Engineering Mechanics : Statics

Chapter 12: kinematics of a particle

Chapter outline

- Introduction
- Rectilinear Kinematics: Continuous Motion
- Rectilinear Kinematics: Erratic Motion
- General Curvilinear
- Curvilinear Motion: Rectangular Motion
- Projectile

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$$v = \frac{ds}{dt}$$

$$a = \frac{dv}{dt}$$

$$a = \frac{d^2s}{dt^2}$$

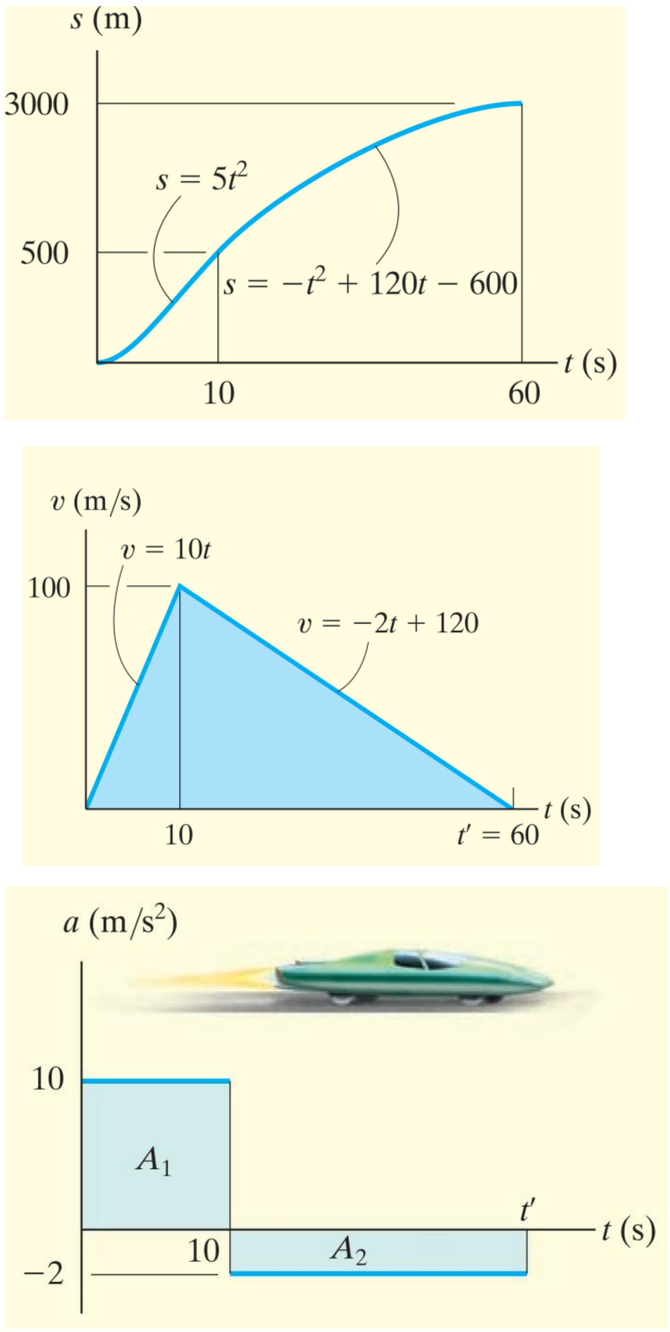
Constant Acceleration, $a = a_c$.

$$v = v_0 + a_c t$$
Constant Acceleration

$$s = s_0 + v_0 t + \tfrac{1}{2} a_c t^2$$
Constant Acceleration

$$v^2 = v_0^2 + 2a_c(s - s_0)$$
Constant Acceleration

The s–t, v–t, and a–t Graphs.



Kinematics of a Particle

12. 4 GENERAL CURVILINEAR

Curvilinear Motion

The motion of an object along a curved path is called a curvilinear motion.

12. 4 GENERAL CURVILINEAR

Curvilinear motion in a plane

Motion of car along a curved road

Motion of cable car along a steel cable

12. 4 GENERAL CURVILINEAR

Curvilinear motion in a space

Motion of roller coaster along its track.

Motion of fighter jets during national parade.

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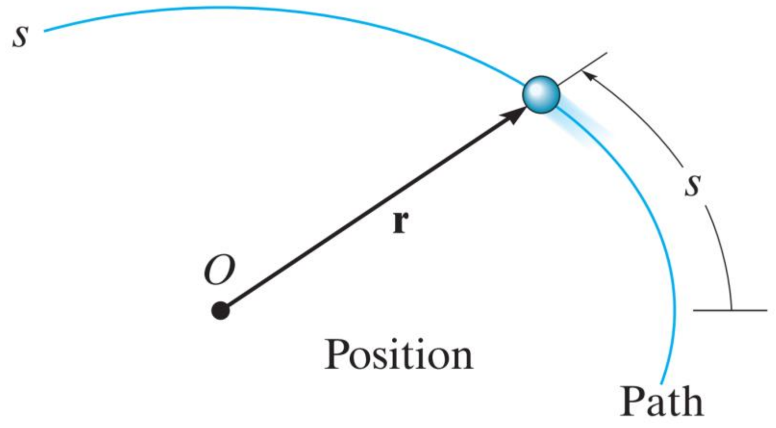
General Curvilinear Motion

Position.

Position. Consider a particle located at a point on a space curve defined by the path function $s(t)$, Fig. 12–16*a*. The position of the particle, measured from a fixed point O, will be designated by the *position vector* $\mathbf{r} = \mathbf{r}(t)$. Notice that both the magnitude and direction of this vector will change as the particle moves along the curve.

$$\mathbf{r} = \mathbf{r}(t).$$



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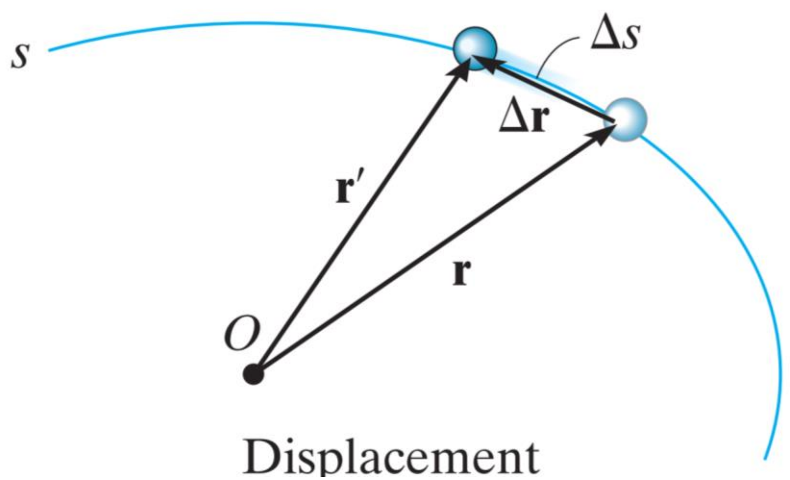
General Curvilinear Motion

Velocity.

Displacement. Suppose that during a small time interval Δt the particle moves a distance Δs along the curve to a new position, defined by $\mathbf{r}' = \mathbf{r} + \Delta\mathbf{r}$, Fig. 12–16*b*. The *displacement* $\Delta\mathbf{r}$ represents the change in the particle's position and is determined by vector subtraction; i.e.,

$$\Delta\mathbf{r} = \mathbf{r}' - \mathbf{r}.$$



Displacement

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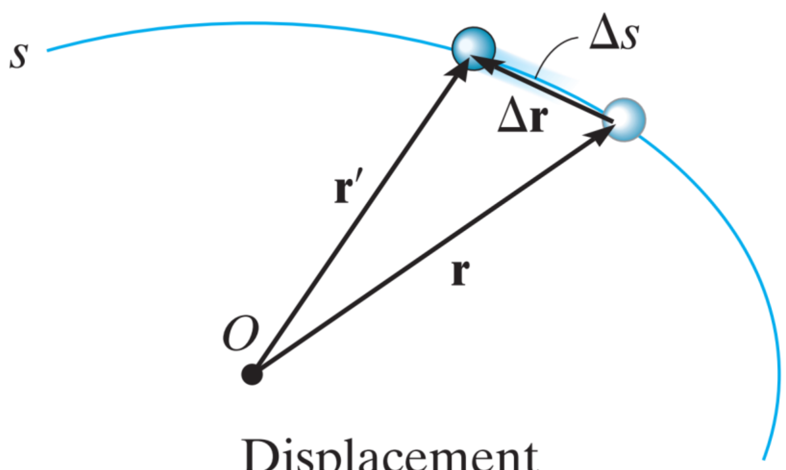
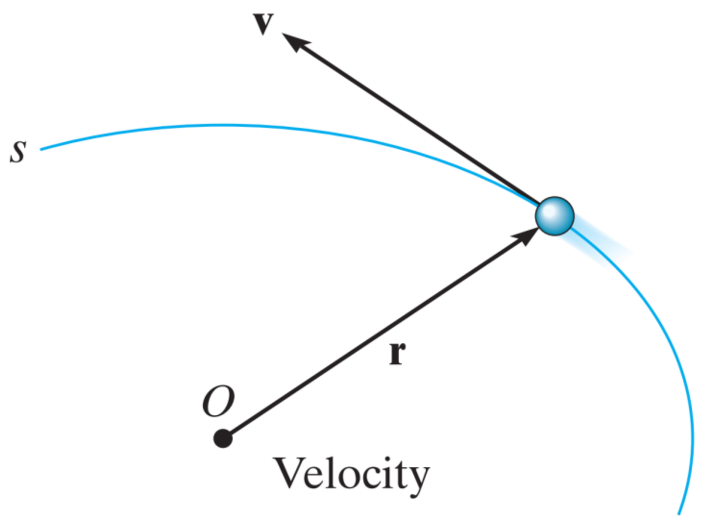
General Curvilinear Motion

Velocity.

$$\mathbf{v}_{\text{avg}} = \frac{\Delta \mathbf{r}}{\Delta t}$$



Displacement



Velocity

The *instantaneous velocity* is determined from this equation by letting $\Delta t \to 0$, and consequently the direction of $\Delta \mathbf{r}$ *approaches* the *tangent* to the curve. Hence, $\mathbf{v} = \lim_{\Delta t \to 0} (\Delta \mathbf{r}/\Delta t)$ or

$$\boxed{\mathbf{v} = \frac{d\mathbf{r}}{dt}}$$

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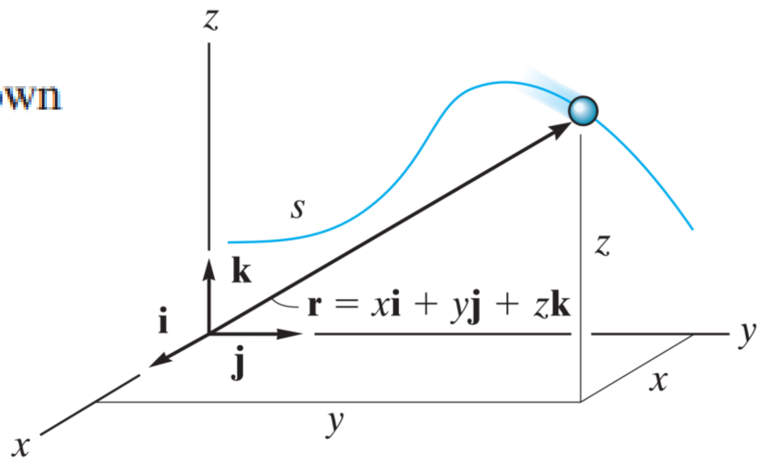
Curvilinear Motion: Rectangular Components

Position. If the particle is at point (x, y, z) on the curved path s shown in Fig. 12–17*a*, then its location is defined by the *position vector*

$$\boxed{\mathbf{r} = x\mathbf{i} + y\mathbf{j} + z\mathbf{k}}$$

$$r = \sqrt{x^2 + y^2 + z^2}$$

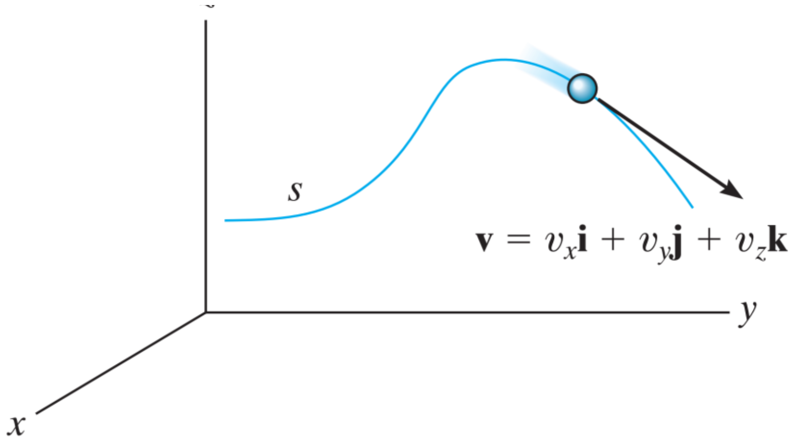


Position

Velocity. The first time derivative of **r** yields the velocity of the particle. Hence,

$$\mathbf{v} = \frac{d\mathbf{r}}{dt} = \frac{d}{dt}(x\mathbf{i}) + \frac{d}{dt}(y\mathbf{j}) + \frac{d}{dt}(z\mathbf{k})$$

$$\boxed{\mathbf{v} = \frac{d\mathbf{r}}{dt} = v_x\mathbf{i} + v_y\mathbf{j} + v_z\mathbf{k}}$$



Velocity

$$v = \sqrt{v_x^2 + v_y^2 + v_z^2}$$

$$\boxed{v_x = \dot{x} \quad v_y = \dot{y} \quad v_z = \dot{z}}$$

$$\mathbf{u}_v = \mathbf{v}/v.$$

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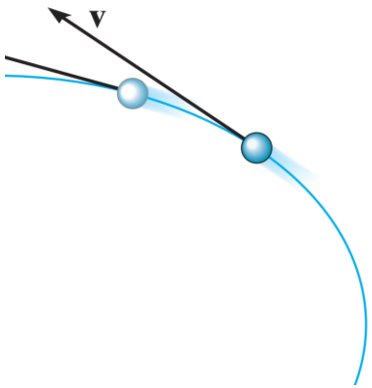
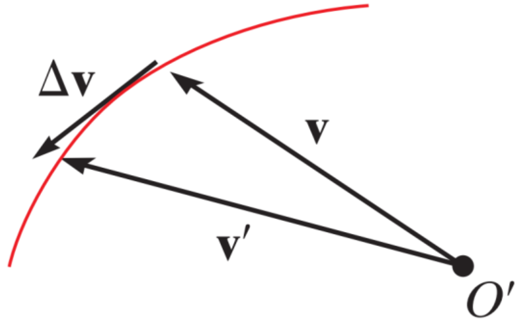
General Curvilinear Motion

Acceleration.

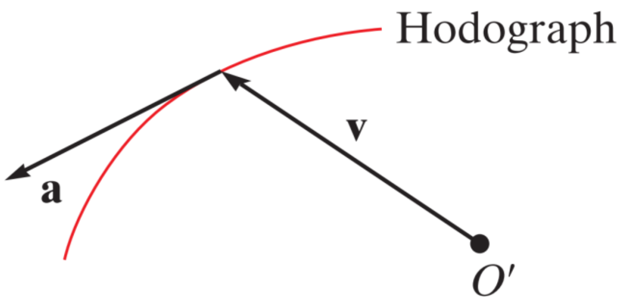
Acceleration. If the particle has a velocity $\mathbf{v}$ at time t and a velocity $\mathbf{v}' = \mathbf{v} + \Delta\mathbf{v}$ at $t + \Delta t$, Fig. 12–16*d*, then the *average acceleration* of the particle during the time interval Δt is

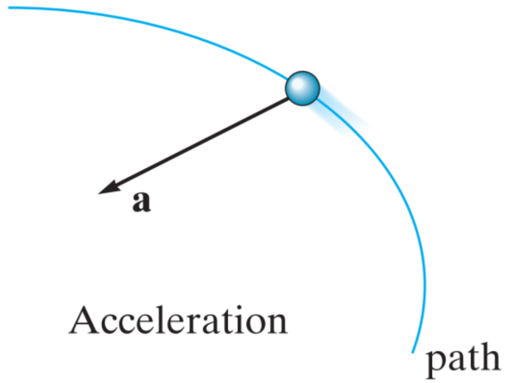




$$\mathbf{a}_{avg} = \frac{\Delta \mathbf{v}}{\Delta t} \qquad \Delta\mathbf{v} = \mathbf{v}' - \mathbf{v}.$$

$$\boxed{\mathbf{a} = \frac{d\mathbf{v}}{dt}} \qquad \mathbf{a} = \frac{d^2\mathbf{r}}{dt^2}$$





such that their tails are located at the fixed point O’ and their arrowheads touch points on a curve. This curve is called a *hodograph*,

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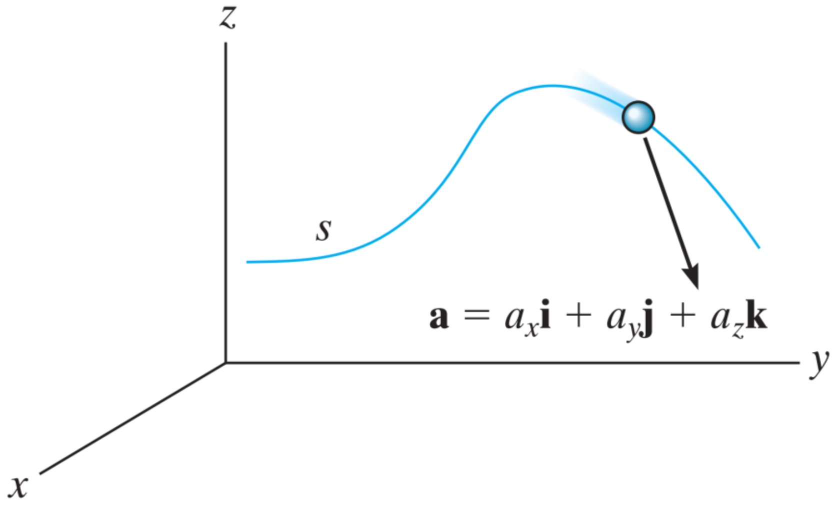
Curvilinear Motion: Rectangular Components

Acceleration. The acceleration of the particle is obtained by taking the first time derivative of Eq. 12–11 (or the second time derivative of Eq. 12–10). We have

$$\boxed{\mathbf{a} = \frac{d\mathbf{v}}{dt} = a_x\mathbf{i} + a_y\mathbf{j} + a_z\mathbf{k}}$$

$$\boxed{\begin{aligned} a_x &= \dot{v}_x = \ddot{x} \\ a_y &= \dot{v}_y = \ddot{y} \\ a_z &= \dot{v}_z = \ddot{z} \end{aligned}}$$



Acceleration

and a *direction* specified by the unit vector $\mathbf{u}_a = \mathbf{a}/a$. Since $\mathbf{a}$ represents the time rate of *change* in both the magnitude and direction of the velocity, in general $\mathbf{a}$ will *not* be tangent to the path, Fig. 12–17*c*.

$$a = \sqrt{a_x^2 + a_y^2 + a_z^2}$$

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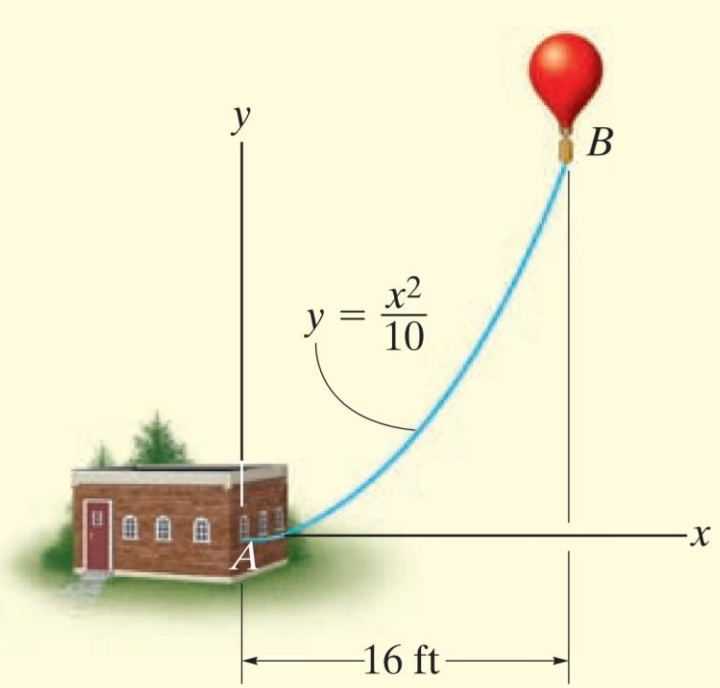
At any instant the horizontal position of the weather balloon in Fig. 12–18*a* is defined by $x = (8t)$ ft, where t is in seconds. If the equation of the path is $y = x^2/10$, determine the magnitude and direction of the velocity and the acceleration when $t = 2$ s.



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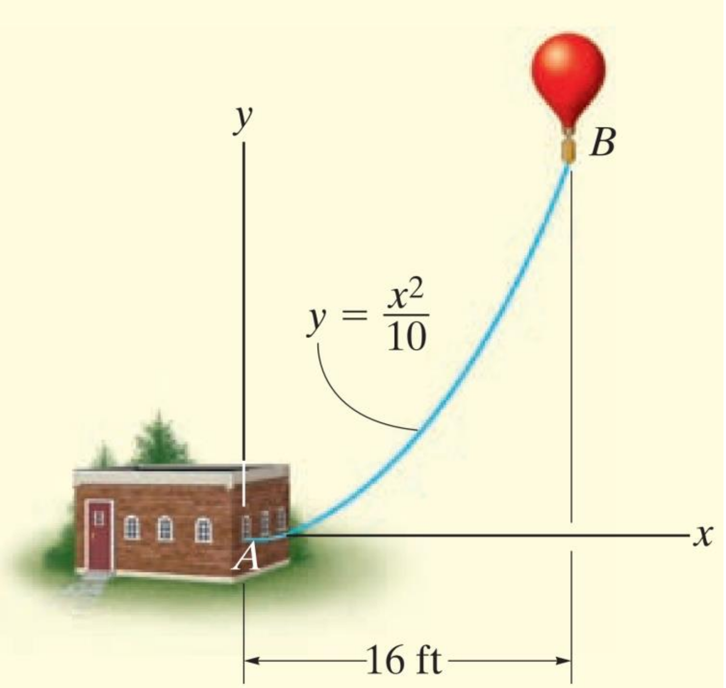


Velocity. The velocity component in the x direction is

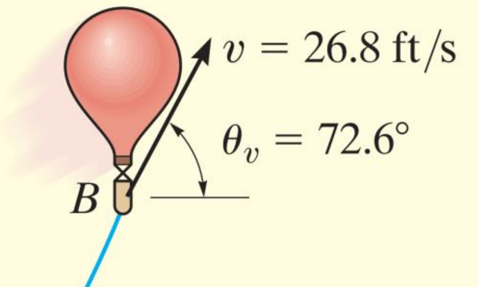
$$v_x = \dot{x} = \frac{d}{dt}(8t) = 8 \text{ ft/s} \rightarrow$$

To find the relationship between the velocity components we will use the chain rule of calculus. When $t = 2$ s, $x = 8(2) = 16$ ft, Fig. 12–18*a*, and so

$$v_y = \dot{y} = \frac{d}{dt}(x^2/10) = 2x\dot{x}/10 = 2(16)(8)/10 = 25.6 \text{ ft/s} \uparrow$$

When $t = 2$ s, the magnitude of velocity is therefore

$$v = \sqrt{(8 \text{ ft/s})^2 + (25.6 \text{ ft/s})^2} = 26.8 \text{ ft/s} \qquad \textit{Ans.}$$



The direction is tangent to the path, Fig. 12–18*b*, where

$$\theta_v = \tan^{-1}\frac{v_y}{v_x} = \tan^{-1}\frac{25.6}{8} = 72.6° \qquad \textit{Ans.}$$

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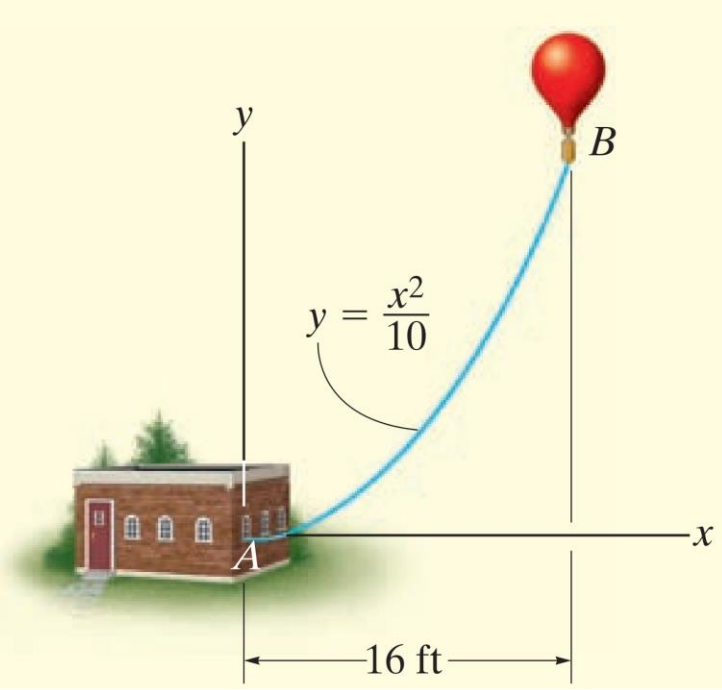

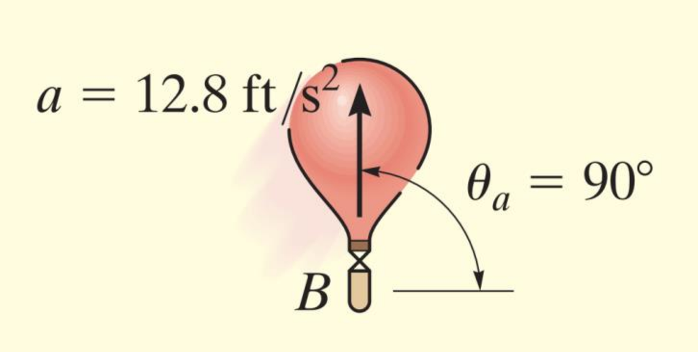

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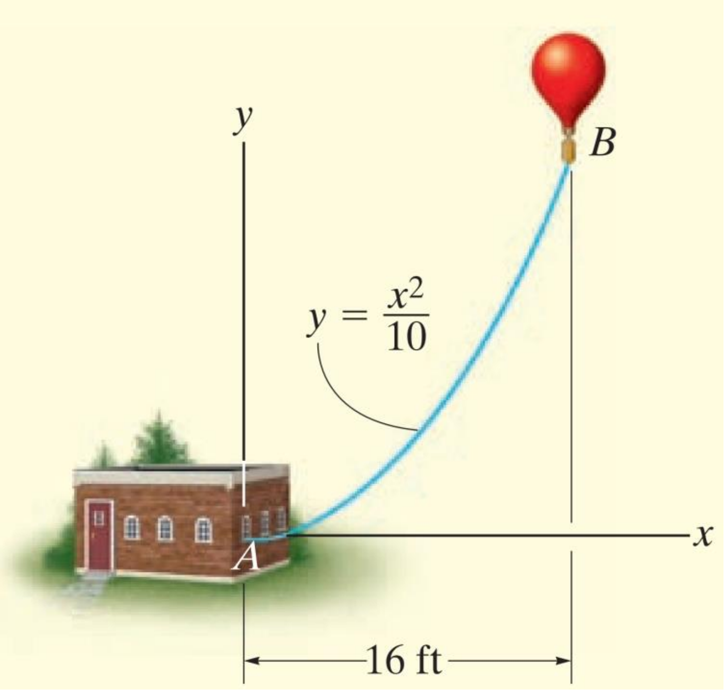


Acceleration. The relationship between the acceleration components is determined using the chain rule. (See Appendix C.) We have

$$a_x = \dot{v}_x = \frac{d}{dt}(8) = 0$$

$$a_y = \dot{v}_y = \frac{d}{dt}(2x\dot{x}/10) = 2(\dot{x})\dot{x}/10 + 2x(\ddot{x})/10$$

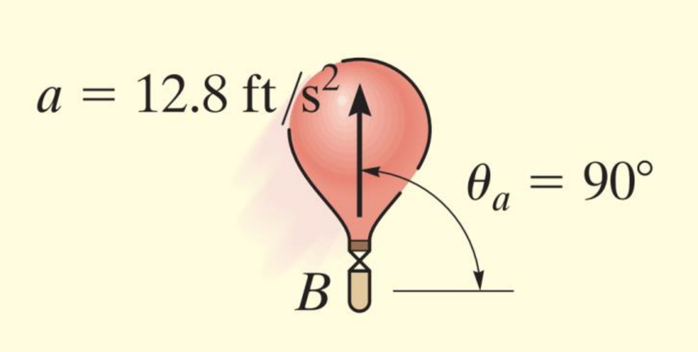
$$= 2(8)^2/10 + 2(16)(0)/10 = 12.8 \text{ ft/s}^2 \uparrow$$

Thus,

$$a = \sqrt{(0)^2 + (12.8)^2} = 12.8 \text{ ft/s}^2 \qquad \textit{Ans.}$$

The direction of **a**, as shown in Fig. 12–18*c*, is

$$\theta_a = \tan^{-1}\frac{12.8}{0} = 90^\circ \qquad \textit{Ans.}$$



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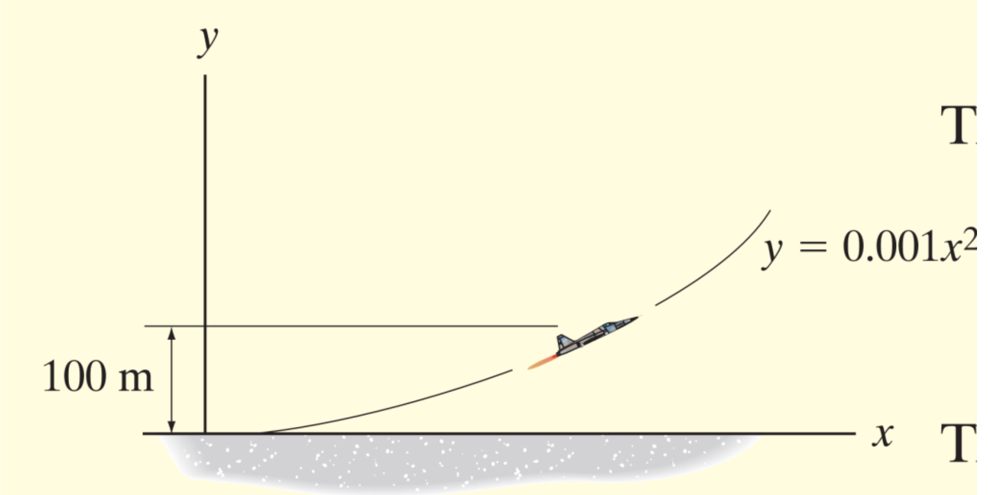
For a short time, the path of the plane in Fig. 12–19*a* is described by $y = (0.001x^2)$ m. If the plane is rising with a constant upward velocity of 10 m/s, determine the magnitudes of the velocity and acceleration of the plane when it reaches an altitude of $y = 100$ m.



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When $y = 100$ m, then $100 = 0.001x^2$ or $x = 316.2$ m. Also, due to constant velocity $v_y = 10$ m/s, so

$$y = v_y t; \qquad 100\text{ m} = (10\text{ m/s})\,t \qquad t = 10\text{ s}$$

Velocity. Using the chain rule (see Appendix C) to find the relationship between the velocity components, we have

$$y = 0.001x^2$$

$$v_y = \dot{y} = \frac{d}{dt}(0.001x^2) = (0.002x)\dot{x} = 0.002xv_x \qquad (1)$$

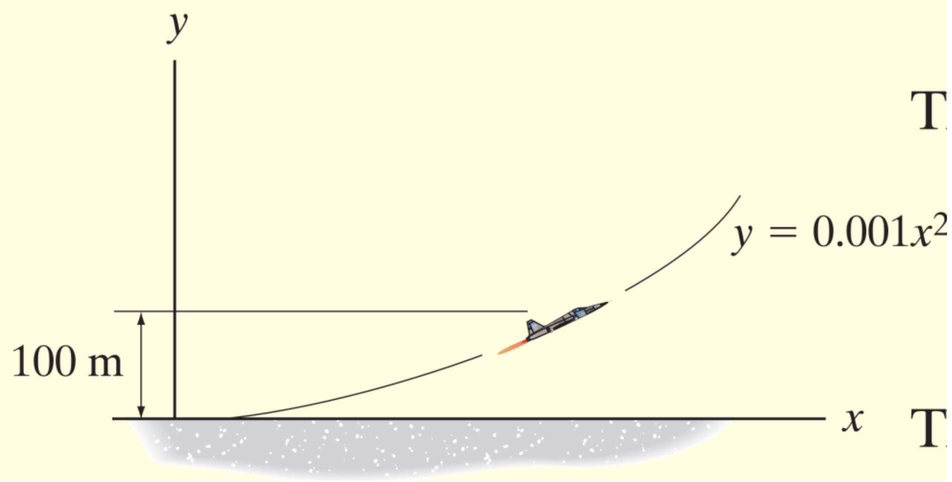
Thus

$1x^2$

$$10\text{ m/s} = 0.002(316.2\text{ m})(v_x)$$
$$v_x = 15.81\text{ m/s}$$

The magnitude of the velocity is therefore

$$v = \sqrt{v_x^2 + v_y^2} = \sqrt{(15.81\text{ m/s})^2 + (10\text{ m/s})^2} = 18.7\text{ m/s} \qquad \textit{Ans.}$$



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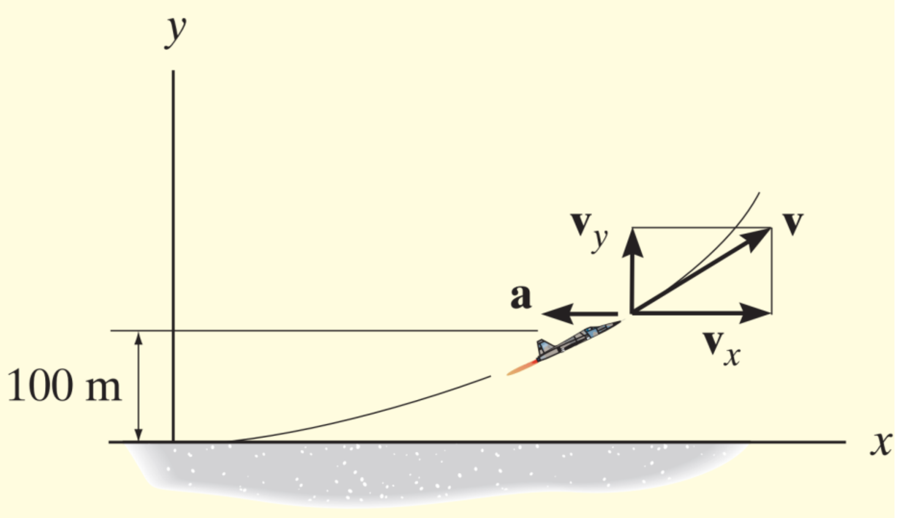

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Acceleration. Using the chain rule, the time derivative of Eq. (1) gives the relation between the acceleration components.

$$a_y = \dot{v}_y = (0.002\dot{x})\dot{x} + 0.002x(\ddot{x}) = 0.002(v_x^2 + xa_x)$$

When $x = 316.2$ m, $v_x = 15.81$ m/s, $\dot{v}_y = a_y = 0$,

$$0 = 0.002\left[(15.81 \text{ m/s})^2 + 316.2 \text{ m}(a_x)\right]$$
$$a_x = -0.791 \text{ m/s}^2$$

The magnitude of the plane's acceleration is therefore

$$a = \sqrt{a_x^2 + a_y^2} = \sqrt{(-0.791 \text{ m/s}^2)^2 + (0 \text{ m/s}^2)^2}$$
$$= 0.791 \text{ m/s}^2 \qquad \textit{Ans.}$$

These results are shown in Fig. 12–19*b*.