Engineering Mechanics : Statics

Chapter 12: kinematics of a particle

Chapter outline

- Introduction
- Rectilinear Kinematics: Continuous Motion
- Rectilinear Kinematics: Erratic Motion
- General Curvilinear
- Curvilinear Motion: Rectangular Motion
- Projectile

$$\boxed{v = \frac{ds}{dt}}$$

$$\boxed{a = \frac{dv}{dt}}$$

$$a = \frac{d^2 s}{dt^2}$$

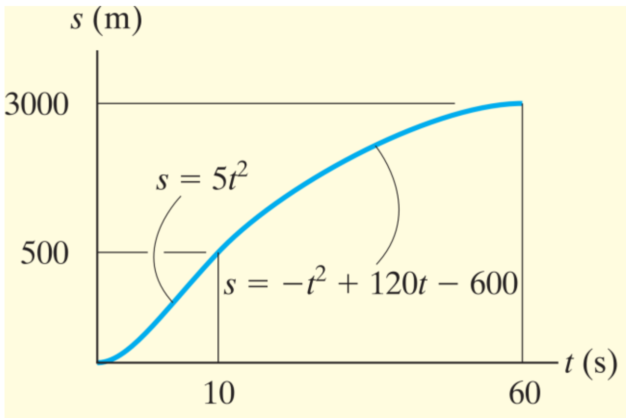
Constant Acceleration, $a = a_c$.

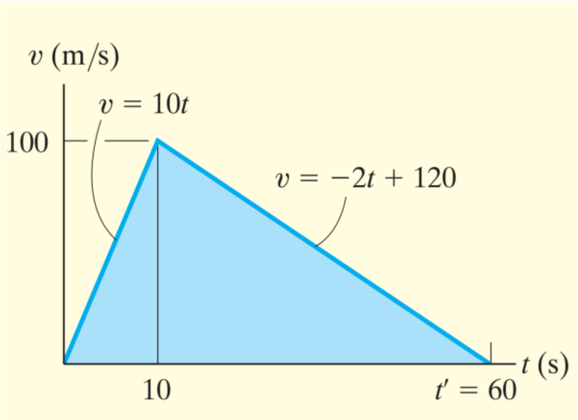
$$\boxed{\begin{gathered} v = v_0 + a_c t \\ \text{Constant Acceleration} \end{gathered}}$$

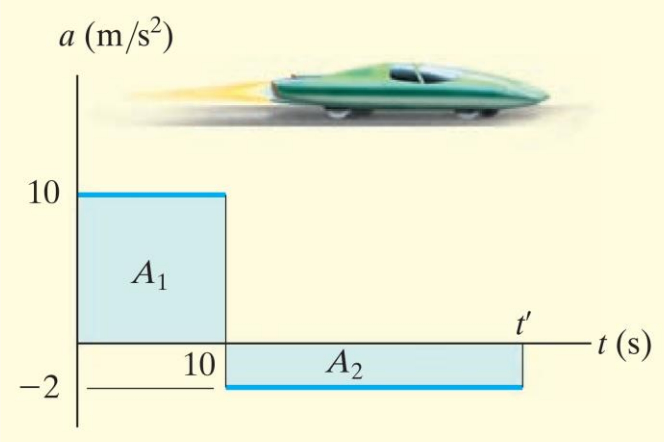
$$\boxed{\begin{gathered} s = s_0 + v_0 t + \tfrac{1}{2} a_c t^2 \\ \text{Constant Acceleration} \end{gathered}}$$

$$\boxed{\begin{gathered} v^2 = v_0^2 + 2a_c(s - s_0) \\ \text{Constant Acceleration} \end{gathered}}$$

The s–t, v–t, and a–t Graphs.

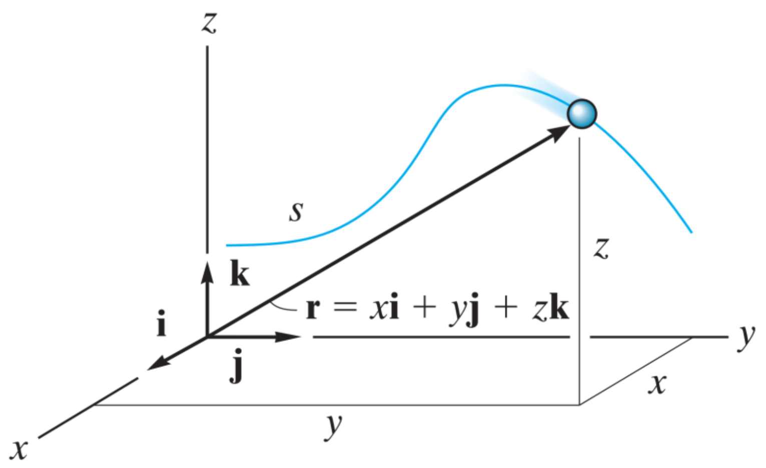






$$\boxed{\mathbf{r} = x\mathbf{i} + y\mathbf{j} + z\mathbf{k}}$$

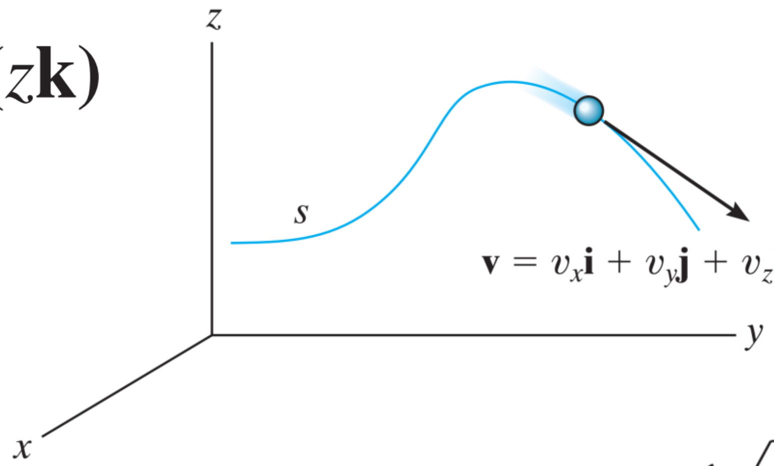
$$r = \sqrt{x^2 + y^2 + z^2}$$



Position

$$\mathbf{v} = \frac{d\mathbf{r}}{dt} = \frac{d}{dt}(x\mathbf{i}) + \frac{d}{dt}(y\mathbf{j}) + \frac{d}{dt}(z\mathbf{k})$$

$$\boxed{\mathbf{v} = \frac{d\mathbf{r}}{dt} = v_x\mathbf{i} + v_y\mathbf{j} + v_z\mathbf{k}}$$



Velocity

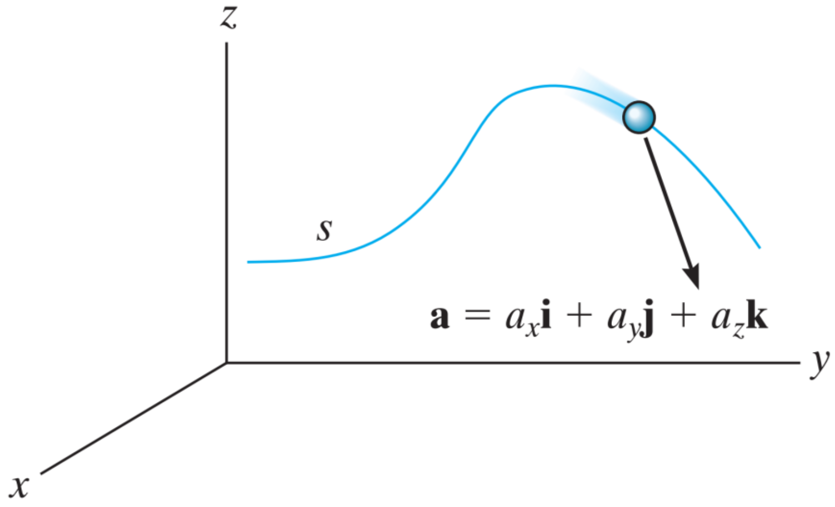
$$v = \sqrt{v_x^2 + v_y^2 + v_z^2}$$

$$\boxed{v_x = \dot{x} \quad v_y = \dot{y} \quad v_z = \dot{z}}$$

$$\mathbf{a} = \frac{d\mathbf{v}}{dt} = a_x\mathbf{i} + a_y\mathbf{j} + a_z\mathbf{k}$$

$$a_x = \dot{v}_x = \ddot{x}$$
$$a_y = \dot{v}_y = \ddot{y}$$
$$a_z = \dot{v}_z = \ddot{z}$$

$$a = \sqrt{a_x^2 + a_y^2 + a_z^2}$$



Acceleration

12. 6 MOTION OF A PROJECTILE

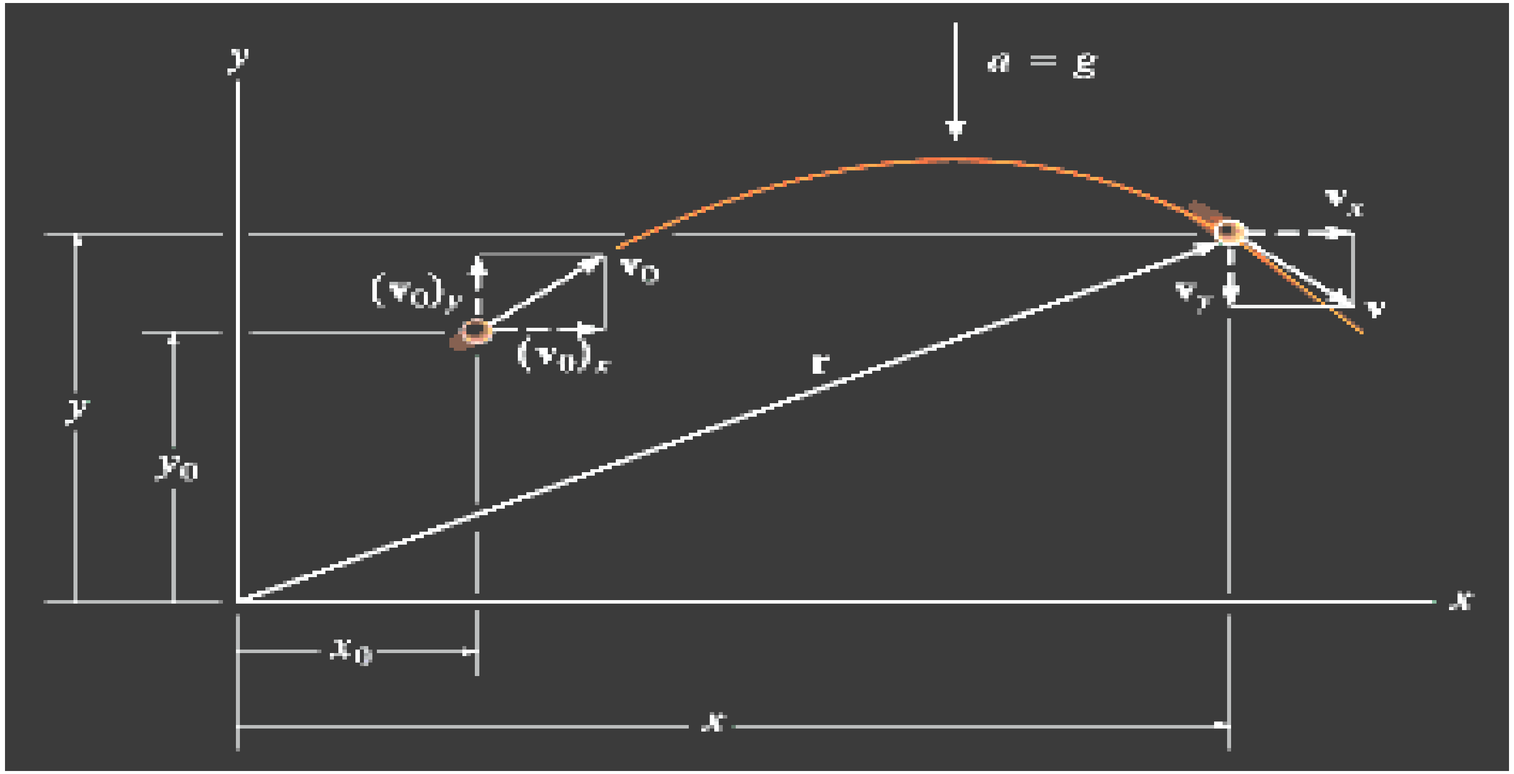
Motion of a Projectile

- **Projectile motion** is one of the traditional branches of classical mechanics, with applications to ballistics. A projectile is any body that is given an initial velocity and then follows a path determined by the effect of the gravitational acceleration and by air resistance. Projectile motion is the motion of such a projectile.

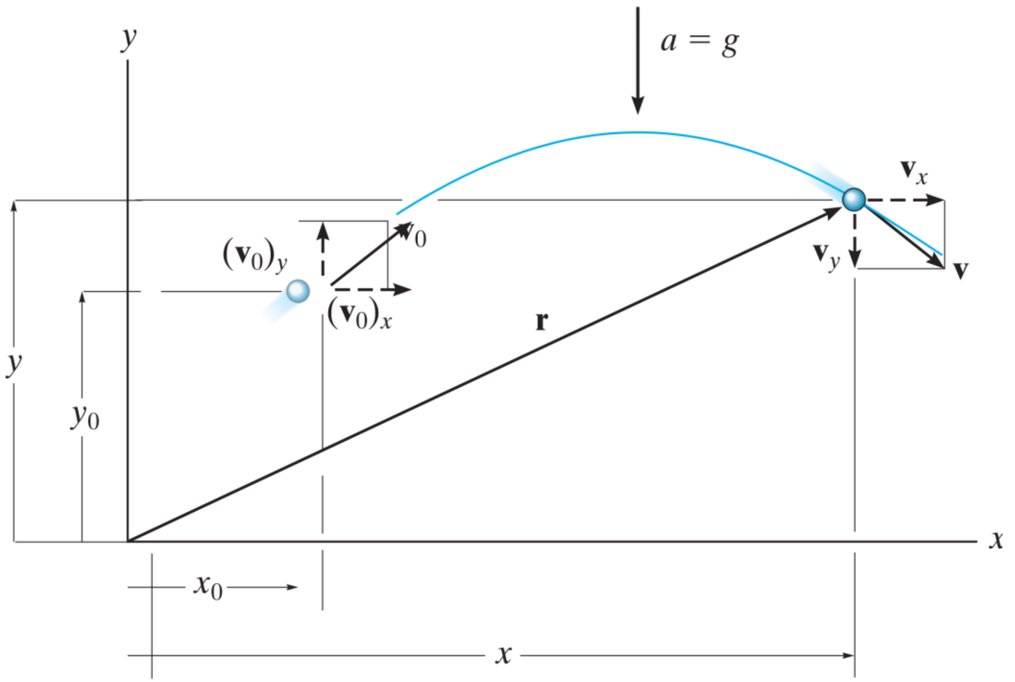
- The free-flight motion of a projectile is often studied in terms of its rectangular components. To illustrate the kinematic analysis, consider a projectile launched at point (Xo, Yo), with an initial velocity of Vo, having components (Vo)x and (Vo)y.

Motion of a Projectile

- In the figure, when air resistance is neglected, the only force acting on the projectile is its weight, which causes the projectile to have a constant downward direction of approximately a = g = 9.81m/s² or g = 32.2ft/s².



Motion of a Projectile

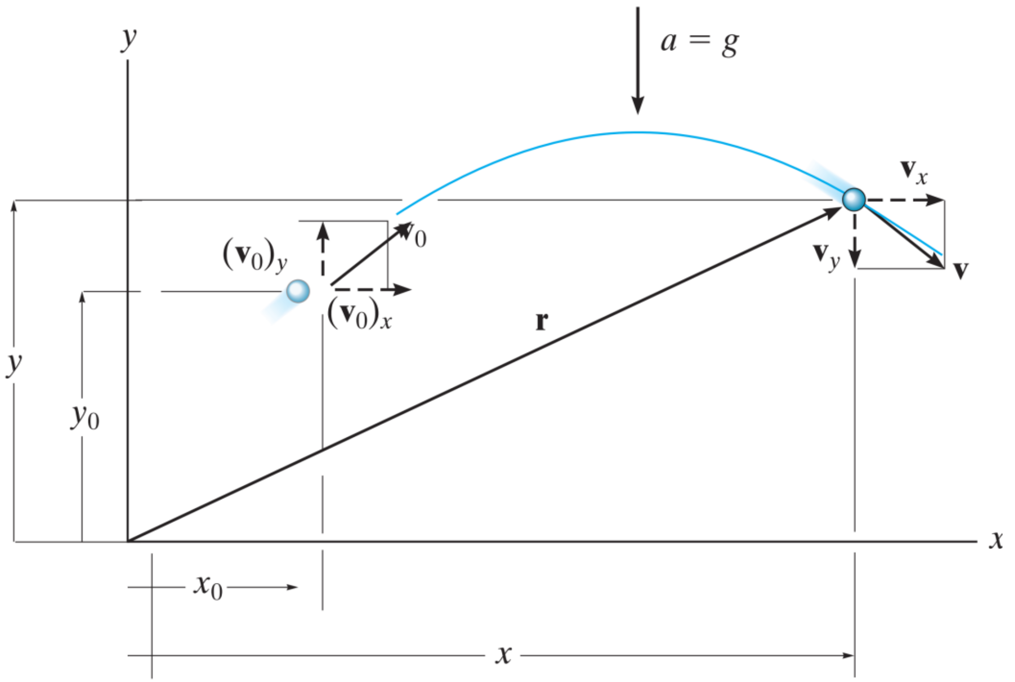


Horizontal Motion. Since $a_x = 0$, application of the constant acceleration equations, 12–4 to 12–6, yields

$$(\xrightarrow{+}) \qquad v = v_0 + a_c t \qquad v_x = (v_0)_x$$

$$(\xrightarrow{+}) \qquad x = x_0 + v_0 t + \tfrac{1}{2} a_c t^2; \qquad x = x_0 + (v_0)_x t$$

Motion of a Projectile



Vertical Motion. Since the positive y axis is directed upward, then $a_y = -g$. Applying Eqs. 12–4 to 12–6, we get

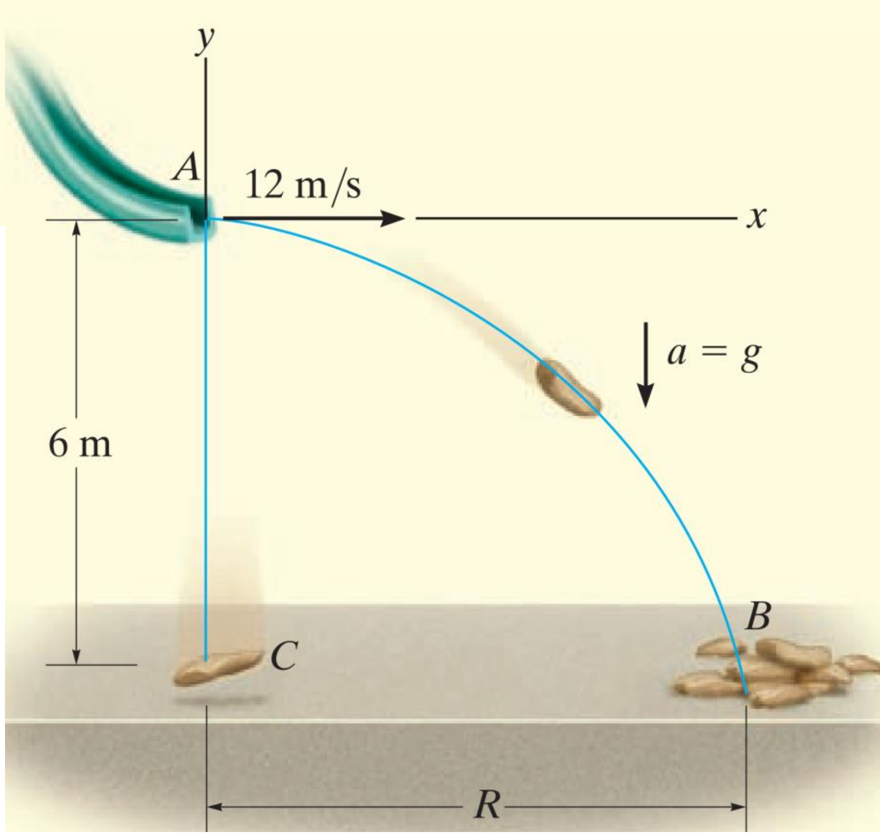
$$(+\uparrow) \qquad v = v_0 + a_c t; \qquad v_y = (v_0)_y - gt$$

$$(+\uparrow) \qquad y = y_0 + v_0 t + \tfrac{1}{2} a_c t^2; \qquad y = y_0 + (v_0)_y t - \tfrac{1}{2} g t^2$$

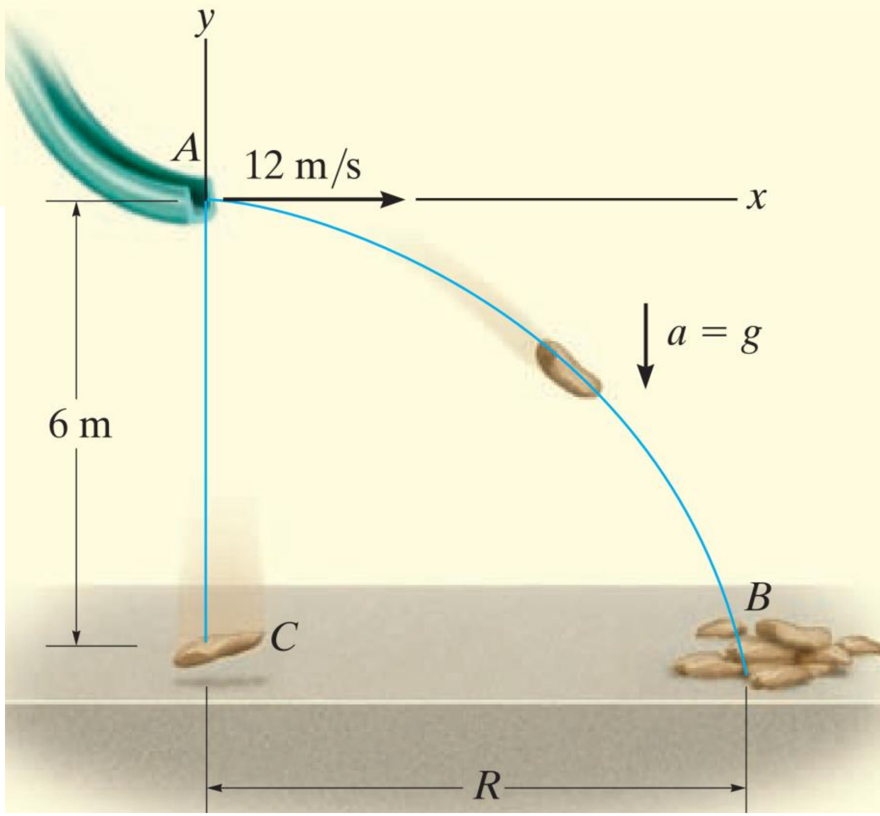
$$(+\uparrow) \qquad v^2 = v_0^2 + 2a_c(y - y_0); \qquad v_y^2 = (v_0)_y^2 - 2g(y - y_0)$$

A sack slides off the ramp, shown in Fig. 12–21, with a horizontal velocity of 12 m/s. If the height of the ramp is 6 m from the floor, determine the time needed for the sack to strike the floor and the range R where sacks begin to pile up.

$$v_y = (v_0)_y - gt$$
$$y = y_0 + (v_0)_y t - \tfrac{1}{2} g t^2$$
$$v_y^2 = (v_0)_y^2 - 2g(y - y_0)$$
$$v_x = (v_0)_x$$
$$x = x_0 + (v_0)_x t$$



A sack slides off the ramp, shown in Fig. 12–21, with a horizontal velocity of 12 m/s. If the height of the ramp is 6 m from the floor, determine the time needed for the sack to strike the floor and the range R where sacks begin to pile up.



Coordinate System. The origin of coordinates is established at the beginning of the path, point A, Fig. 12–21. The initial velocity of a sack has components $(v_A)_x = 12$ m/s and $(v_A)_y = 0$. Also, between points A and B the acceleration is $a_y = -9.81\text{ m/s}^2$. Since $(v_B)_x = (v_A)_x = 12$ m/s, the three unknowns are $(v_B)_y$, R, and the time of flight t_{AB}. Here we do not need to determine $(v_B)_y$.

Vertical Motion. The vertical distance from A to B is known, and therefore we can obtain a direct solution for t_{AB} by using the equation

$(+\uparrow)$

$$y_B = y_A + (v_A)_y t_{AB} + \tfrac{1}{2} a_c t_{AB}^2$$

$$-6\text{ m} = 0 + 0 + \tfrac{1}{2}(-9.81\text{ m/s}^2)t_{AB}^2$$

$$t_{AB} = 1.11\text{ s} \qquad \textit{Ans.}$$

Horizontal Motion. Since t_{AB} has been calculated, R is determined as follows:

$(\overset{+}{\rightarrow})$

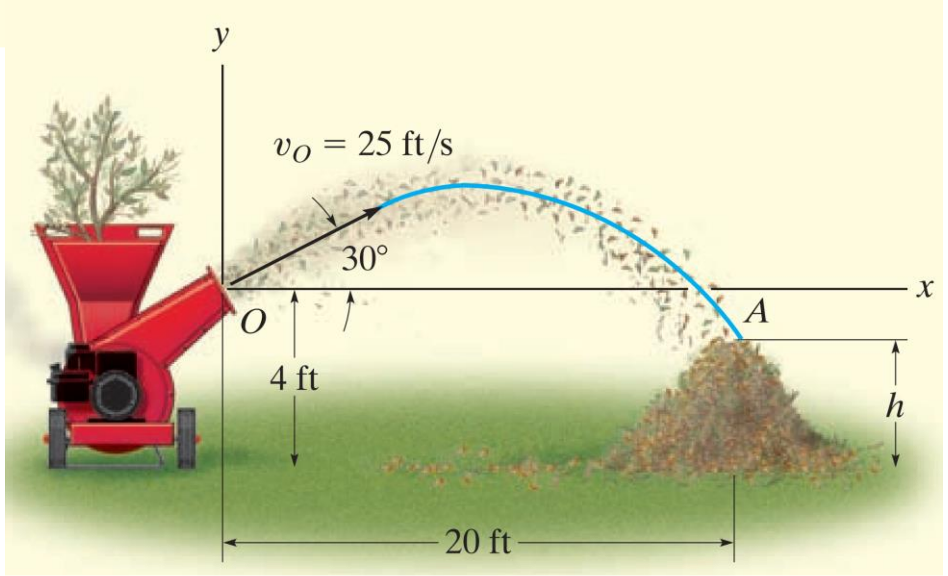
$$x_B = x_A + (v_A)_x t_{AB}$$

$$R = 0 + 12\text{ m/s}\,(1.11\text{ s})$$

$$R = 13.3\text{ m} \qquad \textit{Ans.}$$

NOTE: The calculation for t_{AB} also indicates that if a sack were released *from rest* at A, it would take the same amount of time to strike the floor at C, Fig. 12–21.

The chipping machine is designed to eject wood chips at $v_O = 25$ ft/s as shown in Fig. 12–22. If the tube is oriented at 30° from the horizontal, determine how high, h, the chips strike the pile if at this instant they land on the pile 20 ft from the tube.



Coordinate System. When the motion is analyzed between points O and A, the three unknowns are the height h, time of flight t_{OA}, and vertical component of velocity $(v_A)_y$. [Note that $(v_A)_x = (v_O)_x$.] With the origin of coordinates at O, Fig. 12–22, the initial velocity of a chip has components of

$$(v_O)_x = (25 \cos 30°) \text{ ft/s} = 21.65 \text{ ft/s} \rightarrow$$

$$(v_O)_y = (25 \sin 30°) \text{ ft/s} = 12.5 \text{ ft/s} \uparrow$$

Also, $(v_A)_x = (v_O)_x = 21.65$ ft/s and $a_y = -32.2$ ft/s^2. Since we do not need to determine $(v_A)_y$, we have

Horizontal Motion.

$(\overset{+}{\rightarrow})$

$$x_A = x_O + (v_O)_x t_{OA}$$
$$20 \text{ ft} = 0 + (21.65 \text{ ft/s}) t_{OA}$$
$$t_{OA} = 0.9238 \text{ s}$$

Vertical Motion. Relating t_{OA} to the initial and final elevations of a chip, we have

$$(+\uparrow) \quad y_A = y_O + (v_O)_y t_{OA} + \tfrac{1}{2} a_c t_{OA}^2$$
$$(h - 4 \text{ ft}) = 0 + (12.5 \text{ ft/s})(0.9238 \text{ s}) + \tfrac{1}{2}(-32.2 \text{ ft/s}^2)(0.9238 \text{ s})^2$$
$$h = 1.81 \text{ ft}$$

Ans.

The track for this racing event was designed so that riders jump off the slope at 30°, from a height of 1 m. During a race it was observed that the rider shown in Fig. 12–23*a* remained in mid air for 1.5 s. Determine the speed at which he was traveling off the ramp, the horizontal distance he travels before striking the ground, and the maximum height he attains. Neglect the size of the bike and rider.

Coordinate System. As shown in Fig. 12–23*b*, the origin of the coordinates is established at *A*. Between the end points of the path *AB* the three unknowns are the initial speed v_A, range *R*, and the vertical component of velocity $(v_B)_y$.

Vertical Motion. Since the time of flight and the vertical distance between the ends of the path are known, we can determine v_A.

$(+\uparrow)$
$$y_B = y_A + (v_A)_y t_{AB} + \tfrac{1}{2} a_c t_{AB}^2$$
$$-1\text{ m} = 0 + v_A \sin 30°(1.5\text{ s}) + \tfrac{1}{2}(-9.81\text{ m/s}^2)(1.5\text{ s})^2$$
$$v_A = 13.38\text{ m/s} = 13.4\text{ m/s} \qquad \textit{Ans.}$$

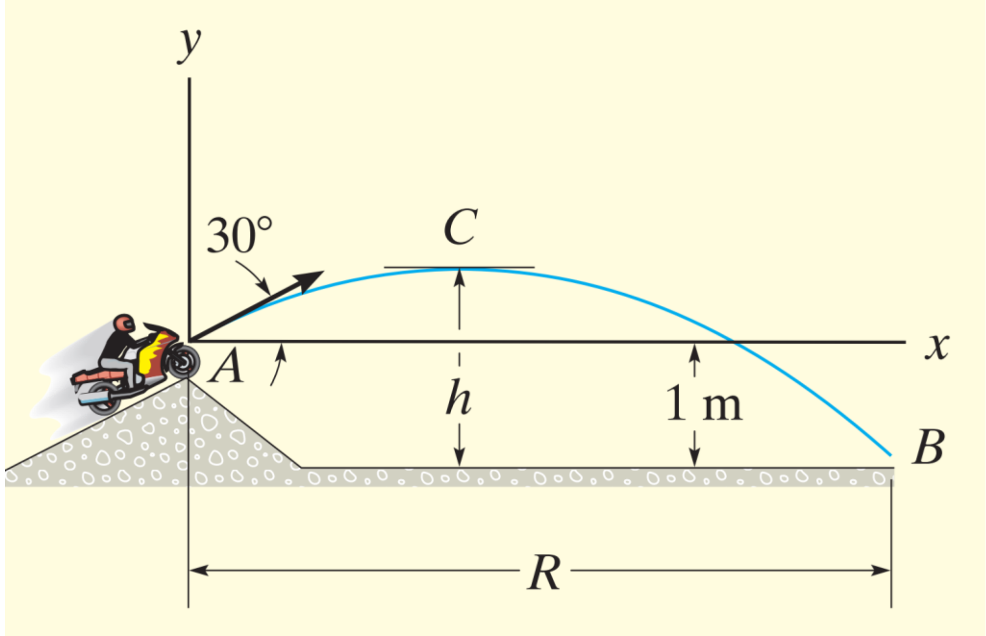
Horizontal Motion. The range *R* can now be determined.

$(\overset{+}{\rightarrow})$
$$x_B = x_A + (v_A)_x t_{AB}$$
$$R = 0 + 13.38 \cos 30°\text{ m/s}(1.5\text{ s})$$
$$= 17.4\text{ m} \qquad \textit{Ans.}$$

$$(v_C)_y^2 = (v_A)_y^2 + 2a_c[y_C - y_A]$$
$$0^2 = (13.38 \sin 30°\text{ m/s})^2 + 2(-9.81\text{ m/s}^2)[(h - 1\text{ m}) - 0]$$
$$h = 3.28\text{ m} \qquad \textit{Ans.}$$

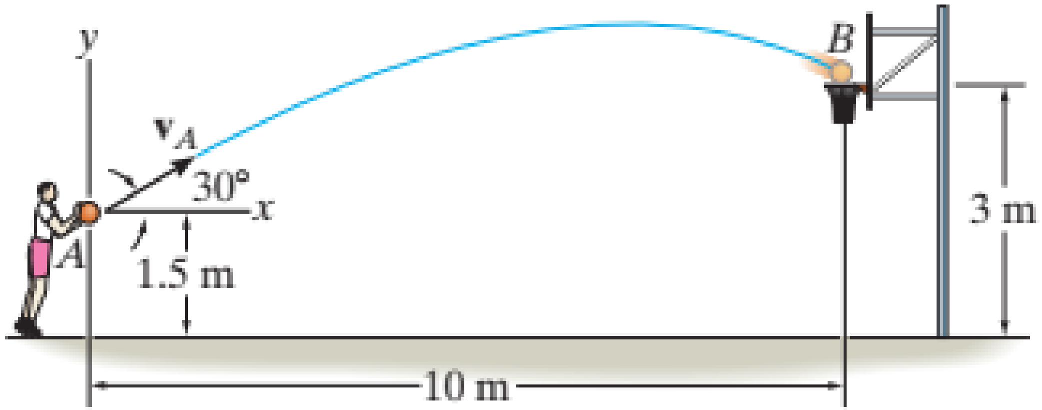
NOTE: Show that the bike will strike the ground at *B* with a velocity having components of

$$(v_B)_x = 11.6\text{ m/s} \rightarrow, \quad (v_B)_y = 8.02\text{ m/s}\downarrow$$



Sample problems(Practice)

F12–23. Determine the speed at which the basketball at A must be thrown at the angle of 30° so that it makes it to the basket at B.



$$v_x = (v_0)_x$$
$$x = x_0 + (v_0)_x t$$

$$v_y = (v_0)_y - gt$$
$$y = y_0 + (v_0)_y t - \tfrac{1}{2} g t^2$$
$$v_y^2 = (v_0)_y^2 - 2g(y - y_0)$$

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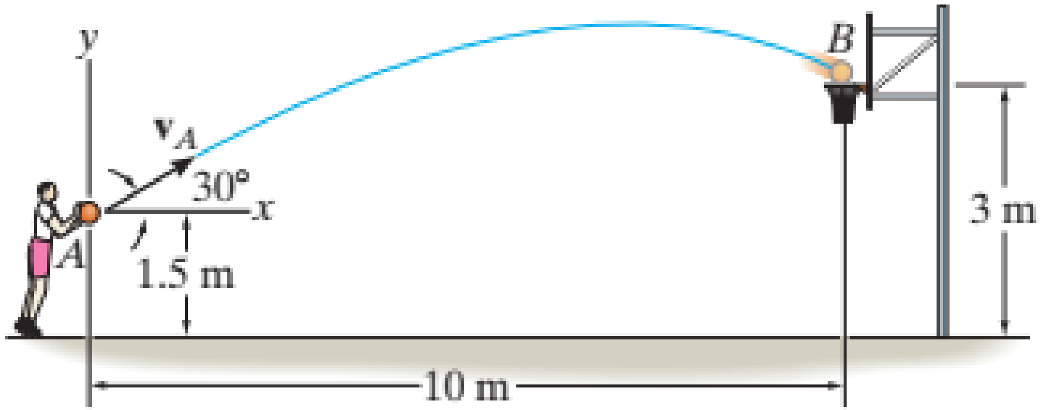


The initial velocity in x and y direction is:

$$(v_0)_x = v_A \cos 30^\circ$$
$$(v_0)_y = v_A \sin 30^\circ$$

Since form the figure we know the coordinates of point A (0 m, 1.5 m) and of the basket at B (10 m, 3 m), we can use equations $x = x_0 + (v_0)_x t$ and $y = y_0 + (v_0)_y t - \frac{1}{2} g t^2$ to determine speed v_A:

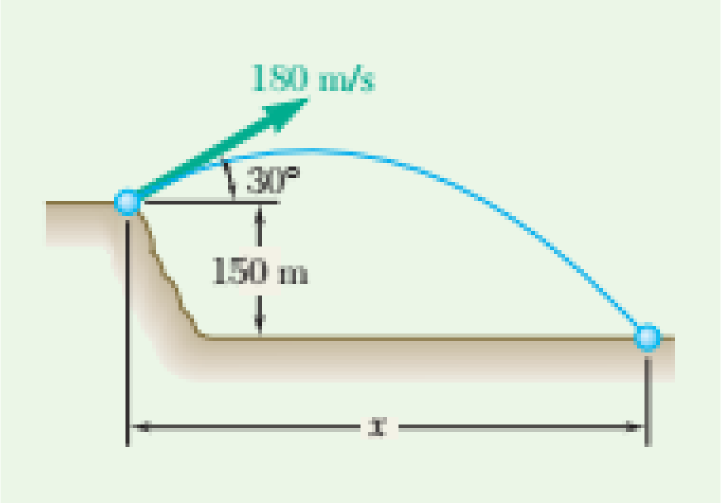
$$x_B = x_A + v_A \cos 30^\circ t$$
$$\rightarrow t = \frac{x_B}{v_A \cos 30^\circ} = \frac{10}{v_A \cos 30^\circ}$$

$$y_B = y_A + v_A \sin 30^\circ \frac{10}{v_A \cos 30^\circ} - \frac{1}{2} g \left(\frac{10}{v_A \cos 30^\circ} \right)^2$$
$$3 = 1.5 + 5.77 - \frac{654}{v_A^2}$$
$$\rightarrow v_A = \sqrt{\frac{654}{4.27}} = \boxed{12.4\,\mathrm{m/s}}$$

$$v_x = (v_0)_x$$
$$x = x_0 + (v_0)_x t$$

$$v_y = (v_0)_y - gt$$
$$y = y_0 + (v_0)_y t - \tfrac{1}{2} g t^2$$
$$v_y^2 = (v_0)_y^2 - 2g(y - y_0)$$

A projectile is fired from the edge of a 150-m cliff with an initial velocity of 180 m/s at an angle of 30° with the horizontal. Neglecting air resistance, find (*a*) the horizontal distance from the gun to the point where the projectile strikes the ground, (*b*) the greatest elevation above the ground reached by the projectile.



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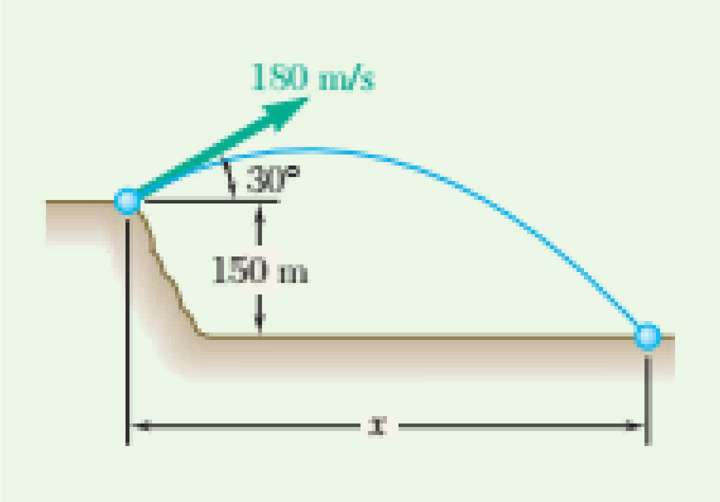


STRATEGY: This is a projectile motion problem, so you can consider the vertical and horizontal motions separately. First determine the equations governing each direction, and then use them to find the distances.

MODELING and ANALYSIS: Model the projectile as a particle and neglect the effects of air resistance. The vertical motion has a constant acceleration. Choosing the positive sense of the y axis upward and placing the origin *O* at the gun (Fig. 1), you have

$$(v_y)_0 = (180 \text{ m/s}) \sin 30° = +90 \text{ m/s}$$
$$a = -9.81 \text{ m/s}^2$$

Substitute these values into the equations for motion with constant acceleration. Thus,

$$v_y = (v_y)_0 + at \qquad v_y = 90 - 9.81t \qquad (1)$$
$$y = (v_y)_0 t + \tfrac{1}{2}at^2 \qquad y = 90t - 4.90t^2 \qquad (2)$$
$$v_y^2 = (v_y)_0^2 + 2ay \qquad v_y^2 = 8100 - 19.62y \qquad (3)$$

The horizontal motion has zero acceleration. Choose the positive sense of the *x* axis to the right (Fig. 2), which gives you

$$(v_x)_0 = (180 \text{ m/s}) \cos 30° = +155.9 \text{ m/s}$$

Substituting into the equation for constant acceleration, you obtain

$$x = (v_x)_0 t \qquad x = 155.9t \qquad (4)$$

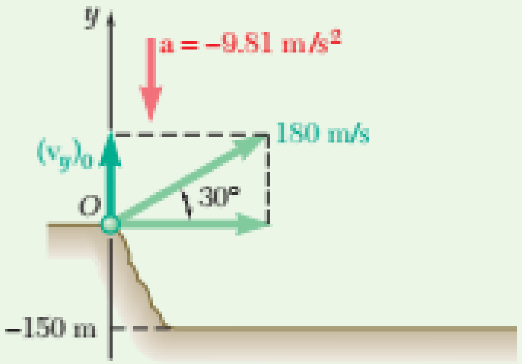


Fig. 1 Acceleration and initial velocity of the projectile in the *y*-direction.

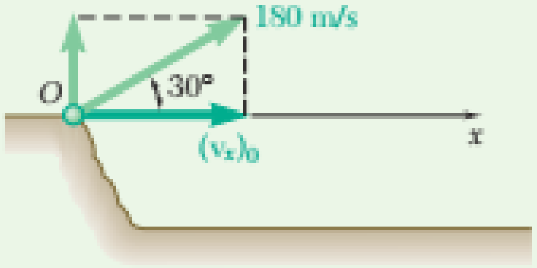
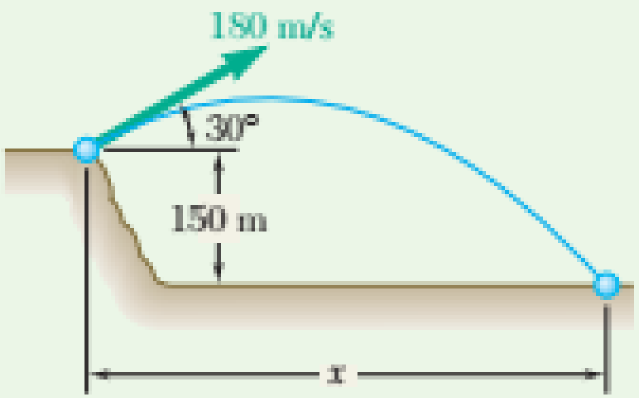


Fig. 2 Initial velocity of the projectile in the *x*-direction.

A projectile is fired from the edge of a 150-m cliff with an initial velocity of 180 m/s at an angle of 30° with the horizontal. Neglecting air resistance, find (*a*) the horizontal distance from the gun to the point where the projectile strikes the ground, (*b*) the greatest elevation above the ground reached by the projectile.



a. Horizontal Distance. When the projectile strikes the ground,

$$y = -150 \text{ m}$$

Substituting this value into Eq. (2) for the vertical motion, you have

$$-150 = 90t - 4.90t^2 \qquad t^2 - 18.37t - 30.6 = 0 \qquad t = 19.91 \text{ s}$$

Substituting $t = 19.91$ s into Eq. (4) for the horizontal motion, you obtain

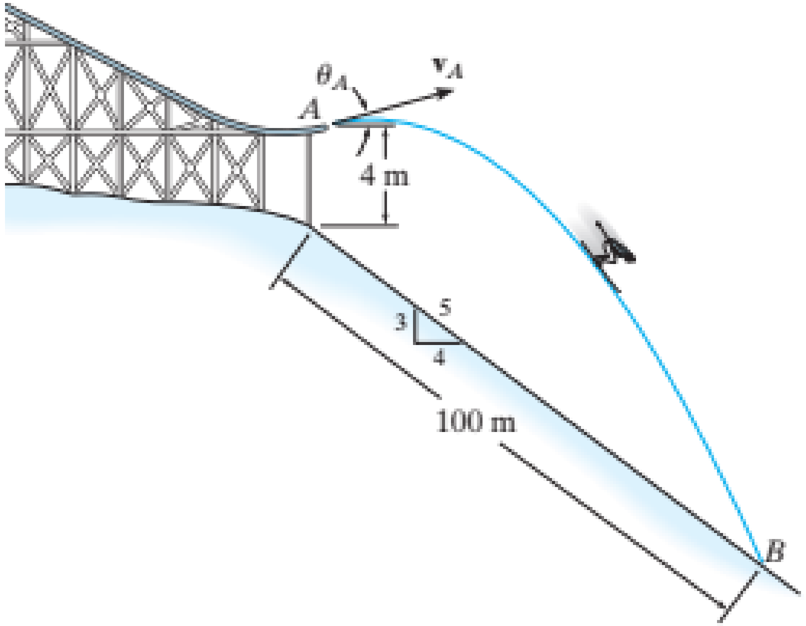
$$x = 155.9(19.91) \qquad x = 3100 \text{ m} \quad ◀$$

b. Greatest Elevation. When the projectile reaches its greatest elevation, $v_y = 0$; substituting this value into Eq. (3) for the vertical motion, you have

$$0 = 8100 - 19.62y \qquad y = 413 \text{ m}$$

$$\text{Greatest elevation above ground} = 150 \text{ m} + 413 \text{ m} = 563 \text{ m} \quad ◀$$

12–97. It is observed that the skier leaves the ramp A at an angle $\theta_A = 25°$ with the horizontal. If he strikes the ground at B, determine his initial speed v_A and the speed at which he strikes the ground.



Coordinate System: $x-y$ coordinate system will be set with its origin to coincide with point A as shown in Fig. *a*.

x-motion: Here, $x_A = 0$, $x_B = 100\left(\frac{4}{5}\right) = 80$ m and $(v_A)_x = v_A \cos 25°$.

$$(\overset{+}{\rightarrow}) \qquad x_B = x_A + (v_A)_x t$$

$$80 = 0 + (v_A \cos 25°)t$$

$$t = \frac{80}{v_A \cos 25°} \qquad \textbf{(1)}$$

y-motion: Here, $y_A = 0$, $y_B = -[4 + 100\left(\frac{3}{5}\right)] = -64$ m and $(v_A)_y = v_A \sin 25°$ and $a_y = -g = -9.81 \text{ m/s}^2$.

$$(+\uparrow) \qquad y_B = y_A + (v_A)_y t + \frac{1}{2} a_y t^2$$

$$-64 = 0 + v_A \sin 25°\, t + \frac{1}{2}(-9.81)t^2$$

$$4.905t^2 - v_A \sin 25°\, t = 64 \qquad \textbf{(2)}$$

Substitute Eq. (1) into (2) yieldS

$$4.905\left(\frac{80}{v_A \cos 25°}\right)^2 = v_A \sin 25°\left(\frac{80}{v_A \cos 25°}\right) = 64$$

$$\left(\frac{80}{v_A \cos 25°}\right)^2 = 20.65$$

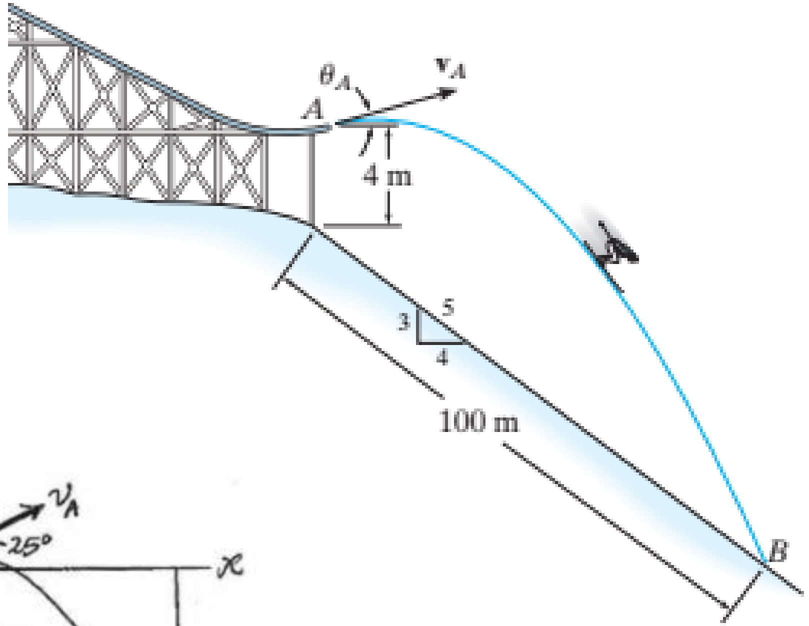
$$\frac{80}{v_A \cos 25°} = 4.545$$

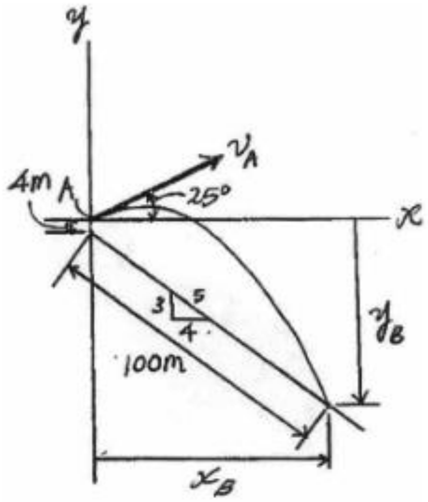
$$v_A = 19.42 \text{ m/s} = 19.4 \text{ m/s} \qquad \textbf{Ans.}$$

Substitute this result into Eq. (1),

$$t = \frac{80}{19.42 \cos 25°} = 4.54465$$

12–97. It is observed that the skier leaves the ramp A at an angle $\theta_A = 25°$ with the horizontal. If he strikes the ground at B, determine his initial speed v_A and the speed at which he strikes the ground.





(a)