Faraday's Law-I

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In a commercial electric power plant, large generators produce energy that is transferred out of the plant by electrical transmission. These generators use magnetic induction to generate a potential difference when coils of wire in the generator are rotated in a magnetic field. The source of energy to rotate the coils might be falling water, burning fossil fuels, or a nuclear reaction. (Michael Melford/Getty Images)

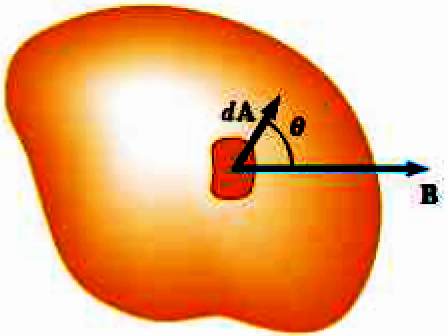
Three Experimental Pillars of Electromagnetism

- *Electric charges attract / repel each other as described by* ***Coulomb's law.***
- *Current-carrying wires attract / repel each other as described by* ***Ampere's law of force.***
- *Magnetic fields that change with time induce electromotive force as described by* ***Faraday's law.***

Gauss's Law for Magnestism

Magnetic flux is the number of magnetic field lines passing though a surface and is defined as

$$\Phi_B = \int \vec{B} \bullet d\vec{A}$$

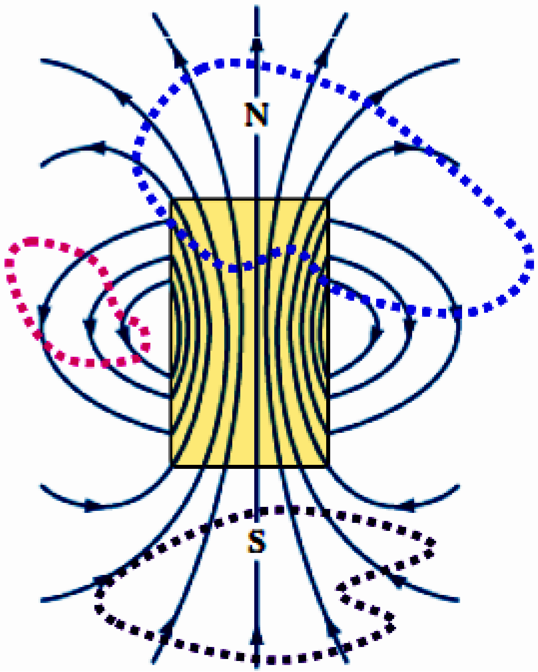


SI unit of flux is weber (Wb); $1\,Wb = 1T.m^2$

Gauss's law in magnetism states that:

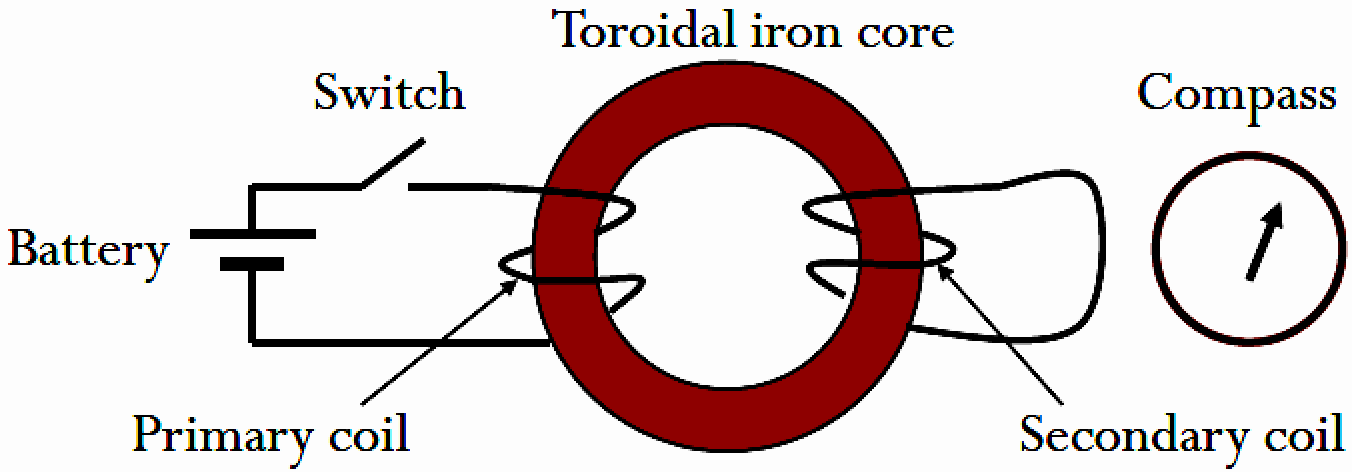
"the net magnetic flux through any closed surface is always zero"

$$\boxed{\int \vec{B} \bullet d\vec{A} = 0}$$



This statement is based on the experimental fact that isolated magnetic poles (monopoles) have never been detected and perhaps do not exist.

Faraday's Experiment



- Upon closing the switch, current begins to flow in the primary coil.
- A momentary deflection of the compass needle indicates a brief surge of current flowing in the secondary coil.
- The compass needle quickly settles back to zero.
- Upon opening the switch, another brief deflection of the compass needle is observed.

Faraday's Law of Electromagnetic Induction

The emf induced in a circuit is directly proportional to the time rate of change of the magnetic flux through the circuit.

$$\boxed{\xi = -\frac{d\Phi_B}{dt}}$$

It shows that emf can be induced in the circuit in several ways:

- By changing magnitude of B with time. } *Transformer emf*
- By changing the area enclosed by the loop with time.
- By changing the angle between B and the normal to the loop with time.

} *Motional emf*

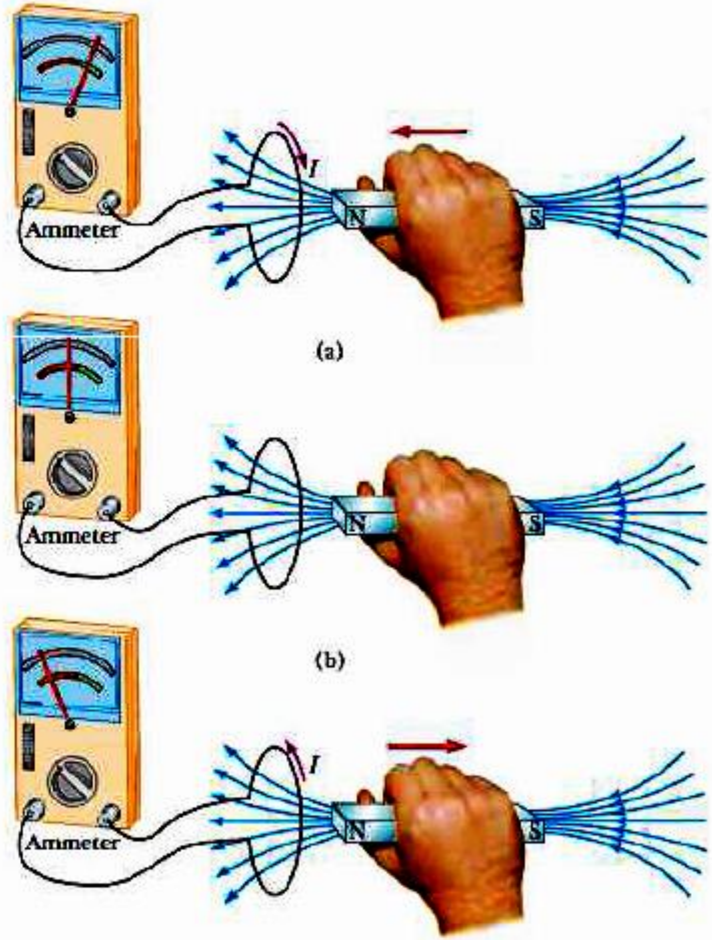
- Any combination of the above can occur. } *Transformer + Motional*

If the circuit is a coil consisting of N loops all of the same area and if B is the flux through one loop, an emf is induced in every loop; thus, the total induced emf in the coil is given by the expression

$$\boxed{\xi = -N\frac{d\Phi_B}{dt}}$$

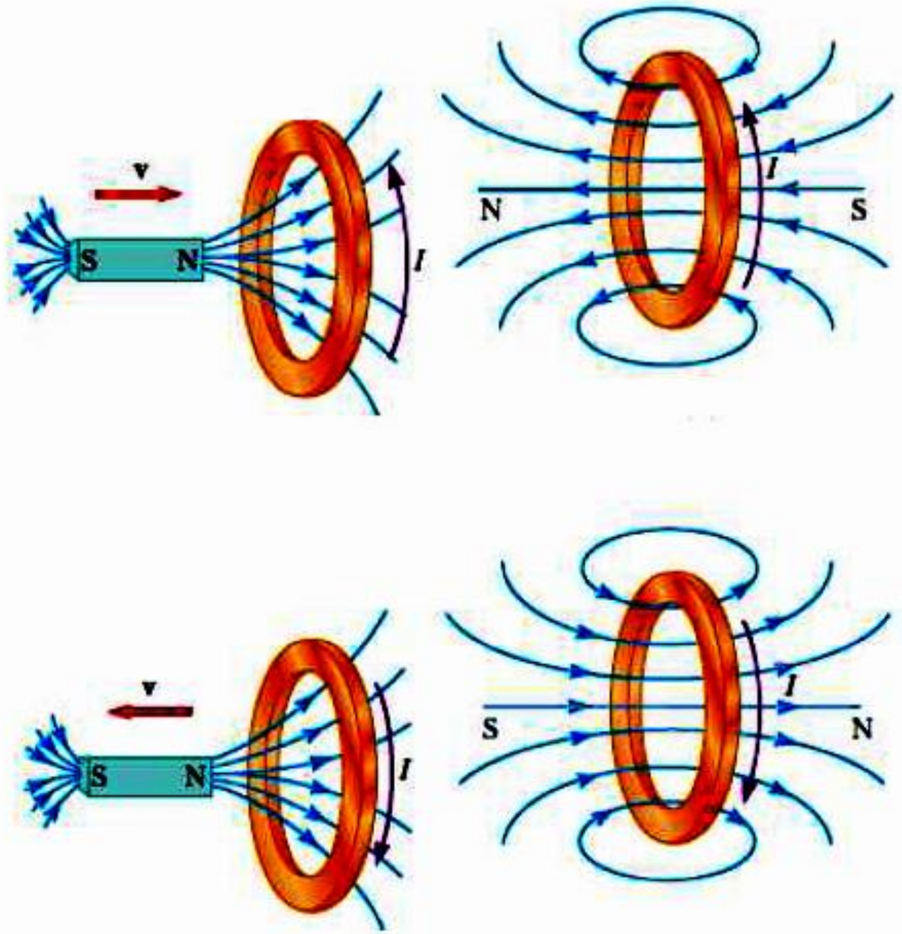
The corresponding *induced current* in the loop will be

$$\boxed{i_{ind} = \frac{\xi}{R}}$$

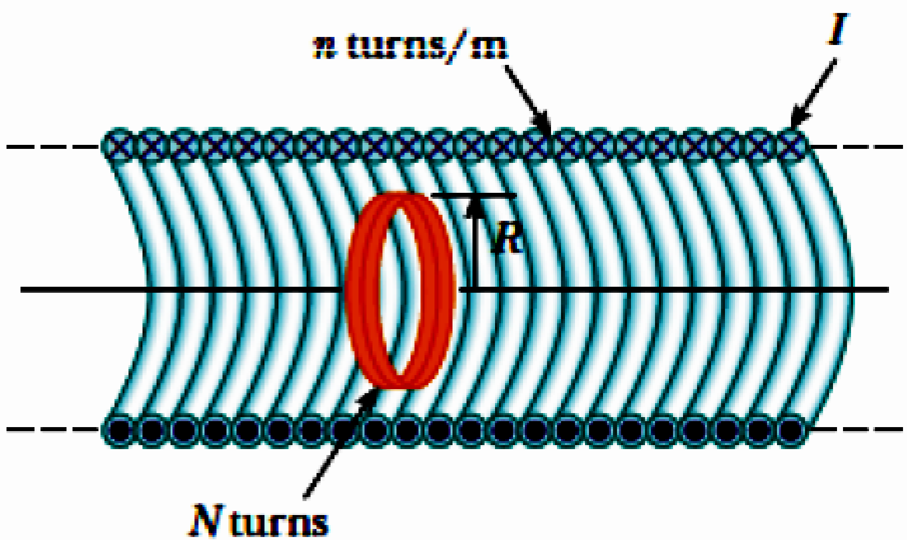


Lenz's Law

- Lenz's law explains the minus sign in Faraday's law.
- Lenz's law is a consequence of conservation of energy.
- The sense of the emf induced by the time-varying magnetic flux is such that any current it produces tends to set up a magnetic field that opposes the change in the original magnetic field.



A long solenoid of radius r=10cm and 400 turns per meter carries a current given by $I = (30.0\ A)(1 - \exp(-1.60t))$. Inside the solenoid and coaxial with it is a coil that has a radius of R=6.00 cm and consists of a total of 250 turns of fine wire as shown in the figure below. What emf is induced in the coil by the changing current in the solenoid? What will be the direction of induced current in the loop?



Here magnetic field inside the solenoid is

$$B = \mu_\circ n i = \mu_\circ n(30)(1 - e^{-1.6t})$$

So the magnetic flux through the coil of 250 turns will be

$$\Phi_B = \int \vec{B} \bullet d\vec{A} = B\int dA = \mu_\circ n(30)(1 - e^{-1.6t})\pi R^2$$

So the induced emf in the coil of 250 turns is

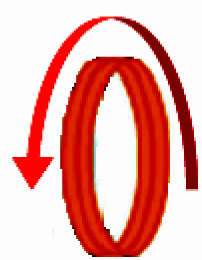
$$|\xi| = N\frac{d\Phi_B}{dt} = N\frac{d}{dt}\left(\mu_\circ n(30)(1 - e^{-1.6t})\pi R^2\right)$$

$$|\xi| = N\mu_\circ n\pi R^2(30)(1.6e^{-1.6t})$$

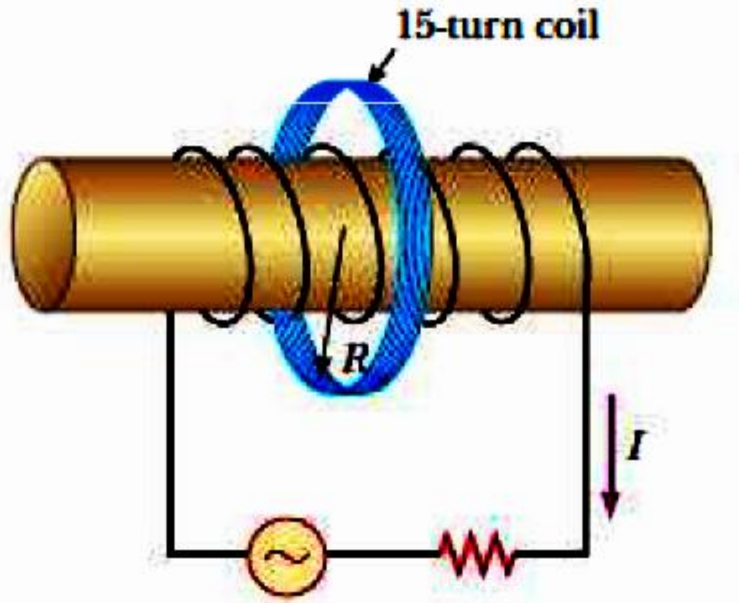
$$|\xi| = 250(4\pi \times 10^{-7})(400)\pi(0.06)^2(30)(1.6e^{-1.6t})$$

$$|\xi| = 68.2e^{-1.6t} mV$$

By Lens's law, induced current will be counter clockwise



A long solenoid of radius r =2cm and 1000 turns per meter carries a current given by $I = (5.0\ A)\sin(120t)$. A 15 turn coil of radius R=10cm and coaxial with solenoid surround the solenoid as shown in figure. What emf is induced in the coil by the changing current in the solenoid?



Here magnetic field inside the solenoid is

$$B = \mu_\circ ni = \mu_\circ n(5)\sin(120t)$$

So the magnetic flux through the coil of 15 turns will be

$$\Phi_B = \int \vec{B} \bullet d\vec{A} = B\int dA = \mu_\circ n(5)\sin(120t)\pi r^2$$

So the induced emf in the coil of 15 turns is

$$|\xi| = N\frac{d\Phi_B}{dt} = N\frac{d}{dt}\left(\mu_\circ n(5)\sin(120t)\pi r^2\right)$$

$$|\xi| = N\mu_\circ n\pi r^2(5)(120\cos(120t))$$

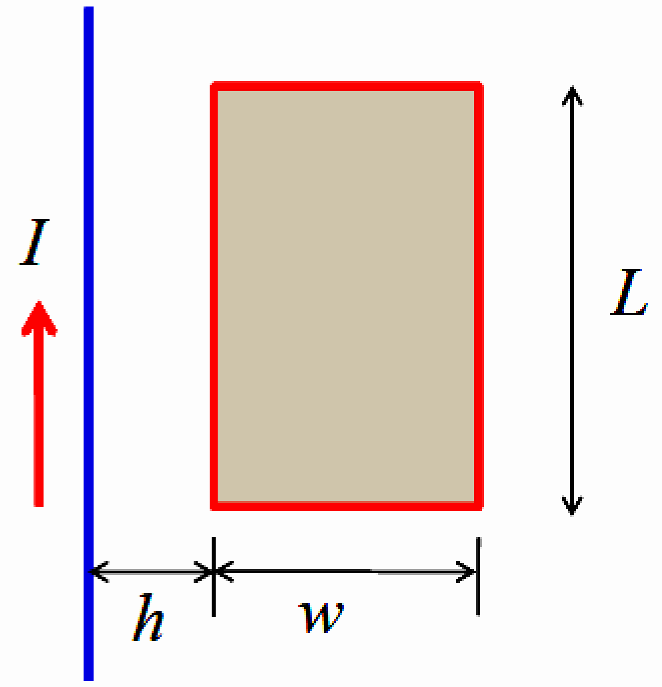
$$|\xi| = 15(4\pi\times10^{-7})(1000)\pi(0.02)^2(5)(120\cos(120t))$$

$$|\xi| = 14.2\cos(120t)mV$$

A loop of wire in the shape of a rectangle of width w and length L and a long, straight wire carrying a current I lie on a tabletop as shown in Figure.

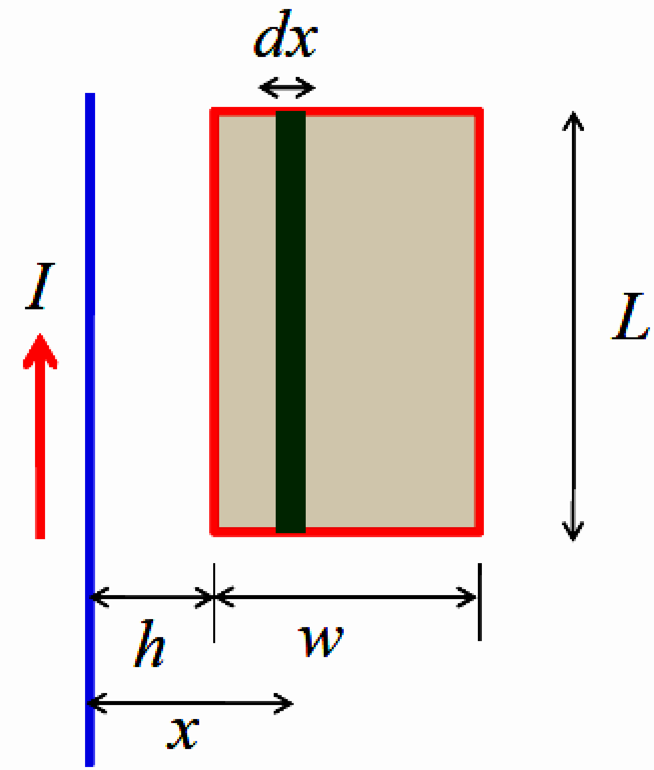
(a) Determine the magnetic flux through the loop due to the current I.

(b) Suppose the current is changing with time according to $I = a + bt$, where a and b are constants. Determine the emf that is induced in the loop if $b = 10.0$ A/s, $h = 1.00$ cm, $w = 10.0$ cm, and $L = 100$ cm. What is the direction of the induced current in the rectangle?



(a) Here magnetic field due to the wire carrying current I is not uniform throughout the loop, so let's divide loop into small strips of width dx and length L. So magnetic flux through a strip at a distance x from the wire is

$$d\Phi_B = \vec{B} \bullet d\vec{A} = BLdx = \frac{\mu_\circ I}{2\pi x} Ldx$$



So the net magnetic flux through the loop will be

$$\Phi_B = \int_h^{h+w} d\Phi_B = \int_h^{h+w} \frac{\mu_\circ I}{2\pi x} Ldx$$

$$\Phi_B = \frac{\mu_\circ IL}{2\pi} \int_h^{h+w} \frac{1}{x} dx$$

$$\Phi_B = \frac{\mu_\circ IL}{2\pi} \ln\left(\frac{h+w}{h}\right)$$

(b) Here $I = a + bt$

So $$\Phi_B = \frac{\mu_\circ (a+bt)L}{2\pi} \ln\left(\frac{h+w}{h}\right)$$

Induced emf in the loop (N=1) is

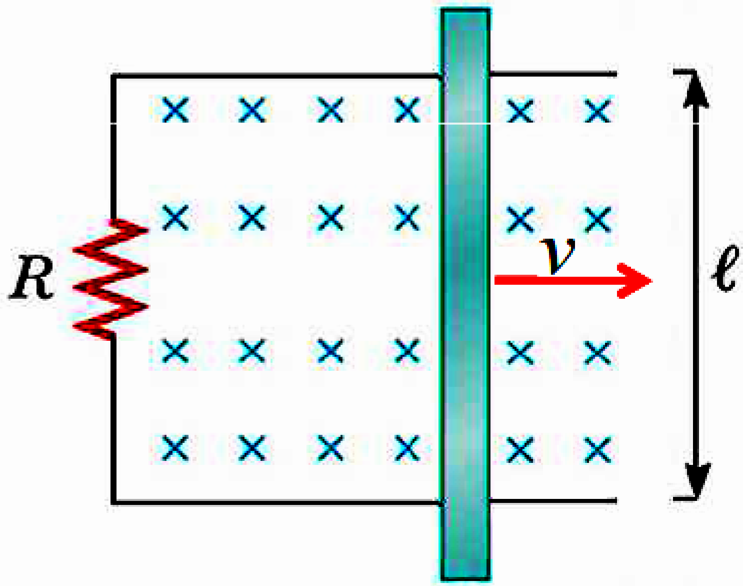
$$|\xi| = \frac{d\Phi_B}{dt} = \frac{\mu_\circ L}{2\pi} \ln\left(\frac{h+w}{h}\right) \frac{d(a+bt)}{dt}$$

$$|\xi| = \frac{d\Phi_B}{dt} = \frac{\mu_\circ Lb}{2\pi} \ln\left(\frac{h+w}{h}\right) = 4.8\mu V$$

By right hand rule, the long wire produces magnetic flux into the page through the rectangle. As the magnetic flux increases, so by Lens' law the rectangle will produce its own magnetic field out of the page, which it does by carrying **counterclockwise current.**

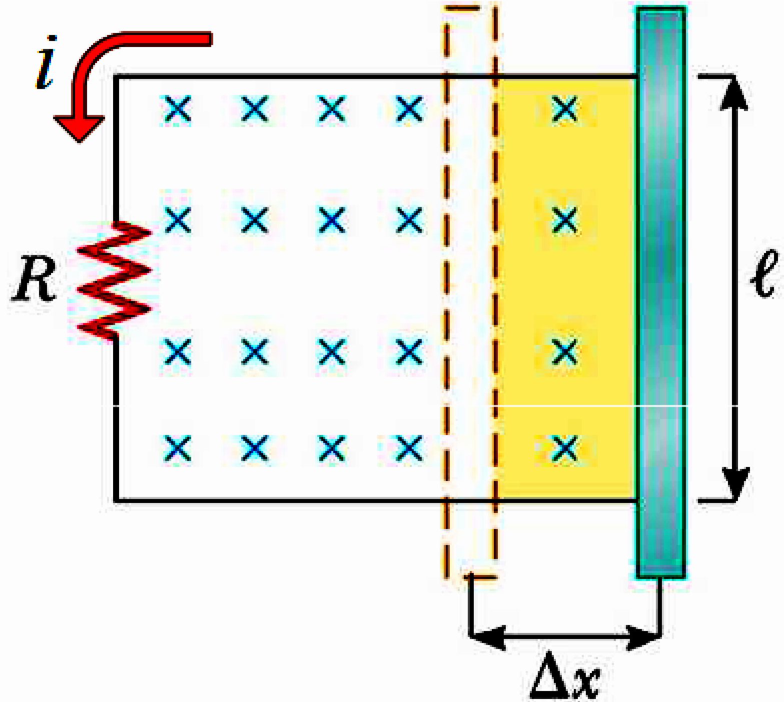
Motional EMF

Consider the arrangement shown in Figure below where a constant magnetic field is applied directed into the page. If the bar is moved with velocity v along x-axis, what will be the induced current in the resistor?



Let bar moves a displacement Δx in the time interval Δt then change in the area of loop will be $l\Delta x$. So change in magnetic flux through he loop is

$$\Delta \Phi_B = B\Delta A = B\ell\Delta x$$



Induced emf will be

$$|\xi| = \frac{\Delta \Phi_B}{\Delta t} = B\ell\frac{\Delta x}{\Delta t}$$

$$|\xi| = vBl$$

Induced current though R will be

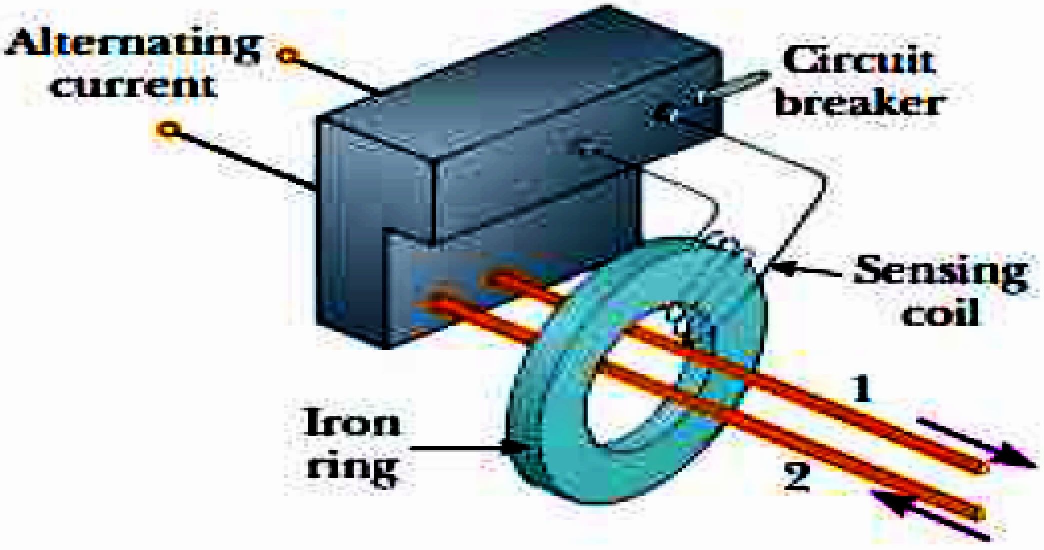
$$i_{ind} = |\xi| / R = vBl/R$$

As the magnetic flux increases by motion of bar, so by Lens' law the loop will produce its own magnetic field out of the page, which it does by carrying **counterclockwise current.**

The Ground fault Interrupter (GFI)

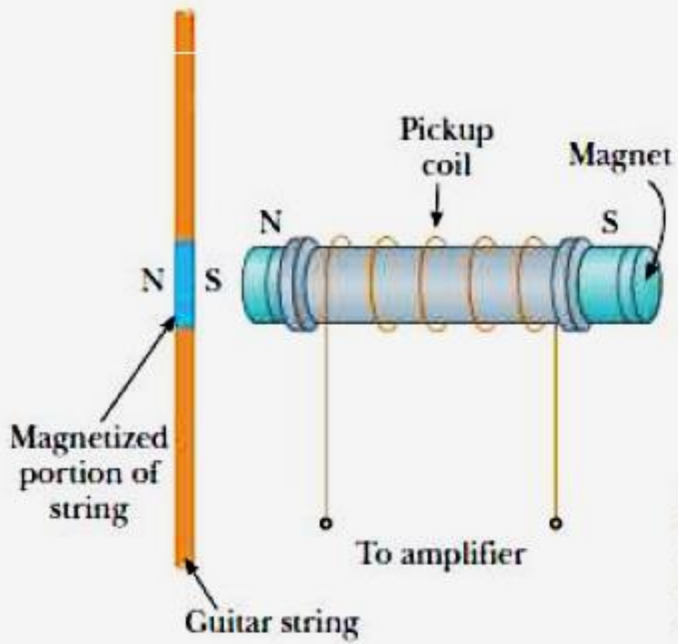
The ground fault interrupter (GFI) is an interesting safety device that protects users of electrical appliances against electric shock. Its operation makes use of Faraday's law. In the GFI shown in Figure, wire 1 leads from the wall outlet to the appliance to be protected, and wire 2 leads from the appliance back to the wall outlet. An iron ring surrounds the two wires, and a sensing coil is wrapped around part of the ring. In general the currents in the wires are equal and in opposite directions, the net magnetic flux through the sensing coil due to the currents is zero. However, if the return current in wire 2 changes, the net magnetic flux through the sensing coil is no longer zero. (This can happen, for example, if the appliance becomes wet, enabling current to leak to ground.) Because household current is alternating, the magnetic flux through the sensing coil changes with time, inducing an emf in the coil.

This induced emf is used to trigger a circuit breaker, which stops the current before it is able to reach a harmful level.



Electric Guitar

Another interesting application of Faraday's law is the production of sound in an electric guitar. The coil in this case, called the pickup coil, is placed near the vibrating guitar string, which is made of a metal that can be magnetized. A permanent magnet inside the coil magnetizes the portion of the string nearest the coil. When the string vibrates at some frequency, its magnetized segment produces a changing magnetic flux through the coil. The changing flux induces an emf in the coil that is fed to an amplifier. The output of the amplifier is sent to the loudspeakers, which produce the sound waves we hear.



(a)

(b)