Ampere' Law-I

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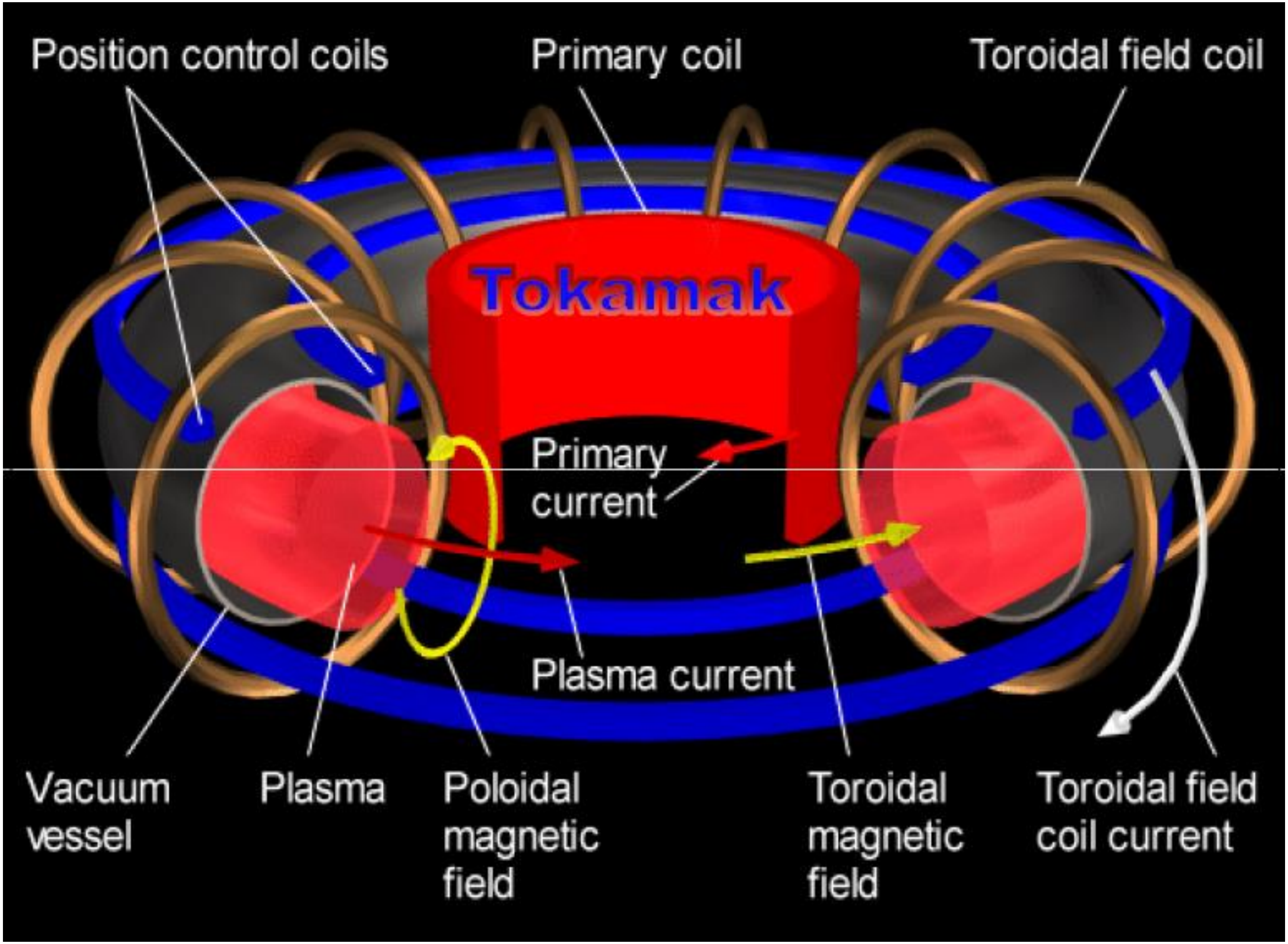
Position control coils
Primary coil
Toroidal field coil
Tokamak
Primary current
Plasma current
Vacuum vessel
Plasma
Poloidal magnetic field
Toroidal magnetic field
Toroidal field coil current

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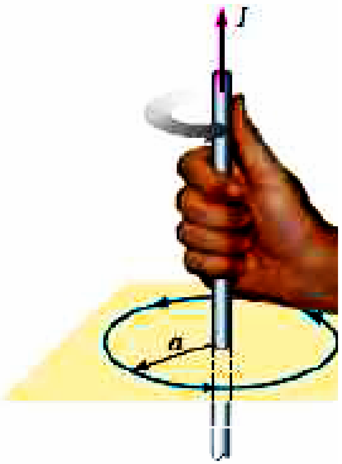
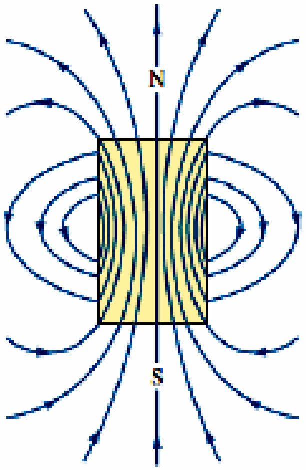
Sources of Magnetic Field

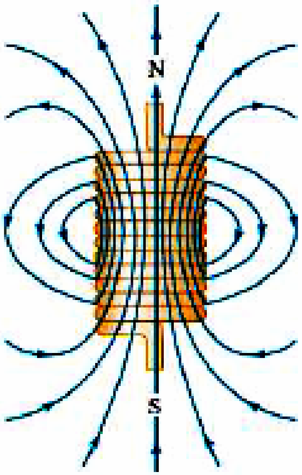
(1) Permanent Magnets (2) Electric Current

Oersted's (1819) demonstrated that a current-carrying conductor produces a magnetic field and lines of B form circles around the wire. If you grasp wire in your right hand with thumb along the direction of current, magnetic field lines will be directed along your fingers.

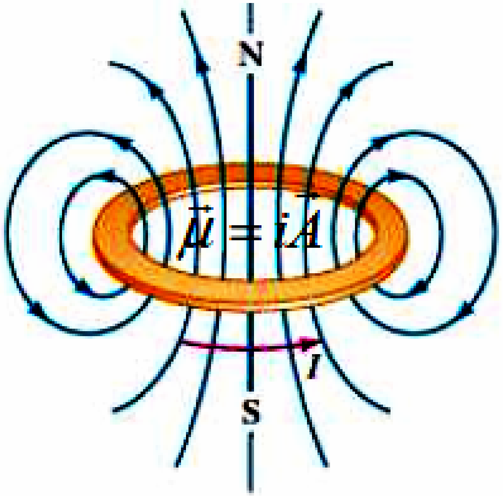




Bar Magnet



Solenoid



Current loop
Magnetic dipole

Ampere’s Law

The line integral of **B.ds** around any closed path equals $\mu_0 i$, where i is the total continuous current passing through any surface bounded by the closed path.

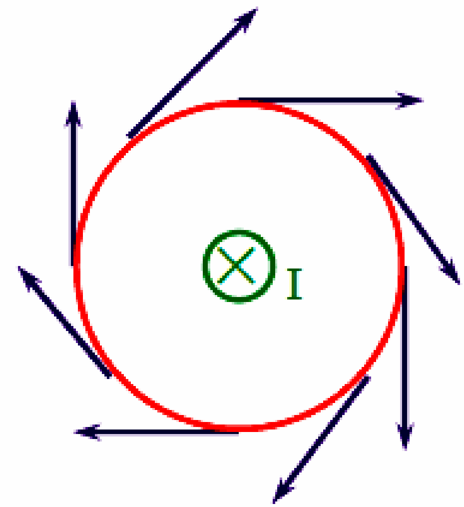
Magnetic field at every point of Amperian Path

Small length element of Amperian Loop

$$\oint_C \vec{B} \bullet d\vec{l} = \mu_0 i_{encl}$$

Integral around an Amperian closed path

Current “enclosed” by Amperian path

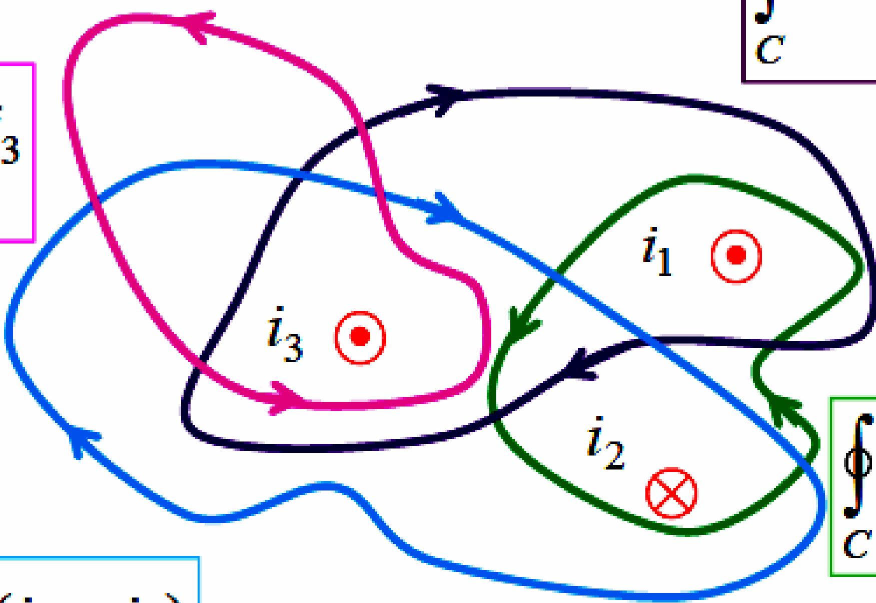


Ampere’s Law

The right hand rule is used to assign signs to the enclosed currents:
With the fingers of your right hand in the direction in which loop is travelled, currents in the direction of your thumb are taken to be positive , whereas currents in the opposite direction are taken to be negative.

$$\oint_C \vec{B} \bullet d\vec{l} = -\mu_0 (i_1 + i_3)$$

$$\oint_C \vec{B} \bullet d\vec{l} = \mu_0 i_3$$



$$\oint_C \vec{B} \bullet d\vec{l} = \mu_0 (i_1 - i_2)$$

$$\oint_C \vec{B} \bullet d\vec{l} = \mu_0 (i_2 - i_3)$$

Consider a cylindrical wire of radius R in which a total current i is distributed uniformly over its cross sectional area with constant current density

$$J = i / \pi R^2$$

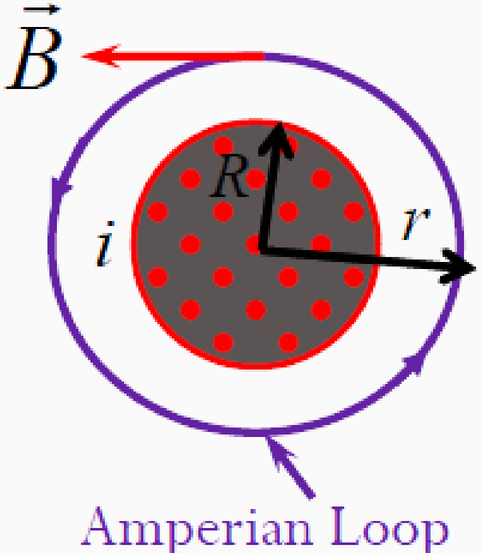
B outside the wire

Consider an Amperian loop of radius r such that r > R. Symmetry suggest that B is constant in magnitude everywhere on the Amperian loop and tangent to the loop.

$$\oint \vec{B} \bullet d\vec{l} = \mu_\circ i_{encl}$$

$$B\oint dl = \mu_\circ i$$

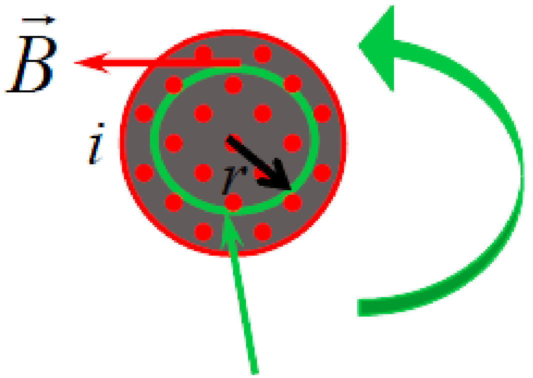
$$B(2\pi r) = \mu_\circ i \quad \Rightarrow \quad \boxed{B = \frac{\mu_\circ i}{2\pi r}}$$



B inside the wire

Consider an Amperian loop of radius r such that r < R. Symmetry suggest that B is constant in magnitude everywhere on the Amperian loop and tangent to the loop. The enclosed current will be the current passing through the Amperian loop, that is,

$$i_{encl} = JA' = \frac{i}{\pi R^2}\pi r^2 = ir^2 / R^2$$

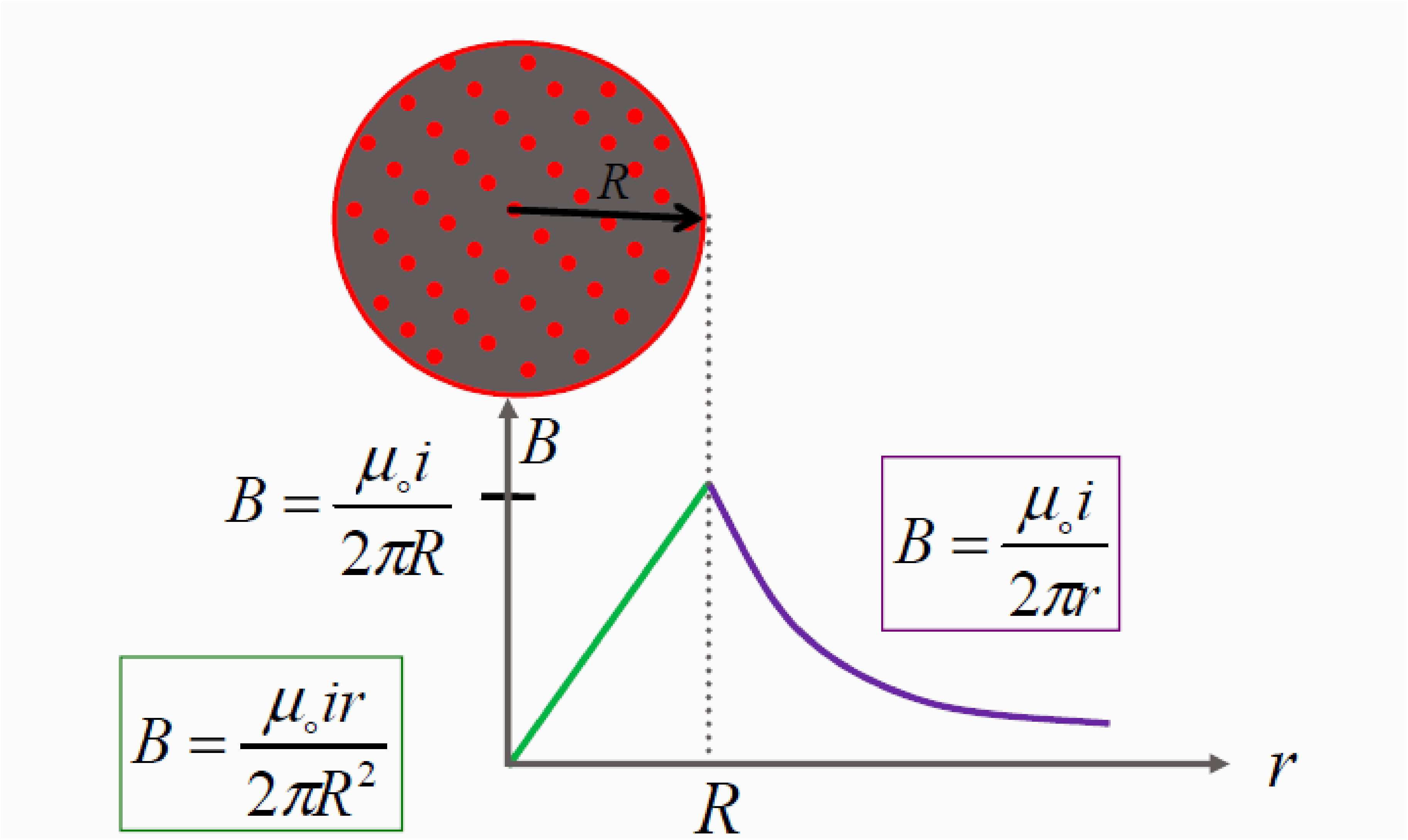


$$\oint \vec{B} \bullet d\vec{l} = \mu_\circ i_{encl}$$

$$B\oint dl = \mu_\circ ir^2 / R^2$$

$$B(2\pi r) = \mu_\circ ir^2 / R^2 \Rightarrow \boxed{B = \frac{\mu_\circ ir}{2\pi R^2}}$$

Magnetic field due to a long current carrying wire



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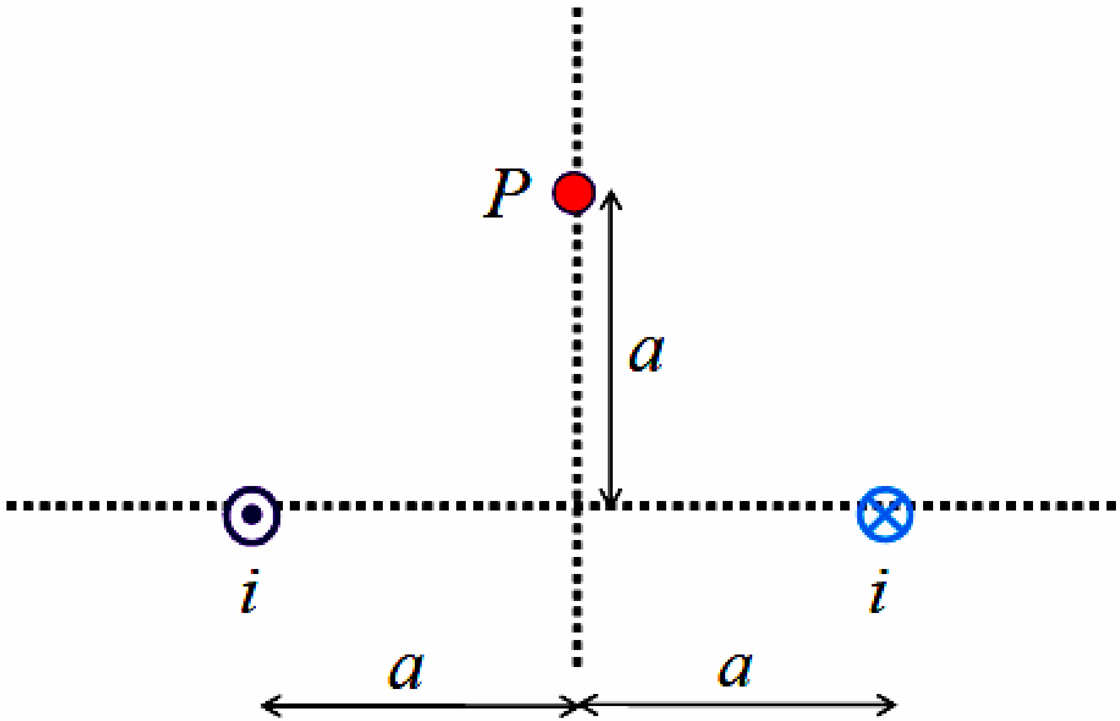
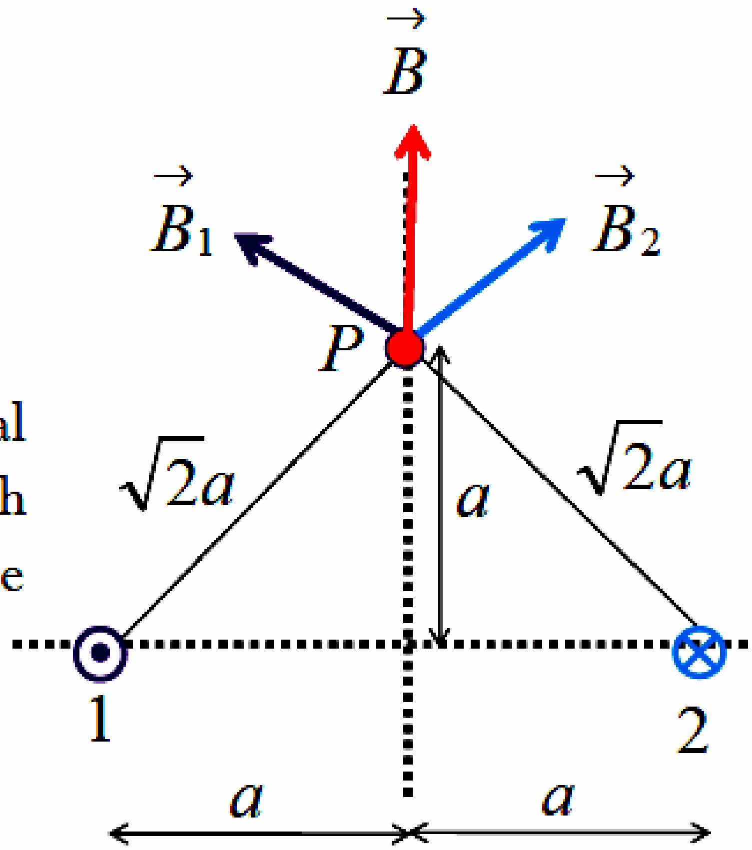
In the figure shown, Find total magnetic field at point P at the perpendicular bisector of the line connecting two wires.



Magnitudes of magnetic fields due to both wires are

$$B_1 = \frac{\mu_o i}{2\pi r_1} = \frac{\mu_o i}{2\sqrt{2}\pi a}$$

$$B_2 = \frac{\mu_o i}{2\pi r_2} = \frac{\mu_o i}{2\sqrt{2}\pi a}$$



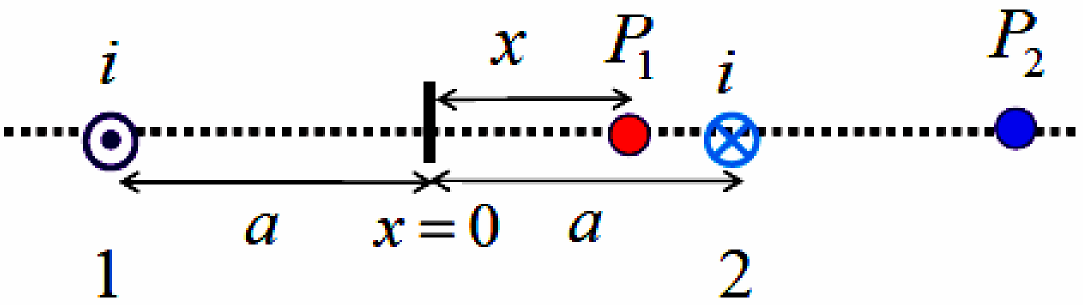
Since B1=B2, By symmetry, horizontal components of two fields will cancel each other while vertical add up. So net field will be

$$\vec{B} = B_1 \sin 45\,\hat{j} + B_2 \sin 45\,\hat{j}$$

$$= \frac{\mu_o i}{2\sqrt{2}\pi a}\frac{1}{\sqrt{2}}\hat{j} + \frac{\mu_o i}{2\sqrt{2}\pi a}\frac{1}{\sqrt{2}}\hat{j}$$

$$= \frac{\mu_o i}{2\pi a}\hat{j}$$

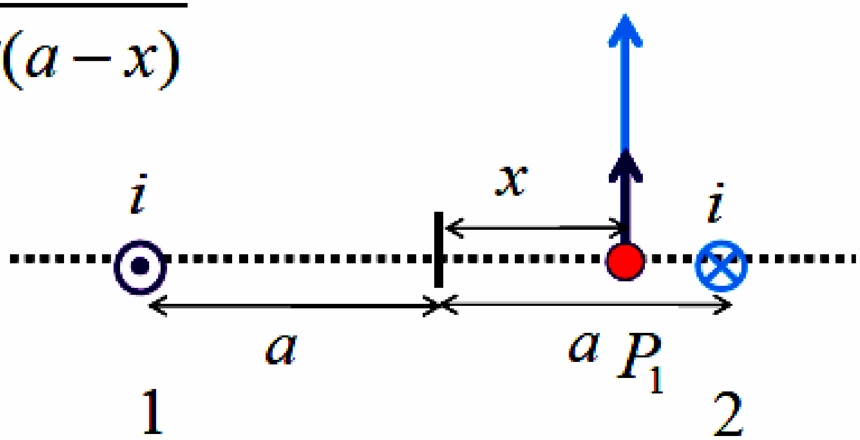
In the figure shown, Find total magnetic field at point P1 and P2 on the line connecting the wires as shown.



Magnitudes of magnetic fields due to both wires at point P1 are

$$B_1 = \frac{\mu_\circ i}{2\pi r_1} = \frac{\mu_\circ i}{2\pi(a+x)}$$

$$B_2 = \frac{\mu_\circ i}{2\pi r_2} = \frac{\mu_\circ i}{2\pi(a-x)}$$



Net field at P1 will be

$$\vec{B}_{P1} = \vec{B}_1 + \vec{B}_2$$

$$= \frac{\mu_\circ i}{2\pi(a+x)}\hat{j} + \frac{\mu_\circ i}{2\pi(a-x)}\hat{j}$$

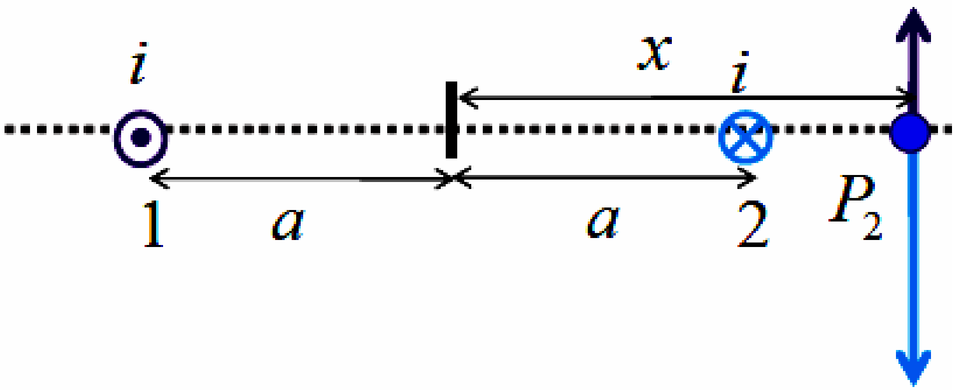
$$= \frac{\mu_\circ i a}{\pi(a^2-x^2)}\hat{j}$$

Magnitudes of magnetic fields due to both wires at point P2 are

$$B_1 = \frac{\mu_o i}{2\pi r_1} = \frac{\mu_o i}{2\pi(x+a)}$$

$$B_2 = \frac{\mu_o i}{2\pi r_2} = \frac{\mu_o i}{2\pi(x-a)}$$

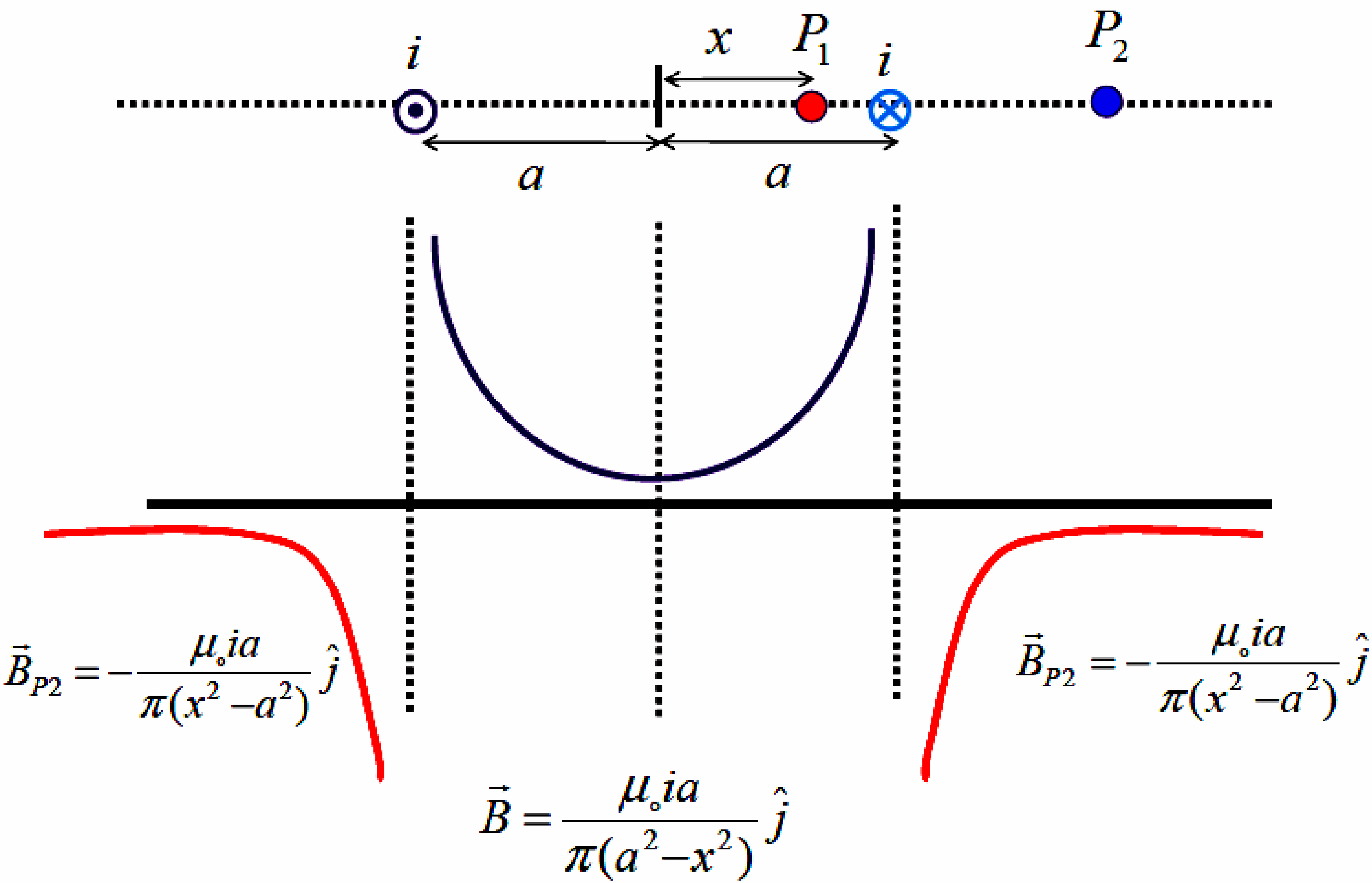
Net field at P2 will be



$$\vec{B}_{P2} = \vec{B}_1 + \vec{B}_2$$

$$= \frac{\mu_o i}{2\pi(x+a)}\hat{j} - \frac{\mu_o i}{2\pi(x-a)}\hat{j}$$

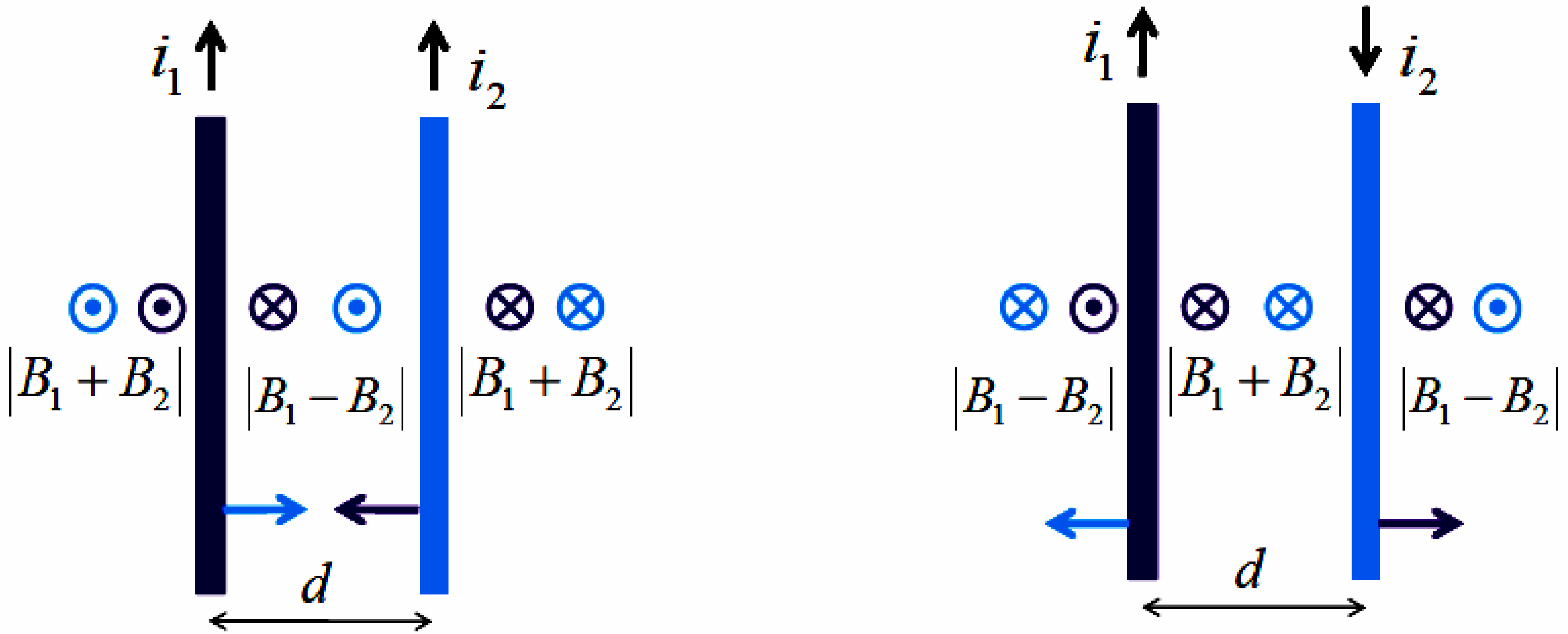
$$= -\frac{\mu_o i a}{\pi(x^2 - a^2)}\hat{j}$$



i
x
P_1
i
P_2
a
a
$\vec{B}_{P2} = -\dfrac{\mu_o ia}{\pi(x^2 - a^2)}\hat{j}$
$\vec{B}_{P2} = -\dfrac{\mu_o ia}{\pi(x^2 - a^2)}\hat{j}$
$\vec{B} = \dfrac{\mu_o ia}{\pi(a^2 - x^2)}\hat{j}$

Interaction between parallel current

Let's consider two wires of same lengths $L_1=L_2$ and carrying currents i_1 and i_2 parallel and antiparallel as shown in figures. Wires are placed parallel and separated by distance d.



Parallel currents attracts, and antiparallel currents repel.

Force on wire 2 due to wire 1; F_{21}

wire 1 generates a magnetic fields at the location of wire 2 as

$$B_1 = \mu_\circ i_1 / 2\pi d$$

wire 2 carrying current i_2 is placed inside field B_1 generated by wire 1, so it will experience a force F_{21}

$$\vec{F}_{21} = i_2 \vec{L}_2 \times \vec{B}_1 = i_2 L_2 B_1 \hat{n} \qquad \therefore \vec{L}_2 \perp \vec{B}_1$$

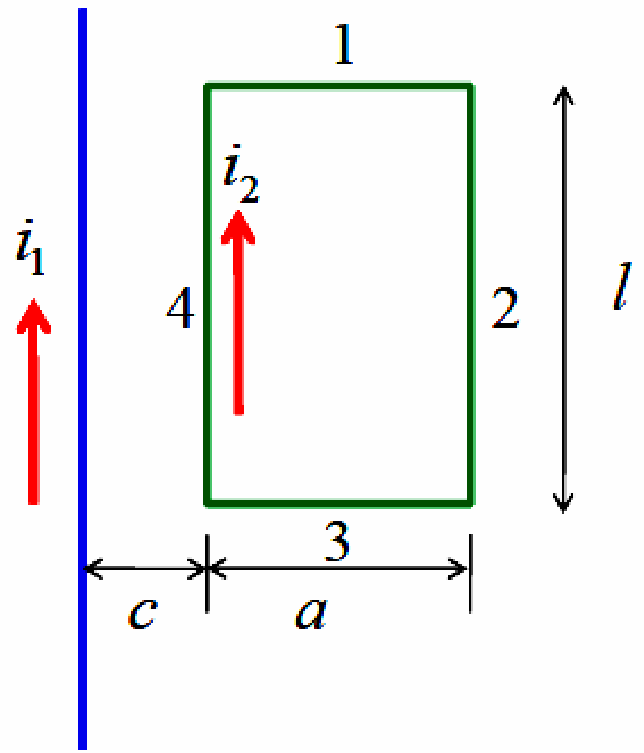
$$F_{21} = i_2 L_2 \frac{\mu_\circ i_1}{2\pi d}$$

$$\boxed{F_{21} = \frac{\mu_\circ i_1 i_2 L}{2\pi d}} \qquad \therefore L_2 = L_1 = L$$

Force on wire 1 due to wire 2; F_{12} ,will be equal and opposite to F_{21}

$$\boxed{\vec{F}_{21} = -\vec{F}_{12}}$$

In Figure , the current in the long, straight wire is $i_1 = 5.00\,A$ and the wire lies in the plane of the rectangular loop, which carries the current $i_2 = 10.0\,A$. The dimensions are $c = 0.100$ m, $a = 0.150$ m, and $l = 0.450$ m. (a) Find the magnitude and direction of the net force exerted on the sides 1,2,3 and 4 of loop and (b) Total force on the loop by the magnetic field created by the wire.



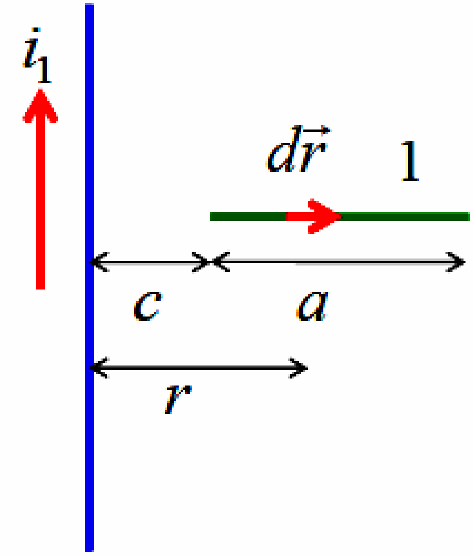
Let's consider a small element dr of the segment 1 at a distance r from the wire. Magnetic force on this small element due to magnetic field created by current carrying wire will be

$$d\vec{F}_1 = i_2 d\vec{r} \times \vec{B}_1 = i_2 dr B_1 \hat{j}$$

Where B1 is the magnetic field of wire at the location of dr.

$$d\vec{F}_1 = i_2 dr \frac{\mu_\circ i_1}{2\pi r} \hat{j}$$

$$\therefore B_1 = \frac{\mu_\circ i_1}{2\pi r}$$



Net force on segment 1 will be

$$\vec{F}_1 = \int_c^{c+a} d\vec{F} = \int_c^{c+a} i_2 dr \frac{\mu_\circ i_1}{2\pi r} \hat{j}$$

$$\vec{F}_1 = \frac{\mu_\circ i_1 i_2}{2\pi} [\ln r]_c^{c+a} \hat{j} = \frac{\mu_\circ i_1 i_2}{2\pi} \ln \frac{c+a}{a} \hat{j}$$

Similarly force on segment 3 will be

$$\vec{F}_3 = -\frac{\mu_\circ i_1 i_2}{2\pi} \ln \frac{c+a}{a} \hat{j}$$

Magnetic forces on segment 2 and 4 will be

$$\vec{F}_2 = \frac{\mu_\circ i_1 i_2 l}{2\pi(c+a)} \hat{i} \qquad \vec{F}_4 = -\frac{\mu_\circ i_1 i_2 l}{2\pi c} \hat{i}$$

The net force on the loop is

$$\vec{F} = \cancel{\vec{F}_1} + \vec{F}_2 + \cancel{\vec{F}_3} + \vec{F}_4$$

$$= \frac{\mu_\circ i_1 i_2 l}{2\pi(c+a)} \hat{i} - \frac{\mu_\circ i_1 i_2 l}{2\pi c} \hat{i}$$

$$= -2.7 \times 10^{-5}\, N \hat{i}$$