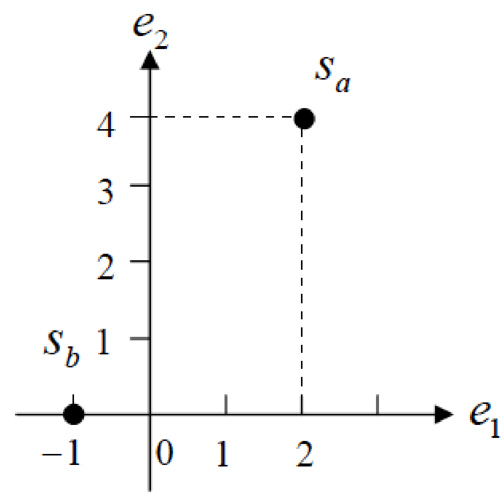
Two signal points S_a and S_b are shown below



a) Suppose the noise spectral height is $N_0/2=25/16$. Find the BER if these two signals are used in a wireless communication link.

$BER = Q\left(\frac{d}{\sqrt{2N_o}}\right)$, where d is the distance between the signal points in signal space and $N_o/2$ is the spectral height of the thermal noise. From the figure, d=5. From the given information, $N_o = 25/8$. Therefore,

$$BER = Q\left(\frac{5}{\sqrt{2\frac{25}{8}}}\right) = Q\left(\frac{5}{\sqrt{\frac{25}{4}}}\right) = Q\left(\frac{5}{\frac{5}{2}}\right) = Q(2)$$

From the Table below, BER=2.275E-2

b)

Suppose the two basis functions are

$$e_1(t) = \begin{cases} \sqrt{\frac{2}{T_s}}\cos(2\pi f_c t) & 0 < t < T_s \\ 0 & otherwise \end{cases}$$

$$e_2(t) = \begin{cases} K\sqrt{\frac{2}{T_s}}\cos\left(\frac{2\pi}{T_s}t\right)\cos(2\pi f_c t) & 0 < t < T_s \\ 0 & otherwise \end{cases}$$

Give an expression of signal labeled S_a in terms of t and Ts.

$$s_a(t) = 2e_1(t) + 4e_2(t)$$

$$= \begin{cases} 2\sqrt{\dfrac{2}{T_s}}\cos(2\pi f_c t) + 4K\sqrt{\dfrac{2}{T_s}}\cos\left(\dfrac{2\pi}{T_s}t\right)\cos(2\pi f_c t) & 0 < t < T_s \\ 0 & otherwise \end{cases}$$

c) Find the value of K such that $e_2(t)$ has a unit norm.

$$1 = \int_0^{T_s} e_2^2(t)dt$$

$$= K^2 \frac{2}{T_s}\int_0^{T_s} \cos^2\left(\frac{2\pi}{T_s}t\right)\cos^2(2\pi f_c t)dt$$

$$= K^2 \frac{2}{4T_s}\int_0^{T_s} \left[1 + \cos\left(\frac{4\pi}{T_s}t\right)\right][1 + \cos(4\pi f_c t)]dt$$

$$= K^2 \frac{1}{2T_s}\int_0^{T_s} 1 + \cos\left(\frac{4\pi}{T_s}t\right) + \cos(4\pi f_c t) + \cos\left(\frac{4\pi}{T_s}t\right)\cos(4\pi f_c t)dt$$

Dropping the "double frequency" terms, we have

$$1 = K^2 \frac{1}{2T_s}\left[T_s + \int_0^{T_s} \cos\left(\frac{4\pi}{T_s}t\right)dt\right]$$

Observe the integral directly above = 0. Therefore, $1 = K^2\dfrac{1}{2}$ and $K = \sqrt{2}$.