Engineering Mechanics

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Research Interest: Photovoltaic systems, Power & Control, Sliding mode control (SMC)

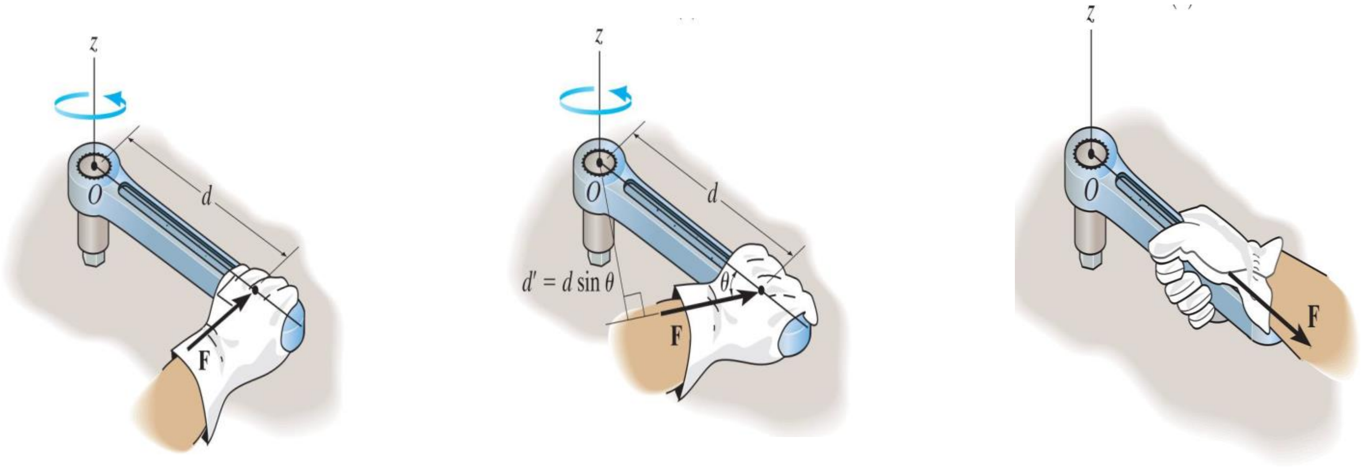
Contents (Section 4.7& 4.8)

- Recap
- Simplificationof a Force and Couple System
- Further Simplification of a Force and Couple System

RECAP

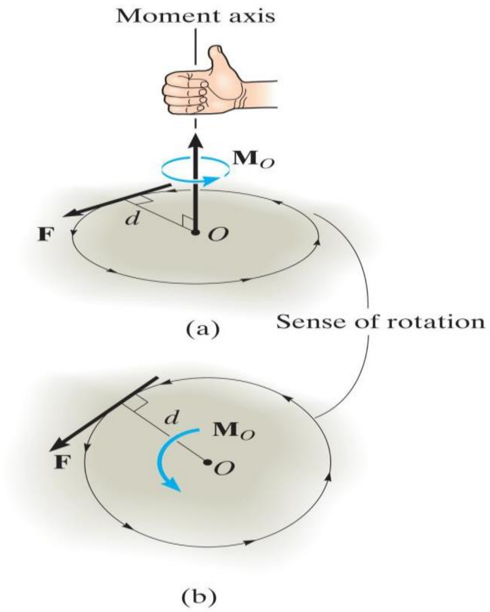
Moment of a Force/Moment/Torque (Scalar)

Definition



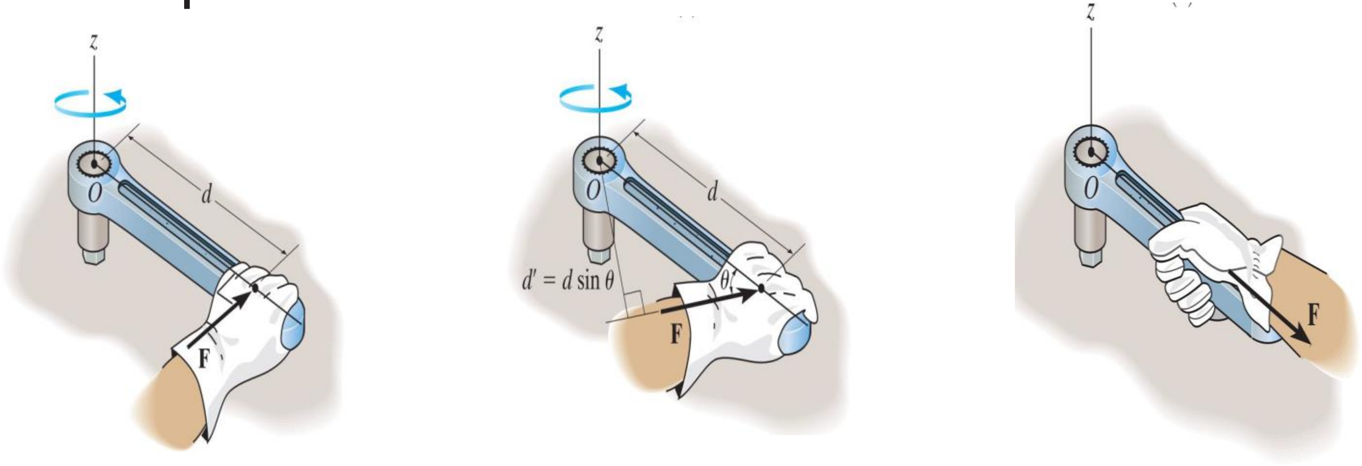
Magnitude

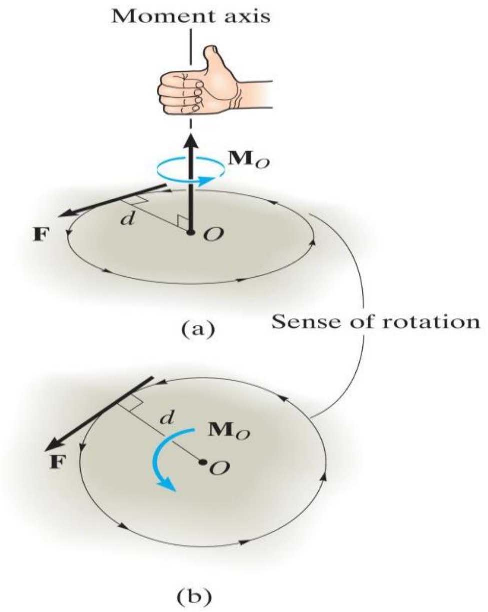
Direction



Moment of a Force/Moment/Torque (Scalar)

When a force is applied to a body it will produce a tendency for the body to rotate about a point that is not on the line of action of the force





Magnitude. The magnitude of $\mathbf{M}_O$ is

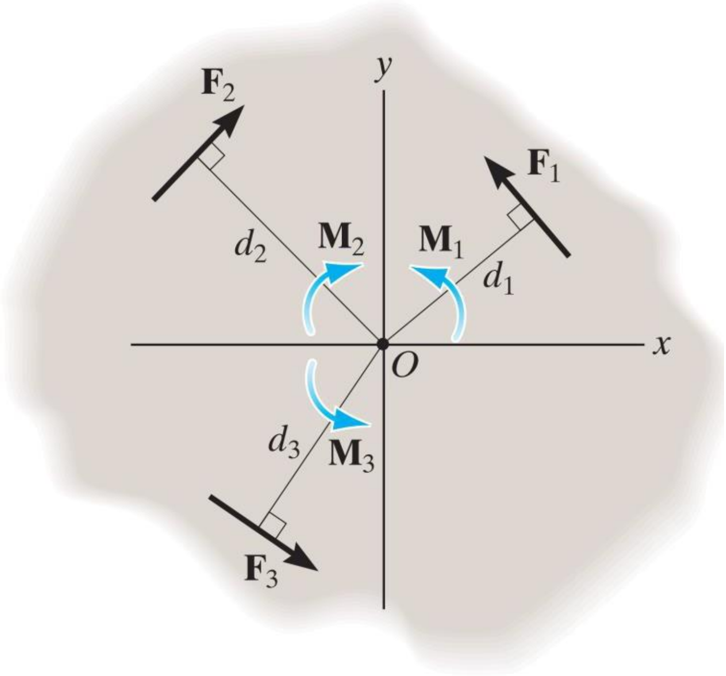
$$\boxed{M_O = Fd}$$

Direction: Direction using "right hand rule"

the thumb of the right hand will give the directional sense Mo

Moment of a Force/Moment/Torque (Scalar)

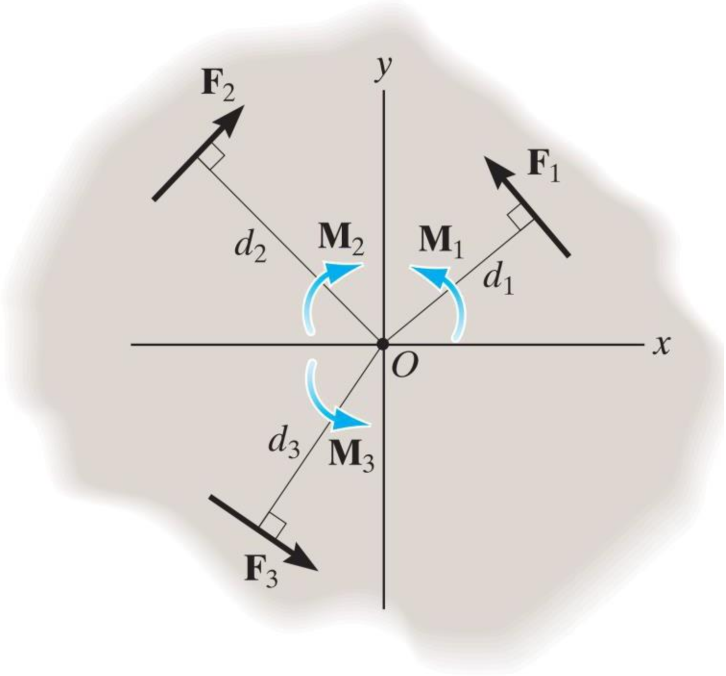
Resultant Moment.



Moment of a Force/Moment/Torque (Scalar)

Resultant Moment.

$$\circlearrowleft + (M_R)_O = \Sigma Fd; \qquad (M_R)_O = F_1 d_1 - F_2 d_2 + F_3 d_3$$

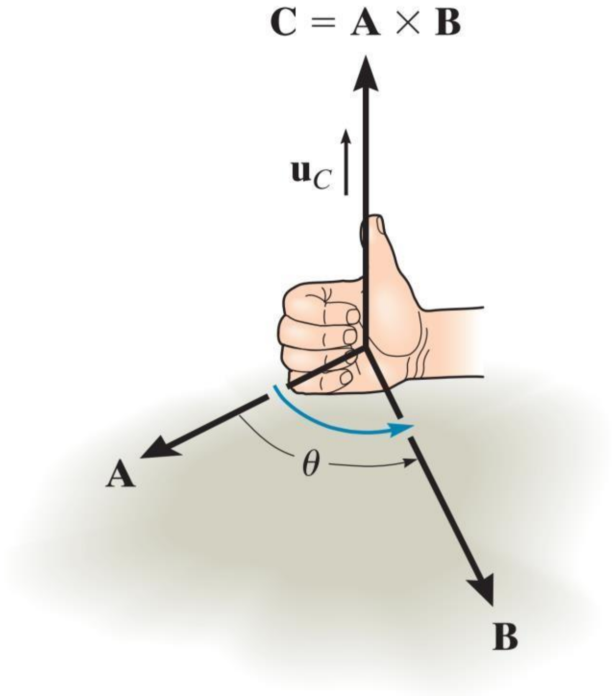


Cross Product

The ***cross product*** of two vectors **A** and **B** yields the vector **C**

Magnitude.

Direction.



Cross Product

The *cross product* of two vectors **A** and **B** yields the vector **C**

$$\mathbf{C} = \mathbf{A} \times \mathbf{B}$$

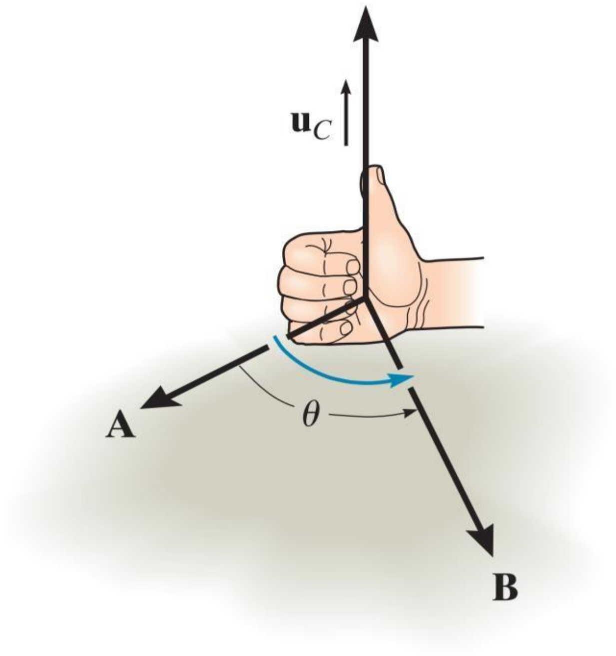
$$\mathbf{A} \times \mathbf{B} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$$

Magnitude.

$$C = AB \sin \theta.$$

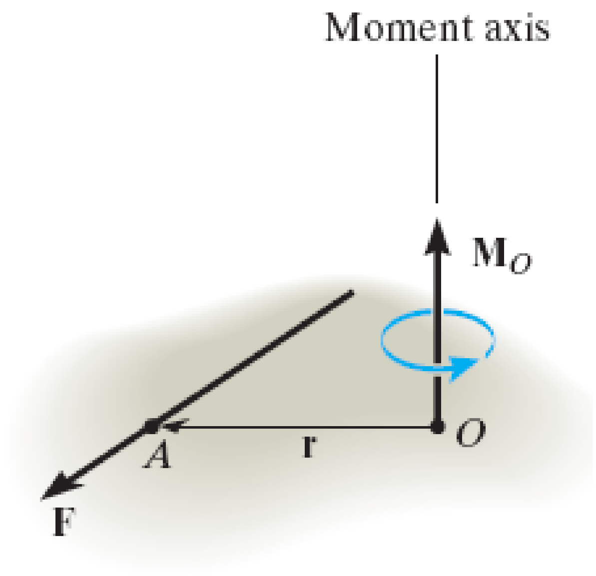
Direction.

$$\mathbf{C} = \mathbf{A} \times \mathbf{B} = (AB \sin \theta)\mathbf{u}_C$$



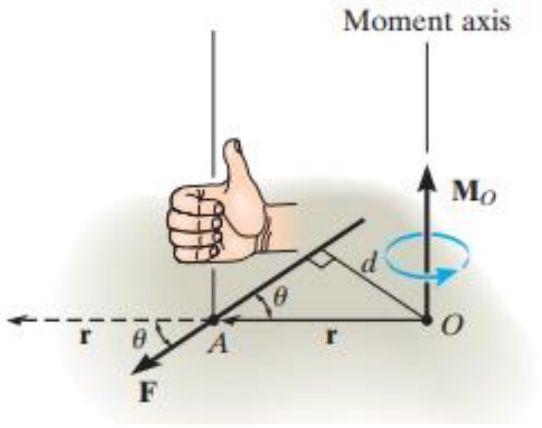
Moment of Force - Vector Formulation

- Moment of force **F** about point O can be expressed using cross product



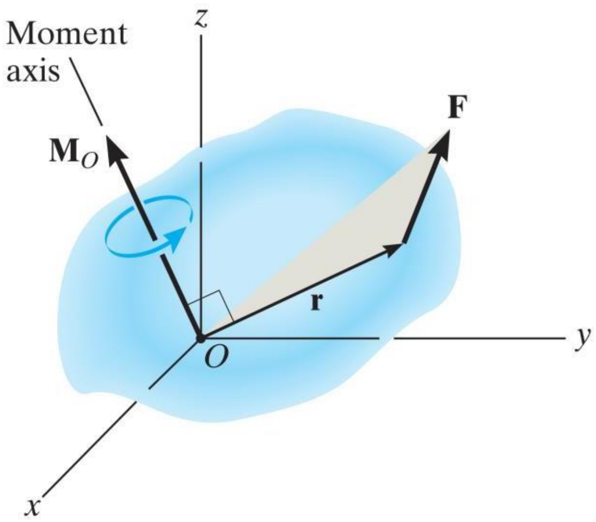
Magnitude….

Direction…



Moment of a Force (Vector)

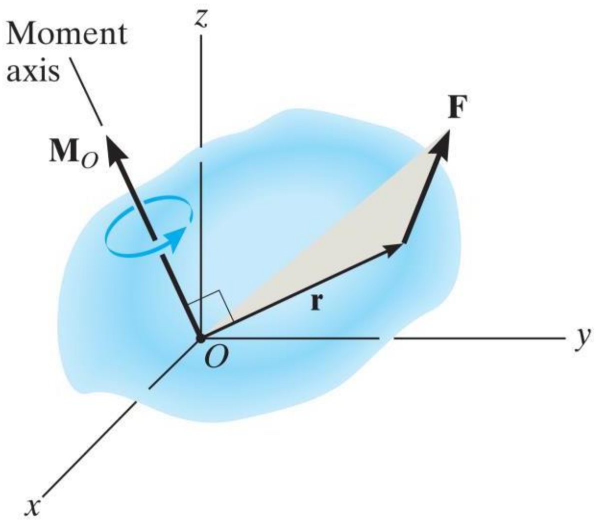
Cartesian Vector Formulation.



Moment of a Force (Vector)

Cartesian Vector Formulation.

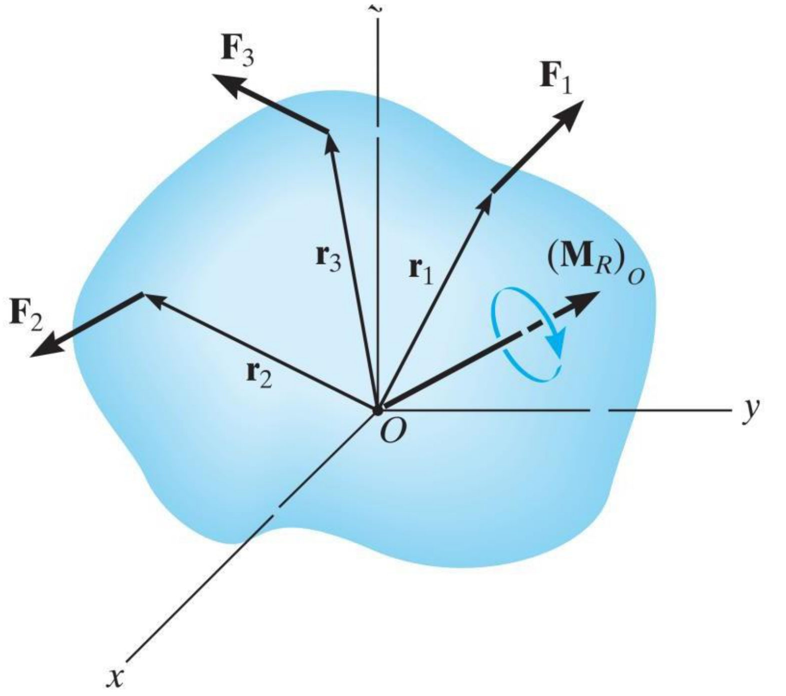
$$\mathbf{M}_O = \mathbf{r} \times \mathbf{F} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ r_x & r_y & r_z \\ F_x & F_y & F_z \end{vmatrix}$$



$$\mathbf{M}_O = (r_y F_z - r_z F_y)\mathbf{i} - (r_x F_z - r_z F_x)\mathbf{j} + (r_x F_y - r_y F_x)\mathbf{k}$$

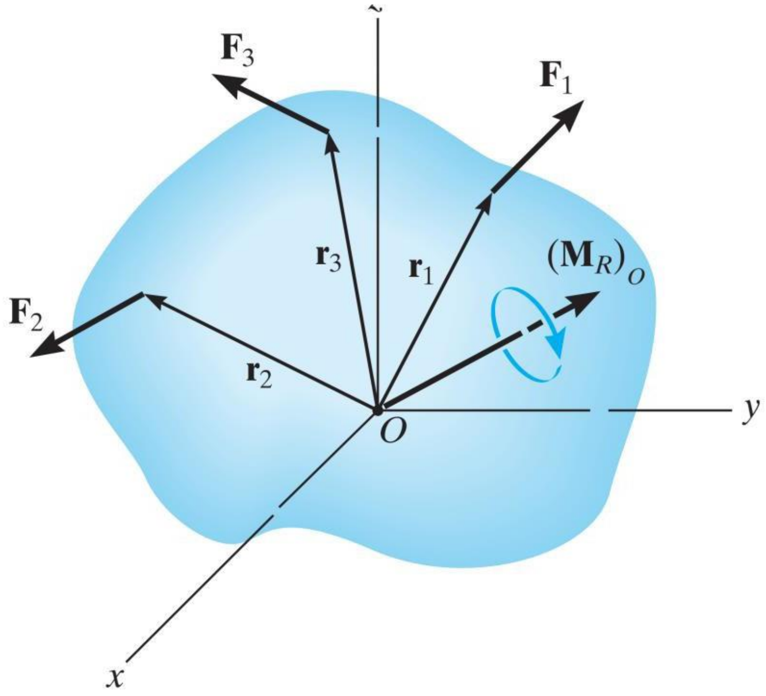
Moment of a Force (Vector)

Resultant Moment of a System of Forces.



Moment of a Force (Vector)

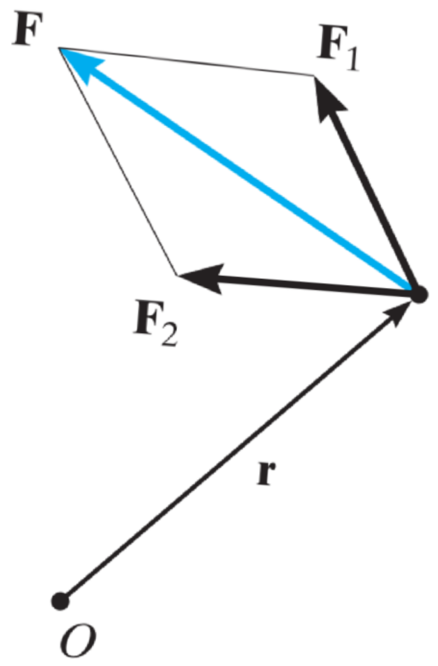
Resultant Moment of a System of Forces.



$$(\mathbf{M}_R)_O = \Sigma(\mathbf{r} \times \mathbf{F})$$

Principle of Moments (Varignon's Theorem)

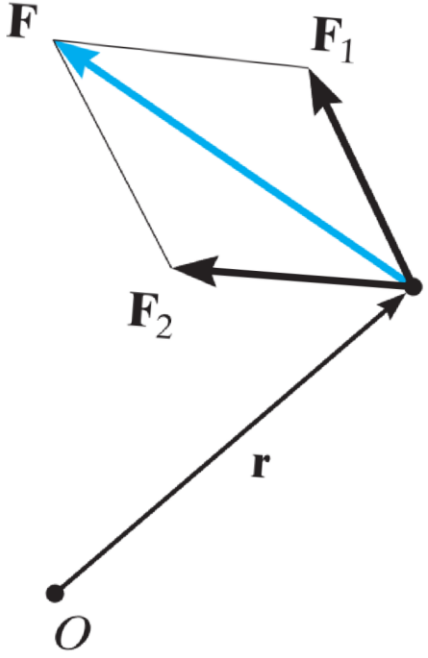
Definition/Statement



Principle of Moments (Varignon's Theorem)

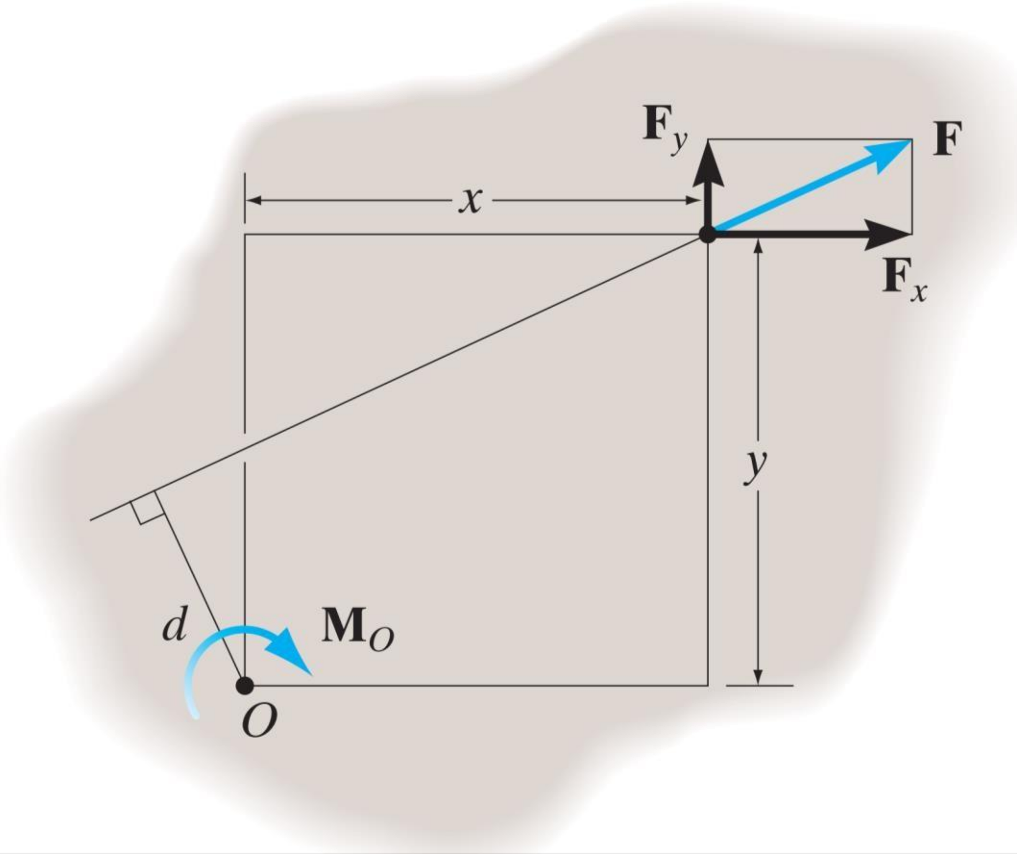
Definition/Statement

The moment of a force about a point is equal to the sum of the moments of the components of the force about the point



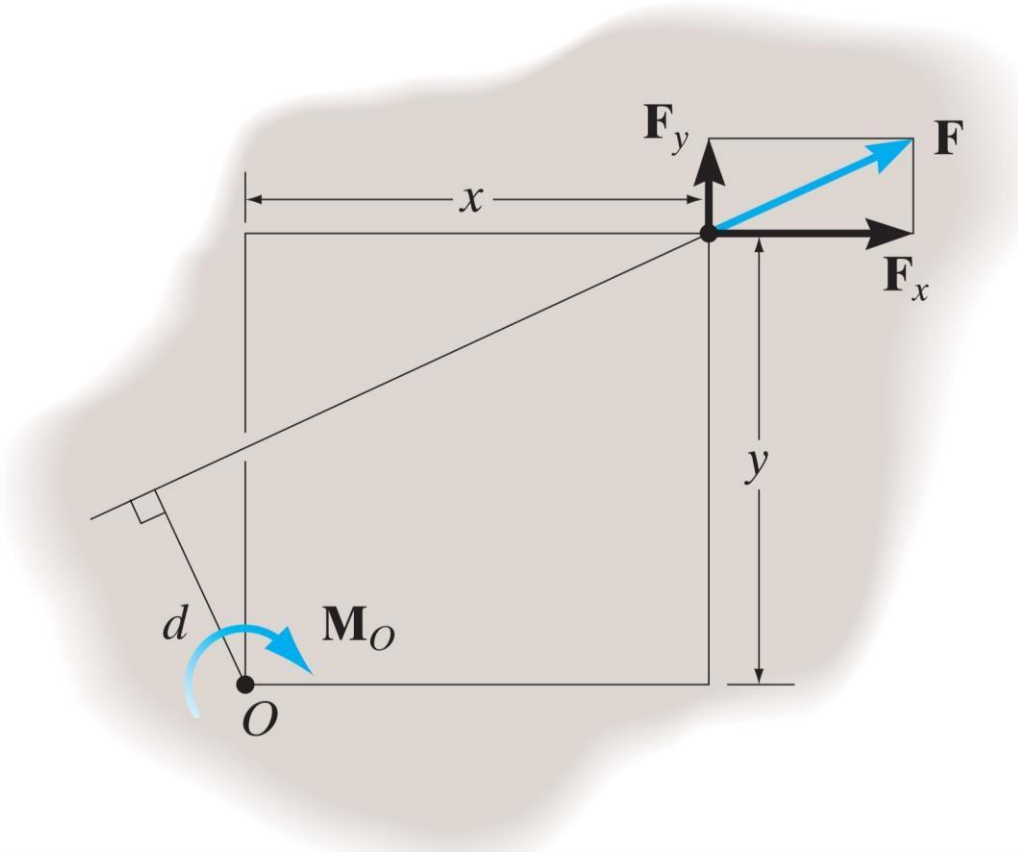
$$\mathbf{M}_O = \mathbf{r} \times \mathbf{F} = \mathbf{r} \times (\mathbf{F}_1 + \mathbf{F}_2) = \mathbf{r} \times \mathbf{F}_1 + \mathbf{r} \times \mathbf{F}_2$$

Principle of Moments (Varignon's Theorem)



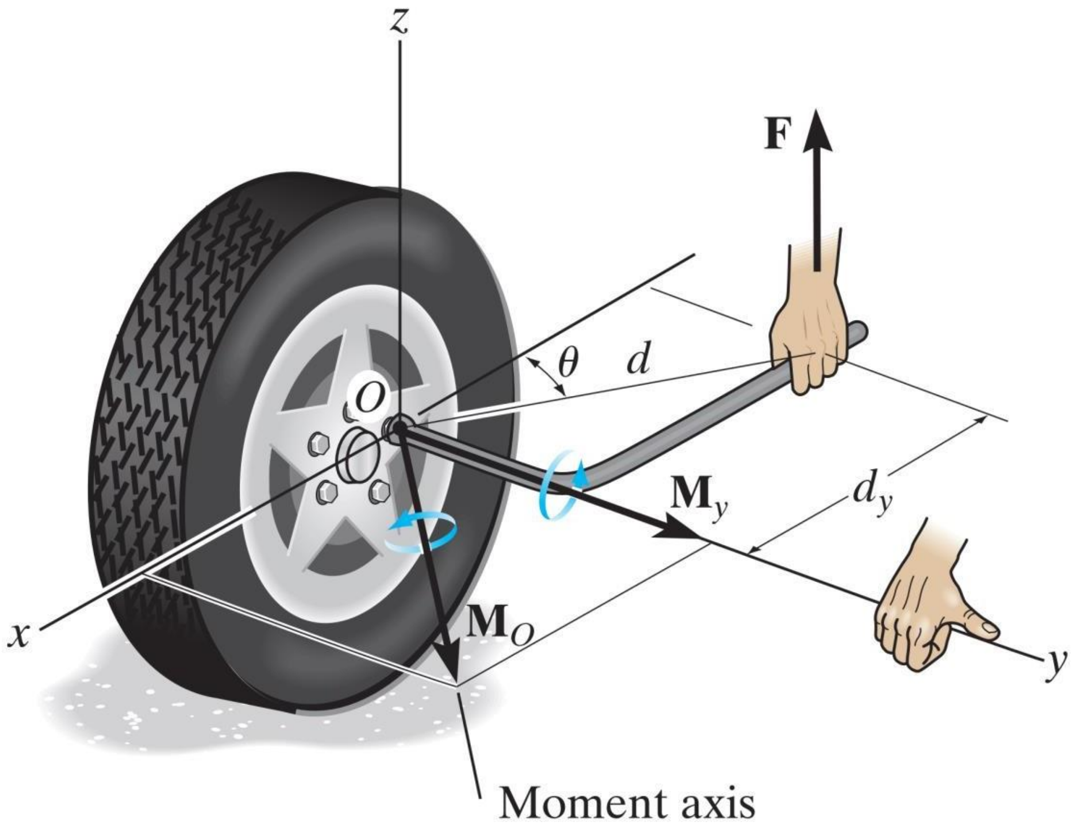
Principle of Moments (Varignon's Theorem)

$$M_O = F_x y - F_y x$$



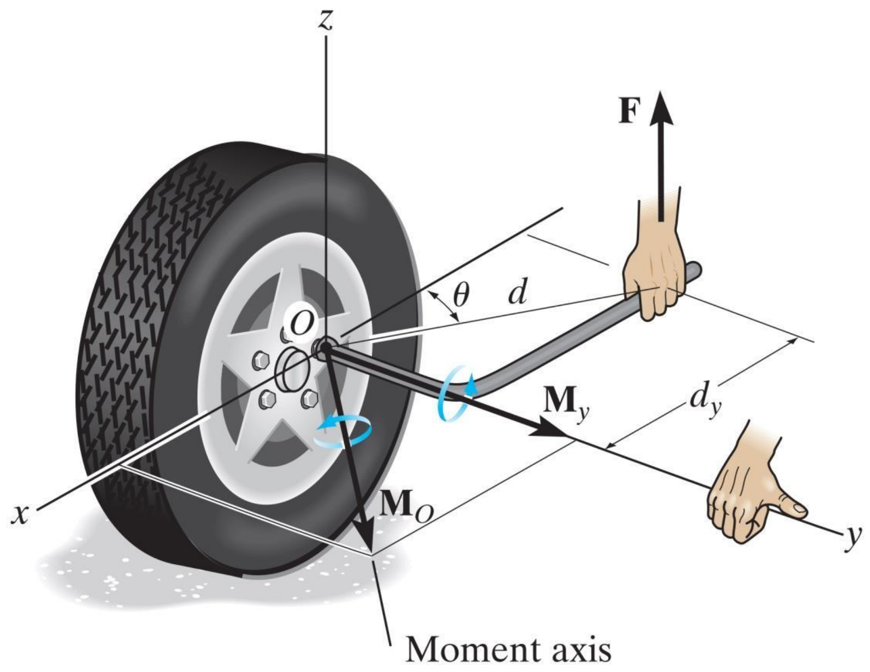
Moment of a Force about a Specified Axis

Scalar Analysis.



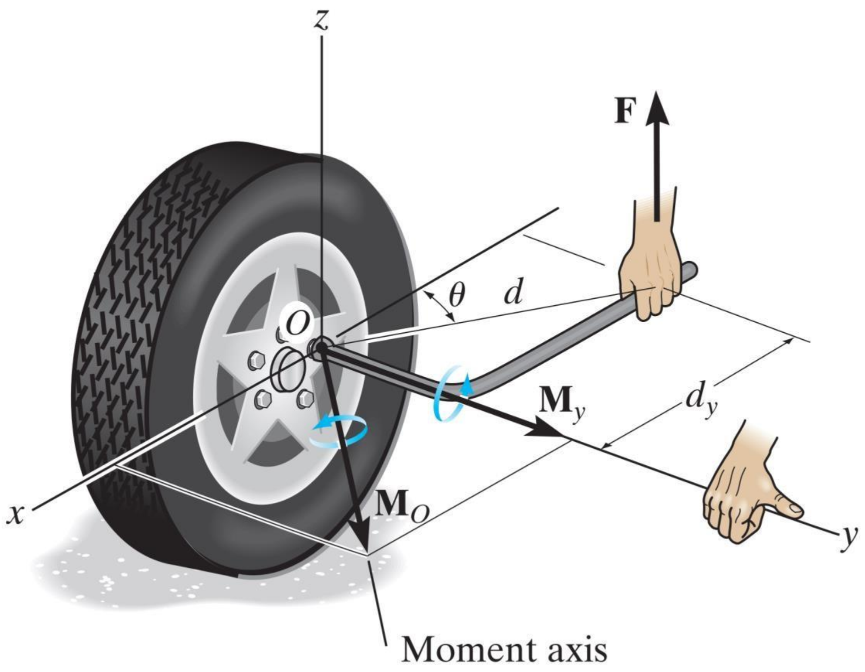
Moment of a Force about a Specified Axis

Vector Analysis.



Moment of a Force about a Specified Axis

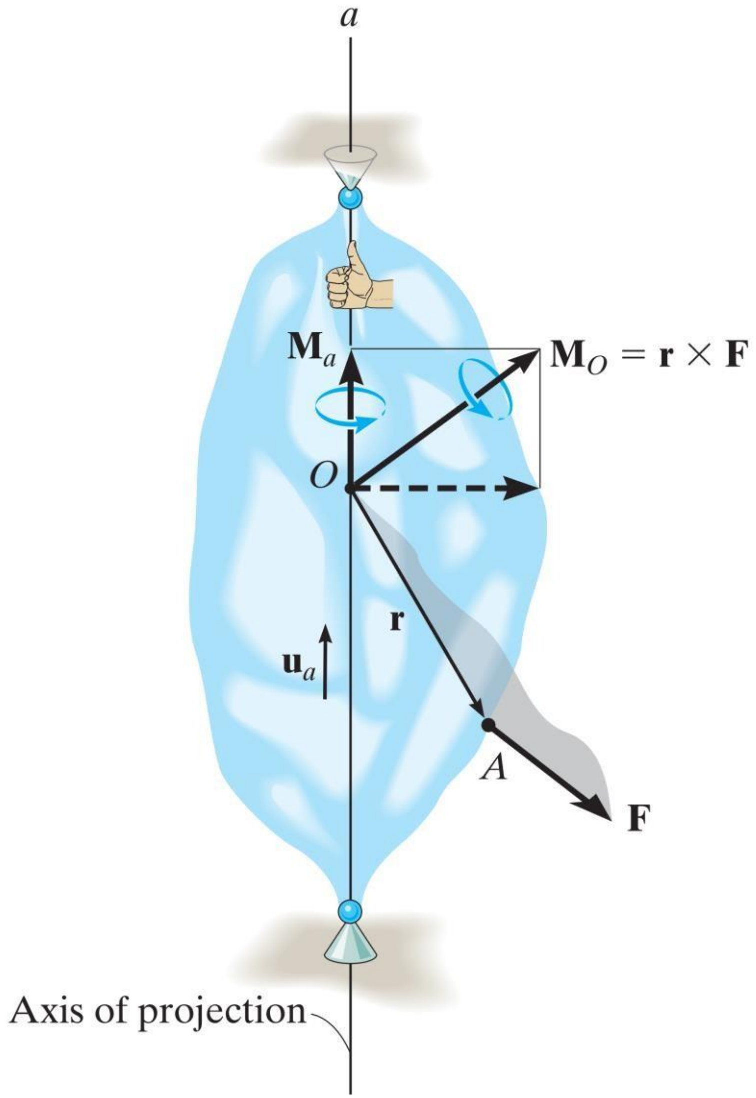
Vector Analysis.



$$M_a = [u_{a_x}\mathbf{i} + u_{a_y}\mathbf{j} + u_{a_z}\mathbf{k}] \cdot \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ r_x & r_y & r_z \\ F_x & F_y & F_z \end{vmatrix}$$

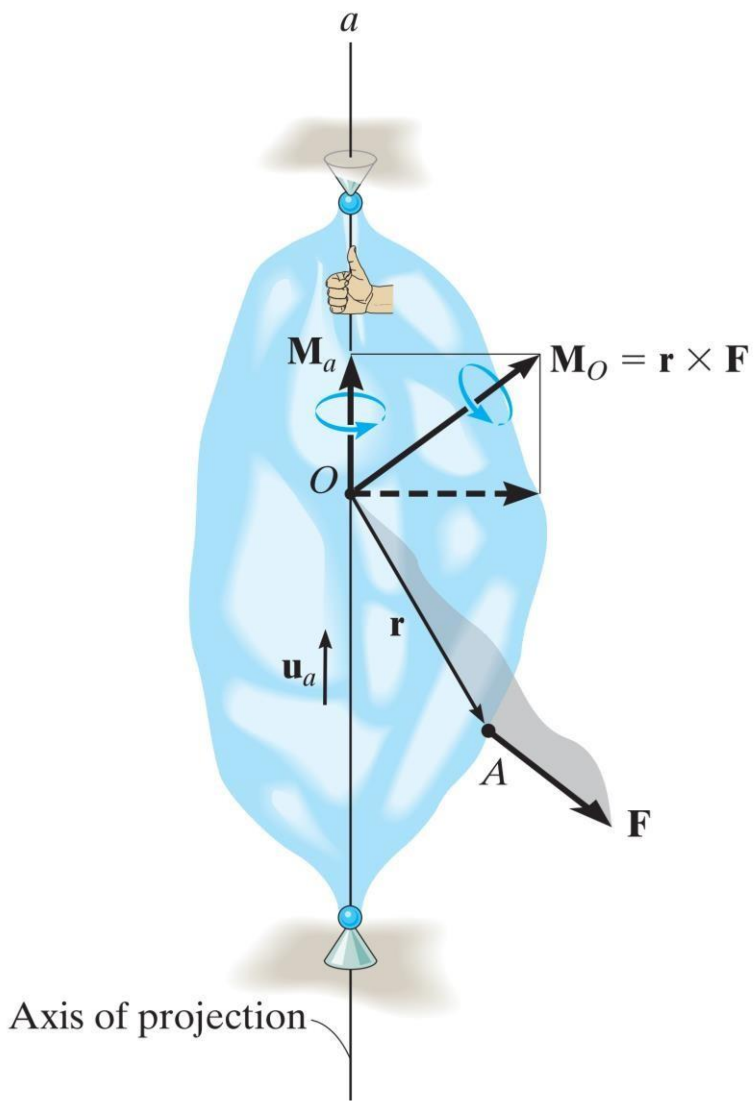
$$= u_{a_x}(r_yF_z - r_zF_y) - u_{a_y}(r_xF_z - r_zF_x) + u_{a_z}(r_xF_y - r_yF_x)$$

Moment of a Force about a Specified Axis



Moment of a Force about a Specified Axis

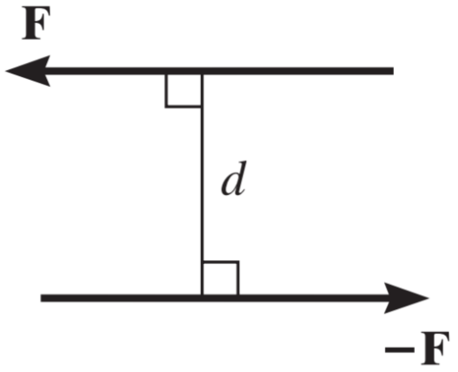
$$M_a = \mathbf{u}_a \cdot (\mathbf{r} \times \mathbf{F}) = \begin{vmatrix} u_{a_x} & u_{a_y} & u_{a_z} \\ r_x & r_y & r_z \\ F_x & F_y & F_z \end{vmatrix}$$

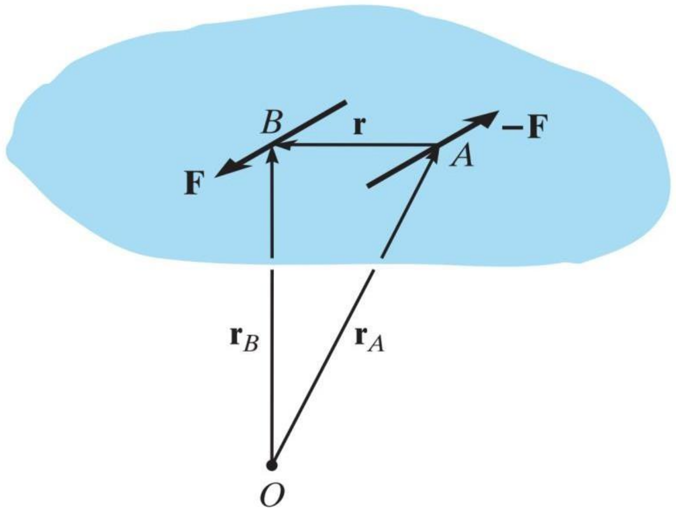


$u_{a_x}, u_{a_y}, u_{a_z}$	represent the x, y, z components of the unit vector defining the direction of the a axis
r_x, r_y, r_z	represent the x, y, z components of the position vector extended from *any point* O on the a axis to *any point* A on the line of action of the force
F_x, F_y, F_z	represent the x, y, z components of the force vector.

Moment of a Couple

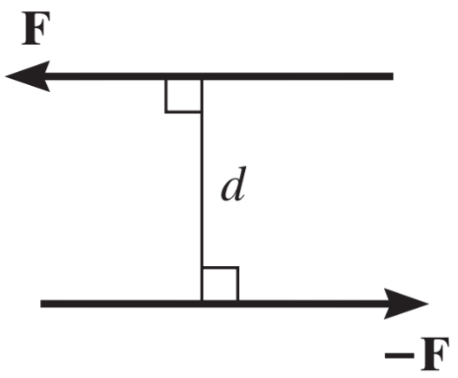
Couple:……

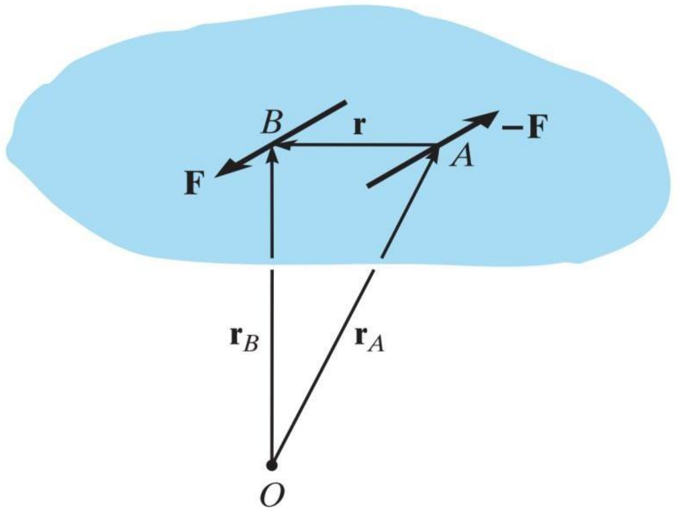




Moment of a Couple

Couple: Two equal and parallel but opposite forces separated by a distance





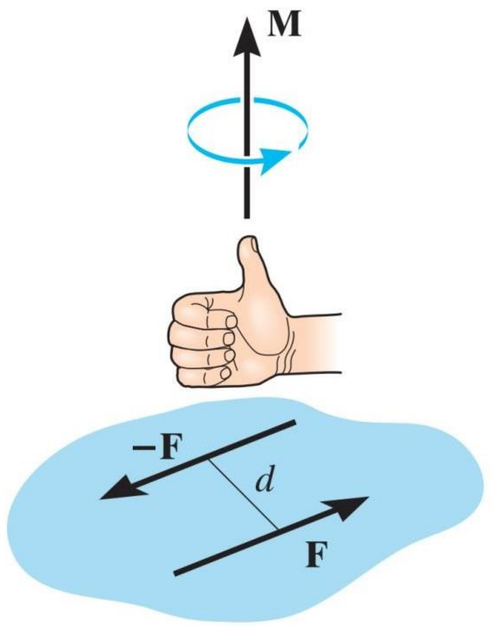
$$\mathbf{M} = \mathbf{r}_B \times \mathbf{F} + \mathbf{r}_A \times -\mathbf{F} = (\mathbf{r}_B - \mathbf{r}_A) \times \mathbf{F}$$

However $\mathbf{r}_B = \mathbf{r}_A + \mathbf{r}$ or $\mathbf{r} = \mathbf{r}_B - \mathbf{r}_A$, so that

$$\mathbf{M} = \mathbf{r} \times \mathbf{F}$$

Moment of a Couple

Scalar Formulation.



Vector Formulation.

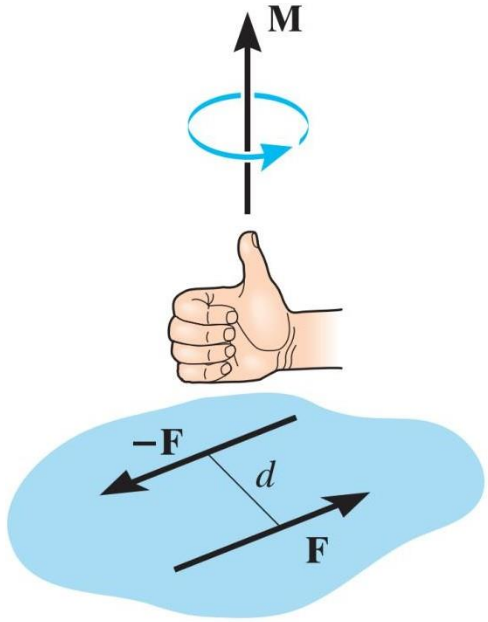
Moment of a Couple

Scalar Formulation.

$$\boxed{M = Fd}$$

Vector Formulation.

$$\boxed{\mathbf{M} = \mathbf{r} \times \mathbf{F}}$$



4.7 Simplification of a Force and Couple System

A system is equivalent if the *external effects* it produces on a body are the same as those caused by the original force and couple moment system.

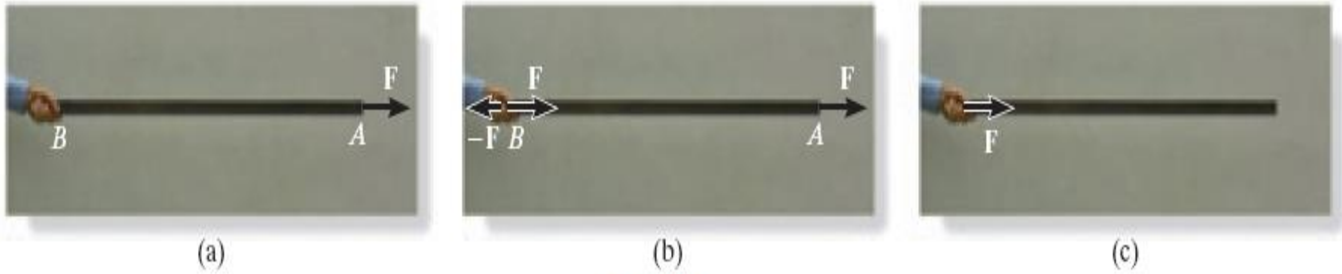


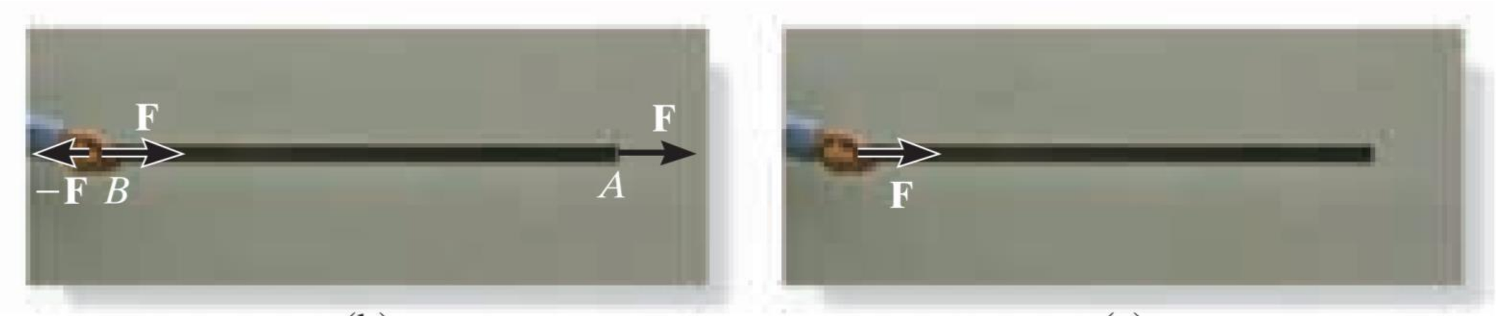
Fig. 4-34

For example, consider holding the stick in Fig. 4–34*a*, which is subjected to the force **F** at point *A*. If we attach a pair of equal but opposite forces **F** and **–F** at point *B*, which is *on the line of action* of **F**, Fig. 4–34*b*, we observe that **–F** at *B* and **F** at *A* will cancel each other, leaving only **F** at *B*, Fig. 4–34*c*. Force **F** has now been moved from *A* to *B* without modifying its *external effects* on the stick; i.e., the reaction at the grip remains the same. This demonstrates the *principle of transmissibility*, which states that a force acting on a body (stick) is a *sliding vector* since it can be applied at any point along its line of action.

Simplification of a Force and Couple System

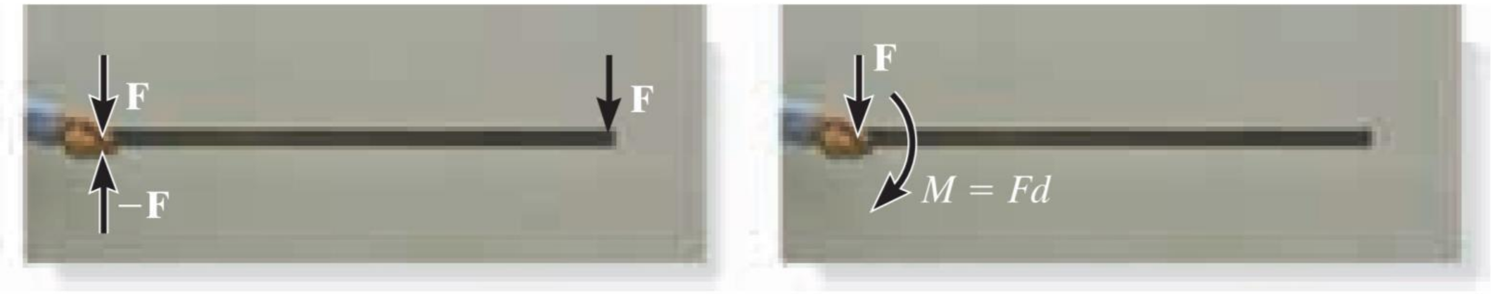
- **Sometimes it is convenient to reduce a system of forces and couple moments acting on a body to a simpler form by replacing it with an *equivalent system***, consisting of a single resultant force acting at a specific point and a resultant couple moment.
- An equivalent system is when the *external effects* are the same as those caused by the original force and couple moment system
- External effects of a system is the *translating and rotating motion* of the body
- Or refers to the *reactive forces* at the supports if the body is held fixed

Simplification of a Force and Couple System



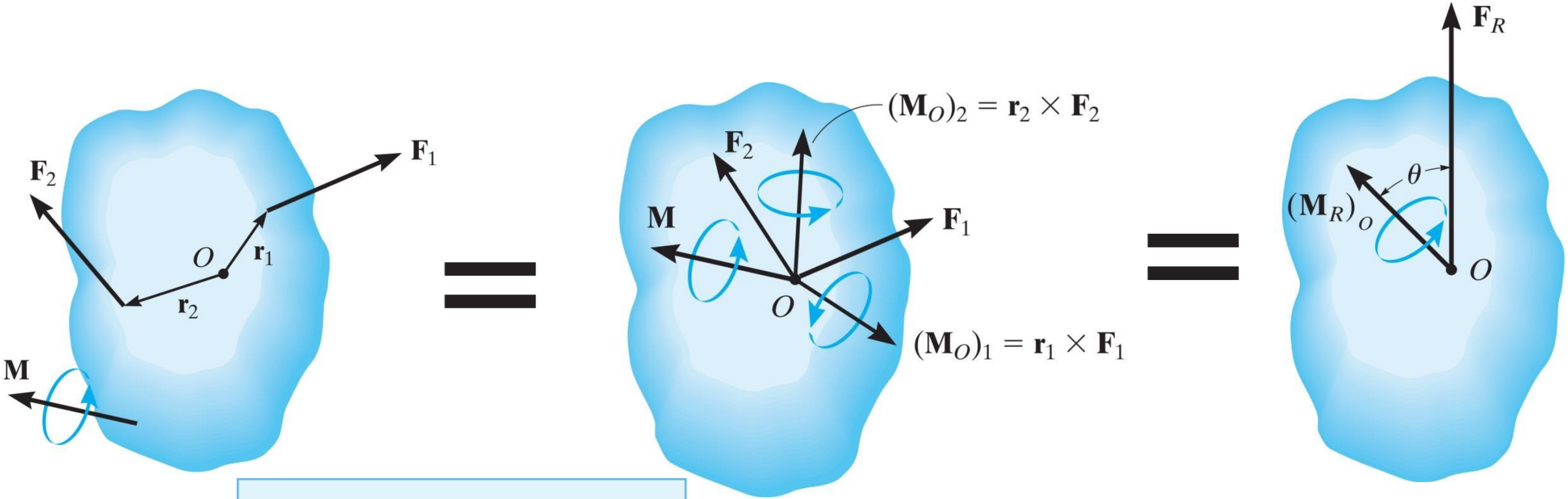
(b) (c)

Fig. 4–34 (© Russell C. Hibbeler)



Simplification of a Force and Couple System

System of Forces and Couple Moments.



$$\mathbf{F}_R = \Sigma \mathbf{F}$$
$$(\mathbf{M}_R)_O = \Sigma \mathbf{M}_O + \Sigma \mathbf{M}$$

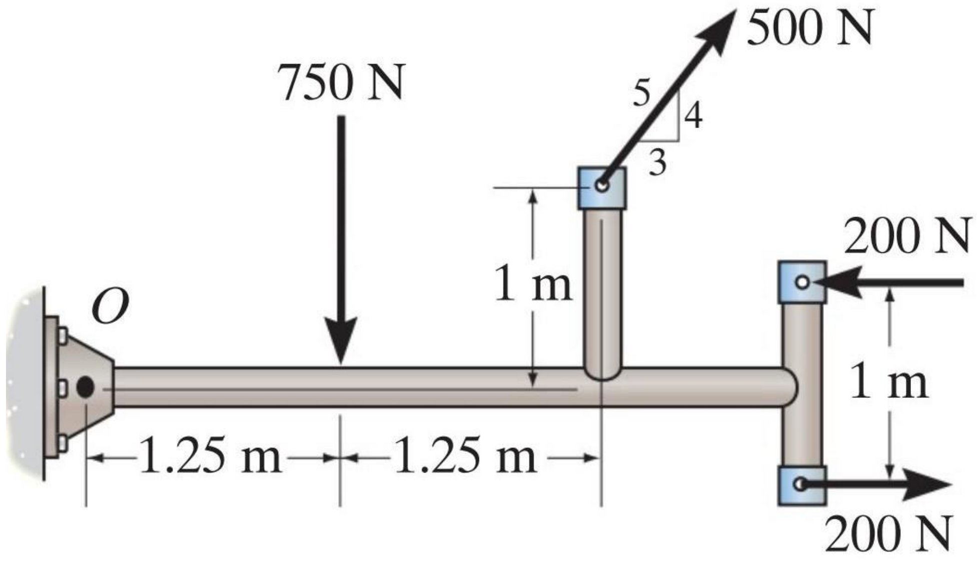
$$(F_R)_x = \Sigma F_x$$
$$(F_R)_y = \Sigma F_y$$
$$(M_R)_O = \Sigma M_O + \Sigma M$$

Simplification of a Force and Couple System

The weights of these traffic lights can be replaced by their equivalent resultant force $W_R = W_1 + W_2$ and a couple moment $(M_R)_O = W_1 d_1 + W_2 d_2$ at the support, O. In both cases the support must provide the same resistance to translation and rotation in order to keep the member in the horizontal position.

Example

Replace the force and couple system acting on the member in Fig. 4–38*a* by an equivalent resultant force and couple moment acting at point *O*.



Example

Replace the force and couple system acting on the member in Fig. 4–38*a* by an equivalent resultant force and couple moment acting at point O.

Force Summation.

$$\overset{+}{\rightarrow} (F_R)_x = \Sigma F_x; \; (F_R)_x = \left(\tfrac{3}{5}\right)(500 \text{ N}) = 300 \text{ N} \rightarrow$$

$$+\uparrow (F_R)_y = \Sigma F_y; \; (F_R)_y = (500 \text{ N})\left(\tfrac{4}{5}\right) - 750 \text{ N} = -350 \text{ N} = 350 \text{ N}\downarrow$$

From Fig. 4–15*b*, the magnitude of $\mathbf{F}_R$ is

$$F_R = \sqrt{(F_R)_x^2 + (F_R)_y^2}$$

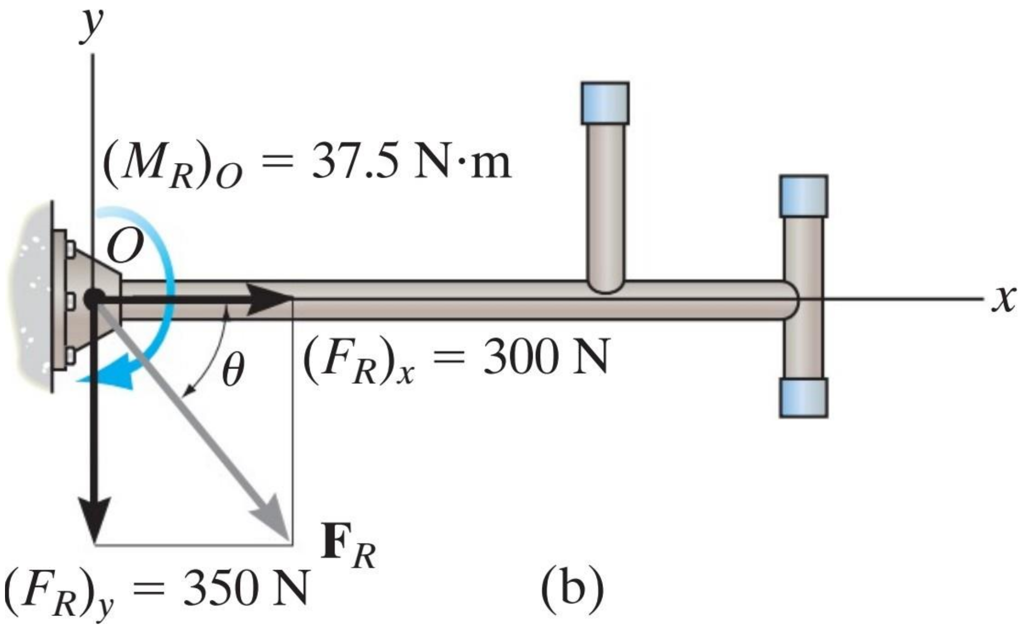
$$= \sqrt{(300 \text{ N})^2 + (350 \text{ N})^2} = 461 \text{ N}$$

Ans.

And the angle θ is

$$\theta = \tan^{-1}\left(\frac{(F_R)_y}{(F_R)_x}\right) = \tan^{-1}\left(\frac{350 \text{ N}}{300 \text{ N}}\right) = 49.4^\circ$$

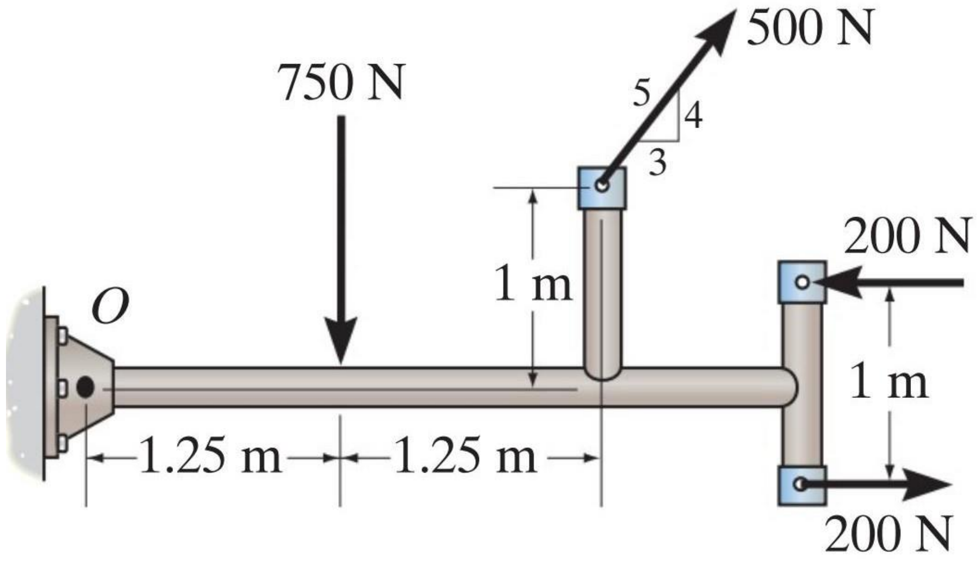
Ans.



(b)

Example

Replace the force and couple system acting on the member in Fig. 4–38*a* by an equivalent resultant force and couple moment acting at point *O*.

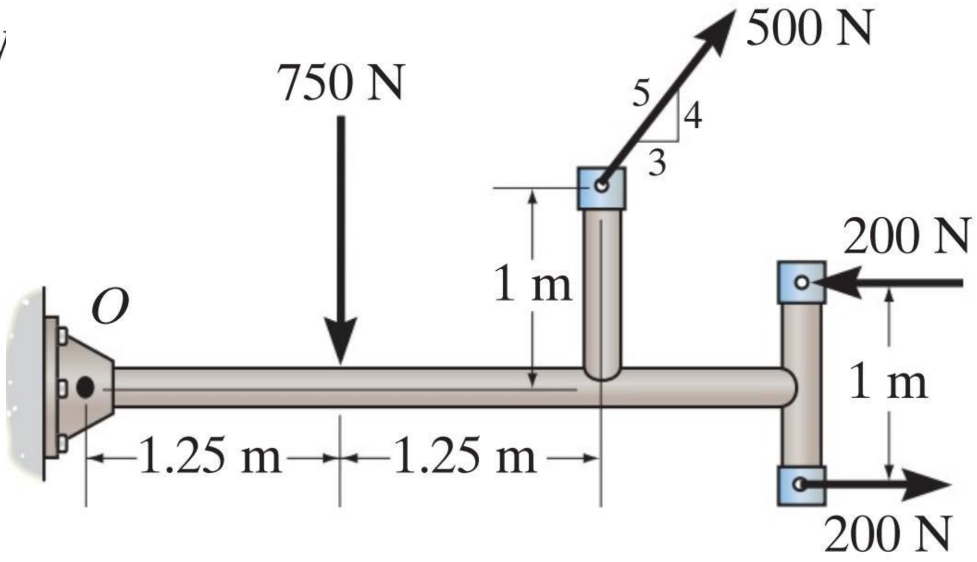


Example

Replace the force and couple system acting on the member in Fig. 4–38*a* by an equivalent resultant force and couple moment acting at point *O*.

Moment Summation. Since the couple moment is a fre

act at any point on the member. Referring to Fig. 4–38*a*, w



$$\circlearrowleft + (M_R)_O = \Sigma M_O + \Sigma M$$

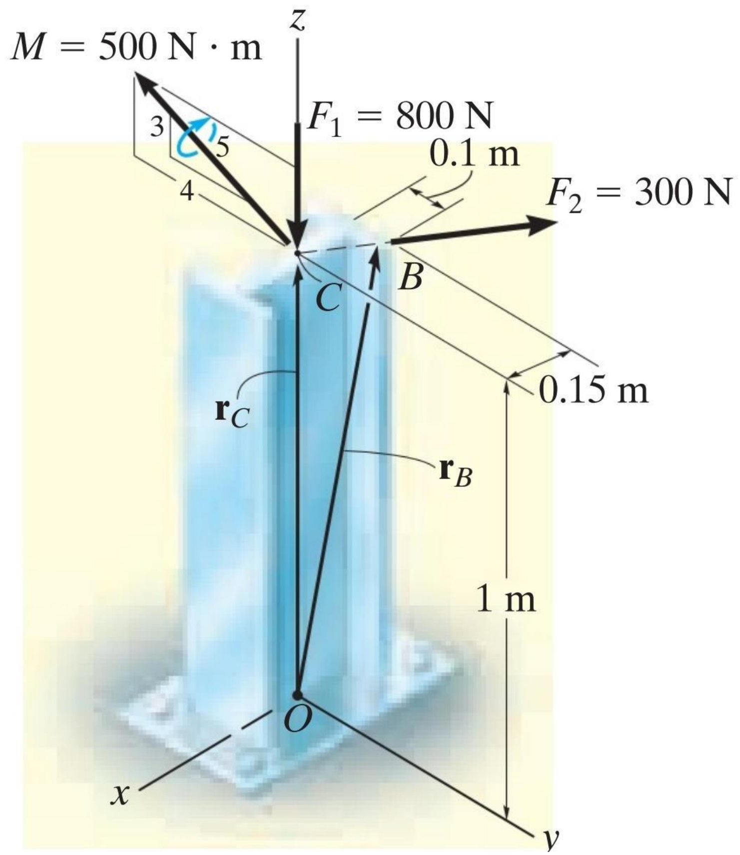
$$(M_R)_O = (500\text{ N})\left(\tfrac{4}{5}\right)(2.5\text{ m}) - (500\text{ N})\left(\tfrac{3}{5}\right)(1\text{ m})$$

$$- (750\text{ N})(1.25\text{ m}) + 200\text{ N}\cdot\text{m}$$

$$= -37.5\text{ N}\cdot\text{m} = 37.5\text{ N}\cdot\text{m} \circlearrowright$$

Example

The structural member is subjected to a couple moment **M** and forces $\mathbf{F}_1$ and $\mathbf{F}_2$ in Fig. 4–39*a*. Replace this system by an equivalent resultant force and couple moment acting at its base, point O.



Example

The structural member is subjected to a couple moment **M** and forces $\mathbf{F}_1$ and $\mathbf{F}_2$ in Fig. 4–39*a*. Replace this system by an equivalent resultant force and couple moment acting at its base, point *O*.

SOLUTION *(VECTOR ANALYSIS)*

The three-dimensional aspects of the problem can be simplified by using a Cartesian vector analysis. Expressing the forces and couple moment as Cartesian vectors, we have

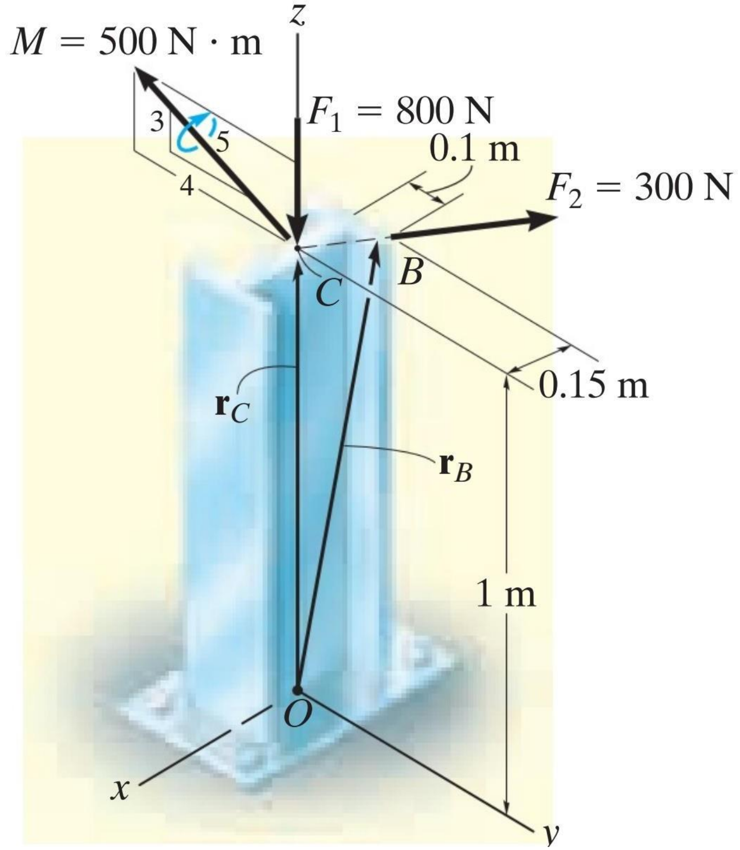
$$\mathbf{F}_1 = \{-800\mathbf{k}\}\text{ N}$$

$$\mathbf{F}_2 = (300\text{ N})\mathbf{u}_{CB}$$

$$= (300\text{ N})\left(\frac{\mathbf{r}_{CB}}{r_{CB}}\right)$$

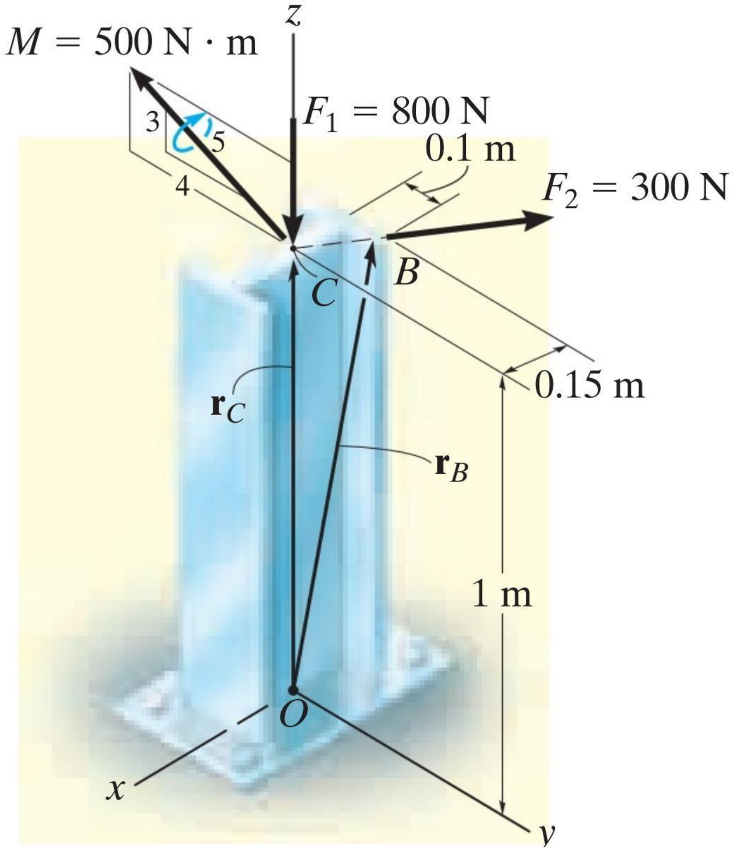
$$= 300\text{ N}\left[\frac{\{-0.15\mathbf{i} + 0.1\mathbf{j}\}\text{ m}}{\sqrt{(-0.15\text{ m})^2 + (0.1\text{ m})^2}}\right] = \{-249.6\mathbf{i} + 166.4\mathbf{j}\}\text{ N}$$

$$\mathbf{M} = -500\left(\tfrac{4}{5}\right)\mathbf{j} + 500\left(\tfrac{3}{5}\right)\mathbf{k} = \{-400\mathbf{j} + 300\mathbf{k}\}\text{ N}\cdot\text{m}$$



Example

The structural member is subjected to a couple moment **M** and forces $\mathbf{F}_1$ and $\mathbf{F}_2$ in Fig. 4–39*a*. Replace this system by an equivalent resultant force and couple moment acting at its base, point *O*.



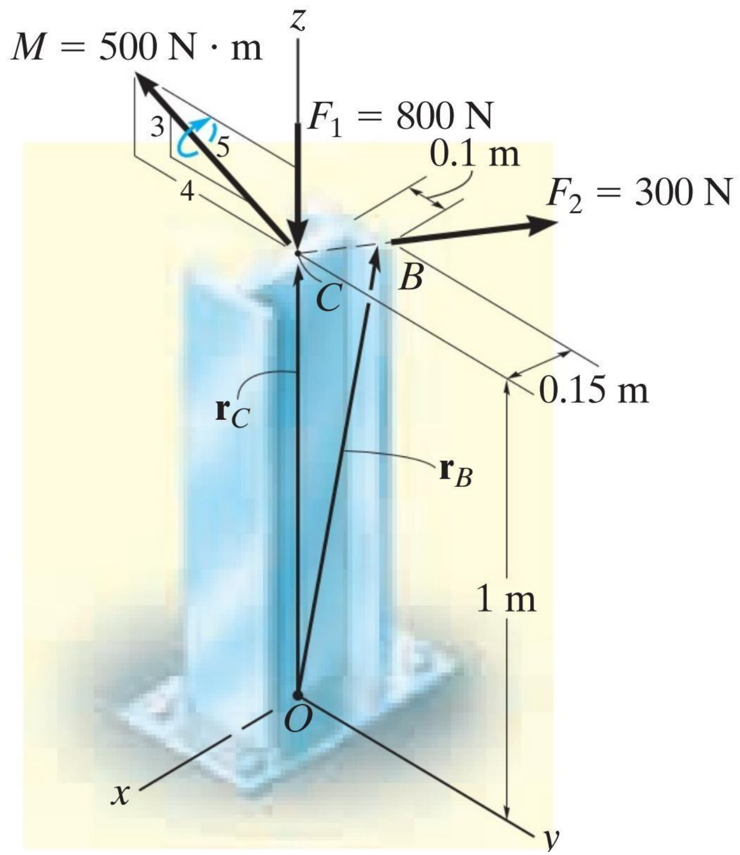
Force Summation.

$$\mathbf{F}_R = \Sigma\mathbf{F}; \qquad \mathbf{F}_R = \mathbf{F}_1 + \mathbf{F}_2 = -800\mathbf{k} - 249.6\mathbf{i} + 166.4\mathbf{j}$$

$$= \{-250\mathbf{i} + 166\mathbf{j} - 800\mathbf{k}\}\ \text{N}$$

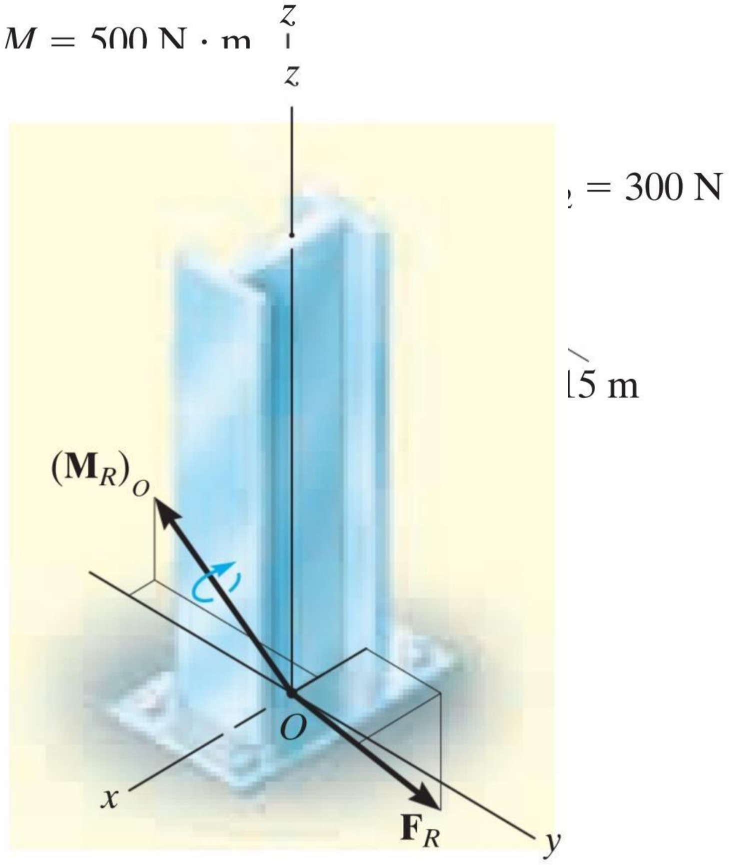
Example

The structural member is subjected to a couple moment **M** and forces $\mathbf{F}_1$ and $\mathbf{F}_2$ in Fig. 4–39*a*. Replace this system by an equivalent resultant force and couple moment acting at its base, point O.



Example

The structural member is subjected to a couple moment **M** and forces $\mathbf{F}_1$ and $\mathbf{F}_2$ in Fig. 4–39*a*. Replace this system by an equivalent resultant force and couple moment acting at its base, point *O*.



$$(\mathbf{M}_R)_o = \Sigma\mathbf{M} + \Sigma\mathbf{M}_O$$

$$(\mathbf{M}_R)_o = \mathbf{M} + \mathbf{r}_C \times \mathbf{F}_1 + \mathbf{r}_B \times \mathbf{F}_2$$

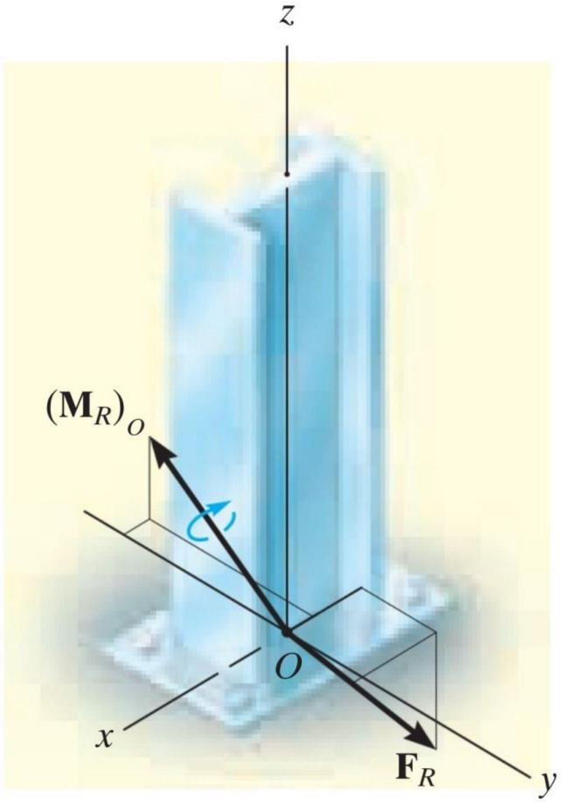
$$(\mathbf{M}_R)_o = (-400\mathbf{j} + 300\mathbf{k}) + (1\mathbf{k}) \times (-800\mathbf{k}) + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -0.15 & 0.1 & 1 \\ -249.6 & 166.4 & 0 \end{vmatrix}$$

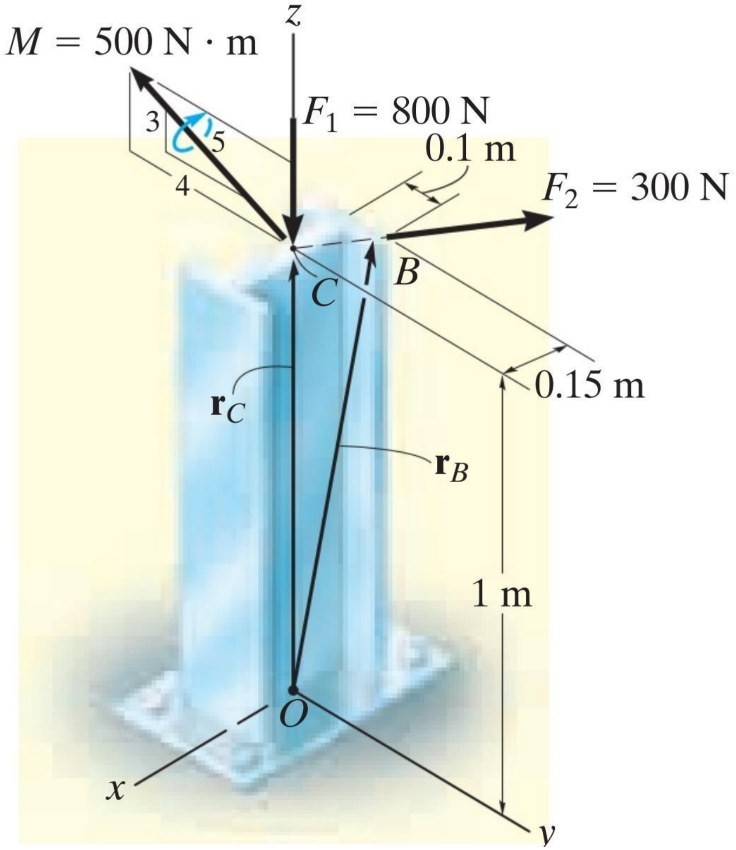
$$= (-400\mathbf{j} + 300\mathbf{k}) + (0) + (-166.4\mathbf{i} - 249.6\mathbf{j})$$

$$= \{-166\mathbf{i} - 650\mathbf{j} + 300\mathbf{k}\}\ \mathrm{N \cdot m}$$

Example

The structural member is subjected to a couple moment **M** and forces $\mathbf{F}_1$ and $\mathbf{F}_2$ in Fig. 4–39*a*. Replace this system by an equivalent resultant force and couple moment acting at its base, point *O*.





Home Assignment

- Example 4.14.