9.6 Complete Response of the RLC Circuit

(PP351 8th Ed HKD)

The complete response is the sum of the :-

— forced response $v_f(t) = V_f$

— which is a <u>constant</u> for dc excitation

and

— natural response

$$v_n(t) = Ae^{s_1 t} + Be^{s_2 t}$$

— we assume that V_f, s_1 and s_2 have already been determined.

— A and B remain to be found from the complete response

$$v(t) = V_f + Ae^{s_1 t} + Be^{s_2 t}$$

— For this we need two initial conditions:-

$$v(0^+) = V_f + A + B$$

and

$$\frac{dv}{dt} = 0 + s_1 Ae^{s_1 t} + s_2 Be^{s_2 t}$$

Note $\frac{dV_f}{dt} = 0$ for dc excitation

Note: Also true for $i(t)$.

(Optional)

A Quick Summary : The RLC circuit

(pp 355 7th Ed HKD)

- To determine the behaviour of a simple three element RLC circuit, determine whether

  i, Series or

  ii, parallel

- $\alpha = \frac{1}{2RC}$ (parallel RLC) (exponential damping coefficient)

  $\alpha = \frac{R}{2L}$ (Series RLC)

- In both cases $\omega_0 = \frac{1}{\sqrt{LC}}$ (resonant frequency)

- If $\alpha > \omega_0$, the circuit is overdamped and the natural response has the form:

$$f_n(t) = A_1 e^{s_1 t} + A_2 e^{s_2 t}$$

where $s_{1,2} = -\alpha \pm \sqrt{\alpha^2 - \omega_0^2}$

- If $\alpha = \omega_0$, then the circuit is critically damped and

$$f_n(t) = e^{-\alpha t}(A_1 t + A_2)$$

- And finally if $\alpha < \omega_0$, the circuit has underdamped response. and

$$f_n(t) = e^{-\alpha t}(A_1 \cos \omega_d t + A_2 \sin \omega_d t)$$

_______ contd

(Optional)

— Contd (355)

where $\omega_d = \sqrt{\omega_0^2 - \alpha^2}$

— If independant sources are present, then the complete response is :-

$$f(t) = f_f(t) + f_n(t)$$

— This is applicable to any current or voltage in the circuit.

— The final step is to solve for unknown constants given the initial conditions; using $i_C = C\frac{dv_C}{dt}$ and $v_L = L\frac{di_L}{dt}$.