y/date

Robotics - I

- Representation
  - Linear ~ Prismatic {P}
  - Rotation ~ Revolute {R}

- Work Envelope / Space ~ Range of ... movements

∴ Revise Control Systems
  - Open-loop ; Closed-loop
  - Controllers (P, PI, PD, PID) ☹

∴ Review PPP (cartesian / rectilinear)
RPP (cylindrical)
RRP (spherical)
RRR (articulated)
RRP (SCARA)

- Reference frames
  - global / world / base — base joint
  - each joint also has a reference frame (joint) i.e., $\boxed{R}$RR ↳ base
  - tool ; local reference frame of end-effector

- Sensors
  - Proprioceptive {internal to the robot}
  - Exteroceptive {external / "observes the world"}

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Robotics - I

- Transformation
  - translation (d^i_j) location of frame j expressed in frame i

i.e., $d^0_1 = \underbrace{O_1 - O_0}_{\text{in independent / distinct frames}} = \begin{bmatrix} 3 \\ 0 \end{bmatrix} - \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

  - rotation (R^i_j) j: of frame j, i: with respect to i

i.e., $R^0_1 = \begin{bmatrix} x_1 \\ y_1 \end{bmatrix} \cdot \begin{bmatrix} x_0 \\ y_0 \end{bmatrix}$

$= \begin{bmatrix} x_1 x_0 & y_1 x_0 \\ x_1 y_0 & y_1 y_0 \end{bmatrix}$

OR ; $R^0_1 = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$

- Properties of R
  - $R^i_j = (R^j_i)^T$; $(R^i_j)^T = (R^i_j)^{-1}$
  - $R(-\theta) = R(\theta)^T$
  - $R(\theta_1) R(\theta_2) = R(\theta_1 + \theta_2)$
  - $\det(R) = 1$

- Example | $p_0 = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$ $\theta = \pi/2$ about y_0
  - $p_1 = R_{y_0, \frac{\pi}{2}} p_0$ $\quad \cos(\pi/2) = 0$; $\sin(\pi/2) = 1$

$\quad // = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix}$

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Robotics - I

d^A_B : translation of frame B wrt A

R^A_B : rotation of " " "

- Homogeneous Representation for reprsenting translation as matmul

Mappings

$$P_B = \begin{bmatrix} 0 \\ 2 \\ 0 \end{bmatrix} \; ; \; R^A_B = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad \theta = 30^\circ$$

Rotational Mapping

$$P_A = R^A_B P_B$$

$$= \begin{bmatrix} \sqrt{3}/2 & -1/2 & 0 \\ 1/2 & \sqrt{3}/2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 2 \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} -1 \\ \sqrt{3} \\ 0 \end{bmatrix}$$

→ Homogeneous transformation ⟹ $\underbrace{\begin{bmatrix} P_A \\ 1 \end{bmatrix}}_{\tilde{P}_A} = \underbrace{\begin{bmatrix} R^A_B & d^A_B \\ 0 & 1 \end{bmatrix}}_{T^A_B} \underbrace{\begin{bmatrix} P_B \\ 1 \end{bmatrix}}_{\tilde{P}_B}$

$\underbrace{T = \begin{bmatrix} R & 0 \\ 0 & 1 \end{bmatrix}}_{\text{Pure rotation}} \qquad \underbrace{T = \begin{bmatrix} I & d \\ 0 & 1 \end{bmatrix}}_{\text{Pure translation}}$

I → identity

date

Example

correct $\begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta \\ 0 & \sin\theta & \cos\theta \end{bmatrix}_{\theta = 90°}$

$d = (6, -2, 10)$ $\quad R_{x,90°} = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}_{\theta = 90°}$ ×

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix}$$

$$T^A_B = \begin{bmatrix} R_{x,90} & d \\ 0 & 1 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 6 \\ 0 & 1 & 0 & -2 \\ 0 & 0 & 1 & 10 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$\tilde{P}_A = T^A_B \tilde{P}_B$

$\hookrightarrow$ inverse of $T^A_B = (T^A_B)^{-1} = T^B_A$

$$= \begin{bmatrix} R^T & -R^T d \\ 0 & 1 \end{bmatrix}$$

$$T^B_A = \begin{array}{c} \qquad R^T \qquad\qquad -R^T d \\ \begin{bmatrix} 1 & 0 & 0 & -6 \\ 0 & 0 & 1 & -10 \\ 0 & -1 & 0 & -2 \\ 0 & 0 & 0 & 1 \end{bmatrix} \end{array}$$

- Compound Transformation

From $P_C \rightarrow P_A$: $\quad P_A = \underbrace{T^A_C}\, P_C$

$$P_A = T^A_B \underbrace{T^B_C P_C}_{\boxed{P_B}}$$

day/date

- Finding unknown transforms

» $T^U_A T^A_D = T^U_B T^B_C T^C_D$

T^B_C can be found as:

$$T^B_C = (T^U_B)^{-1} T^U_A T^A_D (T^C_D)^{-1}$$

Example

$\{T^W_F, T^W_B, T^F_O, T^B_E\}$ known

Find $\{T^B_O, T^E_O\}$ to find

$\rightarrow T^B_O = (T^W_B)^{-1}((T^W_F)(T^F_O))$

$\rightarrow T^E_O = (T^B_E)^{-1} T^B_O$

Example

{0} base : world {1} corner of table

{2} center of cube {3} camera

~ From description

$$T_1^0 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad T_2^0 = \begin{bmatrix} 1 & 0 & 0 & -0.5 \\ 0 & 1 & 0 & 1.5 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

For {3}

$$R^0_3 = \begin{bmatrix} x_1 x_0 & y_1 x_0 & z_1 x_0 \\ x_1 y_0 & y_1 y_0 & z_1 y_0 \\ x_1 z_0 & y_1 z_0 & z_1 z_0 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & -1 \end{bmatrix}$$

$$T_3^0 = \begin{bmatrix} 0 & 1 & 0 & -0.5 \\ 1 & 0 & 0 & 1.5 \\ 0 & 0 & -1 & 3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_3^2 = (T_2^0)^{-1} T_3^0 \quad \Big| \quad T_2^3 = (T_3^0)^{-1} T_2^0$$

- Euler's Rotation
  - any rotation can be represented by not more than three rotations about coordinate axis

s.t. no two rotations are about the same axis

- Singularity
  - when axis align, a degree of freedom is lost

- Rotation Example

$$R = \begin{bmatrix} 0 & -\sqrt{3}/2 & 1/2 \\ 1/2 & -\sqrt{3}/4 & -3/4 \\ \sqrt{3}/2 & 1/4 & \sqrt{3}/4 \end{bmatrix}$$

$$\theta = \cos^{-1}((r_{11} + r_{22} + r_{33} - 1)/2)$$
$$= 120^\circ$$

$$k = \left(1/\sqrt{3},\ 1/2\sqrt{3} - 1/2,\ 1/2\sqrt{3} + 1/2\right)^T$$

Robotics - I

- Quaternion
  - extension of complex numbers

$$Q = \underbrace{w}_{\text{scalar}} + \varepsilon_x i + \varepsilon_y j + \varepsilon_z k$$

$$\cancel{ijk = -1} = i^2 = j^2 = k^2$$

$\rightarrow$ $Q = (w, \varepsilon) = (\underbrace{\cos \theta/2}_{1\times1}, \underbrace{\overset{(x,y,z)}{r} \sin \theta/2}_{1\times3})$

$\rightarrow$ $Q^{-1} = Q^* = w - \varepsilon_x i - \varepsilon_y j - \varepsilon_z k$

$Q_1 = (w_1, \varepsilon_1)$ $\quad$ $Q_2 = (w_2, \varepsilon_2)$

$Q_1 \circ Q_2 = (w_1 w_2 - \varepsilon_1^T \varepsilon_2, \; w_1 \varepsilon_2 + w_2 \varepsilon_1 + \underbrace{\varepsilon_1 \times \varepsilon_2}_{\text{cross product}})$

Example

rotation of $p = (3, 5, 2)$ by $\theta = 60°$ about $(1, 0, 0)$

$\tilde{p} = (0, 3, 5, 2)$

$\tilde{p}_{rot} = Q \circ \underbrace{(0, 3, 5, 2) \circ Q^*}$

$\rightarrow$ $Q = (\cos \theta/2, \; r \sin \theta/2)$

$Q^* = (0.866, -0.5, 0, 0)$

$\rightarrow$ $(0,3,5,2) \circ (0.866, -0.5, 0, 0)$

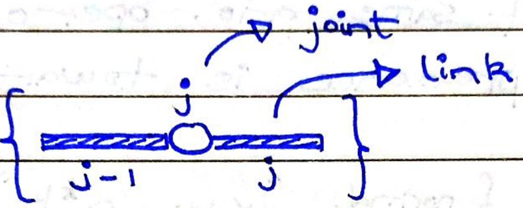
$\rightarrow$ $(1.5, \; [2.6 \;\; 4.33 \;\; 1.732] + [0 \;\; -1 \;\; 2.5])$

$\rightarrow$ $(1.5, \; [2.6, 3.33, 4.232])$

Do same for first two to get $\boxed{\tilde{p}_{rot}}$

$p_{rot} = [3, 0.768, 5.33]$

Kinematics



[Links ?]

$q_j = d_j$: translation of j wrt $j-1$

$\theta_j \longrightarrow$ rotation of j wrt $j-1$

- Forward Kinematics

→ Two Joint : $R(q_1)\,T_x(a_1)\,R(q_2)\,T_x(a_2)$

$$E = \begin{bmatrix} \cos q_1 & -\sin q_1 & 0 \\ \sin q_1 & \cos q_1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & a_1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos q_2 & -\sin q_2 & 0 \\ \sin q_2 & \cos q_2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & a_2 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

→ Three Joints : $R(q_1)\,T_x(a_1)\,R(q_2)\,T_x(a_2)\,\underline{R(q_3)\,T_x(a_3)}$

→ Configuration Space

$e \in \mathbb{R}^N$ } number of joints   task space ξ = {dim J}

└ to reach all of task space {dim e ≥ dim J}

Robotics - I

2 DoF Example

$\xi_E \sim (x, y)$ » configuration string → RR

$C \in \mathbb{R}^2$
$J \in \mathbb{R}^2$] Do the rest from slides (buncha examples)

4 DoF

Given $\xi_E \sim (x, y, z, \theta_y)$

$C \in \mathbb{R}^4$ → configuration space

$J \in \mathbb{R}^3 \times S$ → task space

task space ≤ configuration space

Denavit - Hartenberg (DH) Notation

only valid for serial link robots

x: α_{i-1}, a_{i-1} ; z: d_i, θ_i ~ d_i := translation along z, θ_i := rotation about z

link	α_{i-1}	a_{i-1}	d_i	θ_i
1				
2				
3				

${}^2T_3$

$$\text{lm}: \quad \underbrace{{}^{i-1}T_i = R_x(\alpha_{i-1})\, D_x(a_{i-1})\, R_z(\theta_i)\, D_z(d_i)}_{\text{forward kinematics}}$$

for 2D ; rotation is always about z axis

L :-:

Example

{0} wrt. base

	α_{i-1}	a_{i-1}	d_i	θ_i
1	0	0	0	θ_1
2	0	l_1	0	θ_2
3	0	l_2	0	θ_3

For prismatic joints; translation in DH table is always along the z-axis

→ Example {Link Transformation}

SCARA

$$^2T_3 = \boxed{D_x(a_2)\, D_z(q_4)}$$

Quiz 2

can be ignored clockwise

$$E = R_z(\theta_1)\, T_z(L_1 + d_2)\, T_z(L_2)\, \underline{R_x(-\pi/2)}\, T_z(d_3)$$

└ can do DH table & make E after it if needed

└ 'RPP' → configuration string

forward kinematics
(done?)
inv. kinematics
next lec.

March / 2024

Robotics - I

- Inverse Kinematics
  - geometric
  - analytical
  - numeric

  } approach

→ geometric {trigonometry}
- make triangles ^^
- goal is to find joint angles (no matter in what way)

→ analytical {relies on forward kinematics}
- for 2 joint arm
- → find ξ_E from forward kinematics

$$\begin{cases} x = a_2\cos(q_1+q_2) + a_1\cos(q_1) \\ y = a_2\sin(q_1+q_2) + a_1\sin(q_1) \end{cases}$$

square & add → x^2+y^2 → get q_2

$x^2 = a_2^2\cos^2(q_1+q_2) + a_1^2\cos^2(q_1) + \ldots$
$2a_1a_2\cos(q_1+q_2)\cos(q_1)$

- do same for y^2, add, use $\cos^2+\sin^2=1$
  [$\cos(a+b)$, $\sin(a+b)$]

- $x^2+y^2 = a_1^2 + a_2^2 + 2a_1a_2\cos(q_2)$

find q_2

date

- with q_2, find $x + y$; use q_2 terms as constants ~~get~~ q_1

$$x = a_2 \cos(q_1 + q_2) + a_1 \cos(q_1)$$
$$= a_2 [\cos q_1 \underbrace{\cos q_2}_{C_2} - \sin q_1 \underbrace{\sin q_2}_{S_2}] + \dots$$
$$a_1 \cos q_1$$
$$= a_2 \cos q_1 C_2 - a_2 \sin q_1 S_2 + a_1 \cos q_1$$
$$= \cos q_1 (a_2 C_2 + a_1) - a_2 \sin q_1 S_2$$

do same for y and add

$$y = a_2 \sin(q_1 + q_2) + a_1 \sin q_1$$
$$= a_2 [\sin q_1 \underbrace{\cos q_2}_{C_2} + \cos q_1 \underbrace{\sin q_2}_{S_2}] + \dots$$
$$a_1 \sin q_1$$
$$= a_2 \sin q_1 C_2 + a_2 \cos q_1 S_2 + a_1 \sin q_1$$
$$= \sin q_1 (a_2 C_2 + a_1) + a_2 \cos q_1 S_2$$

- ok, maybe not $\{x + y\}$:-:

take $x \rightarrow$ use $a\cos\theta + b\sin\theta = c$

$$\theta = \tan^{-1}\left[\frac{c}{\pm\sqrt{a^2+b^2-c^2}}\right] - \tan^{-1}\left[\frac{a}{b}\right]$$

both x & y are in $a\cos\theta + b\sin\theta$ form !

EXCELLENT

/date

March / 2024

Robotics - I

- Inverse Kinematics
  - analytical is fast but does not scale well to complex robots
  - {NUMERICAL IK}

- Numerical
  - dependent on initialization
  - more info → less iterations

Example(s)

- $R_B^A = R_{z,30°}$ $\quad p_1^B = \begin{bmatrix} 0 \\ 2 \\ 0 \end{bmatrix}$

  $\rightarrow p_A = R_B^A p^B$

  $(3\times1) \quad (3\times3)\ (3\times1)$

- $R_B^A = R_{z,30°}$ $\quad d_B^A = \begin{bmatrix} 10 \\ 5 \\ 0 \end{bmatrix}$

  $\rightarrow T_B^A = \begin{bmatrix} \cos 30 & -\sin 30 & 0 & 10 \\ \sin 30 & \cos 30 & 0 & 5 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

  $p^A = T_B^A p^B$ or $p^A = R_B^A p^B + d_B^A$

  $\rightarrow T_B^A = \begin{bmatrix} R_B^A & d_B^A \\ 0 & 1 \end{bmatrix}$

  $(T_B^A)^{-1} = T_A^B = \begin{bmatrix} R_B^{A\,T} & -R_B^{A\,T} d_B^{A'} \\ 0 & 1 \end{bmatrix}$

day/date I was here !!!

pre - multiplication → fixed frame
post - multiplication → mobile frame

- $T^A_O = Trans(6, -2, 10)\ Rot_x(90°)$

Velocity [Linear]

derivative := $V = \frac{dx}{dt}$;

$V^B_Q = \frac{d^B Q}{dt}$ or $\frac{dQ^B}{dt}$

$= \lim_{\Delta t \to 0} \frac{Q^B(t + \Delta t) - Q^B(t)}{\Delta t}$

~~velocity of vector Q differentiated in frame B~~

$(V^B_Q)^A$ | same velocity vector but expressed in frame A

conversion $(V^B_Q) \to (V^A_Q)$ } if A & B are related by R

$V^A_Q = R^A_B V^B_Q$

in terms of universal frame

$(V^B_Q)^U = v^B$

$v_C^A = (V_{CORG})^A$ → $V^U_{CORG} = v_C$

origin of C

Example {Do from (slides)}

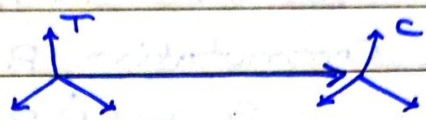
late

April / 2024

Robotics

→ Find $^C({}^T V_{CORG})$ or $^C(V^T_{CORG})$

└ V^T_{CORG}



↓

⇒ $(30-100)\hat{X}$ $\quad V_C - V_T = V^T_C$

⇒ → $R^C_U R^U_T V^T_C$

└ $R^C_U = (R^U_C)^{-1}$

└→ $-(R^U_C)^{-1} R^U_T \, 70\hat{X}$

Velocity [Angular]

└ linear velocity {of a point}

angular velocity {of a body}

└ denote with Ω

Ω^A_B → rotation of B wrt. A

$|\Omega^A_B|$ → speed of rotation

direction (Ω^A_B) → axis of rotation (instantaneous)

$\Omega^U_C = \omega_C \simeq$ in A; $\omega^A_C = (\Omega^U_C)^A$

Recap → $^A V_Q = {}^A V_{BORG} + {}^A R_B V^B_Q$

↳ note: $^A Q = {}^A P_B + {}^A R_B Q^B$

$\frac{dQ^A}{dt} = \frac{d}{dt}{}^A P_B$ ... so on

For any $n \times n$ orthonormal matrix:

→ $RR^T = I_n$

Differentiating

$$\dot{R}R^T + R\dot{R}^T = 0_n$$

Commutative Property

skew symmetric

$$S + (\dot{R}R^T)^T = 0_n \qquad \therefore S = \dot{R}R^T$$

$$S + S^T = 0$$

- Velocity due to rotating reference frame

$$^{A}P = R_B^A P^B \Rightarrow P^B = (R_B^A)^{-1} P^A$$

P^B fixed

$$\dot{P}^A = \dot{R}_B^A P^B \Rightarrow \dot{R}_B^A {R_B^A}^{-1} P^A$$

→ Skew-Symmetric Property

$S_{(3\times3)}$ and $\Omega_{(3\times1)}$

then $SP = \Omega \times P$

did this in CV

(epipolar geometry)

$$V_P^A = \dot{R}_B^A (R_B^A)^T P^A$$

- Rotational Velocity

$$^{A}V_Q = {}^{A}\Omega_B \times {}^{A}Q \qquad \therefore {}^{A}Q = {}^{A}R_B {}^{B}Q$$

$$^{A}V_Q = \underbrace{^{A}V_{BORG}}_{\text{translation}} + \underbrace{^{A}R_B {}^{B}V_Q}_{\text{linear}} + \underbrace{^{A}\Omega_B \times {}^{A}R_B Q_B}_{\text{rotational}}$$

velocity

Robotics - I

- Motion of the Links

  - angular velocity of revolute joint (i)

$${}^{i}\omega_{i+1} = {}^{i}\omega_{i} + {}^{i}R_{i+1}\,\dot{\theta}_{i+1}\,\hat{Z}_{i+1}$$

&

$${}^{i+1}\omega_{i+1} = {}^{i+1}R_{i}\,{}^{i}\omega_{i} + \underbrace{\dot{\theta}_{i+1}}\,\hat{Z}_{i+1}$$

$\to {}^{i+1}R_{\cancel{i}}\,{}^{\cancel{i}}R_{i+1} \to$ NULL (1)

cancels out

- linear velocity ...

$${}^{i}v_{i+1} = {}^{i}v_{i} + {}^{i}\omega_{i} \times {}^{i}P_{i+1}$$

&

$${}^{i+1}v_{i+1} = {}^{i+1}R_{i}\left[{}^{i}v_{i} + {}^{i}\omega_{i} \times {}^{i}P_{i+1}\right]$$

  - for prismatic joint
    - ${}^{i+1}\omega_{i+1} = {}^{i+1}R_{i}\,{}^{i}\omega_{i}$
    - ${}^{i+1}v_{i+1} = {}^{i+1}R_{i}\left({}^{i}v_{i} + {}^{i}\omega_{i} \times {}^{i}P_{i+1}\right) + \dot{d}_{i+1}\,{}^{i+1}\hat{Z}_{i+1}$

Note: $\hat{Z}$ just represents $\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$

Example

$$^{i+1}v_{i+1} = {}^{i+1}R_i\left[{}^iv_i + {}^i\omega_i \times {}^iP_{i+1}\right]$$

$$^{i+1}\omega_{i+1} = {}^{i+1}R_i\,{}^i\omega_i + \dot{\theta}_{i+1}\,{}^{i+1}\hat{Z}_{i+1}$$

$i = 0$

$$^1V_1 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$^1\omega_1 = \begin{bmatrix} 0 \\ 0 \\ \dot{\theta}_1 \end{bmatrix}$$

$i = 1$

cross product

$$^2V_2 = {}^2R_1({}^1V_1 + {}^1\omega_1 \times {}^1P_2)$$

$$^2\omega_2 = {}^2R_1\,{}^1\omega_1 + \dot{\theta}_2\hat{Z}_2$$

$i = 2$

$$^3V_3 = {}^3R_2({}^2V_2 + {}^2\omega_2 \times {}^2P_3)$$

$$^3\omega_3 = {}^3R_2\,{}^2\omega_2 + \dot{\theta}_3\hat{Z}_3$$

$$\rightarrow\ {}^2V_2 = \begin{bmatrix} c_2 & s_2 & 0 \\ -s_2 & c_2 & 0 \\ 0 & 0 & 1 \end{bmatrix}\left(\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \dot{\theta}_1 \end{bmatrix} \times \begin{bmatrix} l_1 \\ 0 \\ 0 \end{bmatrix}\right)$$

$$= \begin{bmatrix} c_2 & s_2 & 0 \\ -s_2 & c_2 & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 0 \\ l_1\dot{\theta}_1 \\ 0 \end{bmatrix} = \begin{bmatrix} s_2\dot{\theta}_1 l_1 \\ c_2\dot{\theta}_1 l_1 \\ 0 \end{bmatrix}$$

$$\rightarrow\ {}^2\omega_2 = \begin{bmatrix} 0 \\ 0 \\ \dot{\theta}_1 + \dot{\theta}_2 \end{bmatrix}$$

$$\begin{array}{ccc} i & j & k \\ 0 & 0 & \dot{\theta}_1+\dot{\theta}_2 \\ l_2 & 0 & 0 \end{array}$$

$j(l_2\dot{\theta}_1 + l_2\dot{\theta}_2)$

$$\rightarrow\ {}^3V_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}\left(\begin{bmatrix} s_2\dot{\theta}_1 l_1 \\ c_2\dot{\theta}_1 l_1 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \dot{\theta}_1+\dot{\theta}_2 \end{bmatrix} \times \begin{bmatrix} l_2 \\ 0 \\ 0 \end{bmatrix}\right)$$

$$= \left(\begin{bmatrix} s_2\dot{\theta}_1 l_1 \\ c_2\dot{\theta}_1 l_1 \\ 0 \end{bmatrix} + \begin{bmatrix} \\ l_2\dot{\theta}_1 + l_2\dot{\theta}_2 \\ 0 \end{bmatrix}\right)$$

$$= \begin{bmatrix} s_2\dot{\theta}_1 l_1 \\ l_2\dot{\theta}_1 + l_2\dot{\theta}_2 + c_2 l_1\dot{\theta}_1 \\ 0 \end{bmatrix}$$

ate

$$^{3}\omega_3 = \begin{bmatrix} 0 \\ 0 \\ \dot\theta_1 + \dot\theta_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \cancel{\dot\theta_3}\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} 0 \\ 0 \\ \dot\theta_1 + \dot\theta_2 + \cancel{\dot\theta_3}\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ \dot\theta_1 + \dot\theta_2 \end{bmatrix}$$

Jacobians

└ multidimensional form of derivative

$$f(x,y) = 2x^2 + 2y^2 + 2xy$$

$$\nabla f(x,y) = \begin{bmatrix} \partial f/\partial x \\ \partial f/\partial y \end{bmatrix}$$

(when multiple funcs →

$$J[f_1, f_2] = \begin{bmatrix} \partial f_1/\partial x & \partial f_1/\partial y \\ \partial f_2/\partial x & \partial f_2/\partial y \end{bmatrix}$$

→ $Y = F(X)$

$$\frac{\partial Y}{\partial t} = \frac{\partial F(X)}{\partial X}\frac{\partial X}{\partial t}$$

$$\dot Y = \boxed{J(X)}\,\dot X$$

Jacobian

Six Jointed Robot $J_{6\times6} \mid \theta_{6\times1} \mid {}^{0}v_{6\times1}$ → $\begin{bmatrix} {}^{0}v_{3\times1} \\ {}^{0}\omega_{3\times1} \end{bmatrix}$

Invertibility of Jacobian

$|J(\theta)| \neq 0$ for $J_{n \times m}$

└ if $|\cdot|$ is zero ; a singularity has occurred , i.e, $n > m$ (non-invertible)

Example

two-link robot → ee along X at 1 m/s

$$\dot{\theta} = {}^{0}J^{-1}(\theta) v$$

$$\rightarrow \frac{1}{l_1 l_2 s_2} \begin{bmatrix} l_2 c_{12} & l_2 s_{12} \\ -l_1 c_1 - l_2 c_{12} & -l_1 s_1 - l_2 s_{12} \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$\dot{\theta} = \frac{1}{l_1 l_2 s_2} \begin{bmatrix} l_2 c_{12} \\ -l_1 c_1 - l_2 c_{12} \end{bmatrix}$$

$$\dot{\theta}_1 = \frac{l_2 c_{12}}{l_1 l_2 s_2} \qquad \dot{\theta}_2 = \frac{-l_1 c_1 - l_2 c_{12}}{l_1 l_2 s_2}$$

{ At $\theta_2 = 0$ and prove }

Forces on Manipulators

$${}^{i}f_i = {}^{i}f_{i+1} = {}^{i}R_{i+1} \, {}^{i+1}f_{i+1}$$

$${}^{i}n_i = {}^{i}R_{i+1} \, {}^{i+1}n_{i+1} + {}^{i}P_{i+1} \times {}^{i}f_i$$

$\tau_i = {}^{i}n_i^{T} \, {}^{i}\hat{Z}_i$ } Rotational Motion | Torque

$\tau_i = {}^{i}f_i^{T} \, {}^{i}\hat{Z}_i$ } Joint is prismatic | Moment

Example

$\rightarrow\ {}^{i}f_i = {}^{i}R_{i+1}\,{}^{i+1}f_{i+1}$

$\rightarrow\ {}^{i}n_i = {}^{i}R_{i+1}\,{}^{i+1}n_{i+1} + {}^{i}P_{i+1} \times {}^{i}f_i$

$i=2$ | ${}^{2}f_2 = \begin{bmatrix} f_x \\ f_y \\ 0 \end{bmatrix}$ $\quad \Bigg\{ \quad {}^{2}n_2 = I\,\overset{0}{\cancel{{}^{3}n_3}} + l_2\hat{X}_2 \times \begin{bmatrix} f_x \\ f_y \\ 0 \end{bmatrix}$

$= \begin{bmatrix} 0 \\ 0 \\ l_2 f_y \end{bmatrix}$

$i=1$ | ${}^{1}f_1 = \begin{bmatrix} c_2 & -s_2 & 0 \\ s_2 & c_2 & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} f_x \\ f_y \\ 0 \end{bmatrix} = \begin{bmatrix} c_2 f_x - s_2 f_y \\ s_2 f_x + c_2 f_y \\ 0 \end{bmatrix}$

$${}^{1}n_1 = \begin{bmatrix} c_2 & -s_2 & 0 \\ s_2 & c_2 & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 0 \\ 0 \\ l_2 f_y \end{bmatrix} + l_1\hat{X}_1 \times \begin{bmatrix} c_2 f_x - s_2 f_y \\ s_2 f_x + c_2 f_y \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} 0 \\ 0 \\ l_2 f_y \end{bmatrix} + (\dots)$$

rest from slides

new lecture

$$= \begin{bmatrix} 0 \\ 0 \\ l_2 f_y \end{bmatrix} + \begin{bmatrix} l_1 \\ 0 \\ 0 \end{bmatrix} \times \begin{array}{c} \begin{bmatrix} c_2 f_x - s_2 f_y \\ s_2 f_x + c_2 f_y \\ 0 \end{bmatrix} \end{array}\begin{array}{l} \alpha \\ \beta \\ \end{array}$$

$$= \quad // \quad \begin{vmatrix} i & j & k \\ l_1 & 0 & 0 \\ \alpha & \beta & 0 \end{vmatrix}$$

$$= \begin{bmatrix} 0 \\ 0 \\ l_2 f_y \end{bmatrix} + l_1(s_2 f_x + c_2 f_y)\,\hat{k}$$

$$= \begin{bmatrix} 0 \\ 0 \\ l_2 f_y + l_1 s_2 f_x + l_1 c_2 f_y \end{bmatrix}$$

ay/date

23 April / 24

Robotics - I

- Jacobians in Force Domain

$W = F.d$

$\Rightarrow F^T \delta x = \tau^T \delta\theta$

Jacobian $\delta x = J \delta\theta$

$\rightarrow F^T J \delta\theta = \tau^T \delta\theta$

$\rightarrow F^T J = \tau^T$

$\rightarrow \tau = J^T F$

when written wrt. frame {0}

$\tau = {}^0J^T \, {}^0F$

→ when Jacobian loses a full rank; it cannot exert static forces in certain direction even if desired ~

- Locomotion

→ rolling is most efficient {not found in nature}

→ for static walking, at least three legs are required

└ at least two DoF is required to move a leg forward

Robotics - I

Classification of Sensors

- proprio / extero
- passive / active

- Linearity — superposition / addition i.e., $f(ax+by) = af(x)+bf(y)$
- Sensitivity → Δoutput / Δinput
- Precision → range / σ → standard deviation

- systematic and random erros might be well defined in a controlled environment but not dynamic / mobile environments.

- Sensors
[IC?] converts electro mechanical to electrical signals
→ optical Encoders

- to find direction ; two encoders are used using first / rising and Falling edge
- typical cycles / rev = 2000 CPR

- band width } i.e., can relate with carrier bandwidth of tower.

im tired :(cant write for shit

i miss danial :(

day/date

May / 2024

Robotics - I

- Quiz 3 Recap

- Accelerometer
  - └ measures external forces

$$F_{applied} = F_{inertial} + F_{damping} \ldots + F_{spring} = m\ddot{x} + c\dot{x} + kx$$

mass, damping coeff., spring coeff.

$$a_{applied} = \frac{kx}{m}$$

- inertial measurement unit (IMU)
  - └ sensitive to errors
  - └ as we integrate accel. to get velocity and position ; error becomes quadratic (double integration) with presence of error in accel.

- Read rest from slides (InM vibes)

Time of flight → $t_{t\to r} + t_{r\to t}$

Elapsed time → $t/2$ {one way}

Probabilistic Localization
- └ belief
- └ sensor model
- └ motion model

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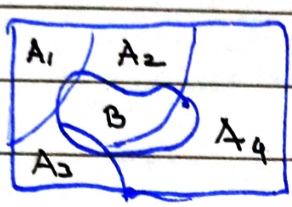
Robotics - I

Probabilic Localization



$\therefore \; P(A \cap B) = P(B|A)P(A)$

$P(B) \propto \sum_i B \cap A_i$

$= \sum_i P(B|A_i)P(A_i)$ } Bayes Rule (Unnormalized)

→ Terminology
- Position X_t
- Control inputs U_t
- Past observations Z_t