NUST

Electric Potential-I

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Processes occurring during thunderstorms cause large differences in electric potential between a thundercloud and the ground. The result of this potential difference is an electrical discharge that we call lightning, such as this display over Tucson, Arizona. (© Keith Kent/ Photo Researchers, Inc.)….

Conservative forces, Work done, and Potential Energy

- The work done by a conservative force on a particle moving between any two points is independent of the path taken by the particle.
- The work done by a conservative force on a particle moving through any closed path is zero.
- Potential energy can be defined only for conservative forces.
- Work done in moving an object from some initial position i to certain final position f stores as potential energy in the system.
- Potential energy is related to work done as

$$\Delta U = U_f - U_i = -W$$

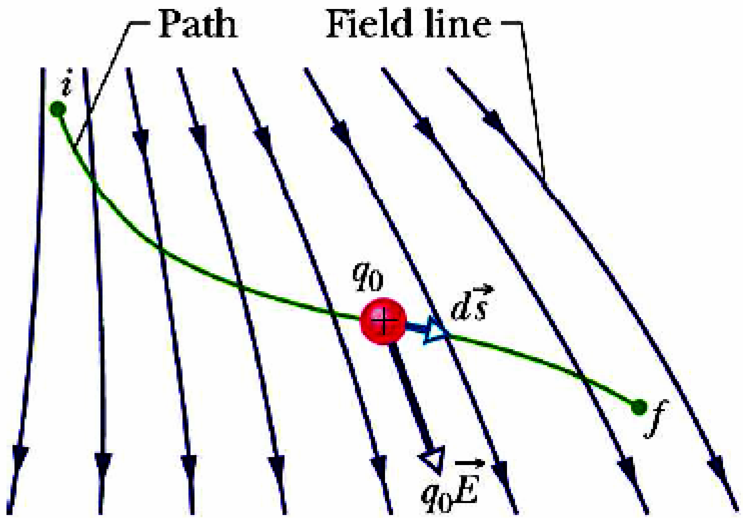
Electrostatic force is a conservative force

Electric Potential Energy

Let a charge particles moves from i to f in the electric field E generated by some source charge distribution as shown.

Work done by the electric force is

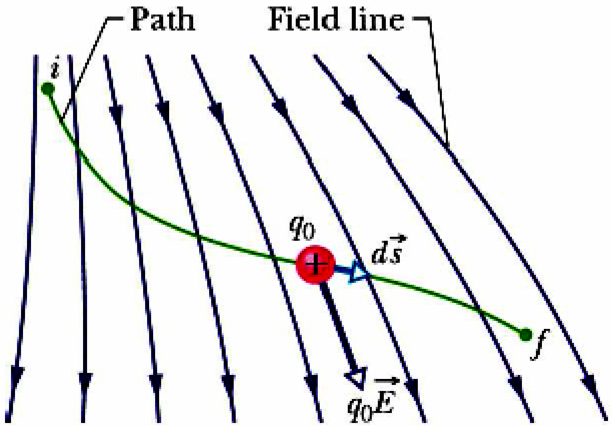
$$W_{if} = q_0 \int_i^f \vec{E} \bullet d\vec{s}$$



Change in potential energy of the system (source charge distribution plus test charge)will be

$$\Delta U = U_f - U_i = -W_{if} = -q_0 \int_i^f \vec{E} \bullet d\vec{s}$$

Electric potential difference between points i and f, $V_f - V_i$, due to source charge distribution is defined as the change in potential energy of the system as a test charge q_0 moves from i to f divided by the size of the test charge.



$$\Delta V = V_f - V_i \equiv \frac{U_f - U_i}{q_0} = -\frac{W_{if}}{q_0}$$

$$\boxed{V_f - V_i = -\int_i^f \vec{E} \bullet d\vec{s}}$$

Potential difference depends only on the source charge distribution

(Consider points i and f without the presence of the test charge)

The difference in potential energy exists only if a test charge is moved between the points i and f.

So far we have been discussing the difference in potential between two point *i* and *f*. We can extend the discussion to define electric potential at a single point *P* a distance r from the source distribution and setting reference value $V_{ref} = 0$.

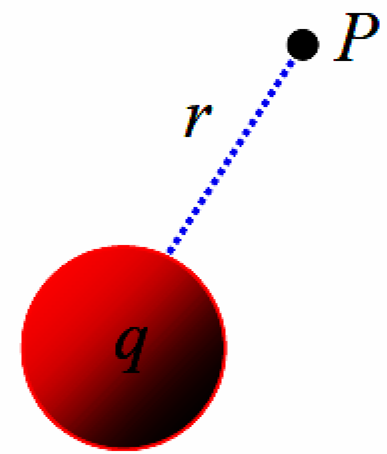
Potential due to Source Charge at point P

Line element along r

$$V = -\int_{ref}^{P} \vec{E} \bullet d\vec{s}$$

Reference point

Field due to Source Charge



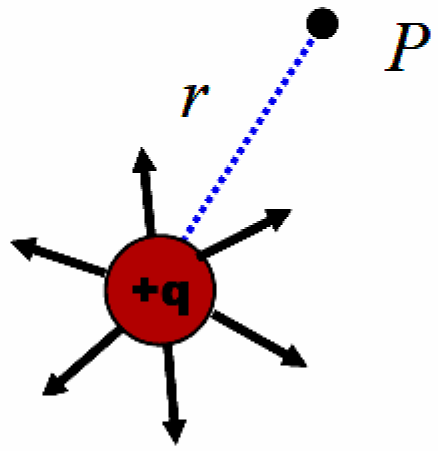
Source Charge

- Electric potential is a scalar quantity.
- The SI units of electric potential are joules per coulomb.
- The unit of potential is the volt.
- A ***VOLT*** is defined: $1\,V = 1\,J/C$

Electric Potential: Point Charge

Electric potential at point P a distance r from point charge will be

$$V = -\int_{ref}^{P} \vec{E} \bullet d\vec{s}$$

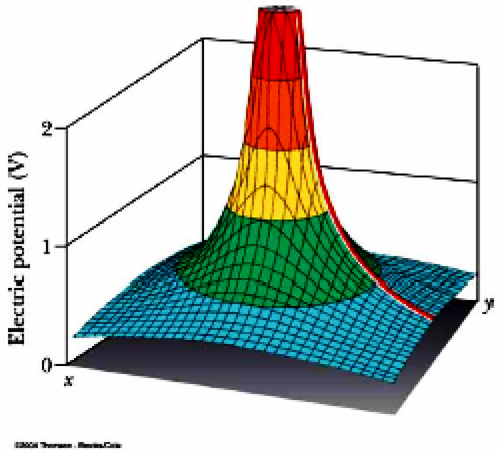


For point charge $\vec{E} = \dfrac{kq}{r^2}\hat{r}$

Let's set reference point to be at infinity

$$V = -\int_{\infty}^{r} \frac{kq}{r^2}\hat{r} \bullet dr\hat{r} = -\int_{\infty}^{r} \frac{kq}{r^2} dr$$

$$V = -kq\left[-1/r\right]_{\infty}^{r} \qquad V = \frac{kq}{r}$$



$$\boxed{V = \frac{kq}{r}}$$

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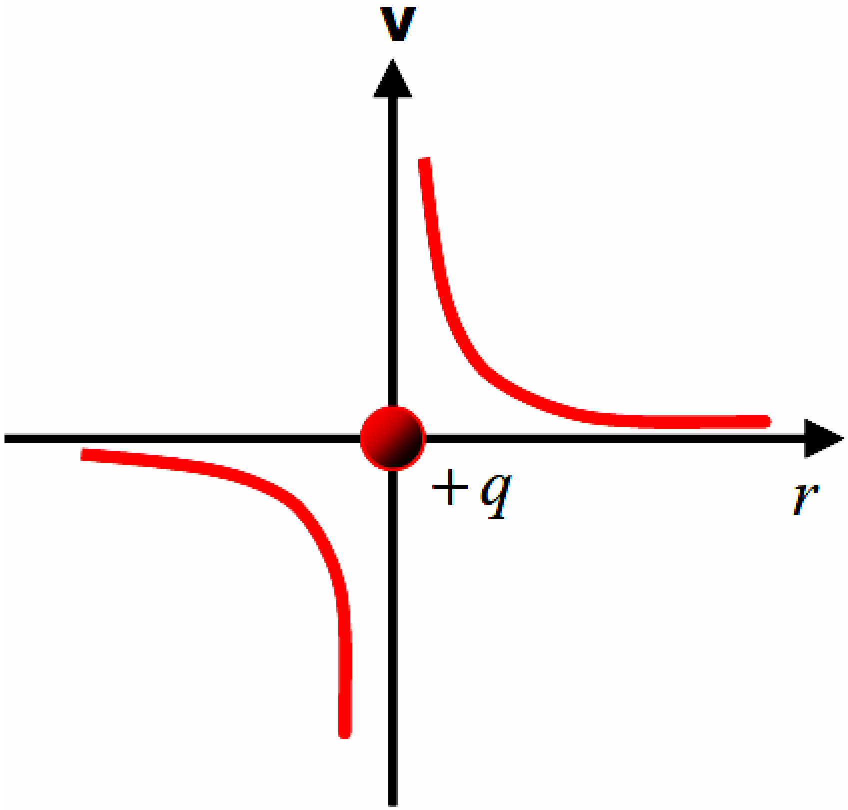
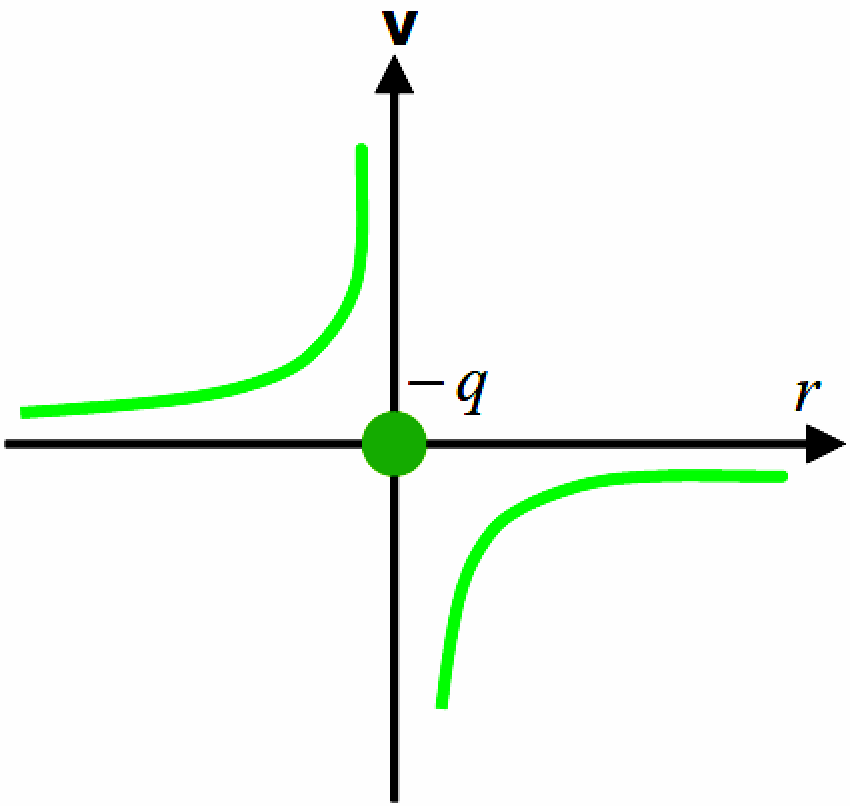
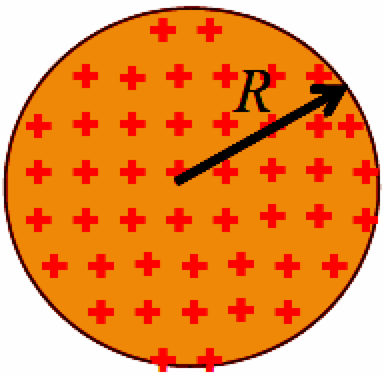




Consider an insulating solid sphere of radius R having a uniform volume charge density ρ and carries a total positive charge q. Electric field inside and outside of charged sphere is

$$E_{in} = \frac{kq}{R^3} r \qquad E_{out} = \frac{kq}{r^2}$$

Find electric potential inside (at a distance r from origin) and outside the charged sphere.



Outside: Let's set reference point to be at infinity

$$V_{out} = -\int_{\infty}^{r} \vec{E} \bullet d\vec{s}$$

$$V_{out} = -\int_{\infty}^{r} \frac{kq}{r^2} \hat{r} \bullet dr\hat{r} = -kq \int_{\infty}^{r} \frac{1}{r^2} dr$$

$$= -kq(-1/r)_{\infty}^{r} = kq/r$$

Inside: Let's set reference point to be at origin

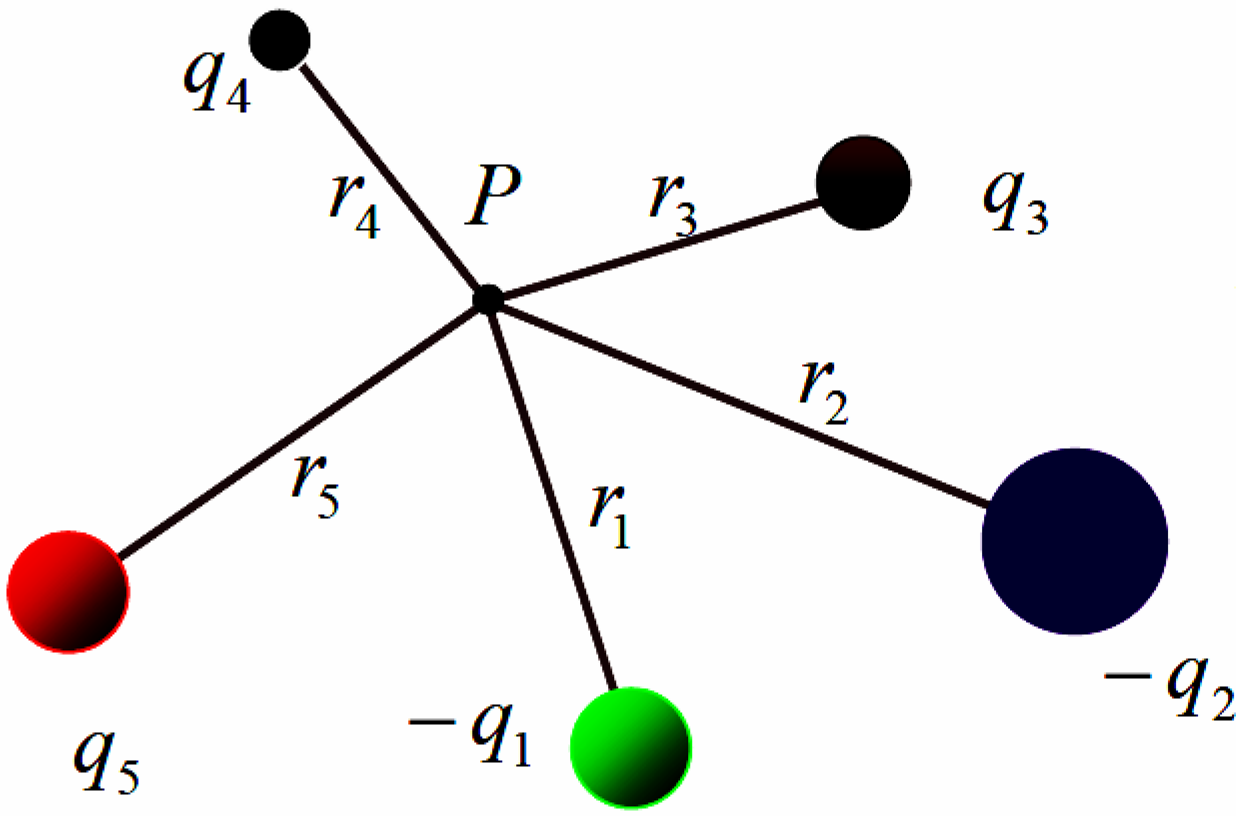
$$V_{in} = -\int_{0}^{r} \vec{E} \bullet d\vec{s}$$

$$V_{in} = -\int_{0}^{r} \frac{kqr}{R^3} \hat{r} \bullet dr\hat{r} = -\frac{kq}{R^3} \int_{0}^{r} r dr$$

$$= -\frac{kq}{R^3} \left(\frac{r^2}{2} \right)_{0}^{r} = -\frac{kqr^2}{2R^3}$$

Superposition

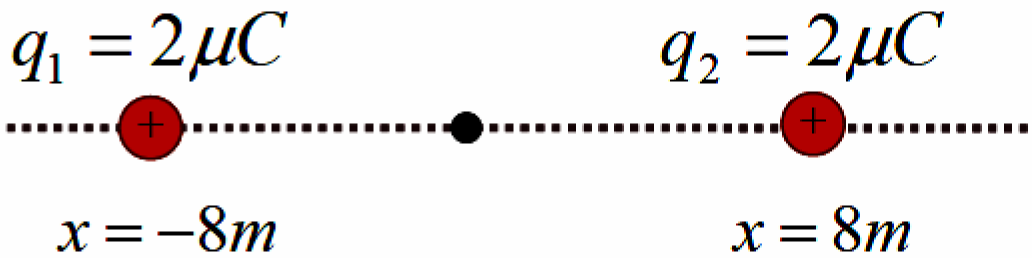
$$\boxed{V = V_1 + V_2 + V_3 + V_4 + V_5}$$



$$V = \sum_i \frac{kq_i}{r_i}$$

Electric Potential at a point P is scalar sum of electric potentials from all charges

Given two 2.00 μC charges, as shown in Figure . (a) What is the electric potential at the origin? (b) What is the electric field at the origin?



(a)

$$V = V_1 + V_2$$

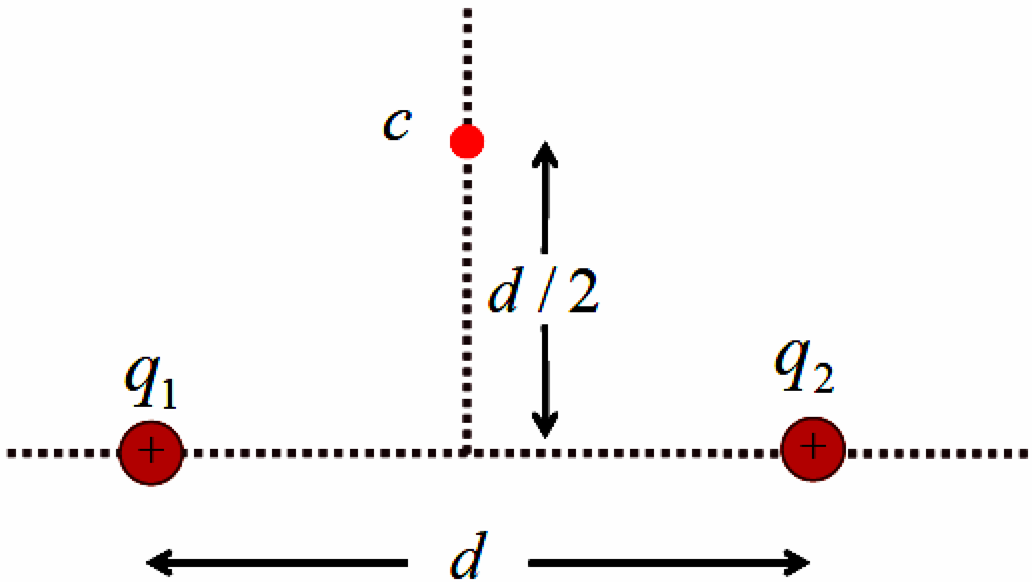
$$V = \frac{kq_1}{r_1} + \frac{kq_2}{r_2}$$

$$V = 2\frac{kq}{r}$$

$$V = 2\frac{(9\times10^{9})(2\times10^{-6})}{8} = 45kV$$

(b) $$E = 0$$

In the configuration shown below **$q1=q2=2.13\mu C$** and **$d=1.96cm$**. (a) What is the electric potential at the point C? (b) Take $V = 0$ at infinity, how much work is required to bring a third charge **$q = 1.91\mu C$** from infinity to point C?



(a) The distance from C to either charge is

$$r = \sqrt{d^2/4 + d^2/4} = d/\sqrt{2}$$

Electric potential at point C will be

$$V = 2\frac{kq}{r} = 2.76\times10^6 V$$

(b) As

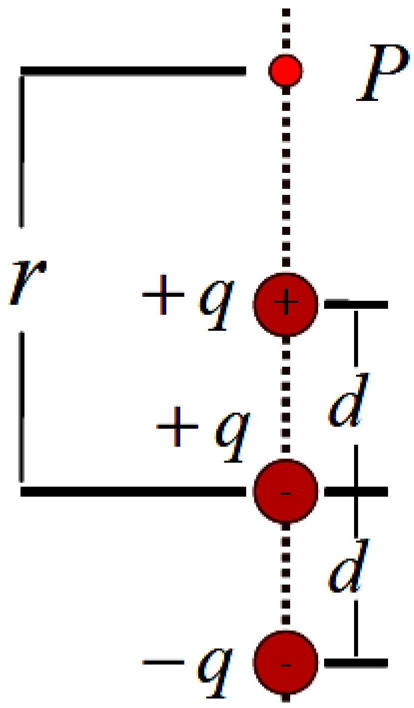
$$\Delta V = V_f - V_i = -\frac{W_{if}}{q_0}$$

$$W = -q\Delta V$$

$$= -(1.91\times10^{-6})(2.76\times10^6) = -5.27J$$

For the configuration shown below show that V(r) for the points on vertical axis . Assuming r>>d, is given by

$$V = \frac{kq}{r}\left(1 + \frac{2d}{r}\right)$$



Net potential will be

$$V = \frac{kq}{r-d} + \frac{kq}{r} - \frac{kq}{(r+d)}$$

$$= \frac{kq}{r(1-d/r)} + \frac{kq}{r} - \frac{kq}{r(1+d/r)}$$

$$= \frac{kq}{r}\left[(1-d/r)^{-1} + 1 - (1+d/r)^{-1}\right]$$

$$= \frac{kq}{r}\left[(1+d/r) + 1 - (1-d/r)\right]$$

$$= \frac{kq}{r}\left[1 + 2d/r\right]$$

Special Case: E Uniform

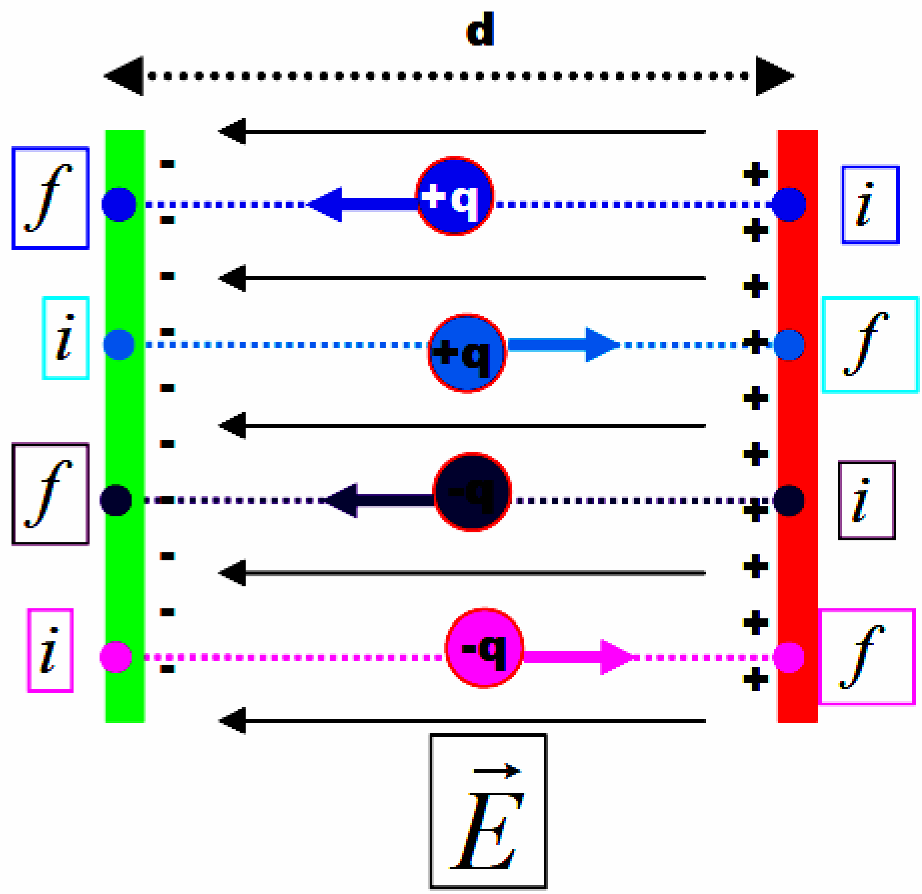
$$\Delta V = V_f - V_i = -\vec{E} \bullet \vec{d} \qquad \Delta U = q_\circ \Delta V$$

$$\boxed{\Delta U = -qEd}$$

$$\boxed{\Delta U = qEd}$$

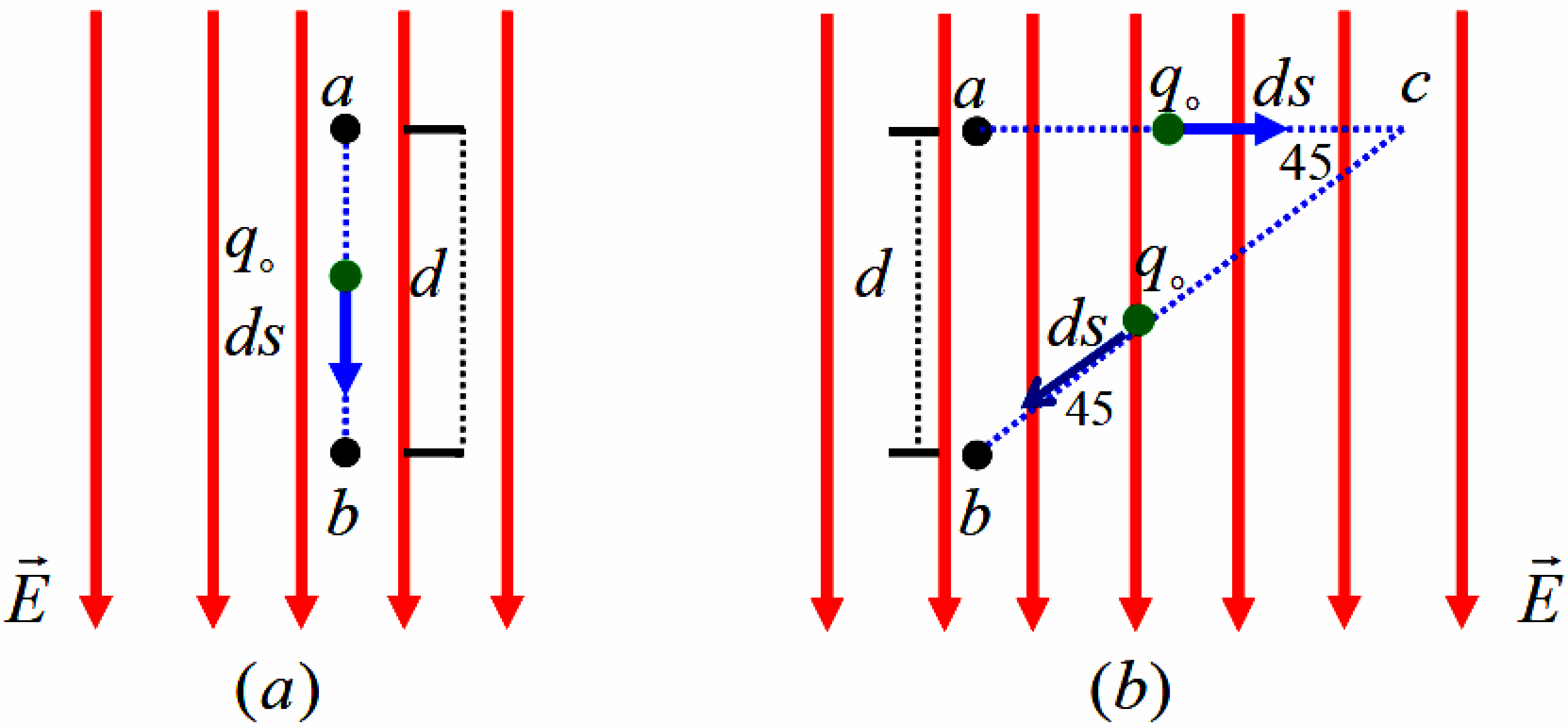
$$\boxed{\Delta U = qEd}$$

$$\boxed{\Delta U = -qEd}$$



- Electric field lines always point in the direction of decreasing electric potential.
- A system consisting of a ***positive charge*** and an electric field ***loses*** electric potential energy when the charge moves in the ***direction of the field.***
- A system consisting of a ***positive charge*** and an electric field ***gains*** electric potential energy when the charge moves in the ***direction opposite to that of the field.***
- A system consisting of a ***negative charge*** and an electric field ***loses*** electric potential energy when the charge moves in the ***direction opposite to that of the field .***
- A system consisting of a ***negative charge*** and an electric field ***gains*** electric potential energy when the charge moves in the ***direction of the field .***
- Potential difference does not depend on the path connecting them

In the figure, a test charge q_o moves through a uniform electric field E through the path (a) ab and (b) acb. Find the potential difference between points a and b and change in potential energy of the system.



(a)

$$\Delta V = V_b - V_a = -Ed$$

$$\Delta U = q_0 \Delta V = -q_0 Ed$$

(b) Along path ac

$$V_c - V_a = -E\left|ac\right|\cos 90° = 0$$

Along path cb

$$V_b - V_c = -E\left|cb\right|\cos 45°$$

$$V_b - V_c = -E\cos 45° \frac{d}{\sin 45°} = -Ed \qquad \therefore \left|cb\right| = \frac{d}{\sin 45°}$$

So

$$V_b - V_a = (V_b - V_c) + (V_c - V_a) = -Ed$$

$$\Delta U = q_0 \Delta V = -q_0 Ed$$

Electric Potential: Electric Dipole

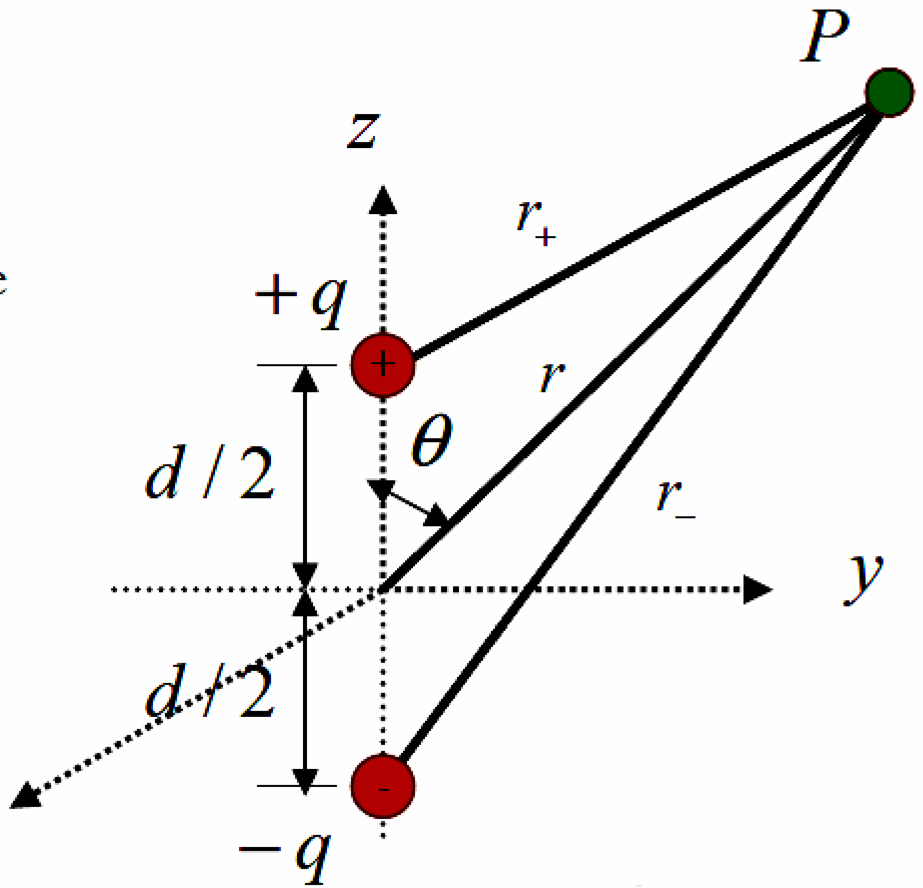
At P, the potentials V+ and V- due to the two charges +q and –q are

$$V_+ = \frac{kq}{r_+} \qquad V_- = -\frac{kq}{r_-}$$

Net potential due to dipole will be

$$V = V_+ + V_-$$

$$V = kq\left[\frac{1}{r_+} - \frac{1}{r_-}\right]$$



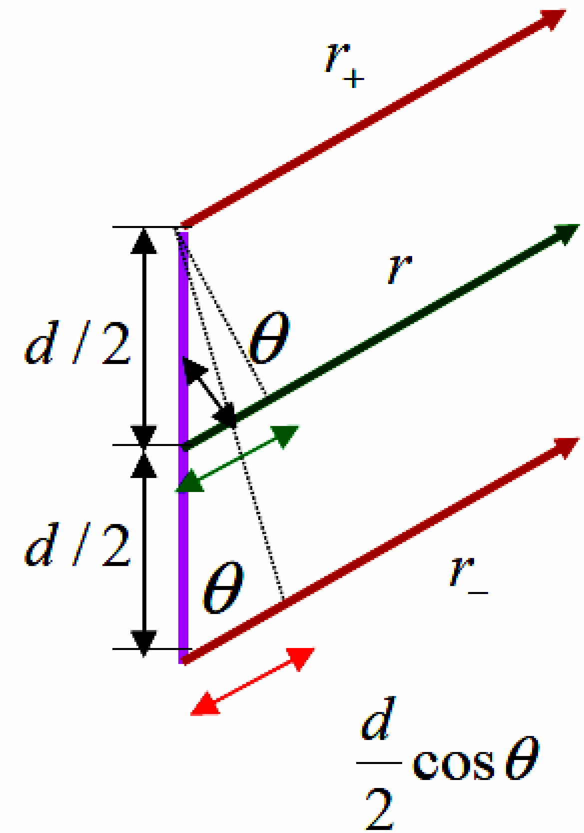
If r >> d $\quad r_+ \approx r - \frac{d}{2}\cos\theta \quad r_- \approx r + \frac{d}{2}\cos\theta$

$$V = kq\left[\left\{r - \frac{d}{2}\cos\theta\right\}^{-1} - \left\{r + \frac{d}{2}\cos\theta\right\}^{-1}\right]$$

$$= kq\left[r^{-1}\left\{1 - \frac{d}{2r}\cos\theta\right\}^{-1} - r^{-1}\left\{1 + \frac{d}{2r}\cos\theta\right\}^{-1}\right]$$

$$= \frac{kq}{r}\left[\left\{1 + \frac{d}{2r}\cos\theta\right\} - \left\{1 - \frac{d}{2r}\cos\theta\right\}\right]$$

$$= \frac{kqd\cos\theta}{r^2}$$



$$\boxed{V = \frac{kp\cos\theta}{r^2}}$$

$$\therefore p = qd$$

Line are parallel approximately

Field from Potential

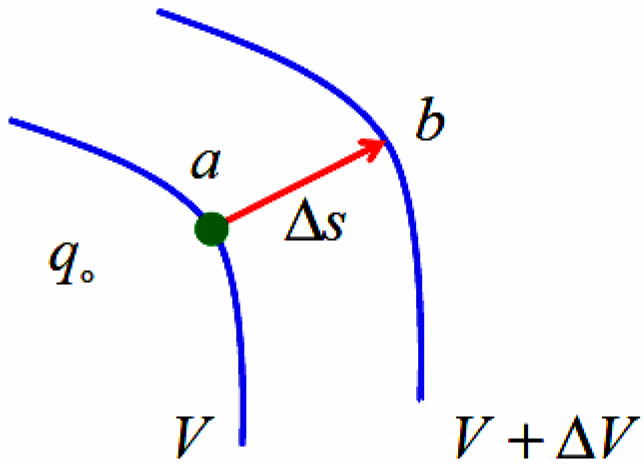
Let's consider a test charge q0 moves from point a (at potential V) to the point b (at potential V+ΔV). In the process, electric potential energy will change as

$$\Delta U = q_\circ \Delta V$$

In the language of forces, we would say that there is an electric field E that exerts a force F=qE on the particle. The work done by this force as the particle moves from a to b is

$$W = F_s \Delta s = q_\circ E_s \Delta s$$

Where Δs is the displacement and assumed to be very small that both field and force are constant along ab.



As $$W = -\Delta U$$

$$q_\circ E_s \Delta s = -q_\circ \Delta V$$

$$\boxed{E_s = -\frac{\Delta V}{\Delta s}}$$

Discrete

If $\Delta s \rightarrow 0$

$$\boxed{E_s = -\frac{dV}{ds}}$$

Continuous

The component of electric field in any direction is the negative of change in electric potential with displacement in that direction.

In general $E_x = -\Delta V / \Delta x \quad E_y = -\Delta V / \Delta y \quad E_z = -\Delta V / \Delta z$

$$E_x = -\frac{\partial V}{\partial x} \qquad E_y = -\frac{\partial V}{\partial y} \qquad E_z = -\frac{\partial V}{\partial z}$$

If electric potential V(x,y,z) is known at all points in space for a particular charge distribution, we can find electric field as

$$\vec{E} = E_x\hat{i} + E_y\hat{j} + E_z\hat{k}$$

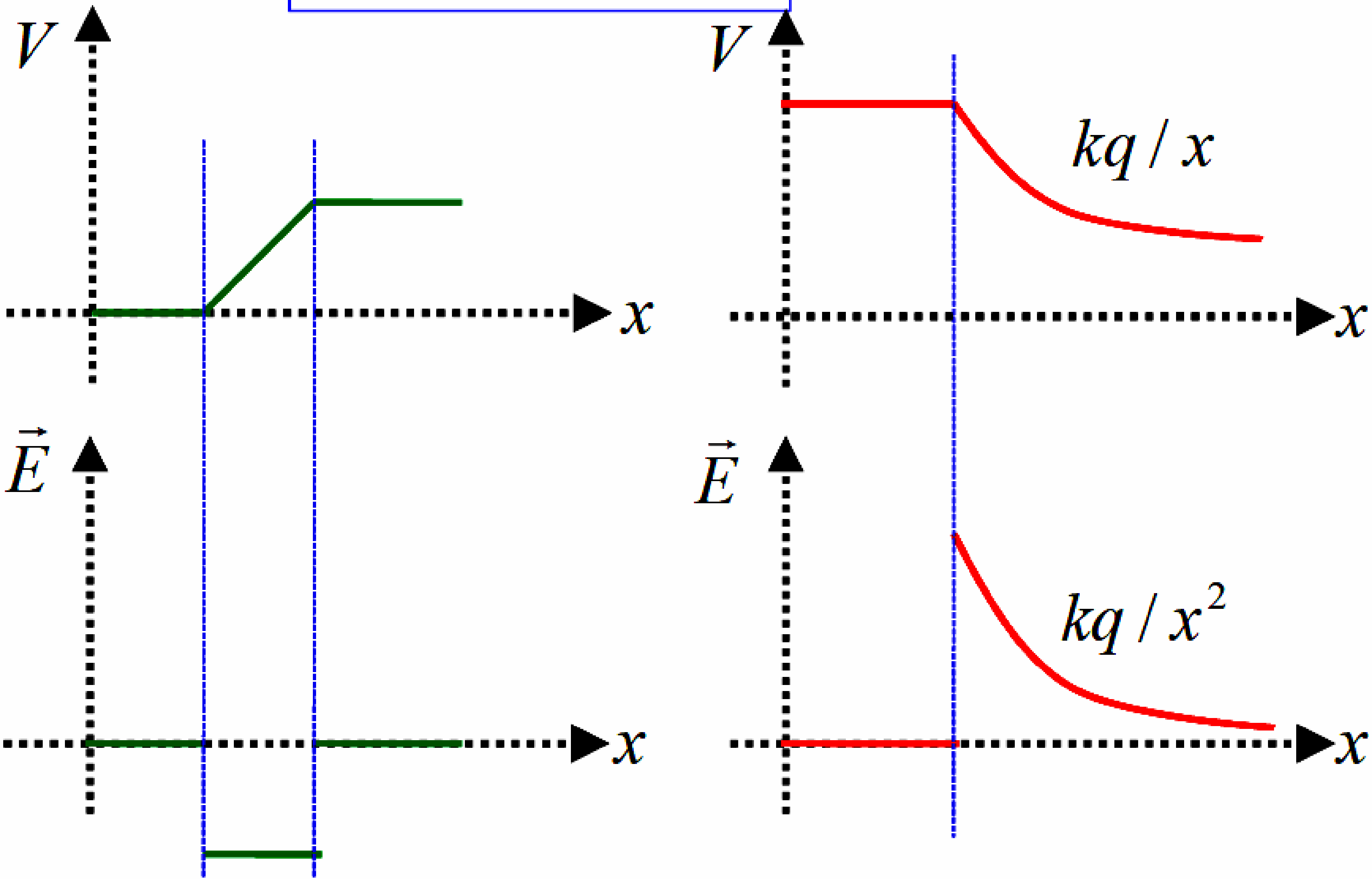
$$\boxed{\vec{E} = -\frac{\Delta V}{\Delta x}\hat{i} - \frac{\Delta V}{\Delta y}\hat{j} - \frac{\Delta V}{\Delta z}\hat{k}}$$

Discrete

$$\boxed{\vec{E} = -\frac{\partial V}{\partial x}\hat{i} - \frac{\partial V}{\partial y}\hat{j} - \frac{\partial V}{\partial z}\hat{k}}$$

Continuous

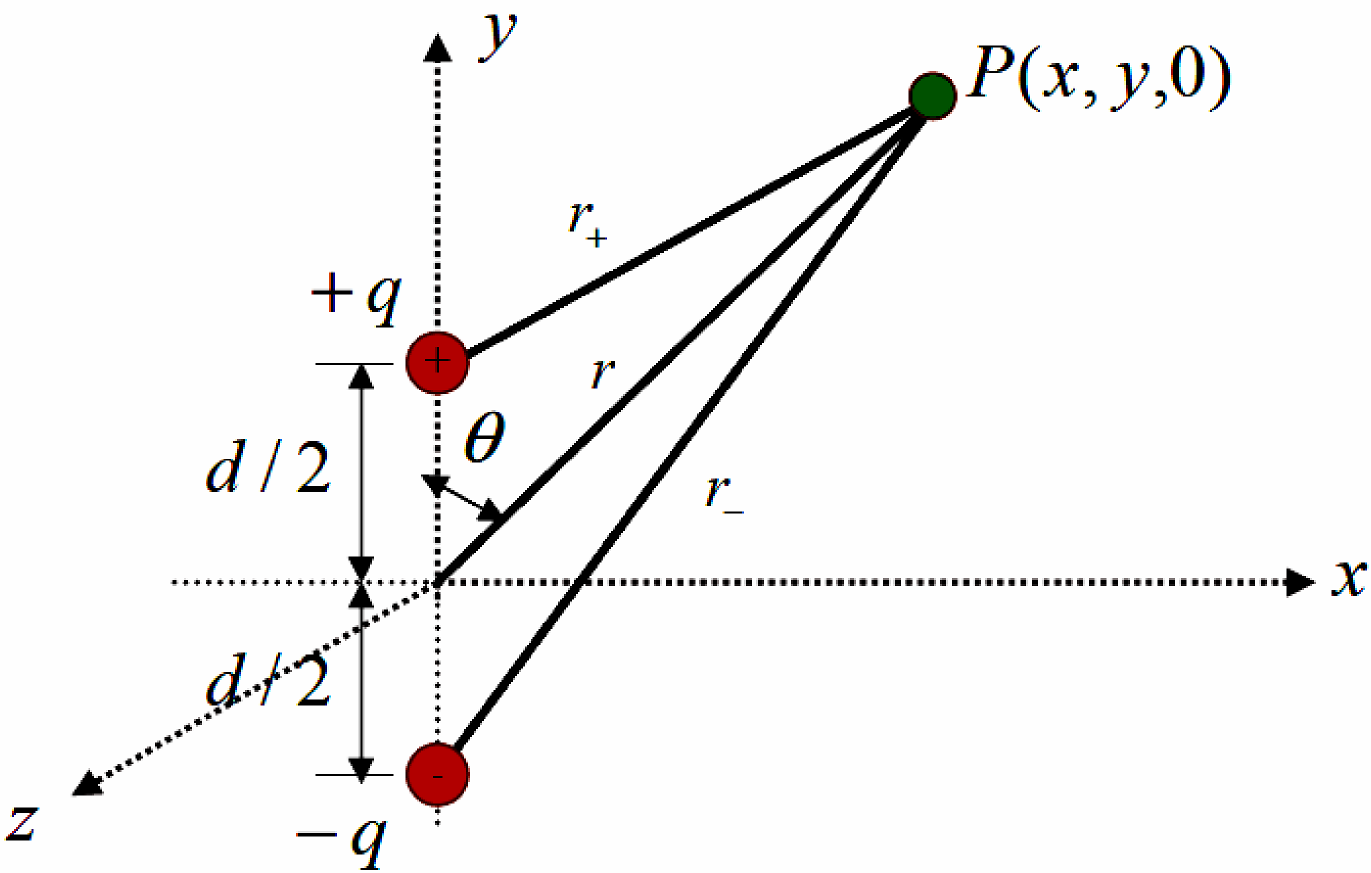
$$\vec{E} = -\frac{dV}{dx} = -\frac{\partial V}{\partial x}$$



For electric dipole shown below, electric potential at point P is given by

$$V = \frac{kp\cos\theta}{r^2}$$

Where p =qd is dipole moment and r >> d. electric field at points P(x,y,0).



At point P

$$V = \frac{kp\cos\theta}{r^2}$$

Here $r^2 = x^2 + y^2$ $\cos\theta = y/r = y/\sqrt{x^2+y^2}$

$$V = \frac{kp}{x^2+y^2}\frac{y}{\sqrt{x^2+y^2}}$$

$$V = kpy\left(x^2+y^2\right)^{-3/2}$$

Electric field point P $\vec{E} = E_x\hat{i} + E_y\hat{j}$

where

$$E_x = -\frac{\partial V}{\partial x} \qquad E_y = -\frac{\partial V}{\partial y}$$

$$E_x = -\frac{\partial V}{\partial x} = 3kpyx\left(x^2 + y^2\right)^{-5/2}$$

$$E_y = -\frac{\partial V}{\partial y}$$

$$= -kp\left[\left(x^2 + y^2\right)^{-3/2} - 3y^2\left(x^2 + y^2\right)^{-5/2}\right]$$

$$= -kp\left(x^2 + y^2\right)^{-5/2}\left[\left(x^2 + y^2\right) - 3y^2\right]$$

$$= kp\left[2y^2 - x^2\right]\left(x^2 + y^2\right)^{-5/2}$$

$$\boxed{\vec{E} = \frac{3kpyx}{\left(x^2 + y^2\right)^{5/2}}\hat{i} + \frac{kp(2y^2 - x^2)}{\left(x^2 + y^2\right)^{5/2}}\hat{j}}$$

$$\vec{E} = -\frac{3kpyx}{\left(x^2 + y^2\right)^{5/2}}\hat{i} + \frac{kp(2y^2 - x^2)}{\left(x^2 + y^2\right)^{5/2}}\hat{j}$$

Electric field at perpendicular bisector (x-axis); y=0

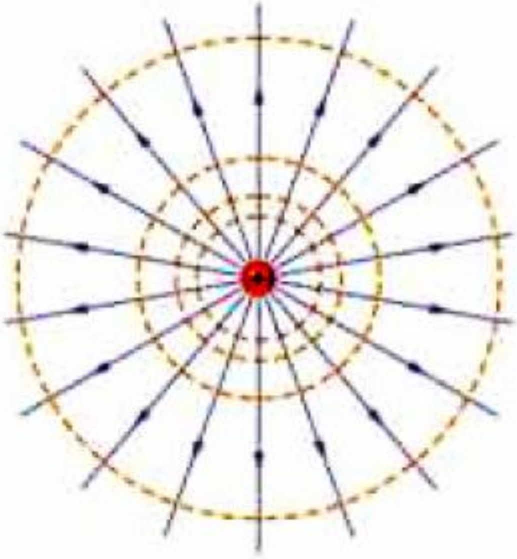
$$\vec{E} = -\frac{kp}{x^3}\hat{j}$$

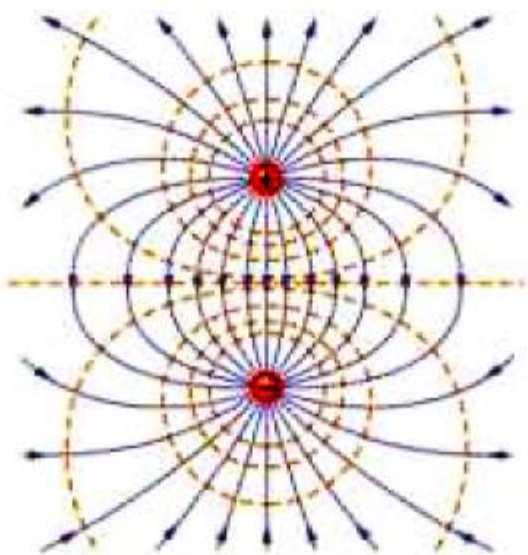
Electric field at axis of dipole (y-axis); x=0

$$\vec{E} = \frac{2kp}{y^3}\hat{j}$$

Equipotential Surfaces

- The name Equipotential surface is given to any surface consisting of a continuous distribution of points having the same electric potential.
- No work is done by the electric field on a charged particle while moving the particle along an Equipotential surface.
- Equipotential surfaces are always perpendicular to electric field lines.





The right figure shows a family of equipotential surfaces associated with the electric field due to some distribution of charges. V1=100 V, V2=80 V, V3=60 V, V4=40 V. W_{I}, W_{II}, W_{III} and W_{IV} are the work done by the electric field on a charged particle q=2C as the particle moves from one end to the other.

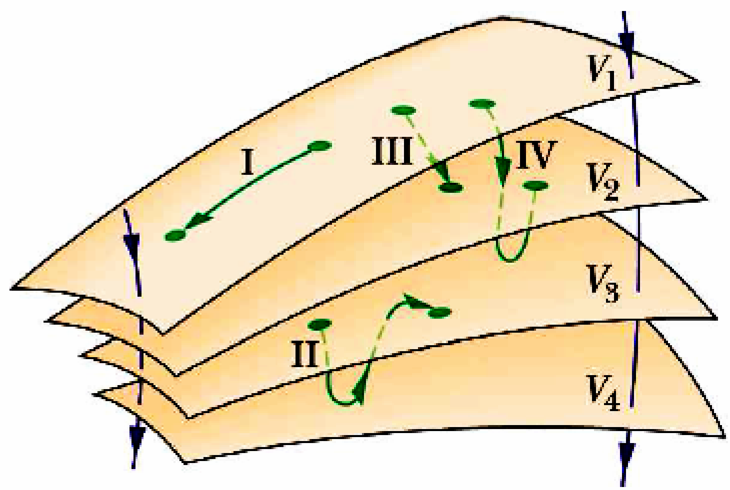
$$W_{ab} = -q_{\circ}\Delta V$$

$$W_{I} = -2(V_1 - V_1) = 0$$

$$W_{II} = -2(V_3 - V_3) = 0$$

$$W_{III} = -2(V_2 - V_1) = 40J$$

$$W_{IV} = -2(V_2 - V_1) = 40J$$



Potential of a Charged Conductor

As we have discussed, for electrostatic, :

- ***If an isolated conductor carries a charge, the charge resides on its surface and conductor remain in equilibrium.***
- ***The electric field is zero everywhere inside the charged conductor.***
- ***The electric field just outside a charged isolated conductor is perpendicular to the surface of the conductor.***

If the charges are in equilibrium on the surface of conductor, then its surface must be an equipotential surface. If this were not so, some parts of the surface would be at higher or lower potential than others. Positive charges would then migrate towards region of lower potential and negative towards region of high potential. However, this contradicts our assertion that the charges are in equilibrium, and therefore the surface must be an equipotential surface.

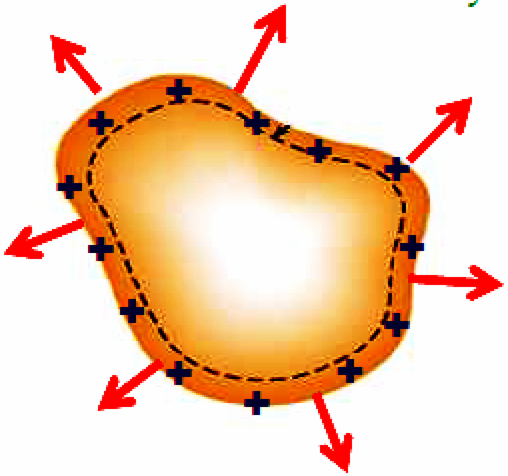
If the electric field is zero in the interior of the conductor, then we may move a test charge along any path in the interior or from the surface to the interior and the net work done on the test charge by the surface charges will be zero.

$$W_{if} = q_0(V_f - V_i) = q_0\int_i^f \vec{E} \bullet d\vec{s}$$

This means that the potential difference between any two points is zero and thus potential has same value at all points in the conductor.

The statement "The electric field just outside a charged isolated conductor is perpendicular to the surface of the conductor" is consistent with the "Equipotential surfaces are always perpendicular to electric field lines."

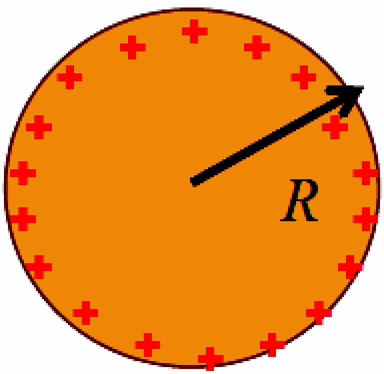
The entire conductor remains at the same potential irrespective of shape

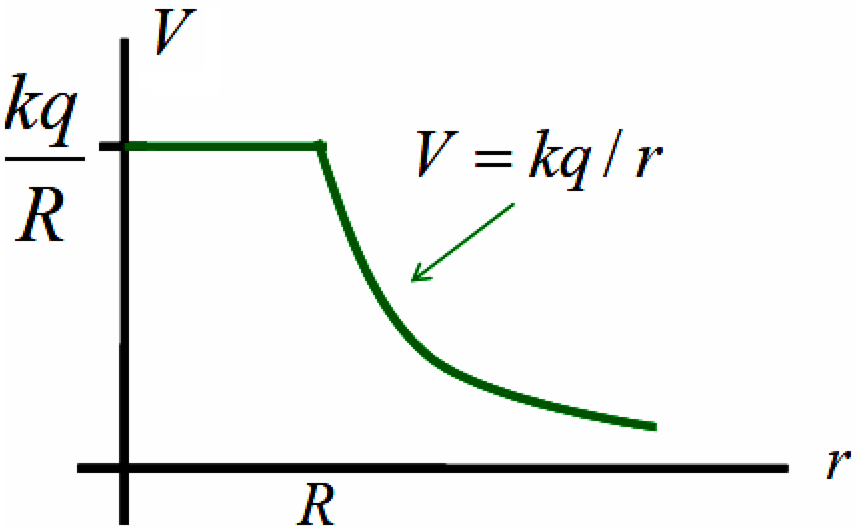


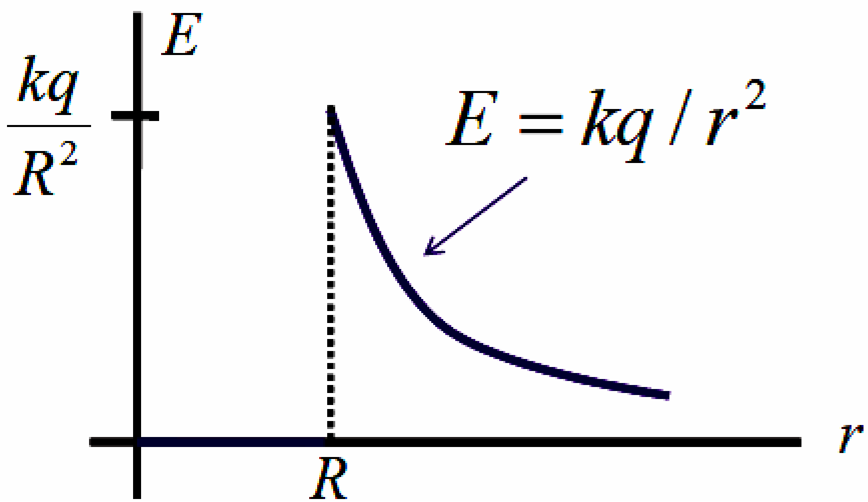
For a spherical conducting sphere of radius R and total charge q

$$E_{in} = 0 \qquad E_{out} = \frac{kq}{r^2}$$

$$V_{Cond} = \frac{kq}{R} \qquad V_{out} = \frac{kq}{r}$$







Two conducting spheres, one having twice the diameter of the other, are separated by a distance large compared to their diameters. The smaller sphere (1) has charge q and the larger sphere (2) is uncharged. If the spheres are then connected by a long thin wire:

A. 1 and 2 have the same potential
B. 2 has twice the potential of 1
C. 2 has half the potential of 1
D. 1 and 2 have the same charge
E. all of the charge is dissipated

Two conducting spheres, one having twice the diameter of the other, are separated by a distance large compared to their diameters. The smaller sphere (1) has charge q and the larger sphere (2) is uncharged. If the spheres are then connected by a long thin wire:

A. 1 and 2 have the same potential

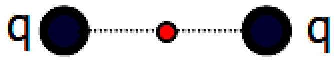
B. 2 has twice the potential of 1

C. 2 has half the potential of 1

D. 1 and 2 have the same charge

E. all of the charge is dissipated

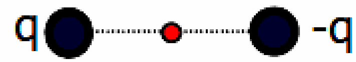
Which of the following figures have V=0 and E=0 at red point?



A

$$V \neq 0$$

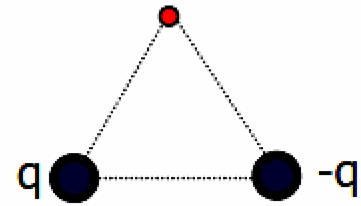
$$E = 0$$



B

$$V = 0$$

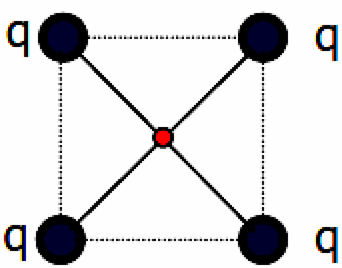
$$E \neq 0$$



C

$$V = 0$$

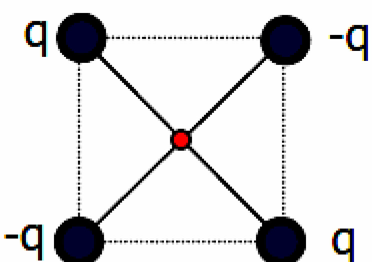
$$E \neq 0$$



D

$$V \neq 0$$

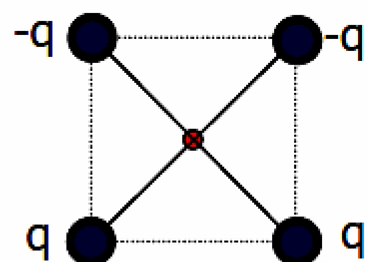
$$E = 0$$



E

$$V = 0$$

$$E = 0$$



F

$$V = 0$$

$$E \neq 0$$

Properties of electric charge

	Vector Description	Scalar Description
Interaction b/w Charges	$\boxed{\vec{F}}$	$\boxed{U}$
Effect of charges at a point in space	$\boxed{\vec{E}}$	$\boxed{V}$