Expected Value and Variance

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Expected Value and Variance

- Expected Value of Discrete Random Variables
- Variance and Standard Deviation of Discrete Random Variables
- Expected Value of continuous Random Variables
- Variance and Standard Deviation of Continuous Random Variables

Expected Value of Discrete Random Variables

The mean, or expected value, of a discrete random variable is

$$\mu = E(x) = \sum xp(x).$$

Mean=$\frac{\sum fx}{\sum f}$=$\frac{Sum\ of\ all\ observation}{total\ no\ of\ observation}$

Mean= $\sum \frac{f}{\sum f} . x$

Mean of discrete p.d=$\sum xp(x)$

x	f(x)	xf(x)
x_1	$f(x_1)$	$x_1 f(x_1)$
x_2	$f(x_2)$	$x_2 f(x_2)$
$\ldots$	…	…
x_n	$f(x_n)$	$x_n f(x_n)$
Total	1	$E(X) = \sum x_i f(x_i)$

Example: die

Mean=3.5

x	*p(x)*	*xp(x)*
1	1/6	1/6
2	1/6	2/6
3	1/6	3/6
4	1/6	4/6
5	1/6	5/6
6	1/6	6/6
Total	1	21/6=3.5

Variance and Standard Deviation

- The **variance** of a discrete random variable x is

$$\sigma^2 = E[(x-\mu)^2] = \sum (x-\mu)^2 p(x).$$

- The **standard deviation** of a **discrete random variable** *x* is

$$\sqrt{\sigma^2} = \sqrt{E[(x-\mu)^2]} = \sqrt{\sum (x-\mu)^2 p(x)}.$$

Example: die

Variance= 2.91

x	$p(x)$	$xp(x)$	$(x\text{-}\mu)$	$(x-\mu)^2p(x)$
1	1/6	1/6	-2.5	1.041
2	1/6	2/6	-1.5	0.3751
3	1/6	3/6	-0.5	.04167
4	1/6	4/6	0.5	.01467
5	1/6	5/6	1.5	0.3751
6	1/6	6/6	2.5	1.041
Total	1	21/6=3.5		2.9

Example: die

Variance= $\sigma^2 = E(X^2) - (E(X))^2$

Variance=$\frac{91}{6} - \left(\frac{21}{6}\right)^2 = \frac{105}{36}$=2.91

x	p(x)	xp(x)	x^2	x^2p(x)
1	1/6	1/6	1	1/6
2	1/6	2/6	4	4/6
3	1/6	3/6	9	9/6
4	1/6	4/6	16	16/6
5	1/6	5/6	25	25/6
6	1/6	6/6	36	36/6
Total	1	21/6=3.5		91/6

Question

The repair cost of electrical components is \$50, \$200, and \$250 with respective probability values of 0.3, 0.2, and 0.5. Find the expected value and variance of following distribution?

Solution

Expected Value: $180

Variance: $7600

Standard Deviation= $87.17

x	*f(x)*	*xf(x)*	*(x-μ)*	$(x-\mu)^2$f(x)
50	0.3	15	-130	5070
200	0.2	40	20	80
250	0.5	125	70	2450
Total	1	180		7600

Question 2

A multiple-choice quiz has 5 question, each with 4 possible answers of which only one is correct.

- Let X denotes the number of correct answers, what is the expected number of correct answers.
- If Y denotes the marks and each question have 5 marks, then find the expected marks of a student.

Solution

Expected number of correct answers= 1.25

Expected Marks= 6.25

X	p(x)	xp(x)	Y	yp(y)
0	0.237305	0	0	0
1	0.395508	0.395508	5	1.977539
2	0.263672	0.527344	10	2.636719
3	0.087891	0.263672	15	1.318359
4	0.014648	0.058594	20	0.292969
5	0.000977	0.004883	25	0.024414
	1	1.25	75	6.25

Expected Values of continuous Random Variables

If the X is continuous with probability density function is f(x) thus

$$\mu = E(x) = \int_{-\infty}^{\infty} x f(x) dx$$

Provided the integral converges

Variance and Standard Deviation

$\sigma^2 = E(X^2) - (E(X))^2$ = variance

Where

$$\mu = E(x) = \int_{-\infty}^{\infty} x f(x)\, dx$$

$E(X^2) = \int_{-\infty}^{\infty} x^2 f(x)\, dx$

S.D $= \sigma = \sqrt{E(X^2) - (E(X))^2}$

Question 3

The arrival time of a student in a class is uniformly distributed with first 10 minutes. Find the mean and standard deviation.

Solution

The p.d.f of X is

$$f(x) = \begin{cases} \dfrac{1}{10} & \mathit{for} \quad 0 \le x \le 10 \\ 0 & \mathit{for} \quad \text{all other values} \end{cases}$$

$$E(x) = \int_{-\infty}^{0} x.0dx + \int_{0}^{10} x.\frac{1}{10}dx + \int_{10}^{\infty} x.0d(x)$$

$$E(x) = 0 + 5 + 0$$

$$E(x) = 5$$

Mean= average arrival time of a student in a class is 5 minutes

Solution

Standard deviation= $\sqrt{E(X^2)-(E(X))^2}$

Here we know that, E(X)=5

$$E(x^2)$$

$$=\int_{-\infty}^{0} x^2.0dx+\int_{0}^{10} x^2.\frac{1}{10}dx+\int_{10}^{\infty} x^2.0dx$$

$$E(x^2)=0+\frac{1000}{30}+0= \frac{100}{3}$$

Standard deviation= $\sqrt{\frac{100}{3}-5^2}= \frac{25}{3}$=8.33