Using mappings to Solve a Dirichlet problem:

we know that

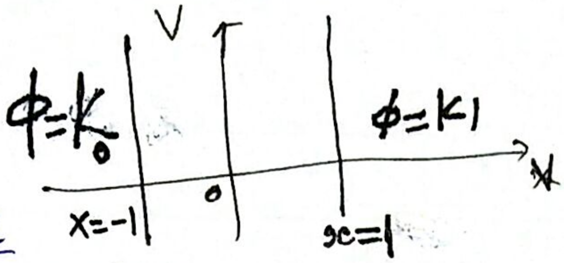


Fig 1

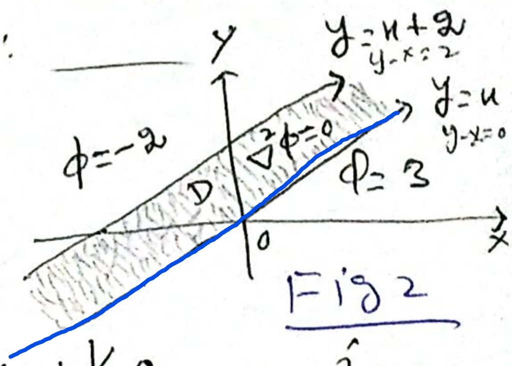
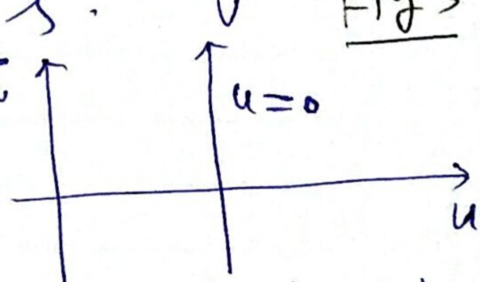


Fig 2

Solution is $\phi(x) = \dfrac{K_1-K_0}{2}x + \dfrac{K_1+K_0}{2}$ — (i)

We rotate the lines by an angle $\frac{\pi}{4}$ radians to reach at vertical lines.

Fig 3



$R(z) = \sqrt{2}\, e^{i\pi/4}$

$w = e^{i\frac{\pi}{4}} z = \left(\frac{1}{\sqrt{2}} + i\frac{1}{\sqrt{2}}\right)(x+iy)$

$= \frac{1}{\sqrt{2}}(x-y) + \frac{1}{\sqrt{2}}(x+y)i$

giving transformation equations as $u = \frac{1}{\sqrt{2}}(x-y)$, $v = \frac{1}{\sqrt{2}}(x+y)$

maps $y=x$ on $u=0$ & $y=x+2$ on $u=-\sqrt{2}$

To make map $u=-1$, we take $R(z) = \sqrt{2}\, e^{i\pi/4}$, we define translation by $z=-1$, $f(z) = \sqrt{2}\, e^{i\pi/4} + 1 = (1+i)z + 1$.

transformation of BC's: $w = f(z) = (1+i)(x+iy) = (x-y+1) + i(x+y)$. (ii)

$\underline{y=x+2:}$ $w = u+iv = x-(x-2)+1+(x+(x+2))i = -1 + 2(x+1)i$

which is the line $u=-1$. Similarly,

$\underline{y=x:}$ $w = u+iv = x-x+1+(x+x)i = 1+2xi$

which is the line $u=1$.

$\phi(x, x+2) = -2 = \Phi(-1, v)$, $\phi(x,x) = 3 = \Phi(1, v)$.

The solution due to equation (i) is

$\Phi(u,v) = \dfrac{3-(-2)}{2}u + \dfrac{-2+3}{2} = \dfrac{5}{2}u + \dfrac{1}{2}$

Due to (ii), we have $u(x,y) = x-y+1$, and $v(x,y) = x+y$

$\phi(x,y) = \Phi(u(x,y), v(x,y)) = \frac{5}{2}(x-y+1) + \frac{1}{2} = \frac{5}{2}x - \frac{5}{2}y + 3$

Example 2, page 168-169, Example 1, page 225-227

Sec 2.7, 3.4, 4.5

<u>Check:</u> $\phi(x,x) = 3$, $\phi(x, x+2) = \frac{5}{2}x - \frac{5}{2}(x+2) + 3 = \frac{5}{2}x - \frac{5}{2}x - 5 + 3 = -2$.

using CREs, the harmonic conjugate the standard problem of Fig(i) is $\psi = \frac{K_1-K_0}{2}y$, the complex potential function is $\Omega(z) = \frac{K_1-K_0}{2}x + \frac{K_1+K_0}{2} + i\,\frac{K_1-K_0}{2}y$.