Thermodynamics I

Lecture 17

Energy Balance & Specific Heats (C_p and C_v) (Ch-4)

Dr. Ahmed Rasheed

ENERGY BALANCE FOR CLOSED SYSTEMS

Energy balance for any system undergoing any kind of process was expressed as

$$\underbrace{E_{\text{in}} - E_{\text{out}}}_{\substack{\text{Net energy transfer} \\ \text{by heat, work, and mass}}} = \underbrace{\Delta E_{\text{system}}}_{\substack{\text{Change in internal, kinetic,} \\ \text{potential, etc., energies}}}$$

or, in the rate form, as

$$\underbrace{\dot{E}_{\text{in}} - \dot{E}_{\text{out}}}_{\substack{\text{Rate of net energy transfer} \\ \text{by heat, work, and mass}}} = \underbrace{dE_{\text{system}}/dt}_{\substack{\text{Rate of change in internal,} \\ \text{kinetic, potential, etc., energies}}}$$

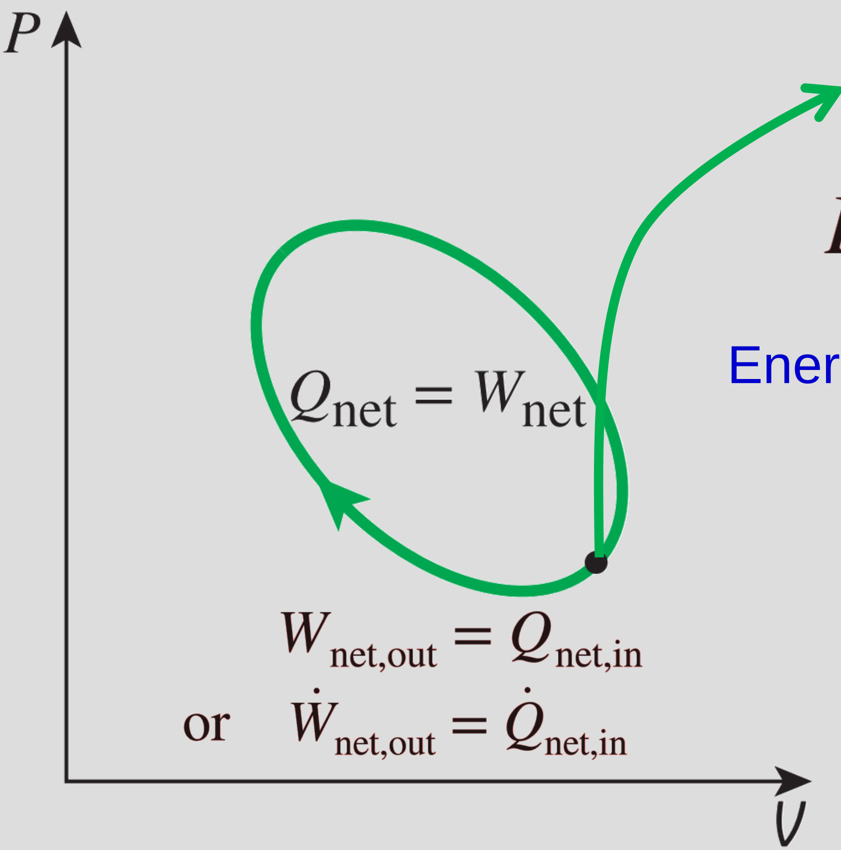
For constant rates, the total quantities during a time interval Δt are related to the quantities per unit time as

$$Q = \dot{Q}\,\Delta t, \quad W = \dot{W}\,\Delta t, \quad \text{and} \quad \Delta E = (dE/dt)\Delta t$$

ENERGY BALANCE FOR CLOSED SYSTEMS

$$e_{in} - e_{out} = \Delta e_{system}$$ (on a per unit mass basis)

$$\delta E_{in} - \delta E_{out} = dE_{system} \quad \text{or} \quad \delta e_{in} - \delta e_{out} = de_{system}$$ (on a differential basis)



$$\Delta E_{system} = E_2 - E_1 = 0$$

$$E_{in} - E_{out} = 0 \text{ or } E_{in} = E_{out}$$

Energy balance relation in case for closed system

$$Q_{net,in} - W_{net,out} = \Delta E_{system}$$

$$\text{or} \quad Q - W = \Delta E$$

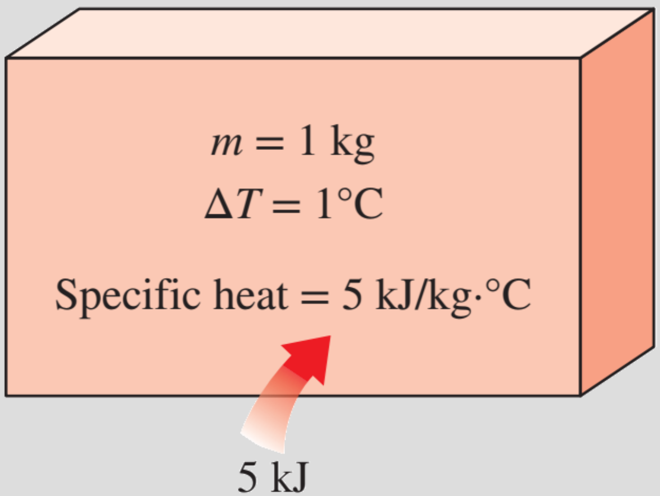
$$Q = Q_{net,in} = Q_{in} - Q_{out}$$

$$W = W_{net,out} = W_{out} - W_{in}$$

SPECIFIC HEATS

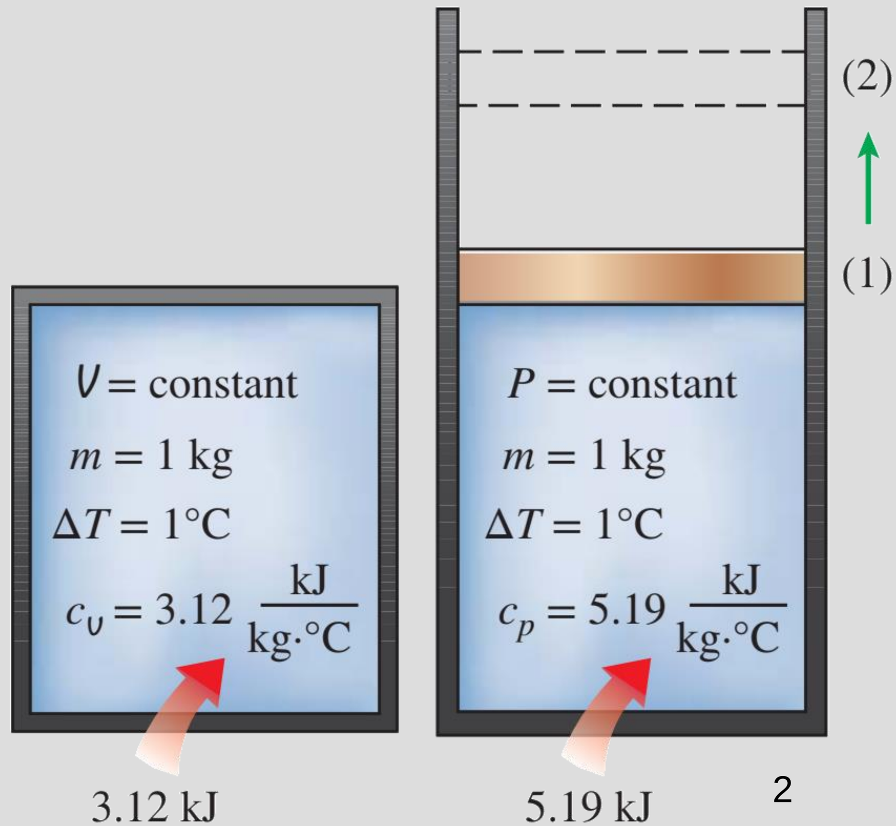
Specific heat at constant volume, c_v: The energy required to raise the temperature of the unit mass of a substance by one degree as the volume is maintained constant.

Specific heat at constant pressure, c_p: The energy required to raise the temperature of the unit mass of a substance by one degree as the pressure is maintained constant.



Specific heat is the energy required to raise the temperature of a unit mass of a substance by one degree in a specified way.

Constant-volume and constant-pressure specific heats c_v and c_p (values are for helium gas).



SPECIFIC HEATS

Now, express the specific heats in terms of other thermodynamic properties, first consider constant-volume process

Conservation of energy principal

$$e_{\text{in}} - e_{\text{out}} = \Delta e_{\text{system}}$$

$$\delta e_{\text{in}} - \delta e_{\text{out}} = du$$

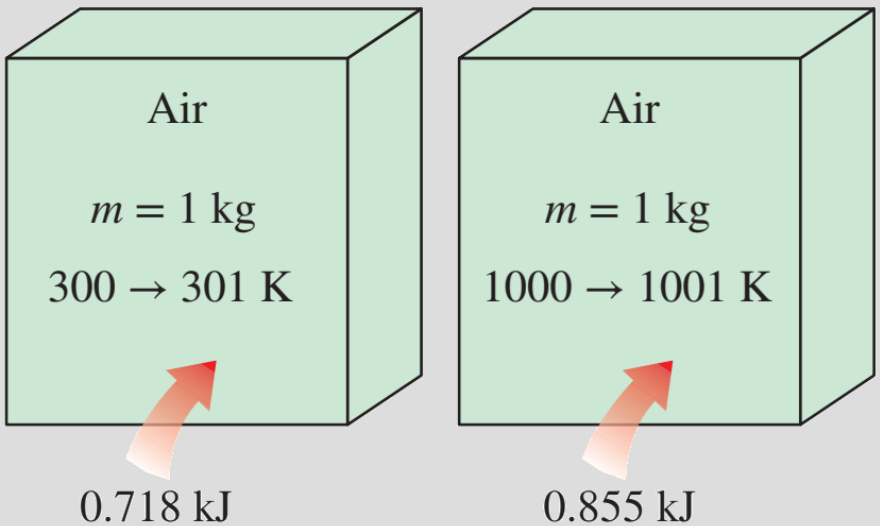
Left side represents net amount of energy transferred to the system
From def. of C_v , this energy must be equal to $C_v dT$, Thus

$$c_\upsilon dT = du \qquad \text{at constant volume} \qquad \textbf{or} \qquad c_\upsilon = \left(\frac{\partial u}{\partial T}\right)_\upsilon$$

Similarly, an expression for specific heat at constant pressure C_p

$$c_p = \left(\frac{\partial u}{\partial T}\right)_p$$

- The equations in the figure are valid for any substance undergoing any process.
- c_v and c_p are properties.
- c_v is related to the changes in *internal energy* and c_p to the changes in *enthalpy*.
- A common unit for specific heats is kJ/kg·°C or kJ/kg·K. **Are these units identical?**



The specific heat of a substance changes with temperature.

True or False?
c_p is always greater than c_v.

$$c_v = \left(\frac{\partial u}{\partial T}\right)_v$$

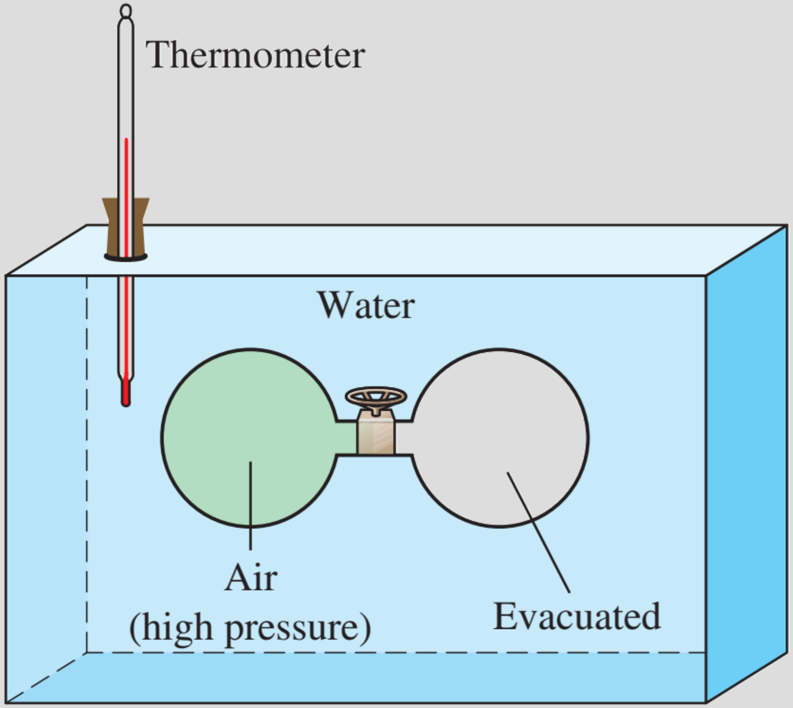
= the change in internal energy with temperature at constant volume

Formal definitions of c_v and c_p.

$$c_p = \left(\frac{\partial h}{\partial T}\right)_p$$

= the change in enthalpy with temperature at constant pressure

INTERNAL ENERGY, ENTHALPY, AND SPECIFIC HEATS OF IDEAL GASES



$$\left.\begin{aligned} h &= u + Pv \\ Pv &= RT \end{aligned}\right\} \quad h = u + RT$$

$$u = u(T) \qquad h = h(T)$$

$$du = c_v(T)\,dT \qquad dh = c_p(T)\,dT$$

$$\Delta u = u_2 - u_1 = \int_1^2 c_v(T)\,dT$$

$$\Delta h = h_2 - h_1 = \int_1^2 c_p(T)\,dT$$

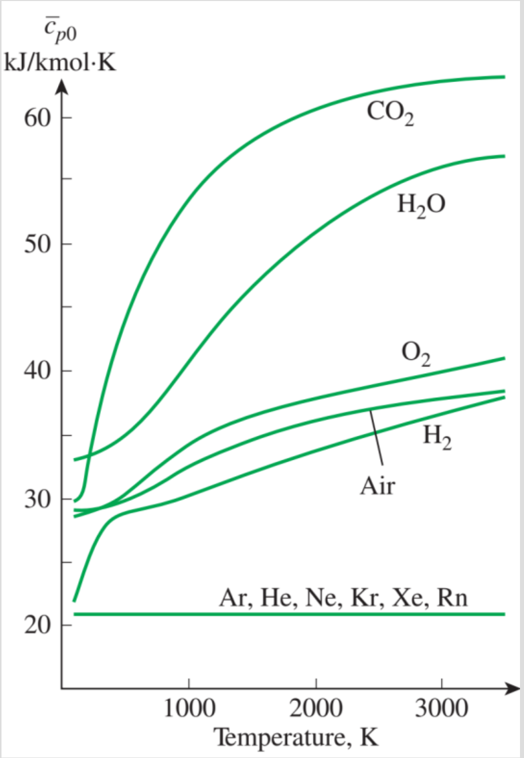
Joule showed using this experimental apparatus that $u=u(T)$

$$\boxed{\begin{aligned} u &= u(T) \\ h &= h(T) \\ c_v &= c_v(T) \\ c_p &= c_p(T) \end{aligned}}$$

For ideal gases, u, h, c_v, and c_p vary with temperature only.

Internal energy and enthalpy change of an ideal gas

- At low pressures, all real gases approach ideal-gas behavior, and therefore their specific heats depend on temperature only.
- The specific heats of real gases at low pressures are called *ideal-gas specific heats*, or *zero-pressure specific heats*, and are often denoted c_{p0} and c_{v0}.
- The *u* and *h* data are given in kJ/kg for air in table A – 17 and usually in kJ/kmol for other gases.



- *u* and *h* data for a number of gases have been tabulated.
- These tables are obtained by choosing an arbitrary reference point and performing the integrations by treating state 1 as the reference state.

AIR

T, K	*u*, kJ/kg	*h*, kJ/kg
0	0	0
.	.	.
.	.	.
300	214.07	300.19
310	221.25	310.24
.	.	.
.	.	.

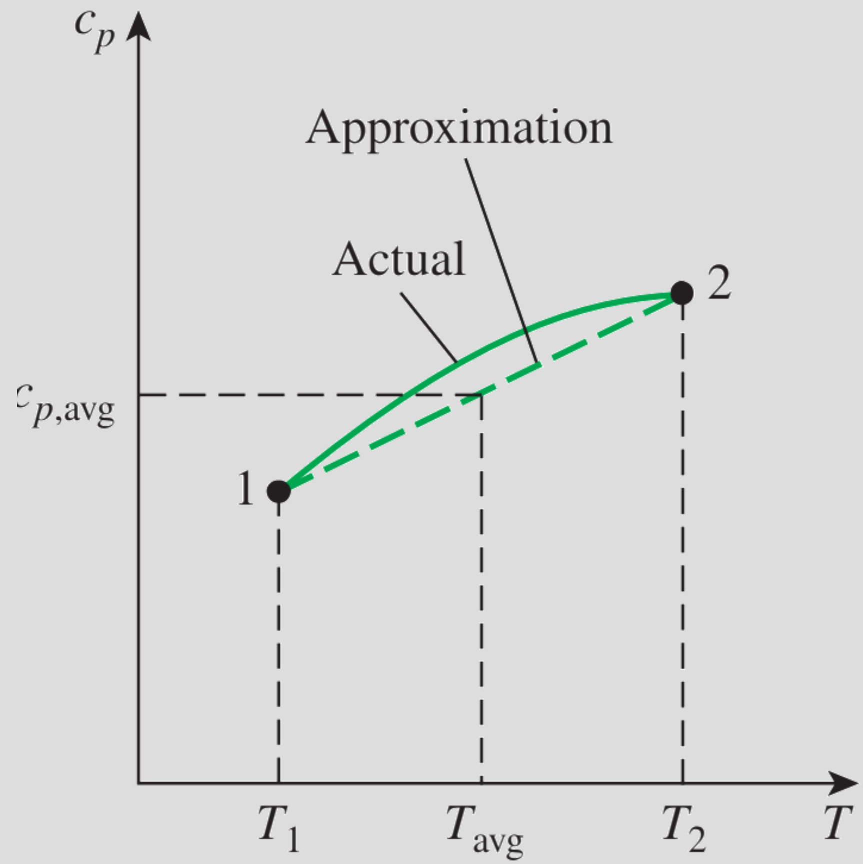
In the preparation of ideal-gas tables, 0 K is chosen as the reference temperature.

Internal energy and enthalpy change when specific heat is taken constant at an average value

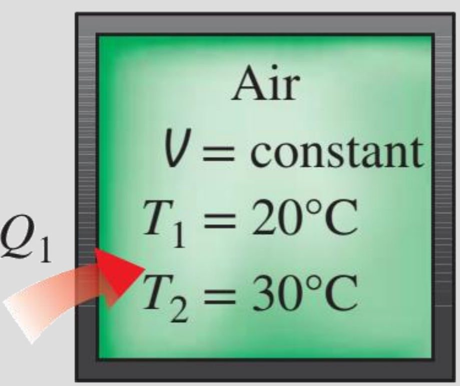
$$u_2 - u_1 = c_{v,\mathrm{avg}}(T_2 - T_1)$$

$$h_2 - h_1 = c_{p,\mathrm{avg}}(T_2 - T_1)$$

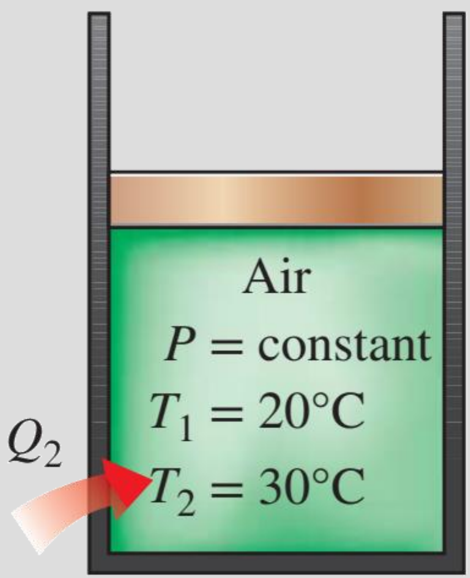
(kJ/kg)



For small temperature intervals, the specific heats may be assumed to vary linearly with temperature.



$$\Delta u = c_v \Delta T$$
$$= 7.18 \text{ kJ/kg}$$



$$\Delta u = c_v \Delta T$$
$$= 7.18 \text{ kJ/kg}$$

The relation $\Delta u = c_v \Delta T$ is valid for *any* kind of process, constant-volume or not.

Three ways of calculating *Δu and Δh*

1. By using the tabulated *u* and *h* data. This is the easiest and **most accurate** way when tables are readily available.
2. By using the c_v or c_p relations (Table A-2c) as a function of temperature and performing the integrations. This is very inconvenient for hand calculations but quite desirable for computerized calculations. The results obtained are **very accurate**.
3. By using average specific heats. This is very simple and certainly very convenient when property tables are not available. The results obtained are **reasonably accurate** if the temperature interval is not very large.

$$\Delta u = u_2 - u_1 \text{ (table)}$$

$$\Delta u = \int_1^2 c_v(T)\, dT$$

$$\Delta u \cong c_{v,\text{avg}}\, \Delta T$$

Three ways of calculating Δ*u*.

Problem: An insulated rigid tank is divided into two equal parts by a partition. Initially, one part contains 4 kg of an ideal gas at 800 kPa and 50°C, and the other part is evacuated. The partition is now removed, and the gas expands into the entire tank. Determine the final temperature and pressure in the tank.

$$\underbrace{E_{in} - E_{out}}_{\substack{\text{Net energy transfer}\\ \text{by heat, work, and mass}}} = \underbrace{\Delta E_{\text{system}}}_{\substack{\text{Change in internal, kinetic,}\\ \text{potential, etc. energies}}}$$

$$0 = \Delta U = m(u_2 - u_1)$$

$$u_2 = u_1$$

$$T_2 = T_1 = \mathbf{50°C}$$

$$\frac{P_1 V_1}{T_1} = \frac{P_2 V_2}{T_2} \longrightarrow P_2 = \frac{V_1}{V_2} P_1 = \frac{1}{2}(800 \text{ kPa}) = \mathbf{400 \text{ kPa}}$$

Problem: A 4-m × 5-m × 6-m room is to be heated by a baseboard resistance heater. It is desired that the resistance heater be able to raise the air temperature in the room from 5 to 25°C within 11 min. Assuming no heat losses from the room and an atmospheric pressure of 100 kPa, determine the required power of the resistance heater. Assume constant specific heats at room temperature.

$$\underbrace{E_{in} - E_{out}}_{\substack{\text{Net energy transfer}\\ \text{by heat, work, and mass}}} = \underbrace{\Delta E_{system}}_{\substack{\text{Change in internal, kinetic,}\\ \text{potential, etc. energies}}}$$

$$W_{e,in} = \Delta U \cong mc_{v,avg}(T_2 - T_1) \quad (\text{since } Q = \text{KE} = \text{PE} = 0)$$

$$\dot{W}_{e,in}\Delta t = mc_{v,avg}(T_2 - T_1)$$

$$V = 4 \times 5 \times 6 = 120 \text{ m}^3$$

$$m = \frac{P_1 V}{RT_1} = \frac{(100 \text{ kPa})(120 \text{ m}^3)}{(0.287 \text{ kPa}\cdot\text{m}^3/\text{kg}\cdot\text{K})(278 \text{ K})} = 150.6 \text{ kg}$$

$$\dot{W}_{e,in} = \frac{(150.6 \text{ kg})(0.718 \text{ kJ/kg}\cdot{}^\circ\text{C})(25-5)^\circ\text{C}}{11 \times 60 \text{ s}} = \mathbf{3.28\ kW}$$

Homework Activity:
Example 4-4,
Example 4-5,
and
Example 4-6.