BOUNDARY CONDITIONS-II

Boundary Conditions

- We shall consider the boundary conditions at an interface separating:

I. Dielectric (ε_{r1}) and dielectric (ε_{r2})

II. Conductor and dielectric

III. Conductor and free space

Conductor-Dielectric

- The conductor is assumed to be perfect (i.e. $\sigma \rightarrow \infty$)

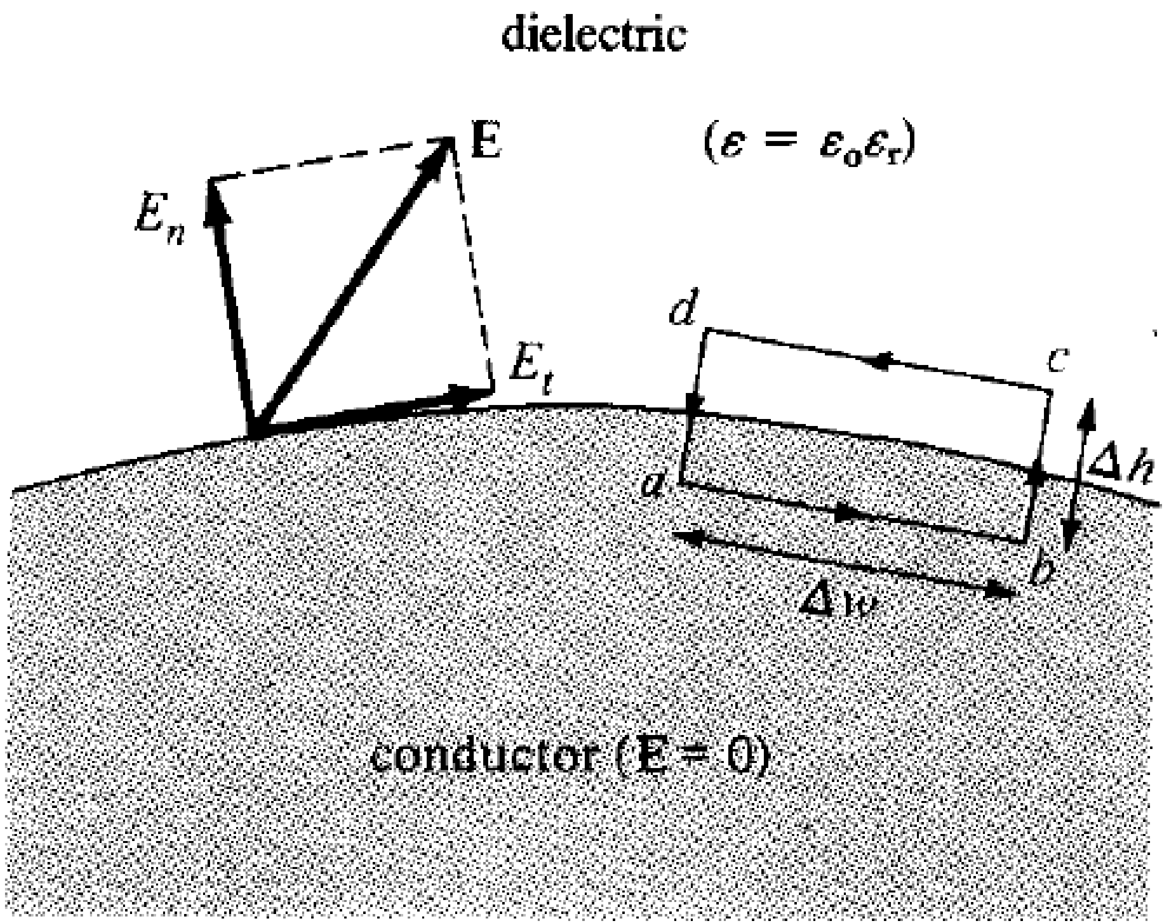
- Although such a conductor is not practically realizable, we may regard conductors such as copper and silver as though they were perfect conductors

- To determine the boundary conditions for a conductor-dielectric interface, we follow the same procedure used for dielectric-dielectric interface

- For conductors, there is a difference which is the fact that $\mathbf{E} = 0$ inside the conductor as mentioned previously

Conductor-Dielectric

- The case for conductor-dielectric interface is shown below



Conductor-Dielectric

- For the tangential components, we apply the following equation:

$$\oint \mathbf{E} \cdot d\mathbf{l} = 0$$

- For the closed path abcda, we get:

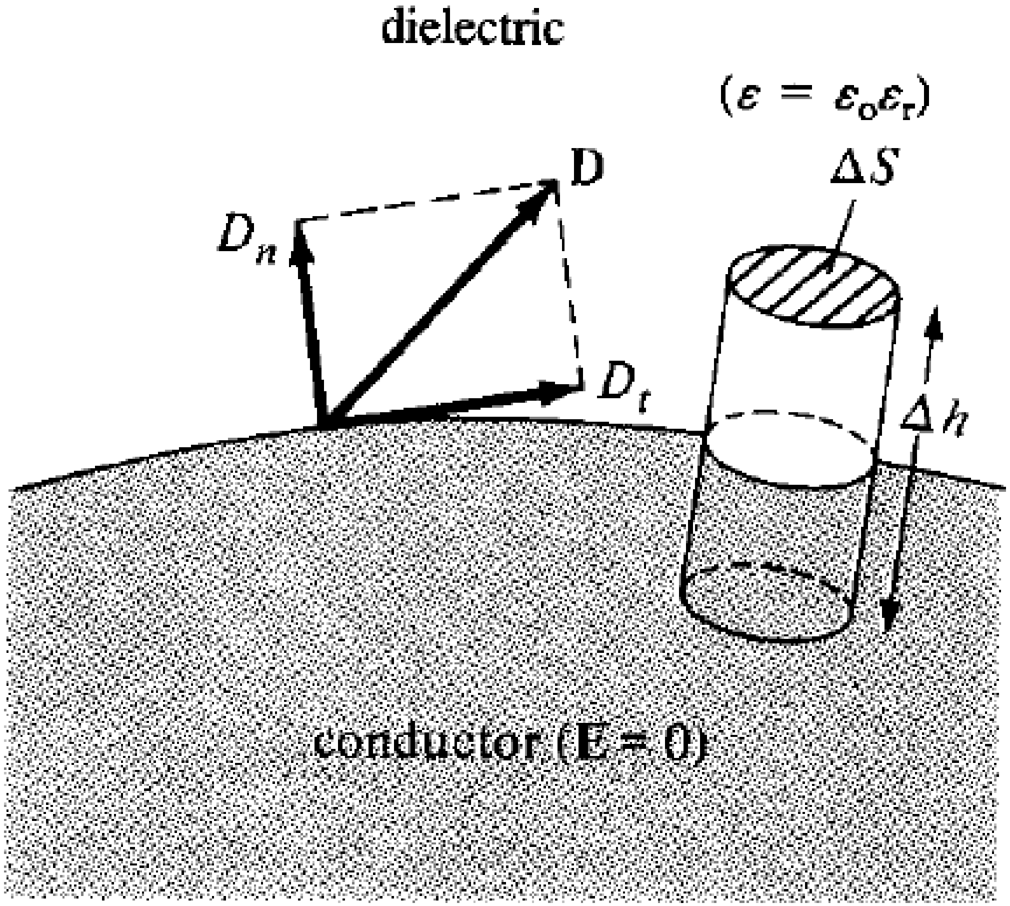
$$0 = 0 \cdot \Delta w + 0 \cdot \frac{\Delta h}{2} + E_n \cdot \frac{\Delta h}{2} - E_t \cdot \Delta w - E_n \cdot \frac{\Delta h}{2} - 0 \cdot \frac{\Delta h}{2}$$

- As $\Delta h \rightarrow 0$ at the interface, therefore: $$E_t = 0$$

- Therefore, no tangential component of **E** exists outside the conductor

Conductor-Dielectric

- For the normal components, we use the Gaussian surface shown in figure below



Conductor-Dielectric

- Charge enclosed by the Gaussian surface is found using the following equation:

$$\oint \mathbf{D} \cdot d\mathbf{S} = Q_{\text{enc}}$$

- By applying the above equation and making $\Delta h \rightarrow 0$ at the interface, we get:

$$\Delta Q = D_n \cdot \Delta S - 0 \cdot \Delta S$$

- Since $\mathbf{D} = \varepsilon \mathbf{E} = 0$ inside the conductor

- The above equation may be written as:

$$D_n = \frac{\Delta Q}{\Delta S} = \rho_S \qquad \text{OR} \qquad D_n = \rho_S$$

Conductor-Dielectric

➢From the previous results, the following conclusions can be made about a perfect conductor under static conditions:

1. No electric field may exist within a conductor; that is:

$$\rho_v = 0, \qquad \mathbf{E} = 0$$

2. Since $\mathbf{E} = -\nabla V = 0$, there can be no potential difference between any two points in the conductor; that is, a conductor is an equipotential body

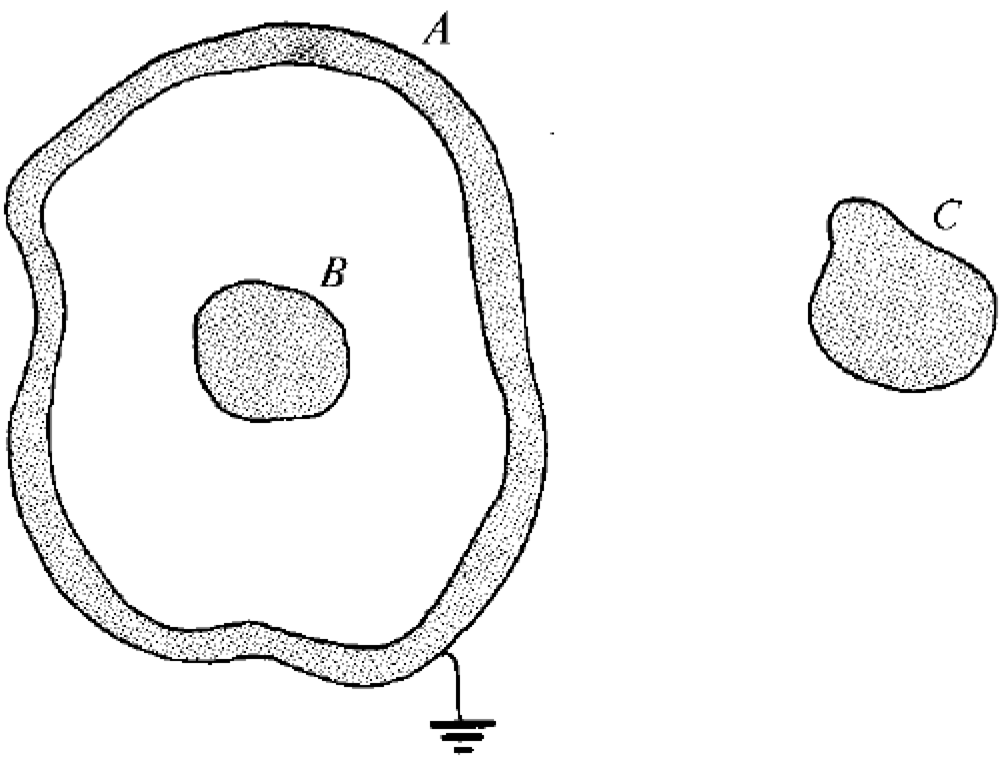
3. The electric field **E** can be external to the conductor and normal to its surface; that is:

$$D_t = \varepsilon_o \varepsilon_r E_t = 0, \qquad D_n = \varepsilon_o \varepsilon_r E_n = \rho_S$$

Conductor-Dielectric

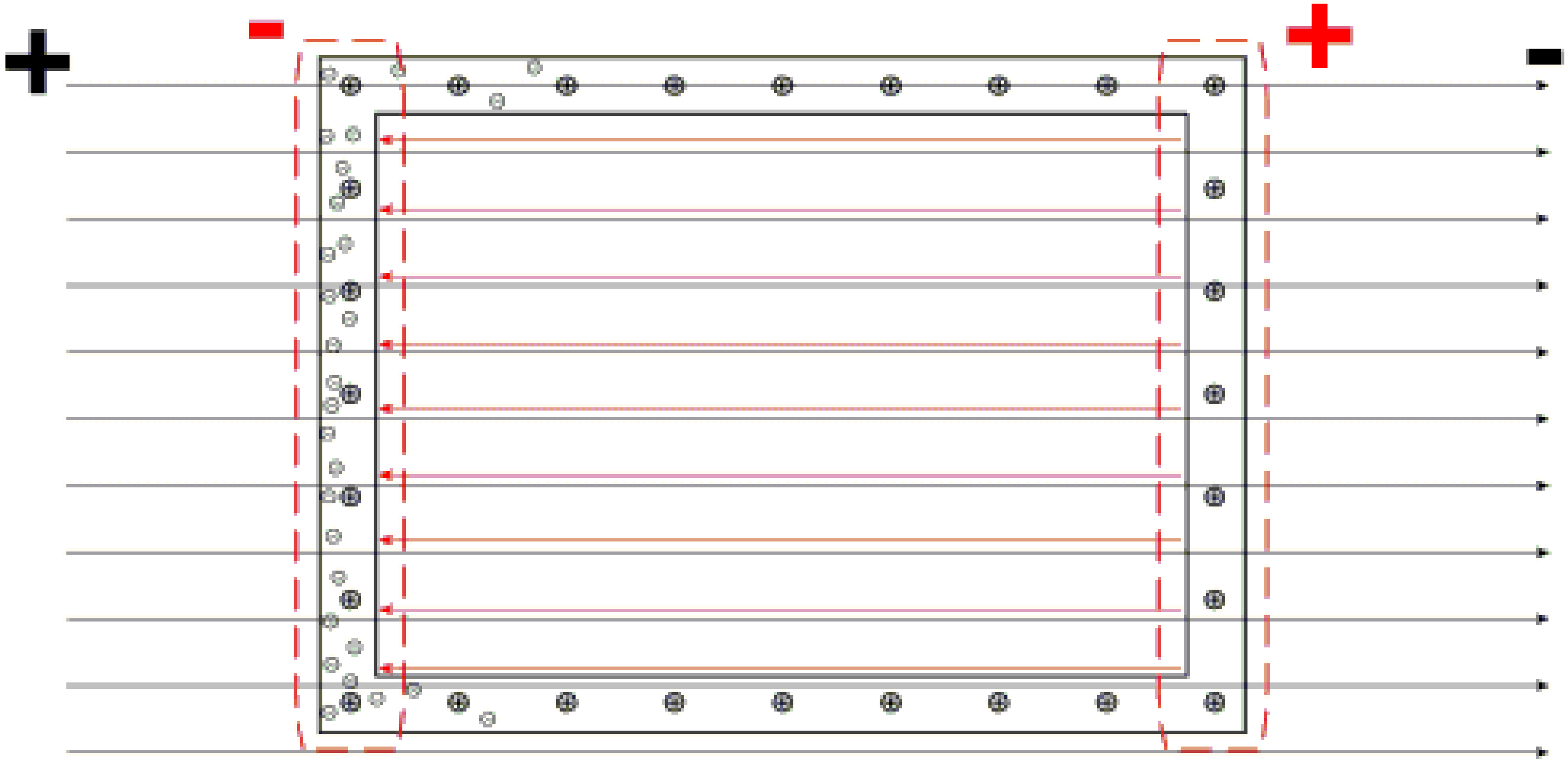
- An important application of the fact that **E** = 0 inside a conductor is in ***electrostatic screening or shielding***

- If conductor A kept at zero potential surrounds conductor B as shown in Figure, B is said to be electrically screened by A from other electric systems, such as conductor C, outside A
- Similarly, conductor C outside A is screened by A from B



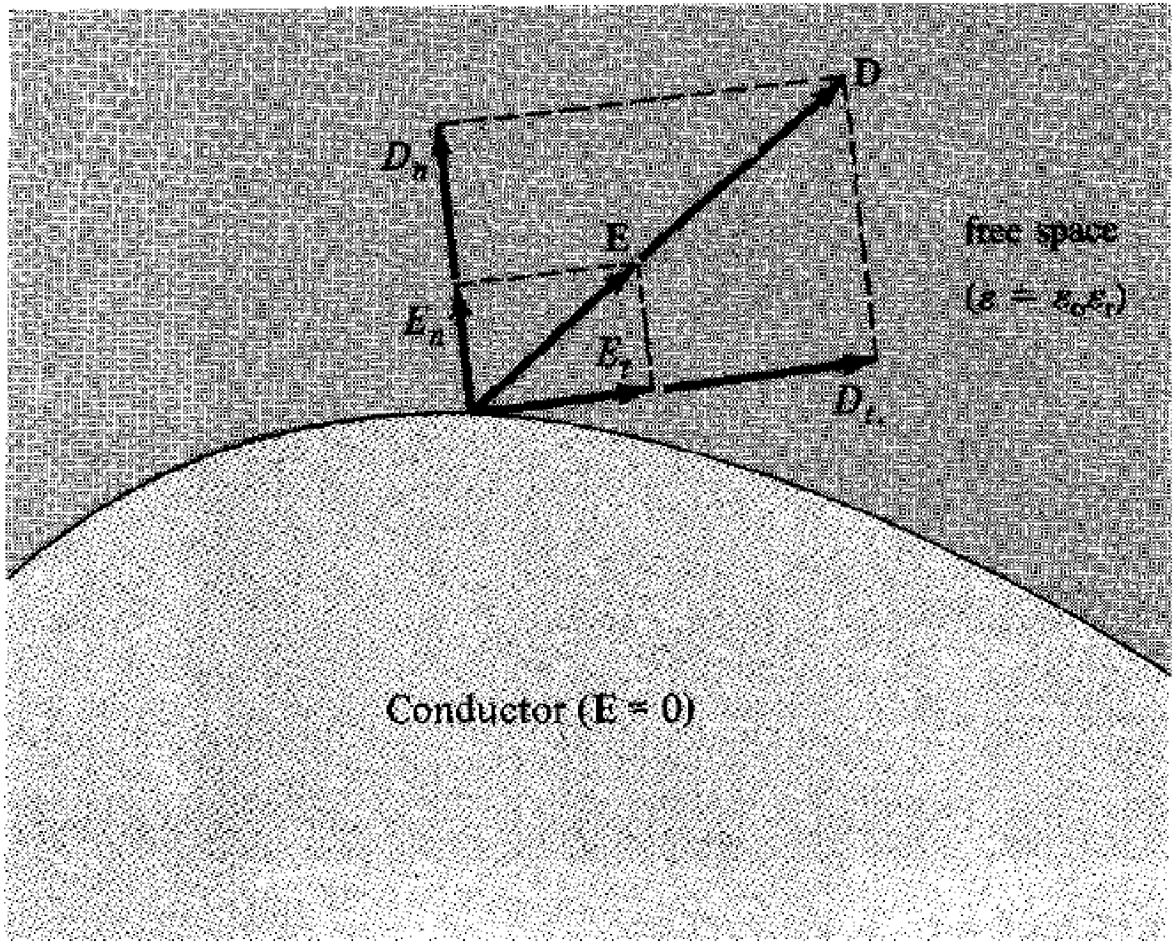
Electrostatic Shielding

- Faraday Cage



Conductor-Free Space

- The conductor to free space boundary condition is a special case of the conductor-dielectric conditions as shown in figure



Conductor-Free Space

- The boundary conditions at the interface between a conductor and free space can be obtained from the equation obtained previously for conductor-dielectric:

$$D_t = \varepsilon_o \varepsilon_r E_t = 0, \qquad D_n = \varepsilon_o \varepsilon_r E_n = \rho_S$$

- By replacing ε_r by 1, because free space may be regarded as a special dielectric for which $\varepsilon_r = 1$, we get:

$$D_t = \varepsilon_o E_t = 0, \qquad D_n = \varepsilon_o E_n = \rho_S$$

- We expect the electric field **E** to be external to the conductor and normal to its surface

Problem-1

- A homogeneous dielectric ($\varepsilon_r = 2.5$) fills region 1 ($x \leq 0$) while region 2 ($x \geq 0$) is free space.

- (a) If $\mathbf{D_1} = 12\mathbf{a_x} - 10\mathbf{a_y} + 4\mathbf{a_z}$ nC/m^2, find $\mathbf{D_2}$ and θ_2.
- (b) If E_2 = 12 V/m and θ_2 = 60°, find E_1 , and θ_1