NUST School of Electrical Engineering and Computer Science

Faculty Member:_______________ Date: _______________

Semester:______________ Section: _______________

Department of Electrical Engineering

EE-397: Control Systems

LAB 9: Root Locus based Design

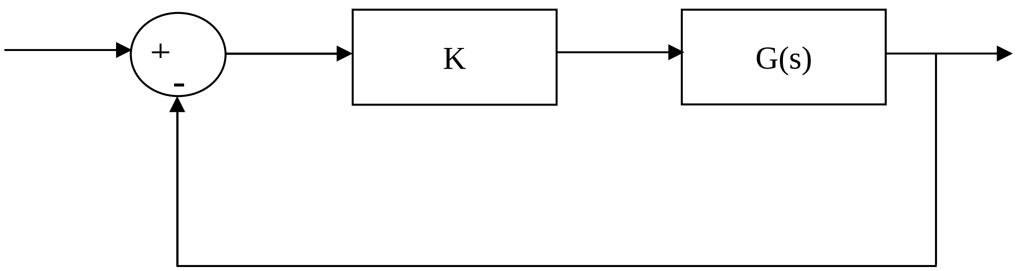
Student name	Reg. No.	Log book Marks / 5	Lab completion Marks / 5	Lab report Marks / 5	Total/15

1. Objectives

- Learn how to plot root locus in MATLAB
- Learn how to use MATLAB to design a simple controller using the root locus

2. Root Locus

Root locus is a plot of the closed loop poles of the system that shown in the figure below. The closed loop poles vary as the values of the gain K is changed. The root locus shows the plot of the closed loop poles as the gain K is varied between zero and positive infinity.



In the lectures, we have learnt how to sketch a root locus and also how to find exact points of interest e.g. the imaginary axis crossing, the break away/in points, the centroid of asymptotes. Plotting a rough sketch of the root locus is considerably simple; however, finding exact points on the root locus requires some mathematical calculations. In this handout, we will see that MATLAB can make this task easier.

3. Root Locus in MATLAB

In MATLAB, the root locus can be plotted with the command `rlocus()`.

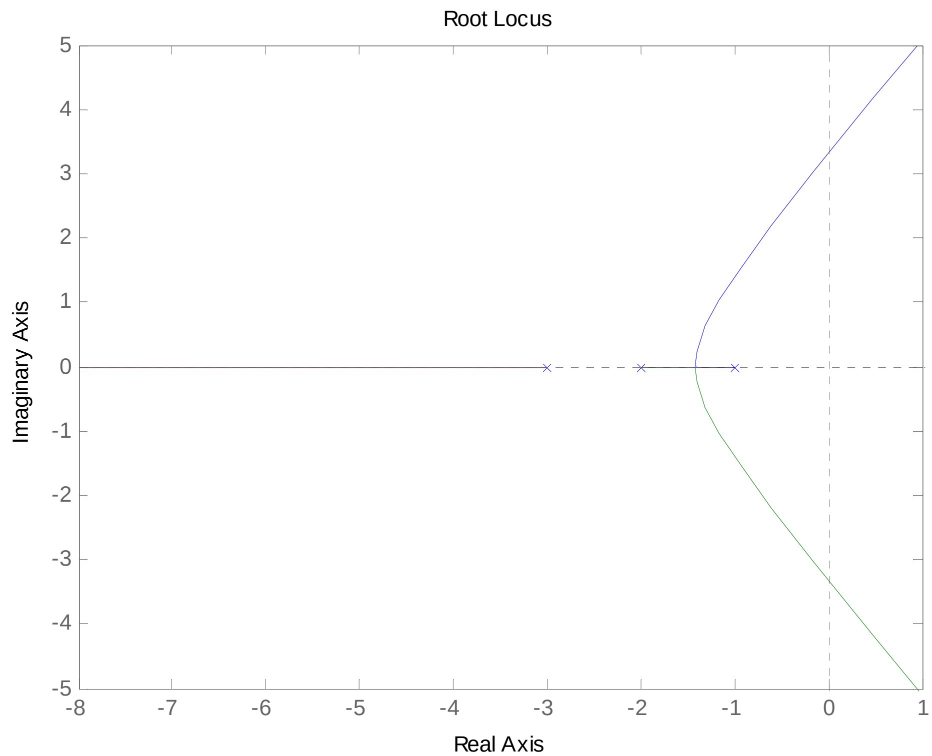
Consider the open loop system

$$G(s) = \frac{1}{(s+1)(s+2)(s+3)}$$

The root locus of the above system can be plotted in MATLAB using the following code:

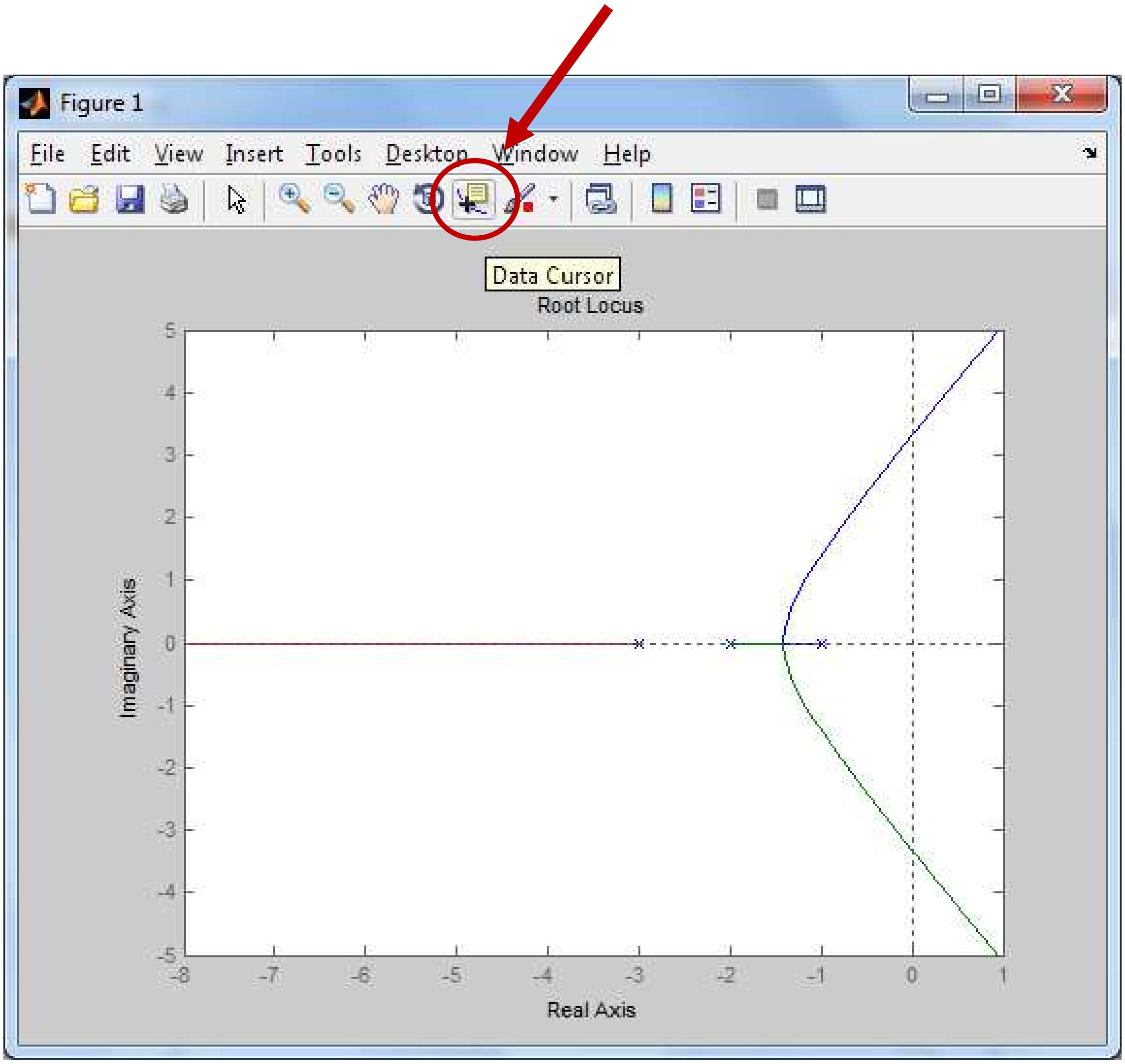
```
G=zpk([],[-1,-2,-3],1)
rlocus(G)
```

After executing the instructions, you should get the following plot



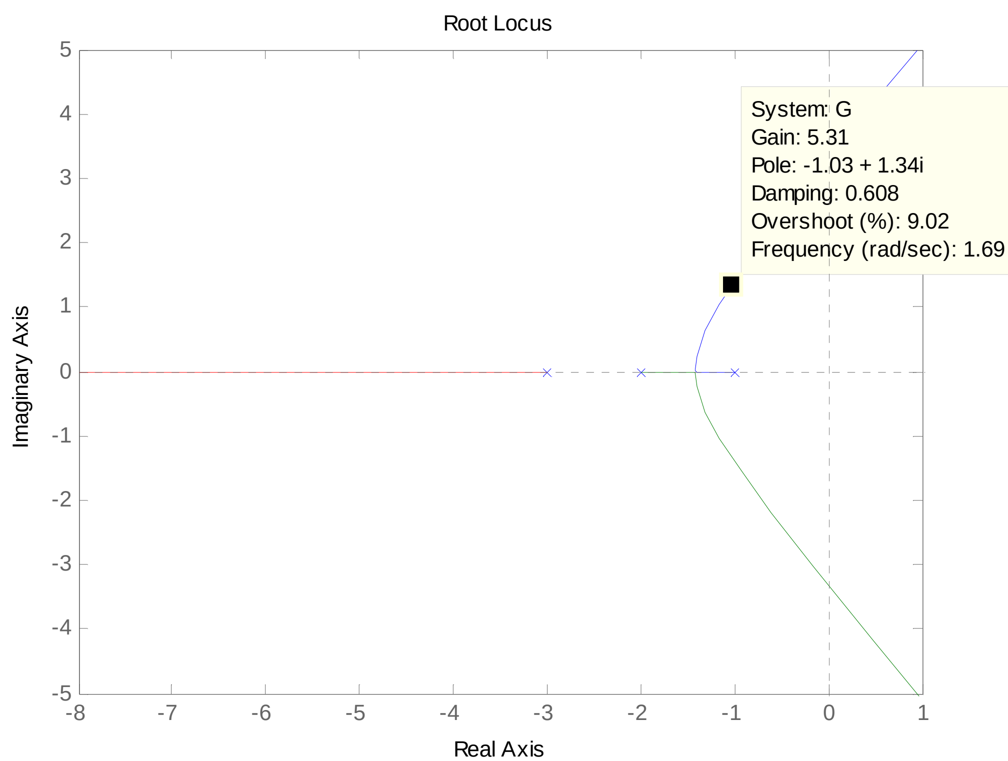
As you can see, plotting the root locus is very easy in MATLAB.

Now we would like to find some points exactly e.g. the imaginary axis crossing, the breakaway point, the centroid of asymptotes. This can be done with the help of the Data cursor tool, which is shown in the figure below.



Select the Data Cursor tool and click anywhere on the root locus. It should give the location of the closed loop pole, the gain K required to have that closed loop pole, and some other useful information.

The overshoot, damping ratio and natural frequency indicated with the data cursor are only w.r.t. to that specific pole, i.e. if the system only had that pole (or the pole pair in case of a complex pole). This overshoot is not for the whole closed loop system.



Once a data cursor is inserted, you can move it around by dragging it. If you cannot move the data cursor in fine steps, then right click on the data cursor, go to Interpolation and select Linear. Sometimes you get a different right click menu. In that case, go to Selection Style and select Mouse Position.

As an exercise move the data cursor, to find

i. The gain at which you have imaginary axis crossing
ii. The value of the pole and the gain at the breakaway point

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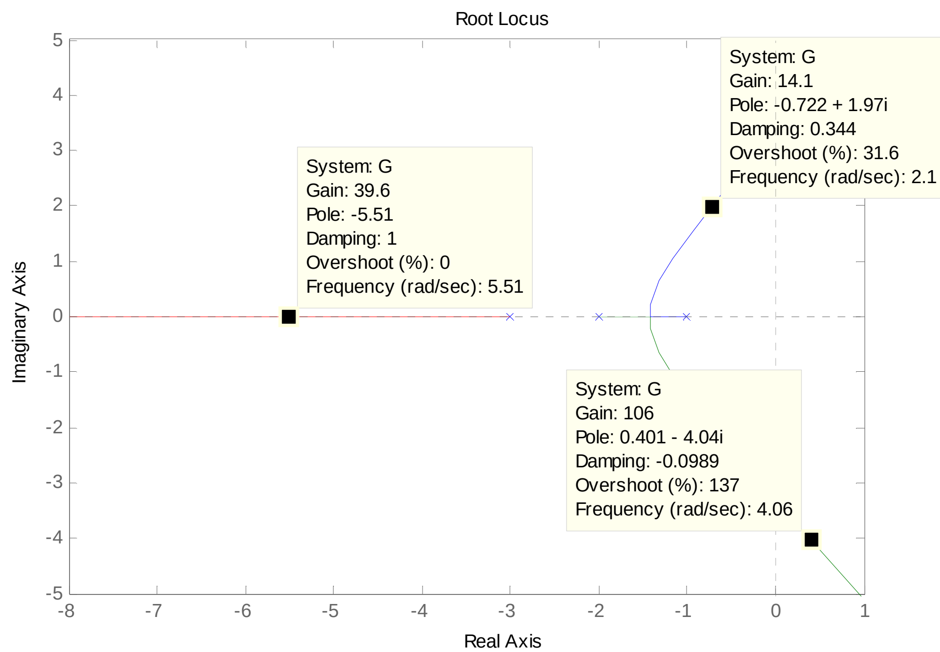
You can insert a separate data cursor on each branch of the root locus. Insert a data cursor on all branches of the root locus.



Move the data cursor for one of the complex poles so that you have an overshoot of 40%. Find the value of K required to get that closed loop pole.

Move the rest of the data cursors such that they are showing the same value of the gain K. This will allow us to see the location of all the closed loop poles of the system for a specific value of the gain.

For the value of gain that is required for 40% overshoot, can we approximate the systems to a second order system with no zeros?

Once you have decided a value for the gain K, you can find the closed loop transfer function with the following code

```
K=30
Gcl = feedback(series(K,G),1)
```

Exercise 1

Consider the open loop system

$$G(s) = \frac{s^2 + 4s + 4.25}{s^3 - s^2 - s + 15}$$

Plot the root locus for this system in MATLAB. Using the MATLAB root locus:

i. Find the values of the gain K for which the closed loop system will be stable.

ii. Design a controller gain K such that the closed system is stable and the dominant poles have an overshoot of 80%. Is a second order approximation valid for this value of K. Find the transfer function of the closed loop system.

Exercise 2

Consider the first order model of the DC motor speed vs voltage, which you have derived in your earlier labs. You get a first order model if you neglect the inductance, i.e. L=0. Using the root locus, choose a value of K such that your settling time is 1/20 secs. Find the transfer function of the closed loop system.

Exercise 3

Consider the first order model of the DC motor position vs voltage that you have derived in your earlier labs. Using the root locus, choose a value of K such that your closed loop system is stable and has a settling time of 1/5 secs. Find the transfer function of the closed loop system.

Exercise 4

Without using MATLAB, make a rough sketch of the root locus of the following transfer functions

$$G(s) = \frac{(s^2 - 2s + 5)}{(s + 1)(s + 2)(s + 3)}$$

$$G(s) = \frac{(s + 7)(s^2 - 2s + 5)}{(s + 1)(s + 2)(s + 3)(s^2 - 4s + 13)}$$

$$G(s) = \frac{(s + 2)}{(s + 1)(s^2 + 12 + 61)(s^2 - 4s + 13)}$$

Check your answers with the help of MATLAB