EE-381 Robotics-1
UG ELECTIVE

Lecture 7

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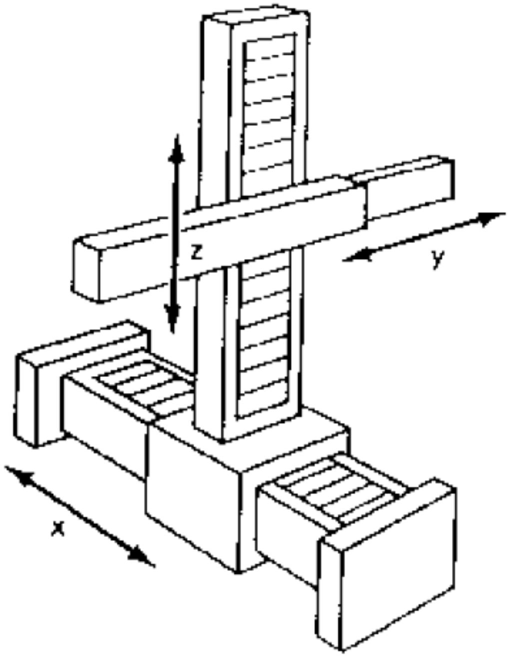
1-Cartesian Robot (PPP)

- 3 Prismatic Joints that orient the end effector, which are usually followed by additional revolute joints



Configuration of Cartesian Robot

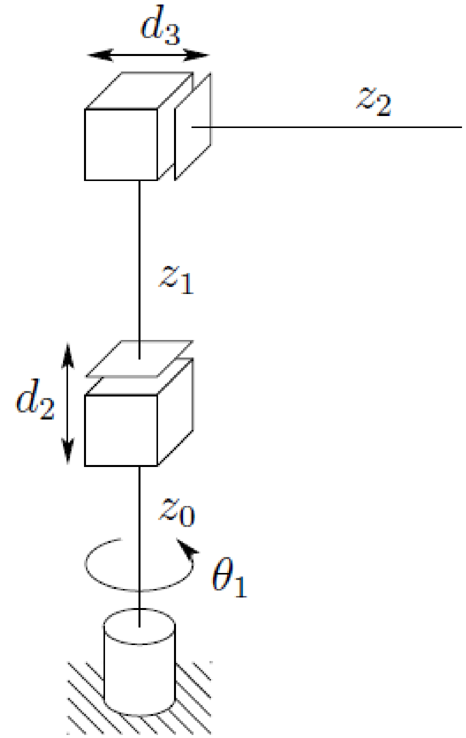
https://www.youtube.com/watch?v=ci_mpRERMog

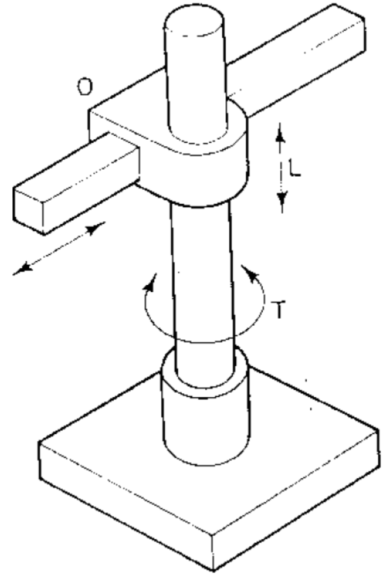
• Guess!

https://www.youtube.com/watch?v=_canCYWZ

2-Cylindrical Robot (RPP)

- First joint is revolute and produces a rotation about the base, second and third joints are prismatic



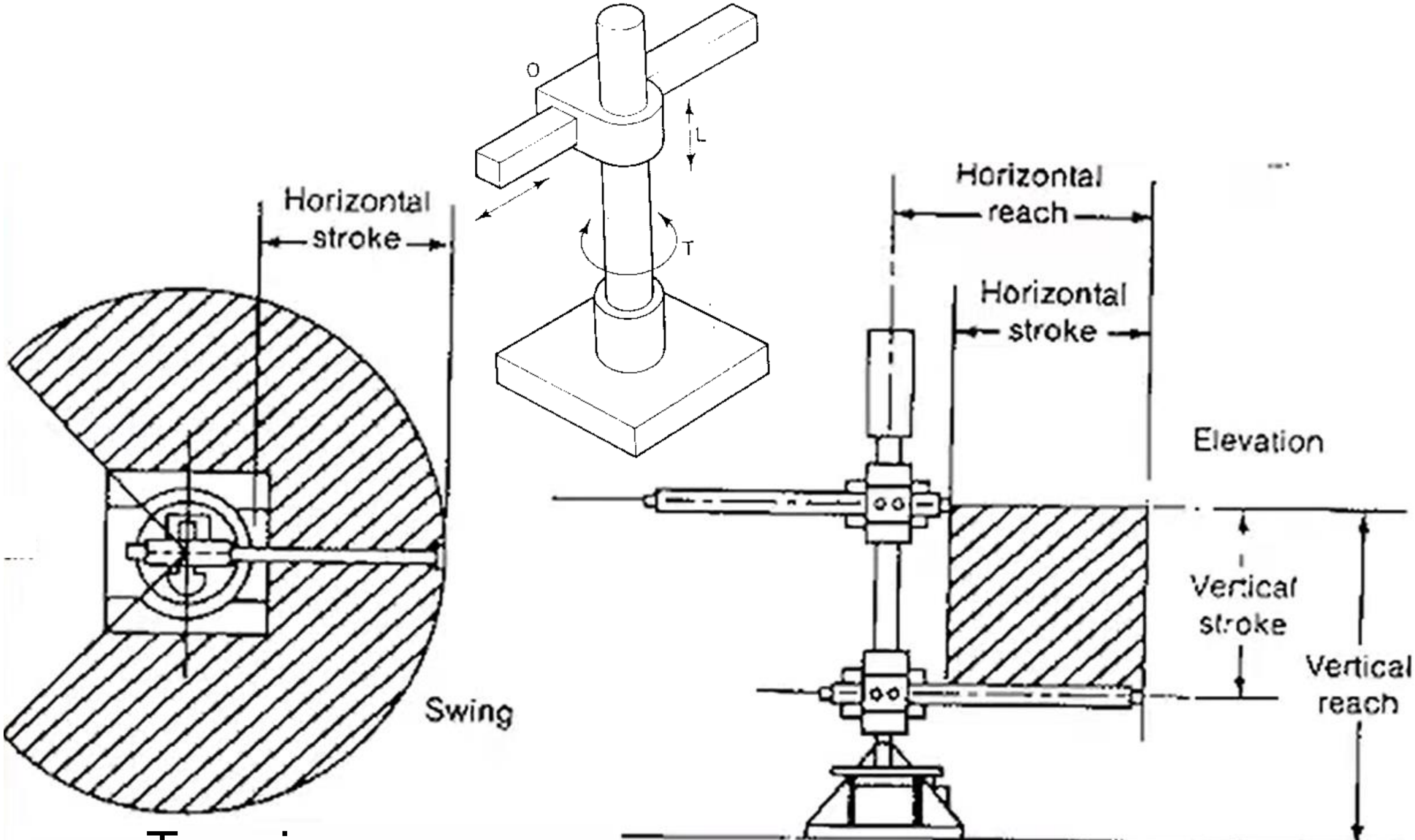


Configuration of Cylindrical Robot

Seiko RT3300 Robot

https://www.youtube.com/watch?v=Vt84DdK5kTg

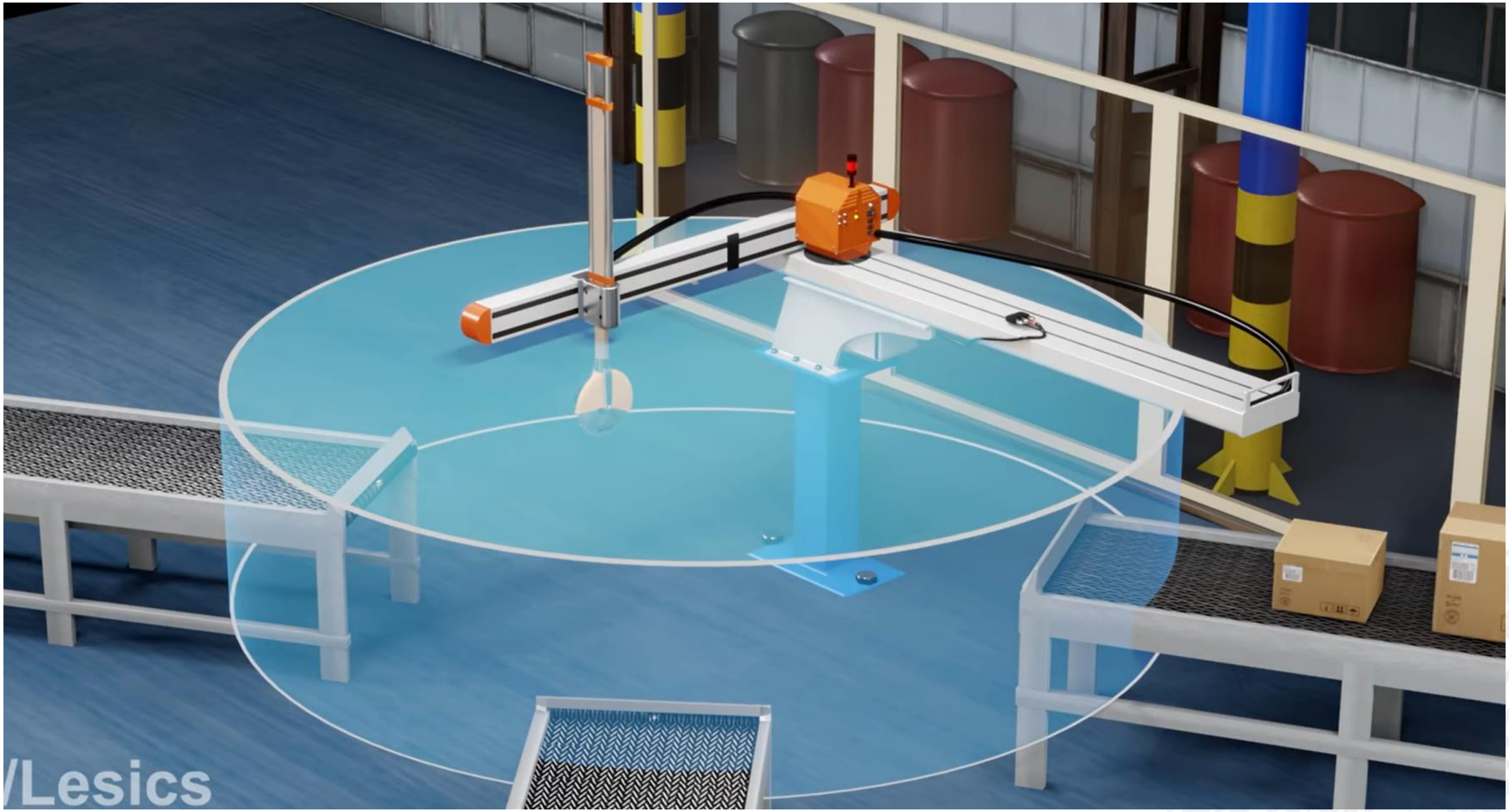
2-Cylindrical Robot- Work Envelop



Top view

Front view

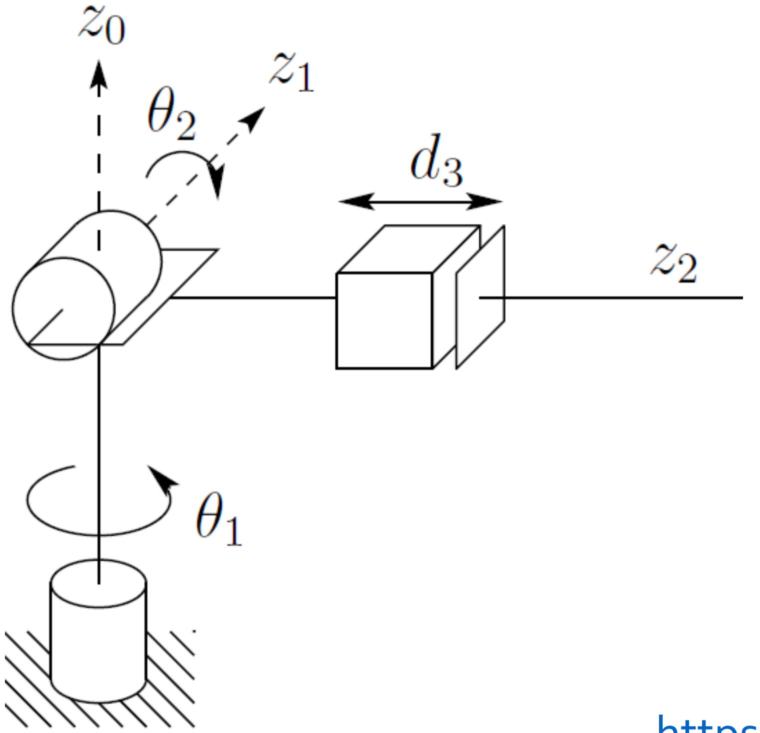
Electromechanical limit switches

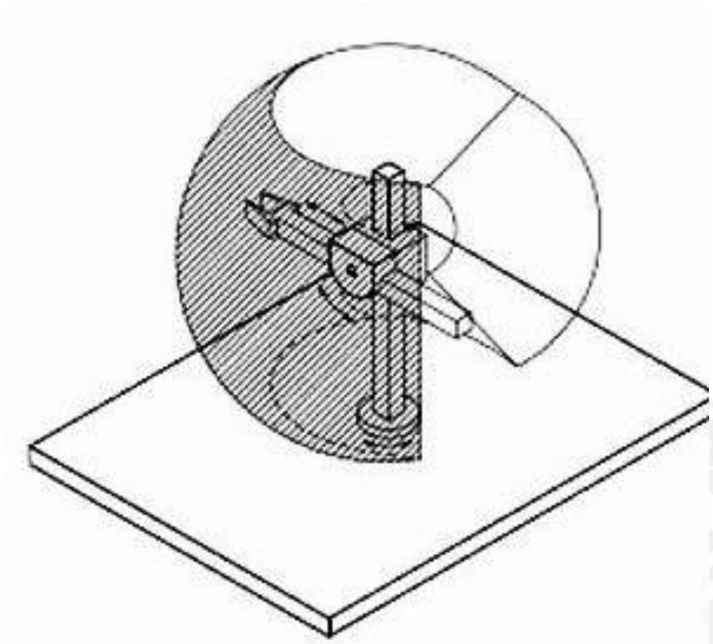


https://www.youtube.com/watch?v=_canCYWZPsc

3-Spherical Robot (RRP)

- Also known as Polar Coordinate Robot
- 2 Revolute and 1 prismatic joint

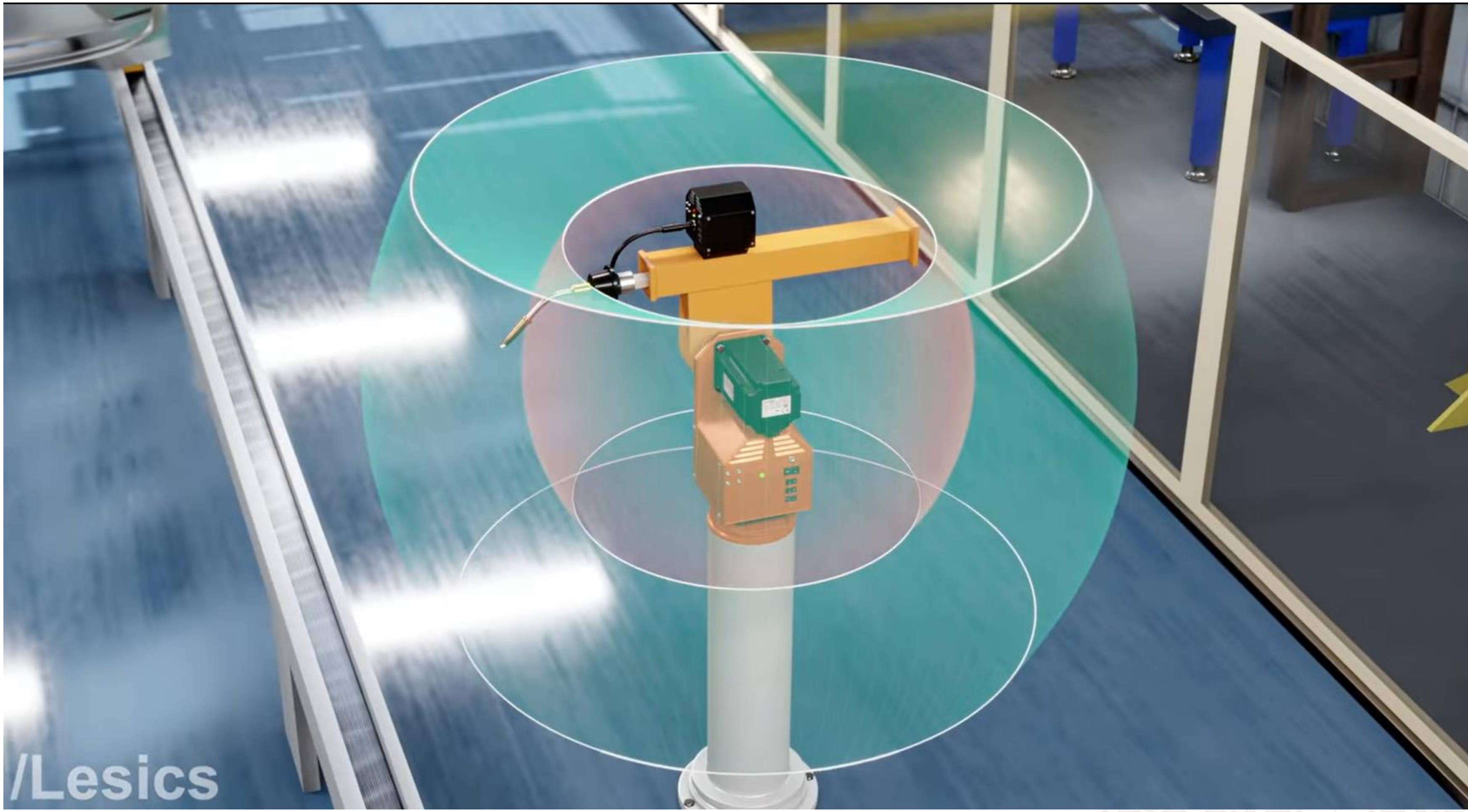




https://www.youtube.com/watch?v=jrF5Dl6ntAc

Configuration of spherical manipulator

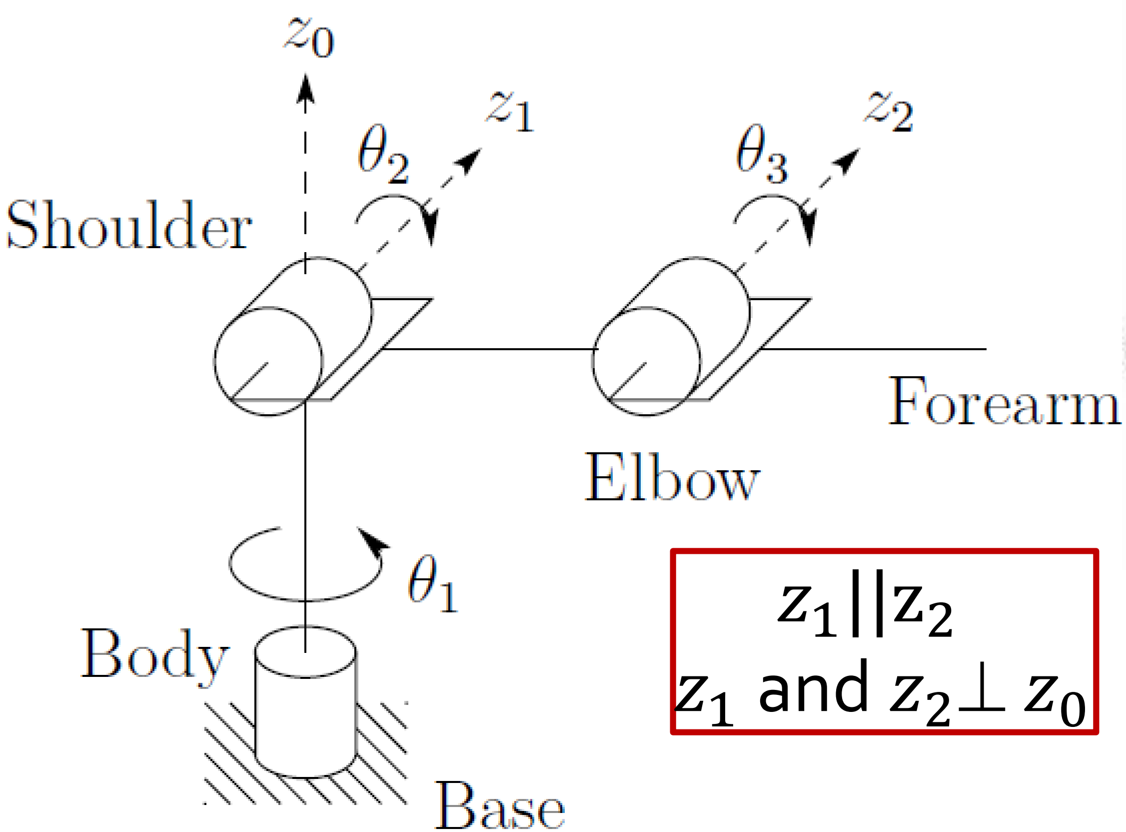
3-Spherical Robot (RRP)-Work Envelop

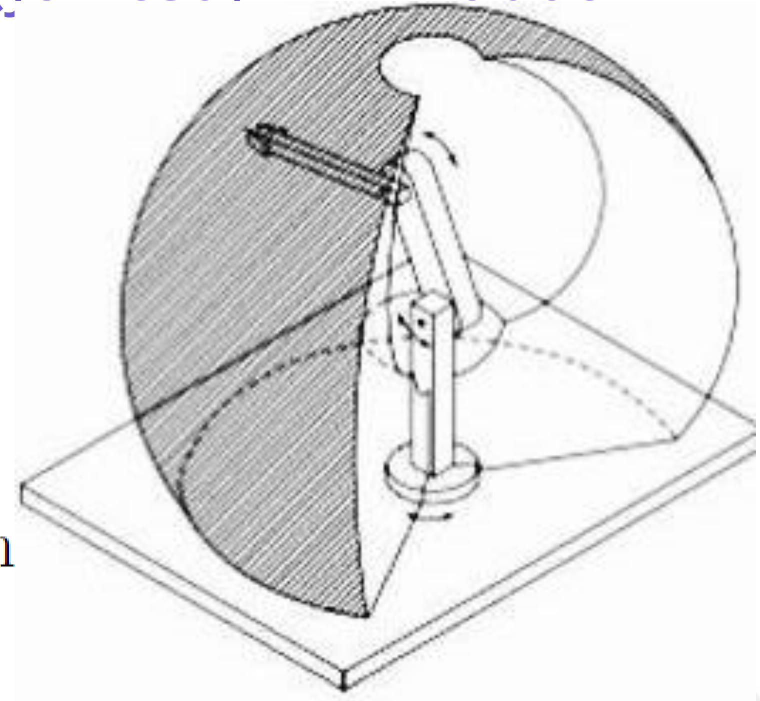


https://www.youtube.com/watch?v=_canCYWZPsc

4-Articulated Robot (RRR)

- Also known as anthropomorphic (jointed) Arm Robot
- 3 revolute joints

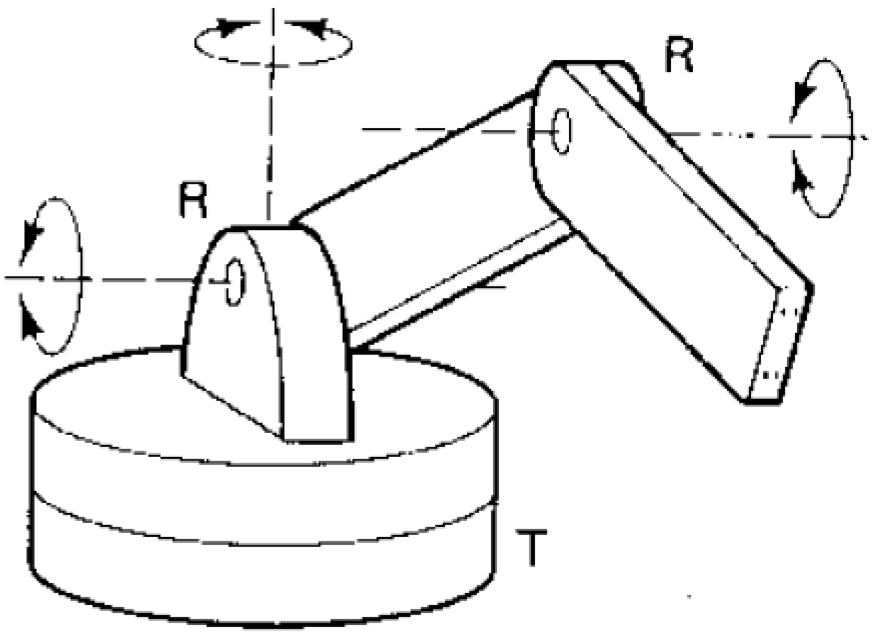




$$z_1 || z_2$$
$$z_1 \text{ and } z_2 \perp z_0$$

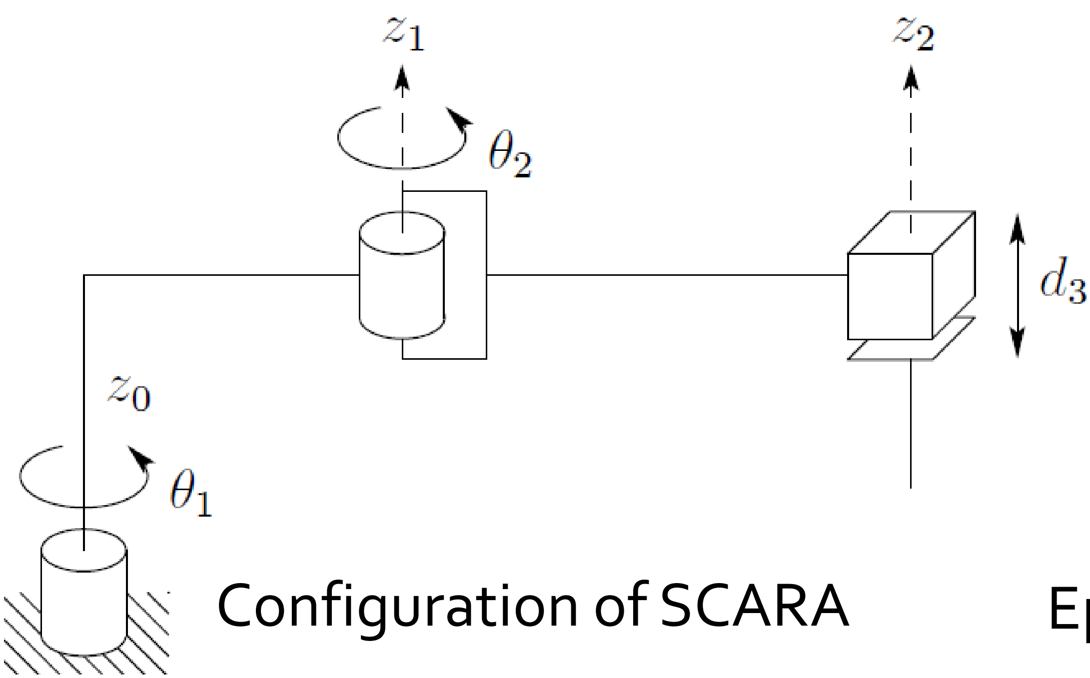
Configuration of articulated robot

4-Articulated Robot (RRR)



5-SCARA (RRP)

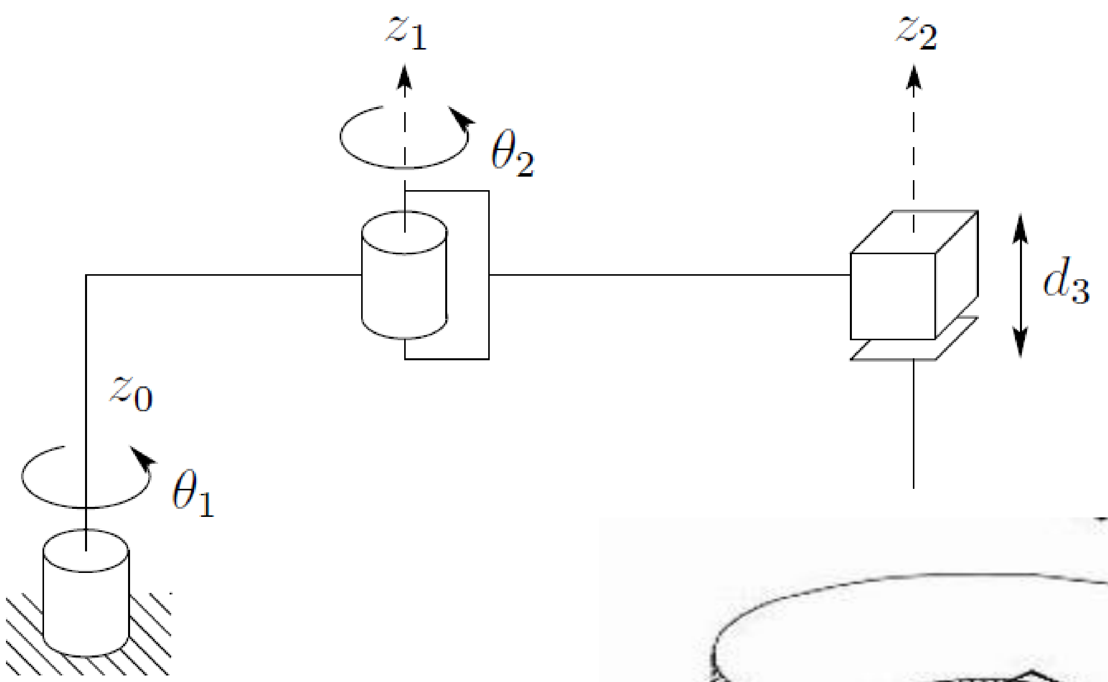
- Selective Compliant Articulated Robot Assembly
- 2 parallel revolute joint that allows the horizontal movement of robot and 1 prismatic that moves vertically
- 4DOF, 3 for Arm and 1 for wrist (roll)

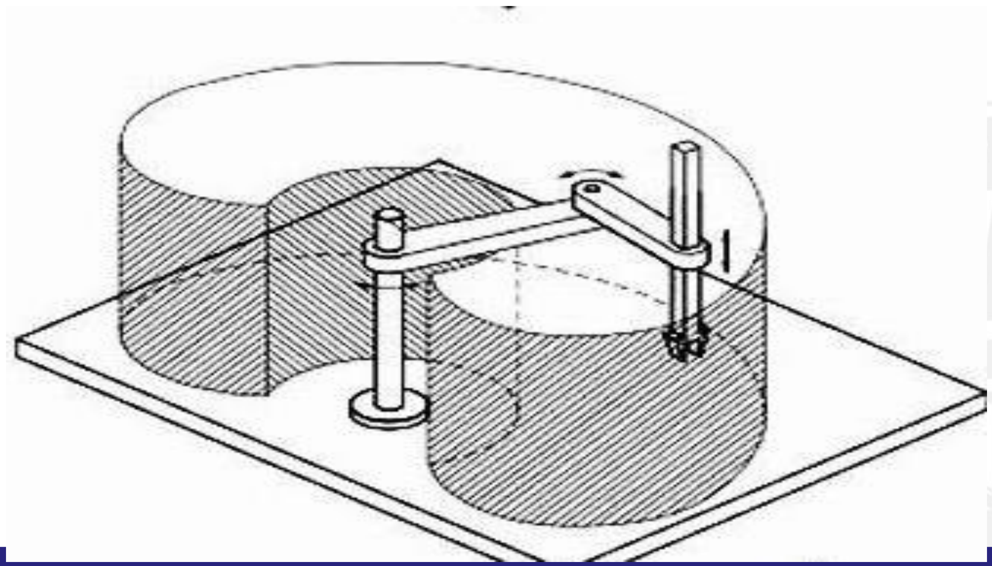


Configuration of SCARA

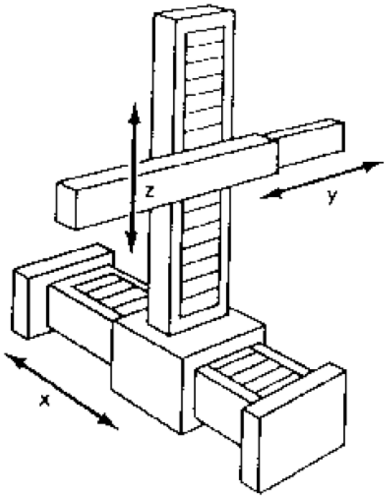
Epson E2L653S SCARA Robot

5-SCARA (RRP)

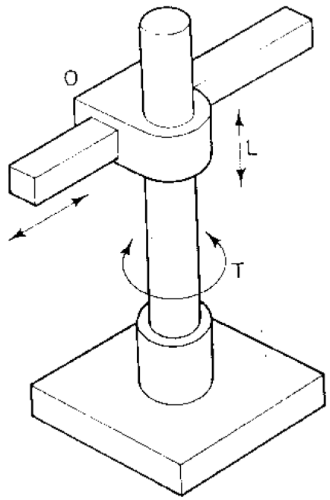




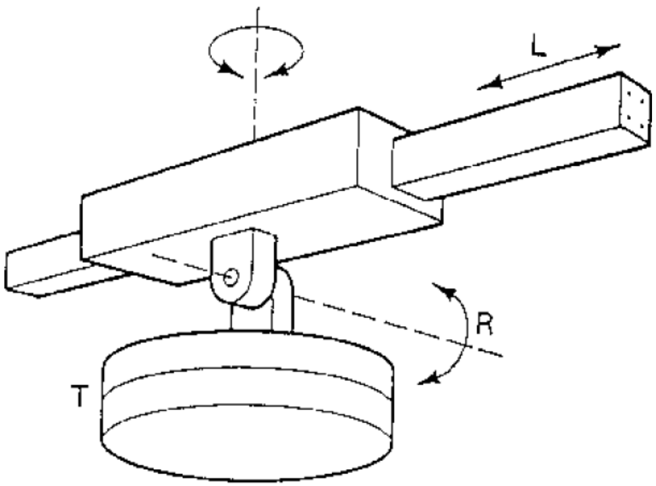
Robot Configurations: Summary



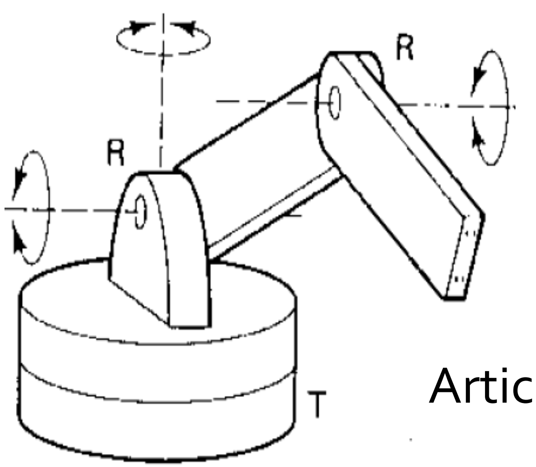
Cartesian



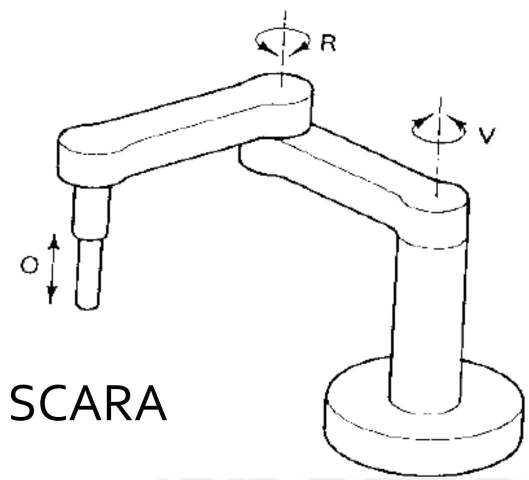
Cylindrical



Spherical



Articulated

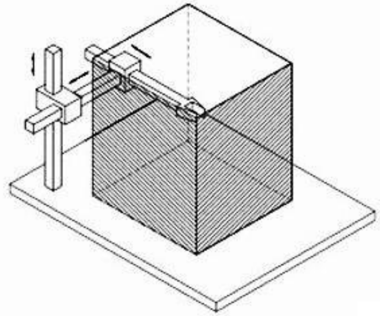


SCARA

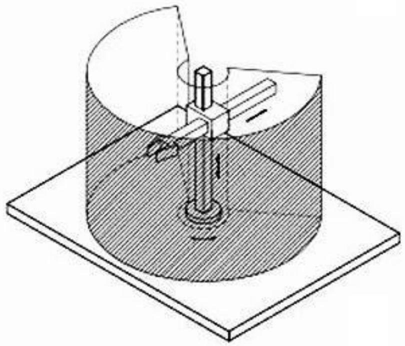
Work Space: Summary

- The region in space a robot can fully interact with

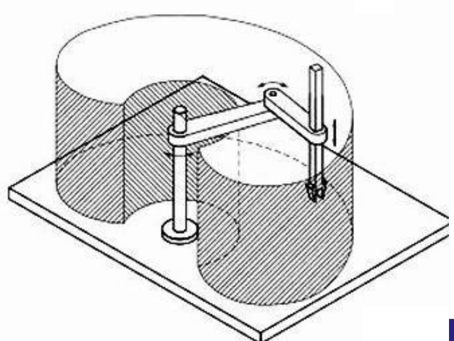
Rectangular/ Cartesian ($3P$)

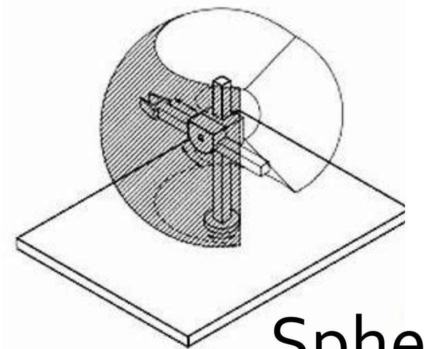


Cylindrical ($1R2P$)

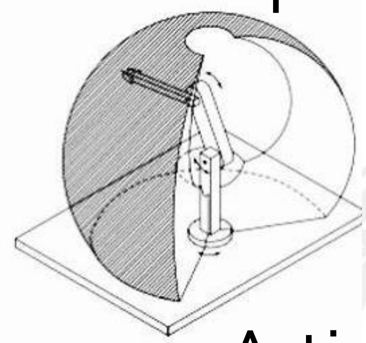


SCARA($2R1P$)





Spherical ($2R1P$)



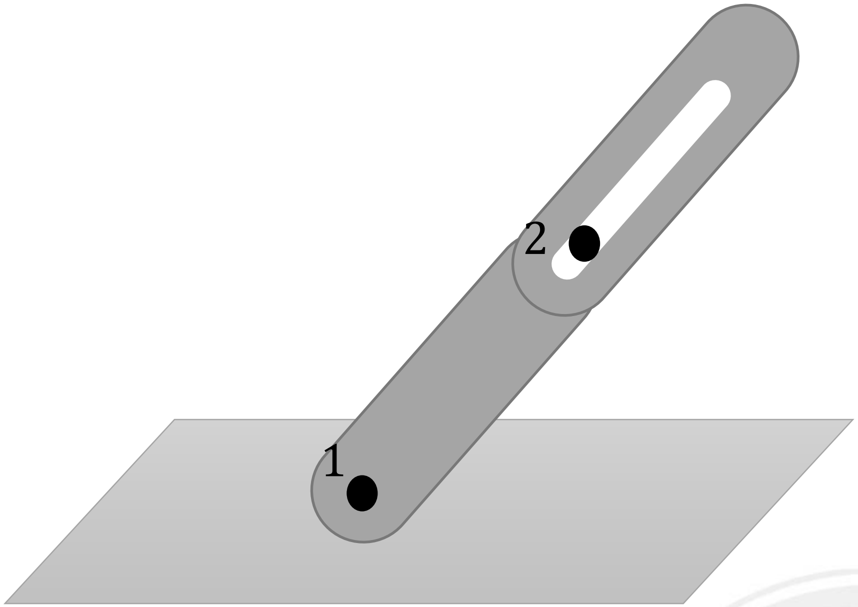
Articulated ($3R$)

Work Envelope

- Link ?
- Joint ?

Work envelop of link 1

Work envelop of link 2



- Angular motion is $[0,2\pi]$

Mappings

Example: Figure shows a frame {B} that is rotated relative to frame {A} about Z by 30 degrees. Given P^B is $\begin{bmatrix} 0.0 \\ 2.0 \\ 0.0 \end{bmatrix}$, Find P_A?

- **Solution:**

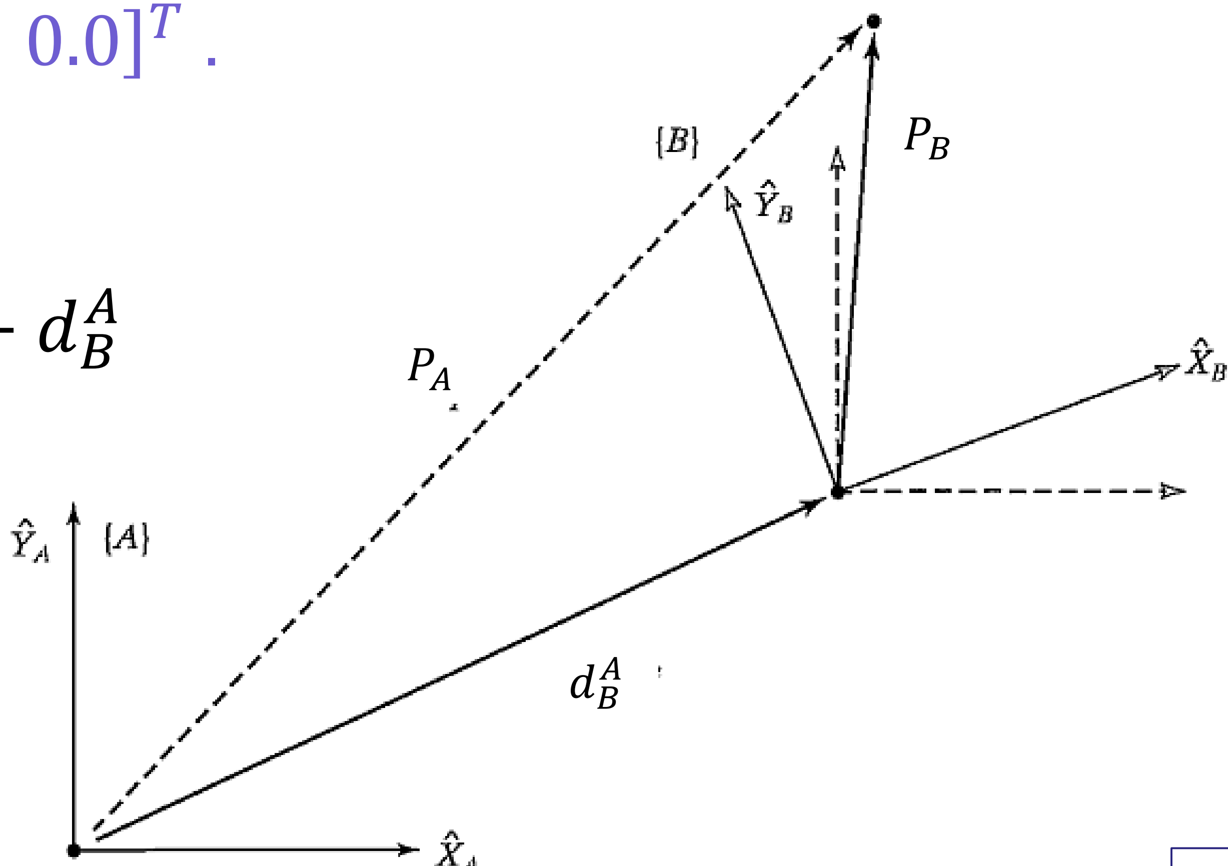
$$P^A = R_B^A P^B$$

Mappings

Example: Figure shows a frame {B}, which is rotated relative to frame {A} about Z by 30 degrees, translated 10 units in X_A, and translated 5 units in Y_A. Find P_A, where $P_B = [3.0, \quad 7.0 , \quad 0.0]^T$.

- **Solution:**

$$P_A = R_B^A P_B + d_B^A$$



Composition

- Composition of transformations

- When a transformation is applied with respect to the **fixed** frame:
  - A **pre-multiplication** is used

- When a transformation is applied with respect to the **mobile** frame (**current new**)
  - A **post-multiplication** is used

Example 1

- A frame {A} is rotated 90^o about x-axis, and then it is translated a vector $(6,-2,10)$ with respect to the **fixed** (initial) frame. Find the homogeneous transformation that describes {B} with respect to {A}.
- **Solution**

$$T_B^A = Trans(6,-2,10)Rot_x(90^o)$$

pre-multiplication

Example 2

- Find the homogeneous transformations matrix that represents a rotation of an angle α about the x −axis, followed by a translation of b units along the **new** x-axis, followed by a translation of d units along the **new** z-axis, followed by a rotation of an angle θ about the **new** z-axis
- **Solution**

$$T_B^A = Rot_x(\alpha)Trans_x(b)Trans_z(d)Rot_z(\theta)$$

post-multiplication

Example 2

- A frame {A} is rotated 90^o about x, and then it is translated a vector $(6,-2,10)$ with respect to the **fixed** (initial) frame. Consider a point $P=(-5,2,-12)$ with respect to the new frame {B}. Determine the coordinates of that point with respect to the initial frame.

Solution

pre-multiplication

- Homogeneous transformation

$$T_B^A = Trans(6,-2,10)Rot_x(90^0)$$

- Point after transformation ? $\tilde{P}^A = T_B^A \tilde{P}^B$

Example 3

- A frame {A} is translated a vector $(6,-2,10)$ and then it is rotated 90^o about x-axis of the fixed (initial) frame. Consider a point $P=(-5,2,-12)$ with respect to the new frame {B}. Find the coordinates of that point with respect to the initial frame.

Solution

pre-multiplication

- Homogeneous transformation

$$T_B^A = Rot_x(90^0)Trans(6,-2,10)$$

- Transformed point? $\tilde{P}^A = T_B^A \tilde{P}^B$

Compound Transformations

Example: A frame {A} is translated a vector $(6, -2, 10)$ and then it is rotated 90^o about x-axis of the **fixed** (initial) frame. Thus, we have a description of T_B^A. Find T_A^B.

Solution

- Homogeneous transformation

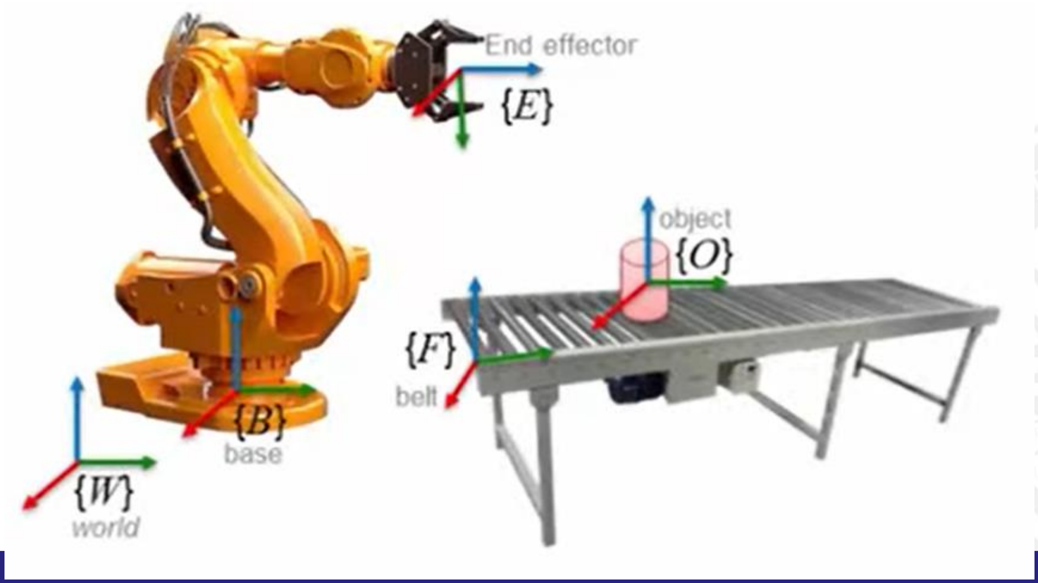
$$T_B^A = Rot_x(90^0)Trans(6, -2, 10)$$

- T_A^B?

$$(T_B^A)^{-1} = T_A^B$$

Example

- Consider that the transformations of the belt and of the robot base with respect to a reference frame {W} are known. The transformation of the object with respect to the belt, as well as the transformation of the end effector with respect to the robot base are also known.
  a) Find the pose of the object with respect to the base of the robot
  b) Find the pose of the object with respect to the end effector



Example

- Consider that the transformations of the belt and of the robot base with respect to a reference frame {W} are known. The transformation of the object with respect to the belt, as well as the transformation of the end effector with respect to the robot base are also known.
  a) Find the pose of the object with respect to the base of the robot
  b) Find the pose of the object with respect to the end effector

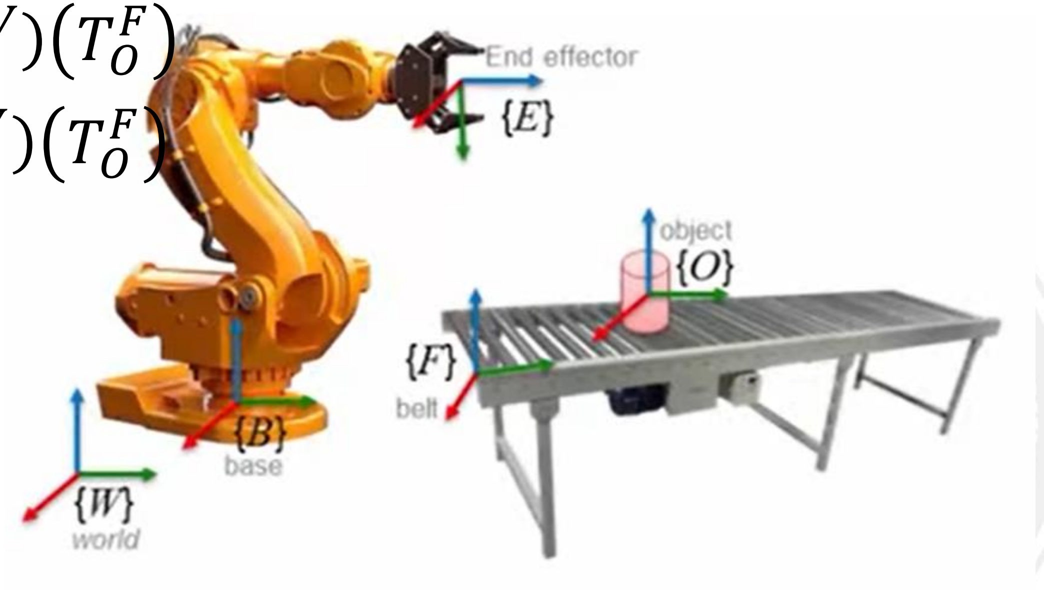
- **Solution**
- Known transformations: $T_F^W, T_B^W, T_O^F, T_E^B,$
  a) Desired pose (in terms of the known transformations): T_O^B

$$T_O^B = (T_W^B)(T_O^W) = (T_B^W)^{-1}\left((T_F^W)(T_O^F)\right)$$

Example

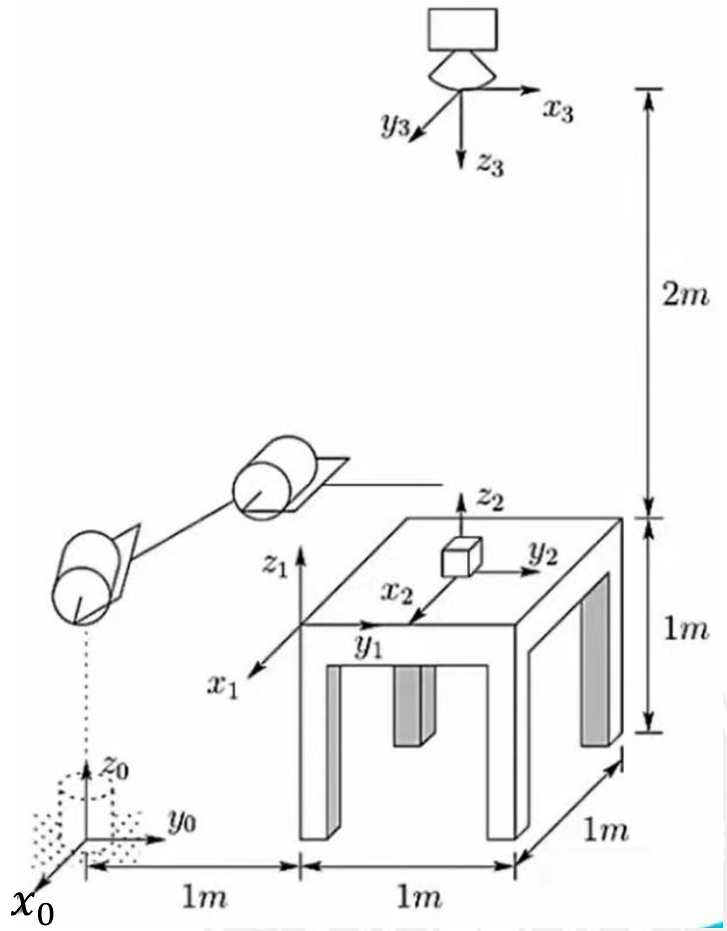
b) Desired pose (in terms of the known transformations): T_O^E

- Known transformations: $T_F^W, T_B^W, T_O^F, T_E^B,$
  - $T_O^E = (T_W^E)(T_O^W)$
  - $= (T_E^W)^{-1}(T_F^W)(T_O^F)$
  - $= \left((T_B^W)(T_E^B)\right)^{-1}(T_F^W)(T_O^F)$
  - $= (T_E^B)^{-1}(T_B^W)^{-1}(T_F^W)(T_O^F)$



Example

- The figure shows a robot whose base is 1m away from the base of the table. The table is 1m height and its surface is a square. Frame {1} is fixed on a corner of the table. A 20cm cube is located on the middle of the table, and it has frame {2} attached to its center. A camera is located 2m above the table, just over the cube, and it has frame {3} attached to it.
  - Find the homogeneous transformations that relate each of these frames with the base system {0}.
  - Find the homogeneous transformations that relates the cube frame {2} wrt the camera frame {3}.



Robot Modeling and Control, Chapter 2, problem 37

Example

- Solution

  a) By inspection, the homogeneous transformations that relate each of the frames wrt the base frame {0} are:

$$ {}^0T_1 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad {}^0T_2 = \begin{bmatrix} 1 & 0 & 0 & -0.5 \\ 0 & 1 & 0 & 1.5 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad {}^0T_3 = \begin{bmatrix} 0 & 1 & 0 & -0.5 \\ 1 & 0 & 0 & 1.5 \\ 0 & 0 & -1 & 3 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

  b) Using the composition of transformations:

$$ R_3^0 = \begin{bmatrix} x_0 . x_3 & x_0 . y_3 & x_0 . z_3 \\ y_0 . x_3 & y_0 . y_3 & y_0 . z_3 \\ z_0 . x_3 & z_0 . y_3 & z_0 . z_3 \end{bmatrix} $$

$$ {}^3T_2 = {}^3T_0 \, {}^0T_2 = {}^0T_3^{-1} \, {}^0T_2 $$

$$ = \begin{bmatrix} 0 & 1 & 0 & -1.5 \\ 1 & 0 & 0 & 0.5 \\ 0 & 0 & -1 & 3 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & -0.5 \\ 0 & 1 & 0 & 1.5 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 2 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

Unit Quaternions

- **Example:**
  a) Find the quaternions that represents a rotation of 60^0 about $(1,0,0)$.

$$Q = \left(\cos\left(\frac{60^\circ}{2}\right), \sin\left(\frac{60^\circ}{2}\right)(1,0,0) \right) = (0.866,\ 0.5,\ 0,\ 0)$$

  b) Find the conjugate and inverse of the previous quaternions Q

$$Q^* = Q^{-1} = (0.866,\ -0.5,\ 0,\ 0)$$

Unit Quaternions

Application of a rotation Q to a vector $\boldsymbol{v}$:

1. Convert vector $\boldsymbol{v}$ to a quaternion (0 scalar component): $\widetilde{\boldsymbol{v}} = (\mathbf{0}, \boldsymbol{v})$

2. Apply the rotation Q: $\widetilde{\boldsymbol{v}}_{\boldsymbol{q_{rot}}} = \boldsymbol{Q} \circ \widetilde{\boldsymbol{v}} \circ \boldsymbol{Q}^*$

3. The rotation vector $\boldsymbol{v}_{\mathrm{rot}}$ is the vector component $\boldsymbol{v}_{\mathrm{rot}} = (\mathbf{0}, \boldsymbol{v}_{\boldsymbol{rot}})$

Unit Quaternions

Example : Find the rotation of point p=(3,5,2) by an angle of 60^0 about (1,0,0) **(a)** using quaternions, **(b)** using a rotation matrix.

Solution

a) Using quaternions

$$\tilde{p} = (0,\ 3,\ 5,\ 2)$$

$$\tilde{p}_{rot} = Q \circ (0,\ 3,\ 5,\ 2) \circ Q^*$$

$$= (0.866,\ 0.5,\ 0,\ 0) \circ (0,\ 3,\ 5,\ 2) \circ (0.866,\ -0.5,\ 0,\ 0)$$

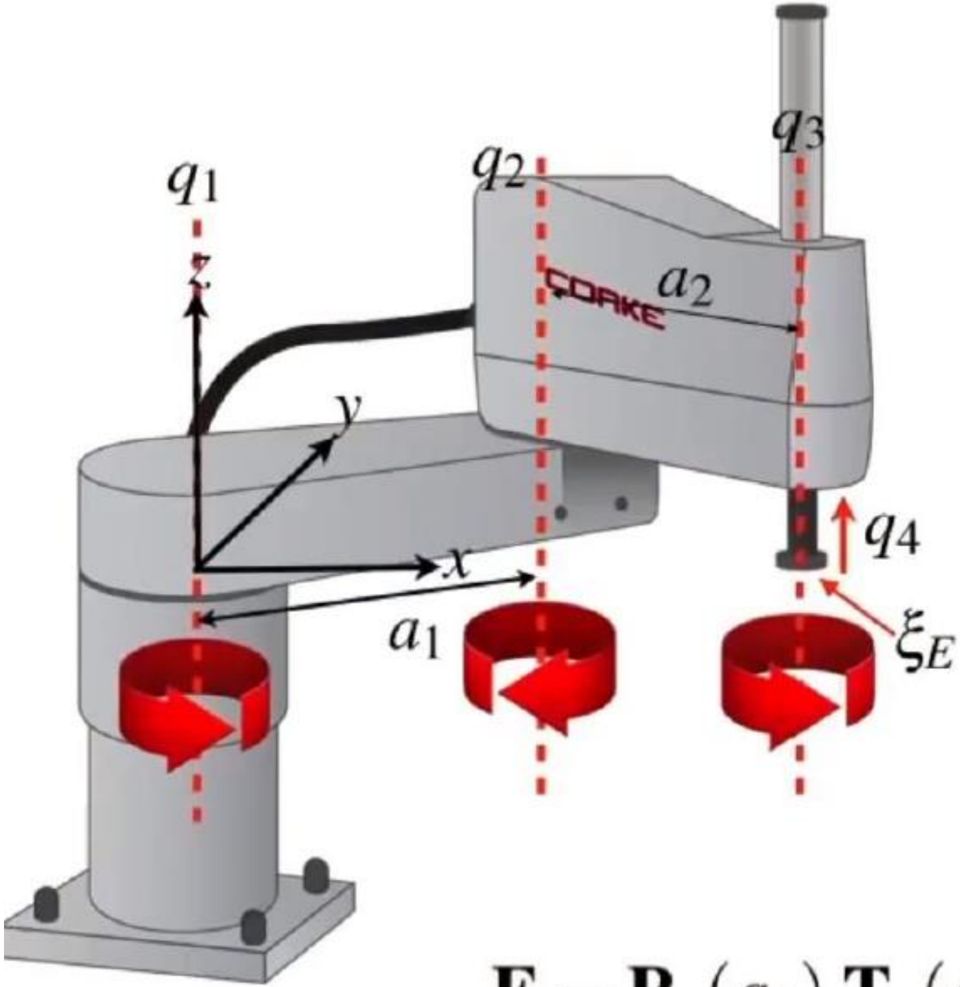
$$= (-1.5,\ 2.6,\ 3.33,\ 4.23) \circ (0.866,\ -0.5,\ 0,\ 0)$$

$$= (0,\ 3,\ 0.768,\ 5.33) \longrightarrow p_{rot} = (3,\ 0.768,\ 5.33)$$

b) Using a rotation matrix

$$p_{rot} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos 60° & -\sin 60° \\ 0 & \sin 60° & \cos 60° \end{bmatrix} \begin{bmatrix} 3 \\ 5 \\ 2 \end{bmatrix} = \begin{bmatrix} 3 \\ 0.768 \\ 5.33 \end{bmatrix}$$

SCARA



	α_{i-1}	a_{i-1}	d_i	θ_i
1	0	0	0	q_1
2	0	a_1	0	q_2
3	0	a_2	q_4	0
4	0	0	0	q_3

$$\mathbf{E} = \mathbf{R}_z(q_1)\,\mathbf{T}_x(a_1)\,\mathbf{R}_z(q_2)\,\mathbf{T}_x(a_2)\,\mathbf{R}_z(q_3)\,\mathbf{T}_z(q_4)$$

Quiz 2

Given the manipulator, compute the following:

a) End-effector pose using forward kinematics.

b) DH parameters.