NATIONAL UNIVERSITY OF SCIENCES & TECHNOLOGY

Force System Resultants

Conceptualization and Implementation of Turning Forces

Group C

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ABSTRACT

Background: The theorems used to relate the concept to moment of force have been explicitly explained using diagrams. The theorems proofs have also been added as part of research for a greater comprehensive approach. The first five sections of chapter 4 from “Hibbeler’s Engineering Mechanics: Statics and Dynamics” have been explained. Moment about a specified axis is explained.
Methods: The research deals with the use of software MATLAB to visualize vector product. Moreover, the participants have introduced the **vector card experiment** for practical understanding of direction of vector product. Both the vector analysis and scalar analysis are used. All methodologies are familiar and relatable to the topic and have been added for arousing interest in readers as well as undertaking a conceptual approach towards the topic of the moment of force.
Results: The comparison of like-wise problems have been thoroughly done to explain the consistency and authenticity of both methods. The authors’ motto is to deliver the study in an easily comprehensible way and mention all key points and key topics of the assigned portion. The objective of study is to see the topic from a conceptual approach which will soon be evident from the study.

I. Introduction

Physics aims to describe the function of everything around us, from the movement of tiny charged particles to the motion of people, cars, and spaceships. In fact, almost everything around you can be described quite accurately by the laws of physics. Although physics is more oriented towards getting a firm grasp of conceptual aspects of studies that come under it, we will be treating mathematical calculations with equal importance in this report.

Generally, in physics, moment of force (often shortened to moment) is a measure of the affinity/tendency of a body to rotate. In order for a moment to arise in a system, the force(s) must act at a point such that it may twist the body i.e. when the line of action of force does not pass through the centroid of the body/ does not have forces equal and opposite to each other on said line of action.

II. Moment of a Force

Definition: Scalar Formulation

The moment of a **force** or ***torque*** is defined as the turning effect of the force about a pivot and is the product of the

force (F) and the **perpendicular distance** (d) from the line of action of the force to the pivot.

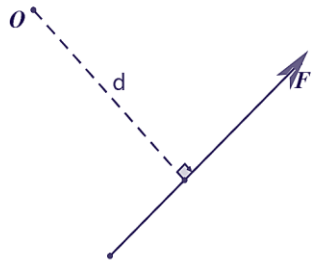
- The SI unit of moment is Newton-metre (Nm)
- The direction of moment is given by right hand grip rule

Mathematically, the magnitude of moment is calculated by;

$$M_0 = Fd$$

Definition: Vector Formulation

- Where d is the perpendicular distance from the point O to the line of action of Force



Or in vector formulation, it is expressed as a cross product of the **position vector r** and the **force F**;

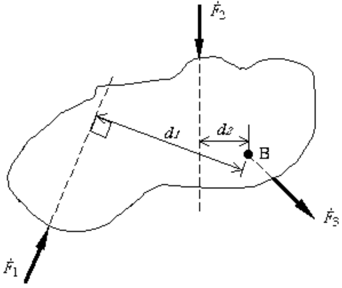
$$M_0 = r \times F$$

- Where **r** is a position vector extending from the point O to any point on the line of action of force
- Moment is always perpendicular to the plane containing the position vector and the force

Resultant Moment

Often a times, a system may have more than just a single force action on it. In a system where all forces are acting on an x-y plane, the resultant moment is simply the algebraic sum of all the moments pertaining to the system. The following expression is applicable to all three-dimensional systems for the resultant moment:

$$\circlearrowleft \text{ or } \circlearrowright + (M_R)_0 = \sum Fd$$



The resultant moment about point B in the given figure is:

$$\circlearrowleft + (M_R)_B = \sum Fd = -F_1d_1 + F_2d_2$$

III. Cross Product

Before starting this topic, we assume that readers have sound knowledge about vector quantities and their representations on Cartesian plane. Moreover, the author has basic understanding of mathematical techniques and operations that can be applied to vectors.

Need of Cross Product

First, we shall provide some motivation for the cross product because at first glance the definition will seem rather ugly and arbitrary.

Question 1

Can area of the space spanned by A and B be represented as vector? Given two vectors a and b, they define a parallelogram. Can we construct an "area product"? In other words, can we multiply a and b in a way that gives us the area of the parallelogram they span?

Question 2

Can we define a useful product that returns a vector?
Other than dot product, we need another quantity which represents the product as a vector quantity.

Question 3

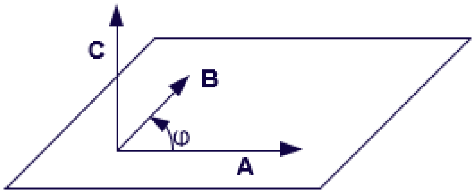
In applied physics and mathematics, orthogonality has such a vast application. In particular, since we know that orthogonality is a nice property, can our product return a vector that is orthogonal to both a and b?

Each of these questions will be good to keep in mind as we move forward, because the cross product is the answer to all of them.

Definition and Geometric Intuition:

The cross product of two vectors **a** and **b** is defined only in three-dimensional space and is denoted by **a** × **b.**
The cross product **a** × **b** is defined as a vector **c** that is perpendicular (orthogonal) to both **a** and **b**, with a direction given by the right-hand rule and a magnitude equal to the area of the parallelogram that the vectors span
.
The cross product is defined by the formula:

$$\mathbf{a} \times \mathbf{b} = \|\mathbf{a}\| \, \|\mathbf{b}\| \sin(\theta) \, \mathbf{n}$$



where θ is the angle between **a** and **b** in the plane containing them. (hence, it is between 0° and 180°), ‖**a**‖ and ‖**b**‖ are the magnitudes of vectors **a** and **b**, and **n** is a unit vector perpendicular to the plane containing **a** and **b**, in the direction given by the right-hand rule.

$$\begin{aligned}\mathbf{A}\times\mathbf{B} &= (A_x\mathbf{i}+A_y\mathbf{j}+A_z\mathbf{k})\times(B_x\mathbf{i}+B_y\mathbf{j}+B_z\mathbf{k})\\ &= A_xB_x(\mathbf{i}\times\mathbf{i})+A_xB_y(\mathbf{i}\times\mathbf{j})+A_xB_z(\mathbf{i}\times\mathbf{k})\\ &\quad+A_yB_x(\mathbf{j}\times\mathbf{i})+A_yB_y(\mathbf{j}\times\mathbf{j})+A_yB_z(\mathbf{j}\times\mathbf{k})\\ &\quad+A_zB_x(\mathbf{k}\times\mathbf{i})+A_zB_y(\mathbf{k}\times\mathbf{j})+A_zB_z(\mathbf{k}\times\mathbf{k})\end{aligned}$$

$$\mathbf{A}\times\mathbf{B}=(A_yB_z-A_zB_y)\mathbf{i}-(A_xB_z-A_zB_x)\mathbf{j}+(A_xB_y-A_yB_x)\mathbf{k}$$

Direction

The direction of the vector product can be visualized with the right-hand rule. If you curl the fingers of your right hand so that they follow a rotation from vector A to vector B, then the thumb will point in the direction of the vector product. The vector product of A and B is always **perpendicular to both A and B**.

IV. Introduction to MATLAB for Vector Products [vi]

MATLAB

MATLAB, short for MATRIX LABORATORY is a programming package specifically designed for quick and easy scientific calculations. It has many built-in functions for a wide variety of computations and many toolboxes designed for specific research disciplines, including statistics, plotting of graphs, optimization, solution of partial differential equations, matrix calculations, data analysis.

Let us use MATLAB commands to calculate vector product.

Objectives of Task 01

1. Proving parallel vectors give null vector as vector product.
2. Finding angle between 2 vectors using dot product and vector product.

Command Window

```
>> %Let us prove parallel vectors give
%NULL VECTOR as vector product
A=[2 0 0];
B=[1 0 0];
%computing cross product
C = cross(A,B)

C =

     0     0     0

>> %Let us find angle between two vectors using vector product:
>> a=[2 3 4];
>> b=[5 7 9]

b =

     5     7     9

>> angle = atan2(norm(cross(a,b)), dot(a,b))

angle =

    0.0365

>> %Covert to degrees
>> angle=angle*180/pi;
>> angle

angle =

    2.0938
```

Proof of Parallel Vector giving Null Result & Angle Formulation

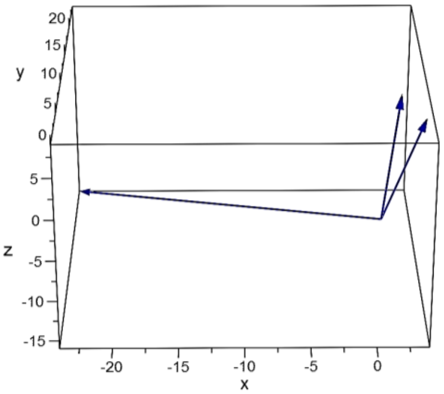
Objectives of Task 02

1. Calculating vector product of two vectors and finding the magnitude.
2. Graphically representing the vector product using MuPAD.

```
Command Window
>> %Graphical representation of vector product
A=[2 8 9]

B=[4 8 6];
C=cross(A,B)


%Let us calculate magnitude of Vector product C
magnitude_C=norm(C)


%Graphical representation


A =

     2     8     9


C =

   -24    24   -16


magnitude_C =

   37.5233

>> |
```

Graphical Representation of Vector Product

```
plot(plot::Arrow3d([2,8,9]), plot::Arrow3d([4,8,6]), plot::Arrow3d([-24,24,-16]))
```



Graphical Plot

An Easy Tool for Vector Product

MATLAB is the easiest computing environment for handling arithmetic operations, computational functions of matrices and other elements of linear algebra, technical computation. Different features and input functions offered by this programming tool have helped us in our lab to efficiently show graphical representations of functions. MATLAB behaves like a calculator and also helped me to use matrix operations.

Efficiency

MATLAB will report an error when I use a reserved word as variable or commit a syntax error.

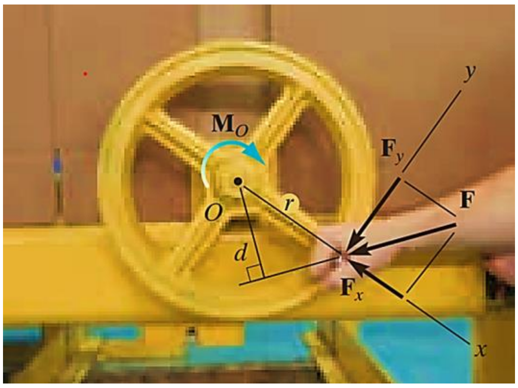
It is interpreted, errors are easier to fix. It is relatively easier to learn where problems and solutions are expressed in familiar mathematical notation.

V. Applications of Moment of Force in our Daily Lives

When a force is applied to a body it will produce a tendency for the body to rotate about a point that is not on the line of action of the force. This tendency to rotate is sometimes called a ***torque***, but most often it is called the moment of a force or simply the ***moment***. This concept comes to our very own use in our daily lives and makes our work far easier. Following are some applications of moment of force used in our daily life:

Heavy Vehicle Steering

The concept of torque has helped a lot in case of a steering of a truck. A truck or any other heavy vehicle has a lot of its own weight besides the one it carries on its rear. It becomes difficult for a truck driver to turn a steering n a sharp cut or a U-turn if it has a steering similar in size to the ones used in cars. For a large amount of force, a steering of larger radius is used to easily rotate the steering and turn the wheels of the vehicle. Hence grater the radius lesser the force is needed to produce the required torque.



System of Seesaw

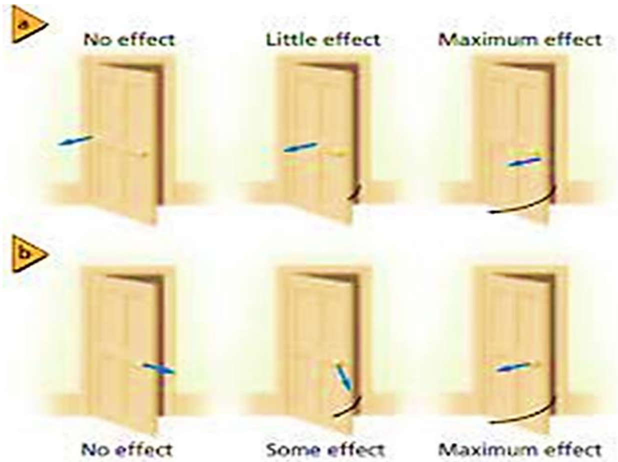
The seesaw seen in the parks rotates on and off the ground due to the torque imbalance. The greater the torque, the greater the tendency of the object to be put in rotation. A seesaw can never give a close rotation to 360°, but the board or the plank gives a relatively narrow parameter.

Example:

Suppose the child weighs 50 lb (11.24 N) and sits 3 ft (0.91 m) from the pivot point, giving her side of the seesaw a torque of 150 pound-feet (10.28 Nm). On the other side, her teenage sister weighs 100 lb (22.48 N) and sits 6 ft (1.82 m) from the centre, creating a torque of 600 pound-feet (40.91 Nm). As a result of the torque imbalance, the side holding the teenager will rotate clockwise, toward the ground, causing the child's side to also rotate clockwise—off the ground.

Door Knob

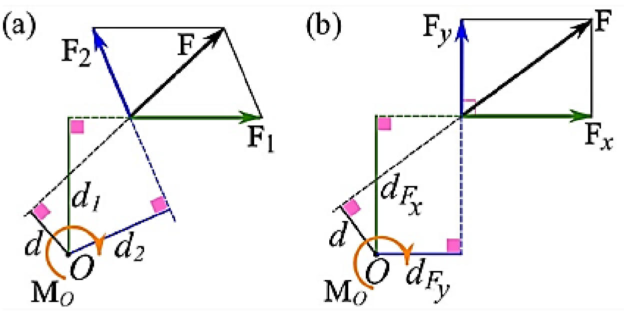
A door knob is placed as far from the pivot point as much it can to give greater moment of force and the door can easily be opened and close. Suppose if the moment arm is too small for the door it would need an extreme amount of force or otherwise lame person cannot easily close the door. Neither will we be applying a force parallel to the moment arm as it will cause no movement or rotation across the pivot.



VI. Moment of a Force: Vector Formulation

Having both a magnitude and a direction, a moment is in fact a vector quantity. For two-dimensional (or coplanar) problems, where all forces (and pivot points) lie in a plane, all moment vectors are perpendicular to the plane (and along the z axis). To make a moment vector easily seen; it is shown by a curled arrow. By convention, a counter clockwise rotation is associated with a positive moment, and a clockwise rotation with a negative moment. This sign convention is not mandatory and either direction can be set as the positive direction. However, it is important that the positive direction selected is consistent through solving a problem.

Calculating the moment of a force about a point usually becomes simpler when using the Cartesian components of the force and with principle of moments. For example, the following equation holds for the force and its Cartesian components shown. In **(a)** *a number of forces produce moment about O while in* **(b)** *resolving forces into components makes easier the*
calculation of moment.



The formula used is:

$$\circlearrowleft +\boldsymbol{M_o} = -\boldsymbol{Fd} = -\boldsymbol{F_x d_{F_x}} + \boldsymbol{F_y d_{F_y}}$$

Principle of Transmissibility

It indicates that the situation of balance or movement of a rigid body does not change if a certain force acting on a particular point of the body is replaced by another. For this to be considered, two premises must be fulfilled.

The first premise is that the new force is of the same magnitude, and the second is that the same direction is applied, even if it is on a different point of the body. The two forces have the same result on a rigid body; therefore, they are equivalent forces.

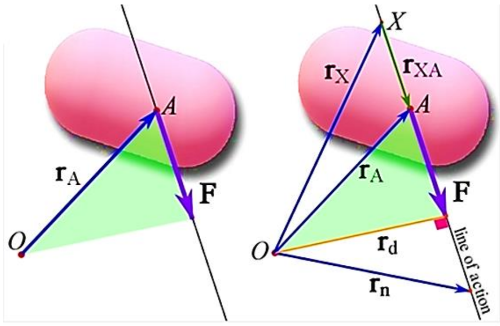
Proof vii

Suppose a body and a force F acting at a point A of the body. Consider a point O anywhere in the space (may also be a point of the body) but not on the line of action of F. If the moment arm vector $\boldsymbol{r_a}$ is considered, the moment is vector product of r and F. Similarly, the moment of F about point O is $\boldsymbol{M_{OA}}$. If any other point x on the line of action is chosen to construct a moment arm vector as $\boldsymbol{r_x}$. Our purpose is now to prove:

$$\mathrm{M}_O^X = \mathrm{M}_O^A.$$

The key to this proof is noting that there exist a position (displacement) vector $\boldsymbol{r_{XA}}$ on the line of action such that:

$$\mathbf{r}_A = \mathbf{r}_X + \mathbf{r}_{XA}$$



The proof uses the distributivity of the cross product.

$$\mathrm{M}_O^A = \mathbf{r}_A \times \mathbf{F} = (\mathbf{r}_X + \mathbf{r}_{XA}) \times \mathbf{F} =$$

$$(\mathbf{r}_X \times \mathbf{F}) + (\mathbf{r}_{XA} \times \mathbf{F}) = (\mathbf{r}_X \times \mathbf{F}) + (0) = \mathrm{M}_O^X$$

The term $\boldsymbol{r_A} \times \boldsymbol{F}$ is zero because the vectors are collinear, or the angle between them is zero. Since the above proof holds for any two moment arm vectors (Figure shows some possibilities), the moment is independent of the location of O being on the line of action.

VII. Visualization of Vector Product Using Vector Card Experiment

Abstract of Experiment

The objective of this research was to offer the way to improve engineering students in Physics topic of vector product. Since topic of vector product is abstract, we have devised a way to visualize the vector product of two vectors using three-dimension vector card.

Introduction

We may not find it interesting or useful because vector product in physics is very abstract. The right-hand rule can thoroughly be understood by this method. Let us solve the ambiguities that arise in topic of vector product.

1. Misconception in the magnitude of the unit vector of vector (say A) and in the *x*- and *y*-components of this vector A.
2. Confusion between the unit vector in the direction of vector **A** with the two component vectors of the vector **A** written in the unit-vector notation.

Three dimensions vector cards assist students to understand in the Right-Hand Rule and it is the aid for the direction of the vector product.

Methods and Materials

The cross-product **A x B** is defined as a vector **C** that is perpendicular to plane containing A and B given by the right-hand rule and a magnitude equal to the area of the parallelogram that the vectors span. viii

The cross product is defined by the formula

$$\mathbf{A x B = | A| |B |sin}\ \boldsymbol{\alpha}$$

where $\boldsymbol{\alpha}$ is the angle between vectors in the plane, it lies between 0° and 180°.

The vector product is not commutative; for any two vectors.

$$\boldsymbol{A \times B} = -\boldsymbol{B \times A}$$

In Cartesian form:

$$\boldsymbol{C} = \boldsymbol{AB \sin\theta}$$

$$\boldsymbol{C_x} = \boldsymbol{A_yB_z - A_zB_y}$$

$$\boldsymbol{C_y} = \boldsymbol{A_zB_x - A_xB_z}$$

$$\boldsymbol{C_z} = \boldsymbol{A_xB_y - A_yB_x}$$

Required Setup is:

Let us make a three-dimensional size card. The three dimensions, size card is:

1) *Paper in 9 cm x 12 cm (double size of ID card)*
2) *Paper in 11.5 cm x 16 cm (half of note book)*
3) *Paper in 15 cm x 21 cm (half of A-4 paper size)*

Methodology and Procedure

In experiments to determine the dimensions of the three dimensions vector card by choosing from three sizes and the most popular will be used to create a three dimensions vector card to use in the vector product classroom. The methodology and experimental is divided into following stages:

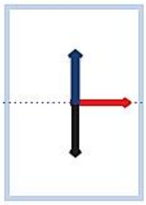
Stage 1: Assembling required set up

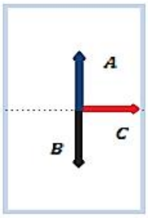
Stage 2: Making the three dimensions vector card and using it for visualization.

The steps make three dimensions vector card.

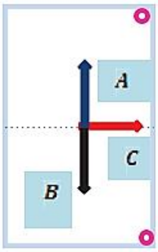
1. Cut a paper from reused candy box notebook etc. and half along it.

2. Draw arrows to represent vectors in three-dimensional space and label as **A, B, C.**





3. The pink dots are placed on cards to help understand orientation of axes.



The students can fold the card in the direction from vector A (blue arrow) to vector B (black arrow) the result is the direction of the vector C (red arrow), can used in the angle α between 0° to 90°. The pink dots are placed on the cards to assist students in orienting the axis.

Learning Objectives

The objective of experiment is to help improve teaching and learning in the physics in vector product lesson. The Right-Hand Rule is an important mnemonic technique to understand in three dimensions vector but difficult to use and difficult to alignment of their hand and fingers, unusual to watch and difficult to apply.

- In this experiment the three dimensions vector card that the student created can help the student understand in vector product lesson. The three dimensions vector size card with 11.5 cm x 16 cm is better useful and applied.
- This study was taken from the remaining material to create a tool to assist students understand the three dimensions vector better , in future could have to a reusable material used in other lessons of physics, such as wood cube or metal cube used to make three dimensions vector. The instructor uses repeats it in many times for the sake of the classroom environment and reduce global warming.
- The three dimensions vector card is simply created, low cost and uses recycled materials. In this reason

the instruction in physics and mathematics is used the three dimensions cards to assistant in the Right-Hand Rule and vectors in three dimensions.

- ❖ The three dimensions vector card is applied to the study of three dimensions vector in the other, for example finding the direction of the magnetic field and force, the concepts of torque and angular momentum.

Inferences

Effect of a force not only depends on its magnitude and direction, but also depends on its point of application. Force is responsible for translational as well as rotational motion of an object. Let a force F be applied at a point A as shown in figure above. Position vector of A with respect to B is given by $\boldsymbol{r_{AB}}$. Therefore, moment of the force F at A with respect to point B is given by :

$$M_B = r_{AB} \times F$$

The magnitude of MB is a measurement of rotating tendency of the object about B. Its SI unit is Newton-metre (Nm).

Key Points:

- M_B is a vector perpendicular to the plane containing $\mathbf{r_{AB}}$ and **F**.
- Direction determined by right hand thumb rule.
- Rotation is anti-clockwise if $\mathbf{M_B}$ points outward to the plane and vice-versa.
- $|\mathbf{M_B}| = |\mathbf{F}||\mathbf{r_{AB}}| \sin\alpha$, where α is the angle between force and position vector.
- In case of multiple forces acting at point B, moment of the resultant force is given by sum of individual moment of forces about point B.

$$M_B = r \times (F_1 + F_2 \ldots) = r \times F_1 + r \times F_2 + \ldots$$

- For an object with zero rotational motion the net moment of forces about every point should be zero.

VIII. Principle of Moments

A concept often used in mechanics is the principle of moments. Which is sometimes referred to as Varignon's theorem since it was originally developed by the French mathematician Pierre Varignon (1654–1722).

Statement 1

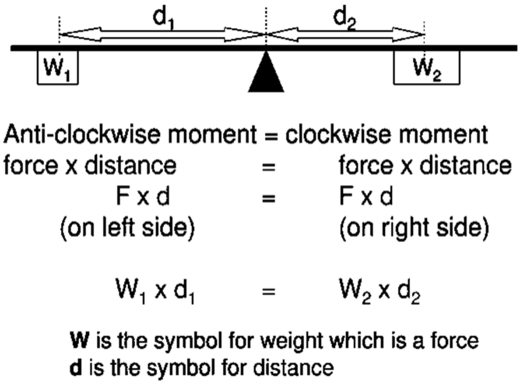
The first and foremost part of the Varignon's Theorem is proceeded as:

"The moment of a force about a point is equal to the sum of all the moments of the components of the force about that point."

Statement 2

The other part of the principle is specified as:

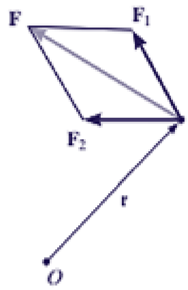
"A body is balanced/in Equilibrium if Sum of clockwise moments is equal to sum of anti-clockwise moments."



This theorem can be proven easily using the vector cross product since the cross product obeys the distributive law. There are two methods of analysis which are discusses below:

Scalar Approach

For example, consider the moments of the force F and two of its components about point O, in the given figure:



Since $F = F_1 + F_2$

$$M_0 = r * F = r * (F_1 + F_2) = r * F_1 + r * F_2$$

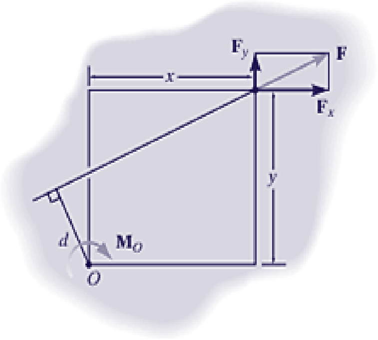
Vector Approach

For two-dimensional problems, Fig. 4–17, we can use the principle of moments by resolving the force into its rectangular components and then determine the moment using a scalar analysis.

Thus,

$$M_0 = F_X\, y - F_Y\, x$$

This method is generally easier than finding the same moment using $\boldsymbol{M_O = Fd}$.



Tips to Solve Examples:

• The moment of a force creates the tendency of a body to turn about an axis passing through a specific point O.

• Using the right-hand rule, the sense of rotation is indicated by the curl of the fingers, and the thumb is directed along the moment axis, or line of action of the moment.

• The magnitude of the moment is determined from **M_O = Fd**, where d is called the moment arm, which represents the perpendicular or shortest distance from point O to the line of action of the force.

• In three dimensions the vector cross product is used to determine the moment, i.e., **M_O = r * F**. Remember that r is directed from point O to any point on the line of action of F.

IX. Moment of a Force: Specified Axis

The moment of a force does not always line up along the desired axis that we are concerned about. Consider the vault in the figure, the moment does not fully line up along the axis of the hinge. Only a specific component of the total moment acts along the hinge axis which is responsible for producing the rotation. Whereas the ones that do not lie on the hinge axis are essentially producing reaction moments in the hinge. Thus, to find only the rotation of the vault, we may find the moment acting on the hinge axis. This is best done through techniques specified in this section, that is, moment about a specific axis.

To determine the magnitude of the moment along the desired axis, we can either use scalar or vector analysis. It is generally preferred to use vector analysis when given information is in cartesian coordinates, but either option is viable and ultimately leads to the same result.

Scalar Analysis

We utilize the same concepts that we used in finding scalar analysis i.e., multiplying force with the perpendicular distance to the axis along which we want to find the moment.

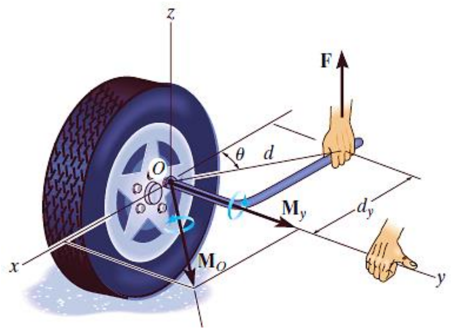
In the given figure,

Magnitude of M_O is simply calculated by multiplying the perpendicular **distance d** with the **force F**. However, since we desire to find the moment causing the rotation, we calculate the moment along Y-axis. The perpendicular distance between the line of action of force and Y-axis can be calculated through laws of trigonometry. That is,

$$d_Y = d\cos(\theta)$$

which when multiplied with the **force F** gives us the magnitude of component of moment along Y-axis. s

$$M_Y = Fd_Y = Fd\cos(\theta)$$



Vector Analysis

Moving on to the vector analysis to find the component moment, we will find that it is advantageous to use this analysis when angles are not given or difficult to find. In this method, we first determine the Moment about any given point O on the Y-axis through cross product of the **position vector r** and the force **F**. This is actually giving us the Moment about point O. We can find the projection of M_O along the Y-axis by using a simple dot product. Surmising the aforementioned steps, we get:

$$\boldsymbol{M_A} = \boldsymbol{u_A} \cdot (\boldsymbol{r} \times \boldsymbol{F})$$

$$M_a = [u_{a_x}\mathbf{i} + u_{a_y}\mathbf{j} + u_{a_z}\mathbf{k}] \cdot \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ r_x & r_y & r_z \\ F_x & F_y & F_z \end{vmatrix}$$

Expanding this scalar triple product gives us the following expression:

$$\boldsymbol{M_A} = u_{a_x}(r_yF_z - r_zF_y) - u_{a_y}(r_xF_z - r_zF_x) + u_{a_z}(r_xF_y - r_yF_x)$$

Direction

The above analysis gives us the magnitude of the moment and not the direction. We can express the **M_A** in cartesian form by multiplying ***M_A*** (*magnitude*) with the unit vector in the direction of the specified axis.

Mathematically,

$$\mathbf{M}_A = M_A * \mathbf{u}_A$$

X. Summary

Lastly, for the culmination of this report we aim to discuss and make deductions that briefly sum up whole of data and information given in the previous sections.

Conclusions

Considering the vast applications of moment of force, the report highlights all mathematical and practical aspects of the turning effect of forces. Each section includes a relevant example that directs the focus on what the reader may expect while on that particular section, along with this aspect, theorems have also been proved mathematically and figuratively to devise a better and comprehensive approach for problem solving.

Discussions

All of the aforementioned results suggest that moment of a force can be solved for a system using distinct and various methods with each being highly consistent towards yielding a similar solution. It is observed vector product plays a significant role.

Using MATLAB and practical experiments, vector product can be conveniently visualized. Our research focuses on all the key topics such as visualization of vector product calculation of moment about a specified axis.

Vectors in 2D can be conveniently solved by scalar analysis. However, once we deal with vectors in three dimensions, vector product is a better approach to tackle problems.

In real life situations and more often than not, a moment is not always acting completely and fully along the axis about which it is rotating. Such a situations calls for finding Moment about specified axis and the methods that deal with it have already been discussed earlier. This particularly helps in reinforcing the hinge when the amount of force may be near or exceeding the amount it can bear without becoming defective and dented.

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