Engineering Mechanics

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Contents (Section 4.6 and 4.7)

- Recap
- Moment of a Force about a Specified Axis
- Moment of a Couple
- Simplification of a Force and Couple System

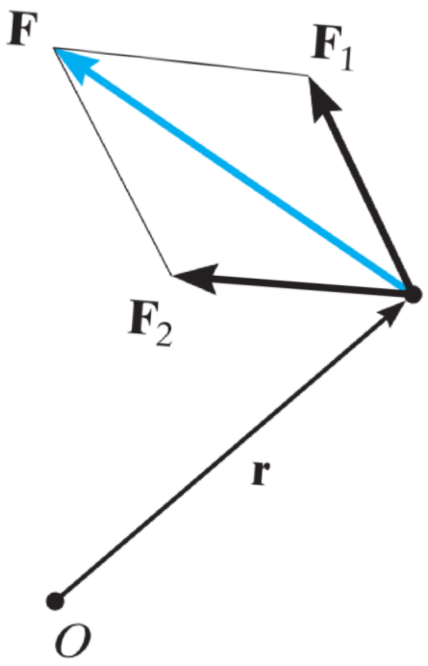
RECAP

Principle of Moments (Varignon's Theorem)

The moment of a force about a point is equal to the sum of the moments of the components of the force about the point

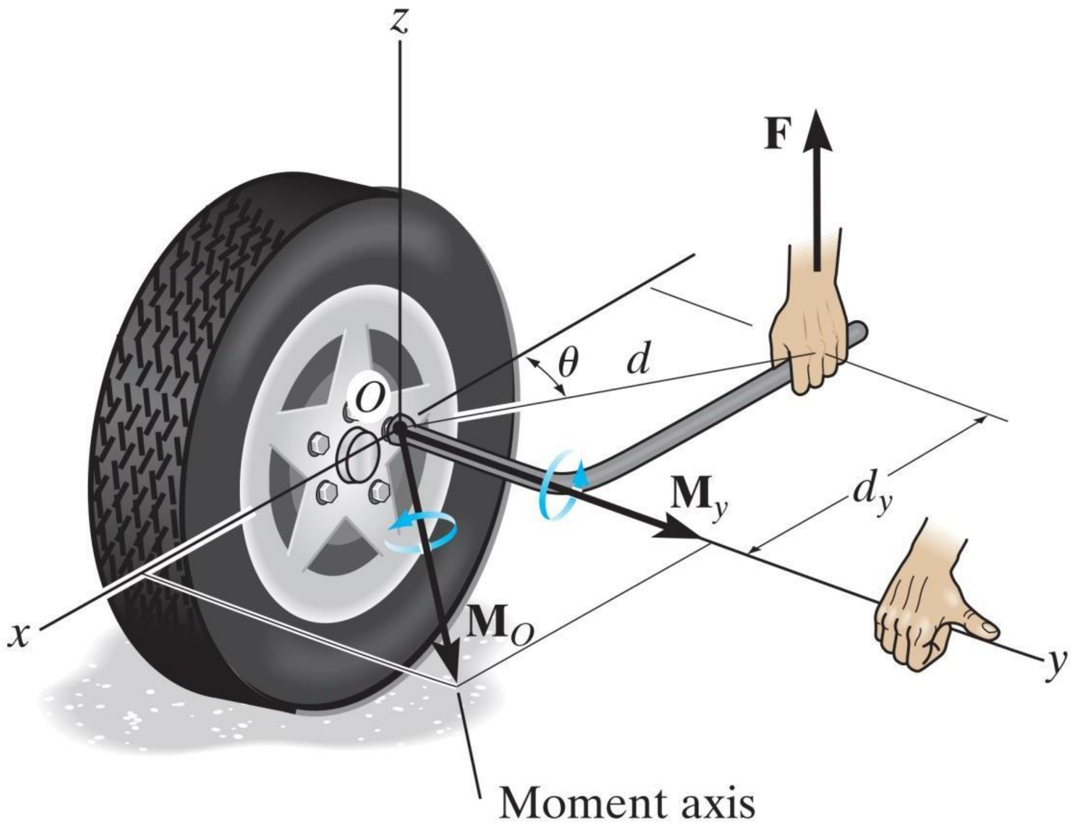
$$\mathbf{M}_O = \mathbf{r} \times \mathbf{F} = \mathbf{r} \times (\mathbf{F}_1 + \mathbf{F}_2) = \mathbf{r} \times \mathbf{F}_1 + \mathbf{r} \times \mathbf{F}_2$$

$$M_O = F_x y - F_y x$$



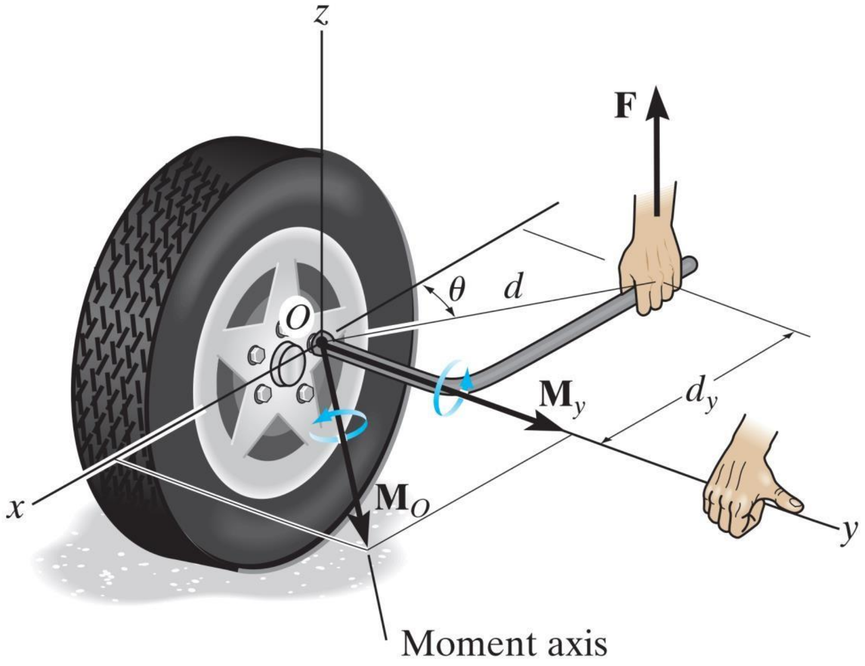
Moment of a Force about a Specified Axis

Scalar Analysis.



Moment of a Force about a Specified Axis

Vector Analysis.

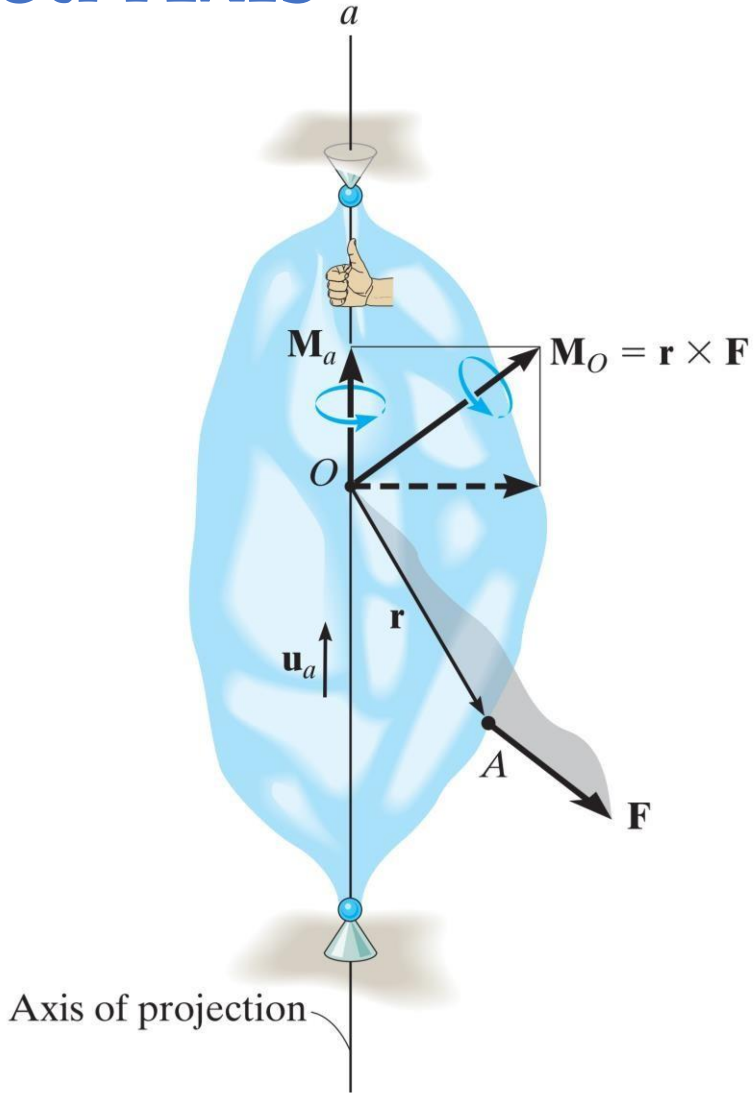


$$M_a = [u_{a_x}\mathbf{i} + u_{a_y}\mathbf{j} + u_{a_z}\mathbf{k}] \cdot \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ r_x & r_y & r_z \\ F_x & F_y & F_z \end{vmatrix}$$

$$= u_{a_x}(r_yF_z - r_zF_y) - u_{a_y}(r_xF_z - r_zF_x) + u_{a_z}(r_xF_y - r_yF_x)$$

Moment of a Force about a Specified Axis

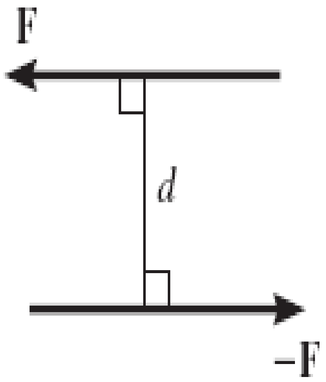
$$M_a = \mathbf{u}_a \cdot (\mathbf{r} \times \mathbf{F}) = \begin{vmatrix} u_{a_x} & u_{a_y} & u_{a_z} \\ r_x & r_y & r_z \\ F_x & F_y & F_z \end{vmatrix}$$



$u_{a_x}, u_{a_y}, u_{a_z}$	represent the *x, y, z* components of the unit vector defining the direction of the *a* axis
r_x, r_y, r_z	represent the *x, y, z* components of the position vector extended from *any point O* on the *a* axis to *any point A* on the line of action of the force
F_x, F_y, F_z	represent the *x, y, z* components of the force vector.

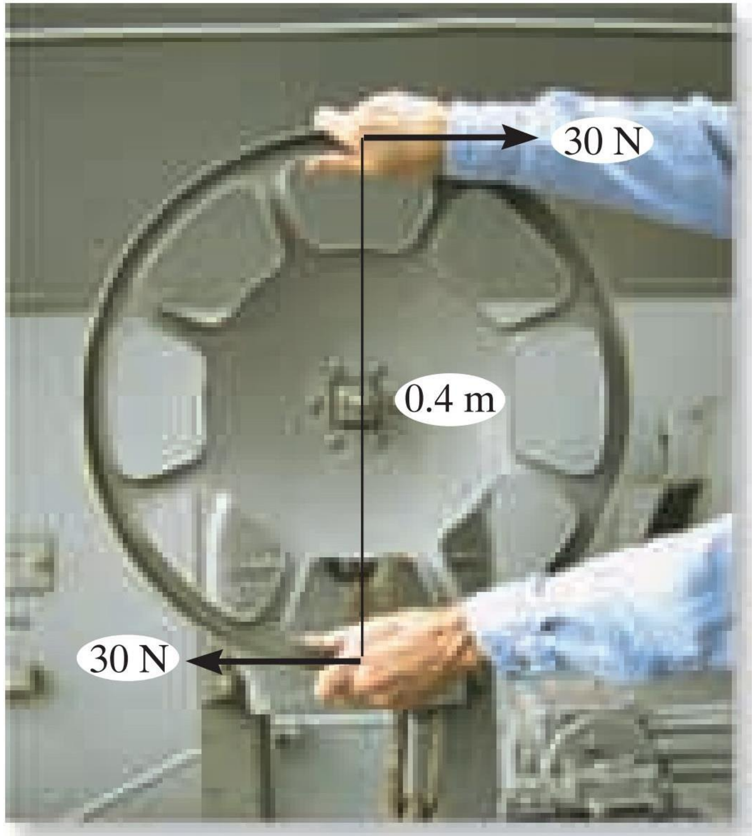
4.6 Moment of a Couple

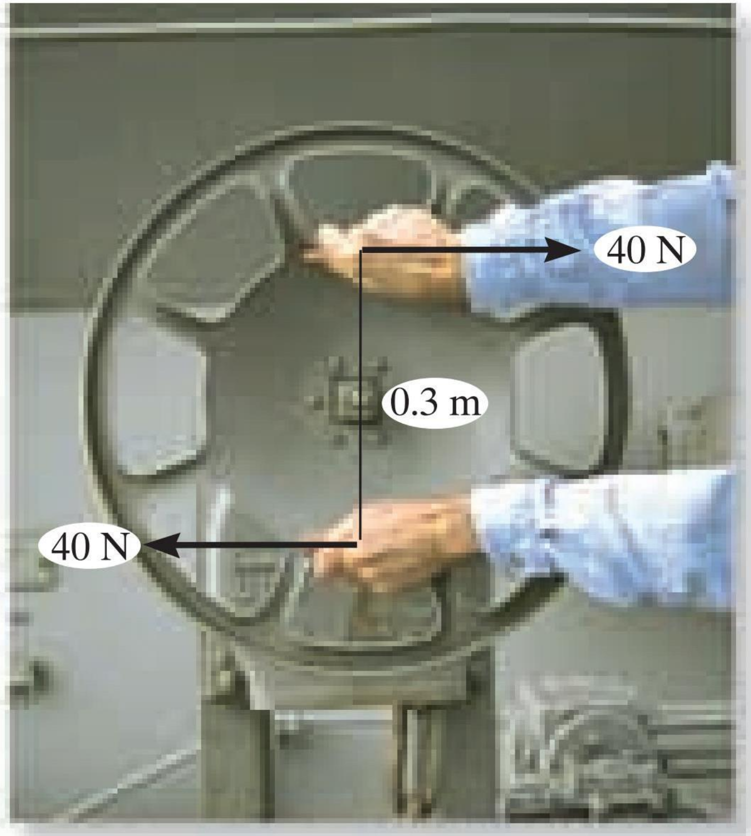
- Couple
  - two parallel forces
  - same magnitude but opposite direction
  - separated by perpendicular distance d
- Resultant force = 0
- Rotation or Tendency to rotate in specified direction
- Couple moment = sum of moments of both couple forces about any arbitrary point

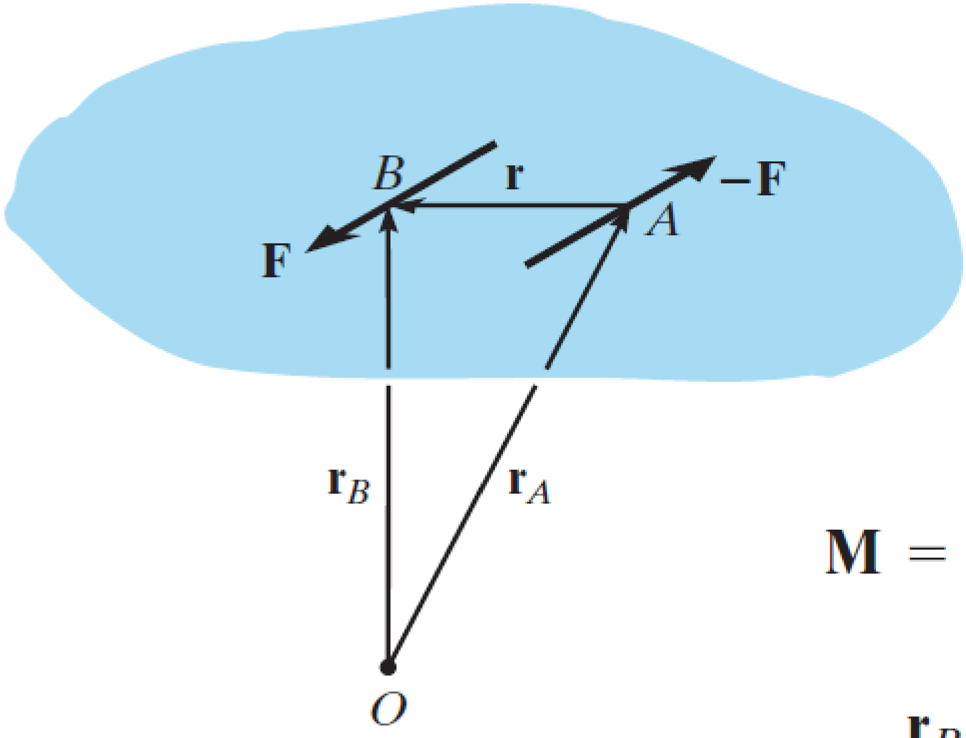


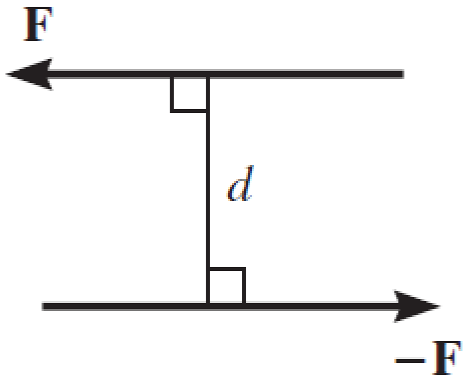
For example, imagine that you are driving a car with both hands on the steering wheel and you are making a turn. One hand will push up on the wheel while the other hand pulls down, which causes the steering wheel to rotate.

Equivalent Couples.









$$\mathbf{M} = \mathbf{r}_B \times \mathbf{F} + \mathbf{r}_A \times -\mathbf{F}$$
$$= (\mathbf{r}_B - \mathbf{r}_A) \times \mathbf{F}$$
$$\mathbf{r}_B = \mathbf{r}_A + \mathbf{r}$$
$$\mathbf{M} = \mathbf{r} \times \mathbf{F}$$

This result indicates that a couple moment is a *free vector*, i.e., it can act at *any point* since **M** depends *only* upon the position vector **r** directed *between* the forces and *not* the position vectors $\mathbf{r}_A$ and $\mathbf{r}_B$, directed from the arbitrary point O to the forces. This concept is unlike the moment of a force, which requires a definite point (or axis) about which moments are determined.

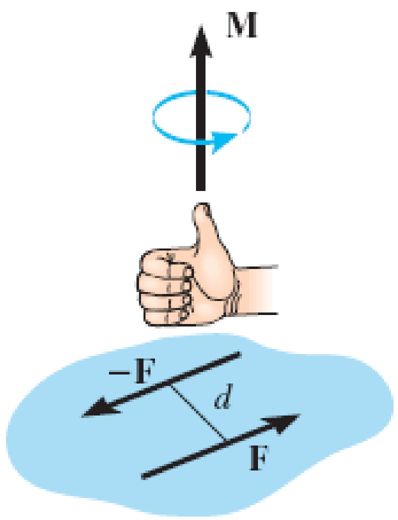
The sum of the moments of both couple forces about *any* arbitrary point.

Scalar Formulation

- Magnitude of couple moment

$$M = Fd$$

- Where F is the magnitude of one of the forces and d is the perpendicular distance or moment arm between the forces.



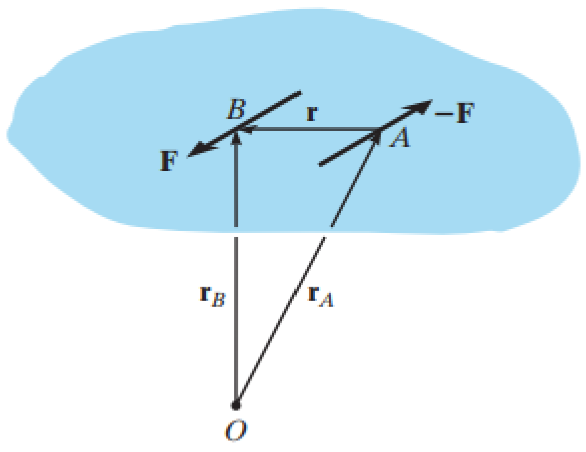
- Direction and sense are determined by right hand rule
- Where the thumb indicates this direction when the fingers are curled with the sense of rotation caused by the couple forces.
- **M** acts perpendicular to plane containing the forces

Vector Formulation

- For couple moment,

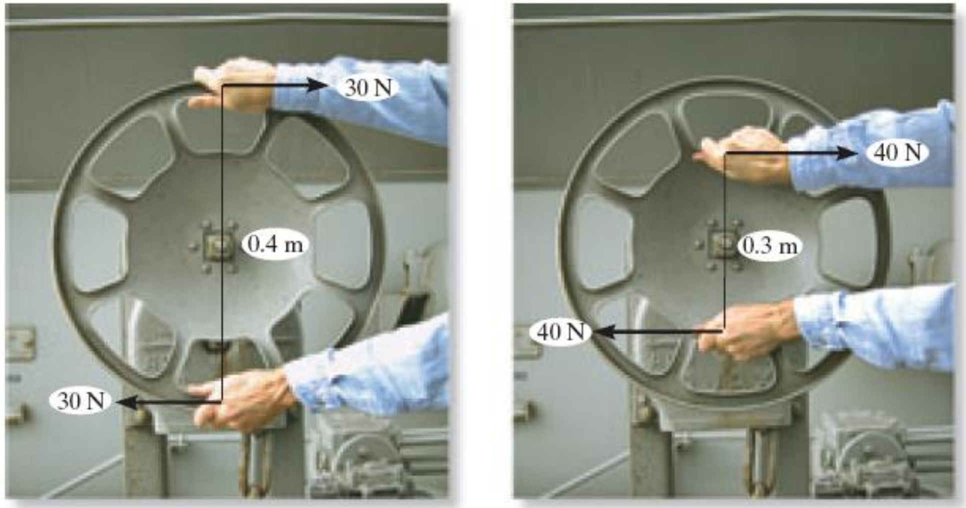
$$\mathbf{M} = \mathbf{r} \times \mathbf{F}$$

- Application of this equation is easily remembered if one thinks of taking the moments of both forces about a point lying on the line of action of one of the forces.
- If moments are taken about point A, moment of **–F** is zero about this point
- **r** is crossed with the force to which it is directed



Equivalent Couples

- If two couples produce a moment with the *same magnitude and direction,* then these two couples are *equivalent*.
- 2 couples are equivalent if they produce the same moment as $M = 30\ N(0.4\ m) = 40\ N(0.3\ m) = 12\ N\ .\ m$
- Forces of equal couples lie on the same plane or plane parallel to one another



Notice that larger forces are required in the second case to create the same turning effect because the hands are placed closer together.

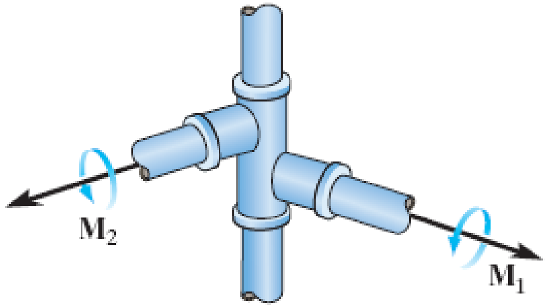
Resultant Couple Moment

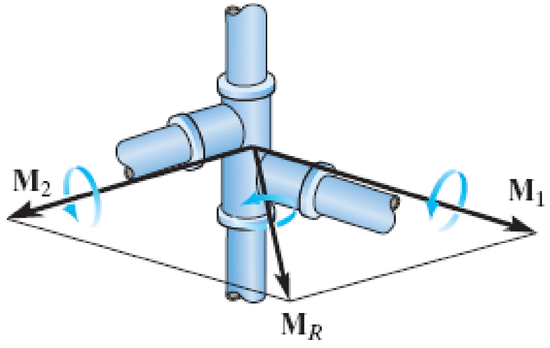
- Since couple moments are vectors, their resultant can be determined by vector addition.
- For resultant moment of two couples at point P,

$$\mathbf{M}_R = \mathbf{M}_1 + \mathbf{M}_2$$

- Since each couple moment is a free vector, we can join their tails at any arbitrary point and find the resultant couple moment
- For more than 2 moments,

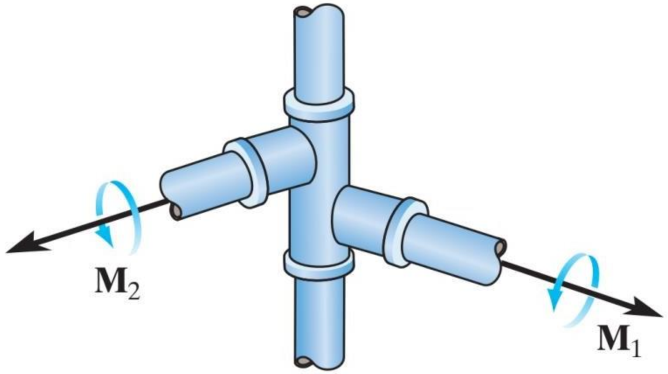
$$\mathbf{M}_R = \sum(\mathbf{r} \times \mathbf{F})$$



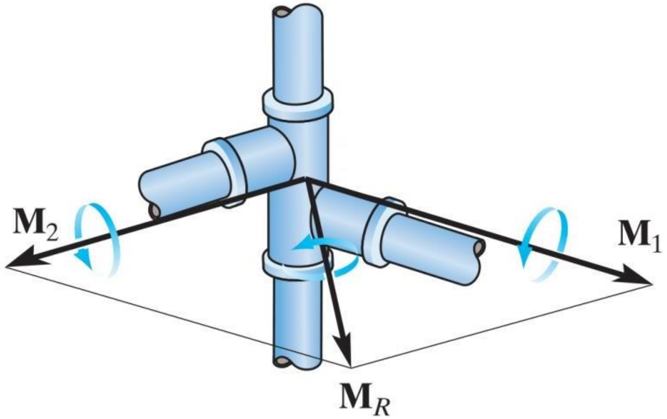


Resultant Couple Moment.

$$\mathbf{M}_R = \Sigma(\mathbf{r} \times \mathbf{F})$$



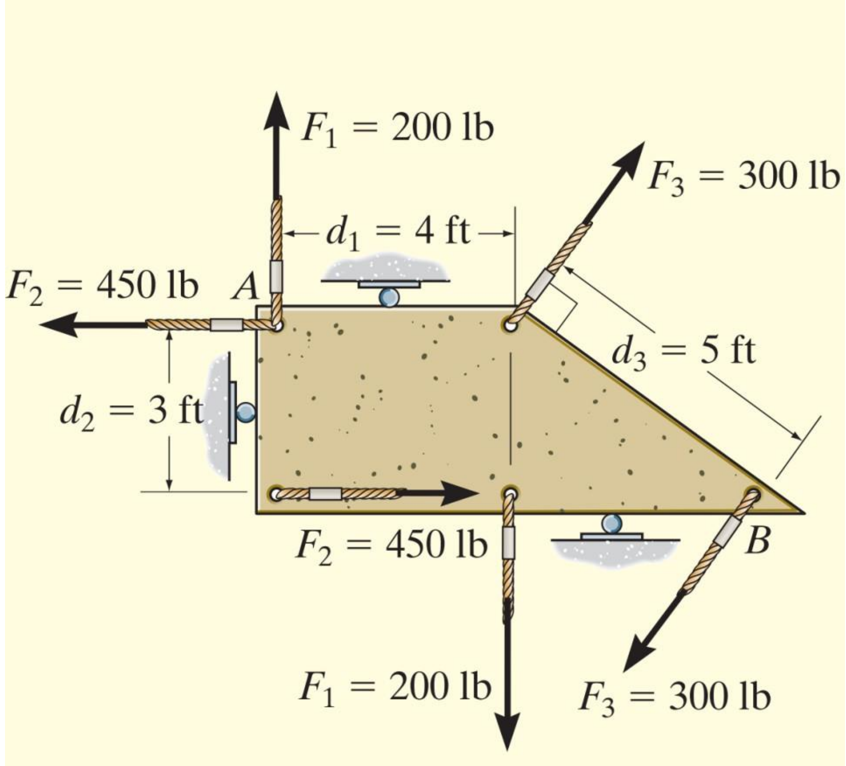
(a)



(b)

Example

Determine the resultant couple moment of the three couples acting on the plate in Fig. 4–30.



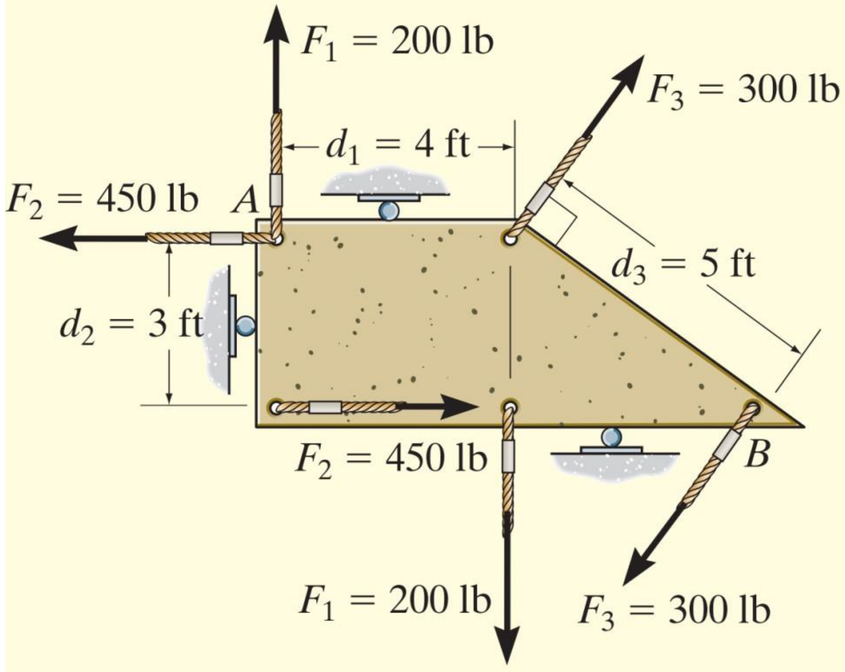
Determine the resultant couple moment of the three couples acting on the plate in Fig. 4–30.

$$\circlearrowleft + M_R = \Sigma M;\ M_R = -F_1 d_1 + F_2 d_2 - F_3 d_3$$

$$= -(200 \text{ lb})(4 \text{ ft}) + (450 \text{ lb})(3 \text{ ft}) - (300 \text{ lb})(5 \text{ ft})$$

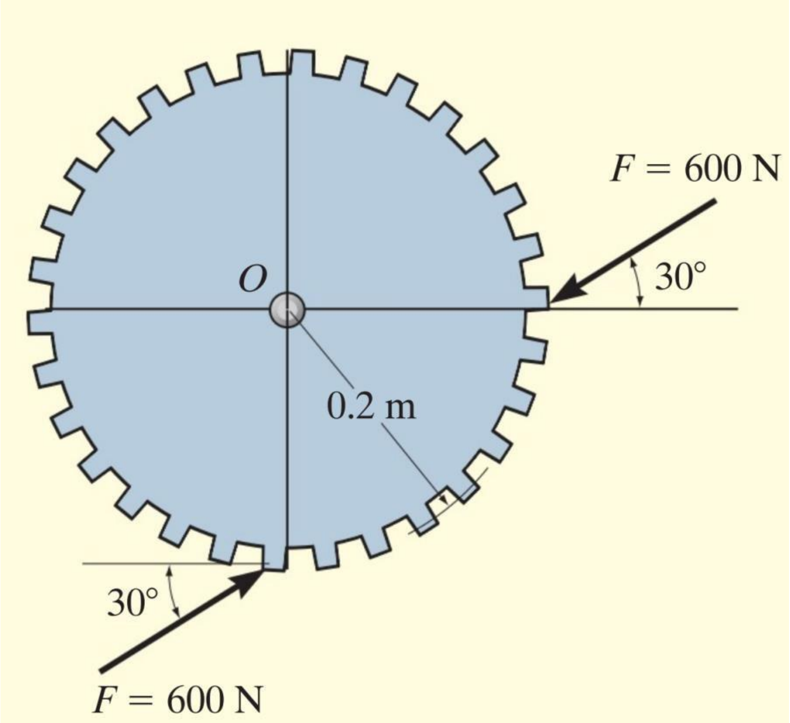
$$= -950 \text{ lb} \cdot \text{ft} = 950 \text{ lb} \cdot \text{ft} \circlearrowright$$

Ans.



Example

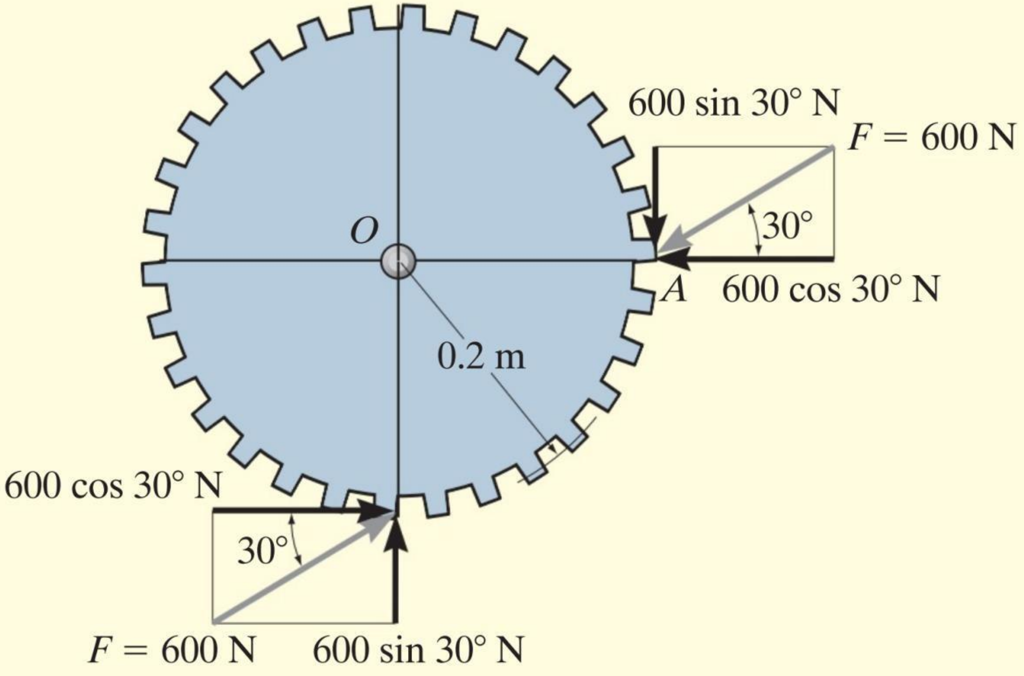
Determine the magnitude and direction of the couple moment acting on the gear in Fig. 4–31*a*.



Determine the magnitude and direction of the couple moment acting on the gear in Fig. 4–31*a*.

$$\circlearrowleft + M = \Sigma M_O;\; M = (600 \cos 30^\circ \text{ N})(0.2 \text{ m}) - (600 \sin 30^\circ \text{ N})(0.2 \text{ m})$$

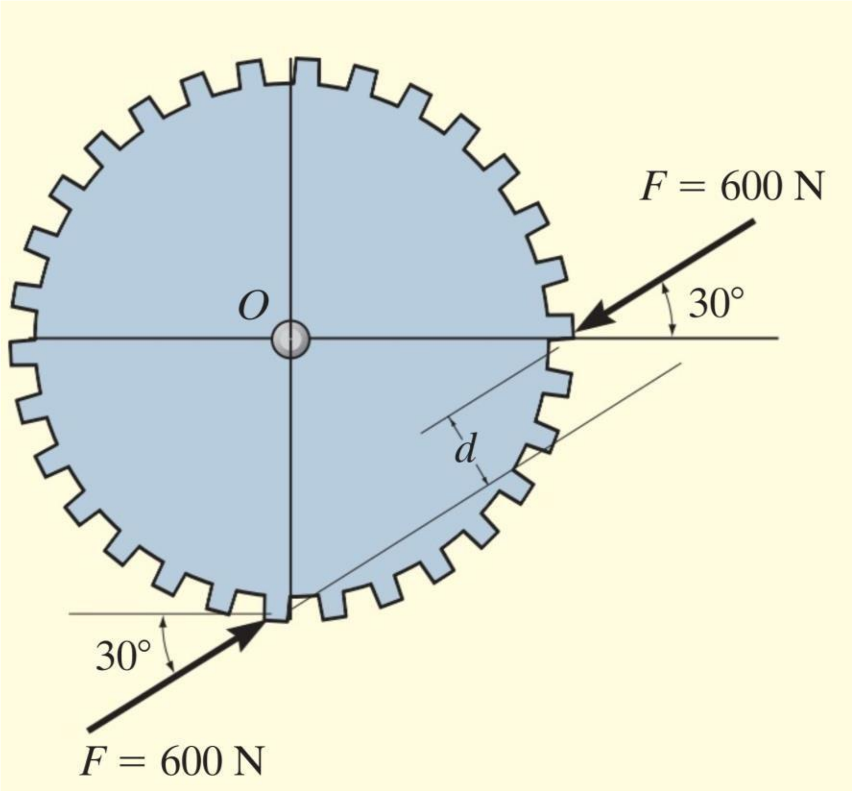
$$= 43.9 \text{ N} \cdot \text{m} \circlearrowright \qquad \textit{Ans.}$$



(b)

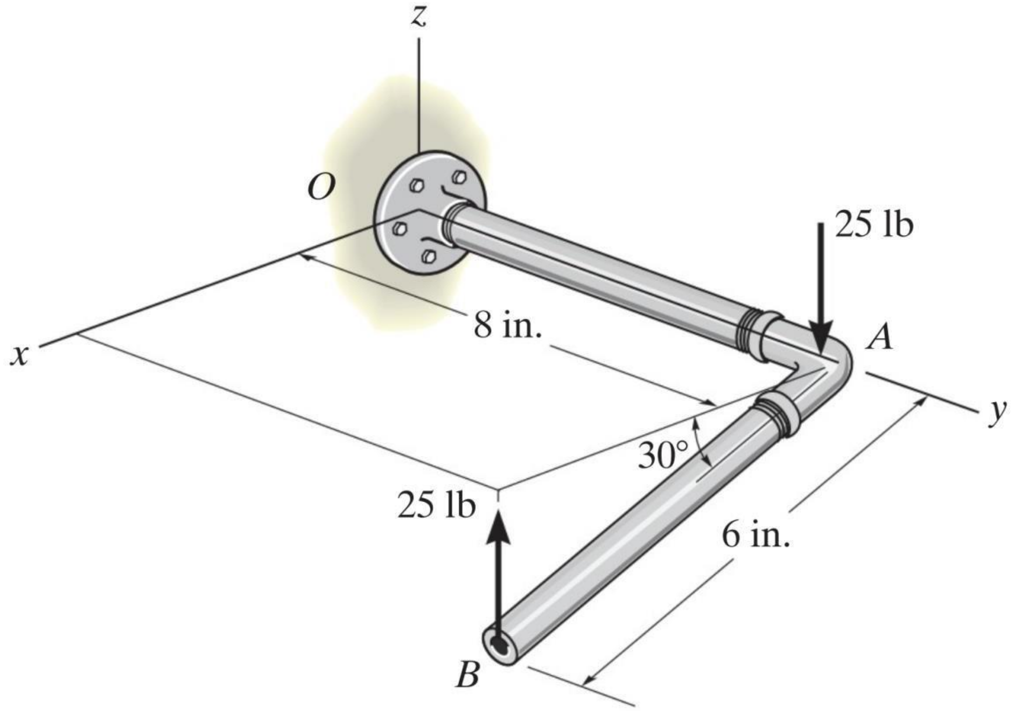
Example

Determine the magnitude and direction of the couple moment acting on the gear in Fig. 4–31*a*.



Example

Determine the couple moment acting on the pipe shown in Fig. 4–32*a*. Segment *AB* is directed 30° below the *x*–*y* plane.



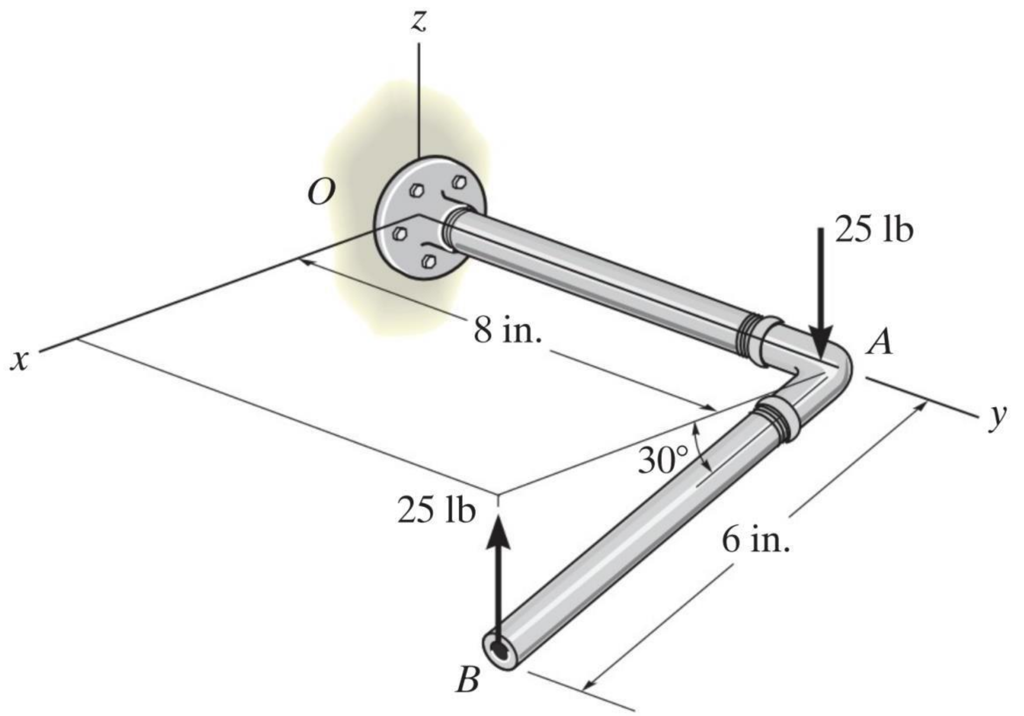
Determine the couple moment acting on the pipe shown in Fig. 4–32*a*. Segment *AB* is directed 30° below the *x–y* plane.

SOLUTION I *(VECTOR ANALYSIS)*

The moment of the two couple forces can be found about *any point*. If point *O* is considered, Fig. 4–32*b*, we have

$$\begin{aligned}\mathbf{M} &= \mathbf{r}_A \times (-25\mathbf{k}) + \mathbf{r}_B \times (25\mathbf{k}) \\ &= (8\mathbf{j}) \times (-25\mathbf{k}) + (6\cos 30°\mathbf{i} + 8\mathbf{j} - 6\sin 30°\mathbf{k}) \times (25\mathbf{k}) \\ &= -200\mathbf{i} - 129.9\mathbf{j} + 200\mathbf{i} \\ &= \{-130\mathbf{j}\}\ \text{lb}\cdot\text{in.}\end{aligned}$$

Ans.



Determine the couple moment acting on the pipe shown in Fig. 4–32*a*. Segment *AB* is directed 30° below the *x*–*y* plane.

SOLUTION I *(VECTOR ANALYSIS)*

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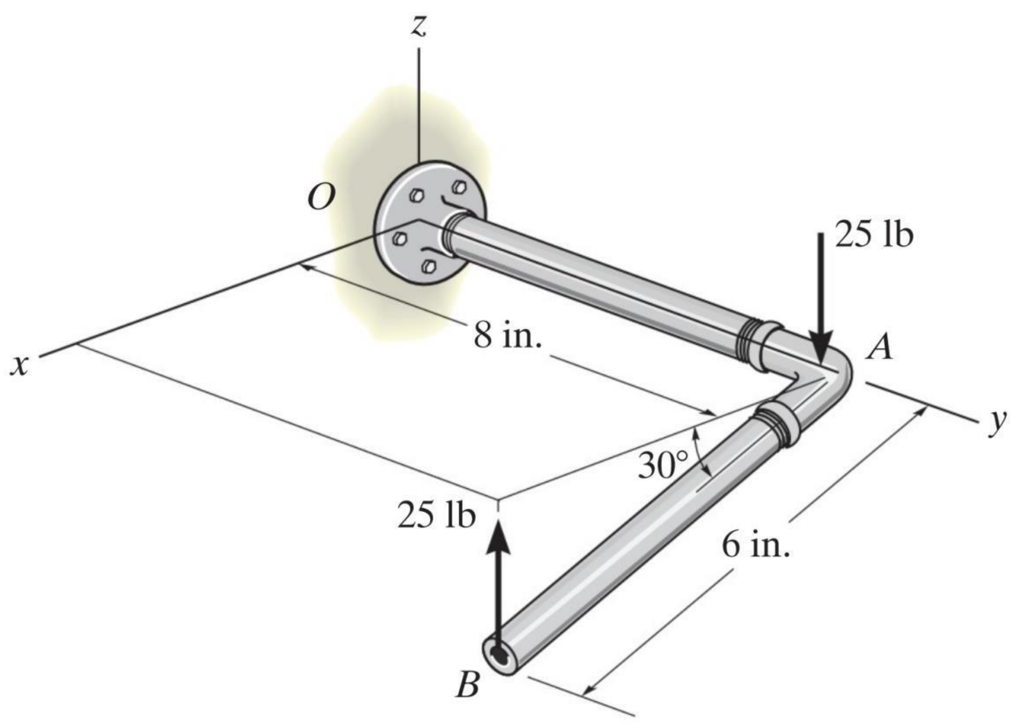
$$\begin{aligned}
\mathbf{M} &= \mathbf{r}_A \times (-25\mathbf{k}) + \mathbf{r}_B \times (25\mathbf{k}) \\
&= (8\mathbf{j}) \times (-25\mathbf{k}) + (6 \cos 30°\mathbf{i} + 8\mathbf{j} - 6 \sin 30°\mathbf{k}) \times (25\mathbf{k}) \\
&= -200\mathbf{i} - 129.9\mathbf{j} + 200\mathbf{i} \\
&= \{-130\mathbf{j}\} \text{ lb} \cdot \text{in.}
\end{aligned}$$

Ans.

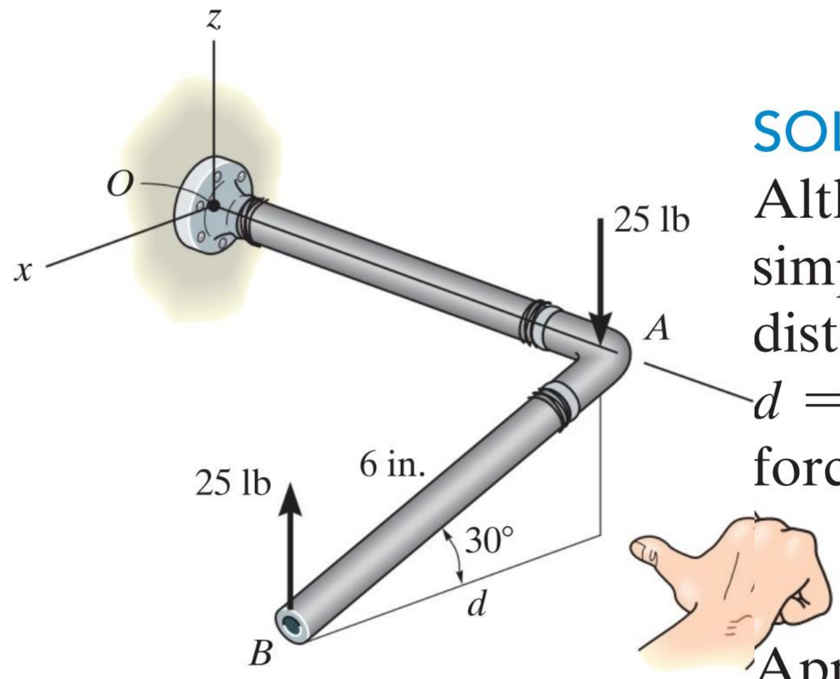


It is *easier* to take moments of the couple forces about a point lying on the line of action of one of the forces, e.g., point *A*, Fig. 4–32*c*. In this case the moment of the force at *A* is zero, so that

$$\begin{aligned}
\mathbf{M} &= \mathbf{r}_{AB} \times (25\mathbf{k}) \\
&= (6 \cos 30°\mathbf{i} - 6 \sin 30°\mathbf{k}) \times (25\mathbf{k}) \\
&= \{-130\mathbf{j}\} \text{ lb} \cdot \text{in.}
\end{aligned}$$

Ans.

Determine the couple moment acting on the pipe shown in Fig. 4–32a. Segment AB is directed 30° below the x–y plane.



SOLUTION II *(SCALAR ANALYSIS)*

Although this problem is shown in three dimensions, the geometry is simple enough to use the scalar equation $M = Fd$. The perpendicular distance between the lines of action of the couple forces is $d = 6 \cos 30° = 5.196$ in., Fig. 4–32d. Hence, taking moments of the forces about either point A or point B yields

$$M = Fd = 25 \text{ lb } (5.196 \text{ in.}) = 129.9 \text{ lb} \cdot \text{in.}$$

Applying the right-hand rule, **M** acts in the $-\mathbf{j}$ direction. Thus,

$$\mathbf{M} = \{-130\mathbf{j}\} \text{ lb} \cdot \text{in.} \qquad \textit{Ans.}$$

Home Assignment

- Example 4.13.