Chapter1: Digital Systems and Binary Numbers

Lecture2- Number Base Conversions, Binary Arithmetic and Determine Unknown Radix

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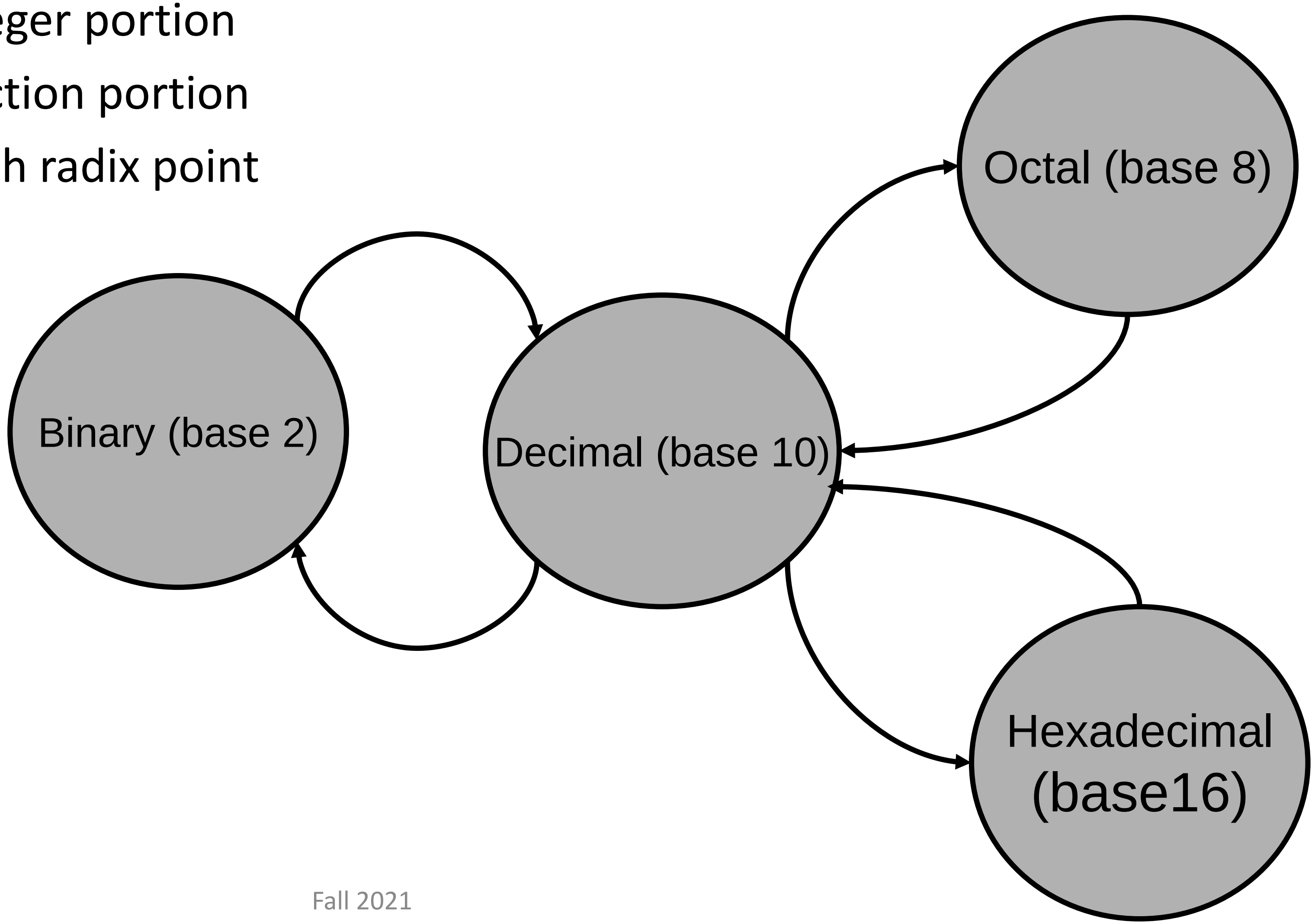
Objectives

- Study Number Base Conversions
- Perform Binary Arithmetic
- Determine Unknown Radix

Conversion between bases

- To convert from one base to other:
  - Convert the integer portion
  - Convert the fraction portion
  - Join the two with radix point



Decimal-r Conversion

- Conversion of a number in base r to decimal is done by expanding the number in a power series and adding all the terms.
- For example, $(C34D)_{16}$ is converted to decimal:

  $12 \text{ X } 16^3 + 3 \text{ X } 16^2 + 4 \text{ X } 16^1 + 13 \text{ X } 16^0 = (49997)_{10}$
- $(11010.11)_2$ is converted to decimal:

  $1 \text{ X } 2^4 + 1 \text{ X } 2^3 + 0 \text{ X } 2^2 + 1 \text{ X } 2^1 + 0 \text{ X } 2^0 + 1 \text{ X } 2^{-1} + 1 \text{ X } 2^{-2} = 26.75$
- In general $N=(\text{Number})_r =$ $$\left(\sum_{i=0}^{i=n-1} a_i \bullet r^i\right) + \left(\sum_{j=-m}^{j=-1} a_j \bullet r^j\right)$$

  (Integer Portion) + (Fraction Portion)

Decimal-r Conversion

- If a decimal number has a radix point, it is necessary to separate the number into an integer part and a fraction part.
- The conversion of a decimal integer into a number in base-r is done by dividing the number and all successive quotients by r and accumulating the remainders in reverse order of computation.
- For example, to convert decimal 13 to binary:

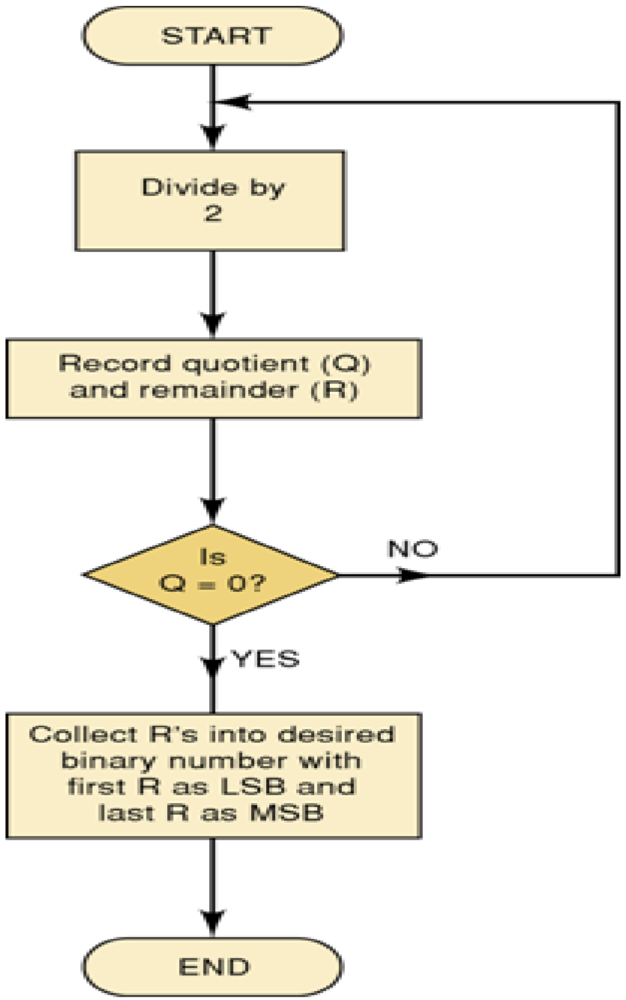
	Integer Quotient		Remainder	Coefficient
13/2 =	6	+	½	$a_0 = 1$
6/2 =	3	+	0	$a_1 = 0$
3/2 =	1	+	½	$a_2 = 1$
1/2 =	0	+	½	$a_3 = 1$

Answer $(13)_{10} = (a_3\, a_2\, a_1\, a_0)_2 = (1101)_2$

Decimal to Binary Conversion

Repeated Division

This flowchart describes the process and can be used to convert from decimal to any other number system.



Decimal to Binary Conversion Example

- Convert $(37)_{10}$ to binary

Division	Quotient	Remainder
37 / 2	18	1
18 / 2	9	0
9 / 2	4	1
4 / 2	2	0
2 / 2	1	0
1 / 2	0	1

$(37)_{10} = 100101_2$

Decimal-r Conversion (Converting Fractions)

- To convert the fraction portion repeatedly multiply the fraction by the radix and save the integer digits that result. The process continued until the fraction becomes 0 or the number of digits have sufficient accuracy. The new radix fraction digits are the integer digits in computed order.
- For example, convert fraction $(0.6875)_{10}$ to base 2

$0.6875 * 2 = 1.3750$	integer = 1
$0.3750 * 2 = 0.7500$	integer = 0
$0.7500 * 2 = 1.5000$	integer = 1
$0.5000 * 2 = 1.0000$	integer = 1

Answer = $(0.1011)_2$

Converting Fractions Cont…

- When converting fractions, we must use multiplication rather than division. The new radix fraction digits are the integer digits in *computed order*.

	Integer		Fraction	Coefficent
0.8432 X 2 =	1	+	0.6864	$a_{-1} = 1$
0.6864 X 2 =	1	+	0.3728	$a_{-2} = 1$
0.3728 X 2 =	0	+	0.7456	$a_{-3} = 0$
0.7456 X 2 =	1	+	0.4912	$a_{-4} = 1$
0.4912 X 2 =	0	+	0.9824	$a_{-5} = 0$
0.9824 X 2 =	1	+	0.9648	$a_{-6} = 1$
0.9648 X 2 =	1	+	0.9296	$a_{-7} = 1$

Continue until fraction becomes 0 or until sufficient accuracy.

$$(0.8432)_{10} = (0.a_{-1}a_{-2}a_{-3}a_{-4}a_{-5}a_{-6}a_{-7})_2 = (0.1101011)_2$$

Another example:

- **Convert 0.8125 decimal to binary.**
  - To convert the decimal 0.8125 to binary, we multiply by the radix 2.
  - **$(0.1101)_2$**

$$\begin{array}{r} .8125 \\ \times\quad 2 \\ \hline 1.6250 \end{array}$$

$$\begin{array}{r} .6250 \\ \times\quad 2 \\ \hline 1.2500 \end{array}$$

$$\begin{array}{r} .2500 \\ \times\quad 2 \\ \hline 0.5000 \end{array}$$

$$\begin{array}{r} .5000 \\ \times\quad 2 \\ \hline 1.0000 \end{array}$$

Decimal to Octal Conversion

- In converting decimal to octal we must divide integer part by 8 till quotient becomes lesser than divisor.

	Integer Quotient		Remainder	Coefficent
35 / 8 =	4	+	3/8	$a_0 = 3$
4 / 8 =	0	+	4/8	$a_1 = 4$

$$(35)_{10} = (a_1 a_0)_8 = (43)_8$$

Converting Fractions (Decimal to Octal)

- Decimal to Octal fraction conversion takes the same approach but it multiplies by the base 8.

	Integer		Fraction	Coefficent
0.8432 X 8 =	6	+	0.7456	$a_{-1} = 6$
0.7456 X 8 =	5	+	0.9648	$a_{-2} = 5$
0.9648 X 8 =	7	+	0.7184	$a_{-3} = 7$
0.7184 X 8 =	5	+	0.7472	$a_{-4} = 5$
0.7472 X 8 =	5	+	0.9776	$a_{-5} = 5$
0.9776 X 8 =	7	+	0.8208	$a_{-6} = 7$
0.8208 X 8 =	6	+	0.5664	$a_{-7} = 6$

Continue until fraction becomes 0 or until sufficient accuracy.

$$(0.8432)_{10} = (0.a_{-1}a_{-2}a_{-3}a_{-4}a_{-5}a_{-6}a_{-7})_8 = (0.6575576)_8$$

Converting Decimal to Hexadecimal

- The conversion of a decimal integer into hexadecimal is done by dividing the number and all successive quotients by 16 and accumulating the remainders in reverse order of computation.

Division	Quotient	Remainder
422 / 16	26	6
26 / 16	1	A
1 / 16	0	1

$$(422)_{10} = (1A6)_{16}$$

Binary, Octal and Hexadecimal

- Conversions between binary, octal and hexadecimal have an easier conversion method.
  - Each octal digit represents 3 binary digits.
  - Each hexadecimal digit represents 4 binary digits.

$$(11\ 010\ 101\ 111\ 111\ .\ 101\ 110\ 01)_2 = (32577.561)_8$$
$$3\quad 2\quad 5\quad 7\quad 7\qquad 5\quad 6\quad 1$$

$$(11\ 0101\ 0111\ 1111\ .\ 1011\ 1001)_2 = (357F.B9)_{16}$$
$$3\quad 5\quad 7\quad F\qquad B\quad 9$$

Binary to Octal and back

- Binary to Octal:
  - Group the binary digits into three-bit groups starting at the radix point and going both ways, padding with zeros as needed (at the ends).
  - Convert each group of three bits to an equivalent octal digit.
- Octal to Binary:
  - It is done by reversing the preceding procedure
  - Restate the octal as three binary digits
  - Start at the radix point and go both ways, padding with zeros as needed.

Examples

- Convert $(10110001101011.11110000011)_2$ to Octal

  = 010 110 001 101 011 . 111 100 000 110

  = 2 6 1 5 3 . 7 4 0 6

  = $(26153.7406)_8$
- Convert $(673.124)_8$ to binary

  = 110 111 011 . 001 010 100

  = $(110111011.001010100)_2$
- Convert $(11010100011011)_2$ to Octal

011	010	100	011	011
3	2	4	3	3

Binary to Hexadecimal and back

- Binary to Hexadecimal:
  - Group the binary digits into four-bit groups starting at the radix point and going both ways, padding with zeros as needed (at the ends)
  - Convert each group of four bits to an equivalent hexadecimal digit
- Hexadecimal to Binary:
  - It is done by reversing the preceding procedure
  - Restate the hexadecimal as four binary digits
  - Start at the radix point and go both ways, padding with zeros as needed

Examples

- Convert $(10110001101011.11110010)_2$ to hexadecimal

  = 0010 1100 0110 1011 . 1111 0010

  = 2 C 6 B . F 2

  = $(2C6B.F2)_{16}$
- Convert $(306.D)_{16}$ to binary

  = 0011 0000 0110. 1101

  = $(001100000110.1101)_2$
- Convert $(11010100011011)_2$ to hexadecimal

0011	0101	0001	1011
3	5	1	B

Your Turn

- Convert $(757.25)_{10}$ to Binary, Octal, Hexadecimal, and Base6.
- Find Decimal Equivalent of the following:-
  - $(1011.11)_2$
  - $(147.3)_8$
  - $(A2F)_{16}$
  - $(3301.13)_6$
- Convert $(231.3)_4$ to Base7
- Convert $(175.6)_8$ to Hexadecimal

Solution

- Convert $(757.25)_{10} = (1011110101.01)_2$

  $= (1365.2)_8$

  $= (2F5.4)_{16}$

  $= (3301.13)_6$

- Decimal Equivalent is:-

➢ $(1011.11)_2 = (11.75)_{10}$

➢ $(147.3)_8 = (103.375)_{10}$

➢ $(A2F)_{16} = (2607)_{10}$

➢ $(3301.13)_6 = (757.25)_{10}$

- Convert $(231.3)_4 = (63.5151\ldots)_7$
- Convert $(175.6)_8 = (7D.C)_8$

Base-r Arithmetic

- Arithmetic operations with numbers in base r follow the same rules as for decimal numbers.
- When a base other than 10 is used, one must remember to use only the r-allowable digits.
- The following are some examples:

```
augend:   110011    minuend:       110101    multiplicand:    1011
addend:  +100011    subtrahend:   -100111      multiplier:   X 101
         -------                  -------                    -----
         1010110                   001110                     1011
                                                             0000
                                                            1011
                                                            ----
                                                            110111
```

Arithmetic Rules

- The sum of two digits are calculated as expected but the digits of the sum can only be from the r-allowable coefficients.
- Any carry in a sum is passed to the next significant digits to be summed.
- In subtraction the rules are the same but a borrow adds r (where r is the base) to the minuend digit.
- The examples of addition and subtraction of binary numbers are presented in the next slides.

Binary Addition Rules

Given two binary digits (X,Y), a carry in (Z) we get the following sum (S) and carry (C):

Carry in (Z) of 0:

Z	0	0	0	0
X	0	0	1	1
+ Y	+ 0	+ 1	+ 0	+ 1
C S	0 0	0 1	0 1	1 0

Carry in (Z) of 1:

Z	1	1	1	1
X	0	0	1	1
+ Y	+ 0	+ 1	+ 0	+ 1
C S	0 1	1 0	1 0	1 1

Binary Multiplication

- Multiplication table

  0 x 0 = 0
  0 x 1 = 0
  1 x 0 = 0
  1 x 1 = 1

```
              1   0   1   1   1
           X      1   0   1   0
----------------------------------
             0   0   0   0   0
         1   0   1   1   1
     0   0   0   0   0
 1   0   1   1   1
----------------------------------
 1   1   1   0   0   1   1   0
```

Binary Division

- Binary division is similar to decimal division

```
          1101
     ________
1011 )10010001
      1011
      ----
       1110
       1011
       ----
         1101
         1011
         ----
           10
```

The quotient is 1101 with a remainder of 10.

Determine Unknown Radix

Example: Determine the base of the number for the following operation to be correct

$$54/4=13$$

Solution: Both sides of the given expression carry unknown radices that we must determine. Convert both sides into decimal as we have learned previously

$5xr^1+4xr^0/4xr^0=1xr^1+3xr^0$

$=5r+4/4=r+3$

$=5r+4=4r+12$

Simplification gives

$r=8$

After you substitute r=8 in the given expression LHS=RHS. So the required radix is 8.

Your Turn

Example: Determine the unknown radix for the following operation to be correct

$$(365)_r = (194)_{10}$$

Solution

LHS of the given expression carries unknown radix that we must determine whereas RHS is known here. Convert LHS into decimal as we have learned previously

$3xr^2+6xr^1+5xr^0=194$

$=3r^2+6r^1-189=0$

Simplification gives

r=7 & r=-9 ; Discard r=-9 since radix can't be –ve.

After you substitute r=7 in the given expression LHS=RHS. So the required radix is 7.

The End