_d = \sqrt{\omega_0^2 - \alpha^2} = \sqrt{9 - 0} = 3$ rad/s

(damped natural frequency)

contd

— contd (357)

— In the absence of exponential damping;

$$v = (A\cos 3t + B\sin 3t)\cancel{e^{-\alpha t}}^{\,=1}$$

Since $v(0) = 0$

$$0 = A \times 1 + B \times 0$$

$$A = 0$$

So $\quad v = B\sin 3t$

Now $\quad \dfrac{dv}{dt} = 3B\cos 3t \Big|_{t=0} = 3B$

Also $\quad \dfrac{dv}{dt}\Big|_{t=0} = -\dfrac{1}{C}\, i(0^+) \quad$ (The −ve sign because $i(0^+)$ enters −ve terminal of v of capacitor)

In our case due passive sign convention

$$\frac{dv}{dt}\Big|_{t=0} = -\frac{-1/6}{1/36} = 6 \qquad \text{because } i(0^+) = -\tfrac{1}{6}$$

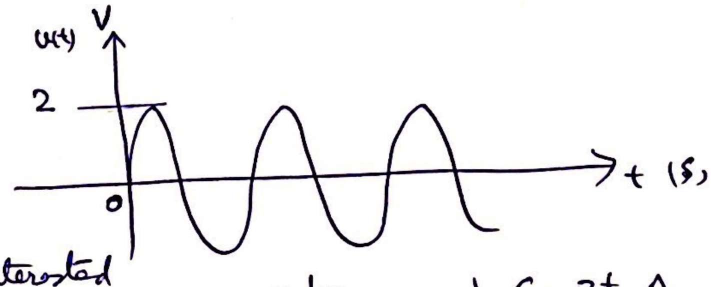
So $\quad 3B = 6$

$$B = 2$$

Hence

$$v(t) = 2\sin 3t \ \ V$$

Graphically;



If we are interested in current i it is $= -C\dfrac{dv}{dt} = -\dfrac{1}{6}\cos 3t$ A

contd

— contd (357)

Alternately we can start with

$$i(t) = A \cos 3t + B \sin 3t$$

$$i(0) = -\frac{1}{6}$$

$$-\frac{1}{6} = A + B \times 0$$

$$\text{So } A = -\frac{1}{6} \quad \text{———— (1)}$$

And $$\left.\frac{di}{dt}\right|_{t=0} = -3A \sin 3t + 3B \cos 3t$$

$$\left.\frac{di}{dt}\right|_{t=0} = 3B$$

Now $$v_L = L\frac{di}{dt}$$

$$\frac{v_L(0^+)}{L} = \left.\frac{di}{dt}\right|_{t=0^+}$$ $\left\{ \text{because } v_L(0^+) = v_c(0^+) \text{ in parallel} \right\}$

$$\frac{0}{L} = \left.\frac{di}{dt}\right|_{t=0^+}$$

$$\left.\frac{di}{dt}\right|_{t=0^+} = 0$$

So $$3B = 0$$
$$B = 0$$

Hence $$i(t) = -\frac{1}{6} \cos 3t \text{ A}$$

as before.