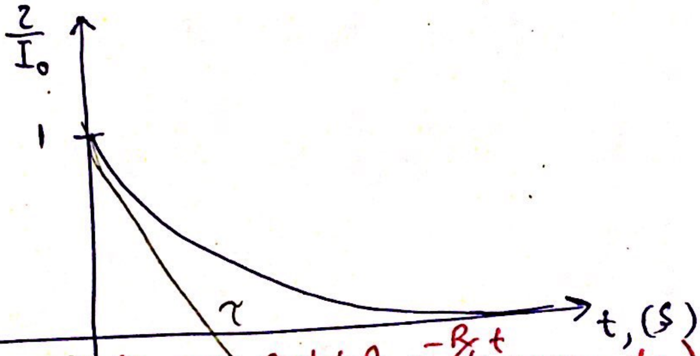
8.2 Properties of the Exponential Response

(pp 262 7th Ed HKD) (pp 268 8th Ed)

In a source-free RL circuit, the functional form of the response is:

$$i(t) = I_0 e^{-\frac{Rt}{L}} \quad \text{for } t \geq 0$$

Graphically:



(Fig 8.7 A plot of $e^{-\frac{R}{L}t}$ versus t.)

— At $t=0$ $\quad i(0) = I_0$

So $\dfrac{i}{I_0} = 1$

— The time current would take to drop to zero if it continued to drop at its initial rate can be found by evaluating the derivative at zero time.

$$\text{So } \frac{d}{dt}\left(\frac{i}{I_0}\right)\Bigg|_{t=0} = -\frac{R}{L} e^{-\frac{Rt}{L}}\Bigg|_{t=0} = -\frac{R}{L}$$

— The rapidity with which the current decreases is expressed in terms of the time constant, τ.

——— contd

— contd (268)

"The time constant of a circuit is the time required for the response to decay to a factor e^{-1} or $\frac{1}{e}$ or 36.8% of its initial value".

— This implies that at $t = \tau$

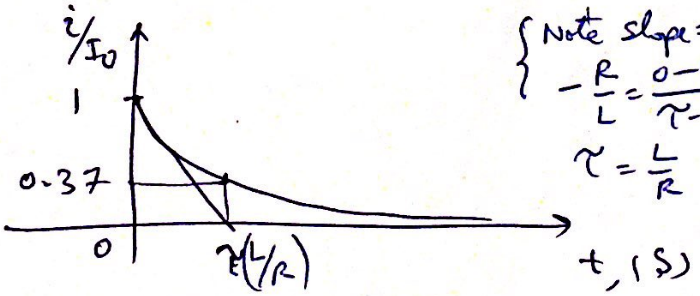
$$I_0 e^{-\frac{R}{L}t} = I_0 e^{-1} = 0.368 I_0.$$

— Hence $\tau = \frac{L}{R}$

— So $i(t) = I_0 e^{-t/\tau}$ A

— $\tau = \frac{L}{R}$ has the unit of seconds.

— The time constant can be found graphically:



{Note slope = $-\frac{R}{L} = \frac{0-1}{\tau - 0}$

$\tau = \frac{L}{R}$

$-\frac{R}{L}$

$\frac{\text{fin} - \text{ini}}{\text{fin} - \text{init}}$

— Thus $t = 1\tau$ drops to 37%

$= 2\tau$ " " 14%

$= 3\tau$ " " 5%

$= 4\tau$ " " 2%

$= 5\tau$ " " 0.7%

contd.

— contd (268)

— Generally, we assume that it takes 5τ to decay to zero.