The magnitude-squared response of Chebyshev-I filter

$$|H_a(j\Omega)|^2 = \frac{1}{1+\varepsilon^2 T_N^2\left(\dfrac{\Omega}{\Omega_c}\right)}$$

N is the order of the filter,

Epsilon is the passband ripple factor

$$T_N(x) = \begin{cases} \cos(N\cos^{-1}(x)), & 0 \le x \le 1 \\ \cosh(\cosh^{-1}(x)), & 1 < x < \infty \end{cases}$$

Nth-order Chebyshev polynomial

(a) For 0<x<1, $T_N(x)$ oscillates between −1 and 1, and

(b) For 1<x<infinity, $T_N(x)$ increases monotonically to infinity

Figure on P.314 (two possible shapes)

Observations: P.315

Observations

At x=0 (or Ω=0); $|Ha(j0)|^2$ = 1; for N odd;

= 1/(1+epson^2); for N even

At x=1 (or Ω= Ωc); $|Ha(j1)|^2$= 1/(1+epson^2) for all N.

For 0<=x<=1 (or 0<= Ω<= Ωc)

$|Ha(jx)|^2$ oscillates between 1 and 1/(1+epson^2)

For x>1 (or Ω > Ωc), $|Ha(jx)|^2$ decreases monotonically to 0 .

At x= Ωr, $|Ha(jx)|^2$ = 1/(A^2).

Causal and stable Ha(s)

To determine a *causal and stable* Ha(s), we must find the poles of Ha(s)Ha(-s) and select the *left half-plane* poles for Ha(s).

The poles of Ha(s)Ha(-s) are obtained by finding the roots of

$$1+\varepsilon^2 T_N^2\left(\frac{s}{j\Omega_c}\right)$$

It can be shown that if $p_k = \sigma_k + j\Omega_k, k = 0,1,\cdots,N-1$

are the (left half-plane) roots of the above polynomial, then

$$p_k = \sigma_k + j\Omega_k, k = 0,1,\cdots, N-1$$

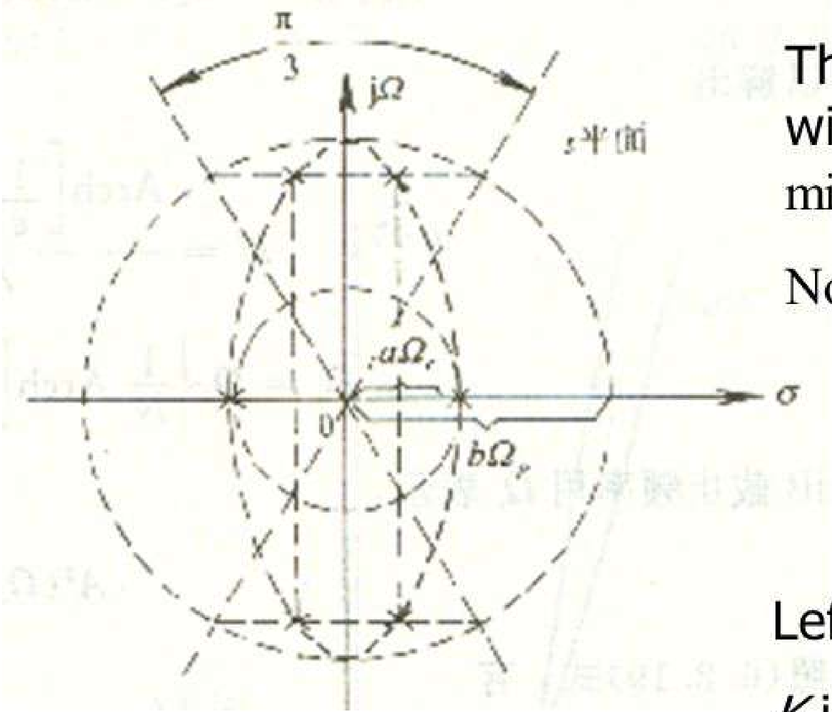
$$\sigma_k = (a\Omega_c)\cos\left[\frac{\pi}{2} + \frac{(2k+1)\pi}{2N}\right]$$

$$\Omega_k = (b\Omega_c)\sin\left[\frac{\pi}{2} + \frac{(2k+1)\pi}{2N}\right]$$

$$a = \frac{1}{2}\left(\sqrt[N]{\alpha} - \sqrt[N]{1/\alpha}\right), b = \frac{1}{2}\left(\sqrt[N]{\alpha} + \sqrt[N]{1/\alpha}\right),$$

$$\alpha = \frac{1}{\varepsilon} + \sqrt{1 + \frac{1}{\varepsilon^2}}$$

The poles of $H_a(s)H_a(-s)$



The poles fall on an ellipse with major axis b Ωc and minor axis a Ωc .

Now the *system function* is

$$H_a(s) = \frac{K}{\prod_k (s - p_k)}$$

Left half-plane

K is a normalizing factor

Chebyshev-II filter

Related to the Chebyshev-I filter through a simple transformation.

It has a monotone passband and an equiripple stopband, which implies that this filter has both poles and zeros in the s-plane.

Therefore the group delay characteristics are better (and the phase response more linear) in the passband than the Chebyshev-I prototype.

$$|H_a(j\Omega)|^2 = \frac{1}{1+\left(\varepsilon^2 T_N^2(\Omega_c/\Omega)\right)^{-1}}$$

The magnitude-squared response

$$|H_a(j\Omega)|^2 = \frac{1}{1+\varepsilon^2 U_N^2\left(\dfrac{\Omega}{\Omega_c}\right)}$$

N: the order; epsilon: passbang ripple; $U_N()$ is the Nth order Jacobian elliptic function

Typical responses for odd and even N are shown on P.323

Computation of filter order N:

$$N = \frac{K(k)K\left(\sqrt{1-k_1^2}\right)}{K(k_1)K\left(\sqrt{1-k^2}\right)}, k = \frac{\Omega_p}{\Omega_s}, k_1 = \frac{\varepsilon}{\sqrt{A^2-1}}, K(x) = \int_0^{\pi/2} \frac{d\theta}{\sqrt{1-x^2\sin^2\theta}}$$