Engineering Mechanics

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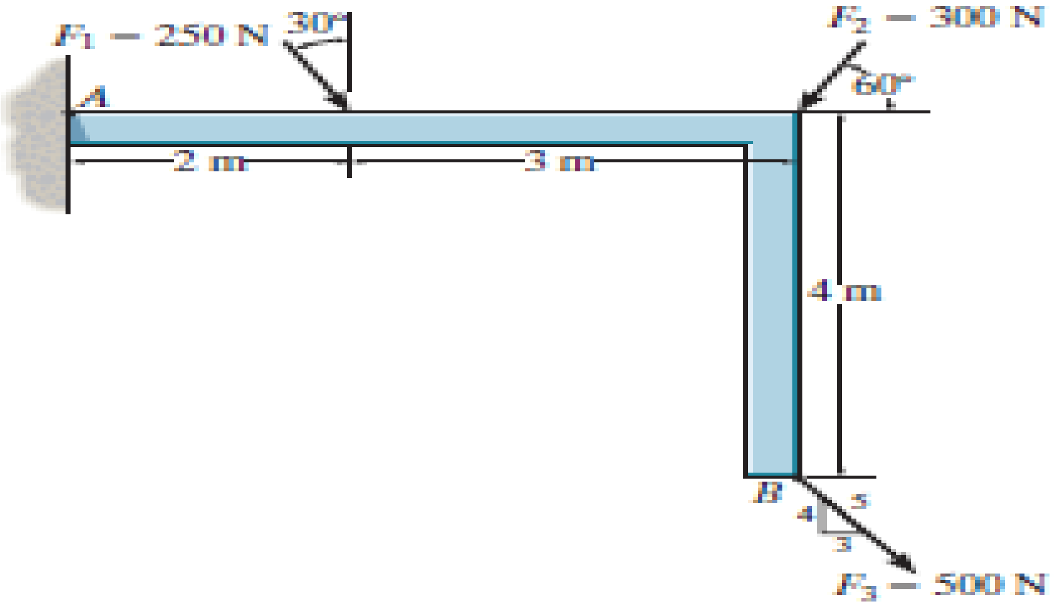
Research Interest: Photovoltaic systems, Power & Control, Sliding mode control (SMC)

Problem

4–7. Determine the moment of each of the three forces about point A.

***4–8.** Determine the moment of each of the three forces about point B.

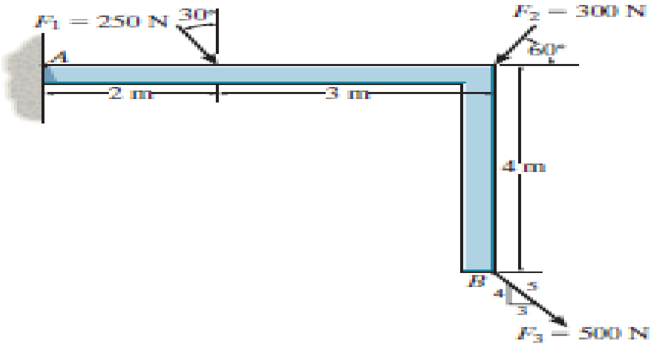


Probs. 4–7/8

Solution

4–7.

Determine the moment of each of the three forces about point A.



SOLUTION

The moment arm measured perpendicular to each force from point A is

$$d_1 = 2 \sin 60° = 1.732 \text{ m}$$

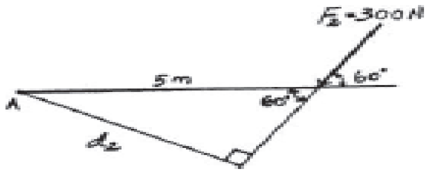
$$d_2 = 5 \sin 60° = 4.330 \text{ m}$$

$$d_3 = 2 \sin 53.13° = 1.60 \text{ m}$$

Using each force where $M_A = Fd$, we have

$\curvearrowleft + (M_{F_1})_A = -250(1.732)$

$= -433 \text{ N} \cdot \text{m} = 433 \text{ N} \cdot \text{m}$ *(Clockwise)* **Ans.**

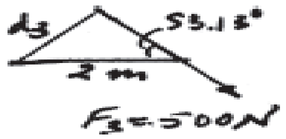


$\curvearrowleft + (M_{F_2})_A = -300(4.330)$

$= -1299 \text{ N} \cdot \text{m} = 1.30 \text{ kN} \cdot \text{m}$ *(Clockwise)* **Ans.**

$\curvearrowleft + (M_{F_3})_A = -500(1.60)$

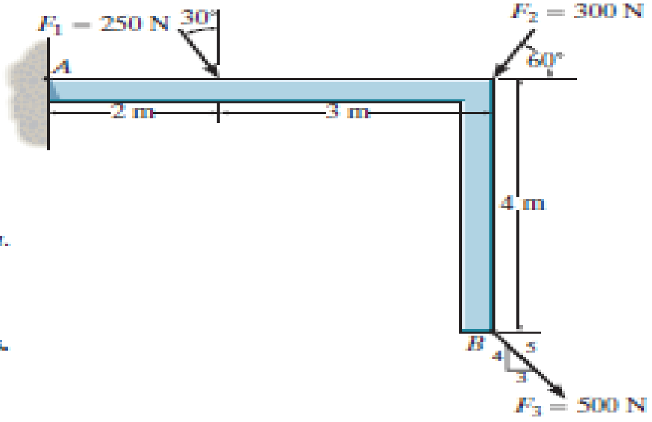
$= -800 \text{ N} \cdot \text{m} = 800 \text{ N} \cdot \text{m}$ *(Clockwise)* **Ans.**



Solution

***4–8.**

Determine the moment of each of the three forces about point B.



SOLUTION

The forces are resolved into horizontal and vertical component as shown in Fig. *a*.
For $\mathbf{F}_1$,

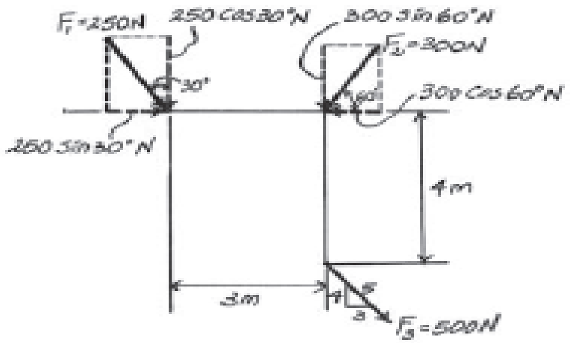
$$\circlearrowleft + M_B = 250 \cos 30°(3) - 250 \sin 30°(4)$$
$$= 149.51 \text{ N} \cdot \text{m} = 150 \text{ N} \cdot \text{m} \circlearrowleft \qquad \textbf{Ans.}$$

For $\mathbf{F}_2$,

$$\circlearrowleft + M_B = 300 \sin 60°(0) + 300 \cos 60°(4)$$
$$= 600 \text{ N} \cdot \text{m} \circlearrowleft \qquad \textbf{Ans.}$$

Since the line of action of $\mathbf{F}_3$ passes through B, its moment arm about point B is zero. Thus

$$M_B = 0 \qquad \textbf{Ans.}$$

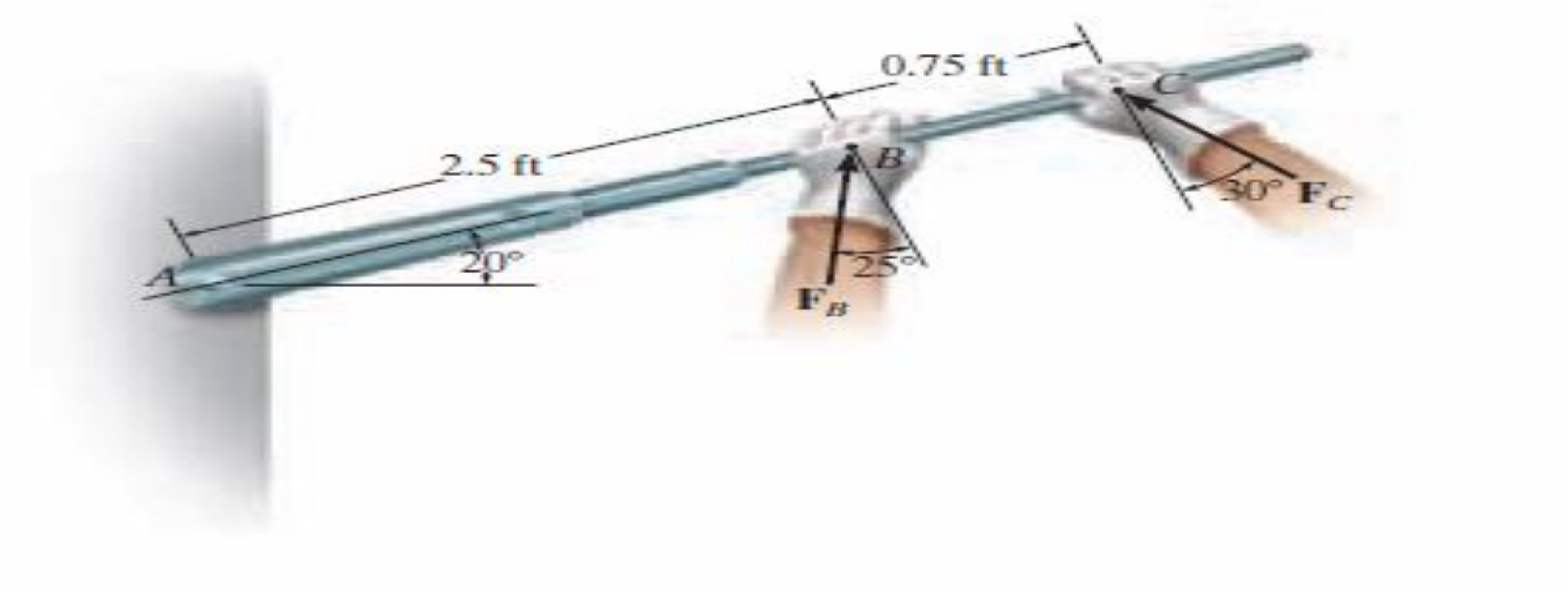


(a)

Problem

4–9. Determine the moment of each force about the bolt located at A. Take $F_B = 40$ lb, $F_C = 50$ lb.

4–10. If $F_B = 30$ lb and $F_C = 45$ lb, determine the resultant moment about the bolt located at A.

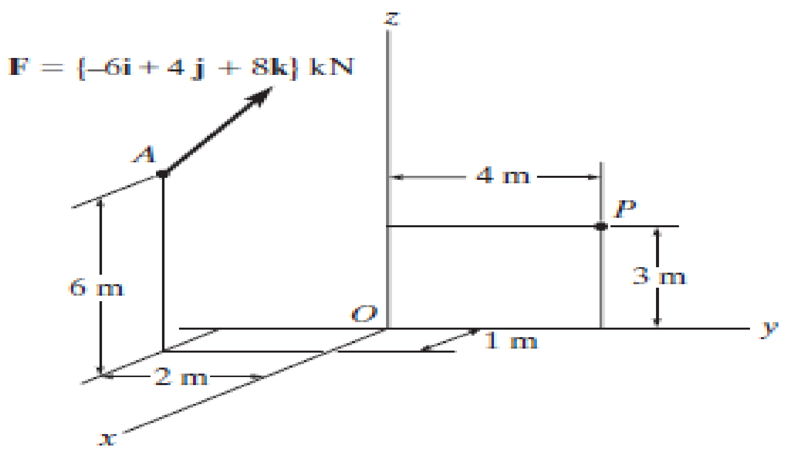


Probs. 4–9/10

Solution

4–27.

Determine the moment of the force **F** about point *O*. Express the result as a Cartesian vector.



SOLUTION

Position Vector. The coordinates of point *A* are (1, −2, 6) m.

Thus,

$$\mathbf{r}_{OA} = \{\mathbf{i} - 2\mathbf{j} + 6\mathbf{k}\}\text{ m}$$

The moment of *F* About Point *O*.

$$\mathbf{M}_O = \mathbf{r}_{OA} \times \mathbf{F}$$

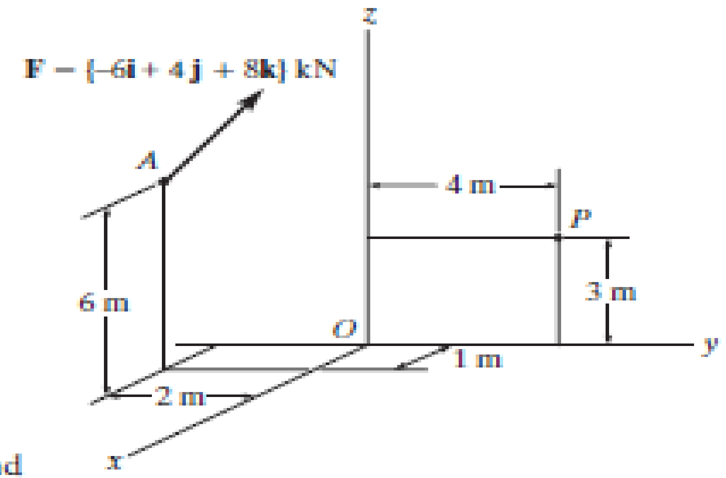
$$= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & -2 & 6 \\ -6 & 4 & 8 \end{vmatrix}$$

$$= \{-40\mathbf{i} - 44\mathbf{j} - 8\mathbf{k}\}\text{ kN}\cdot\text{m} \qquad \textbf{Ans.}$$

Solution

***4–28.**

Determine the moment of the force **F** about point *P*. Express the result as a Cartesian vector.



SOLUTION

Position Vector. The coordinates of points A and P are $A\ (1, -2, 6)$ m and $P\ (0, 4, 3)$ m, respectively. Thus

$$\mathbf{r}_{PA} = (1 - 0)\mathbf{i} + (-2 - 4)\mathbf{j} + (6 - 3)\mathbf{k}$$

$$= \{\mathbf{i} - 6\mathbf{j} + 3\mathbf{k}\}\ \text{m}$$

The moment of *F* About Point *P*.

$$M_P = \mathbf{r}_{PA} \times \mathbf{F}$$

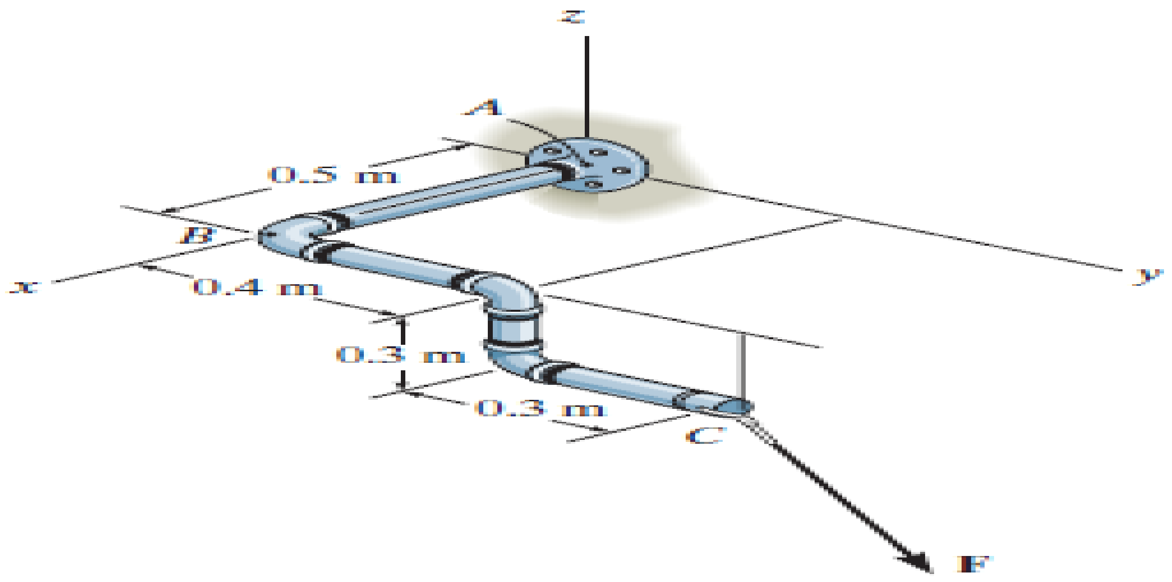
$$= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & -6 & 3 \\ -6 & 4 & 8 \end{vmatrix}$$

$$= \{-60\mathbf{i} - 26\mathbf{j} - 32\mathbf{k}\}\ \text{kN} \cdot \text{m}$$ **Ans.**

Problem

*4–32. The pipe assembly is subjected to the force of $\mathbf{F} = \{600\mathbf{i} + 800\mathbf{j} - 500\mathbf{k}\}$ N. Determine the moment of this force about point A.

4–33. The pipe assembly is subjected to the force of $\mathbf{F} = \{600\mathbf{i} + 800\mathbf{j} - 500\mathbf{k}\}$ N. Determine the moment of this force about point B.

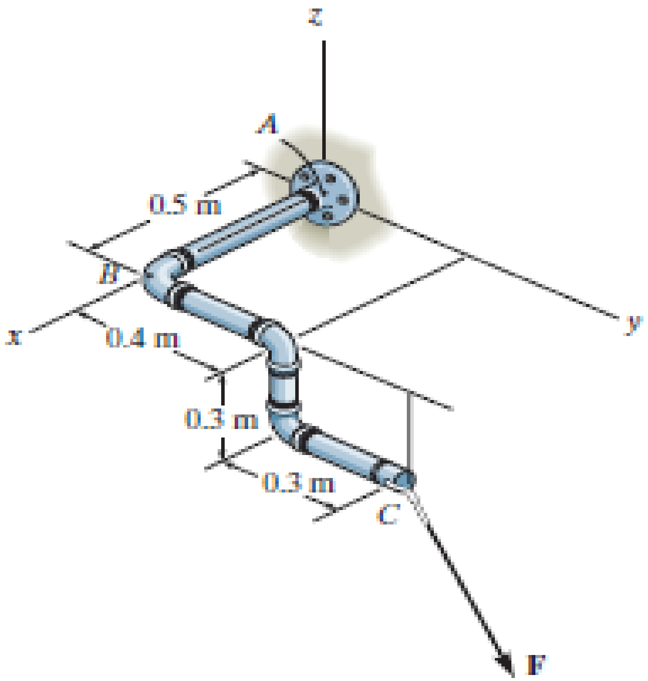


Probs. 4–32/33

Solution

***4–32.**

The pipe assembly is subjected to the force of $\mathbf{F} = \{600\mathbf{i} + 800\mathbf{j} - 500\mathbf{k}\}$ N. Determine the moment of this force about point A.



SOLUTION

Position Vector. The coordinates of point C are C (0.5, 0.7, −0.3) m. Thus

$$\mathbf{r}_{AC} = \{0.5\mathbf{i} + 0.7\mathbf{j} - 0.3\mathbf{k}\}\text{ m}$$

Moment of Force *F* About Point A.

$$\mathbf{M}_A = \mathbf{r}_{AC} \times \mathbf{F}$$

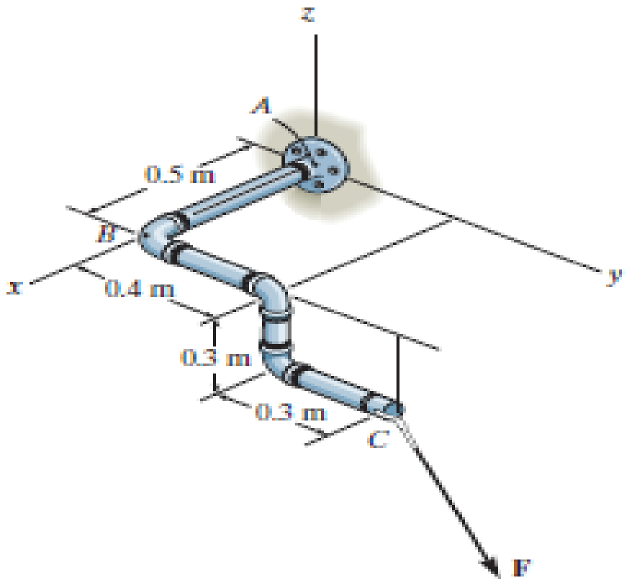
$$= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0.5 & 0.7 & -0.3 \\ 600 & 800 & -500 \end{vmatrix}$$

$$= \{-110\mathbf{i} + 70\mathbf{j} - 20\mathbf{k}\}\text{ N}\cdot\text{m} \qquad \textbf{Ans.}$$

Solution

4–33.

The pipe assembly is subjected to the force of $\mathbf{F} = \{600\mathbf{i} + 800\mathbf{j} - 500\mathbf{k}\}$ N. Determine the moment of this force about point B.



SOLUTION

Position Vector. The coordinates of points B and C are $B\,(0.5, 0, 0)$ m and $C\,(0.5, 0.7, -0.3)$ m, respectively. Thus,

$$\mathbf{r}_{BC} = (0.5 - 0.5)\mathbf{i} + (0.7 - 0)\mathbf{j} + (-0.3 - 0)\mathbf{k}$$

$$= \{0.7\mathbf{j} - 0.3\mathbf{k}\}\ \text{m}$$

Moment of Force F About Point B. Applying Eq. 4

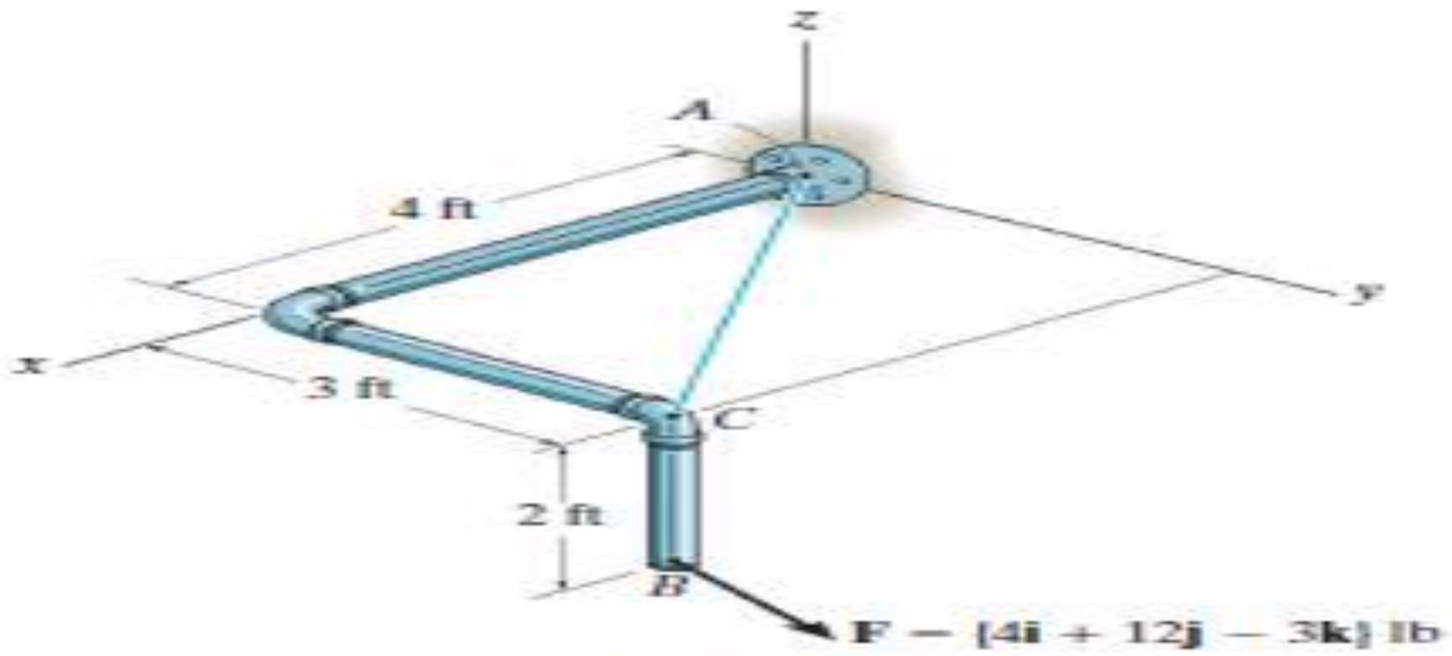
$$\mathbf{M}_B = \mathbf{r}_{BC} \times \mathbf{F}$$

$$= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0.7 & -0.3 \\ 600 & 800 & -500 \end{vmatrix}$$

$$= \{-110\mathbf{i} - 180\mathbf{j} - 420\mathbf{k}\}\ \text{N}\cdot\text{m} \qquad \textbf{Ans.}$$

Problem

4–57. Determine the moment of this force **F** about an axis extending between *A* and *C*. Express the result as a Cartesian vector.

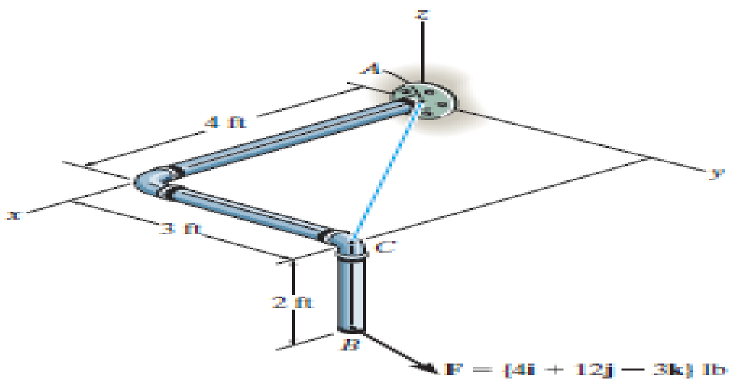


Probs. 4–56/57

Solution

4–57.

Determine the moment of the force **F** about an axis extending between *A* and *C*. Express the result as a Cartesian vector.



SOLUTION

Position Vector:

$$\mathbf{r}_{CB} = \{-2\mathbf{k}\}\text{ ft}$$

$$\mathbf{r}_{AB} = \{(4-0)\mathbf{i} + (3-0)\mathbf{j} + (-2-0)\mathbf{k}\}\text{ ft} = \{4\mathbf{i} + 3\mathbf{j} - 2\mathbf{k}\}\text{ ft}$$

Unit Vector Along AC Axis:

$$\mathbf{u}_{AC} = \frac{(4-0)\mathbf{i} + (3-0)\mathbf{j}}{\sqrt{(4-0)^2 + (3-0)^2}} = 0.8\mathbf{i} + 0.6\mathbf{j}$$

Moment of Force F About AC Axis: With $\mathbf{F} = \{4\mathbf{i} + 12\mathbf{j} - 3\mathbf{k}\}$ lb, applying Eq. 4–7, we have

$$M_{AC} = \mathbf{u}_{AC} \cdot (\mathbf{r}_{CB} \times \mathbf{F})$$

$$= \begin{vmatrix} 0.8 & 0.6 & 0 \\ 0 & 0 & -2 \\ 4 & 12 & -3 \end{vmatrix}$$

$$= 0.8[(0)(-3) - 12(-2)] - 0.6[0(-3) - 4(-2)] + 0$$

$$= 14.4 \text{ lb} \cdot \text{ft}$$

Or

$$M_{AC} = \mathbf{u}_{AC} \cdot (\mathbf{r}_{AB} \times \mathbf{F})$$

$$= \begin{vmatrix} 0.8 & 0.6 & 0 \\ 4 & 3 & -2 \\ 4 & 12 & -3 \end{vmatrix}$$

$$= 0.8[(3)(-3) - 12(-2)] - 0.6[4(-3) - 4(-2)] + 0$$

$$= 14.4 \text{ lb} \cdot \text{ft}$$

Expressing $\mathbf{M}_{AC}$ as a Cartesian vector yields

$$\mathbf{M}_{AC} = M_{AC}\,\mathbf{u}_{AC}$$

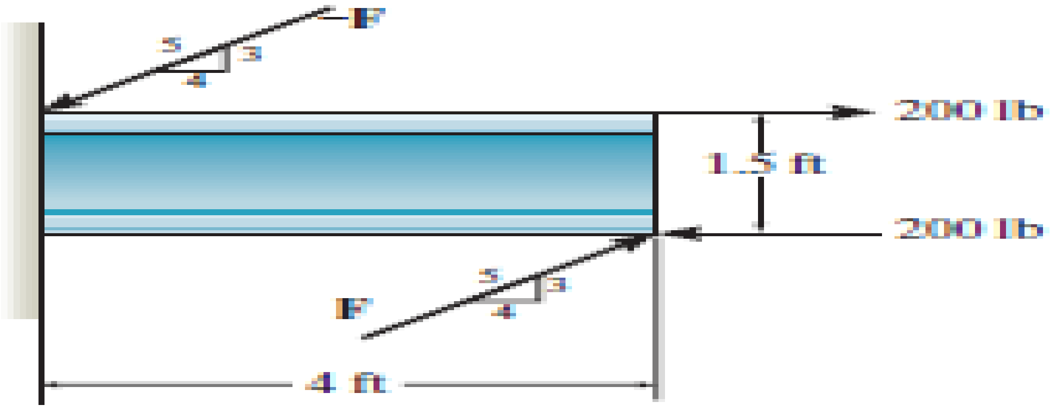
$$= 14.4(0.8\mathbf{i} + 0.6\mathbf{j})$$

$$= \{11.5\mathbf{i} + 8.64\mathbf{j}\} \text{ lb} \cdot \text{ft}$$ **Ans.**

Problem

4–77. Two couples act on the beam as shown. If $F = 150$ lb, determine the resultant couple moment.

4–78. Two couples act on the beam as shown. Determine the magnitude of **F** so that the resultant couple moment is $300\ \text{lb} \cdot \text{ft}$ counterclockwise. Where on the beam does the resultant couple act?

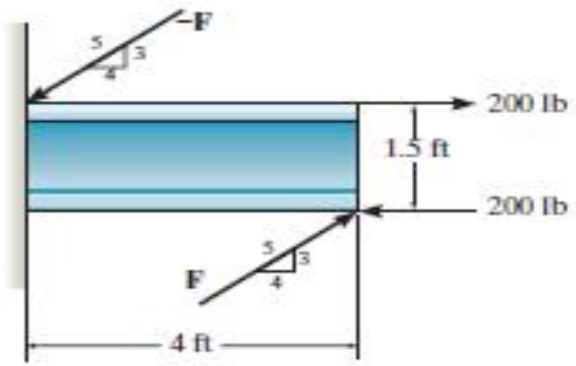


Probs. 4–77/78

Solution

4–77.

Two couples act on the beam as shown. If $F = 150$ lb, determine the resultant couple moment.

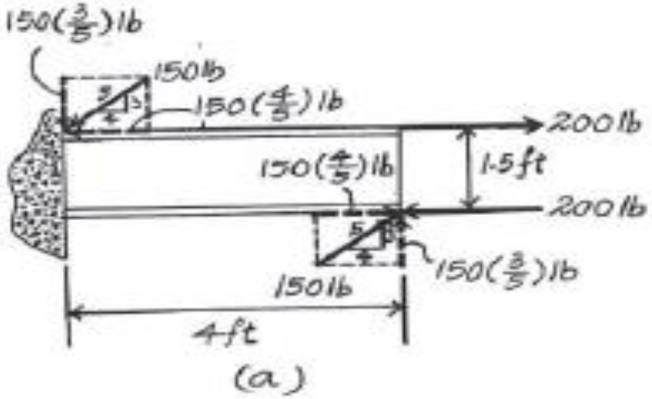


SOLUTION

150 lb couple is resolved into their horizontal and vertical components as shown in Fig. *a*

$$\zeta + (M_R)_c = 150\left(\frac{4}{5}\right)(1.5) + 150\left(\frac{3}{5}\right)(4) - 200(1.5)$$

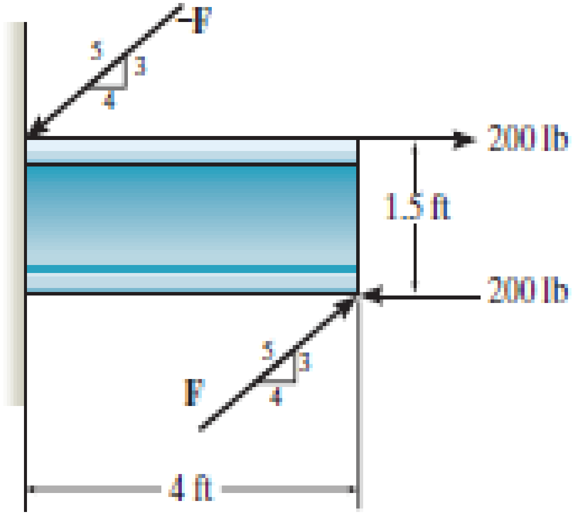
$$= 240 \text{ lb} \cdot \text{ft} \circlearrowleft$$ **Ans.**



Solution

4–78.

Two couples act on the beam as shown. Determine the magnitude of **F** so that the resultant couple moment is 300 lb · ft counterclockwise. Where on the beam does the resultant couple act?



SOLUTION

$$\circlearrowleft + (M_C)_R = \frac{3}{5}F(4) + \frac{4}{5}F(1.5) - 200(1.5) = 300$$

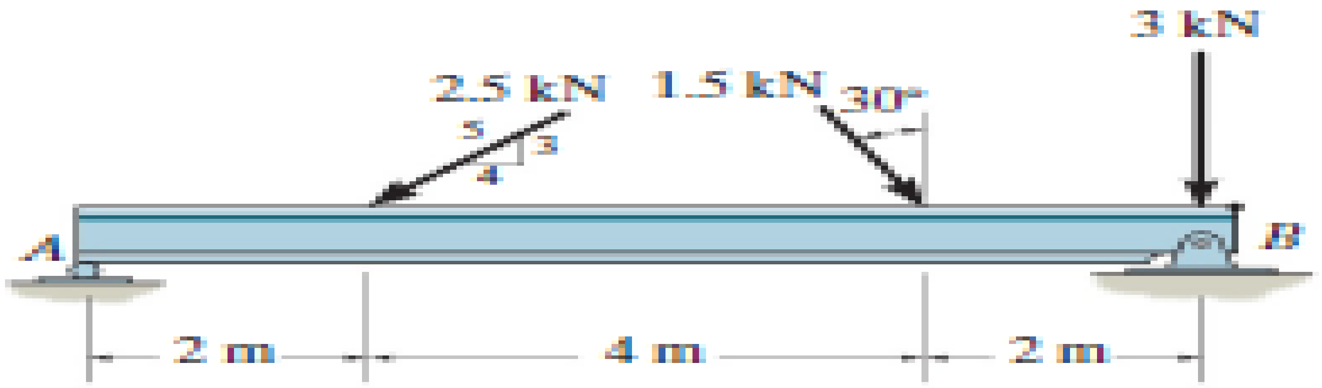
$$F = 167 \text{ lb}$$ **Ans.**

Resultant couple can act anywhere. **Ans.**

Problem

4–99. Replace the force system acting on the beam by an equivalent force and couple moment at point A.

*4–100. Replace the force system acting on the beam by an equivalent force and couple moment at point B.

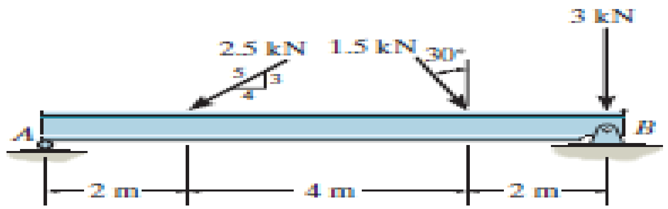


Probs. 4–99/100

Solution

4–99.

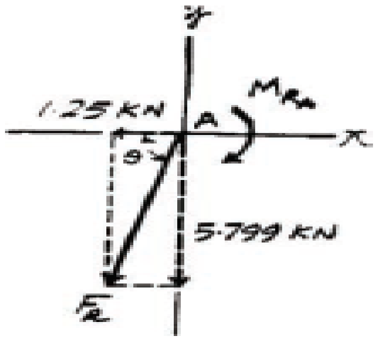
Replace the force system acting on the beam by an equivalent force and couple moment at point A.



SOLUTION

$$\xrightarrow{+} F_{R_x} = \Sigma F_x; \qquad F_{R_x} = 1.5 \sin 30° - 2.5\left(\frac{4}{5}\right)$$
$$= -1.25 \text{ kN} = 1.25 \text{ kN} \leftarrow$$

$$+\uparrow F_{R_y} = \Sigma F_y; \qquad F_{R_y} = -1.5 \cos 30° - 2.5\left(\frac{3}{5}\right) - 3$$
$$= -5.799 \text{ kN} = 5.799 \text{ kN} \downarrow$$



Thus,

$$F_R = \sqrt{F_{R_x}^2 + F_{R_y}^2} = \sqrt{1.25^2 + 5.799^2} = 5.93 \text{ kN}$$ **Ans.**

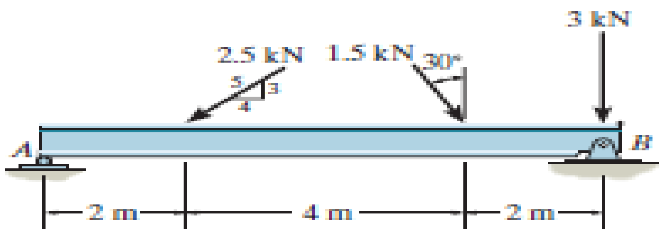
and

$$\theta = \tan^{-1}\left(\frac{F_{R_y}}{F_{R_x}}\right) = \tan^{-1}\left(\frac{5.799}{1.25}\right) = 77.8°$$ **Ans.**

$$\circlearrowleft + M_{R_A} = \Sigma M_A; \qquad M_{R_A} = -2.5\left(\frac{3}{5}\right)(2) - 1.5 \cos 30°(6) - 3(8)$$
$$= -34.8 \text{ kN} \cdot \text{m} = 34.8 \text{ kN} \cdot \text{m} \quad \textit{(Clockwise)}$$ **Ans.**

***4–100.**

Replace the force system acting on the beam by an equivalent force and couple moment at point B.



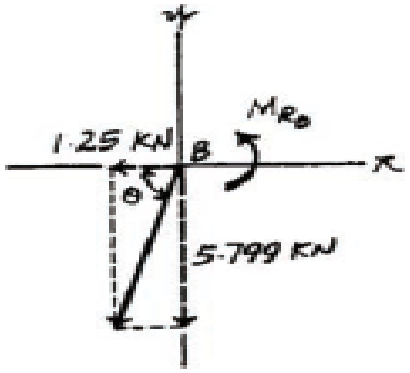
SOLUTION

$$\xrightarrow{+} F_{R_x} = \Sigma F_x; \qquad F_{R_x} = 1.5 \sin 30^\circ - 2.5\left(\frac{4}{5}\right)$$

$$= -1.25 \text{ kN} = 1.25 \text{ kN} \leftarrow$$

$$+\uparrow F_{R_y} = \Sigma F_y; \qquad F_{R_y} = -1.5 \cos 30^\circ - 2.5\left(\frac{3}{5}\right) - 3$$

$$= -5.799 \text{ kN} = 5.799 \text{ kN} \downarrow$$



Thus,

$$F_R = \sqrt{F_{R_x}^2 + F_{R_y}^2} = \sqrt{1.25^2 + 5.799^2} = 5.93 \text{ kN} \qquad \textbf{Ans.}$$

and

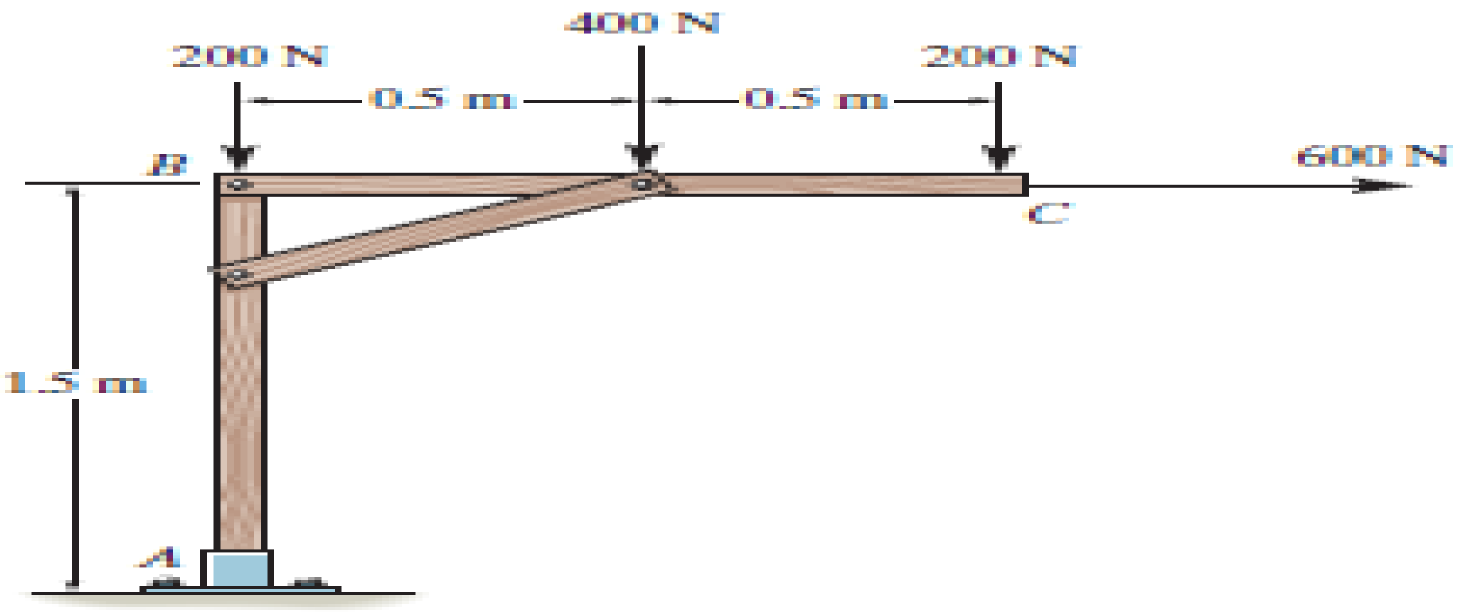
$$\theta = \tan^{-1}\left(\frac{F_{R_y}}{F_{R_x}}\right) = \tan^{-1}\left(\frac{5.799}{1.25}\right) = 77.8^\circ \qquad \textbf{Ans.}$$

$$\circlearrowleft + M_{R_B} = \Sigma M_{R_B}; \qquad M_B = 1.5\cos 30^\circ(2) + 2.5\left(\frac{3}{5}\right)(6)$$

$$= 11.6 \text{ kN} \cdot \text{m} \quad \textit{(Counterclockwise)} \qquad \textbf{Ans.}$$

Problem

4–119. Replace the loading on the frame by a single resultant force. Specify where its line of action intersects a vertical line along member AB, measured from A.

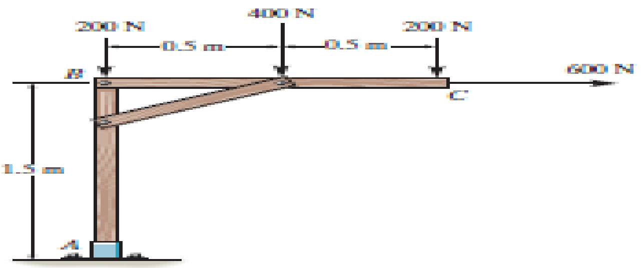


Prob. 4–119

Solution

4–119.

Replace the loading on the frame by a single resultant force. Specify where its line of action intersects a vertical line along member AB, measured from A.



SOLUTION

Equivalent Resultant Force. Referring to Fig. a,

$\xrightarrow{+} (F_R)_x = \Sigma F_x;\qquad (F_R)_x = 600 \text{ N} \rightarrow$

$+\uparrow (F_R)_y = \Sigma F_y;\qquad (F_R)_y = -200 - 400 - 200 = -800 \text{ N} = 800 \text{ N}\downarrow$

As indicated in Fig. a,

$$F_R = \sqrt{(F_R)_x^2 + (F_R)_y^2} = \sqrt{600^2 + 800^2} = 1000 \text{ N} \qquad \textbf{Ans.}$$

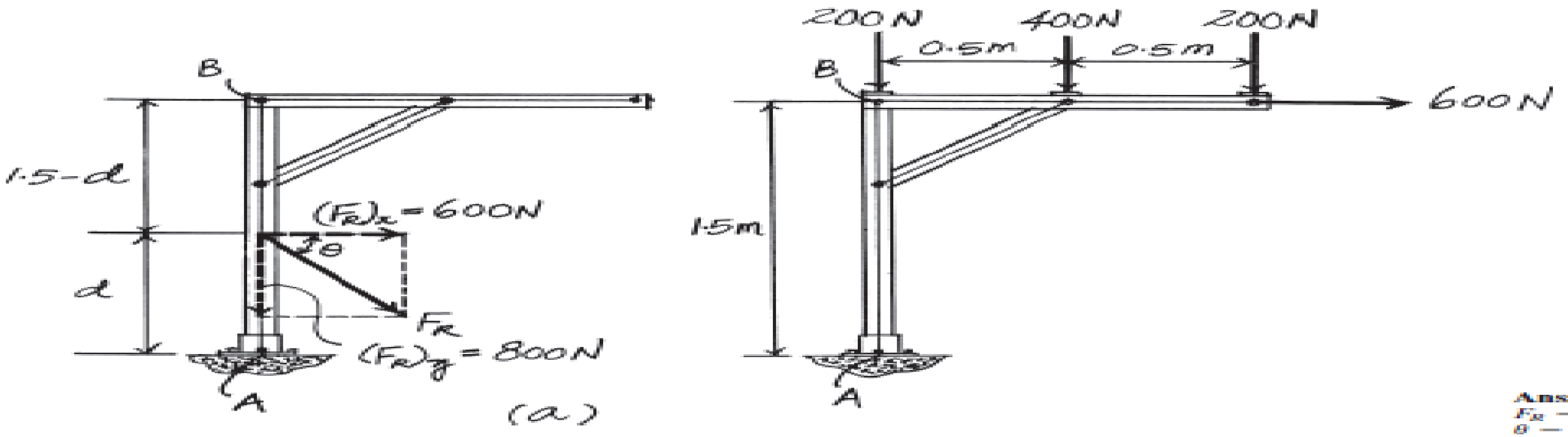
And

$$\theta = \tan^{-1}\left[\frac{(F_R)_y}{(F_R)_x}\right] = \tan^{-1}\left(\frac{800}{600}\right) = 53.13^\circ = 53.1^\circ \qquad \textbf{Ans.}$$

Location of Resultant Force. Along AB,

$\zeta+ (M_R)_B = \Sigma M_B;\qquad 600(1.5 - d) = -400(0.5) - 200(1)$

$$d = 2.1667 \text{ m} = 2.17 \text{ m} \qquad \textbf{Ans.}$$



Ans:
$F_R = 1000$ N
$\theta = 53.1^\circ$
$d = 2.17$ m