Magnetic Field-I

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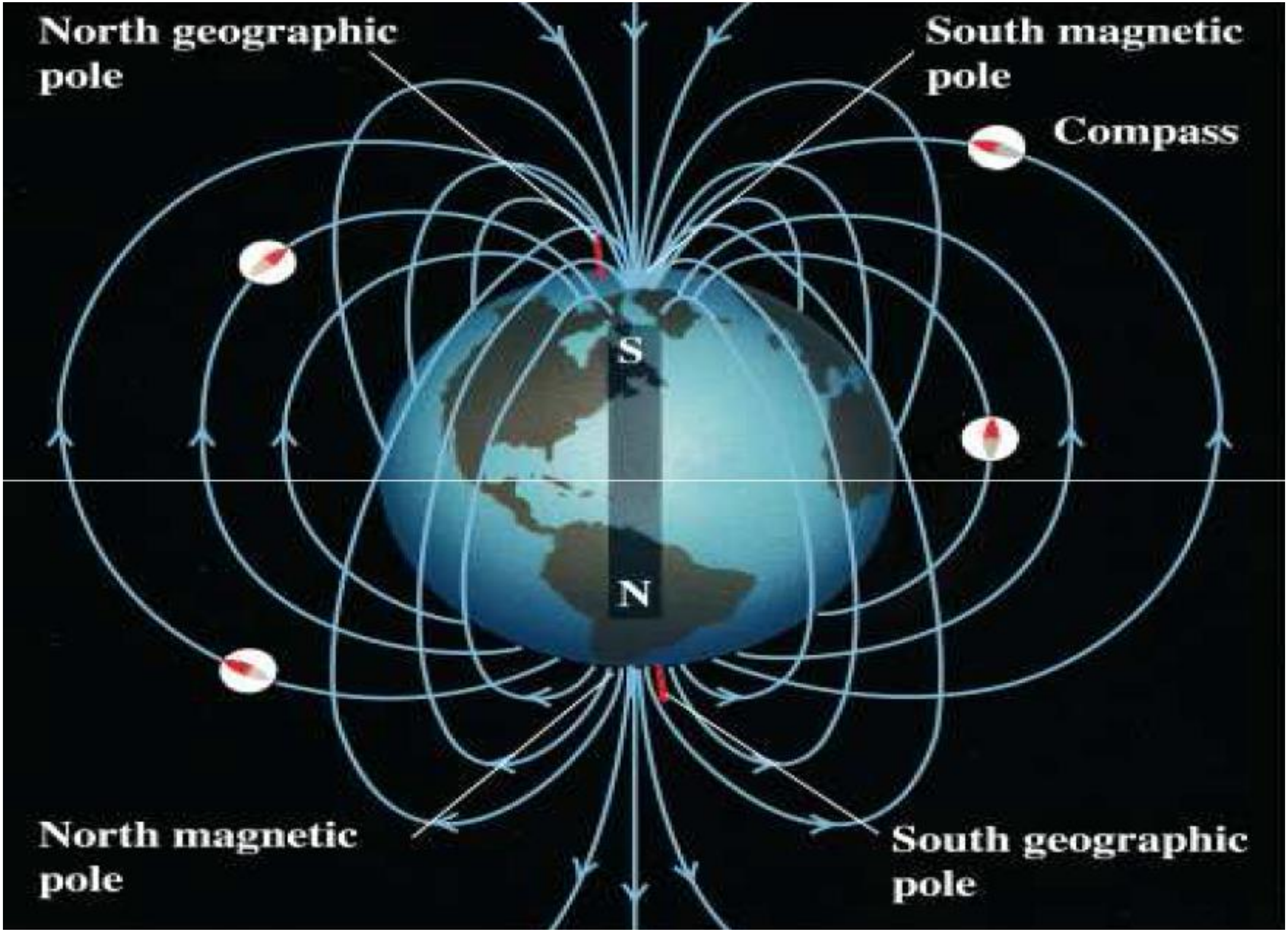

North geographic pole
South magnetic pole
Compass
S
N
North magnetic pole
South geographic pole

Magnestism

- Many historians of science believe that the compass, which uses a magnetic needle, was used in China as early as the 13th century B.C., its invention being of Arabic or Indian origin.
- The early Greeks knew about magnetism as early as 800 B.C. by discovering that the stone magnetite (Fe3O4) attracts pieces of iron.
- Every magnet, regardless of its shape, has two poles, called *north and south poles, that exert forces on other* magnetic poles just as electric charges exert forces on one another.
- Like poles repel each other, and unlike poles attract each other.
- A single magnetic pole has never been isolated. That is, magnetic poles are always found in pairs.

- In 1819, Hans Christian Oersted discovered the relationship between magnetism and electricity.
- Hans Christian Oersted found that an electric current in a wire deflects a nearby compass needle.
- Shortly thereafter, André Ampere (1775–1836) formulated quantitative laws for calculating the magnetic force exerted by one current-carrying electrical conductor on another.
- He also suggested that on the atomic level, electric current loops are responsible for *all magnetic phenomena*.
- In the 1820s, Faraday and Joseph Henry (1797–1878) showed that an electric current can be produced in a circuit either by moving a magnet near the circuit or by changing the current in a nearby circuit.
- These observations demonstrate that a changing magnetic field creates an electric field.
- Years later, *theoretical work* by Maxwell showed that the reverse is also true: A changing electric field creates a magnetic field.

Electric Field and Magnetic Field

Electric forces acting at a distance through electric field **E**.

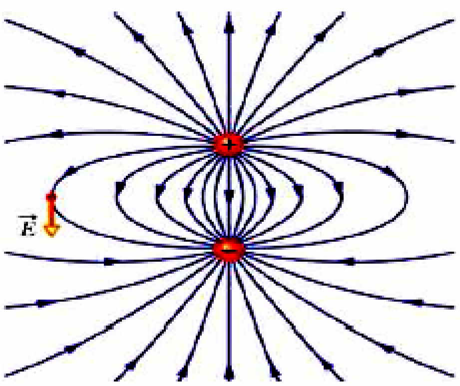
Source: electric charge.

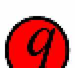
Positive (+) and negative charge (-)

Opposite charges attract, like repel.

Isolated charge exists

Electric field lines begin and end on charges.





Magnetic forces acting at a distance through Magnetic field **B.**

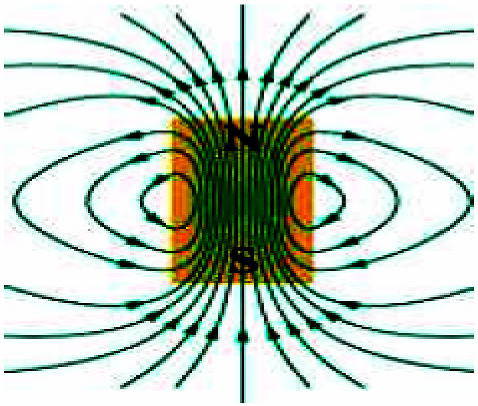
Source: current or permanent magnet

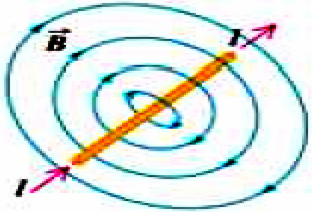
North pole (N) and south pole (S)

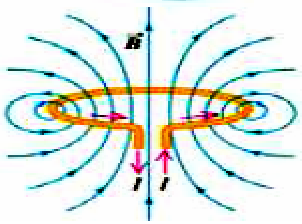
Opposite poles attract, like poles repel.

No isolated pole exists

Magnetic field lines are continuous, these never begin or end as there are nc magnetic charges (monopoles).







Test charge and electric field

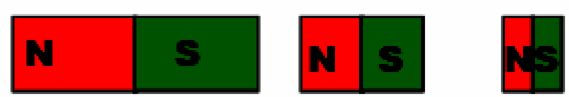
$$\vec{E} = \frac{\vec{F}_E}{q}$$

Test monopole and magnetic field ?

$$\cancel{\vec{B} = \frac{\vec{F}_B}{p}}$$

So what we can do?

Magnetic poles are always found in pairs.
A single magnetic pole has never been isolated.



Force on Charged Particle in Magnetic Field

Let's define **B** at some point in space in terms of the magnetic force $\mathbf{F_B}$ that the field exerts on a ***charged*** particle ***moving*** with a velocity ***v*** :

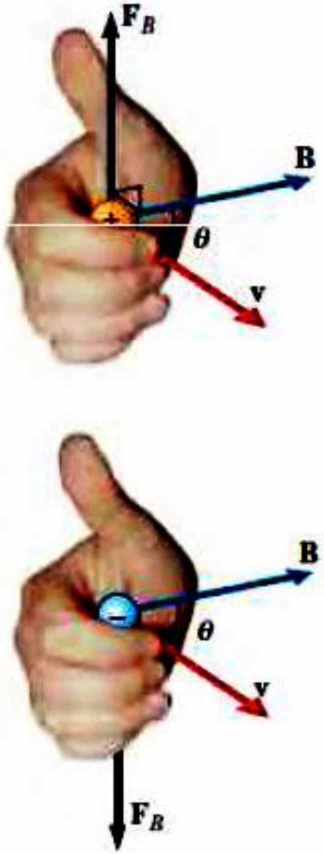
- The magnitude F_B is proportional to the charge q.
- The magnitude F_B is proportional to the speed v of particle.
- $F_B = 0$ when the charged particle moves parallel to the magnetic field.
- When velocity makes any angle $\theta \neq 0$ with the magnetic field, $\boldsymbol{F_B}$ is perpendicular to both ***B*** and ***v***.
- F_B on a positive charge is opposite to that on a negative charge.
- The magnitude F_B is proportional to $\sin\theta$.

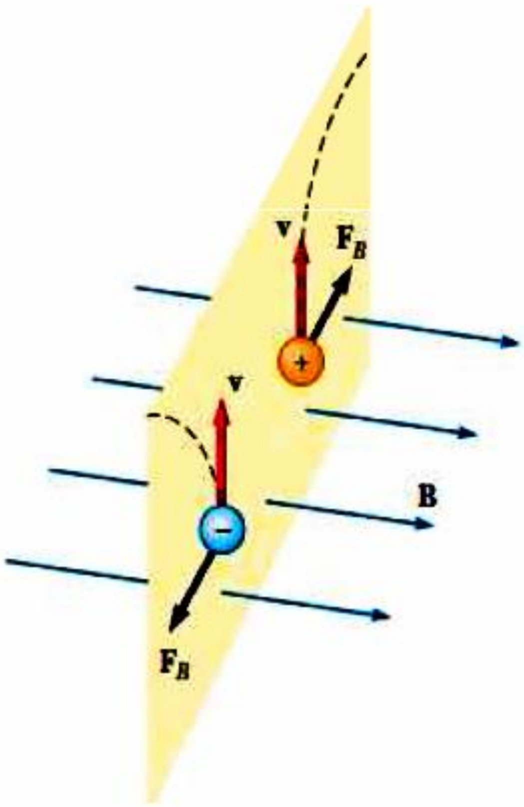
$$\vec{F}_B = q\,\vec{v} \times \vec{B}$$

$$F_B = |q| v B \sin\theta$$

Direction of Magnetic Force

Right-hand rule determine the direction of magnetic force. So the magnetic force is always perpendicular to **v** and **B**.





$$\vec{F}_E = q\,\vec{E} \quad \longleftrightarrow \quad \vec{F}_B = q\,\vec{v} \times \vec{B}$$

The electric force is along the direction of the electric field

The electric force acts on a charged particle regardless of whether the particle is moving or not.

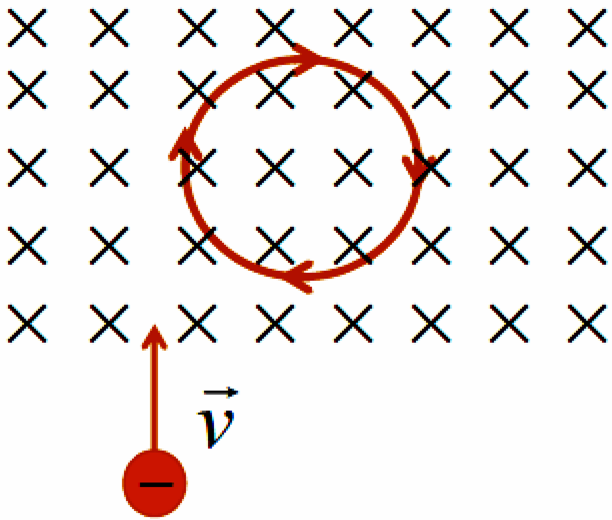
The electric force does work in displacing a charged particle.

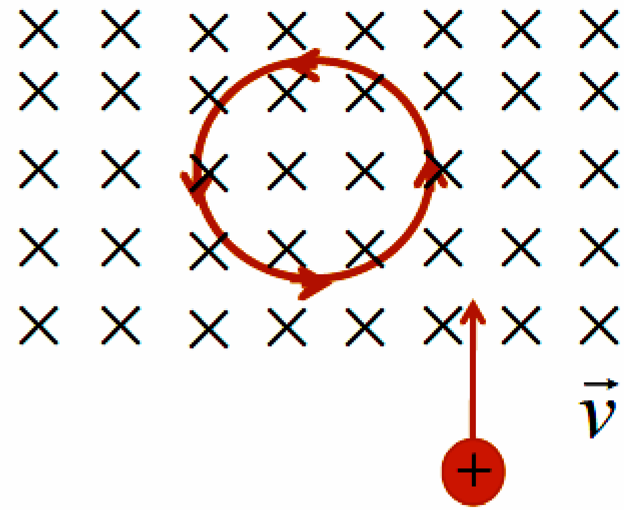
The magnetic force is perpendicular to the magnetic field.

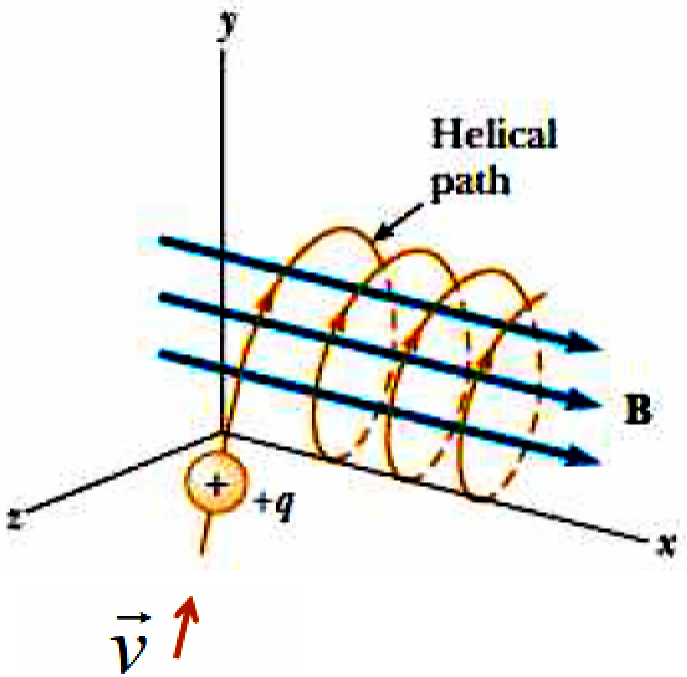
The magnetic force acts on a charged particle only when the particle is in motion.

The magnetic force does no work when a particle is displaced ($F \perp v$).

Trapped Particle







Magnetic Fields

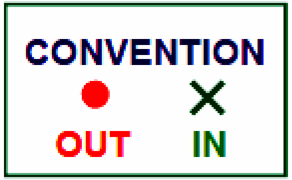
$$B = \frac{F_B}{|q|v}$$

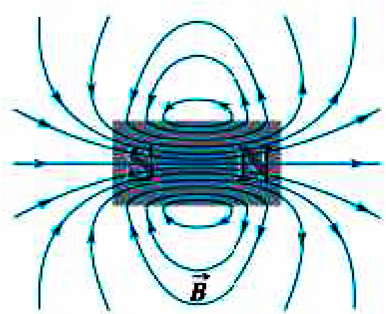
SI unit of magnetic field: tesla (T)

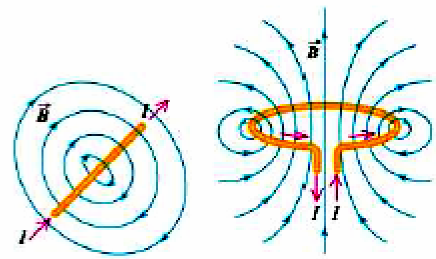
1T = 1 N/[Cm/s] = 1 N/[Am] = 10^4 gauss

The direction of the tangent to a magnetic field line at any point gives the direction of **B** at that point;

The spacing of the lines represents the magnitude of **B** – the magnetic field is stronger where the lines are closer together, and conversely.







Typical Magnetic Field Values

- Brain (at scalp) @ **1fT**
- Earth's magnetic field @ **0.5μT**
- Typical Refrigerator magnetic @ **5mT**
- Typical Loudspeaker magnet @ **2.4T**
- Medical MRI @ **3T**
- High resolution research MRI @ **18T**
- strongest continuous magnetic field yet produced in a laboratory (FSU,NHMFLT, USA) @ **45T**
- strongest pulsed magnetic field yet obtained non-destructively in a laboratory (FDR) @ **91.4T**
- strongest pulsed magnetic field yet obtained in a laboratory, destroying the used equipment(ISSP, Tokyo) @ **730T**
- strongest pulsed magnetic field ever obtained (with explosives) in a laboratory (VNIIEF in Sarov, Russia, 1998) @ **2.8KT**

An electron in the uniform magnetic field has the velocity $\vec{v} = 2\hat{i} + 3\hat{j}$

If magnetic field B is $\vec{B} = -4\hat{i} + 8\hat{j}$ Calculate the magnetic force.

$$\vec{F}_B = q\,\vec{v} \times \vec{B}$$

$$\vec{F}_B = e\begin{vmatrix} i & j & k \\ 2 & 3 & 0 \\ -4 & 8 & 0 \end{vmatrix}$$

$$\vec{F}_B = 28e\hat{k}$$

An electron in the uniform magnetic field has the velocity $\vec{v} = 40\hat{i} + 35\hat{j}$ in Km/s. If it experiences a force $\vec{F} = -4.2\hat{i} + 4.8\hat{j}$ in fN, Calculate the magnetic field B

As magnetic force remains always perpendicular to both velocity and magnetic field, so magnetic field will be directed along z-axis, that is

$$\vec{B} = B\hat{k}$$

Magnitude of B is $$B = F / ev$$

$$F = 6.38\,fN \quad v = 53.1\,Km/s \quad e = 1.6\times10^{-19}\,C$$

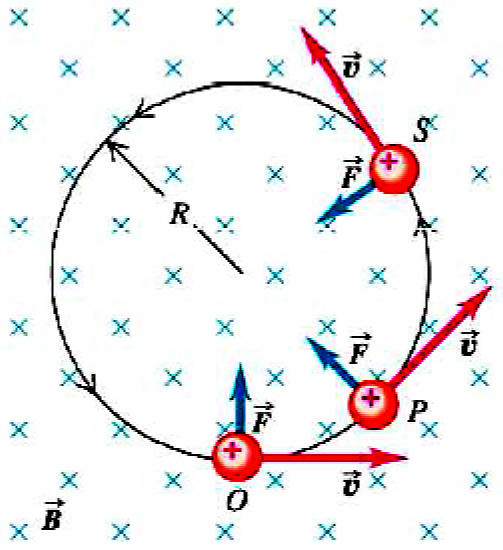
$$B = 0.75T$$

$$\vec{B} = 0.75T\hat{k}$$

Motion of a Charged Particle in a Uniform Magnetic Field

As magnetic force is always perpendicular to the velocity of the particle so, the work done on the particle by the magnetic force is zero. Magnetic force changes only the direction of v and not its magnitude.

Let us now consider the special case of a positively charged particle moving in a uniform magnetic field with the initial velocity vector of the particle perpendicular to the field. Let us assume that the direction of the magnetic field is into the page. Magnetic force provides centripetal force. As a result particle moves in a circle in a plane perpendicular to the magnetic field.



$$qvB = \frac{mv^2}{r}$$

Radius of circular path is

$$\boxed{r = \frac{mv}{qB}}$$

The angular speed of the particle is

$$\boxed{\omega = \frac{v}{r} = \frac{qB}{m}}$$

Period of the motion is

$$\boxed{T = \frac{2\pi r}{v} = \frac{2\pi}{\omega} = \frac{2\pi m}{qB}}$$

T and ω do not depend on v of the particle. Fast particles move in large circles and slow ones in small circles, but all particles with the same charge-to-mass ratio take the same time T to complete one round trip.

Velocity Selector

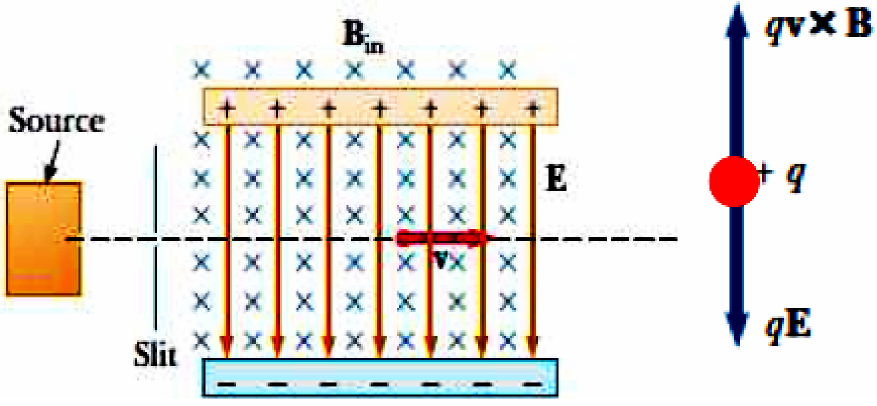
A uniform electric field is directed vertically downward and a uniform magnetic field is applied into the page (perpendicular to the electric field). For *q positive*, the magnetic force is upward and the electric force *qE is downward*. When the magnitudes of the two fields are chosen so that

$$qE = qvB$$

The particle will move in a straight horizontal line through the region of the fields with velocity,

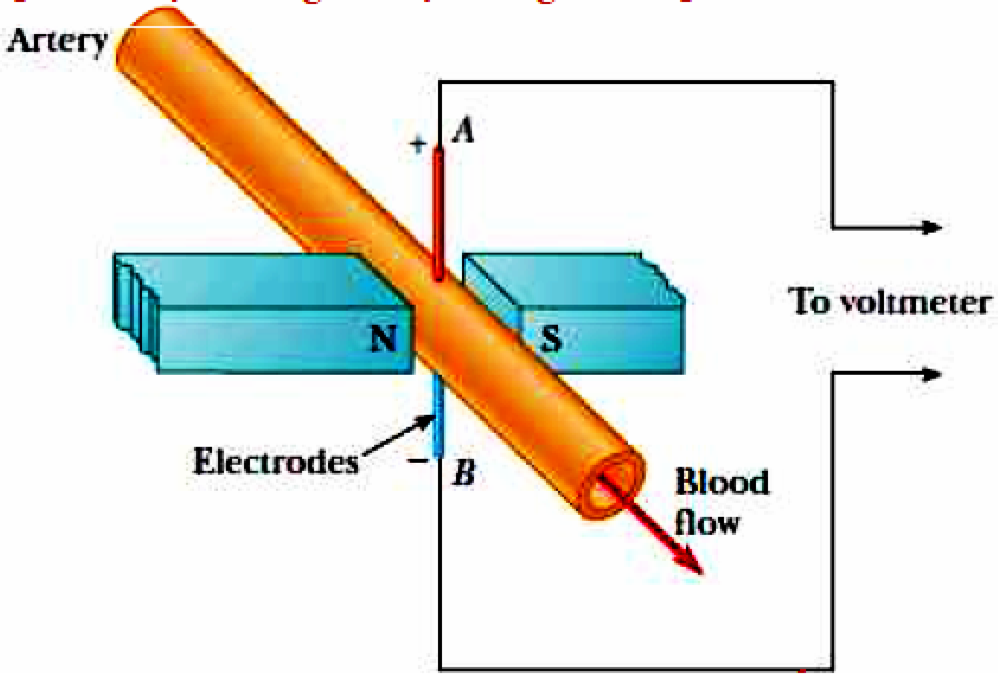
$$v = \frac{E}{B}$$

Only those particles having speed v pass undeflected through the mutually perpendicular electric and magnetic fields. The magnetic force exerted on particles moving at speeds greater than this is stronger than the electric force, and the particles are deflected upward. Those moving at speeds less than this are deflected downward.



Electromagnetic Flowmeter

A heart surgeon monitors the flow rate of blood through an artery using an electromagnetic flowmeter (Shown in Fig. below). Electrodes *A and B make contact with the outer* surface of the blood vessel, which has interior diameter 3.00 mm. (a) For a magnetic field magnitude of 0.040 0 T, an emf of 160 μV appears between the electrodes. Calculate the speed of the blood. (b) Verify that electrode *A is* positive, as shown. Does the sign of the emf depend on whether the mobile ions in the blood are predominantly positively or negatively charged? Explain.

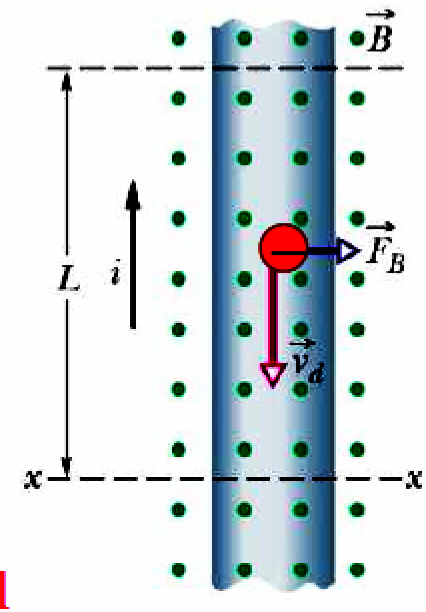


Magnetic Force on a Current Carrying Wire

A conductor of length L and cross sectional area A is placed in magnetic field B.

Free electrons move with drift velocity v_d opposite to the current. Electrons in this section feel magnetic force:

$$\vec{F}_B = q\vec{v}_d \times \vec{B}$$



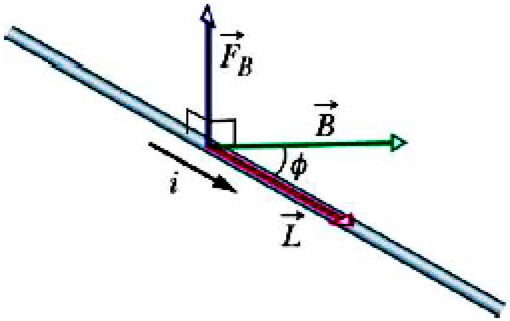
If n is the electron number density then total number of electrons in the conductor of length L and cross sectional area A will be nAL. So

$$\vec{F}_B = (q\vec{v}_d \times \vec{B})nAL$$

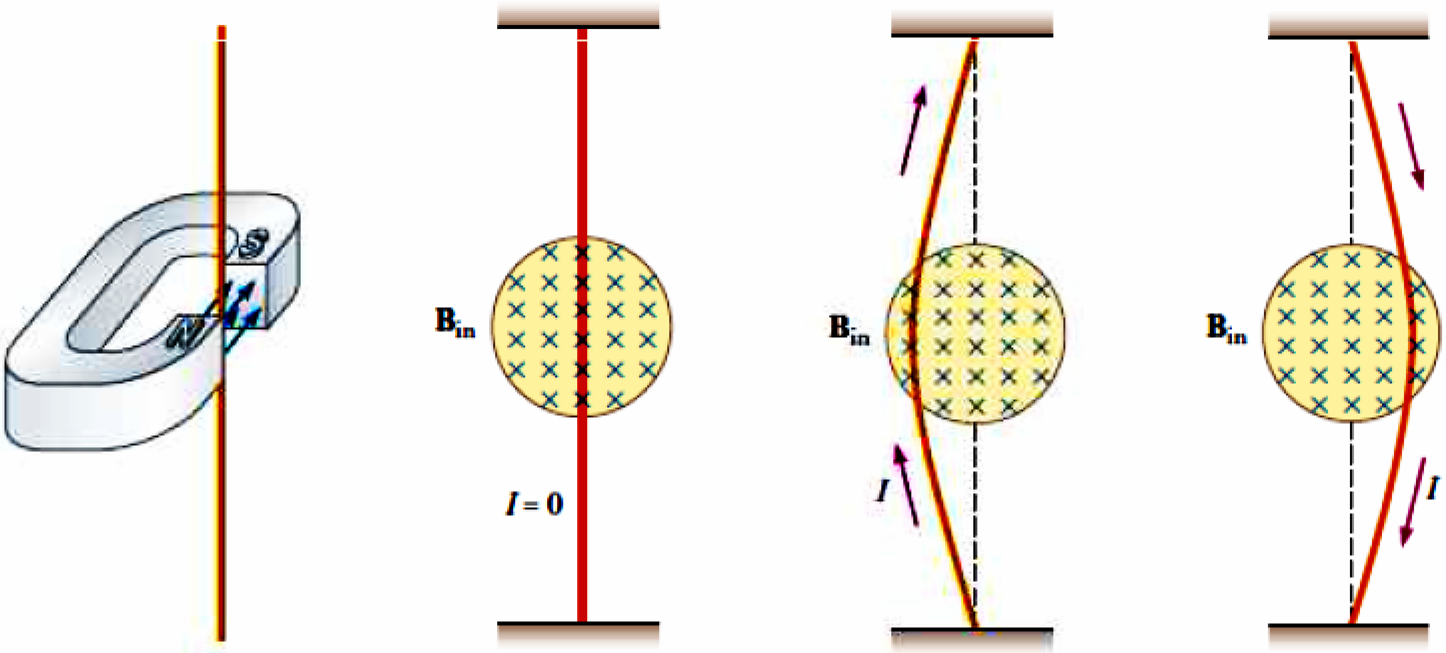
As

$$i = nqv_d A$$

$$\boxed{\vec{F}_B = i\,\vec{L} \times \vec{B}}$$



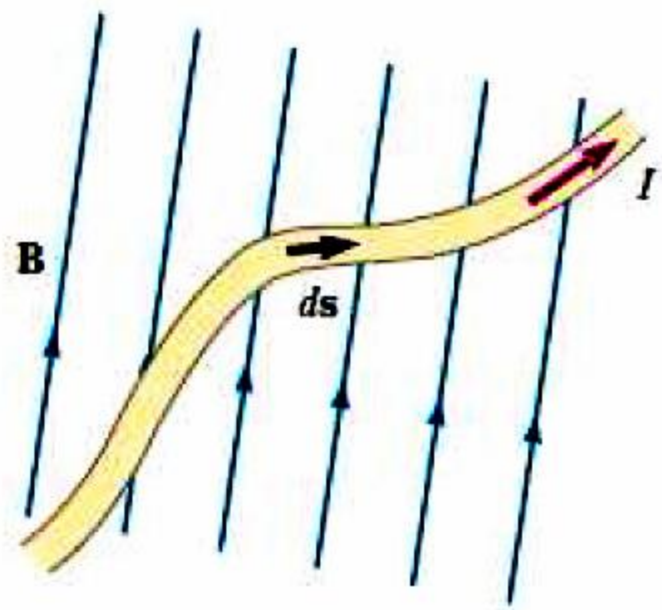
L is a length vector that points in the direction of *i* and has a magnitude equal to the length. Direction of F_B can be found by right hand rule



For arbitrarily shaped wire segment of uniform cross section in a magnetic field.

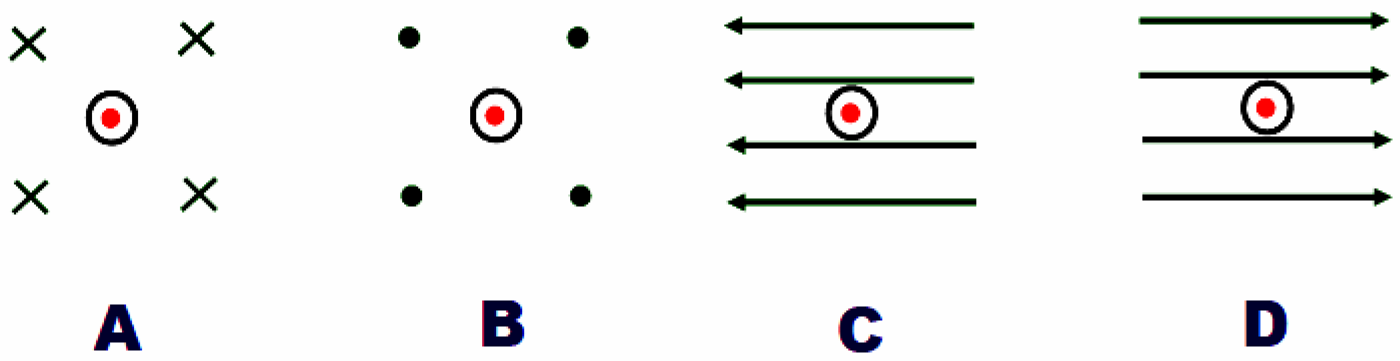
$$d\vec{F}_B = i\,d\vec{s} \times \vec{B}$$

$$\vec{F}_B = \int_a^b d\vec{F}_B = i\int_a^b d\vec{s} \times \vec{B}$$

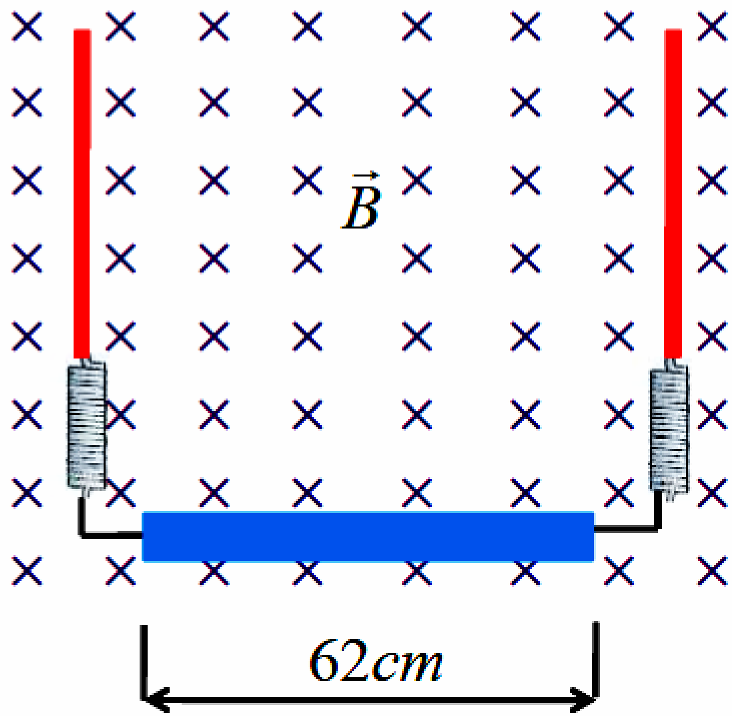


Suspend a Wire

A straight, horizontal length of copper wire is immersed in a uniform magnetic field. The current through the wire is out of page. Which magnetic field can possibly suspend this wire to balance the gravity?



A wire of length 62cm and mass 13g is suspended by a pair of flexible leads in a magnetic field of 440mT. Find the magnitude and direction of the current in the wire required to remove the tension in the supporting leads.



Since L is perpendicular to B can use

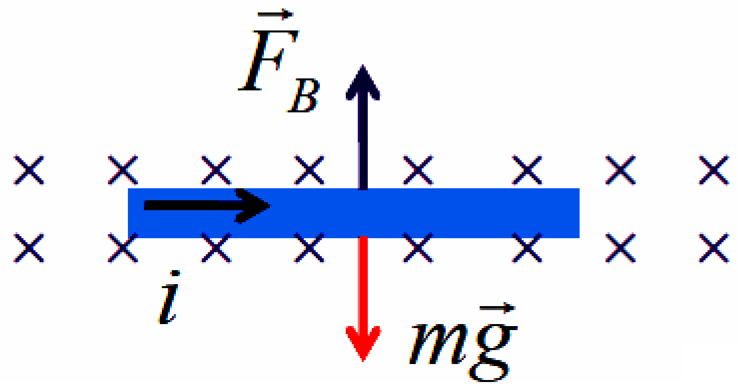
$$F_B = iBL$$

There will be no tension in the supporting leads if magnetic force is directed upward and equal to the weight of the wire, that is

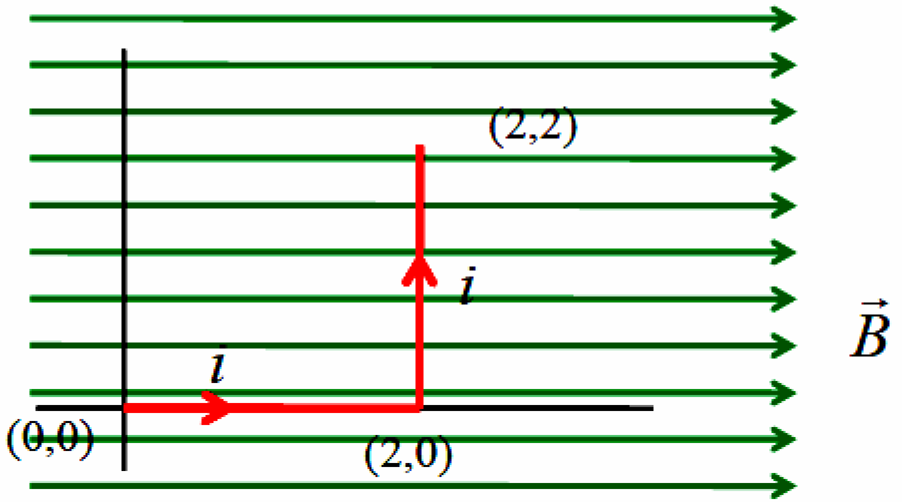
$$iBL = mg$$

$$i = 0.46A$$

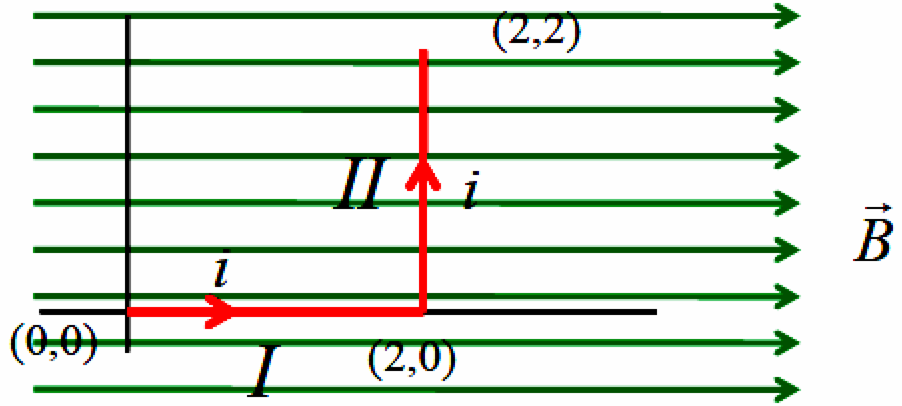
Right hand rule indicates that the current must be directed from left to right in order to have a magnetic force directed upward.



A wire segment is placed in a uniform magnetic field B as shown in the figure. If the segment carries a current i, what resultant magnetic force acts on it?



Divide the wire into two segments as shown

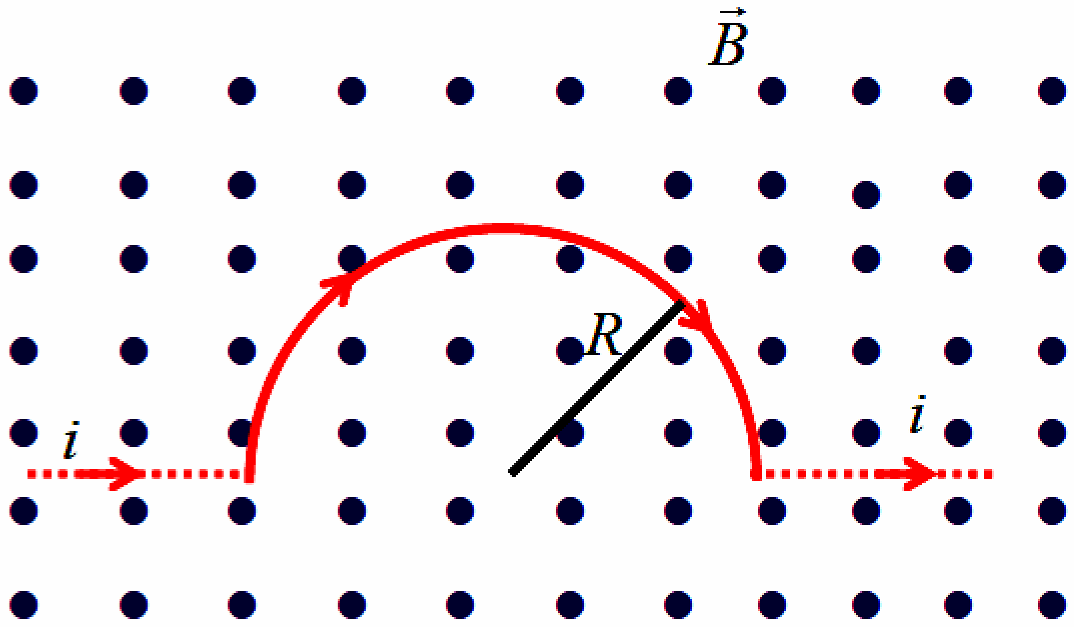


$$\vec{F}_1 = 0$$

$$\vec{F}_2 = -2iB\hat{k}$$

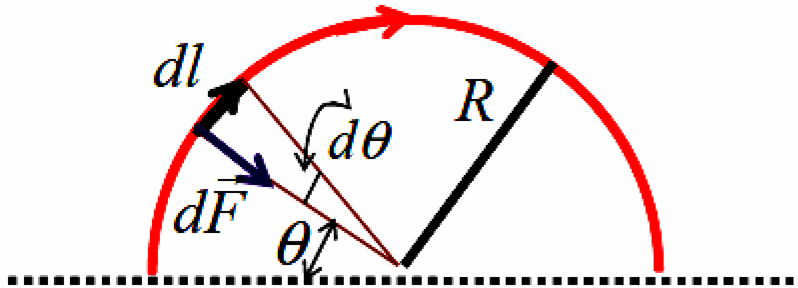
$$\boxed{\vec{F} = \vec{F}_1 + \vec{F}_2 = -2iB\hat{k}}$$

Figure shows a wire segment placed in a uniform magnetic field B directed out of the plane of figure. If the segment carries a current i, find the magnetic force acting on the semicircular part of wire.



To find the magnetic force on semicircular segment , let's consider a small length element dl. Force on this element will be directed radially towards origin and have magnitude

$$dF_2 = iBdl = iBRd\theta$$



Note that the horizontal component, dFcosθ, of this force element will be cancelled out by an oppositely directed horizontal component due to a symmetrically located small length segment on the opposite side of the arc.

$$d\vec{F}_2 = -dF_2 \sin\theta \ \hat{j}$$

So net force on the segment 3 will be

$$\vec{F}_2 = -\int_0^{\pi} dF_2 \sin\theta \ \hat{j} = -\int_0^{\pi} iBR \sin\theta d\theta \ \hat{j}$$

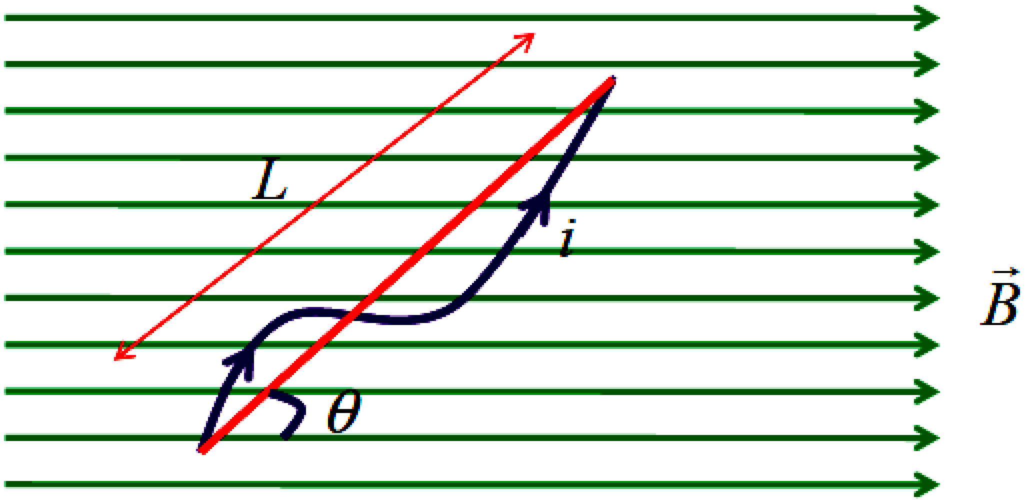
$$= -iBR\left[-\cos\theta\right]_0^{\pi} \ \hat{j} = -2iRB\hat{j}$$

Theorem

If a current carrying wire of arbitrary shape is placed inside a magnetic field B, magnetic force acting on it will be

$$\boxed{\vec{F}_B = i\,\vec{L}\times\vec{B} = iLB\sin\theta\,\hat{n}}$$

Where L in the length of straight line joining ends of current carrying wire and θ is the angle between B and L.



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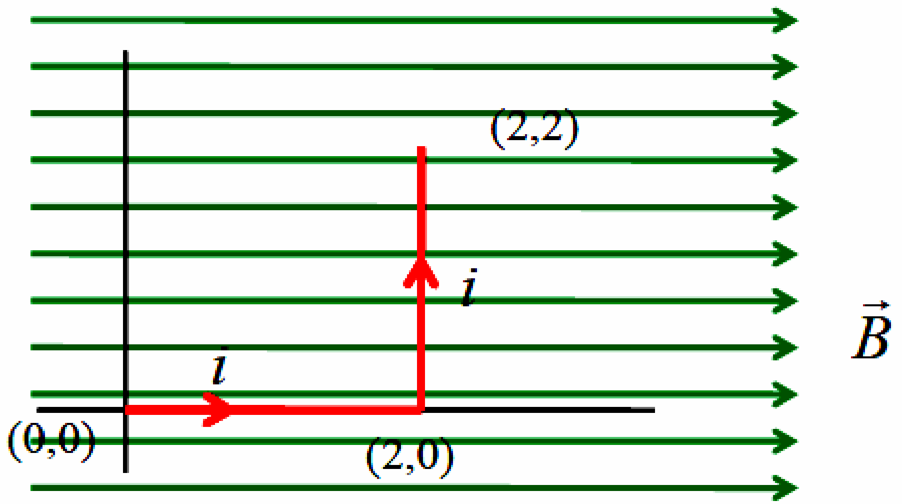
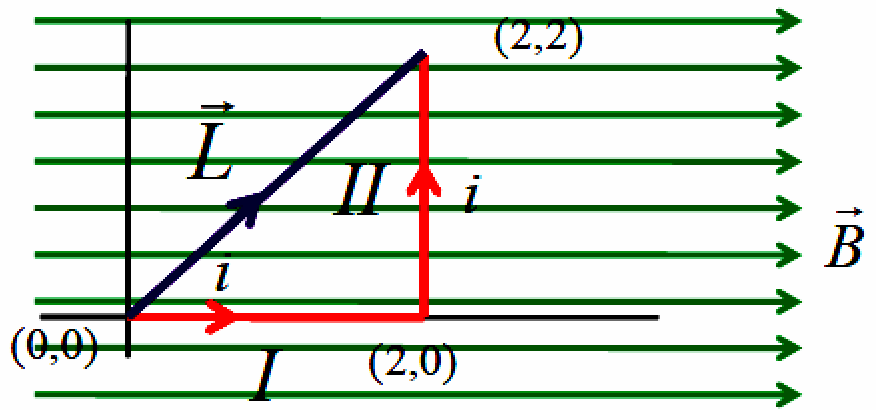


Connect the end points of wire and find length and direction of



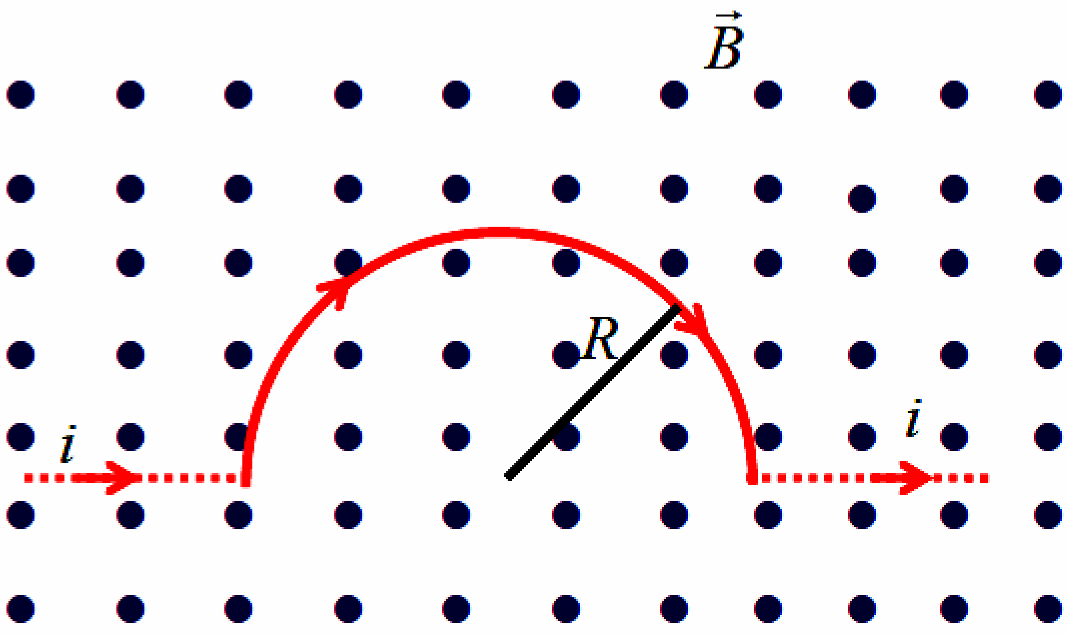
$$L = \sqrt{2^2 + 2^2} = 2\sqrt{2}$$

$$\theta = \tan^{-1}(y/x) = \tan^{-1}(1) = 45^\circ$$

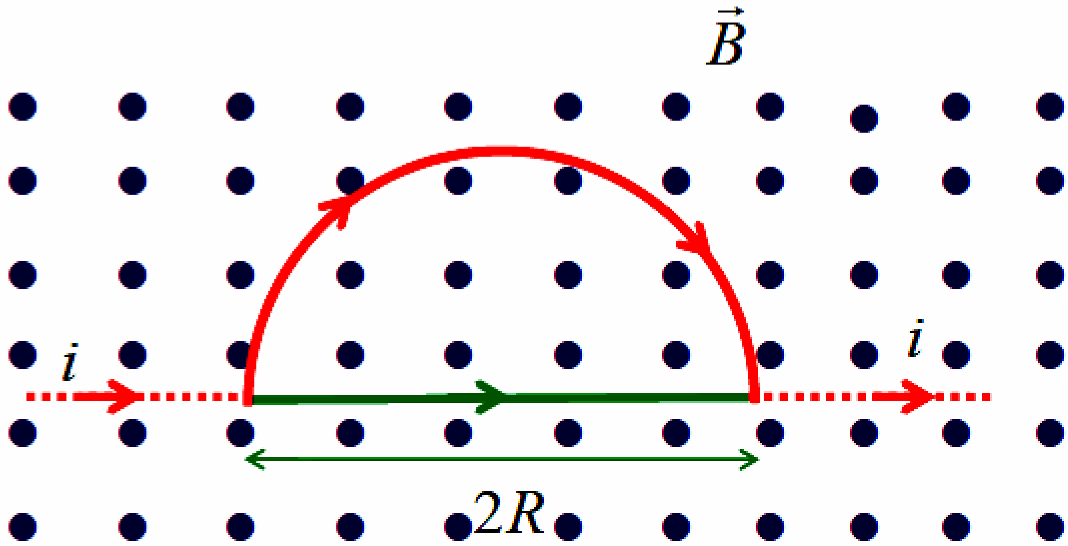
$$\vec{F} = i\,\vec{L} \times \vec{B} = iLB\sin\theta\,\hat{n}$$

$$\vec{F} = -i(2\sqrt{2})B\sin 45\,\hat{k} = -2iB\hat{k}$$

Figure shows a wire segment placed in a uniform magnetic field B directed out of the plane of figure. If the segment carries a current i, find the magnetic force acting on the semicircular part of wire.



Connect the end points of wire and find length and direction of $\vec{L}$



$$\vec{F} = i\,\vec{L} \times \vec{B} = iLB \sin\theta \,\hat{n}$$

$$\vec{F} = -i(2R)B \sin 90 \,\hat{j} = -2iRB \,\hat{j}$$