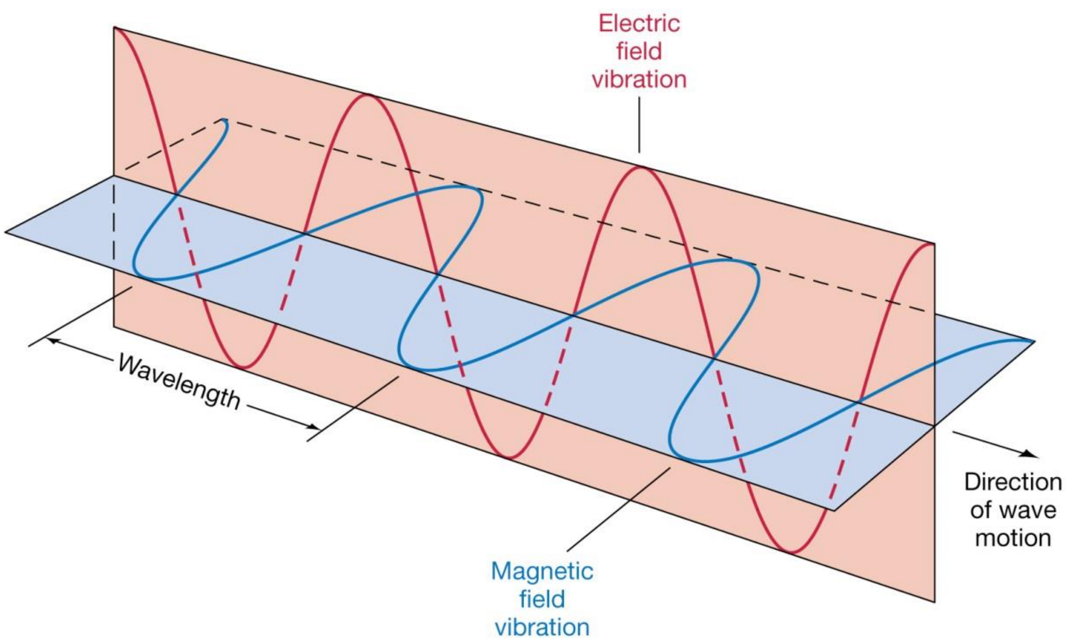
COURSE INTRODUCTION AND VECTOR ALGEBRA

Electromagnetics

- Electromagnetics (EM) is a branch of physics or electrical engineering in which electric and magnetic phenomena are studied
- (EM) may be regarded as the study of the interactions between electric charges at rest and in motion

- EM principles find applications in various disciplines such as microwaves, antennas, electric machines, satellite communications, fibre optics, electromagnetic interference and compatibility ….



Scalars and Vectors

- Vector analysis is a mathematical tool with which EM concepts are most conveniently expressed and comprehended
- A **scalar** is a quantity that has only magnitude (time, mass, distance ....)
- A **vector** is a quantity that has both magnitude and direction (velocity, force, electric field intensity …)
- EM theory is essentially a study of some particular fields
- A **field** can be scalar or vector and is a function that specifies a particular quantity everywhere in a region

- Examples of **scalar fields** are temperature distribution in a building, electric potential in a region …
- The gravitational force on a body in space is an example of **vector field**

Scalars and Vectors

- A vector **A** has both magnitude and direction
- A unit vector $\mathbf{a}_A$ along **A** is defined as a vector whose magnitude is unity and its direction is along **A**, that is

$$\boldsymbol{a}_A = \frac{\mathbf{A}}{|\mathbf{A}|}$$

- A vector **A** in Cartesian (or rectangular) coordinates may be represented as:

$$\boldsymbol{A} = A_x\boldsymbol{a}_x + A_y\boldsymbol{a}_y + A_z\boldsymbol{a}_z$$

- Where A_x, A_y and A_z are called the components of **A** in the x, y, and z directions, respectively
- $\mathbf{a}_x$, $\mathbf{a}_y$ and $\mathbf{a}_z$ are the unit vectors in the x, y, and z directions, respectively
- Therefore, the unit vector along **A** may be written as:

$$\mathbf{a}_A = \frac{A_x\mathbf{a}_x + A_y\mathbf{a}_y + A_z\mathbf{a}_z}{\sqrt{A_x^2 + A_y^2 + A_z^2}}$$

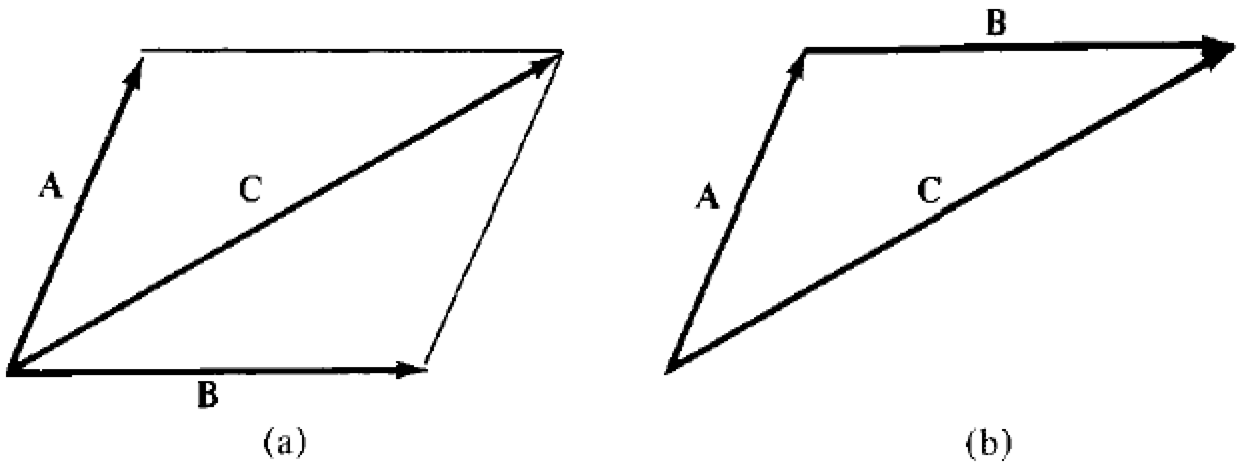
Vector Addition and Subtraction

- Two vectors **A** and **B** can be added together to give another vector **C**, that is: $\mathbf{C} = \mathbf{A} + \mathbf{B}$
- The vector addition is carried out component by component
- Thus, if **A** = (A_x, A_y, A_z) and **B** = (B_x, B_y, B_z), then

$$\mathbf{C} = (A_x + B_x)\mathbf{a}_x + (A_y + B_y)\mathbf{a}_y + (A_z + B_z)\mathbf{a}_z$$

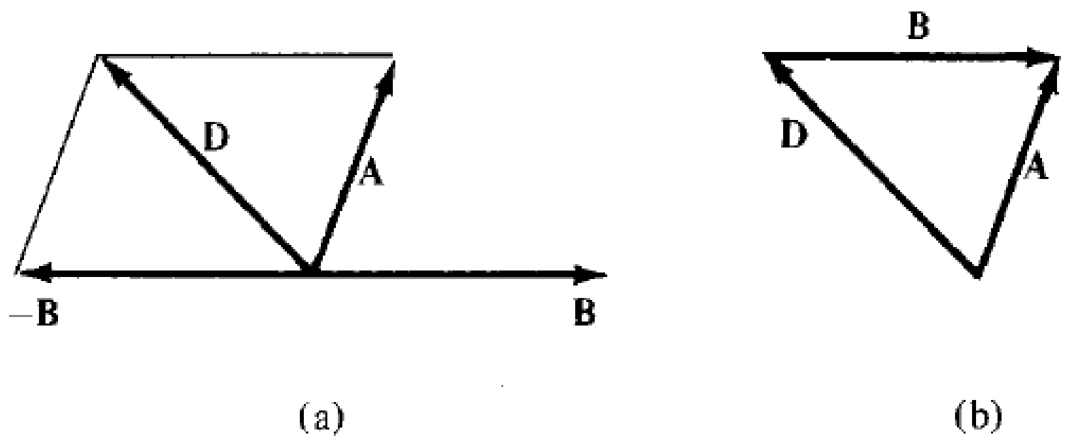
- Vector subtraction is similarly carried out as:

$$\begin{aligned}\mathbf{D} &= \mathbf{A} - \mathbf{B} = \mathbf{A} + (-\mathbf{B}) \\ &= (A_x - B_x)\mathbf{a}_x + (A_y - B_y)\mathbf{a}_y + (A_z - B_z)\mathbf{a}_z\end{aligned}$$



Vector addition **C** = **A** + **B**: **(a)** parallelogram rule,
(b) head-to-tail rule.

Vector Addition and Subtraction



Vector subtraction $\mathbf{D} = \mathbf{A} - \mathbf{B}$: (**a**) parallelogram rule,
(**b**) head-to-tail rule.

- The three basic laws of algebra obeyed by any given vectors **A**, **B**, and **C**, are summarized as follows:

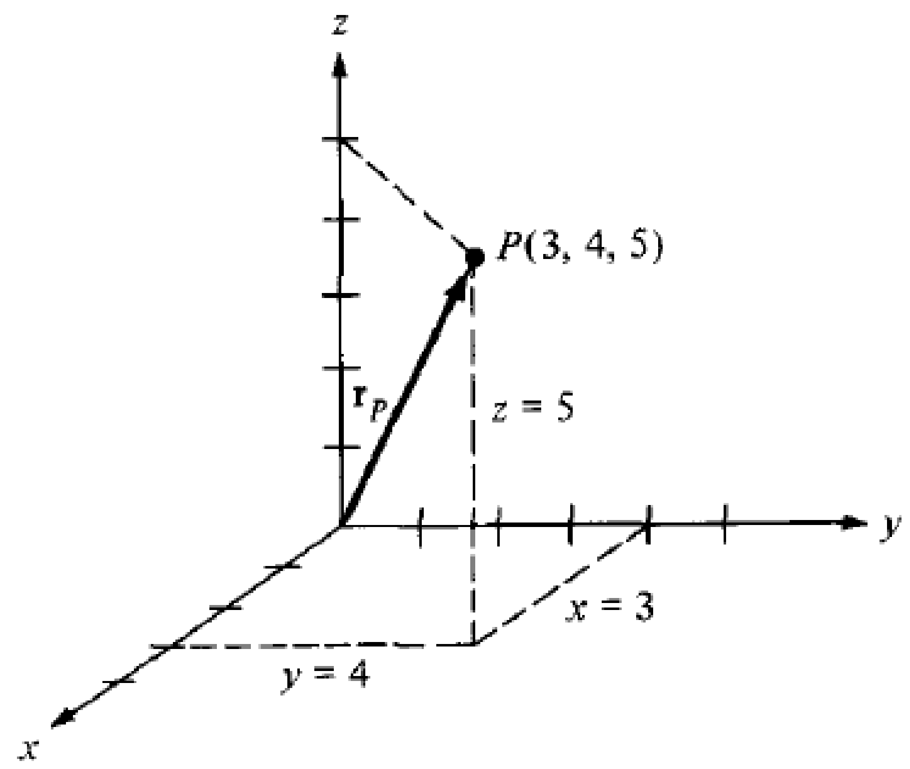
Law	Addition	Multiplication
Commutative	$\mathbf{A} + \mathbf{B} = \mathbf{B} + \mathbf{A}$	$k\mathbf{A} = \mathbf{A}k$
Associative	$\mathbf{A} + (\mathbf{B} + \mathbf{C}) = (\mathbf{A} + \mathbf{B}) + \mathbf{C}$	$k(\ell\mathbf{A}) = (k\ell)\mathbf{A}$
Distributive	$k(\mathbf{A} + \mathbf{B}) = k\mathbf{A} + k\mathbf{B}$	

Position Vector

- Point P in Cartesian coordinates may be represented by (x, y, z)
- The **position vector $\mathbf{r}_P$** (or radius vector) of point P is defined as the directed distance from the origin O to P, i.e.

$$\mathbf{r}_P = OP = x\mathbf{a}_x + y\mathbf{a}_y + z\mathbf{a}_z$$

- For point (3, 4, 5), the position vector is shown in the figure

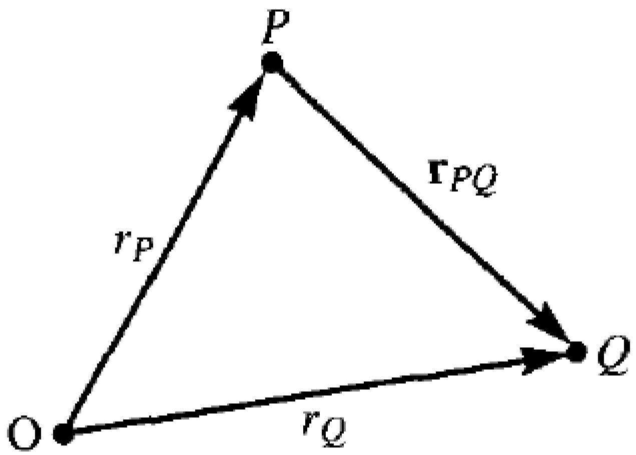


Distance Vector

- The **distance vector** is the displacement from one point to another

- For two points P and Q given by (x_P, y_P, z_p) and (x_Q, y_Q, z_Q), the distance vector (or separation vector) is the displacement from P to Q, that is:

$$
\begin{aligned}
\mathbf{r}_{PQ} &= r_Q - r_P \\
&= (x_Q - x_P)\mathbf{a}_x + (y_Q - y_P)\mathbf{a}_y + (z_Q - z_P)\mathbf{a}_z
\end{aligned}
$$



Distance Vector

- Both P and **A** may be represented in the same manner as (x, y, z) and (A_x, A_y, A_z), respectively

- However, the point P is not a vector; only its position vector $\mathbf{r}_P$ is a vector

- A vector field is said to be constant or uniform if it does not depend on space variables x, y, and z

- For example, vector $\mathbf{B} = 3\mathbf{a}_x - 2\mathbf{a}_y + 10\mathbf{a}_z$ is a uniform vector while vector $\mathbf{A} = 2xy\mathbf{a}_x + y^2\mathbf{a}_y - xz^2\mathbf{a}_z$ is not uniform

Vector Multiplication – Dot Product

- The dot product of two vectors **A** and **B**, written as **A • B**, is defined geometrically as the product of the magnitudes of **A** and **B** and the cosine of the angle between them

$$\mathbf{A} \cdot \mathbf{B} = AB \cos \theta_{AB}$$

- Also called **scalar product**

- If **A** = (Ax, Ay, Az) and **B** = (Bx, By, Bz), then

$$\mathbf{A} \cdot \mathbf{B} = A_x B_x + A_y B_y + A_z B_z$$

- Note that:

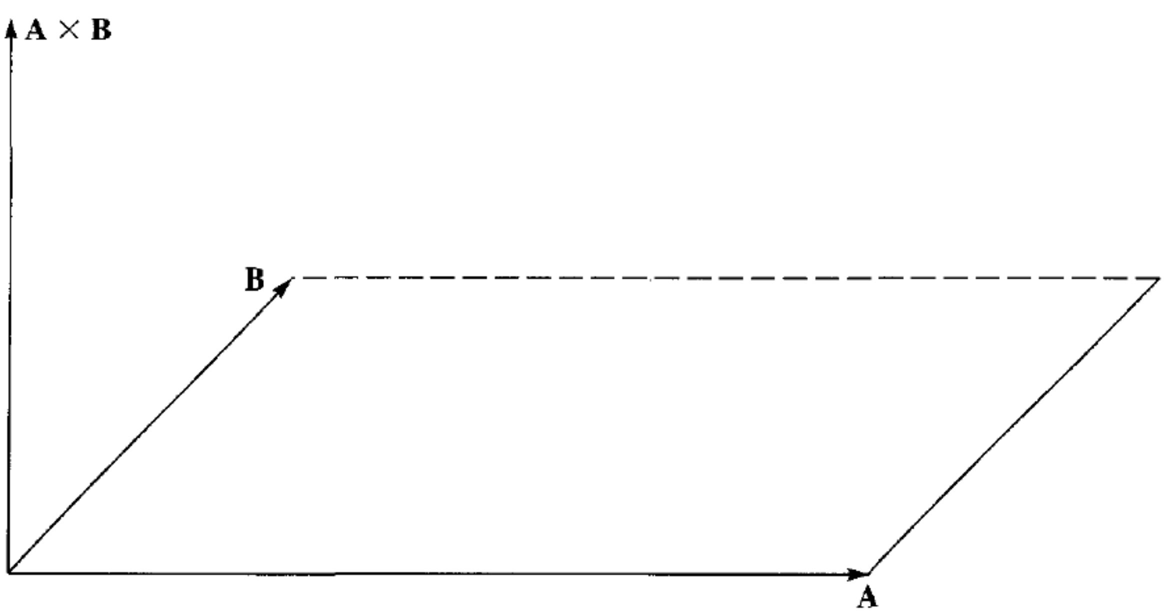
$$\mathbf{a}_x \cdot \mathbf{a}_y = \mathbf{a}_y \cdot \mathbf{a}_z = \mathbf{a}_z \cdot \mathbf{a}_x = 0$$

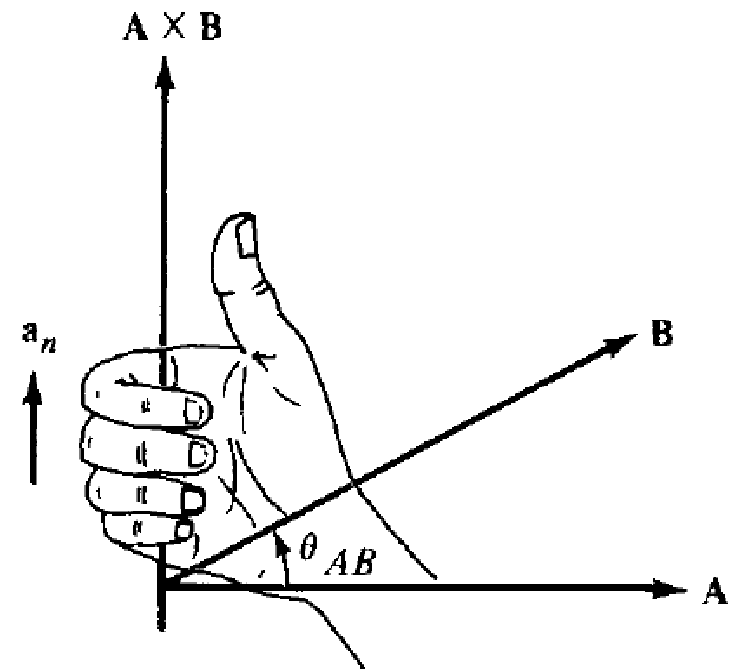
$$\mathbf{a}_x \cdot \mathbf{a}_x = \mathbf{a}_y \cdot \mathbf{a}_y = \mathbf{a}_z \cdot \mathbf{a}_z = 1$$

Vector Multiplication – Cross Product

- The cross product of two vectors **A** and **B**, written as **A** x **B**, is a vector quantity whose magnitude is the area of the parallelopiped formed by **A** and **B** and is in the direction of the right thumb when the fingers of the right hand rotate from **A** to **B**

$$\mathbf{A} \times \mathbf{B} = AB \sin \theta_{AB} \mathbf{a}_n$$





Vector Multiplication – Cross Product

- If **A** = (Ax, Ay, Az) and **B** = (Bx, By, Bz), then

$$\boxed{\mathbf{A} \times \mathbf{B} = \begin{vmatrix} \mathbf{a}_x & \mathbf{a}_y & \mathbf{a}_z \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}}$$

$$= (A_yB_z - A_zB_y)\mathbf{a}_x + (A_zB_x - A_xB_z)\mathbf{a}_y + (A_xB_y - A_yB_x)\mathbf{a}_z$$

- Note that:

$$\mathbf{a}_x \times \mathbf{a}_y = \mathbf{a}_z$$

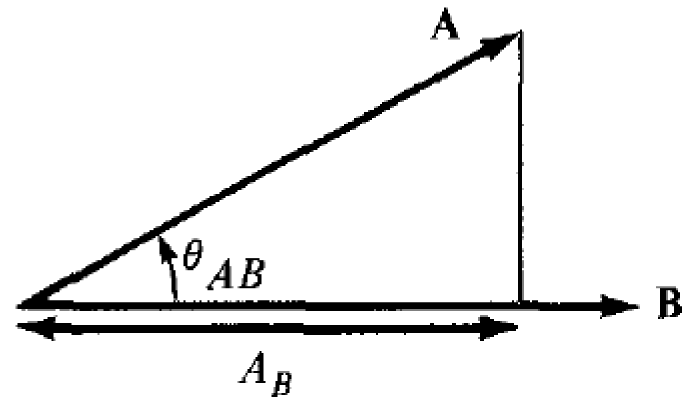
$$\mathbf{a}_y \times \mathbf{a}_z = \mathbf{a}_x$$

$$\mathbf{a}_z \times \mathbf{a}_x = \mathbf{a}_y$$

Components of a Vector

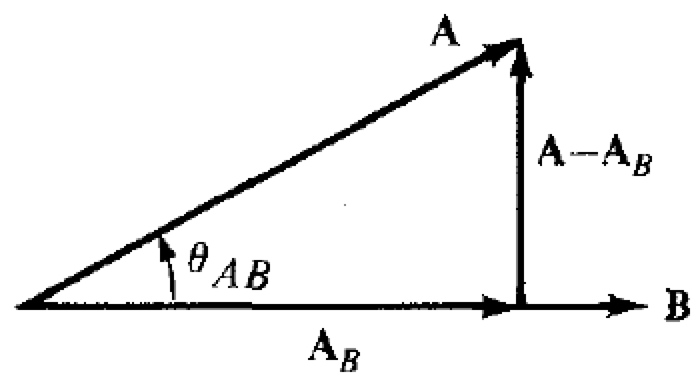
- Scalar Component:

$$A_B = \mathbf{A} \cdot \mathbf{a}_B$$



- Vector Component:

$$\mathbf{A}_B = A_B\mathbf{a}_B = (\mathbf{A} \cdot \mathbf{a}_B)\mathbf{a}_B$$



Problem- 1

- Let $\mathbf{E} = 3\mathbf{a}_y + 4\mathbf{a}_z$, and $\mathbf{F} = 4\mathbf{a}_x - 10\mathbf{a}_y + 5\mathbf{a}_z$

(a) Find the vector component of **E** along **F**

(b) Determine a unit vector perpendicular to both **E** and **F**