Engineering Mechanics

Dr. Hina Gohar Ali

Hina.gohar@seecs.edu.pk

Office : IAEC building

Office Hours: Appointment through emails

Research Interest: Photovoltaic systems, Power & Control, Sliding mode control (SMC)

Contents (Section 2.7-2.9)

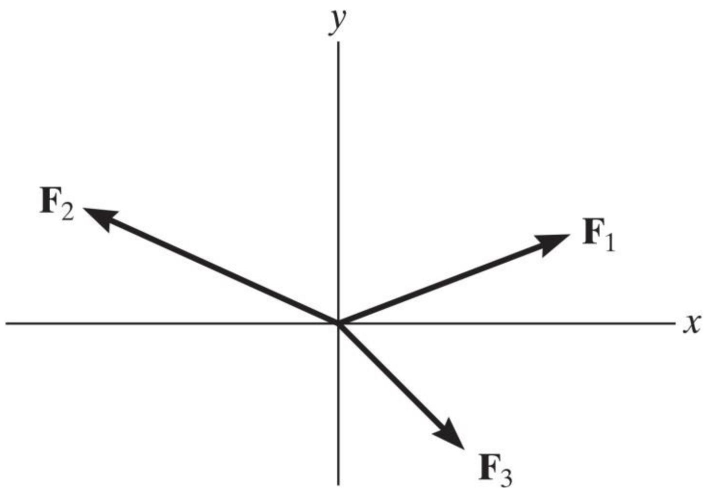
- Recap
- Position Vector
- Force along a line
- Dot Product

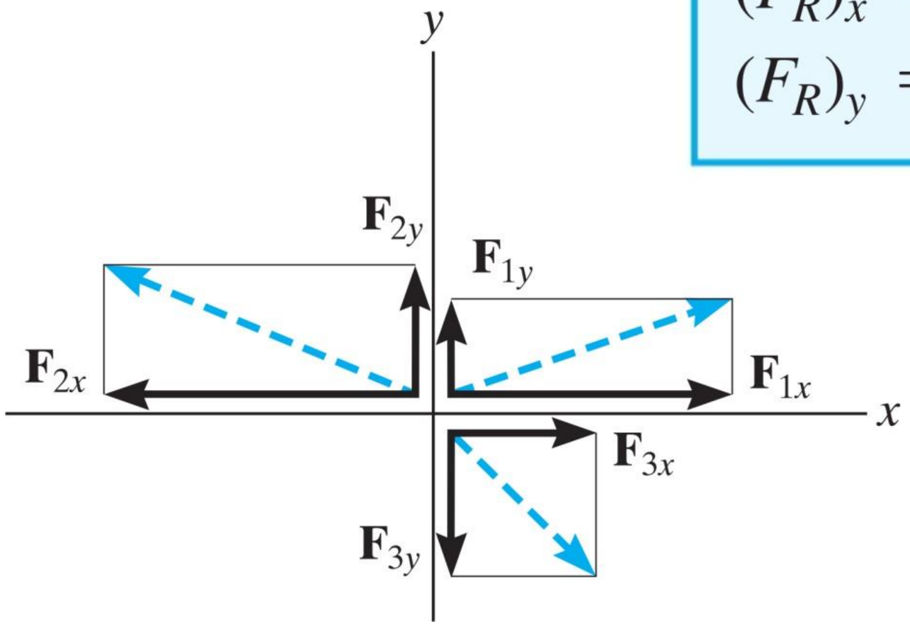
RECAP

Engineering Mechanics

Cartesian Vector Notation

$$(F_R)_x = \Sigma F_x$$
$$(F_R)_y = \Sigma F_y$$

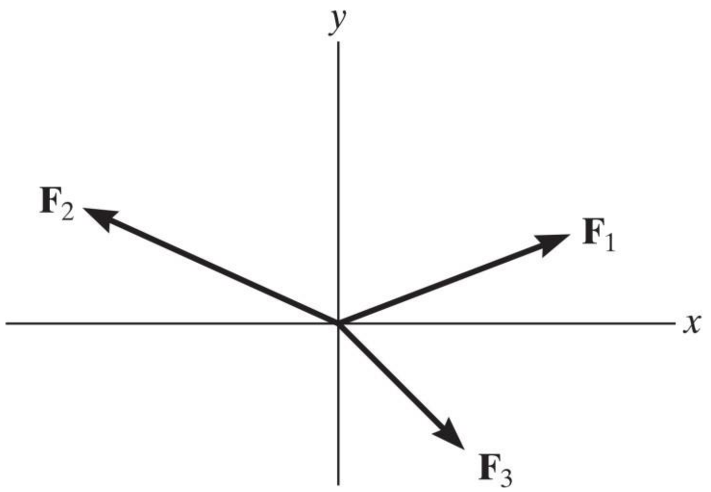


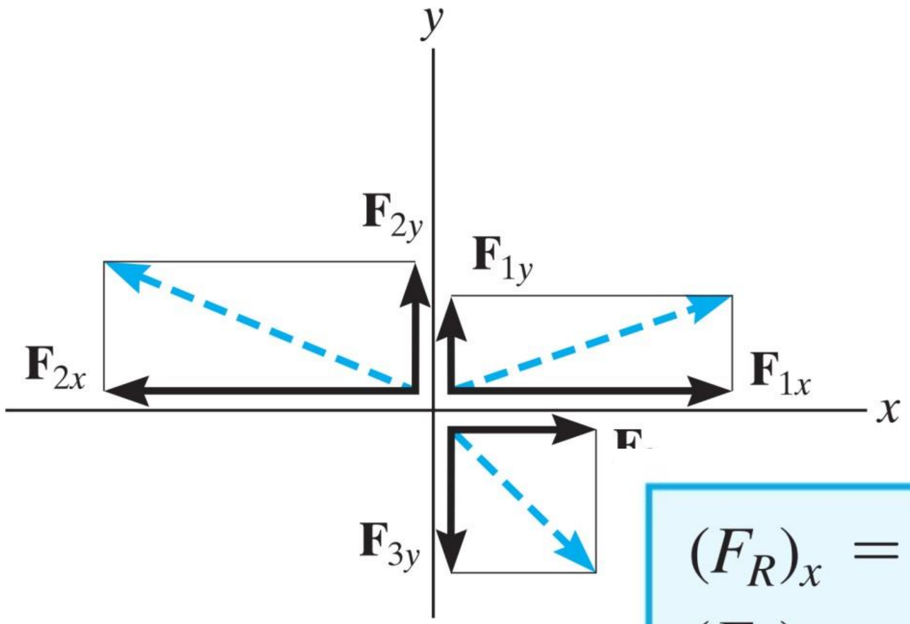


$$\mathbf{F}_1 =$$
$$\mathbf{F}_2 =$$
$$\mathbf{F}_3 =$$

$$\begin{aligned} \mathbf{F}_R &= \\ &= \\ &= \\ &= \end{aligned}$$

Cartesian Vector Notation





$$(F_R)_x = \Sigma F_x$$
$$(F_R)_y = \Sigma F_y$$

$$\mathbf{F}_1 = F_{1x}\,\mathbf{i} + F_{1y}\,\mathbf{j}$$
$$\mathbf{F}_2 = -F_{2x}\,\mathbf{i} + F_{2y}\,\mathbf{j}$$
$$\mathbf{F}_3 = F_{3x}\,\mathbf{i} - F_{3y}\,\mathbf{j}$$

$$\begin{aligned}\mathbf{F}_R &= \mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3\\ &= F_{1x}\,\mathbf{i} + F_{1y}\,\mathbf{j} - F_{2x}\,\mathbf{i} + F_{2y}\,\mathbf{j} + F_{3x}\,\mathbf{i} - F_{3y}\,\mathbf{j}\\ &= (F_{1x} - F_{2x} + F_{3x})\mathbf{i} + (F_{1y} + F_{2y} - F_{3y})\mathbf{j}\\ &= (F_{Rx})\mathbf{i} + (F_{Ry})\mathbf{j}\end{aligned}$$

Cartesian Representation in 3D

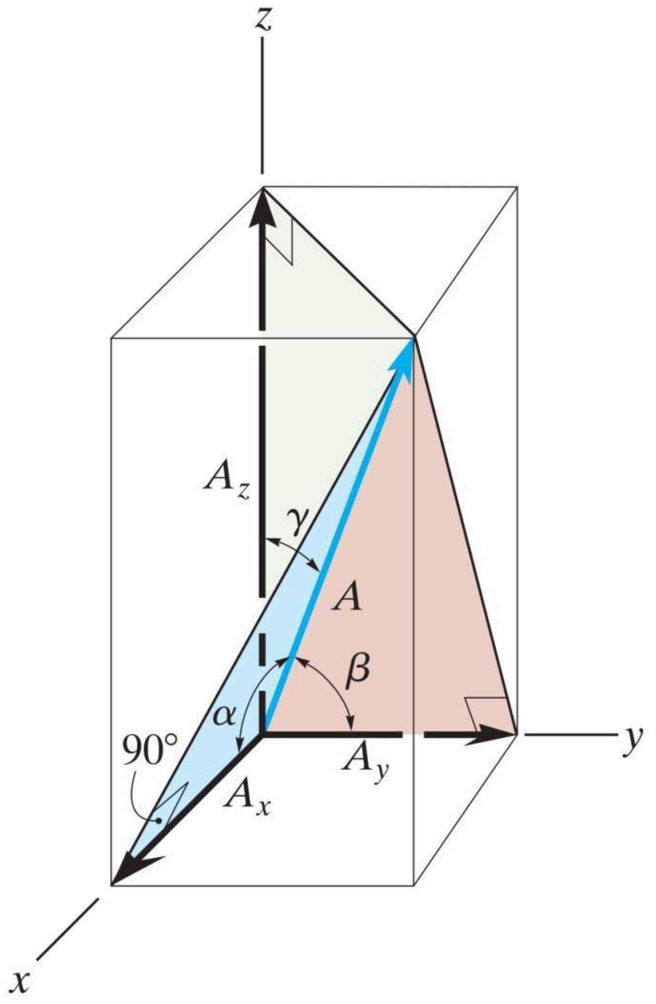
- Magnitude

$$A =$$

- Angles

$$\cos \alpha =$$

$$\mathbf{A} =$$
$$=$$
$$=$$



Cartesian Representation in 3D

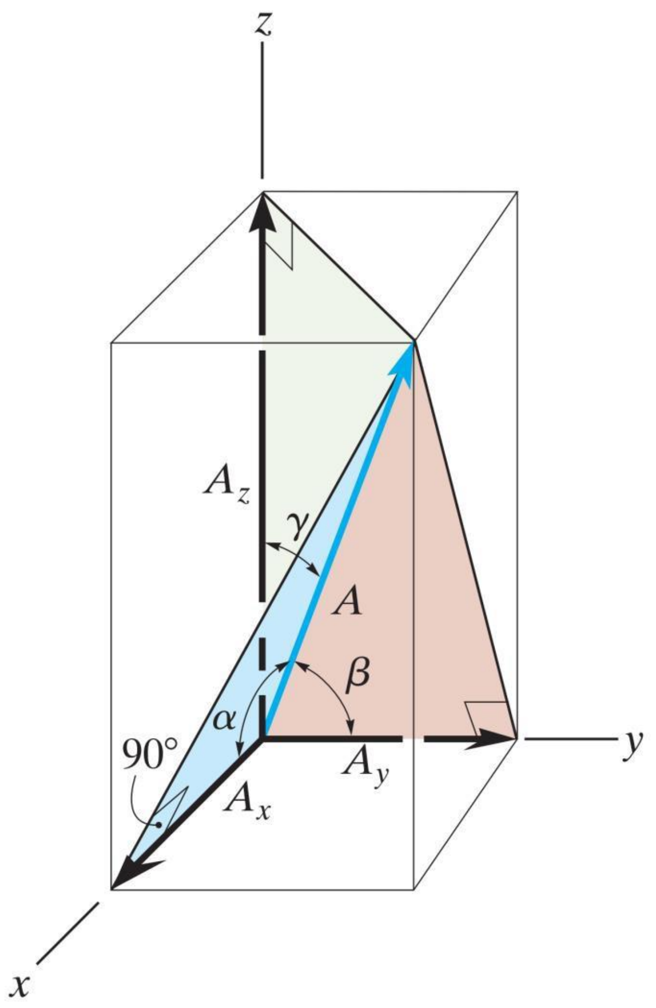
- Magnitude

$$A = \sqrt{A_x^2 + A_y^2 + A_z^2}$$

- Angles

$$\cos\alpha = \frac{A_x}{A} \quad \cos\beta = \frac{A_y}{A} \quad \cos\gamma = \frac{A_z}{A}$$

$$\begin{aligned} \mathbf{A} &= A\,\mathbf{u}_A \\ &= A\cos\alpha\,\mathbf{i} + A\cos\beta\,\mathbf{j} + A\cos\gamma\,\mathbf{k} \\ &= A_x\mathbf{i} + A_y\mathbf{j} + A_z\mathbf{k} \end{aligned}$$



Transverse and Azimuth Angle Representation

- Sometimes

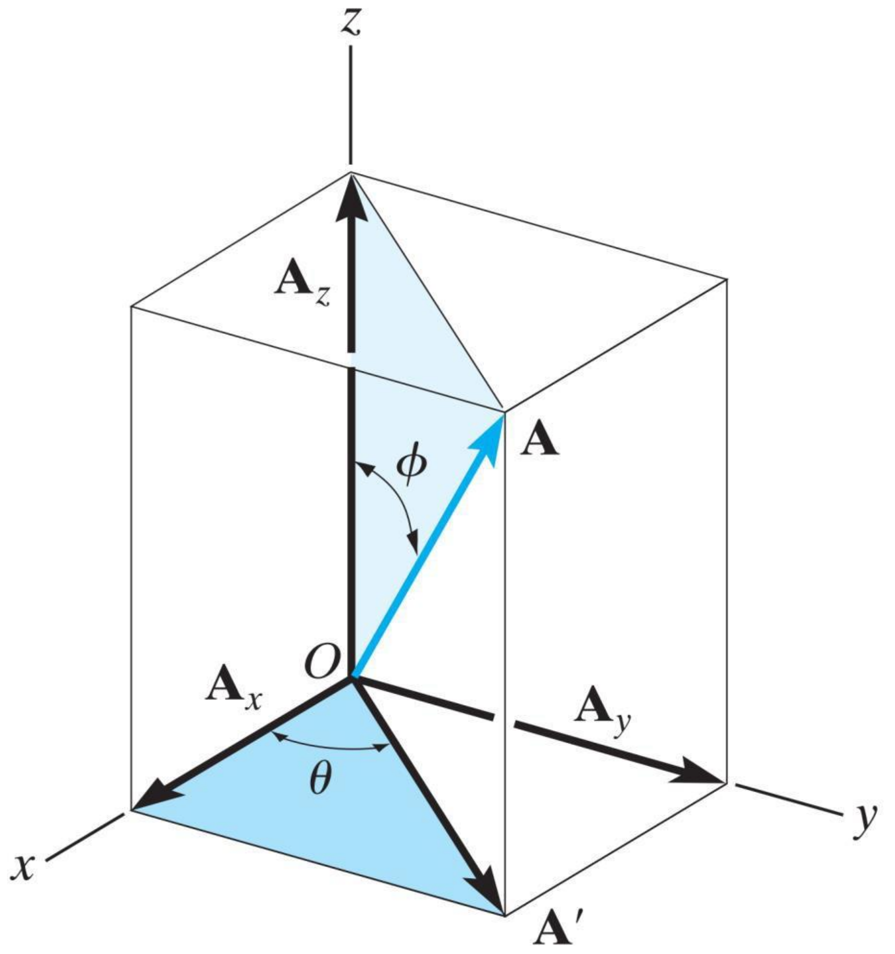
$$A_z = A \cos\phi$$

$$A' = A \sin\phi$$

$$A_x = A' \cos\theta = A \sin\phi \cos\theta$$

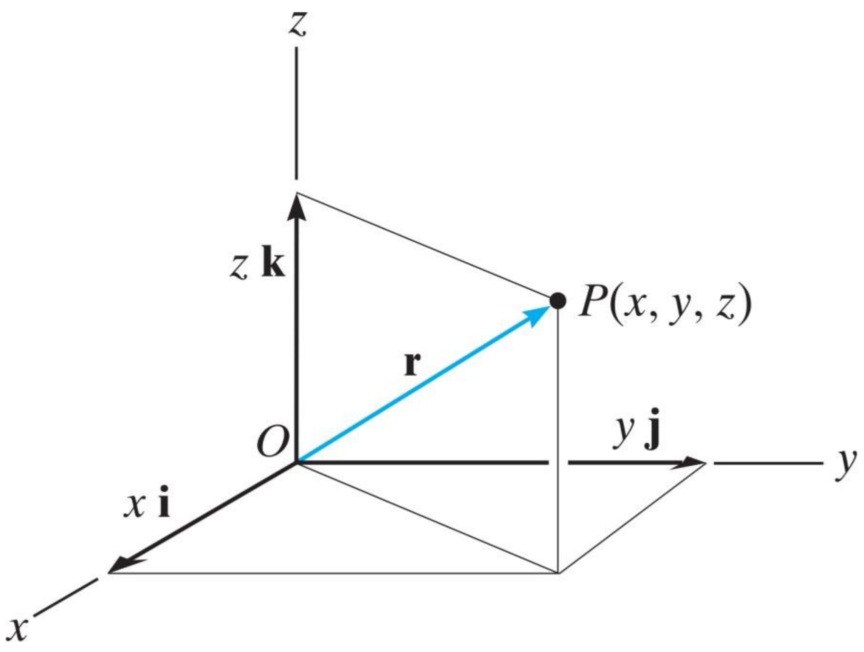
$$A_y = A' \sin\theta = A \sin\phi \sin\theta$$

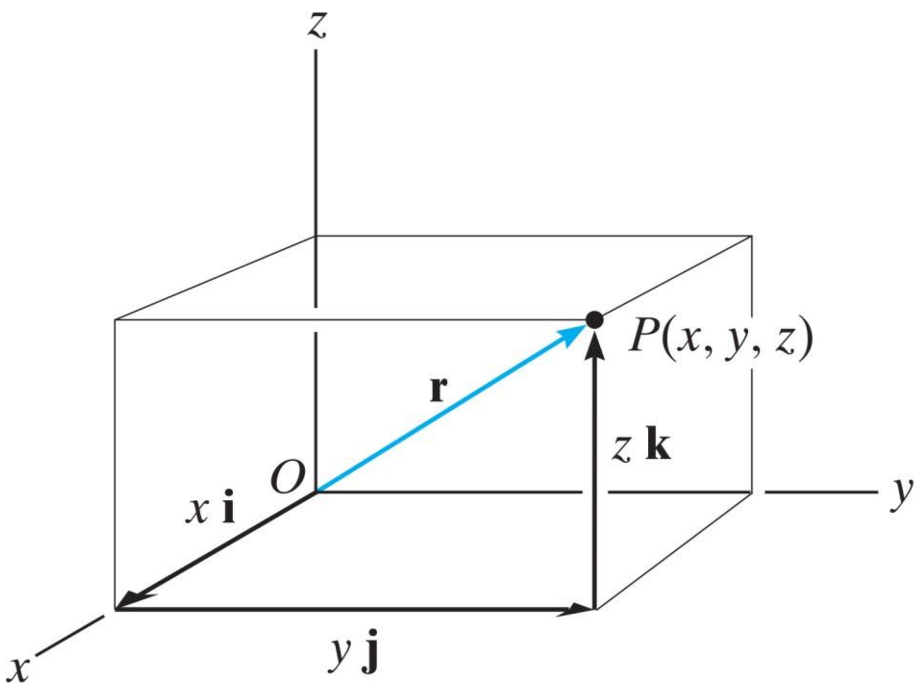
$$\mathbf{A} = A \sin\phi \cos\theta\, \mathbf{i} + A \sin\phi \sin\theta\, \mathbf{j} + A \cos\phi\, \mathbf{k}$$



Position Vector

- A fixed vector which locates a point in space relative to another point

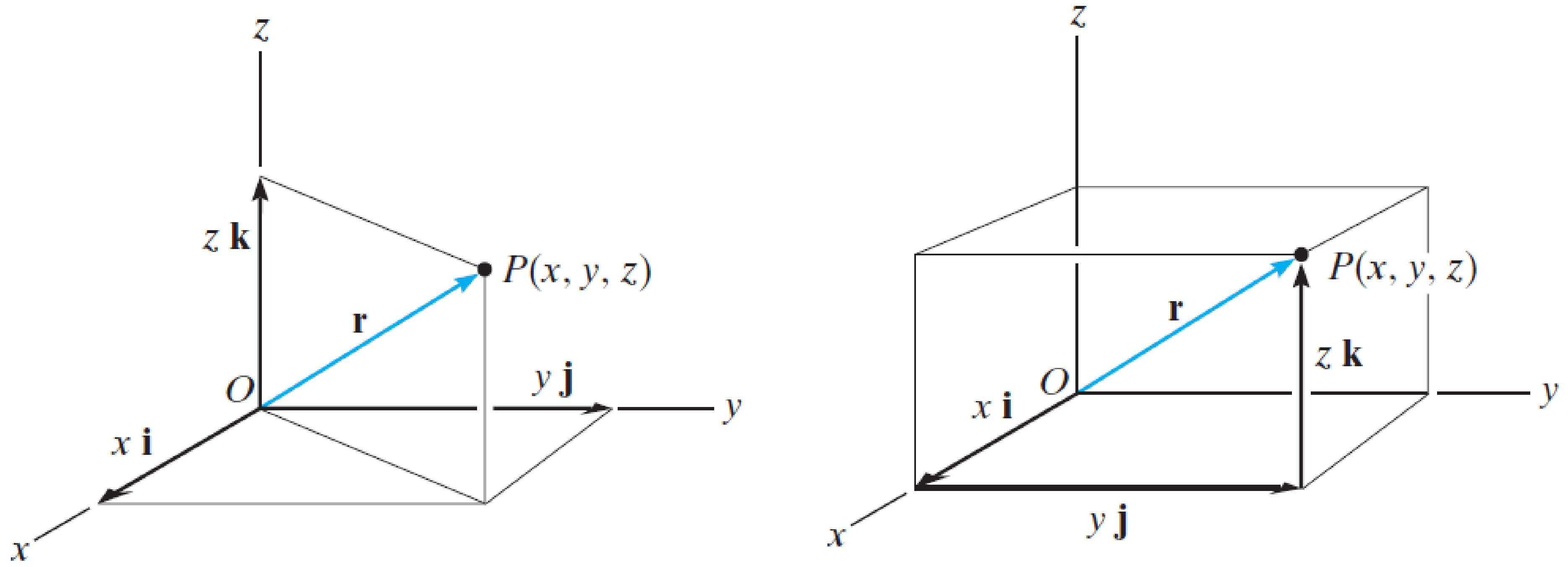




Position Vector. A *position vector* $\mathbf{r}$ is defined as a fixed vector which locates a point in space relative to another point. For example, if $\mathbf{r}$ extends from the origin of coordinates, O, to point $P(x, y, z)$, Fig. 2–35a, then $\mathbf{r}$ can be expressed in Cartesian vector form as

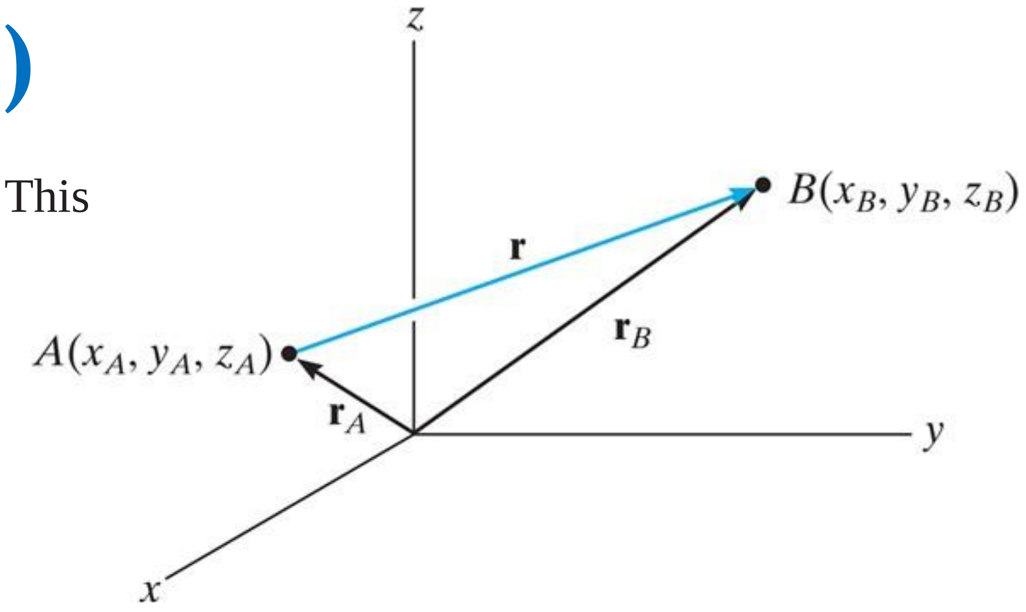
$$\mathbf{r} = x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$$

Note how the head-to-tail vector addition of the three components yields vector $\mathbf{r}$, Fig. 2–35b. Starting at the origin O, one "travels" x in the $+\mathbf{i}$ direction, then y in the $+\mathbf{j}$ direction, and finally z in the $+\mathbf{k}$ direction to arrive at point $P(x, y, z)$.



Position Vector (General Case)

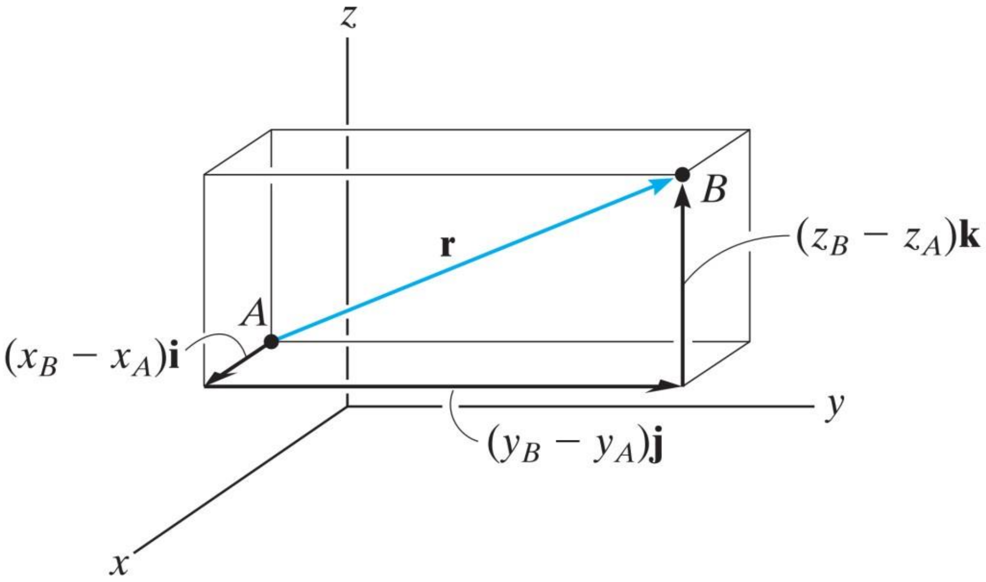
The position vector may be directed from point A to point B in space, Fig. This vector is also designated by the symbol $\mathbf{r}$.



$$\mathbf{r}_A + \mathbf{r} = \mathbf{r}_B$$

$$\mathbf{r} = \mathbf{r}_B - \mathbf{r}_A = (x_B\mathbf{i} + y_B\mathbf{j} + z_B\mathbf{k}) - (x_A\mathbf{i} + y_A\mathbf{j} + z_A\mathbf{k})$$

$$\boxed{\mathbf{r} = (x_B - x_A)\mathbf{i} + (y_B - y_A)\mathbf{j} + (z_B - z_A)\mathbf{k}}$$



We can also form these components *directly*, Fig. 2–36*b*, by starting at A and moving through a distance of **($x_B - x_A$)** along the positive x axis (+**i**), then **($y_B - y_A$)** along the positive y axis (+**j**), and finally ($z_B - z_A$) along the positive z axis (+**k**) to get to B

Example

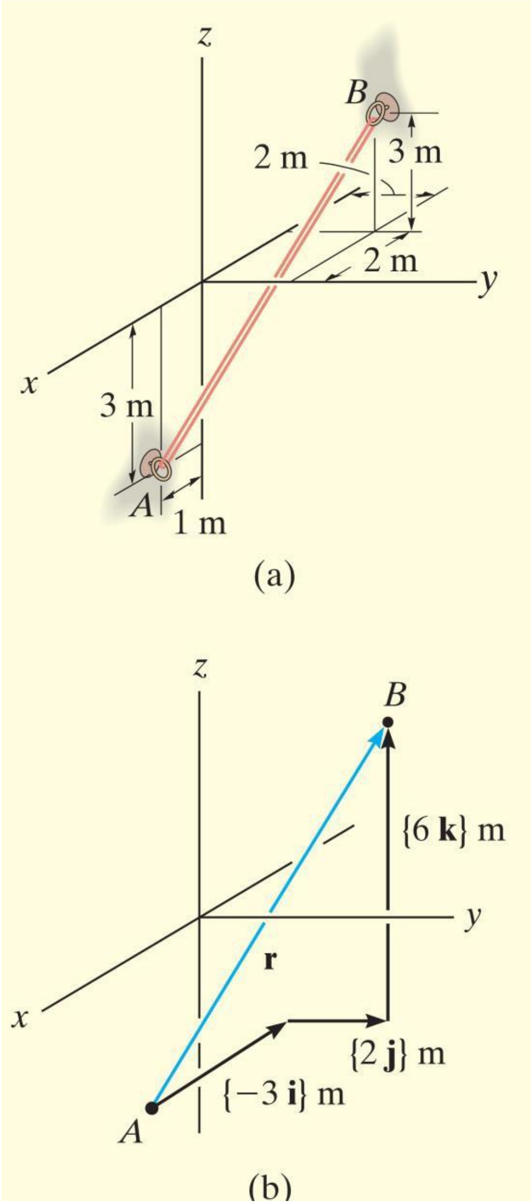
An elastic rubber band is attached to points *A* and *B*. Determine its length and its direction measured from *A* toward *B*.

$$\mathbf{r} =$$

$$=$$

$$r =$$

$$\mathbf{u} =$$



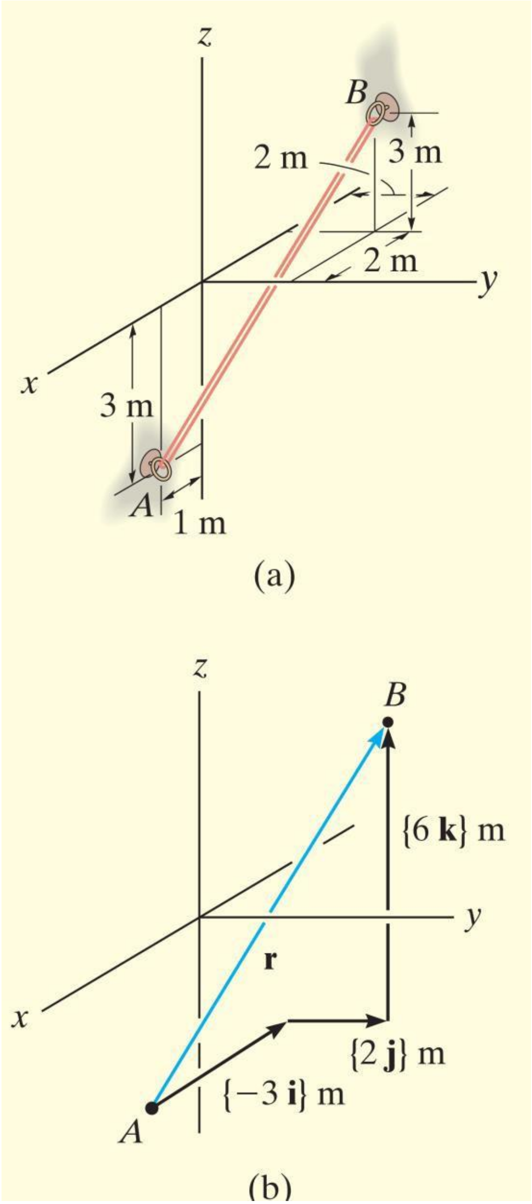
Example

An elastic rubber band is attached to points *A* and *B.* Determine its length and its direction measured from *A* toward *B*.

$$\mathbf{r} = [-2\text{ m} - 1\text{ m}]\mathbf{i} + [2\text{ m} - 0]\mathbf{j} + [3\text{ m} - (-3\text{ m})]\mathbf{k}$$
$$= \{-3\mathbf{i} + 2\mathbf{j} + 6\mathbf{k}\}\text{ m}$$

$$r = \sqrt{(-3\text{ m})^2 + (2\text{ m})^2 + (6\text{ m})^2} = 7\text{ m}$$

$$\mathbf{u} = \frac{\mathbf{r}}{r} = -\frac{3}{7}\mathbf{i} + \frac{2}{7}\mathbf{j} + \frac{6}{7}\mathbf{k}$$

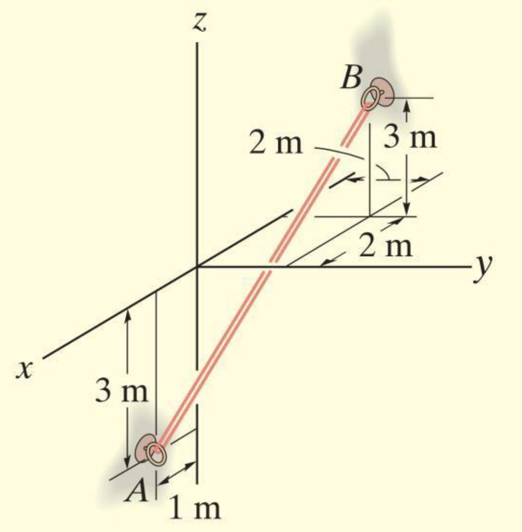


Example

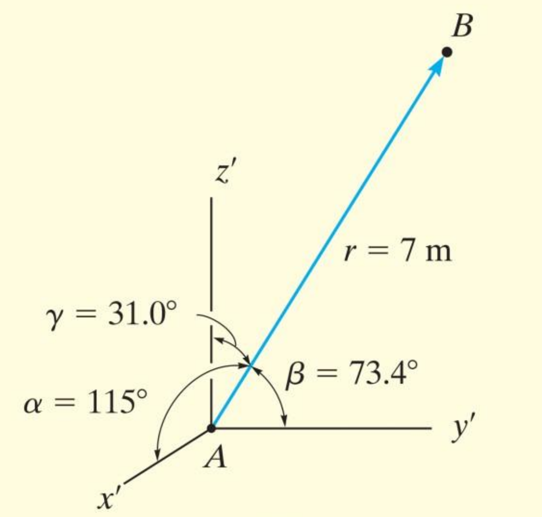
$$\alpha =$$

$$\beta =$$

$$\gamma =$$



(a)



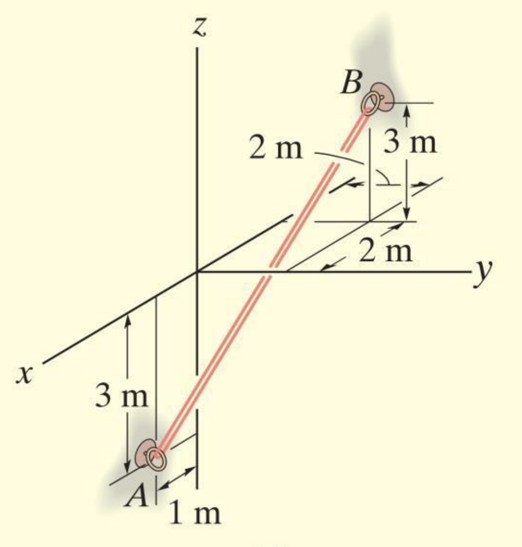
(c)

Example

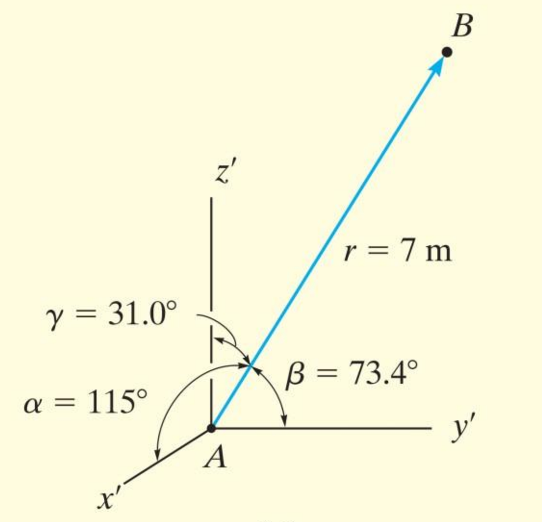
$$\alpha = \cos^{-1}\left(-\frac{3}{7}\right) = 115^\circ$$

$$\beta = \cos^{-1}\left(\frac{2}{7}\right) = 73.4^\circ$$

$$\gamma = \cos^{-1}\left(\frac{6}{7}\right) = 31.0^\circ$$



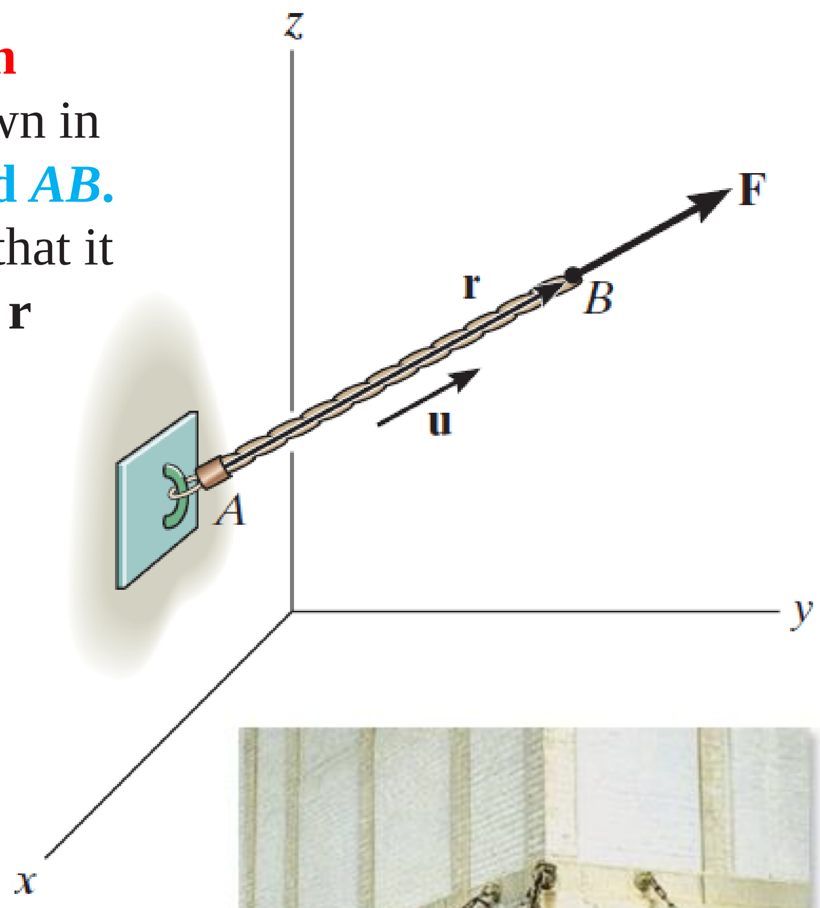
(a)



(c)

Force Vector Directed Along a Line

Quite often in three-dimensional statics problems, **the direction of a force is specified by two points through which its line of action passes.** Such a situation is shown in Fig. 2–38, where **the force F is directed along the cord *AB*.** We can formulate **F** as a Cartesian vector by realizing that it has the ***same direction* and *sense*** as the position vector **r**



$$\mathbf{F} = F\mathbf{u}$$

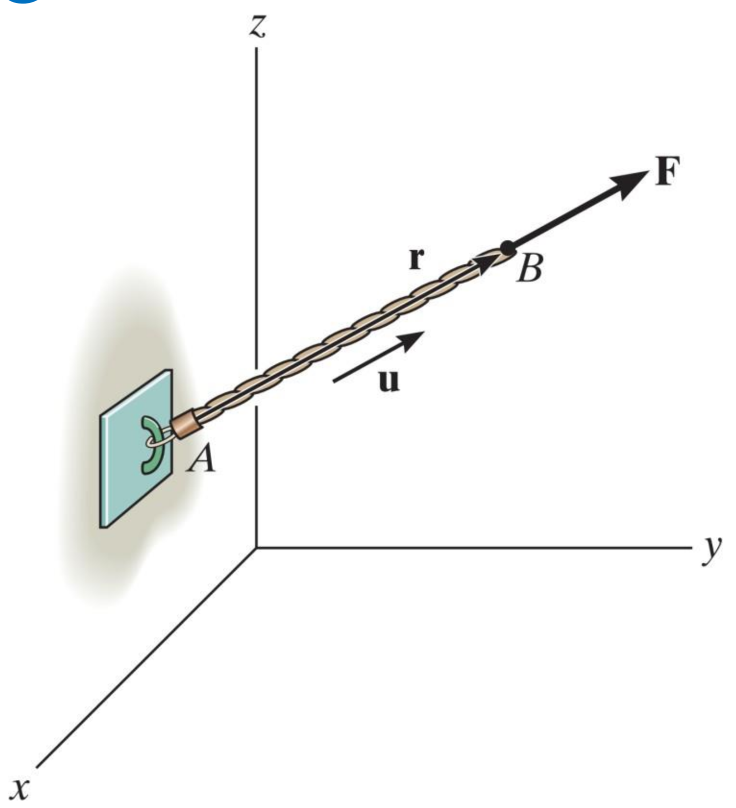
$$\mathbf{u} = \mathbf{r}/r$$

$$\mathbf{F} = F\mathbf{u} = F\left(\frac{\mathbf{r}}{r}\right)$$

$$F\left(\frac{(x_B - x_A)\mathbf{i} + (y_B - y_A)\mathbf{j} + (z_B - z_A)\mathbf{k}}{\sqrt{(x_B - x_A)^2 + (y_B - y_A)^2 + (z_B - z_A)^2}}\right)$$

Force Vector Directed Along A Line

$$\mathbf{F} = F\,\mathbf{u} = F\left(\frac{\mathbf{r}}{r}\right) = F\left(\frac{(x_B - x_A)\mathbf{i} + (y_B - y_A)\mathbf{j} + (z_B - z_A)\mathbf{k}}{\sqrt{(x_B - x_A)^2 + (y_B - y_A)^2 + (z_B - z_A)^2}}\right)$$



Example

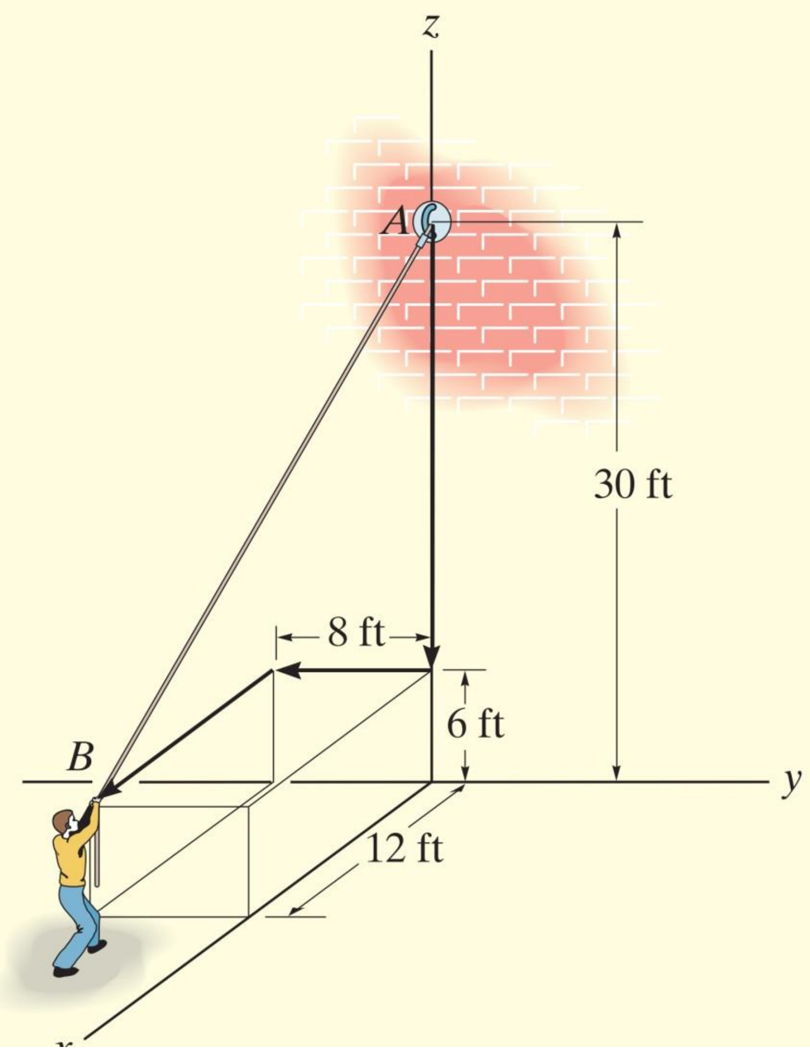
The man pulls on the cord with a force of 70 lb. represent this force acting on the support A as a Cartesian vector and determine its direction.

$\mathbf{r} =$

$r =$

$\mathbf{u} =$

$\mathbf{F} =$



Example

The man pulls on the cord with a force of 70 lb. represent this force acting on the support A as a Cartesian vector and determine its direction.

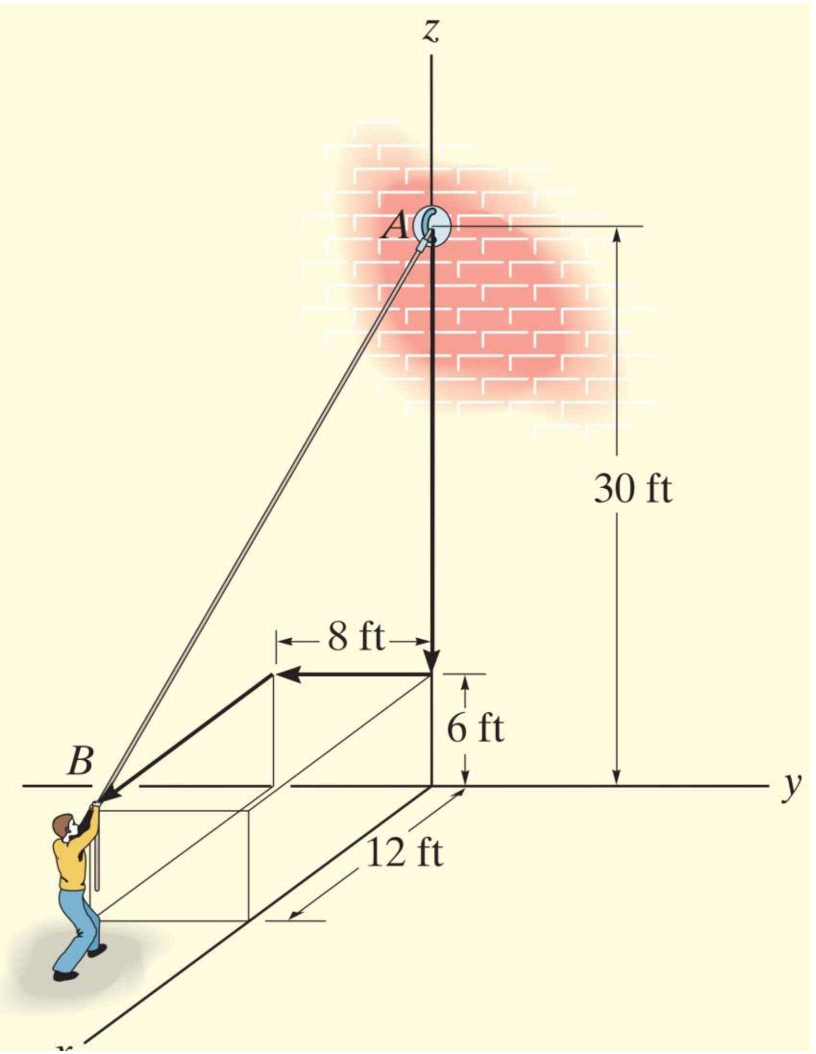
$$\mathbf{r} = \{12\mathbf{i} - 8\mathbf{j} - 24\mathbf{k}\} \text{ ft}$$

$$r = \sqrt{(12 \text{ ft})^2 + (-8 \text{ ft})^2 + (-24 \text{ ft})^2} = 28 \text{ ft}$$

$$\mathbf{u} = \frac{\mathbf{r}}{r} = \frac{12}{28}\mathbf{i} - \frac{8}{28}\mathbf{j} - \frac{24}{28}\mathbf{k}$$

$$\mathbf{F} = F\mathbf{u} = 70 \text{ lb}\left(\frac{12}{28}\mathbf{i} - \frac{8}{28}\mathbf{j} - \frac{24}{28}\mathbf{k}\right)$$

$$= \{30\mathbf{i} - 20\mathbf{j} - 60\mathbf{k}\} \text{ lb}$$



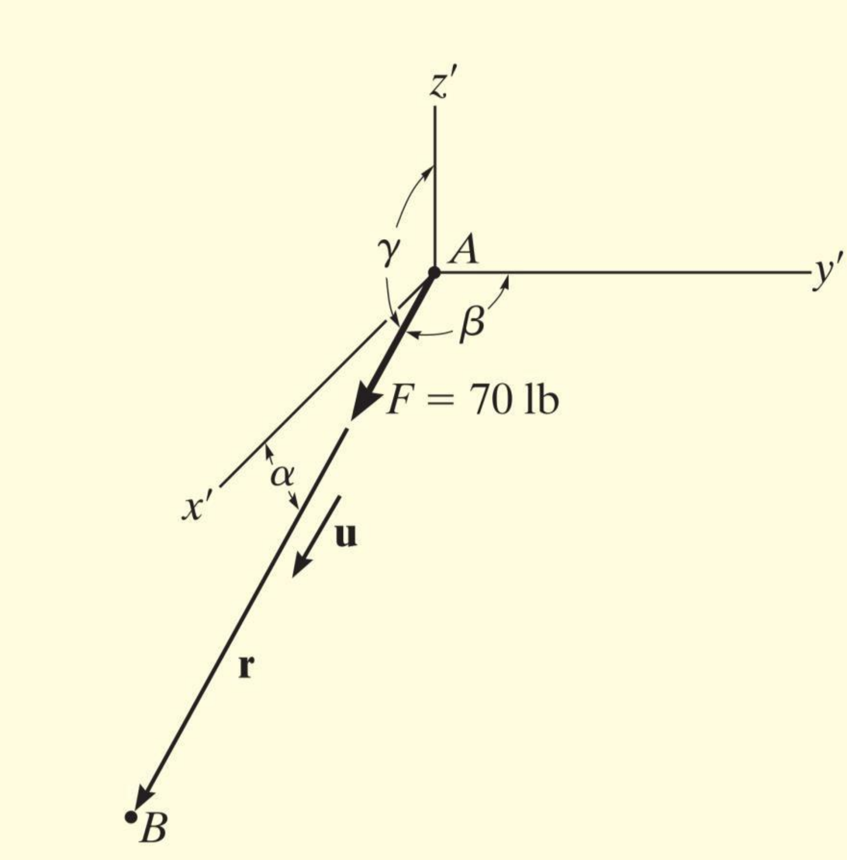
Example

The man pulls on the cord with a force of 70 lb. represent this force acting on the support A as a Cartesian vector and determine its direction.

$$\alpha =$$

$$\beta =$$

$$\gamma =$$



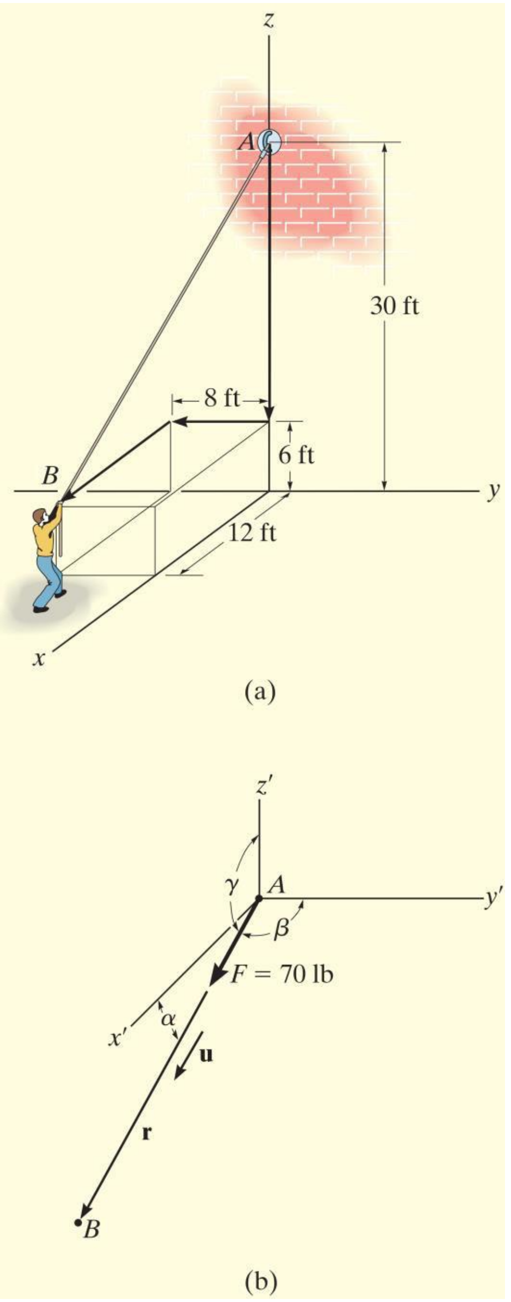
Example

The man pulls on the cord with a force of 70 lb. represent this force acting on the support A as a Cartesian vector and determine its direction.

$$\alpha = \cos^{-1}\left(\frac{12}{28}\right) = 64.6^\circ$$

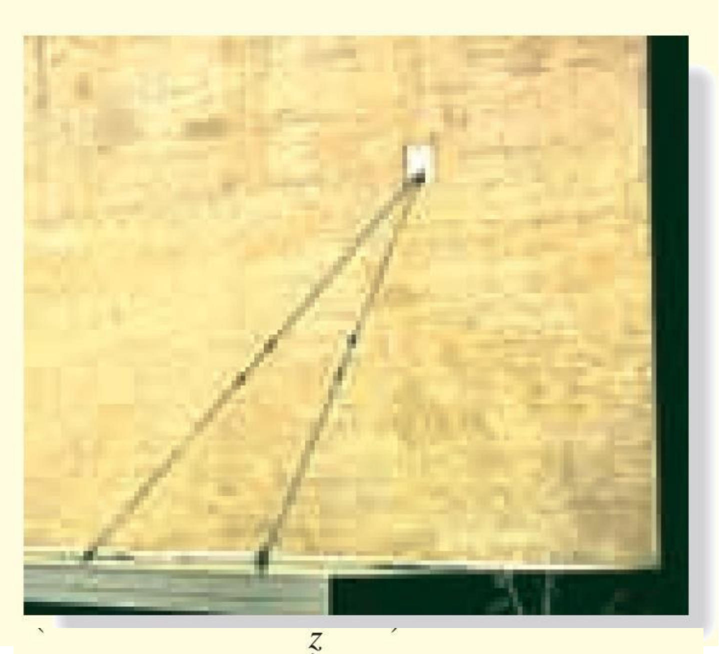
$$\beta = \cos^{-1}\left(\frac{-8}{28}\right) = 107^\circ$$

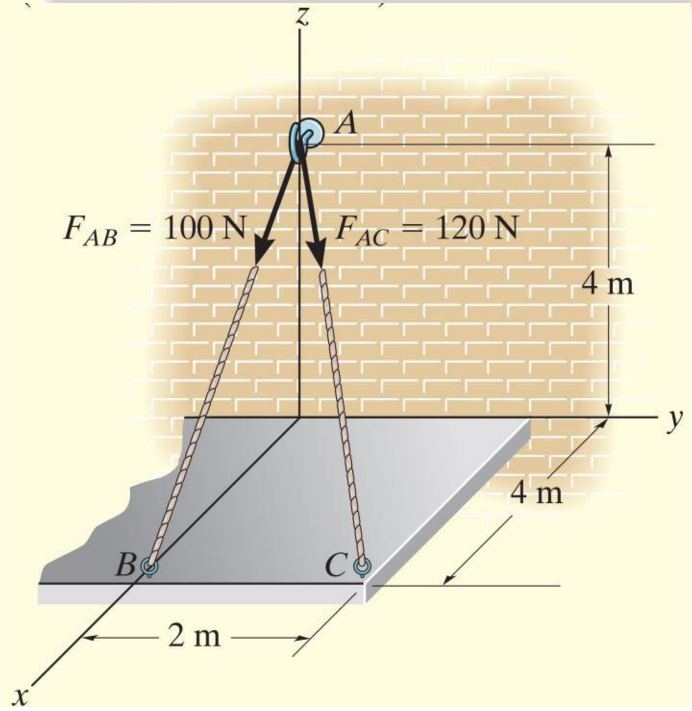
$$\gamma = \cos^{-1}\left(\frac{-24}{28}\right) = 149^\circ$$



Example

The roof is supported by cables as shown in the photo. If the cables exert forces $F_{AB} = 100$ N and $F_{AC} = 120$ N on the wall hook at A, determine the resultant force acting at A. Express the result as a Cartesian vector.





$\mathbf{r}_{AB} =$

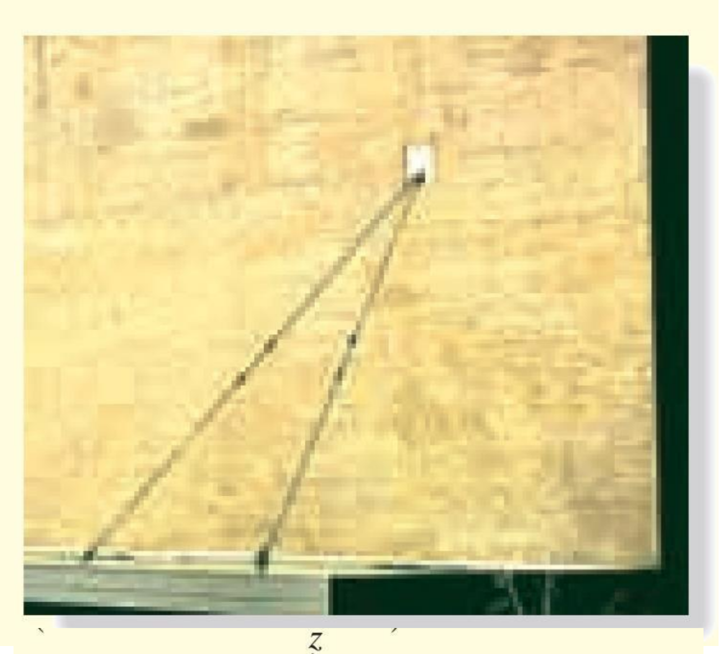
$r_{AB} =$

$\mathbf{F}_{AB} =$

$\mathbf{F}_{AB} =$

Example

The roof is supported by cables as shown in the photo. If the cables exert forces F_{AB} = 100 N and F_{AC} = 120 N on the wall hook at A, determine the resultant force acting at A. Express the result as a Cartesian vector.

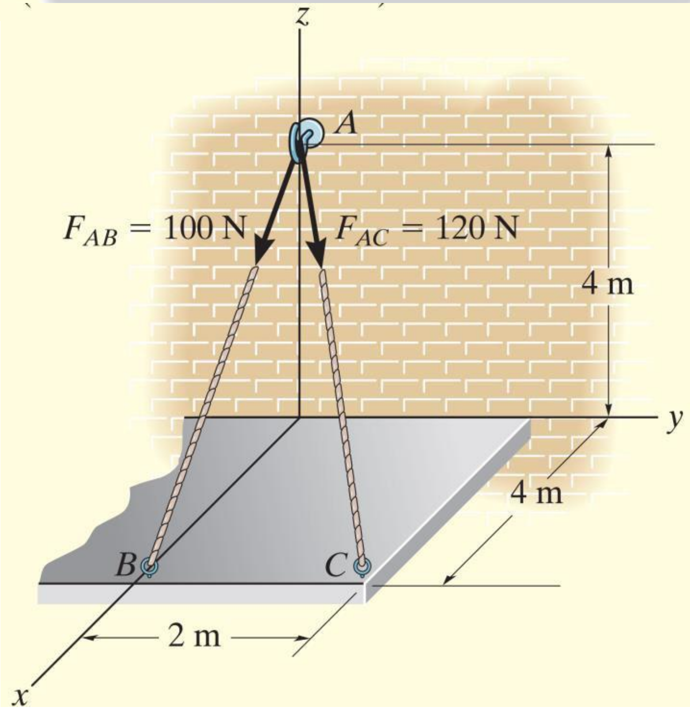


$$\mathbf{r}_{AB} = \{4\mathbf{i} - 4\mathbf{k}\}\ \text{m}$$

$$r_{AB} = \sqrt{(4\ \text{m})^2 + (-4\ \text{m})^2} = 5.66\ \text{m}$$

$$\mathbf{F}_{AB} = F_{AB}\left(\frac{\mathbf{r}_{AB}}{r_{AB}}\right) = (100\ \text{N})\left(\frac{4}{5.66}\mathbf{i} - \frac{4}{5.66}\mathbf{k}\right)$$

$$\mathbf{F}_{AB} = \{70.7\mathbf{i} - 70.7\mathbf{k}\}\ \text{N}$$



Example

$$\mathbf{r}_{AC} =$$

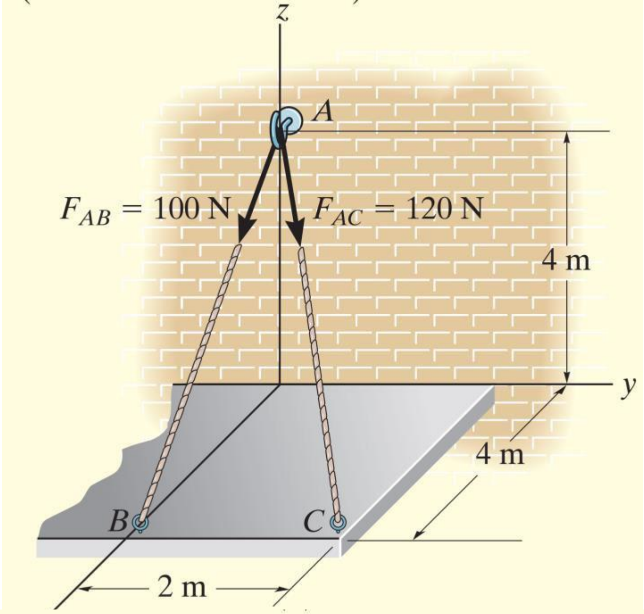
$$r_{AC} =$$

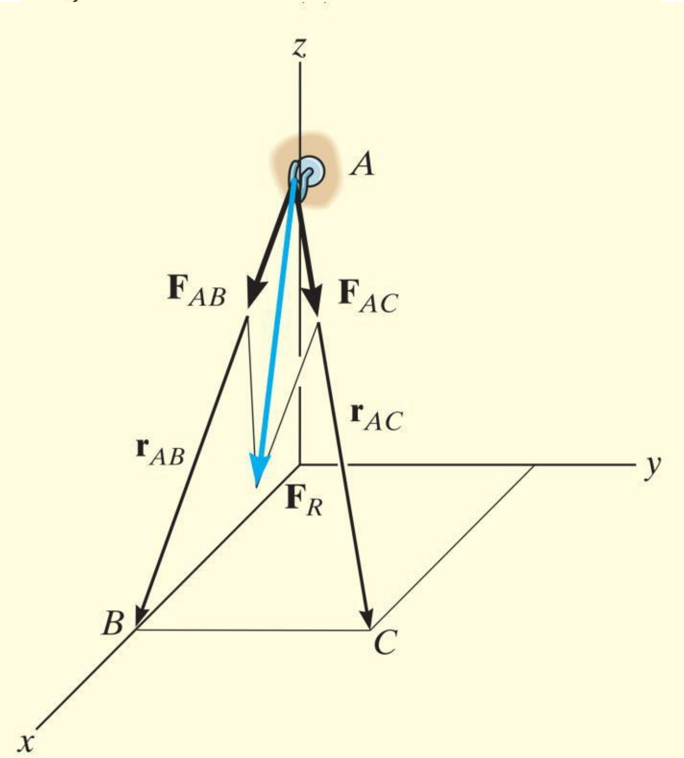
$$\mathbf{F}_{AC} =$$

$$=$$

$$\mathbf{F}_R = \mathbf{F}_{AB} + \mathbf{F}_{AC} =$$

$$=$$





Example

$$\mathbf{r}_{AC} = \{4\mathbf{i} + 2\mathbf{j} - 4\mathbf{k}\}\ \text{m}$$

$$r_{AC} = \sqrt{(4\ \text{m})^2 + (2\ \text{m})^2 + (-4\ \text{m})^2} = 6\ \text{m}$$

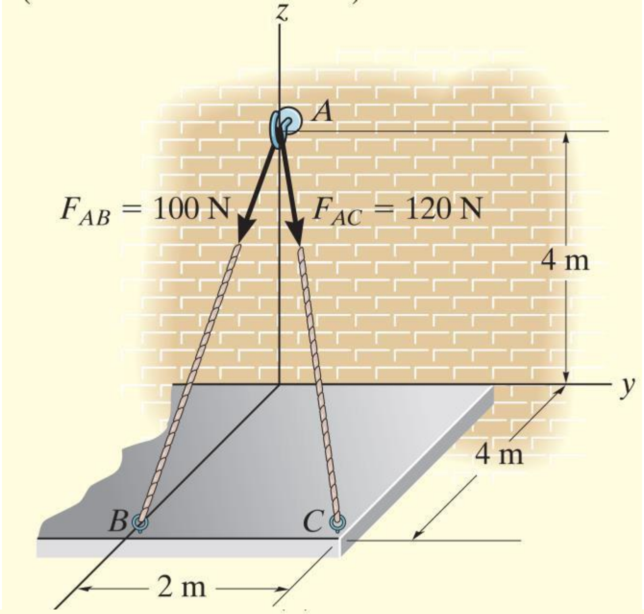
$$\mathbf{F}_{AC} = F_{AC}\left(\frac{\mathbf{r}_{AC}}{r_{AC}}\right) = (120\ \text{N})\left(\frac{4}{6}\mathbf{i} + \frac{2}{6}\mathbf{j} - \frac{4}{6}\mathbf{k}\right)$$

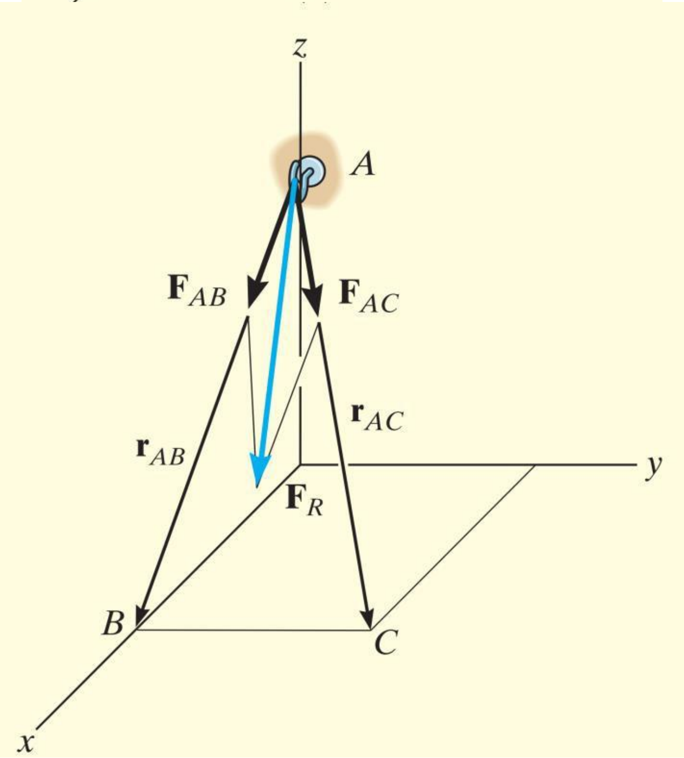
$$= \{80\mathbf{i} + 40\mathbf{j} - 80\mathbf{k}\}\ \text{N}$$

The resultant force is therefore

$$\mathbf{F}_R = \mathbf{F}_{AB} + \mathbf{F}_{AC} = \{70.7\mathbf{i} - 70.7\mathbf{k}\}\ \text{N} + \{80\mathbf{i} + 40\mathbf{j} - 80\mathbf{k}\}\ \text{N}$$

$$= \{151\mathbf{i} + 40\mathbf{j} - 151\mathbf{k}\}\ \text{N} \qquad \textit{Ans.}$$





Home Assignment

- Example 2.14