NATIONAL UNIVERSITY OF SCIENCES & TECHNOLOGY

Mobile Communication Systems (EE-451)

Homework 3

Submission Details

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Class:	BEE-12
Semester:	7^{th}
Due:	21/12/2023

Problem 1 (CLO-1):

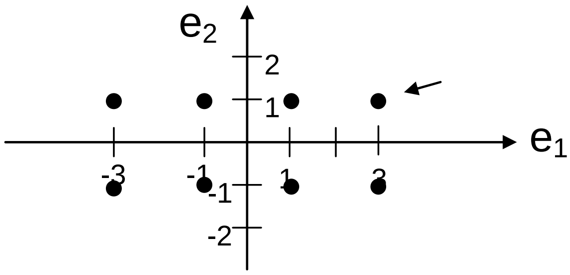
Consider a linearly modulated signal of the form $s(t) = \text{Re}\{s_l(t)e^{j2\pi f_c t}\}$, where

$$s_l(t) = \sum_{n=-\infty}^{\infty} b_n p(t - nT_s)$$

and where all pairs of symbols b_n and b_m are independent for $n \neq m$ and each b_n ε {-3, -1, 1, 3}, taking the values with equal probability. Furthermore, assume the pulse p(t) is rectangular with unit height and width Ts. Sketch the power spectral density (PSD), including labeling its peak height.

Problem 2:

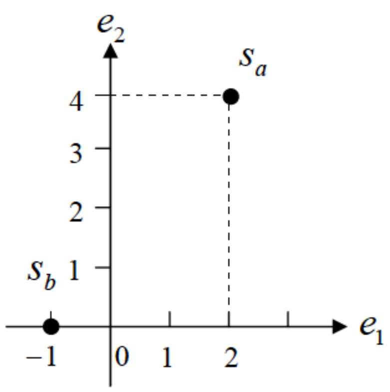
Suppose the following constellation of 8 signals for a communication system is given:



a) Find the energy of each symbol and the average symbol energy of this constellation
b) Give a complete union bound for the probability of symbol error.
c) If the basis functions are the usual ones for QPSK, give the expression of the symbol waveform as a function of time for the symbol that is indicated by the arrow.

Problem 3:

Two signal points S_a and S_b are shown below:



a) Suppose the noise spectral height is $N_0/2=25/16$. Evaluate the BER if these two signals are used in a wireless communication link.

b) Suppose the two basis functions are:

$$e_1(t) = \begin{cases} \sqrt{\dfrac{2}{T_s}} \cos(2\pi f_c t) & 0 < t < T_s \\ 0 & otherwise \end{cases}$$

$$e_2(t) = \begin{cases} K\sqrt{\dfrac{2}{T_s}} \cos\left(\dfrac{2\pi}{T_s} t\right) \cos(2\pi f_c t) & 0 < t < T_s \\ 0 & otherwise \end{cases}$$

Construct an expression of signal labeled S_a in terms of t and Ts.

Problem 4:

Let the carrier frequency be 100 MHz and let the symbol period be 1 microsecond. Consider a transmitted BPSK signal that uses the 25% excess bandwidth Root Raised Cosine pulses (you can use the definition in Wikipedia, where beta = 0.25). Using MATLAB or your favorite programming language, plot the RF modulated BPSK signal,

$$x(t) = \left[\sum_{n=1}^{8} x_n p_{rrc}(t - nT_s)\right] \cos\,(2\pi f_c t)$$

Assume the symbol sequence x_n be [1, -1, -1, 1, -1, -1, -1, 1].

Problem 5:

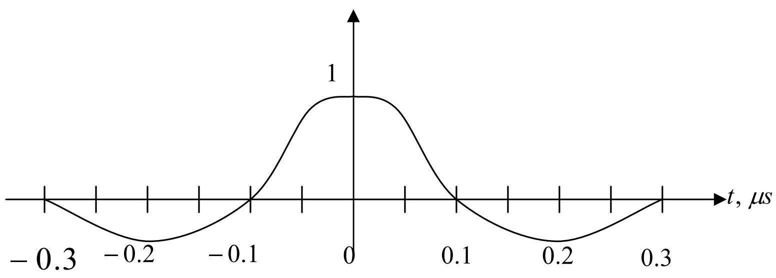
Using the same parameters as in Problem 4, plot the RF modulated QPSK waveform

$$x(t) = \left[\sum_{n=1}^{8} x_n p_{rrc}(t - nT_s)\right] \cos(2\pi f_c t) - \left[\sum_{n=1}^{8} y_n p_{rrc}(t - nT_s)\right] \sin(2\pi f_c t)$$

Use the same x_n sequence as in Problem 4, and let the quadrature symbols of the signal, y_n, be [-1, -1, 1, 1, 1, -1, 1, -1].

Problem 6:

Consider the pulse below, plotted versus time in microseconds. Could this pulse be a Nyquist pulse for a binary transmission with a 10MHz data rate? Why or why not?



Problem 4:

Let the carrier frequency be 100 MHz and let the symbol period be 1 microsecond. Consider a transmitted BPSK signal that uses the 25% excess bandwidth Root Raised Cosine pulses (you can use the definition in Wikipedia, where beta = 0.25). Using MATLAB or your favorite programming language, plot the RF modulated BPSK signal,

$$x(t) = \left[\sum_{n=1}^{8} x_n p_{rrc}(t - nT_s)\right] \cos(2\pi f_c t)$$

Assume the symbol sequence x_n be [1, -1, -1, 1, -1, -1, -1, 1].

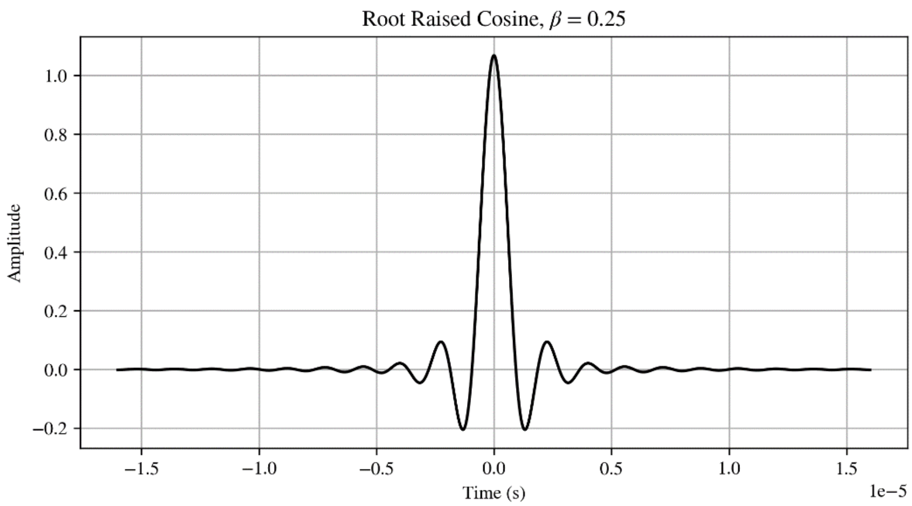
We start by making the necessary imports, definitions, and important indices.

```
import numpy as np
import matplotlib.pyplot as plt

plt.rcParams["mathtext.fontset"] = "stix"
plt.rcParams["font.family"] = "STIXGeneral"

fc = 100e6
Ts = 1e-6
beta = 0.25
N = 8
t = np.arange(-2 * N * Ts, 2 * N * Ts, 1 / (2 * fc))
x_n = np.array([1, -1, -1, 1, -1, -1, -1, 1])

# Zero crossing and shift for +-Ts/(4*beta)
zc = len(t) // 2
shift = len(np.arange(0, Ts / (4 * beta), 1 / (2 * fc))) + 1
```

Next, we define a function for Root Raised Cosine (RRC) pulse and substitute the zero crossing and shifts about the zero crossing with appropriate values.

```
# Root Raised Cosine (Ts * 1/Ts cancels); from Wikipedia
p_rrc = lambda t: (
    np.sin(np.pi * t / Ts * (1 - beta))
    + 4 * beta * t / Ts * np.cos(np.pi * t / Ts * (1 + beta))
) / (np.pi * t / Ts * (1 - (4 * beta * t / Ts) ** 2))
p_rrc_zc = Ts * 1 / Ts * (1 + beta * (4 / np.pi - 1))
p_rrc_shift = (beta / np.sqrt(2)) * (
    (1 + 2 / np.pi) * np.sin(np.pi / (4 * beta))
    + (1 - 2 / np.pi) * np.cos(np.pi / (4 * beta))
)
```

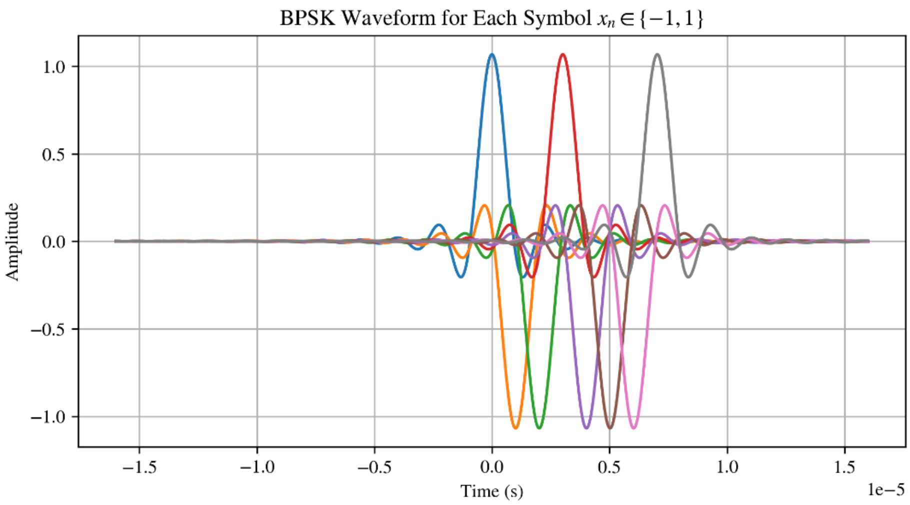
```
p = p_rrc(t)
p[zc] = p_rrc_zc
p[zc + shift] = p_rrc_shift
p[zc - shift] = p_rrc_shift

# Plot
plt.figure(figsize=(8, 4)), plt.tight_layout()

plt.plot(t, p, "k-", linewidth=1.5)
plt.xlabel("Time (s)")
plt.ylabel("Amplitude")
plt.title(r"Root Raised Cosine, $\beta$ = 0.25")
plt.grid()
plt.savefig("p4a.png", dpi=300)
plt.show()
```

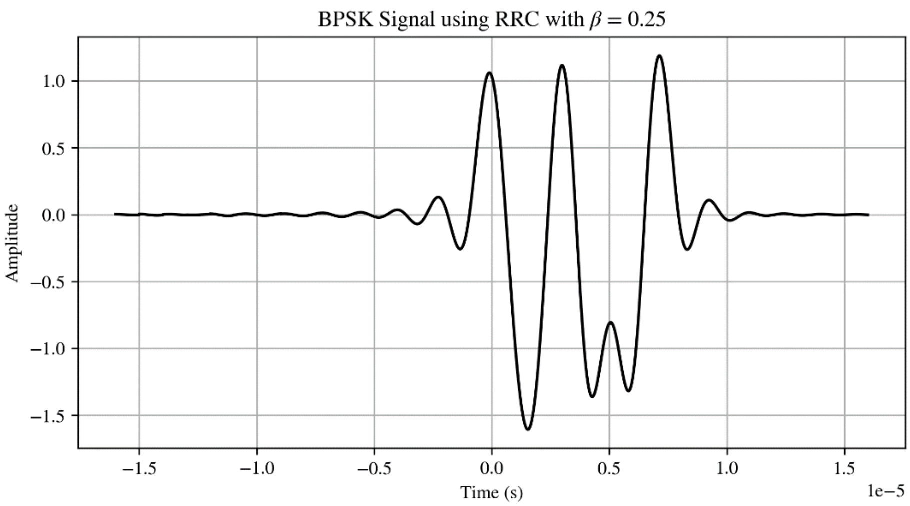


Now, we just define and plot the aggregated BPSK waveform, i.e., the complex envelope.

```
shifted_p = np.zeros(len(t))
interval = len(np.arange(0, Ts, 1 / (2 * fc))) + 1
x_t = np.zeros(len(t))

plt.figure(figsize=(8, 4))
plt.tight_layout()

for i in range(N):
    shifted_p[interval * i : len(p)] = p[0 : len(p) - interval * i]
    x_t += x_n[i] * shifted_p
    plt.plot(t, x_n[i] * shifted_p, linewidth=1.5)

plt.xlabel("Time (s)")
plt.ylabel("Amplitude")
```

```
plt.title(r"BPSK Waveform for Each Symbol $x_n \in \{-1, 1\}$")
plt.grid()
plt.savefig("p4b.png", dpi=300)
plt.show()

plt.figure(figsize=(8, 4))
plt.plot(t, x_t, "k-", linewidth=1.5)
plt.xlabel("Time (s)")
plt.ylabel("Amplitude")
plt.title("BPSK Signal using RRC with $\\beta$ = 0.25")
plt.grid()
plt.savefig("p4c.png", dpi=300)
plt.show()
```

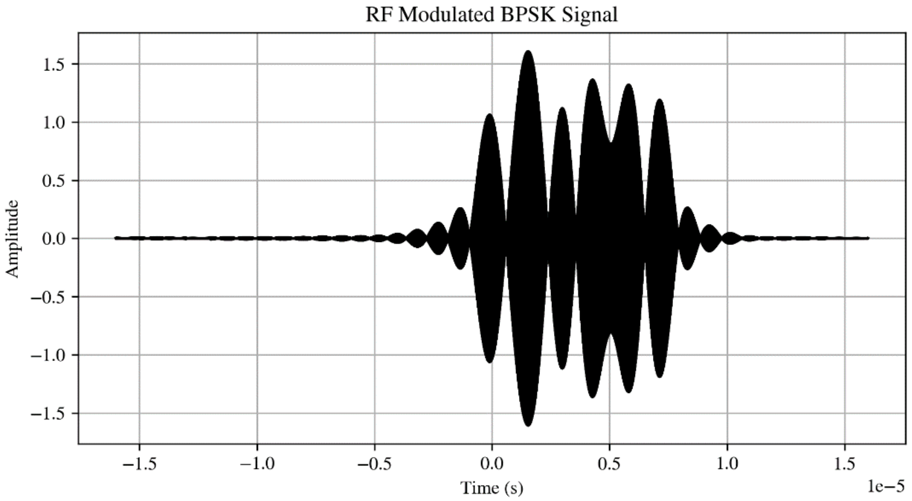




Lastly, we multiply the complex envelope with the necessary carrier wave as follows:

$$s(t) = x(t) \times \cos(2\pi f_c t)$$

```
modulated_p4 = x_t * np.cos(2 * np.pi * fc * t)

plt.figure(figsize=(8, 4))
plt.plot(t, modulated_p4, "k-", linewidth=1.5)
plt.xlabel("Time (s)")
plt.ylabel("Amplitude")
plt.title("RF Modulated BPSK Signal")
plt.grid()
plt.savefig("p4d.png", dpi=300)
plt.show()
```



Problem 5:

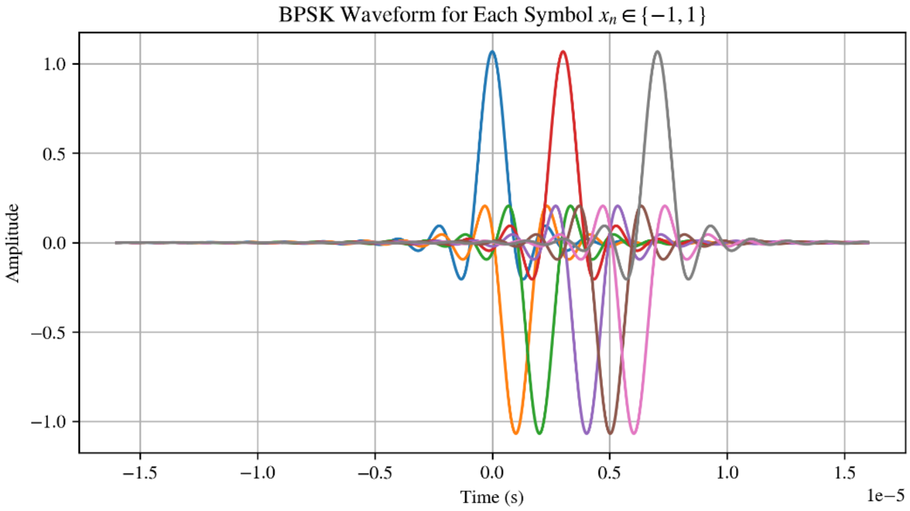
Using the same parameters as in Problem 4, plot the RF modulated QPSK waveform

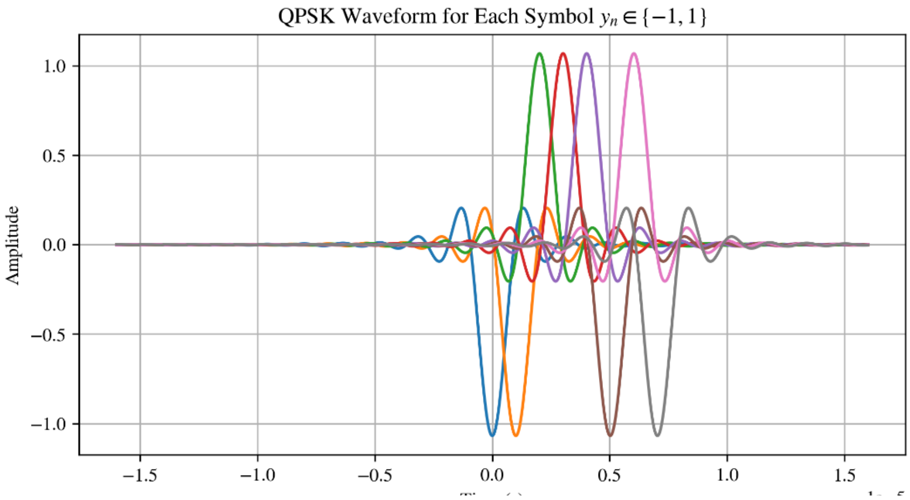
$$x(t) = \left[\sum_{n=1}^{8} x_n p_{rrc}(t - nT_s)\right] \cos(2\pi f_c t) - \left[\sum_{n=1}^{8} y_n p_{rrc}(t - nT_s)\right] \sin(2\pi f_c t)$$

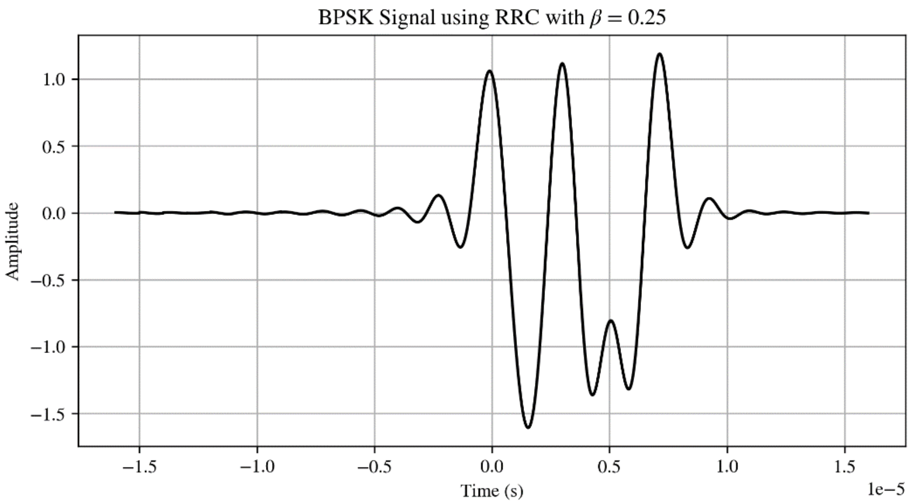
Use the same x_n sequence as in Problem 4, and let the quadrature symbols of the signal, y_n, be [-1, -1, 1, 1, 1, -1, 1, -1].

Continuing from the previous problem, we define y_n and perform the same steps as before.

```
y_n = np.array([-1, -1, 1, 1, 1, -1, 1, -1])
shifted_p = np.zeros(len(t))
interval = len(np.arange(0, Ts, 1 / (2 * fc))) + 1
x_t = np.zeros(len(t))
y_t = np.zeros(len(t))
ax, fig = plt.subplots(2, 1, figsize=(7, 8))

for i in range(N):
    shifted_p[interval * i : len(p)] = p[0 : len(p) - interval * i]
    x_t += x_n[i] * shifted_p
    y_t += y_n[i] * shifted_p
    fig[0].plot(t, x_n[i] * shifted_p, linewidth=1.5)
    fig[1].plot(t, y_n[i] * shifted_p, linewidth=1.5)

fig[0].set_xlabel("Time (s)")
fig[0].set_ylabel("Amplitude")
fig[0].set_title(r"BPSK Waveform for Each Symbol $x_n \in \{-1, 1\}$")
fig[0].grid()
fig[1].set_xlabel("Time (s)")
fig[1].set_ylabel("Amplitude")
fig[1].set_title(r"QPSK Waveform for Each Symbol $y_n \in \{-1, 1\}$")
fig[1].grid()
plt.tight_layout()
plt.savefig("p5a.png", dpi=300)
plt.show()
plt.figure(figsize=(8, 4))
plt.plot(t, x_t, "k-", linewidth=1.5)
plt.xlabel("Time (s)")
plt.ylabel("Amplitude")
plt.title("BPSK Signal using RRC with $\\beta$ = 0.25")
plt.grid()
plt.savefig("p5b.png", dpi=300)
plt.show()
```







Now, we can define the modulated signal in accordance with the following formula, and visualize it:

$$x(t) = \left[\sum_{n=1}^{8} x_n p_{rrc}(t - nT_s)\right] \cos(2\pi f_c t) - \left[\sum_{n=1}^{8} y_n p_{rrc}(t - nT_s)\right] \sin(2\pi f_c t)$$

```
modulated_i = x_t * np.cos(2 * np.pi * fc * t)
modulated_q = y_t * np.sin(2 * np.pi * fc * t)
modulated_p5 = modulated_i - modulated_q

plt.figure(figsize=(8, 4))
plt.plot(t, modulated_p5, "k-", linewidth=1.5)
plt.xlabel("Time (s)")
plt.ylabel("Amplitude")
plt.title("RF Modulated QPSK Signal")
plt.grid()
plt.savefig("p5c.png", dpi=300)
plt.show()
```