Thermodynamics I

# Lecture 19

## Energy Analysis of Cycles (Ch-4)

**Example:** A gas within a piston–cylinder assembly undergoes a thermodynamic cycle consisting of three processes in series, beginning at state 1 where $p_1 = 1$ bar, $V_1 = 1.5\ m^3$, as follows:

**Process 1–2:** Compression with $pV = constant$, $W_{12} = -104$ kJ, $U_1 = 512$ kJ, $U_2 = 690$ kJ.

**Process 2–3:** $W_{23} = 0$, $Q_{23} = -150$ kJ.

**Process 3–1:** $W_{31} = +50$ kJ.

There are no changes in kinetic or potential energy. **(a)** Determine $Q_{12}$, $Q_{31}$, and $U_3$, each in kJ. **(b)** Can this cycle be a power cycle? Explain

ANALYSIS: (a) Process 1-2, $\Delta U + \cancel{\Delta KE + \Delta PE} = Q_{12} - W_{12} \Rightarrow$

$$Q_{12} = [U_2 - U_1] + W_{12} = (690 - 512)\,kJ + (-104\,kJ) = +74\,kJ$$

For any cycle, $W_{cycle} = Q_{cycle}$ (Eq. 2.40). Thus

$$W_{12} + W_{23} + W_{31} = Q_{12} + Q_{23} + Q_{31}$$

$$\Rightarrow \quad Q_{31} = W_{12} + W_{23} + W_{31} - Q_{12} - Q_{23}$$

$$= (-104) + 0 + 50 - 74 - (-150) = +22\,kJ$$

Process 3-1: $\Delta U + \cancel{\Delta KE + \Delta PE} = Q_{31} - W_{31} \Rightarrow U_1 - U_3 = Q_{31} - W_{31}$

$$\Rightarrow \quad U_3 = U_1 - Q_{31} + W_{31} = 512 - 22 + 50 = 540\,kJ$$

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There are no changes in kinetic or potential energy. **(a)** Determine $Q_{12}$, $Q_{31}$, and $U_3$, each in kJ. **(b)** Can this cycle be a power cycle? Explain

(b) A power cycle is one for which $W_{cycle} > 0$.

$$W_{cycle} = W_{12} + W_{23} + W_{31}$$

$$= +(-104) + (0) + (50) = -54\ kJ$$

*

**Example:** A gas undergoes a cycle in a piston–cylinder assembly consisting of the following three processes:

**Process 1–2:** Constant pressure, $p = 1.4$ bar, $V_1 = 0.028$ m$^3$, $W_{12} = 10.5$ kJ

**Process 2–3:** Compression with $pV = constant$, $U_3 = U_2$

**Process 3–1:** Constant volume, $U_1 - U_3 = -26.4$ kJ

There are no significant changes in kinetic or potential energy.

**(a)** Sketch the cycle on a p–V diagram.

**(b)** Calculate the net work for the cycle, in kJ.

**(c)** Calculate the heat transfer for process 1–2, in kJ

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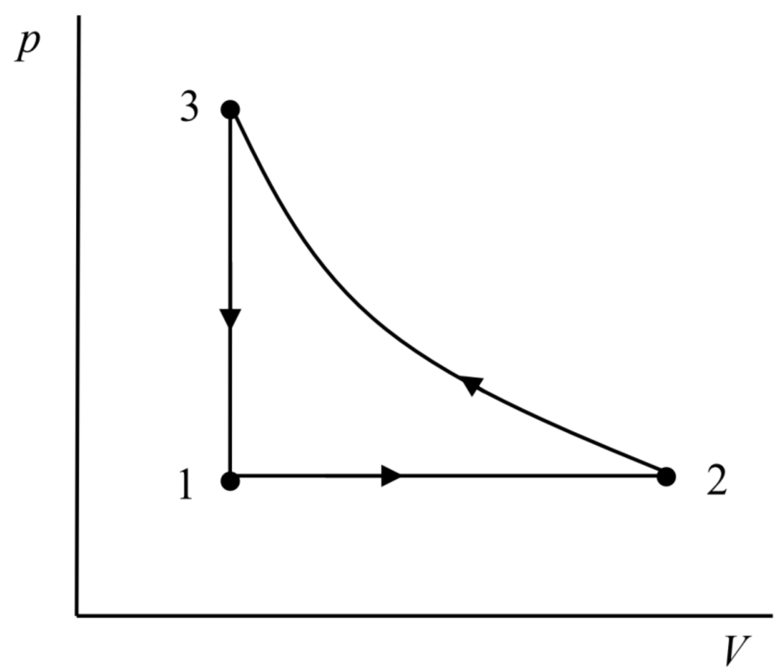
**(a)** Sketch the cycle on a p–V diagram.

(a) Since $W_{12} > 0$, the process is an expansion. Thus



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**(b)** Calculate the net work for the cycle, in kJ.

$$W_{\text{cycle}} = W_{12} + W_{23} + \cancelto{0}{W_{31}}.$$

$$W_{23} = \int_{V_2}^{V_3} p dV = \int_{V_2}^{V_3} \frac{const}{V} dV$$

$$= (p_2 V_2) \ln\left(\frac{V_3}{V_2}\right) = (p_2 V_2) \ln\left(\frac{V_1}{V_2}\right)$$

$$W_{cycle} = W_{12} + W_{23} + \cancelto{0}{W_{31}}. \quad W_{12} = 10.5 \text{ kJ},$$

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**For $V_2$**

$$W_{12} = \int_{V_1}^{V_2} p dV = p(V_2 - V_1)$$

$$V_2 = \frac{W_{12}}{p} + V_1 = 0.103 \text{ m}^3$$

$$W_{23} = (1.4 \text{ bar})(0.103 \text{ m}^3) \ln\left(\frac{0.028}{0.103}\right) \left|\frac{10^5 \text{N/m}^2}{1 \text{ bar}}\right| \left|\frac{1 \text{ kJ}}{10^3 \text{N}\cdot\text{m}}\right| = -18.78 \text{ kJ}$$

$$W_{cycle} = 10.5 \text{ kJ} + (-18.78 \text{ kJ}) + 0 = -8.28 \text{ kJ}$$

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**Process 3–1:** Constant volume, $U_1 - U_3 = -26.4$ kJ

There are no significant changes in kinetic or potential energy.

**(c)** Calculate the heat transfer for process 1–2, in kJ

$$\Delta \overset{0}{\cancel{KE}} + \Delta \overset{0}{\cancel{PE}} + (U_2 - U_1) = Q_{12} - W_{12}$$

$$Q_{12} = (U_3 - U_1) + W_{12} = (+26.4 \text{ kJ}) + (10.5 \text{ kJ}) = 36.9 \text{ kJ}$$

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# Example: Energy Balance of Cycles

A gas within a piston-cylinder assembly undergoes a thermodynamic cycle consisting of three processes:

Process 1-2: Compression with pV = constant, from $p_1 = 1$ bar, $V_1 = 1.6\ m^3$ to $V_2 = 0.2\ m^3$, $U_2 – U_1 = 0$
Process 2-3: Constant pressure to $V_3 = V_1$
Process 3-1: Constant volume, $U_1 – U_3 = -3549$ kJ

There are no changes in K.E. and P.E. Determine the heat transfer and work for Process 2-3, in kJ. Is this a power cycle or a refrigeration cycle?