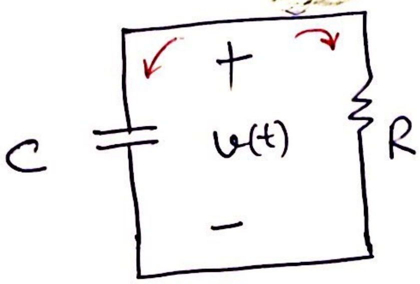
8.3 The Source-Free RC Circuit
(PP 272 8th Ed HKD)

Consider the following circuit:-



Given $v(0) = V_0$

(Fig 8.15 A parallel RC circuit)

— Applying KCL (Sum of all currents leaving upper node is zero)

So $$C\frac{dv}{dt} + \frac{v}{R} = 0$$

or $$\frac{dv}{dt} + \frac{1}{C}\frac{v}{R} = 0 \qquad \text{—— (1)}$$

— Assume, the solution is :-

$$v(t) = Ae^{s_1 t}$$

Taking derivative $$\frac{dv(t)}{dt} = As_1 e^{s_1 t}$$

— Putting in equation (1), it becomes

$$As_1 e^{s_1 t} + \frac{1}{RC}Ae^{s_1 t} = 0$$

—— contd

— Contd (273)

or $s_1 + \frac{1}{RC} = 0$

and $s_1 = -\frac{1}{RC}$

— Now $v(t) = Ae^{s_1 t}$

so $v(0) = A = V_0$

Hence the solution is :-

$v(t) = V_0 e^{-\frac{1}{RC}t}$ Volts.

Notice at $t=0$

$v(0) = V_0$

at $t = \infty$

$v(t) \approx 0$

— The time constant is found by noting the time at which the response has dropped to 37% of its initial value.

So $\frac{\tau}{RC} = 1$ because $e^{-1} = 0.36$

or $\tau = RC$

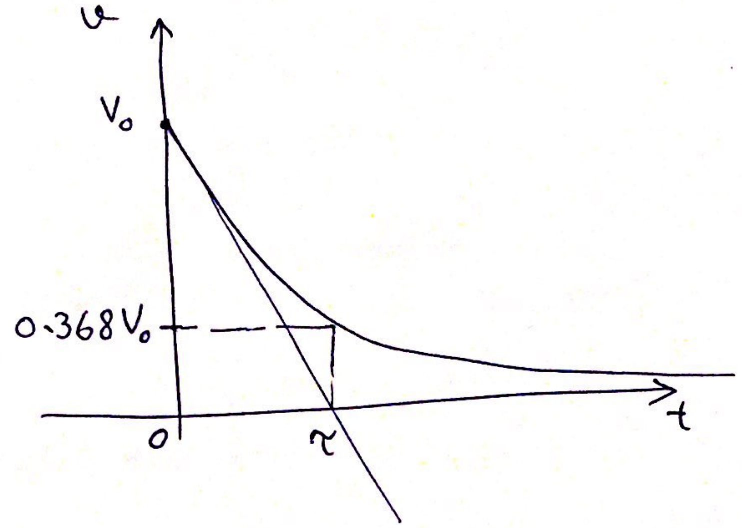
— Recall that for an RL circuit,

$\tau = \frac{L}{R}$

———— contd

— Contd (274)

The response curve can be sketched as:-



Slope:- M_2

$$v(t) = V_0 e^{-\frac{t}{RC}} \text{, volts}$$

$$\left.\frac{d\,\frac{v(t)}{V_0}}{dt}\right|_{t=0} = -\frac{1}{RC}$$

Now slope is equal to

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \frac{0 - 1}{\tau - 0} = -\frac{1}{RC}$$

$$\tau = RC$$

M_2

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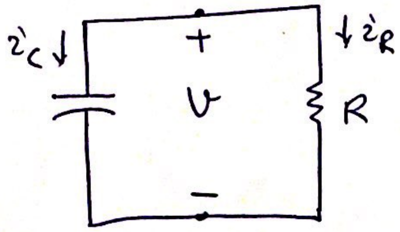
(Optional)
(Academic)

The Source-Free RC circuit: Another approach

(PP 220 6th Ed)

Consider the circuit:-



Assume at $t=0$, the initial voltage is:-

$$v(0) = V_0 \text{ volts}$$

— Applying KCL at top node:-

$$i_R + i_C = 0$$

or $$\frac{v}{R} + C\frac{dv}{dt} = 0$$ — (currents out of node)

So $$\frac{dv}{v} = -\frac{1}{RC}\,dt$$

Integrating; $$\ln v = -\frac{t}{RC} + \ln A$$ ($\ln A$ is constant of integration)

or $$\ln v - \ln A = -\frac{t}{RC}$$

$$\ln\frac{v}{A} = -\frac{t}{RC}$$

Anti-log $$\frac{v(t)}{A} = e^{-t/RC}$$

or $$v(t) = A e^{-\frac{1}{RC}t}$$

As $$v(0) = A = V_0$$

therefore $$v(t) = V_0 e^{-\frac{1}{RC}t}$$

— Exponentially decaying value of initial voltage.

Note: If we choose i as the variable then

$$\frac{1}{C}\int_0^t i\,dt - v_0(t_0) + Ri = 0$$

or $$\frac{1}{C}\times i + R\frac{di}{dt} = 0$$

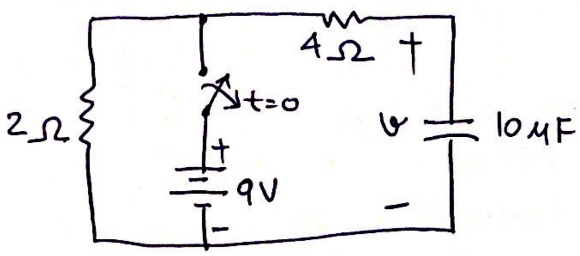
$$\frac{v}{R} = i$$

So $$\frac{v}{RC} + \frac{dv}{dt} = 0$$ as before.

— [signature]

Example 8.3 The Source-Free RC Circuit
(pp 274 8th Ed HKD)

Find the voltage labeled 'v' at $t = 200\ \mu s$.

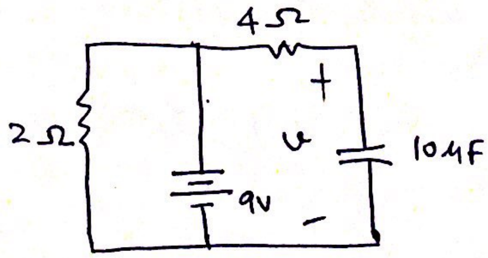


Solution: To determine 'v' we need to draw and analyze:

- one circuit corresponding to before the switch is thrown and
- another circuit corresponding to after the switch is thrown.

— The purpose of analyzing the circuit for $t \le 0$ is to obtain initial capacitor voltage.

— The circuit for $t \le 0$ is: {Note $t \le 0$}



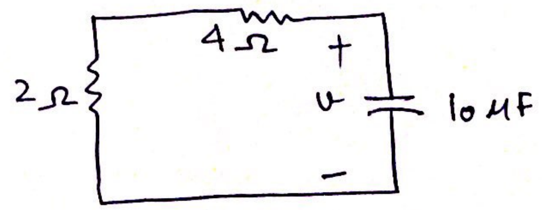
— we assume the transients died long time ago, so we have a purely dc circuit.

— By inspection $v(0) = 9\ V$

contd

— contd (274)

— Now the circuit for $t \geq 0$ is:



and $v(0) = 9\,V$

— We recognize that the time constant:

$$\tau = R_{eq}C = (2+4)(10\times10^{-6})$$

$$\tau = 60\times10^{-6}\ s$$

— We also know that for the source-free circuit, the response is of the form

$$v(t) = V_0\, e^{-t/RC}$$

where $V_0 = v(0) = 9\ V$

So $$v(t) = 9\, e^{-\frac{t}{60\times10^{-6}}}\ V$$

— At $t = 200\ \mu s$

$$v(200\mu s) = 9\, e^{-\frac{20\cancel{0}\times10^{-6}}{6\cancel{0}\times10^{-6}}}\ V$$

or $$v(200\mu s) = 9\, e^{-\frac{10}{3}}\ V$$

$$= 9\times0.03567$$

$$= 321.1\ mV$$