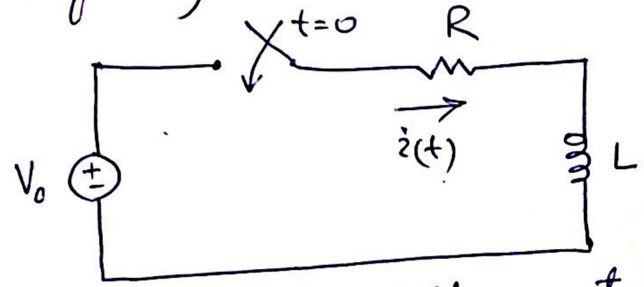
8.6 Driven RL Circuits

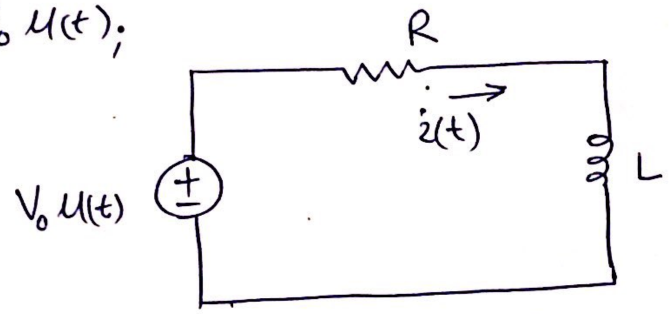
(pp 286 8^{th} Ed HKD)

Consider the following circuit:-



or the application of a voltage-step forcing function

$V_0\, u(t)$;



— $i(t)$ has two components;

— the natural response and

— the forced response.

also known as

— complementary solution (natural) and

— particular solution (forced) wrt a

linear differential equation.

8.7 Natural and Forced Response

(PP 289 8th Ed HKD)

The complete response is composed of two parts, the natural response and the forced response.

— The natural response is a characteristic of the circuit and not of the sources.

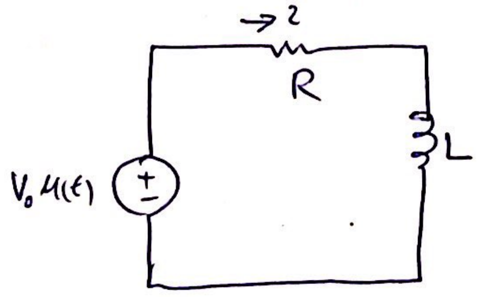
— The forced response has the characteristics of the forcing function.

Imp: (A mathematical reason for considering the complete response to be composed of two parts is based on the fact that the solution of any linear differential equation may be expressed as the sum of two parts:

— the complementary solution (natural response) and

— the particular solution (forced response))

— Let us see the simple RL circuit and explain how to determine the complete response by the addition of the natural and forced responses.



— contd

— contd (291)

— The desired response is the current $i(t)$, and we express this current as the sum of the natural and the forced current,

$$i = i_n + i_f$$

— The functional form of the natural response must be the same as that obtained without any sources.

— So we replace the step-voltage source by a short circuit and observe the source-free RL circuit that had a response

$$i_n = A e^{-R/L\, t}$$

where the amplitude A is yet to be determined.

Note: Since the initial condition applies to the complete response, we cannot assume $A = i(0)$.

— For the forced response, in this problem, must be constant because the source is a constant V_0 for positive values of time.

— After the natural response has died out, there can be no voltage across the inductor, hence, forced response is simply

$$i_f = \frac{V_0}{R}$$

— So $i(t) = \frac{V_0}{R} + A e^{-R/L\, t}$ ——— contd

— contd (292)

If $i(t)\big|_{t=0} = i(0)$

then from $i = i_n + i_f = Ae^{-R/L\,t} + \frac{V_0}{R}$

$$i(0) = Ae^{0} + \frac{V_0}{R} \quad \text{or} \quad A = i(0) - \frac{V_0}{R}$$

where $\frac{V_0}{R} = i(\alpha)$

Hence $A = i(0) - i(\alpha)$

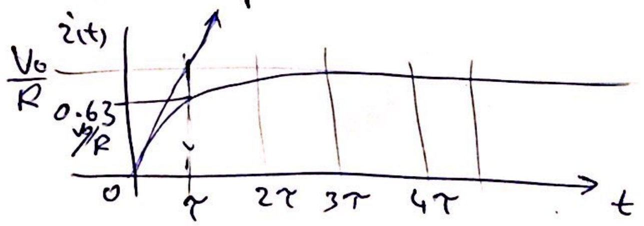
Therefore $i(t) = \frac{V_0}{R} + Ae^{-R/L\,t}$ becomes

$$i(t) = i(\alpha) + \left[i(0^+) - i(\alpha)\right]e^{-R/L\,t}\,u(t),\ A$$

OR $i(t) = \text{Final Value} + \left[\text{initial value} - \text{final value}\right]e^{-t/\tau}u(t)$ A.

(Note: applies only to dc excitation)

A plot of this response is:-



— In one time constant, the current has attained 63.2% of its final value.

— In purely dc circuits, inductor act as short cct and capacitor as open circuit.

— Note: Applying the initial condition to the complete response allows us to determine the unknown constant which multiplies the transient term.