National University of Sciences and Technology (NUST)
School of Electrical Engineering and Computer Science

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Semester: 6th

Section: BEE 12C

EE-330 Digital Signal Processing

Lab 2: Complex Exponentials and Sinusoids

Group Members

		PLO4 - CLO4		PLO5 - CLO5	PLO8 - CLO6	PLO9 - CLO7
Name	Reg. No	Viva / Quiz / Lab Performance	Analysis of data in Lab Report	Modern Tool Usage	Ethics and Safety	Individual and Teamwork
		5 Marks	5 Marks	5 Marks	5 Marks	5 Marks
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2 Complex Exponentials and Sinusoids

2.1 Objectives

The goal of this Part is to gain familiarity with complex numbers and their use in representing sinusoidal signals such as $x(t) = Acos(\omega t + \emptyset)$ As complex exponentials $z(t) = Ae^{j\theta}e^{j\omega t}$. The key is to use the appropriate complex amplitude together with the real part operator as follows:

$$x(t) = Acos(\omega t + \emptyset) = Real\{Ae^{j\theta}e^{j\omega t}\}$$

- How to work with Complex Numbers in MATLAB
- Familiarization with MATLAB function and commands for complex exponentials
- Sinusoid addition using complex exponentials.

2.2 Introduction

Complex numbers are a mathematical concept that involves a combination of a real part and an imaginary part. In signal processing, complex numbers are useful for analyzing signals that have a sinusoidal waveform. These signals can be expressed as complex exponentials $z(t) = Ae^{j\theta}e^{j\omega t}$ by using the appropriate complex amplitude and the real part operator.

We will begin by reviewing the properties of complex numbers and how they are used to represent sinusoidal signals. We will then explore MATLAB functions and commands for complex exponentials, including how to perform basic operations such as addition, subtraction, and multiplication. In addition to gaining familiarity with complex numbers and their use in representing signals, we will also learn how to use MATLAB to perform operations on these signals.

2.3 Software

MATLAB is a high-level programming language and numerical computing environment. Developed by MathWorks, it provides an interactive environment for numerical computation, visualization, and programming. MATLAB is widely used in various fields, including engineering, science, and finance, due to its capabilities for matrix and vector operations, implementation of algorithms, and creation of graphical representations of data.

2.4 Lab Report Instructions

All questions should be answered precisely to get maximum credit. Lab report must ensure following items:

- Lab objectives
- MATLAB codes
- Results (graphs/tables) duly commented and discussed
- Conclusion

3 Lab Procedure

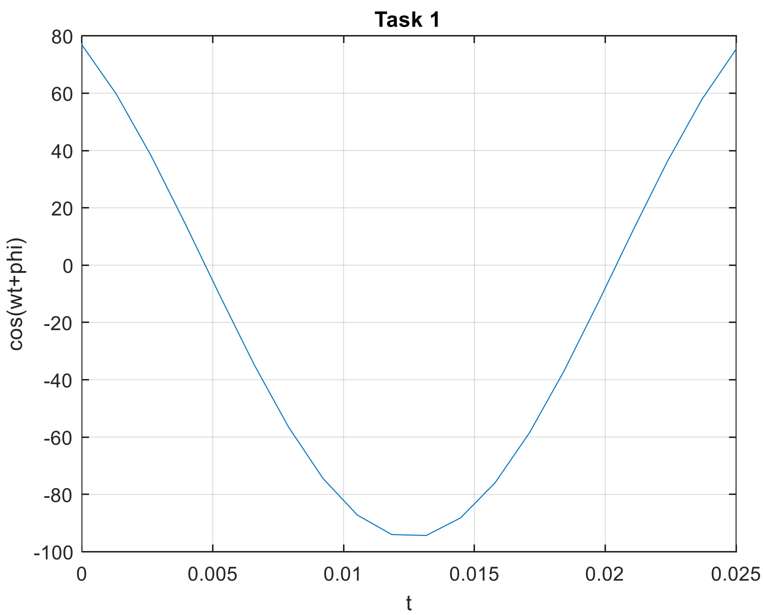
3.1 M-file to Generate a Sinusoid

Write a function that will generate a single sinusoid, $x(t) = A\cos(wt + \phi)$, by using four input arguments: amplitude (A), frequency (ω), phase (ϕ) and duration (dur). The function should return two outputs: the values of the sinusoidal signal (x) and corresponding times (t) at which the sinusoid values are known. Make sure that the function generates 20 values of the sinusoid per period. Call this function one cos().

Demonstrate that your one_cos() function works by plotting the output for the following parameters: A = 95, ω = 200 rad/sec, ϕ = π/5 radians, and dur = 0.025 seconds. Be prepared to explain to the lab instructor features on the plot that indicate how the plot has the correct period and phase. What is the expected period in millisec?

```
function one_cos(A, w, phi, dur)
    t = linspace(0, dur, 20);
    x_t = A * cos(w * t + phi);
    plot(t, x_t)
    xlabel('t')
    ylabel('cos(wt+phi)')
    title('Task 1')
    grid on
end

one_cos(95, 200, pi / 5, 0.025)
```




```
w = 200 rad/sec → w = 2πf → f = 200/2π = 100/π
T = 1/f → 0.0314 → 31.4 ms
```

3.2 Sinusoidal Synthesis with an M-file: Different Frequencies

Write an M-file called syn_sin.m that will synthesize a waveform in the form of:

$$x(t) = \Re e(\sum_{k=1}^{N} X_k e^{j2\pi f_k t})$$

Although 'for' loops are rather inefficient in MATLAB but you must write the function with one loop in this lab. The first few statements of the M-file are the comment lines—they should look like:

```
function [xx,tt] = syn_sin(fk, Xk, fs, dur, tstart)
%SYN_SIN Function to synthesize a sum of cosine waves
% usage:
% [xx,tt] = syn_sin(fk, Xk, fs, dur, tstart)
% fk = vector of frequencies
% (these could be negative or positive)
% Xk = vector of complex amplitudes: Amp*e^(j*phase)
% fs = the number of samples per second for the time axis
% dur = total time duration of the signal
% tstart = starting time (default is zero, if you make this input optional)
% xx = vector of sinusoidal values
% tt = vector of times, for the time axis
% Note: fk and Xk must be the same length.
% Xk(1) corresponds to frequency fk(1),
% Xk(2) corresponds to frequency fk(2), etc.
```

```
function [x_t, t] = syn_sin(fk, Xk, fs, dur, tstart)
    % syn_sin: Function to synthesize a sum of cosine waves
    % If tstart is not specified, default tstart = 0
    if nargin < 5
        tstart = 0;
    end

    t = linspace(tstart, dur - abs(tstart), fs);

    assert(length(fk) == length(Xk), "Input fk and Xk must " + ...
    "have the same length.")

    x_t = 0;

    for k = 1:length(fk)
        x_t = x_t + real(Xk(k) * exp(2 * pi * 1i * fk(k) * t));
    end

end
```

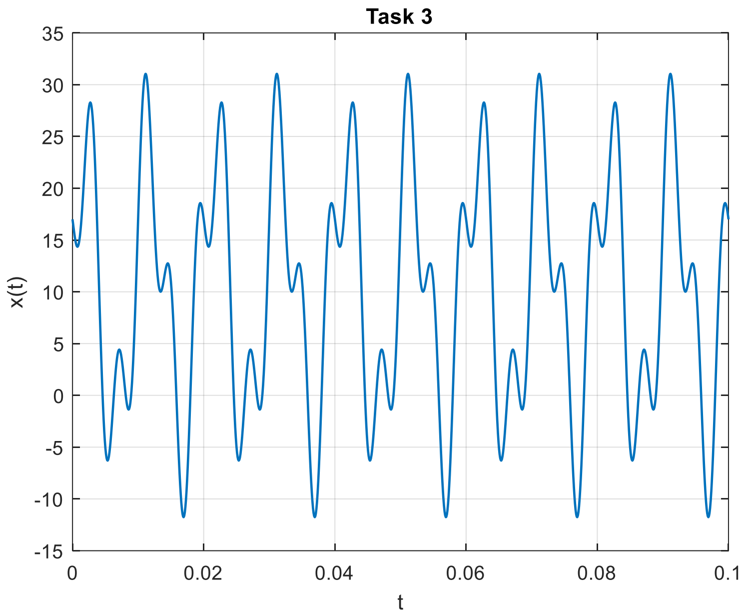
3.3 Testing Sinusoidal Synthesis

To use the M-file Section 0 to synthesize harmonic waveforms, you must choose the entries in the frequency vector to be integer multiples of some desired fundamental frequency.

```
[xx0, tt0] = syn_sin([0, 100, 250], [10, 14*exp(-j*pi/3), 8*j], 10000, 0.1, 0);
%-Period =?
```

Measure the period of xx0 by hand. Then compare the period of xx0 to the periods of the three sinusoids that make up xx0 and write an explanation on the verification sheet of why the period of xx0 is longer.

The period of xx0 is larger than the signals which add up to it; the period of signal with f = 100 Hz is 10 ms, with f = 250 Hz is 4 ms and the resulting signal has a period which is the LCM of the components' fundamental period which is 20m seconds (as is verifiable by the output plot)



3.4 Representation of Sinusoids with Complex Exponentials

a) Generate the signal $x(t) = \Re e\{2e^{j\pi t} + 2e^{j\pi(t-1.25)} + (1-j)e^{j\pi t}\}$ and make a plot versus t. Use the syn sin function and take a range for t that will cover three periods starting at t = −0.5 secs. *Include the* MATLAB *code with your report.*

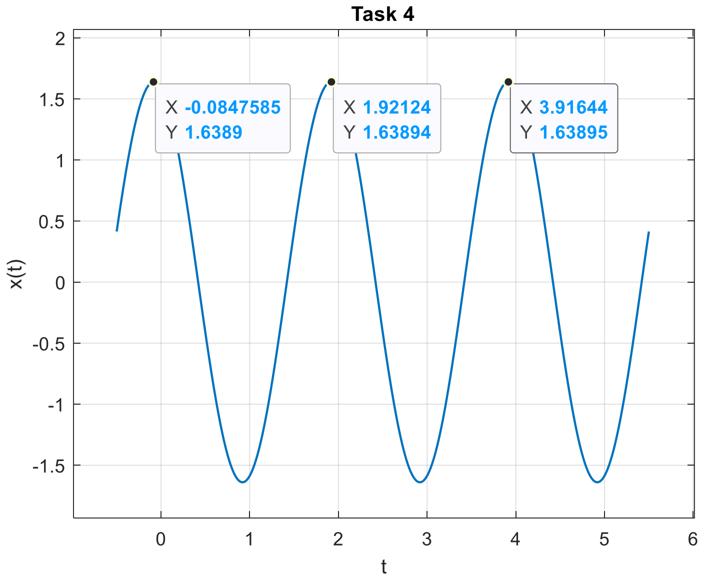
```
f = 1/2 Hz
T = 2 s
tstart = -0.5
range for three periods = 2 s * 3 = 6 s
dur = -0.5 + 6 = 5.5 s
```

b) From the plot of x(t) versus t, measure the frequency, phase and amplitude of the sinusoidal signal by hand. Show annotations on the plots to indicate how these measurements were made and what the values are.

```
X1 = 2; X2 = 2 * exp(1i * pi * (-1.25)); X3 = (1 - 1i);
[x, t] = syn_sin([1/2, 1/2, 1/2], [X1, X2, X3], 10000, 6, -0.5);

plot(t, x)
```

```
ylabel('x(t)')
xlabel('t')
title('Task 4')
grid on
```



$A = 1.64$

$T = T_2 - T_1 = 3.91 - 1.92 = \mathbf{1.99\ s}$

$\phi = Shift\ from\ 0 = 360° \dfrac{\Delta T}{T} = 15.26°, where\ \Delta T = 0.08475\ (from\ the\ plot)$

4 Conclusion

In this lab, we have gained familiarity with complex numbers and their use in representing sinusoidal signals. We have learned how to use complex exponentials to represent sinusoidal signals and how to manipulate them using MATLAB functions and commands. We have also learned how to use MATLAB to add sinusoids using complex exponentials and visualize the resulting signals. By working through the examples provided in this lab, we have gained an understanding of the relationship between complex numbers and sinusoidal signals. In addition, we have learned how to use MATLAB to visualize signals and gain insights into their behavior. This is an important skill in many signal processing applications, where it is essential to understand the behavior of signals to make informed decisions.