Thermodynamics I

Lecture 18

Specific Heat Relations of Ideal Gases (Ch-4)

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Specific Heat Relations of Ideal Gases

$$h = \bar{u} + RT,$$

$$dh = du + R\, dT$$

$$du = c_v(T)\, dT \quad \text{and}$$

$$dh = c_p(T)\, dT$$

The relationship between c_p, c_v and R

$$c_p = c_v + R \qquad (\text{kJ/kg·K})$$

On a molar basis

$$\bar{c}_p = \bar{c}_v + R_u \qquad (\text{kJ/kmol·K})$$

Specific heat ratio

$$k = \frac{c_p}{c_v}$$

Air at 300 K

$$\left. \begin{array}{l} c_v = 0.718 \text{ kJ/kg·K} \\ R = 0.287 \text{ kJ/kg·K} \end{array} \right\} c_p = 1.005 \text{ kJ/kg·K}$$

or

$$\left. \begin{array}{l} \bar{c}_v = 20.80 \text{ kJ/kmol·K} \\ R_u = 8.314 \text{ kJ/kmol·K} \end{array} \right\} \bar{c}_p = 29.114 \text{ kJ/kmol·K}$$

The c_p of an ideal gas can be determined from a knowledge of c_v and R.

- The specific ratio varies with temperature, but this variation is very mild.
- For monatomic gases (helium, argon, etc.), its value is essentially constant at 1.667.
- Many diatomic gases, including air, have a specific heat ratio of about 1.4 at room temperature.

Example 4-8

An insulated rigid tank initially contains 0.7 kg of helium 27°C and 350kPa. A paddle wheel with a power rating of 0.015 kW operated within the tank for 30 min. Determine (a) the final temperature and (b) the final pressure of the helium gas.

Assumptions **1** Helium is an ideal gas since it is at a very high temperature relative to its critical-point value of -268°C. **2** Constant specific heats can be used for helium. **3** The system is stationary and thus the kinetic and potential energy changes are zero, $\Delta KE = \Delta PE = 0$ and $\Delta E = \Delta U$. **4** The volume of the tank is constant, and thus there is no boundary work. **5** The system is adiabatic and thus there is no heat transfer.

(a) The amount of paddle-wheel work done on the system is

$$W_{sh} = \dot{W}_{sh}\,\Delta t = \; \textbf{27 kJ}$$

Example 4-8

Under the stated assumptions and observations, the energy balance on the system can be expressed as

$$\underbrace{E_{\text{in}} - E_{\text{out}}}_{\substack{\text{Net energy transfer} \\ \text{by heat, work, and mass}}} = \underbrace{\Delta E_{\text{system}}}_{\substack{\text{Change in internal, kinetic,} \\ \text{potential, etc., energies}}}$$

$$W_{\text{sh,in}} = \Delta U = m(u_2 - u_1) = mc_{\upsilon,\text{avg}}(T_2 - T_1)$$

As we pointed out earlier, the ideal-gas specific heats of monatomic gases (helium being one of them) are constant. The c_υ value of helium is determined from Table A-2a to be 3.1156 kJ/kg K   Substituting this and other known quantities into the above equation, we obtain
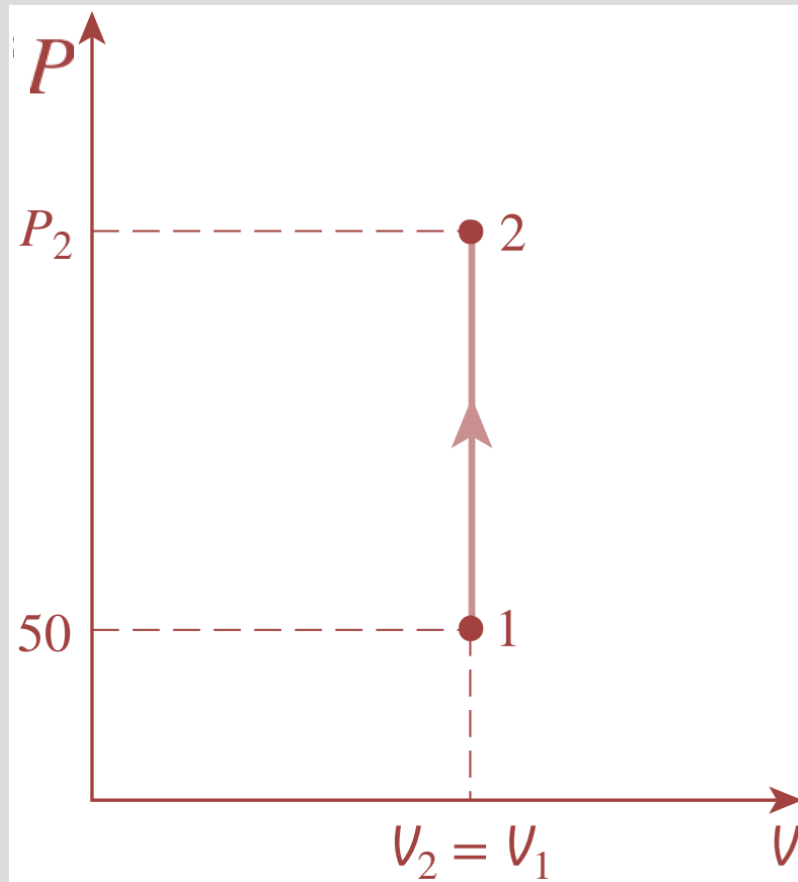
$$T_2 = 39.4 \ ^\circ C$$

Example 4-8

(b) The final pressure is determined from the ideal-gas relation

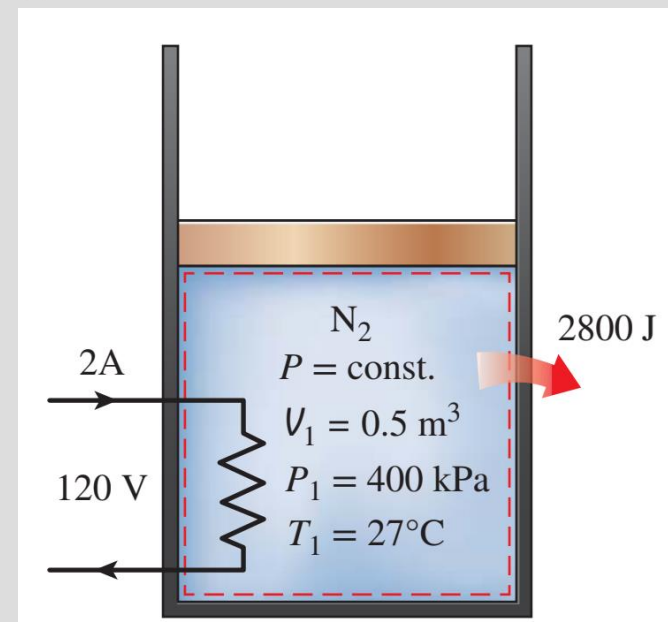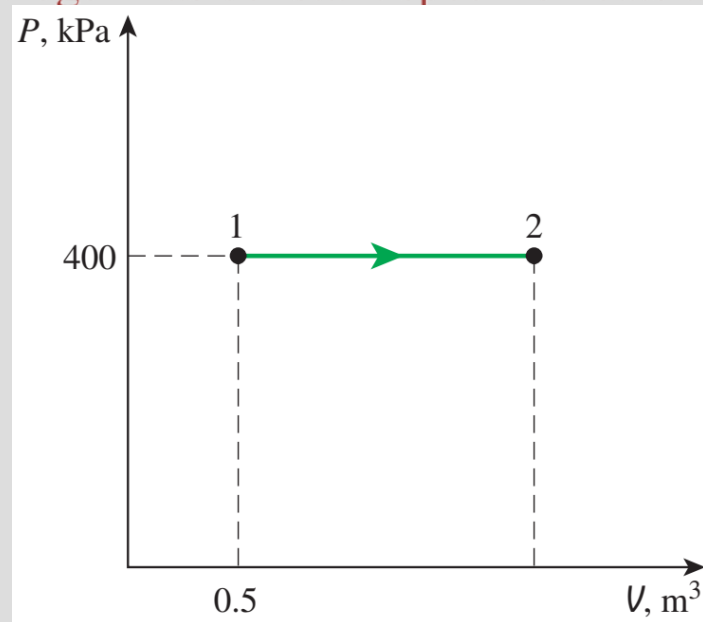$$\frac{P_1 \cancel{V_1}}{T_1} = \frac{P_2 \cancel{V_2}}{T_2}$$

$$P_2 = 364\ kPa$$

Example 4-9

A piston–cylinder device initially contains 0.5 m³ of nitrogen gas at 400 kPa and 27°C. An electric heater within the device is turned on and is allowed to pass a current of 2 A for 5 min from a 120-V source. Nitrogen expands at constant pressure, and a heat loss of 2800 J occurs during the process. Determine the final temperature of nitrogen.

Assumptions **1** Nitrogen is an ideal gas since it is at a high temperature and low pressure relative to its critical-point values of −147°C and 3.39 MPa. **2** The system is stationary, and thus the kinetic and potential energy changes are zero, $\Delta KE = \Delta PE = 0$ and $\Delta E = \Delta U$. **3** The pressure remains constant during the process and thus $P_2 = P_1$. **4** Nitrogen has constant specific heats at room temperature.

P, kPa

400 — 1 ——————→ 2

0.5 V, m³

N₂
$P = $ const.
$V_1 = 0.5$ m³
$P_1 = 400$ kPa
$T_1 = 27°$C

2A

120 V

2800 J

Example 4-9

First, let us determine the electrical work done on the nitrogen:

$$W_e = \mathbf{V}I \, \Delta t$$

$$= (120 \text{ V})(2 \text{ A})(5 \times 60 \text{ s})\left(\frac{1 \text{ kJ/s}}{1000 \text{ VA}}\right) = 72 \text{ kJ}$$

The mass of nitrogen is determined from the ideal-gas relation:

$$m = \frac{P_1 V_1}{R T_1}$$

$$= \frac{(400 \text{ kPa})(0.5 \text{ m}^3)}{(0.297 \text{ kPa·m}^3/\text{kg·K})(300 \text{ K})} = 2.245 \text{ kg}$$

Example 4-9

$$\underbrace{E_{\text{in}} - E_{\text{out}}}_{\substack{\text{Net energy transfer} \\ \text{by heat, work, and mass}}} = \underbrace{\Delta E_{\text{system}}}_{\substack{\text{Change in internal, kinetic,} \\ \text{potential, etc., energies}}}$$

$$W_{e,\text{in}} - Q_{\text{out}} - W_{b,\text{out}} = \Delta U$$

$$W_{e,\text{in}} - Q_{\text{out}} = \Delta H = m(h_2 - h_1) = mc_p(T_2 - T_1)$$

From Table A–2a, **c_p = 1.039 kJ/kg·K** for nitrogen at room temperature. The only unknown quantity in the previous equation is T_2, and it is found to be

$$72 \text{ kJ} - 2.8 \text{ kJ} = (2.245 \text{ kg})(1.039 \text{ kJ/kg·K})(T_2 - 27)°\text{C}$$

$$T_2 = 56.7°\text{C}$$

Practice Problem

A piston–cylinder device initially contains air at 150 kPa and 27°C. At this state, the piston is resting on a pair of stops, as shown in Fig. 4–32, and the enclosed volume is 400 L. The mass of the piston is such that a 350-kPa pressure is required to move it. The air is now heated until its volume has doubled. Determine (a) the final temperature, (b) the work done by the air, and (c) the total heat transferred to the air.
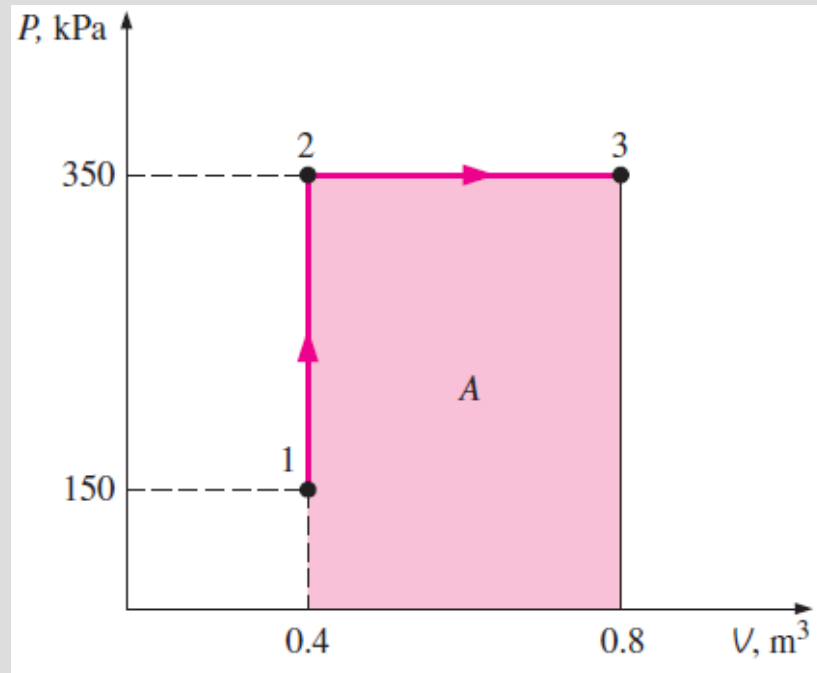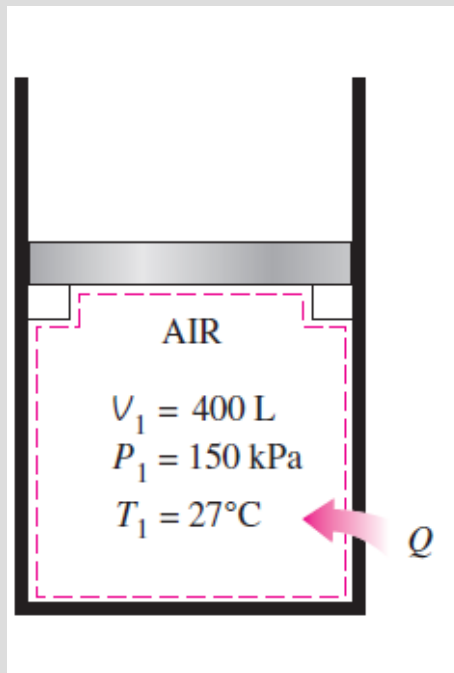
Practice Problem

(*a*) The final temperature can be determined easily by using the ideal-gas relation between states 1 and 3 in the following form:

$$\frac{P_1 V_1}{T_1} = \frac{P_3 V_3}{T_3} \longrightarrow \frac{(150 \text{ kPa})(V_1)}{300 \text{ K}} = \frac{(350 \text{ kPa})(2V_1)}{T_3}$$

$$T_3 = \textbf{1400 K}$$

(*b*) The work done could be determined by integration, but for this case it is much easier to find it from the area under the process curve on a *P-V* diagram, shown in Fig. 4–32:

$$A = (V_2 - V_1)P_2 = (0.4 \text{ m}^3)(350 \text{ kPa}) = 140 \text{ m}^3 \cdot \text{kPa}$$

Therefore,

$$W_{13} = \textbf{140 kJ}$$

The work is done by the system (to raise the piston and to push the atmospheric air out of the way), and thus it is work output.

Practice Problem

$$\underbrace{E_{\text{in}} - E_{\text{out}}}_{\substack{\text{Net energy transfer} \\ \text{by heat, work, and mass}}} = \underbrace{\Delta E_{\text{system}}}_{\substack{\text{Change in internal, kinetic,} \\ \text{potential, etc., energies}}}$$

$$Q_{\text{in}} - W_{b,\text{out}} = \Delta U = m(u_3 - u_1)$$

The mass of the system can be determined from the ideal-gas relation:

$$m = \frac{P_1 V_1}{RT_1} = \frac{(150 \text{ kPa})(0.4 \text{ m}^3)}{(0.287 \text{ kPa·m}^3/\text{kg·K})(300 \text{ K})} = 0.697 \text{ kg}$$

The internal energies are determined from the air table (Table A–17) to be

$$u_1 = u_{@\,300\,\text{K}} = 214.07 \text{ kJ/kg}$$

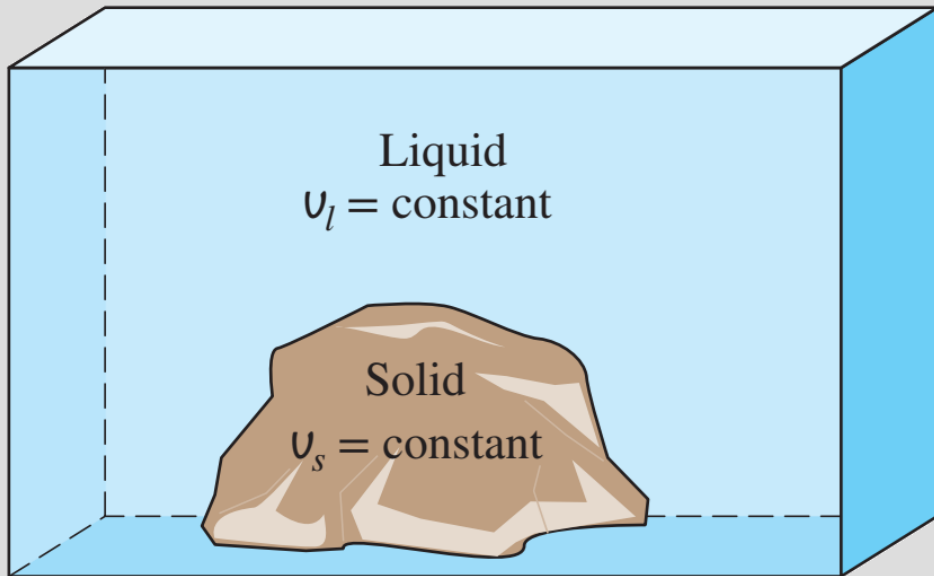$$u_3 = u_{@\,1400\,\text{K}} = 1113.52 \text{ kJ/kg}$$

Thus,

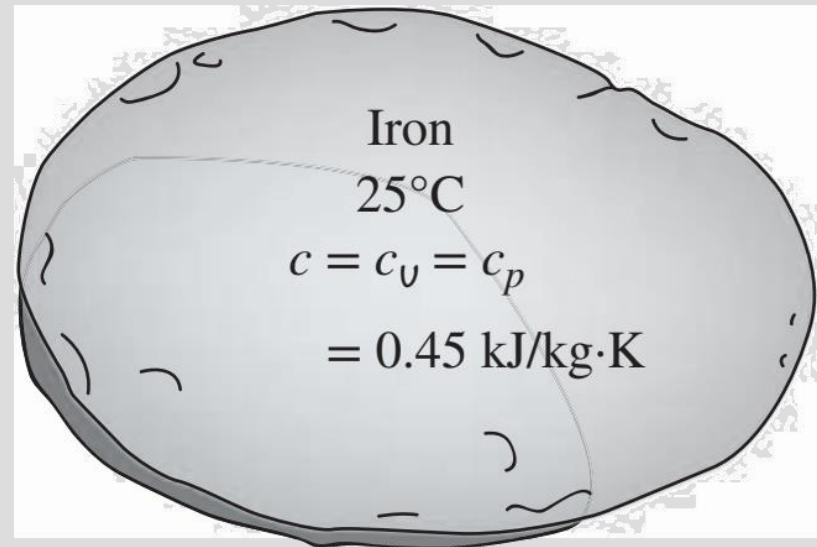$$Q_{\text{in}} - 140 \text{ kJ} = (0.697 \text{ kg})[(1113.52 - 214.07) \text{ kJ/kg}]$$

$$Q_{\text{in}} = \mathbf{767 \text{ kJ}}$$

INTERNAL ENERGY, ENTHALPY, AND SPECIFIC HEATS OF SOLIDS AND LIQUIDS

Incompressible substance: Asubstance whose specific volume (or density) is constant. Solids and liquids are incompressible substances.

The specific volumes of incompressible substances remain constant during a process.

The c_v and c_p values of incompressible substances are identical and are denoted by c.

Internal Energy Changes

$$du = c_v\, dT = c(T)\, dT$$

$$\Delta u = u_2 - u_1 = \int_1^2 c(T)\, dT \qquad (\text{kJ/kg})$$

$$\Delta u \cong c_{\text{avg}}(T_2 - T_1)$$

Enthalpy Changes

$$h = u + Pv \longrightarrow dh = du + v\, dP + P\, dv \overset{0}{\nearrow} \longrightarrow = du + v\, dP$$

$$\Delta h = \Delta u + v\, \Delta P \cong c_{\text{avg}}\, \Delta T + v\, \Delta P \qquad (\text{kJ/kg})$$

For *solids*, the term $v\,\Delta P$ is insignificant, and thus $\Delta h \cong \Delta u \cong c_{\text{avg}}\Delta T$. For *liquids*, two special cases are commonly encountered:

1. *Constant-pressure processes*, as in heaters ($\Delta P = 0$): $\Delta h = \Delta u \cong c_{\text{avg}}\Delta T$

2. *Constant-temperature processes*, as in pumps ($\Delta T = 0$): $\Delta h = v\, \Delta P$

The enthalpy of a compressed liquid $\qquad h_{@\ P,T} \cong h_{f\,@\ T} + v_{f\,@\ T}(P - P_{\text{sat}\,@\ T})$

A more accurate relation than $\qquad h_{@\ P,T} \cong h_{f\,@\ T}$

Example 4-11

Determine the enthalpy of liquid water at 100°C and 15 MPa (a) by using compressed liquid tables, (b) by approximating it as a saturated liquid, and (c) by using the correction given by Eq. 4–38.

(a) From compressed liquid tables, we read

$$\left.\begin{array}{l} P = 15 \text{ MPa} \\ T = 100°C \end{array}\right\} h = \textbf{430.39 kJ/kg} \qquad \text{(Table A–7)}$$

This is the exact value.

(b) Approximating the compressed liquid as a saturated liquid at 100°C, as is commonly done, we obtain

$$h \cong h_{f\,@\,100°C} = \textbf{419.17 kJ/kg}$$

This value is in error by about 2.6 percent.

Example 4-11

(c) From Eq. 4–38,

$$h_{@ P,T} \cong h_{f @ T} + \mathrm{v}_{f @ T}(P - P_{\text{sat} @ T})$$

$$= (419.17 \text{ kJ/kg}) + (0.001 \text{ m}^3/\text{kg})[(15{,}000 - 101.42) \text{ kPa}]\left(\frac{1 \text{ kJ}}{1 \text{ kPa·m}^3}\right)$$

$$= \mathbf{434.07 \text{ kJ/kg}}$$

Discussion   Note that the correction term reduced the error from 2.6 to about 1 percent in this case. However, this improvement in accuracy is often not worth the extra effort involved.

Summary

- Moving boundary work
  - ✓ W_b for an isothermal process
  - ✓ W_b for a constant-pressure process
  - ✓ W_b for a polytropic process
- Energy balance for closed systems
  - ✓ Energy balance for a constant-pressure expansion or compression process
- Specific heats
  - ✓ Constant-pressure specific heat, c_p
  - ✓ Constant-volume specific heat, c_v
- Internal energy, enthalpy, and specific heats of ideal gases
  - ✓ Specific heat relations of ideal gases
- Internal energy, enthalpy, and specific heats of incompressible substances (solids and liquids)