MAGNETIC TORQUE AND MAGNETIZATION

Magnetic Torque

- The concept of a current loop experiencing a torque in a magnetic field is of importance in understanding the behaviour of orbiting charged particles, D.C motors, and generators

- If the loop is placed parallel to a magnetic field, it experiences a force that tends to rotate it

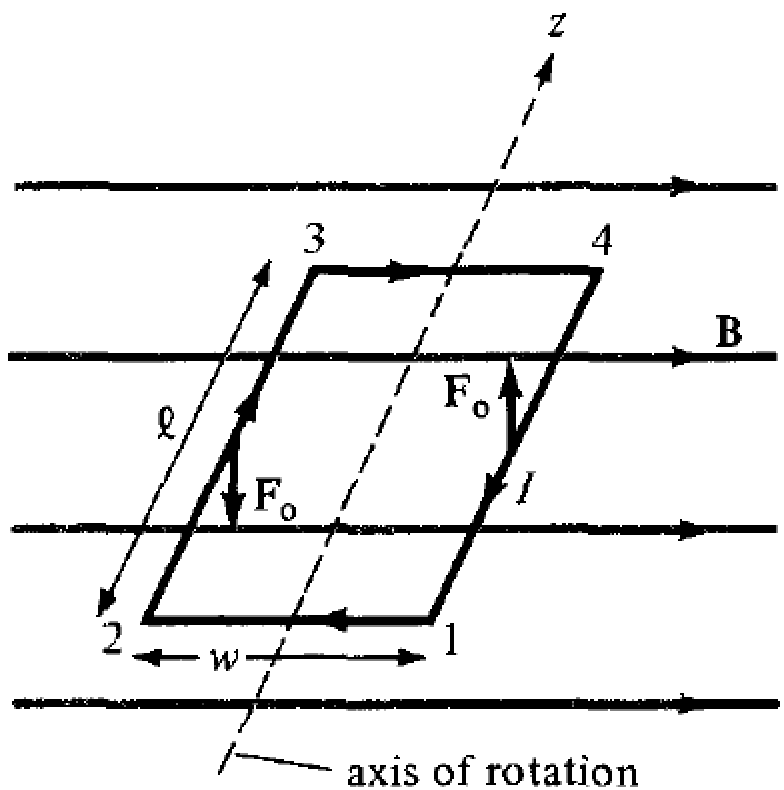
- The torque **T** (or mechanical moment of force) on the loop is the vector product of the moment arm **r** and the force **F**

$$\mathbf{T} = \mathbf{r} \times \mathbf{F}$$

- The units for torque are Newton-meters (N . m)

Magnetic Torque on Rectangular Loop

- We apply this torque equation to a rectangular loop of length "l" and width "w" placed in a uniform magnetic field **B** as shown in figure below



- From this figure, we notice that "d**l**" is parallel to **B** along sides 12 and 34 of the loop, therefore no force is exerted on these sides

Magnetic Torque on Rectangular Loop

- The net force on the rectangular loop is:

$$\mathbf{F} = I \int_2^3 d\mathbf{l} \times \mathbf{B} + I \int_4^1 d\mathbf{l} \times \mathbf{B}$$
$$= I \int_0^\ell dz\, \mathbf{a}_z \times \mathbf{B} + I \int_\ell^0 dz\, \mathbf{a}_z \times \mathbf{B}$$

- Which results in: $\mathbf{F} = \mathbf{F}_o - \mathbf{F}_o = 0$

- where $|\mathbf{F}_o| = IBl$ because **B** is uniform

- Thus, no force is exerted on the loop as a whole!!! So how does it rotate???

Magnetic Torque on Rectangular Loop

- **F**$_o$ and -**F**$_o$ act at different points on the loop, thereby creating a couple

- Hence the torque on the loop is:

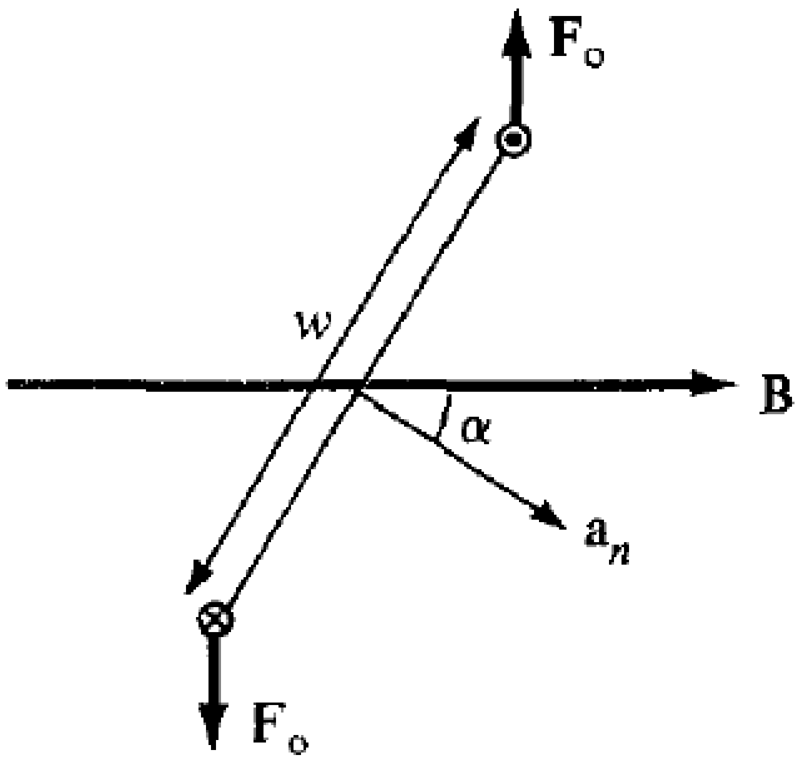
$$|\mathbf{T}| = |\mathbf{F_o}|\frac{w}{2} + |\mathbf{F_o}|\frac{w}{2}$$

- This toque results in a circular motion of the current loop around the axis

Magnetic Torque on Rectangular Loop

- If the normal to the plane of the loop makes an angle "∝" with **B**, as shown in the cross-sectional view in figure below, the torque on the loop is:

$$|\mathbf{T}| = |\mathbf{F}_o|\, w \sin \alpha$$



Magnetic Torque on Rectangular Loop

- Since $|\mathbf{F}_o| = IBl$, we get:

$$T = BI\ell w \sin \alpha$$

- But $lw = S$, the area of the loop, hence:

$$T = BIS \sin \alpha$$

- We define the quantity **m** as the magnetic dipole moment of the loop, where:

$$\mathbf{m} = IS\mathbf{a}_n$$

- $\mathbf{a}_n$ is a unit normal vector to the plane of the loop

- Therefore, the magnetic dipole moment "**m**" is the product of current and area of the loop; its direction is normal to the loop.

Magnetic Torque on Rectangular Loop

- Torque in terms of "**m**" for uniform B can be written as:

$$\mathbf{T} = \mathbf{m} \times \mathbf{B}$$

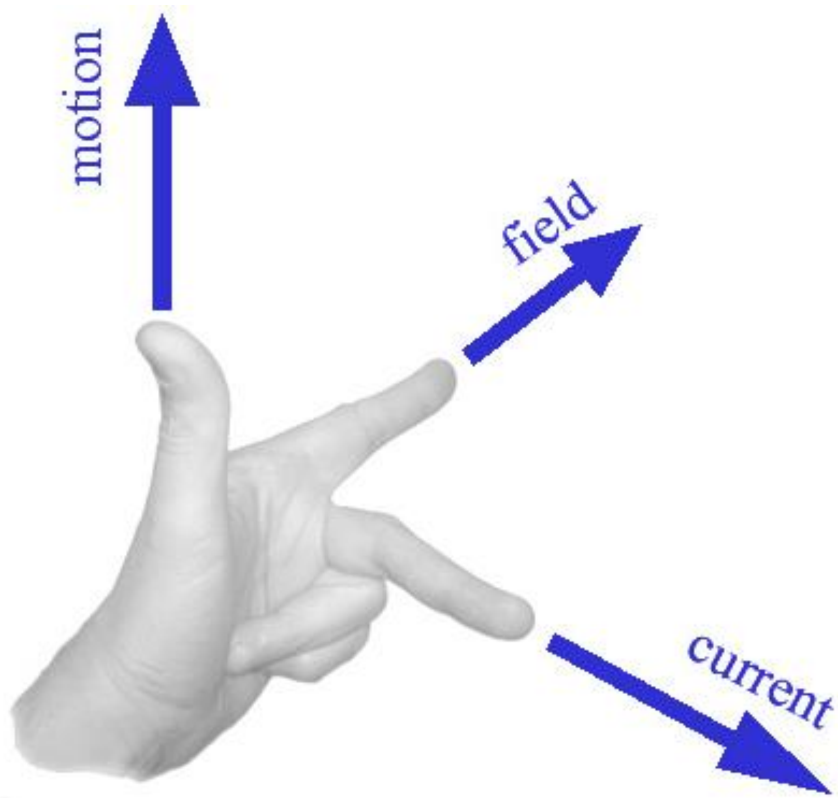
- It should be noted that the torque is in the direction of the axis of rotation (cross product)

- The torque is directed such as to reduce "α" so that **m** and **B** are in the same direction

- In an equilibrium position (when **m** and **B** are in the same direction), the loop is perpendicular to the magnetic field and the torque will be zero

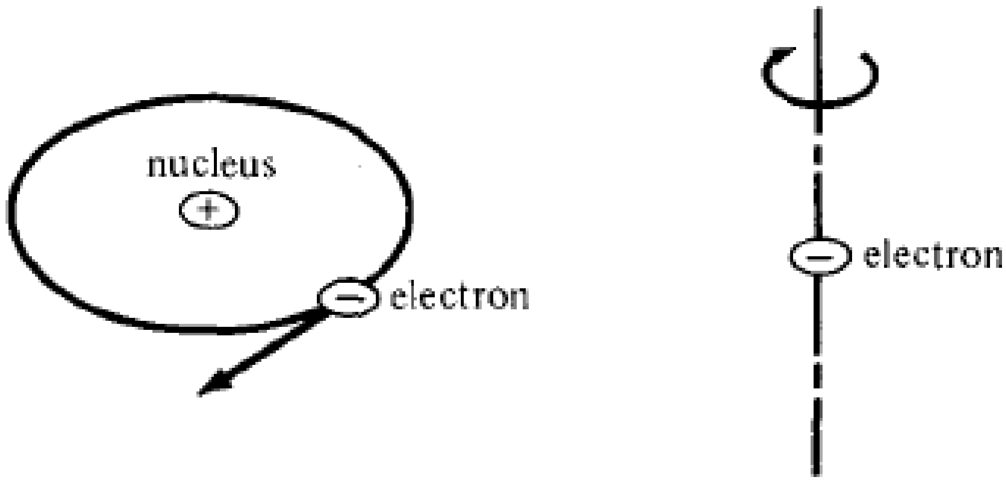
Magnetic Torque on Rectangular Loop

- The direction of motion is determined by the left-hand rule
- This rule is also known as the Motor Rule



Magnetization

- An atom may be regarded as consisting of electrons orbiting about a central positive nucleus, as well as rotate (or spin) about their own axes
- Thus an internal magnetic field is produced by electrons orbiting around the nucleus as in figure (a) or electrons spinning as in figure (b)



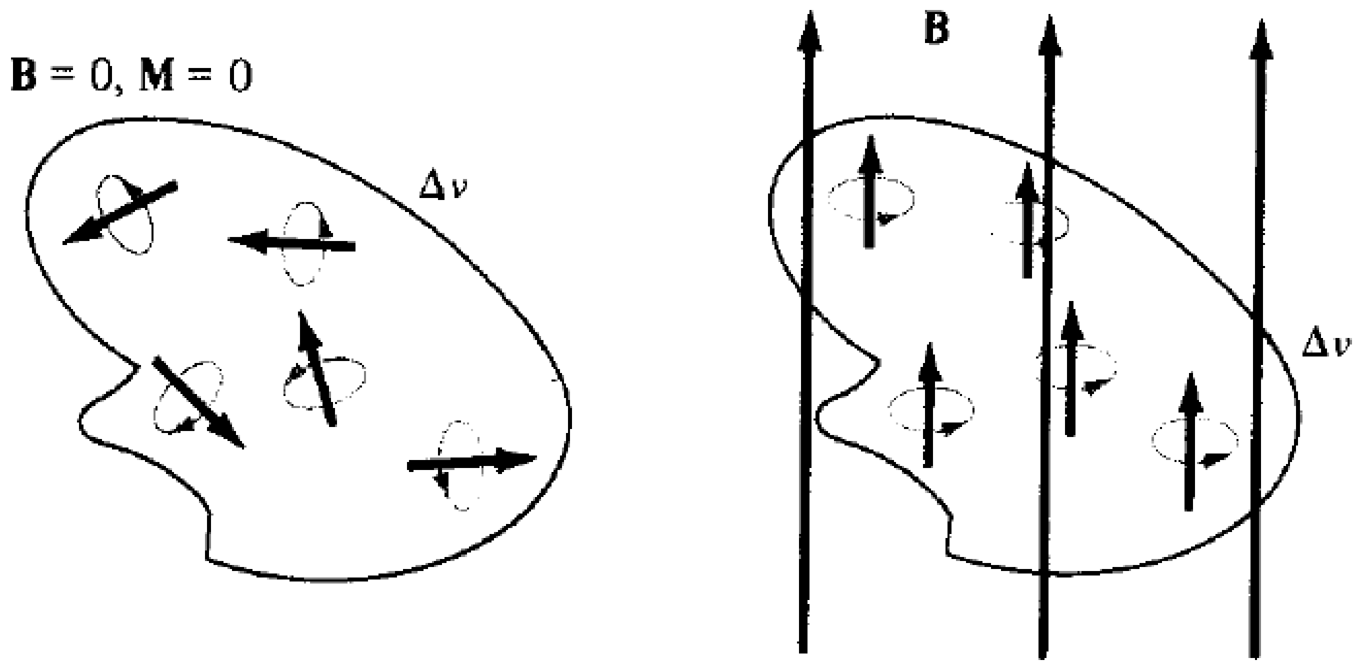
(a) (b)

- Both of these electronic motions produce internal magnetic fields **B**, that are similar to the magnetic field produced by a current loop.

Magnetization

- Without an external **B** field applied to the material, the sum of m's is zero due to random orientation as in figure (a)

- When an external **B** field is applied, the magnetic moments of the electrons more or less align themselves with **B** so that the net magnetic moment is not zero, as illustrated in figure (b)



(a) (b)

Magnetization

- The magnetization **M** (in amperes/meter) is the magnetic dipole moment per unit volume

- If there are *N* atoms in a given volume Δv and the k_{th} atom has a magnetic moment $\mathbf{m}_k$, then:

$$\mathbf{M} = \lim_{\Delta v \to 0} \frac{\sum_{k=1}^{N} \mathbf{m}_k}{\Delta v}$$

- A medium for which **M** is not zero everywhere is said to be magnetized

- For a differential volume *dv'* and differential surface *dS'*, the potential of a magnetic body is given as:

$$\mathbf{A} = \frac{\mu_o}{4\pi} \int_{v'} \frac{\mathbf{J}_b \, dv'}{R} + \frac{\mu_o}{4\pi} \oint_{S'} \frac{\mathbf{K}_b \, dS'}{R}$$

Magnetization

- Where $\mathbf{J}_b = \nabla \times \mathbf{M}$ and $\mathbf{K}_b = \mathbf{M} \times \mathbf{a}_n$
- $\mathbf{J}_b$ is the *bound volume current density* or *magnetization volume current density* (in amperes per meter square)
- $\mathbf{K}_b$ is the *bound surface current density* (in amperes per meter), and $\mathbf{a}_n$ is a unit vector normal to the surface
- Therefore, the potential of a magnetic body is due to a volume current density $\mathbf{J}_b$ throughout the body and a surface current $\mathbf{K}_b$ on the surface of the body
- The vector **M** is analogous to the polarization **P** in dielectrics and is sometimes called the *magnetic polarization density* of the medium

Magnetization

- In free space, **M** = 0 and we have (Maxwell's Third Equation):

$$\nabla \times \mathbf{H} = \mathbf{J}_f \qquad \text{or} \qquad \nabla \times \left(\frac{\mathbf{B}}{\mu_o}\right) = \mathbf{J}_f$$

- where $\mathbf{J}_f$ is the free current volume density

- In a material medium **M** ≠ 0, and as a result, **B** changes so that:

$$\nabla \times \left(\frac{\mathbf{B}}{\mu_o}\right) = \mathbf{J}_f + \mathbf{J}_b = \mathbf{J}$$

$$= \nabla \times \mathbf{H} + \nabla \times \mathbf{M}$$

OR

$$\mathbf{B} = \mu_o(\mathbf{H} + \mathbf{M})$$

Magnetization

- For linear materials, **M** (in A/m) depends linearly on **H** such that:

$$\mathbf{M} = \chi_m \mathbf{H}$$

- Where χ_m is a dimensionless quantity (ratio of *M* to *H)* called *magnetic susceptibility* of the medium

- It is more or less a measure of how susceptible (or sensitive) the material is to a magnetic field

- From above equations, we have:

$$\mathbf{B} = \mu_o(1 + \chi_m)\mathbf{H} = \mu\mathbf{H}$$

Magnetization

- Or:

$$\mathbf{B} = \mu_o \mu_r \mathbf{H}$$

- Where:

$$\mu_r = 1 + \chi_m = \frac{\mu}{\mu_o}$$

- The quantity $\mu = \mu_o \mu_r$ is called the *permeability* of the material and is measured in henrys/meter

- The dimensionless quantity μ_r is the ratio of the permeability of a given material to that of free space and is known as the *relative permeability* of the material.

Problem-1

- A rectangular loop carrying current I_2 is placed parallel to an infinitely long filamentary wire carrying current I_1 as shown in figure below. Determine the force experienced by the loop.