National University of Sciences & Technology
School of Electrical Engineering and Computer Science
Department of Basic Sciences

MATH-101: Calculus and Analytical Geometry (3+0): BEE2k20-ABC Fall 2020

Assignment 5	
CLO: 3 (Comprehend sequence, series and their convergence using miscellaneous tests)	
Maximum Marks: 10	Instructor: Dr. Naila Amir
Announcement Date: 25^{th} January 2021	Due Date: 31^{st} January 2021

Instructions:

- Understanding the question is part of the assignment and copying is not allowed.
- Express your answer in the most simplified form. Direct calculations using calculator are not allowed, you need to show the detail of your work to get the maximum marks.
- This is an individual assignment.
- Assignment must be handwritten and properly scanned in a single pdf file. This page must be part of every assignment.
- Assignment must be properly tagged and is required to be submitted on MS teams.
- Assignment is not acceptable after deadline.

Tasks: Attempt all questions.

Students Name	CMS Id.	Section
Muhammad Umer	**345834**	**BEE-12C**

Total Marks	**Marks Obtained**
10 Marks	

Question # 1: [10 marks]

a) Determine the values of x for which the power series:

$$\sum_{n=1}^{\infty} \frac{2^n(4x-8)^n}{n},$$

1) converges absolutely,
2) converges conditionally,
3) diverges.

b) Using part (a), determine the radius of convergence and interval of convergence of the given series.
(Note: you need to show details of your work to get maximum marks)

Q.

a) $\sum_{n=1}^{\infty} \frac{2^n(4x-8)^n}{n}$

It can be re-written as;

$$\sum_{n=1}^{\infty} \frac{2^n 4^n (x-2)^n}{n}$$

$$\sum_{n=1}^{\infty} \frac{2^n 2^{2n} (x-2)^n}{n}$$

$$\sum_{n=1}^{\infty} \frac{2^{3n} (x-2)^n}{n}$$

$$\boxed{\sum_{n=1}^{\infty} \frac{8^n (x-2)^n}{n}}$$

Applying Ratio Test,

$$\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right|$$

Here $a_n = \frac{8^n(x-2)^n}{n}$

$$\lim_{n\to\infty} \left| \frac{\cancel{8^n}(8)\cancel{(x-2)^n}(x-2)}{n+1} \cdot \frac{n}{\cancel{8^n}\cancel{(x-2)^n}} \right|$$

$$\lim_{n\to\infty} \left| \frac{8(x-2)\,n}{n+1} \right|$$

$$|8(x-2)| \lim_{n\to\infty} \left(\frac{n}{n+1}\right)$$

$$|8(x-2)|$$

Hence, this series;

Converges when $|8(x-2)| < 1$

$$\boxed{|(x-2)| < 1/8}$$

Diverges when $|8(x-2)|>1$

$$\boxed{|x-2|>\frac{1}{8}}$$

- Convergence:

$$|x-2|<\frac{1}{8}$$

$$-\frac{1}{8}<x-2<\frac{1}{8}$$

$$\boxed{\frac{15}{8}<x<\frac{17}{8}}$$ • Converges Absolutely

Now, testing for endpoints:

When $x = 15/8$

Series becomes,

$$\sum_{n=1}^{\infty} \frac{8^n(-1/8)^n}{n}$$

$$\sum_{n=1}^{\infty} \frac{(-1)^n}{n}$$

which is a conditionally convergent series by alternating series test.

When $x = 17/8$

Series becomes,

$$\sum_{n=1}^{\infty} \frac{8^n(1/8)^n}{n}$$

$$\sum_{n=1}^{\infty} \frac{1}{n}$$

which is a divergent series.

• The series $\sum_{n=1}^{\infty} \frac{2^n(4x-8)^n}{n}$;

1) Converges Absolutely:

For $\boxed{\frac{15}{8} < x < \frac{17}{8}}$

2) Converges Conditionally:

For $\boxed{x = \frac{15}{8}}$

3) Diverges :

For $\boxed{\frac{15}{8} > x}$ and $\boxed{x \geq \frac{17}{8}}$

b) Radius of Convergence.

Comparing $|x-2| < \frac{1}{8}$ with $|x-a| < R$

The Radius of Convergence, thus, is $1/8$.

$\boxed{R = 1/8}$

• Interval of Convergence:

$\boxed{\frac{15}{8} \leq x < \frac{17}{8}}$