

Quiz 2: Digital Communication Systems

Suppose you are designing a wireless link from moon to earth (free space path loss exponent) using carrier frequency of 2.3 GHz. The range is 400,000 km. On the moon, the transmit power is 2000 mW and the antenna gain is 4 dBi. Suppose that a received power of -90 dBm is necessary for you to receive BPSK signal at the desired bit error rate. Assume line losses of 1.5 dB at each end.

- Evaluate the EIRP in dBW? (1 Marks)
- If the effective area of a parabolic dish antenna is the actual area times the antenna efficiency, and the efficiency is 0.75, then evaluate the required diameter of the antenna on the earth? (4 Marks)
- If log-normal shadowing is also present with $\sigma=6$ dB, and the outage probability at the receiver is 30%, what is the fraction of area with signal above the threshold? (1 Mark)
- If a BPSK signal is received at the receiver with above outage probability, solve for the threshold power. (2 Marks)
- If the channel experiences multipath fading and the rms delay of the wireless channel is 0.7 microseconds, what would be the data rate of the system that ensures no equalizer is required at the receiver? (2marks)

Moon \rightarrow Earth

Free space path loss exponent, $n=2$

$$f_c = 2.3 \text{ GHz} = 2.3 \times 10^9 \text{ Hz}$$

$$d = 400,000 \text{ km} = 4 \times 10^8 \text{ m}$$

$$P_t = 2000 \text{ mW} = 3.010 \text{ dB}$$

$$G_t = 4 \text{ dBi} = 2.512$$

$$P_r = -90 \text{ dBm} = 1 \times 10^{-12} \text{ W}$$

$$\text{Line loss : } L_t = L_r = 1.5 \text{ dB} = 1.4125$$

$$(a) \text{ EIRP (dBW)} = ?$$

$$\text{EIRP [dBW]} = P_t + G_t - L_t$$

$$= 3.010 + 4 - 1.5 = 5.51 \text{ dBW}$$

$$\text{EIRP [dBW]} = 5.51 \text{ dBW}$$

$$(b) D_r = ?$$

$$\eta = 0.75$$

D_r can be calculated using following formulas:

$$P_r = \frac{P_t \times G_t \times G_r \times \lambda^2}{(4\pi)^2 \times d^2 \times L_t \times L_r}, \quad G_r = \frac{4\pi A_e}{\lambda^2}, \quad A_e = \eta A, \quad A = \frac{\pi D_r^2}{4}$$

$$G_r = \frac{P_r \times (4\pi)^2 \times d^2 \times L_t \times L_r}{P_t \times G_t \times \lambda^2} = \frac{(1 \times 10^{12}) \times (4\pi)^2 \times (4 \times 10^8)^2 \times (1.412)^2}{(2000 \times 10^3) \times (2.512) \times \left(\frac{3 \times 10^8}{2.3 \times 10^9}\right)^2}$$

$$G_r = 0.589 \times 10^9$$

$$A_e = \frac{G_r \times \lambda^2}{4\pi} = \frac{0.589 \times 10^9 \times \left(\frac{3 \times 10^8}{2.3 \times 10^9}\right)^2}{4\pi} = 0.797 \times 10^6 \text{ m}^2$$

$$A = \frac{A_e}{\eta} = \frac{0.797 \times 10^6}{0.75} = 1.063 \times 10^6 \text{ m}^2$$

$$D_r = \sqrt{\frac{4A}{\pi}} = \sqrt{\frac{4 \times 1.063 \times 10^6}{\pi}} = 1163.5 \text{ m}$$

c) $\sigma = 6 \text{ dB}$, $n=2$, outage Probability = 30%.

$$\sigma_n = \frac{6}{2} = 3$$

Boundary Coverage: $P(\text{Pr}(R) > \gamma) = 100 - 30 = 70\%$.

Using the graph in slide 22 of Lecture - 05:

The fraction of area with signal above the threshold $\approx 86\%$.

$$(d) P[\text{Pr}(d) < \gamma] = Q\left[\frac{\overline{\text{Pr}(d)} - \gamma}{\sigma}\right]$$

$$0.3 = Q\left[\frac{-90 - \gamma}{6}\right]$$

Using the Q-function table

$$Q^{-1}(0.3) = 0.5244$$

$$\frac{-90 - \gamma}{6} = 0.5244 \Rightarrow \gamma = -[6 \times 0.5244 + 90] = -93.1464 \text{ dBm}$$

(e) Equalizer is not required if the channel undergoes flat fading

$$\frac{\sigma_c}{T_s} \leq 0.1$$

$$\frac{0.7 \times 10^6}{0.1} \leq T_s$$

$$T_s \geq 7 \times 10^6 \text{ s}$$

$$R_s = \frac{1}{T_s}$$

$$R_s = 1 / 7 \times 10^6 = 142.86 \text{ kbps}$$

For BPSK $R_b = R_s$, $R_b = 142.86 \text{ kbps}$.