NUST School of Electrical Engineering & Computer Science

MATH 232-Complex Variables & transforms - Summer 2019 27-06-19

Problem Sheet No. 1

Q-1 Let $f(z) = \begin{cases} \dfrac{z\,Re(z)}{|z|}, & z \neq 0 \\ 0, & z = 0. \end{cases}$ Discuss continuity of $f(z)$ for all values of z.

Q-2 Define the function $f(z) = \begin{cases} \dfrac{z^5}{|z|^4}, & z \neq 0 \\ 0, & z = 0 \end{cases}$ Prove that the real and imaginary parts of f satisfy CREs at $z=0$ but $f(z)$ is not differentiable at $z=0$.

Q-3 Let $f_1(z) = r^{1/3} e^{i\theta/3}$, $r>0$, $-\pi < \theta \leq \pi$ and $f_2(z) = r^{1/3} e^{i\left(\frac{\theta+2\pi}{3}\right)}$ be two branches of the multivalued cube root function $f(z) = z^{1/3}$. What is the range of f_1 & f_2. Calculate $f_1(i)$ & $f_2(i)$.

Q-4 Consider the complex logarithmic function

$$w = \log z = \ln r + i\theta, \quad -\frac{\pi}{2} < \theta \leq 3\pi/2 .$$

Calculate & sketch the branch cut. With this choice of branch what is the numerical value of $\log(-1-i)$?

Q-5 Find the largest domain of analyticity of $f(z) = Log[z-(3+4i)]$. Find the numerical value of $f(0)$.

Q-6 Consider a branch of $\log z$ analytic in the domain created with the branch cut $x=-y$, $x \geq 0$. If for this branch, $\log 1 = -2\pi i$, find the following: $\log(i)$, $\log(\sqrt{3}+i)$, $\log(-ie)$.

Q-7 Use L'Hopital rule to evaluate $\lim\limits_{z\to 0} \left(\dfrac{\sin z}{z}\right)^{1/z^2}$ Ans: $e^{-1/6}$.

Q-8 Find all solutions to $\cosh z = i$, $\sin z = i+1$, $\cos z = 2i$, $(e^z - 1)^3 = 1$.

Q-9 Find a harmonic function ϕ in the infinite strip

$$\{z : -2 \leq Re(z) - Im(z) \leq 3\}$$

such that $\phi = 0$ on the left edge and $\phi = 4$ on the right edge.

Q-10 Find a harmonic function $\phi(x,y)$ in the region of the first quadrant between the curves $xy=2$ and $xy=4$ and take value 1 on the lower edge and the value 3 on the upper edge.

Q-11 Find a harmonic function ϕ in the annulus $\{z : 1 \leq |z| \leq 2\}$ such that $\phi = 1$ on $\{|z|=1\}$ and $\phi = 2$ on $\{|z| = 2\}$.