Engineering Mechanics

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CHAPTER 4

Force System Resultants

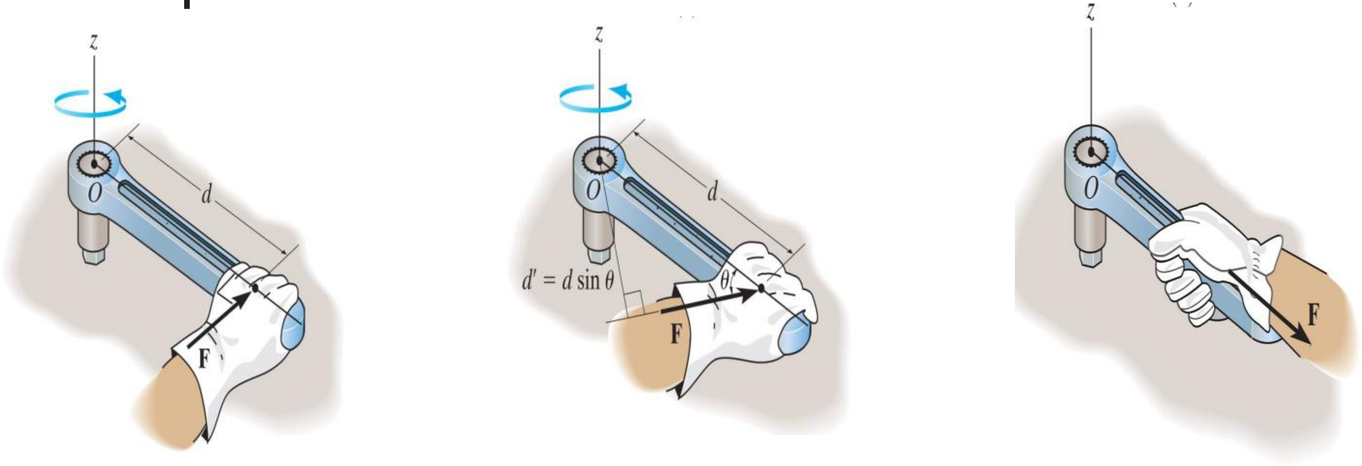
Contents (Section 4.3)

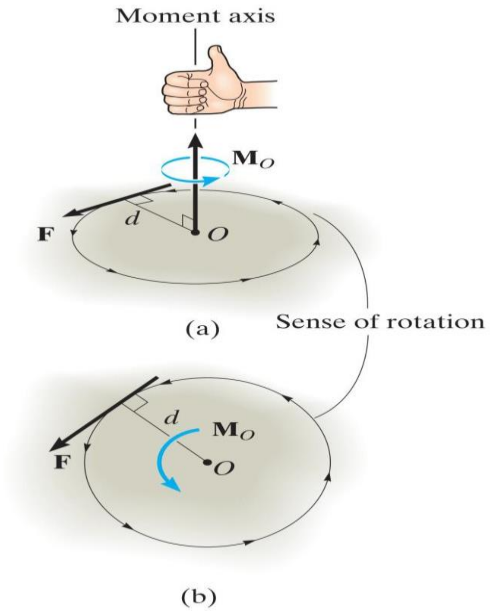
- Recap
- Moment of Force- Vector Formulation

RECAP

Moment of a Force/Moment/Torque (Scalar)

When a force is applied to a body it will produce a tendency for the body to rotate about a point that is not on the line of action of the force





Magnitude. The magnitude of $\mathbf{M}_O$ is

$$\boxed{M_O = Fd}$$

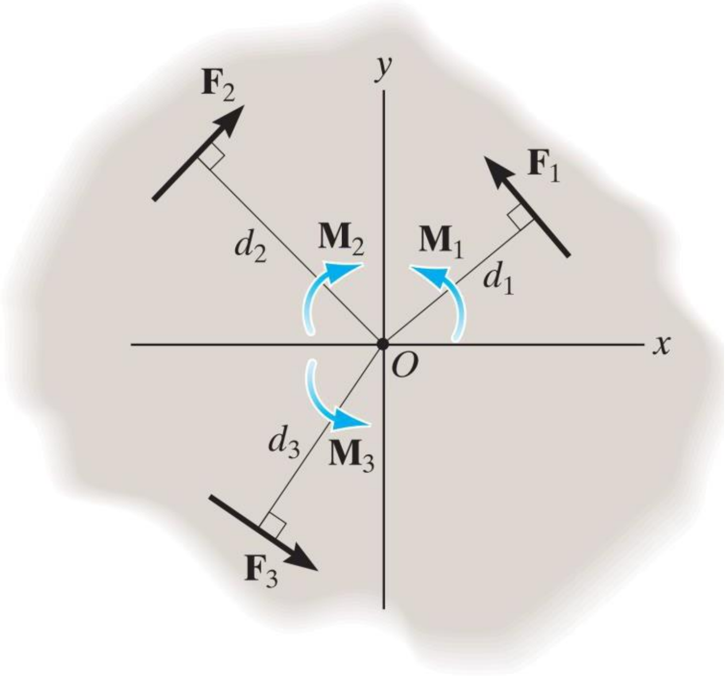
Direction: Direction using "right hand rule"

the thumb of the right hand will give the directional sense Mo

Moment of a Force/Moment/Torque (Scalar)

Resultant Moment.

$$\circlearrowleft + (M_R)_o = \Sigma Fd; \qquad (M_R)_o = F_1 d_1 - F_2 d_2 + F_3 d_3$$



Cross Product

The ***cross product*** of two vectors **A** and **B** yields the vector **C**

$$\mathbf{C} = \mathbf{A} \times \mathbf{B}$$

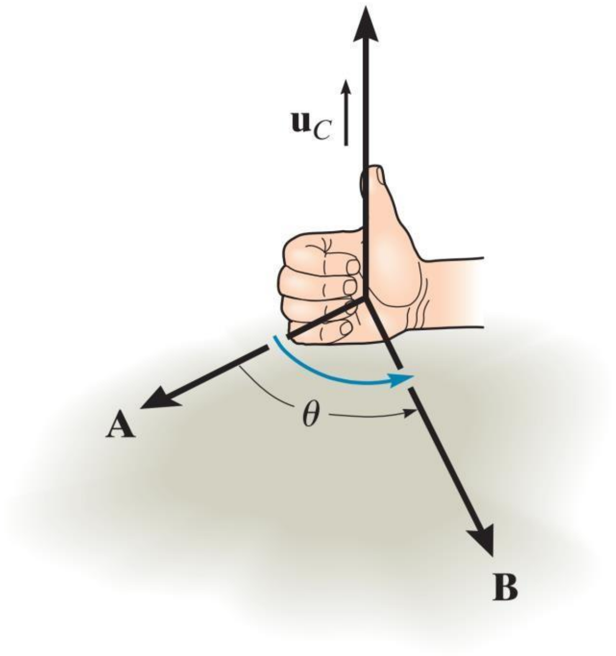
$$\mathbf{A} \times \mathbf{B} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$$

Magnitude.

$$C = AB \sin \theta.$$

Direction.

$$\mathbf{C} = \mathbf{A} \times \mathbf{B} = (AB \sin \theta)\mathbf{u}_C$$

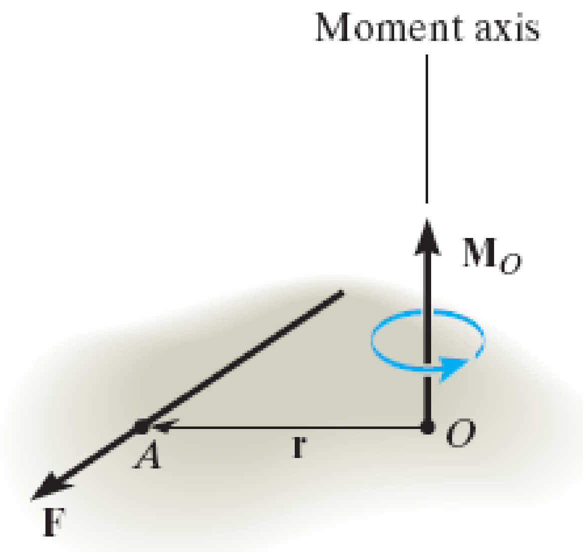


4.3 Moment of Force - Vector Formulation

- Moment of force **F** about point O can be expressed using cross product

$$\mathbf{M}_O = \mathbf{r} \times \mathbf{F}$$

Here **r** represents a position vector directed *from* O to *any point* on the line of action of **F**.



Magnitude

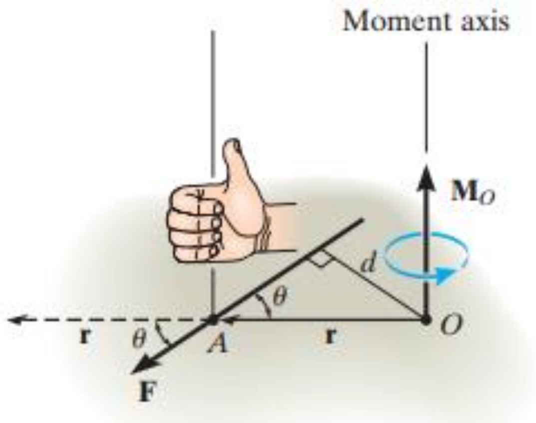
- For magnitude of cross product,

$$M_O = rF \sin\theta$$

where the angle is measured between the *tails* of **r** and **F**.

- Treat **r** as a sliding vector. Since $d = r \sin\theta$,

$$M_O = rF \sin\theta = F\,(r\sin\theta) = Fd$$



Moment of Force - Vector Formulation

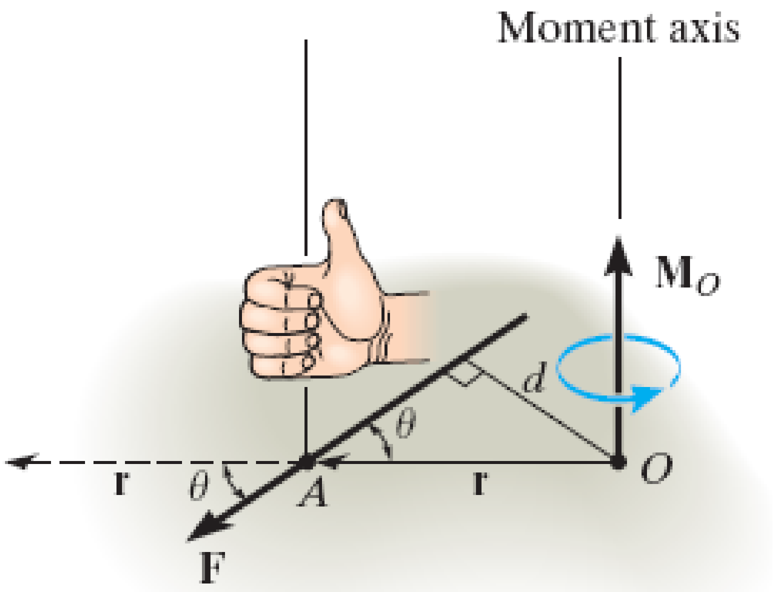
Direction

- Direction and sense of $\mathbf{M}_O$ are determined by right-hand rule

*Note:

- "curl" of the fingers indicates the sense of rotation

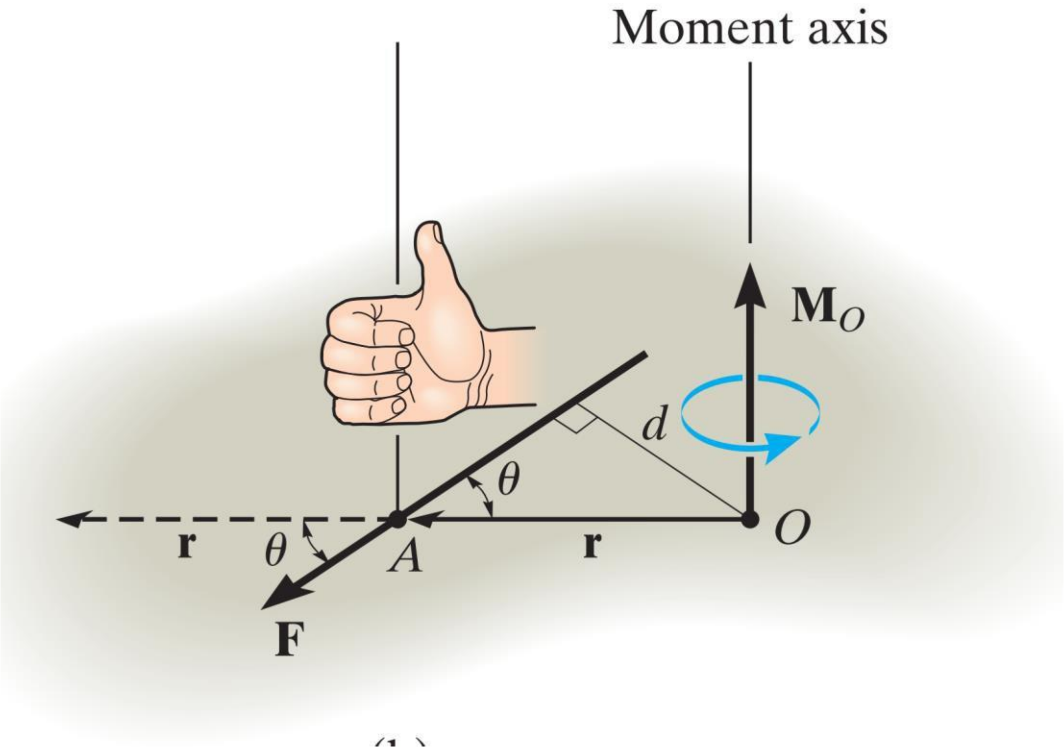
- Maintain proper order of **r** and **F** since cross product is not commutative



Moment of a Force (Vector)

$$\mathbf{M}_O = \mathbf{r} \times \mathbf{F}$$

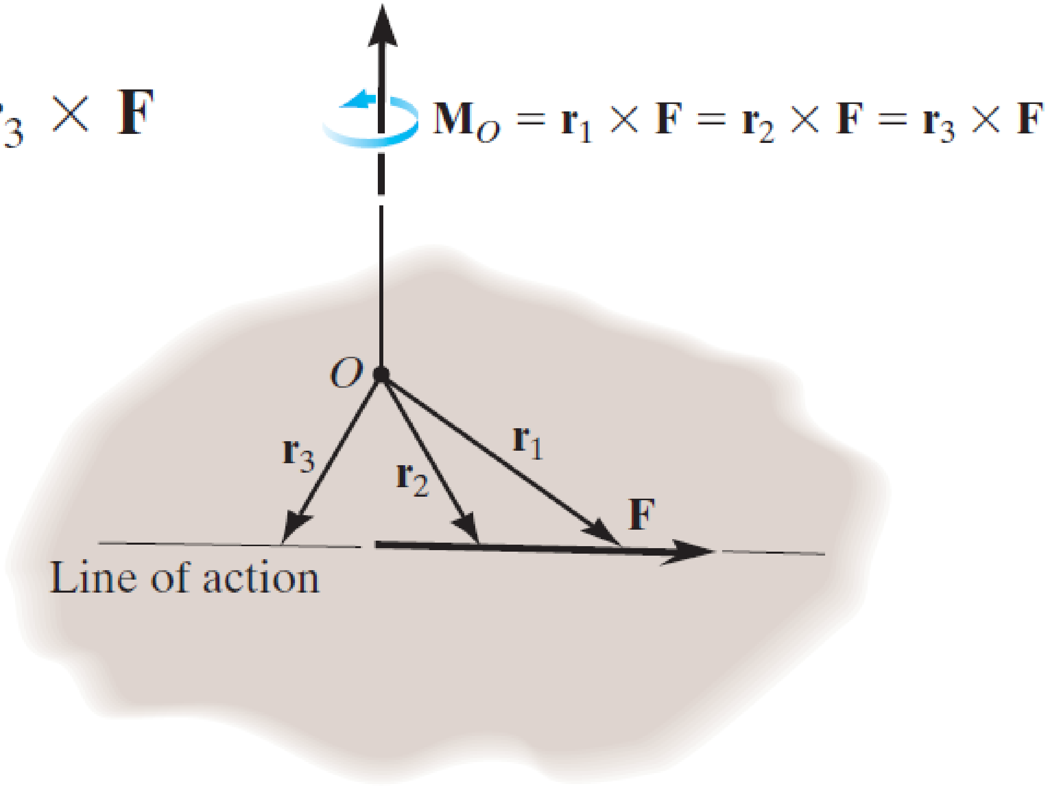
$$M_O = rF\sin\theta = F(r\sin\theta) = Fd$$



Principle of Transmissibility.

we can use any position vector **r** measured from point O to any point on the line of action of the force **F**

$$\mathbf{M}_O = \mathbf{r}_1 \times \mathbf{F} = \mathbf{r}_2 \times \mathbf{F} = \mathbf{r}_3 \times \mathbf{F}$$

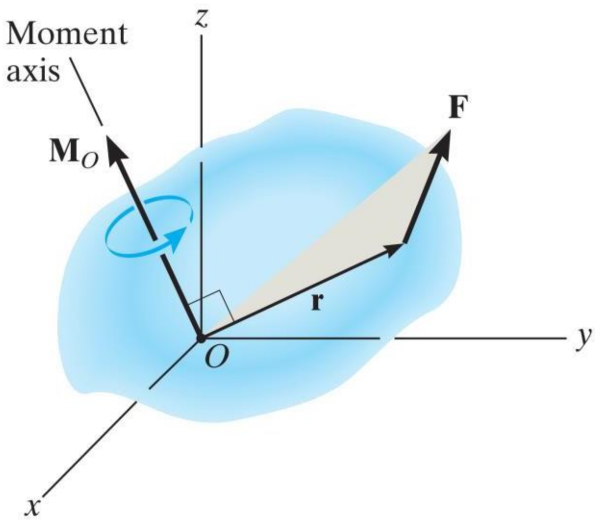


Since **F** can be applied at any point along its line of action and still create this *same moment* about point O, then **F** can be considered a *sliding vector*. This property is called the *principle of transmissibility* of a force.

Moment of a Force (Vector)

Cartesian Vector Formulation.

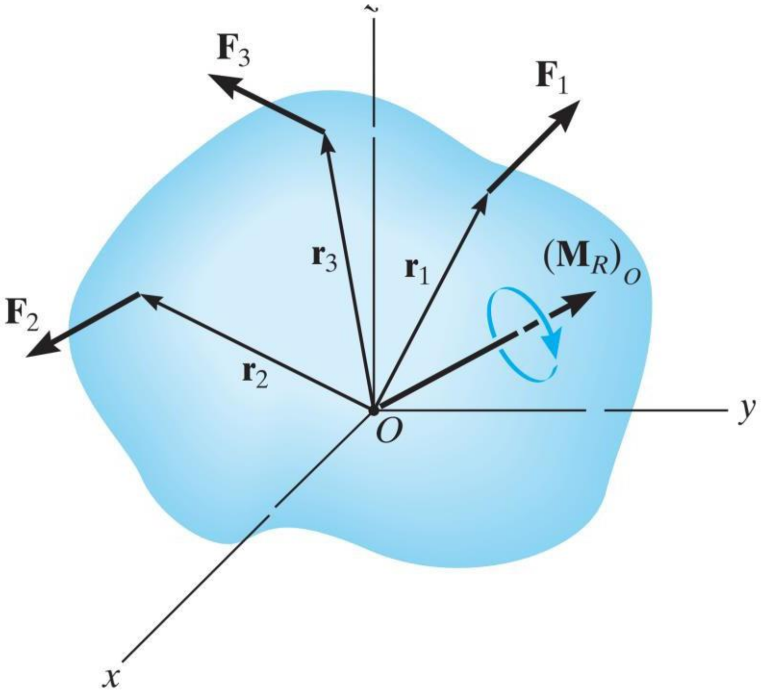
$$\mathbf{M}_O = \mathbf{r} \times \mathbf{F} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ r_x & r_y & r_z \\ F_x & F_y & F_z \end{vmatrix}$$



$$\mathbf{M}_O = (r_y F_z - r_z F_y)\mathbf{i} - (r_x F_z - r_z F_x)\mathbf{j} + (r_x F_y - r_y F_x)\mathbf{k}$$

Moment of a Force (Vector)

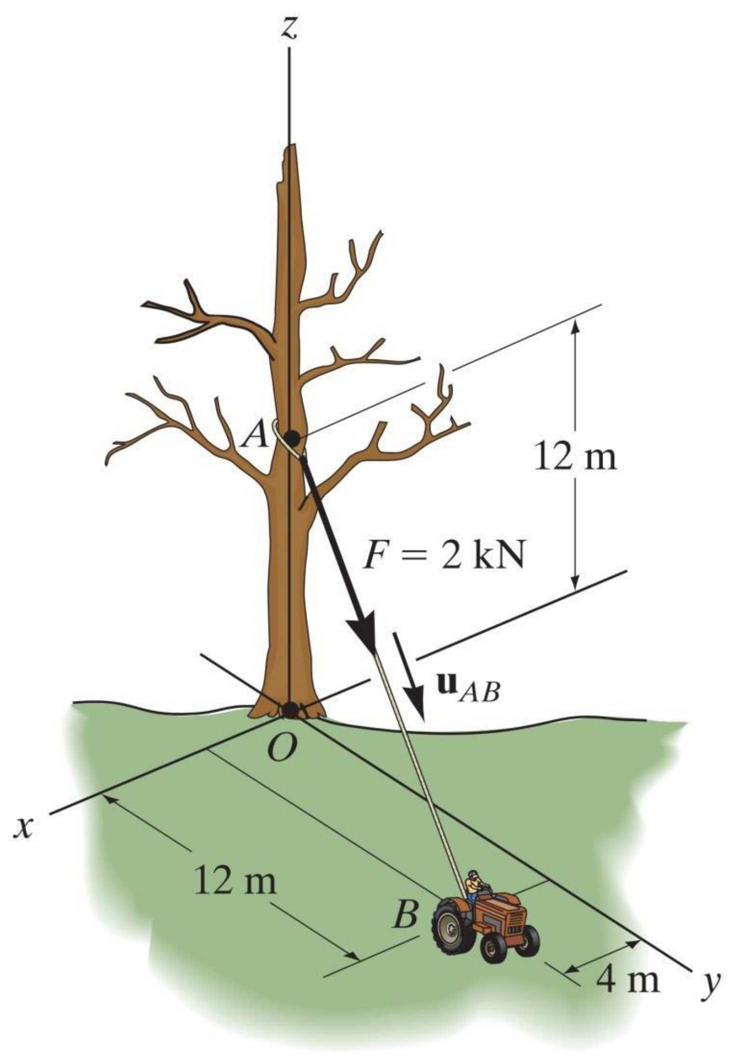
Resultant Moment of a System of Forces.



$$(\mathbf{M}_R)_O = \Sigma(\mathbf{r} \times \mathbf{F})$$

Example

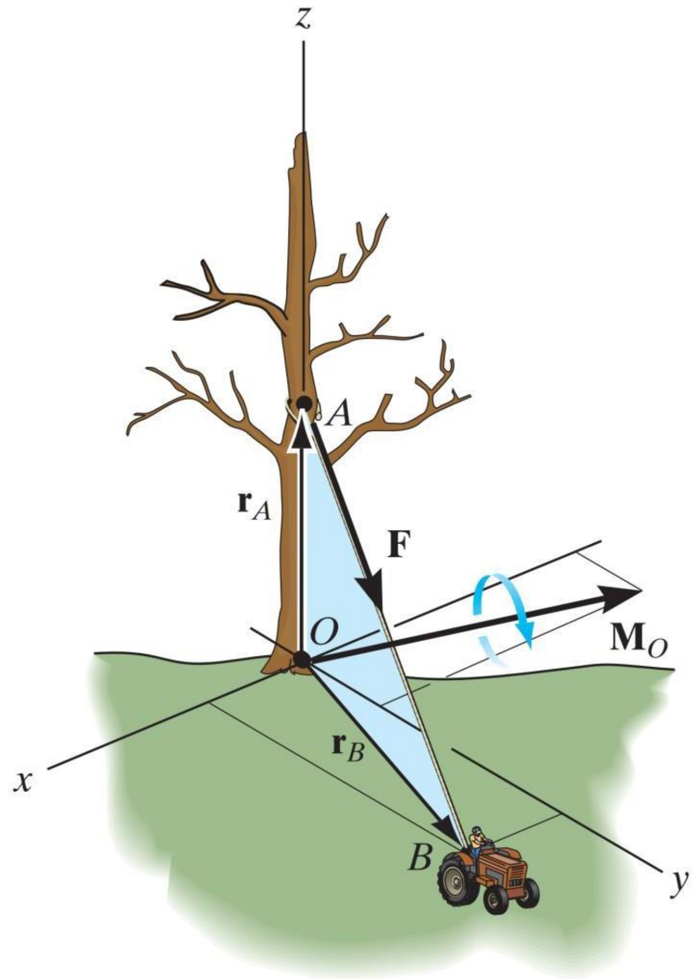
Determine the moment produced by the force **F** in Fig. 4–14*a* about point O. Express the result as a Cartesian vector.



$$\mathbf{r}_A = \{12\mathbf{k}\}\ \text{m} \quad \text{and} \quad \mathbf{r}_B = \{4\mathbf{i} + 12\mathbf{j}\}\ \text{m}$$

$$\mathbf{F} = F\mathbf{u}_{AB} = 2\ \text{kN}\left[\frac{\{4\mathbf{i} + 12\mathbf{j} - 12\mathbf{k}\}\ \text{m}}{\sqrt{(4\ \text{m})^2 + (12\ \text{m})^2 + (-12\ \text{m})^2}}\right]$$

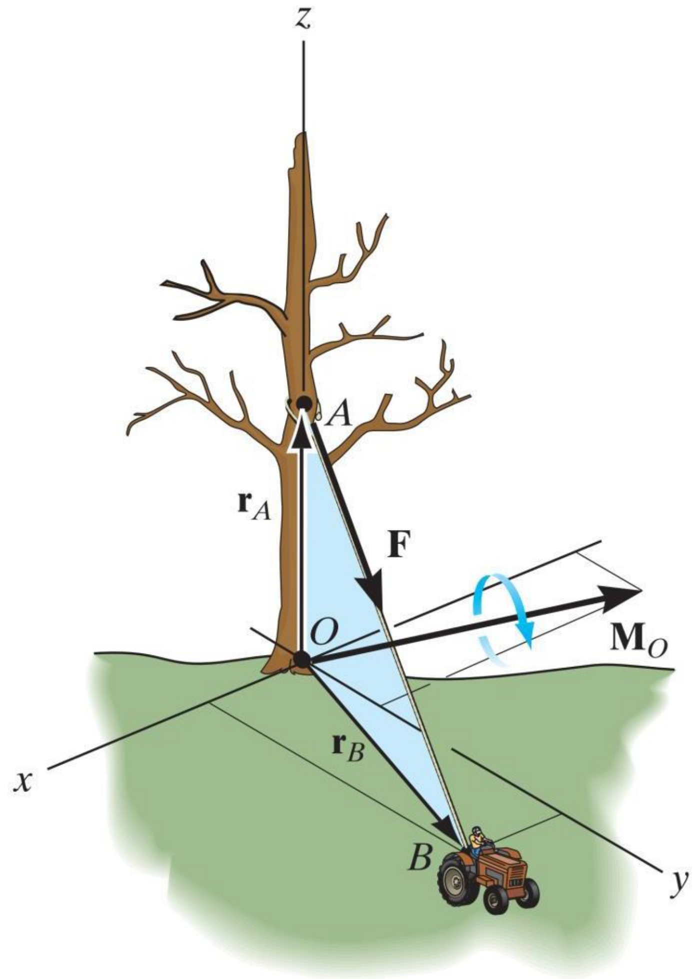
$$= \{0.4588\mathbf{i} + 1.376\mathbf{j} - 1.376\mathbf{k}\}\ \text{kN}$$



$$\mathbf{r}_A = \{12\mathbf{k}\}\text{ m}^{\circ}\text{and}^{\circ}\,\mathbf{r}_B = \{4\mathbf{i} + 12\mathbf{j}\}\text{ m}$$

$$\mathbf{F} = F\mathbf{u}_{AB} = 2\text{ kN}\left[\frac{\{4\mathbf{i} + 12\mathbf{j} - 12\mathbf{k}\}\text{ m}}{\sqrt{(4\text{ m})^2 + (12\text{ m})^2 + (-12\text{ m})^2}}\right]$$

$$= \{0.4588\mathbf{i} + 1.376\mathbf{j} - 1.376\mathbf{k}\}\text{ kN}$$



$$\mathbf{M}_O = \mathbf{r}_A \times \mathbf{F} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & 12 \\ 0.4588 & 1.376 & -1.376 \end{vmatrix}$$

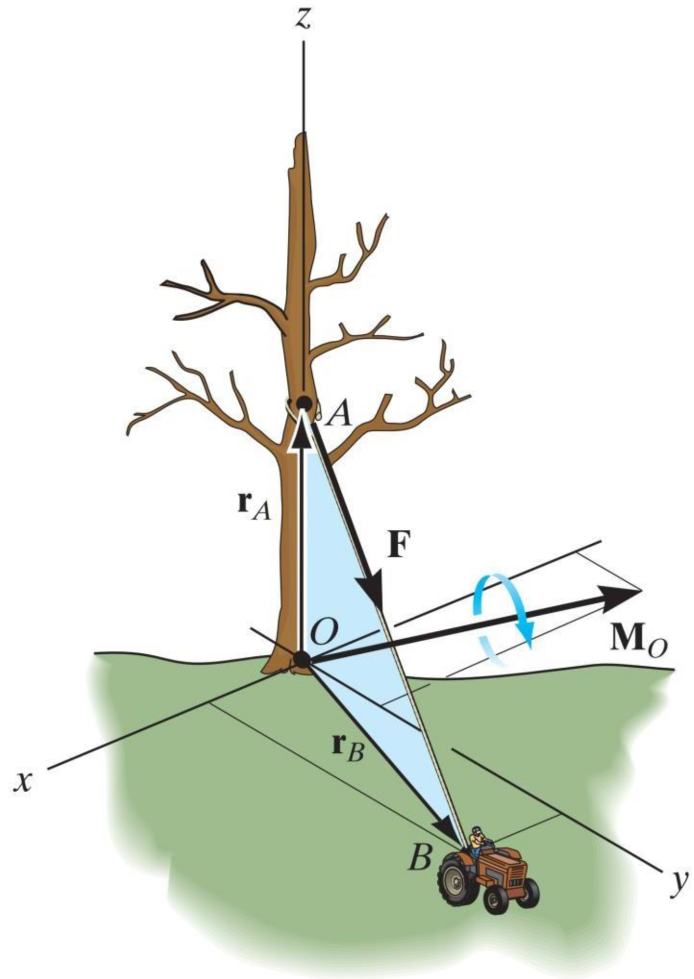
$$= [0(-1.376) - 12(1.376)]\mathbf{i} - [0(-1.376) - 12(0.4588)]\mathbf{j} + [0(1.376) - 0(0.4588)]\mathbf{k}$$

$$= \{-16.5\mathbf{i} + 5.51\mathbf{j}\}\text{ kN}\cdot\text{m}$$ *A*

$$\mathbf{r}_A = \{12\mathbf{k}\}\ \text{m} \quad \text{and} \quad \mathbf{r}_B = \{4\mathbf{i} + 12\mathbf{j}\}\ \text{m}$$

$$\mathbf{F} = F\mathbf{u}_{AB} = 2\ \text{kN}\left[\frac{\{4\mathbf{i} + 12\mathbf{j} - 12\mathbf{k}\}\ \text{m}}{\sqrt{(4\ \text{m})^2 + (12\ \text{m})^2 + (-12\ \text{m})^2}}\right]$$

$$= \{0.4588\mathbf{i} + 1.376\mathbf{j} - 1.376\mathbf{k}\}\ \text{kN}$$



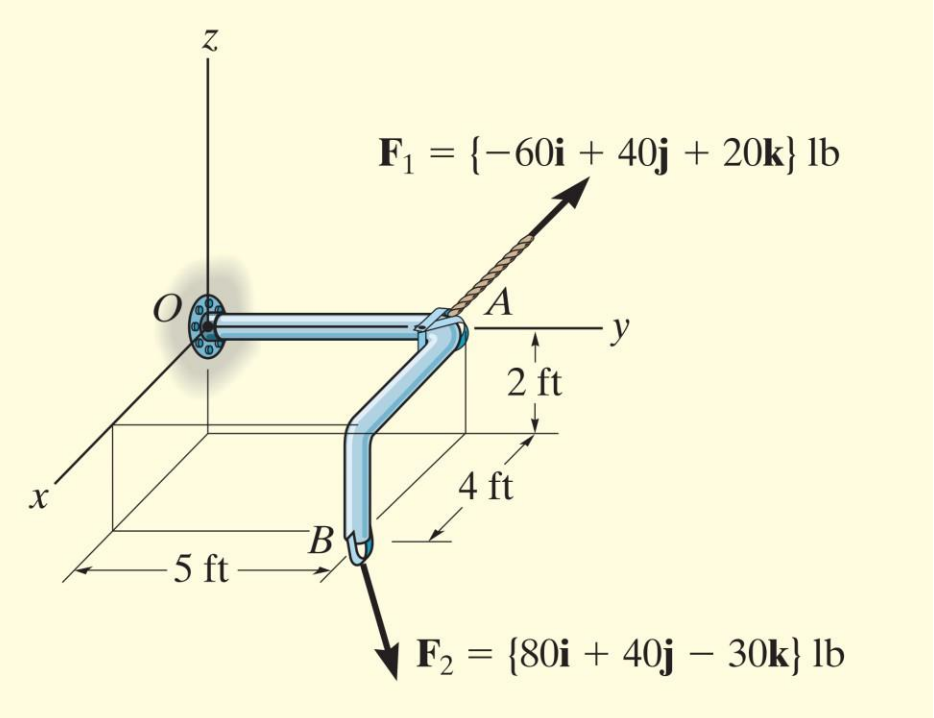
$$\mathbf{M}_O = \mathbf{r}_B \times \mathbf{F} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 4 & 12 & 0 \\ 0.4588 & 1.376 & -1.376 \end{vmatrix}$$

$$= [12(-1.376) - 0(1.376)]\mathbf{i} - [4(-1.376) - 0(0.4588)]\mathbf{j} + [4(1.376) - 12(0.4588)]\mathbf{k}$$

$$= \{-16.5\mathbf{i} + 5.51\mathbf{j}\}\ \text{kN} \cdot \text{m}$$ *Ans.*

Example

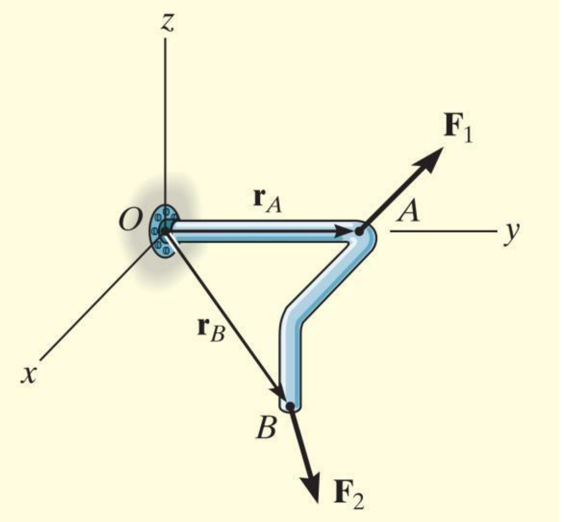
Two forces act on the rod shown in Fig. 4–15*a*. Determine the resultant moment they create about the flange at *O*. Express the result as a Cartesian vector.

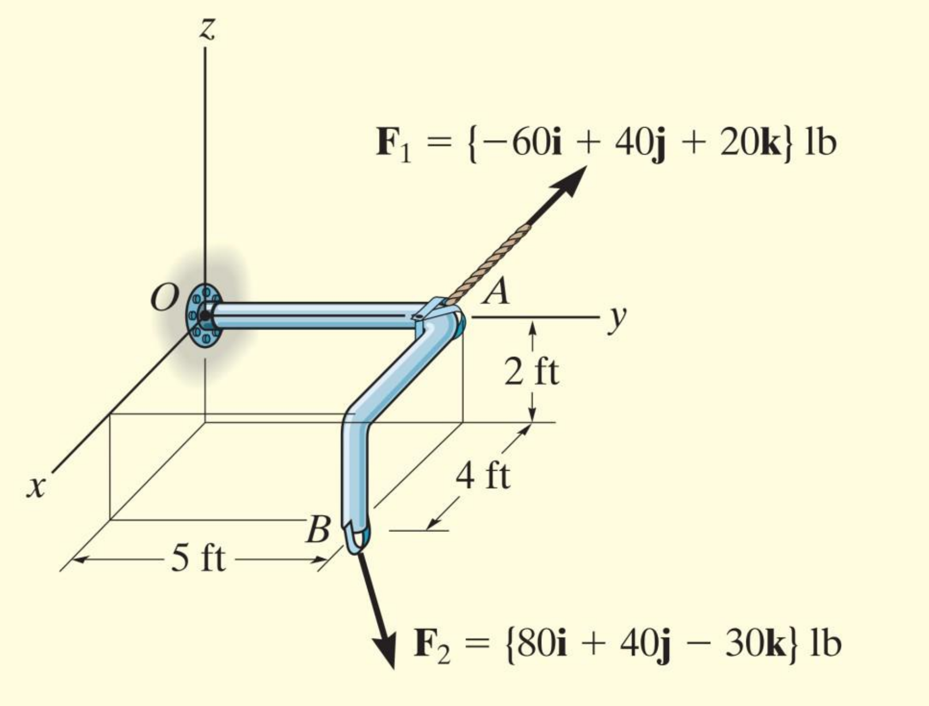


Two forces act on the rod shown in Fig. 4–15*a*. Determine the resultant moment they create about the flange at *O*. Express the result as a Cartesian vector.

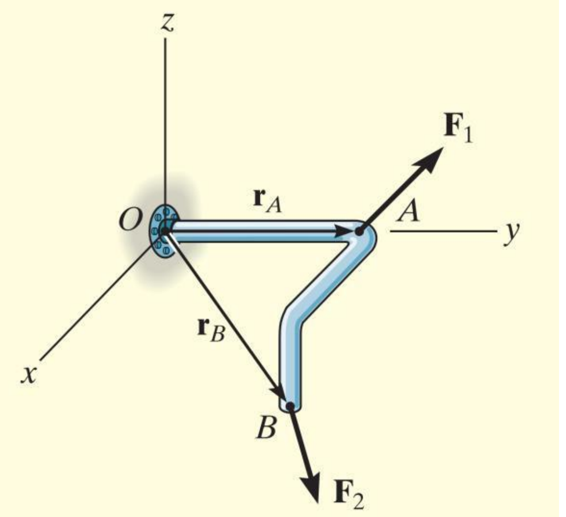
$$\mathbf{r}_A = \{5\mathbf{j}\}\text{ ft}$$

$$\mathbf{r}_B = \{4\mathbf{i} + 5\mathbf{j} - 2\mathbf{k}\}\text{ ft}$$



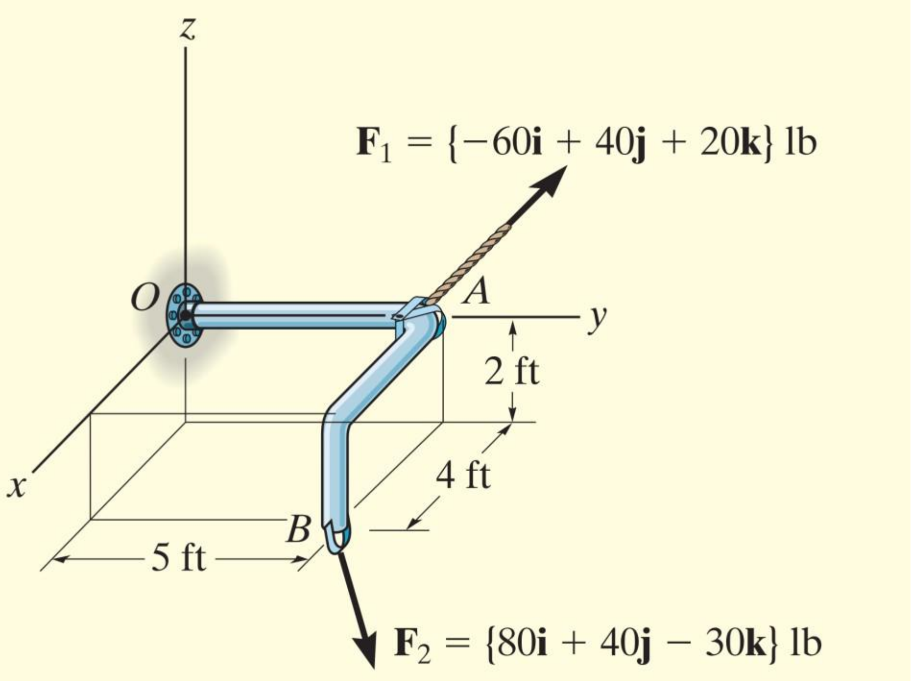


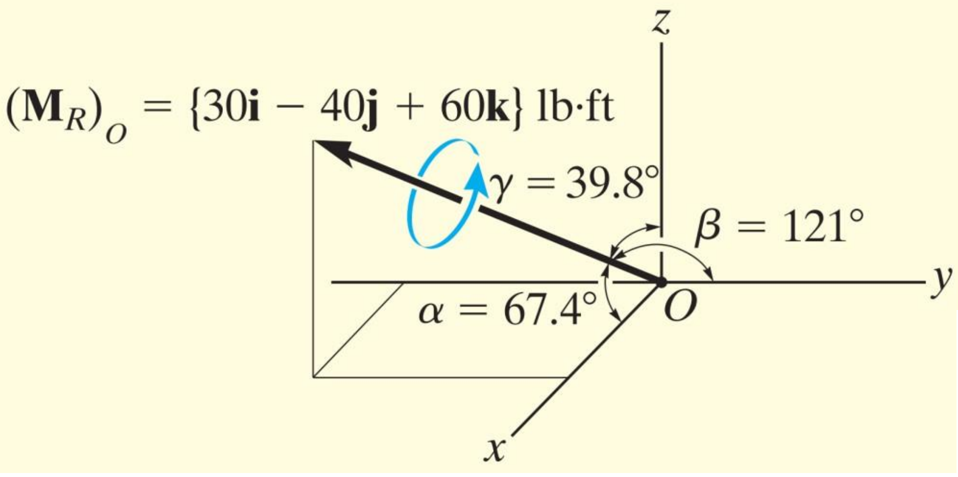
Two forces act on the rod shown in Fig. 4–15*a*. Determine the resultant moment they create about the flange at *O*. Express the result as a Cartesian vector.



$$
\begin{aligned}
(\mathbf{M}_R)_O &= \Sigma(\mathbf{r} \times \mathbf{F}) \\
&= \mathbf{r}_A \times \mathbf{F}_1 + \mathbf{r}_B \times \mathbf{F}_2 \\
&= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 5 & 0 \\ -60 & 40 & 20 \end{vmatrix} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 4 & 5 & -2 \\ 80 & 40 & -30 \end{vmatrix} \\
&= [5(20) - 0(40)]\mathbf{i} - [0]\mathbf{j} + [0(40) - (5)(-60)]\mathbf{k} \\
&\quad + [5(-30) - (-2)(40)]\mathbf{i} - [4(-30) - (-2)(80)]\mathbf{j} + [4(40) - 5(80)]\mathbf{k} \\
&= \{30\mathbf{i} - 40\mathbf{j} + 60\mathbf{k}\}\ \text{lb}\cdot\text{ft}
\end{aligned}
$$

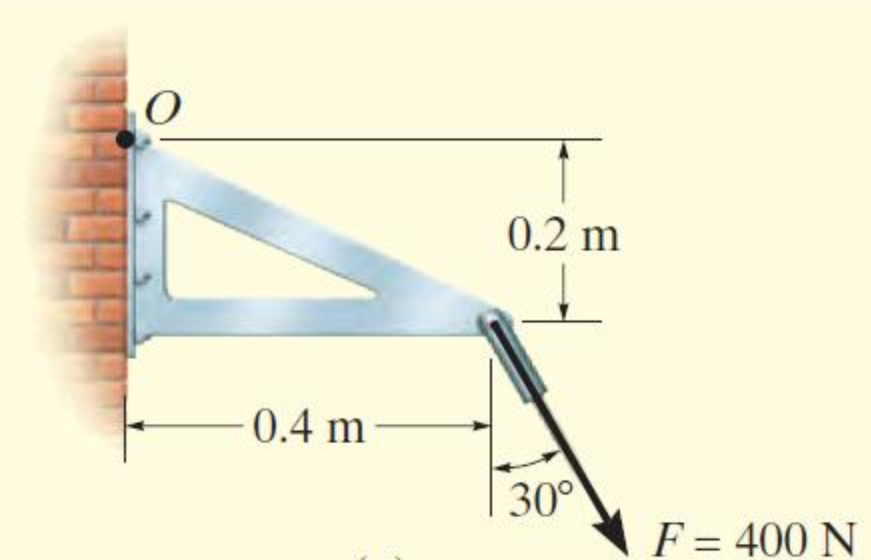
Ans.





Example

Force **F** acts at the end of the angle bracket shown in Fig. 4–19*a*. Determine the moment of the force about point *O*.



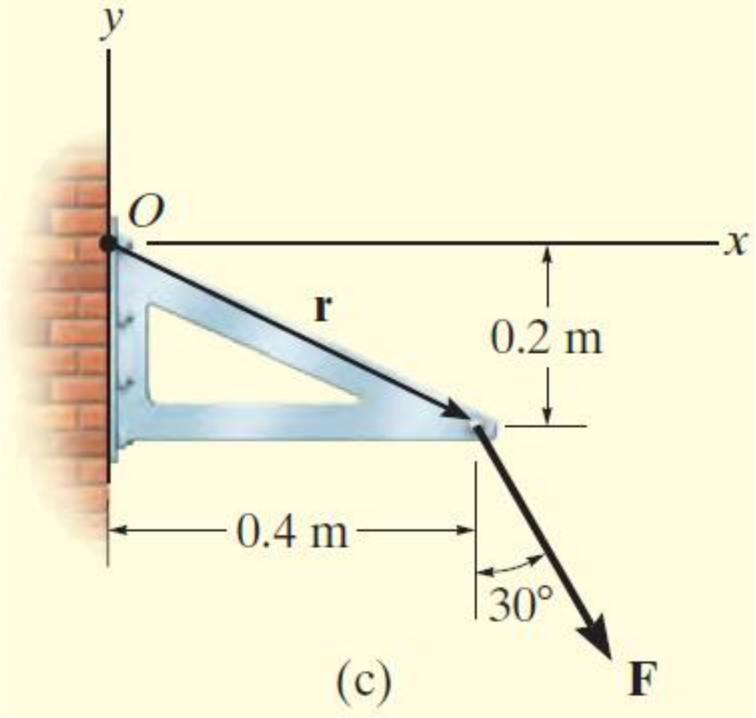
(a)

$$\mathbf{r} = \{0.4\mathbf{i} - 0.2\mathbf{j}\}\ \text{m}$$

$$\mathbf{F} = \{400 \sin 30°\mathbf{i} - 400 \cos 30°\mathbf{j}\}\ \text{N}$$

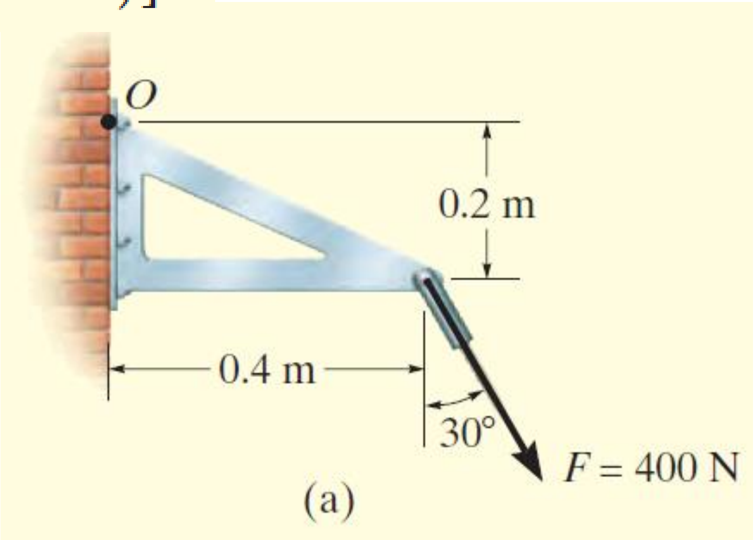
$$= \{200.0\mathbf{i} - 346.4\mathbf{j}\}\ \text{N}$$

$$\mathbf{M}_O = \mathbf{r} \times \mathbf{F} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0.4 & -0.2 & 0 \\ 200.0 & -346.4 & 0 \end{vmatrix}$$



$$= 0\mathbf{i} - 0\mathbf{j} + [0.4(-346.4) - (-0.2)(200.0)]\mathbf{k}$$

$$= \{-98.6\mathbf{k}\}\ \text{N}\cdot\text{m}$$



Home Assignment

- Examples.