PROPERTIES OF LTI SYSTEMS

Commutative Property

$$y[n] = x[n] * h[n] = h[n] * x[n]$$

$$x[n] \longrightarrow \boxed{\quad h[n] \quad} \longrightarrow y[n] \qquad h[n] \longrightarrow \boxed{\quad x[n] \quad} \longrightarrow y[n]$$

$$x(t) * h(t) = h(t) * x(t) = \int_{-\infty}^{\infty} h(\tau) x(t-\tau) d\tau$$

Commutative Property

- The step response of an LTI system is the summation of its impulse responses

Step response $s[n]$ of an LTI system

$$s[n] = u[n] * h[n] = h[n] * u[n]$$

step input

"input"

$$\Downarrow$$

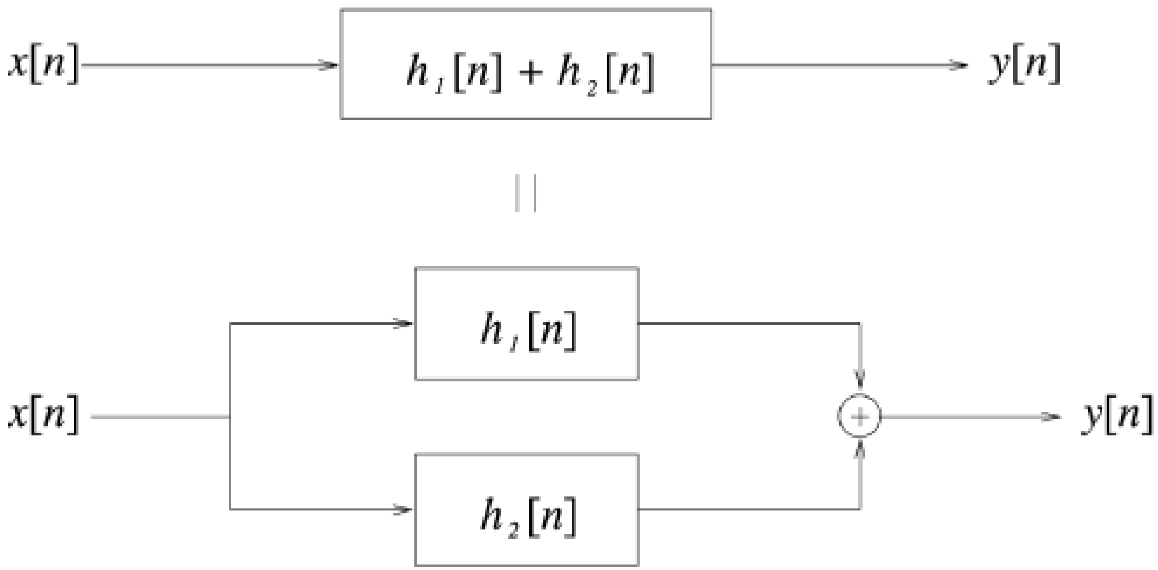
$$s[n] = \sum_{k=-\infty}^{n} h[k]$$

Distributive Property

$$x[n] * (h_1[n] + h_2[n]) = x[n] * h_1[n] + x * h_2[n]$$

$$x(t) * [h_1(t) + h_2(t)] = x(t) * h_1(t) + x(t) * h_2(t)$$

Interpretation



Associative Property

$$x[n] * (h_1[n] * h_2[n]) = (x[n] * h_1[n]) * h_2[n]$$

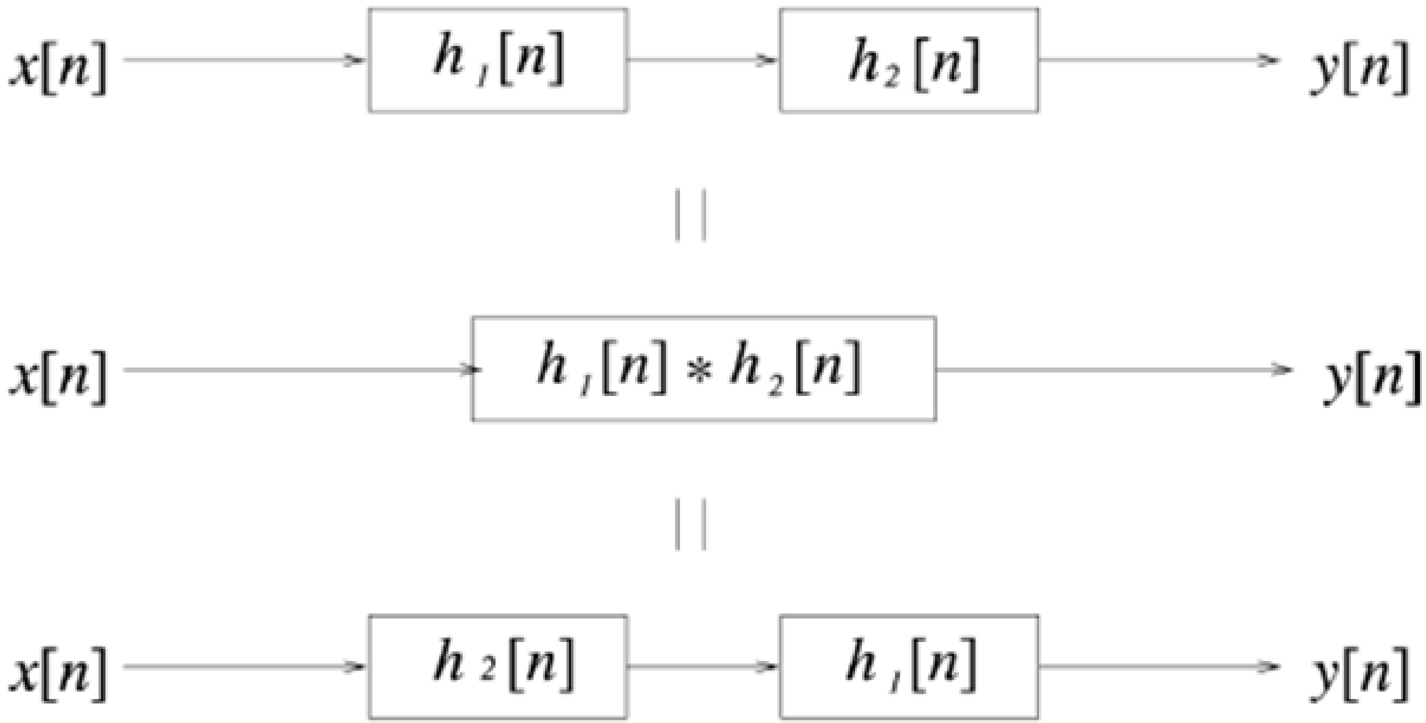
$$\text{(Commutativity)} \Big\|$$

$$x[n] * (h_2[n] * h_1[n]) = (x[n] * h_2[n]) * h_1[n]$$

$$x(t) * (h_1(t) * h_2(t)) = \left[x(t) * h_1(t)) \right] * h_2(t)$$

Associative Property

Implication (Very special to LTI Systems)



Associative Property

- Associative Property special to LTI systems
- Consider the following pair of systems:

  System 1: $y_1[n] = 2x[n]$

  System 2: $y_2[n] = x^2[n]$

- Run System 1 followed by System 2, get:

  $$y[n] = 4x^2[n]$$

- Run System 2 followed by System 1, get:

  $$y[n] = 2x^2[n]$$

- Cannot interchange order of non-LTI systems

Memoryless and Identity LTI System

- a DT system is ***memoryless*** if its output at any time depends ***only*** on the value of the input at that same time $\Rightarrow$ for a discrete-time LTI system $h[n]=0,\ n\neq 0$

  or, equivalently,

  $$h[n]=K\delta[n]$$

  where K is a constant, and the convolution sum becomes:

  $$y[n]=Kx[n]$$

Memoryless and Identity LTI System

- a CT system is ***memoryless*** if $h(t)=0$ for $t \neq 0$ and has the form:

  $$y(t) = Kx(t)$$

- for some constant K and has the impulse response:

  $$h(t) = K\delta(t)$$

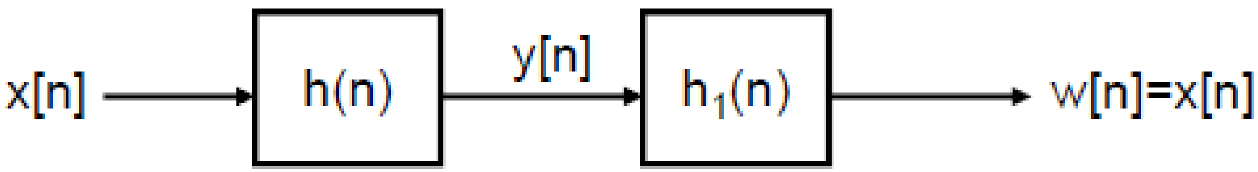
- If $K = 1$ for these systems, they become identity systems with output equal to the input

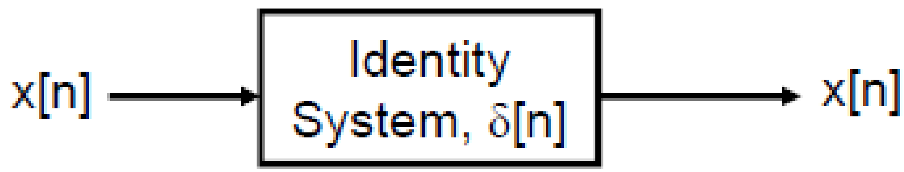
Invertible LTI System - DT

- an LTI system with impulse response $h[n]$ is ***invertible*** only if an inverse system exists that, when connected in series with the original system, produces an output equal to the input of the first system, i.e.,

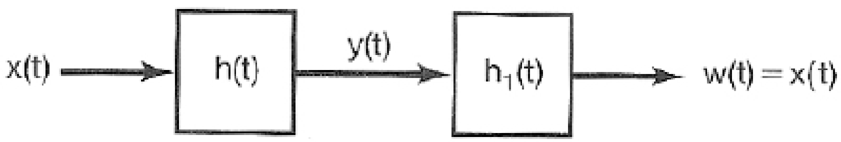
  $$h[n] * h_1[n] = \delta[n]$$

  where $h_1[n]$ is the impulse response of the LTI inverse system

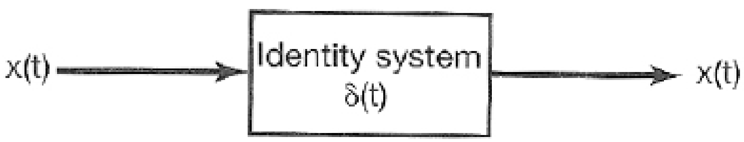




Invertible LTI System - CT



(a)



(b)

- LTI System with impulse response $h(t)$ with output $y(t) = x(t) * h(t)$
- Inverse system with impulse response $h_1(t)$ processes signal giving output $w(t) = y(t) * h_1(t) = x(t)$
- The overall impulse response of the system followed by its inverse is $h(t) * h_1(t) = \delta(t)$

Invertible LTI System - Example

- Consider LTI system of form:

$$y(t) = x(t - t_0)$$

- with $t_0 > 0$ (i.e., positive delay)

➢ What is the impulse response of this system?

➢ What is the inverse system for *y(t)*

Invertible LTI System - Example

- The impulse response of the system is:

$$h(t) = \delta(t - t_0)$$

- Therefore we have the relation:

$$y(t) = x(t - t_0) = x(t) * \delta(t - t_0)$$

- The inverse system is thus of the form:

$$h_1(t) = \delta(t + t_0)$$

- giving the result:

$$h(t) * h_1(t) = \delta(t - t_0) * \delta(t + t_0) = \delta(t)$$

Problem-1

- Consider the LTI system with impulse response:

$$h[n] = u[n]$$

- Find the impulse response of the inverse system

END