EE-381 Robotics-1
UG ELECTIVE

Lecture 5

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Quiz 1

Given a transformation matrix:

$$ {}^{B}_{A}T = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos(\theta) & -\sin(\theta) & 2 \\ 0 & \sin(\theta) & \cos(\theta) & 3 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

Given $\theta = 45^{\circ}$ and $P^B = [4, 5, 6]^T$.

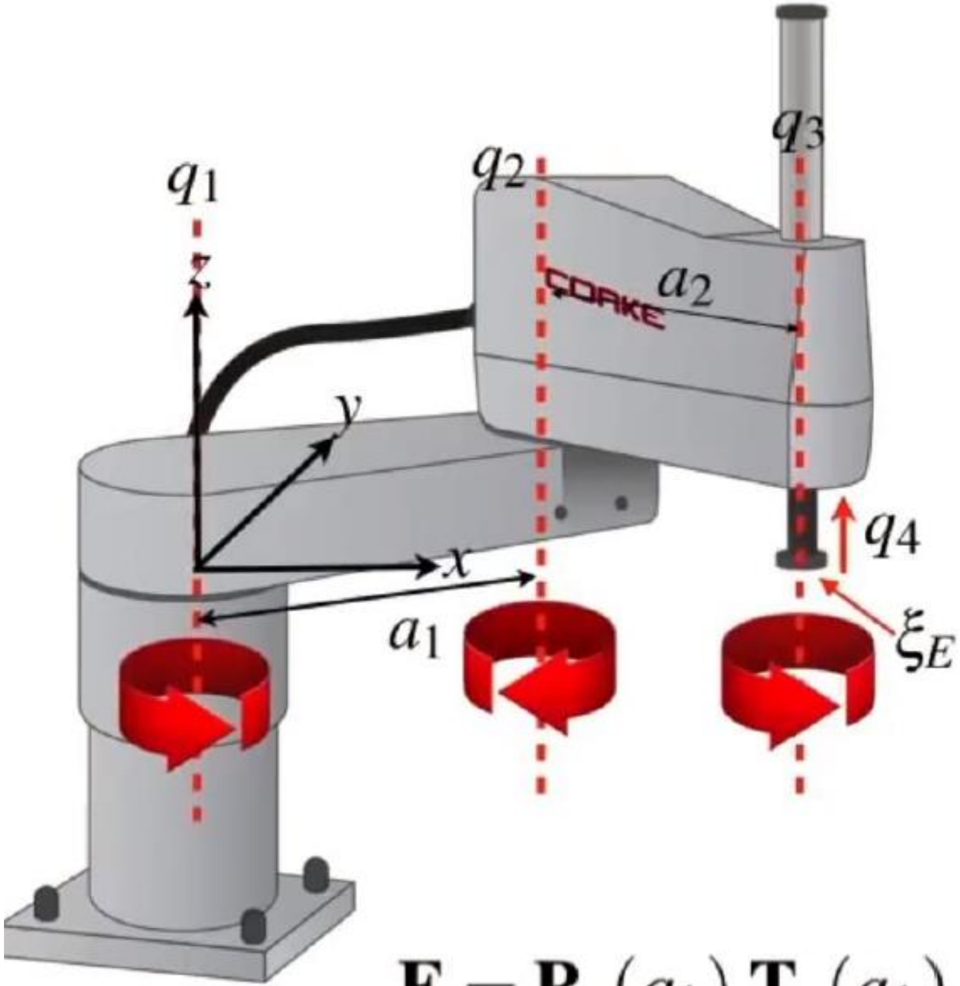
1. Find T_B^A. $(5\,points)$
2. Compute P^A. $(5\,points)$

Last Lecture

- Angle/ axis representation
- Quaternions
- Forward kinematics

Forward Kinematics—$3D$ (SCARA Robot)

https://www.youtube.com/watch?v=-m1oKuFkSTE

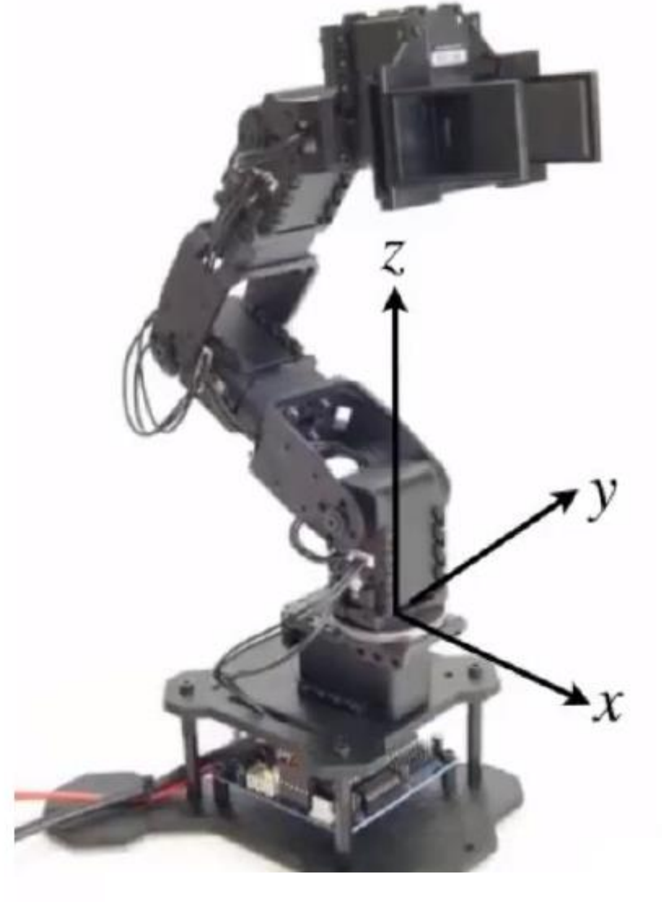


SCARA robot

$$\mathbf{E} = \mathbf{R}_z(q_1)\,\mathbf{T}_x(a_1)\,\mathbf{R}_z(q_2)\,\mathbf{T}_x(a_2)\,\mathbf{R}_z(q_3)\,\mathbf{T}_z(q_4)$$

Forward Kinematics—$3D$

https://www.youtube.com/watch?v=zwTRbiUEVPk



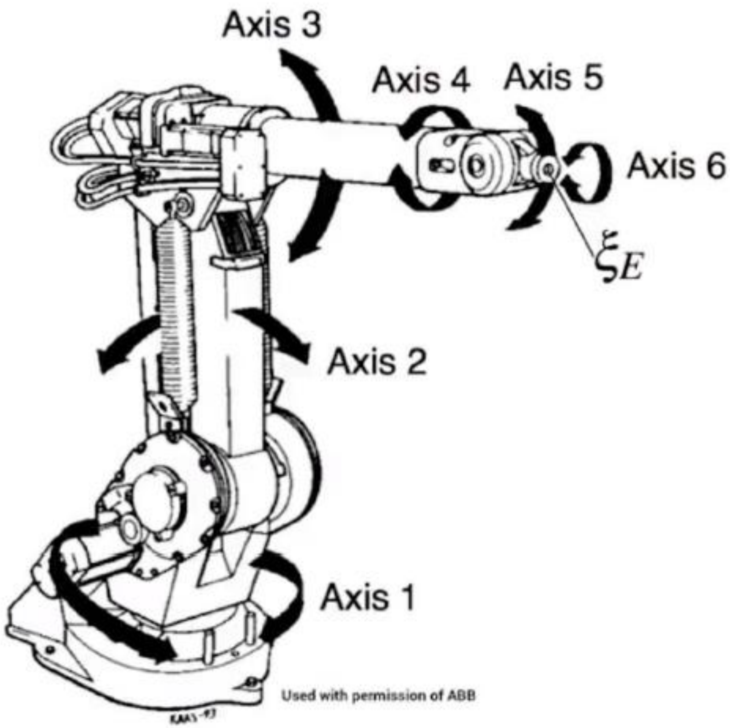
4 joints

PhantomX Pincher Robot Arm 2014

$$\mathbf{E} = \mathbf{R}_z(q_1)\mathbf{T}_z(a_1)\mathbf{R}_y(q_2)\mathbf{T}_z(a_2)\mathbf{R}_y(q_3)\mathbf{T}_z(a_3)\mathbf{R}_y(q_4)\mathbf{T}_z(a_4)$$

Forward Kinematics-General Purpose 3D Robot

Unimate Puma 500
Oussama Khatib | Used with permission



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Spaces

Configuration Space

- Robot configuration is described by a vector of generalized coordinates
- Coordinate is
  - Angle in case of revolute joints
  - Length in case of prismatic joints

Number of joints

$$\boldsymbol{q} = \{q_j, j \in [1 \ldots N]\} \in \mathcal{C}$$

Joint configuration

$$\mathcal{C} \subset \mathbb{R}^N$$

Joint coordinate

Space of all possible configurations

Task Space

- The space of all possible end-effector poses

$$\xi_E \in \mathcal{T}$$

Space of all possible end-effector poses

- In 2D

$$\xi_E \sim (x, y)$$
$$\xi_E \sim (x, y, \theta)$$

- In 3D

$$\xi_E \sim (x, y, z)$$
$$\xi_E \sim (x, y, z, \theta_p)$$
$$\xi_E \sim (x, y, z, \theta_r, \theta_p, \theta_y)$$

Dimensions

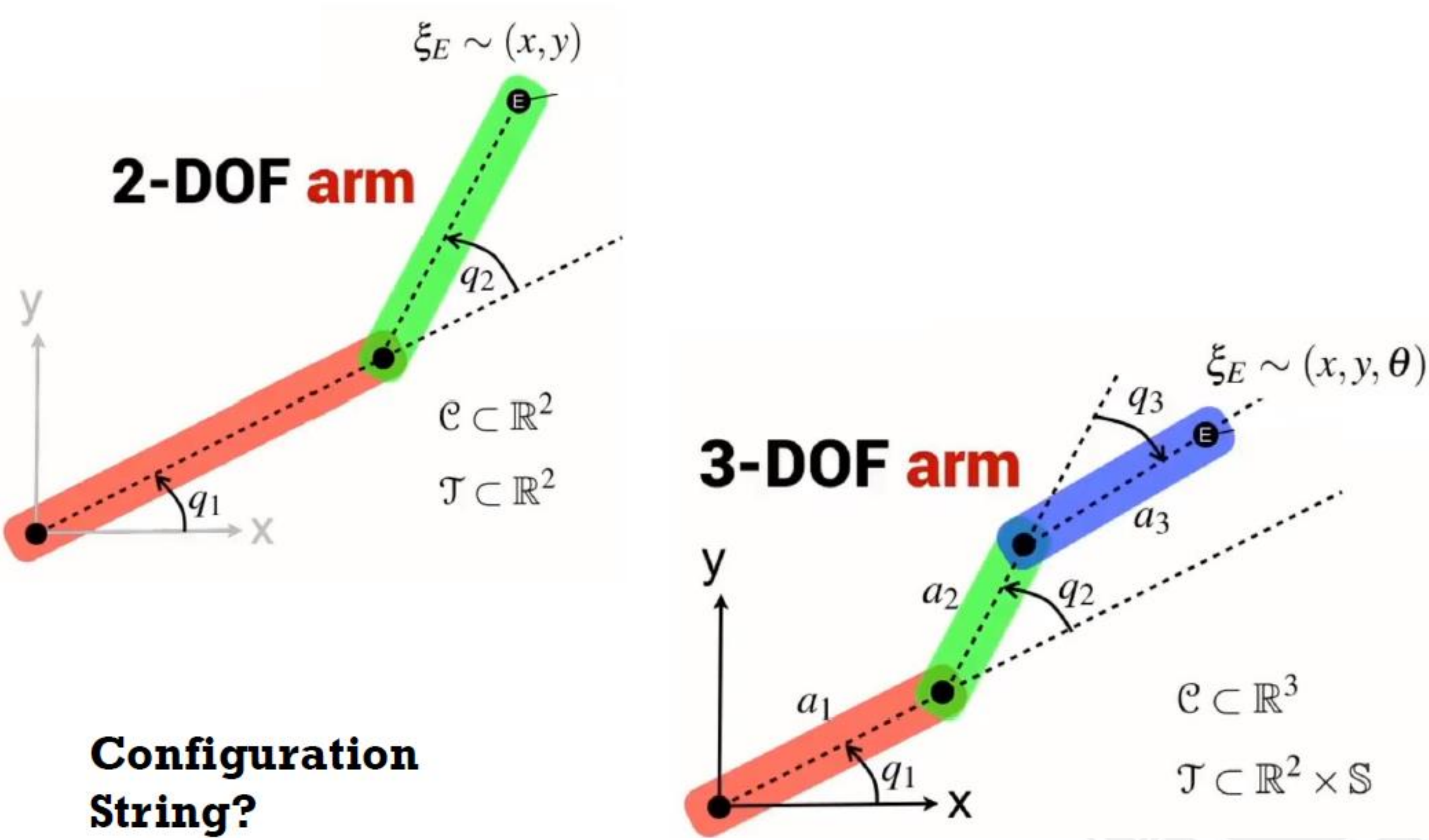
- Robots degree of freedom (number of joints) $\dim \mathcal{C}$
- Task space degrees of freedom $\dim \mathcal{T}$

$$\xi_E \sim (x, y) \quad \rightarrow \dim \mathcal{T} = 2$$
$$\xi_E \sim (x, y, \theta) \quad \rightarrow \dim \mathcal{T} = 3$$
$$\xi_E \sim (x, y, z) \quad \rightarrow \dim \mathcal{T} = 3$$
$$\xi_E \sim (x, y, z, \theta_p) \quad \rightarrow \dim \mathcal{T} = 4$$
$$\xi_E \sim (x, y, z, \theta_r, \theta_p, \theta_y) \quad \rightarrow \dim \mathcal{T} = 6$$

- To reach all of the task space

$$\dim \mathcal{C} \geq \dim \mathcal{T}$$

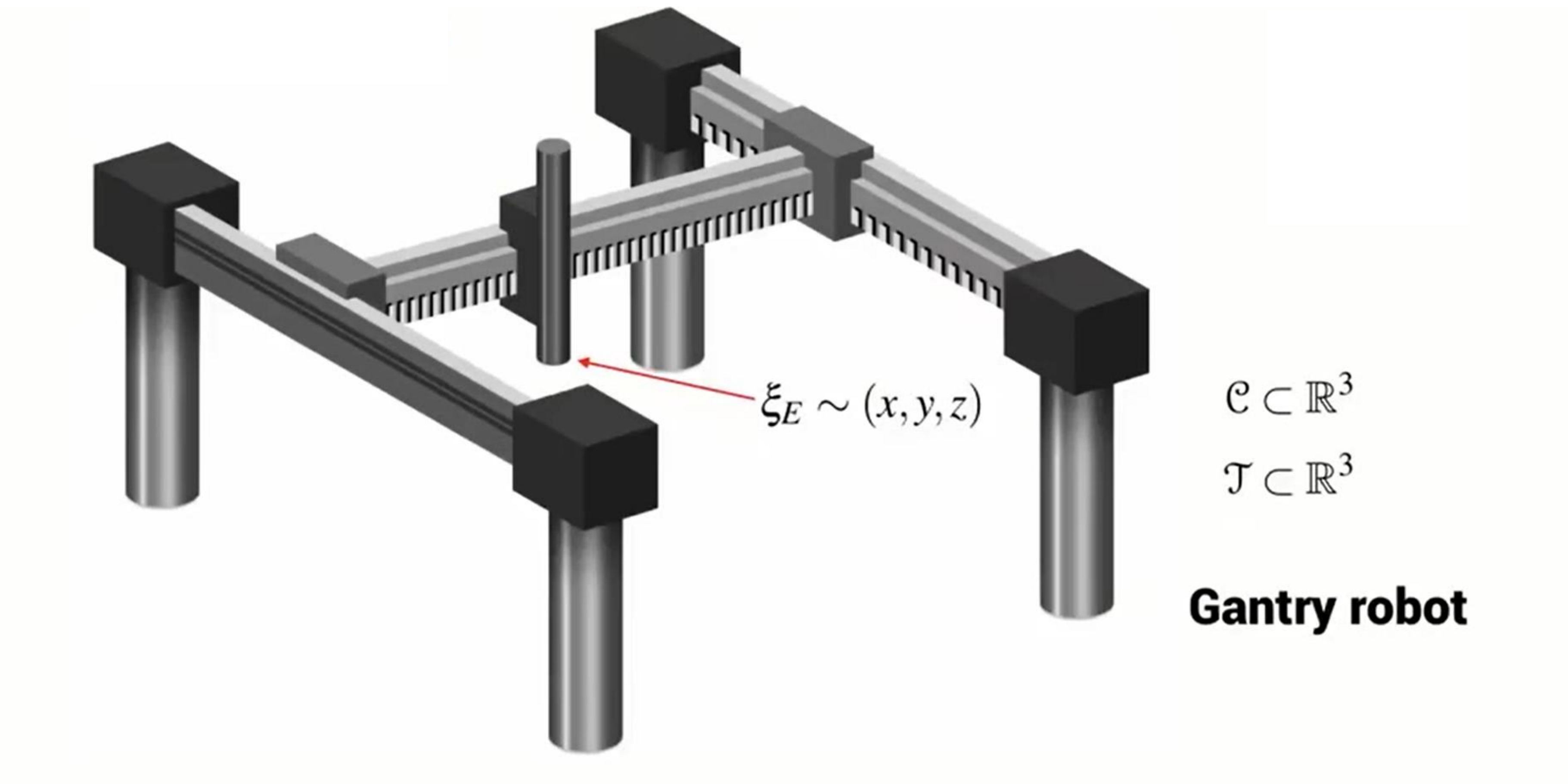
Examples



$\xi_E \sim (x, y)$
2-DOF arm
q_2
y
$\mathcal{C} \subset \mathbb{R}^2$
$\mathcal{T} \subset \mathbb{R}^2$
q_1
x
$\xi_E \sim (x, y, \theta)$
q_3
3-DOF arm
a_3
y
a_2
q_2
a_1
$\mathcal{C} \subset \mathbb{R}^3$
q_1
x
$\mathcal{T} \subset \mathbb{R}^2 \times \mathbb{S}$
Configuration
String?

3-DOF arm

https://www.youtube.com/watch?v=3zVfldaEoBs



4-DOF arm

$$\xi_E \sim (x, y, z, \theta_y)$$

$$\mathcal{C} \subset \mathbb{R}^4$$

$$\mathcal{T} \subset \mathbb{R}^3 \times \mathbb{S}$$

SCARA robot

4-DOF arm

https://www.youtube.com/watch?v=zudMHcIxiw8

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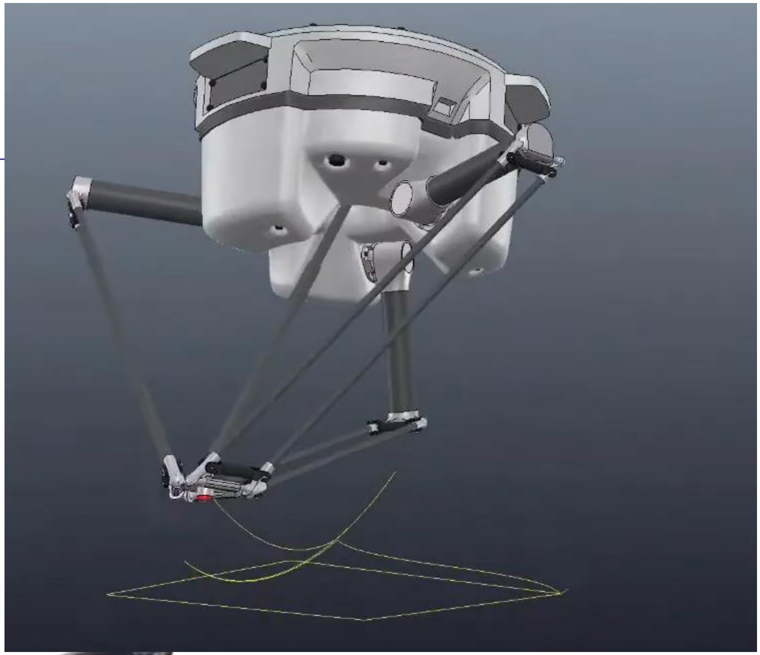


$$\mathcal{C} \subset \mathbb{R}^4$$

$$\mathcal{T} \subset \mathbb{R}^3 \times \mathbb{S}$$

$$\xi_E \sim (x, y, z, \theta_y)$$

Parallel link robot

4-DOF arm

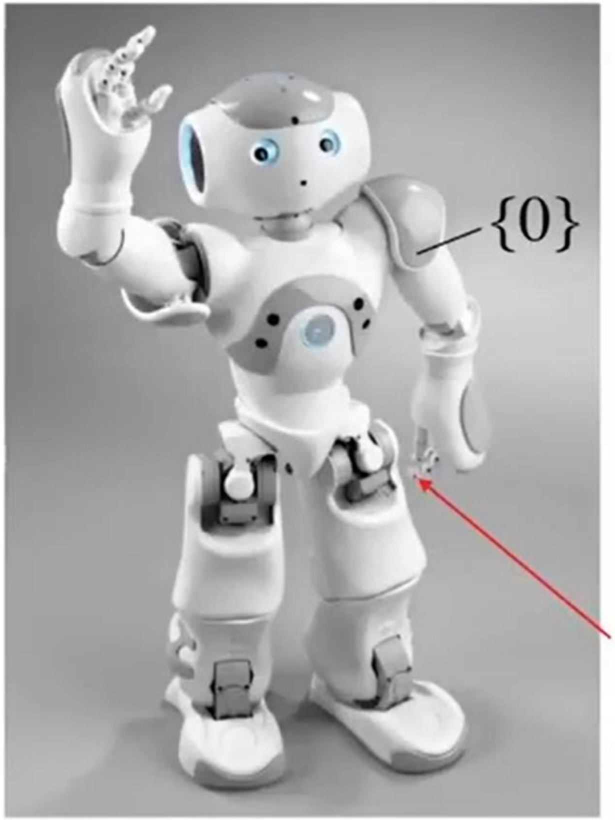
$$\xi_E \sim (x, y, z, \theta_p)$$

PhantomX Pincher Robot Arm 2014
Trossen Robotics | Used with permission

$$\mathcal{C} \subset \mathbb{R}^4$$

$$\mathcal{T} \subset \mathbb{R}^3 \times \mathbb{S}$$

4-DOF arm



By Aldebaran Robotics via Wikimedia Commons

$$\mathcal{C} \subset \mathbb{R}^4$$

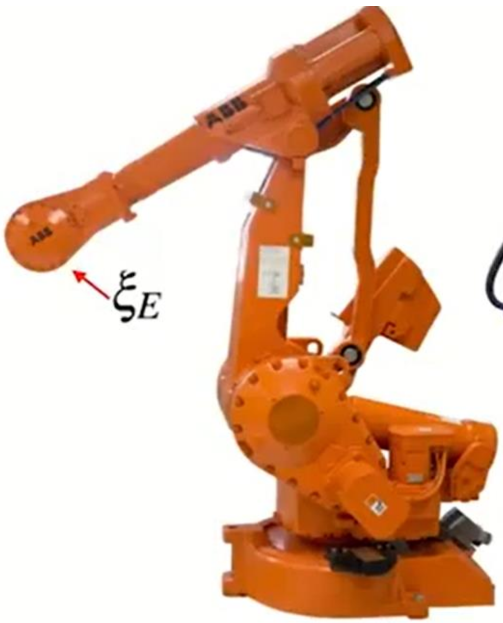
$$\mathcal{T} \subset \mathbb{R}^3 \times \mathbb{S}$$

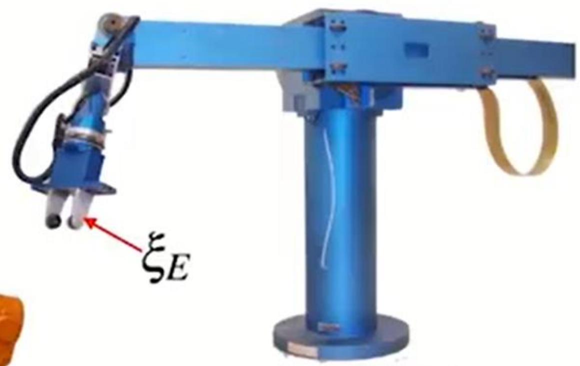
Aldebaran robotics

6-DOF arm

Homework!

$$\mathcal{C} \subset \mathbb{R}^6$$
$$\mathcal{T} \subset \mathbb{R}^3 \times \mathbb{S}(3)$$





Stanford Robotic Arm
Oussama Khatib | Used with permission

$$\xi_E \sim (x, y, z, \theta_r, \theta_p, \theta_y)$$

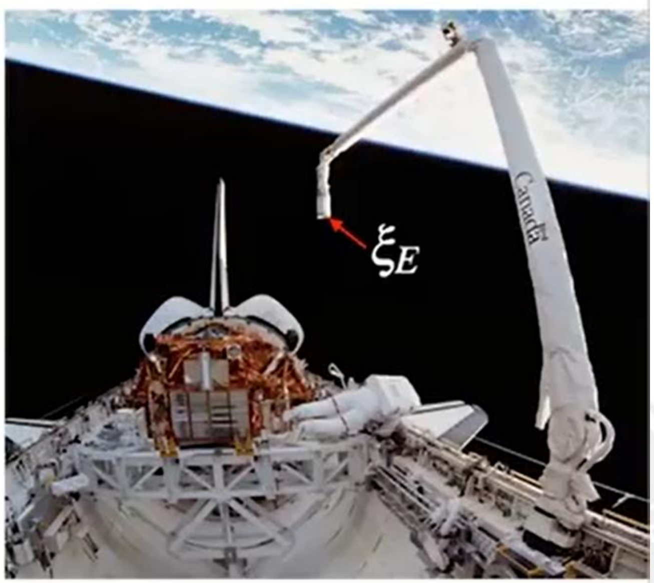


Image: NASA

Very high DOF Robot

$\mathcal{C} \subset \mathbb{R}^n$

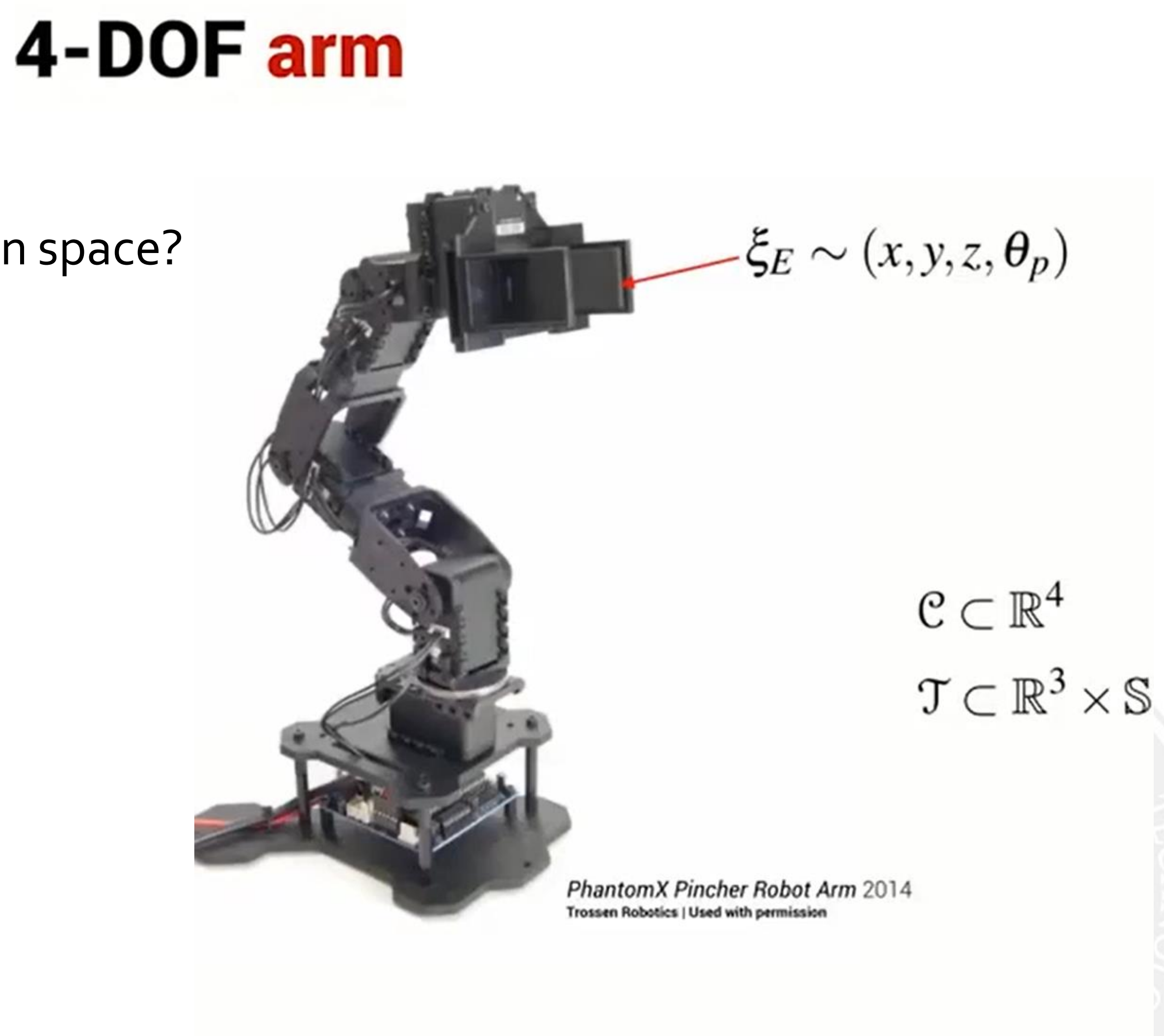
$\mathcal{T} \subset \mathbb{R}^3 \times \mathbb{S}(3)$

$\xi_E \sim (x, y, z, \theta_r, \theta_p, \theta_y)$

Redundant robot

Configuration String

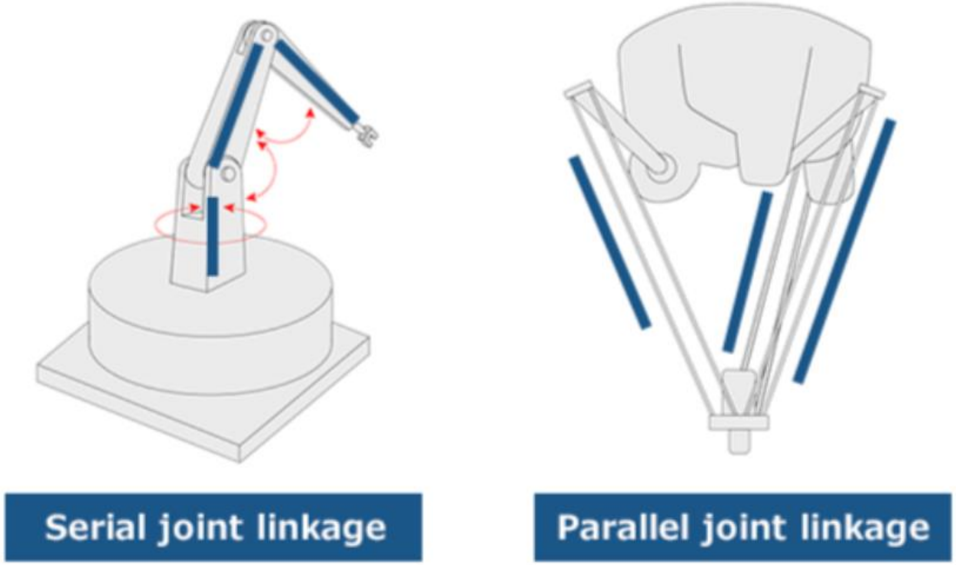
Summary



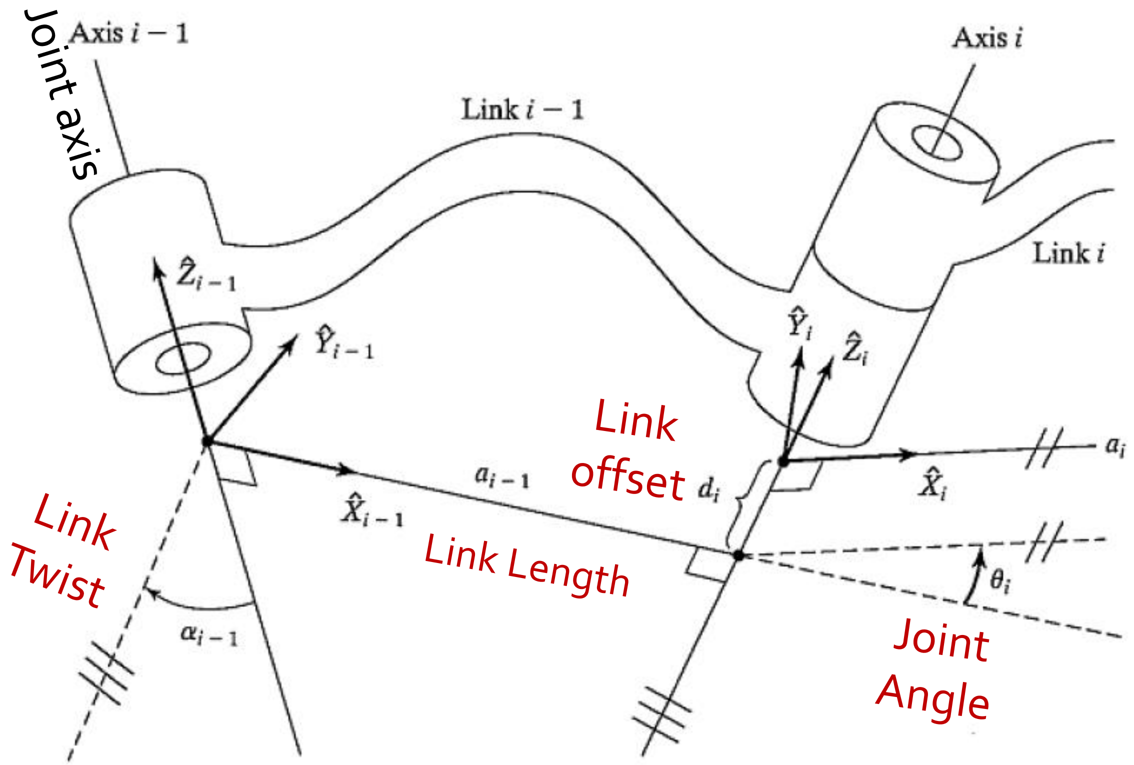
- Links?
- Joints?
- Dimensions of configuration space?
- DOF?
- Configuration string?
- End-effectors' position?
- Dimensions of task space?

Denavit-Hartenberg (DH) Notation

- Developed a general theory to describe an articulated sequence of joints.
- Each joint in the robot is described by **four parameters**.
- Only applicable to serial link mechanisms NOT parallel mechanisms



DH Parameters



a_i = *the distance from* $\hat{Z}_i$ *to* $\hat{Z}_{i+1}$ *measured along* $\hat{X}_i$;

α_i = *the angle from* $\hat{Z}_i$ *to* $\hat{Z}_{i+1}$ *measured about* $\hat{X}_i$;

d_i = *the distance from* $\hat{X}_{i-1}$ *to* $\hat{X}_i$ *measured along* $\hat{Z}_i$; and

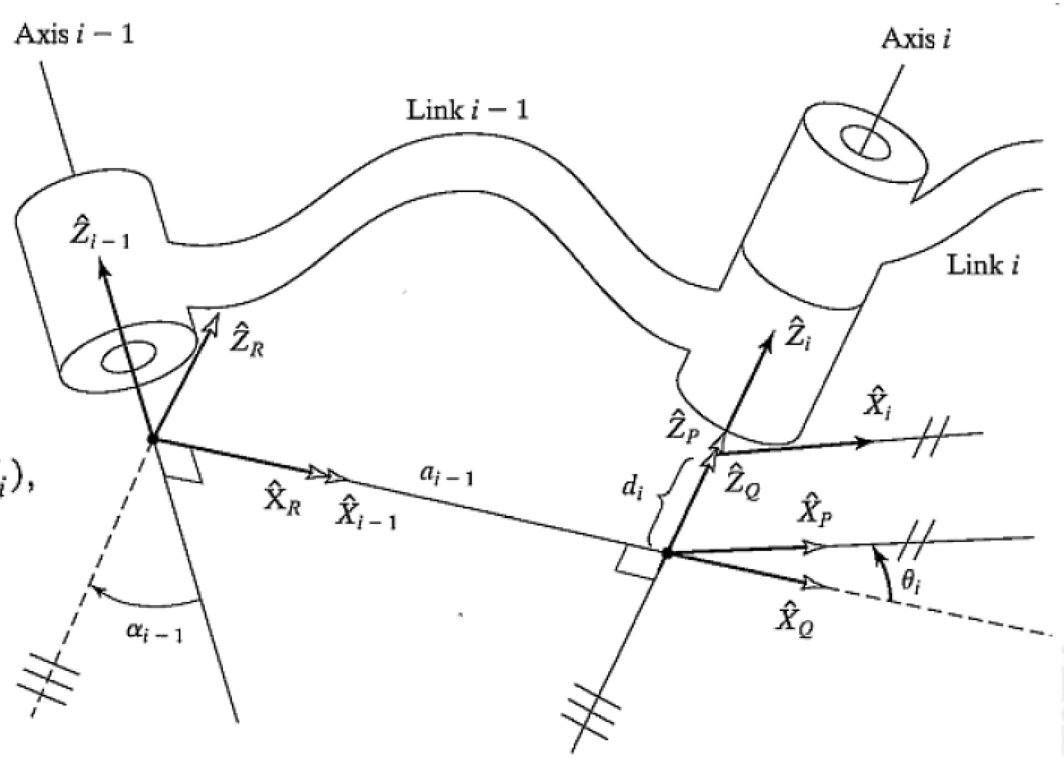
θ_i = *the angle from* $\hat{X}_{i-1}$ *to* $\hat{X}_i$ *measured about* $\hat{Z}_i$.

Summary of link-frame attachment procedure

The following is a summary of the procedure to follow when faced with a new mechanism, in order to properly attach the link frames:

1. Identify the joint axis and imagine (or draw) infinite lines along them. steps 2 through 5 below, consider two of these neighboring lines (at axis i and $i + 1$).
2. Identify the common perpendicular between them, or point of intersection. At the point of intersection, or at the point where the common perpendicular meets the i^{th} axis, assign the link-frame origin.
3. Assign the $\hat{Z}_i$ axis pointing along the i^{th} joint axis.
4. Assign the $\hat{X}_i$ axis pointing along the common perpendicular, or, if the axis intersect, assign $\hat{X}_i$ to be normal to the plane containing the two axis.
5. Assign the $\hat{Y}_i$ axis to complete a right-hand coordinate system.
6. Assign {0} to match {1} when the first joint variable is zero. For {N}, choose an origin location and $\hat{X}_N$ direction freely, but generally so as to cause as many linkage parameters as possible to become zero.

General Form of Link Transformations



$$ {}^{i-1}_{i}T = R_X(\alpha_{i-1})D_X(a_{i-1})R_Z(\theta_i)D_Z(d_i), $$

$$ {}^{i-1}_{i}T = \begin{bmatrix} c\theta_i & -s\theta_i & 0 & a_{i-1} \\ s\theta_i c\alpha_{i-1} & c\theta_i c\alpha_{i-1} & -s\alpha_{i-1} & -s\alpha_{i-1}d_i \\ s\theta_i s\alpha_{i-1} & c\theta_i s\alpha_{i-1} & c\alpha_{i-1} & c\alpha_{i-1}d_i \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

Concatenating Link Transformations

- Single transformation that relates frame {N} to frame {0}

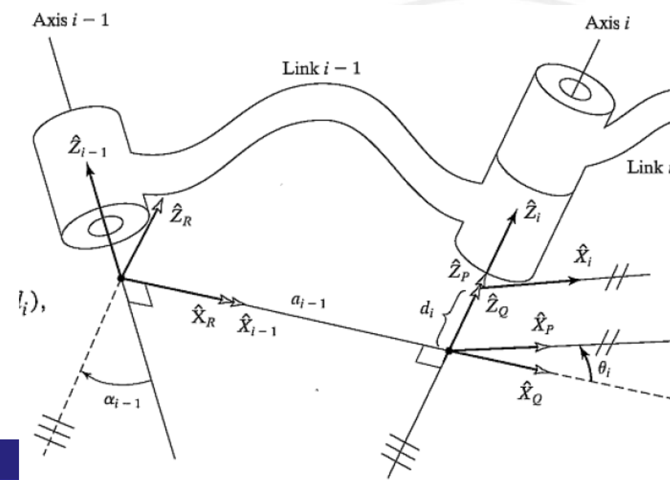
$$T_N^0 = T_1^0 T_2^1 T_3^2 \ldots T_N^{N-1}$$

This transformation, T_N^0, will be a function of all n joints variables. If the robot's joint-position sensors are required, the Cartesian position and orientation of the last link can be computed by T_N^0

Concatenating Link Transformations Figure on slide 6

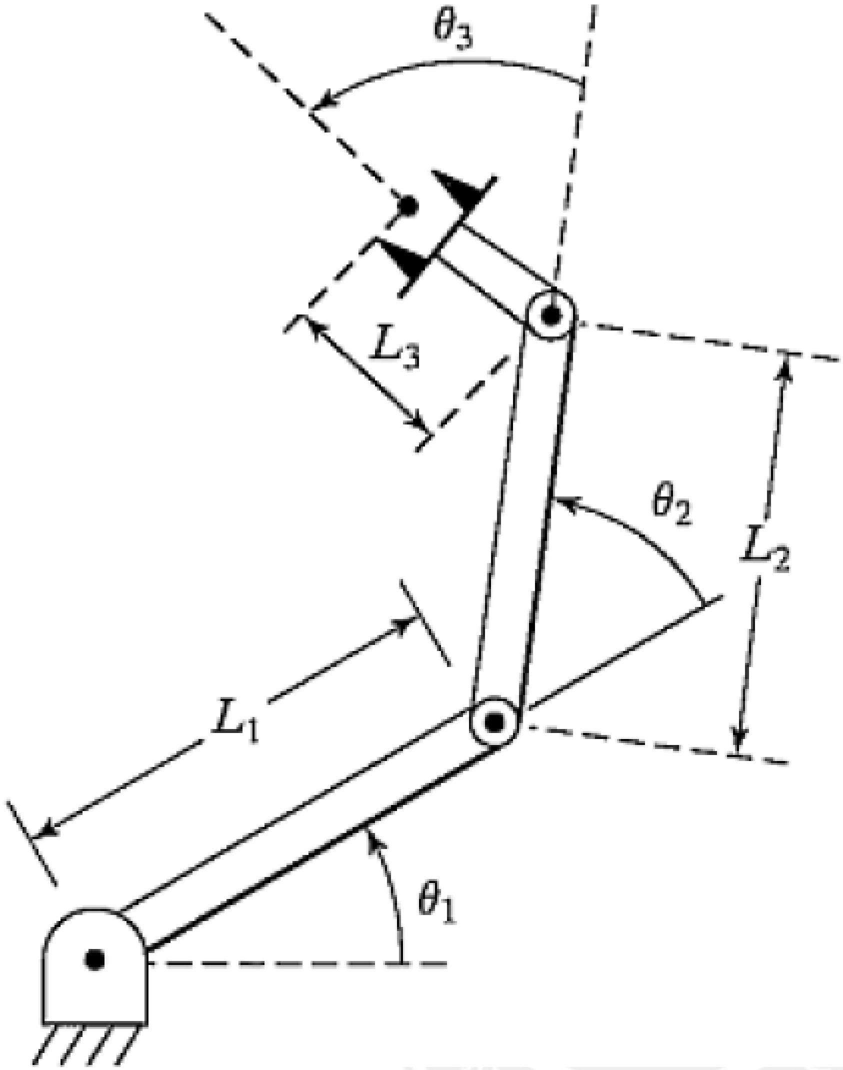
$${}^{i-1}_{i}T = {}^{i-1}_{R}T \; {}^{R}_{Q}T \; {}^{Q}_{P}T \; {}^{P}_{i}T.$$

(Section 3.5; $John\ J.\ Craig$)

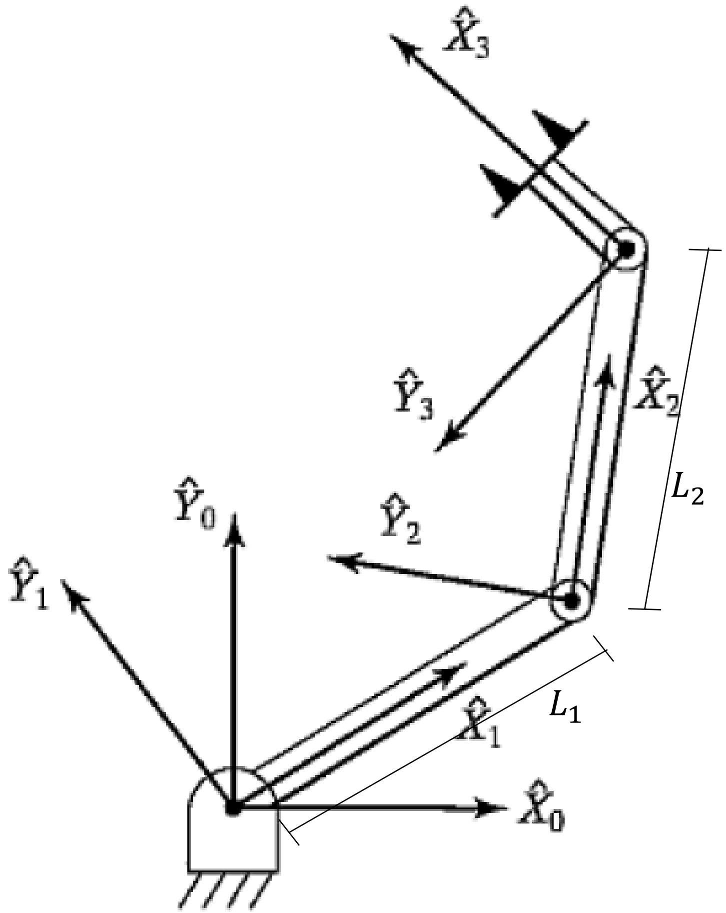


Example 1

Find DH parameters for the manipulator with **RRR** configuration.



Example 1

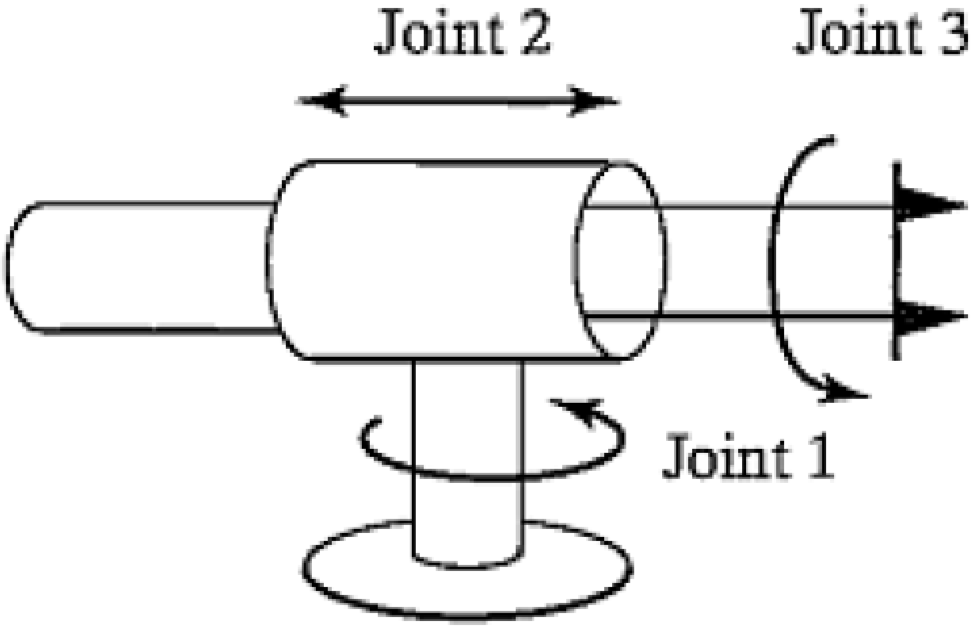


i	α_{i-1}	a_{i-1}	d_i	θ_i
1	0	0	0	θ_1
2	0	L_1	0	θ_2
3	0	L_2	0	θ_3

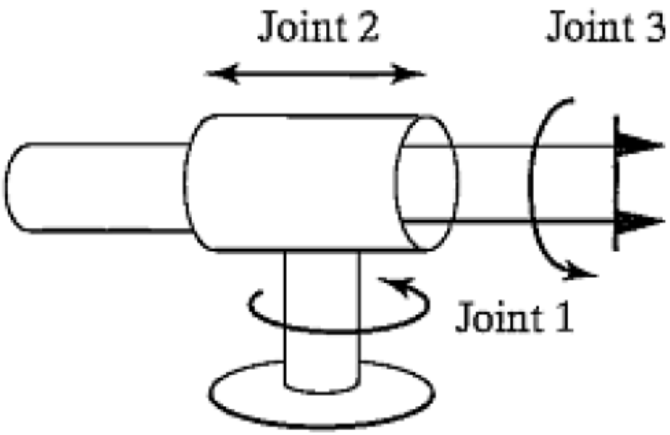
Link: i

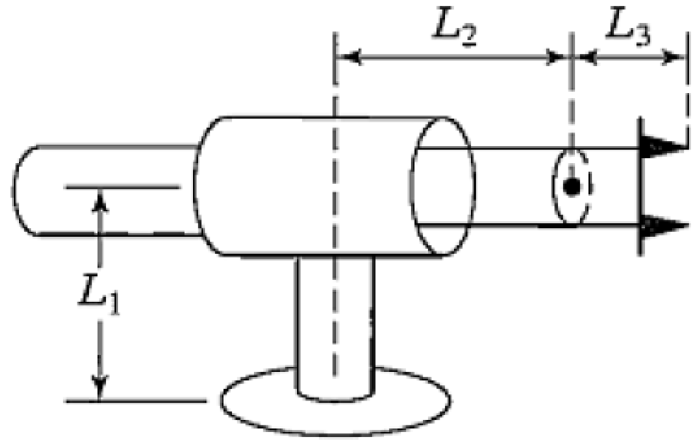
Example 2

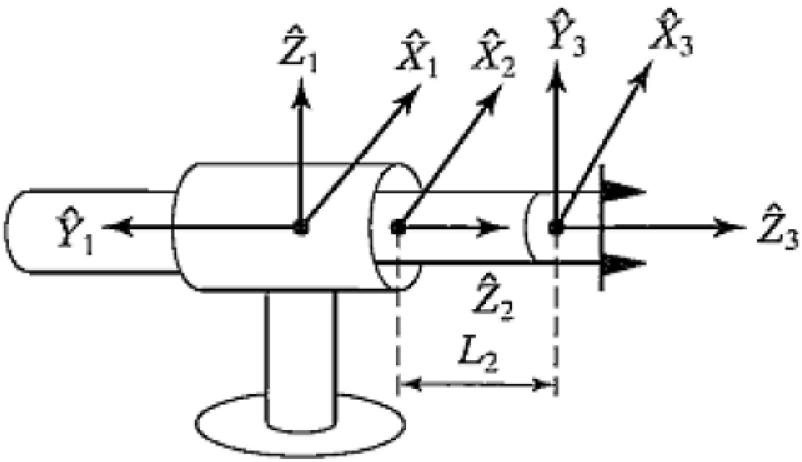
Find DH parameters for the manipulator with **RPR** configuration.



Example 2

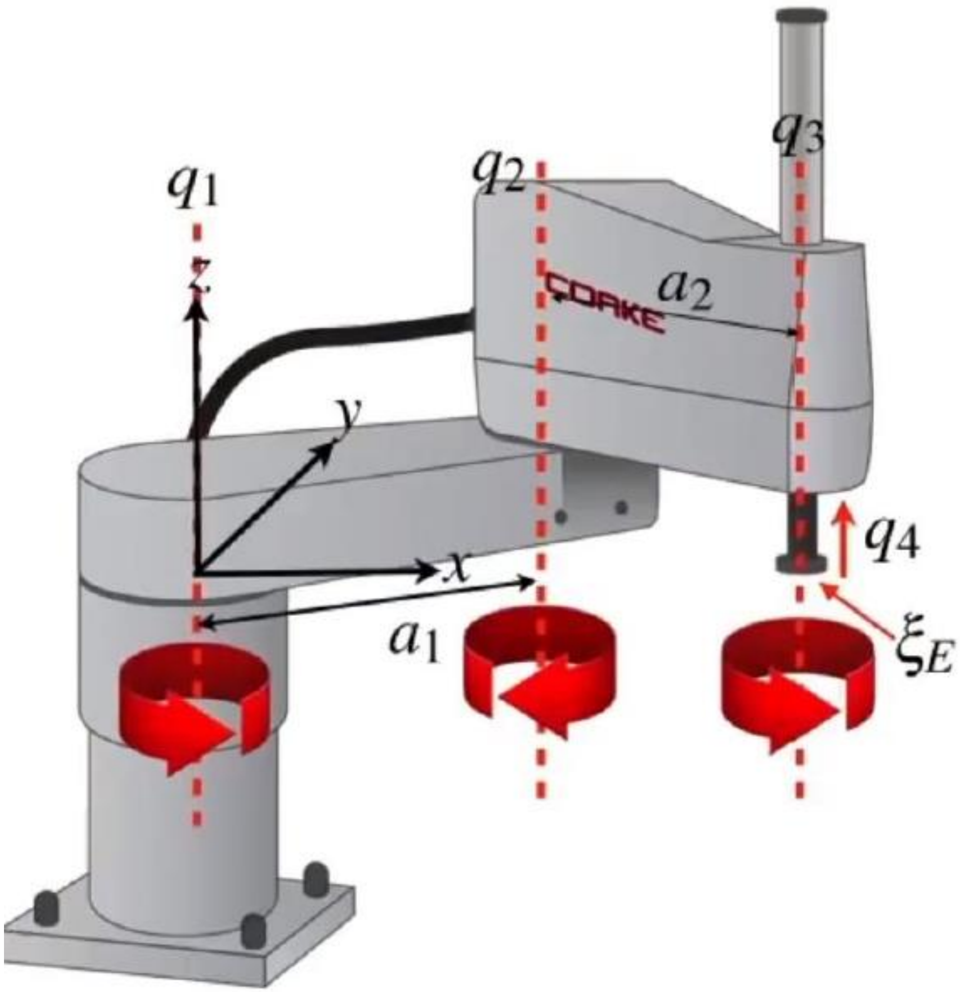






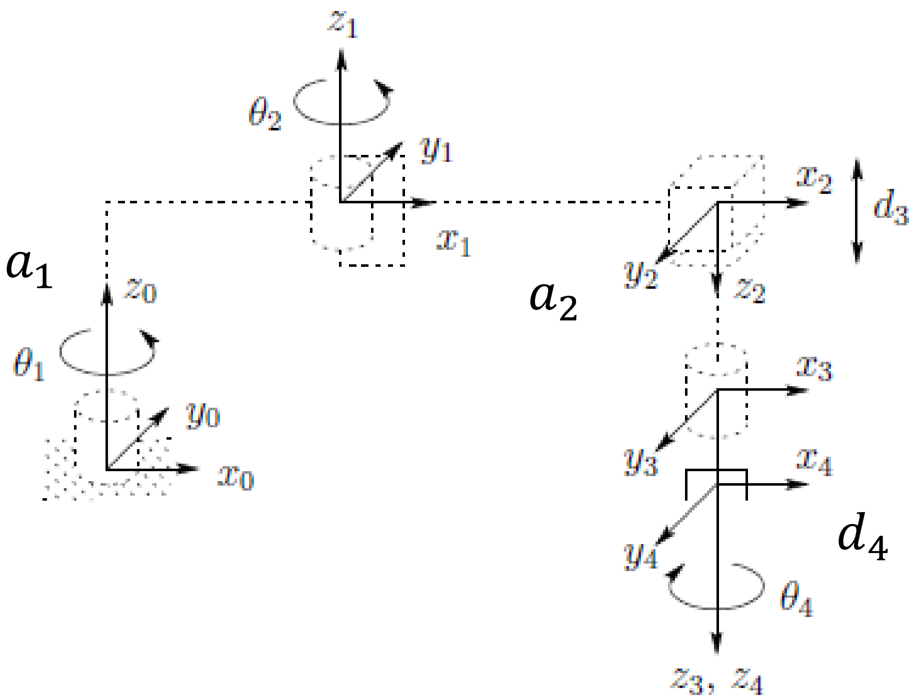
i	α_{i-1}	a_{i-1}	d_i	θ_i
1	0	0	0	θ_1
2	90°	0	d_2	0
3	0	0	L_2	θ_3

SCARA



	α_{i-1}	a_{i-1}	d_i	θ_i
1	0	0	0	q_1
2	0	a_1	0	q_2
3	0	a_2	q_4	0
4	0	0	0	q_3

SCARA



Link	a_i	α_i	d_i	θ_i
1	a_1	0	0	θ_1
2	a_2	180	0	θ_2
3	0	0	d_3	0
4	0	0	d_4	θ_4

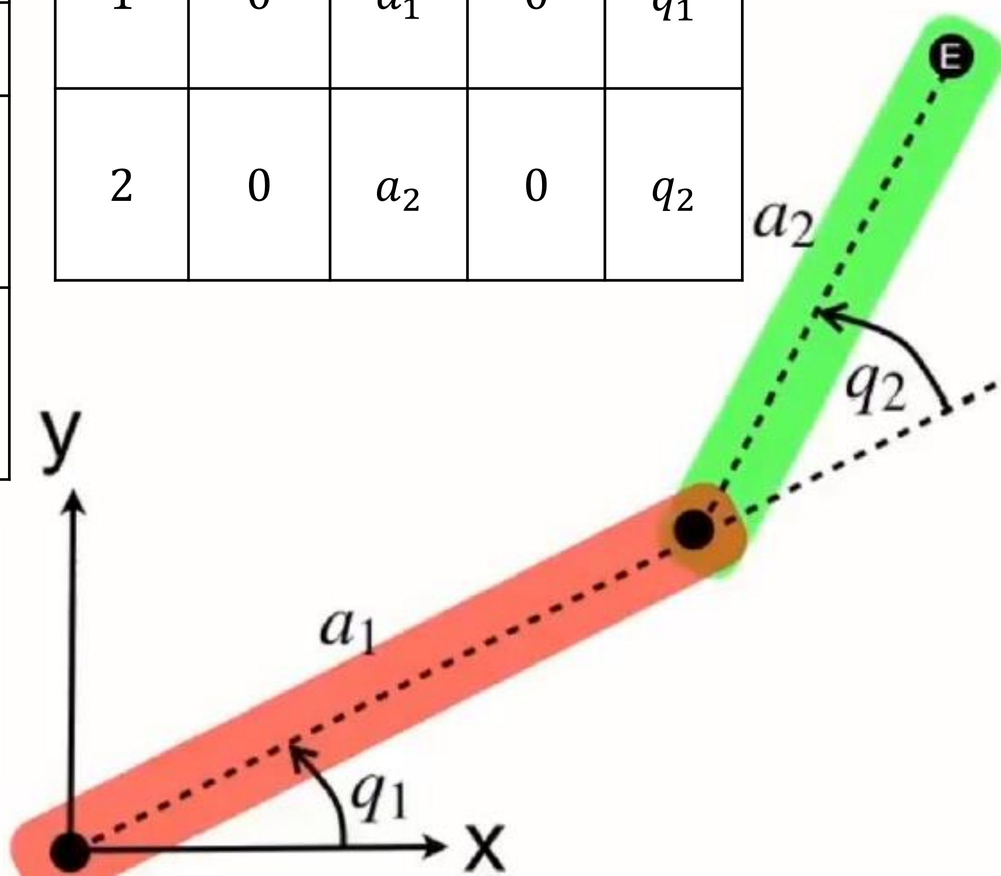
SCARA

- Link Transformation of SCARA

DH Parameters for plane 2-Joint Robot

	α_{i-1}	a_{i-1}	d_i	θ_i
0	0	0	0	q_1
1	0	a_1	0	q_2
2	0	a_2	0	0

	α_{i-1}	a_{i-1}	d_i	θ_i
1	0	a_1	0	q_1
2	0	a_2	0	q_2



***DH Table completely defines the kinematics of the robot**

Reading Assignment

- Section 3.7 of Introduction to Robotics (Craig)
  - The PUMA 560
  - The Yasukawa Motoman $L-3$

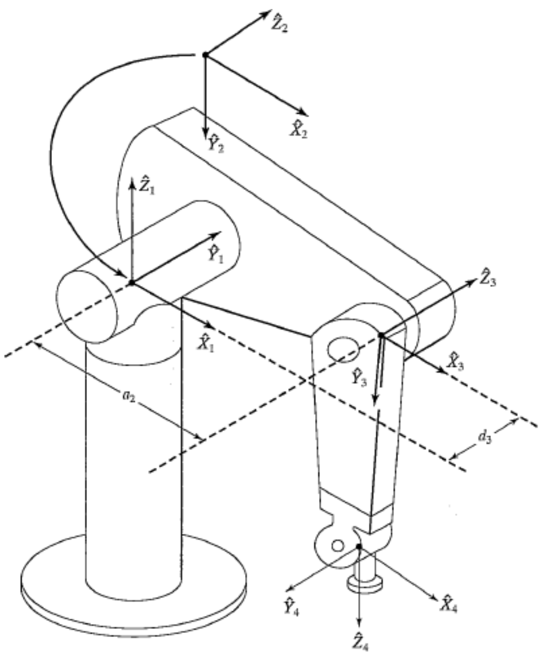


FIGURE 3.18: Some kinematic parameters and frame assignments for the PUMA 560 manipulator.

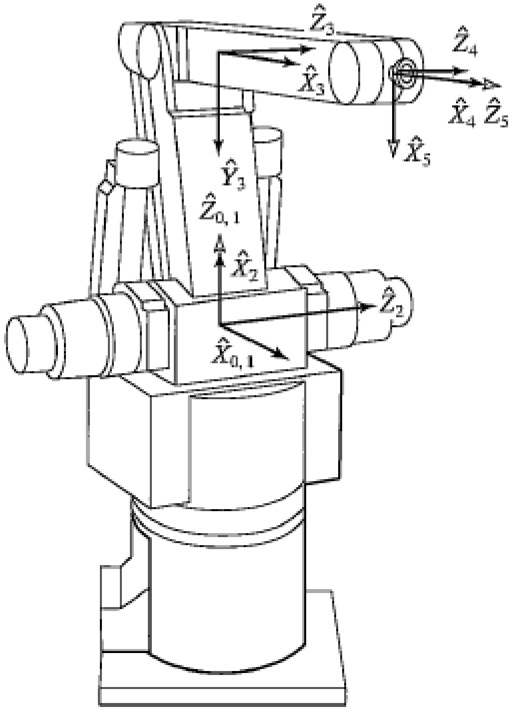


FIGURE 3.25: Assignment of link frames for the Yasukawa L-3

General Form

$$\text{if } \boldsymbol{\sigma}_j = \begin{cases} R \rightarrow \theta_j = q_j \\ P \rightarrow d_j = q_j \end{cases}$$

pose

Joint angles

link offsets

link twists

$$\xi_N = \mathcal{K}(\boldsymbol{q}; \boldsymbol{\theta}, \boldsymbol{d}, \boldsymbol{a}, \boldsymbol{\alpha}, \boldsymbol{\sigma})$$

joint configuration

link lengths

Joint types

Degree of Freedom (DOF)

- DOF is defined as the number of independent parameters required to specify the location of every link within a mechanism
- 6-DOF$\rightarrow$ $maximum$
- Each joint has 1 $-$DOF
- Mobile robots, Airplanes

Inverse Kinematics (IK)

- How to compute the position of each joint given the end –effector pose?
- How to generate smooth paths/trajectories for the end-effector?

Inverse Kinematics (IK)

- What joint angles to set to achieve a certain end-effector pose.

$$\xi_N = \mathcal{K}(\boldsymbol{q})$$

$$\boldsymbol{q} = \{q_j, j \in [1 \dots N]\}$$

$$\boldsymbol{q} = \mathcal{K}^{-1}(\xi_N)$$

IK for 2 Joint Arm- Geometric Approach