PROPERTIES OF SYSTEMS

System Properties

1. Memory
2. Invertible
3. Causal
4. Stability
5. Time Variance
6. Linearity

System Properties - Stability

- A system is said to be bounded-input bounded-output stable (BIBO stable or just stable) if the output signal is bounded for all input signals that are bounded

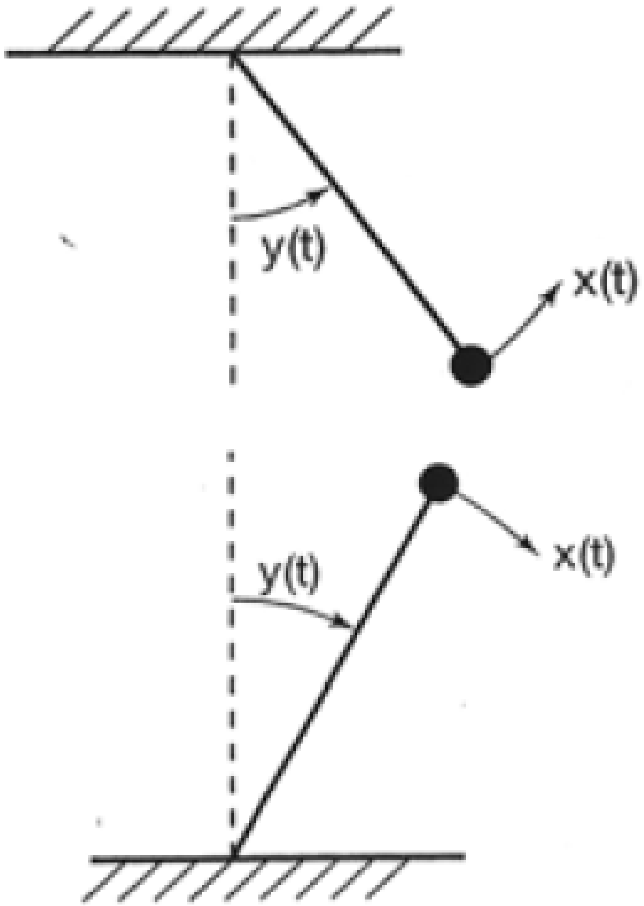
- Consider a discrete-time system with input x and output y

- An input is bounded if there is a real number $M < \infty$ such that $|x(k)| \leq M$ for all $k \in Integers$

- An output is bounded if there is a real number $N < \infty$ such that $|y(n)| \leq N$ for all $n \in Integers$

System Properties - Stability

- A stable system is one in which small inputs lead to responses that do not diverge
- Consider cases of pendulum and inverted pendulum – one is stable, the other not stable



System Properties – Time Invariance

Informally, a system is **time-invariant** (TI) if its behavior does not depend on what time it is.

- Mathematically (in DT): A system $x[n] \rightarrow y[n]$ is TI if for <u>any</u> input $x[n]$ and <u>any</u> time shift n_0,

If	$x[n] \rightarrow y[n]$
then	$x[n-n_0] \rightarrow y[n-n_0]$.

- Similarly for a CT time-invariant system,

If	$x(t) \rightarrow y(t)$
then	$x(t-t_o) \rightarrow y(t-t_o)$.

System Properties – Time Invariance

- Consider the continuous-time system defined by:

$$y(t) = \sin\left[x(t)\right]$$

- To check for time invariance, we must determine whether the time-invariance property holds for any input and for any time shift t_0.

System Properties – Time Invariance

- Let $x_1(t)$ be an arbitrary input to this system, and let:

  $$y_1(t) = \sin\left[x_1(t)\right]$$

- be the corresponding output.
- Consider a second input obtained by shifting $x_1(t)$ in time:

  $$x_2(t) = x_1(t - t_0)$$

- with output:

  $$y_2(t) = \sin\left[x_2(t)\right] = \sin\left[x_1(t - t_0)\right]$$

- From the system definition we get:

  $$y_1(t - t_0) = \sin\left[x_1(t - t_0)\right] = y_2(t)$$

- Hence the system is time invariant.

System Properties – Time Invariance

- Consider the discrete-time system:

$$y[n] = nx[n]$$

- Is this system time invariant?

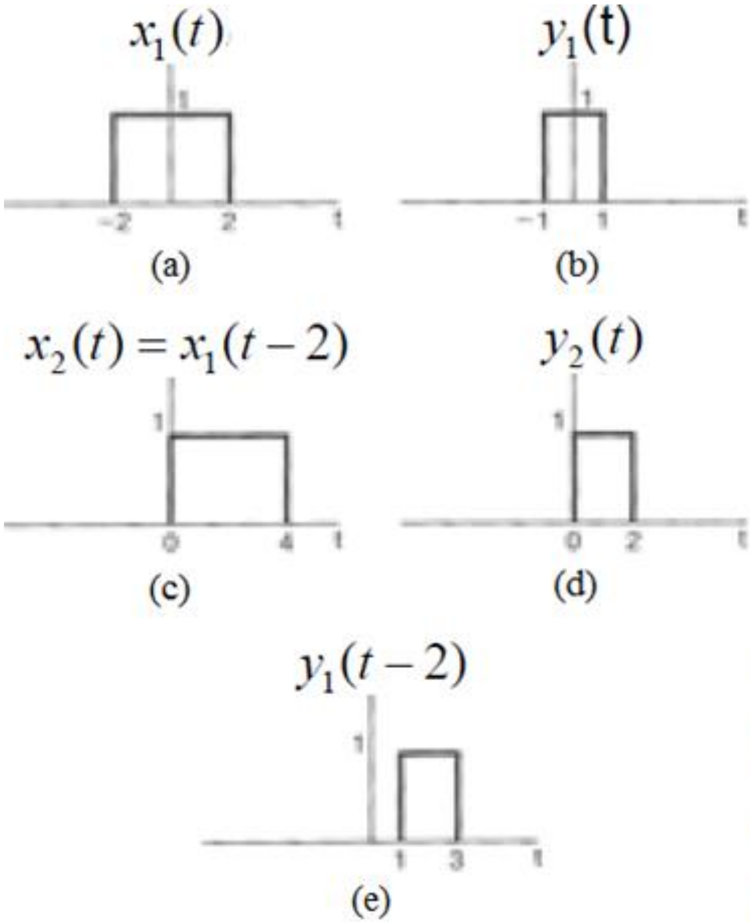
System Properties – Time Invariance

- Consider the system:

$$y(t) = x(2t)$$

➢ Is the system time-variant?

System Properties – Time Invariance



- This system represents a time scaling.
- Intuitively any time shift in the input will also be compressed by a factor of 2 and thus the system is not time invariant.

(a) Input to system, $x_1(t)$.

(b) Output of system, y_1(t) from given input $x_1(t)$.

(c) Shifted input $x_2(t) = x_1(t-2)$

(d) Output $y_2(t)$ corresponding to $x_2(t)$

(e) Shifted signal $y_1(t-2) \neq y_2(t)$

System Properties – Time Invariance

If the input to a TI System is periodic, then the output is periodic with the same period.

"Proof": Suppose $x(t+T) = x(t)$

and $x(t) \to y(t)$

then by TI

$$\underset{\uparrow}{x(t+T)} \to \underset{\uparrow}{y(t+T)}.$$

These are the same input!

So these must be the same output, i.e., $y(t) = y(t + T)$.

System Properties – Time Invariance

EX 1 $$y(t) = x^2(t+1)$$

- Does the behaviour change with time?

EX 2 $$y[n] = \left(\frac{1}{2}\right)^{n+1} x^3[n-1]$$

- Does the behaviour change with time?

System Properties – Linearity

A (CT) system is linear if it obeys the superposition property:

If $x_1(t) \rightarrow y_1(t)$ and $x_2(t) \rightarrow y_2(t)$

then $ax_1(t) + bx_2(t) \rightarrow ay_1(t) + by_2(t)$

$y[n] = x^2[n]$ Nonlinear, TI, Causal

$y(t) = x(2t)$ Linear, <u>not</u> TI, Noncausal

System Properties – Linearity

- Superposition is combination of Additivity and Homogeneity

- **Additivity:** $$x_1(t) + x_2(t) = y_1(t) + y_2(t)$$

- **Homogeneity:** $$ax(t) = ay(t)$$

- Superposition

  If $$x_k[n] \quad \rightarrow \quad y_k[n]$$

  Then $$\sum_k a_k x_k[n] \quad \rightarrow \quad \sum_k a_k y_k[n]$$

- For linear systems, zero input $\rightarrow$ zero output

System Properties – Linearity

- Many systems are nonlinear, for example circuit elements such as diodes and transistors

- However, in this course we focus mainly on linear systems

- WHY?

- Linear models provide accurate representations of the behaviour of many systems such as resistors and capacitors

- We can often linearize models to examine "small signal" perturbations around "operating points"

Problem-1

- Consider a system S whose input $x(t)$ and output $y(t)$ are related by:

$$y(t) = x^2(t)$$

- Is this system Linear? Prove your answer.

Problem-2

Consider the system:

$$y[n] = 2x[n] + 3$$

- Is this system Linear? Prove your answer.