MAGNETOSTATICS – BIOT-SAVART'S LAW

Introduction

- We have studied that an electrostatic field is produced by static or stationary charges

- If the charges are moving with constant velocity, a static magnetic (or magnetostatic) field is produced

- A magnetostatic field is produced by a constant current flow (or direct current)

- This current flow may be due to magnetization currents as in permanent magnets, electron-beam currents as in vacuum tubes, or conduction currents as in current-carrying wires

- Applications: motors, transformers, microphones, compasses ….

Introduction

- There are two major laws governing magneto static fields:

- (1) Biot-Savart's law (2) Ampere's circuit law

- Like Coulomb's law, Biot-Savart's law is the general law of magneto statics

- Like Gauss's law is a special case of Coulomb's law, Ampere's law is a special case of Biot-Savart's law

- Ampere’s law is applied in problems involving symmetrical current distribution

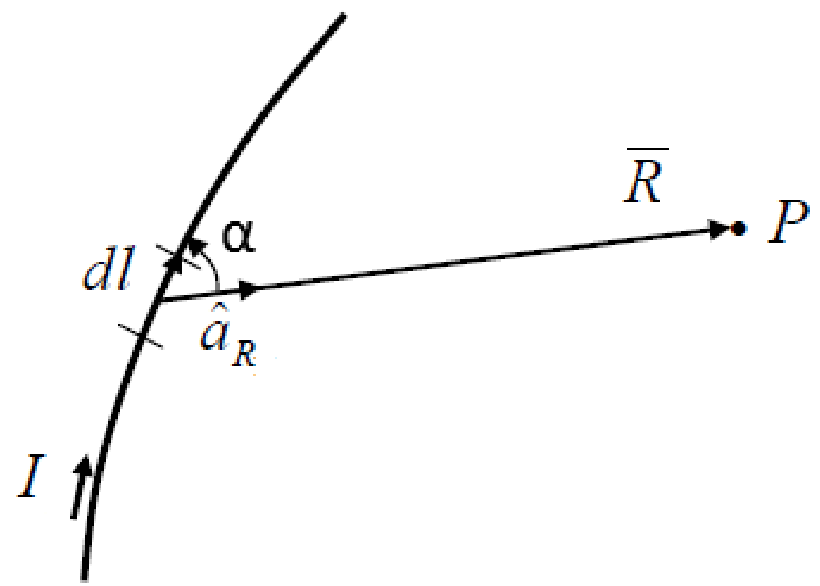
Biot Savart Law

- The contribution to the magnetic field (*dH*) at a point *P* is directly proportional to:

1. The current ***I*** flowing through the wire,
2. The differential length ***dl***,
3. The sine of the angle between the differential length and the direction to the observation point **α**

- Inversely proportional to:

1. The square of the distance between the current element and the observation point ***R***



Biot Savart Law – Mathematical Form

$$dH \propto \frac{I\,dl \sin\alpha}{R^2} \qquad \text{or} \qquad dH = \frac{kI\,dl \sin\alpha}{R^2}$$

- where *k* is the constant of proportionality
- In SI units, $k = 1/4\pi$, so:

$$dH = \frac{I\,dl \sin\alpha}{4\pi R^2}$$

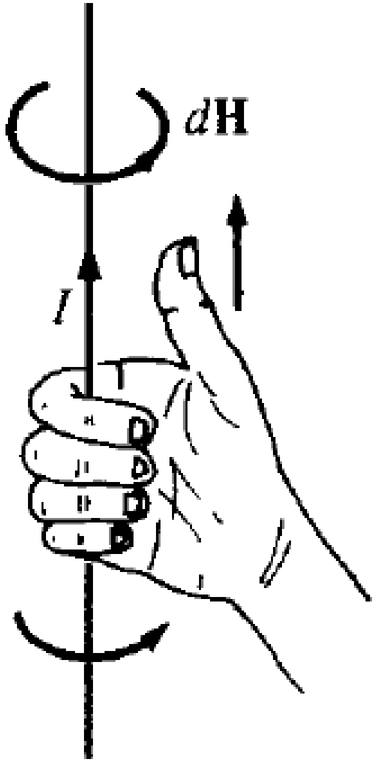
- From the definition of cross product:

$$d\mathbf{H} = \frac{I\,d\mathbf{l} \times \mathbf{a}_R}{4\pi R^2} = \frac{I\,d\mathbf{l} \times \mathbf{R}}{4\pi R^3}$$

- where $R = |\mathbf{R}|$ and $\mathbf{a}_R = \mathbf{R}/R$

Direction of *dH*

- The direction of *dH* can be determined by the right-hand rule with the right-hand thumb pointing in the direction of the current, the right-hand fingers encircling the wire in the direction of *dH*



Distributed Current Sources

- Just as we can have different charge configurations, we can have different current distributions:

1. Line current,
2. Surface current, and
3. Volume current

- The source elements are related as:

$$I\, d\mathbf{l} \equiv \mathbf{K}\, dS \equiv \mathbf{J}\, dv$$

- **K** is the surface current density (amperes/meter)

- **J** is the volume current density (amperes/meter square)

Distributed Current Sources

- So in terms of the distributed current sources, the Biot-Savart law becomes:

$$\mathbf{H} = \int_L \frac{I\,d\mathbf{l} \times \mathbf{a}_R}{4\pi R^2} \qquad \text{(line current)}$$

$$\mathbf{H} = \int_S \frac{\mathbf{K}\,dS \times \mathbf{a}_R}{4\pi R^2} \qquad \text{(surface current)}$$

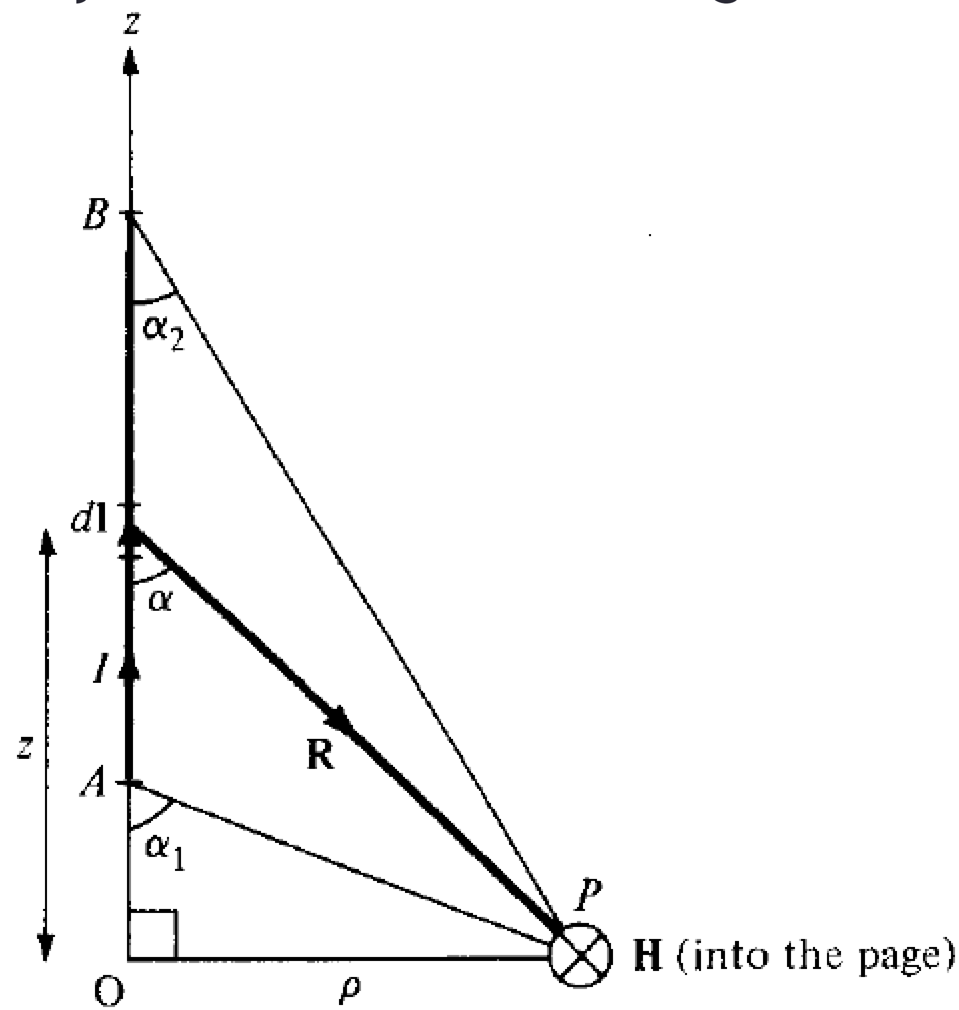
$$\mathbf{H} = \int_v \frac{\mathbf{J}\,dv \times \mathbf{a}_R}{4\pi R^2} \qquad \text{(volume current)}$$

Current Carrying Filament

- Let us apply Biot-Savart law to determine the field due to a straight current carrying filamentary conductor of finite length AB

- We assume that the conductor is along the z-axis with its upper and lower ends respectively subtending angles $\propto_2$ and $\propto_1$ at *P*, the point at which **H** is to be determined

- Particular note should be taken of this assumption as the formula to be derived will have to be applied accordingly



Current Carrying Filament

- We consider the contribution d**H** at *P* due to an element d**l** at (0, 0, z)

$$d\mathbf{H} = \frac{I\,d\mathbf{l} \times \mathbf{R}}{4\pi R^3}$$

- Since $d\mathbf{l} = dz\mathbf{a_z}$ and $\mathbf{R} = \rho\mathbf{a_\rho} - z\mathbf{a_z}$, so:

$$d\mathbf{l} \times \mathbf{R} = \rho\, dz\, \mathbf{a}_\phi$$

- Hence:

$$\mathbf{H} = \int \frac{I\rho\, dz}{4\pi[\rho^2 + z^2]^{3/2}}\,\mathbf{a}_\phi$$

Current Carrying Filament

- Let $z = \rho cot \propto, dz = -\rho cosec^2 \propto d \propto$

- We get:

$$\mathbf{H} = -\frac{1}{4\pi}\int_{\alpha_1}^{\alpha_2} \frac{\rho^2 \operatorname{cosec}^2 \alpha \, d\alpha}{\rho^3 \operatorname{cosec}^3 \alpha} \mathbf{a}_\phi$$

$$= -\frac{I}{4\pi\rho} \mathbf{a}_\phi \int_{\alpha_1}^{\alpha_2} \sin \alpha \, d\alpha$$

- Or:

$$\mathbf{H} = \frac{I}{4\pi\rho}(\cos \alpha_2 - \cos \alpha_1)\mathbf{a}_\phi$$

Current Carrying Filament

- This expression is generally applicable for any straight filamentary conductor of finite length

- Notice that **H** is always along the unit vector $\mathbf{a}_\phi$ (i.e., along concentric circular paths) irrespective of the length of the wire or the point of interest *P*

- As a special case, when the conductor is semi-infinite (with respect to *P*) so that point A is now at *O(0, 0, 0)* while B is at $(0, 0, \infty)$; $\alpha_1 = 90^\circ$, $\alpha_2 = 0^\circ$, the above becomes:

$$\mathbf{H} = \frac{I}{4\pi\rho}\mathbf{a}_\phi$$

Current Carrying Filament

- Another special case is when the conductor is infinite in length

- For this case, point A is at $(0, 0, -\infty)$ while B is at $(0, 0, \infty)$; $\propto_1 = 180°$, $\propto_2 = 0°$, so the equation reduces to:

$$\mathbf{H} = \frac{I}{2\pi\rho}\mathbf{a}_\phi$$

- A simple method to determine the unit vector $\mathbf{a}_\emptyset$ is to use the relation below:

$$\mathbf{a}_\phi = \mathbf{a}_\ell \times \mathbf{a}_\rho$$

- $\mathbf{a}_l$ is a unit vector along the line current and $\mathbf{a}_\rho$ is a unit vector along the perpendicular from the line current to the field point

Problem-1

- The conducting triangular loop in the figure carries a current of 10 A. Find H at (0, 0, 5) due to side 1 of the loop