FORMULA SHEET

$$r = \sqrt{x^2 + y^2 + z^2}, \quad \theta = \tan^{-1}\frac{\sqrt{x^2 + y^2}}{z}, \quad \phi = \tan^{-1}\frac{y}{x}$$

$$x = r\sin\theta\cos\phi, \quad y = r\sin\theta\sin\phi, \quad z = r\cos\theta$$

$$\begin{bmatrix} A_r \\ A_\theta \\ A_\phi \end{bmatrix} = \begin{bmatrix} \sin\theta\cos\phi & \sin\theta\sin\phi & \cos\theta \\ -\cos\theta\cos\phi & \cos\theta\sin\phi & -\sin\theta \\ -\sin\phi & \cos\phi & 0 \end{bmatrix} \begin{bmatrix} A_x \\ A_y \\ A_z \end{bmatrix}$$

$$\begin{bmatrix} A_\rho \\ A_\phi \\ A_z \end{bmatrix} = \begin{bmatrix} \cos\phi & \sin\phi & 0 \\ -\sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} A_x \\ A_y \\ A_z \end{bmatrix}$$

$$\begin{bmatrix} A_x \\ A_y \\ A_z \end{bmatrix} = \begin{bmatrix} \sin\theta\cos\phi & \cos\theta\cos\phi & -\sin\phi \\ \sin\theta\sin\phi & \cos\theta\sin\phi & \cos\phi \\ \cos\theta & -\sin\theta & 0 \end{bmatrix} \begin{bmatrix} A_r \\ A_\theta \\ A_\phi \end{bmatrix}$$

$$\rho = \sqrt{x^2 + y^2}, \quad \phi = \tan^{-1}\frac{y}{x}, \quad z = z$$

$$\varepsilon_o = 8.854 \times 10^{-12} \simeq \frac{10^{-9}}{36\pi} \text{ F/m}$$

$$\text{or } k = \frac{1}{4\pi\varepsilon_o} \simeq 9 \times 10^9 \text{ m/F}$$

$$\mathbf{E} = \int \frac{\rho_L \, dl}{4\pi\varepsilon_o R^2} \mathbf{a}_R \quad \text{(line charge)}$$

$$\mathbf{E} = \int \frac{\rho_S \, dS}{4\pi\varepsilon_o R^2} \mathbf{a}_R \quad \text{(surface charge)}$$

$$\mathbf{E} = \int \frac{\rho_v \, dv}{4\pi\varepsilon_o R^2} \mathbf{a}_R \quad \text{(volume charge)}$$

$$\mathbf{E} = \frac{\rho_L}{4\pi\varepsilon_o\rho}[-(\sin\alpha_2 - \sin\alpha_1)\mathbf{a}_\rho + (\cos\alpha_2 - \cos\alpha_1)\mathbf{a}_z]$$

$$\mathbf{E} = \frac{\rho_L}{2\pi\varepsilon_o\rho}\mathbf{a}_\rho$$

$$\nabla = \mathbf{a}_r \frac{\partial}{\partial r} + \mathbf{a}_\theta \frac{1}{r}\frac{\partial}{\partial\theta} + \mathbf{a}_\phi \frac{1}{r\sin\theta}\frac{\partial}{\partial\phi}$$

$$\nabla V = \frac{\partial V}{\partial x}\mathbf{a}_x + \frac{\partial V}{\partial y}\mathbf{a}_y + \frac{\partial V}{\partial z}\mathbf{a}_z$$

$$\nabla V = \frac{\partial V}{\partial \rho}\mathbf{a}_\rho + \frac{1}{\rho}\frac{\partial V}{\partial \phi}\mathbf{a}_\phi + \frac{\partial V}{\partial z}\mathbf{a}_z$$

$$\nabla V = \frac{\partial V}{\partial r}\mathbf{a}_r + \frac{1}{r}\frac{\partial V}{\partial \theta}\mathbf{a}_\theta + \frac{1}{r\sin\theta}\frac{\partial V}{\partial \phi}\mathbf{a}_\phi$$

$$\nabla \cdot \mathbf{A} = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}$$

$$\nabla \cdot \mathbf{A} = \frac{1}{\rho}\frac{\partial}{\partial\rho}(\rho A_\rho) + \frac{1}{\rho}\frac{\partial A_\phi}{\partial\phi} + \frac{\partial A_z}{\partial z}$$

$$W_E = \frac{1}{2}\int \mathbf{D}\cdot\mathbf{E}\, dv = \frac{1}{2}\int \varepsilon_o E^2 \, dv$$

$$R = \frac{V}{I} = \frac{\int \mathbf{E}\cdot d\mathbf{l}}{\int \sigma\mathbf{E}\cdot d\mathbf{S}}$$

$$P = \int \mathbf{E}\cdot\mathbf{J}\, dv$$

$$w_P = \frac{dP}{dv} = \mathbf{E}\cdot\mathbf{J} = \sigma|\mathbf{E}|^2$$

$$\rho_{ps} = \mathbf{P}\cdot\mathbf{a}_n$$

$$\rho_{pv} = -\nabla\cdot\mathbf{P}$$

$$\mathbf{D} = \varepsilon_o\mathbf{E} + \mathbf{P}$$

$$\nabla \times \mathbf{A} = \left[\frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z}\right]\mathbf{a}_x + \left[\frac{\partial A_x}{\partial z} - \frac{\partial A_z}{\partial x}\right]\mathbf{a}_y + \left[\frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y}\right]\mathbf{a}_z$$

$$\nabla \times \mathbf{A} = \frac{1}{r \sin\theta}\left[\frac{\partial(A_\phi \sin\theta)}{\partial\theta} - \frac{\partial A_\theta}{\partial\phi}\right]\mathbf{a}_r + \frac{1}{r}\left[\frac{1}{\sin\theta}\frac{\partial A_r}{\partial\phi} - \frac{\partial(rA_\phi)}{\partial r}\right]\mathbf{a}_\theta + \frac{1}{r}\left[\frac{\partial(rA_\theta)}{\partial r} - \frac{\partial A_r}{\partial\theta}\right]\mathbf{a}_\phi$$

$$\nabla \times \mathbf{A} = \left[\frac{1}{\rho}\frac{\partial A_z}{\partial\phi} - \frac{\partial A_\phi}{\partial z}\right]\mathbf{a}_\rho + \left[\frac{\partial A_\rho}{\partial z} - \frac{\partial A_z}{\partial\rho}\right]\mathbf{a}_\phi + \frac{1}{\rho}\left[\frac{\partial(\rho A_\phi)}{\partial\rho} - \frac{\partial A_\rho}{\partial\phi}\right]\mathbf{a}_z$$

$$\nabla^2 V = \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2}$$

$$\nabla^2 V = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial V}{\partial r}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial V}{\partial\theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2 V}{\partial\phi^2}$$

$$\nabla^2 V = \frac{1}{\rho}\frac{\partial}{\partial\rho}\left(\rho\frac{\partial V}{\partial\rho}\right) + \frac{1}{\rho^2}\frac{\partial^2 V}{\partial\phi^2} + \frac{\partial^2 V}{\partial z^2}$$

$$\mathbf{E} = \frac{\rho_S}{2\varepsilon_o}\mathbf{a}_n$$

$$\mathbf{P} = \chi_e\varepsilon_o\mathbf{E}$$

$$V_{AB} = \frac{W}{Q} = -\int_A^B \mathbf{E}\cdot d\mathbf{l}$$

$$\varepsilon_r = 1 + \chi_e = \frac{\varepsilon}{\varepsilon_o}$$

$$d\mathbf{H} = \frac{I\,d\mathbf{l}\times\mathbf{a}_R}{4\pi R^2} = \frac{I\,d\mathbf{l}\times\mathbf{R}}{4\pi R^3}$$

$$\mathbf{B} = \frac{\mu_o m}{4\pi r^3}(2\cos\theta\,\mathbf{a}_r + \sin\theta\,\mathbf{a}_\theta)$$

$$\mathbf{H} = \int_L \frac{I\,d\mathbf{l}\times\mathbf{a}_R}{4\pi R^2} \quad \text{(line current)}$$

$$\mathbf{A} = \frac{\mu_o\,\mathbf{m}\times\mathbf{a}_r}{4\pi r^2}$$

$$\mathbf{H} = \int_S \frac{\mathbf{K}\,dS\times\mathbf{a}_R}{4\pi R^2} \quad \text{(surface current)}$$

$$V_{\text{emf}} = \oint_L \mathbf{E}_m\cdot d\mathbf{l} = \oint_L (\mathbf{u}\times\mathbf{B})\cdot d\mathbf{l}$$

$$\mathbf{H} = \int_v \frac{\mathbf{J}\,dv\times\mathbf{a}_R}{4\pi R^2} \quad \text{(volume current)}$$

$$V_{\text{emf}} = \oint_L \mathbf{E}\cdot d\mathbf{l} = -\int_S \frac{\partial\mathbf{B}}{\partial t}\cdot d\mathbf{S}$$

$$\mathbf{H} = \frac{I}{4\pi\rho}(\cos\alpha_2 - \cos\alpha_1)\mathbf{a}_\phi$$

$$\mathbf{J}_b = \nabla\times\mathbf{M}$$

$$\mathbf{H} = \frac{I}{2\pi\rho}\mathbf{a}_\phi$$

$$\mathbf{K}_b = \mathbf{M}\times\mathbf{a}_n$$

$$\alpha = \omega\sqrt{\frac{\mu\varepsilon}{2}\left[\sqrt{1 + \left[\frac{\sigma}{\omega\varepsilon}\right]^2} - 1\right]}$$

$$1\ \text{Np} = 20\log_{10} e = 8.686\ \text{dB}$$

$$\mathbf{H} = \frac{1}{2}\mathbf{K}\times\mathbf{a}_n$$

$$\beta = \omega\sqrt{\frac{\mu\varepsilon}{2}\left[\sqrt{1 + \left[\frac{\sigma}{\omega\varepsilon}\right]^2} + 1\right]}$$

$$\mu_o = 4\pi\times 10^{-7}\ \text{H/m}$$

$$\mathbf{F} = m\frac{d\mathbf{u}}{dt} = Q(\mathbf{E} + \mathbf{u}\times\mathbf{B})$$

$$\eta = \sqrt{\frac{j\omega\mu}{\sigma + j\omega\varepsilon}} = |\eta|\underline{/\theta_\eta} = |\eta|e^{j\theta_\eta}$$

$$\mathbf{F}_1 = \frac{\mu_o I_1 I_2}{4\pi}\oint_{L_1}\oint_{L_2}\frac{d\mathbf{l}_1\times(d\mathbf{l}_2\times\mathbf{a}_{R_{21}})}{R_{21}^2}$$