

# EE-381 Robotics-1

## UG ELECTIVE COURSE



### Lecture 2

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# Last Lecture

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Enrollment Code: **983675410**

- Introduction to Robotics: Definition, history
- Robot accessories; joints
- Classification of Robots; power source, application, control systems, geometry, method of control



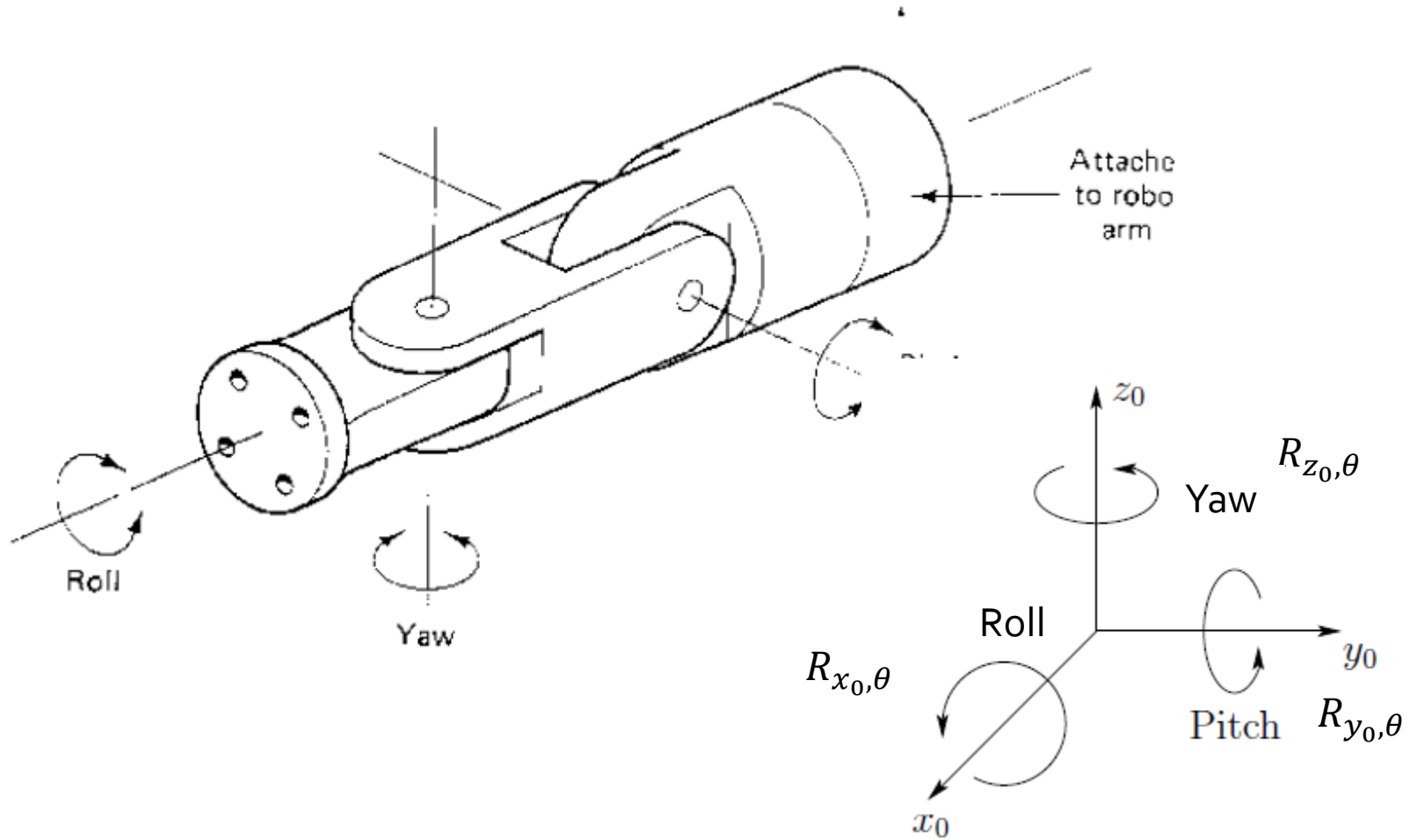
# Today's Lecture Agenda

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- Robot Configurations
- Robot Programming



# Wrist



# Robot Configurations

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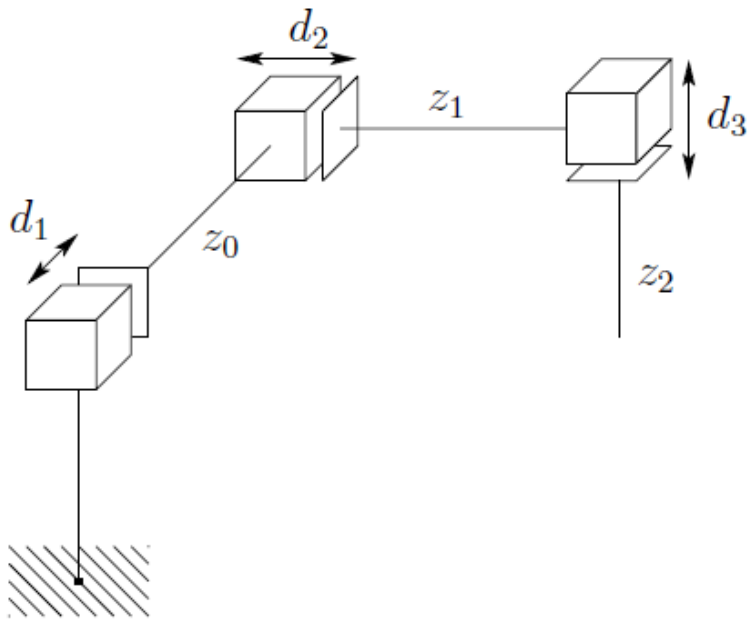
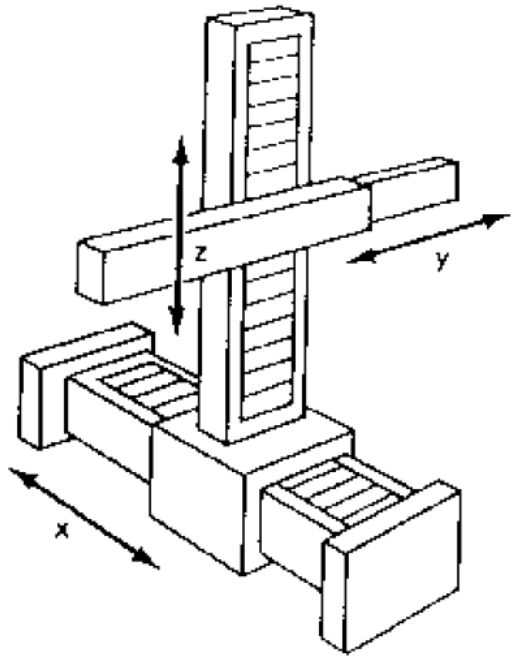
Based on coordinate system

1. Cartesian/Rectangular Robot (PPP)
2. Cylindrical Robot (RPP)
3. Spherical Robot (RRP)
4. Articulated Robot (RRR)
5. SCARA (special types of spherical) (RRP)



# 1-Cartesian Robot (PPP)

- 3 Prismatic Joints that orient the end effector, which are usually followed by additional revolute joints



Configuration of Cartesian Robot

[https://www.youtube.com/watch?v=ci\\_mpRERMog](https://www.youtube.com/watch?v=ci_mpRERMog)

# 1-Cartesian Robot

- **Advantages**

- Simple configuration
- Equal & constant spatial resolution
- Use for assembly applications and transfer of material or cargo

- **Disadvantages**

- Lacks mechanical flexibility
- Cannot reach objects on the floor
- Speed of operation in horizontal plane is slower than the robots with rotary base

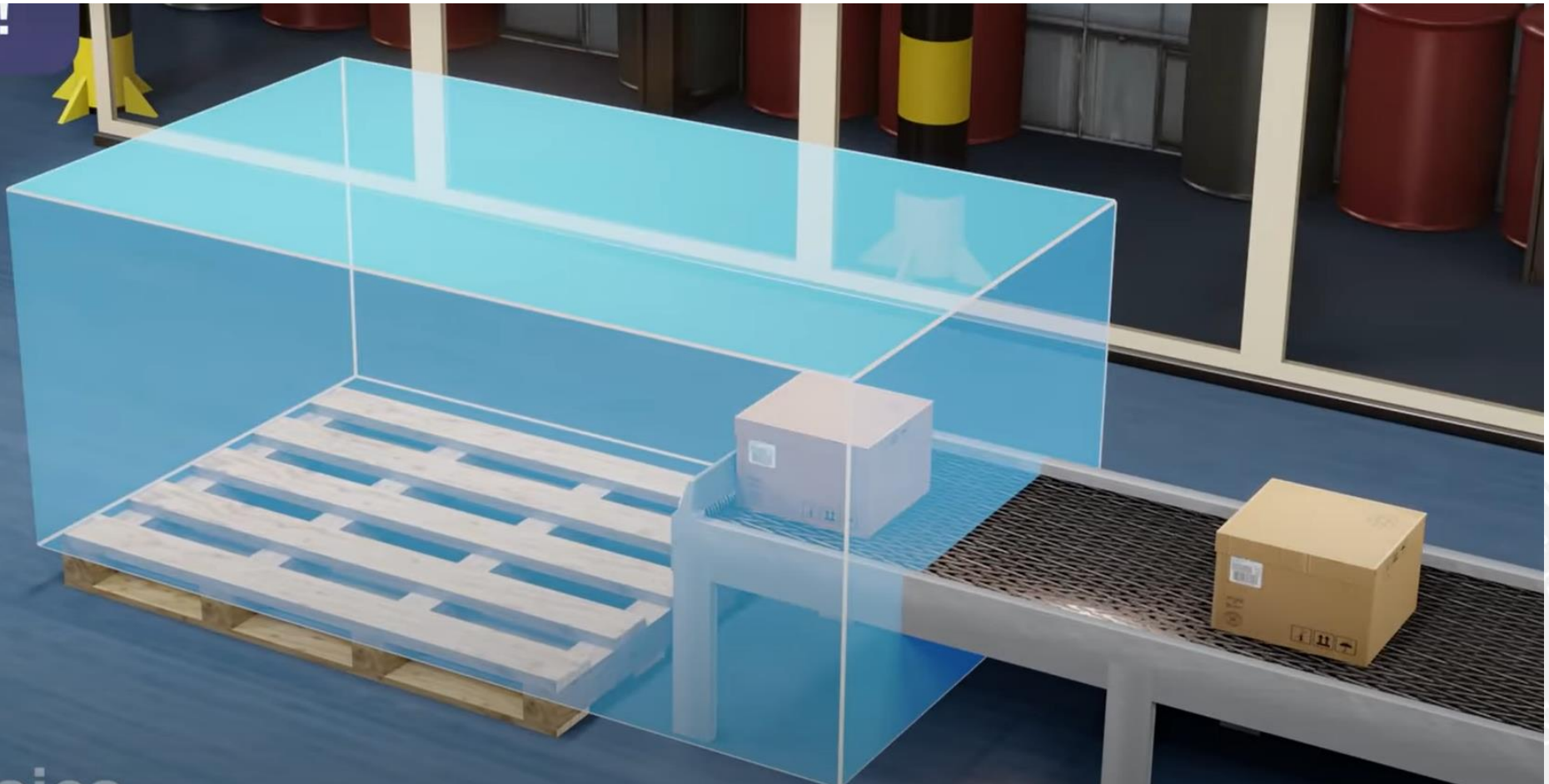


Epson Cartesian Robot



- Guess!

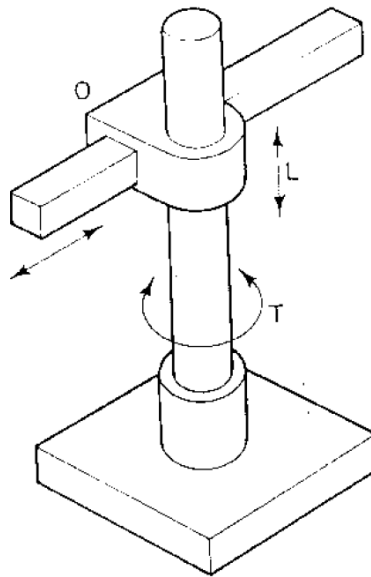
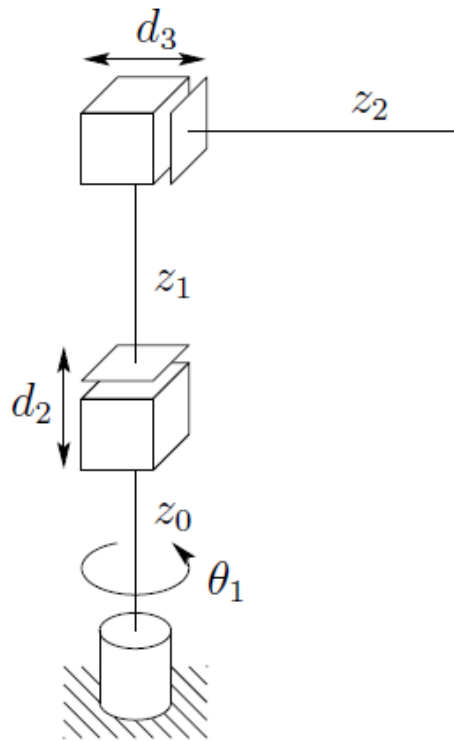
[https://www.youtube.com/watch?v=\\_canCYWZPsc](https://www.youtube.com/watch?v=_canCYWZPsc)



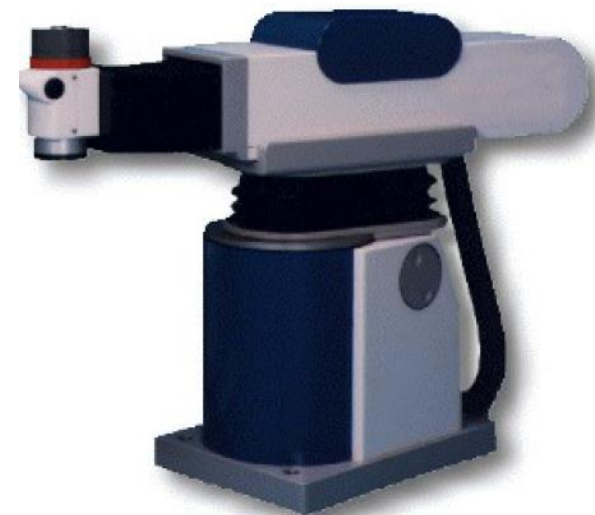


# 2-Cylindrical Robot (RPP)

- First joint is revolute and produces a rotation about the base, second and third joints are prismatic

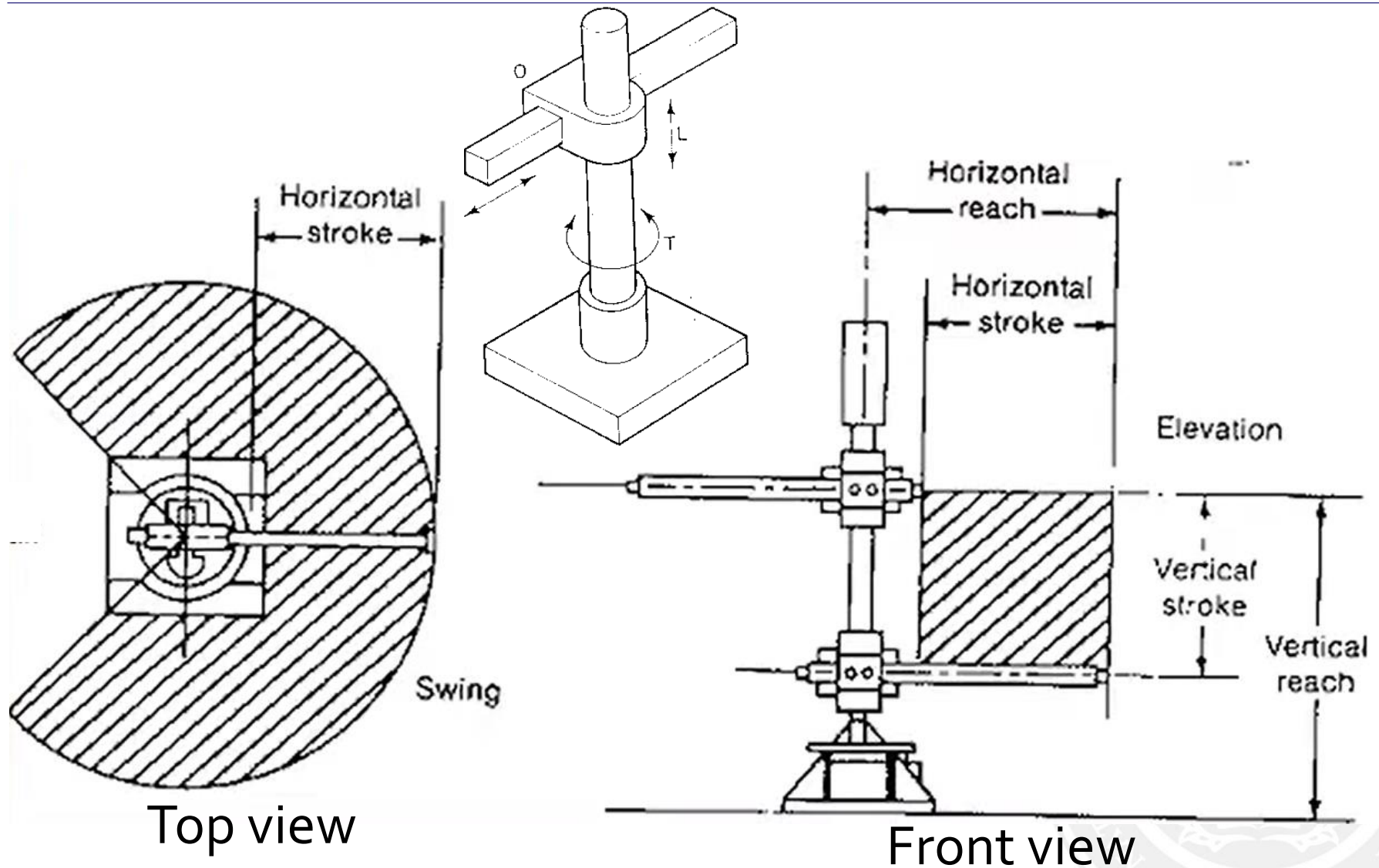


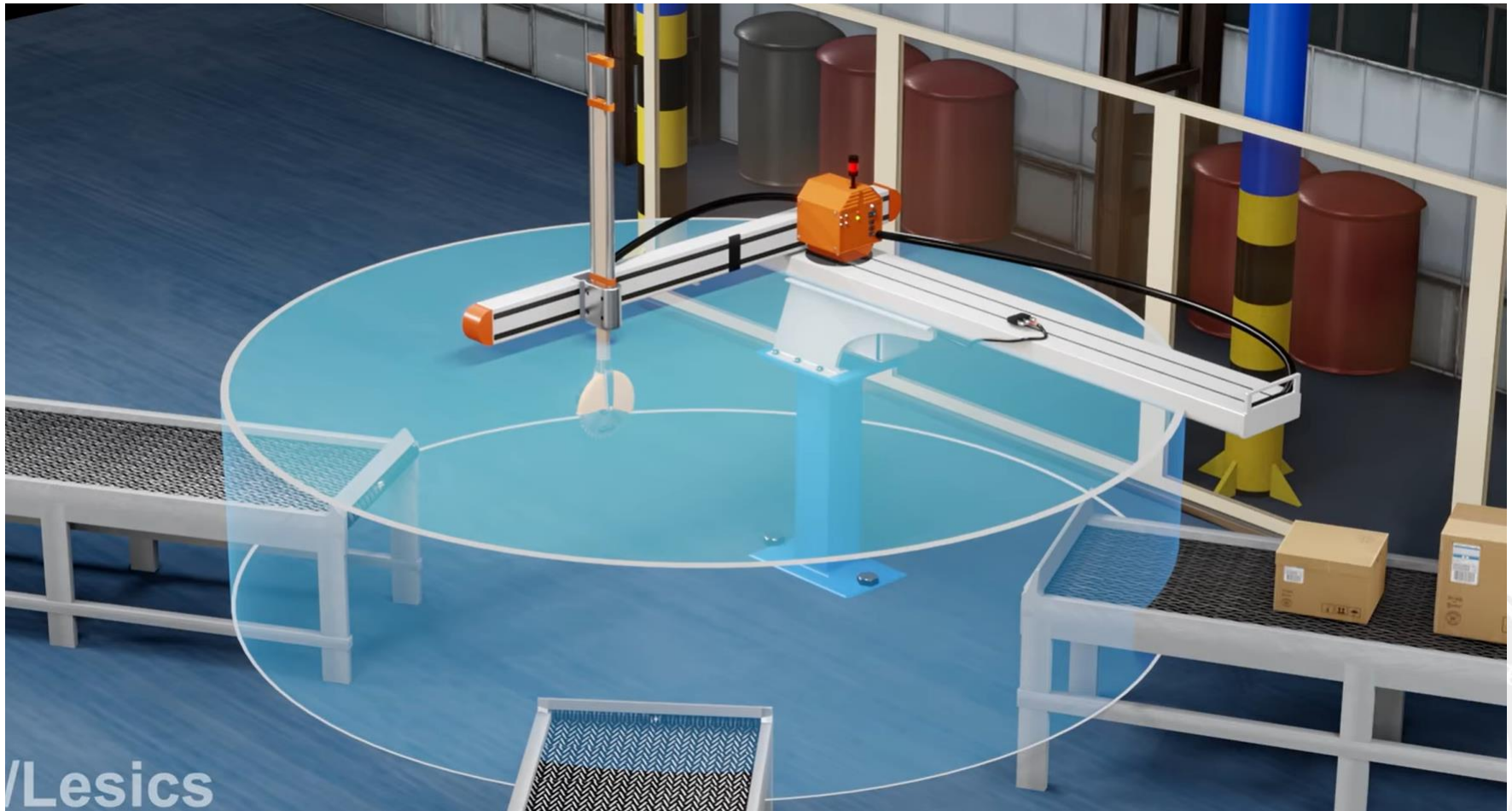
Configuration of Cylindrical Robot



Seiko RT3300 Robot

# 2-Cylindrical Robot- Work Envelop





[https://www.youtube.com/watch?v=\\_canCYWZPsc](https://www.youtube.com/watch?v=_canCYWZPsc)

# 2-Cylindrical Robot- Advantages

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- Results in a larger work envelope than a rectangular robot
- Suited for pick-and-place operations
- Vertical structure preserves the floor space
- Deep horizontal reach is useful for far-reaching operations



## 2-Cylindrical Robot- Disadvantages

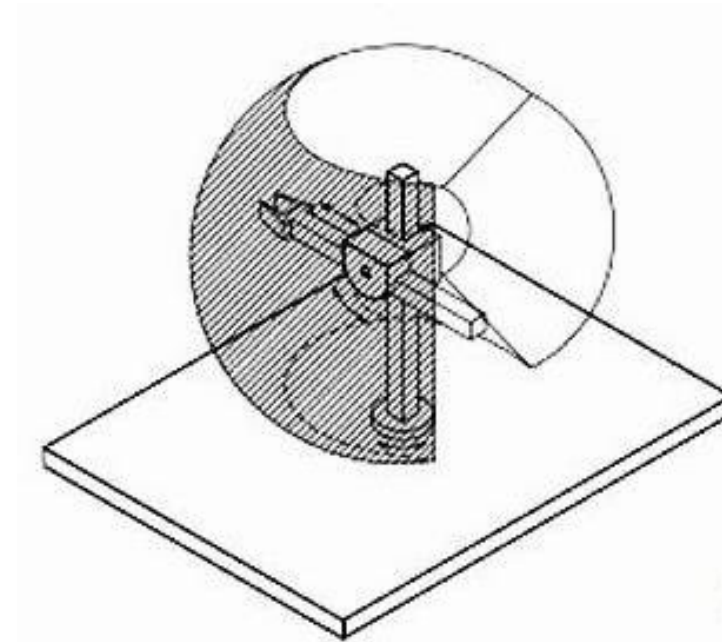
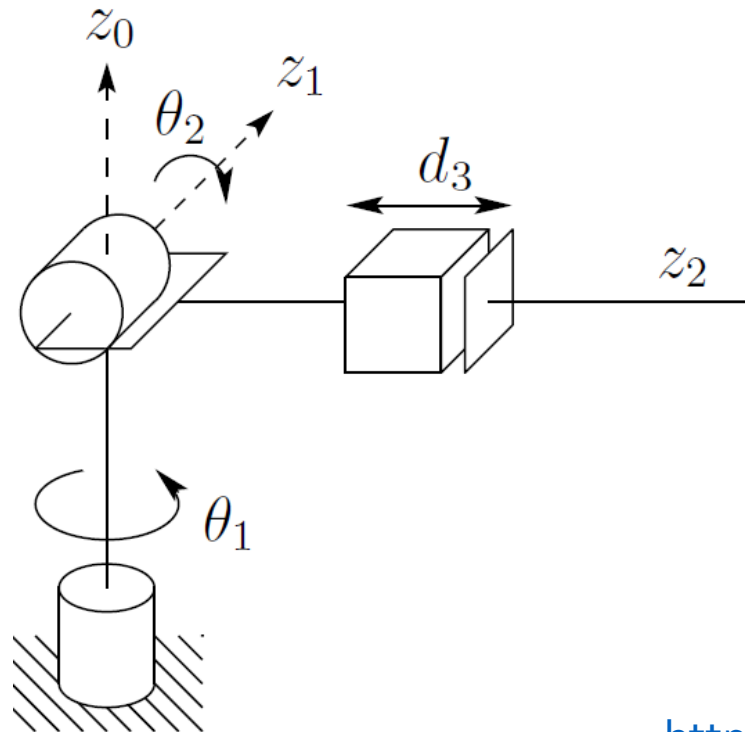
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- Overall mechanical rigidity is lower than that of the rectilinear robots.
- Repeatability and accuracy are also lower in the direction of rotary motion.
- Configuration requires a more sophisticated control system than the rectangular robots.



# 3-Spherical Robot (RRP)

- Also known as Polar Coordinate Robot
- 2 Revolute and 1 prismatic joint

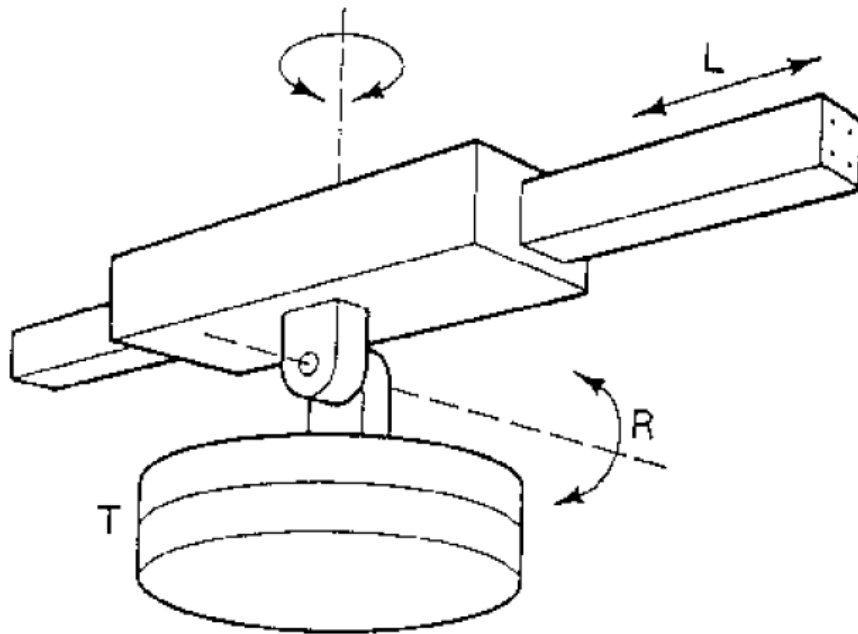


<https://www.youtube.com/watch?v=jrF5DI6ntAc>

Configuration of spherical manipulator



# 3-Spherical Robot (RRP)

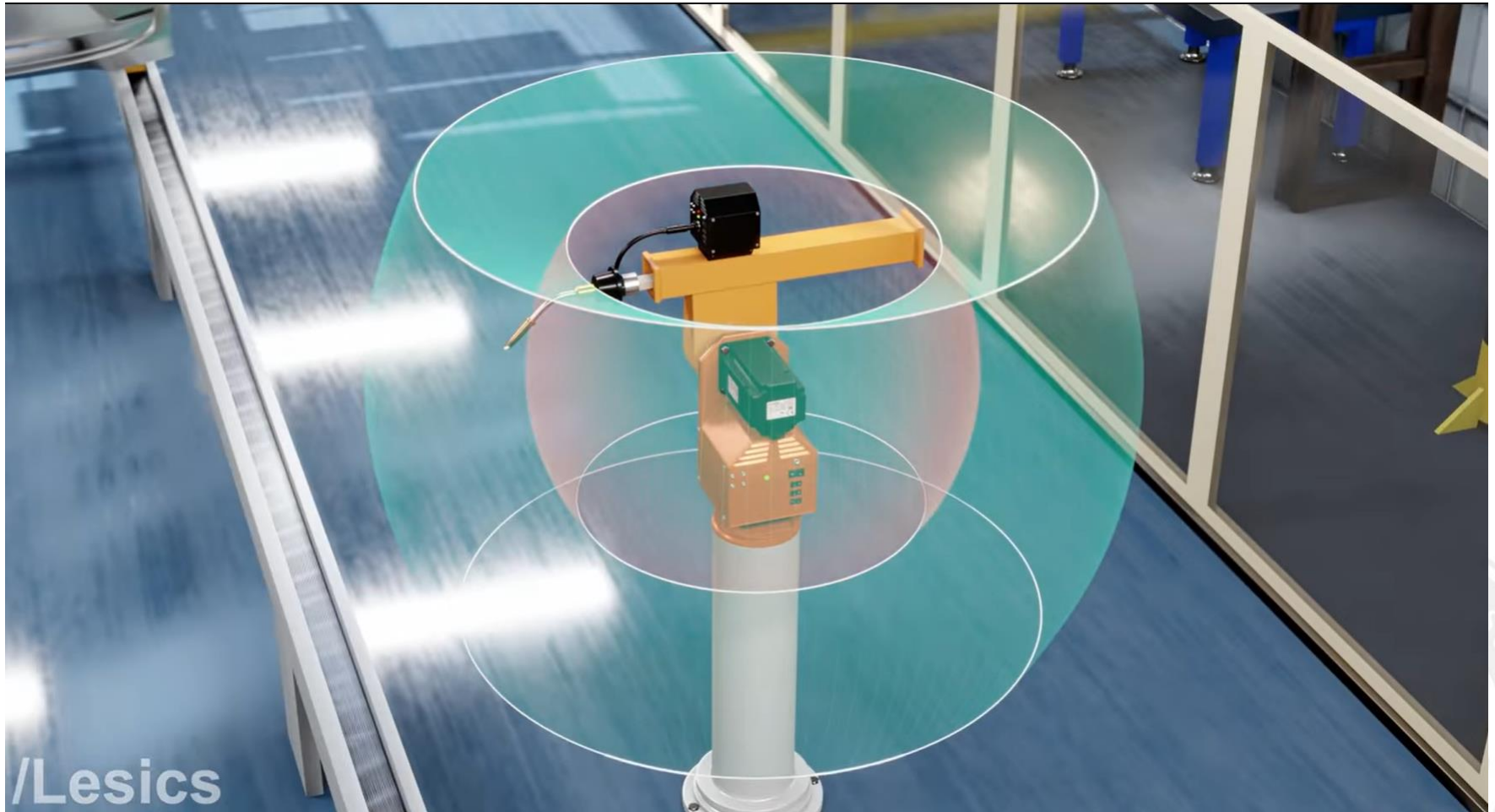


Workspace of spherical manipulator



Stanford Arm

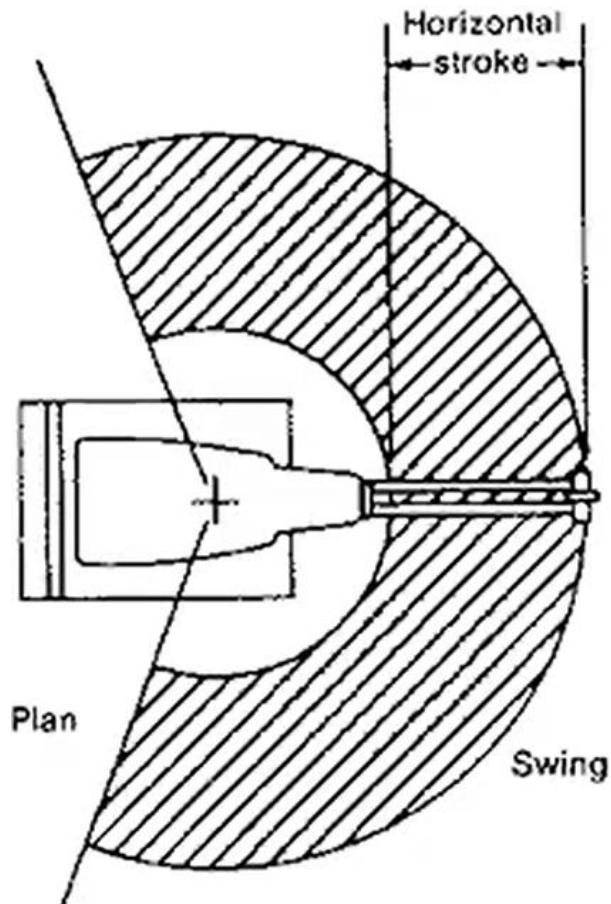
# 3-Spherical Robot (RRP)-Work Envelop



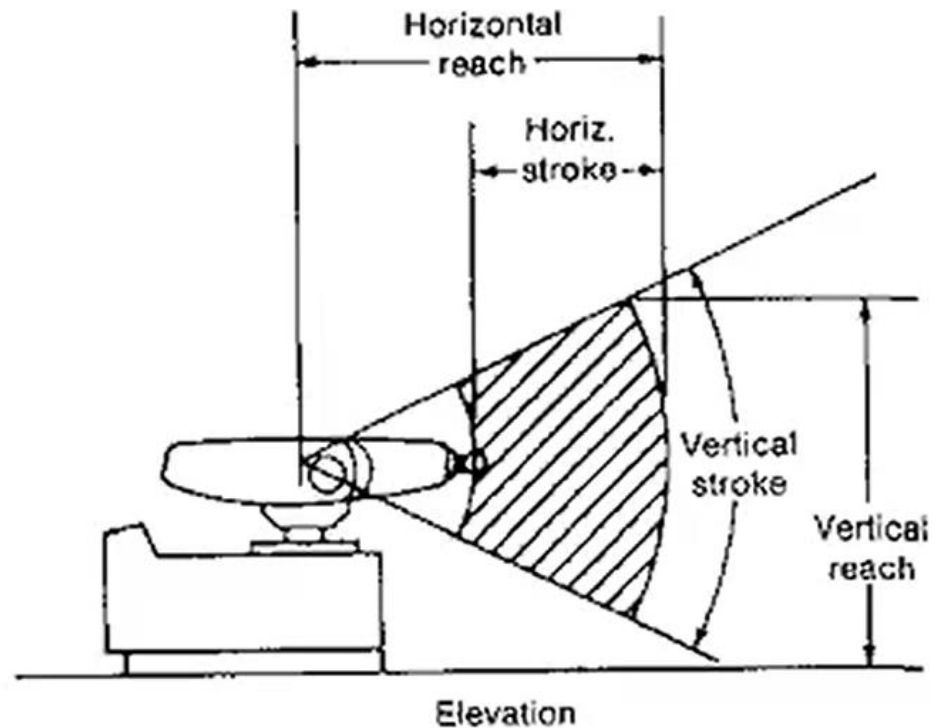
[https://www.youtube.com/watch?v=\\_canCYWZPsc](https://www.youtube.com/watch?v=_canCYWZPsc)



# 3-Spherical Robot (RRP)-Work Envelop



Top view



Front view

# 3-Spherical Robot (RRP)

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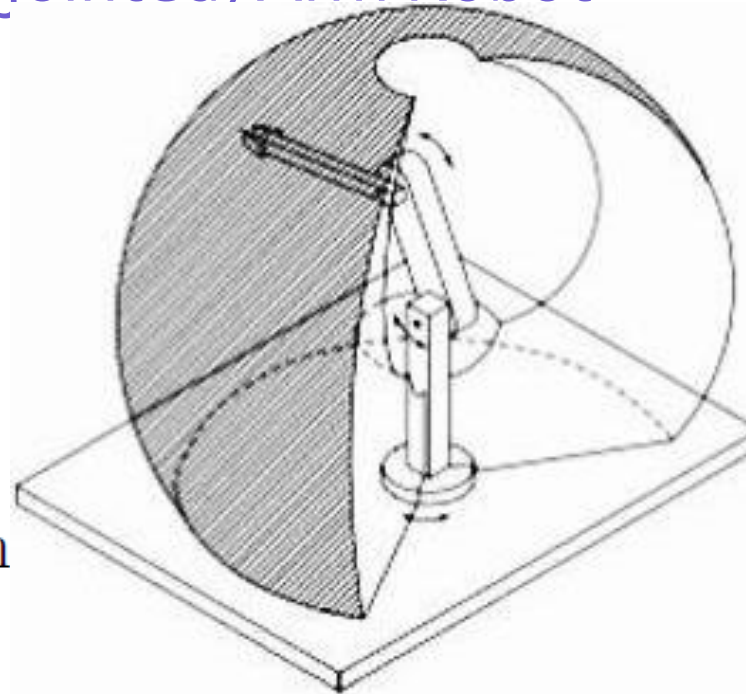
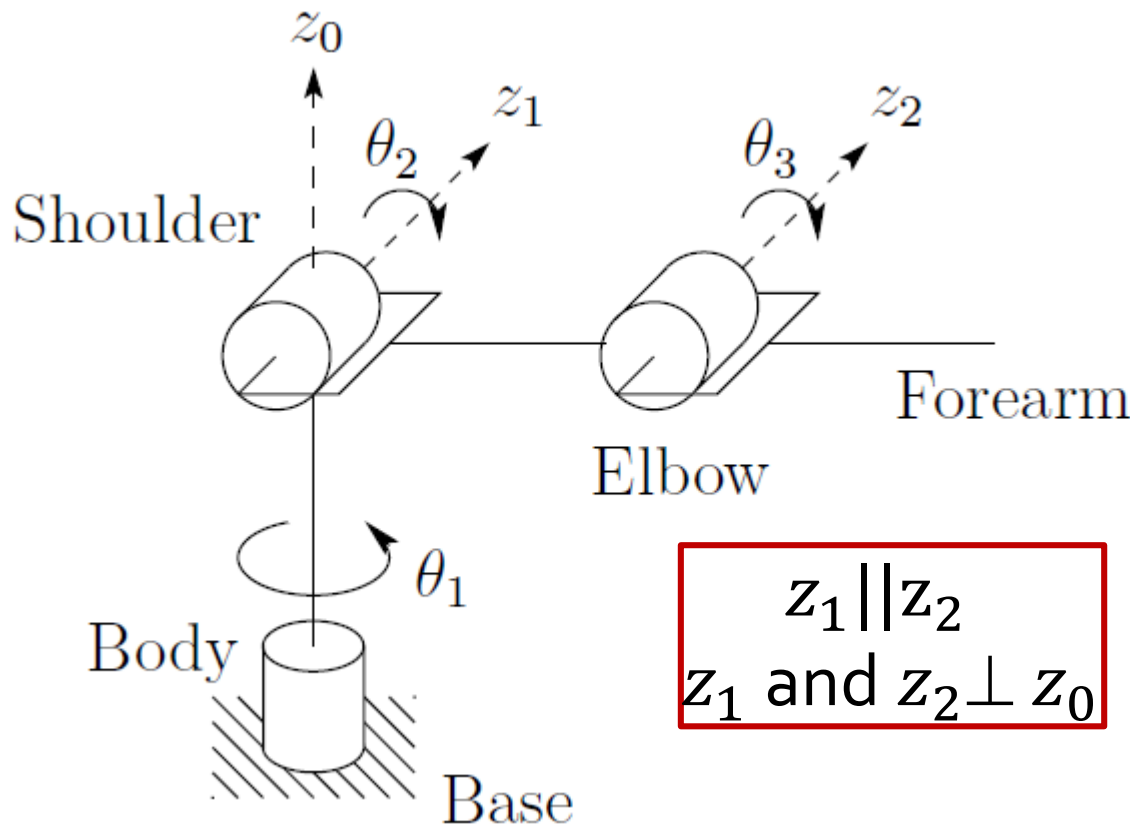
- Provides a larger work envelope than the rectilinear or cylindrical robot
- Design gives weight lifting capabilities
- Advantages and disadvantages same as cylindrical-coordinated robot

<https://www.youtube.com/watch?v=jrF5DI6ntAc>



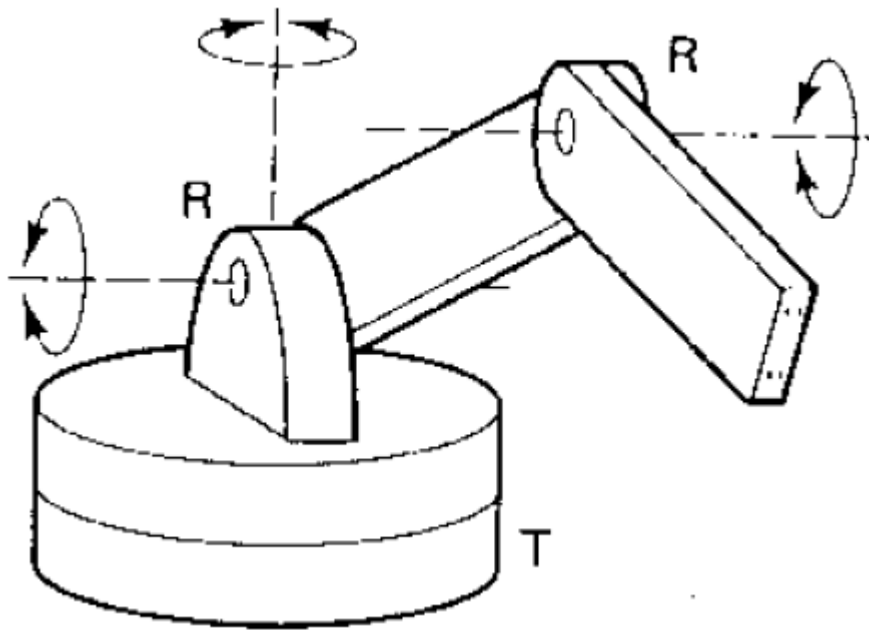
# 4-Articulated Robot (RRR)

- Also known as anthropomorphic (jointed) Arm Robot
- 3 revolute joints



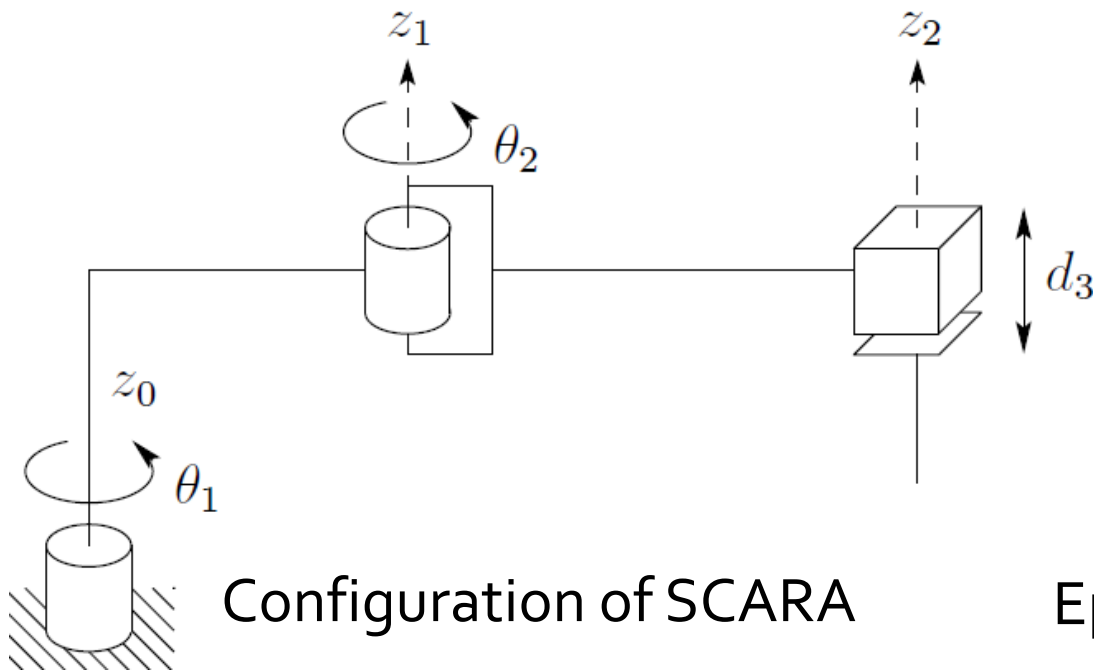
Configuration of articulated robot

# 4-Articulated Robot (RRR)



# 5-SCARA (RRP)

- Selective Compliant Articulated Robot Assembly
- 2 parallel revolute joint that allows the horizontal movement of robot and 1 prismatic that moves vertically
- 4DOF, 3 for Arm and 1 for wrist (roll)

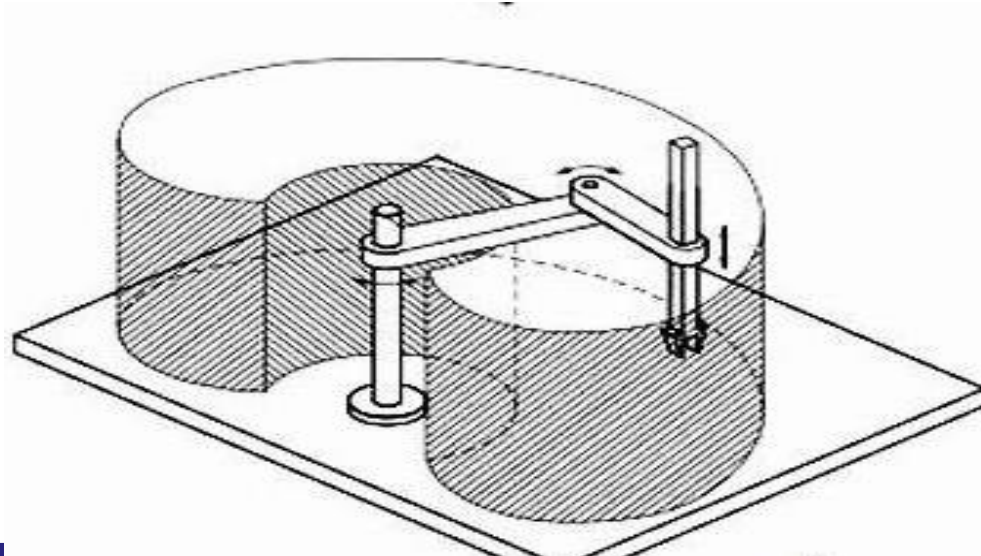
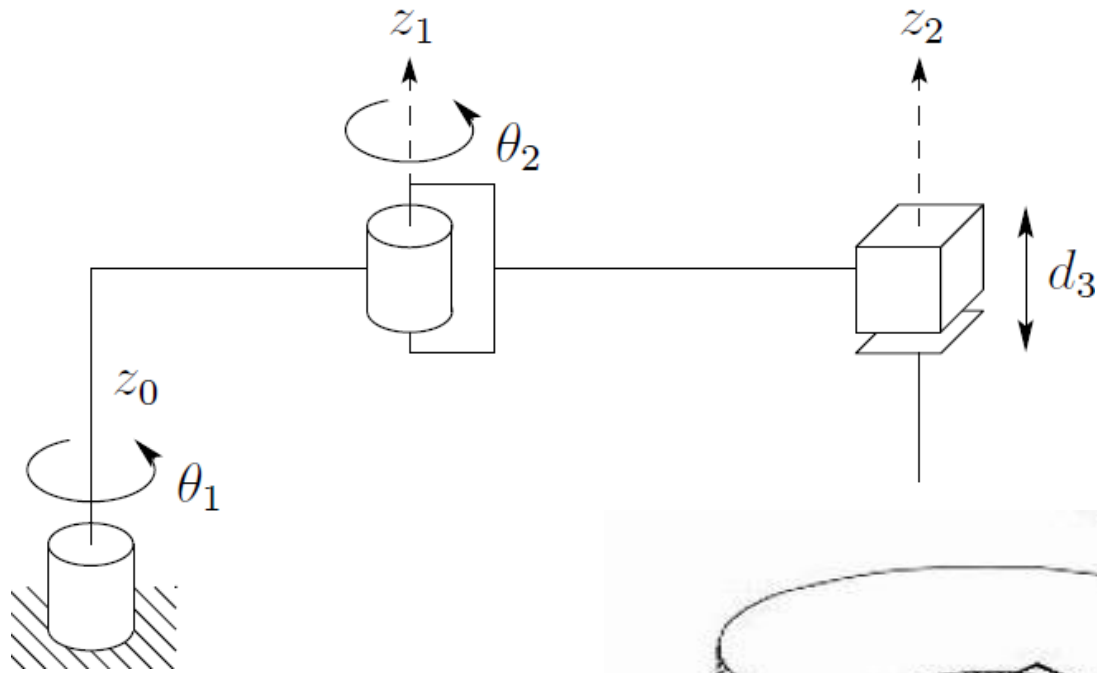


Configuration of SCARA

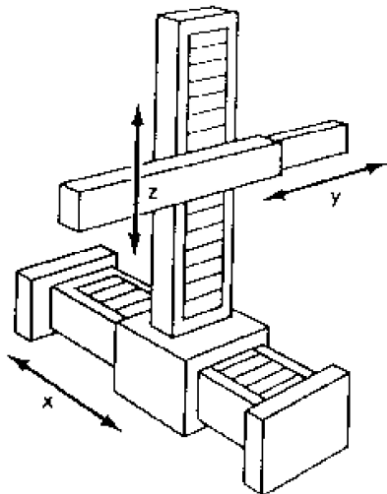


Epson E2L653S SCARA Robot

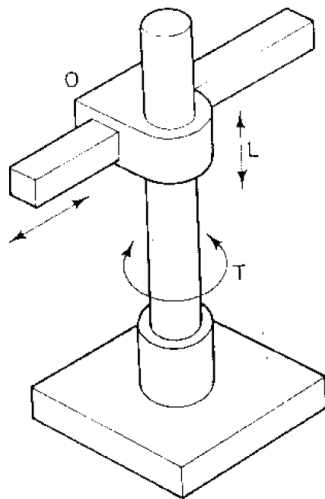
# 5-SCARA (RRP)



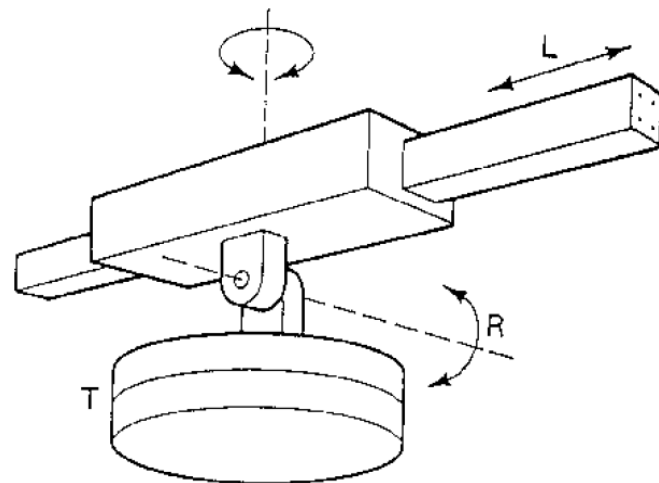
# Robot Configurations: Summary



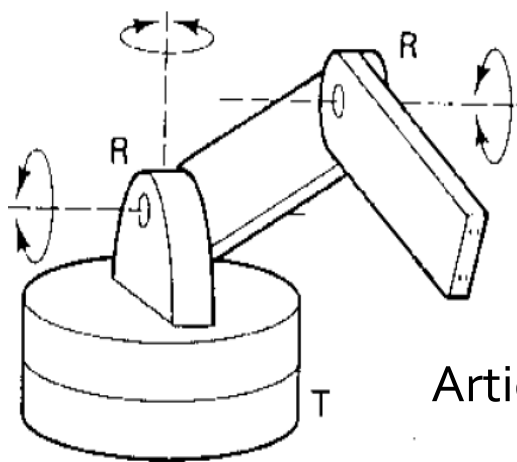
Cartesian



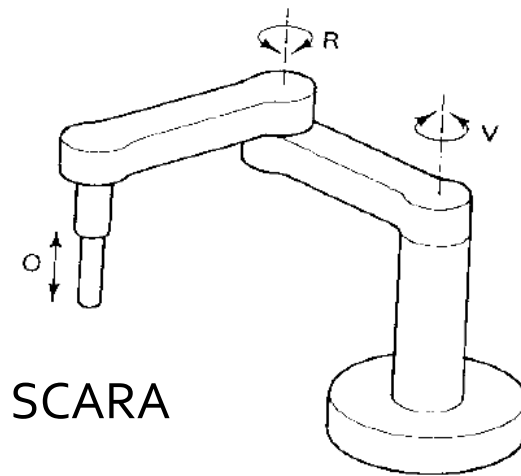
Cylindrical



Spherical



Articulated



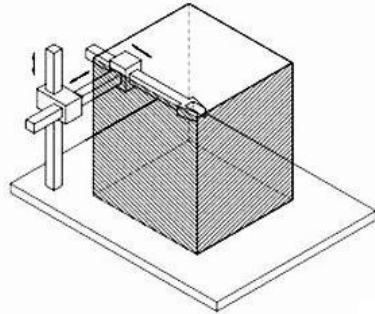
SCARA



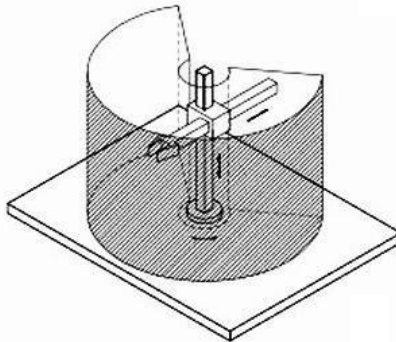
# Work Space: Summary

- The region in space a robot can fully interact with

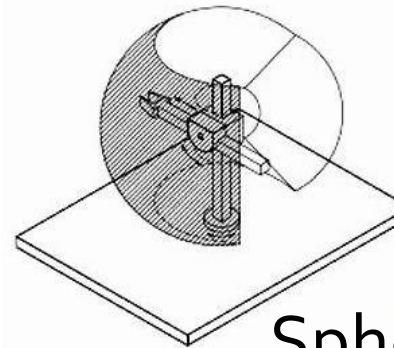
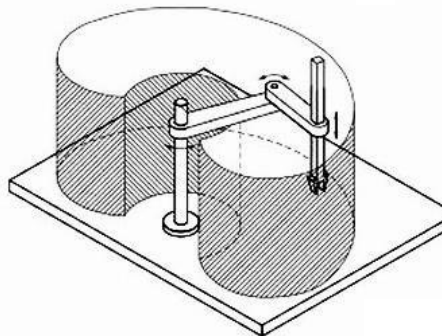
Rectangular/  
Cartesian ( $3P$ )



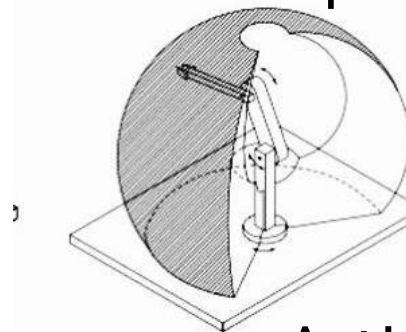
Cylindrical ( $1R2P$ )



SCARA ( $2R1P$ )



Spherical ( $2R1P$ )



Articulated ( $3R$ )



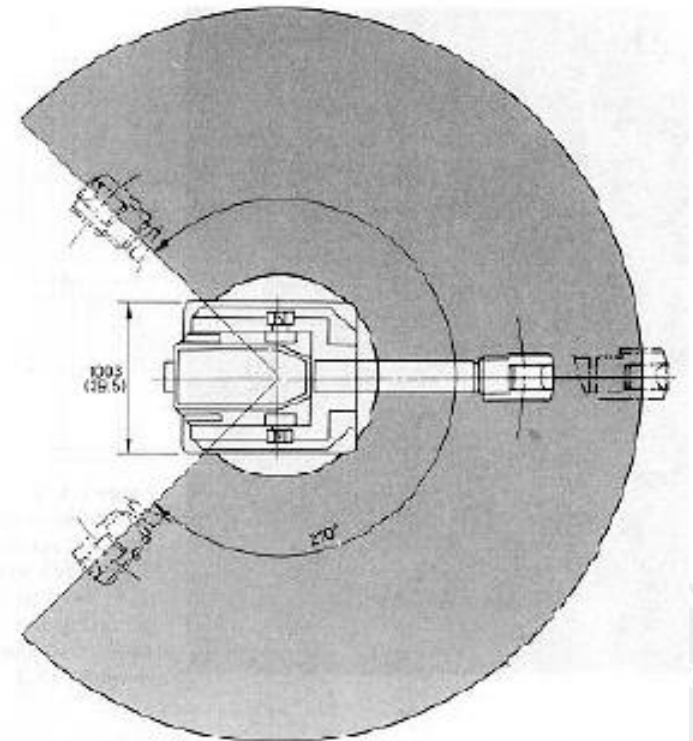
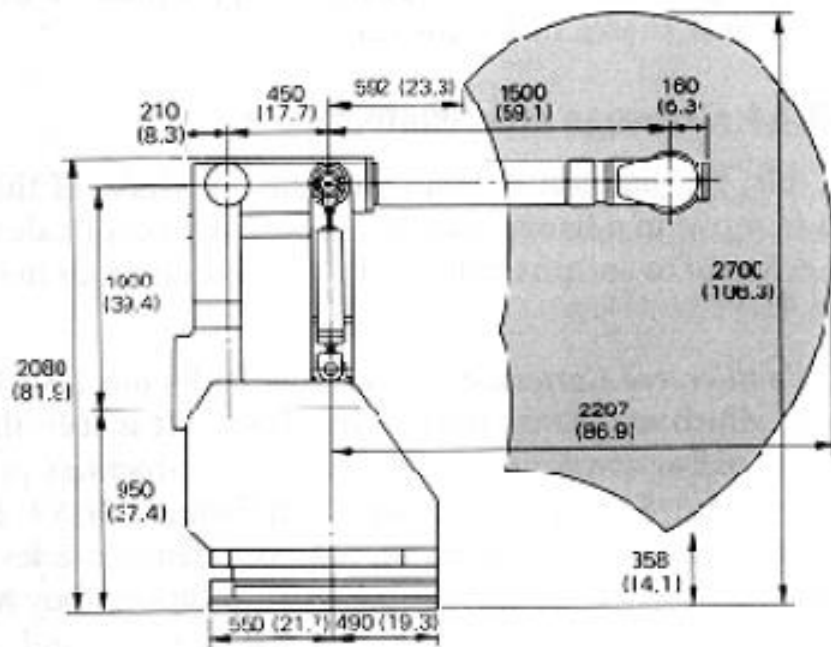
# Workspace

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- Depending on the configuration and size of the links and wrist joints, robots can reach a collection of points called a **Workspace**.
- Alternately Workspace may be found empirically, by moving each joint through its range of motions and combining all space it can reach and subtracting what space it cannot reach

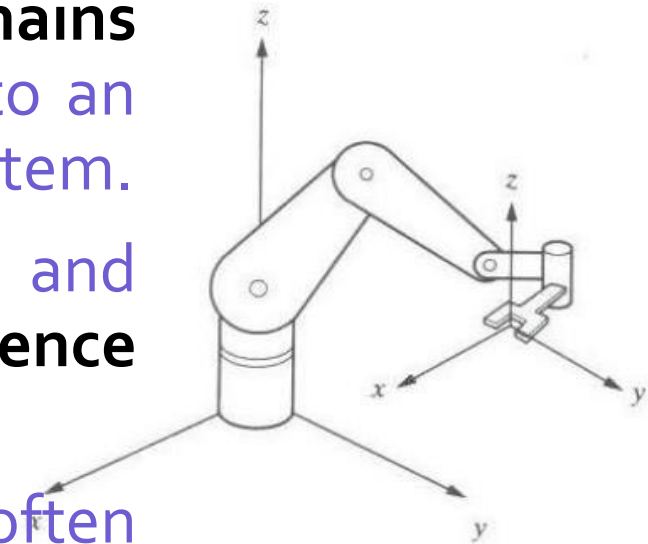


# Work Envelope



# Reference Frames

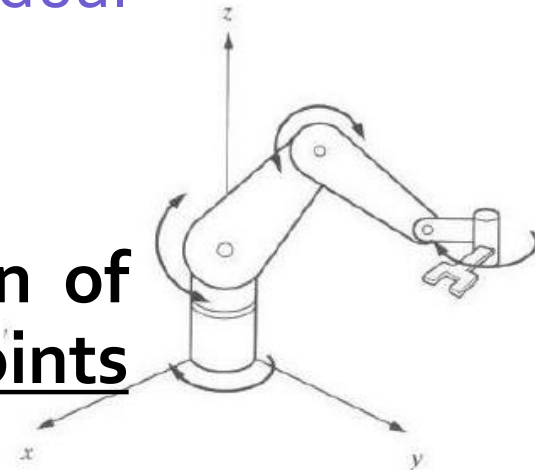
- World reference frame aka global reference frame or base reference frame, is an inertial frame of reference fixed in space.
- The world reference frame **remains stationary** and does not move relative to an external reference point or coordinate system.
- All other frames move simultaneously and defined **relative to the world reference frame**.
- In robotics, the world reference frame is often located at the **base of the robot** or at a fixed point in the workspace.



World reference frame

# Reference Frames

- Joint reference frame aka local reference frame or joint coordinate frame, is a reference frame attached to each individual joint of the robotic system.
- It defines the **position and orientation of the joint relative to its neighboring joints or links.**
- The joint reference frame moves and rotates as the joint moves during operation.

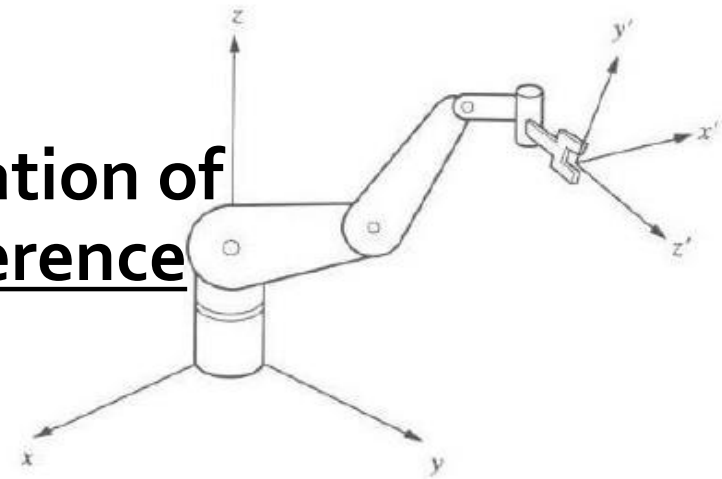


Joint reference frame



# Reference Frame

- Tool reference frame aka TCP (Tool Center Point), is a local reference frame attached to the end-effector or tool of the robotic system.
- It defines the **position and orientation of the tool relative to the world reference frame.**
- The tool reference frame moves and changes orientation as the end-effector moves and rotates during operation.



Tool reference frame

# Robot Programming/training

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Typically performed using one of the following

- Online

- Teach pendant
- Lead through programming

- Offline

- Programming languages
- Task level programming



# Teach Pendant Programming

- Hand held device with switches used to control the robot motions
- End points are recorded in controller memory
- Sequentially played back to execute robot actions
- Trajectory determined by robot controller
- Suited for point-to-point control applications



[https://www.youtube.com/watch?v=EA6pWwNI\\_wg](https://www.youtube.com/watch?v=EA6pWwNI_wg)

# Lead Through Programming

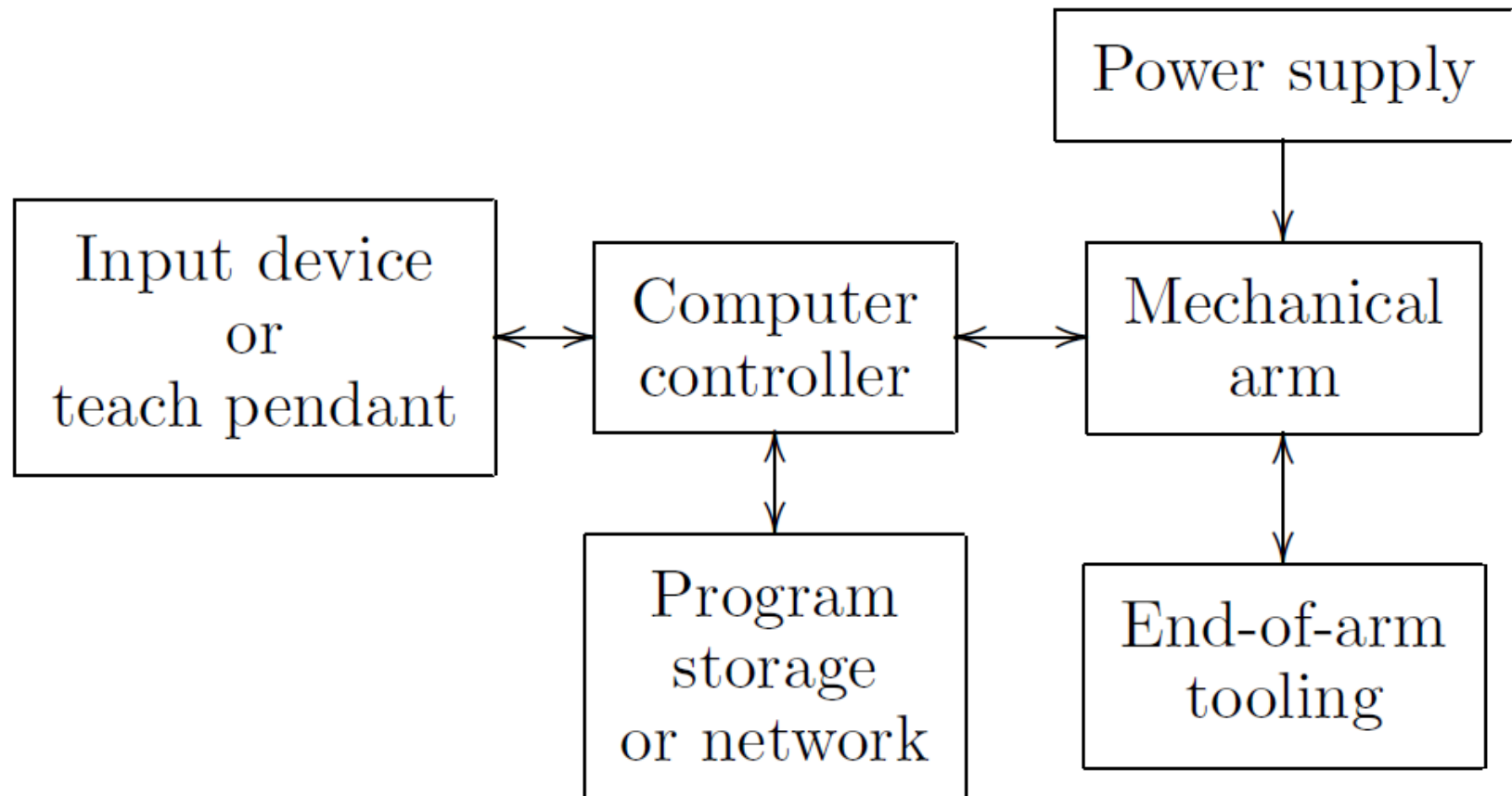
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- Lead the robot physically through the required sequences of motions
- Trajectory and endpoints are recorded, using a sampling routine which record points at 60 – 80 times in a second
- When played back results in a smooth continuous motion
- Large memory requirements





# Robotic System



Component of Robotic system

# Online Programming

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- Advantages

- Easy to use
- No special programming skills required
- Useful when programming the robots for wide range of repetitive tasks for long production runs

- Disadvantages

- Required production line shutdown
- Technician programming inside work envelope



# Programming Languages

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- Motivation:

- Need to interface robot control system to external sensors to provide “real-time” changes based on **sensory equipment**
- Commuting based on geometry of environment
- Ability to interface with CAD/CAM systems
- Meaningful task descriptions
- Offline programming capability



# Decision Making in Autonomous Mobile Robots

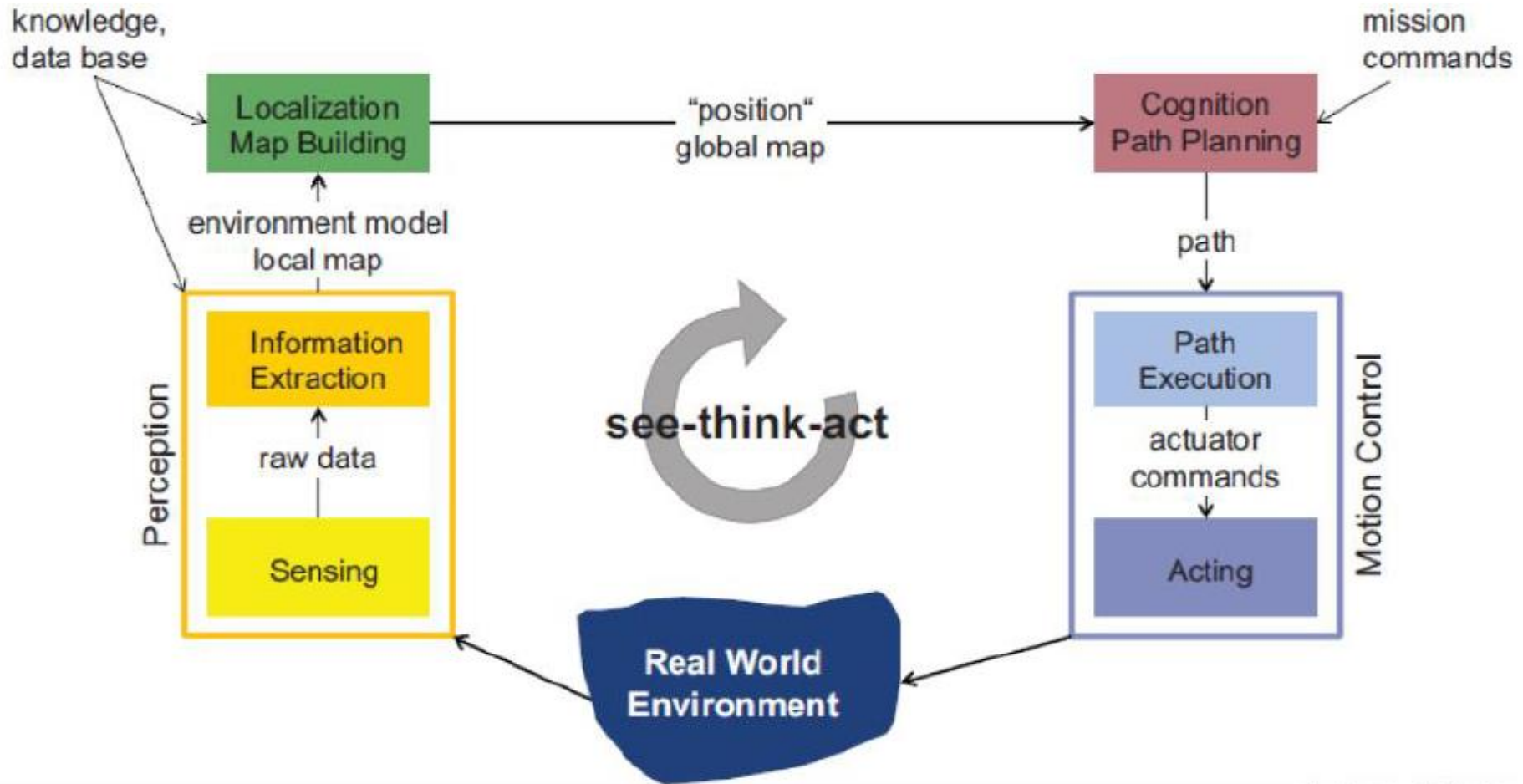
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- Sensing
  - **Proprioceptive sensors:** internal measurement of robot; such as, angle of the joints of robotic arm, wheel revolutions, current drawn by an electric motor etc.
  - **Exteroceptive sensors:** measure the external state of the world with respect to the robot; such as, detect collision, distance between robot and the surrounding objects.
- Given an example of exteroceptive sensor
- Perceiving
- Planning



# Autonomous Mobile Robots (Vehicles)

## THE SEE THINK AND ACT CYCLE!



# Programming Languages

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- Wide range of robot's programming languages are available such as: AML, VAL, AL, RAIL, RobotStudio (200+)
- Each robot manufacturer has their own robot programming language
- No standards exist
- Portability of programs virtually non-existent



# Agenda

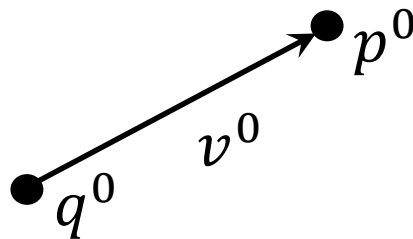
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- Orientation, Spaces and Transformations (SPONG, chp 2-3)
  - Representing robot position
  - Transforms
  - Mappings
  - Representations of Orientation
  - Joints and Spaces



# Point and Vectors

- **Point:** A Point has position in space. The only characteristic that distinguishes one point from another is its position.
  - Draw point as dot
- **Vector:** A Vector has both magnitude and direction, but no fixed position in space.
  - Draw vector as line



$$p^0 = \begin{bmatrix} 4 \\ 2.5 \end{bmatrix}$$

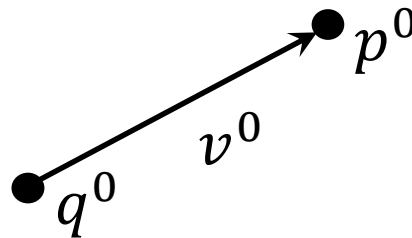
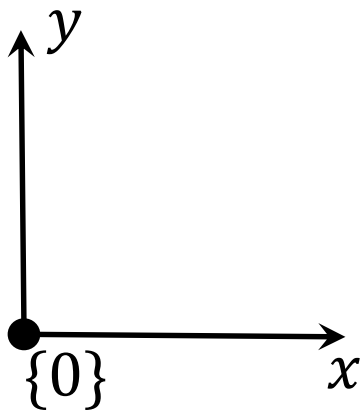
$$q^0 = \begin{bmatrix} 2 \\ 1.5 \end{bmatrix}$$

$$v^0 = p^0 - q^0 = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$$



# Coordinate Frames

- A **coordinate frame** in two-dimensional space is a set of two vectors having unit length and that are perpendicular to each other.
- Allow us to assign the coordinates to the point



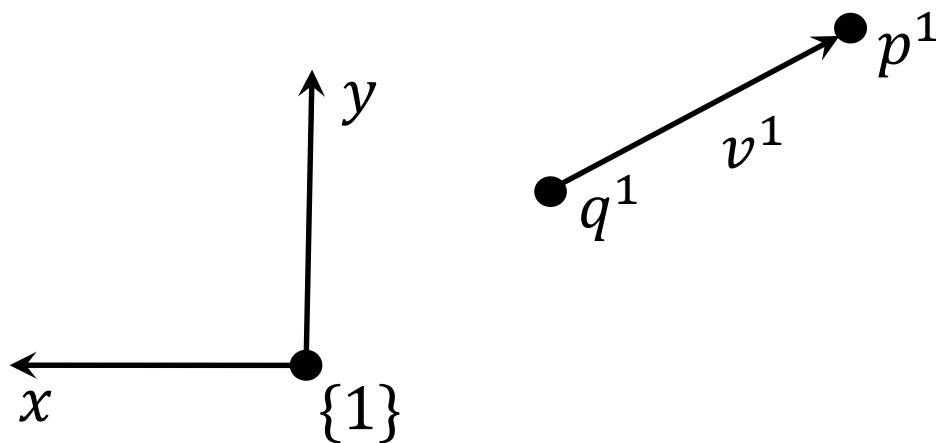
$$p^0 = \begin{bmatrix} 4 \\ 2.5 \end{bmatrix}$$

$$q^0 = \begin{bmatrix} 2 \\ 1.5 \end{bmatrix}$$

$$v^0 = p^0 - q^0 = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$$

# Choice of Coordinate Frames

- Coordinates change depending on the choice of frame



$$p^1 = \begin{bmatrix} -0.5 \\ 4 \end{bmatrix}$$

$$q^1 = \begin{bmatrix} 0.5 \\ 2 \end{bmatrix}$$

$$v^1 = p^1 - q^1 = \begin{bmatrix} -1 \\ 2 \end{bmatrix}$$

# Dot Product

- Dot product of two vectors gives the projection of one onto the other

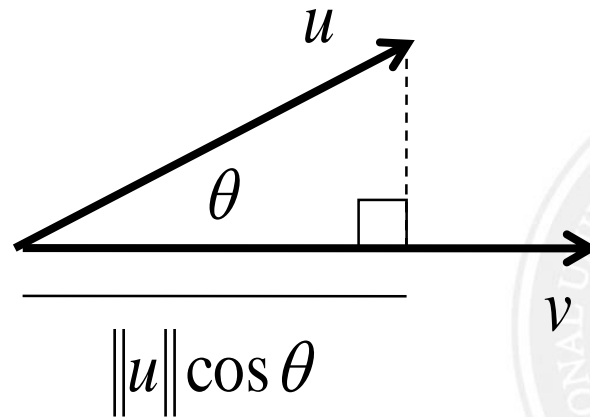
$$u = \begin{bmatrix} u_1 \\ u_2 \\ \vdots \\ u_n \end{bmatrix} \quad v = \begin{bmatrix} v_1 \\ v_2 \\ \vdots \\ v_n \end{bmatrix}$$

$$u \cdot v = u_1 v_1 + u_2 v_2 + \dots + u_n v_n = u^T v$$

$$u \cdot v = \|u\| \|v\| \cos \theta$$

Orthogonal vectors

$$u \cdot v = 0$$



# Home Work!

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- List the application of dot product in AI, gaming etc. with examples.



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# Transformations



# Translation

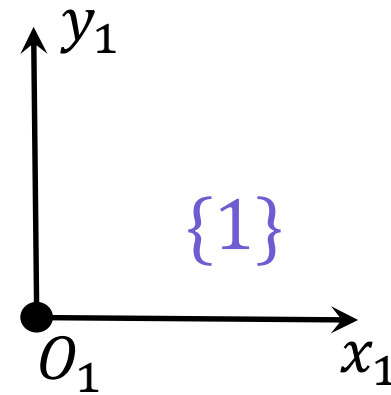
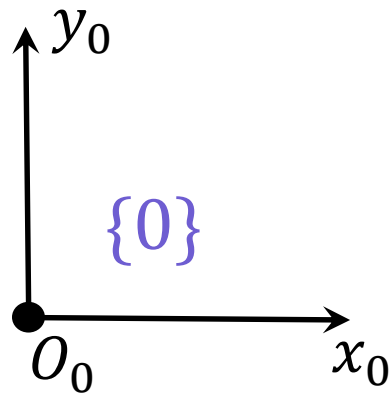
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1. The translation vector  $d_j^i$  can be interpreted as the **location** of frame {j} expressed in frame {i}.



# Translation

- **Example 1**

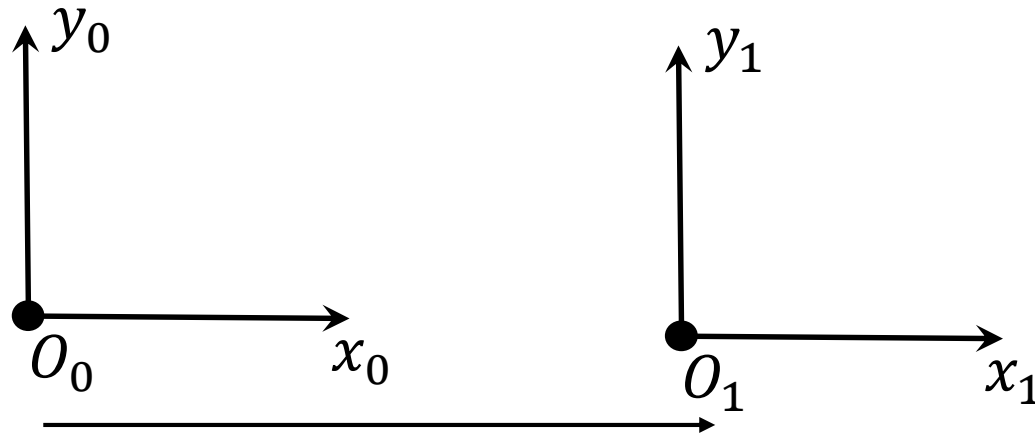


- Suppose  $O_0$  is **zero vector** and  $O_1 = \begin{bmatrix} 3 \\ 0 \end{bmatrix}$



# Translation

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- The location of  $\{1\}$  is expressed in  $\{0\}$

$$d_1^0 = O_1 - O_0 = \begin{bmatrix} 3 \\ 0 \end{bmatrix} - \begin{bmatrix} 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 3 \\ 0 \end{bmatrix}$$





# Translation

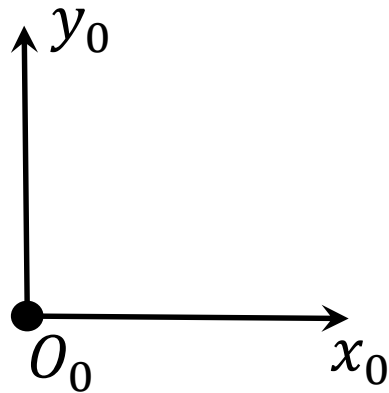
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1. The translation vector  $d_j^i$  can be interpreted as the **location** of frame {j} expressed in frame {i}.
2. The translation vector  $d_j^i$  can be interpreted as a **coordinate transformation of a point** from frame {j} to frame {i}.

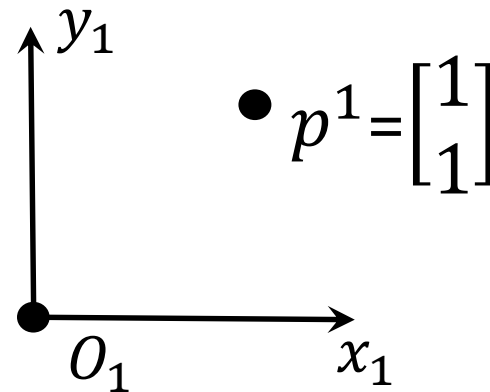


# Translation: Example 2

- A point expressed in frame  $\{0\}$  when  $d_1^0 = \begin{bmatrix} 3 \\ 0 \end{bmatrix}$ .



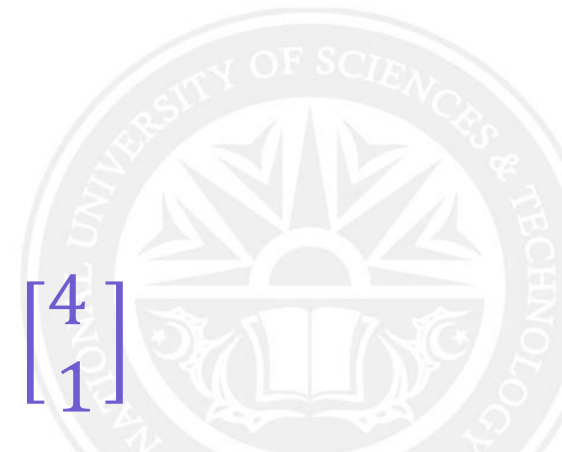
$\{0\}$



$\{1\}$

- $p^1$  is expressed in  $\{0\}$

$$p^0 = d_1^0 + p^1 = \begin{bmatrix} 3 \\ 0 \end{bmatrix} + \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 4 \\ 1 \end{bmatrix}$$



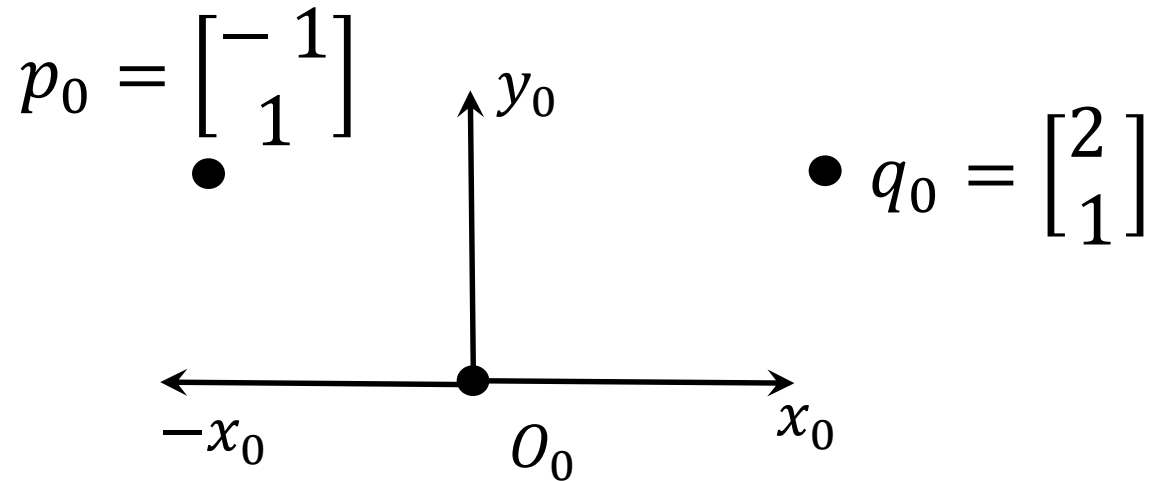
# Translation

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1. The translation vector  $d_j^i$  can be interpreted as the **location** of frame {j} expressed in frame {i}.
2. The translation vector  $d_j^i$  can be interpreted as a **coordinate transformation of a point** from frame {j} to frame {i}.
3. Translation vector  $d$  can be interpreted as an operator that takes a point and **moves it to a new point in the same frame**.



# Translation: Example 3



- $q_0$  expressed in  $\{0\}$ , given is  $d = \begin{bmatrix} 3 \\ 0 \end{bmatrix}$

$$q_0 = d + p_0 = \begin{bmatrix} 3 \\ 0 \end{bmatrix} + \begin{bmatrix} -1 \\ 1 \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$$



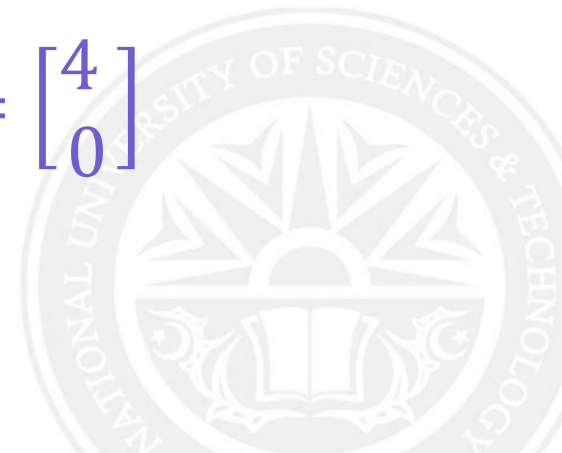
# Translation: Example 3

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- $p_0 = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$  and  $p_1 = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$

- The location of  $p_1$  is expressed in  $p_0$

$$d_1^0 = p_1 - p_0 = \begin{bmatrix} 2 \\ 1 \end{bmatrix} - \begin{bmatrix} -1 \\ 1 \end{bmatrix} = \begin{bmatrix} 4 \\ 0 \end{bmatrix}$$



# Rotation

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1. The rotation matrix  $R_j^i$  can be interpreted as the **orientation** of frame {j} expressed in frame {i}.

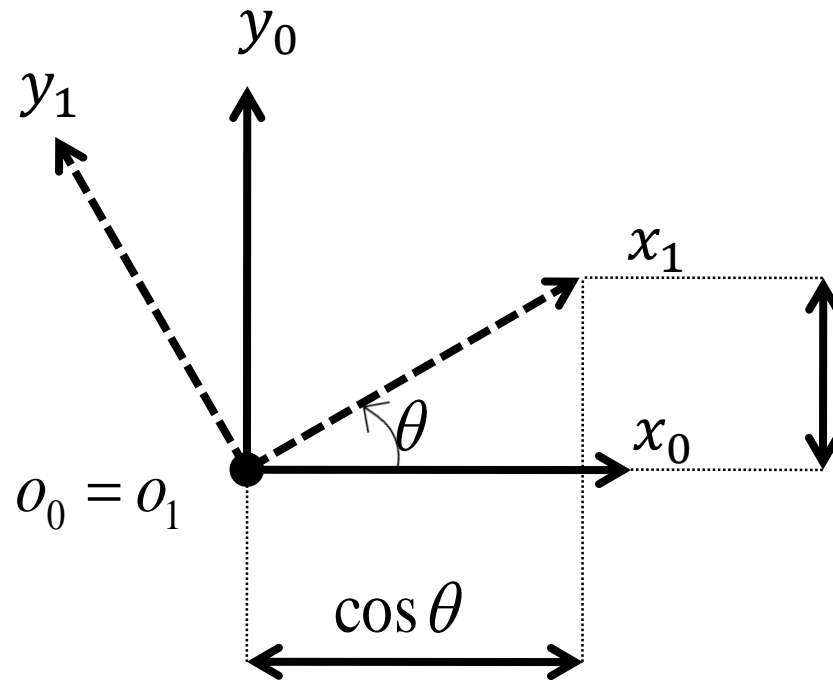


# Rotation

- Suppose that frame  $\{1\}$  is rotated relative to frame  $\{0\}$

$$R_1^0 = [x_1^0 | y_1^0]$$

- In 2D case



# Rotation-I

- Project frame {1} onto the frame {0} (Dot product)

$$R_1^0 = \begin{bmatrix} x_1 \\ y_1 \end{bmatrix} \cdot \begin{bmatrix} x_0 \\ y_0 \end{bmatrix}$$

$$\rightarrow \begin{bmatrix} x_1 \cdot x_0 \\ x_1 \cdot y_0 \end{bmatrix}, \begin{bmatrix} y_1 \cdot x_0 \\ y_1 \cdot y_0 \end{bmatrix}$$

Projection of {1} over {0}



|       |       |
|-------|-------|
| $x_1$ | $y_1$ |
|-------|-------|

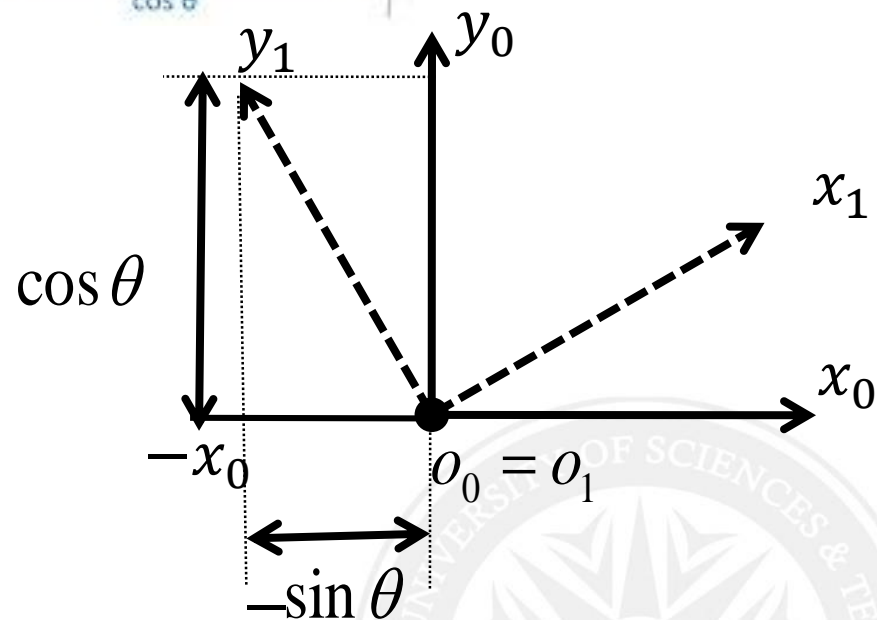
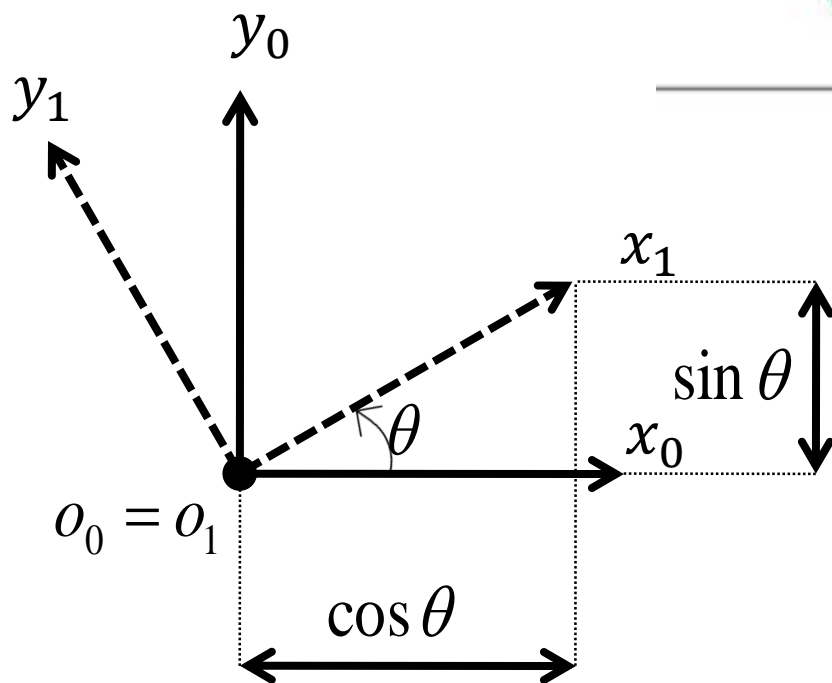
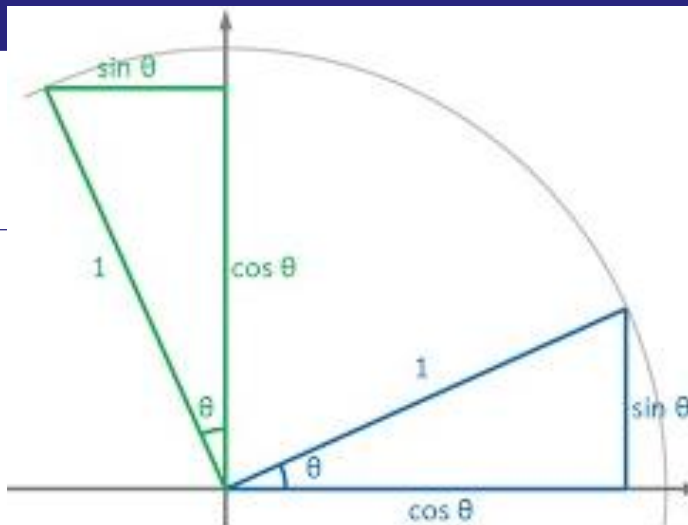
$$R_1^0 = \begin{bmatrix} x_1 \cdot x_0 & y_1 \cdot x_0 \\ x_1 \cdot y_0 & y_1 \cdot y_0 \end{bmatrix}$$

|       |
|-------|
| $x_0$ |
| $y_0$ |



# Rotation-II

- Alternate approach



$$x_1^0 = \begin{bmatrix} \cos \theta \\ \sin \theta \end{bmatrix},$$

$$y_1^0 = \begin{bmatrix} -\sin \theta \\ \cos \theta \end{bmatrix}$$

# Rotation-III

- The orientation of frame {1} is expressed in frame {0}

$$x_1^0 = \begin{bmatrix} \cos \theta \\ \sin \theta \end{bmatrix}, \quad y_1^0 = \begin{bmatrix} -\sin \theta \\ \cos \theta \end{bmatrix}$$

$$\Downarrow R_1^0 = [x_1^0 | y_1^0] \Downarrow$$

$$R_1^0 = \begin{bmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{bmatrix}$$



# Rotation

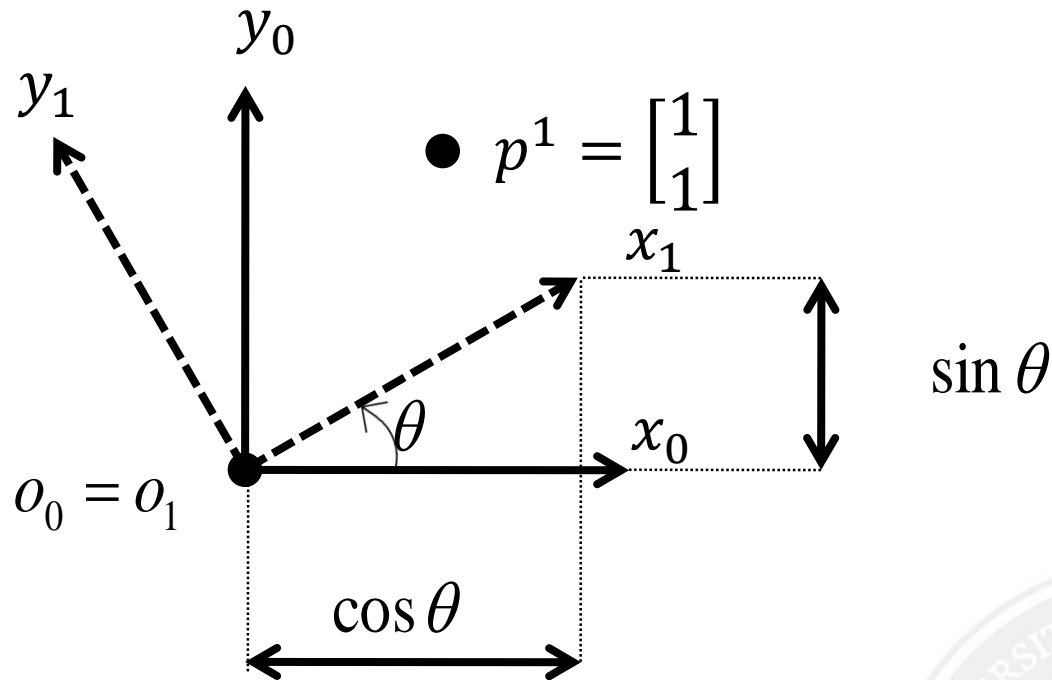
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1. The rotation matrix  $R_j^i$  can be interpreted as the **orientation** of frame {j} expressed in frame {i}.
2. The rotation matrix  $R_j^i$  can be interpreted as a **coordinate transformation of a point** from frame {j} to frame {i}.



# Rotation: Example

- $p^1$  expressed in  $\{0\}$



$$p^0 = R_1^0 p^1 = \begin{bmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

# Rotation

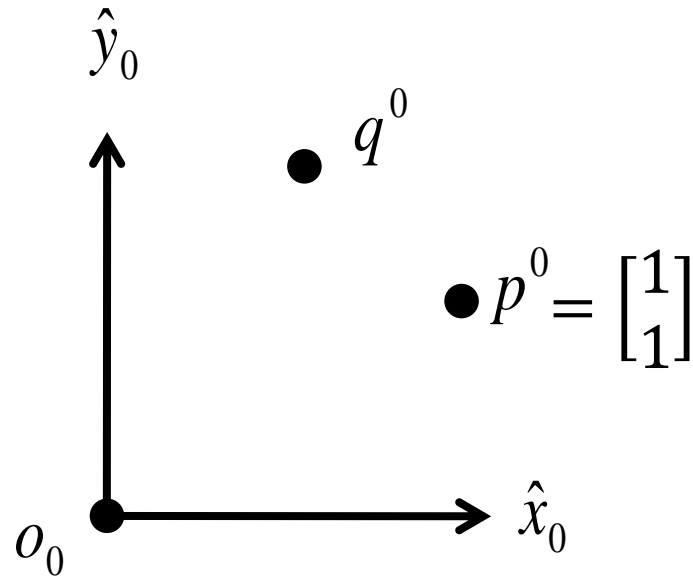
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1. The rotation matrix  $R_j^i$  can be interpreted as the **orientation** of frame {j} expressed in frame {i}.
2. The rotation matrix  $R_j^i$  can be interpreted as a **coordinate transformation of a point** from frame {j} to frame {i}.
3. Rotation matrix  $R$  can be interpreted as an operator that takes a point and **moves it to a new point in the same frame**.



# Rotation: Example

- $q^0$  expressed in frame  $\{0\}$



$$q^0 = R p^0 = \begin{bmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$



# Properties of Rotation Matrix

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## Properties of rotation matrix

- $R_j^i = \left(R_i^j\right)^T$
- $\left(R_j^i\right)^T = \left(R_j^i\right)^{-1}$
- $R(\theta_1)R(\theta_2) = R(\theta_1 + \theta_2)$
- $R(-\theta) = R(\theta)^T$
- Columns/rows of R are mutually orthogonal
- Each column/row of R is a unit vector
- Determinant of R is equal to 1 ( $\det(R) = 1$ )



# Rotation in 3D

Projection of {1} over {0} →

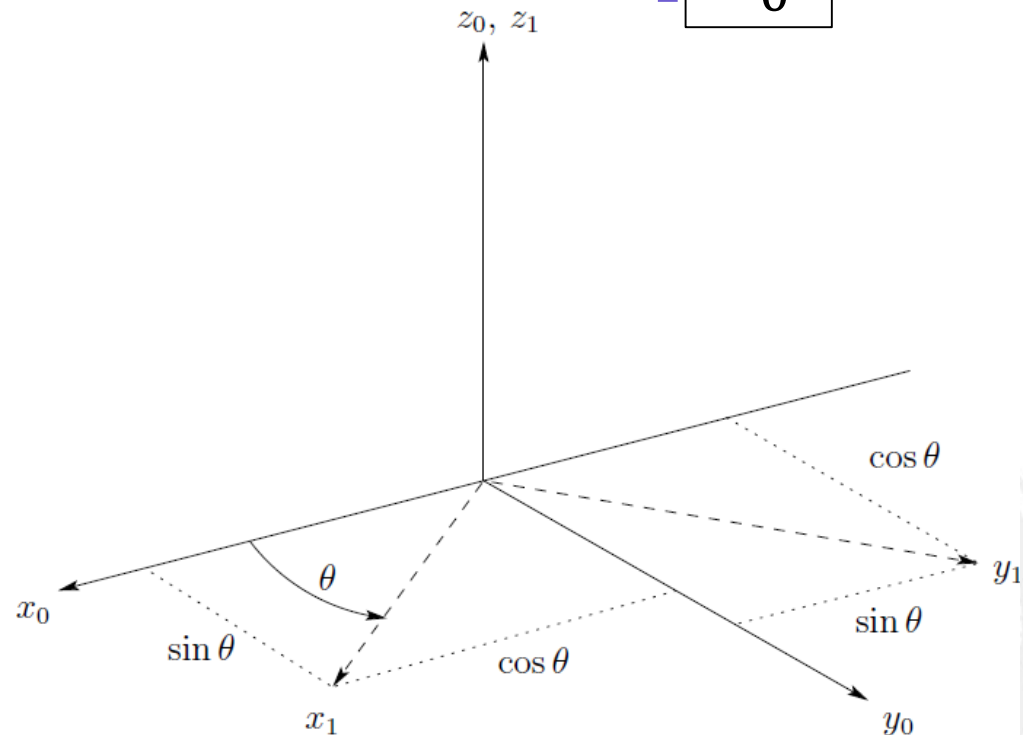
$$\begin{bmatrix} x_1 & y_1 & z_1 \end{bmatrix}$$

$$R_1^0 = \begin{bmatrix} x_1 \cdot x_0 & y_1 \cdot x_0 & z_1 \cdot x_0 \\ x_1 \cdot y_0 & y_1 \cdot y_0 & z_1 \cdot y_0 \\ x_1 \cdot z_0 & y_1 \cdot z_0 & z_1 \cdot z_0 \end{bmatrix} \begin{bmatrix} x_0 \\ y_0 \\ z_0 \end{bmatrix}$$

## Example 2.1: $(R_{z,\theta})$

Rotation of {1} about z-axis

$$R_1^0 = \begin{bmatrix} \cos \theta & -\sin \theta & 0 \\ \sin \theta & \cos \theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$



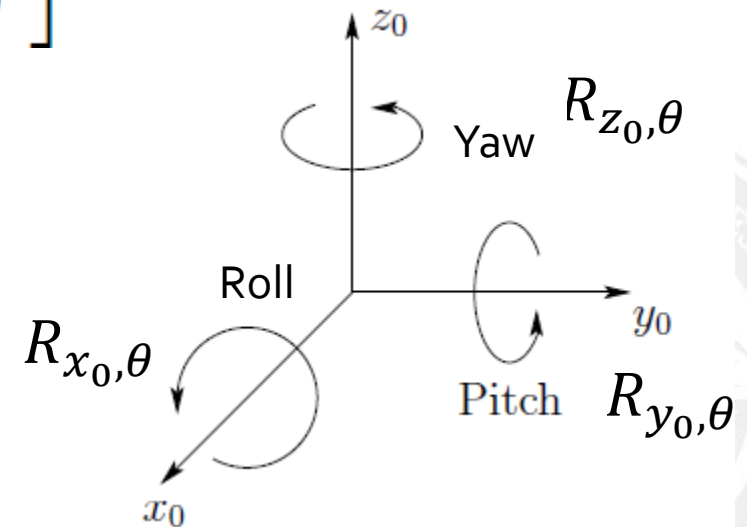


# Rotation in 3D

Rotation of {1} about x and y-axis are as;

$$R_{x,\theta} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos \theta & -\sin \theta \\ 0 & \sin \theta & \cos \theta \end{bmatrix}$$

$$R_{y,\theta} = \begin{bmatrix} \cos \theta & 0 & \sin \theta \\ 0 & 1 & 0 \\ -\sin \theta & 0 & \cos \theta \end{bmatrix}$$



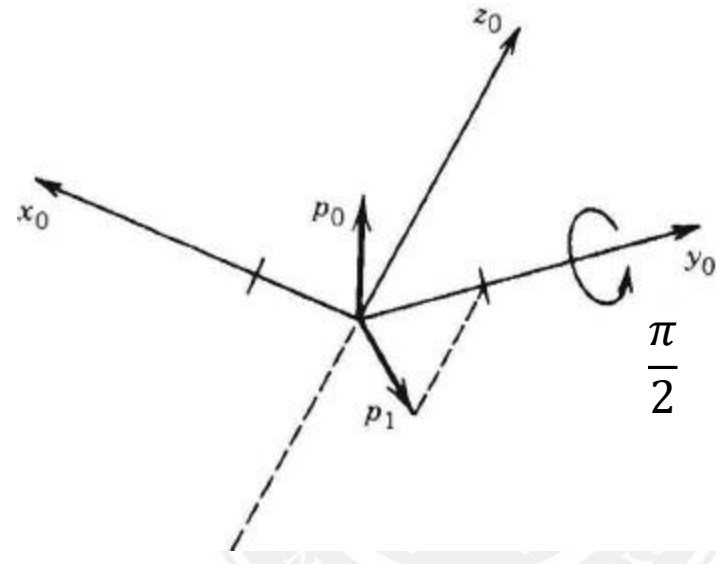
# Rotation in 3D

**Example:** Vector  $p_0 = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$  is rotated about  $y_0$ -axis by  $\frac{\pi}{2}$  as shown in figure. What will be the resulting vector  $p_1$ ?

• **Solution:**

$$p_1 = R_{y_0, \frac{\pi}{2}} p_0$$

$$R_{y, \theta} = \begin{bmatrix} \cos \theta & 0 & \sin \theta \\ 0 & 1 & 0 \\ -\sin \theta & 0 & \cos \theta \end{bmatrix}$$



# Properties of Rotation Matrix

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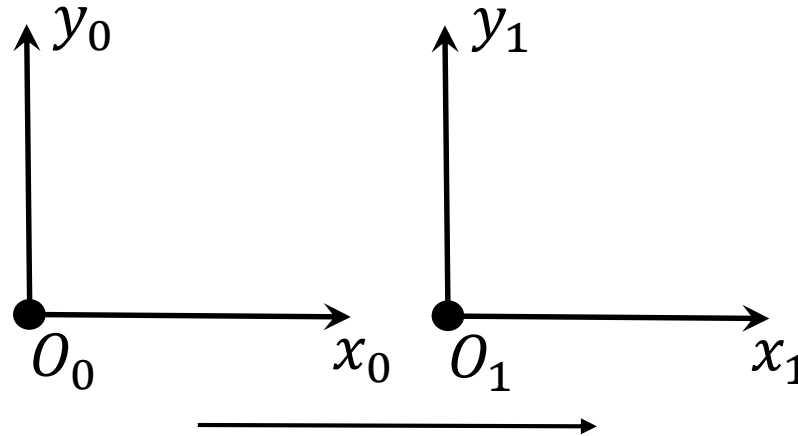
## Properties of rotation matrix

- $R_{z,0} = R_{y,0} = R_{x,0} = I$
- $R_{z,\theta} R_{z,\phi} = R_{z,\theta+\phi}$
- $R_{z,\theta}^{-1} = R_{z,-\theta}$

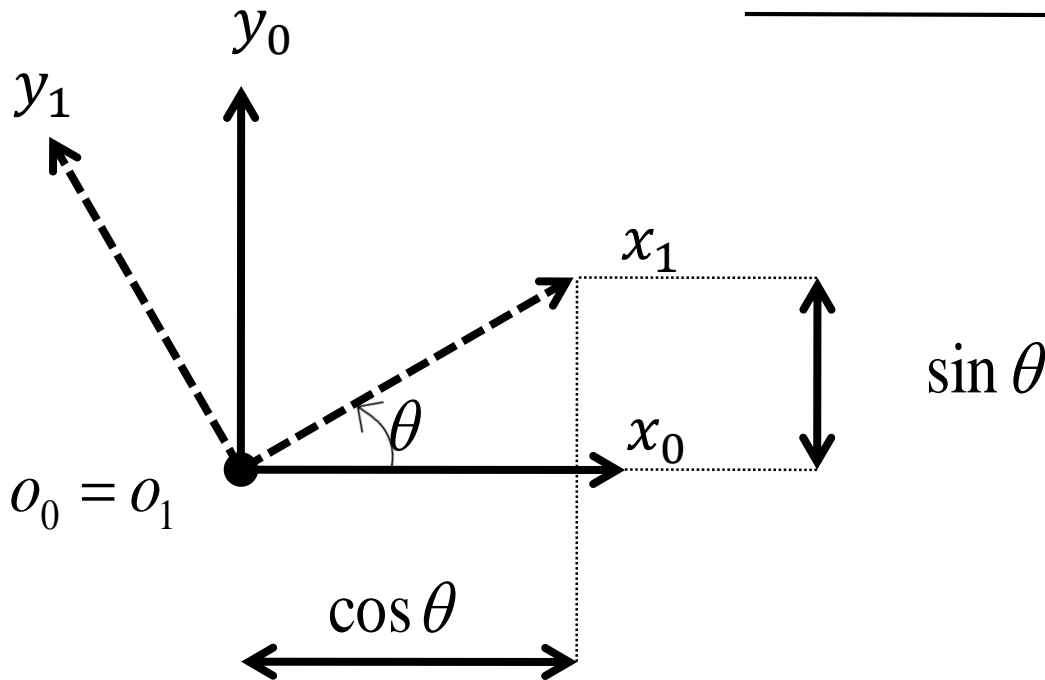


# Summary

- Translation



- Rotation



# Pose

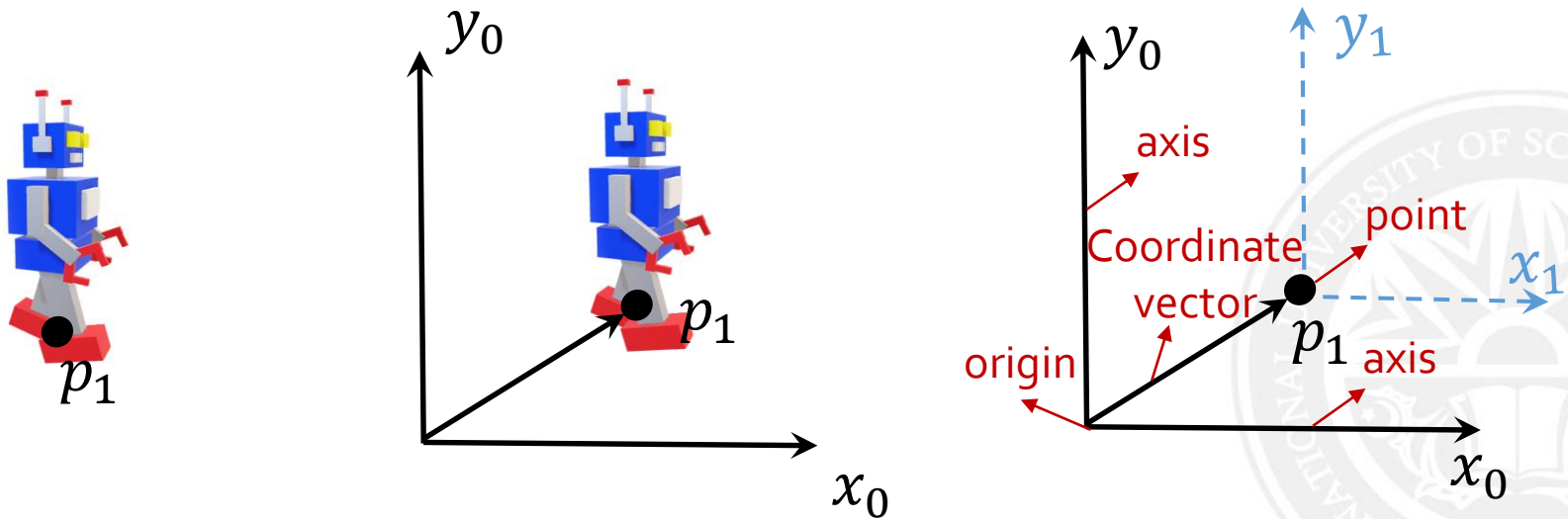
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- How to define the pose of an object in space?
- **Pose**: combination of **position and orientation**
- A point in space?
- Coordinate frame/ Cartesian coordinate system?



# Pose

- **Convention:** Attach the coordinate frame to the object. It enables us to describe the pose of the object with respect to reference/universal coordinate frame.
- **Assumption:** Object has rigid body
- What should be the required dimension to define the pose of an object?



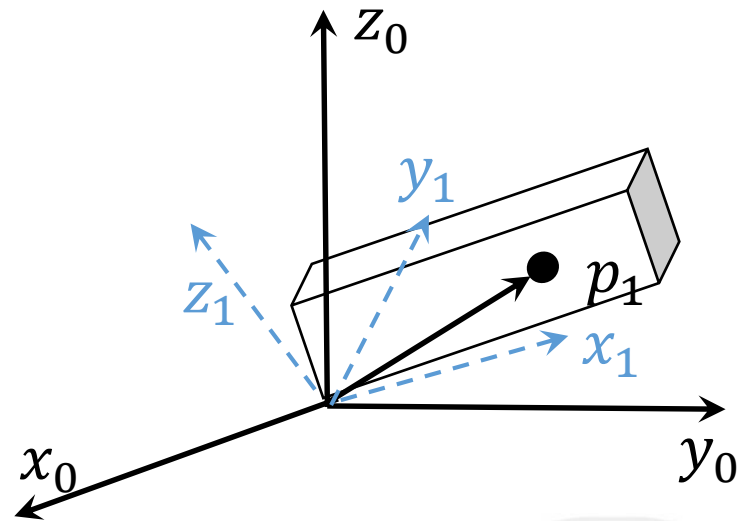
# Pose: Position

- **Position:** we can locate any point in space with 3D position vector

$$p_1 = \begin{bmatrix} u \\ v \\ w \end{bmatrix}$$

or

$$p_1 = ux_1 + vy_1 + wz_1$$



# Pose: Position

- Project the point  $p_1$  on reference frame  $\{0\}$

$$p_0 = (ux_1 + vy_1 + wz_1) \cdot \begin{bmatrix} x_0 \\ y_0 \\ z_0 \end{bmatrix} = \begin{bmatrix} (ux_1 + vy_1 + wz_1) \cdot x_0 \\ (ux_1 + vy_1 + wz_1) \cdot y_0 \\ (ux_1 + vy_1 + wz_1) \cdot z_0 \end{bmatrix}$$

$$p_0 = \begin{bmatrix} ux_1 \cdot x_0 + vy_1 \cdot x_0 + wz_1 \cdot x_0 \\ ux_1 \cdot y_0 + vy_1 \cdot y_0 + wz_1 \cdot y_0 \\ ux_1 \cdot z_0 + vy_1 \cdot z_0 + wz_1 \cdot z_0 \end{bmatrix}$$

$$p_0 = \underbrace{\begin{bmatrix} x_1 \cdot x_0 + y_1 \cdot x_0 + z_1 \cdot x_0 \\ x_1 \cdot y_0 + y_1 \cdot y_0 + z_1 \cdot y_0 \\ x_1 \cdot z_0 + y_1 \cdot z_0 + z_1 \cdot z_0 \end{bmatrix}}_{R_1^0} \underbrace{\begin{bmatrix} u \\ v \\ w \end{bmatrix}}_{p_1}$$

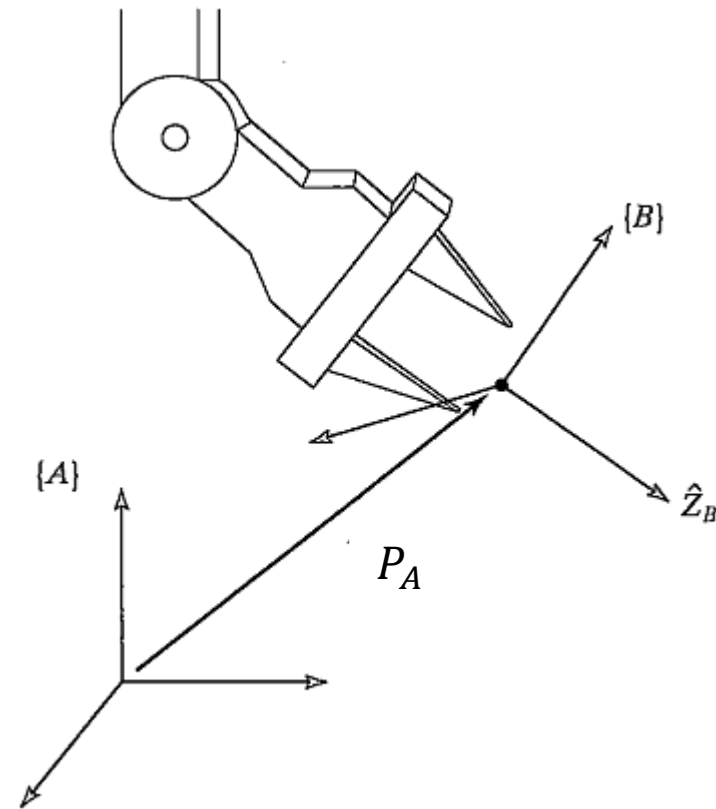
$$p_0 = R_1^0 p_1$$



# Pose: Rotation

- To describe the orientation of a body, we attach a coordinate system to the body and then give a description of this coordinate system relative to the reference system.

- $$R_B^A = \begin{bmatrix} x_B \cdot x_A & y_B \cdot x_A & z_B \cdot x_A \\ x_B \cdot y_A & y_B \cdot y_A & z_B \cdot y_A \\ x_B \cdot z_A & y_B \cdot z_A & z_B \cdot z_A \end{bmatrix}$$



# Summary: Pose

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- Position of point are described with vectors
- Orientation of bodies are described with an attached coordinate system using Rotation matrix

