

### Assignment 3

$$y'' + 2y' - 8y = 2e^{-2x} - e^{-x}$$

$$L \quad y(0) = 0, \quad y'(0) = 0, \quad 0 \leq x \leq 2 \quad h = 0.5$$

$$\text{Let } y' = \frac{dy}{dx} = z, \quad y'' = z'$$

System of equations:

$$y' = z = f(x, y, z) \quad - i$$

$$z' = -2z + 8y + 2e^{-2x} - e^{-x} \quad - ii \\ = g(x, y, z)$$

RK 4 Method

$$y_{n+1} = y_n + \frac{1}{6} (k_1 + 2k_2 + 2k_3 + k_4)$$

$$z_{n+1} = z_n + \frac{1}{6} (k_1^* + 2k_2^* + 2k_3^* + k_4^*)$$

$$\text{where } \left[ \begin{array}{l} K_1 = hf(x_n, y_n, z_n) \\ K_2 = hf(x_n + h/2, y_n + k_1/2, z_n + k_1^*/2) \\ K_3 = hf(x_n + h/2, y_n + k_2/2, z_n + k_2^*/2) \\ K_4 = hf(x_n + h, y_n + k_3, z_n + k_3^*) \\ K_1^* = hg(x_n, y_n, z_n) \\ K_2^* = hg(x_n + h/2, y_n + k_1/2, z_n + k_1^*/2) \\ K_3^* = hg(x_n + h/2, y_n + k_2/2, z_n + k_2^*/2) \\ K_4^* = hg(x_n + h, y_n + k_3, z_n + k_3^*) \end{array} \right]$$

Iteration 1:

$$K_1 = hf(x_0, y_0, z_0) = 0.5f(0, 0, 0) = 0$$

$$K_1^* = hg(x_0, y_0, z_0) = 0.5g(0, 0, 0) = 0.5$$

$$K_2 = hf(x_0 + h/2, y_0 + k_1/2, z_0 + k_1^*/2) = 0.125$$

$$K_2^* = hg(x_0 + h/2, y_0 + k_1/2, z_0 + k_1^*/2) = -0.03287$$

$$K_3 = hf(x_0 + h/2, y_0 + k_2/2, z_0 + k_2^*/2) = -0.00822$$

$$K_3^* = hg(x_0 + h/2, y_0 + k_2/2, z_0 + k_2^*/2) = 0.48357$$

$$K_4 = h f(x_0 + h, y_0 + k_3, z_0 + k_3^*) = 0.24178$$

$$K_4^* = h g(x_0 + h, y_0 + k_3, z_0 + k_3^*) = -0.45182$$

$$\Rightarrow y_1 = y(0.5) = y_0 + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4) = \boxed{0.07922}$$

$$\Rightarrow z_1 = y'(0.5) = z_0 + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4) = \boxed{0.15826}$$

Errors:

$$\text{Absolute} = |y_1 - \bar{y}_1| = 0.01326$$

$$\text{Relative} = \frac{\text{Absolute}}{y_1} = 0.20103$$

$$\% \text{ age} = \text{Relative} \times 100\% = 20.10309\%$$

Iteration 2

$$K_1 = h f(x_1, y_1, z_1) = 0.07913$$

$$K_1^* = h g(x_1, y_1, z_1) = 0.22325$$

$$K_2 = h f(x_1 + h/2, y_1 + k_1/2, z_1 + k_1^*/2) = 0.13494$$

$$K_2^* = h g(x_1 + h/2, y_1 + k_1/2, z_1 + k_1^*/2) = 0.19222$$

$$K_3 = h f(x_1 + h/2, y_1 + k_2/2, z_1 + k_2^*/2) = 0.12719$$

$$K_3^* = h g(x_1 + h/2, y_1 + k_2/2, z_1 + k_2^*/2) = 0.31936$$

$$K_4 = h f(x_1 + h, y_1 + k_3, z_1 + k_3^*) = 0.23881$$

$$K_4^* = h g(x_1 + h, y_1 + k_3, z_1 + k_3^*) = 0.29941$$

$$\Rightarrow y_2 = y(1) = y_1 + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4) = \boxed{0.21959}$$

$$\Rightarrow z_2 = y'(1) = z_1 + \frac{1}{6}(k_1^* + 2k_2^* + 2k_3^* + k_4^*) = \boxed{0.41590}$$



### Errors

$$\text{Absolute} = |y_2 - \bar{y}_2| = 0.00526$$

$$\text{Relative} = \text{Absolute} / y_2 = 0.02454$$

$$\% \text{ age} = \text{Relative} \times 100\% = 2.45416 \%$$

### Iteration 3

$$K_1 = hf(x_2, y_2, z_2) = 0.20795$$

$$K_1^* = hg(x_2, y_2, z_2) = 0.41386$$

$$K_2 = hf(x_2 + h/2, y_2 + k_1/2, z_2 + k_1^*/2) = 0.31142$$

$$K_2^* = hg(x_2 + h/2, y_2 + k_1/2, z_2 + k_1^*/2) = 0.61027$$

$$K_3 = hf(x_2 + h/2, y_2 + k_2/2, z_2 + k_2^*/2) = 0.36052$$

$$K_3^* = hg(x_2 + h/2, y_2 + k_2/2, z_2 + k_2^*/2) = 0.71900$$

$$K_4 = hf(x_2 + h, y_2 + k_3, z_2 + k_3^*) = 0.56745$$

$$K_4^* = hg(x_2 + h, y_2 + k_3, z_2 + k_3^*) = 1.12376$$

$$\Rightarrow y_3 = y(1.5) = y_2 + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4) = 0.57280$$

$$\Rightarrow z_3 = y'(1.5) = z_2 + \frac{1}{6}(k_1^* + 2k_2^* + 2k_3^* + k_4^*) = 1.11526$$

### Errors

$$\text{Absolute} = |y_3 - \bar{y}_3| = 0.00225$$

$$\text{Relative} = \text{Absolute} / y_3 = 0.00394$$

$$\% \text{ age} = \text{Relative} \times 100\% = 0.39436 \%$$

### Iteration 4

$$K_1 = hf(x_3, y_3, z_3) = 0.55763$$

$$K_1^* = hg(x_3, y_3, z_3) = 1.11417$$

$$\begin{aligned}
K_2 &= h f(x_3 + h/2, y_3 + k_1/2, z_3 + k_1^*/2) = 0.83167 \\
K_2^* &= h g(x_3 + h/2, y_3 + k_1/2, z_3 + k_1^*/2) = 1.67743 \\
K_3 &= h f(x_3 + h/2, y_3 + k_2/2, z_3 + k_2/2) = 0.97699 \\
K_3^* &= h g(x_3 + h/2, y_3 + k_2/2, z_3 + k_2^*/2) = 1.95289 \\
K_4 &= h f(x_3 + h, y_3 + k_3, z_3 + k_3^*) = 1.53407 \\
K_4^* &= h g(x_3 + h, y_3 + k_3, z_3 + k_3^*) = 3.08166
\end{aligned}$$

$$\begin{aligned}
\Rightarrow y_4 = y(2) &= y_3 + \frac{1}{6} (k_1 + 2k_2 + 2k_3 + k_4) \\
&= \boxed{1.52580}
\end{aligned}$$

$$\begin{aligned}
\Rightarrow z_4 = y'(2) &= z_3 + \frac{1}{6} (k_1^* + 2k_2^* + 2k_3^* + k_4^*) \\
&= 2.11709
\end{aligned}$$

### Errors

$$\text{Absolute} = |y_4 - \bar{y}_4| = 0.00131$$

$$\text{Relative} = \text{Absolute} / y_4 = 0.00086$$

$$\% \text{ age} = \text{Relative} \times 100 \% = 0.08586 \%$$

Answer :

$$\Rightarrow y(2) = 1.52580$$