Binomial Distribution

Ansar Shahzadi

School of Electrical Engineering & Computer Science

National University of Science and Technology(NUST)

Binomial Distribution

A binomial experiment is a probability experiment that satisfies the following conditions:

1) There are only two possible outcomes of each trial. The outcomes can be classified as a success (S) or as a failure (F).
2) The experiment is repeated for a fixed number of times, say n.
3) The successive trial are all independent.
4) The probability of a success, denoted by p, is the same for each trial.

Notation for Binomial Experiments

Symbol	*Description*
n	The number of times a trial is repeated
$p=P(S)$	The probability of success in a single trial.
$q=P(F)$	The probability of failure in a single trial.
x	The random variable represents a count of the number of successes in n trials: x = 0, 1, 2, 3, . . . n.

Binomial Probability Formula

In a binomial experiment, the probability of exactly x successes in n trials is:

$$P(X = x) = \binom{n}{x} p^x q^{n-x} = \frac{n!}{(n-x)!\,x!} p^x q^{n-x}$$

$$where \qquad x = 0,1,2,\ldots\ldots\ldots n$$

Binomial Distribution

Binomial Distribution has two parameters n and p. We write: ***X ~ Bin (n, p)*** *{reads: "X is distributed binomially with parameters n and p}*

Mean=$\mu = np$

Variance=$\sigma^2 = npq$

Standard Deviation=$\sqrt{npq}$

Definitions: Bernoulli

Bernoulli trial: If there is only 1 trial with probability of success *p* and probability of failure *1-p*, this is called a Bernoulli distribution. (special case of the binomial with *n*=1)

Probability of success:

$$P(X=1)=\binom{1}{1}p^{1}(1-p)^{1-1}=p$$

Probability of failure:

$$P(X=0)=\binom{1}{0}p^{0}(1-p)^{1-0}=1-p$$

Question#1

A die is rolled five times and 5 or 6 is considers as success. Find the probability of

- No Success
- At least two Successes
- At least one but not more than 3

Solution Hints

Here n=5 and p=1/3

P(X=x)= $\binom{5}{x}\left(\frac{1}{3}\right)^{x}\cdot\left(\frac{2}{3}\right)^{5-x}$ *for x=0,1 2, 3, 4, and,5*

- No Success

P(x=0)= $\binom{5}{0}\left(\frac{1}{3}\right)^{0}\cdot\left(\frac{2}{3}\right)^{4-0}$

- At least two Successes

P(x≥2)=1-P(x=0)-P(x=1)

- At least one but not more than 3

P(1≤x≤3)=P(x=1)+P(x=2)+P(x=3)

Question#2

If on the average rain falls on twelve days in every thirty, find the probability that

- Rain will fall on just three days of a given week.
- The first three days of a given week will be fine and the remaining wet.

Solution Hints

Here n=7 and p=12/30=2/5

P(X=x)= $\binom{7}{x}\left(\frac{2}{5}\right)^{x}\cdot\left(\frac{3}{5}\right)^{7-x}$ *for x=0,1 2, 3, 4, 5, 6 and,7*

Rain will fall on just three days of a given week.

P(X=3)= $\binom{7}{3}\left(\frac{2}{5}\right)^{3}\cdot\left(\frac{3}{5}\right)^{7-3}$

The first three days of a given week will be fine and the remaining wet.

P(FFFWWWW)= $\left(\frac{3}{5}\right)\left(\frac{3}{5}\right)\left(\frac{3}{5}\right)\left(\frac{2}{5}\right)\left(\frac{2}{5}\right)\left(\frac{2}{5}\right)\left(\frac{2}{5}\right)=\left(\frac{3}{5}\right)^{3}\left(\frac{2}{5}\right)^{4}$

Question#3

Suppose that the death rate of malaria is 20%, find the probability that the no of death in a particular village is more than half out of 8?

n = 8; p = 0.2; q = 0.8

P(X = x) = 8Cx * (0.2)^x * (0.8)^(8 - x)

More than half out of 8;
P(x > 4) = P(X=5) + P(X=6) + P(X=7) + P(X=8)

Question#4

A communication system transmit binary information over a channel that introduced random bit errors with probability $\varepsilon=10^{-1}$. The transmitter transmits each information bit three times, and decoder takes a majority vote of the received bits to decide on what the transmitted bit was. Find the probability that the receiver will make an incorrect decision.

Question#5

A random variable *x* is binomially distributed with mean *3* and variance *2*. find the probability distribution of *X*, and also draw the graph of Distribution Function.

mean (np) = 3; var (npq) = 2

Putting np in npq >> (3)q=2 >> q = 2/3
>> p = 1 - 2/3 = 1/3
Putting p in np >> (1/3)p = 3 >> n = 9

Use binomial distribution formula P(X=x) = nCx * p^x * q^(n-x)
, where n = 9, p = 1/3, q = 2/3 and x ranges from 0 to 9

Graph will be a histogram with x-axis being 0 to 9 (n) and y-axis the probabilities associated with that particular value of x.