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Coulomb's Law

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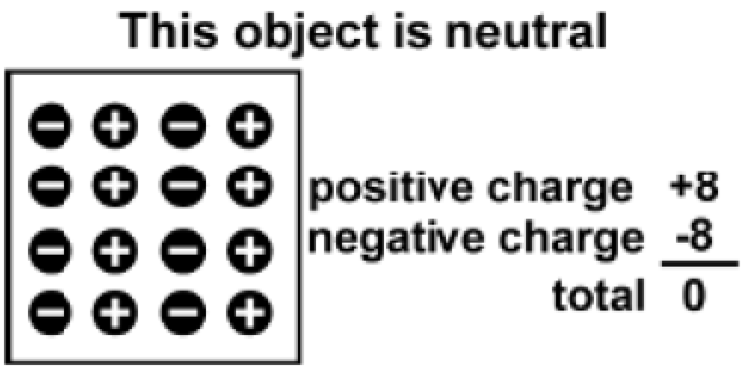
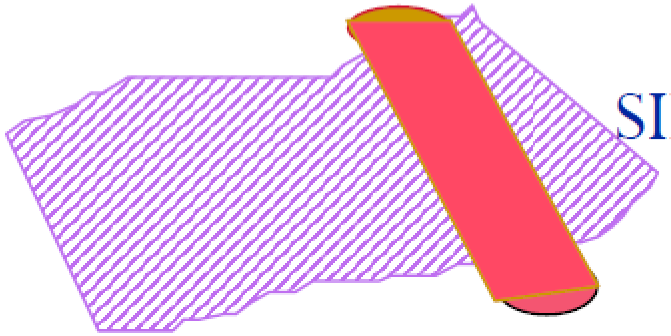
Electric Charges

- All ordinary matter contains both positive and negative charge.
- You do not usually notice the charge because most matter contains the exact same number of positive and negative charges.
- An object is electrically neutral when it has equal amounts of both types of charge.

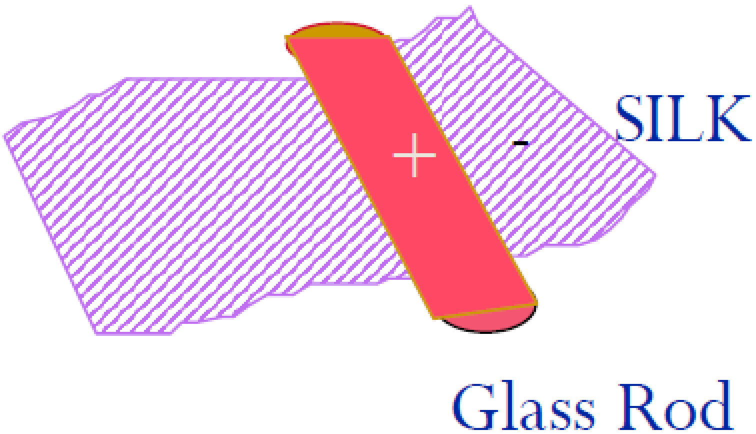


The Transfer of Charge



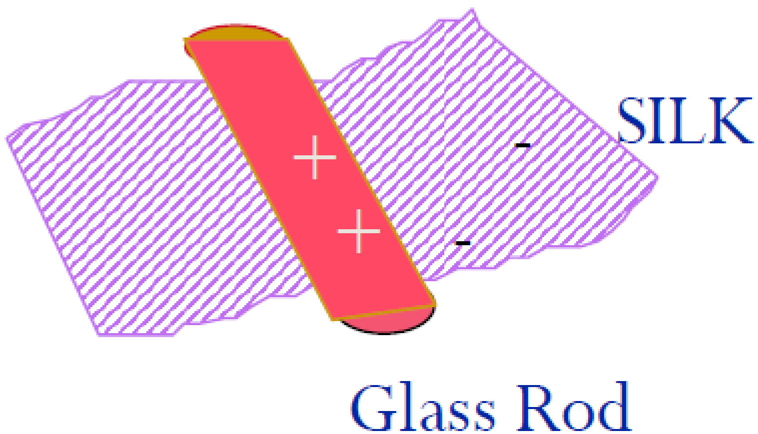
Some materials attract electrons more than others.

The Transfer of Charge



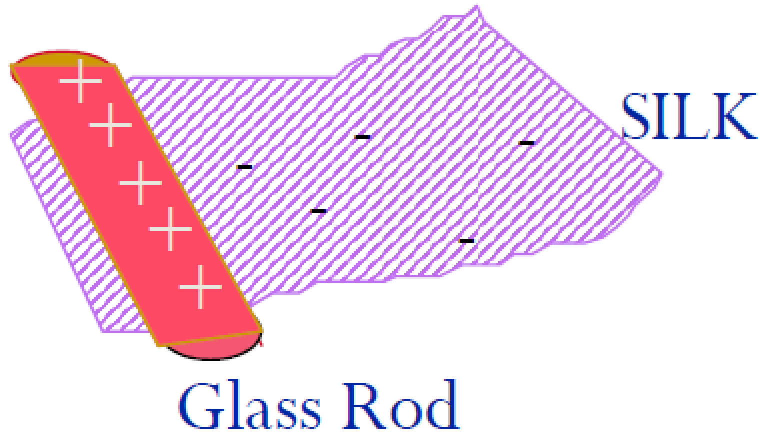
- As the glass rod is rubbed against silk, electrons are pulled off the glass onto the silk.
- Glass and silk are insulators: charges stuck on them stay put.

The Transfer of Charge



•Usually matter is charge neutral, because the number of electrons and protons are equal. But here the silk has an excess of electrons and the rod a deficit.

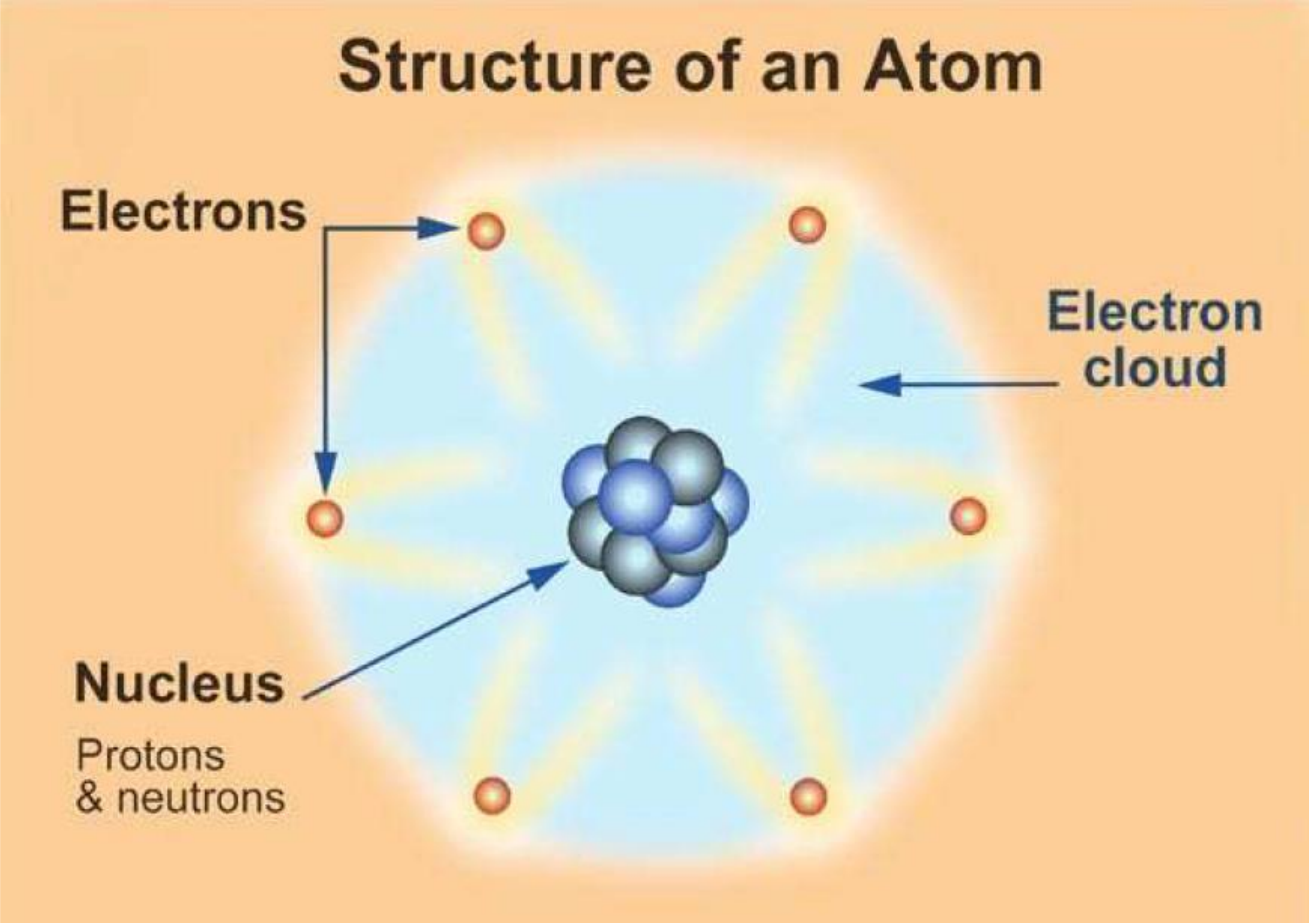
The Transfer of Charge

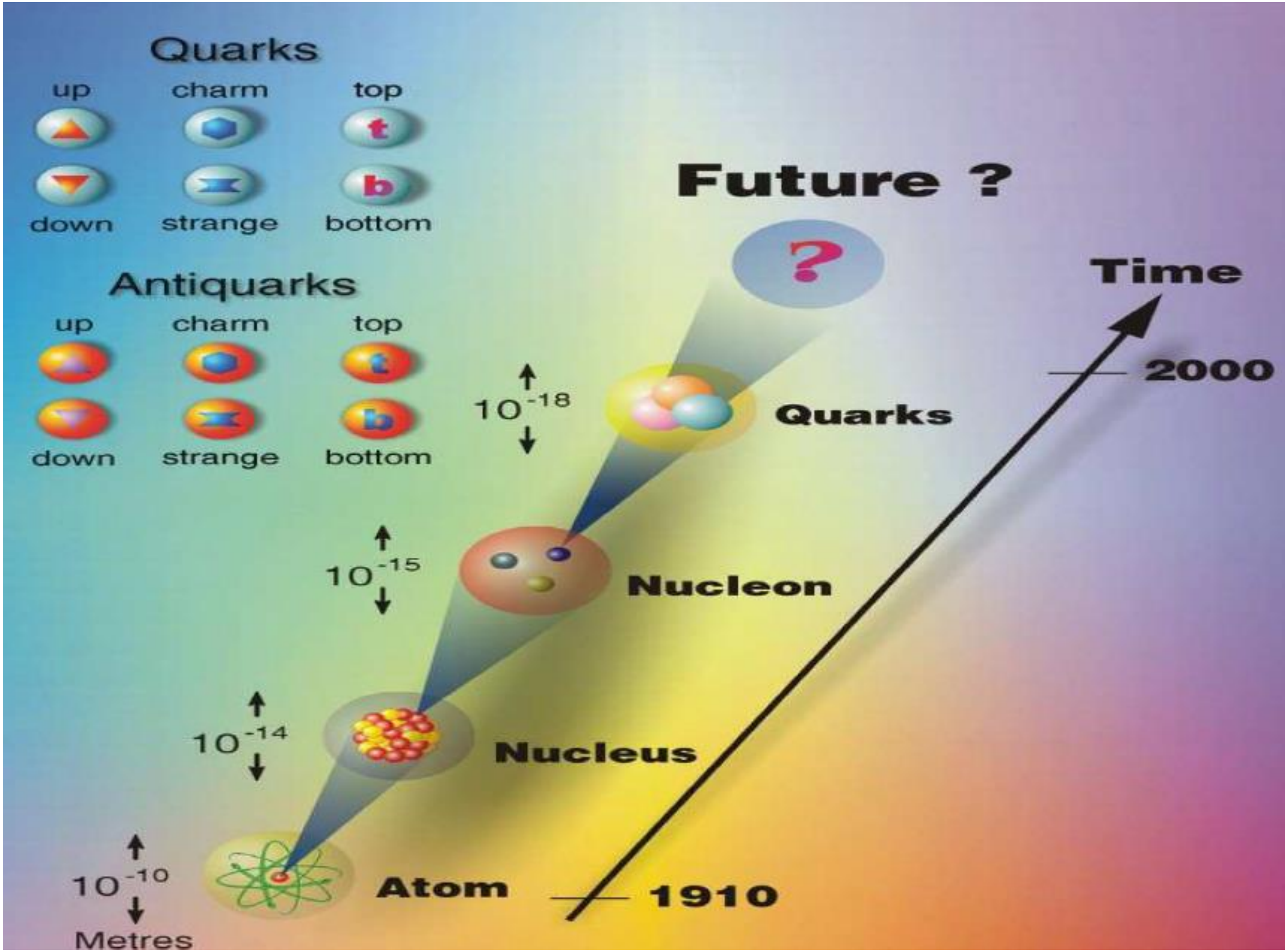


Electric charge is always conserved. That is, when one object is rubbed against another, charge is not created in the process. The electrified state is due to a *transfer of charge from one object to the other. One object gains some amount of* negative charge while the other gains an equal amount of positive charge.

Like charges repel, unlike charges attract.

- Attractive electric forces are responsible for the behavior of a wide variety of commercial products.
- For example, the plastic in many contact lenses, *etafilcon, is* made up of molecules that electrically attract the protein molecules in human tears. These protein molecules are absorbed and held by the plastic so that the lens ends up being primarily composed of the wearer's tears. Because of this, the wearer's eye does not treat the lens as a foreign object, and it can be worn comfortably.


Structure of an Atom
Electrons
Electron cloud
Nucleus
Protons & neutrons



Quarks
up
charm
top
down
strange
bottom
Antiquarks
up
charm
top
down
strange
bottom
Future ?
Time
2000
10^{-18}
Quarks
10^{-15}
Nucleon
10^{-14}
Nucleus
10^{-10}
Metres
Atom
1910

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History

600 BC	Greeks first discover attractive properties of amber when rubbed.
1600 AD	Electric bodies repel as well as attract
1735 AD	du Fay: Two distinct types of electricity
1750 AD	Franklin: Positive and Negative Charge
1770 AD	Coulomb: "Inverse Square Law"
1890 AD	J.J. Thompson: Quantization of electric charge - "Electron"

Summary of things we know:

- There is a property of matter called electric charge. (In the SI system its units are Coulombs.)
- Charges can be negative (like electrons) or positive (like protons).
- In matter, the positive charges are stuck in place in the nuclei and electrons revolve around it. Matter is negatively charged when extra electrons are added, and positively charged when electrons are removed.
- Like charges repel, unlike charges attract.
- Charges travel in conductors, not in insulators
- Force of attraction or repulsion $\sim 1 / r^2$

Charge is Quantized

- q = multiple of an elementary charge e: $\boxed{q = Ne}$
- e = 1.6 x 10^{-19} Coulombs

	Charge	Mass	Diameter
electron	- e	1	0
proton	+e	1836	$\sim 10^{-15}$m
neutron	0	1839	$\sim 10^{-15}$m

Protons and neutrons are made up of quarks, whose charge is quantized in multiples of e/3. Quarks can't be isolated.

Electromagnetism

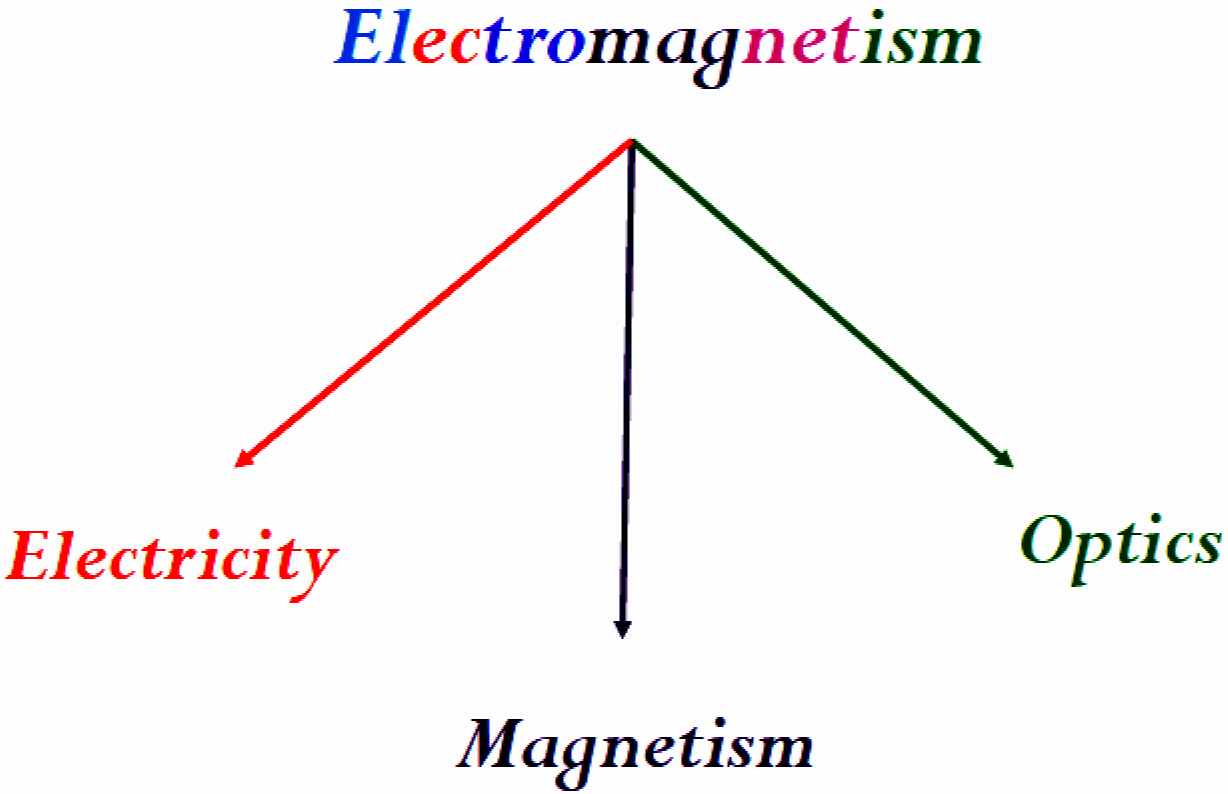
- Electromagnetism is one of the fundamental forces in nature, and the dominant force in a vast range of natural and technological phenomena

- The electromagnetic force is solely responsible for the structure of matter, organic, or inorganic

→ Physics, chemistry, biology, materials science

- The operation of most technological devices is based on electromagnetic forces. From lights, motors, and batteries, to communication and broadcasting systems, as well as microelectronic devices.

→ Engineering


Electromagnetism
Electricity
Magnetism
Optics

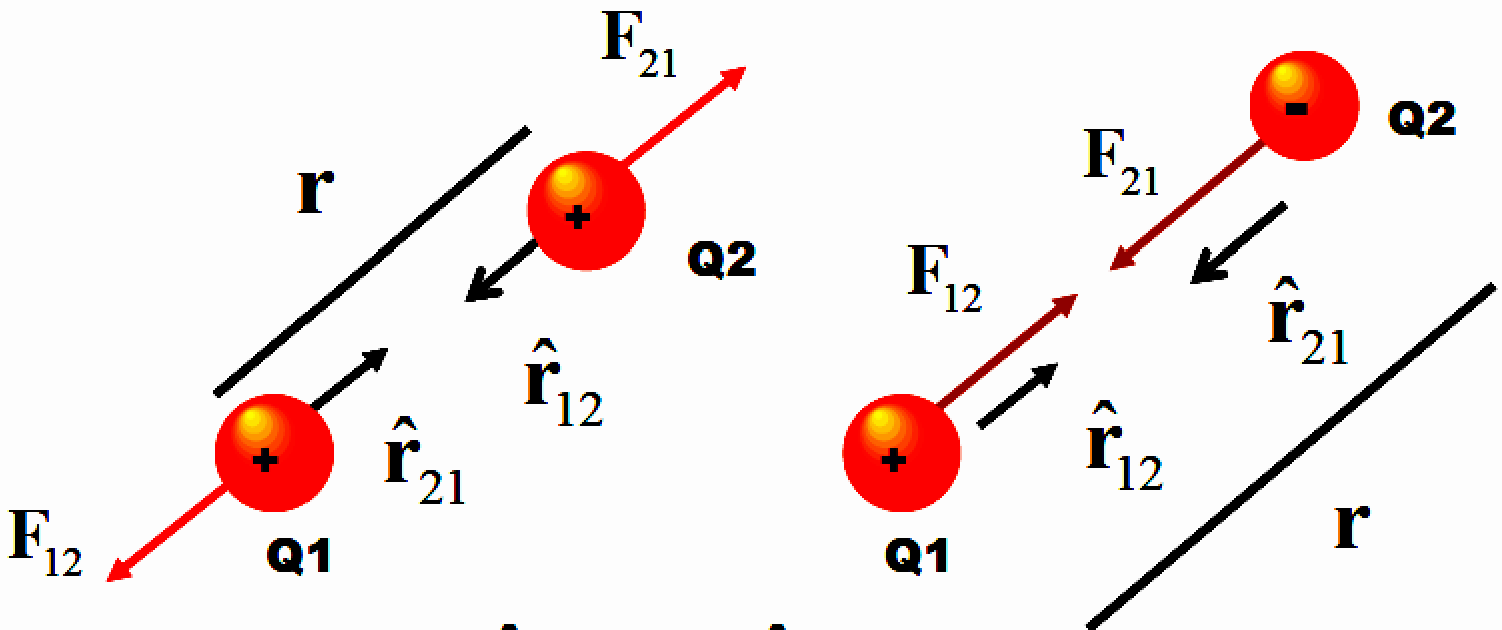
Coulomb's Law

- Coulomb's showed that the electric force between two stationary charged particles is:
  - inversely proportional to the square of the separation r *between the particles* and directed along the line joining them;
  - proportional to the product of the charges *q1 and q2 on the two particles;*
  - attractive if the charges are of opposite sign and repulsive if the charges have the same sign.

$$F = \frac{k|q_1||q_2|}{r^2} \qquad K = 1/4\pi\varepsilon_\circ$$

$$K = 9 \times 10^9 \frac{Nm^2}{C^2} \qquad \varepsilon_\circ = 8.85 \times 10^{-12} \frac{C^2}{Nm^2}$$

Vector form of Coulomb's Law



$$\hat{r}_{12} = -\hat{r}_{21}$$

Force on 1 due to 2

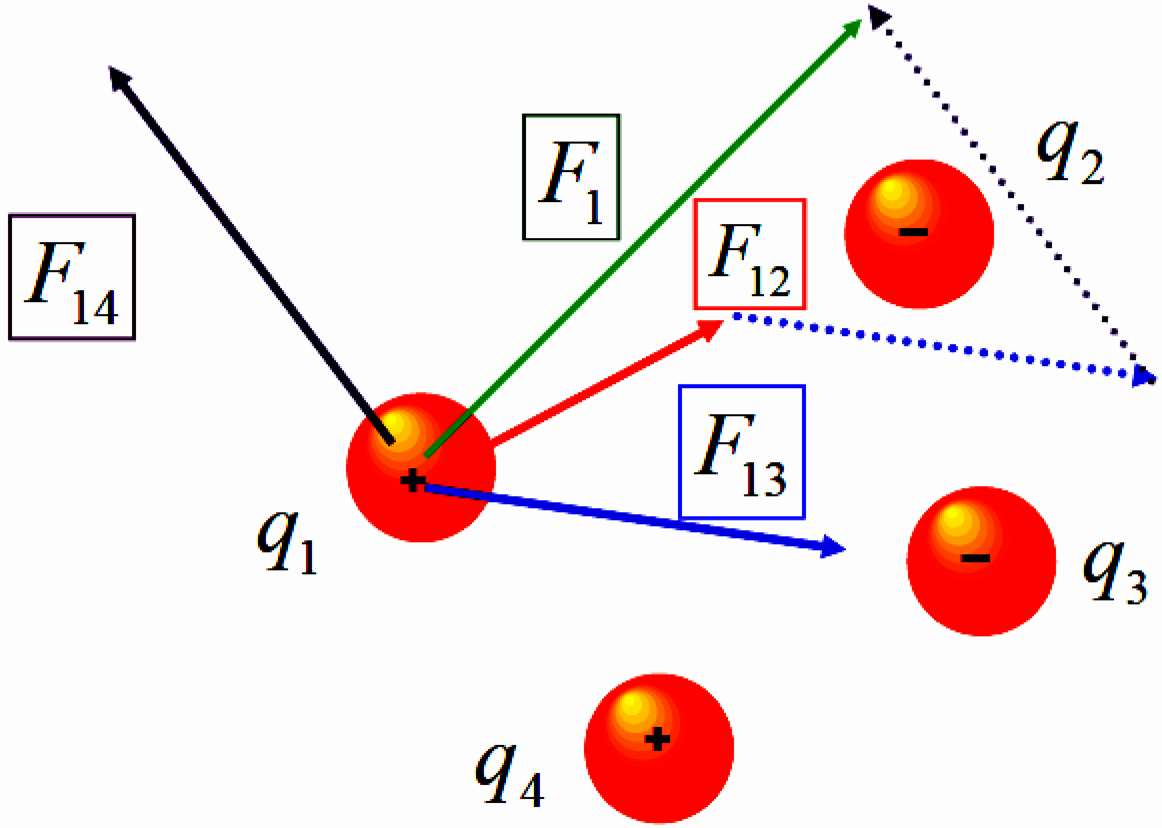
$$F_{12} = \frac{kq_1q_2}{r^2}\hat{r}_{12}$$

Force on 2 due to 1

$$F_{21} = \frac{kq_1q_2}{r^2}\hat{r}_{21}$$

Superposition

Force on charge is vector sum of forces from all charges.



$$\vec{F}_1 = \vec{F}_{12} + \vec{F}_{13} + \vec{F}_{14}$$

Coulomb vs. Newton

$$\boxed{\mathbf{F}_{12} = \frac{1}{4\pi\varepsilon_0}\frac{Q_1 Q_2}{\mathbf{r}_{12}^2}\hat{\mathbf{r}}_{12}}$$

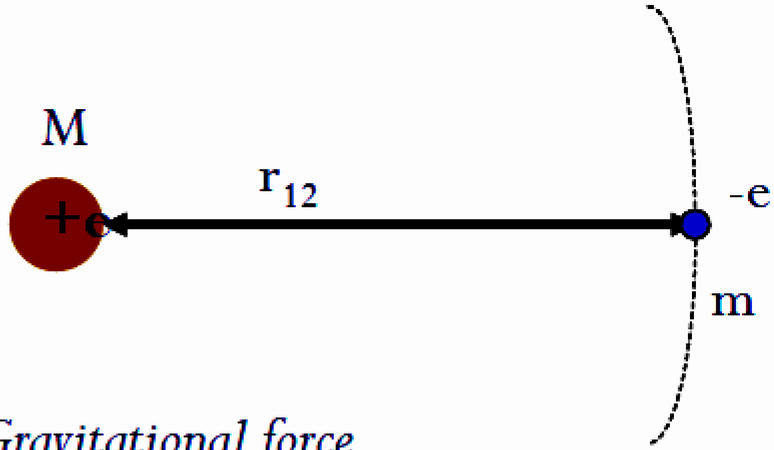
- Attractive or repulsive
- $1/r^2$
- very strong
- only relevant relatively local scales

$$\boxed{\mathbf{F}_{12} = -G\frac{m_1 m_2}{\mathbf{r}_{12}^2}\hat{\mathbf{r}}_{12}}$$

- Always attractive
- $1/r^2$
- very weak
- important on very large scales, planets, the Universe

$$\boxed{\frac{e^2}{4\pi\varepsilon_0} >> -Gm^2}$$

Gravitational and Electric Forces in the Hydrogen Atom



$m = 9.1\ 10^{-31}$ kg
$M = 1.7\ 10^{-27}$ kg
$r_{12} = 5.3\ 10^{-11}$ m

Gravitational force

$$\vec{F}_g = G\frac{Mm}{r_{12}^2}\hat{r}$$

$$\boxed{F_g = 3.6\ 10^{-47}\ \mathrm{N}}$$

Electric Force

$$\vec{F}_e = \left(\frac{1}{4\pi\varepsilon_0}\right)\frac{Qq}{r_{12}^2}\hat{r}$$

$$\boxed{F_e = 3.6\ 10^{-8}\ \mathrm{N}}$$

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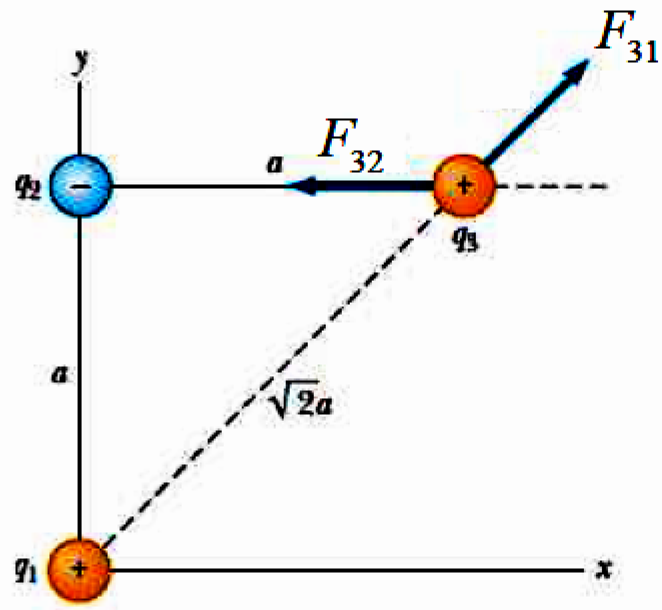
Consider three point charges located at the corners of a right triangle as shown in Figure, where $q_1=q_3=5\mu C$, $q_2=-2\mu C$ and $a=0.1m$. Find the resultant force exerted on q_3 .



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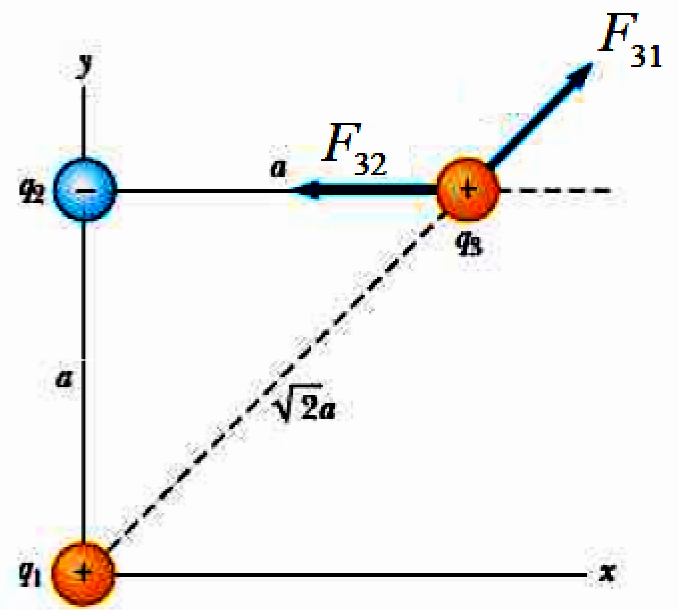


$$F_{31} = \frac{k|q_1||q_3|}{(\sqrt{2}a)^2} = 11\,N$$

The force F_{31} is repulsive and makes an angle of $45°$ with the *x axis. Therefore,*

$$F_{31x} = F_{31}\cos 45 = 7.9$$

$$F_{31y} = F_{31}\sin 45 = 7.9$$

$$\vec{F}_{31} = 7.9\hat{i} + 7.9\hat{j}$$

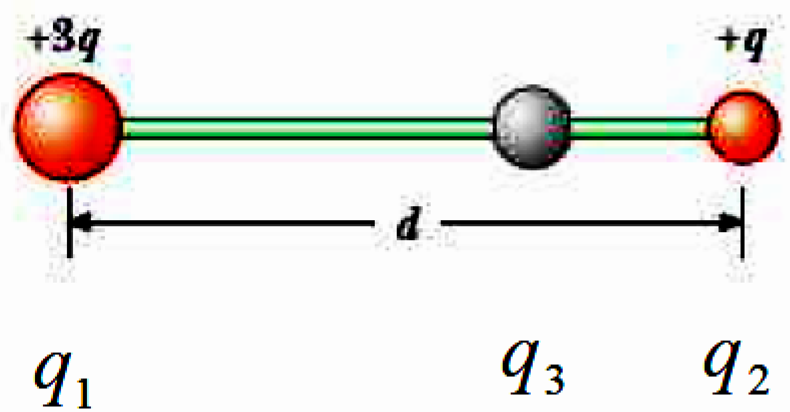
$$F_{32} = \frac{k|q_2||q_3|}{(a)^2} = 9\,N$$

$$\vec{F}_{32} = -9\hat{i}$$

Net force will be

$$\begin{aligned}\vec{F}_3 &= \vec{F}_{32} + \vec{F}_{31} \\ &= (7.9 - 9)\hat{i} + 7.9\,\hat{j} \\ &= -1.1\hat{i} + 7.9\,\hat{j}\end{aligned}$$

Two small beads having positive charges 3q and q are fixed at the opposite ends of a horizontal, insulating rod, extending from the origin to the point x =d. As shown in Figure, a third small charged bead is free to slide on the rod. At what position is the third bead in equilibrium? Can it be in stable equilibrium?



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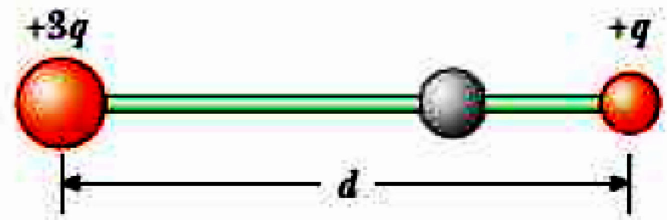


$$q_1 \qquad q_3 \qquad q_2$$

$$x = 0$$

$$x$$

At equilibrium, net force on q_3 is zero,

$$F_{31} = F_{32}$$

$$\frac{k|3q||q_3|}{(x)^2} = \frac{k|q||q_3|}{(d-x)^2}$$

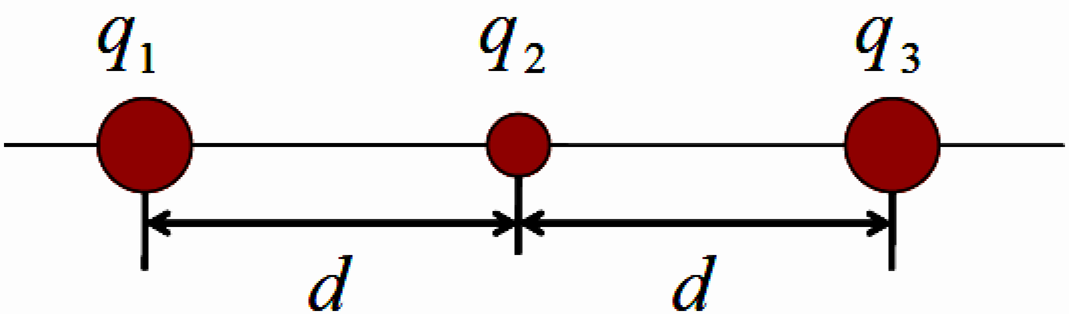
$$3(d-x)^2 = x^2$$

$$2x^2 - 6dx + 3d^2 = 0$$

$$\boxed{x = 0.7d} \qquad \cancel{x = 2.28d}$$

System can be in stable equilibrium ifq3 is +ve

Three charged particles lie on a straight line and are separated by a distance d as shown. Charges q1 and q2 are held fixed while charge q3 is free to move. If the charge q3 is found to be in equilibrium under the action of electric forces, find q1 in terms of q2.



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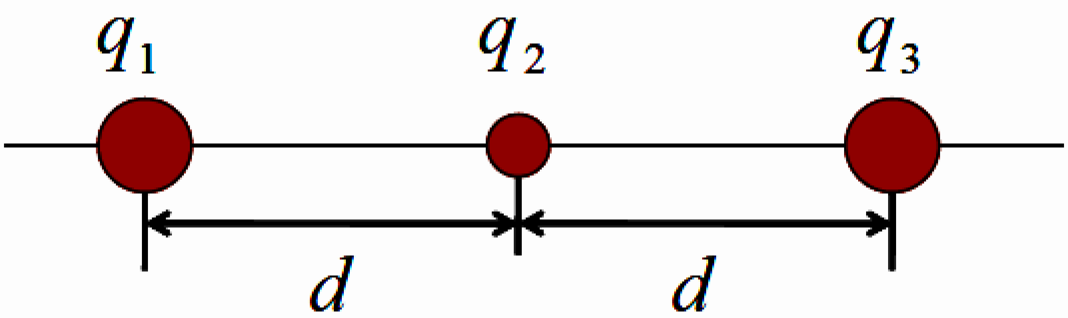


$$F_{31} = F_{32}$$

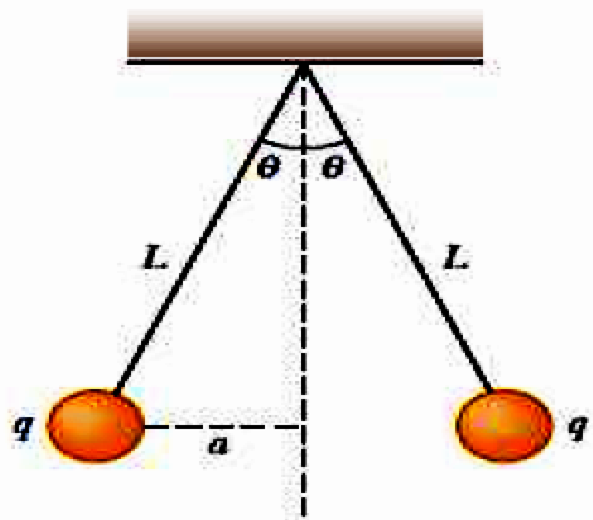
$$\frac{kq_1q_3}{(2d)^2} = \frac{kq_2q_3}{(d)^2}$$

$$\frac{q_1}{4} = \frac{q_2}{1}$$

$$q_1 = 4q_2$$

q_1 and q_2 must be of opposite signs, that is, $q_1 = -4q_2$

Two identical small charged spheres, each having a charge q and mass of 0.03 kg, hang in equilibrium as shown in Figure. The length of each string is 0.15 m, and the angle θ is 5°. Find the magnitude of the charge on each sphere.



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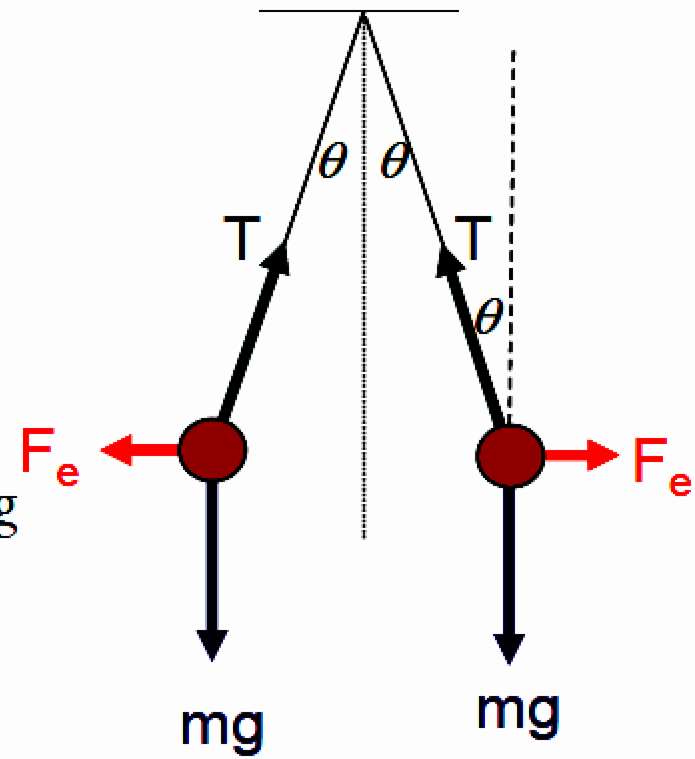


Repulsive electric force between charges is

$$F_e = kq^2/(2a)^2 \qquad \text{(I)}$$

$$a = L\sin\theta$$

$$= (0.15)\sin(5) = 0.013$$

System is in equilibrium, equating forces acting on one charge

$$T\cos\theta = mg \qquad \text{(II)}$$

$$T\sin\theta = F_e \qquad \text{(III)}$$

$$F_e = mg\tan\theta = (0.03)(9.8)\tan(5) = 0.025$$

From (I)

$$q = \sqrt{\frac{4a^2 F_e}{K}}$$

$$q = \sqrt{\frac{4(0.013)^2(0.026)}{9\times10^9}}$$

$$= 4.4\times10^{-8}\,C$$

A certain charge Q is to be divided into two parts, Q-q and q. What is the relation of Q to q if the two parts, placed a given distance apart, are to have a maximum coulomb's repulsion?

A certain charge Q is to be divided into two parts, Q-q and q. What is the relation of Q to q if the two parts, placed a given distance apart, are to have a maximum coulomb's repulsion?

The repulsive force between two charges Q-q and q , if they are distance r apart, will be

$$F = \frac{k(Q-q)(q)}{r^2}$$

We want to maximize this force with respect to variation in q. it can be done by setting

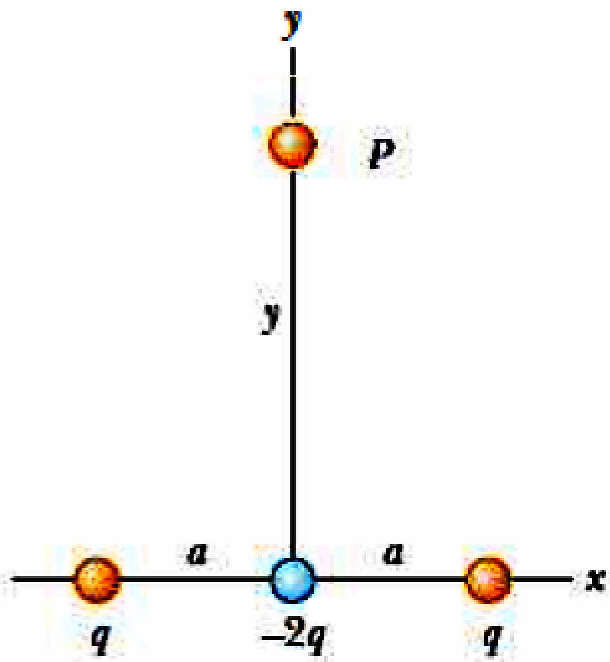
$$\frac{dF}{dq} = 0$$

$$\frac{k(Q-2q)}{r^2} = 0$$

$$q = Q/2$$

Three point charges q, -2q, and q are located along the x axis and a fourth charge Q is placed on y-axis, as shown in Figure. If y>>a, show that the net electric force experience by Q is

$$\boxed{F = -\frac{3kQqa^2}{y^4}\hat{j}}$$



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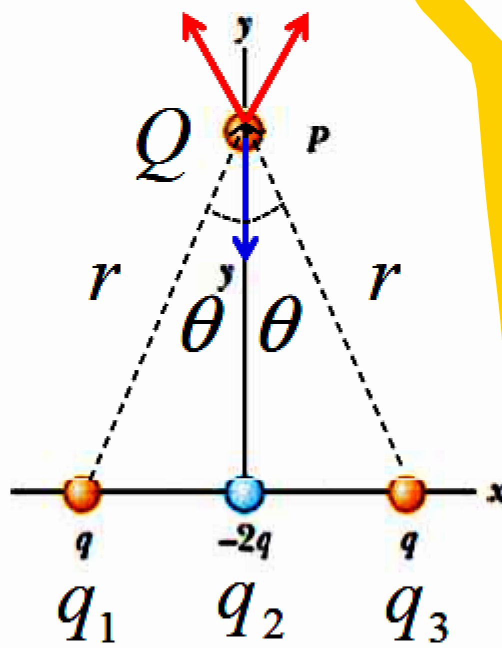

Force due to q1

$$F_1 = \frac{kQq}{r^2}$$

Force due to q2

$$F_2 = \frac{2kQq}{y^2}$$

Force due to q3

$$F_3 = \frac{kQq}{r^2}$$

X-component of F1 and F3 will be cancelled out; equal and opposite. Hence net force on Q will be directed along y-axis

$$\vec{F} = \vec{F}_{1y} + \vec{F}_{2y} + \vec{F}_{3y}$$

$$= \frac{kQq}{r^2} \cos\theta \ \hat{j} - \frac{2kQq}{y^2} \hat{j} + \frac{kQq}{r^2} \cos\theta \ \hat{j}$$

$$= \frac{2kQq}{r^2} \cos\theta \ \hat{j} - \frac{2kQq}{y^2} \hat{j}$$

From fig.

$$r = \sqrt{a^2 + y^2} \qquad \cos\theta = \frac{y}{r} = \frac{y}{\sqrt{a^2 + y^2}}$$

$$\vec{F} = \frac{2kQq}{a^2 + y^2} \frac{y}{\sqrt{a^2 + y^2}} \hat{j} - \frac{2kQq}{y^2} \hat{j}$$

$$= \frac{2kQqy}{(a^2 + y^2)^{3/2}} \hat{j} - \frac{2kQq}{y^2} \hat{j}$$

$$\vec{F} = 2kQq \left[yy^{-3}(1 + a^2 / y^2)^{-3/2} - y^{-2} \right] \hat{j}$$

$if \quad y >> a$

$$\vec{F} = 2kQq \left[y^{-2}(1 - 3a^2 / 2y^2) - y^{-2} \right] \hat{j}$$

$$\vec{F} = -\frac{3kQqa^2}{y^4} \hat{j}$$