Problem-3

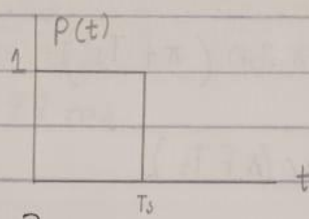
Problem 1

$$s(t) = Re\left\{ s_1(t)\, e^{j2\pi f_c t} \right\}$$

$$s_1(t) = \sum_{n=-\infty}^{\infty} b_n\, p(t - nT_s)$$

$b_n \in \{-3, 1, 1, 3\}$

$p(t)$ is pulse with unit height and width T_s



PSD $S_s(f) = ?$

Solution:

$$S_s(f) = \frac{1}{2}\left[S_g(f - f_c) + S_g(-f - f_c) \right]$$

$$S_g(f) = \frac{A^2\, E\{|x_n|^2\}}{2T_s} |P(f)|^2$$

$$E\{|x_n|^2\} = \frac{(-3)^2 + 1^2 + 1^2 + 3^2}{4} = 5$$

$$A^2 = 1$$

$$P(f) = \int_0^{T_s} \exp(-j2\pi f t)\, dt$$

$$P(f) = \left| \frac{\exp(-j2\pi f t)}{-j2\pi f} \right|_0^{T_s}$$

$$P(f) = \frac{e^{-j2\pi f T_s} - e^{-j2\pi f(0)}}{-j2\pi f}$$

$$P(f) = \frac{e^{-j2\pi f T_s} - 1}{-j2\pi f} = \frac{e^{-j\pi f T_s}\left[e^{-j\pi f T_s} - e^{j\pi f T_s}\right]}{-j2\pi f}$$

$$= \frac{e^{-j\pi f T_s}}{\pi f}\left[\frac{e^{j\pi f T_s} - e^{-j\pi f T_s}}{2j}\right]$$

$$= \frac{e^{-j\pi f T_s}}{\pi f} \sin(\pi f T_s)$$

$$= \frac{e^{-j\pi f T_s}}{\pi f} \times \frac{T_s}{T_s} \times \sin(\pi f T_s)$$

$$= T_s e^{-j\pi f T_s} \operatorname{sinc}(\pi f T_s)$$

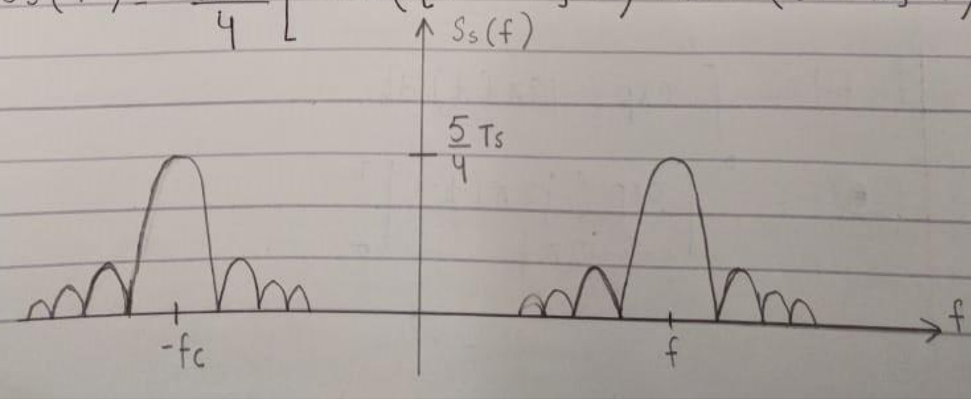
$$|P(f)|^2 = T_s^2 \operatorname{sinc}^2(f T_s)$$

Putting $|P(f)|^2$ and $E\{|x|^2\}$ in $S_g(f)$

$$S_g(f) = \frac{5}{2T_s} T_s^2 \operatorname{sinc}^2(f T_s)$$

$$S_g(f) = \frac{5}{2} T_s \operatorname{sinc}^2(f T_s)$$

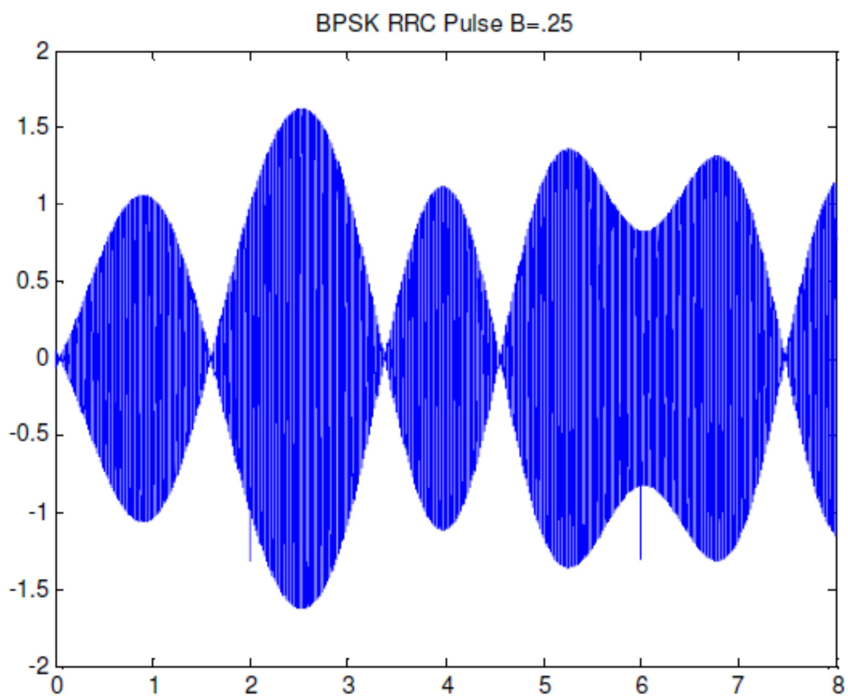
$$S_s(f) = \frac{5T_s}{4}\left[\operatorname{sinc}^2([f - f_c]T_s) + \operatorname{sinc}^2([f + f_c]T_s)\right]$$



Problem 4:
Let the carrier frequency be 100 MHz and let the symbol period be 1 microsecond. Consider a transmitted BPSK signal that uses the 25% excess bandwidth **Root Raised Cosine pulses** (you can use the definition in Wikipedia, where beta=0.25). Using MATLAB or your favorite programming language, plot the RF modulated BPSK signal,

$$x(t) = \left[\sum_{n=1}^{8} x_n p_{rrc}(t - nT_s)\right] \cos(2\pi f_c t)$$

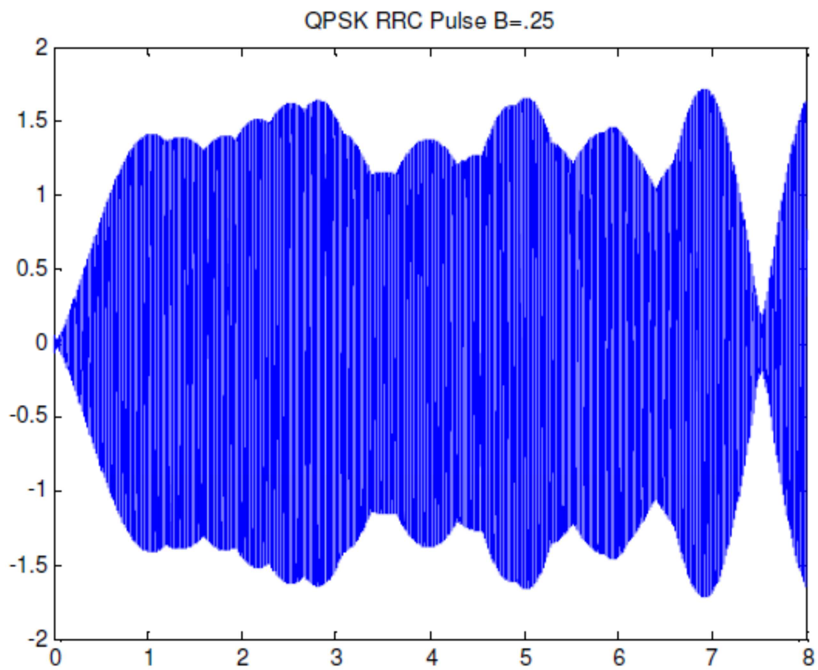
assuming the symbol sequence 1, -1, -1, 1, -1, -1, -1, 1.



Problem 5:
Using the same parameters as in Problem 1, plot the RF modulated QPSK waveform

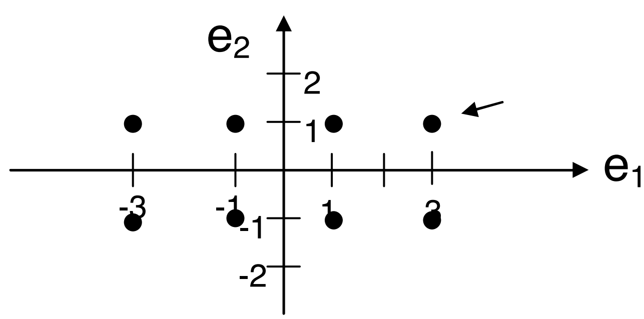
$$x(t) = \left[\textstyle\sum_{n=1}^{8} x_n p_{rrc}(t - nT_s)\right] \cos(2\pi f_c t) - \left[\textstyle\sum_{n=1}^{8} y_n p_{rrc}(t - nT_s)\right] \sin(2\pi f_c t)$$

Use the same x_n sequence as in Problem 1, and let the quadrature symbols of the signal, y_n, be -1, -1, 1, 1, 1, -1, 1, -1.



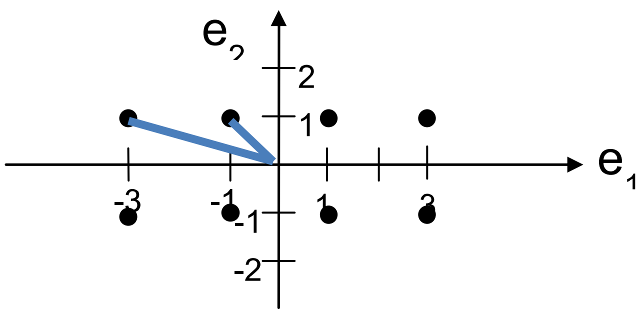
Problem 2**:**

Suppose the constellation of 8 signals for a communication system is given below



a) Find the energy of each symbol and the average symbol energy of this constellation

The energy of a single signal is its squared distance from the origin. The average symbol energy is the sum of the squared distances for all of the symbols, divided by the number of symbols. There are only two distinct distances in this constellation, shown by the thick lines:



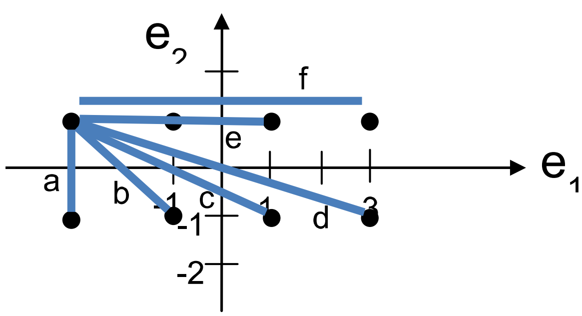
The square of the shorter distance is 2 and the square of the longer is 10. There are four of each, so the average squared distance is 6.

b) Give a complete union bound for the probability of symbol error.

$$P(\text{symbolerror}) = \sum_{i=1}^{M} P(\text{symbolerror} \mid s_i \text{ sent}) P(s_i \text{ sent})$$

$$= \sum_{i=1}^{M} \sum_{\substack{j=1 \\ j\neq i}}^{M} P(s_j \text{ detected} \mid s_i \text{ sent}) P(s_i \text{ sent}) \leq \frac{1}{M} \sum_{i=1}^{M} \sum_{\substack{j=1 \\ j\neq i}}^{M} \mathrm{Q}\left(\frac{\mathrm{d}_{ij}}{\sqrt{2\mathrm{N}_o}}\right)$$

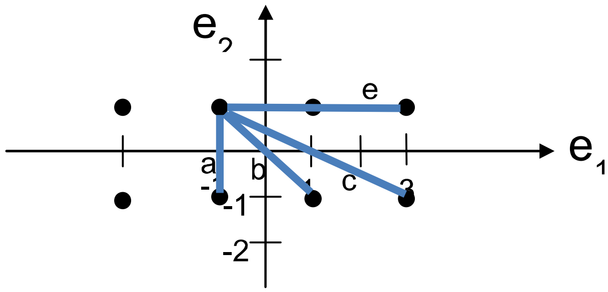
If i is one of the outer corners, the thick lines indicate the different distances involved: a,b,c,d,e, and f. $a^2=4$, $b^2=8$, $c^2=20$, $d^2=40$, e=4, and f=6.



$$\sum_{\substack{j=1 \\ j\neq i}}^{M} \mathrm{Q}\left(\frac{\mathrm{d}_{ij}}{\sqrt{2\mathrm{N}_o}}\right) = 2\mathrm{Q}\left(\frac{\mathrm{a}}{\sqrt{2\mathrm{N}_o}}\right) + \mathrm{Q}\left(\frac{\mathrm{b}}{\sqrt{2\mathrm{N}_o}}\right) + \mathrm{Q}\left(\frac{\mathrm{c}}{\sqrt{2\mathrm{N}_o}}\right) + \mathrm{Q}\left(\frac{\mathrm{d}}{\sqrt{2\mathrm{N}_o}}\right)$$

$$+ \mathrm{Q}\left(\frac{\mathrm{e}}{\sqrt{2\mathrm{N}_o}}\right) + \mathrm{Q}\left(\frac{\mathrm{f}}{\sqrt{2\mathrm{N}_o}}\right)$$

If i is one of the inner corners, the thick lines indicate the different distances involved: a,b,c,and e.



$$\sum_{\substack{j=1\\ j\neq i}}^{M} Q\left(\frac{d_{ij}}{\sqrt{2N_o}}\right) = 3Q\left(\frac{a}{\sqrt{2N_o}}\right) + 2Q\left(\frac{b}{\sqrt{2N_o}}\right) + Q\left(\frac{c}{\sqrt{2N_o}}\right) + Q\left(\frac{e}{\sqrt{2N_o}}\right)$$

Combining these gives:

$$\frac{1}{M}\sum_{i=1}^{M}\sum_{\substack{j=1\\ j\neq i}}^{M} Q\left(\frac{d_{ij}}{\sqrt{2N_o}}\right) = \frac{4}{8}\left\{2Q\left(\frac{a}{\sqrt{2N_o}}\right) + Q\left(\frac{b}{\sqrt{2N_o}}\right) + Q\left(\frac{c}{\sqrt{2N_o}}\right) + Q\left(\frac{d}{\sqrt{2N_o}}\right)\right.$$

$$\left. + Q\left(\frac{e}{\sqrt{2N_o}}\right) + Q\left(\frac{f}{\sqrt{2N_o}}\right)\right\}$$

$$+\frac{4}{8}\left\{3Q\left(\frac{a}{\sqrt{2N_o}}\right) + 2Q\left(\frac{b}{\sqrt{2N_o}}\right) + Q\left(\frac{c}{\sqrt{2N_o}}\right) + Q\left(\frac{e}{\sqrt{2N_o}}\right)\right\}$$

Grouping like terms yields:

$$\frac{1}{M}\sum_{i=1}^{M}\sum_{\substack{j=1\\ j\neq i}}^{M} Q\left(\frac{d_{ij}}{\sqrt{2N_o}}\right) = \frac{1}{2}\left\{5Q\left(\frac{2}{\sqrt{2N_o}}\right) + 3Q\left(\frac{2\sqrt{2}}{\sqrt{2N_o}}\right) + 2Q\left(\frac{2\sqrt{5}}{\sqrt{2N_o}}\right) + Q\left(\frac{2\sqrt{10}}{\sqrt{2N_o}}\right)\right.$$

$$\left. + 2Q\left(\frac{4}{\sqrt{2N_o}}\right) + Q\left(\frac{6}{\sqrt{2N_o}}\right)\right\}$$

c) If the basis functions are the usual ones for QPSK, give the expression of the symbol waveform as a function of time for the symbol that is indicated by the arrow.

The basis functions can be $\varphi_1(t)$ *and* $\varphi_2(t)$ for QPSK

$$\varphi_1(t) = \sqrt{\frac{2}{T_s}}\cos(2\pi f_c t)$$

$$\varphi_2(t) = \sqrt{\frac{2}{T_s}}\cos\left(2\pi f_c t + \frac{\pi}{2}\right)$$

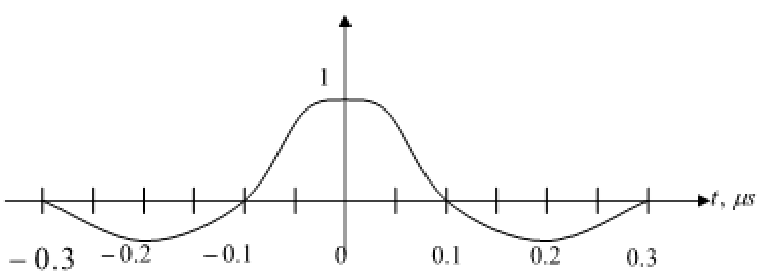
The highlighted symbol can be expressed as

$$x(t) = 3\,\varphi_1(t) + \varphi_2(t)$$

$$= \begin{cases} 3\sqrt{\frac{2}{T_s}}\cos(2\pi f_c t) + \sqrt{\frac{2}{T_s}}\cos\left(2\pi f_c t + \frac{\pi}{2}\right) & , 0 < t < T_s \\ 0 & , otherwise \end{cases}$$

Problem 6:

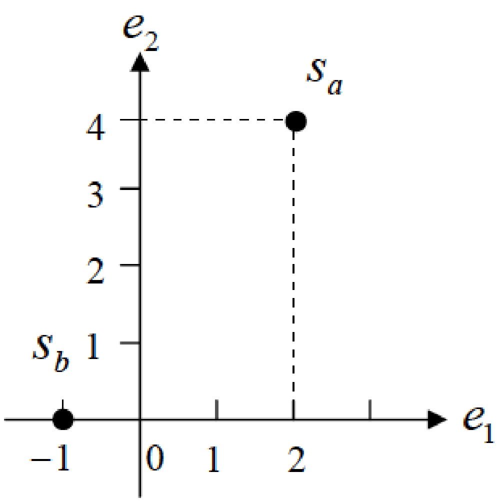
Consider the pulse below, plotted versus time in microseconds. Could this pulse be a Nyquist pulse for a binary transmission with a 10MHz data rate? Why or why not?



A 10Mbps data rate implies a bit period of T=0.1μs. A Nyquist pulse must be zero at all multiples of T (except at 0). Since this pulse is not zero at 0.2μs, it cannot be a Nyquist pulse.

Problem:3

Two signal points Sa and Sb are shown below



Suppose the two basis functions are

$$e_1(t) = \begin{cases} \sqrt{\dfrac{2}{T_s}}\cos(2\pi f_c t) & 0 < t < T_s \\ 0 & otherwise \end{cases}$$

$$e_2(t) = \begin{cases} K\sqrt{\dfrac{2}{T_s}}\cos\left(\dfrac{2\pi}{T_s}t\right)\cos(2\pi f_c t) & 0 < t < T_s \\ 0 & otherwise \end{cases}$$

a) Suppose the noise spectral height is $N_0/2=25/16$. Evaluate the BER if these two signals are used in a wireless communication link.

We can use the BER expression for BPSK modulation here

$$P(bit\ error) = Q\left\{\frac{d_{xy}}{\sqrt{2N_0}}\right\}$$

where d_{xy} is the distance between these 2 points and is given by

$$d_{xy} = \sqrt{(2+1)^2 + (4-0)^2} = 5$$

$$P(bit\ error) = Q\left\{\frac{5}{\sqrt{2\left(\frac{25}{16}\right)}}\right\} = Q(2) = 0.023$$

b) Construct an expression of signal labeled as S_a in terms of t and Ts.

We can use basis functions to express S_a in terms of t and T_s

$$S_a = 2e_1(t) + 4e_2(t)$$

$$S_a = 2\sqrt{\frac{2}{T_s}}\cos(2\pi f_c t) + 4K\sqrt{\frac{2}{T_s}}\cos\left(\frac{2\pi}{T_s}t\right)\cos(2\pi f_c t)$$