NUST School of Electrical Engineering & computer Science

MATH 232 - Complex Variables & transforms - BEE 8AB - Problem Sheet No. 2

Q-1 For each of the following functions, describe the domain of definition:

i) $f(z) = \text{Arg}(z^{-1})$ ii) $f(z) = \dfrac{1}{z^2+1}$ iii) $f(z) = \dfrac{1}{|z|^2+1}$ (iv) $f(z) = \dfrac{z}{z+\bar{z}}$.

Q-2 Let $w = e^{i2\pi/3}$ and define $f(z) = wz$. what type of geometric transformation is f?

Q-3 Let $f(z) = \begin{cases} \bar{z}^3/z^2 & \text{if } z \neq 0 \\ 0 & \text{if } z = 0 \end{cases}$

Show that $f(z)$ is continuous everywhere in $\mathbb{C}$ but the complex derivative $f'(0)$ does not exist.

Q-4 In each case below evaluate the limit or show that lt DNE:

(a) $\lim\limits_{z \to -i} \dfrac{z^2 - (\bar{z})^2}{z^2+1}$ (b) $\lim\limits_{z \to 0} \left(\dfrac{\bar{z}}{z}\right)^2$ (c) $\lim\limits_{z \to 0} z^2/|z|^2$.

Q-5 Let $f(z) = \bar{z}\, e^{-|z|^2}$, determine all points where CREs are satisfied, points where f is differentiable, find a formula for $f'(z)$ at these points.

Q-6. Discuss differentiability of $f(z) = 1/\bar{z}$.

Q-7. Let $f(z) = e^{\bar{z}}$, determine the set on which the Cauchy-Riemann equations are satisfied, and explain what you conclude about the differentiability of $f(z)$.

Q-8. Verify whether the following equations hold:

(a) $\text{Log}[(1+i)^3] = 3\,\text{Log}(1+i)$ (b) $\text{Log}[(-1+i)^2] = 2\,\text{Log}(-1+i)$.

Q-9 Consider the function $f(z) = z^{1/3} = (re^{i\theta})^{1/3}$, $(r > 0, \alpha \leq \theta < \alpha + 2\pi)$ where α is a fixed real number. Prove that f is differentiable everywhere

Q-10 Show that the function $f(z) = z\,\text{Re}(z)$ is nowhere differentiable except at the origin; hence calculate $f'(0)$.

Q-11. Find an sketch the image of the semi-infinite strip $S = \{ z = x+iy,\ 0 \leq x,\ 0 \leq y \leq \pi \}$ under the transformation $w = e^z$.

Q-12 Find and sketch the image of lines of constant x $(x = a)$ and lines of constant y $(y = b)$ under the transformation $w = f(z) = \sin z$.