ELECTRIC DIPOLE AND ENERGY DENSITY

Electric Dipole

- An electric dipole is formed when two point charges of equal magnitude but opposite sign are separated by a small distance

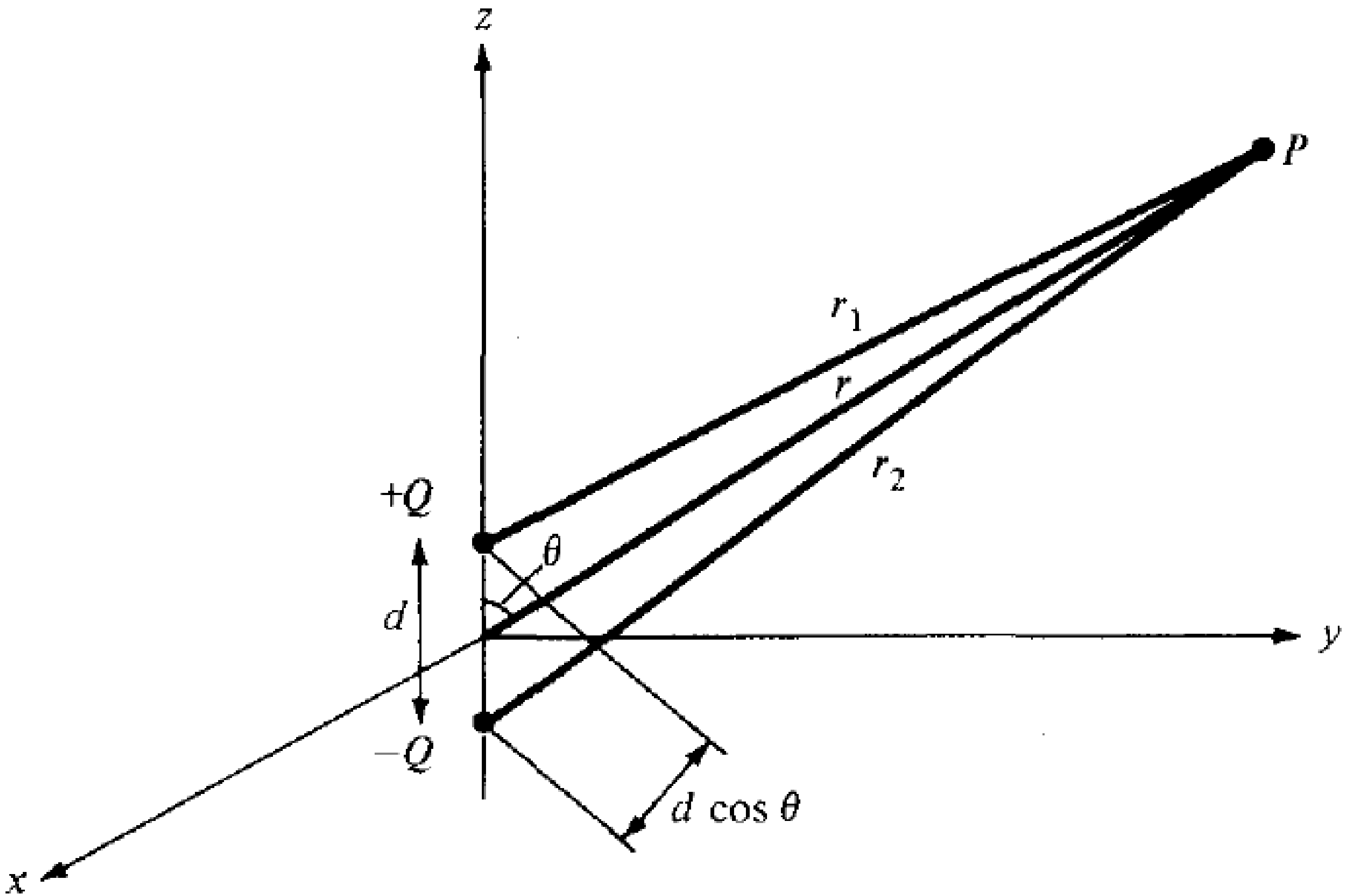
- Example is the formation of dipoles in dielectric materials in the presence of an electric field

- Dipole moment is a measure of system's overall polarity

- The distance between the point charges is small compared to the distance to the point *P* at which we want to know the electric potential and potential fields

Electric Dipole

- Figure below shows an electric dipole:



Potential due to Electric Dipole

- The potential at point $\boldsymbol{P(r, \theta, \emptyset)}$ is given by:

$$V = \frac{Q}{4\pi\varepsilon_o}\left[\frac{1}{r_1} - \frac{1}{r_2}\right] = \frac{Q}{4\pi\varepsilon_o}\left[\frac{r_2 - r_1}{r_1 r_2}\right]$$

- where r_1 and r_2 are the distances between P and $+Q$ and P and $-Q$ respectively

- If $r \gg d, r_2 - r_1 \approx d\cos\theta,\ r_1 r_2 \cong r^2$, the above becomes:

$$V = \frac{Q}{4\pi\varepsilon_o}\frac{d\cos\theta}{r^2}$$

Potential due to Electric Dipole

- Since $d\cos\theta = \mathbf{d}\cdot\mathbf{a_r}$, where $\mathbf{d} = d\mathbf{a_z}$, if we define:

$$\mathbf{p} = Q\mathbf{d}$$

- as the dipole moment, the equation for potential becomes:

$$V = \frac{\mathbf{p}\cdot\mathbf{a}_r}{4\pi\varepsilon_o r^2}$$

- Note that the dipole moment **p** is directed from – Q to +Q
- If the dipole center is not at the origin but at r', the above equation becomes:

$$V(\mathbf{r}) = \frac{\mathbf{p}\cdot(\mathbf{r}-\mathbf{r}')}{4\pi\varepsilon_o|\mathbf{r}-\mathbf{r}'|^3}$$

Electric Field due to Electric Dipole

- The electric field due to the dipole with center at the origin, can be obtained readily as:

$$\mathbf{E} = -\nabla V = -\left[\frac{\partial V}{\partial r}\mathbf{a}_r + \frac{1}{r}\frac{\partial V}{\partial \theta}\mathbf{a}_\theta\right]$$

$$= \frac{Qd\cos\theta}{2\pi\varepsilon_o r^3}\mathbf{a}_r + \frac{Qd\sin\theta}{4\pi\varepsilon_o r^3}\mathbf{a}_\theta$$

OR

$$\mathbf{E} = \frac{p}{4\pi\varepsilon_o r^3}(2\cos\theta\,\mathbf{a}_r + \sin\theta\,\mathbf{a}_\theta)$$

$$\text{where } p = |\mathbf{p}| = Qd$$

Electric Dipole

- Notice that a point charge is a monopole and its electric field varies inversely as r^2 while its potential field varies inversely as r

- From the equations for electric dipole, we notice that the electric field due to a dipole varies inversely as r^3 while its potential varies inversely as r^2

- The electric fields due to successive higher-order multi-poles (such as a *quadrupole* consisting of two dipoles or an *octupole* consisting of two quadrupoles) vary inversely as r^4, r^5, r^6,…. while their corresponding potentials vary inversely as r^3, r^4, r^5,….

Electric Flux Lines

- The idea of *electric flux lines* (or electric lines of force as they are sometimes called) was introduced by Michael Faraday in his experimental investigation as a way of visualizing the electric field

- *An electric flux line is an imaginary path or line drawn in such a way that its direction at any point is the direction of the electric field at that point*

Equipotential Surfaces

- Any surface on which the potential is the same throughout is known as an equipotential surface

- The intersection of an equipotential surface and a plane results in a path or line known as an equipotential line (surface would be like a sphere)

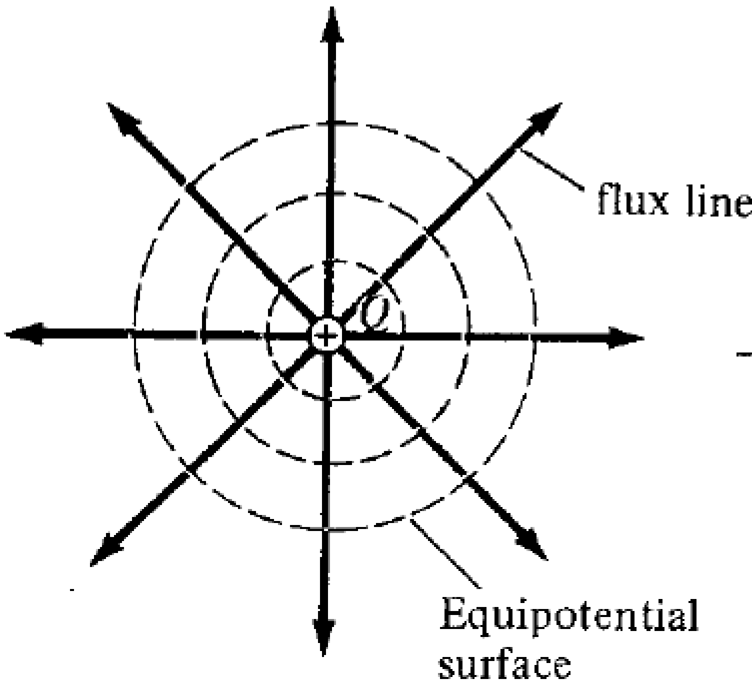
- No work is done in moving a charge from one point to another along an equipotential line or surface (V_A - V_B = 0) and hence:

$$\int \mathbf{E} \cdot d\mathbf{l} = 0$$

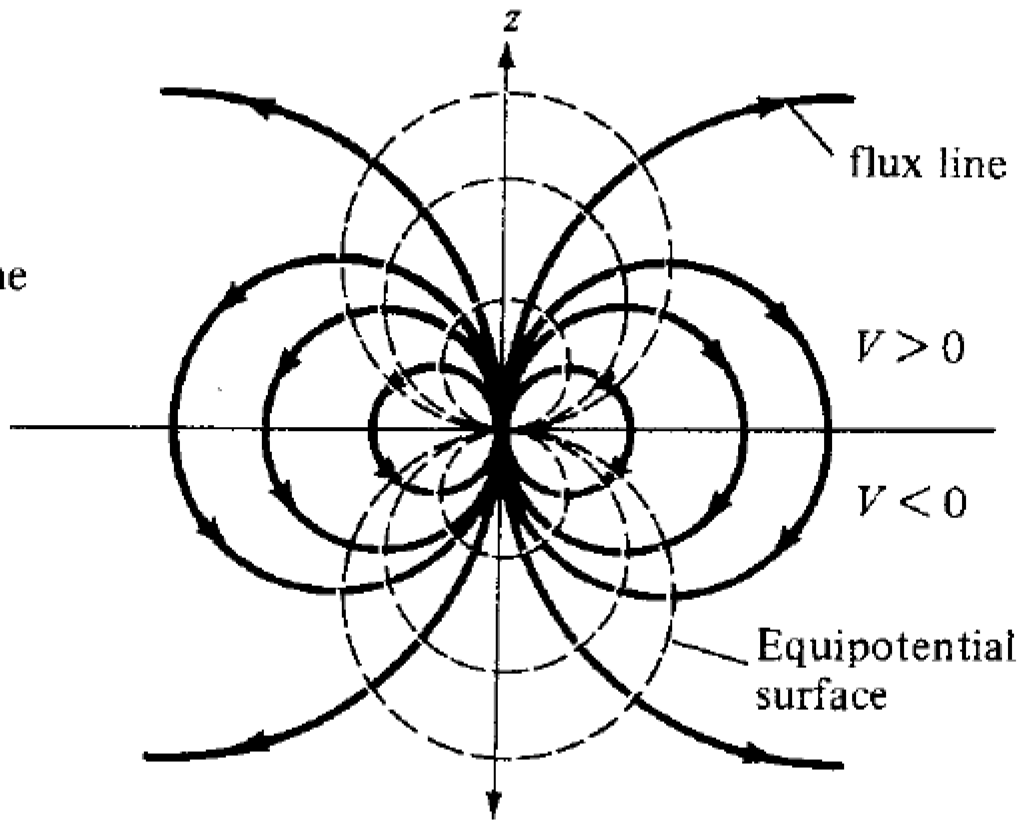
- From the above equation, we may conclude that the lines of force or flux lines (or the direction of **E**) are always normal to equipotential surfaces

Equipotential Surfaces

- Examples of equipotential surfaces for point charge and a dipole are shown in figure (a) and (b), respectively



(a)

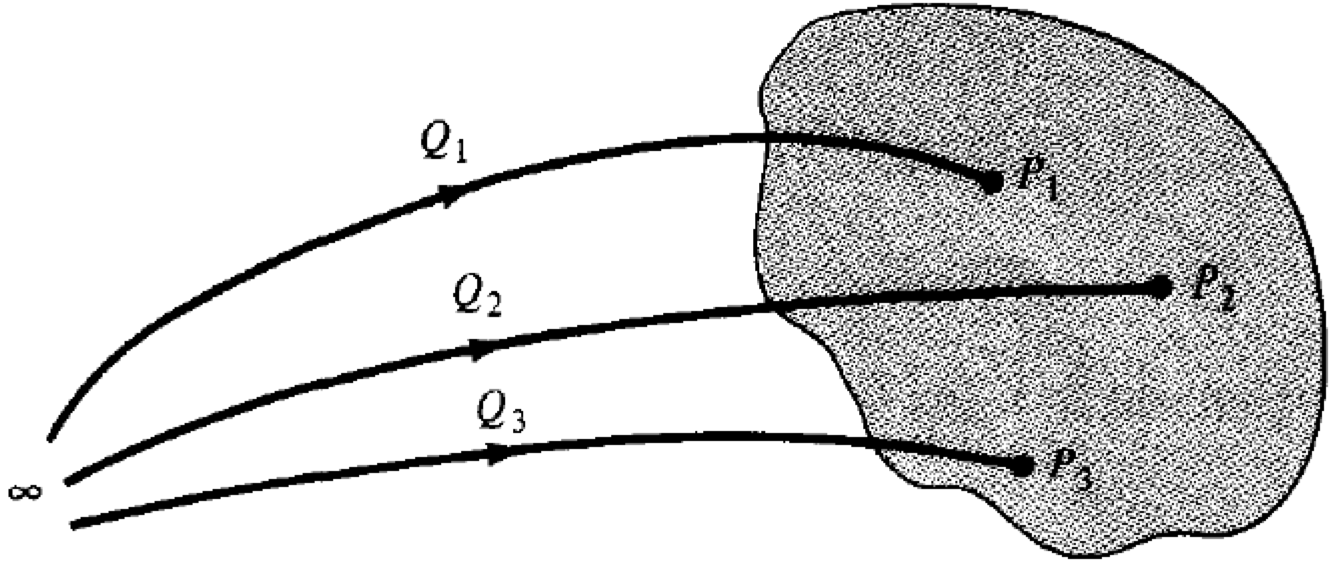


(b)

Energy Density in Electrostatic Fields

- To determine the energy present in an assembly of charges, we must first determine the amount of work necessary to assemble them

- Suppose we wish to position three point charges Q_1, Q_2, and Q_3 in an initially empty space shown shaded in figure below



Energy Density in Electrostatic Fields

- No work is required to transfer Q_1 from infinity to P_1 because the space is initially charge free and there is no electric field

- The work done in transferring Q_2 from infinity to P_2 is equal to the product of Q_2 and the potential V_{21} at P_2 due to Q_1

- Similarly, the work done in positioning Q_3 at P_3 is equal to $Q_3(V_{32} + V_{31})$, where V_{32} and V_{31} are the potentials at P_3 due to Q_2 and Q_1, respectively

- Hence the total work done in positioning the three charges is:

$$\begin{aligned} W_E &= W_1 + W_2 + W_3 \\ &= 0 + Q_2 V_{21} + Q_3(V_{31} + V_{32}) \end{aligned}$$

Energy Density in Electrostatic Fields

- If the charges were positioned in reverse order, we get:

$$
\begin{aligned}
W_E &= W_3 + W_2 + W_1 \\
&= 0 + Q_2V_{23} + Q_1(V_{12} + V_{13})
\end{aligned}
$$

- where V_{23} is the potential at P_2 due to Q_3, V_{12} and V_{13} are, respectively, the potentials at P_1 due to Q_2 and Q_3

- Adding the two equations above, we get:

$$
\begin{aligned}
2W_E &= Q_1(V_{12} + V_{13}) + Q_2(V_{21} + V_{23}) + Q_3(V_{31} + V_{32}) \\
&= Q_1V_1 + Q_2V_2 + Q_3V_3
\end{aligned}
$$

Energy Density in Electrostatic Fields

- Or

$$W_E = \frac{1}{2}(Q_1V_1 + Q_2V_2 + Q_3V_3)$$

- where V_1, V_2, and V_3 are total potentials at P_1, P_2, and P_3, respectively

- In general, if there are *n* point charges, we have:

$$W_E = \frac{1}{2}\sum_{k=1}^{n} Q_kV_k$$

Energy Density in Electrostatic Fields

- If, instead of point charges, the region has a continuous charge distribution, the summation in previous equation becomes integration; that is:

$$W_E = \frac{1}{2} \int \rho_L V \, dl \qquad \text{(line charge)}$$

$$W_E = \frac{1}{2} \int \rho_S V \, dS \qquad \text{(surface charge)}$$

$$W_E = \frac{1}{2} \int \rho_v V \, dv \qquad \text{(volume charge)}$$

Energy Density in Electrostatic Fields

- As $\rho_v = \nabla \cdot \mathbf{D}$, the volume charge equation may be written as:

$$W_E = \frac{1}{2} \int_v (\nabla \cdot \mathbf{D})\, V\, dv$$

- By using vector identity, we get:

$$W_E = -\frac{1}{2} \int_v (\mathbf{D} \cdot \nabla V)\, dv = \frac{1}{2} \int_v (\mathbf{D} \cdot \mathbf{E})\, dv$$

- Since $\mathbf{E} = -\nabla V$ and $\mathbf{D} = \varepsilon_o \mathbf{E}$, therefore:

$$W_E = \frac{1}{2} \int \mathbf{D} \cdot \mathbf{E}\, dv = \frac{1}{2} \int \varepsilon_o E^2\, dv$$

Problem-1

- Point charges Q_1 = 1 nC, Q_2 = - 2 nC, Q_3 = 3 nC, and Q_4 = - 4 nC are positioned one at a time and in that order at (0,0,0), (1,0,0), (0,0,-1), and (0,0,1), respectively. Calculate the energy in the system after each charge is positioned