The multiplication theorem:-

If $f(t)$ and $g(t)$ are two periodic functions having the same period T, c_n and d_n are the coefficients in the complex Fourier series expansion of $f(t)$ & $g(t)$ respectively, then

$$\frac{1}{T}\int_d^{d+T} f(t)g(t)\,dt = \sum_{n=-\infty}^{\infty} c_n d_n^* \quad \text{——— (i)}$$

where d_n^* is the conjugate of d_n.

Parseval's theorem:-

If $f(t)$ is a periodic function with period T then

$$\frac{1}{T}\int_d^{d+T} [f(t)]^2\,dt = \sum_{n=-\infty}^{\infty} c_n c_n^* = \sum_{n=-\infty}^{\infty} |c_n|^2 \quad \text{——— (ii)}$$

where the c_n are the coefficients in the complex Fourier series expansion of $f(t)$.

The root mean square (RMS) value f_{RMS} of a periodic function $f(t)$ of period T, defined by

$$f_{RMS}^2 = \frac{1}{T} = \int_d^{d+T} [f(t)]^2\,dt \quad \text{——— (iii)}$$

(ii) & (iii) give, $f_{RMS}^2 = \frac{1}{T}\int_d^{d+T} [f(t)]^2\,dt = \sum_{n=-\infty}^{+\infty} |c_n|^2$ ——— (iv)

The average power P associated with a periodic signal $f(t)$, of period T, is defined as the mean square value; i.e.,

$$P = f_{RMS}^2 = \frac{1}{T}\int_d^{d+T} [f(t)]^2\,dt = \sum_{n=-\infty}^{\infty} |c_n|^2 \quad \text{——— (v)}$$

(F.S. 34)

Conclusion, $P = \frac{1}{T}\int_{d}^{d+T}[f(t)]^2 dt = \sum_{n=-\infty}^{\infty}|c_n|^2$

Total power delivered Sum of the power associated with each harmonic.

For example, if $f(t)$ represents a voltage waveform applied to a resistor then P represents the average power, measured in watts, dissipated by a 1Ω resistor.

The component $e^{jn\omega_0 t}$ at frequency $\omega_n = n\omega_0$, must be considered alongside the component $e^{-jn\omega_0 t}$ at the corresponding negative frequency $-\omega_n$, in order to form the actual nth harmonic of the function $f(t)$. Since $|c_n^*|^2 = |c_{-n}|^2 = |c_n|^2$, it follows that the power associated with the nth harmonic is the sum of the power associated with $e^{jn\omega_0 t}$ and $e^{-jn\omega_0 t}$, i.e., $P_n = 2|c_n|^2$ ——— (vi).

EX:- RMS of a Sinusoid:

Consider, $V = V_p \sin\omega t$, $\omega = \frac{2\pi}{T} = 2\pi f$, $T = 1/f$, V_p is the peak value.

$$V_{RMS}^2 = \frac{1}{T}\int_0^T v^2 dt = \frac{1}{T}\int_0^T V_p^2 \sin^2\omega t\, dt = \frac{V_p^2}{T}\int_0^T \sin^2\omega t\, dt$$

$$= \frac{V_p^2}{T}\int_0^T \frac{1}{2}(1-2\cos\omega t)\,dt = \frac{V_p^2}{T}\left(\frac{t}{2} - \frac{1}{4\omega}\sin 2\omega t\right)\Big|_0^T$$

$$= \frac{V_p^2}{T}\left(\frac{T}{2} - \frac{1}{4\omega}\sin 2\omega T\right) = \frac{V_p^2}{T}\left(\frac{T}{2} - 0\right), T = \frac{2\pi}{\omega}$$

The integral value of a sine wave over an integral number of cycles is zero. Thus,

$$V_{RMS}^2 = \frac{V_p^2}{2}.$$

Hence, if the sine wave of peak value V_p then

$$V_{RMS} = \frac{V_{peak}}{\sqrt{2}}$$

(F.S.35)

Ex:- A periodic voltage $V(t)$ (in volts) period 5 ms and specified by

$$V(t) = \begin{cases} 60, & 0 < t < 5/4 \text{ ms} \\ 0, & \frac{5}{4}\text{ms} < t < 5 \text{ ms} \end{cases} \qquad V(t+5\text{ms}) = V(t)$$

is applied across the terminals of a $15\,\Omega$ resistor. (a) obtain expression for the coefficients c_n of the complex Fourier series representation of $V(t)$, and write down the values of the first five non-zero terms.
(b) Calculate the power associated with each of the first five non-zero terms of the Fourier expansion. (c) Calculate the total power delivered to the $15\,\Omega$ resistor. (d) what is the percentage of the total power delivered to the resistor by the first five non-zero terms of F.S?

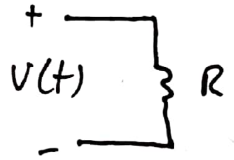
Sol:- $c_n = \frac{1}{5}\int_0^{5/4} 60\, e^{-j\frac{2n\pi}{5}t}\,dt = 12\left[\frac{-5}{2n\pi j}\, e^{-j\frac{2n\pi}{5}t}\right]_0^{5/4} = \frac{30}{jn\pi}\left[1 - e^{-jn\pi/2}\right], n \neq 0,$

$c_0 = \frac{1}{5}(60)(5/4) = 15$. First five non-zero terms are $c_0 = 15$, $c_1 = \frac{30}{\pi j}(1+j)$,
$c_2 = \frac{30}{j\pi} = -\frac{30}{\pi}j$, $c_3 = \frac{10}{j\pi}(1-j) = \frac{10}{\pi}(-1-j)$, $c_4 = 0$, $c_5 = \frac{6}{j\pi}(1+j) = \frac{6}{\pi}(1-j)$.

Power associated with the first five non-zero terms, $P_0 = \frac{1}{15}[15^2] = 15W$,
$P_1 = \frac{1}{15}[2|c_1|^2] = \frac{2}{15}(13.5)^2 = 24.3W$, $P_2 = \frac{1}{15}[2|c_2|^2] = \frac{1}{15}(9.55)^2 = 12.16W$,
$P_3 = \frac{1}{15}[2|c_3|^2] = \frac{2}{15}(4.5)^2 = 2.70W$, $P_4 = 0$, $P_5 = \frac{1}{15}[2|c_5|^2] = \frac{2}{15}(2.70)^2 = 0.97W$

The total power delivered by first five non-zero terms $= P_0 + \dots + P_5 = 55.13W$
Total power delivered to $15\,\Omega$ is $P = \frac{1}{15}\left[\frac{1}{5}\int_0^{5/4}(60)^2 dt\right] = \frac{1}{15}\left(\frac{1}{5}\right)(60^2)\left(\frac{5}{4}\right) = 60W$
% of total power delivered by the first five non-zero terms is $\frac{55.13}{60} \times 100\% = 91.9\%$.

Problems:- 1. Let $V(t) = \sin(4\pi t)$ be the input signal to the circuit shown. Find the average power delivered to the resistor $R = 1\,\Omega$.



2. Let $V(t) = 3 - 5\sin(4t - \frac{\pi}{3}) - 4\cos(3t + \pi/3)$ be the input signal to the circuit shown. Find the average power delivered to the resistor $R = 1\,\Omega$ by using both sides of Parseval's theorem.

3. A periodic function $f(t)$, of period 2π, is defined in the range $-\pi < t < \pi$ by $f(t) = \sin\frac{1}{2}t$. Show that the complex form of Fourier series expansion for $f(t)$ is $f(t) = \sum_{n=-\infty}^{+\infty} \frac{j\,4n(-1)^n}{\pi(4n^2-1)} e^{jnt}$. Sketch 2-sided discrete Fourier spectra and obtain trigonometric Fourier series.
[Ref/Keep this question: Example 7.18, page 610, Glyn James at LMS].

(F.S.36)