- Course: **EE383 Instrumentation and Measurements**
- Session: Fall 2022

- **Lectures: Week 4**
- Course Instructor: Dr. Shahzad

INSTRUMENTATION
& MEASUREMENT

SCHOOL OF ELECTRICAL ENGINEERING
& COMPUTER SCIENCE

Week 4

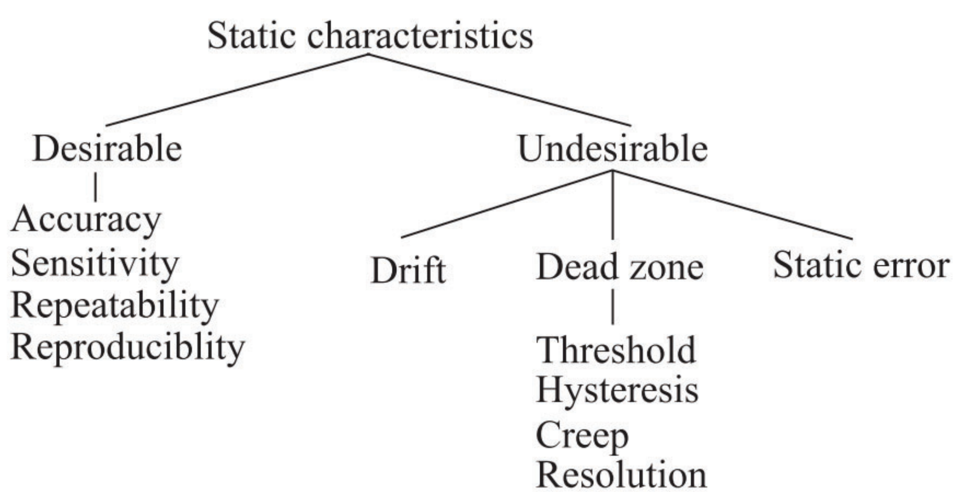
- **Chapter 2**

  Static Characteristics of Instruments
- **Chapter 3**

  Estimation of Static Errors & Reliability

Static Characteristics Of Instrument

- **Characteristics relating the steady-state (achieved) of an instrument**
- **Measurement of quantities which are constant or vary very slowly with time**
- ***Example: EMF, resistance at constant T***



Ch. 2: Introduction to Measurements and Instrumentation, Arun K. Ghosh

Static Errors

- ❑ **Classification of errors**

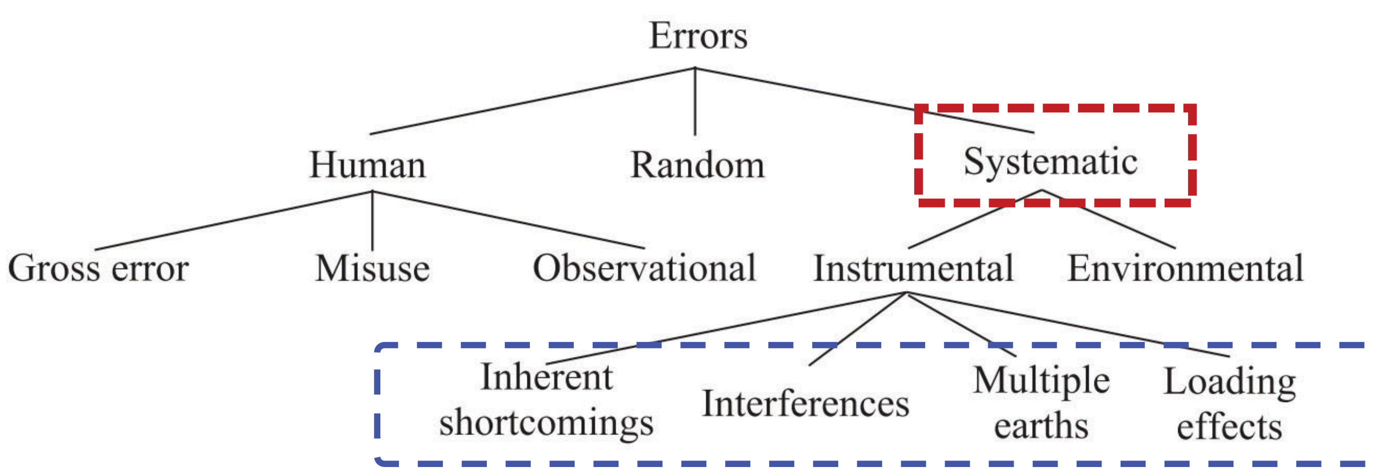


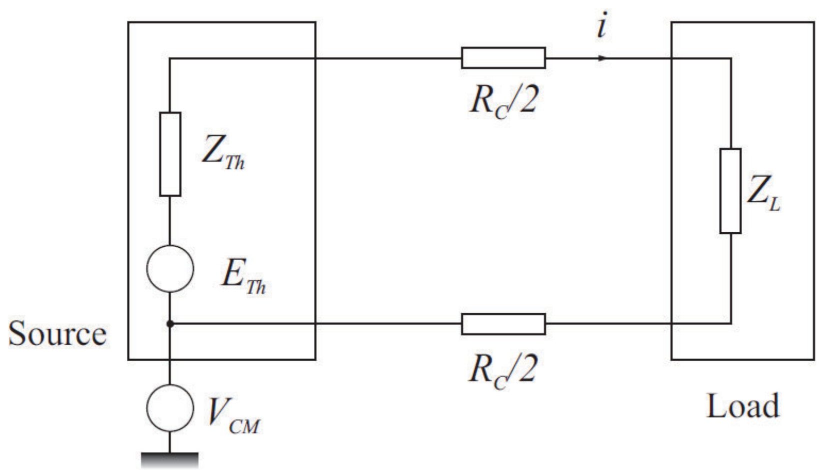
Fig. 2.7 The error tree.

Instrumental Errors

1. **Inherent shortcomings**
2. **Interference**
3. **Multiple earths**
4. **Loading effects**

Instrumental Errors: Interference

Common mode interference



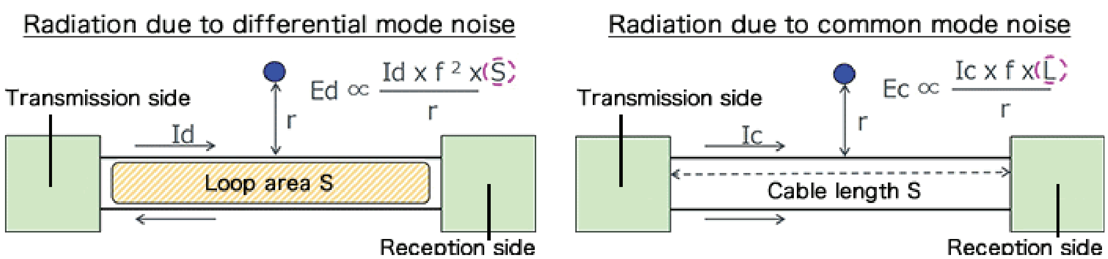
- A common mode interference is caused by a potential offset on both sides of the circuit, relative to the common ground.
- If the impedance of the load is much higher than of the equivalent circuit, the voltage across the load is not significantly affected.

Common Mode Interference and Radio Reception

01-06-2020 | | By Nnamdi Anyadike

The high switching frequency of current data lines has made common mode (CM) interference a problem for radio reception. In normal or differential (single) mode, current travels on one line in one direction from the source to the load and in the opposite direction on the return line. This completes the circuit. However, in CM, the noise current travels on both lines in the same direction. Inductors create magnetic fields that oppose changes in current. An electrical filter, the CM choke, blocks the high frequency noise common to two or more data or power lines while allowing the desired DC or low-frequency signal to pass.

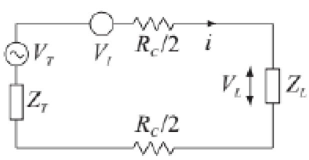
CM noise current is typically radiated from sources such as unwanted radio signals, unshielded electronics, inverters and motors. If left unfiltered, this noise presents interference problems in electronics and electrical circuits. Key players in the global CM choke market include AKEMET Corporation, EPCOS, Murata,



2.2 Undesirable Characteristics

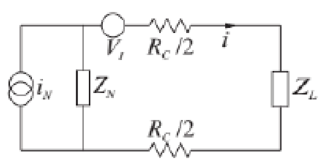
- **Instrumental Error**
  - Interference

Series Mode Interference



$$V_L = V_T + V_I$$

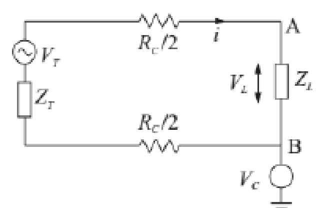
Affects voltage transmission



$$V_L = i_N Z_L$$

Do not affect current transmission

Common Mode Interference



$$V_A = V_c + V_T$$
$$V_B = V_c$$
$$V_L = V_A - V_B = V_T$$

Have no effect

Instrumental Errors

1. **Inherent shortcomings**
2. **Interference**
3. **Multiple earths**
4. **Loading effects**

Instrumental Errors: Multiple earths

Consider the situation as shown in Fig. 2.12. As before, we assume, $Z_L \gg (Z_T + R_C)$. Hence negligible current flows through the ABCD loop.

But leakage impedances at the source Z_S and receiver Z_R exist and therefore a voltage difference V_E exists between the earth points at the source and the receiver. As a consequence, a current i_E flows in the circuit CFEB. It is given by

$$i_E = \frac{V_E}{Z_S + Z_E + Z_R + R_C/2} \tag{2.7}$$

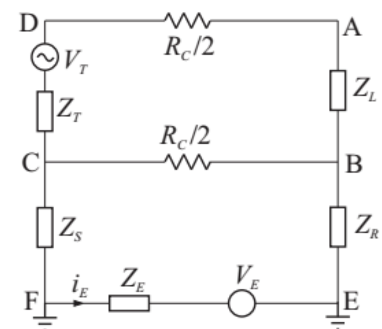


Fig. 2.12 Multiple earth situation.

Previously, potentials between (A and D) and (C and B) were equal because almost no current flowed through the ABCD loop. Now, because of i_E, potentials at these points are

$$\begin{aligned} V_C &= V_E - i_E(Z_E + Z_S) \\ V_B &= i_E Z_R \\ V_D = V_A &= V_E - i_E(Z_E + Z_S) + V_T \end{aligned} \tag{2.8}$$

V_B is common mode interference and its value is found from Eqs. (2.8) and (2.7) as

$$V_B = \frac{Z_R V_E}{Z_S + Z_E + Z_R + R_C/2}$$

The series mode interference can be obtained by figuring out the voltage across the load as

$$\begin{aligned} V_L = V_A - V_B &= V_E - i_E(Z_E + Z_S) + V_T - i_E Z_R \\ &= [V_E - i_E(Z_E + Z_S + Z_R)] + V_T \\ &= \frac{i_E R_C}{2} + V_T \end{aligned} \tag{2.9}$$

Thus, we find from Eq. (2.9) that the series mode interference is

$$V_I = \frac{i_E R_C}{2} = \frac{V_E R_C}{2(Z_S + Z_E + Z_R) + R_C}$$

- ✓ **To minimize V_I as well as i_E, Z_S and Z_R should be as large as possible**
  - ✓ **But this may not always be possible in an industry**

Instrumental Errors: Multiple earths

- ❑ **Practically there may be leakage currents**
- ❑ **If there are multiple earths, there is a potential difference**
  - ❑ **series and common mode interference arises**

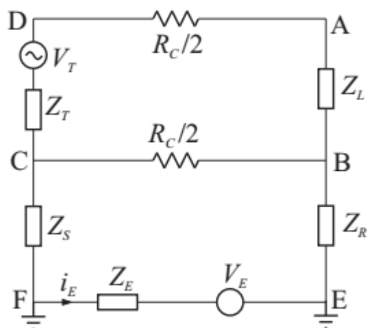


Fig. 2.12 Multiple earth situation.

$$V_L = \frac{i_E R_C}{2} + V_T \qquad V_I = \frac{i_E R_C}{2} = \frac{V_E R_C}{2(Z_S + Z_E + Z_R) + R_C}$$

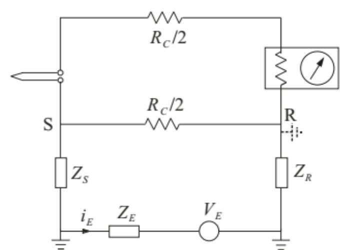
- ✓ **To minimise V_I as well as i_E , Z_S and Z_R should be as large as possible**
  - ✓ **But this may not always be possible in an industry**

Instrumental Errors: Multiple earths

- **Example**

Example 2.7
In a thermocouple installation the tip of the thermocouple touches the thermowell and the thermowell is bolted to a metal pipe which in turn is connected to one point in the earth plane. The emf generated is taken to a receiver 500 metres away by means of a cable of resistance 0.1 Ω per metre. The receiver is isolated from earth but a 1 V potential difference exists between the source earth and the receiver earth. If this 1 V source has 1 Ω impedance and the impedance between receiver and earth is 10^6 Ω while that between the source and the earth is 10 Ω, calculate the series mode interference voltage. What changes will be observed if the receiver is earthed?

Solution
The diagram for the problem is shown below.



Given,

$$R_C/2 = (500 \times 0.1)\ \Omega = 50\ \Omega$$

$V_E = 1$ V $\qquad Z_E = 1\ \Omega$

$Z_S = 10\ \Omega \qquad Z_R = 10^6\ \Omega$

Instrumental Errors: Multiple earths

- **Example**

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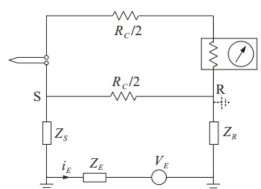

Given,

$$R_C/2 = (500 \times 0.1)\ \Omega = 50\ \Omega$$
$$V_E = 1\text{ V} \qquad Z_E = 1\ \Omega$$
$$Z_S = 10\ \Omega \qquad Z_R = 10^6\ \Omega$$

Therefore,

$$i_E = \frac{V_E}{Z_S + Z_R + Z_E + R_C/2}$$
$$= \frac{1}{10^6 + 10 + 1 + 50} = 0.999 \times 10^{-6}\text{ A}$$

Series mode interference voltage, $V_I = i_E \cdot R_C/2 = 0.999 \times 10^{-6} \times 50 \cong 50\ \mu$V. Now, if the receiver is earthed, $Z_R = 0$. Then,

$$i_E = \frac{V_E}{Z_S + Z_E + R_C/2} = \frac{1}{10 + 1 + 50} = 0.0164\text{ A}$$

Therefore,

$$V_I = i_E \cdot R_C/2 = 0.0164 \times 50 = 0.82\text{ V}$$

Thus, the series mode interference voltage increases from 50 μV to 820 mV when the receiver is earthed.

Instrumental Errors

1. **Inherent shortcomings**
2. **Interference**
3. **Multiple earths**
4. **Loading effects**

Instrumental Errors

- **Loading effect:-** These errors are committed by beginners. e.g. a well calibrated voltmeter may give a wrong reading when connected across a high resistance circuit, the same voltmeter give a more dependable reading when connected across a low resistance circuit

Instrumental Errors: Loading effects

- **Instruments extract energy from the measured medium thereby changing the measurand from its pristine undisturbed state**
- **Extraction of energy by a measurement system from the measured medium is known as the loading effect**

Instrumental Errors: Loading effects

- **Parallel or shunt connection (voltage measurement)**
  - **As voltmeter is attached to the terminals A and B, the circuit is changed and the value of E_o is altered**

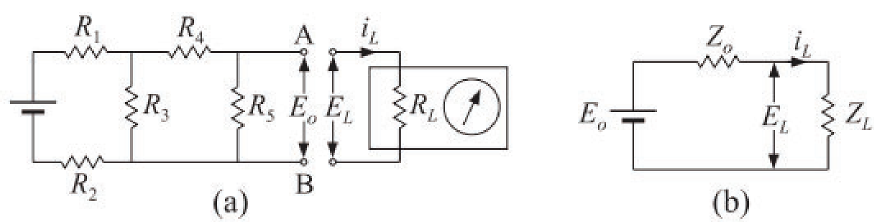

Fig. 2.13 Loading effect in voltage measurement: (a) schematic arrangement, (b) its Thevenin equivalent.

Instrumental Errors: Loading effects

- ❑ **Parallel or shunt connection (voltage measurement)**
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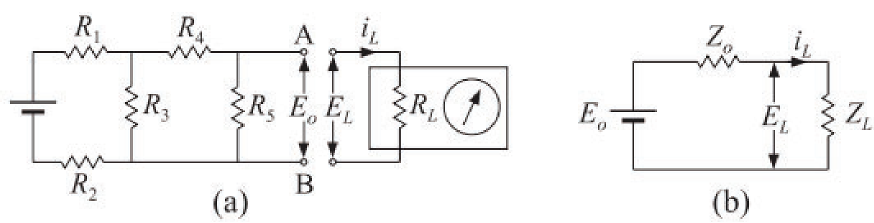

$$E_o = i_L(Z_o + Z_L)$$

Fig. 2.13 Loading effect in voltage measurement: (a) schematic arrangement, (b) its Thevenin equivalent.

$$E_L = \frac{E_o}{1 + (Z_o/Z_L)}$$

✓ **For ac, the voltage is modified both in magnitude and phase**
✓ **Instrument will give true result if $Z_L \rightarrow \infty$, and a reasonably accurate one if $Z_L \gg Z_o$**

If Z_o is the true impedance between A and B, Z_L is the impedance of the loading circuit (here, the voltmeter), E_o is the true voltage between A and B, and E_L is the voltage seen by the loading circuit, their values may be derived as follows:

$$Z_o = \frac{\left[\dfrac{(R_1 + R_2)R_3}{R_1 + R_2 + R_3} + R_4\right] R_5}{\dfrac{(R_1 + R_2)R_3}{R_1 + R_2 + R_3} + R_4 + R_5}$$

$$Z_L = R_m, \qquad E_L = E_o - i_L Z_o = i_L Z_L \qquad E_o = i_L(Z_o + Z_L)$$

Therefore,

$$\frac{E_L}{E_o} = \frac{i_L Z_L}{i_L(Z_o + Z_L)} = \frac{1}{1 + (Z_o/Z_L)}$$

or

$$E_L = \frac{E_o}{1 + (Z_o/Z_L)} \tag{2.10}$$

Instrumental Errors: Loading effects

- **Series connection (current measurement)**
  - **When the current is measured by an ammeter of impedance Z_L, the current changes from i_o (E_o/Z_o) to i_L**

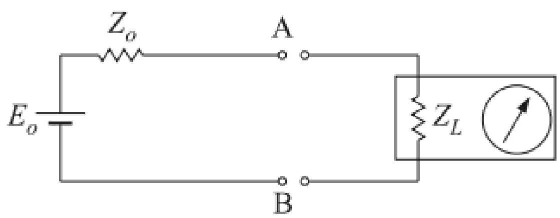

Fig. 2.14 Loading effect in current measurement.

Instrumental Errors: Loading effects

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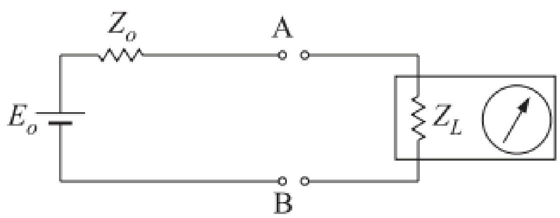

Fig. 2.14 Loading effect in current measurement.

$$i_L = \frac{E_o}{Z_o + Z_L} = \frac{i_o Z_o}{Z_o + Z_L} = \frac{i_o}{1 + (Z_L/Z_o)}$$

✓ **To minimise the measurement error, i.e. to make $i_L \to i_o$, $Z_o \gg Z_L$**

Instrumental Errors: Loading effects

Example 2.9

What is the true value of the voltage across the terminals A and B (Fig. 2.16)? What would a voltmeter of 20 kΩ/V sensitivity read on the 50 V and 10 V ranges?

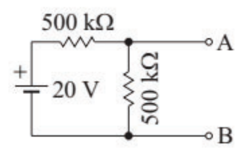

Fig. 2.16 Example 2.9.

Instrumental Errors: Loading effects

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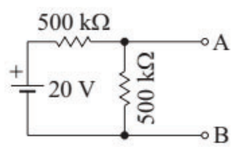


Fig. 2.16 Example 2.9.

Solution

Current in the circuit, $I = 20/(500 + 500)$ mA $= 0.02$ mA. Therefore, $E_o = (0.02 \times 10^{-3} \times 500 \times 10^3)$ V $= 10$ V. But the voltmeter offers different load resistances in its different ranges.

In the 50 V range: The load resistance, $R_L = (20 \times 10^3 \times 50)\ \Omega = 10^6\ \Omega$. Therefore,

$$\boxed{E_L = \frac{E_o}{1 + (Z_o/Z_L)}}$$

$$E_L = \frac{10}{1 + \dfrac{250 \times 10^3}{10^6}} = 8.0 \text{ V}$$

In the 10 V range: $R_L = (20 \times 10^3 \times 10) = 2 \times 10^5\ \Omega$. So,

$$E_L = \frac{10}{1 + \dfrac{250 \times 10^3}{2 \times 10^5}} \approx 4.4 \text{ V}$$

Note: How a wrong setting of the instrument can introduce a huge error!

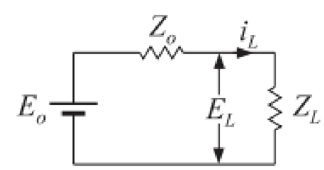
Instrumental Errors: Loading effects

- **Maximum power transfer theorem**
  - **for maximum power transfer to the load, the Thevenin-equivalent resistance R_o of the circuit should be equal to the load resistance, R_L**

$$R_L = R_o$$

- **For ac circuits,**

$$Z_L = Z_o^*$$



Instrumental Errors: Loading effects

- ❑ **Maximum power transfer theorem**
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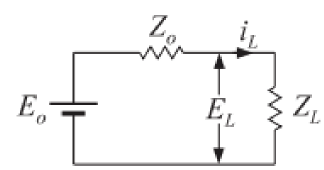

Instrumental Errors: Loading effects

❑ Maximum power transfer theorem

Proof: Power transferred to the load,

$$P = \frac{E_L^2}{R_L} = \frac{E_o^2 R_L}{(R_o + R_L)^2} \tag{2.11}$$

$$= \frac{E_o^2}{R_L[1 + (R_o/R_L)]^2} \tag{2.12}$$

For maximum power transfer,

$$\frac{dP}{dR_L} = 0$$

Therefore,

$$\frac{E_o^2}{(R_o + R_L)^2}\left(1 - \frac{2R_L}{R_o + R_L}\right) = 0$$

or

$$R_L = R_o$$

4. At $R_L = R_o$, the current in the circuit $I = E_o/(2R_o)$. Therefore, the power absorbed by the load is

$$P_L = I^2 R_o = \frac{E_o^2}{4R_o}$$

But, the total available power is

$$P_T = I^2(2R_o) = \frac{E_o^2}{2R_o}$$

Instrumental Errors: Loading effects

- ❑ **Maximum power transfer theorem**
  - ❑ **for maximum power transfer to the load, the Thevenin-equivalent resistance Ro of the circuit should be equal to the load resistance, R_L**

$$R_L = R_o$$

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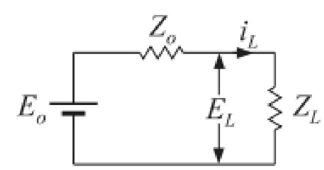


1. ***$R_L \rightarrow 0$, $P \rightarrow 0$*** *(Eq 2.11)* **and as** ***$R_L \rightarrow \infty$, $P \rightarrow 0$*** *(Eq 2.12)*
2. ***$P_{max} = E_o$^2 /4R_o***
3. **Impedance matching is not critical, because**
   - **For a 10% deviation, i.e. R_L / R_o = 1.1 or 0.9, Power transfer is ~100%**
   - **for a 100% deviation, the power transfer is ~89%**

$$\frac{P}{P_{\max}} = \frac{4(R_L/R_o)}{[1+(R_L/R_o)]^2}$$

4. **Efficiency at maximum power transfer condition is 50%**

$$\frac{P_L}{P_T} = \frac{1}{2} = 50\%$$

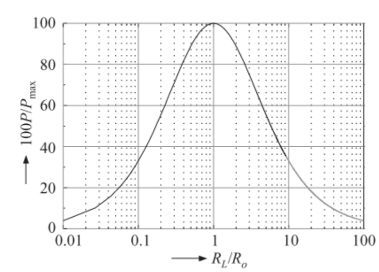


Fig. 2.19 Impedance matching characteristics.

Instrumental Errors: Loading effects

- **Maximum power transfer theorem**
  - **for maximum power transfer to the load, the Thevenin-equivalent resistance Ro of the circuit should be equal to the load resistance, R_L**

Example 2.13

In a series circuit if E_o, R_o and R_L are the source voltage, source resistance and load resistance respectively, and P and $P_{\max}$ are power transferred to the load and the maximum power that can be transferred to the load respectively. Find the value of the ratio $P/P_{\max}$ in per cent when the source resistance is 50% of the load resistance.

Instrumental Errors: Loading effects

- **Maximum power transfer theorem**
  - **for maximum power transfer to the load, the Thevenin-equivalent resistance Ro of the circuit should be equal to the load resistance, *R_L***

Example 2.13

In a series circuit if E_o, R_o and R_L are the source voltage, source resistance and load resistance respectively, and P and $P_{\max}$ are power transferred to the load and the maximum power that can be transferred to the load respectively. Find the value of the ratio $P/P_{\max}$ in per cent when the source resistance is 50% of the load resistance.

Solution

Given, $R_L/R_o = 2$. Hence from Eq. (2.13), we have

$$\boxed{\frac{P}{P_{\max}} = \frac{4(R_L/R_o)}{[1+(R_L/R_o)]^2}} \qquad \frac{P}{P_{\max}} = \frac{4\times 2}{(1+2)^2} = \frac{8}{9} = 88.9\%.$$

Static Errors

- ❑ **Classification of errors**

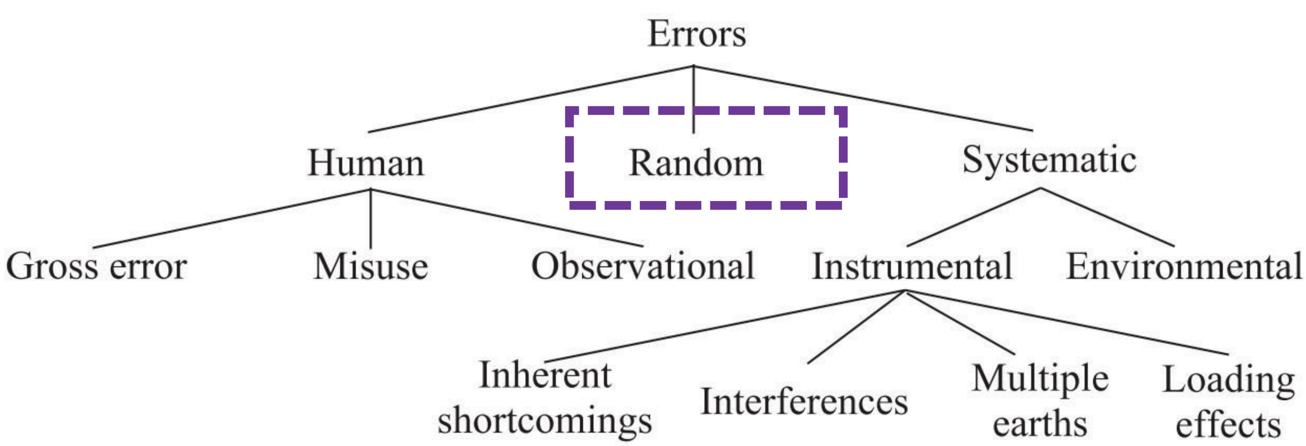


Fig. 2.7 The error tree.

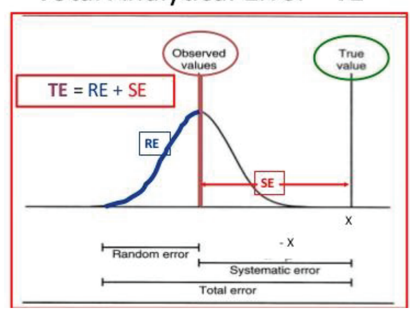
- **Random Error:-** These error exists due to unknown causes and occurs even when all the errors have been accounted for. (***Also known as residual error***)

- These errors are caused due to variation on position of setting standard and work piece.
- Due to Displacement of level joints of instrument, due to backlash and friction, these error are induced.
- Random errors in experimental measurements are caused by unknown and unpredictable changes in the experiment. These changes may occur in the measuring instruments or in the environmental conditions.

Random (Residual) Error

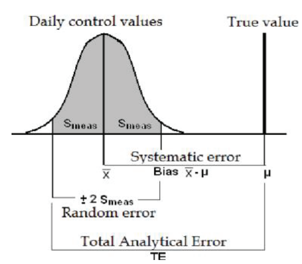
- ❑ **Unpredictable variations in the measurement system**
- ❑ **Small <u>perturbations/fluctuations</u> on either side of the true value**
- ❑ **Random environmental changes**
  - ❑ **sudden draught of air**
  - ❑ **cosmic ray showers**
  - ❑ **changes in geomagnetism**
  - ❑ **thunder-cloud activities, minor earth tremors etc.**

Total Analytical Error - TE



Remedy?

Estimation of error from statistical analysis of measurement data.
Mean, standard deviation, variance.

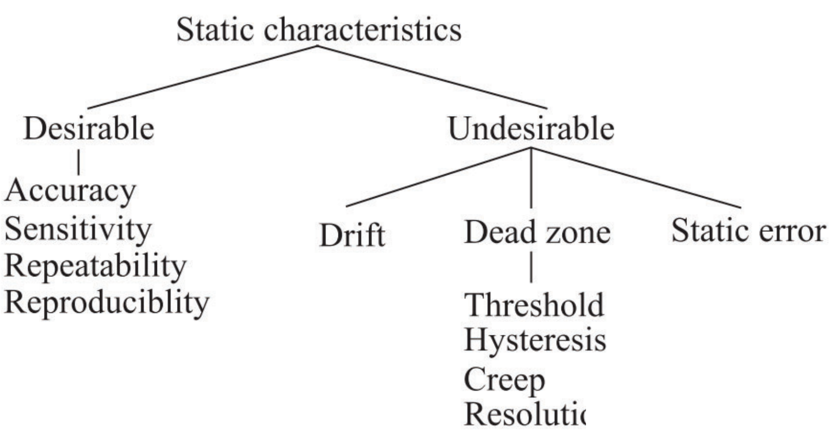


NEXT

- Estimation of Static Errors and Reliability

Review

- Characteristics relating the steady-state (achieved) of an instrument
- Measurement of quantities which are constant or vary very slowly with time



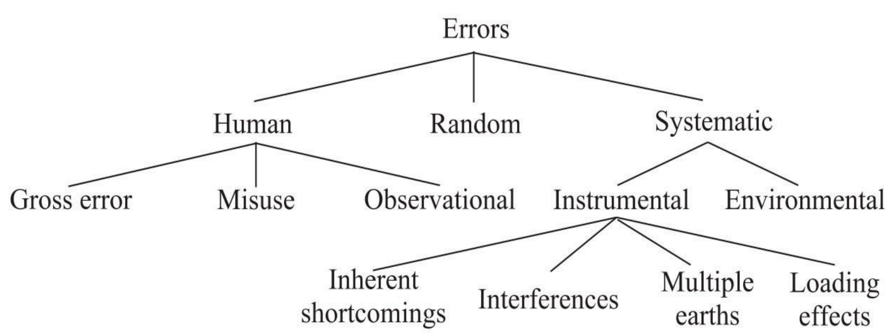


Fig. 2.7 The error tree.

Chapter-3

Estimation of Static Errors & Reliability

Arun K. Ghosh

Estimation of Static Errors

- While reporting a measured value of a quantity, it is necessary to indicate the possible error in the measurement
  - **How do estimate the measurement error?**

Why estimation of static errors?

Measurements involve error.

While reporting the measured values it is necessary to indicate a ***range of possible error***.

How to estimate the range of possible error?

In this chapter we will find the answer

Contents

Definition of Parameters

1. Error
2. Scale Range
3. Scale Span
4. Limiting Error
5. Probable Error

Parameters: Error

- **Absolute static error**
  - **If X_m is the measured value and X_t is the true value,**

$$\varepsilon_0 = |X_m - X_t|$$

  - **Same unit as the measurand**

Parameters: Error

- **Absolute static error**
  - **If X_m is the measured value and X_t is the true value,**

$$\varepsilon_0 = |X_m - X_t|$$

  - **Same unit as the measurand**

- **Relative static error**

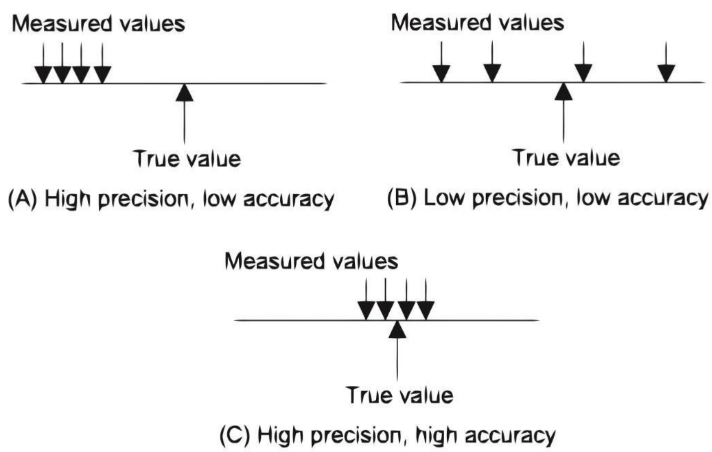
$$\varepsilon_r = \frac{|X_m - X_t|}{X_t} = \frac{\varepsilon_0}{X_t}$$

  - **Mostly, an error is much less than the true value, $\varepsilon_r < 1$**
  - **Unit less**

Definition of Parameters

Question. What is True Value?

True Value: If infinite number of measurements are made with the help of calibrated measuring instruments and individual measurements agree between themselves within a specified degree of accuracy, measured value can be assumed to be the true value.



Parameters: Scale Range

- **Scale Range**
  - **If X_{min} and X_{max} are the minimum and maximum values that an instrument can measure**

$$\text{Scale range} = \text{Between } X_{\min} \text{ and } X_{\max}$$

Parameters: Scale Range

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- **Dynamic Range**

$$\text{Dynamic range } N = \frac{\text{Range of operation}}{\text{Resolution}}$$

$$\text{Dynamic range} = 20 \log_{10} N$$

Parameters: Scale Range

- **Dynamic Range**

$$\text{Dynamic range } N = \frac{\text{Range of operation}}{\text{Resolution}}$$

$$\text{Dynamic range} = 20 \log_{10} N$$

Example 3.1
A voltmeter has a range of [4 V, 20 V] and a resolution of 1 mV. The dynamic range of the instrument is

(a) 21 dB (b) 60 dB (c) 72 dB (d) 84 dB

Parameters: Scale Range

- **Dynamic Range**

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$$\text{Dynamic range} = 20 \log_{10} N$$

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A voltmeter has a range of [4 V, 20 V] and a resolution of 1 mV. The dynamic range of the instrument is

(a) 21 dB (b) 60 dB (c) 72 dB (d) 84 dB

Solution
The range of operation of the instrument is $(20 - 4) = 16$ V and the resolution is 1×10^{-3} V. So,

$$\text{Dynamic range} = 20 \log \frac{16}{1 \times 10^{-3}} = 84\,\text{dB}$$

Therefore, the answer is (d).

Parameters: Scale Range

- **Dynamic Range**

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Example: Another voltmeter has a range of [0V, 30V] = 30 V, resolution of 1mV. Then dynamic range will be ?

Dynamic range = 20 log $30/10^{-3}$ = 88 dB

Parameters: Scale Range

- **Dynamic Range**

$$\text{Dynamic range } N = \frac{\text{Range of operation}}{\text{Resolution}} \qquad \text{Dynamic range} = 20\log_{10} N$$

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Example: Another voltmeter has a range of [0V, 30V] = 30 V, resolution of 1mV. Then dynamic range will be ?

Dynamic range = 20 log 30/10^{-3} = 88 dB

Thus, an instrument having a 40 dB dynamic range means that it can handle input sizes of 100 to 1.

Parameters: Scale Range

Other Usages of dynamic range (DR/DYR/DNR)

→Photography: range of luminance that a given camera can capture (low light scenes→ bright scenes)

→Music: range of quietest to loudest volumes that a recording instrument can capture

Dynamic Range Application

Dynamic range is the "ratio between the largest and smallest values of a changeable quantity." This range between the largest and smallest can be measured with light or sound. With sound, it's the measurement between the "noise floor and the maximum sound pressure level" and what a microphone can capture. In terms of light, it revolves around the **ratio between the brightest to the darkest**.

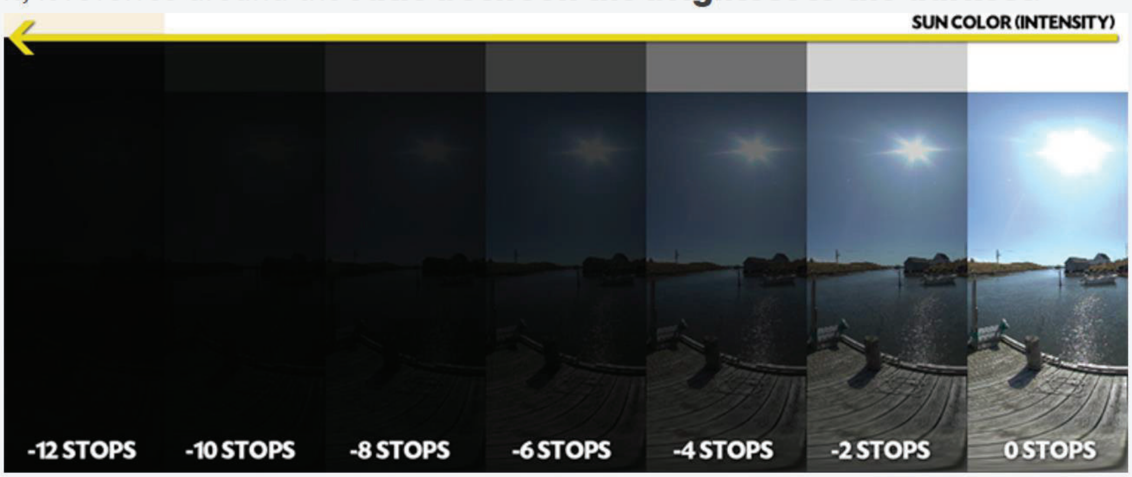


Image via Aversis 3D

In terms of photo and video, this is the range in which a camera can capture the **brightest and darkest areas of an image** without the loss of details in the image. If the camera moves beyond this dynamic range, then that's when you'll see the brights being washed out or the darks becoming noisy.

World Most Expensive Dynamic Range Camera?

Is It Really That Important?

Camera manufacturers are constantly upgrading their technology with the goal of getting closer to the capability of the human eye. While they end up falling short, it's not for a lack of trying. The human eye is an amazing optical receiver — and if it was a camera, it would be the most expensive camera ever.

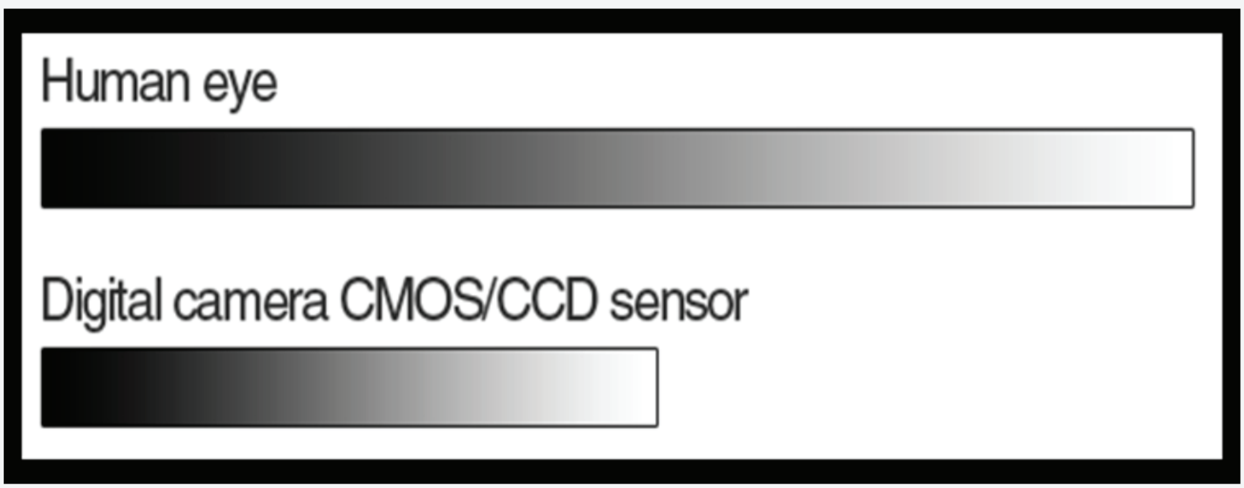


Image via Photography-101

The **dynamic range** of the human eye is something that current camera technology just can't match.

Parameters: Scale Span

- **Scale Span**
  - **If X_{min} and X_{max} are the minimum and maximum values that an instrument can measure**

$$\text{Scale span} = X_{\max} - X_{\min}$$

Example 3.2
A voltmeter is calibrated between 10 V and 250 V. The scale span and scale range are respectively

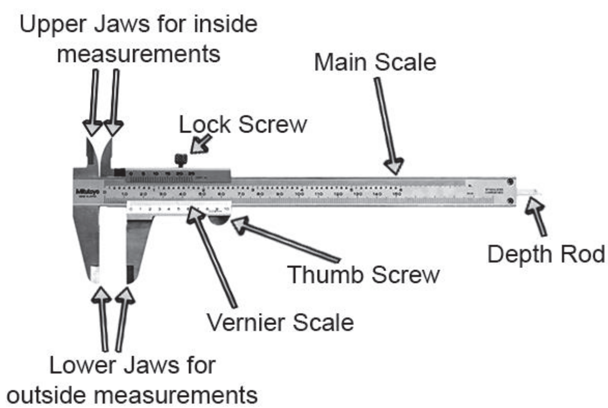
(a) 250, 250 (b) 240, 250 (c) 250, 240 (d) 240, 240

Solution
The nearest answer is (d). But it is better said that the scale range is 10 to 250 V.

Parameters: Limiting Error

Example

Suppose the length of a rod is being measured with the help of a vernier scale which has a vernier constant of 0.1 mm. One may measure the length only once by the vernier scale and report value as $L \pm 0.1$ mm, if the measured value is L mm. This reported error is called the *limiting error* (or *guarantee error*), because this is the maximum error which might have occurred during the measurement, assuming that the vernier scale has no calibration error. Many components (e.g. resistor, capacitor) or instruments are sold by manufacturers with some limits in their values or readings and indicated by gold or silver bands. These are limiting errors of the components.



Definition of Parameters

4. Limiting Error/Guarantee Error

Maximum error that might occur during measurement if there is no calibration error.

Limiting error is reported as a certain % of full scale reading in indicating instruments.

Limiting error of circuit components is certain percentage of full rated value.

E.g. Resistance specified as 500 Ω ± 10 %, implies manufacturer guarantee that resistance lies between 450 Ω & 550 Ω.

Parameters: Limiting Error

- **Indicating instrument**
  - **Accuracy is guaranteed to a certain percentage of the full-scale deflection/reading**

Parameters: Limiting Error

- **Indicating instrument**
  - **Accuracy is guaranteed to a certain percentage of the full-scale deflection/reading**

Example 3.5
A 0–10 ampere ammeter has a guaranteed accuracy of 1% of the full-scale deflection. The limiting error while reading 2.5 A is

(a) 1% (b) 2%
(c) 4% (d) none of the above

Solution
The full-scale deflection is 10 A. Therefore, the guaranteed accuracy is $10 \times 1\% = 0.1$ A. A 0.1 amp error in 2.5 amp amounts to

$$\frac{0.1}{2.5} \times 100 = 4\%$$

Therefore, the correct answer is (c).

Parameters: Limiting Error

- Indicating instrument
  - Accuracy is guaranteed to a certain percentage of the full-scale deflection/reading
- Components
  - Accuracy is guaranteed to a certain percentage of the rated value
    - Resistor: $R = 500\ \Omega \pm 10\%$
    - The R falls between 450 Ω and 550 Ω

Parameters: Limiting Error

- **Indicating instrument**
  - **Accuracy is guaranteed to a certain percentage of the full-scale deflection/reading**
- **Components**
  - **Accuracy is guaranteed to a certain percentage of the rated value**

- **Limiting errors or guarantee errors**
  - **Limits of the deviations from the specified value**
  - **Maximum error which might occur during a measurement**

Limiting Error/ Guarantee Error

Estimation of limiting error for a single measurand is pretty simple.

When many measurands are involved (e.g. a series or parallel combination of resistors) the calculation of resultant limiting error becomes difficult.

There are rules for combination of limiting errors.

Parameters: Limiting Error

- **Combination of Limiting Errors**
  - **when final result is a function of individual measurands**
  - **how to combine errors?**

Combination of Limiting Errors

Suppose *u* and *v* are two measurands and *X* is the final result.

Addition and subtraction. Here, $X = u \pm v$. Then

$$\frac{\delta X}{X} = \pm \left(\frac{u}{X} \frac{\delta u}{u} + \frac{v}{X} \frac{\delta v}{v} \right)$$

Parameters: Limiting Error

- **Combination of Limiting Errors**
  - **when final result is a function of individual measurands**
  - **how to combine errors?**
- **Addition and subtraction**
  - **For *X* = *u* $\pm$ *v*,**

$$\frac{dX}{X} = \frac{du}{X} \pm \frac{dv}{X} = \frac{u}{X}\frac{du}{u} \pm \frac{v}{X}\frac{dv}{v}$$

  - **Errors:** $\pm\delta u$ **and** $+\delta v$

$$\frac{\delta X}{X} = \pm\left(\frac{u}{X}\frac{\delta u}{u} + \frac{v}{X}\frac{\delta v}{v}\right)$$

Parameters: Limiting Error

- **Combination of Limiting Errors**

- **Addition and subtraction**

Example 3.3

Three resistors have the following values: $R_1 = 200\ \Omega \pm 10\%$, $R_2 = 100\ \Omega \pm 5\%$ and $R_3 = 50\ \Omega \pm 5\%$. Determine the magnitude of the resultant resistances and the limiting errors if they are connected in (a) series, and (b) parallel.

Parameters: Limiting Error

- **Combination of Limiting Errors**
- **Addition and subtraction**

Example 3.3

Three resistors have the following values: $R_1 = 200\ \Omega \pm 10\%$, $R_2 = 100\ \Omega \pm 5\%$ and $R_3 = 50\ \Omega \pm 5\%$. Determine the magnitude of the resultant resistances and the limiting errors if they are connected in (a) series, and (b) parallel.

Solution

(a) $R = R_1 + R_2 + R_3 = 350\ \Omega$. $\dfrac{\delta R_1}{R_1} = 0.1$ and $\dfrac{\delta R_2}{R_2} = \dfrac{\delta R_3}{R_3} = 0.05$.

Therefore,

$$\begin{aligned}\frac{\delta R}{R} &= \frac{R_1}{R}\frac{\delta R_1}{R_1} + \frac{R_2}{R}\frac{\delta R_2}{R_2} + \frac{R_3}{R}\frac{\delta R_3}{R_3} \\ &= \frac{200}{350}(0.1) + \frac{100}{350}(0.05) + \frac{50}{350}(0.05) \cong 0.079\end{aligned}$$

Thus, $R = 350\ \Omega \pm 7.9\%$.

Reference

$$\frac{dX}{X} = \frac{du}{X} \pm \frac{dv}{X} = \frac{u}{X}\frac{du}{u} \pm \frac{v}{X}\frac{dv}{v}$$

$$\boxed{\frac{\delta X}{X} = \pm\left(\frac{u}{X}\frac{\delta u}{u} + \frac{v}{X}\frac{\delta v}{v}\right)}$$

Parameters: Limiting Error

- **Combination of Limiting Errors**
  - **when final result is a function of individual measurands**
  - **how to combine errors?**
- **Multiplication and division**
  - **For *X* = *uv or u/v***
  - **Taking log and differentiating**

$$\ln X = \ln u \pm \ln v$$

$$\frac{dX}{X} = \frac{du}{u} \pm \frac{dv}{v}$$

  - **Errors:** $\pm\delta u$ **and** $\pm\delta v$

$$\frac{\delta X}{X} = \pm\left(\frac{\delta u}{u} + \frac{\delta v}{v}\right)$$

Parameters: Limiting Error

- **Combination of Limiting Errors**
  - **when final result is a function of individual measurands**
  - **how to combine errors?**
- **Multiplication and division**
  - **In case of indices, e.g., $X = u^m v^n$**
  - **On logarithmic differentiation**

$$\frac{dX}{X} = m\frac{du}{u} + n\frac{dv}{v}$$

$$\frac{\delta X}{X} = \pm\left(m\frac{\delta u}{u} + n\frac{\delta v}{v}\right)$$

Parameters: Limiting Error

- **Combination of Limiting Errors**
- **Multiplication and division**

Example 3.6
What is the percentage error in the measurement of kinetic energy of a body if percentage errors in the measurement of mass and speed are 2% and 3% respectively?

Parameters: Limiting Error

- ❑ **Combination of Limiting Errors**
- ❑ **Multiplication and division**

Example 3.6
What is the percentage error in the measurement of kinetic energy of a body if percentage errors in the measurement of mass and speed are 2% and 3% respectively?

Solution
We know that the expression for kinetic energy is

$$E = \frac{1}{2}mv^2$$

Taking logarithm of both sides and then differentiating, we get

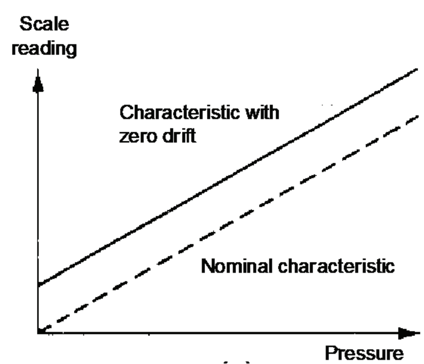
$$\begin{aligned}\frac{\delta E}{E} &= \frac{\delta m}{m} + 2\frac{\delta v}{v} \\ &= 0.02 + 2(0.03) = 0.08\end{aligned}$$

Thus, the required error is 8%.

Statistical Treatment

Statistical Treatment

- ❑ **Systematic errors**
  - ❑ **Systematic errors can be removed by calibration (e.g. removing bias) and controlling environmental conditions**
  - ✓ **For example, zero drift or bias**



Statistical Treatment

- ❑ **Random errors**
  - ❑ **Random variations of in the measured quantity due to unknown causes**
  - ❑ **Cannot be corrected by calibration or control**

Statistical Treatment

- ❑ **Random errors**
  - ❑ **Random variations of in the measured quantity due to unknown causes**
  - ❑ **Cannot be corrected by calibration or control**

  - ❑ **Solution: take a number of readings and apply statistics to obtain the best approximation of the true value**

➢ **Statistical treatment**

Statistical Treatment

- multi-sample test
    - different instruments, different observers and different methods
- single-sample test
    - instrument, observer and method remaining the same, the data have been acquired at different times

Statistical Treatment

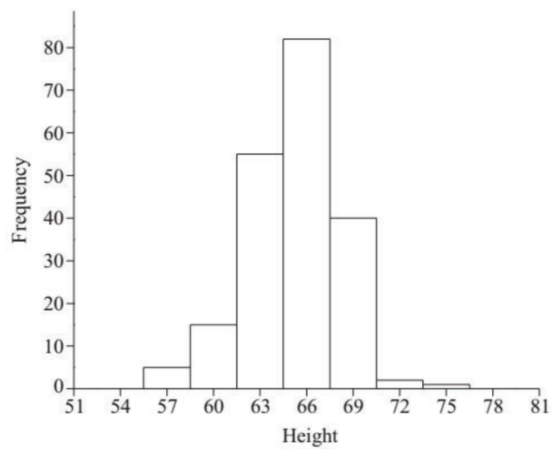
Statistical analysis of measurement data help us to get a better insight into it and estimate errors.

Suppose heights of 200 college students is measured and recorded in inches. How to conveniently summarize that data?

By making a frequency distribution graph

Statistical Treatment

- ❑ **Statistical methods are employed to estimate random errors**
  - **Frequency distribution of the height of 200 students.**
- **Height is divided in classes or cells**



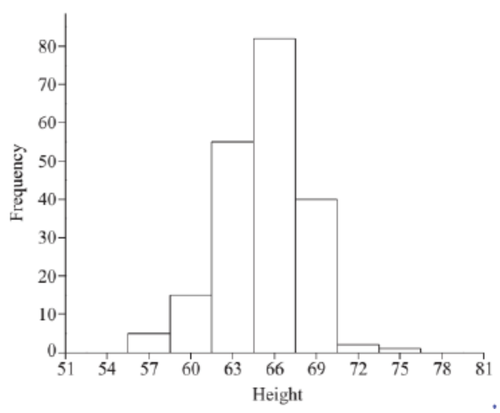
➢ **Histogram or frequency distribution**

Statistical Treatment

Height of one student maybe 62.35 inches. Another may be 58.75 inches.

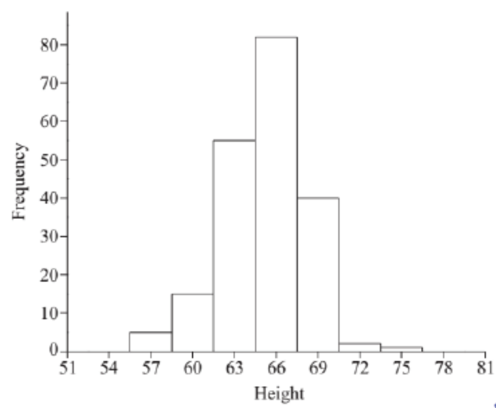
No two students may have the exact same height.

To make a histogram we divide the height of students within a few classes or cells such as 56-58 inches, 58-61 inches and so on.



How to characterize the frequency distribution data?

Statistical Treatment



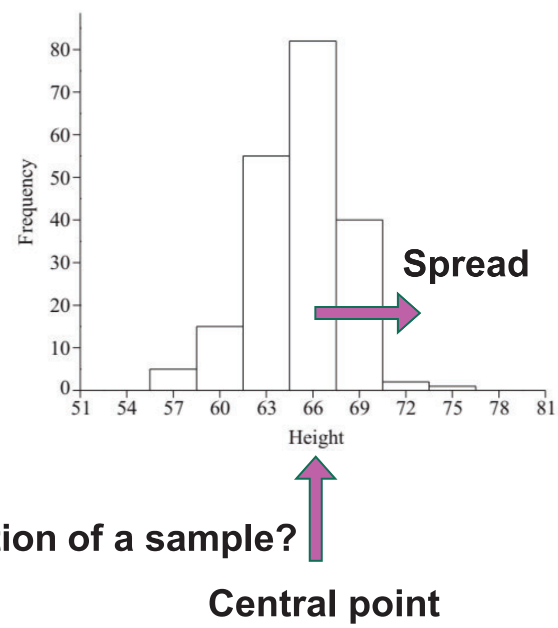
We can characterize the frequency distribution with two descriptive measures.

Central point/ tendency of distribution

Spread

Statistical Treatment

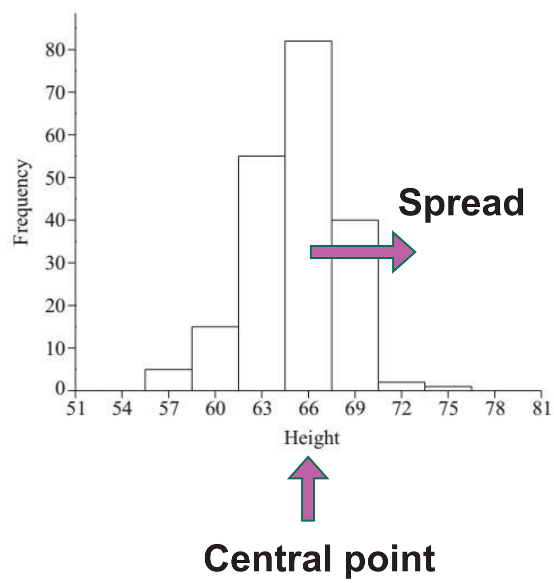
- ❑ **Statistical methods are employed to estimate random errors**
  - ▪ **Frequency distribution of the height of 200 students.**
- ▪ **Height is divided in classes or cells**



- ❑ **Histogram or frequency distribution**
  - ❑ **how to characterize the frequency distribution of a sample?**
    1. **Central point of the distribution**
    2. **Spread**

Statistical Treatment

- Histogram or frequency distribution of the height of 200 students.



1. Measures of central tendency
2. Measures of spread

Measures of Central Tendency

- **A central point or average is a value which is a representative of a set of data**

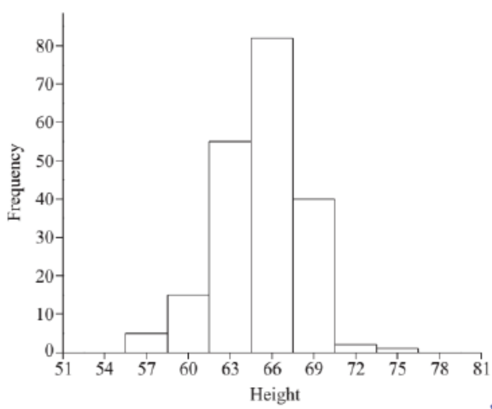
- **Six types of averages**

  1. Mode
  2. Median
  3. Arithmetic mean or simply, mean
  4. Geometric mean
  5. Harmonic mean
  6. Root mean square

Measures of Central Tendency

Mode

Mode is the most frequent value amongst set of measurements.



What is the mode in this set of measurements?

Measures of Central Tendency

Mode

Mode may not necessarily exist.

Mode may not necessarily be unique.

Example 3.7
What are the mode values for the following three sets?

(a) 2, 3, 5, 6, 9, 10, 10, 10, 11, 12, 12, 14, 18
(b) 3, 5, 6, 9, 10, 11, 12, 14, 18
(c) 2, 3, 5, 5, 5, 6, 9, 10, 11, 11, 11, 12, 12, 14, 18

Measures of Central Tendency: Mode

- **The most frequent value**

Example 3.7
What are the mode values for the following three sets?

(a) 2, 3, 5, 6, 9, 10, 10, 10, 11, 12, 12, 14, 18
(b) 3, 5, 6, 9, 10, 11, 12, 14, 18
(c) 2, 3, 5, 5, 5, 6, 9, 10, 11, 11, 11, 12, 12, 14, 18

Solution
The mode of set (a) is 10. Set (b) has no mode. Set (c) has two modes, 5 and 11.

Measures of Central Tendency: Median

- **The value below which half of the values in the sample fall**
  - **if the *N* number of data is arranged according to magnitude**
    - **if *N* is odd, it is the value corresponding to the (*N* ÷ 2 + 0.5)th data**
    - **If *N* is even, the median is represented by the average of the (*N* ÷ 2)th and (N ÷ 2 + 1)th points**

Measures of Central Tendency

Median

The median is the value below which half the values in the sample fall. So, if the number of data N, arranged according to magnitude, is odd, it is the value corresponding to the $(N \div 2 + 0.5)$th data. If N is even, the median is represented by the average of the $(N \div 2)$th and $(N \div 2 + 1)$th points.

Example 3.8

Find the medians of the given sets of data:

(a) 2, 3, 4, 4, 6, 7, 7, 7, 9

(b) 3, 3, 7, 8, 12, 13, 16, 19

Measures of Central Tendency: Median

- **The value below which half of the values in the sample fall**
  - **if the *N* number of data is arranged according to magnitude**
    - **if *N* is odd, it is the value corresponding to the (*N* ÷ 2 + 0.5)th data**
    - **If *N* is even, the median is represented by the average of the (*N* ÷ 2)th and (N ÷ 2 + 1)th points**

Example 3.8

Find the medians of the given sets of data:

(a) 2, 3, 4, 4, 6, 7, 7, 7, 9

(b) 3, 3, 7, 8, 12, 13, 16, 19

Solution

(a) The number of data points is 9 which is odd. So, the $(9 \div 2) + 0.5 = 5$th point, i.e. 6, is the median.

(b) Here the number of data points, 8, is even. So, the median is the average of the $8 \div 2 = 4$th and 5th points, i.e. $(8 + 12) \div 2 = 10$.

Measures of Central Tendency

Arithmetic Mean

The arithmetic mean is considered as most probable value of the measurand.

$$\mu = \frac{1}{n}(x_1 + x_2 + \cdots + x_n) = \frac{1}{n}\sum_{i=1}^{n} x_i$$

Measures of Central Tendency: Arithmetic Mean

- **If x_i's are individual data and n is the number of measurements**

$$\mu = \frac{1}{n}(x_1 + x_2 + \cdots + x_n) = \frac{1}{n}\sum_{i=1}^{n} x_i$$

- **If the data x_1 , x_2 , . . . , x_k occur f_1 , f_2, . . . , f_k times respectively**

$$\mu = \frac{f_1x_1 + f_2x_2 + \cdots + f_kx_k}{f_1 + f_2 + \cdots + f_k} = \frac{1}{n}\sum_{i=1}^{n} f_ix_i$$

- **Most probable value of the measured variable**
  - **Best approximation is at a large theoretically infinite number of readings**

Measures of Central Tendency: Arithmetic Mean

Example 3.9

If 10, 16, 12 and 4 occur with frequencies 5, 3, 4 and 2 respectively then what is the arithmetic mean?

Solution

The arithmetic mean is

$$\mu = \frac{(5)(10) + (3)(16) + (4)(12) + (2)(4)}{5 + 3 + 4 + 2} = 11$$

Measures of Central Tendency: Geometric Mean

- **The geometric mean g_m of a set of n numbers $x_1, x_2, \ldots, x_n$ is the nth root of the product of the numbers**

$$g_m = \sqrt[n]{x_1 x_2 x_3 \ldots x_n}$$

Measures of Central Tendency

Geometric Mean

Example 3.10 $g_m = \sqrt[n]{x_1 x_2 x_3 \ldots x_n}$

The mass of a substance is being measured in a faulty common balance having unequal arm lengths. Show that the true mass of the substance is the geometric mean of the masses determined by placing the substance once on the left pan and next time on the right pan of the balance.

Measures of Central Tendency: Geometric Mean

Example 3.10 $g_m = \sqrt[n]{x_1 x_2 x_3 \ldots x_n}$

The mass of a substance is being measured in a faulty common balance having unequal arm lengths. Show that the true mass of the substance is the geometric mean of the masses determined by placing the substance once on the left pan and next time on the right pan of the balance.

Solution

Let the true mass of the substance be m and the lengths of the left and right arms of the balance be x_1 and x_2 respectively. Initially, the substance is placed on the left pan and a mass m_1 on the right pan balances it. Then

$$m x_1 = m_1 x_2$$

or

$$m = m_1 \frac{x_2}{x_1}$$

Next, the mass is placed on the right pan and a mass m_2 on the left pan balances it. Then

$$m x_2 = m_2 x_1$$

or

$$x_2 = \frac{m_2}{m} x_1$$

Substituting this value of x_2 in the first equation, we get

$$m = m_1 \frac{x_2}{x_1} = \frac{m_1}{x_1} \cdot \frac{m_2}{m} x_1$$

or

$$m^2 = m_1 m_2$$

or

$$m = \sqrt{m_1 m_2}$$

Measures of Central Tendency

Geometric Mean

The Geometric Mean is useful when we want to compare things with very different properties.

Example: you want to buy a new camera.

- One camera has a zoom of 200 and gets an 8 in reviews,
- The other has a zoom of 250 and gets a 6 in reviews.

Comparing using the usual arithmetic mean gives (200+8)/2 = **104** vs (250+6)/2 = **128**. The zoom is such a big number that the user rating gets lost.

But the geometric means of the two cameras are:

- $\sqrt{(200 \times 8)}$ = **40**
- $\sqrt{(250 \times 6)}$ = **38.7...**

So, even though the zoom is 50 bigger, the lower user rating of 6 is still important.

Measures of Central Tendency: Harmonic Mean

- **The harmonic mean, h_m of a set of n numbers x_1 , x_2 , . . . , x_n is the reciprocal of the arithmetic mean of the reciprocals of number**

$$h_m = \frac{1}{\frac{1}{n}\sum_{i=1}^{n}\frac{1}{x_i}} = \frac{n}{\sum_{i=1}^{n}\frac{1}{x_i}}$$

The harmonic mean is best used for fractions such as rates or multiples.

Measures of Central Tendency

Harmonic Mean

$$h_m = \frac{1}{\frac{1}{n}\sum_{i=1}^{n}\frac{1}{x_i}} = \frac{n}{\sum_{i=1}^{n}\frac{1}{x_i}}$$

Example 3.11
A person travels from X to Y at an average speed of 60 km/h and returns by the same route at an average speed of 50 km/h. Find the average speed for the round trip.

Measures of Central Tendency: Harmonic Mean

$$h_m = \frac{1}{\frac{1}{n}\sum_{i=1}^{n}\frac{1}{x_i}} = \frac{n}{\sum_{i=1}^{n}\frac{1}{x_i}}$$

Example 3.11

A person travels from X to Y at an average speed of 60 km/h and returns by the same route at an average speed of 50 km/h. Find the average speed for the round trip.

Solution

Let the distance between the two places be x km. Then, if t_1 and t_2 be the time (in h) taken for the onward and return trips, we have

$$t_1 = \frac{x}{60}$$

$$t_2 = \frac{x}{50}$$

S= d//t

Therefore, the average speed v_{av} for the round trip is

$$v_{\mathrm{av}} = \frac{\text{total distance}}{\text{total time}} = \frac{2x}{t_1 + t_2} = \frac{2x}{\frac{x}{60} + \frac{x}{50}} = \frac{2}{\frac{1}{60} + \frac{1}{50}}$$

We observe that v_{av} is nothing but the harmonic mean of the two speeds.

Measures of Central Tendency: Root Mean Square

- **The root mean square (rms) or quadratic mean of a set of *n* numbers x_1 , x_2 , . . . , x_n**

$$\text{rms} = \sqrt{\frac{1}{n}\sum_{i=1}^{n} x_i^2}$$

- **Such averaging is quite common in engineering and physical applications**
  - **Any example?**

Electrical engineers often need to know the power, *P*, dissipated by an electrical resistance, *R*. It is easy to do the calculation when there is a constant current, *I*, through the resistance. For a load of *R* ohms, power is defined simply as:

$$P = I^2 R.$$

However, if the current is a time-varying function, $I(t)$, this formula must be extended to reflect the fact that the current (and thus the instantaneous power) is varying over time.

$$= I_{\text{RMS}}^2 R \qquad \text{by definition of root-mean-square}$$

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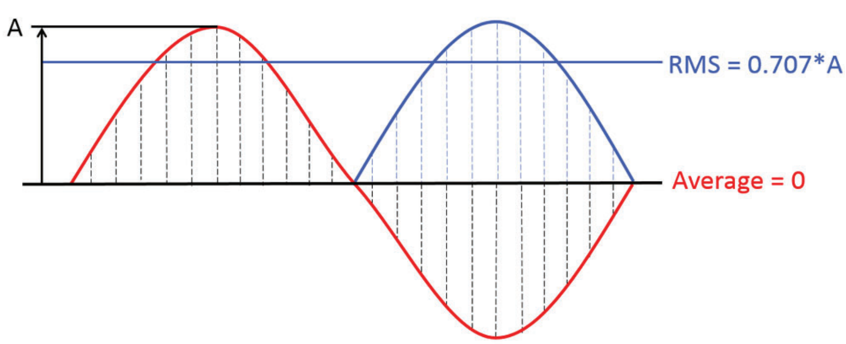
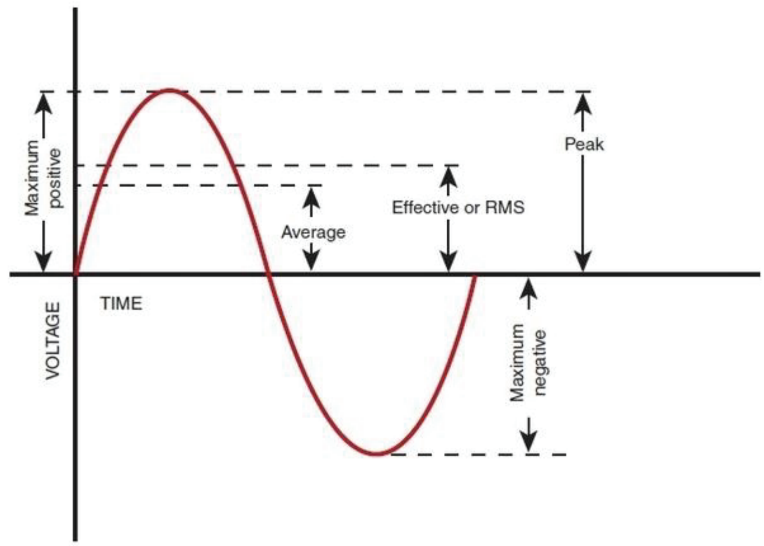


RMS Concepts

General equation for the RMS value of a periodic function.

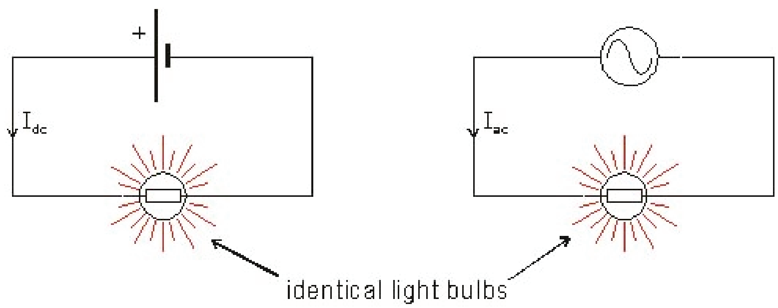
$$V_{RMS} = \underbrace{\sqrt{\underbrace{\frac{1}{T}\int_0^T \underbrace{[V(t)]^2}_{\text{Square}}\, dt}_{\text{Mean}}}}_{\text{Root}}$$

In everyday **use**, **AC** voltages (and currents) are always given as **RMS values** because this allows a sensible comparison to be made with steady DC voltages (and currents), such as from a battery. For example, a 6V **AC** supply means 6V **RMS** with the peak voltage about 8.6V.

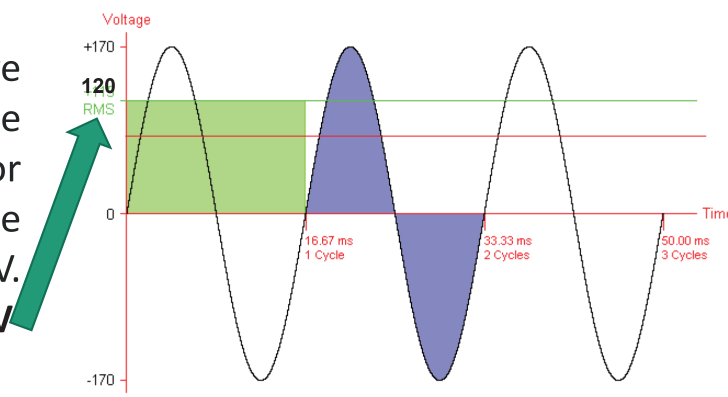


Attempts to find an average value of AC would directly provide you the answer zero... Hence, **RMS** values are used. They help to find the effective value of AC (voltage or current). This **RMS** is a mathematical quantity (used in many math fields) used to compare both alternating and direct currents (or voltage).

RMS Concepts



In order to relate both, we have nothing to use better than the RMS value. The direct voltage for the bulb is 115 V while the alternating voltage is 170 V. Hence, Vrms=Vdc=Vac/√2= **120 V**



Queries

Thanks!