Normal Distribution

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Normal Distribution

- Continuous Distribution
- Bell shaped curve
- Uni-model single peak at the center, symmetrical
- Normal distribution has two parameters mean and variance.
- The total area under the curve is one.
- The normal curve approaches, but never touches the x-axis.

Normal Distribution

The p.d.f of normal distribution is

$$f(x)=\frac{1}{\sigma\sqrt{2\pi}}e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2} \quad \text{for} \quad -\infty \leq x \leq \infty$$

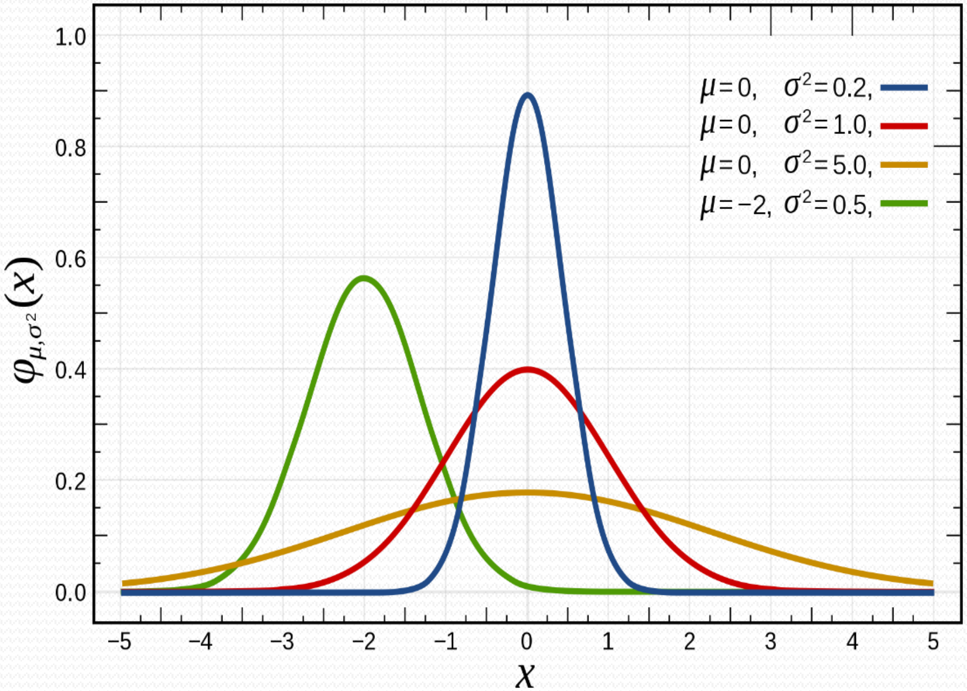
Here

Mean=μ

Standard deviation= σ

e=2.7281

$\pi = 3.14$



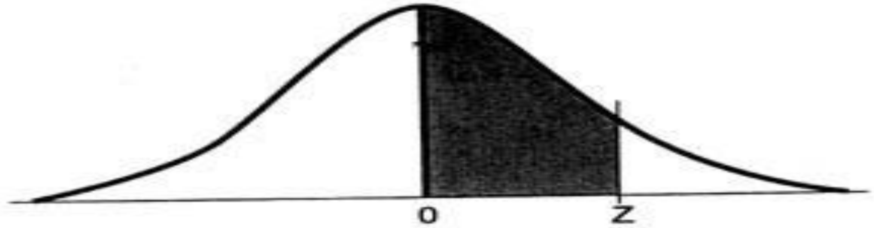
Standard Normal Distribution

A distribution of mean zero and standard deviation one is called standard normal distribution. The p.d.f of standard normal distribution is

f(z)=$\frac{1}{1.\sqrt{2\pi}}\, e^{-\frac{1}{2}\left(\frac{x-0}{1}\right)^2}$ for $-\infty \le z \le \infty$

f(z)=$\frac{1}{\sqrt{2\pi}}\, e^{-\frac{1}{2}(z)^2}$ for $-\infty \le z \le \infty$

Here z=$\frac{x-\mu}{\sigma}$ and E(z)=0, and S.D(z)=1



This table presents the area between the mean and the Z score . When Z=1.96, the shaded area is 0.4750.

Areas Under the Standard Normal Curve

Z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.0000	0.0040	0.0080	0.0120	0.0160	0.0199	0.0239	0.0279	0.0319	0.0359
0.1	.0398	.0438	.0478	.0517	.0557	.0596	.0636	.0675	.0714	.0753
0.2	.0793	.0832	.0871	.0910	.0948	.0987	.1026	.1064	.1103	.1141
0.3	.1179	.1217	.1255	.1293	.1331	.1368	.1406	.1443	.1480	.1517
0.4	.1554	.1591	.1628	.1664	.1700	.1736	.1772	.1808	.1844	.1879
0.5	.1915	.1950	.1985	.2019	.2054	.2088	.2123	.2157	.2190	.2224
0.6	.2257	.2291	.2324	.2357	.2389	.2422	.2454	.2486	.2517	.2549
0.7	.2580	.2611	.2642	.2673	.2704	.2734	.2764	.2794	.2823	.2852
0.8	.2881	.2910	.2939	.2967	.2995	.3023	.3051	.3078	.3106	.3133
0.9	.3159	.3186	.3212	.3238	.3264	.3289	.3315	.3340	.3365	.3389
1.0	.3413	.3438	.3461	.3485	.3508	.3531	.3554	.3577	.3599	.3621
1.1	.3643	.3665	.3686	.3708	.3729	.3749	.3770	.3790	.3810	.3830
1.2	.3849	.3869	.3888	.3907	.3925	.3944	.3962	.3980	.3997	.4015
1.3	.4032	.4049	.4066	.4082	.4099	.4115	.4131	.4147	.4162	.4177
1.4	.4192	.4207	.4222	.4236	.4251	.4265	.4279	.4292	.4306	.4319
1.5	.4332	.4345	.4357	.4370	.4382	.4394	.4406	.4418	.4429	.4441
1.6	.4452	.4463	.4474	.4484	.4495	.4505	.4515	.4525	.4535	.4545
1.7	.4554	.4564	.4573	.4582	.4591	.4599	.4608	.4616	.4625	.4633
1.8	.4641	.4649	.4656	.4664	.4671	.4678	.4686	.4693	.4699	.4706
1.9	.4713	.4719	.4726	.4732	.4738	.4744	.4750	.4756	.4761	.4767
2.0	.4772	.4778	.4783	.4788	.4793	.4798	.4803	.4808	.4812	.4817
2.1	.4821	.4826	.4830	.4834	.4838	.4842	.4846	.4850	.4854	.4857
2.2	.4861	.4864	.4868	.4871	.4875	.4878	.4881	.4884	.4887	.4890
2.3	.4893	.4896	.4898	.4901	.4904	.4906	.4909	.4911	.4913	.4916
2.4	.4918	.4920	.4922	.4925	.4927	.4929	.4931	.4932	.4934	.4936
2.5	.4938	.4940	.4941	.4943	.4945	.4946	.4948	.4949	.4951	.4952
2.6	.4953	.4955	.4956	.4957	.4959	.4960	.4961	.4962	.4963	.4964
2.7	.4965	.4966	.4967	.4968	.4969	.4970	.4971	.4972	.4973	.4974
2.8	.4974	.4975	.4976	.4977	.4977	.4978	.4979	.4979	.4980	.4981
2.9	.4981	.4982	.4982	.4983	.4984	.4984	.4985	.4985	.4986	.4986
3.0	.4987	.4987	.4987	.4988	.4988	.4989	.4989	.4989	.4990	.4990
3.1	.4990	.4991	.4991	.4991	.4992	.4992	.4992	.4992	.4993	.4993
3.2	.4993	.4993	.4994	.4994	.4994	.4994	.4994	.4995	.4995	.4995
3.3	.4995	.4995	.4995	.4996	.4996	.4996	.4996	.4996	.4996	.4997

Normal Distribution

A random variable X is a normally distributed with μ=100 and σ^2=225, find the following probabilities

- $P(X\leq 92.5)$
- $P(X< 107.5)$
- $P(X\geq 124)$
- $P(112 \leq X \leq 128.5)$
- $P(91 \leq X \leq 127)$
- $P(X \geq 76)$

Normal Distribution

Let $Y = 5X + 10$ and X be normally distributed with a $\mu=10$ and $\sigma^2=25$, find the following probabilities

- $P(Y \leq 54.5)$
- $P(Y \geq 68)$
- $P(52 \leq Y \leq 67)$

Normal Distribution

The fill volume of an automated filling machine used for filling cans of carbonated beverage is normally distributed with a mean of 12.4 fluid ounces and a standard deviation of 0.1 fluid ounce.

- What is the probability a fill volume is less than 12.1 fluid ounces?
- If all cans less than 12.2 or greater than 12.5 ounces are scrapped, what proportion of cans is scrapped?
- Determine specification that are symmetric about the mean include 95% of all cans.

Normal Distribution

A Man goes by car to his office, and the route through the city centers takes him, on the average, 27 minutes with a standard deviation of 5 minutes. With the opening of a new ring road, the man can bypass the congestion of the city center, but the journey now takes, on the average, 29 minutes with the standard deviation of 2 minutes. Assuming that both journey times are normally distributed, determine which route is the better one in the man has (i) 28 minutes, and (ii) 32 minutes to reach his office for an appointment

Normal Distribution

In a normal Distribution, 31% of the items are under 45 and 8% are over 64. Find the mean and standard deviation of the distribution.

Normal Distribution

An athlete finds that in a high jump he can clear a height of 1.68m once in five attempts and a height of 1.52m nine times out of ten attempts. Assuming the height he can clear in various jumps from a normal distribution, estimate the mean and standard deviation of the distribution.

Normal Distribution

Assuming that the number of marks scored by a candidate is normally distributed, find the mean and the standard deviation, if the number of first class students(60% or more marks) is 25, the number of failed students(less than 30%marks) is 90 and the total number of candidates appearing for the examination is 450.

Normal Approximation to the Binomial Distribution

The binomial distribution can be approximated by a normal distribution, when n is sufficiently large and neither p nor q is close to zero. Before you apply a normal approximation you have to check the following conditions $np>5$ or $n(1-p)>5$.

Normal Approximation to the Binomial Distribution

The probability for a binomial random variable X to take the value x is

$$P(X = x) = \binom{n}{x} p^{x} q^{n-x} \quad where \qquad x = 0,1,2,..........n$$

The variable X has the mean=np and variance=npq.

The binomial random variable in term of z is

$$z = \frac{(x \pm 0.5) - np}{\sqrt{npq}}$$

Here E(z)=0, and S.D(z)=1

Continuity Correction

We are approximating a discrete distribution with a continuous one, and so we must use a continuity correction. **A** discrete value x becomes the interval from x-0.5 to x+0.5; and this sort of adjustment is called continuity correction.

To do this,

- Draw the diagram and shade the required area.
- Use a broken line for the "boundary" if the probability is an inequality>, greater than or <, less than and a solid line otherwise ≥, greater than or equal to or≤, less than or equal to
- Adjust any inequality by extending to the next "included" integer For example, >2 becomes ≥3, < 5 becomes ≤4
- The shaded area is now extended by half a unit

Normal Approximation to the Binomial Distribution

So, continuing, the process is still:

- Draw and label a diagram
- Shade required area
- Change raw score to standard (Z score):
- Look up z value In tables
- Adjust probability from table to required area

Normal Approximation to the Binomial Distribution

A pair of fair dice rolled 180 times. Use the normal approximation method to find the probability that a total of 7 occurs

- At least 25 times
- Between 33 and 34 times inclusive
- Less than 27 times

Normal Approximation to the Binomial Distribution

In a digital communication channel, assume that the number of bits received in error can be modeled by a binomial random variable, and assumed that a bit is received in error is 1×10^{-5} . if 16 million bits are transmitted,

- What is the probability that more than 150 errors occur?
- Find the median and mode of the distribution.

Normal Approximation to the Poisson Distribution

A telephone exchange receives, on average, 5 calls per minute. Find the probability that in a 20 minute period

- No more than 102 calls are received
- Exactly 90 calls are received