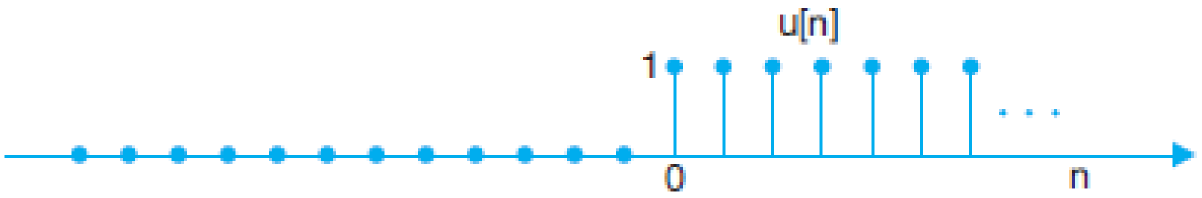
IMPORTANT SIGNALS - DISCRETE TIME

Discrete Time Unit Functions

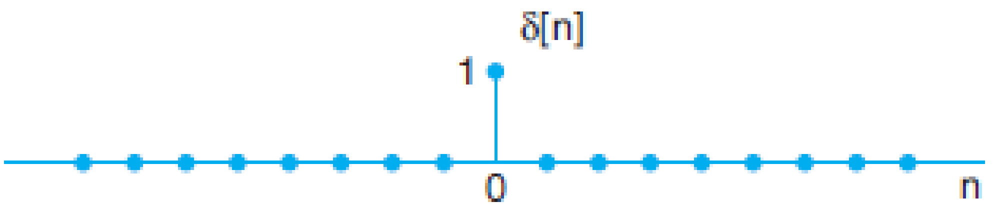
- The discrete-time unit step, is defined as:

$$u[n] = \begin{cases} 0, & n < 0 \\ 1, & n \geq 0 \end{cases}$$



- The discrete time unit impulse is defined as:

$$\delta[n] = \begin{cases} 0, & n \neq 0 \\ 1, & n = 0 \end{cases}$$



- The discrete-time unit impulse is the first difference of the discrete-time step as:

$$\delta[n] = u[n] - u[n-1]$$

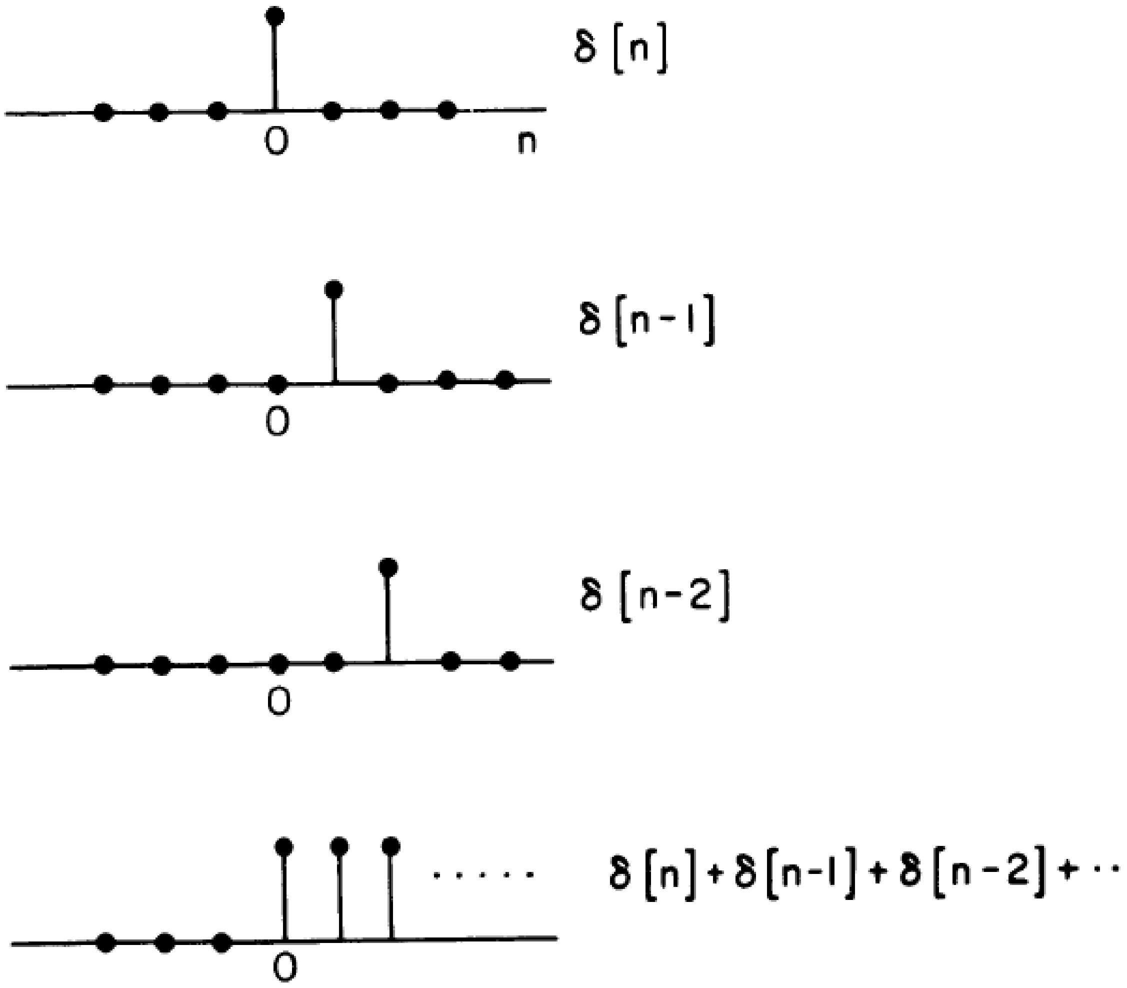
- The discrete-time unit step is the running sum of the unit sample as:

$$u[n] = \sum_{m=-\infty}^{n} \delta[m]$$

Discrete Time Unit Functions

$$u[n] = \sum_{k=0}^{\infty} \delta[n-k]$$

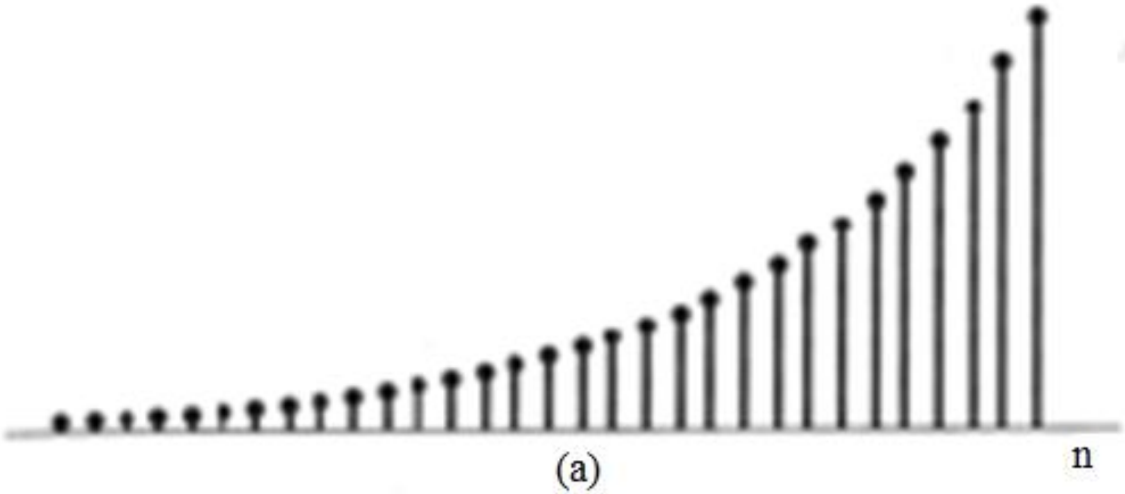
- A unit step can be written as an infinite sum of time-delayed unit impulses



Exponential Signals

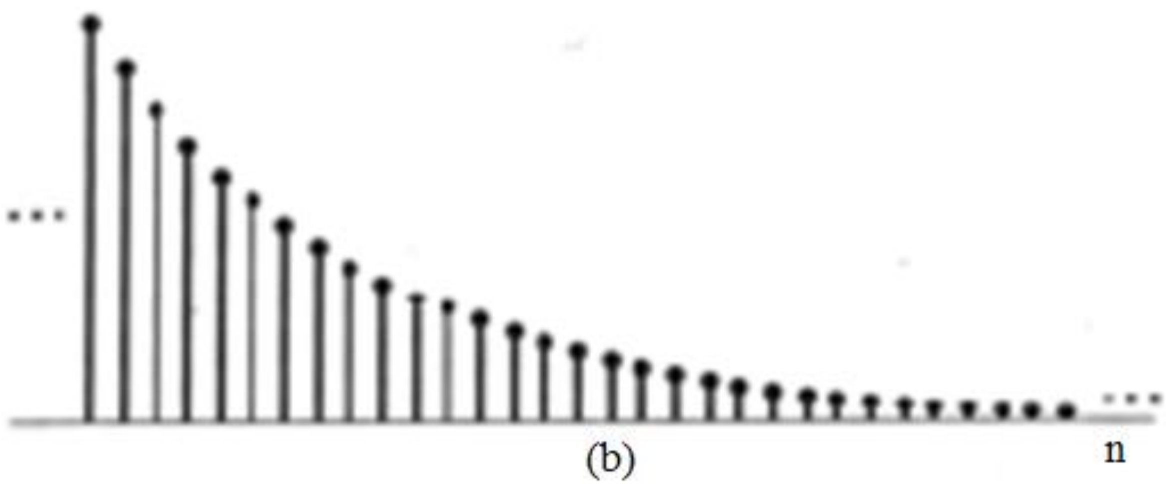
- Real Exponential signal $x[n] = Ae^{an}$; A and a are Real

a) $a > 0$ and $A > 0$;
exponential rise



b) $a < 0$ and $A > 0$;
exponential decay

c) $a > 0$ and $A < 0$;



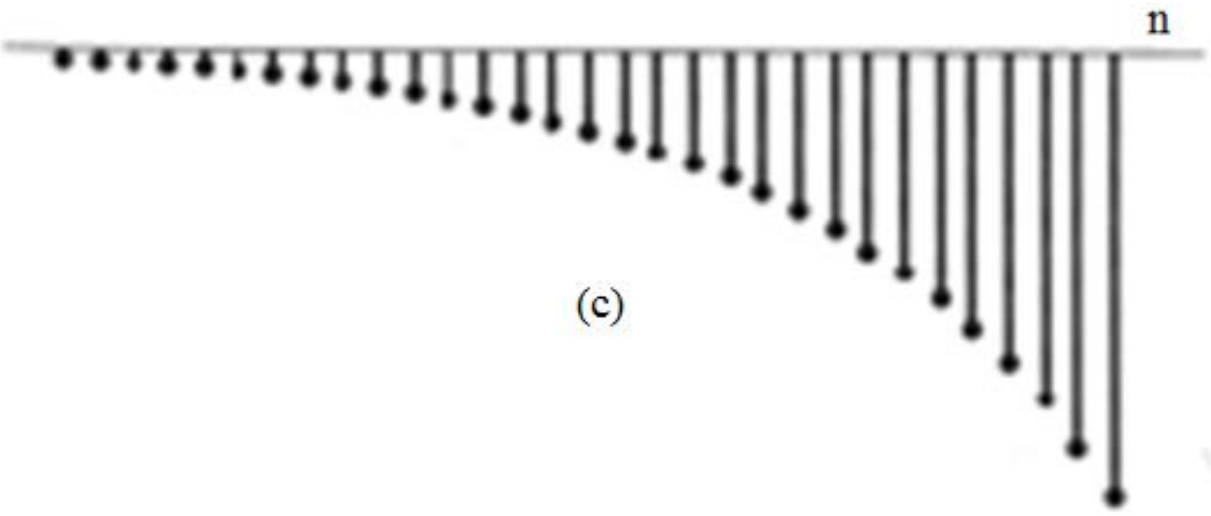
d) $a < 0$ and $A < 0$;

- How will (c) and (d) look like ???

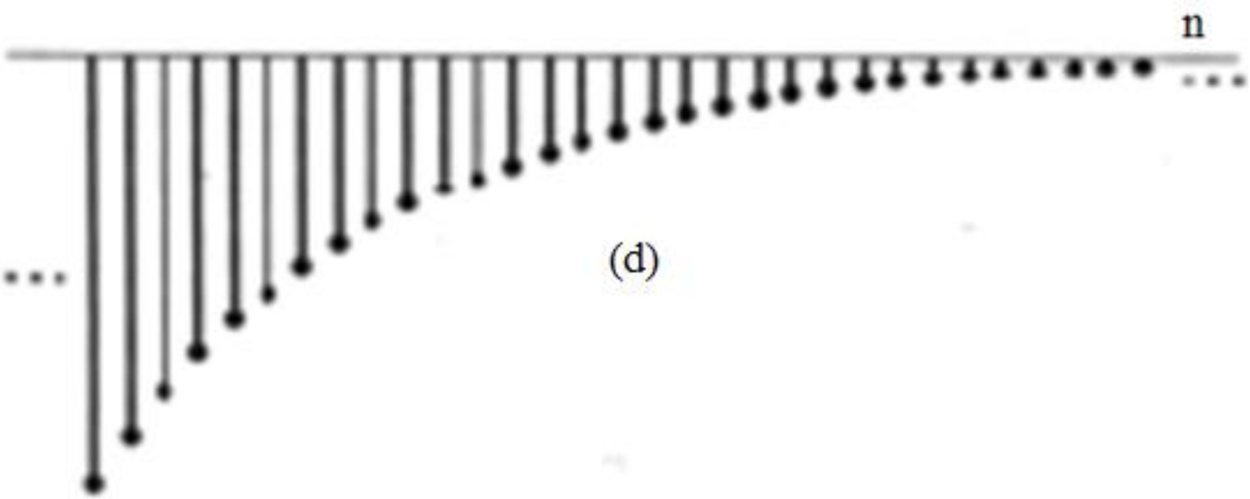
Exponential Signals

- Real Exponential signal $x[n] = Ae^{an}$; A and a are Real

c) $a > 0$ and $A < 0$;

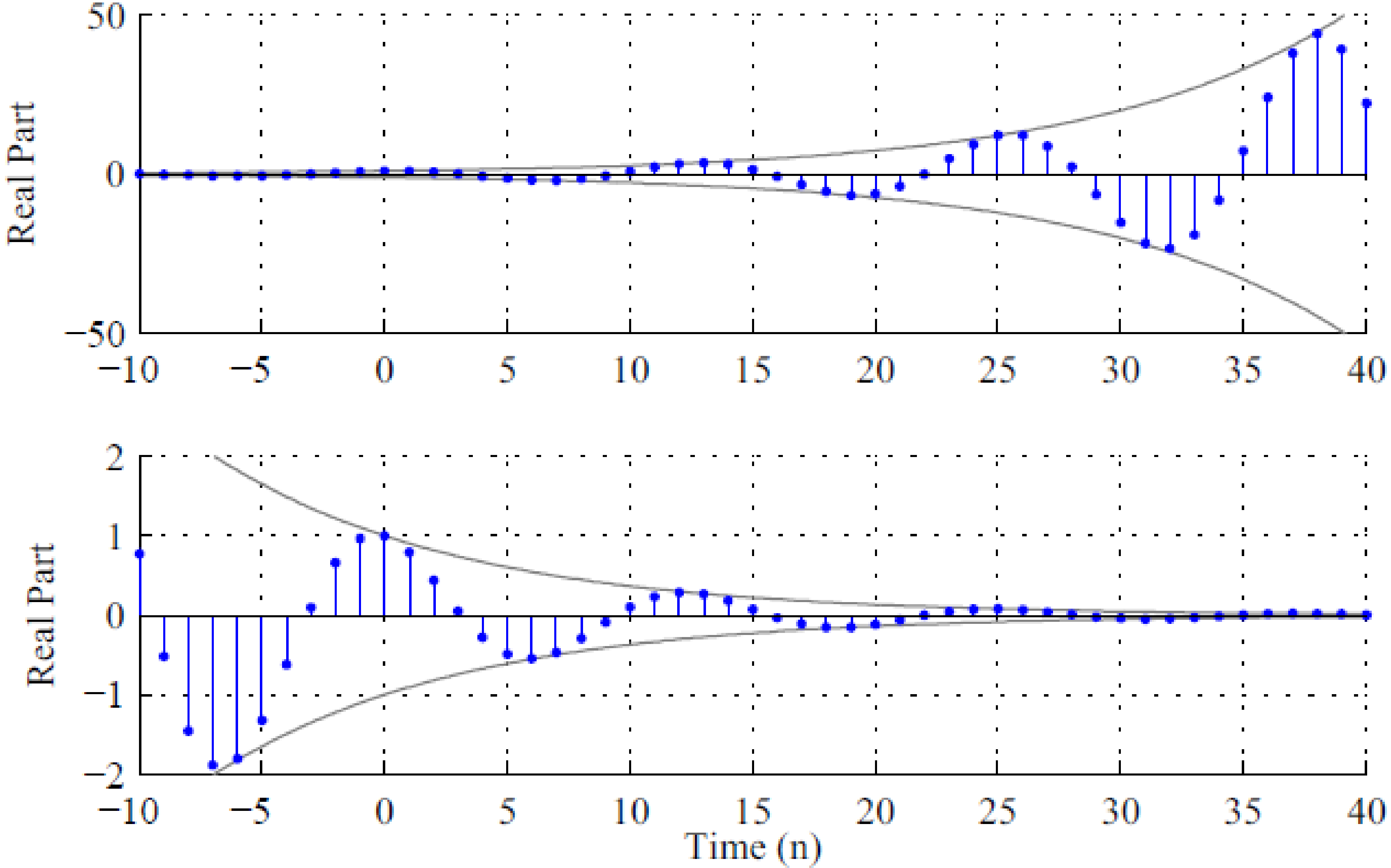


d) $a < 0$ and $A < 0$;



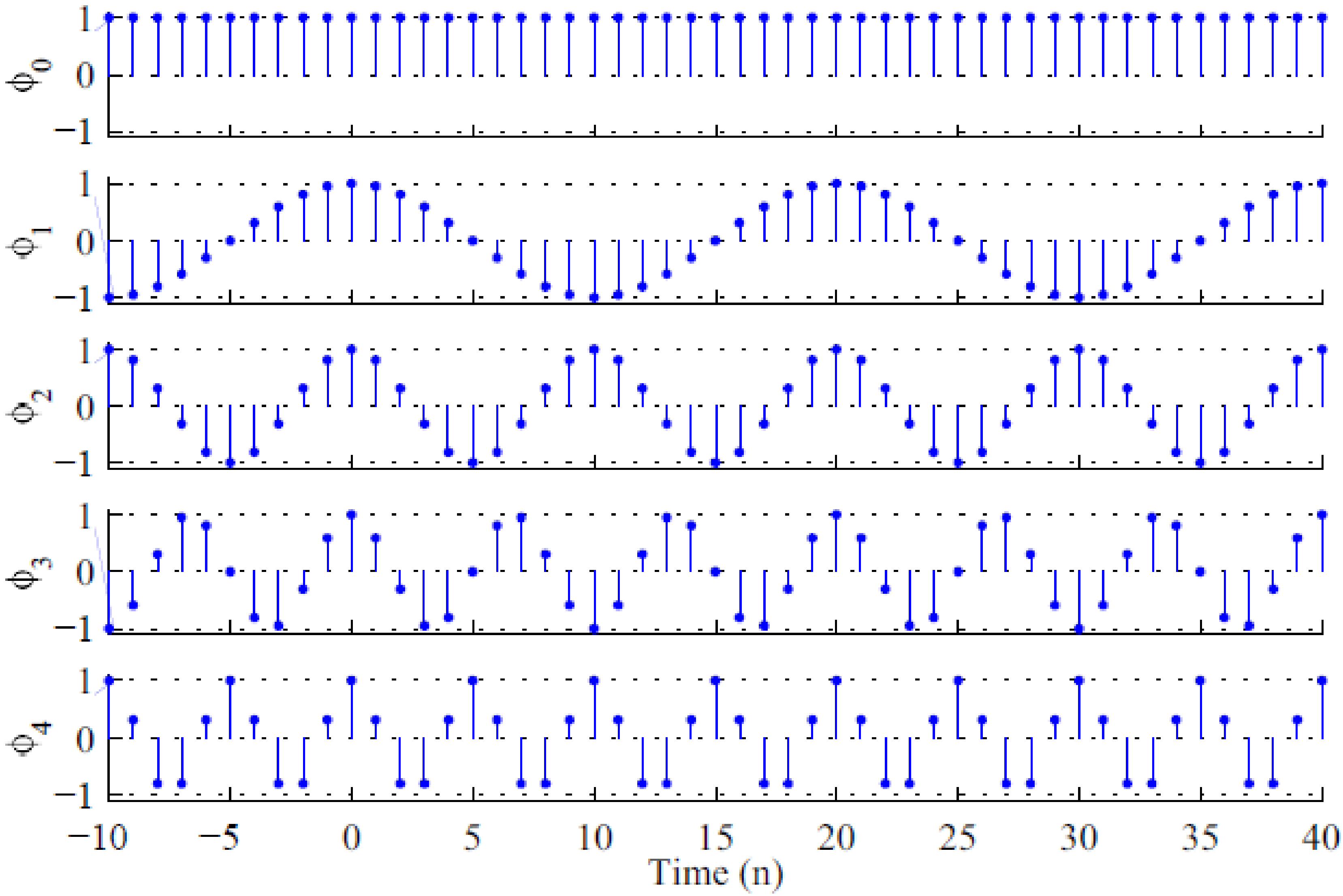
Damped Complex Sinusoidal Exponentials

Ae^{an}, $A = 1$ **and** $a = \pm 0.1 + j0.5$



Sinusoidal Exponential Harmonics

- Example of Discrete-time harmonics



Periodicity of Discrete-Time Sinusoids

- A periodic discrete-time signal x[n] has the property that for a positive integer N,

  $x[n] = x[n + N]$, for all values of n.

- The discrete time signal x[n] is periodic with period N if it is unchanged by a time shift of N

$$x[n] = Ae^{\omega n}; \text{ then:}$$

$$x[n+N] = Ae^{j\omega(n+N)} = Ae^{j\omega n} \,.\, e^{j\omega N} = x[n];$$

Possible if:

$$e^{j\omega N} = 1$$

Periodicity of Discrete-Time Sinusoids

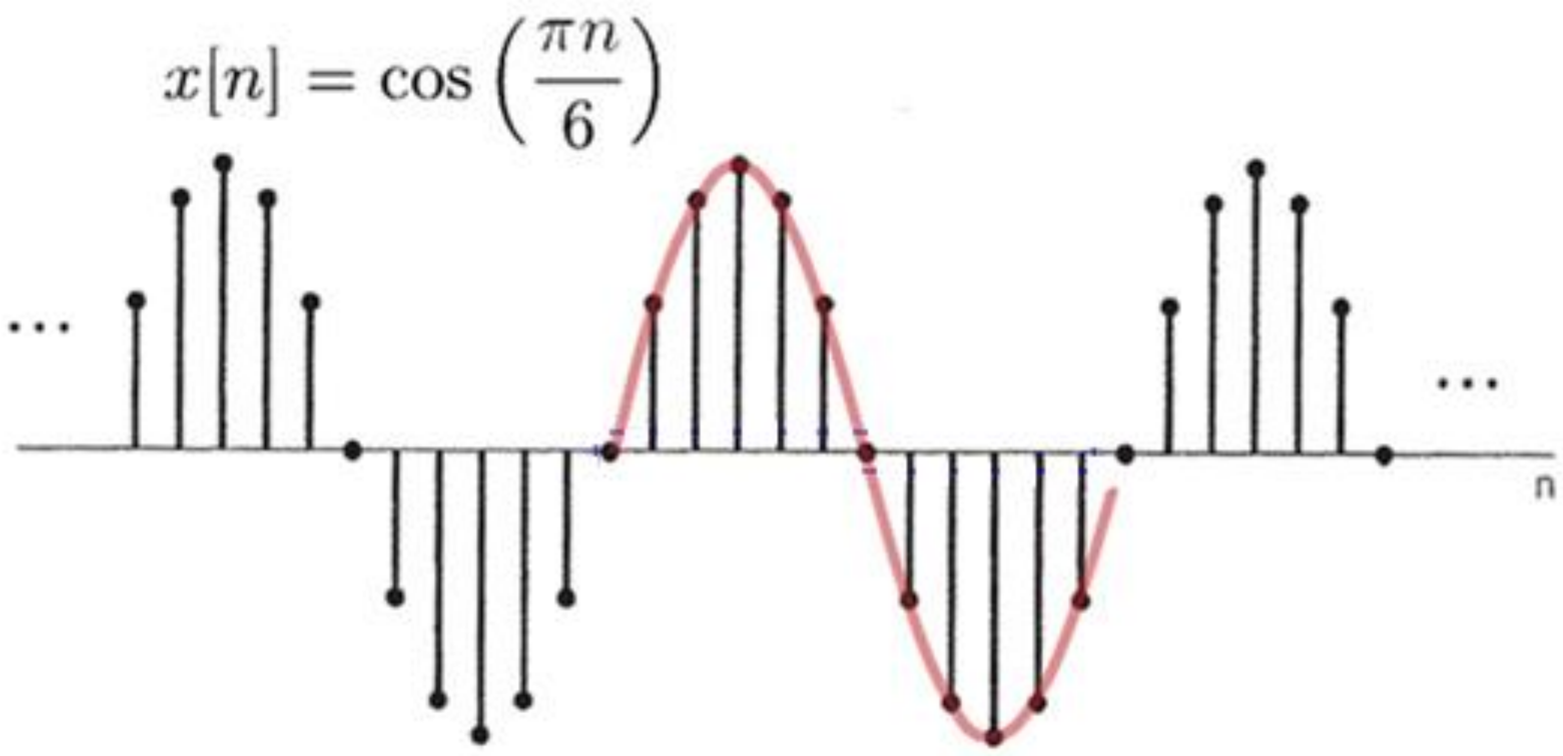
- Or $\omega N = 2\pi m$ or $N = 2\pi m/\omega$
- Here m and N should be integers
- We can also write the above as:

$$\omega = (2m/N) \, . \, \pi$$

- Therefore, a discrete-time sinusoid is periodic if its radian frequency ω is a rational multiple (integer/integer) of π

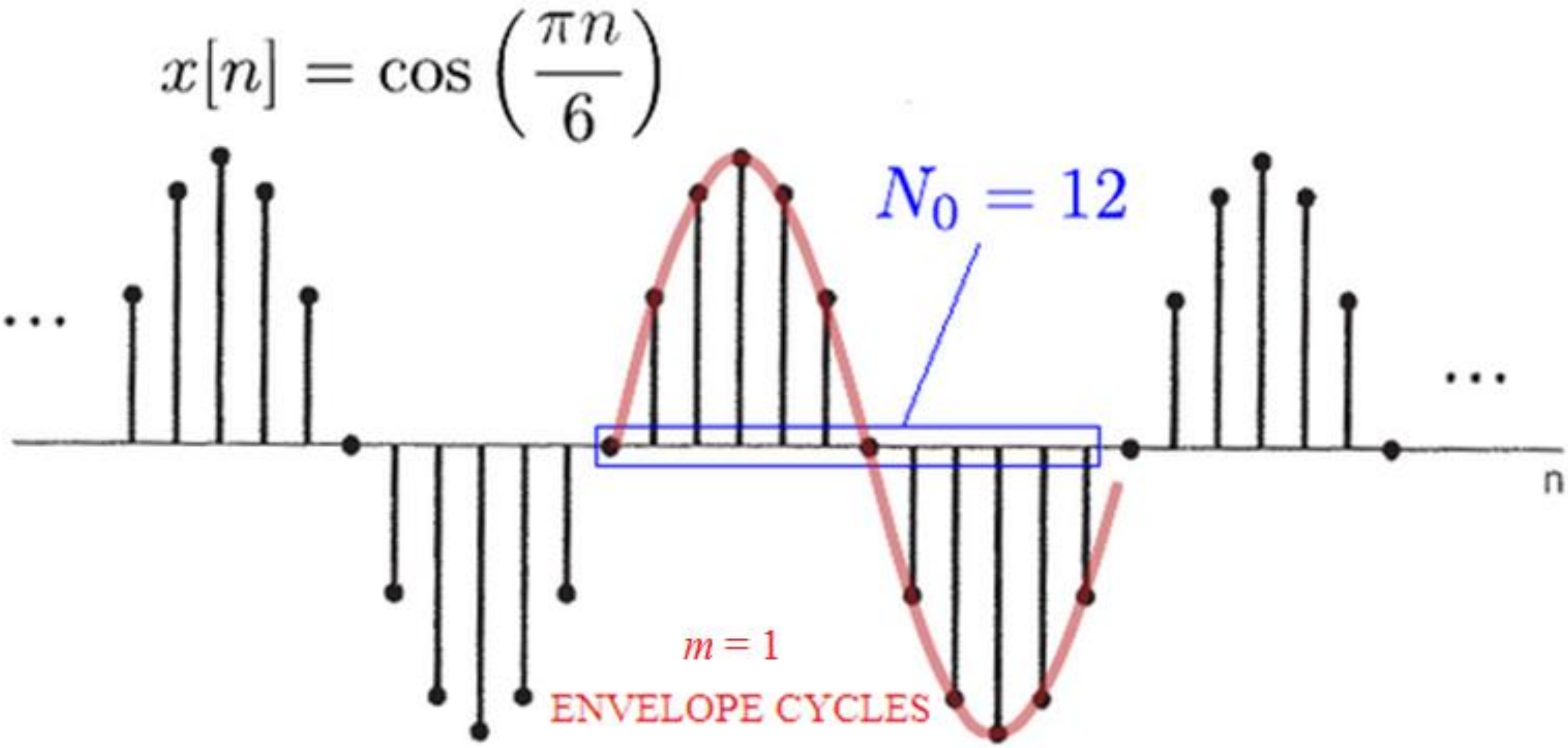
Periodicity of Discrete-Time Sinusoids

- EXAMPLE: Is the sinusoid periodic? What is the fundamental period?



Periodicity of Discrete-Time Sinusoids

- Periodic with fundamental period 12.

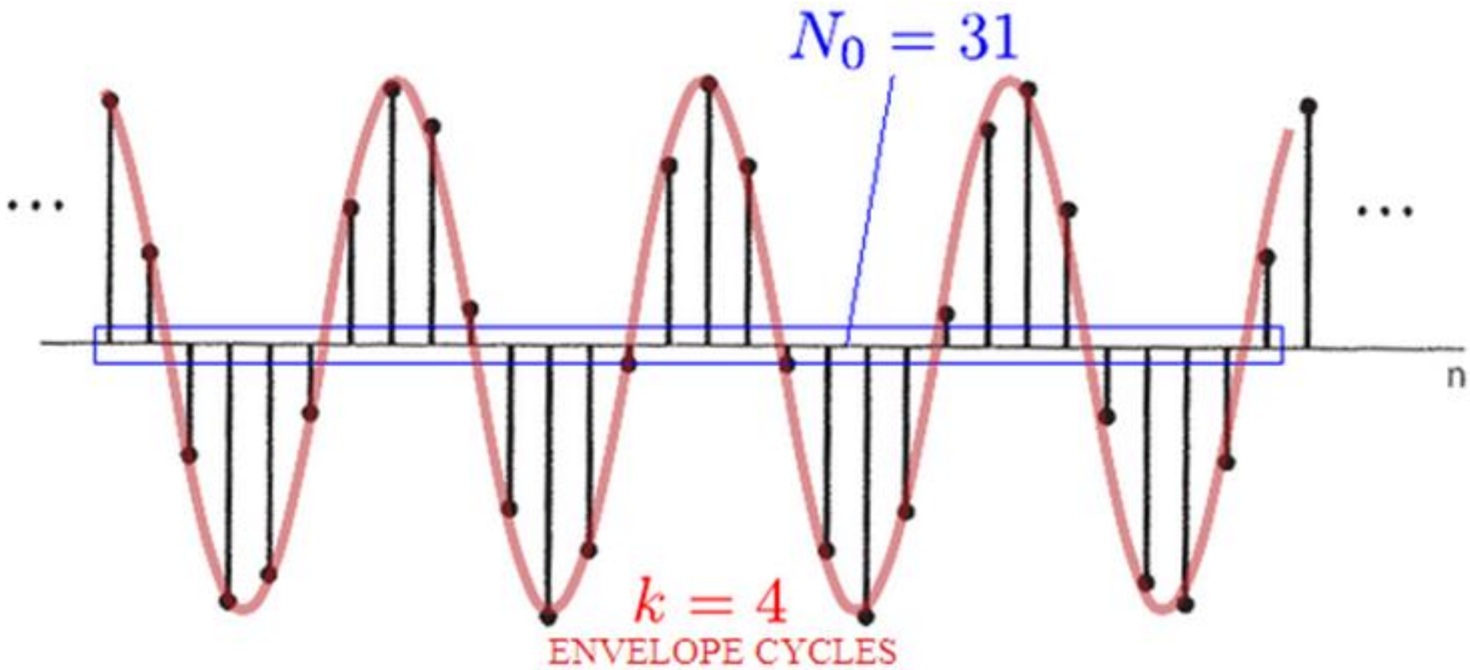


Periodicity of Discrete-Time Sinusoids

- EXAMPLE: Is the sinusoid periodic? What is the fundamental period?

$$x[n] = \cos\left(\frac{8\pi n}{31}\right)$$

Periodicity of Discrete-Time Sinusoids

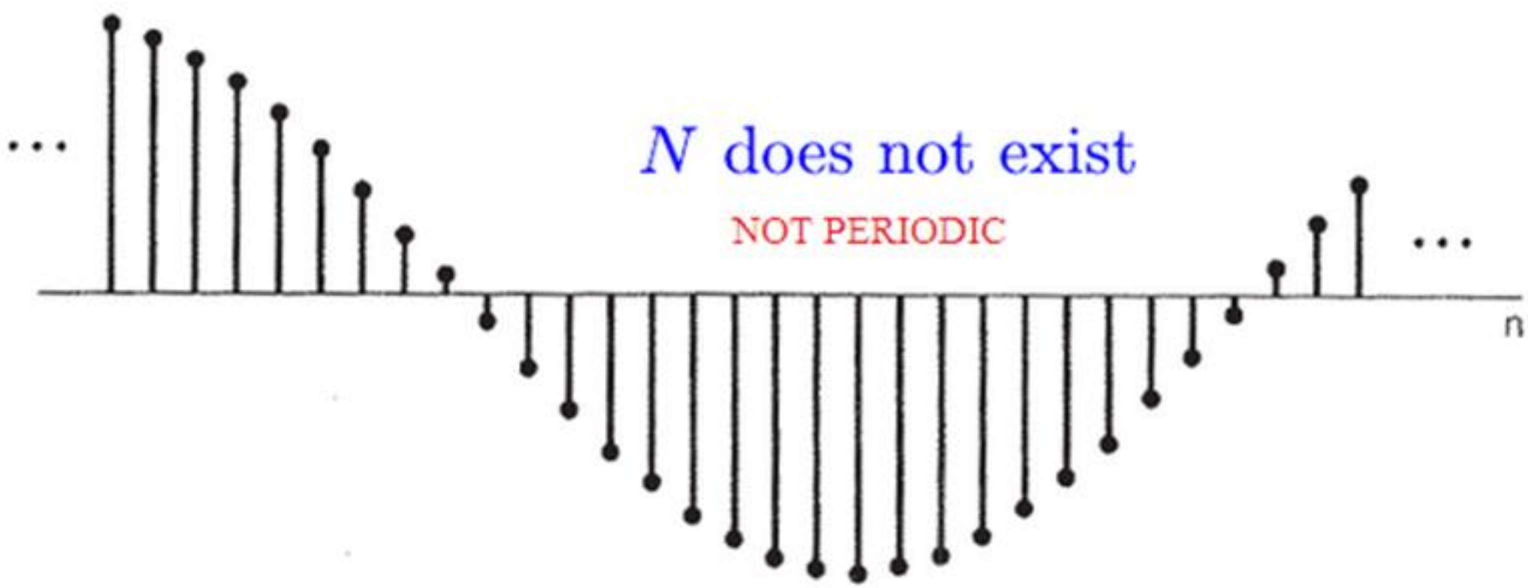


Periodicity of Discrete-Time Sinusoids

- EXAMPLE: Is the sinusoid periodic? What is the fundamental period?

$$x[n] = \cos\left(\frac{n}{6}\right)$$

Periodicity of Discrete-Time Sinusoids



DT Sinusoids Frequency & Rate of Oscillation

- for the CT signal $x(t) = e^{j\omega_0 t}$ we have the following two properties:
  1. the larger the magnitude of ω_0, the higher the rate of oscillation in the signal
  2. $e^{j\omega_0 t}$ is periodic for any value of ω_0

DT Sinusoids Frequency & Rate of Oscillation

- for the DT signal $x[n]=e^{j\omega_0 n}$ these properties don't hold for the following reason:

  $$e^{j(\omega_0+2\pi)n}=e^{j2\pi n}e^{j\omega_0 n}=e^{j\omega_0 n}$$

- thus the exponential at frequency $\omega_0+2\pi$ is the same as the exponential at frequency $\omega_0 \Rightarrow$ we only need to consider the frequency interval $-\pi\leq\omega<\pi$
- a DT sinusoid, $e^{j\omega_0 n}$, is periodic of period N only when:

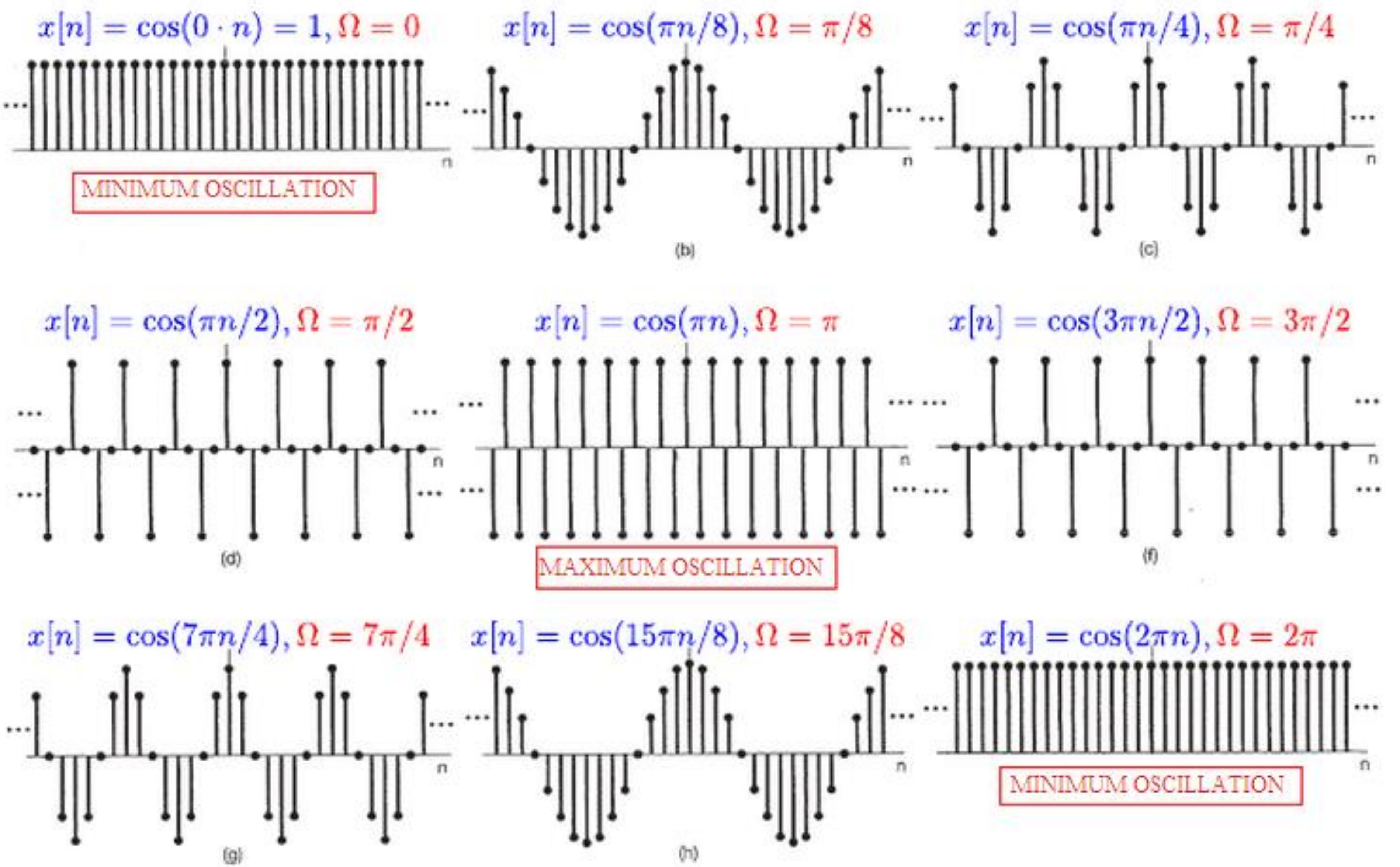
  $$e^{j\omega_0(n+N)}=e^{j\omega_0 n} \Rightarrow e^{j\omega_0 N}=1$$

  $\omega_0 N=2\pi m$ for some integer m

  $$\frac{\omega_0}{2\pi}=\frac{m}{N}$$

- if the above condition is not met, the DT sinusoid is not periodic

DT Sinusoids Frequency & Rate of Oscillation



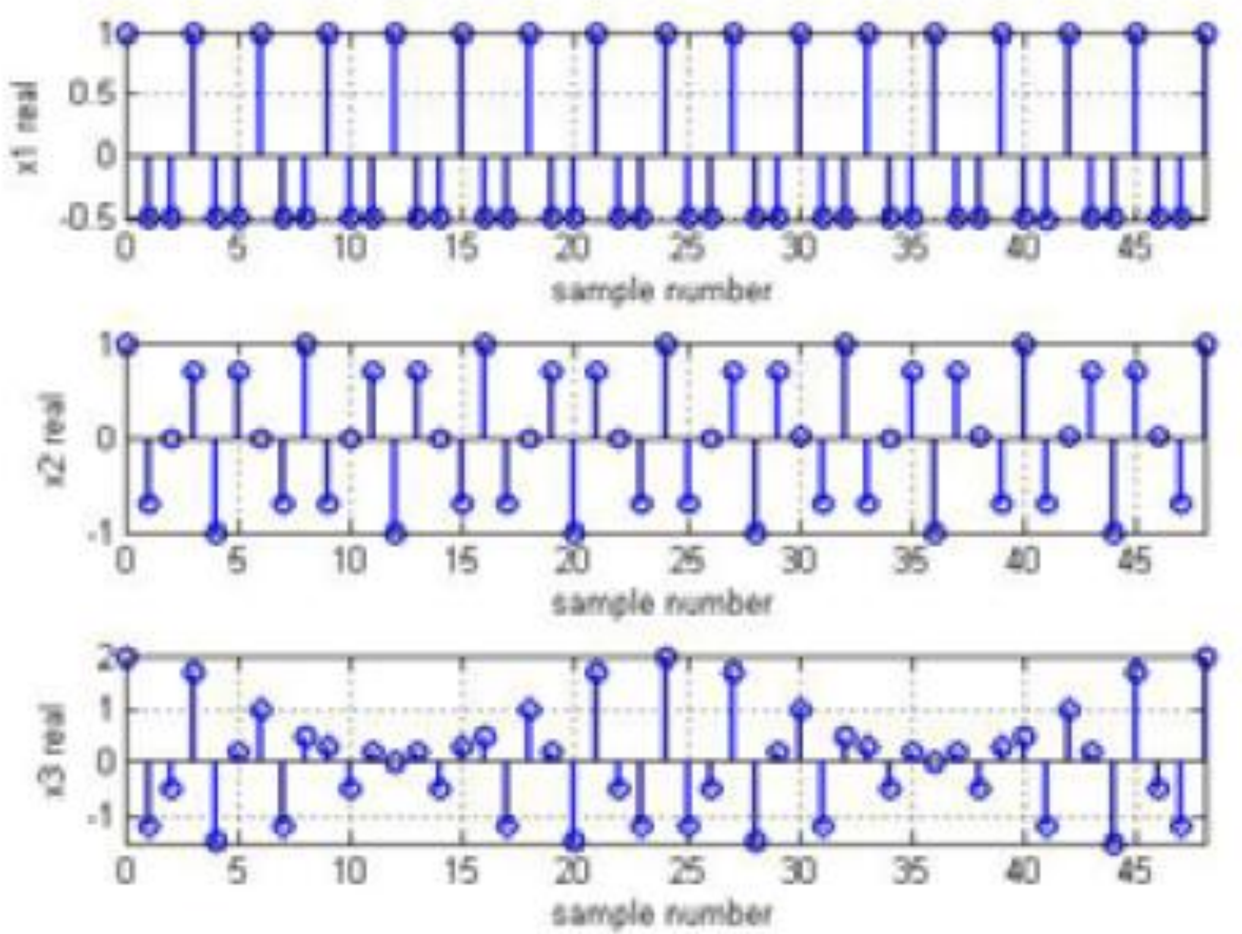
Problem-1

- Determine the fundamental period of the DT signal:

$$x[n] = e^{j(2\pi/3)n} + e^{j(3\pi/4)n}$$

DT Sinusoids Frequency & Rate of Oscillation

$$x[n] = e^{j(2\pi/3)n} + e^{j(3\pi/4)n} = x_1[n] + x_2[n]$$



period: 3 samples

period: 8 samples

period: 24 samples