(Supplementary)

C

* Critical Damping: Proof

Proceeding from: $\frac{d^2v}{dt^2} + 2\alpha\frac{dv}{dt} + \alpha^2 v = 0$

or $\frac{d}{dt}\left(\frac{dv}{dt} + \alpha v\right) + \alpha\left(\frac{dv}{dt} + \alpha v\right) = 0$

Let $\frac{dv}{dt} + \alpha v = f$ —— (A)

— Then $\frac{df}{dt} + \alpha f = 0$

— This is a 1st order eqn so

$f = A_1 e^{-\alpha t}$ is the solution

— So we get from (A)

$$\frac{dv}{dt} + \alpha v = A_1 e^{-\alpha t}$$

— or $e^{\alpha t}\frac{dv}{dt} + e^{\alpha t}(\alpha v) = A_1$

— This can be written as

$$\frac{d}{dt}\left(e^{\alpha t} v\right) = A_1$$

— Integrating both sides

$$e^{\alpha t} v = A_1 t + A_2$$

or $v = e^{-\alpha t}(A_1 t + A_2)$, V