NATIONAL UNIVERSITY OF SCIENCES & TECHNOLOGY

Instrumentation and Measurements (EE-383)

Assignment # 2

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3.1 Discuss how limiting errors in y can be computed from the measurement of two quantities u and v, each having limiting errors when (a) $y = u + v$ and (b) $y = u/v$.

- Addition : $y = u + v$

» Limiting error : Taking differentials

$$\gg \frac{\delta y}{y} = \frac{u}{y}\frac{\delta u}{u} + \frac{v}{y}\frac{\delta v}{v}$$

Due to error nature $\gg \underline{\frac{\delta y}{y} = \pm\left(\frac{u}{y}\frac{\delta u}{u} + \frac{v}{y}\frac{\delta v}{v}\right)}$

- Division : $y = u/v$

» Limiting error : Taking natural log on both sides

$$\gg \ln(y) = \ln(u) - \ln(v)$$

$$\gg \frac{\delta y}{y} = \frac{\delta u}{u} - \frac{\delta v}{v}$$

Due to error nature $\gg \underline{\frac{\delta y}{y} = \pm\left(\frac{\delta u}{u} + \frac{\delta v}{v}\right)}$

3.3 The measured value of a capacitor is 205 μF, whereas its true value is 200.4 μF. Determine the relative error.

- Measured $A_m = 205\ \mu F$
- True $A_t = 200.4\ \mu F$

Relative error $\varepsilon_r = \frac{|A_m - A_t|}{A_t} = 0.0229$

$$\gg \underline{\varepsilon_r = 2.29\ \%}$$

3.4 Three resistors have the following ratings: $R_1 = 47\ \Omega \pm 4\%$, $R_2 = 65\ \Omega \pm 4\%$ and $R_3 = 55\,\Omega \pm 4\%$. Determine the magnitude and limiting error in ohms and in percentage of the resistance if these resistances are connected in series.

- $R_1 = 47\,\Omega \pm 4\%$; $R_2 = 65\,\Omega \pm 4\%$
  $R_3 = 55\,\Omega \pm 4\%$

» Magnitude $= R_1 + R_2 + R_3$

$$R_T = 167\ \Omega$$

» Limiting error $\frac{\delta R_T}{R_T} = \pm\left(\frac{R_1}{R_T}\frac{\delta R_1}{R_1} + \frac{R_2}{R_T}\frac{\delta R_2}{R_2} \dots + \frac{R_3}{R_T}\frac{\delta R_3}{R_3}\right)$

$$= \pm\left(\frac{47}{167}(0.04) + \frac{65}{167}(0.04) \dots + \frac{55}{167}(0.04)\right)$$

$$= \pm(0.04)$$

» $\frac{\delta R_T}{R_T}_{(\%)} = \pm 4\%$; $\frac{\delta R_T}{R_T}_{(\Omega)} = \pm 6.68\,\Omega$

3.7 The following 10 observations were recorded while measuring a voltage: 41.7, 42.0, 41.8, 42.0, 42.1, 41.9, 42.5 and 41.8 volts. Find (a) the mean, (b) the standard deviation, (c) the probable error of one reading, and (d) the probable error of the mean.

a) Arithmetic mean $\bar{x} / \mu = \frac{41.7 + \dots + 41.8}{8}$

$$\mu = 41.975$$

b) Standard Deviation $\sigma = \sqrt{\frac{\Sigma(x_i - \bar{x})^2}{n-1}}$ $\therefore n < 20$

$$= \sqrt{\frac{(41.7 - 41.975)^2 + \dots}{8-1}}$$

$$\sigma = 0.249$$

c) Probable error of one reading $r = \pm 0.6745\,\sigma$

$$r = \pm 0.1679$$

d) Probable error of mean $r_m = \frac{r}{\sqrt{n-1}}$ $\therefore n \leq 20$

$$= \frac{\pm 0.1679}{\sqrt{7}}$$

$$r_m = \pm 0.0634$$

3.8 The following values were recorded during the measurement of a resistance: 147.2, 147.4, 147.9, 148.1, 147.1, 147.5, 147.6 and 147.5 ohms. Taking arithmetic mean as the central value, calculate (a) standard deviation, (b) probable error of one reading, and (c) probable error of the mean.

a) Standard Deviation $6 = \sqrt{\frac{\Sigma(x_i - \bar{x})^2}{n-1}}$ $\therefore n \leq 20$

Mean $\bar{x} / \mu = 147.5375$

$6 = \sqrt{\frac{(147.2 - 147.5375)^2 + \dots}{8-1}}$

$6 = 0.3335$

b) Probable error of one reading $r = \pm 0.6745\, 6$

$r = \pm 0.2249$

c) Probable error of mean $r_m = \frac{r}{\sqrt{n-1}}$ $\therefore n \leq 20$

$r_m = \frac{\pm 0.2249}{\sqrt{7}}$

$r_m = \pm 0.0850$

3.10 Indicate the correct choice:

(a) The current through a resistance is measured with uncertainties: $I = 4\text{ A} \pm 0.5\%$, $R = 100\,\Omega \pm 0.2\%$. The uncertainty in the measurement of power is

(i) 1600 W ± 0.01%

(ii) 1600 W ± 0.2%

(iii) 1600 W ± 0.5%

(iv) 1600 W ± 1.02%

(b) To measure 2 volts, if one selects a 0–100 volt range voltmeter which is accurate to within ±1%, the error in one's measurement may be up to

(i) ±0.02%

(ii) ±1%

(iii) ±2%

(iv) ±50%

(c) A thermometer is calibrated from 150 to 200°C. The accuracy specified is ±0.25%. The maximum static error in measurement is

(i) ±0.5°C

(ii) ±0.375°C

(iii) ±0.125°C

(iv) ±0.0125°C

(d) The radius of a sphere is given as 40.0 ± 0.5 mm. The estimated error in its mass is:

(i) ± 3.75%
(ii) ± 1.25%
(iii) ± 12.5%
(iv) ± 0.125%

(e) A large number of 230 Ω resistors are obtained by combining 120 Ω resistors with a standard deviation of 4.0 Ω and 110 Ω resistors with a standard deviation of 3.0 Ω. The standard deviation of the 230 Ω resistors thus formed will be

(i) 3.5 Ω
(ii) 5.0 Ω
(iii) 7.0 Ω
(iv) 12.0 Ω

(f) The calibration data for a pressure compensator of a pump is given below

Input x	0	1	2	3	4	5	6	7	8	9	10
Output y	9.5	8.4	7.8	7.4	6.1	5.4	5.2	4.6	3.2	1.9	1.1

For the given data, the slope of the best-fit line applying the least squares method is:

(i) 0.921
(ii) −0.803
(iii) 0.819
(iv) −0.945

(g) The measurements of a source voltage are 5.9 V, 5.7 V and 6.1 V. The sample standard deviation of the readings is:

(i) 0.013
(ii) 0.04
(iii) 0.115
(iv) 0.2

(h) The reliability of an instrument refers to:

(i) measurement changes due to temperature variation
(ii) degree to which repeatability continues to remain within specified limits
(iii) the life of the instrument
(iv) the extent to which the characteristics remain linear

(i) Using the given data points tabulated below, a straight line passing through the origin is fitted using the least squares method. The slope of the line is

x	1.0	2.0	3.0
y	1.5	2.2	2.7

(i) 0.9
(ii) 1.0
(iii) 1.1
(iv) 1.5

3.11 For the following given data,
$x_1 = 49.7$, $x_2 = 50.1$, $x_3 = 50.2$, $x_4 = 49.6$, $x_5 = 49.7$
calculate the following

(a) Arithmetic mean
(b) Deviation of each value
(c) Algebraic sum of the deviations
(d) Average deviation
(e) Standard deviation

a) Arithmetic mean $\bar{x} / \mu = \dfrac{49.7 + \dots + 49.7}{5}$

$$\underline{\mu = 49.86}$$

b) Deviation of each value :

$$\left.\begin{aligned} d_1 &= x_1 - \bar{x} = 0.16 \\ d_2 &= x_2 - \bar{x} = 0.24 \\ d_3 &= x_3 - \bar{x} = 0.34 \\ d_4 &= x_4 - \bar{x} = 0.26 \\ d_5 &= x_5 - \bar{x} = 0.16 \end{aligned}\right\}$$

c) Algebraic sum of deviations : $d_1 + d_2 + d_3 + d_4 + d_5$

$$\underline{d_T = 1.16}$$

d) Average deviation $= \dfrac{1}{n} \Sigma |d_i| = \dfrac{1.16}{5} = \underline{0.232}$

e) Standard deviation $\sigma = \sqrt{\dfrac{\Sigma (d_i)^2}{n-1}}$ $\therefore n \leq 20$

$$= \sqrt{\frac{(0.16)^2 + \dots + (0.16)^2}{4}}$$

$$\underline{\sigma = 0.270}$$

3.16 Two resistors R_1 and R_2 are connected in series and then in parallel. The values of resistances are: $R_1 = 100.0\,\Omega \pm 0.1\%$, $R_2 = 50\,\Omega \pm 0.06\%$. Calculate the uncertainty in the combined resistance of both series and parallel arrangements.

- $R_1 = 100.0\,\Omega \pm 0.1\%$; $R_2 = 50\,\Omega \pm 0.06\%$

» Series ($R_T = R_1 + R_2$)

$$R_T = 100 + 50 = 150\,\Omega$$

» $\dfrac{\partial R_T}{\partial R_1} = 1$; $\dfrac{\partial R_T}{\partial R_2} = 1$

» Uncertainty $U_r = \sqrt{(1)^2(0.1\% \times 100)^2 + (1)^2(0.06\% \times 50)^2}$

$$U_r = \pm 0.1\,\Omega$$

» Parallel $(R_T = R_1R_2/R_1+R_2)$

$$R_T = \frac{(100)(50)}{150} = 33.33\,\Omega$$

» $\dfrac{\partial R_T}{\partial R_1} = \dfrac{R_2}{R_1+R_2} - \dfrac{R_1R_2}{(R_1+R_2)^2} = \dfrac{R_2^{\,2}}{(R_1+R_2)^2}$

$$\frac{\partial R_T}{\partial R_1} = \frac{50^2}{150^2} = \frac{1}{9}$$

» $\dfrac{\partial R_T}{\partial R_2} = \dfrac{R_1^{\,2}}{(R_1+R_2)^2} = \dfrac{100^2}{150^2} = \dfrac{4}{9}$

» Uncertainty $U_r = \sqrt{\left(\frac{1}{9}\right)^2(0.1\% \times 100)^2 + \left(\frac{4}{9}\right)^2(0.06\% \times 50)^2}$

$$U_r = \pm 0.02\,\Omega$$