Quiz-2 CLO-2
Machine Learning
BEE-12(E)

Name: Solution

Given two normal distributions $p(x|C_1) \sim \mathcal{N}(\mu_1, \sigma_1^2)$ and $p(x|C_2) \sim \mathcal{N}(\mu_2, \sigma_2^2)$ and $P(C_1)$ and $P(C_2)$, calculate the Bayes' discriminant points analytically.

Assume $m_1 = m_2 = m$, $\mu_i \approx m_i$ *and* $\sigma_i^2 \approx s_i^2$
In this two-class problem, C_1 has data samples twice as much as C_2.

$$p(x|C_i) = \frac{1}{\sqrt{2\pi}\sigma_i} \exp\left[-\frac{(x-\mu_i)^2}{2\sigma_i^2}\right]$$

$$g_i(x) = \log(p(x|C_i)P(C_i))$$

For discriminant Points:

$$g_1(x) = g_2(x)$$

$$\cancel{-\tfrac{1}{2}\log 2\pi} - \log S_1 - \frac{(x-m)^2}{2S_1^2} + \log P(C_1) =$$

$$\cancel{-\tfrac{1}{2}\log 2\pi} - \log S_2 - \frac{(x-m)^2}{2S_2^2} + \log P(C_2).$$

$$P(C_1) = 2/3$$

$$P(C_2) = 1/3$$

$$-\log S_1 - \frac{(x-m)^2}{2S_1^2} + \log(2/3) = -\log S_2 - \frac{(x-m)^2}{2S_2^2} + \log(1/3)$$

$$\log(S_2/S_1) + \log\left[(2/3)/1/3\right]$$

$$= \frac{(x-m)^2}{2S_1^2} - \frac{(x-m)^2}{2S_2^2}$$

$$\log(S_2/S_1) + 0.3 = (x-m)^2\left[\frac{S_2^2 - S_1^2}{2S_1^2S_2^2}\right]$$

$$(x-m)^2 = \log\left(0.3\, S_2/S_1\right)\left[\frac{2S_1^2 S_2^2}{S_2^2 - S_1^2}\right]$$

$$x = \pm\sqrt{\log\left(0.3\, S_2/S_1\right)\left[\frac{2S_1^2 S_2^2}{S_2^2 - S_1^2}\right]} + m$$