**NUST School of Electrical Engineering and Computer Science**

Faculty Member:_______________ Date: _______________

Semester:______________ Section: _______________

Department of Electrical Engineering

EE-379: Control Systems

# LAB 7: Performance of systems

| Student name | Reg. No. | Log book Marks / 5 | Lab completion Marks / 5 | Lab report Marks / 5 | Total/15 |
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# LAB 7: Performance criterion

## 1. Objectives

- Learn how to compute the transient and steady state characteristics of a system in MATLAB

## 2. Performance

As you should have studied in the lectures and we also mentioned in earlier lab handouts, the step input is very common in control systems, e.g., if you want the elevator to go from the ground floor to the fourth floor, then the desired behavior is a step of magnitude 4. Therefore, the performance of a control system is often based on the response of the system to a step input.

The step response has two categories of performance measures: the performance of transient response and the performance of steady state response. The characteristics of the transient response include the rise time, settling time, peak time and the maximum overshoot. For the steady state we are interested in the steady state error. In this handout we will learn how to calculate these performance measures in MATLAB.

## 3. Step response characteristics in MATLAB

You already know that the step response of a system can be found in MATLAB using the function step(). The information about the transient response of a system when excited by a step input, can be found by the function stepinfo(). There is no function to find the steady state error in response to a step input. However, you can use the code

```
abs(1-dcgain(sys))
```

to find the steady state error for a step input. An example code is given below

```
my_tf = tf(1,[1 2]);
stepinfo(my_tf)
steady_state_error = abs(1-dcgain(my_tf))
```

**Exercise 1**

Find the rise time, peak time, peak value, overshoot, settling time and the steady state error for step input of the following systems

$$\frac{2s+2}{s^2+9s+20}, \qquad \frac{s+1}{s^3+12s^2+47s+60}, \qquad \frac{1}{s+10}$$

## 4. First order systems

By definition first order systems have a single pole. Moreover, proper single order systems have no/one zero. In this section we will try to find out the effect of the location of the poles, the location of the zeros and the gain of the transfer function on the step response. Note that the gain we are mentioning here is not the feedback gain. It is the gain when the system is written in zero/pole/gain form.

**Exercise 2**

Consider the systems of the following form

$$\frac{p}{s-p}$$

This system has a pole at $p$, it has no zeros and the gain is equal to the negative of the pole i.e. $-p$. Using Matlab, plot the step response of systems of this form for p= $-1, -2, -5, -10$. Plot all the step responses on a single figure. A sample code is given below.

```
clc
clear
close

sys = zpk([],-1,1);
step(ss(sys));
sys = zpk([],-2,2);
hold on
step(ss(sys));
sys = zpk([],-5,5);
step(ss(sys));
sys = zpk([],-10,10);
step(ss(sys));
legend('p=-1','p=-2','p=-5', ‘p=-10’);
```

For each system also find the values of the various performance characteristics (rise time, overshoot, steady state error, etc.).

Comment on how does the pole of a first order system affects the step response of the system. In particular, change in which parameters of transient response is the most prominent.

Note: In the sample code we have used the legend() function. Use this function to properly label your signal in all the exercises in this handout.

**Exercise 3**

Now fix the pole to a constant value and let's see the effect of changing the gain 'k'

$$\frac{k}{s-p}$$

Using Matlab, plot the step response of systems of this form for $p = -5$ and k= 1,2,5,10. Plot all the step responses on a single figure. For each system also find the values of the various performance characteristics. Comment on the effects of changing the gain.

**Exercise 4**

Now we will introduce a zero and see the effect of changing it. Consider the system

$$\frac{k(s-z)}{s-p}$$

Using Matlab, plot the step response of systems of this form for $p = -5$, k= $1$ and $z = -1, -2, -5, -10$. Also have a plot for no zero. Plot all the step responses on a single figure. For each system also find the values of the various performance characteristic. Comment on the effects of changing the zeros.

## 4.1 Formulas for transient response characteristics

As you may have noticed, if the first ordered system is strictly proper, then it does not have any overshoot. The rise time $T_r$ and the settling time $T_s$ of first order systems are related to the value of pole by the following relations

$$T_r = \frac{2.2}{-p}$$

$$T_s = \frac{4}{-p}$$

**Exercise 5**

Use the formulas given above to find the values of the pole of a first order system that would give

- rise times of 0.1, 0.5 and 1
- settling times of 1, 1.5 and 2

## 5. Second order systems with complex conjugate poles and no zeros

In this section we will consider systems of the following type

$$\frac{b}{s^2 + as + b} \tag{1}$$

Where a and b are real constants. For a second order system with complex conjugate poles we can find the damping ratio and the natural frequency. In MATLAB, these values can be found by using the function damp().

```
my_tf = tf(1,[1 -4 5]);
damp(my_tf)
```

**Exercise 6**
Find the damping ratio and the natural frequency of the following systems

$$\frac{5}{\mathrm{s}^2 - 4\mathrm{s} + 5}, \quad \frac{2}{\mathrm{s}^2 - 2\mathrm{s} + 2}, \quad \frac{5}{\mathrm{s}^2 - 2\mathrm{s} + 5}$$

Using the formulas for the damping ratio and the natural frequency, it can be shown that we can write system (1) as follows

$$\frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2} \tag{2}$$

**Exercise 7**
Write a MATLAB function that takes the damping ratio and natural frequency as arguments and returns a transfer function of the form given in equation (2).

**Exercise 8**
Write a MATLAB function that takes the damping ratio and natural frequency as arguments and returns a transfer function of the form given in equation (2). Call this function my_second_order_tf.

**Exercise 9**

Using the function that you have just created, my_second_order_tf, make transfer functions for the following sets of damping ratios and natural frequencies

Set 1: $\zeta = 0,\ \omega_n = 1,2,5$ (See the note given below)
Set 2: $\zeta = 1,\ \omega_n = 1,2,5$
Set 3: $\zeta = 0,0.5, 1,2,\ \omega_n = 1$

For each set of values plot the step responses on a single figure. For each system also find the values of the various performance characteristic. Comment on the effects of changing the natural frequency and the damping ratio.

Classify each of the above systems as undamped, underdamped, critically damped or overdamped.

Note: when plotting the step response for a system with $\zeta = 0$, limit the time of the step response output to 10 seconds. This can be done easily by giving a second argument to the step function e.g. step(sys,10).

## 5.1 Formulas for transient response characteristics

The settling time $T_s$ , peak time $T_p$ , percent overshoot %OS, damping ratio and the natural frequency are related by the following expressions:

$$T_p = \frac{\pi}{\omega_n\sqrt{1-\zeta^2}}, \qquad T_s = \frac{4}{\zeta\omega_n}, \qquad T_p = \frac{4}{\zeta\omega_n}$$

$$\%OS = 100e^{-(\zeta\pi/\sqrt{1-\zeta^2})}, \qquad \zeta = \frac{-\ln{(\%OS/100)}}{\sqrt{\pi^2 + \ln^2(\%OS/100)}}$$

A precise expression for the rise time cannot be found in terms of the damping ratio and the natural frequency.

**Exercise 10**

Using the formulas given above, find the values of damping ratio and natural frequency that result in %OS=10 and $T_s = 1$.

## 6. Second order systems with zeros

For second order systems that have zeros, the transient response characteristics are not completely defined by the damping ratio and the natural frequency.

**Exercise 11**
For the systems given below, find the natural frequency, the damping ratio and transient characteristics. Also plot their step responses on a single graph

$$\frac{1}{s^2+s+1}, \qquad \frac{s+1}{s^2+s+1}$$

Comment on your observations.

## 7. Higher order systems

For higher order systems you can also use the MATLAB functions dcgain(), stepinfo() and damp().

If in a stable higher order system, only a single real pole is dominant i.e. all other poles are much more negative, then the response is similar to first order system. Similarly if in a stable higher order system, only a complex conjugate pair of poles is dominant, then the response is similar to a simple second order system. Therefore, even if we are working with higher order systems, we can have a good idea about the response of the system by just looking at the pole of most right of the complex plane.

**Exercise 12**
For the systems given below, find the natural frequency, the damping ratio and transient characteristics. Also plot their step responses on a single graph.

$$\frac{1}{(s+1)}, \quad \frac{2}{(s+1)(s+2)}, \quad \frac{10}{(s+1)(s+10)}, \quad \frac{20}{(s+1)(s+20)},$$
$$\frac{125}{(s+1)(s+10+5i)(s+10-5i)}$$

Comment on your observations.

Note that some of the systems in the above exercise are second order systems with real zeros.

**Exercise 13**

For the systems given below, find the natural frequency, the damping ratio and transient characteristics. Also plot their step responses on a single graph

$$\frac{5}{(s+1+2i)(s+1-2i)}$$

$$\frac{5}{(s+1)(s+1+2i)(s+1-2i)}$$

$$\frac{25}{(s+5)(s+1+2i)(s+1-2i)}$$

$$\frac{100}{(s+20)(s+1+2i)(s+1-2i)}$$

Comment on your observations.