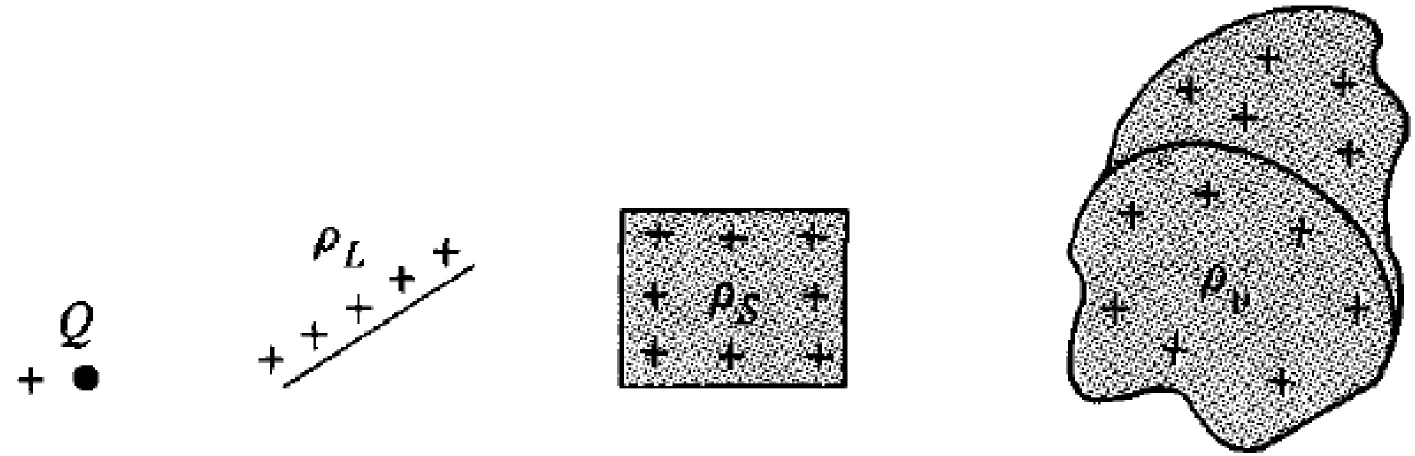
ELECTRIC FIELDS DUE TO CONTINUOUS CHARGE DISTRIBUTIONS - LINE CHARGE

Introduction

- So far we have only considered forces and electric fields due to point charges, which are essentially charges occupying very small physical space

- It is also possible to have continuous charge distribution along a line, on a surface, or in a volume, as shown below:



Point charge | Line charge | Surface charge | Volume charge

Introduction

- It is customary to denote the line charge density, surface charge density, and volume charge density by ρ_L (in C/m), ρ_s (in C/m^2), and ρ_v (in C/m^3), respectively

- These must not be confused with ρ (without subscript) used for radial distance in cylindrical coordinates

- The charge element dQ and the total charge Q due to line charge distribution is obtained as:

$$dQ = \rho_L \, dl \rightarrow Q = \int_L \rho_L \, dl \qquad \text{(line charge)}$$

Line Charge Distribution

- Practical example of a line charge distribution is a charged conductor of very small radius and a sharp beam in a cathode-ray tube

- In the case of the electron beam the charges are in motion and it is true that we do not have an electrostatic problem

- However,

1. If the electron motion is steady and uniform (a DC beam) and
2. If we ignore for the moment the magnetic field which is produced

- The electron beam may be considered as composed of stationary electrons

Line Charge Distribution

- The equation for electric field due to point charge is:

$$\mathbf{E} = \frac{Q}{4\pi\varepsilon_o R^2}\mathbf{a}_R = \frac{Q(\mathbf{r} - \mathbf{r}')}{4\pi\varepsilon_o|\mathbf{r} - \mathbf{r}'|^3}$$

- The electric field intensity due to line charge distribution ρ_L may be regarded as the summation of the field contributed by the numerous point charges making up the charge distribution

- Thus by replacing Q in the equation with charge element $dQ = \rho_L\, dl$, we get:

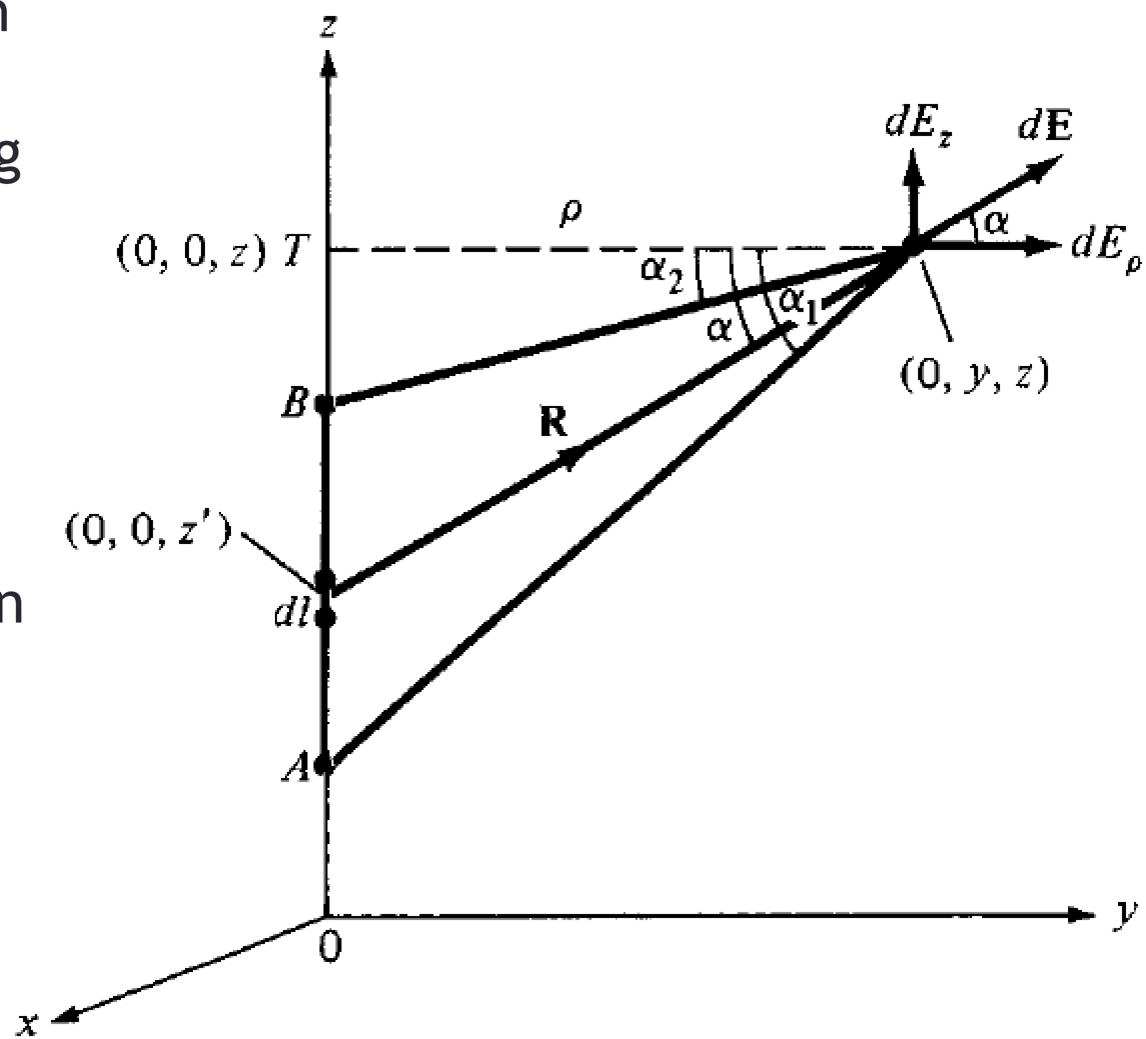
$$\mathbf{E} = \int \frac{\rho_L\, dl}{4\pi\varepsilon_o R^2}\mathbf{a}_R \qquad \text{(line charge)}$$

- We shall now apply this formula to line charge distribution

Line Charge Distribution

- Consider a line charge with uniform charge density ρ_L extending from A to B along the z-axis as shown in figure below:

- Since the field does not vary with a variation in Φ, for simplicity, we choose an arbitrary point *P(0,y,z)* to find the electric field intensity at



Line Charge Distribution

- We will denote the field point by (x, y, z) and the source point by (x', y', z')
- We have from the figure: $dl = dz'$

$$\mathbf{R} = (0, y, z) - (0, 0, z') = y\mathbf{a}_y + (z - z')\mathbf{a}_z$$

- Converting to cylindrical coordinates gives:

$$\mathbf{R} = \rho\mathbf{a}_\rho + (z - z')\,\mathbf{a}_z$$

- Therefore:

$$\frac{\mathbf{a}_R}{R^2} = \frac{\mathbf{R}}{|\mathbf{R}|^3} = \frac{\rho\mathbf{a}_\rho + (z - z')\mathbf{a}_z}{[\rho^2 + (z - z')^2]^{3/2}}$$

- By substitution into the equation for **E**, we get:

$$\mathbf{E} = \frac{\rho_L}{4\pi\varepsilon_o}\int \frac{\rho\mathbf{a}_\rho + (z - z')\,\mathbf{a}_z}{[\rho^2 + (z - z')^2]^{3/2}}\,dz'$$

Line Charge Distribution

- To evaluate the integral, it is convenient to define $\propto$, $\propto_1$ and $\propto_2$ shown in the figure

- We get the following relations from the figure:

$$R = \left[\rho^2 + (z - z')^2\right]^{1/2} = \rho \sec \alpha$$

$$z' = OT - \rho \tan \alpha, \qquad dz' = -\rho \sec^2 \alpha \, d\alpha$$

- By substitution, the integral becomes:

$$\mathbf{E} = \frac{-\rho_L}{4\pi\varepsilon_o} \int_{\alpha_1}^{\alpha_2} \frac{\rho \sec^2 \alpha \, [\cos \alpha \, \mathbf{a}_\rho + \sin \alpha \, \mathbf{a}_z] \, d\alpha}{\rho^2 \sec^2 \alpha}$$

$$= -\frac{\rho_L}{4\pi\varepsilon_o\rho} \int_{\alpha_1}^{\alpha_2} [\cos \alpha \, \mathbf{a}_\rho + \sin \alpha \, \mathbf{a}_z] \, d\alpha$$

Line Charge Distribution

- Thus, for a finite line charge, we have:

$$\mathbf{E} = \frac{\rho_L}{4\pi\varepsilon_o\rho}[-(\sin\alpha_2 - \sin\alpha_1)\mathbf{a}_\rho + (\cos\alpha_2 - \cos\alpha_1)\mathbf{a}_z]$$

- As a special case, for an infinite line charge, point B is at $(0,0,\infty)$ and A at $(0,0,-\infty)$

- So $\propto_1 = \frac{\pi}{2}$, $\propto_2 = -\frac{\pi}{2}$ and the z-component vanishes (How?)

- The above equation reduces to the equation below:

$$\mathbf{E} = \frac{\rho_L}{2\pi\varepsilon_o\rho}\mathbf{a}_\rho$$

- ρ is the perpendicular distance from the line to the point of interest

Problem-1

- A circular ring of radius *a* carries a uniform charge ρ_L C/m and is placed on the xy-plane with axis the same as the z-axis. Calculate the electric field intensity **E** on z-axis at a distance of *h* from the origin.