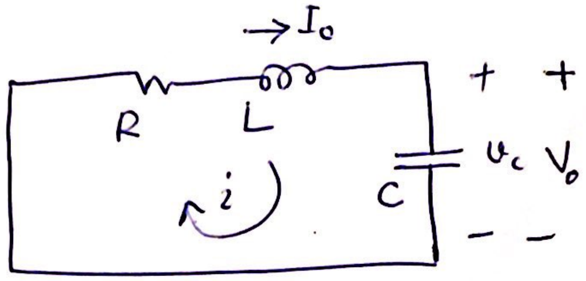
9.5 The Source-Free Series RLC Circuit

(pp346 8th Ed HKD)

The Series RLC circuit is:-



— The fundamental integro-differential equation is:-

$$Ri + L\frac{di}{dt} + \frac{1}{C}\int_{t_0}^{t} i\,dt' + v_c(t_0) = 0$$

— Differentiating wrt time:-

$$R\frac{di}{dt} + L\frac{d^2i}{dt^2} + \frac{1}{C}i = 0$$

— Rearranging:

$$L\frac{d^2i}{dt^2} + R\frac{di}{dt} + \frac{1}{C}i = 0$$

(Recall for parallel RLC circuit

$$C\frac{d^2v}{dt^2} + \frac{1}{R}\frac{dv}{dt} + \frac{1}{L}v = 0$$
)

— The overdamped response is:-

$$i(t) = A_1 e^{s_1 t} + A_2 e^{s_2 t} \qquad (\alpha > \omega_0)$$

contd

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where $s_{1,2} = -\frac{R}{2L} \pm \sqrt{\left(\frac{R}{2L}\right)^2 - \frac{1}{LC}}$

and $\alpha = \frac{R}{2L}$

$$\omega_0 = \frac{1}{\sqrt{LC}}$$

$$s_{1,2} = -\alpha \pm \sqrt{\alpha^2 - \omega_0^2}$$

— The form of the critically damped response is :-

$$i(t) = e^{-\alpha t}(A_1 t + A_2) \qquad (\alpha = \omega_0)$$

— And the underdamped response may be written as :

$$i(t) = e^{-\alpha t}\left(B_1 \cos \omega_d t + B_2 \sin \omega_d t\right) \quad (\alpha < \omega_0)$$

where $\omega_d = \sqrt{\omega_0^2 - \alpha^2}$

ω_d = damped natural frequency.

ω_0 = resonant frequency

also called undamped natural frequency.