# NATIONAL UNIVERSITY OF SCIENES & TECHNOLOGY

**Linear Algebra and ODE (MATH-121)**

**Assignment # 1**

**Submitted to:** Dr. Saira Zainab

**Submitted by:** Muhammad Umer

**Class:** BEE-12C

**Semester:** 2nd

**Dated:** 15/03/2021

**CMS ID:** 345834

National University of Sciences & Technology
School of Electrical Engineering and Computer Science
Department of Humanities and Sciences

MATH-121: Linear Algebra & ODEs (3+0): 2k20-BEE12ABC Spring 2021

<table>
<tr><td colspan="2" align="center">Assignment 1</td></tr>
<tr><td colspan="2">CLO1: Solve the system of linear equations using matrices and determinants.</td></tr>
<tr><td>Maximum Marks: 10</td><td>Instructor: Dr. Saira Zainab</td></tr>
<tr><td>Announcement Date: 11th March 2021</td><td>Due Date: 19th March 2021(Day Scholars)<br>: 18th March 2021(Hostilities)</td></tr>
</table>

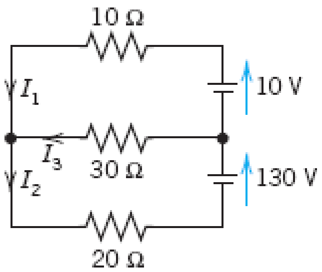
**Instructions:**

1. Assignment should be hand written.
2. Copied assignments will be marked zero.
3. CR will collect all the assignments (on due date) and submit to me in my office till 4:30.
4. Assignment is not acceptable after deadline.
5. Assignment presentation also contains marks (Good presentation does not mean over decoration).
6. Do not use files and folders.

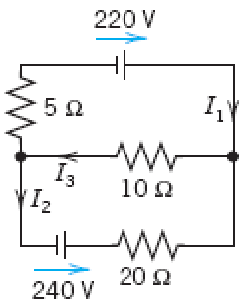
**Tasks: Attempt all questions.**

**Question# 1:** Find the mentioned circuits by using system of linear equations and Kirchhoff's laws of voltage and current.

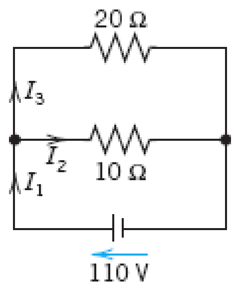
**(a).**




**(b).**

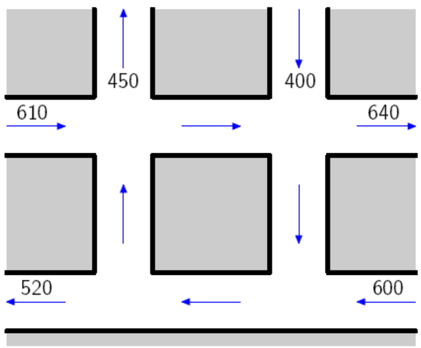



**(c)**




**Question #2:** Determine the amount of traffic for each of the four intersections.

**(a). Label the diagram and then solve.** **(b). vph stands for vehicle per hour.**



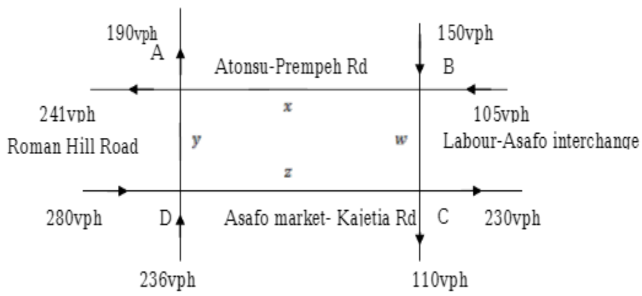
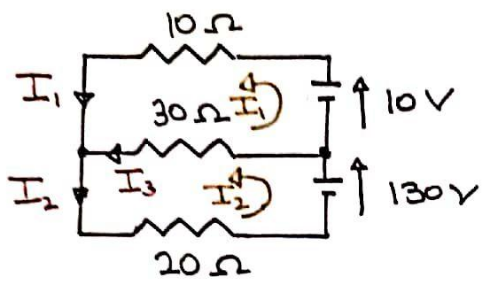




Figure 1: Diagram of the four one-way streets, in Kumasi

# Assignment 1

## Q1:

a)




Applying KVL to Mesh 1

- $-10 + 10I_1 + 30(I_1 - I_2) = 0$

$$40I_1 - 30I_2 = 10 \quad 1$$

Mesh 2:

$$-130 + 30(I_2 - I_1) + 20I_2 = 0$$

$$-30I_1 + 50I_2 = 130 \quad 2$$

Constraint Equation:

$$I_2 - I_1 = I_3$$

$$-I_1 + I_2 = I_3 \quad 3$$

Augmented Matrix of 1, 2, 3 is;

$$[AB] = \begin{bmatrix} 40 & -30 & 0 & 10 \\ -30 & 50 & 0 & 130 \\ -1 & 1 & -1 & 0 \end{bmatrix}$$

Applying Gaussian Elimination

$$= \begin{bmatrix} -1 & 1 & -1 & 0 \\ 40 & -30 & 0 & 10 \\ -30 & 50 & 0 & 130 \end{bmatrix} \begin{matrix} R_1 \leftrightarrow R_3 \\ R_2 \leftrightarrow R_3 \end{matrix}$$

$$= \begin{bmatrix} -1 & 1 & -1 & 0 \\ 0 & 10 & -40 & 10 \\ 0 & 20 & 30 & 130 \end{bmatrix} \begin{matrix} R_2 + 40R_1 \\ R_3 + (-30R_1) \end{matrix}$$

$$= \left[\begin{array}{cccc} -1 & 1 & -1 & 0 \\ 0 & 10 & -40 & 10 \\ 0 & 0 & 110 & 110 \end{array}\right] R_3 + (-2R_2)$$

In Equations,

- $110 I_3 = 110 \quad \Rightarrow \quad \boxed{I_3 = 1A}$
- $10 I_2 - 40 I_3 = 10$

  $10 I_2 - 40(1) = 10$

  $10 I_2 = 50 \quad \Rightarrow \quad \boxed{I_2 = 5A}$
- $-I_1 + I_2 - I_3 = 0$

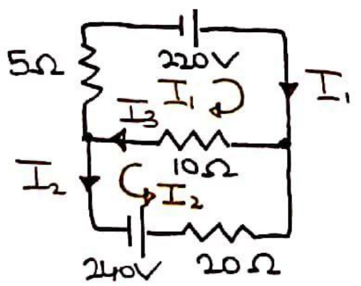
  $-I_1 + 5 - 1 = 0 \quad \Rightarrow \quad \boxed{I_1 = 4A}$

Hence, $I_1 = 4A$

$I_2 = 5A$

$I_3 = 1A$

b)




Mesh 1:

$-220 + 10I_1 + 10I_2 + 5I_1 = 0$

$\underline{15I_1 + 10I_2 + 0I_3 = 220}$

Mesh 2:

$-240 + 20I_2 + 10I_2 + 10I_1 = 0$

$\underline{10I_1 + 30I_2 + 0I_3 = 240}$

Constraint Equation:

$$\underline{I_1 + I_2 - I_3 = 0}$$

Augmented Matrix of above equations

$$[AB] = \begin{bmatrix} 15 & 10 & 0 & 220 \\ 10 & 30 & 0 & 240 \\ 1 & 1 & -1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 1 & -1 & 0 \\ 10 & 30 & 0 & 240 \\ 15 & 10 & 0 & 220 \end{bmatrix} R_1 \longleftrightarrow R_3$$

$$= \begin{bmatrix} 1 & 1 & -1 & 0 \\ 0 & 20 & 10 & 240 \\ 0 & -5 & 15 & 220 \end{bmatrix} \begin{matrix} \\ R_2 + (-10R_1) \\ R_3 + (-15R_1) \end{matrix}$$

$$= \begin{bmatrix} 1 & 1 & -1 & 0 \\ 0 & 20 & 10 & 240 \\ 0 & 0 & 35/2 & 280 \end{bmatrix} R_3 + (1/4R_2)$$

In Equations,

- $\frac{35}{2} I_3 = 280 \quad \Rightarrow \boxed{I_3 = 16A}$
- $20 I_2 + 10 I_3 = 240$

  $20 I_2 + 160 = 240$

  $20 I_2 = 80 \quad \Rightarrow \boxed{I_2 = 4A}$
- $I_1 + I_2 - I_3 = 0$

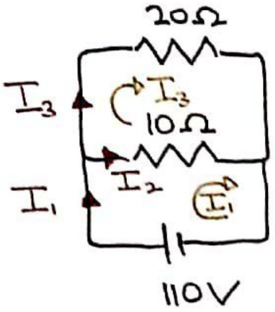
  $I_1 + 4 - 16 = 0 \quad \Rightarrow \boxed{I_1 = 12 A}$

Hence, $I_1 = 12A$

$I_2 = 4A$

$I_3 = 16 A$

c)




Mesh 1:

$-110 + 10I_1 - 10I_3 = 0$

$10I_1 + 0I_2 - 10I_3 = 110$

Mesh 2:

$20I_3 + 10I_3 - 10I_1 = 0$

$-10I_1 + 0I_2 + 30I_3 = 0$

Constraint Equation:

$I_1 - I_3 = I_2$

$I_1 - I_2 - I_3 = 0$

Augmented Matrix of above equations

$$[AB] = \begin{bmatrix} 1 & -1 & -1 & 0 \\ -10 & 0 & 30 & 0 \\ 10 & 0 & -10 & 110 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & -1 & -1 & 0 \\ 0 & -10 & 20 & 0 \\ 0 & 10 & 0 & 110 \end{bmatrix} \begin{matrix} \\ R_2 + R_1(10) \\ R_2 + R_1(-10) \end{matrix}$$

In Equations,

- $10I_2 = 110 \Rightarrow \boxed{I_2 = 11\,A}$
- $-10I_2 + 20I_3 = 0$

  $-110 + 20I_3 = 0 \Rightarrow \boxed{I_3 = 11/2}$
- $I_1 - I_2 - I_3 = 0$

  $I_1 - 11 - 11/2 = 0 \Rightarrow \boxed{I_1 = 33/2}$

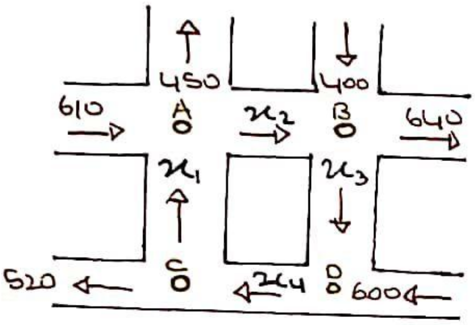
Hence, $I_1 = 33/2$ A

$I_2 = 11$ A

$I_3 = 11/2$ A

Q 2:

a)




At A,

$$x_1 + 610 = x_2 + 450$$

$$x_1 - x_2 = -160$$

At B,

$$x_2 + 400 = x_3 + 640$$

$$x_2 - x_3 = 240$$

At C,

$$x_4 = x_1 + 520$$

$$x_4 - x_1 = 520$$

At D,

$$x_3 + 600 = x_4 \Rightarrow x_4 - x_3 = 600$$

The Augmented Matrix is;

$$[AB] = \begin{bmatrix} 1 & -1 & 0 & 0 & -160 \\ 0 & 1 & -1 & 0 & 240 \\ -1 & 0 & 0 & 1 & 520 \\ 0 & 0 & -1 & 1 & 600 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & -1 & 0 & 0 & -160 \\ 0 & 1 & -1 & 0 & 240 \\ 0 & -1 & 0 & & 360 \\ 0 & 0 & -1 & 1 & 600 \end{bmatrix} R_3 + (R_1)$$

$$= \begin{bmatrix} 1 & -1 & 0 & 0 & -160 \\ 0 & 1 & -1 & 0 & 240 \\ 0 & 0 & -1 & 1 & 600 \\ 0 & 0 & -1 & 1 & 600 \end{bmatrix} R_3 + (R_2)$$

$$= \begin{bmatrix} 1 & -1 & 0 & 0 & -160 \\ 0 & 1 & -1 & 0 & 240 \\ 0 & 0 & -1 & 1 & 600 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} R_4 + (-R_3)$$

Hence, there are infinitely many solutions

Let $\boxed{x_4 = t}$ ; $t$ ; parameter, that can be any real number

- $-x_3 + x_4 = 600$

  $\boxed{x_3 = t - 600}$

- $x_2 - x_3 = 240$

  $x_2 - t + 600 = 240$

  $\boxed{x_2 = t - 360}$

- $x_1 - x_2 = -160$

  $x_1 - t + 360 = -160$

  $\boxed{x_1 = t - 520}$

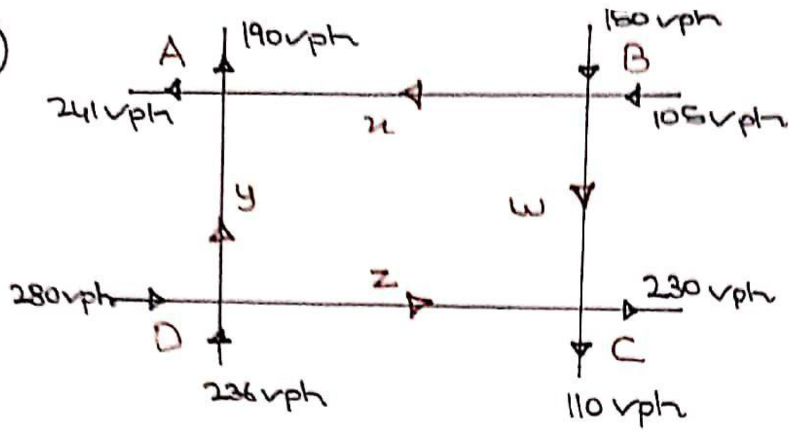
Thus, $x_1 = t - 520$

$x_2 = t - 360$

$x_3 = t - 600$

$x_4 = t$

b)




At A,

$x + y = 431$

At B,

$x + w = 255$

At C,

$z + w = 340$

At D,

$y + z = 516$

Augmented Matrix

$$[AB] = \begin{bmatrix} 1 & 1 & 0 & 0 & 431 \\ 1 & 0 & 0 & 1 & 255 \\ 0 & 0 & 1 & 1 & 340 \\ 0 & 1 & 1 & 0 & 516 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 1 & 0 & 0 & 431 \\ 0 & -1 & 0 & 1 & -176 \\ 0 & 0 & 1 & 1 & 340 \\ 0 & 1 & 1 & 0 & 516 \end{bmatrix} R_2 + R_1(-1)$$

$$= \begin{bmatrix} 1 & 1 & 0 & 0 & 431 \\ 0 & -1 & 0 & 1 & -176 \\ 0 & 0 & 1 & 1 & 340 \\ 0 & 0 & 1 & 1 & 340 \end{bmatrix} R_4 + R_2$$

$$= \begin{bmatrix} 1 & 1 & 0 & 0 & 431 \\ 0 & -1 & 0 & 1 & -176 \\ 0 & 0 & 1 & 1 & 340 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} R_4 - R_3$$

Hence, there are infinitely many solutions

$\boxed{w = t}$; where $t$ can be any real number

- $z + w = 340$

  $\boxed{z = 340 - t}$

- $-y + w = -176$

  $\boxed{y = 176 + t}$

- $x + y = 431$

  $x + 176 + t = 431$

  $\boxed{x = 255 - t}$

Thus, $x = 255 - t$
$y = 176 + t$
$z = 340 - t$
$w = t$