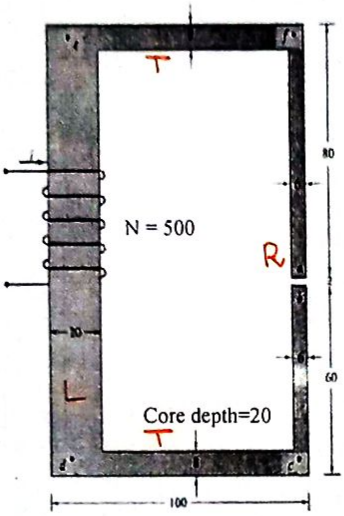
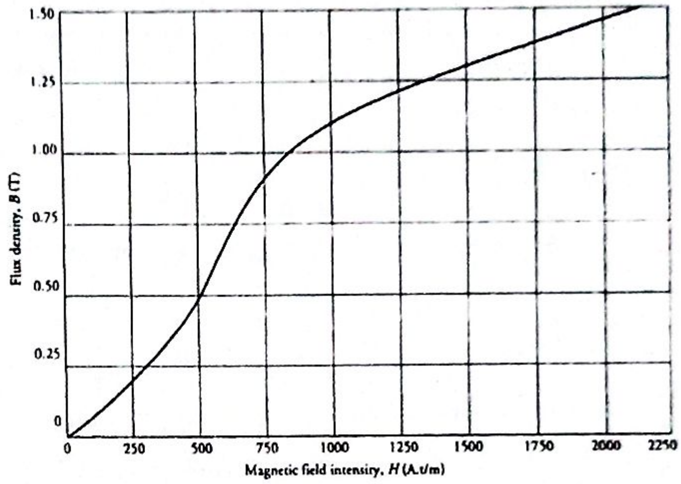
Q1. A magnetic circuit with its pertinent dimensions in millimeters is depicted in Figure 01(a). The magnetization characteristic of the magnetic material is shown in Figure 01(b). The magnetic core has a uniform depth of 20 mm. Determine the following:

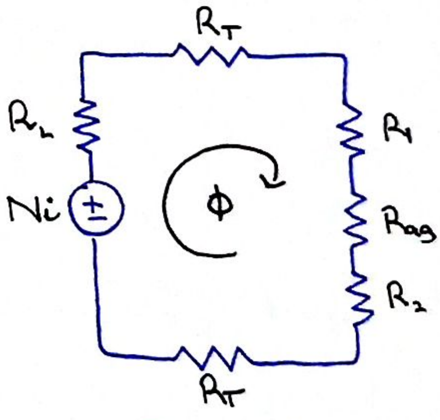
(a) Current I in the coil to establish a flux density of 1.0 T in the air gap.
(b) Relative permeability μ_r of sections *bc*, *cd*, and *de* of the core when flux density through the air gap is 1.0 T.
(c) Magneto motive force (mmf) across the air gap.
(d) Total reluctance of the magnetic circuit when flux density through the air gap is increased to 1.5 T.





- Series Magnetic Circuit

$$\Phi = B_R A_{ag} = (1.0)(0.006)(0.02)$$
$$= 0.12\ mWb$$



Equivalent Circuit

→ Top and Bot are same due to symmetry

$$B_T = \frac{\Phi}{A_T} = \frac{0.12m}{(0.008)(0.02)} = 0.75T$$

$$B_L = \frac{\Phi}{A_L} = \frac{0.12m}{(0.02)(0.02)} = 0.3T$$

$\because B = \mu_r \mu_0 H$

$\because \mathcal{R} = \frac{l_c}{\mu_r \mu_0 A}$

b)

- $$\mu_{bc} = \frac{B_R}{\mu_0 H} = \frac{1.0T}{\mu_0 (850)} = 936.2$$

- $$\mu_{cd} = \frac{B_T}{\mu_0 H} = \frac{0.75}{\mu_0 (650)} = 918.2$$

- $$\mu_{de} = \frac{B_L}{\mu_0 H} = \frac{0.3}{\mu_0 (310)} = 770.1$$

$$R_L = \frac{l_{cL}}{\mu_r \mu_0 A_L} = \frac{132\,m}{(770.1)\mu_0(0.02)(0.02)} = 341\ k\ H^{-1}$$

$$R_T = \frac{l_{cT}}{\mu_r \mu_0 A_T} = \frac{87\,m}{(918.2)\mu_0(0.008)(0.02)} = 471.25\ k\ H^{-1}$$

$$R_{ag} = \frac{l_{cag}}{\mu_0 A_{ag}} = \frac{0.002}{(1)\mu_0(0.006)(0.02)} = 13.2\ M H^{-1}$$

$$R_1 = \frac{l_{c1}}{\mu_r \mu_0 A_r} = \frac{76\,m}{(936.2)\mu_0(0.006)(0.02)} = 538.3\ k H^{-1}$$

$$R_2 = \frac{l_{c2}}{\mu_r \mu_0 A_r} = \frac{56\,m}{(936.2)\mu_0(0.006)(0.02)} = 396.6\ k H^{-1}$$

$$R_{eq} = R_L + 2R_T + R_{ag} + R_1 + R_2$$
$$= 15.41\ M H^{-1}$$

$\rightarrow Ni = \Phi R_{eq} \quad \rightarrow \quad i = \dfrac{(0.12m)(15.41M)}{500}$

$$\boxed{i = 3.67\,A} \quad a)$$

$\rightarrow \mathcal{F}_{ag} = \Phi R_{ag} \quad \rightarrow \quad \boxed{\mathcal{F}_{ag} = 1584\ A.t}$ c)

d) Since, $B_{new} = 1.5\,B \rightarrow \phi_{new} = 1.5\,\phi$

- $\mu_{bc} = \dfrac{1.5\,B_r}{\mu_0 H} = \dfrac{1.5}{\mu_0(2125)} = 561.7$

- $\mu_{cd} = \dfrac{1.5\,B_T}{\mu_0 H} = \dfrac{1.125}{\mu_0(1050)} = 852.6$

- $\mu_{de} = \dfrac{1.5\,B_L}{\mu_0 H} = \dfrac{0.45}{\mu_0(450)} = 795.7$

Following the same procedure as done for the case when $B = 1.0T$:

$$R_L = \frac{l_{cL}}{\mu_r \mu_0 A_L} = \frac{132m}{(795.7)\mu_0(0.02)(0.02)} = 330\ kH^{-1}$$

$$R_T = \frac{l_{cT}}{\mu_0 \mu_r A_T} = \frac{87m}{(852.6)\mu_0(0.008)(0.02)} = 507.5\ kH^{-1}$$

$$R_{ag} = \frac{l_{cag}}{\mu_0 A_{ag}} = \frac{2m}{(1)\mu_0(0.006)(0.02)} = 13.2\ MH^{-1}$$

$$R_1 = \frac{l_{c1}}{\mu_r \mu_0 A_r} = \frac{76m}{(561.7)\mu_0(0.006)(0.02)} = 897.2\ kH^{-1}$$

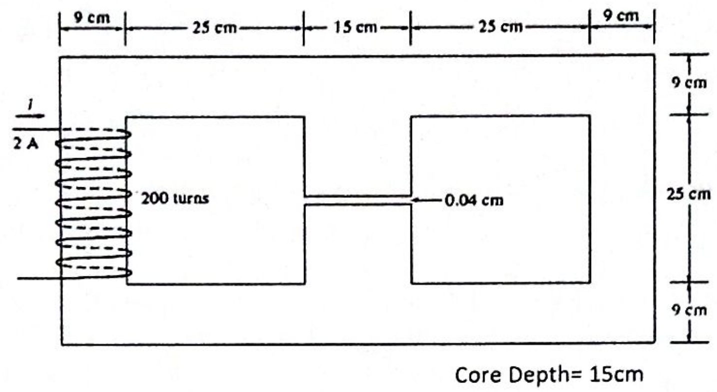
$$R_2 = \frac{l_{c2}}{\mu_r \mu_0 A_r} = \frac{56m}{(561.7)\mu_0(0.006)(0.02)} = 661.1\ kH^{-1}$$

$$R_{eq} = R_L + 2R_T + R_{ag} + R_1 + R_2$$

$$\boxed{R_{eq} = 16.10\ MH^{-1}}$$ d)

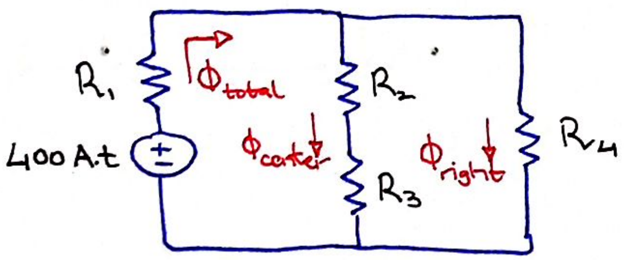
Q2. A core with three legs is shown in Figure 2. Its depth is 5 cm and there are 200 turns on the leftmost leg. The relative permeability of the core is assumed to be 2000 and constant.

(a) Sketch an equivalent magnetic circuit and clearly indicate reluctances and the mmf source using appropriate circuit symbols.
(b) Assuming 5% increase in the effective cross-sectional area of the air gap due to fringing effect, determine the flux in each of the three legs of the core.
(c) Determine mmf drop across the air gap.



a) Equivalent Circuit

$\therefore R_1 = R_4$ (Symmetry)



$\therefore R_3$: air gap reluctance

$\therefore R_{eq} = R_1 + \dfrac{(R_2+R_3)R_4}{R_2+R_3+R_4}$

$$R_1 = \frac{l_1}{\mu_r\mu_0 A_1} = \frac{1.08}{(2000)\mu_0(0.09)(0.15)} = 31.8\ kH^{-1} = R_4$$

$$R_2 = \frac{l_2}{\mu_r\mu_0 A_2} = \frac{0.34}{(2000)\mu_0(0.15)(0.15)} = 6\ kH^{-1}$$

$$R_3 = \frac{l_3}{\mu_r\mu_0 A_3} = \frac{0.0004}{(1)\mu_0(0.15)(0.15)(1.05)} = 13.47\ kH^{-1}$$

$$R_{eq} = R_1 + (R_2+R_3)R_4 / (R_2+R_3+R_4) = 40.6\ kH^{-1}$$

$$\left.\begin{array}{l} \phi_{left} = \phi_{total} = \mathcal{F}/R_{eq} = 9.85\ mWb \\ \phi_{center} = \phi_{total} \times R_4/(R_2+R_3+R_4) = 6.10\ mWb \\ \phi_{right} = \phi_{total} - \phi_{center} = 3.75\ mWb \end{array}\right\}\ b)$$

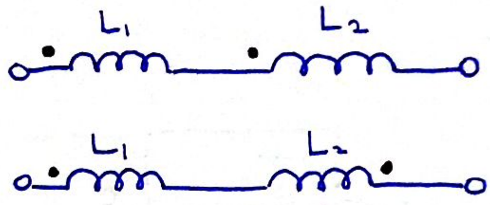
$$\mathcal{F}_{ag} = \Phi_{center} \times R_3$$
$$= (6.10\,m)(13.47k)$$
$$\boxed{\mathcal{F}_{ag} = 82.167\ A.t}$$ c)

Q3. The effective inductances when two coils are connected in series aiding and series opposing are 2.38 H and 1.02 H, respectively. If the inductance of one coil is 16 times the inductance of the other, determine

(Note: Please refer to your ENA's Magnetic Induction (self and mutual) Knowledge)

(a) the inductance of each coil,
(b) the mutual inductance, and
(c) the coefficient of coupling.

$\therefore L_1 = 16 L_2$



Series Aiding
$L_{eq} = 2.38H$

Series Opposing
$L_{eq} = 1.02\ H$

Series Aiding

$$L_{eq_a} = L_1 + L_2 + 2M = 2.38\ H \quad - i$$

Series Opposing

$$L_{eq_o} = L_1 + L_2 - 2M = 1.02\ H \quad - ii$$

→ Subtracting ii from i ;

$$4M = 1.36\ H \quad \Rightarrow \quad \boxed{M = 0.34\ H}$$ b)

→ Using $L_1 = 16L_2$ and M in i ;

$$17L_2 = 17/10 \quad \Rightarrow \quad L_2 = 0.1\ H$$
$$L_1 = 16\,L_2 \quad \Rightarrow \quad L_1 = 1.6\ H$$ a)

→ Coupling Coefficient ;

$$\boxed{k} = \frac{M}{\sqrt{L_1 \times L_2}} = \frac{0.34}{\sqrt{0.1 \times 1.6}} = \boxed{0.85}$$ c)

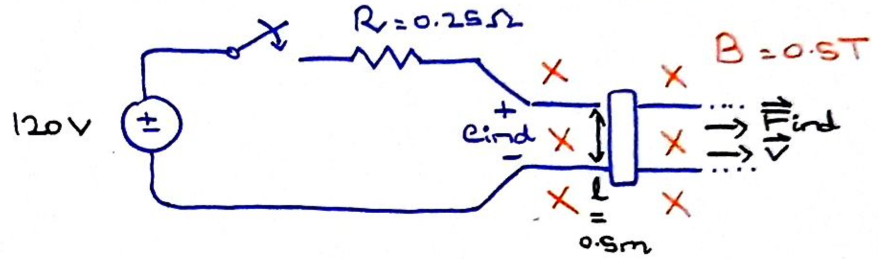
Q4.

A linear machine has the following characteristics:

$\mathbf{B} = 0.5$ T into page $\qquad R = 0.25\ \Omega$

$l = 0.5$ m $\qquad V_B = 120$ V

(a) If this bar has a load of 20 N attached to it opposite to the direction of motion, what is the steady-state speed of the bar?
(b) If the bar runs off into a region where the flux density falls to 0.45 T, what happens to the bar? What is its final steady-state speed?
(c) Suppose VB is now decreased to 100 V with everything else remaining as in part (b). What is the new steady-state speed of the bar?
(d) From the results for parts (b) and (c), what are two methods of controlling the speed of a linear machine (or a real dc motor)?



a) $F_{load} = 20\,N$

→ At steady state,

$F_{ind} = F_{load} = 20\,N$

$$i = \frac{F_{load}}{lB} = \frac{20}{(0.5)(0.5T)} = 80\,A$$

→ From KVL,

$$e_{ind} = V_B - iR = 120 - (80)(0.25)$$
$$= 100\,V$$

$$\boxed{|\vec{V}|} = \frac{e_{ind}}{lB} = \frac{100}{(0.5)(0.5)} = \boxed{400\,m/s}$$

b) B drop will cause a transient until steady state is achieved;

$$i = \frac{F_{load}}{(0.5)(0.45)} = 88.89\,A$$

$$e_{ind} = V_B - iR$$
$$= 120 - (88.89)(0.25)$$
$$= 97.77 \text{ V}$$

$$\boxed{|\vec{v}|} = \frac{e_{ind}}{lB} = \frac{97.77}{(0.5)(0.45)} = \boxed{434.5 \text{ m/s}}$$

c) $V_B = 100$ V

i will remain same, $i = 88.89$ A

$$e_{ind} = V_B - iR$$
$$= 100 - (88.89)(0.25)$$
$$= 77.77 \text{ V}$$

$$\boxed{|\vec{v}|} = \frac{e_{ind}}{lB} = \frac{77.77}{(0.5)(0.45)} = \boxed{345.6 \text{ m/s}}$$

d) From part b) and c), we can infer that we can control the speed of a linear machine by either changing flux density or by changing the applied voltage.

Relations can be expressed as;

- $|\vec{B}| \propto \dfrac{1}{|\vec{v}|}$
- $V_B \propto |\vec{v}|$
