ELECTROMAGNETIC WAVE PROPAGATION-CONTINUED

Wave Propagation in Lossy Dielectrics

- Previously we had derived the following equation for electric field of an EM wave:

$$\mathbf{E}(z, t) = E_o e^{-\alpha z} \cos(\omega t - \beta z)\mathbf{a}_x$$

- And for the magnetic field, we have:

$$\mathbf{H} = \frac{E_o}{|\eta|} e^{-\alpha z} \cos(\omega t - \beta z - \theta_\eta)\, \mathbf{a}_y$$

- Notice from the above two equations that as the wave propagates along $\mathbf{a_z}$, it decreases or attenuates in amplitude by a factor $e^{-\propto z}$

- Hence $\propto$ is known as the attenuation constant or attenuation factor of the medium

Wave Propagation in Lossy Dielectrics

- ∝ is a measure of the spatial rate of decay of the wave in the medium, measured in nepers per meter (Np/m) or in decibels per meter (dB/m)

- As derived earlier:

$$\alpha = \omega\sqrt{\frac{\mu\varepsilon}{2}\left[\sqrt{1+\left[\frac{\sigma}{\omega\varepsilon}\right]^2}-1\right]}$$

- An attenuation of 1 neper denotes a reduction to e^{-1} of the original value whereas an increase of 1 neper indicates an increase by a factor of *e*

- Hence for voltages:

$$1\ \text{Np} = 20\log_{10} e = 8.686\ \text{dB}$$

Wave Propagation in Lossy Dielectrics

- From the relation for attenuation, we notice that if $\sigma = 0$, as is the case for a lossless medium and free space, $\propto = 0$ and the wave is not attenuated as it propagates

- The quantity β is a measure of the phase shift per length and is called the phase constant or wave number

- In terms of β, the wave velocity *u* and wavelength λ are, respectively, given as below:

$$u = \frac{\omega}{\beta}, \qquad \lambda = \frac{2\pi}{\beta}$$

Wave Propagation in Lossy Dielectrics

- The complex quantity η in the relation for **E** and **H** was derived as:

$$\eta = \sqrt{\frac{j\omega\mu}{\sigma + j\omega\varepsilon}} = |\eta| \angle \theta_\eta = |\eta| e^{j\theta_\eta}$$

- Therefore, **E** and **H** are out of phase by θ_η at any instant of time due to the complex intrinsic impedance of the medium

- Thus at any time, **E** leads **H** (or **H** lags **E**) by θ_η

- The ratio of the magnitude of the conduction current density **J** to that of the displacement current density **J$_d$** in a lossy medium is:

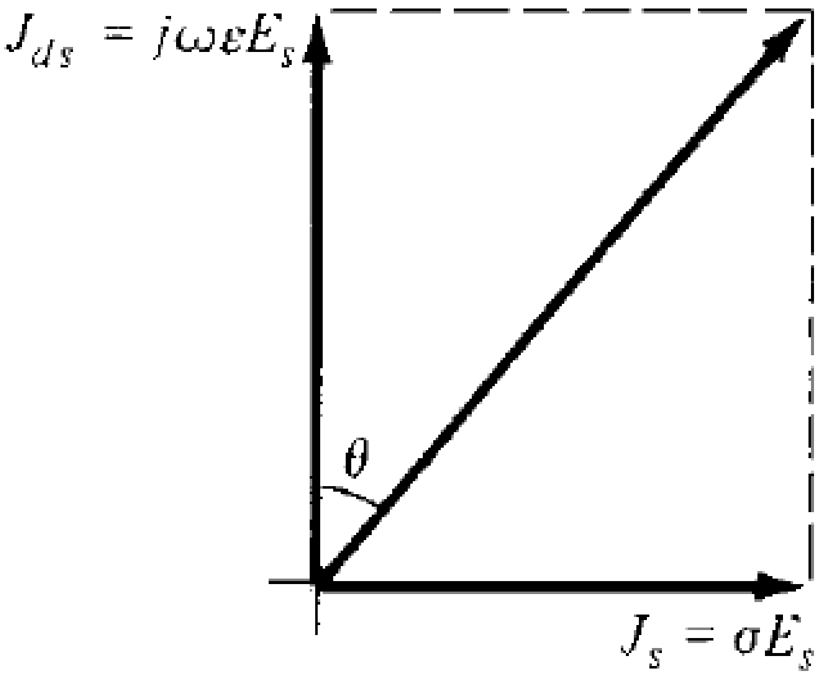
$$\frac{|\mathbf{J}_s|}{|\mathbf{J}_{ds}|} = \frac{|\sigma \mathbf{E}_s|}{|j\omega\varepsilon \mathbf{E}_s|} = \frac{\sigma}{\omega\varepsilon} = \tan\theta$$

Wave Propagation in Lossy Dielectrics

- Or:

$$\tan\theta = \frac{\sigma}{\omega\varepsilon}$$

- Here $\tan\theta$ is known as the loss tangent and θ is the loss angle of the medium as illustrated in figure below



Wave Propagation in Lossy Dielectrics

- Although a line of demarcation between good conductors and lossy dielectrics is not easy to make, $\tan\theta$ or θ may be used to determine how lossy a medium is

- A medium is said to be a good (lossless or perfect) dielectric if $\tan\theta$ is very small ($\sigma \ll \omega\varepsilon$) or a good conductor if $\tan\theta$ is very large ($\sigma \gg \omega\varepsilon$)

- From the viewpoint of wave propagation, the characteristic behavior of a medium depends not only on its parameters σ, ε and μ but also on the frequency of operation

Wave Propagation in Lossless Dielectrics

- In a lossless dielectric $\sigma \ll \omega\varepsilon$

- For lossless dielectric: $\sigma \simeq 0, \qquad \varepsilon = \varepsilon_o\varepsilon_r, \qquad \mu = \mu_o\mu_r$

- Previously, we derived the following equations:

$$\alpha = \omega\sqrt{\frac{\mu\varepsilon}{2}\left[\sqrt{1+\left[\frac{\sigma}{\omega\varepsilon}\right]^2}-1\right]}$$

$$\beta = \omega\sqrt{\frac{\mu\varepsilon}{2}\left[\sqrt{1+\left[\frac{\sigma}{\omega\varepsilon}\right]^2}+1\right]}$$

- For lossless dielectric, we get:

$$\alpha = 0, \qquad \beta = \omega\sqrt{\mu\varepsilon}$$

Wave Propagation in Lossless Dielectrics

- Also:

$$u = \frac{\omega}{\beta} = \frac{1}{\sqrt{\mu\varepsilon}}, \qquad \lambda = \frac{2\pi}{\beta}$$

- And:

$$\eta = \sqrt{\frac{\mu}{\varepsilon}} \angle 0^\circ$$

- Therefore, **E** and **H** are in time phase with each other

Wave Propagation in Free Space

- For free space, we have:

$$\sigma = 0, \qquad \varepsilon = \varepsilon_o, \qquad \mu = \mu_o$$

- Therefore, we have the following relations:

$$\alpha = 0, \qquad \beta = \omega\sqrt{\mu_o\varepsilon_o} = \frac{\omega}{c}$$

$$u = \frac{1}{\sqrt{\mu_o\varepsilon_o}} = c, \qquad \lambda = \frac{2\pi}{\beta}$$

- Where *c* is the speed of light in vacuum

- This shows that light is the manifestation of an EM wave

- In other words, light is characteristically electromagnetic

Wave Propagation in Free Space

- By substitution, we get $\theta_\eta = 0$ and $\eta = \eta_o$ where η_o is called the intrinsic impedance of free space and is given by:

$$\eta_o = \sqrt{\frac{\mu_o}{\varepsilon_o}} = 120\pi \simeq 377\ \Omega$$

$$\mathbf{E} = E_o \cos(\omega t - \beta z)\ \mathbf{a}_x$$

- Then:

$$\mathbf{H} = H_o \cos(\omega t - \beta z)\ \mathbf{a}_y = \frac{E_o}{\eta_o} \cos(\omega t - \beta z)\ \mathbf{a}_y$$

- In general, if $\mathbf{a}_E$, $\mathbf{a}_H$, and $\mathbf{a}_k$ are unit vectors along the **E** field, the **H** field, and the direction of wave propagation

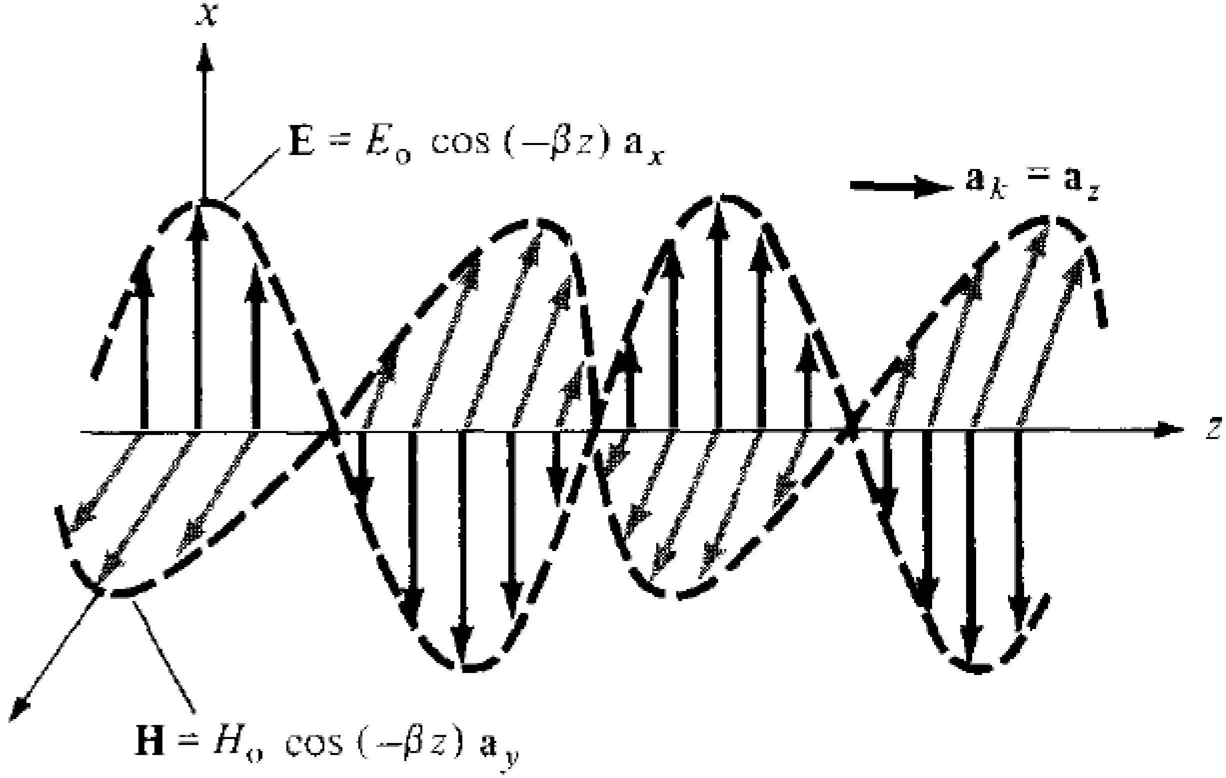
- Therefore:

$$\mathbf{a}_k \times \mathbf{a}_E = \mathbf{a}_H$$

$$\mathbf{a}_k \times \mathbf{a}_H = -\mathbf{a}_E$$

EM Wave Propagation

➢The plots of **E** and **H** are shown in figure below:



➢Both **E** and **H** form an EM wave that has no electric or magnetic field components along the direction of propagation; such a wave is called a transverse electromagnetic (TEM) wave

Wave Propagation in Good Conductors

- A perfect, or good conductor, is one in which $\sigma \gg \omega\varepsilon$ so that $\sigma/\omega\varepsilon \to \infty$; that is:

$$\sigma \simeq \infty, \qquad \varepsilon = \varepsilon_o, \qquad \mu = \mu_o\mu_r$$

- The attenuation and phase constants were derived as:

$$\alpha = \omega\sqrt{\frac{\mu\varepsilon}{2}\left[\sqrt{1+\left[\frac{\sigma}{\omega\varepsilon}\right]^2} - 1\right]}$$

$$\beta = \omega\sqrt{\frac{\mu\varepsilon}{2}\left[\sqrt{1+\left[\frac{\sigma}{\omega\varepsilon}\right]^2} + 1\right]}$$

- Hence for good conductors, the equations are as below:

$$\alpha = \beta = \sqrt{\frac{\omega\mu\sigma}{2}} = \sqrt{\pi f\mu\sigma}$$

Wave Propagation in Good Conductors

- And:

$$u = \frac{\omega}{\beta} = \sqrt{\frac{2\omega}{\mu\sigma}}, \qquad \lambda = \frac{2\pi}{\beta}$$

- We have the intrinsic impedance of the medium as:

$$\eta = \sqrt{\frac{j\omega\mu}{\sigma + j\omega\varepsilon}} = |\eta| \angle\theta_\eta = |\eta| e^{j\theta_\eta}$$

- For good conductors, this becomes:

$$\eta = \sqrt{\frac{\omega\mu}{\sigma}} \angle 45^\circ$$

- Therefore, **E** leads **H** by 45°

Wave Propagation in Good Conductors

- So if:

$$\mathbf{E} = E_o e^{-\alpha z} \cos(\omega t - \beta z)\, \mathbf{a}_x$$

- Then:

$$\mathbf{H} = \frac{E_o}{\sqrt{\dfrac{\omega\mu}{\sigma}}} e^{-\alpha z} \cos(\omega t - \beta z - 45°)\, \mathbf{a}_y$$

- Therefore, as **E** (or **H**) wave travels in a conducting medium, its amplitude is attenuated by the factor $e^{-\alpha z}$

- The distance δ, through which the wave amplitude decreases by a factor e^{-1} (about 37%) is called skin depth or penetration depth of the medium; that is:

$$E_o e^{-\alpha\delta} = E_o e^{-1}$$

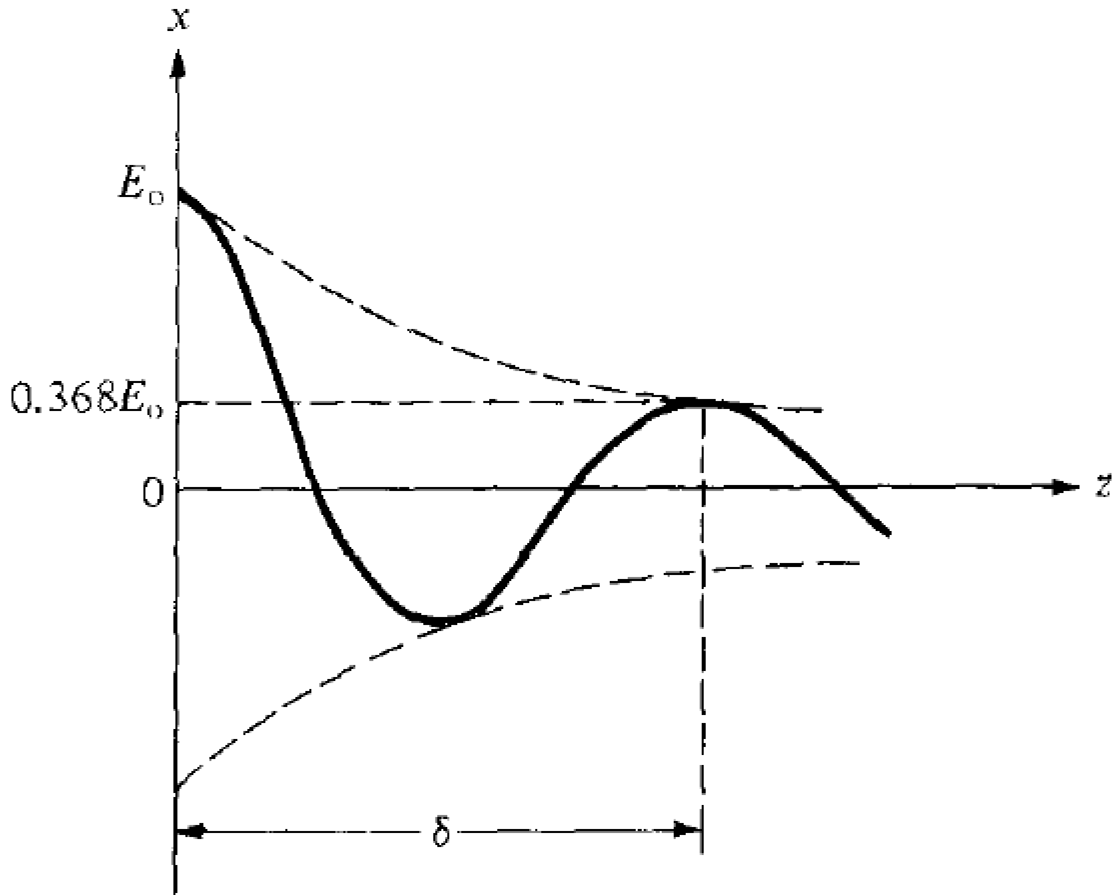
- Or:

$$\delta = \frac{1}{\alpha}$$

Wave Propagation in Good Conductors

- The skin depth is a measure of the depth to which an EM wave can penetrate the medium

- Figure below illustrates skin depth



Problem-1

- A plane wave propagating through a medium with $\varepsilon_r = 8, \mu_r = 2$ has $\mathbf{E} = 0.5e^{-z/3}\sin(10^8 t - \beta z)\mathbf{a_x}$ V/m. Determine:

a) β
b) The loss tangent
c) Intrinsic Impedance
d) Wave velocity
e) **H** field