National University of Sciences & Technology
School of Electrical Engineering and Computer Science

Department of Basic Sciences

MATH-101: Calculus and Analytical Geometry (3+0): BEE2k20-ABC Fall 2020

Assignment 1	
CLO: 1 (Understand the concept of limit, continuity and derivative with its application to find extrema)	
Maximum Marks: 70	Instructor: Dr. Naila Amir
Announcement Date: 5th November 2020	Due Date: 13th November 2020

Instructions:

- Understanding the question is part of the assignment and copying is not allowed.
- Express your answer in the most simplified form. Direct calculations using calculator are not allowed, you need to show the detail of your work to get the maximum marks.
- This is an individual assignment.
- Assignment must be handwritten and properly scanned in a single pdf file. These two pages must be part of every assignment.
- Assignment must be properly tagged and is required to be submitted on MS teams.
- Assignment is not acceptable after deadline.

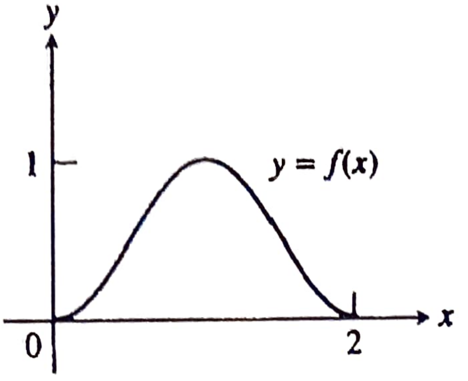
Tasks: Attempt all questions.

Students Name	CMS Id.	Section
Muhammad Umer	345834	12 C

Total Marks	Marks Obtained	Weight in 10
70 Marks		

Q - 1: [CLO-1: 30 marks]

The accompanying figure shows the graph of a function $f(x)$ with domain $[0, 2]$ and range $[0, 1]$. Find the domains and ranges of the following functions and sketch their graphs by clearly mentioning the type of transformation used in each case:



1) $f(x) + 2$
2) $f(x) - 1$
3) $2f(x)$
4) $f(2x)$
5) $\frac{1}{2}f(x)$
6) $f(x + 2)$
7) $f(x - 1)$
8) $-f(x)$
9) $f(-x)$
10) $-f(x + 1) + 1$

Q - 2: [CLO-1: 20 marks]

Draw graphs and determine domain and range of the following functions.

1) $f(x) = |x| + |x - 1|$
2) $f(x) = x - \lfloor x \rfloor$
3) $f(x) = \begin{cases} x^2 - 1, & x \leq 2 \\ \sqrt{x - 1}, & x > 2 \end{cases}$
4) $f(x) = \begin{cases} x^2, & -2 \leq x < 1 \\ x + 1, & 1 \leq x \leq 2 \end{cases}$

Q - 3: [CLO-1: 20 marks]

Determine the formulas and domain for the functions $(f + g)(x)$, $(fg)(x)$, $\left(\frac{f}{g}\right)(x)$, $(fog)(x)$, and $(gof)(x)$, where

1) $f(x) = \frac{1}{\sqrt{4-x^2}}$ and $g(x) = \sqrt{x^2 - 1}$
2) $f(x) = x^2 + 3$ and $g(x) = \sqrt{x^2 - 3}$

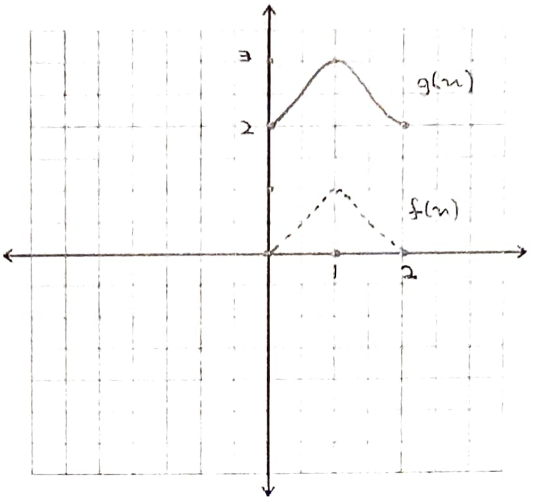
Q1 :

1) $f(x)+2$

Let $y=f(x)$ and $g(x) = f(x)+2$

x	0	1	2
y	0	1	0
$g(x)$	2	3	2

Scale:
x-axis : 2 squares = 1 unit
y-axis : 2 squares = 1 unit



Domain : $[0,2]$
Range : $[2,3]$

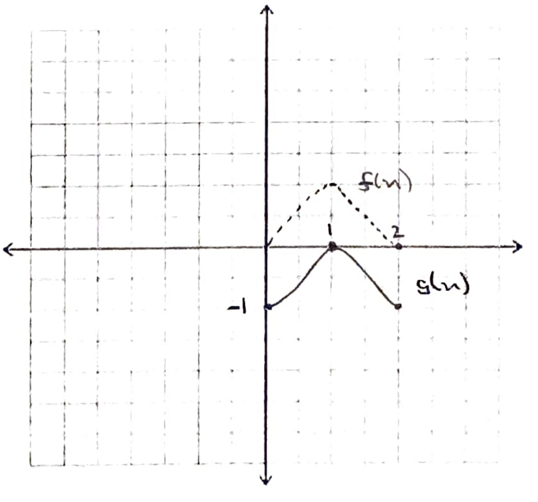
Type of Transformation

It is a vertical shift.

2) $f(x)-1$

Let $y=f(x)$ and $g(x) = f(x)-1$

x	0	1	2
y	0	1	0
$g(x)$	-1	0	-1



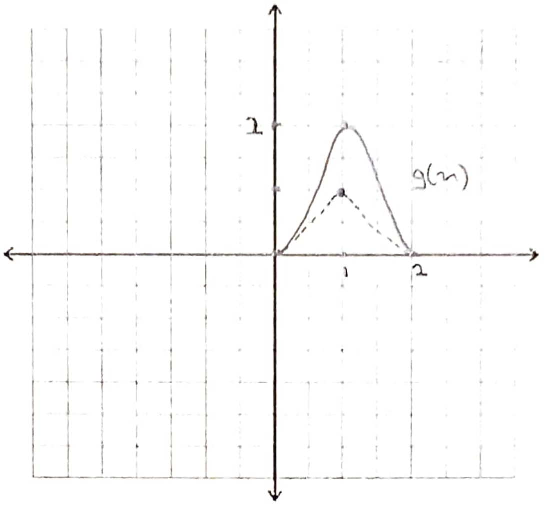
Domain: $[0,2]$
Range: $[-1,0]$

Type:
It is a vertical shift.

3) $2f(x)$

Let $f(x) = y$ and $g(x) = 2f(x)$

x	0	1	2
y	0	1	0
$g(x)$	0	2	0



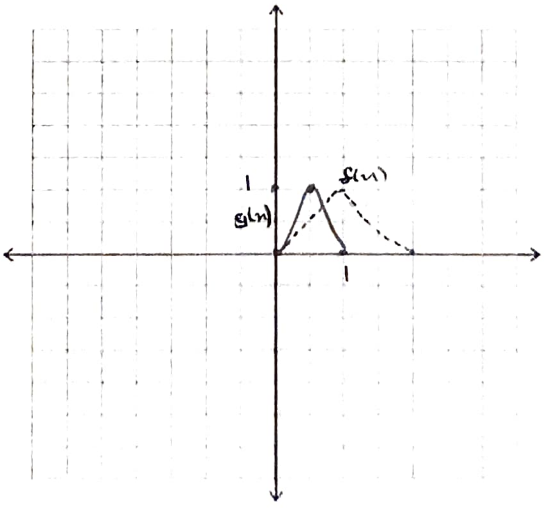
Domain : $[0,2]$
Range : $[0,2]$

Type:
It is a vertical stretch (scaling).

4) $f(2x)$

Let $f(x) = y$ and $g(x) = f(2x)$

x	0	0.5	1
y	0	0.5	1
$g(x)$	0	1	0



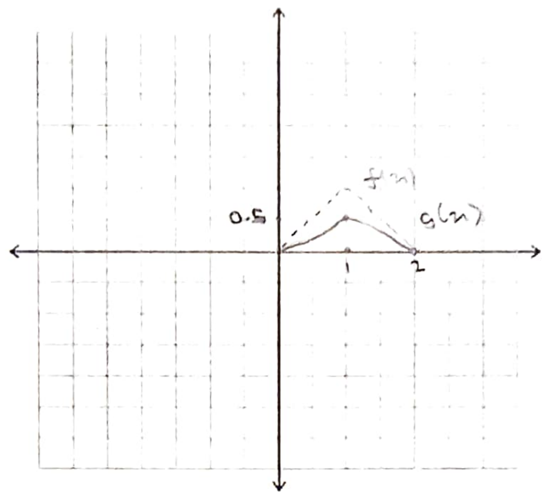
Domain : $[0,1]$
Range : $[0,1]$

Type:
It is a horizontal compress (scaling)

5) $\frac{1}{2} f(x)$:

Let $y = f(x)$ and $g(x) = \frac{1}{2} f(x)$

x	0	1	2
y	0	1	0
$g(x)$	0	0.5	0

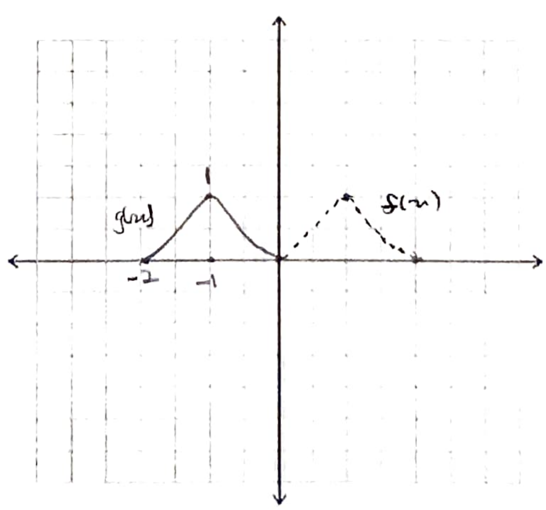


Domain: $[0, 2]$
Range: $[0, 0.5]$
Type:
It is a vertical compress. (Scaling)

6) $f(x+2)$:

Let $y = f(x)$ and $g(x) = f(x+2)$

x	0	-1	-2
y	0	Und.	Und.
$g(x)$	0	1	0

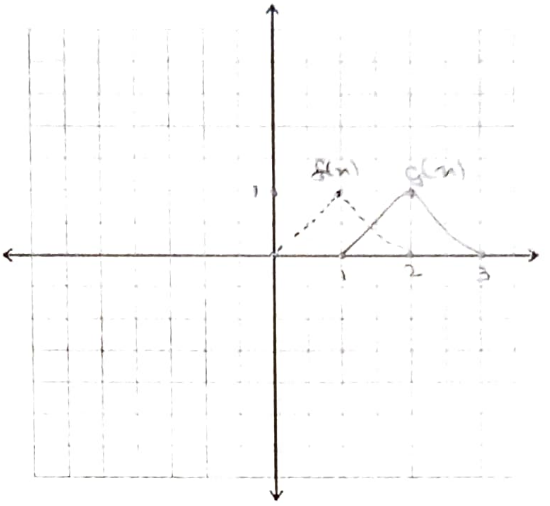


Domain $[-2, 0]$
Range $[0, 1]$
Type:
It is a horizontal shift.

7) $f(x-1)$:

Let $y = f(x)$ and $g(x) = f(x-1)$

x	1	2	3
y	1	0	U.d.
$g(x)$	0	1	0



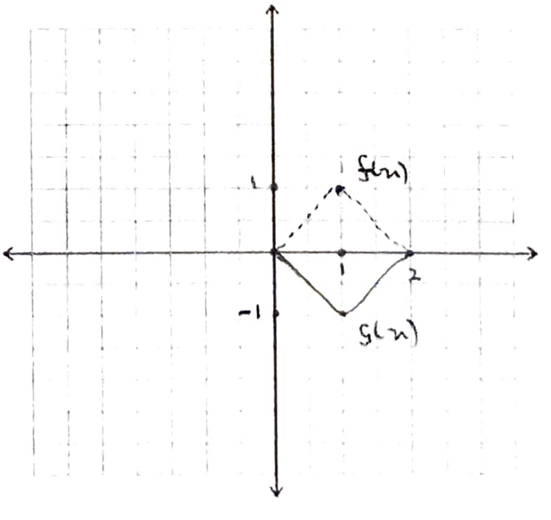
Domain : [1, 3]
Range : [0, 1]

Type:
It is a horizontal shift.

8) $-f(x)$

Let $y = f(x)$ and $g(x) = -f(x)$

x	0	1	2
y	0	1	0
$g(x)$	0	-1	0



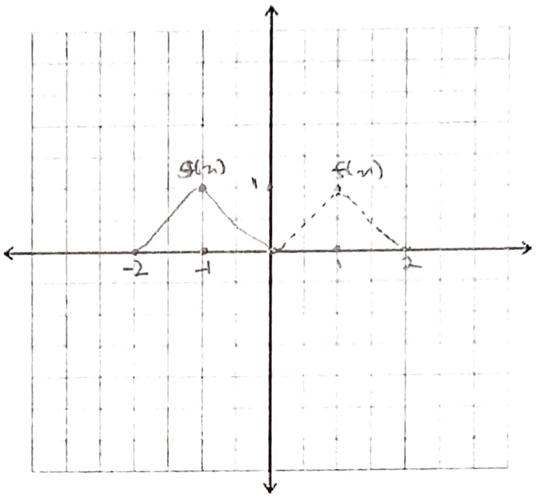
Domain : [0, 2]
Range : [-1, 0]

Type:
It is a vertical reflection.

9) $f(-x)$

Let $y = f(x)$ and $g(x) = f(-x)$

x	0	-1	-2
y	0	Und.	Und.
$g(x)$	0	1	0



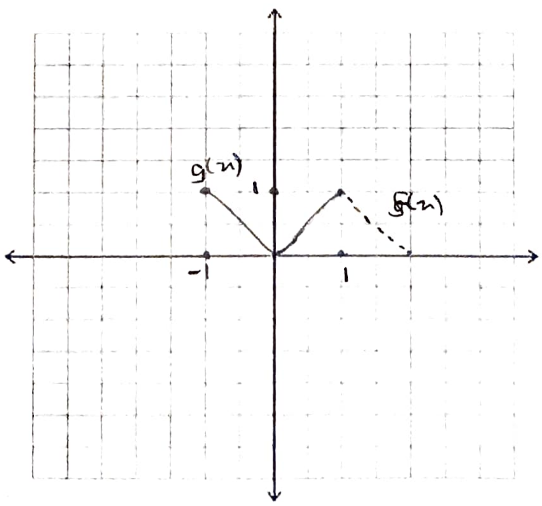
Domain : $[-2, 0]$
Range : $[0, 1]$

Type :
It is a horizontal reflection.

10) $-f(x+1)+1$

Let $y = f(x)$ and $g(x) = -f(x+1)+1$

x	0	-1	1
y	0	Und.	1
$g(x)$	0	1	1



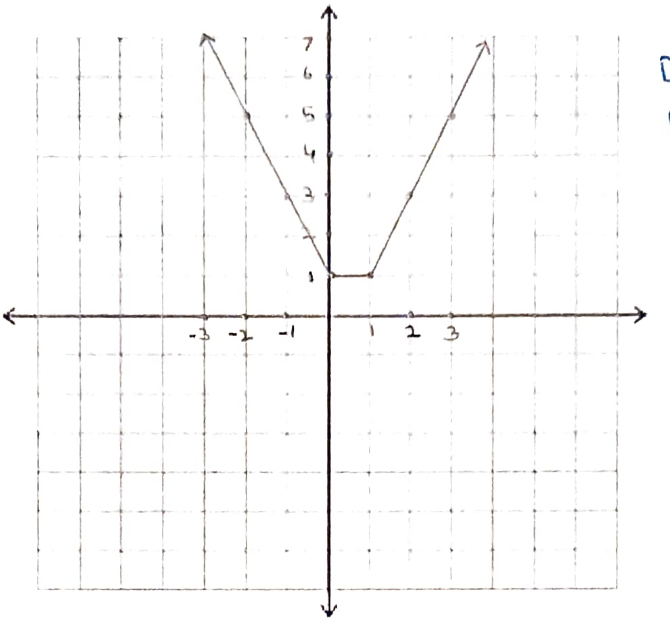
Domain : $[-1, 1]$
Range : $[0, 1]$
Type :
In order:
1. Horizontal Shift
2. Vertical Reflection
3. Vertical Shift

Q2

1) $f(x) = |x| + |x-1|$

x	-3	-2	-1	0	1	2	3
$f(x)$	7	5	3	1	1	3	5

Scale:
x-axis: one square = 1 unit
y-axis: one square = 1 unit

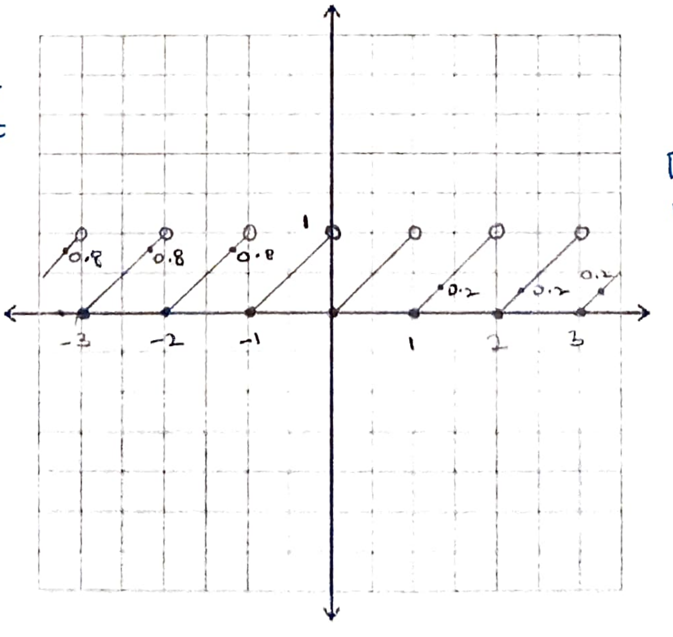


Domain: $(-\infty, \infty)$
Range: $[1, \infty)$

2) $f(x) = x - \lfloor x \rfloor$

x	-3.2	-2.2	-1.2	0	1.2	2.2	3.2
$f(x)$	0.8	0.8	0.8	0	0.2	0.2	0.2

Scale:
x-axis: 2 squares = 1 unit
y-axis: 2 squares = 1 unit

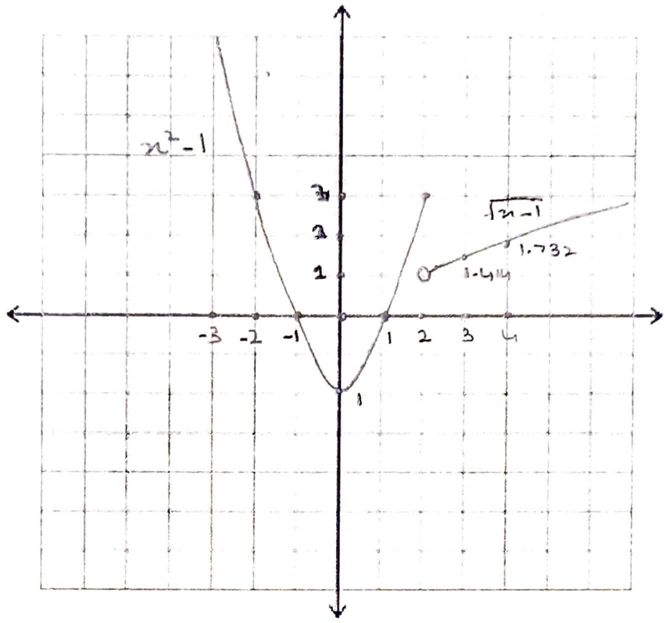


Domain $(-\infty, \infty)$
Range $[0, 1)$

Q2

3) $f(x) = \begin{cases} x^2 - 1 & x \le 2 \\ \sqrt{x-1} & x > 2 \end{cases}$

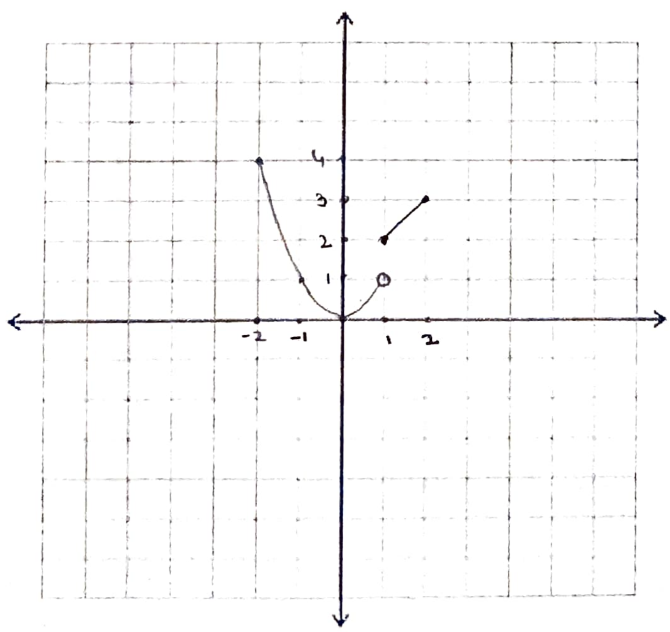
x	-3	-2	-1	1	2	3	4
x^2-1	8	3	0	0	3		
$\sqrt{x-1}$						1.414	1.732



Domain $(-\infty, \infty)$
Range $[-1, \infty)$

4) $f(x) = \begin{cases} x^2 & -2 \le x < 1 \\ x+1 & 1 \le x \le 2 \end{cases}$

x	-3	-2	-1	0	1	2	3
x^2		4	1	0			
$x+1$					2	3	



Domain $[-2, 2]$
Range $[0, 4]$

Q3

1) $f(x) = 1/\sqrt{4-x^2}$ and $g(x) = \sqrt{x^2-1}$

i) $(f+g)(x) = \dfrac{1}{\sqrt{4-x^2}} + \sqrt{x^2-1}$

Domain of $(f+g)(x)$: $(\text{Dom } f(x)) \cap (\text{Dom } g(x))$

: $(4-x^2 > 0) \cap (x^2-1 \geq 0)$

: $(\pm x < 2) \cap (x \geq 1 \text{ or } x \leq -1)$

: $(-2<x<2) \cap (x \geq 1 \text{ or } x \leq -1)$

$\rightarrow$: $(-2,-1] \cup [1,2)$

ii) $(fg)(x) = \dfrac{\sqrt{x^2-1}}{\sqrt{4-x^2}}$

Domain of $(fg)(x)$: $(\text{Dom } f(x)) \cap (\text{Dom } g(x))$

: $(4-x^2 > 0) \cap (x^2-1 \geq 0)$

: $(-2<x<2) \cap (x \geq 1 \text{ or } x \leq -1)$

$\rightarrow$: $(-2,-1] \cup [1,2)$

iii) $(f/g)(x) = \dfrac{1}{\sqrt{4-x^2}\sqrt{x^2-1}}$

Domain of $(f/g)(x)$: $(\text{Dom } f(x)) \cap (\text{Dom } g(x))$

: $(4-x^2 > 0) \cap (x^2-1 > 0)$

: $(-2<x<2) \cap (x>1 \text{ or } x<-1)$

$\rightarrow$: $(-2,-1) \cup (1,2)$

iv) $(f\circ g)(x) = \dfrac{1}{\sqrt{4-(\sqrt{x^2-1})^2}} = \dfrac{1}{\sqrt{4-x^2+1}} = \dfrac{1}{\sqrt{5-x^2}}$

Domain of $(f\circ g)(x)$: $\sqrt{5-x^2} > 0$

$x^2 < 5$

$-\sqrt{5} < x < \sqrt{5}$

$\rightarrow$: $(-\sqrt{5}, \sqrt{5})$

v) $(g\circ f)(x) = \sqrt{(1/\sqrt{4-x^2})^2 - 1} = \sqrt{1/(4-x^2) - 1} = \sqrt{(x^2-3)/(4-x^2)}$

Domain of $(g\circ f)(x)$: $(-\sqrt{3} \geq x \geq \sqrt{3}) \cap (-2<x<2)$

: $(-2,-\sqrt{3}] \cup [\sqrt{3},2)$

2) $f(x) = x^2+3$ and $g(x) = \sqrt{x^2-3}$

i $(f+g)(x) = x^2+3 + \sqrt{x^2-3}$

Domain of $(f+g)(x)$: $(\text{Dom } x^2+3) \cap (\text{Dom } \sqrt{x^2-3})$

: $(\mathbb{R}) \cap (x^2-3 \geq 0)$

: $(-\infty, \infty) \cap (x \geq \sqrt{3}$ or $x \leq -\sqrt{3})$

$\longrightarrow$: $(-\infty, -\sqrt{3}] \cup [\sqrt{3}, \infty)$

ii $(fg)(x) : (x^2+3)(\sqrt{x^2-3})$

Domain of $(fg)(x)$: $(\text{Dom } f(x)) \cap (\text{Dom } g(x))$

: $(-\infty, \infty) \cap (x \geq \sqrt{3}$ or $x \leq -\sqrt{3})$

$\longrightarrow$: $(-\infty, -\sqrt{3}] \cup [\sqrt{3}, \infty)$

iii $(f/g)(x) = \dfrac{x^2+3}{\sqrt{x^2-3}}$

Domain of $(f/g)(x)$: $(\text{Dom } f(x)) \cap (\text{Dom } g(x))$

: $(-\infty, \infty) \cap (x > \sqrt{3}$ or $x < -\sqrt{3})$

$\longrightarrow$: $(-\infty, -\sqrt{3}) \cup (\sqrt{3}, \infty)$

iv $(f \circ g)(x) = (\sqrt{x^2-3})^2 + 3 = x^2-3+3 = x^2$

$\longrightarrow$ Domain of $(f \circ g)(x)$: $(-\infty, \infty)$

v $(g \circ f)(x) = \sqrt{(x^2+3)^2-3} = \sqrt{x^4+9+6x^2-3} = \sqrt{x^4+6+6x^2}$

Domain of $(g \circ f)(x)$: $x^4+6x^2+6 \geq 0$

$\longrightarrow$: $(-\infty, \infty)$