

Quiz 1: Cylinders and Quadric Surfaces

Maximum Marks: $5 + 5 = 10$	Instructor: Dr. Naila Amir
Date: 29 - 09 - 2021	Duration: 10 Minutes
Name: Master Solution	CMS ID:

Question:

- a) Sketch the solid described by the given inequalities and identify the resulting surface.
 $0 \leq \rho \cos \theta \sin \phi \leq 2$; $0 \leq \rho \sin \theta \sin \phi \leq 3$; $0 \leq \rho \cos \phi \leq 4$.
- b) Convert the following equation in rectangular coordinates and identify the resulting surface.
 $r = \sin \theta$.

(a) Consider the given inequalities:

$$0 \leq \rho \cos \theta \sin \phi \leq 2; \quad 0 \leq \rho \sin \theta \sin \phi \leq 3;$$

$$0 \leq \rho \cos \phi \leq 4. \rightarrow (1)$$

We know that:

$$x = \rho \cos \theta \sin \phi$$

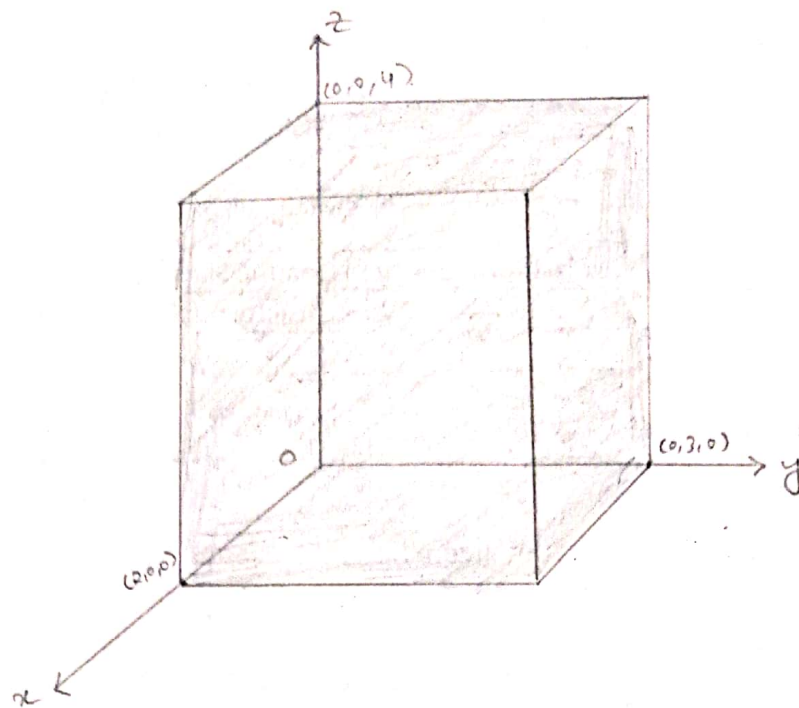
$$y = \rho \sin \theta \sin \phi$$

$$z = \rho \cos \phi$$

Using above in (1) we get

$$0 \leq x \leq 2; \quad 0 \leq y \leq 3; \quad 0 \leq z \leq 4.$$

This represents a rectangular box of dimensions $2 \times 3 \times 4$ along with its interior. This box is in the first octant.



(b) Given that,

$$r = \sin \theta$$

$$\Rightarrow r^2 = r \sin \theta$$

$$\Rightarrow x^2 + y^2 = y$$

$$\Rightarrow x^2 + y^2 - y = 0$$

$$\Rightarrow x^2 + y^2 - 2(y)(\frac{1}{2}) + (\frac{1}{2})^2 - (\frac{1}{2})^2 = 0$$

$$\Rightarrow x^2 + y^2 - 2(y)(\frac{1}{2}) + (\frac{1}{2})^2 = (\frac{1}{2})^2$$

$$\Rightarrow x^2 + (y - \frac{1}{2})^2 = (\frac{1}{2})^2$$

This represents a circular cylinder with z -axis as axis and center at $(0, \frac{1}{2}, 0)$ and radius " $\frac{1}{2}$ ".