→ EE - 371 - Assignment # 1

CLO - 1

Name: Muhammad Umer

Class: BEE - 12C

CMS: 345834

Q # 1

$$\frac{Y(s)}{X(s)} = ? \quad \text{of} \quad \left\{ \frac{d^3y}{dt^3} + 3\frac{d^2y}{dt^2} + 5\frac{dy}{dt} + y = \frac{d^3x}{dt^3} + 4\frac{d^2x}{dt^2} + \ldots \right.$$

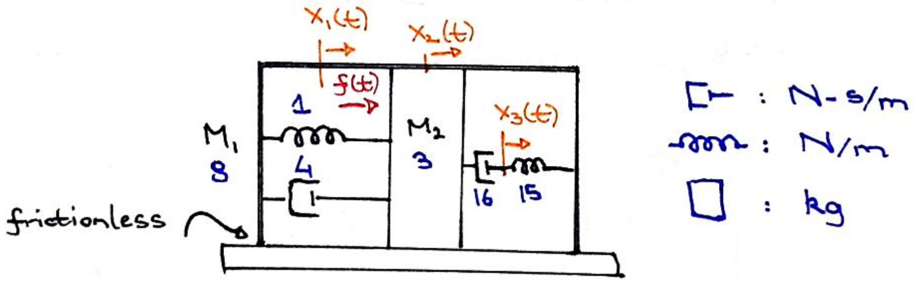
$$\left. \ldots \frac{6dx}{dt} + 8x \right\}$$

Taking the laplace transform;

$$\Rightarrow (s^3 + 3s^2 + 5s + 1)\,Y(s) = (s^3 + 4s^2 + 6s + 8)\,X(s)$$

$$\Rightarrow \boxed{\frac{Y(s)}{X(s)} = \frac{s^3 + 4s^2 + 6s + 8}{s^3 + 3s^2 + 5s + 1}}$$

Q # 2



Degree of freedom = 3 = Number of equations

$$\begin{cases} \text{i}: & X_1(s)(8s^2 + 4s + 16) - X_2(s)(4s + 1) - X_3(s)(15) = 0 \\ \text{ii}: & -X_1(s)(4s + 1) + X_2(s)(3s^2 + 20s + 1) - X_3(s)(16s) = F(s) \\ \text{iii}: & -X_1(s)(15) - X_2(s)(16s) + X_3(s)(16s + 15) = 0 \end{cases}$$

In matrix form.

$$\underbrace{\begin{bmatrix} 8s^2+4s+16 & -(4s+1) & -15 \\ -(4s+1) & 3s^2+20s+1 & -16s \\ -15 & -16s & 16s+15 \end{bmatrix}}_{\Delta} \underbrace{\begin{bmatrix} X_1(s) \\ X_2(s) \\ X_3(s) \end{bmatrix}}_{X} = \underbrace{\begin{bmatrix} 0 \\ F(s) \\ 0 \end{bmatrix}}_{Y}$$

$$|\Delta| : \left[(8s^2+4s+16)\left((3s^2+20s+1)(16s+15) - 256s\right)\right] \ldots$$

$$\ldots - \left[-(4s+1)\left(-(4s+1)(16s+15) - 240s\right)\right] \ldots$$

$$\ldots + \left[-15\left(-(4s+1)(-16s) - (-15)(3s^2+20s+1)\right)\right]$$

$$\Rightarrow (8s^2+4s+16)(48s^3+109s^2+316s+15) \ldots$$

$$+ (4s+1)(-64s^2-316s-15) \ldots$$

$$- 1635s^2 - 4740s - 225$$

$$\Rightarrow 384s^5 + 1064s^4 + 3732s^3 + 3128s^2 + 5116s \ldots$$

$$+240 - 256s^3 - 1328s^2 - 376s - 15 \ldots$$

$$-1635s^2 - 4740s - 225$$

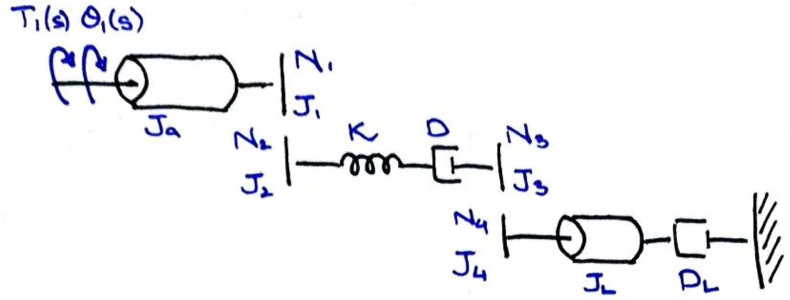
$$\Rightarrow \underbrace{384s^5 + 1064s^4 + 3476s^3 + 165s^2}_{} - |\Delta|$$

$$X_3(s) = \frac{\begin{vmatrix} 8s^2+4s+16 & -(4s+1) & 0 \\ -(4s+1) & 3s^2+20s+1 & F(s) \\ -15 & -16s & 0 \end{vmatrix}}{|\Delta|}$$

$$= \frac{-F(s)\begin{vmatrix} 8s^2+4s+16 & -(4s+1) \\ -15 & -16s \end{vmatrix}}{|\Delta|}$$

$$\boxed{\frac{X_3(s)}{F(s)} = \frac{128s^3+64s^2+316s+15}{384s^5+1064s^4+3476s^3+165s^2}}$$

Q # 3



The system can be equivalently drawn as:

where:

$$\left[\begin{array}{lll} J_{1eq} & = & J_a + J_1 + J_2 (N_1/N_2)^2 \\ K' & = & K (N_1/N_2)^2 \\ D' & = & D (N_1/N_2)^2 \\ J_{2eq} & = & \left[J_3 + (J_4 + J_L)(N_3/N_4)^2 \right] (N_1/N_2)^2 \end{array}\right]$$

Degree of freedom = 3

i: $\theta_1(s)(J_{1eq} s^2 + K') - \theta_2(s)(K') - \theta_3(s)(0) = T_1(s)$

ii: $-\theta_1(s)(K') + \theta_2(s)(D's + K') - \theta_3(s)(D' s) = 0$

iii: $-\theta_1(s)(0) - \theta_2(s)(D's) + \theta_3(s)(J_{2eq} s^2 + D_{eq} s \ldots$

$\ldots + D's) = 0$

In matrix form:

$$\Rightarrow \underbrace{\begin{bmatrix} J_{1eq} s^2 + K' & -K' & 0 \\ -K' & D's + K' & -D's \\ 0 & -D's & J_{2eq} s^2 + (D_{eq} + D')s \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} \theta_1(s) \\ \theta_2(s) \\ \theta_3(s) \end{bmatrix}}_{X} = \underbrace{\begin{bmatrix} T_1(s) \\ 0 \\ 0 \end{bmatrix}}_{Y}$$

$$\Delta = |A| = (J_{1eq} s^2 + K') \begin{vmatrix} D's + K' & -D's \\ -D's & J_{2eq} s^2 + (D_{eq} + D')s \end{vmatrix} \ldots$$

$$\cdots - (-K') \begin{vmatrix} -K' & -D's \\ 0 & J_{2eq}s^2 + (D_{eq} + D')s \end{vmatrix}$$

$$\Rightarrow \quad (J_{1eq}s^2 + K')(D_{eq}K's + D'K's + D_{eq}D's^2 \cdots$$

$$+ J_{2eq}D's^3 + J_{2eq}K's^2) + K'(D_{eq}s \cdots$$

$$+ D's + J_{2eq}s^2)$$

$$\Rightarrow \quad \underbrace{s^5(J_{1eq}J_{2eq}D') + s^4(J_{1eq}D'D_{eq} + J_{1eq}J_{2eq}K') \cdots + s^3(J_{1eq}D_{eq}K' + J_{1eq}D'K' + J_{2eq}D'K') + \cdots \cdots s^2(D_{eq}D'K')} - \Delta$$

$$\Theta_1(s) = \frac{\begin{vmatrix} T_1(s) & -K' & 0 \\ 0 & K' + D's & -D's \\ 0 & -D's & J_{2eq}s^2 + (D_{eq} + D')s \end{vmatrix}}{\Delta}$$

$$= \frac{T_1(s)\begin{vmatrix} K' + D's & -D's \\ -D's & J_{2eq}s^2 + (D_{eq} + D')s \end{vmatrix}}{\Delta}$$

$$\boxed{\frac{\Theta_1(s)}{T_1(s)} = \frac{J_{2eq}D's^2 + (J_{2eq}K' + D_{eq}D')s + D_{eq}K' + D'K'}{s\left[J_{1eq}J_{2eq}D's^3 + (J_{1eq}J_{2eq}K' + J_{1eq}D_{eq}D')s^2 \cdots (J_{1eq}D_{eq}K' + J_{1eq}D'K' + J_{2eq}D'K')s + D_{eq}D'K'\right]}}$$