Q 1 - Part 2 : $(R^i_j)^T = (R^i_j)^{-1}$

Consider the rotation matrix (2D)

L $R^i_j = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$

LHS $(R^i_j)^T$

$$(R^i_j)^T = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}^T$$

$$= \underline{\begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}}$$

RHS $(R^i_j)^{-1}$

inverse of matrix $A \Rightarrow A^{-1} = \frac{\text{adj}(A)}{\det(A)}$

$\rightarrow \det(R^i_j) = \begin{vmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{vmatrix}$

$= \cos^2\theta + \sin^2\theta = 1$ ∴ Pythagorean identity

$\rightarrow \text{adj}(R^i_j) = \text{adj}\left(\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}\right)$

$= \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$

• $(R^i_j)^{-1} = \det(R^i_j)^{-1} \text{adj}(R^i_j)$

$= \underline{\begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}}$

$\rightarrow$ As LHS == RHS ; $(R^i_j)^T = (R^i_j)^{-1}$ is validated.

Q 1 - Part 3 : $R(\theta_1)R(\theta_2) = R(\theta_1+\theta_2)$

LHS $[R(\theta_1)R(\theta_2)]$

$$R(\theta_1) = \begin{bmatrix} \cos\theta_1 & -\sin\theta_1 \\ \sin\theta_1 & \cos\theta_1 \end{bmatrix} \qquad R(\theta_2) = \begin{bmatrix} \cos\theta_2 & -\sin\theta_2 \\ \sin\theta_2 & \cos\theta_2 \end{bmatrix}$$

$$\rightarrow R(\theta_1)R(\theta_2) = \begin{bmatrix} \cos\theta_1 & -\sin\theta_1 \\ \sin\theta_1 & \cos\theta_1 \end{bmatrix}\begin{bmatrix} \cos\theta_2 & -\sin\theta_2 \\ \sin\theta_2 & \cos\theta_2 \end{bmatrix}$$

$$= \underline{\begin{bmatrix} \cos\theta_1\cos\theta_2 - \sin\theta_1\sin\theta_2 & -\sin\theta_2\cos\theta_1 - \sin\theta_1\cos\theta_2 \\ \sin\theta_1\cos\theta_2 + \cos\theta_1\sin\theta_2 & -\sin\theta_1\sin\theta_2 + \cos\theta_1\cos\theta_2 \end{bmatrix}}$$

RHS $[R(\theta_1+\theta_2)]$

$$\rightarrow R(\theta_1+\theta_2) = \begin{bmatrix} \cos(\theta_1+\theta_2) & -\sin(\theta_1+\theta_2) \\ \sin(\theta_1+\theta_2) & \cos(\theta_1+\theta_2) \end{bmatrix}$$

$$= \underline{\begin{bmatrix} \cos\theta_1\cos\theta_2 - \sin\theta_1\sin\theta_2 & -\cos\theta_1\sin\theta_2 - \sin\theta_1\cos\theta_2 \\ \sin\theta_1\cos\theta_2 + \cos\theta_1\sin\theta_2 & -\sin\theta_1\sin\theta_2 + \cos\theta_1\cos\theta_2 \end{bmatrix}}$$

$\therefore \cos(a+b) = \cos a\cos b - \sin a \sin b$

$\therefore \sin(a+b) = \cos a \sin b + \cos b \sin a$

$\rightarrow$ As LHS == RHS $\therefore$ $R(\theta_1)R(\theta_2) = R(\theta_1+\theta_2)$ is validated.