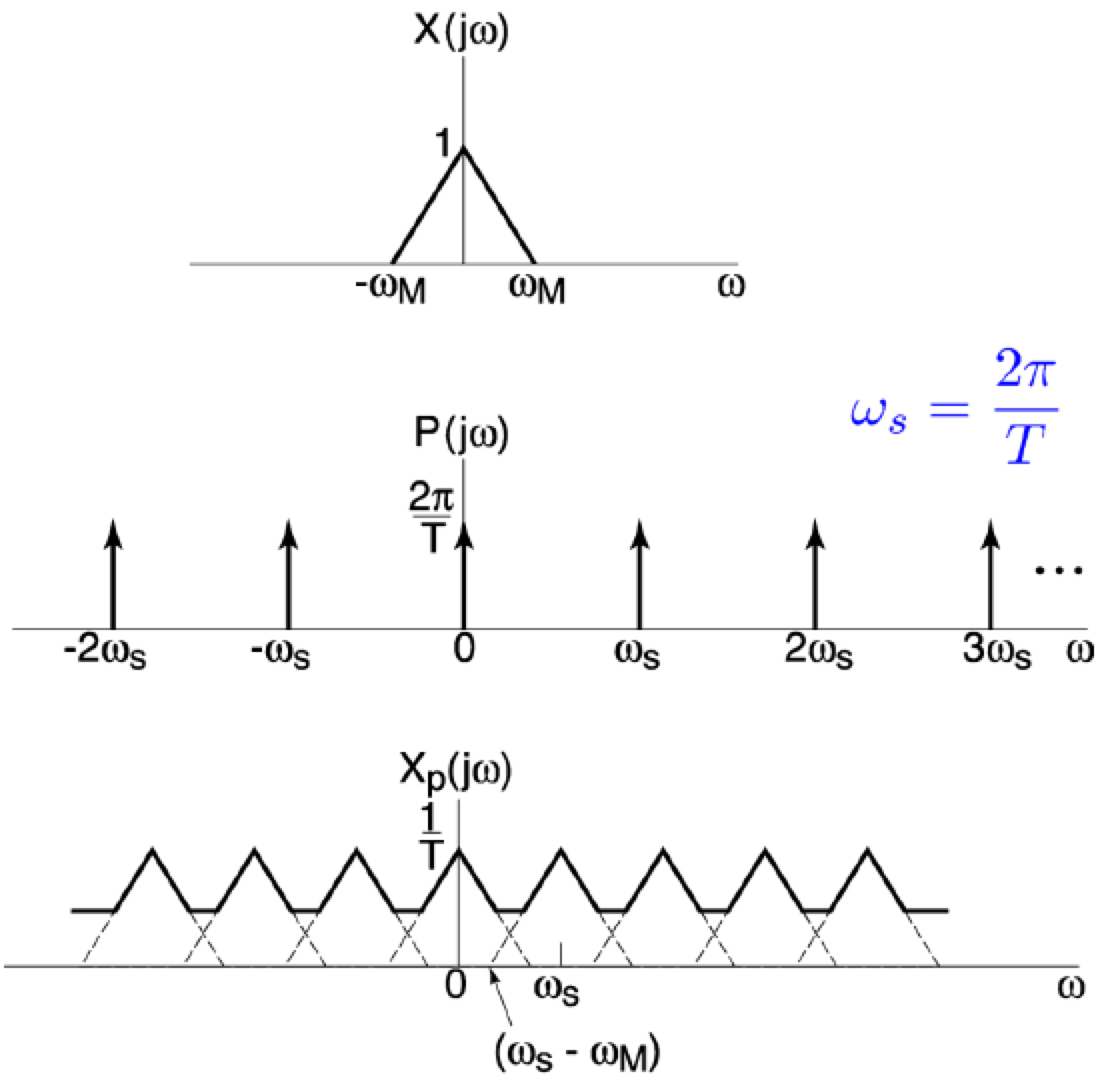
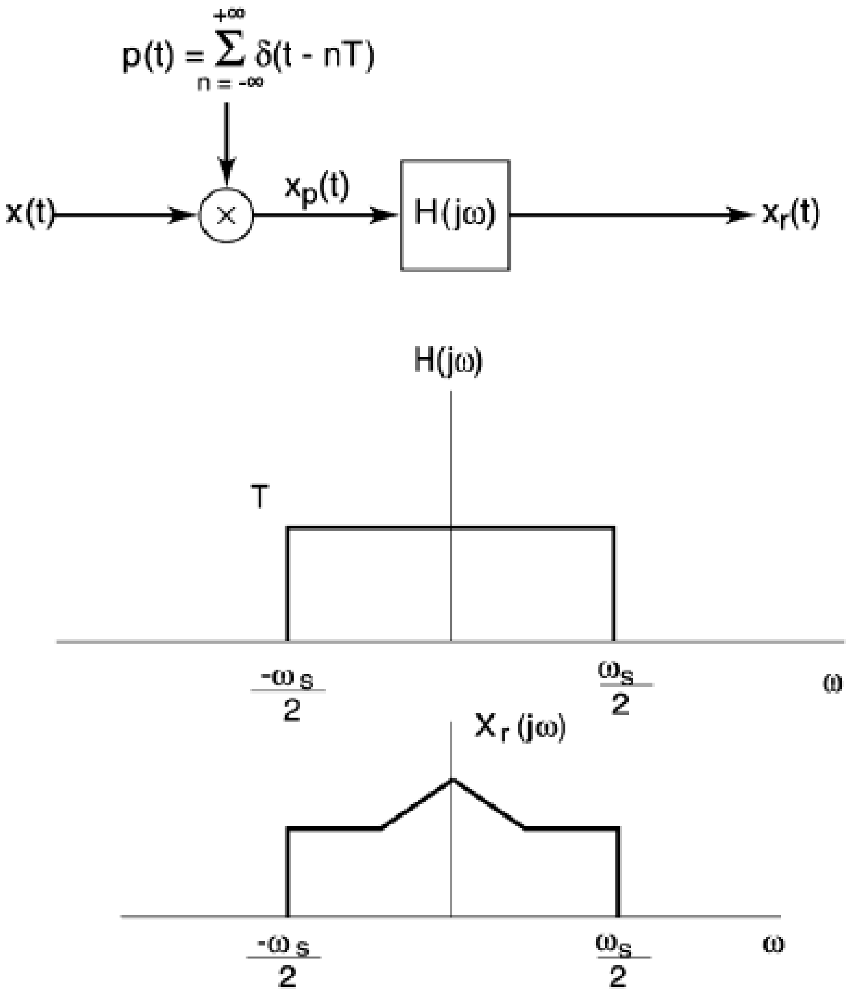
UNDERSAMPLING AND ALIASING

Undersampling and Aliasing

When $\omega_s < 2\,\omega_M$ => Undersampling (and resulting aliasing)



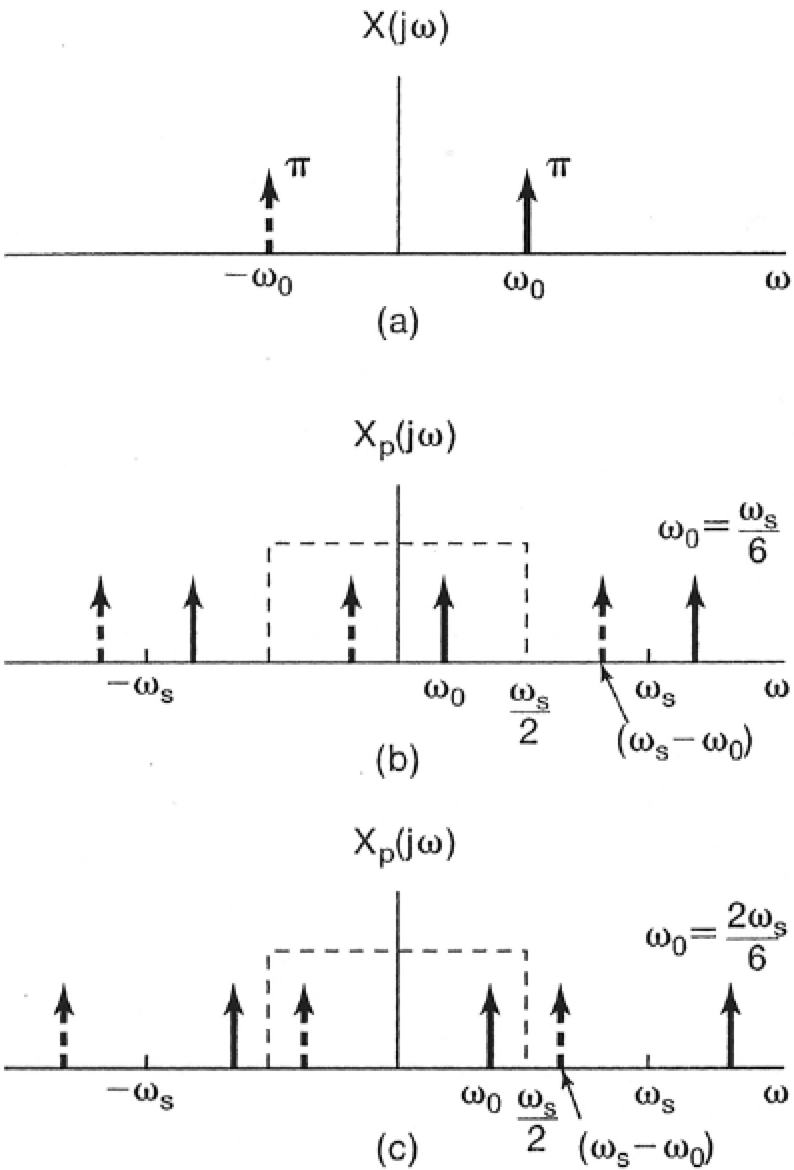
Undersampling and Aliasing

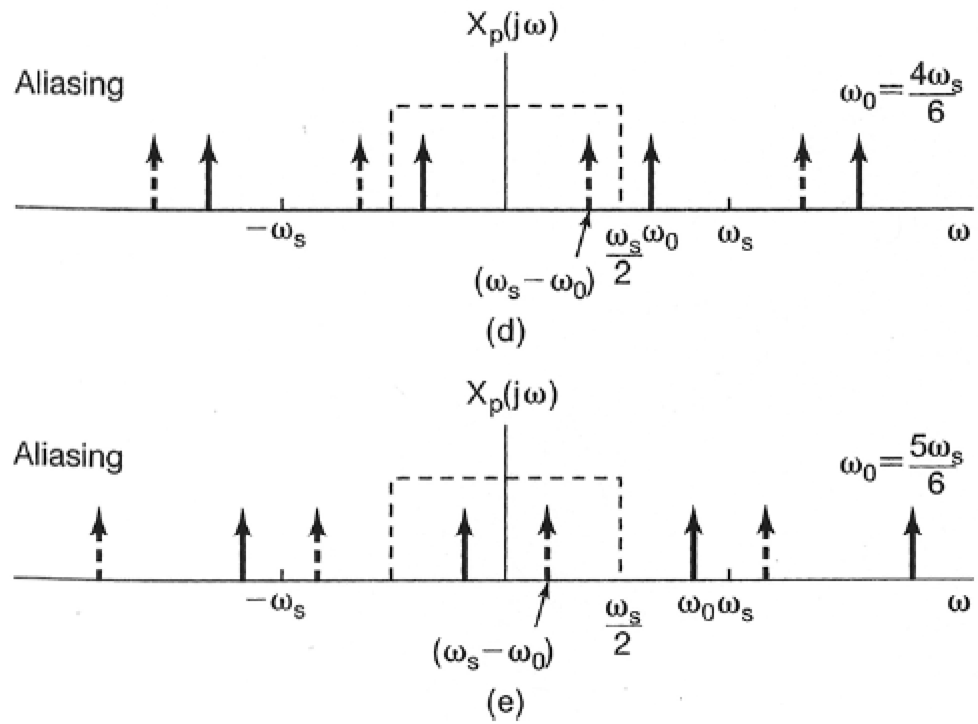


$X_r(j\omega) \neq X(j\omega)$
Distortion because of *aliasing*

— Higher frequencies of $x(t)$ are "folded back" and take on the "aliases" of lower frequencies

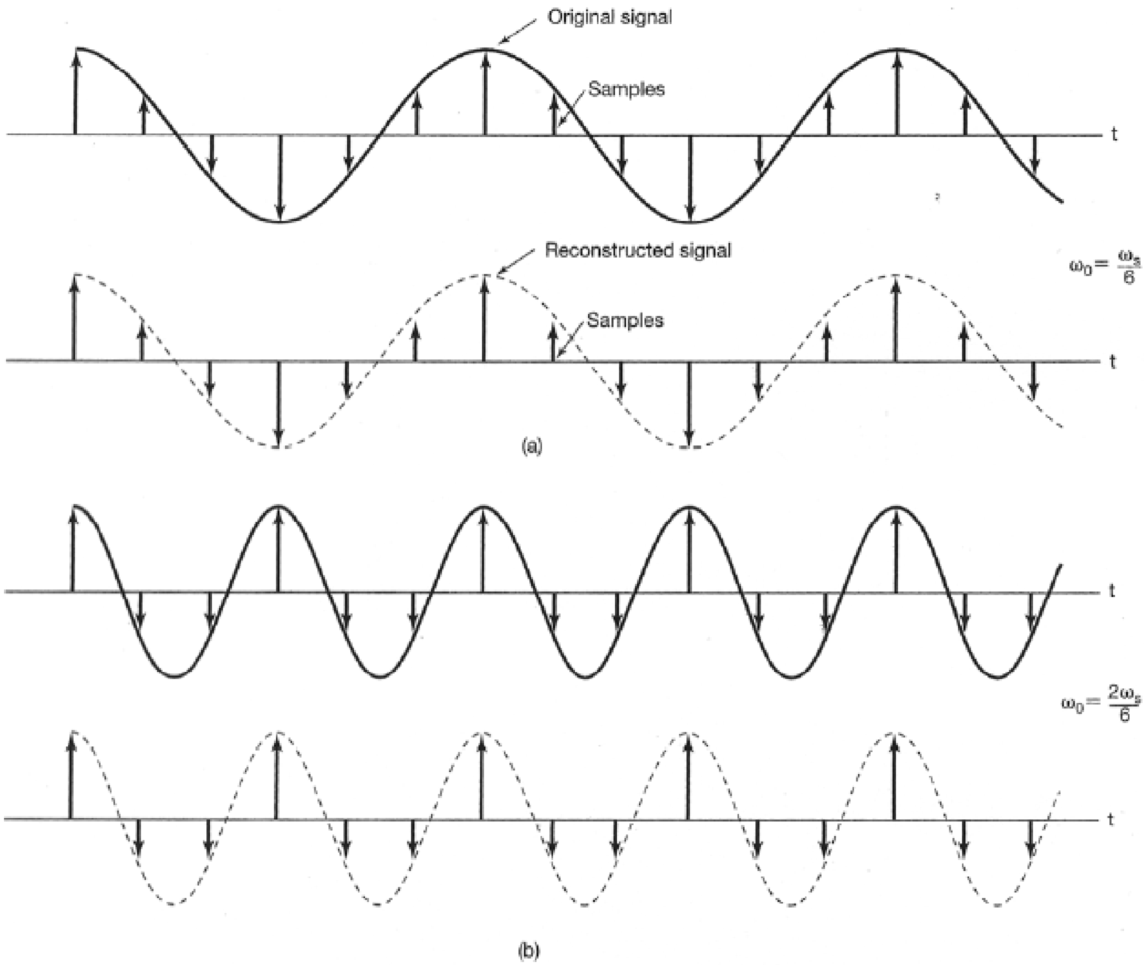
Aliasing Examples





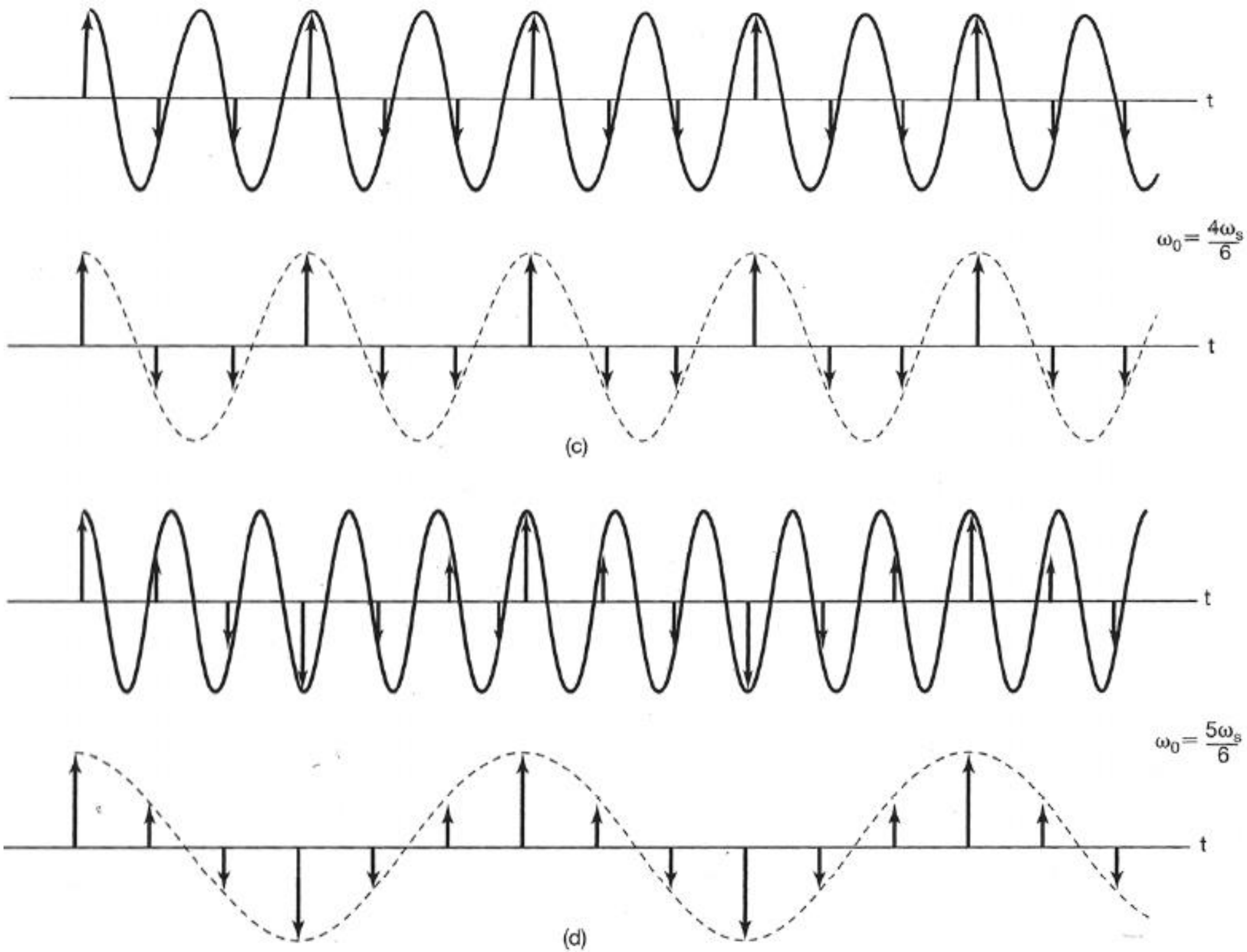
(a) original signal spectrum; (b) spectrum of sampled signal with $\omega_s = 6\omega_0$; (c) spectrum of sampled signal with $\omega_s = 3\omega_0$; (d) spectrum of sampled signal with $\omega_s = (3/2)\ \omega_0$; (e) spectrum of sampled signal with $\omega_s = (6/5)\omega_0$;

Aliasing Examples



Effect of aliasing on a sinusoidal signal for each of four values of ω_0.
For each value of ω_0, the pair of plots show the original sinusoidal samples along with the reconstructed signal (dashed curve); (a) $\omega_0 = \omega_S / 6$ (no aliasing); (b) $\omega_0 = 2\omega_S / 6$ (no aliasing); (c) $\omega_0 = 4\omega_S / 6$ (aliasing); (d) $\omega_0 = 5\omega_S / 6$ (aliasing).

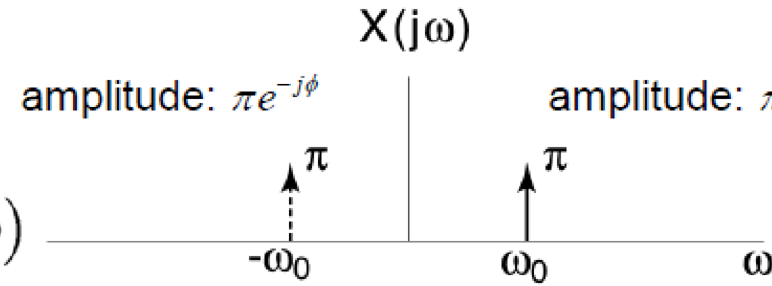
Aliasing Examples

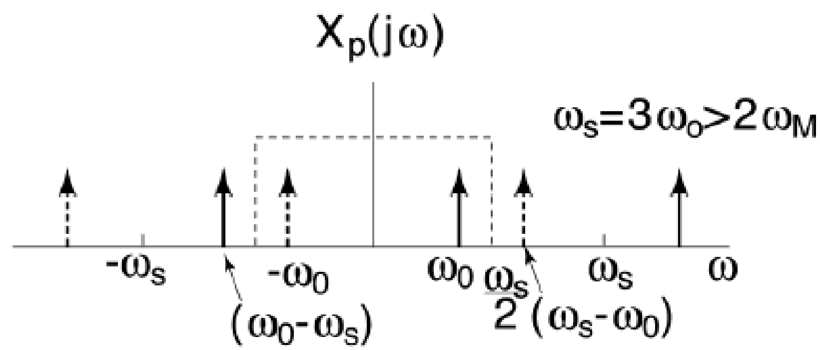


Aliasing Examples

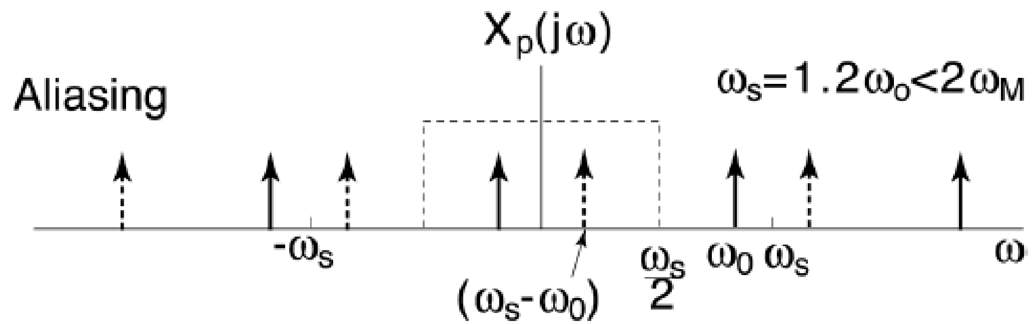
- ω_o takes on the identity or "alias" of a lower frequency (ω_s - ω_o)

$$x(t) = \cos(\omega_0 t + \phi)$$





Picture would be modified…→ phase reversals occur due to aliasing



Sampling - Problem

A signal *x(t)* has a Nyquist rate of ω_o. Find the Nyquist rates of the following signals

a) $y(t) = x(t) + x(t-1)$

b) $y(t) = \frac{dx(t)}{dt}$

c) $y(t) = x^2(t)$

d) $y(t) = x(t) cos w_o t$

END