Engineering Mechanics : Statics

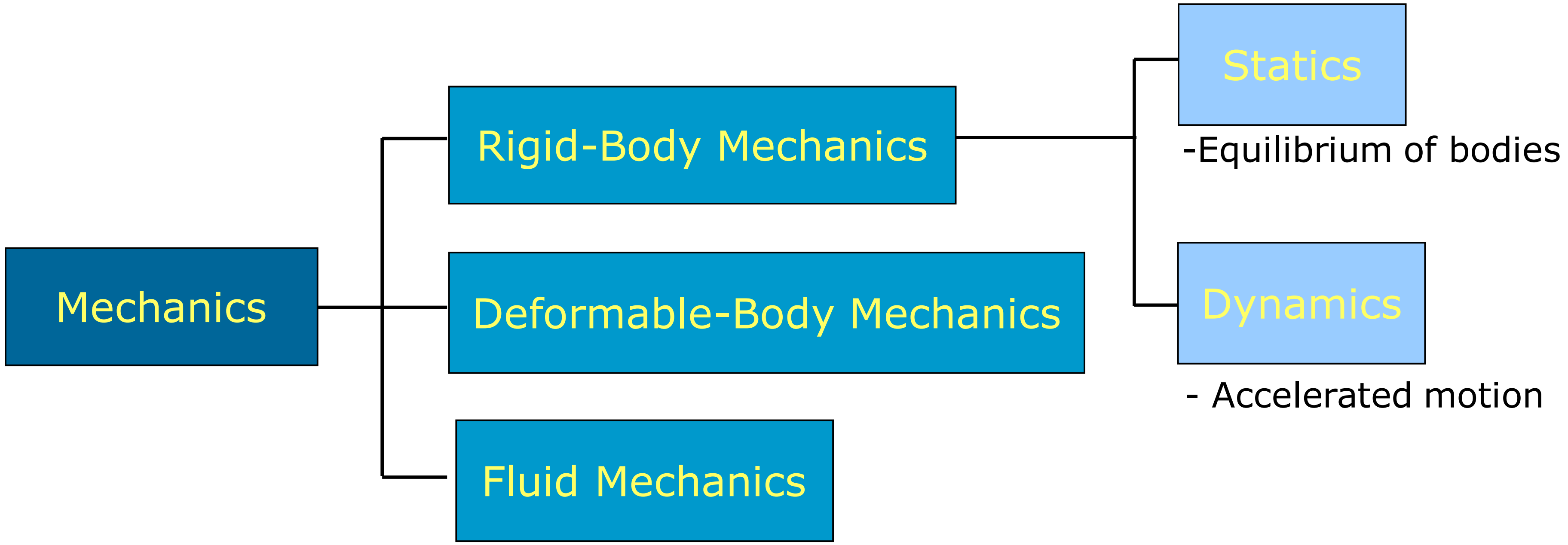
Chapter 12: kinematics of a particle

Chapter outline

- Introduction
- Rectilinear Kinematics: Continuous Motion
- Rectilinear Kinematics: Erratic Motion
- General Curvilinear
- Curvilinear Motion: Rectangular Motion
- Projectile

12.1 INTRODUCTION

Engineering Mechanics



Dynamics: Deals with the accelerated motion of a body

***kinematics*, which treats only the geometric aspects of the motion,**

***kinetics*, which is the analysis of the forces causing the motion.**

Involves calculus

We will study only particle dynamics

Kinematics of a Particle

12.1 INTRODUCTION

- Dynamics includes:
  - *Kinematics*: study of the geometry of motion. Kinematics is used to relate displacement, velocity, acceleration, and time without reference to the cause of motion.
  - *Kinetics*: study of the relations existing between the forces acting on a body, the mass of the body, and the motion of the body. Kinetics is used to predict the motion caused by given forces or to determine the forces required to produce a given motion.

- *Rectilinear* motion: position, velocity, and acceleration of a particle as it moves along a straight line.

- *Curvilinear* motion: position, velocity, and acceleration of a particle as it moves along a curved line in two or three dimensions.

12.1 INTRODUCTION

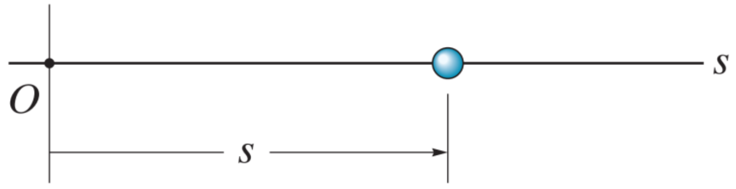
Kinematic relationships are used to help us determine the trajectory of a golf ball, the orbital speed of a satellite, and the accelerations during acrobatic flying.

12. 2 RECTILINEAR KINEMATICS: CONTINUOUS MOTION

Rectilinear Kinematics: Continuous Motion

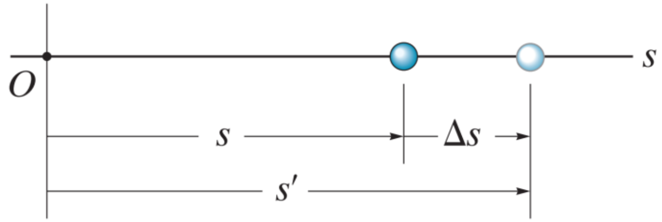
Rectilinear Kinematics. The kinematics of a particle is characterized by specifying, at any given instant, the particle's position, velocity, and acceleration.

Position.



Position

Displacement.



Displacement

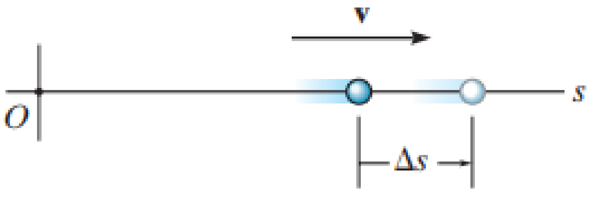
$$\Delta s = s' - s$$

Rectilinear Kinematics: Continuous Motion

Velocity. If the particle moves through a displacement Δs during the time interval Δt, the *average velocity* of the particle during this time interval is

If we take smaller and smaller values of Δt, the magnitude of Δs becomes smaller and smaller. Consequently, the *instantaneous velocity* is a vector defined as $v = \lim_{\Delta t \to 0} (\Delta s/\Delta t)$, or

If the particle is moving to the *right*, Fig. 12–1*c*, the velocity is *positive;* whereas if it is moving to the *left*, the velocity is *negative*.



Velocity

Velocity.

$$v_{avg} = \frac{\Delta s}{\Delta t}$$

it is generally expressed in units of m/s or ft/s.

instantaneous velocity

$v = \lim_{\Delta t \to 0} (\Delta s/\Delta t)$, or

$$(\overset{+}{\rightarrow}) \qquad \boxed{v = \frac{ds}{dt}}$$

Rectilinear Kinematics: Continuous Motion

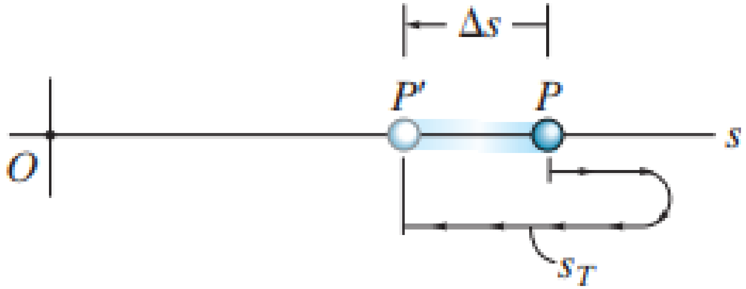
Average Velocity Vs Average Speed

Occasionally, the term "average speed" is used. The *average speed* is always a positive scalar and is defined as the total distance traveled by a particle, s_T, divided by the elapsed time i.e Δt

$$(v_{sp})_{avg} = s_T/\Delta t,$$

For example, the particle in Fig. 12–1*d* travels along the path of length s_T in time Δt, so its average speed is $(v_{sp})_{avg} = s_T/\Delta t$, but its average velocity is $v_{avg} = -\Delta s/\Delta t$.

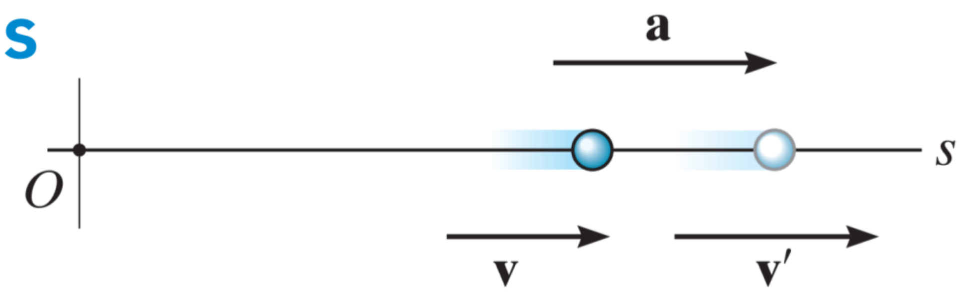
$$v_{avg} = -\Delta s/\Delta t.$$



Average velocity and Average speed

(d)

Rectilinear Kinematics: Continuous Motion

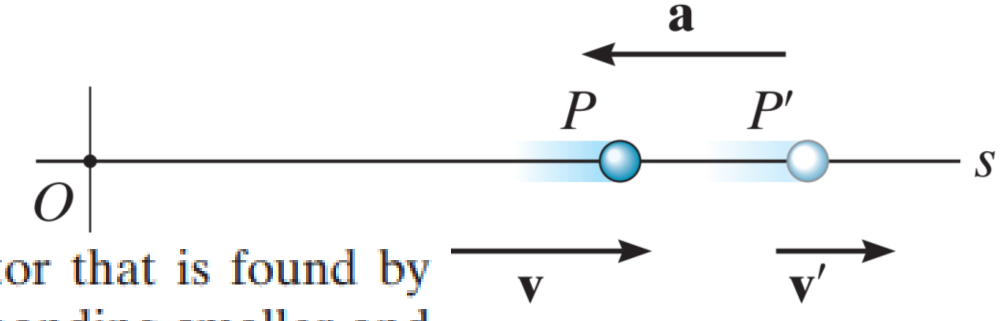


Acceleration.

$$a_{avg} = \frac{\Delta v}{\Delta t}$$

interval Δt, i.e., $\Delta v = v' - v$,

Acceleration



The *instantaneous acceleration* at time t is a vector that is found by taking smaller and smaller values of Δt and corresponding smaller and smaller values of Δv, so that $a = \lim_{\Delta t \to 0} (\Delta v / \Delta t)$, or

$$\boxed{a = \frac{dv}{dt}} \qquad a = \frac{d^2 s}{dt^2}$$

By eliminating the time differential

$$\boxed{a\, ds = v\, dv}$$

Rectilinear Kinematics: Continuous Motion

Constant Acceleration, $a = a_c$. When the acceleration is constant, each of the three kinematic equations $a_c = dv/dt$, $v = ds/dt$, and $a_c\,ds = v\,dv$ can be integrated to obtain formulas that relate a_c, v, s, and t.

Velocity as a Function of Time. Integrate $a_c = dv/dt$, assuming that initially $v = v_0$ when $t = 0$.

$$\int_{v_0}^{v} dv = \int_{0}^{t} a_c\,dt$$

$$(\overset{+}{\rightarrow}) \qquad \boxed{v = v_0 + a_c t} \qquad (12\text{–}4)$$

Constant Acceleration

Rectilinear Kinematics: Continuous Motion

Position as a Function of Time. Integrate $v = ds/dt = v_0 + a_c t$, assuming that initially $s = s_0$ when $t = 0$.

$$\int_{s_0}^{s} ds = \int_{0}^{t} (v_0 + a_c t)\, dt$$

$$(\overset{+}{\rightarrow}) \qquad \boxed{s = s_0 + v_0 t + \tfrac{1}{2} a_c t^2} \qquad (12\text{–}5)$$

Constant Acceleration

Rectilinear Kinematics: Continuous Motion

Velocity as a Function of Position. Either solve for t in Eq. 12–4 and substitute into Eq. 12–5, or integrate $v\,dv = a_c\,ds$, assuming that initially $v = v_0$ at $s = s_0$.

$$\int_{v_0}^{v} v\,dv = \int_{s_0}^{s} a_c\,ds$$

$(\overset{+}{\rightarrow})$
$$\boxed{v^2 = v_0^2 + 2a_c(s - s_0)}$$
Constant Acceleration (12–6)

Rectilinear Kinematics: Continuous Motion

Constant Acceleration, $a = a_c$.

$$v = v_0 + a_c t$$

Constant Acceleration

$$s = s_0 + v_0 t + \tfrac{1}{2} a_c t^2$$

Constant Acceleration

$$v^2 = v_0^2 + 2a_c(s - s_0)$$

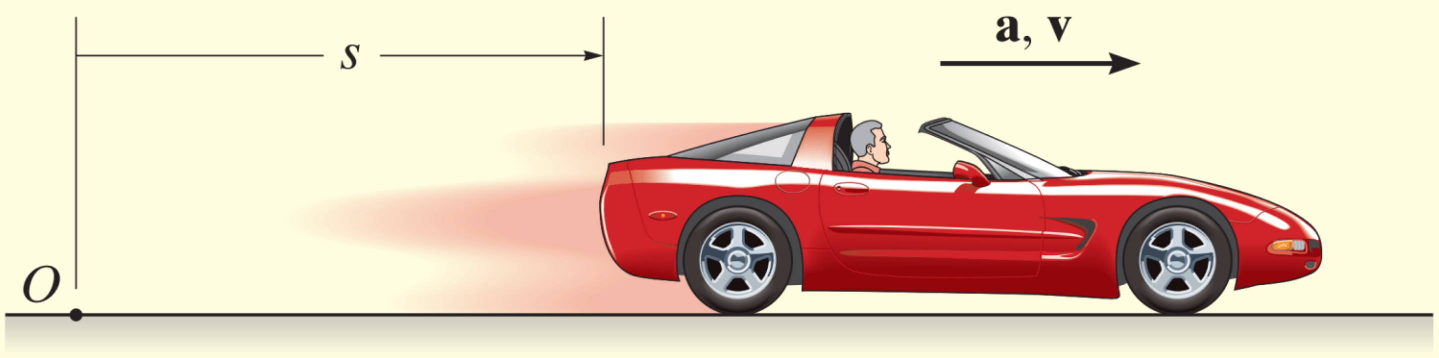
Constant Acceleration

Acceleration as a function of time, position, or velocity

If….	Kinematic relationship	Integrate
$a = a(t)$	$\frac{dv}{dt} = a(t)$	$\int_{v_0}^{v} dv = \int_{0}^{t} a(t)\, dt$
$a = a(x)$	$dt = \frac{dx}{v}$ and $a = \frac{dv}{dt}$ $v\,dv = a(x)\,dx$	$\int_{v_0}^{v} v\,dv = \int_{x_0}^{x} a(x)\,dx$
$a = a(v)$	$\frac{dv}{dt} = a(v)$	$\int_{v_0}^{v} \frac{dv}{a(v)} = \int_{0}^{t} dt$
	$v\frac{dv}{dx} = a(v)$	$\int_{x_0}^{x} dx = \int_{v_0}^{v} \frac{v\,dv}{a(v)}$

Example 12. 1

The car on the left in the photo and in Fig. 12–2 moves in a straight line such that for a short time its velocity is defined by $v = (3t^2 + 2t)$ ft/s, where t is in seconds. Determine its position and acceleration when $t = 3$ s. When $t = 0$, $s = 0$.



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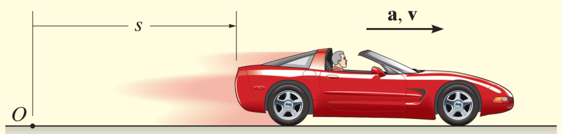


Coordinate System. The position coordinate extends from the fixed origin O to the car, positive to the right.

Position. Since $v = f(t)$, the car's position can be determined from $v = ds/dt$, since this equation relates v, s, and t. Noting that $s = 0$ when $t = 0$, we have*

$$(\overset{+}{\rightarrow}) \qquad v = \frac{ds}{dt} = (3t^2 + 2t)$$

$$\int_0^s ds = \int_0^t (3t^2 + 2t)dt$$

$$s\Big|_0^s = t^3 + t^2\Big|_0^t$$

$$s = t^3 + t^2$$

When $t = 3$ s,

$$s = (3)^3 + (3)^2 = 36 \text{ ft} \qquad \textit{Ans.}$$

$$a = \frac{dv}{dt} = \frac{d}{dt}(3t^2 + 2t)$$

$$= 6t + 2$$

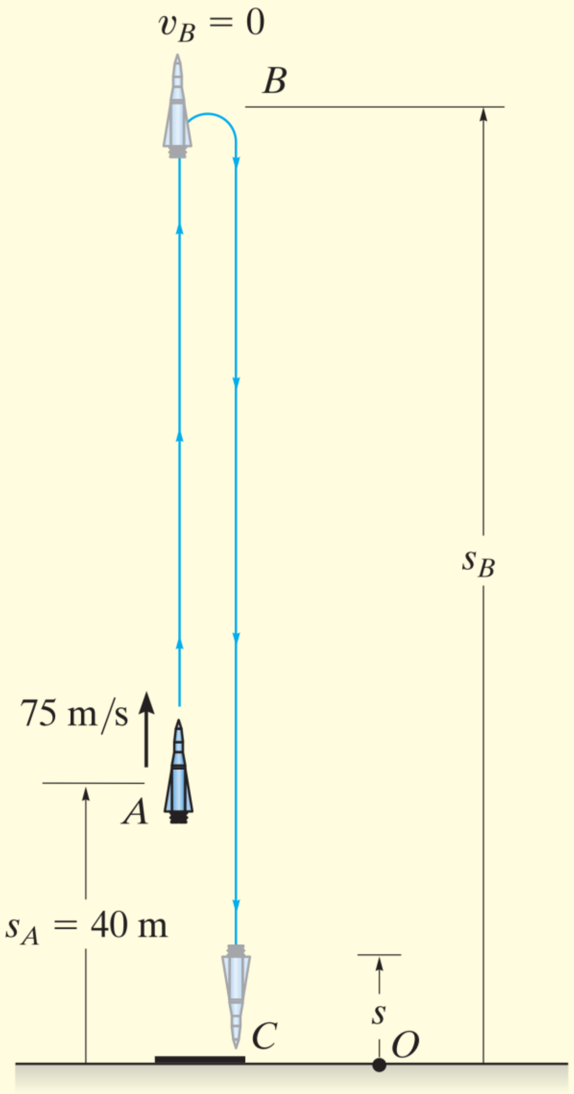
$$a = 6(3) + 2 = 20 \text{ ft/s}^2 \rightarrow$$

NOTE: The formulas for constant acceleration *cannot* be used to solve this problem, because the acceleration is a function of time.

Rectilinear Kinematics: Continuous Motion

Example 12. 3

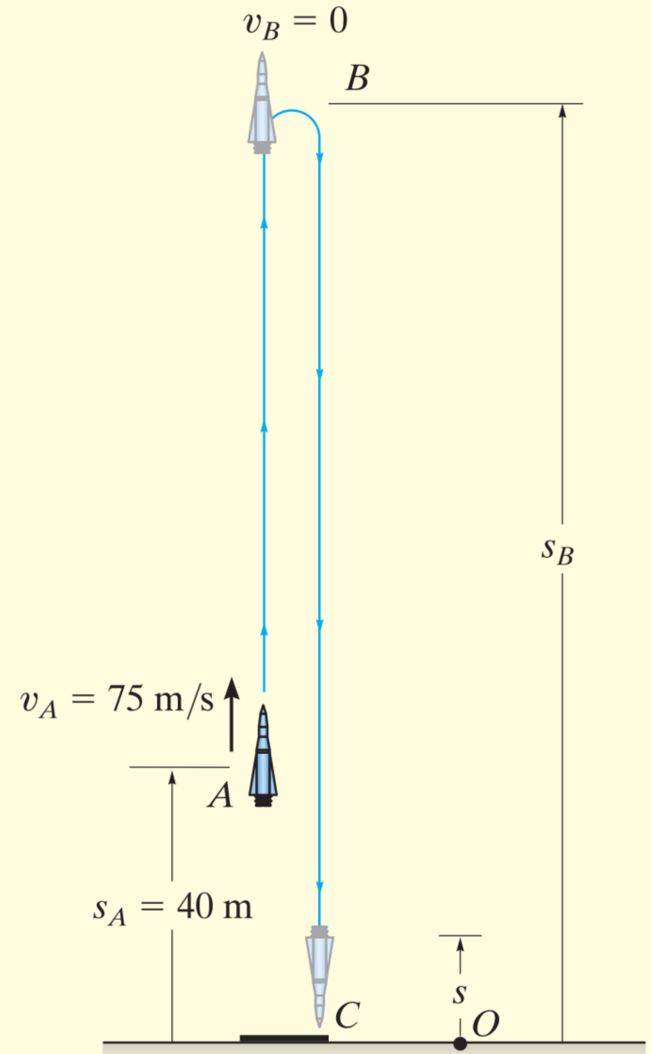
During a test a rocket travels upward at 75 m/s, and when it is 40 m from the ground its engine fails. Determine the maximum height s_B reached by the rocket and its speed just before it hits the ground. While in motion the rocket is subjected to a constant downward acceleration of 9.81 m/s^2 due to gravity. Neglect the effect of air resistance.



SOLUTION

Coordinate System. The origin O for the position coordinate s is taken at ground level with positive upward, Fig. 12–4.

Maximum Height. Since the rocket is traveling *upward*, $v_A = +75\text{m/s}$ when $t = 0$. At the maximum height $s = s_B$ the velocity $v_B = 0$. For the entire motion, the acceleration is $a_c = -9.81\ \text{m/s}^2$ (negative since it acts in the *opposite* sense to positive velocity or positive displacement). Since a_c is *constant* the rocket's position may be related to its velocity at the two points A and B on the path by using Eq. 12–6, namely,



Maximum Height.

$$(+\uparrow) \qquad v_B^2 = v_A^2 + 2a_c(s_B - s_A)$$

$$0 = (75\ \text{m/s})^2 + 2(-9.81\ \text{m/s}^2)(s_B - 40\ \text{m})$$

$$s_B = 327\ \text{m}$$

Velocity.

$$(+\uparrow) \qquad v_C^2 = v_B^2 + 2a_c(s_C - s_B)$$

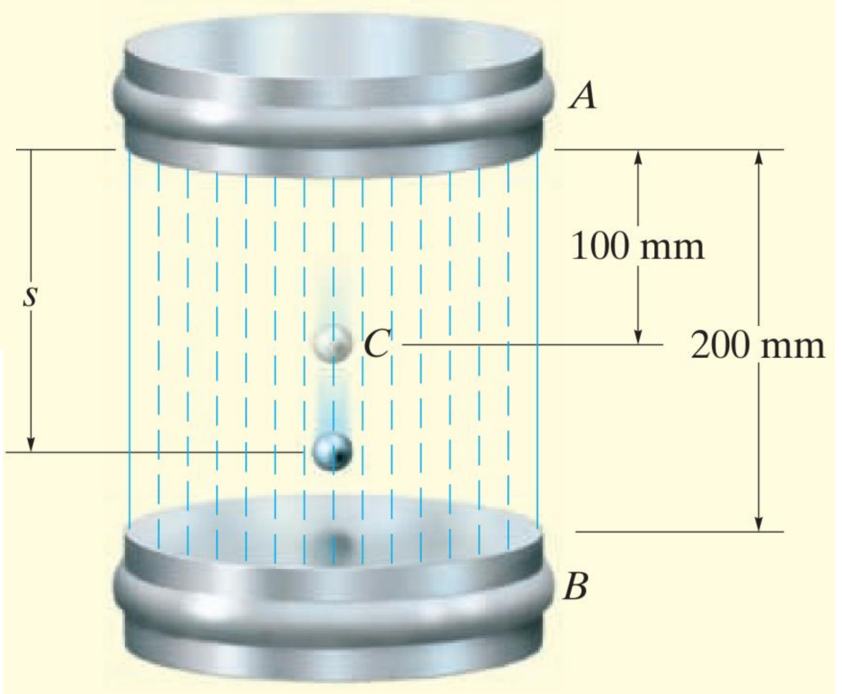
$$= 0 + 2(-9.81\ \text{m/s}^2)(0 - 327\ \text{m})$$

$$v_C = -80.1\ \text{m/s} = 80.1\ \text{m/s} \downarrow$$

NOTE: It should be realized that the rocket is subjected to a *deceleration* from A to B of $9.81\ \text{m/s}^2$, and then from B to C it is *accelerated* at this rate. Furthermore, even though the rocket momentarily comes to *rest* at B ($v_B = 0$) the acceleration at B is still $9.81\ \text{m/s}^2$ downward!

Example 12. 4

A metallic particle is subjected to the influence of a magnetic field as it travels downward through a fluid that extends from plate A to plate B, Fig. 12–5. If the particle is released from rest at the midpoint C, $s = 100$ mm, and the acceleration is $a = (4s)\ \mathrm{m/s^2}$, where s is in meters, determine the velocity of the particle when it reaches plate B, $s = 200$ mm, and the time it takes to travel from C to B.



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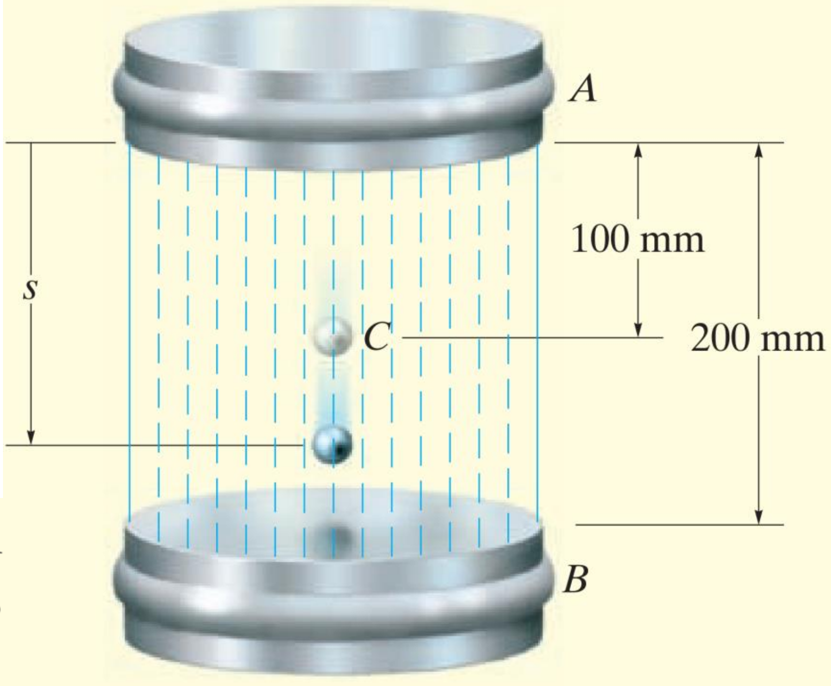


SOLUTION

Coordinate System. As shown in Fig. 12–5, s is positive downward, measured from plate A.

Velocity. Since $a = f(s)$, the velocity as a function of position can be obtained by using $v\,dv = a\,ds$. Realizing that $v = 0$ at $s = 0.1$ m, we have

$$(+\downarrow) \qquad v\,dv = a\,ds$$

$$\int_0^v v\,dv = \int_{0.1\text{ m}}^s 4s\,ds$$

$$\frac{1}{2}v^2\Big|_0^v = \frac{4}{2}s^2\Big|_{0.1\text{ m}}^s$$

$$v = 2(s^2 - 0.01)^{1/2}\ \text{m/s} \qquad (1)$$

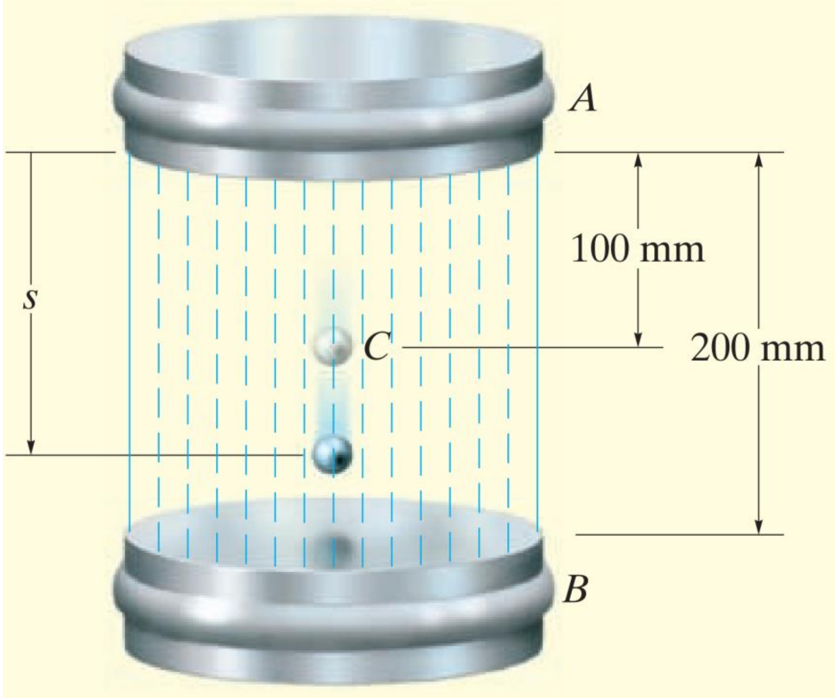
At $s = 200\text{ mm} = 0.2$ m,

$$v_B = 0.346\ \text{m/s} = 346\ \text{mm/s}\downarrow \qquad \textit{Ans.}$$

The positive root is chosen since the particle is traveling downward, i.e., in the $+s$ direction.

Time. The time for the particle to travel from C to B can be obtained using $v = ds/dt$ and Eq. 1, where $s = 0.1$ m when $t = 0$. From Appendix A,

$(+\downarrow)$
$$ds = v\,dt$$
$$= 2(s^2 - 0.01)^{1/2}dt$$
$$\int_{0.1}^{s} \frac{ds}{(s^2 - 0.01)^{1/2}} = \int_0^t 2\,dt$$
$$\ln\left(\sqrt{s^2 - 0.01} + s\right)\Big|_{0.1}^{s} = 2t\Big|_0^t$$
$$\ln\left(\sqrt{s^2 - 0.01} + s\right) + 2.303 = 2t$$



At $s = 0.2$ m,

$$t = \frac{\ln\left(\sqrt{(0.2)^2 - 0.01} + 0.2\right) + 2.303}{2} = 0.658 \text{ s} \qquad \textit{Ans.}$$

NOTE: The formulas for constant acceleration cannot be used here because the acceleration changes with position, i.e., $a = 4s$.

Home assignment

Example 12.2 & 12.5