Conditional Probability, Multiplication Rule of Probability, Dependent and independent Events

Ansar Shahzadi

School of Electrical Engineering & Computer Science

National University of Science and Technology(NUST)

Conditional Probability

The probability of an event given that another event has occurred is called a conditional probability.

If A and B are any events in S and P(B)≠ 0, The conditional probability of *A* given *B* is

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

Multiplication rule Of probability

If A and B are any events in S, then

$$\mathrm{P}(\mathrm{A} \cap B) = P(B).P(A|B)\ if\ P(B) \neq 0$$

$$\mathrm{P}(\mathrm{A} \cap B) = P(A).P(B|A)\ if\ P(A) \neq 0$$

Independent Events

Two events A and B are defined to be independent if the probability that one event occurs, is not affected by the other event has or has not occurred, that is

$$P(A|B)=\text{P(A) and } P(B|A)=\text{P(B)}$$

It then follows the two events A and B are independent if and only if

$$P(A \cap B) = P(A).P(B)$$

Question 1

A pair of fair dice is thrown. If the two number appearing are different, find the probability that

- The sum is six.
- The sum is 4 or less.

Total outcomes = 36 - 6 = 30 (-6 cause numbers must be different)
Case A: Sum is 6
>> S = {(1, 5), (2, 4), (4, 2), (5, 1)}
>> P(A) = 4/30

Same approach for sum is <= 4

Question 2

A bag contains 10 white and 6 black balls. Four balls are drawn out and not replaced, find the probability that they are alternatively of different colors.

Case 1: Balls alternate as B W B W
>> (6/16)(10/15)(5/14)(9/13) = 45/728

Case 2: Balls alternate as W B W B
>> (10/16)(6/15)(9/14)(5/13) = 45/728

P(Alternate) = 45/728 + 45/728 = 45/364

Question 3

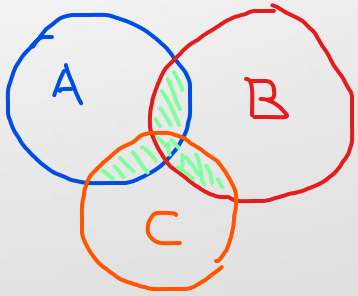
A and B are two independent events. The probability of A is 1/4 and Probability of B is 1/3. Find the Probabilities

- Neither A nor B occurs (P(A)' intersec P(B)')
- Both A and b occurs P(A) intersec P(B)
- Only A occurs P(A) - P(A intersec B)
- Only B occurs P(B) - P(A intersec B)
- At least one occurs P(A) union P(B) = P(A) + P(B) - P(A intersec B)

Question 4

A committee of three members A, B, and C is to make decision on the basis of majority vote. What is the probability of wrong decision by the committee? If the probabilities of wrong decision by each member are 0.05, 0.05, and 0.10.

Basis of majority implies at least two
>> P(A intersec B) + P(B intersec C)
+ P(C intersec A) - 2*(P(A intersec B intersec C))



Question 5

A Satellite Launch system is controlled by a computer (computer I) that has two identical backup computers (computers II and III).Normally computer I control the system, but if it has malfunction, the computer two automatically control the system. If computer II malfunction, then computer III automatically takes over, and if computer III malfunction, there is general system shutdown. The probability of malfunction of each computer is 0.01 and the malfunction of three computers independent of each other. Find the probability of

- Computer I in use? Always in use unless it fails: P(C1) = 1 - 0.01
- Computer III in use? After C1 and C2 fail: P(C3) = 0.01*0.01*(1 - 0.01)
- System failure? When all fail: P(SF) = 0.01*0.01*0.01

Question 6

The table provides an example of 400 parts classified by surface flaws and as (functionally) defective.

	Surface Flaws			
Defective		Yes(event F)	No	Total
	Yes(event D)	10	18	28
	No	30	342	372
	Total	40	360	400

Find the following probabilities

- P(F|D) >> 10/28 = 5/14
- P($\overline{D}|F$) >> 30/40 = 3/4
- P($\overline{F} \cup D$) >> P(F') = 360/400
  >> P(D) = 28/400
  >> P(F' union D) = P(F') + P(D) - P(F' intersec D)
  >> P(F' union D) = 360/400 + 28/400 - 18/400 = 370/400