Engineering Mechanics

Dr. Hina Gohar Ali

Hina.gohar@seecs.edu.pk

Office : IAEC building

Office Hours: Appointment through emails

Research Interest: Photovoltaic systems, Power & Control, Sliding mode control (SMC)

CHAPTER 4

Force System Resultants

Objectives

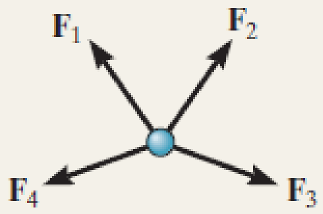
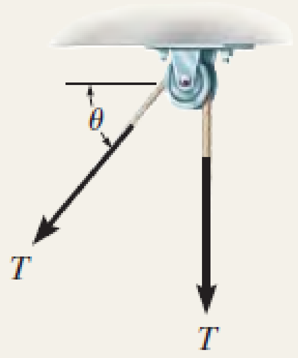
- Concept of moment of a force in two and three dimensions
- Method for finding the moment of a force about a specified axis.
- Define the moment of a couple.
- Determine the resultants of non-concurrent force systems
- Reduce a simple distributed loading to a resultant force having a specified location

Contents (Section 4.1-4.3)

- Recap
- Moment of a Force (Torque)
- Cross Product

RECAP

CHAPTER REVIEW

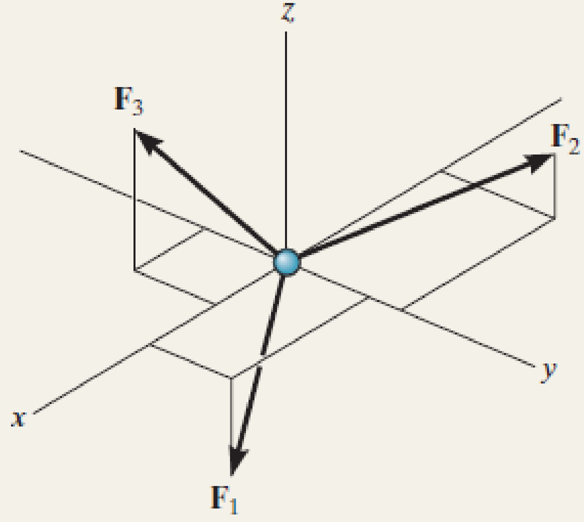
Particle Equilibrium When a particle is at rest or moves with constant velocity, it is in equilibrium. This requires that all the forces acting on the particle form a zero resultant force. In order to account for all the forces that act on a particle, it is necessary to draw its free-body diagram. This diagram is an outlined shape of the particle that shows all the forces listed with their known or unknown magnitudes and directions.	$\mathbf{F}_R = \Sigma \mathbf{F} = \mathbf{0}$	
Two Dimensions The two scalar equations of force equilibrium can be applied with reference to an established *x, y* coordinate system. The tensile force developed in a *continuous cable* that passes over a frictionless pulley must have a *constant* magnitude throughout the cable to keep the cable in equilibrium. If the problem involves a linearly elastic spring, then the stretch or compression *s* of the spring can be related to the force applied to it.	$\Sigma F_x = 0$ $\Sigma F_y = 0$ $F = ks$	 Cable is in tension

Three Dimensions

If the three-dimensional geometry is difficult to visualize, then the equilibrium equation should be applied using a Cartesian vector analysis. This requires first expressing each force on the free-body diagram as a Cartesian vector. When the forces are summed and set equal to zero, then the **i**, **j**, and **k** components are also zero.

$$\Sigma \mathbf{F} = \mathbf{0}$$

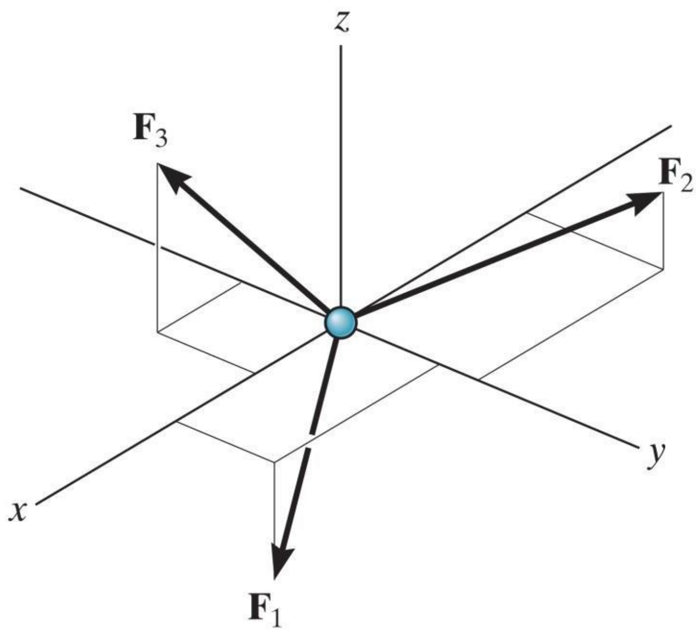
$$\Sigma F_x = 0$$
$$\Sigma F_y = 0$$
$$\Sigma F_z = 0$$



3 Dimensional Force System

$$\Sigma \mathbf{F} = \mathbf{0}$$

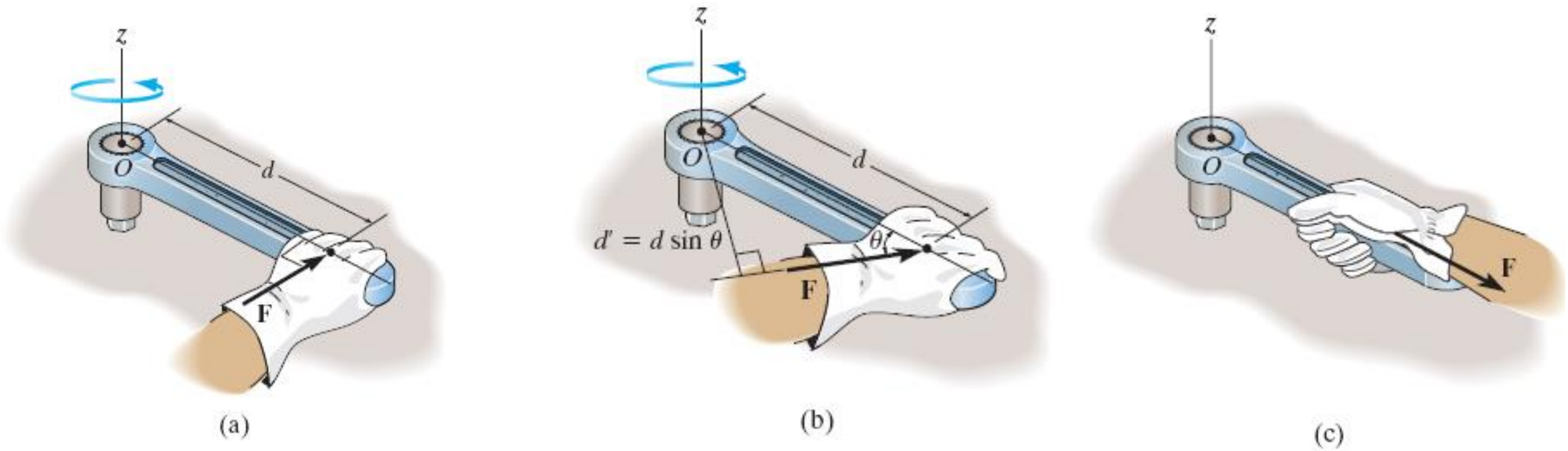
$$\boxed{\begin{aligned} \Sigma F_x &= 0 \\ \Sigma F_y &= 0 \\ \Sigma F_z &= 0 \end{aligned}}$$



Moment of a Force/Moment/Torque (Scalar)

- *Moment* of a force about a point or axis – a measure of the tendency of the force to cause a body to rotate about the point or axis

- Torque – tendency of rotation caused by $\mathbf{F}_x$ or simple moment $(\mathbf{M}_o)_z$
- When a force is applied to a body it will produce a tendency for the body to rotate about a point that is not on the line of action of the force. This tendency to rotate is sometimes called a *torque,* but most often it is called the moment of a force or simply the *moment*.



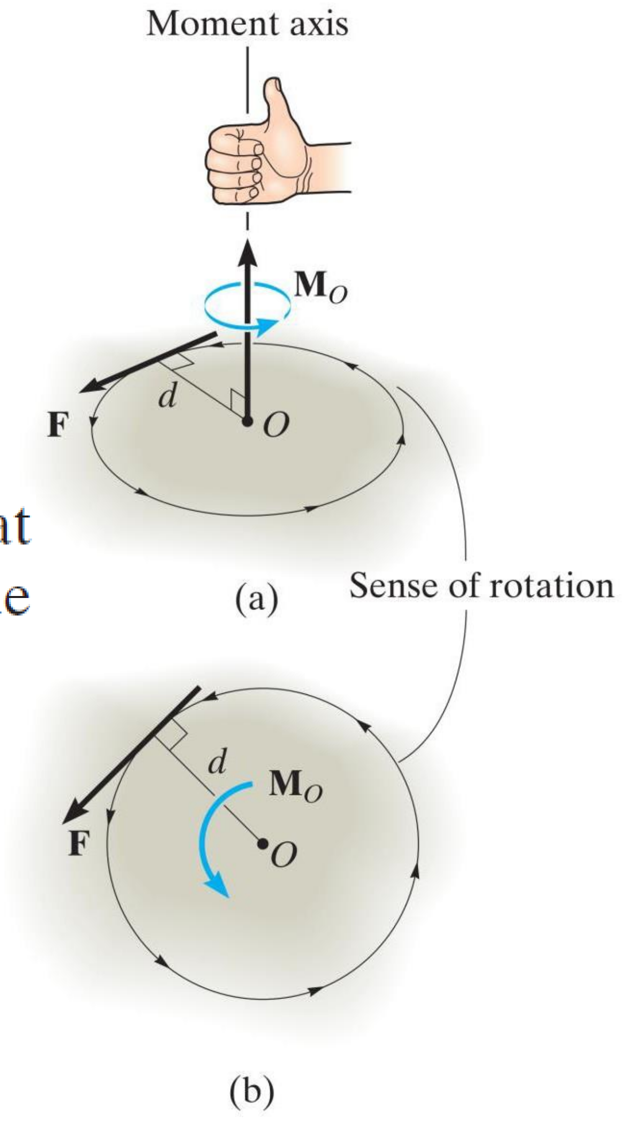
The magnitude of the moment is directly proportional to the magnitude of **F** and the perpendicular distance or *moment arm d.*

Moment of a Force/Moment/Torque (Scalar)

Magnitude. The magnitude of $\mathbf{M}_O$ is

$$\boxed{M_O = Fd}$$

where d is the *moment arm* or *perpendicular distance* from the axis at point O to the line of action of the force. Units of moment magnitude consist of force times distance, e.g., N · m or lb · ft.



Direction.

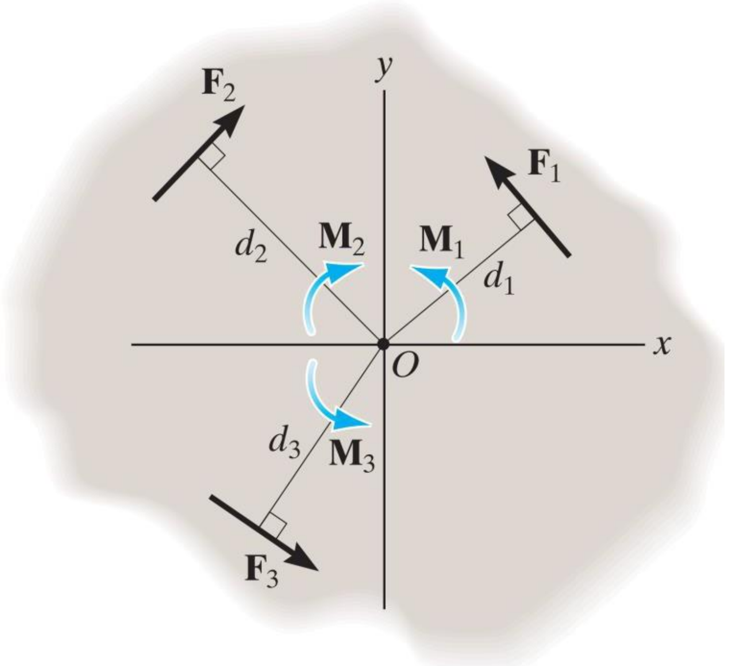
Direction: Direction using "right hand rule"

the thumb of the right hand will give the directional sense Mo

Moment of a Force/Moment/Torque (Scalar)

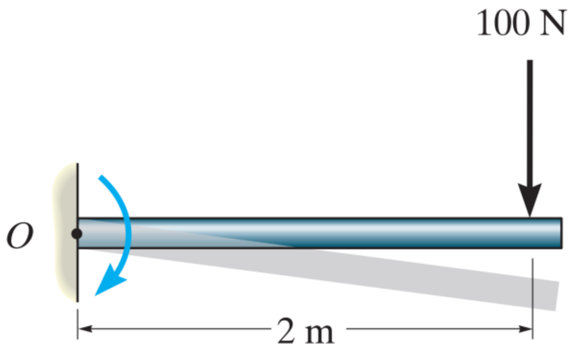
Resultant Moment.

$$\circlearrowleft +(M_R)_o = \Sigma Fd; \qquad (M_R)_o = F_1 d_1 - F_2 d_2 + F_3 d_3$$

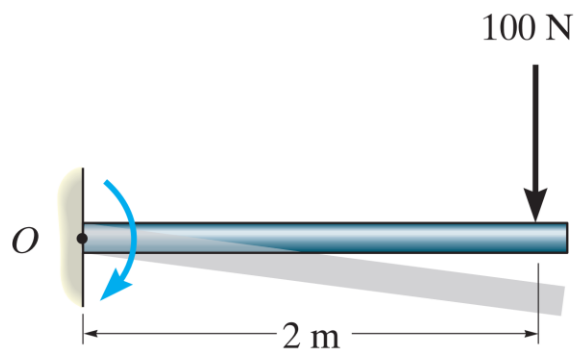


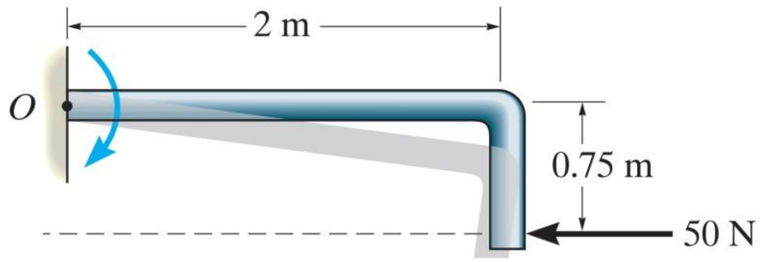
If the numerical result of this sum is a positive scalar, $(\mathbf{M}_R)_o$ will be a counterclockwise moment (out of the page); and if the result is negative, $(\mathbf{M}_R)_o$ will be a clockwise moment (into the page).

Example



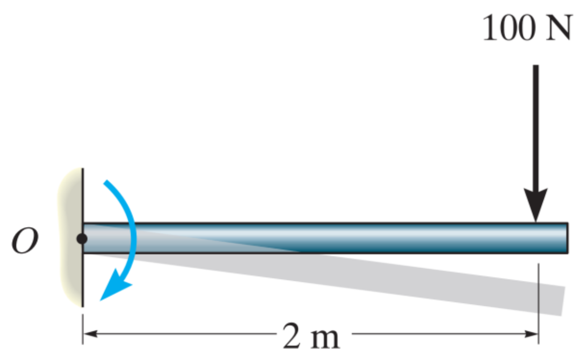
Example



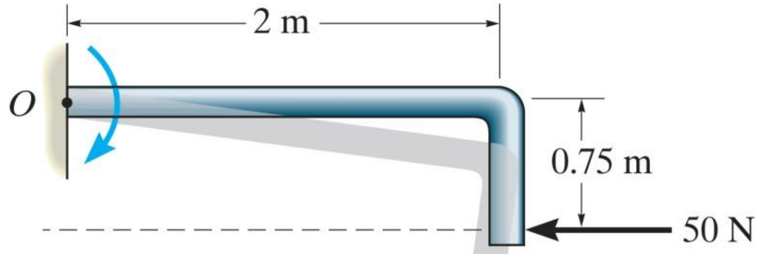


$$M_O = (100\text{ N})(2\text{ m}) = 200\text{ N}\cdot\text{m} \circlearrowright$$

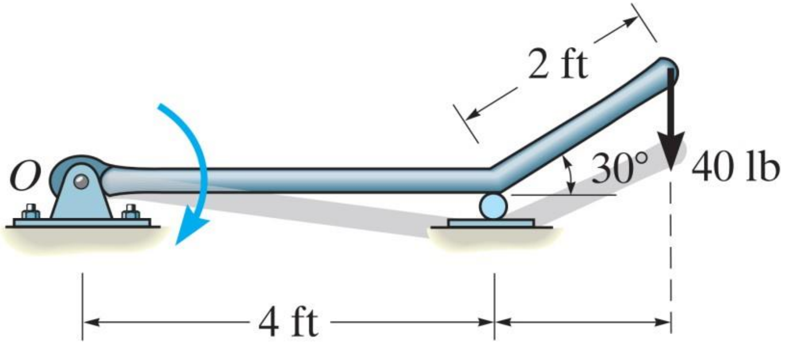
Example



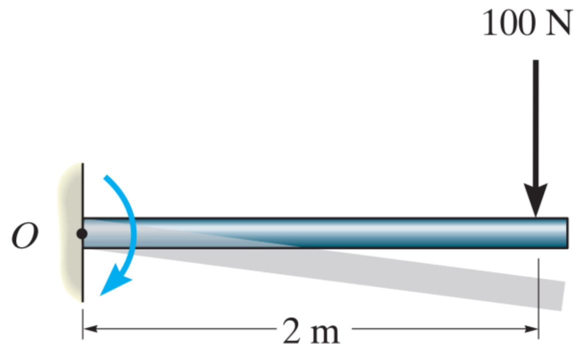
$$M_O = (100\ \text{N})(2\ \text{m}) = 200\ \text{N}\cdot\text{m}$$



$$M_O = (50\ \text{N})(0.75\ \text{m}) = 37.5\ \text{N}\cdot\text{m}$$



Example



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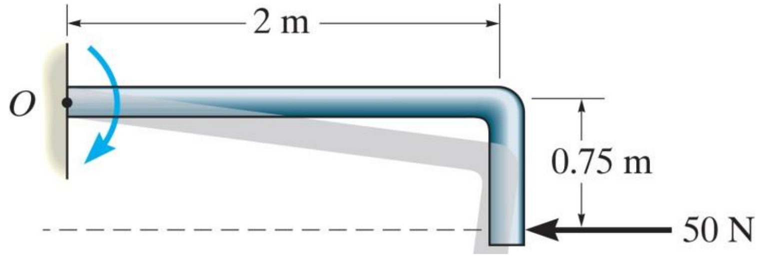

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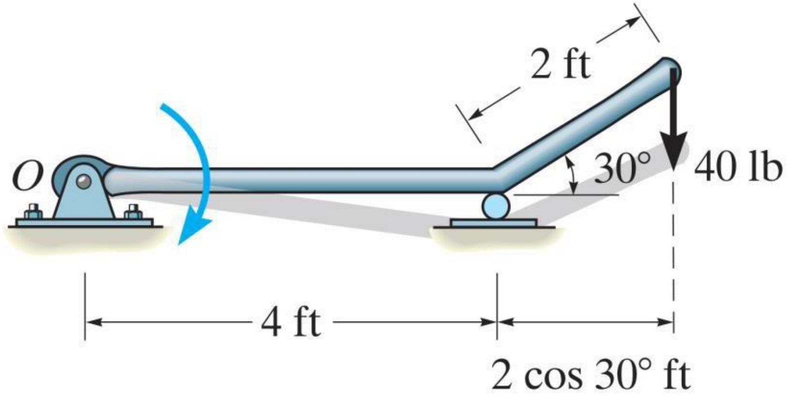
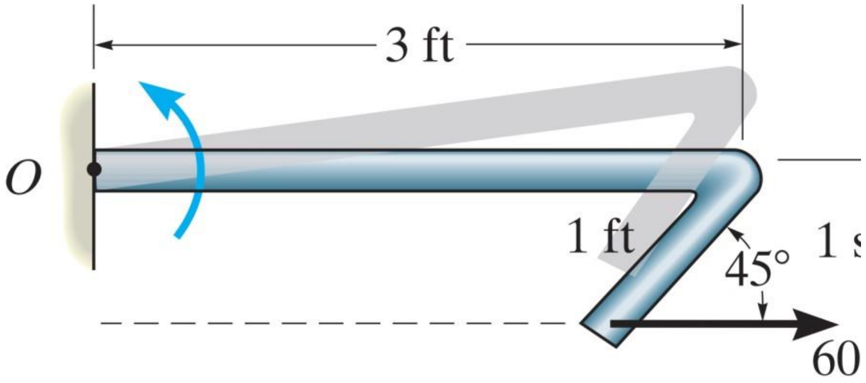
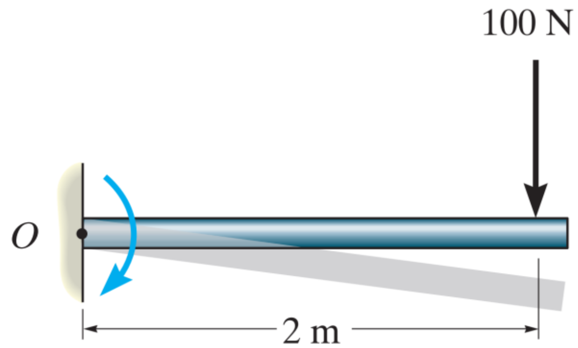


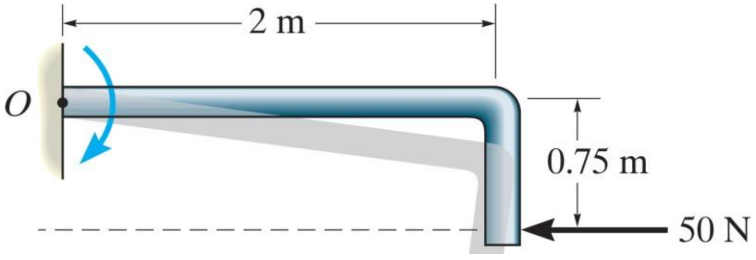
$$M_O = (40\ \text{lb})(4\ \text{ft} + 2\cos 30^\circ\ \text{ft}) = 229\ \text{lb}\cdot\text{ft} \circlearrowright$$



Example



$$M_O = (100\ \text{N})(2\ \text{m}) = 200\ \text{N}\cdot\text{m} \circlearrowright$$



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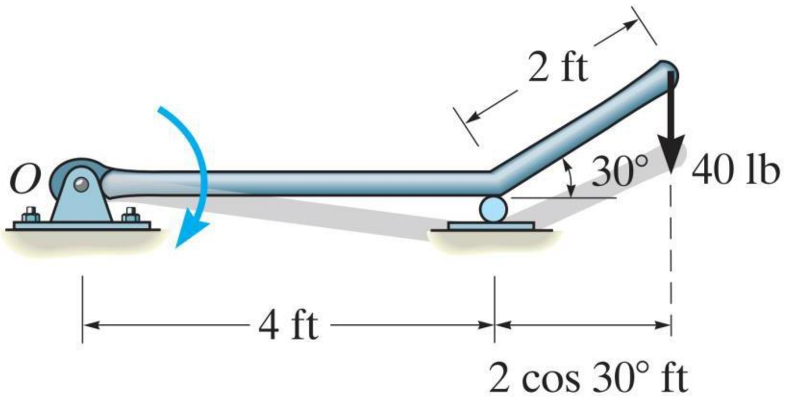
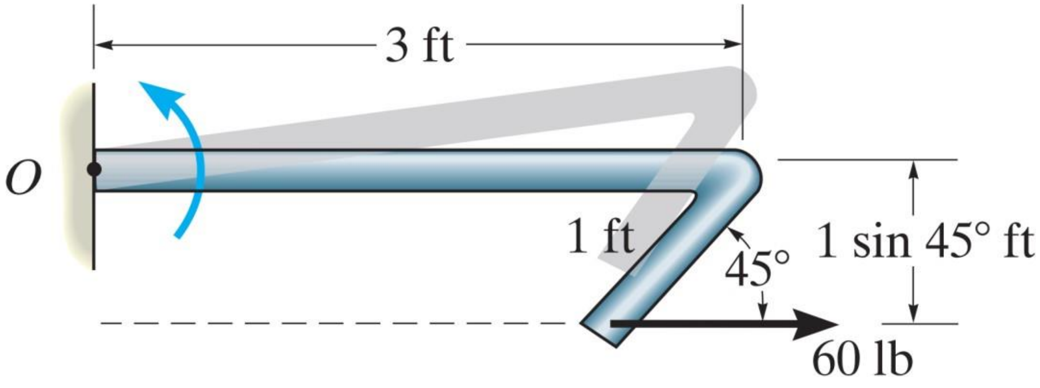
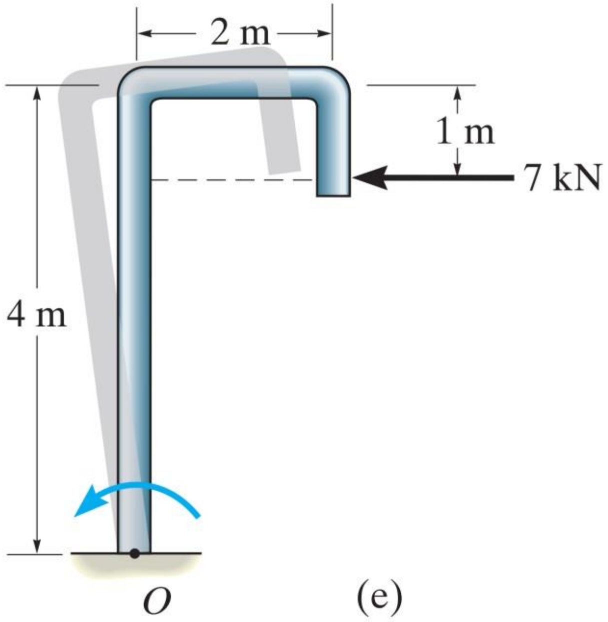


$$M_O = (40\ \text{lb})(4\ \text{ft} + 2\cos 30^\circ\ \text{ft}) = 229\ \text{lb}\cdot\text{ft} \circlearrowright$$

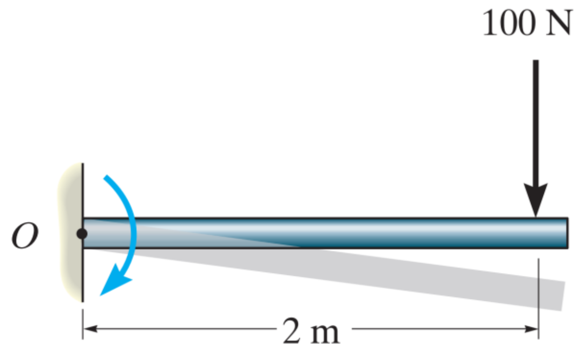


$$M_O = (60\ \text{lb})(1\sin 45^\circ\ \text{ft}) = 42.4\ \text{lb}\cdot\text{ft} \circlearrowleft$$

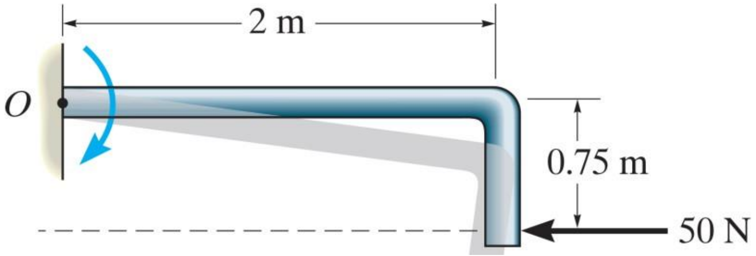


(e)

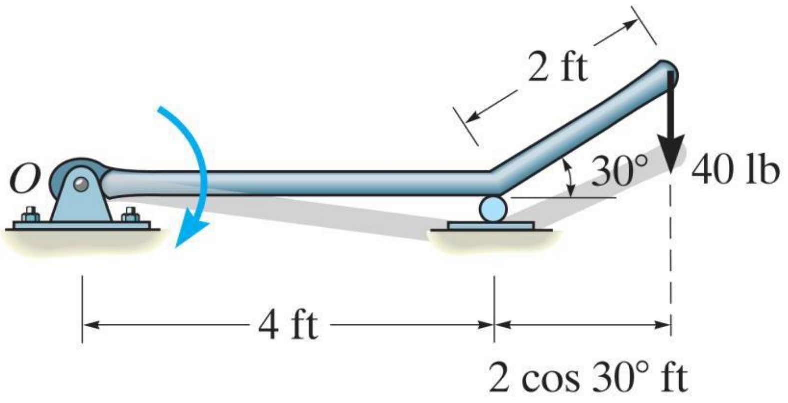
Example



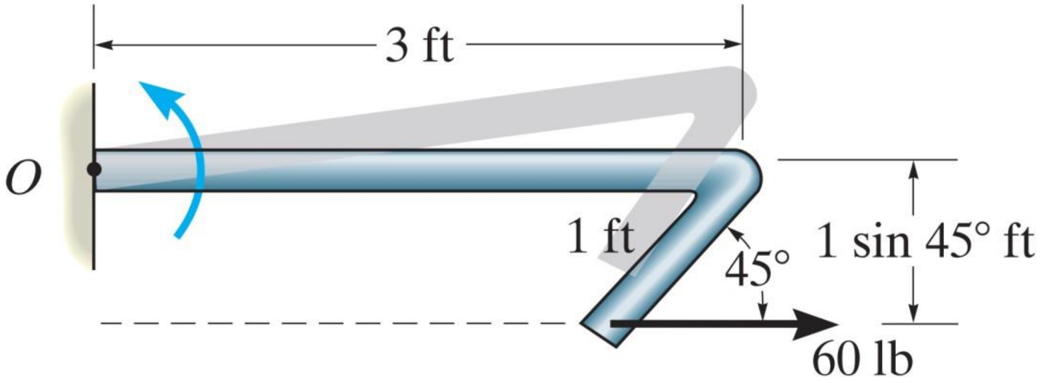
$$M_O = (100\ \text{N})(2\ \text{m}) = 200\ \text{N}\cdot\text{m} \curvearrowright$$



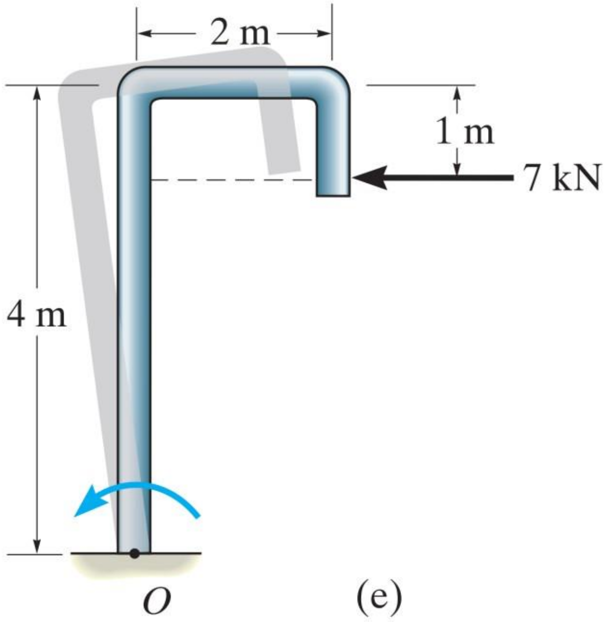
$$M_O = (50\ \text{N})(0.75\ \text{m}) = 37.5\ \text{N}\cdot\text{m} \curvearrowright$$



$$M_O = (40\ \text{lb})(4\ \text{ft} + 2\cos 30^\circ\ \text{ft}) = 229\ \text{lb}\cdot\text{ft} \curvearrowright$$



$$M_O = (60\ \text{lb})(1\sin 45^\circ\ \text{ft}) = 42.4\ \text{lb}\cdot\text{ft} \curvearrowleft$$



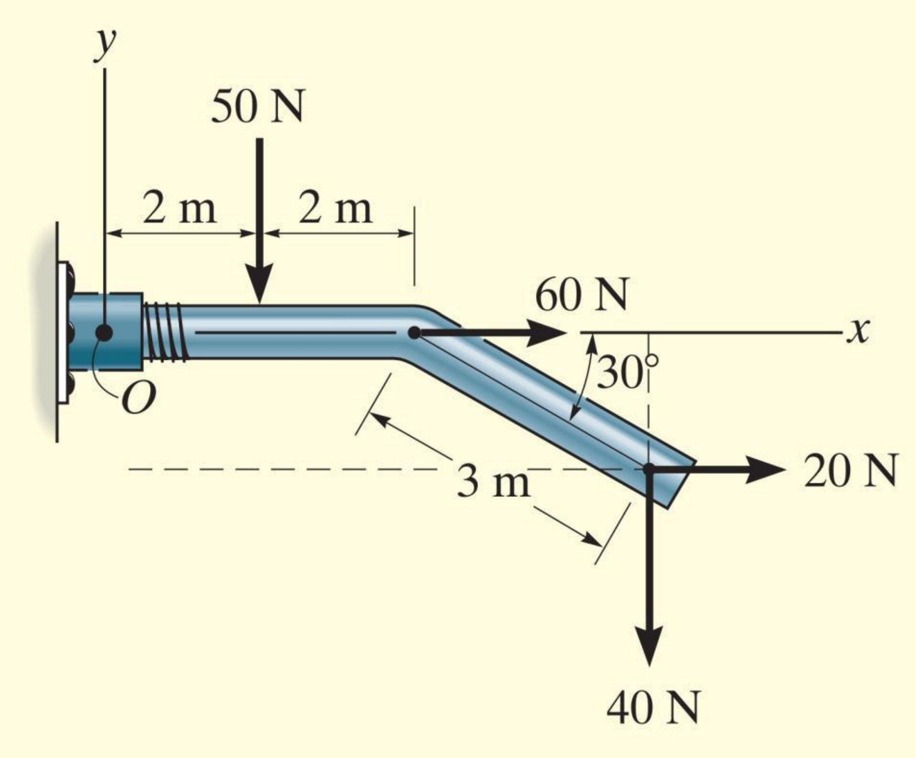
(e)

$$M_O = (7\ \text{kN})(4\ \text{m} - 1\ \text{m}) = 21.0\ \text{kN}\cdot\text{m} \curvearrowleft$$

Example

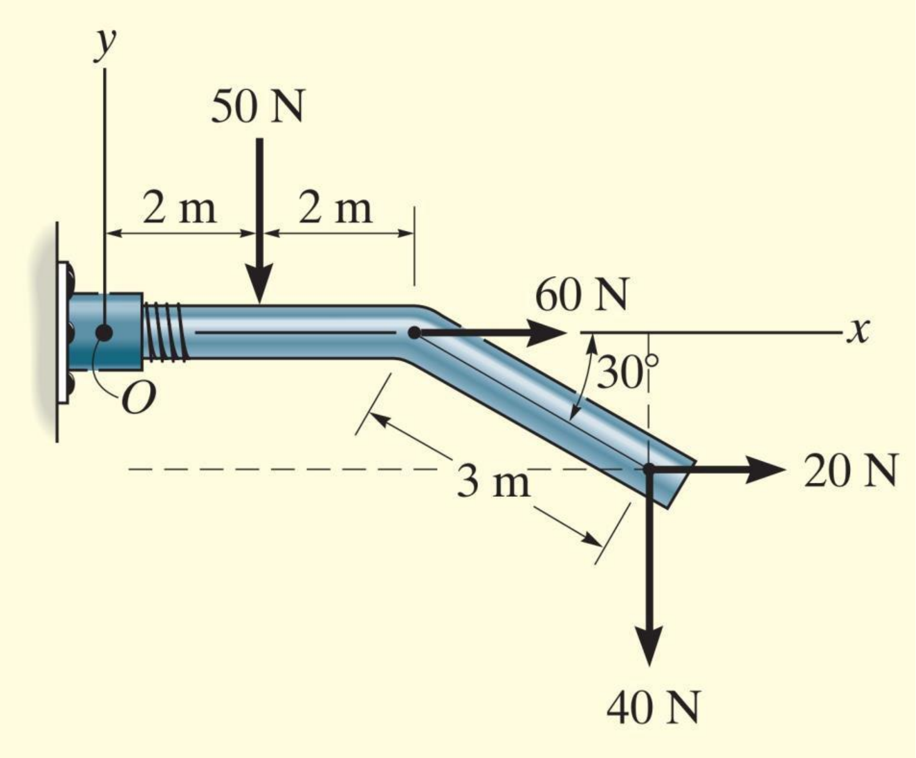
Determine the resultant moment of the four forces acting on the rod

$$\circlearrowleft + (M_R)_o = \Sigma Fd;$$



Example

Determine the resultant moment of the four forces acting on the rod



$$\circlearrowleft + (M_R)_o = \Sigma Fd;$$

$$(M_R)_o = -50\text{ N}(2\text{ m}) + 60\text{ N}(0) + 20\text{ N}(3\sin 30^\circ\text{ m})$$

$$-40\text{ N}(4\text{ m} + 3\cos 30^\circ\text{ m})$$

$$(M_R)_o = -334\text{ N}\cdot\text{m} = 334\text{ N}\cdot\text{m}\circlearrowright$$

Cross Product

The *cross product* of two vectors **A** and **B** yields the vector **C**

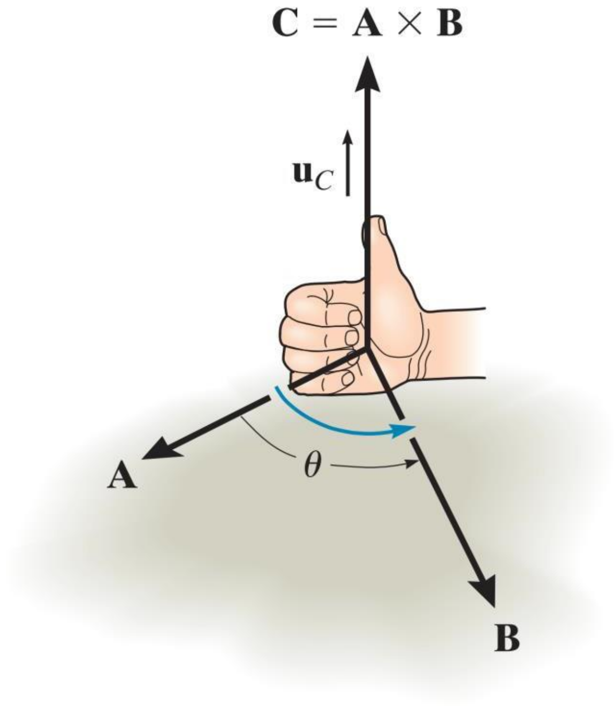
$$\mathbf{C} = \mathbf{A} \times \mathbf{B}$$

Magnitude.

$$C = AB \sin \theta.$$

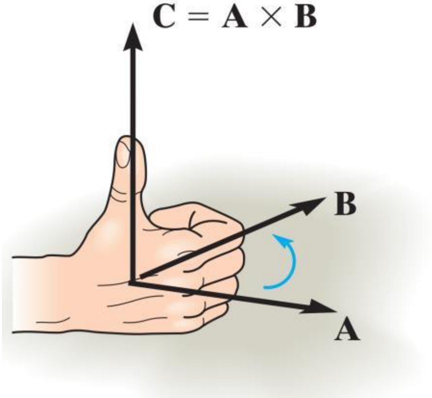
Direction.

$$\mathbf{C} = \mathbf{A} \times \mathbf{B} = (AB \sin \theta)\mathbf{u}_C$$

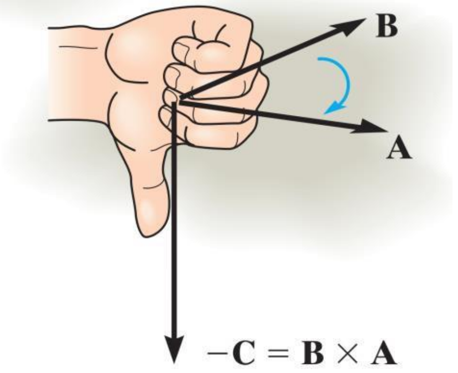


Laws of Operations

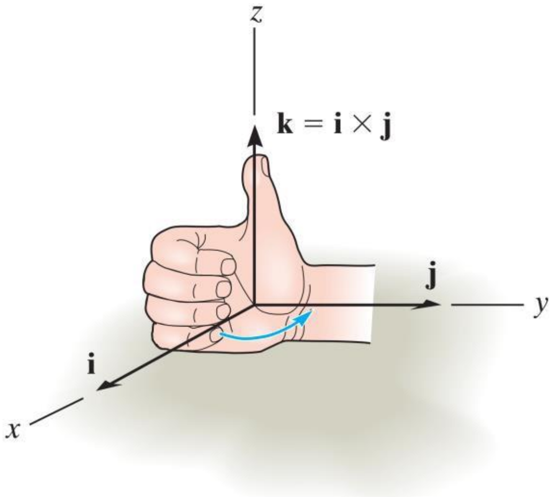
$$\mathbf{A} \times \mathbf{B} = -\mathbf{B} \times \mathbf{A}$$



$$a(\mathbf{A} \times \mathbf{B}) = (a\mathbf{A}) \times \mathbf{B} = \mathbf{A} \times (a\mathbf{B}) = (\mathbf{A} \times \mathbf{B})a$$



$$\mathbf{A} \times (\mathbf{B} + \mathbf{D}) = (\mathbf{A} \times \mathbf{B}) + (\mathbf{A} \times \mathbf{D})$$

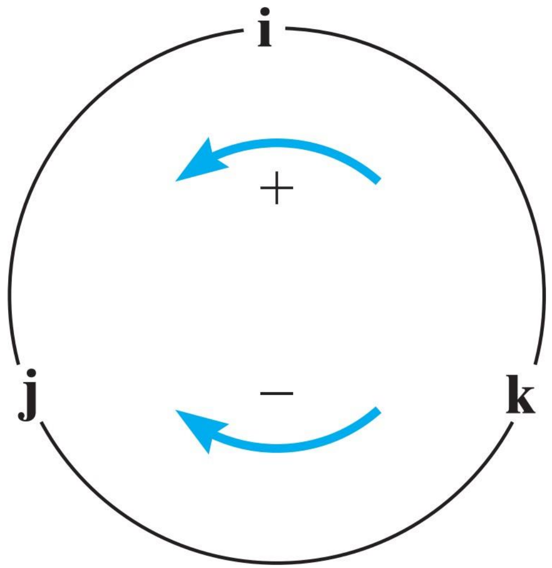


Cartesian Vector Formulation.

$$\mathbf{i} \times \mathbf{j} = \mathbf{k} \quad \mathbf{i} \times \mathbf{k} = -\mathbf{j} \quad \mathbf{i} \times \mathbf{i} = 0$$

$$\mathbf{j} \times \mathbf{k} = \mathbf{i} \quad \mathbf{j} \times \mathbf{i} = -\mathbf{k} \quad \mathbf{j} \times \mathbf{j} = 0$$

$$\mathbf{k} \times \mathbf{i} = \mathbf{j} \quad \mathbf{k} \times \mathbf{j} = -\mathbf{i} \quad \mathbf{k} \times \mathbf{k} = 0$$



Cartesian Vector Formulation.

$$\begin{aligned}\mathbf{A} \times \mathbf{B} &= (A_x\mathbf{i} + A_y\mathbf{j} + A_z\mathbf{k}) \times (B_x\mathbf{i} + B_y\mathbf{j} + B_z\mathbf{k}) \\ &= A_xB_x(\mathbf{i} \times \mathbf{i}) + A_xB_y(\mathbf{i} \times \mathbf{j}) + A_xB_z(\mathbf{i} \times \mathbf{k}) \\ &\quad + A_yB_x(\mathbf{j} \times \mathbf{i}) + A_yB_y(\mathbf{j} \times \mathbf{j}) + A_yB_z(\mathbf{j} \times \mathbf{k}) \\ &\quad + A_zB_x(\mathbf{k} \times \mathbf{i}) + A_zB_y(\mathbf{k} \times \mathbf{j}) + A_zB_z(\mathbf{k} \times \mathbf{k})\end{aligned}$$

$$\mathbf{A} \times \mathbf{B} = (A_yB_z - A_zB_y)\mathbf{i} - (A_xB_z - A_zB_x)\mathbf{j} + (A_xB_y - A_yB_x)\mathbf{k}$$

Cartesian Vector Formulation.

$$\mathbf{A} \times \mathbf{B} = (A_x\mathbf{i} + A_y\mathbf{j} + A_z\mathbf{k}) \times (B_x\mathbf{i} + B_y\mathbf{j} + B_z\mathbf{k})$$
$$= A_xB_x(\mathbf{i} \times \mathbf{i}) + A_xB_y(\mathbf{i} \times \mathbf{j}) + A_xB_z(\mathbf{i} \times \mathbf{k})$$
$$+ A_yB_x(\mathbf{j} \times \mathbf{i}) + A_yB_y(\mathbf{j} \times \mathbf{j}) + A_yB_z(\mathbf{j} \times \mathbf{k})$$
$$+ A_zB_x(\mathbf{k} \times \mathbf{i}) + A_zB_y(\mathbf{k} \times \mathbf{j}) + A_zB_z(\mathbf{k} \times \mathbf{k})$$

$$\mathbf{A} \times \mathbf{B} = (A_yB_z - A_zB_y)\mathbf{i} - (A_xB_z - A_zB_x)\mathbf{j} + (A_xB_y - A_yB_x)\mathbf{k}$$

$$\boxed{\mathbf{A} \times \mathbf{B} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}}$$

How to find Determinant???

Home Assignment

- F4-1, F4-4, Problem 4-1,4-2 & 4-3.