Engineering Mechanics

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Research Interest: Photovoltaic systems, Power & Control, Sliding mode control (SMC)

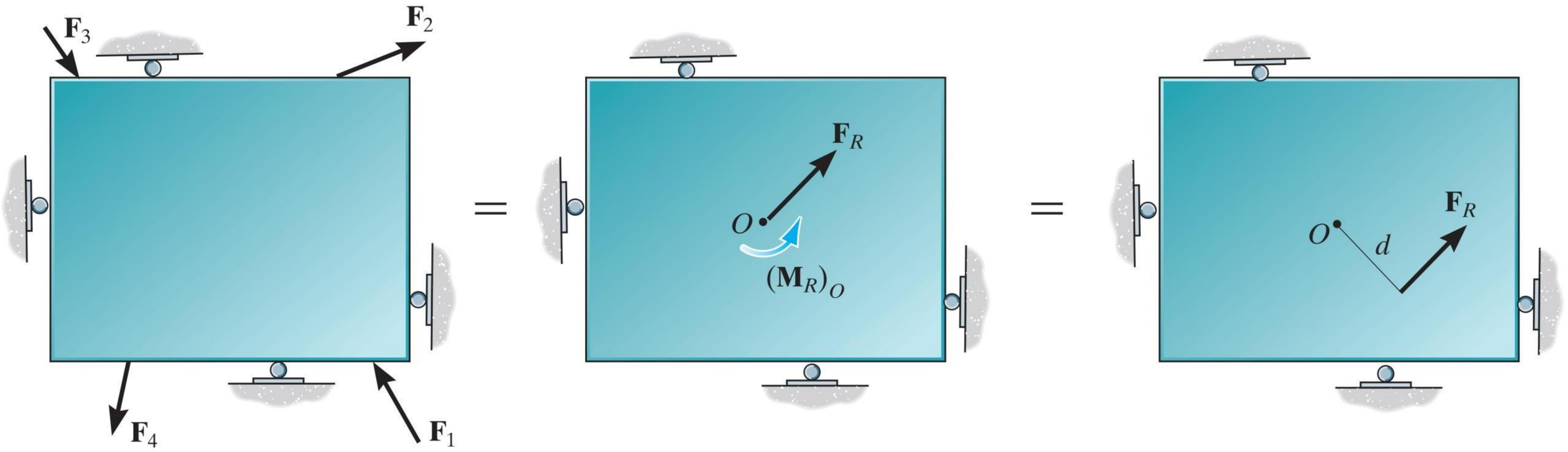
Contents (Section 4.9)

- Recap
- Reduction of Simple Distributed Loading

RECAP

Further Simplification of a Force and Couple System

Coplanar Force System.

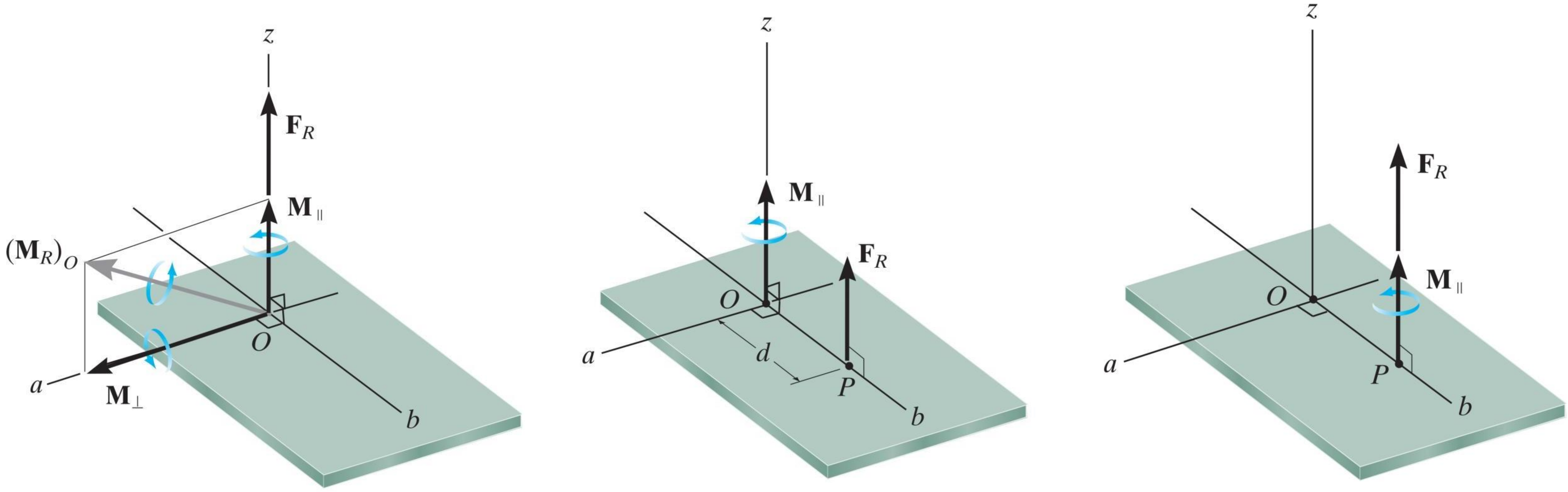


(a) (b) (c)

$$(M_R)_O = F_R d = \Sigma M_O \text{ or } d = (M_R)_O / F_R.$$

Further Simplification of a Force and Couple System

Reduction to a Wrench.



4.9 Reduction of Simple Distributed Loading

Reduction of Simple Distributed Loading

- Large surface area of a body may be subjected to distributed loadings
- For example, the pressure of the wind on the face of a sign, the pressure of water within a tank, or the weight of sand on the floor of a storage container, are all *distributed loadings*.
- Loadings on the surface is defined as pressure
- Pressure is measured in Pascal (Pa): $\underline{1\ \text{Pa} = 1\text{N/m}^2 = \text{lb/ft}^2}$

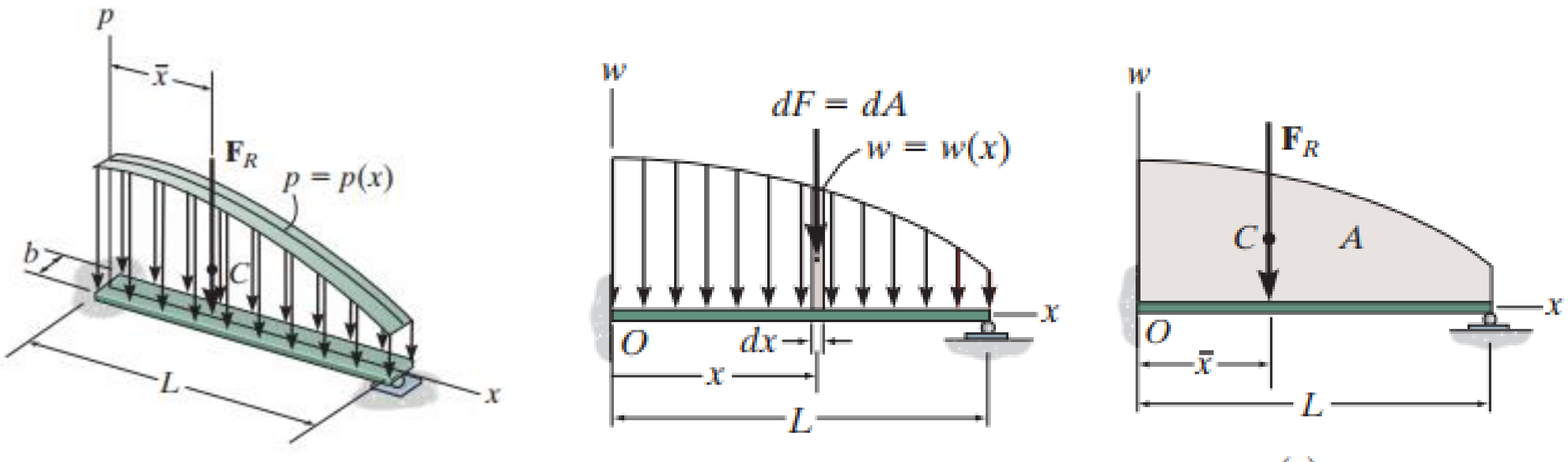
Uniform Loading Along a Single Axis

- Most common type of distributed loading is uniform along a single axis

Reduction of Simple Distributed Loading

Uniform Loading Along a Single Axis

- For example, consider the beam (or plate) in Fig. 4–48*a* that has a **constant width** and is subjected to a pressure loading that varies only along the *x* axis.
- This loading can be described by the function <u>$p = p(x)$ N/m^2</u> . It contains only one variable *x*, and for this reason, we can also represent it as a ***coplanar distributed load***.
- To do so, we multiply the loading function by the width *b* m of the beam, so that <u>$w(x) = p(x)b$ N/m^2</u> , Fig. 4-48*b*.
- Using the methods of Sec. 4.8, we can replace this coplanar parallel force system with a single equivalent resultant force acting at a specific location on the beam, Fig. 4–48*c*.



(a) (b) (c)

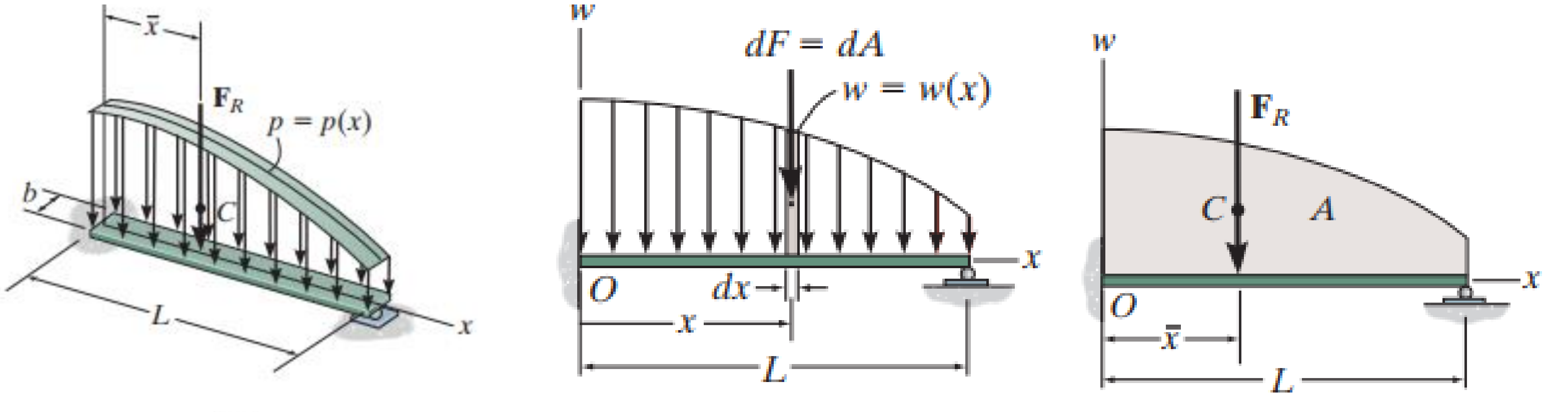
Reduction of Simple Distributed Loading

Magnitude of Resultant Force

- There is an infinite number of parallel forces $d\mathbf{F}$ acting on the beam, Fig. 4–48*b* and the $\mathbf{F}_R = \sum \mathbf{F}$
- Since $d\mathbf{F}$ is acting on an element of length dx, and $w(x)$ is a force per unit length, then $dF = w(x)\,dx = dA$.
- In other words, the magnitude of $d\mathbf{F}$ is determined from the colored differential *area dA* under the loading curve. For the entire length L

$$+\downarrow F_R = \Sigma F; \qquad \boxed{F_R = \int_L w(x)\,dx = \int_A dA = A}$$

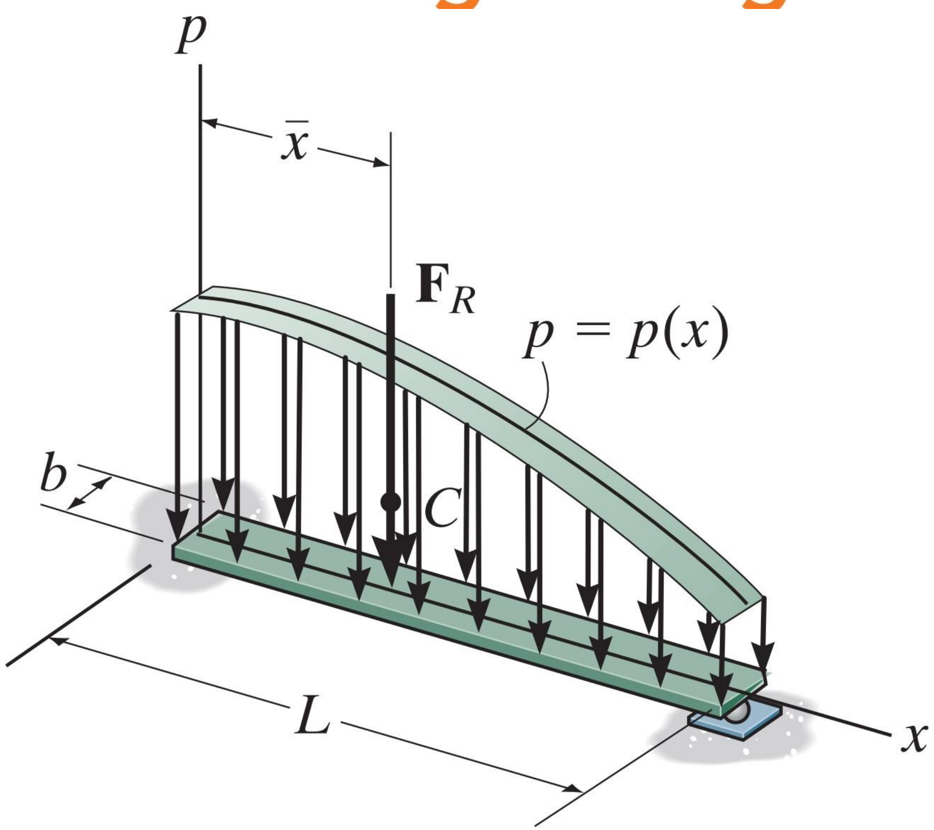
Therefore, the magnitude of the resultant force is equal to the total area A under the loading diagram, Fig. 4–48c.



(a) (b) (c)

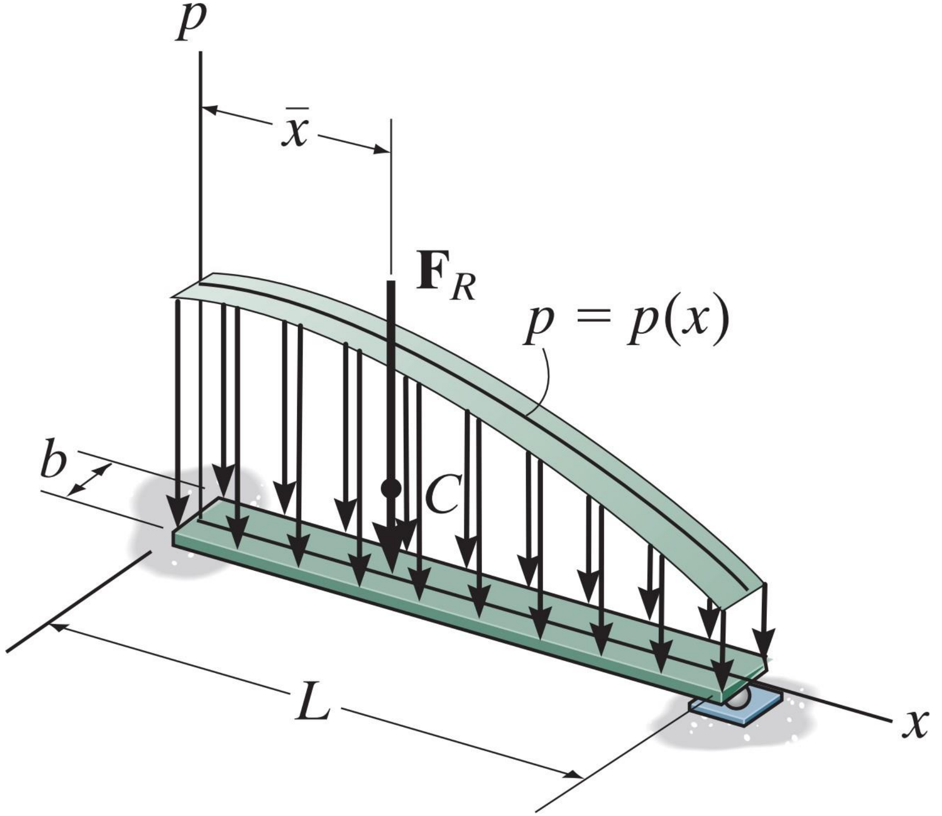
Reduction of Simple Distributed Loading

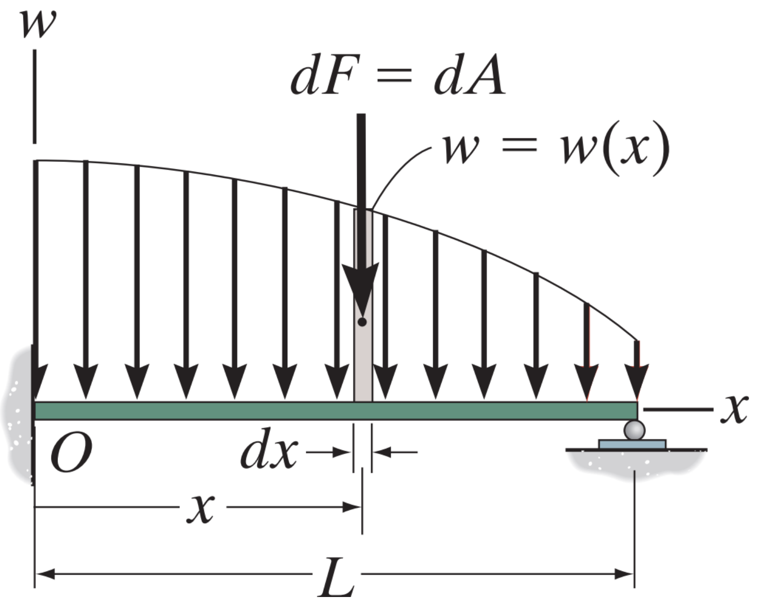
Loading Along a Single Axis.



Reduction of Simple Distributed Loading

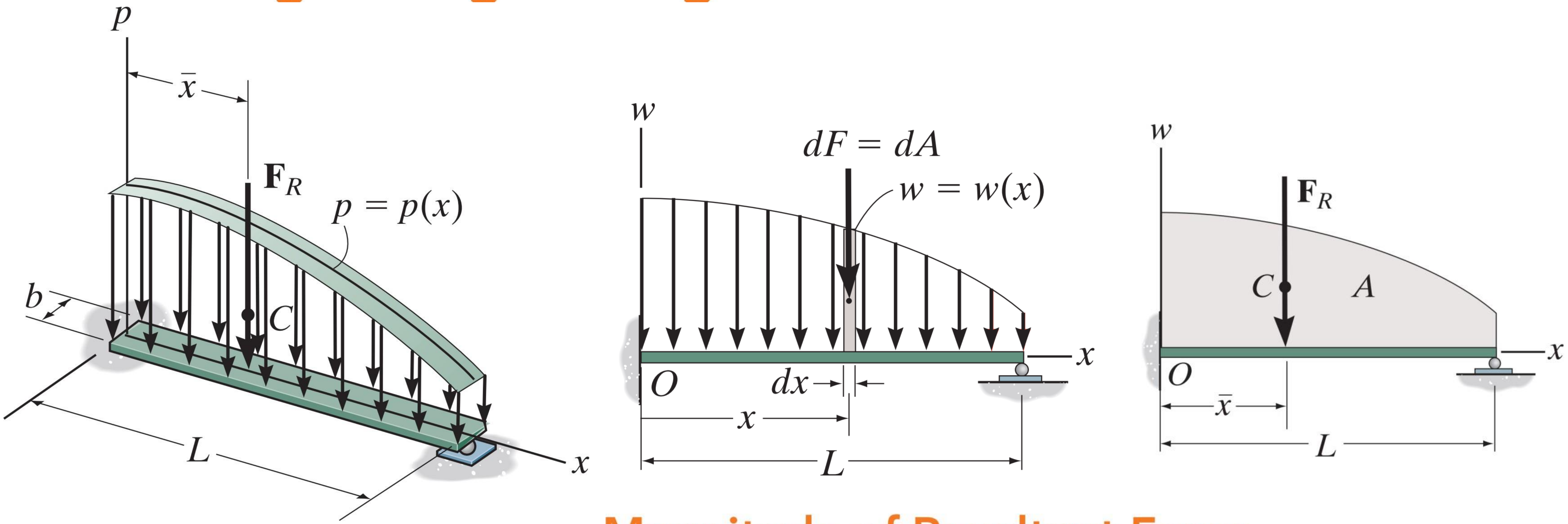
Loading Along a Single Axis.





Reduction of Simple Distributed Loading

Loading Along a Single Axis.



Magnitude of Resultant Force.

$$+\downarrow F_R = \Sigma F; \qquad \boxed{F_R = \int_L w(x)\,dx = \int_A dA = A}$$

Reduction of Simple Distributed Loading

Location of Resultant Force

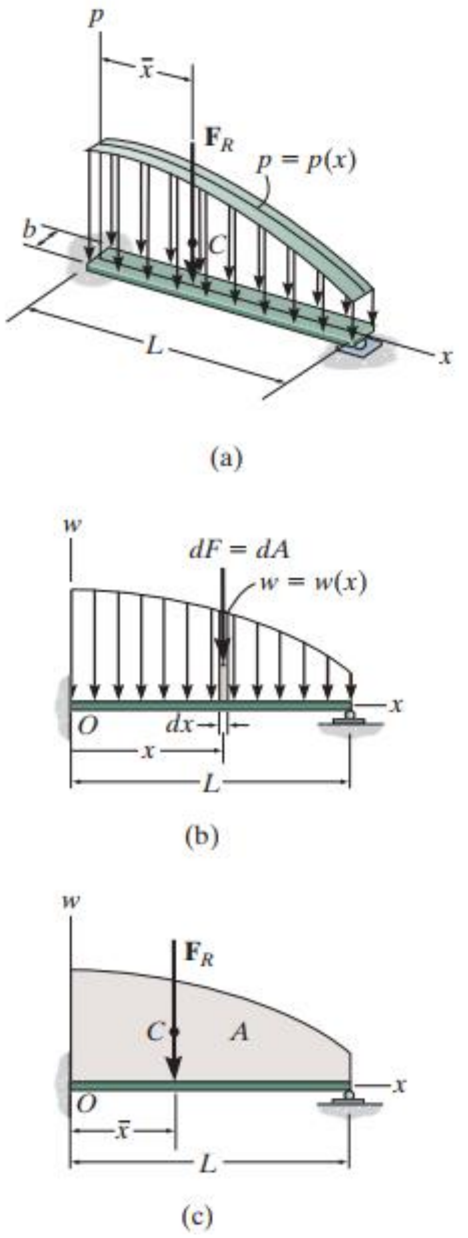
- *Apply* $M_{Ro} = \sum M_O$
- $d\mathbf{F}$ produces a moment of $xdF = x\,w(x)\,dx$ about O
- For the entire plate, Fig. 4–48*c*,

$$M_{Ro} = \Sigma M_O \qquad \bar{x}F_R = \int_L x w(x) dx$$

- Solving for

$$\bar{x} = \frac{\int_L x w(x) dx}{\int_L w(x) dx} = \frac{\int_A x dA}{\int_A dA}$$

This coordinate $\bar{x}$ locates the geometric center or *centroid* of the *area* under the distributed loading.



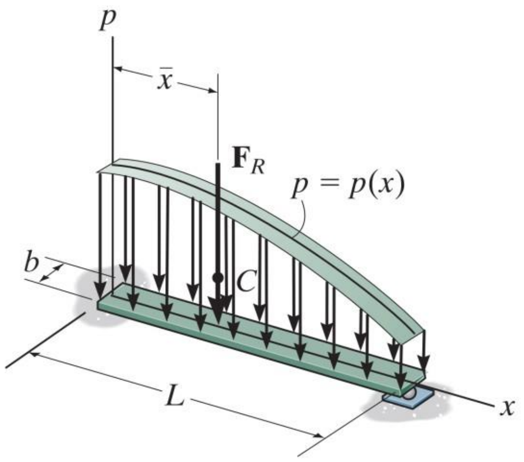
Reduction of Simple Distributed Loading

Location of Resultant Force.

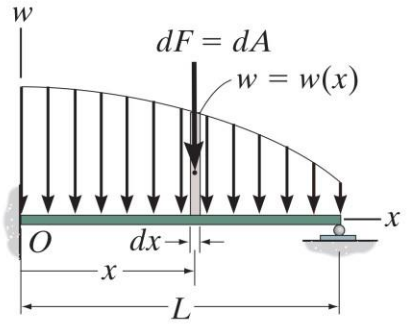
$$\circlearrowleft + (M_R)_O = \Sigma M_O; \qquad -\bar{x}F_R = -\int_L x w(x)\, dx$$

Solving for $\bar{x}$, using Eq. 4–19, we have

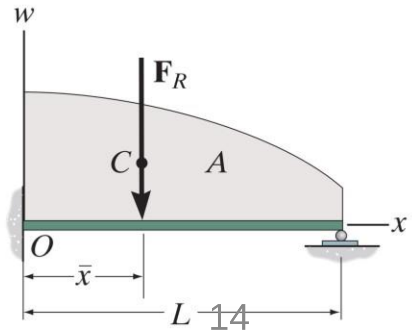
$$\boxed{\bar{x} = \frac{\displaystyle\int_L x w(x)\, dx}{\displaystyle\int_L w(x)\, dx} = \frac{\displaystyle\int_A x\, dA}{\displaystyle\int_A dA}}$$



(a)



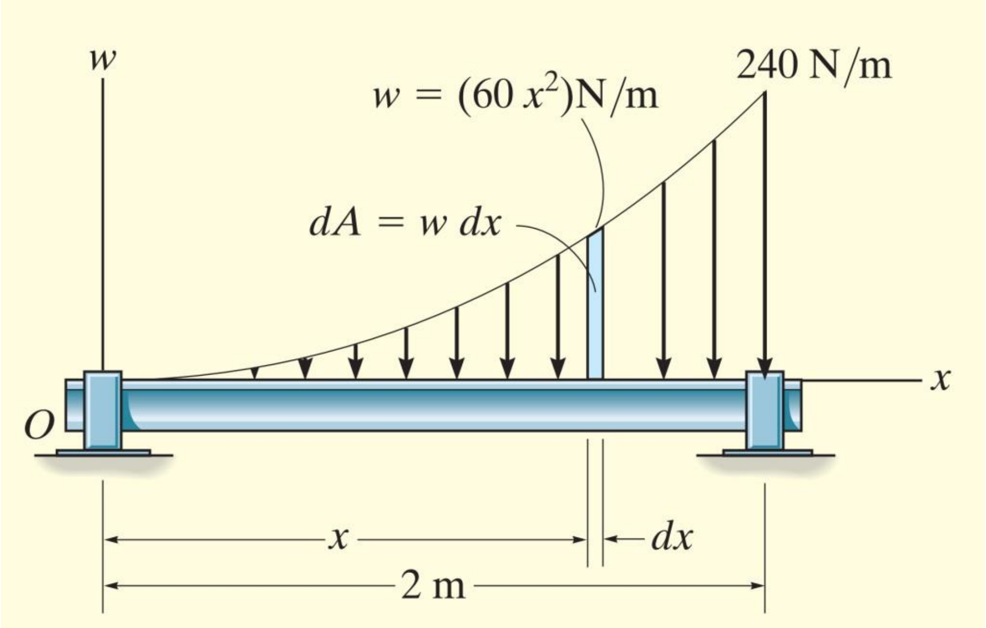
(b)



(c)

Example

Determine the magnitude and location of the equivalent resultant force acting on the shaft

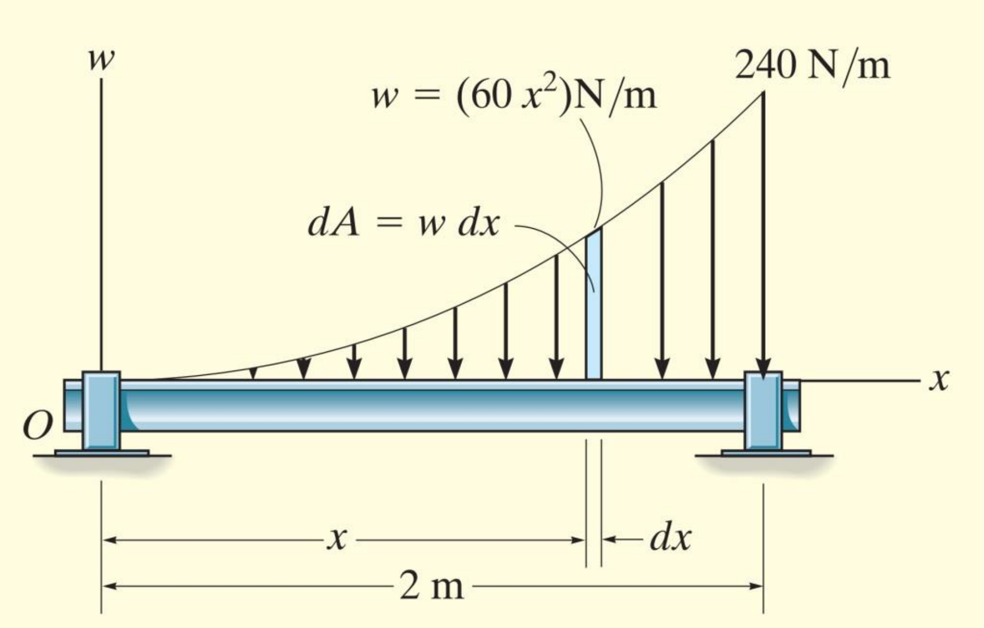


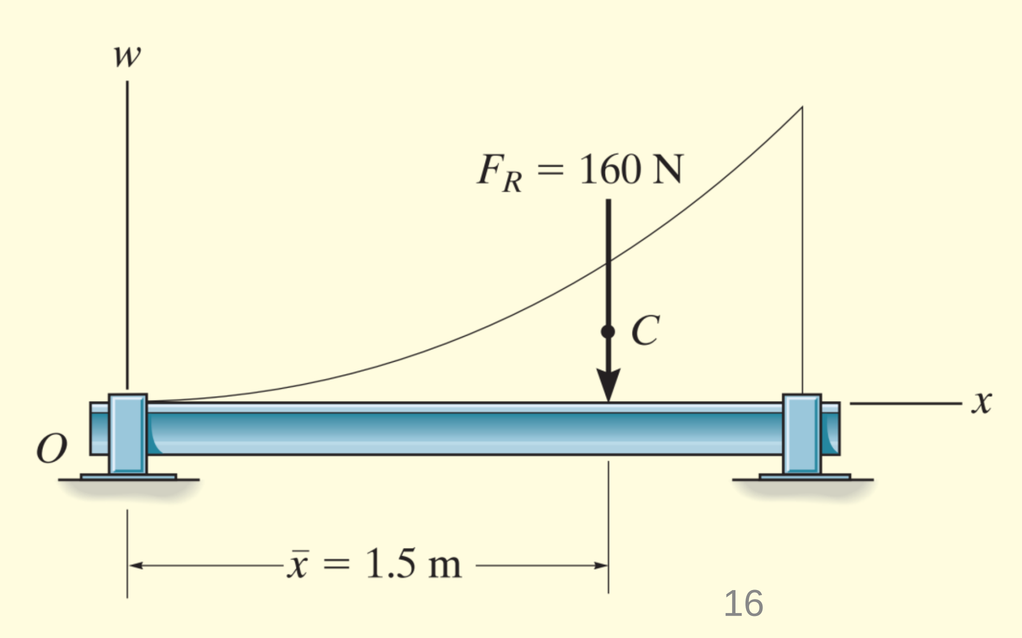
The differential element has an area $dA = w\,dx = 60x^2\,dx$. Applying Eq. 4–19,

$$+\downarrow F_R = \Sigma F;$$

$$F_R = \int_A dA = \int_0^{2\text{ m}} 60x^2\,dx = 60\left(\frac{x^3}{3}\right)\Bigg|_0^{2\text{ m}} = 60\left(\frac{2^3}{3} - \frac{0^3}{3}\right)$$

$$= 160\text{ N}$$ *Ans.*





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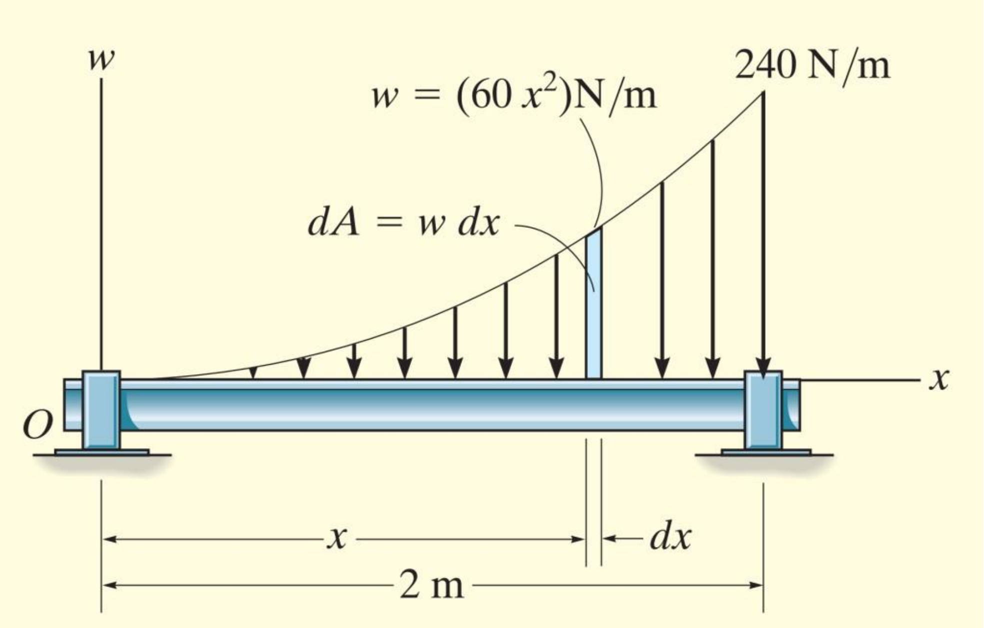
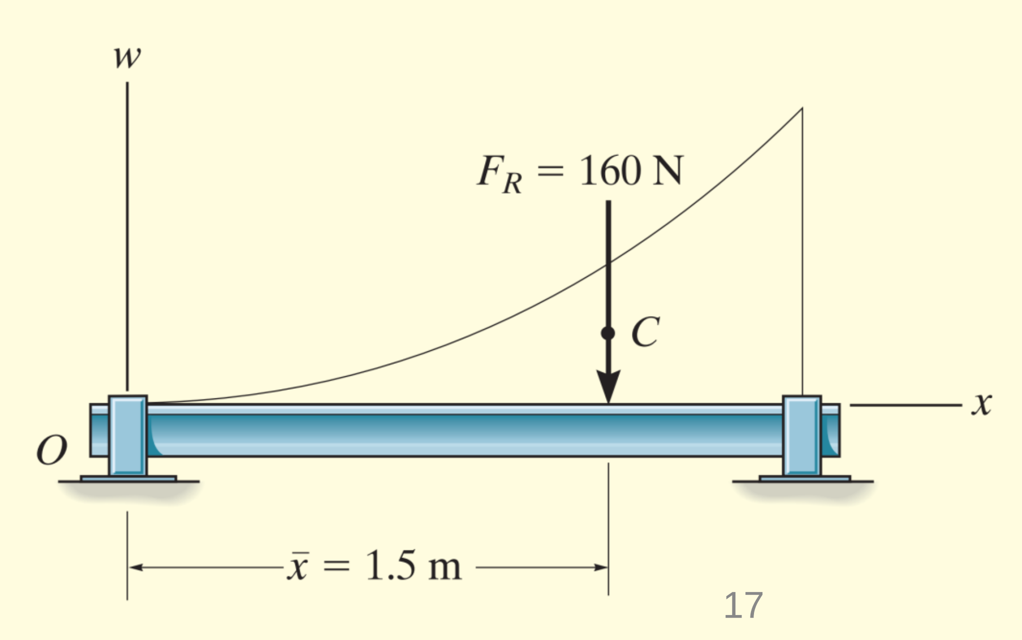
$$= 160\text{ N} \qquad \textit{Ans.}$$

The location $\bar{x}$ of $\mathbf{F}_R$ measured from O, Fig. 4–49*b*, is determined from Eq. 4–20.

$$\bar{x} = \frac{\int_A x\,dA}{\int_A dA} = \frac{\int_0^{2\text{ m}} x(60x^2)\,dx}{160\text{ N}} = \frac{60\left(\frac{x^4}{4}\right)\Big|_0^{2\text{ m}}}{160\text{ N}} = \frac{60\left(\frac{2^4}{4} - \frac{0^4}{4}\right)}{160\text{ N}}$$

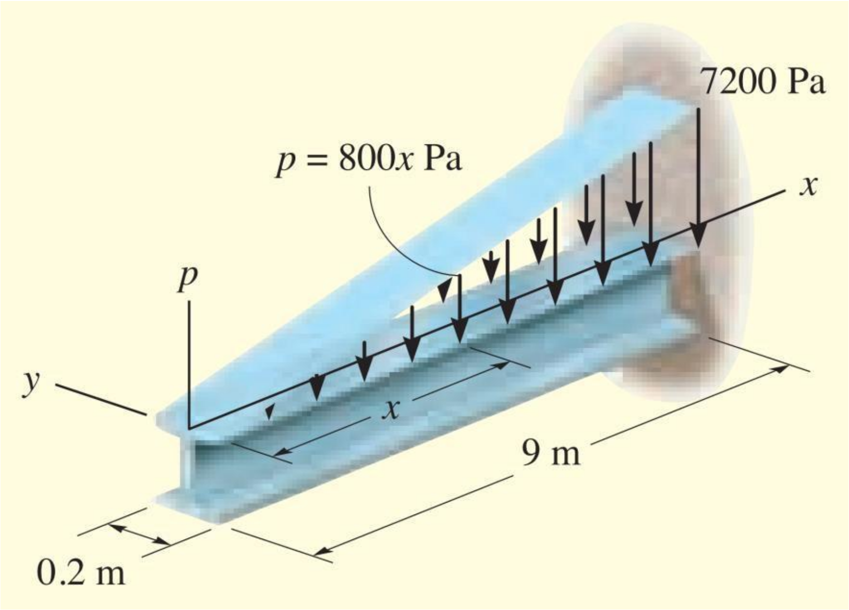
$$= 1.5\text{ m} \qquad \textit{Ans.}$$





Example

A distributed loading of p = (800x) Pa acts over the top surface of the beam. Determine the magnitude and location of the equivalent resultant force.



Since the loading intensity is uniform along the width of the beam (the y axis), the loading can be viewed in two dimensions as shown in Fig. 4–50b. Here

$$w = (800x \text{ N/m}^2)(0.2 \text{ m})$$
$$= (160x) \text{ N/m}$$

At $x = 9$ m, note that $w = 1440$ N/m. Although we may again apply Eqs. 4–19 and 4–20 as in the previous example, it is simpler to use the table on the inside back cover.

The magnitude of the resultant force is equivalent to the area of the triangle.

$$F_R = \tfrac{1}{2}(9 \text{ m})(1440 \text{ N/m}) = 6480 \text{ N} = 6.48 \text{ kN} \qquad \textit{Ans.}$$

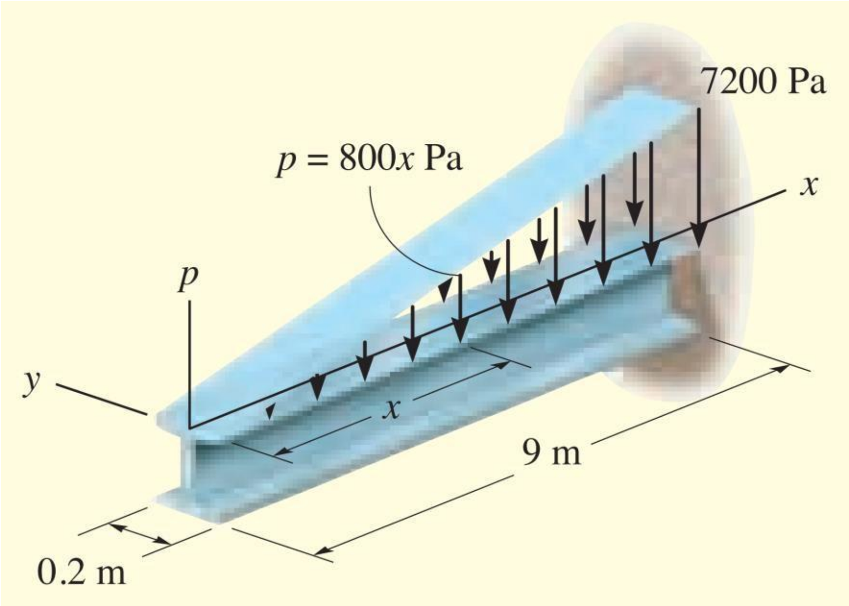
The line of action of $\mathbf{F}_R$ passes through the *centroid C* of this triangle. Hence,

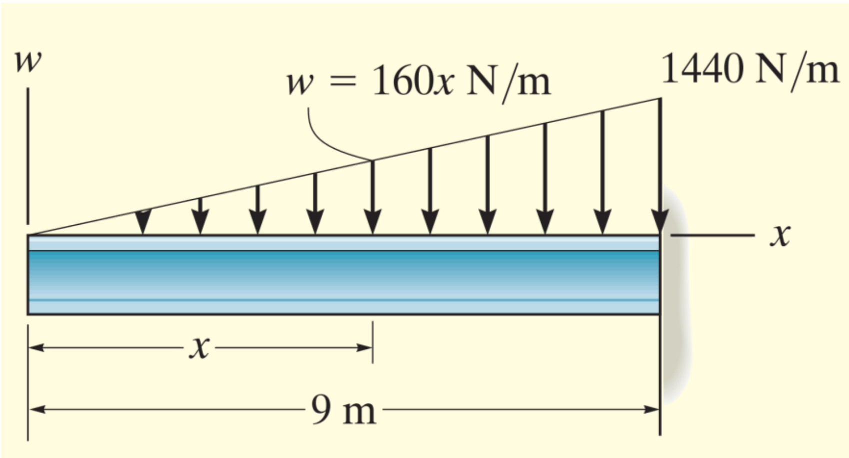
$$\bar{x} = 9 \text{ m} - \tfrac{1}{3}(9 \text{ m}) = 6 \text{ m} \qquad \textit{Ans.}$$

The results are shown in Fig. 4–50c.

NOTE: We may also view the resultant $\mathbf{F}_R$ as *acting* through the *centroid* of the *volume* of the loading diagram $p = p(x)$ in Fig. 4–50a. Hence $\mathbf{F}_R$ intersects the x–y plane at the point (6 m, 0). Furthermore, the magnitude of $\mathbf{F}_R$ is equal to the volume under the loading diagram; i.e.,

$$F_R = V = \tfrac{1}{2}(7200 \text{ N/m}^2)(9 \text{ m})(0.2 \text{ m}) = 6.48 \text{ kN} \qquad \textit{Ans.}$$





Home Assignment

- Example 4.23.