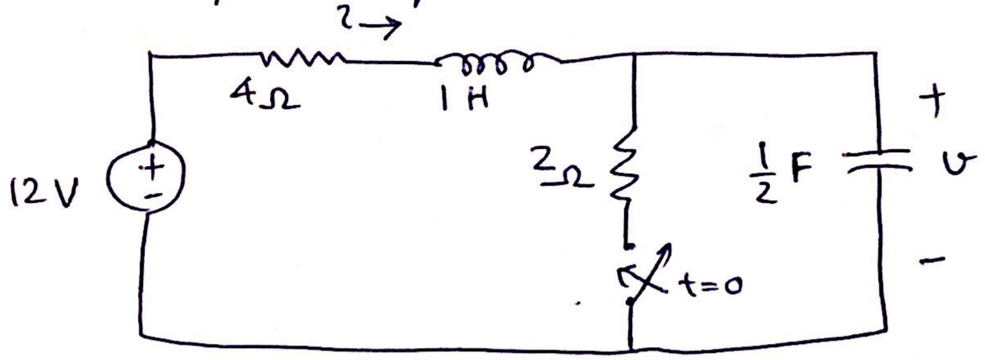
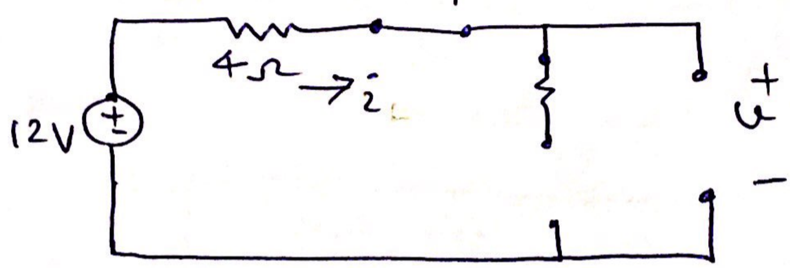
Example 8.9

Find the complete response 'v' and then 'i' for $t > 0$.



Solution: Let us first find the initial and the final values.

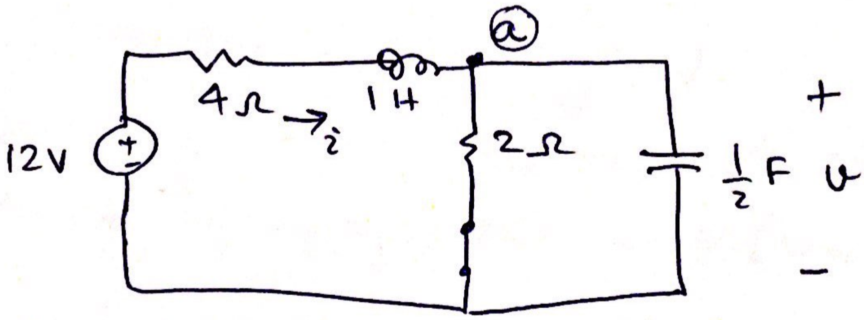
— When the switch is open and at $t < 0$



$i(0^-) = 0$

$v(0^-) = 12\,V$

— When the switch is closed and at $t > 0$



— Due to continuity $i(0^+) = i(0^-) = 0$

and $v(0^+) = v(0^-) = 12\ V$

contd

(neither)

— Contd (340)

For more initial conditions, we apply KCL at node labelled ⓐ.

$$i(0^+) = \frac{v(0^+)}{2} + i_c(0^+)$$

Putting $i(0^+) = 0$

$v(0^+) = 12$ we get

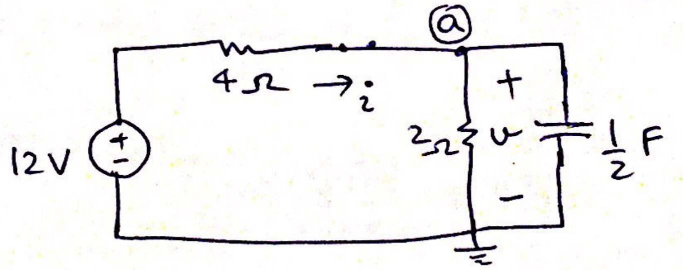
$$0 = \frac{12}{2} + i_c(0^+)$$

$$i_c(0^+) = -6 \text{ A}$$

Now $i_c(0^+) = C\dfrac{dv(0^+)}{dt}$

or $$\frac{dv(0^+)}{dt} = \frac{1}{C} i_c(0^+) = \frac{1}{1/2}(-6) = -12 \text{ V/s}$$

— The final values are obtained



Now $i(\infty) = \dfrac{12}{4+2} = 2 \text{ A}$

So $v(\infty) = 2 \times 2 = 4 \text{ V}$.

— Next for $t > 0$, the transient response form is obtained by applying KCL at ⓐ (turning off 12V source)

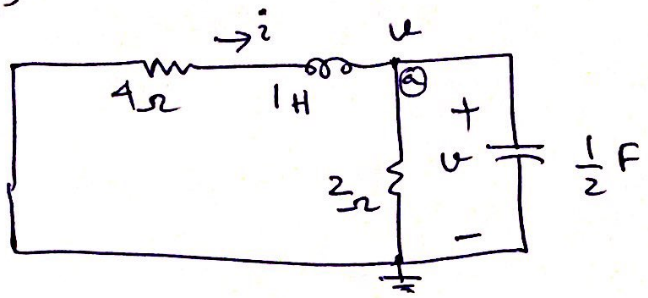
$$i = \frac{v}{2} + C\frac{dv}{dt} = \frac{v}{2} + \frac{1}{2}\frac{dv}{dt}$$

contd

(Optional)
(neither)

— contd (340)

Applying KVL to the left mesh,



(obtaining the form for transient response)

$$4i + 1 \times \frac{di}{dt} + v = 0$$

— because we are interested in v, we substitute i.

Now $i = \frac{v}{2} + \frac{1}{2}\frac{dv}{dt}$

So

$$4\frac{v}{2} + \frac{4}{2}\frac{dv}{dt} + \frac{1}{2}\left(\frac{dv}{dt} + \frac{d^2v}{dt^2}\right) + v = 0 \qquad \times 2$$

$$4v + 4\frac{dv}{dt} + \frac{dv}{dt} + \frac{d^2v}{dt^2} + 2v = 0$$

Arranging

$$\frac{d^2v}{dt^2} + \frac{5dv}{dt} + 6v = 0$$

— The characteristic equation is

$$s^2 + 5s + 6 = 0$$

— The roots are $s = -2$ and $s = -3$.

— Thus the natural response is

$$v_n(t) = Ae^{-2t} + Be^{-3t}$$

———— contd

(Optional)
(neither)

— contd (340)

Now $v_{ss}(t) = v(\infty) = 4$

So the complete response is:-

$$v(t) = v_{ss}(t) + v_t$$

$$v(t) = 4 + Ae^{-2t} + Be^{-3t}$$

— Using initial conditions

$$v(0) = 12$$

So $12 = 4 + A + B$

$$A + B = 8 \quad \text{——— (1)}$$

— And

$$\frac{dv}{dt} = -2Ae^{-2t} - 3Be^{-3t}$$

$\frac{dv}{dt}$ at $t=0$ is -12 V/s

So $-12 = -2A - 3B$

$$2A + 3B = 12 \quad \text{——— (2)}$$

— Solving (1) and (2) $A = 12$

$B = -4$

Hence

$$v(t) = 4 + 12e^{-2t} - 4e^{-3t} \text{ V}, \quad t > 0$$

— Now to obtain i, we know

$$i = \frac{v}{2} + \frac{1}{2}\frac{dv}{dt}$$

——— contd

(Optional)
(neither)

— Cont'd (341)

So $i = \dfrac{4 + 12e^{-2t} - 4e^{-3t}}{2} + \frac{1}{2}\left(-24e^{-2t} + 12e^{-3t}\right)$

$i = 2 + 6e^{-2t} - 2e^{-3t} - 12e^{-2t} + 6e^{-3t}$

$i = 2 - 6e^{-2t} + 4e^{-3t}$ A , $t > 0$

Note $i(0) = 0$