MATH-243: Vector Calculus (3+0): BEE-2k20-A Fall 2021

Quiz - 5: Partial Differential Equations	
CLO-3: Develop analytical solutions of partial differential equations.	
Maximum Marks: 10	Instructor: Dr. Naila Amir
Date: $24 - 12 - 2021$	Duration: 10 Minutes
Name: Master Solution	CMS ID:

Question:

Classify the following partial differential equations in terms of:

(a) **Order & Degree.**

(b) **linearity** (linear/quasi-linear/non-linear).

(c) **homogeneity** (homogeneous/non-homogeneous)

(d) Furthermore, if it is a linear/ quasi-linear 2nd order partial differential equation, **classify it as parabolic, elliptic, or hyperbolic.**

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1. $u_{yy} + u_{xyz} + 2u_x = u^2 \cos(z).$

(a) Order = 3
    Degree = 1

(b) Quasi - linear
    [due to presence of u^2]

(c) Homogeneous
    [no term without u]

(d) Not applicable
    because order of PDE
    is 3.

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2. $(y^2 - 1)u_{xx} - 2xyu_{xy} - (x^2 - 1)u_{yy} = 1; x > 1.$

(a) Order = 2
    Degree = 1

(b) Linear PDE

(c) Non- homogeneous
    [R.H.S $\neq 0$; one term without
    u is there]

(d) $A = y^2 - 1$ & $B = -2xy$, $C = -(x^2 - 1)$

$B^2 - 4AC = 4x^2y^2 + 4(y^2 - 1)(x^2 - 1)$

Note that:

$B^2 - 4AC > 0$ if $x > 1$; $y > 1$
                                      or $y < 1$

$\Rightarrow$ Hyperbolic

$B^2 - 4AC < 0$ if $x > 1$; $0 < y < 1$
                                      or $-1 < y < 0$

$\Rightarrow$ Elliptic