Engineering Mechanics : Statics

Chapter 12: kinematics of a particle

Chapter outline

- Introduction
- Rectilinear Kinematics: Continuous Motion
- Rectilinear Kinematics: Erratic Motion
- General Curvilinear
- Curvilinear Motion: Rectangular Motion
- Projectile

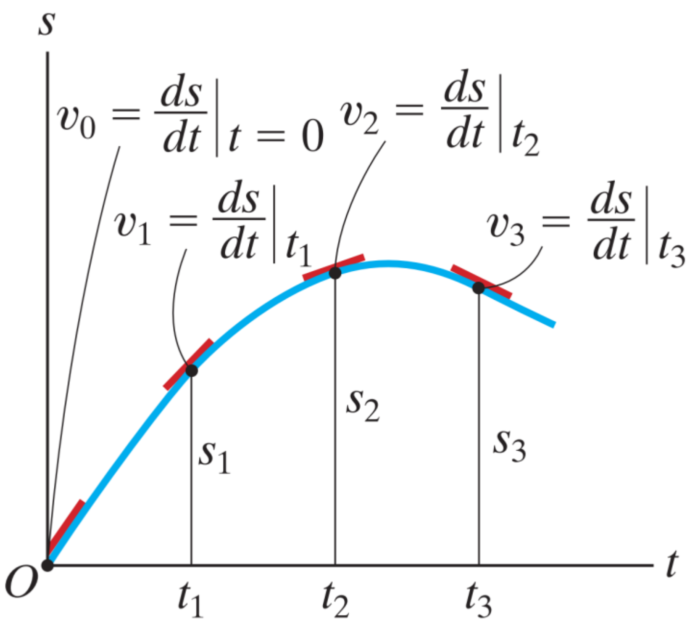
Rectilinear Kinematics: Erratic Motion

changing motion

cannot be described by a single continuous mathematical function

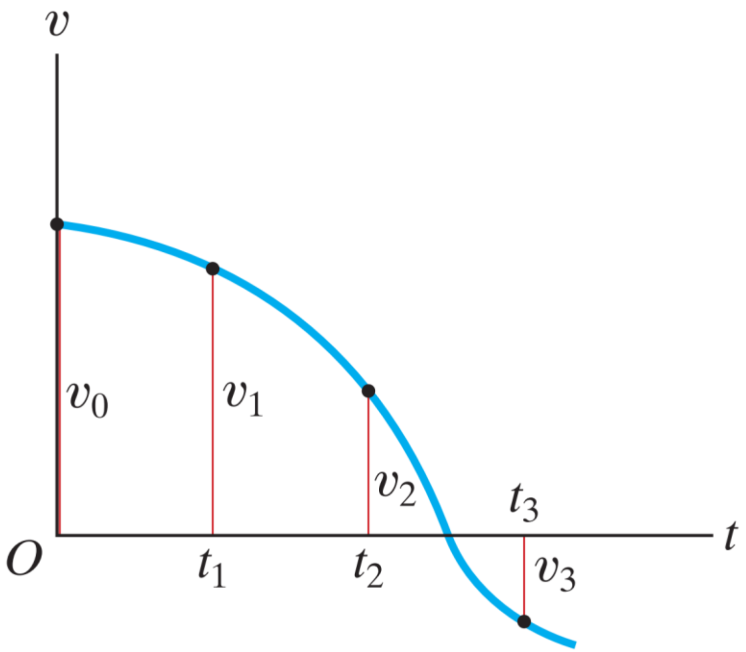
represent the motion as a graph

The *s–t*, *v–t*, and *a–t* Graphs.



$$\frac{ds}{dt} = v$$

slope of *s–t* graph = velocity



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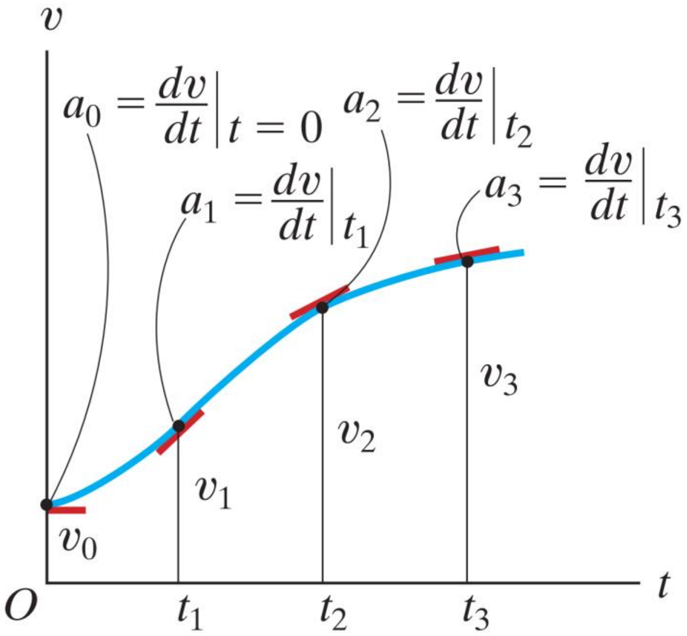
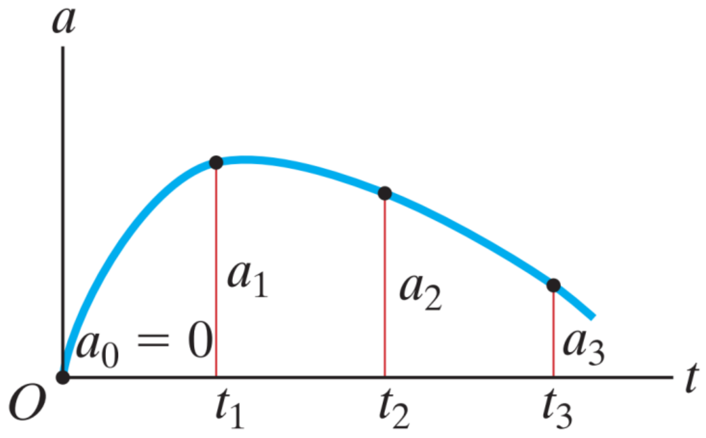


$$\frac{dv}{dt} = a$$

slope of v–t graph = acceleration



Rectilinear Kinematics: Erratic Motion

changing motion

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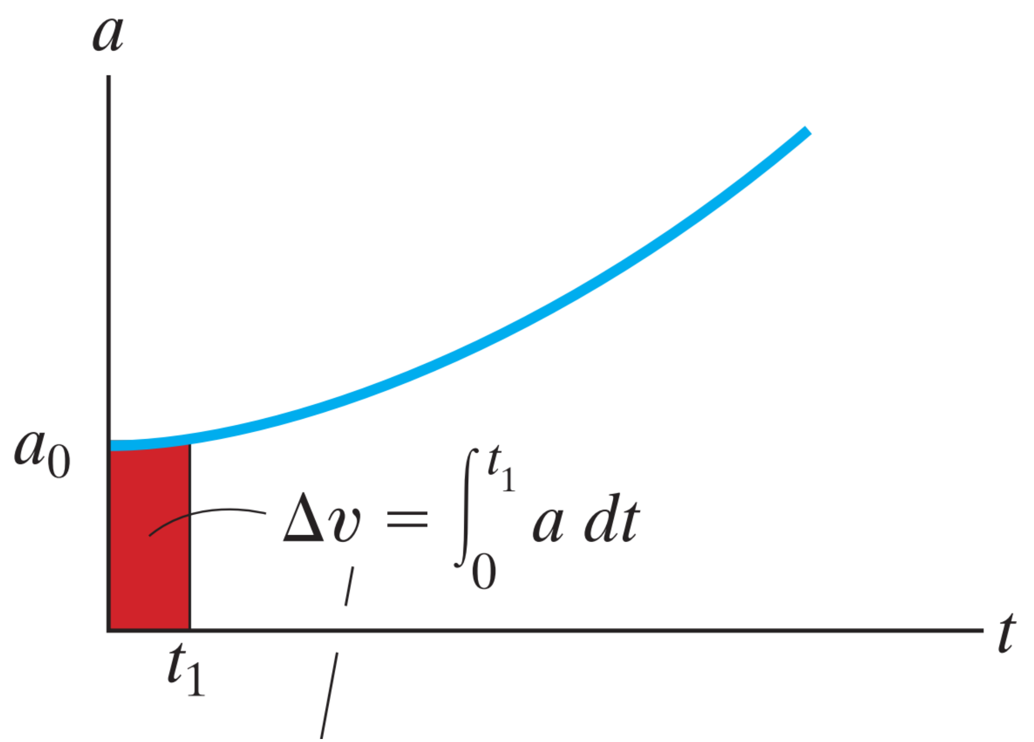
The *s–t*, *v–t*, and *a–t* Graphs.

$$\Delta v = \int a\,dt$$

$$\text{change in velocity} = \text{area under } a\text{–}t \text{ graph}$$



Rectilinear Kinematics: Erratic Motion

changing motion

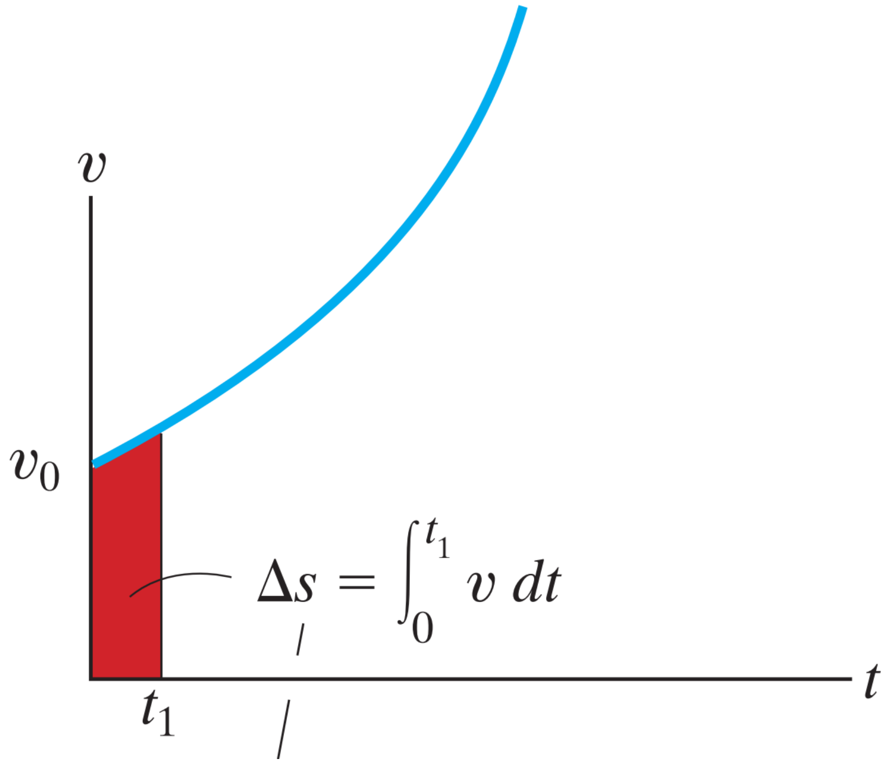
cannot be described by a single continuous mathematical function

represent the motion as a graph

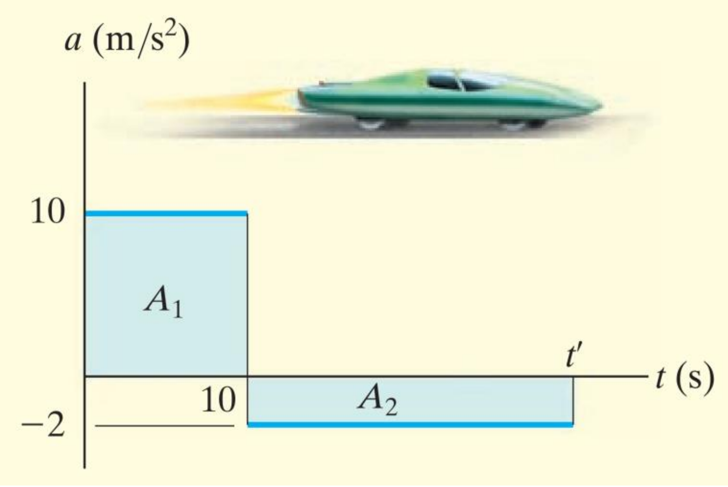
The *s–t*, *v–t*, and *a–t* Graphs.

$$\Delta s = \int v\, dt$$

displacement = area under v–t graph



The car in Fig. 12–14*a* starts from rest and travels along a straight track such that it accelerates at $10\ \text{m/s}^2$ for 10 s, and then decelerates at $2\ \text{m/s}^2$. Draw the v–t and s–t graphs and determine the time t' needed to stop the car. How far has the car traveled?



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v–t Graph. Since $dv = a\,dt$, the v–t graph is determined by integrating the straight-line segments of the a–t graph. Using the *initial condition* $v = 0$ when $t = 0$, we have

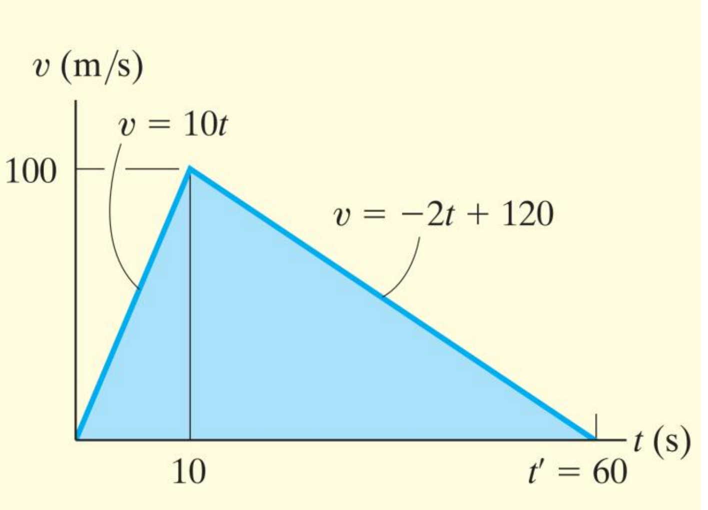
$$0 \le t < 10\ \text{s}; \qquad a = (10)\ \text{m/s}^2; \qquad \int_0^v dv = \int_0^t 10\,dt, \qquad v = 10t$$

When $t = 10$ s, $v = 10(10) = 100$ m/s. Using this as the *initial condition* for the next time period, we have

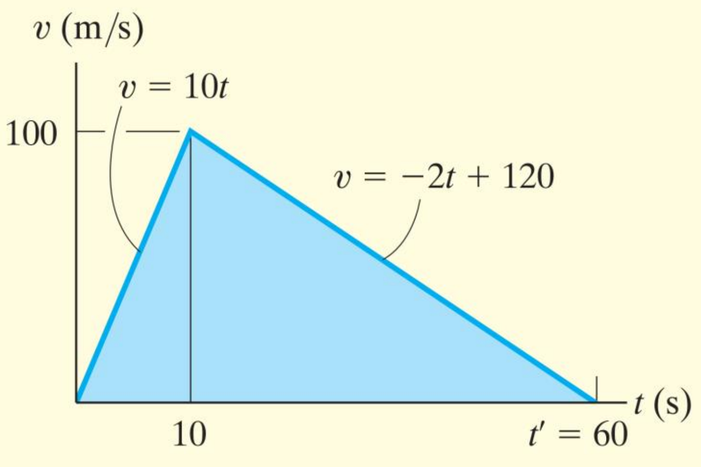
$$10\ \text{s} < t \le t';\ a = (-2)\ \text{m/s}^2;\ \int_{100\ \text{m/s}}^{v} dv = \int_{10\ \text{s}}^{t} -2\,dt,\ v = (-2t + 120)\ \text{m/s}$$

When $t = t'$ we require $v = 0$. This yields, Fig. 12–14*b*,

$$t' = 60\ \text{s}$$ *Ans.*



The car in Fig. 12–14*a* starts from rest and travels along a straight track such that it accelerates at $10\ \mathrm{m/s^2}$ for 10 s, and then decelerates at $2\ \mathrm{m/s^2}$. Draw the v–t and s–t graphs and determine the time t' needed to stop the car. How far has the car traveled?



s–t Graph. Since $ds = v\,dt$, integrating the equations of the v–t graph yields the corresponding equations of the s–t graph. Using the *initial condition* $s = 0$ when $t = 0$, we have

$$0 \le t \le 10\ \mathrm{s};\quad v = (10t)\ \mathrm{m/s};\quad \int_0^s ds = \int_0^t 10t\,dt,\quad s = (5t^2)\ \mathrm{m}$$

When $t = 10$ s, $s = 5(10)^2 = 500$ m. Using this *initial condition*,

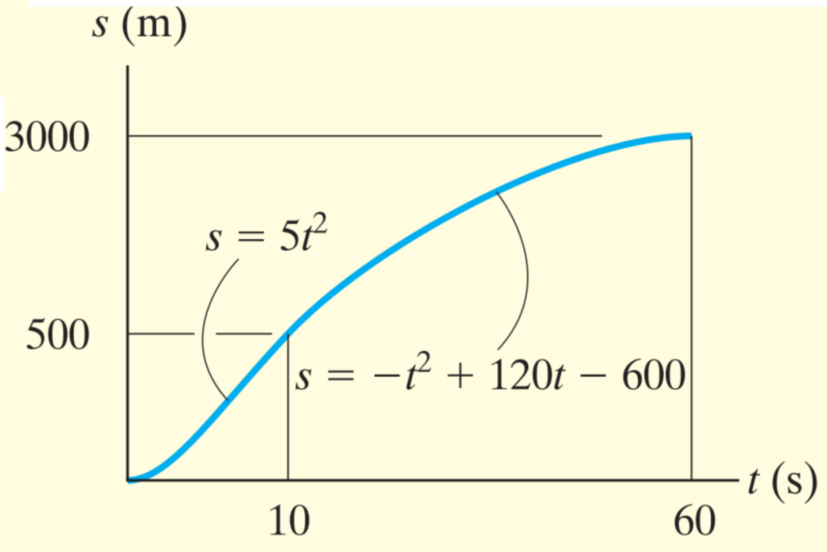
$$10\ \mathrm{s} \le t \le 60\ \mathrm{s};\ v = (-2t + 120)\ \mathrm{m/s};\ \int_{500\ \mathrm{m}}^{s} ds = \int_{10\ \mathrm{s}}^{t} (-2t + 120)\,dt$$

$$s - 500 = -t^2 + 120t - [-(10)^2 + 120(10)]$$

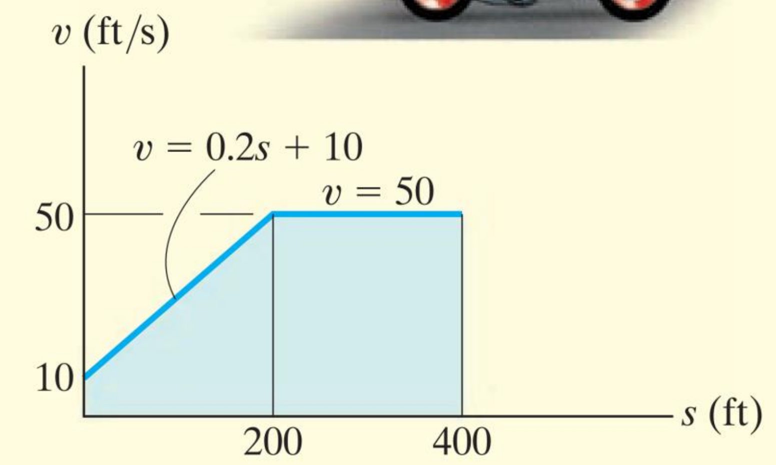
$$s = (-t^2 + 120t - 600)\ \mathrm{m}$$

When $t' = 60$ s, the position is

$$s = -(60)^2 + 120(60) - 600 = 3000\ \mathrm{m} \qquad \textit{Ans.}$$



The v–s graph describing the motion of a motorcycle is shown in Fig. 12–15a. Construct the a–s graph of the motion and determine the time needed for the motorcycle to reach the position $s = 400$ ft.



a–s Graph. Since the equations for segments of the v–s graph are given, the a–s graph can be determined using $a\,ds = v\,dv$.

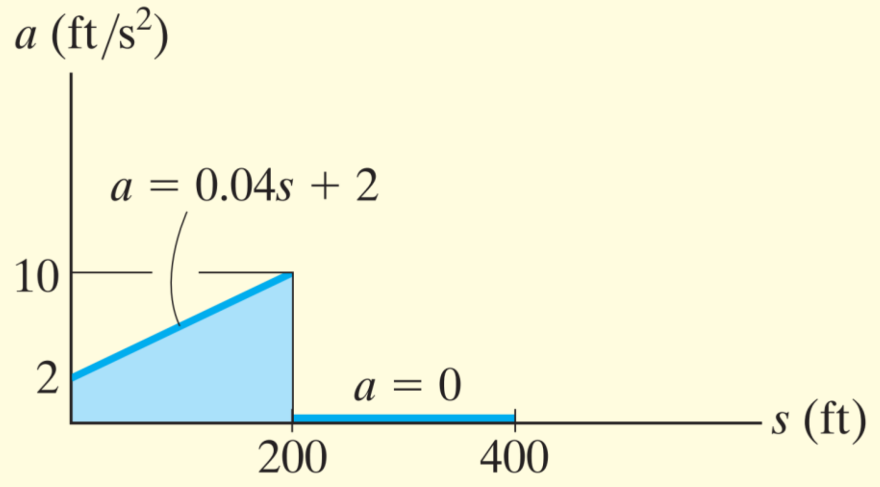
$0 \le s < 200$ ft; $\qquad v = (0.2s + 10)$ ft/s

$$a = v\frac{dv}{ds} = (0.2s + 10)\frac{d}{ds}(0.2s + 10) = 0.04s + 2$$

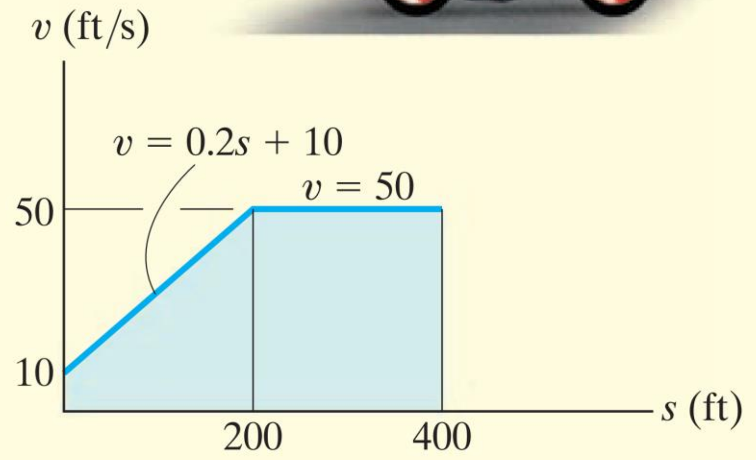
$200 \text{ ft} < s \le 400$ ft; $\qquad v = 50$ ft/s

$$a = v\frac{dv}{ds} = (50)\frac{d}{ds}(50) = 0$$

The results are plotted in Fig. 12–15b.



The v–s graph describing the motion of a motorcycle is shown in Fig. 12–15a. Construct the a–s graph of the motion and determine the time needed for the motorcycle to reach the position $s = 400$ ft.



Time. The time can be obtained using the v–s graph and $v = ds/dt$, because this equation relates v, s, and t. For the first segment of motion, $s = 0$ when $t = 0$, so

$$0 \le s < 200 \text{ ft}; \qquad v = (0.2s + 10) \text{ ft/s}; \qquad dt = \frac{ds}{v} = \frac{ds}{0.2s + 10}$$

$$\int_0^t dt = \int_0^s \frac{ds}{0.2s + 10}$$

$$t = (5 \ln(0.2s + 10) - 5 \ln 10) \text{ s}$$

At $s = 200$ ft, $t = 5 \ln[0.2(200) + 10] - 5 \ln 10 = 8.05$ s. Therefore, using these initial conditions for the second segment of motion,

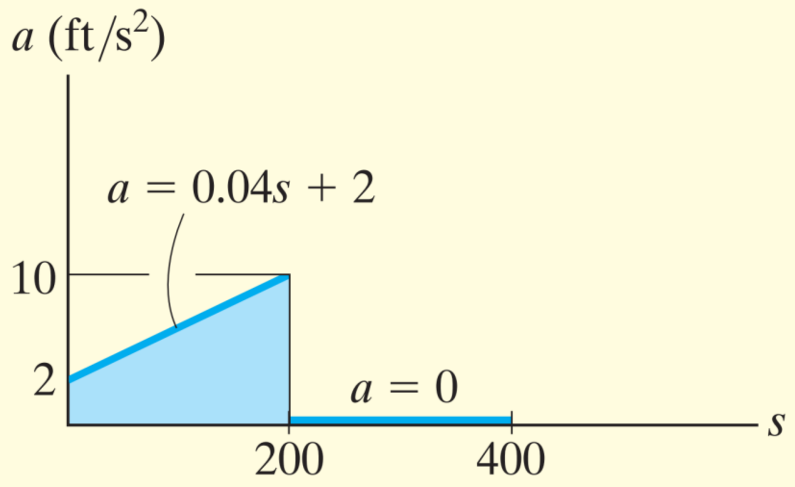
$$200 \text{ ft} < s \le 400 \text{ ft}; \qquad v = 50 \text{ ft/s}; \qquad dt = \frac{ds}{v} = \frac{ds}{50}$$

$$\int_{8.05 \text{ s}}^t dt = \int_{200 \text{ m}}^s \frac{ds}{50};$$

$$t - 8.05 = \frac{s}{50} - 4; \quad t = \left(\frac{s}{50} + 4.05\right) \text{ s}$$

Therefore, at $s = 400$ ft,

$$t = \frac{400}{50} + 4.05 = 12.0 \text{ s} \qquad \textit{Ans.}$$



$$\boxed{v = \frac{ds}{dt}}$$

$$\boxed{a = \frac{dv}{dt}}$$

$$a = \frac{d^2s}{dt^2}$$

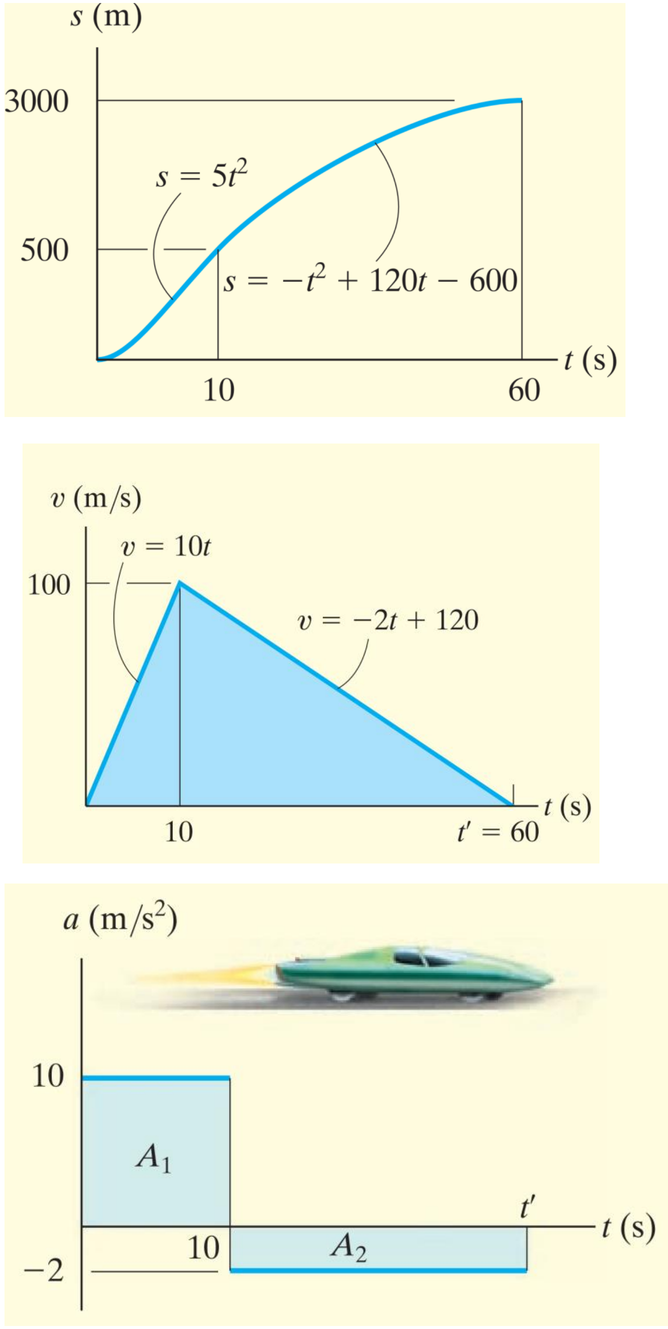
Constant Acceleration, $a = a_c$.

$$\boxed{\begin{array}{c} v = v_0 + a_c t \\ \text{Constant Acceleration} \end{array}}$$

$$\boxed{\begin{array}{c} s = s_0 + v_0 t + \frac{1}{2} a_c t^2 \\ \text{Constant Acceleration} \end{array}}$$

$$\boxed{\begin{array}{c} v^2 = v_0^2 + 2a_c(s - s_0) \\ \text{Constant Acceleration} \end{array}}$$

The s–t, v–t, and a–t Graphs.



Kinematics of a Particle

12. 4 GENERAL CURVILINEAR

Curvilinear Motion

The motion of an object along a curved path is called a curvilinear motion.

12. 4 GENERAL CURVILINEAR

Curvilinear motion in a plane

Motion of car along a curved road

Motion of cable car along a steel cable

12. 4 GENERAL CURVILINEAR

Curvilinear motion in a space

Motion of roller coaster along its track.

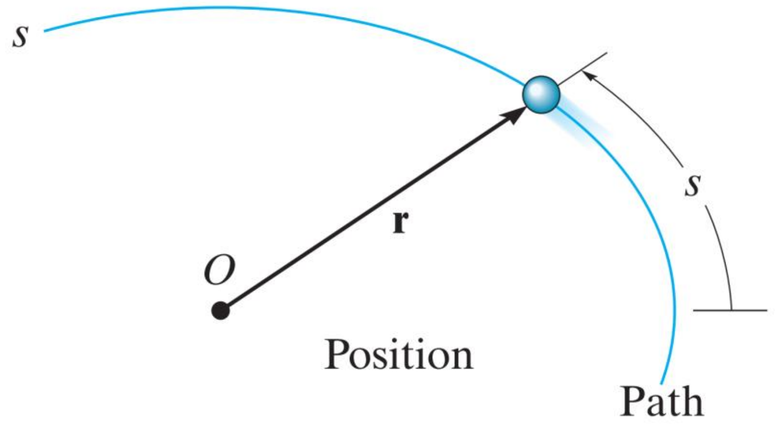
Motion of fighter jets during national parade.

General Curvilinear Motion

Position.

Position. Consider a particle located at a point on a space curve defined by the path function $s(t)$, Fig. 12–16*a*. The position of the particle, measured from a fixed point O, will be designated by the *position vector* $\mathbf{r} = \mathbf{r}(t)$. Notice that both the magnitude and direction of this vector will change as the particle moves along the curve.

$$\mathbf{r} = \mathbf{r}(t).$$

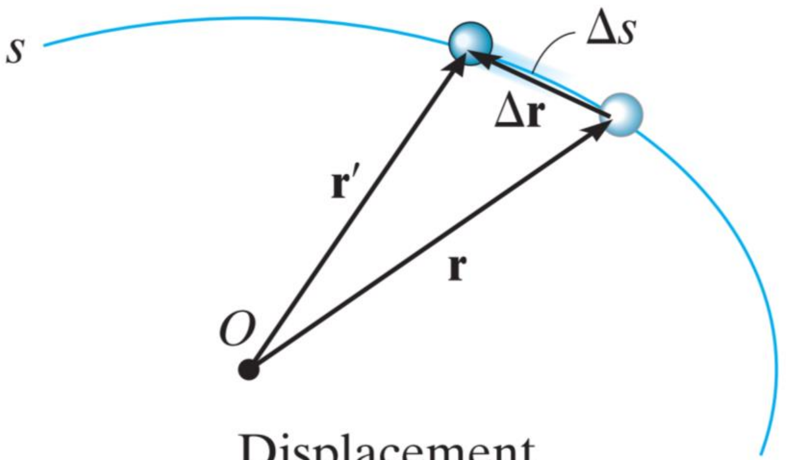


General Curvilinear Motion

Velocity.

Displacement. Suppose that during a small time interval Δt the particle moves a distance Δs along the curve to a new position, defined by $\mathbf{r}' = \mathbf{r} + \Delta\mathbf{r}$, Fig. 12–16*b*. The *displacement* $\Delta\mathbf{r}$ represents the change in the particle's position and is determined by vector subtraction; i.e.,

$$\Delta\mathbf{r} = \mathbf{r}' - \mathbf{r}.$$

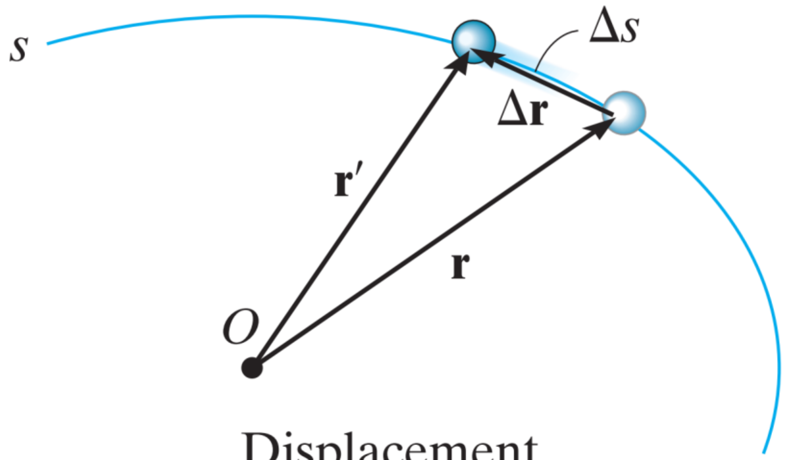


Displacement

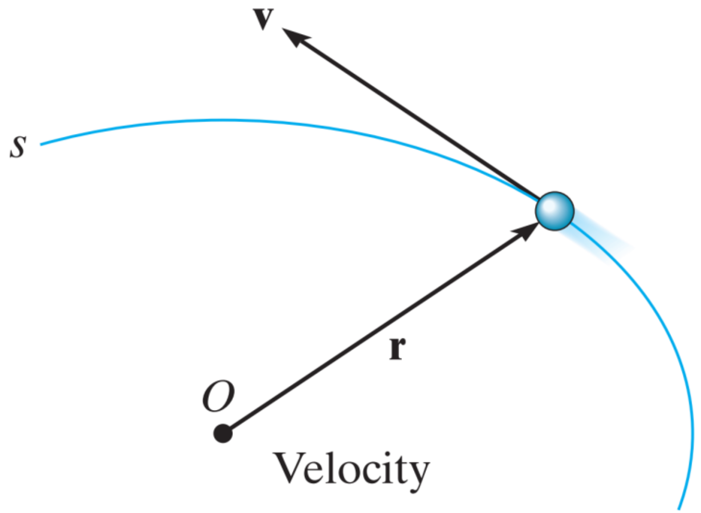
General Curvilinear Motion

Velocity.

$$\mathbf{v}_{avg} = \frac{\Delta \mathbf{r}}{\Delta t}$$



Displacement



Velocity

The *instantaneous velocity* is determined from this equation by letting $\Delta t \to 0$, and consequently the direction of $\Delta \mathbf{r}$ *approaches* the *tangent* to the curve. Hence, $\mathbf{v} = \lim_{\Delta t \to 0} (\Delta \mathbf{r}/\Delta t)$ or

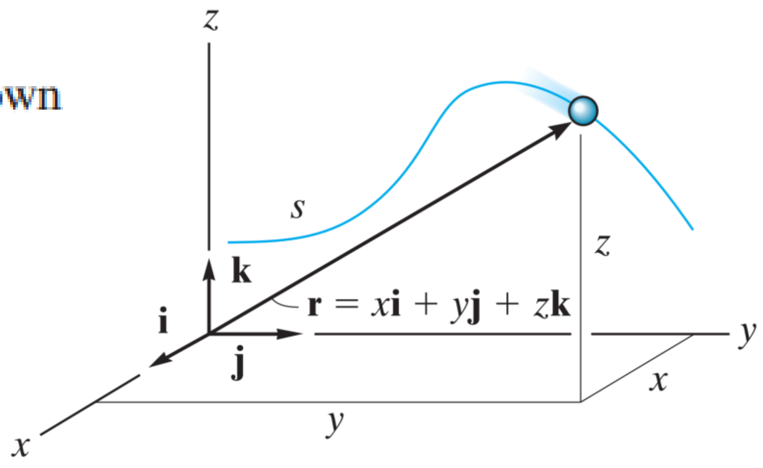
$$\boxed{\mathbf{v} = \frac{d\mathbf{r}}{dt}}$$

Curvilinear Motion: Rectangular Components

Position. If the particle is at point (x, y, z) on the curved path s shown in Fig. 12–17*a*, then its location is defined by the *position vector*

$$\boxed{\mathbf{r} = x\mathbf{i} + y\mathbf{j} + z\mathbf{k}}$$

$$r = \sqrt{x^2 + y^2 + z^2}$$

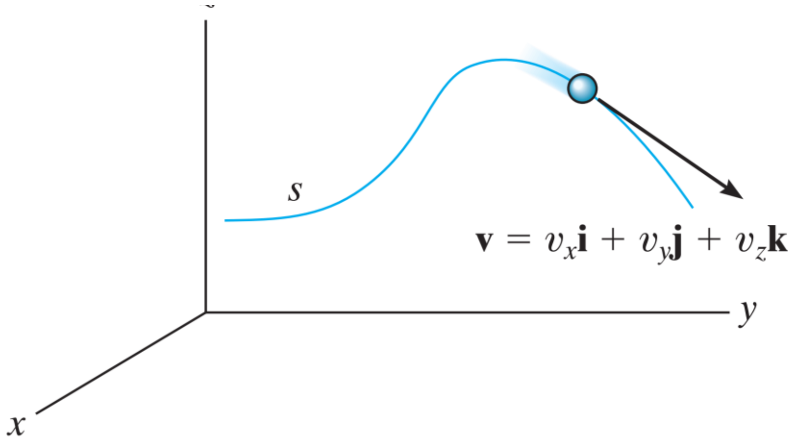


Position

Velocity. The first time derivative of $\mathbf{r}$ yields the velocity of the particle. Hence,

$$\mathbf{v} = \frac{d\mathbf{r}}{dt} = \frac{d}{dt}(x\mathbf{i}) + \frac{d}{dt}(y\mathbf{j}) + \frac{d}{dt}(z\mathbf{k})$$

$$\boxed{\mathbf{v} = \frac{d\mathbf{r}}{dt} = v_x\mathbf{i} + v_y\mathbf{j} + v_z\mathbf{k}}$$



Velocity

$$v = \sqrt{v_x^2 + v_y^2 + v_z^2}$$

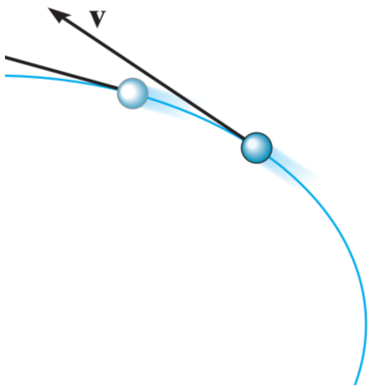
$$\boxed{v_x = \dot{x} \quad v_y = \dot{y} \quad v_z = \dot{z}}$$

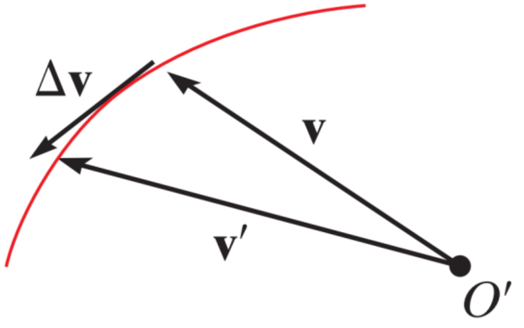
$$\mathbf{u}_v = \mathbf{v}/v.$$

General Curvilinear Motion

Acceleration.

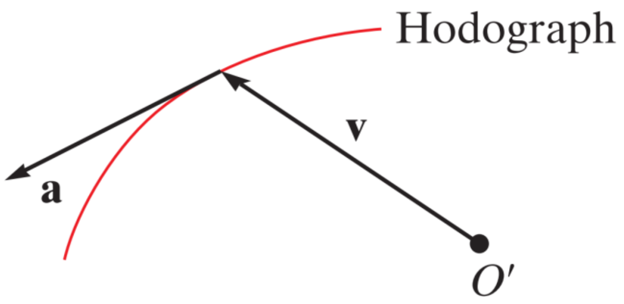
Acceleration. If the particle has a velocity $\mathbf{v}$ at time t and a velocity $\mathbf{v}' = \mathbf{v} + \Delta\mathbf{v}$ at $t + \Delta t$, Fig. 12–16*d*, then the *average acceleration* of the particle during the time interval Δt is

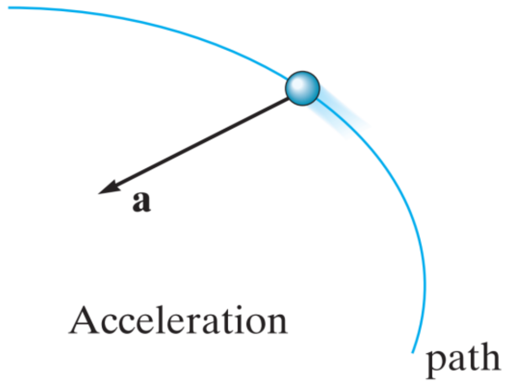




$$\mathbf{a}_{avg} = \frac{\Delta\mathbf{v}}{\Delta t} \qquad \Delta\mathbf{v} = \mathbf{v}' - \mathbf{v}.$$

$$\boxed{\mathbf{a} = \frac{d\mathbf{v}}{dt}} \qquad \mathbf{a} = \frac{d^2\mathbf{r}}{dt^2}$$





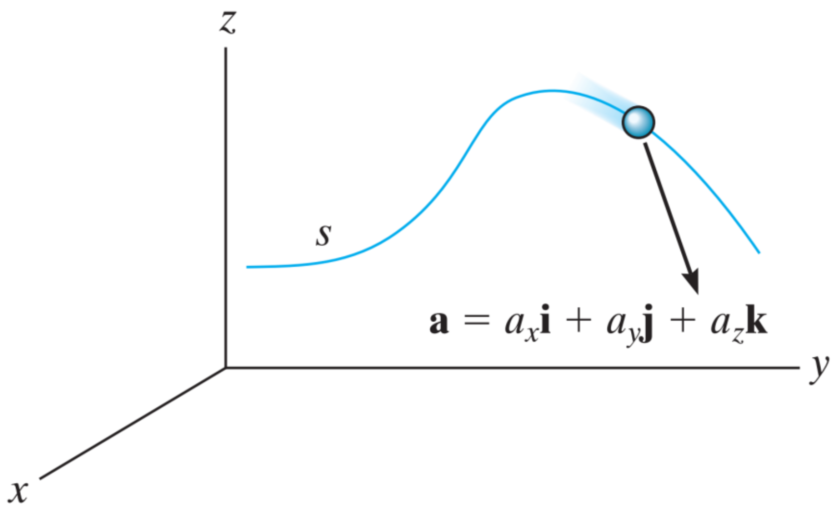
such that their tails are located at the fixed point O’ and their arrowheads touch points on a curve. This curve is called a *hodograph*,

Curvilinear Motion: Rectangular Components

Acceleration. The acceleration of the particle is obtained by taking the first time derivative of Eq. 12–11 (or the second time derivative of Eq. 12–10). We have

$$\boxed{\mathbf{a} = \frac{d\mathbf{v}}{dt} = a_x\mathbf{i} + a_y\mathbf{j} + a_z\mathbf{k}}$$

$$\boxed{\begin{aligned} a_x &= \dot{v}_x = \ddot{x} \\ a_y &= \dot{v}_y = \ddot{y} \\ a_z &= \dot{v}_z = \ddot{z} \end{aligned}}$$



Acceleration

and a *direction* specified by the unit vector $\mathbf{u}_a = \mathbf{a}/a$. Since $\mathbf{a}$ represents the time rate of *change* in both the magnitude and direction of the velocity, in general $\mathbf{a}$ will *not* be tangent to the path, Fig. 12–17*c*.

$$a = \sqrt{a_x^2 + a_y^2 + a_z^2}$$