National University of Sciences & Technology School of Electrical Engineering and Computer Science Department of Basic Sciences

MATH-243: Vector Calculus (3+0): BEE-2k20-ABC Fall 2021

| Assignment # 1 | | | | |
|--|---|--|--|--|
| CLO-1: Interpret the consequences of del (nabla) operator on scalar and vector fields. | | | | |
| Maximum Marks: 10 | Instructor: Dr. Naila Amir | | | |
| Announcement Date: 4 th October 2021 | Due Date: 12 th October 2021 | | | |

Instructions:

- Understanding the question is part of the assignment and copying is not allowed.
- Express your answer in the most simplified form. Direct calculations using calculator are not allowed, you need to show the detail of your work to get the maximum marks.
- This is a group assignment. Each group having 4 members only. All group members are required to
 contribute equally. Each member of group will attempt one question and afterwards will discuss his/her
 attempt with other group members so that all group members have an idea of the solution of whole
 assignment. Name of student should be mentioned in the following table against the question attempted
 by him/her.
- Assignment must be hand written on A4 papers and properly bound.
- There are two pages in this assignment, including this cover page. These three pages should be part of every assignment.
- Assignment is not acceptable after deadline.

Tasks: Attempt all questions.

| Sr. No. | Students Name | CMS Id. | Question Attempted | Marks Obtained |
|---------|-----------------------|---------|--------------------|----------------|
| 1 | Tariq Umar | 334943 | Q#1 | |
| 2 | Danial Ahmad | 331388 | Q#2 | |
| 3 | Muhammad Umer | 345834 | Q#3 | |
| 4 | Muhammad Ahmed Mohsin | 333060 | Q#4 | |

| Total Marks | Marks Obtained | Weight in 10 |
|-------------|----------------|--------------|
| 40 Marks | | |

Q - 1: [10 marks]

Identify the given surface by identify and sketching the traces of the surface.

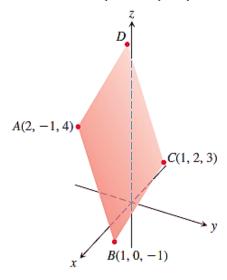
$$y^2 - x^2 - 6y - z + 9 = 0.$$

(Note: You are required to identify the type of surface by explicitly mentioning the axis and vertex of the given surface. Moreover, you are required to complete the following table for identification and sketching of traces. You can either take print of this table or you can make it yourself on your papers.

| Trace | Equation of trace | Description of trace | Sketch of trace |
|--|-------------------|----------------------|---------------------------------------|
| xy —trace | | | x y |
| yz —trace | | | , y |
| xz —trace | | | y |
| Trace on plane parallel to xy —plane, $z = c$ | | | , , , , , , , , , , , , , , , , , , , |
| Trace on plane parallel to xy —plane, $z = -c$ | | | y y |

Q - 2: [10 marks]

The parallelogram shown below has vertices at A(2, -1, 4), B(1, 0, -1), C(1, 2, 3), and D.



Determine the following:

- a) the coordinates of D.
- b) the cosine of the interior angle at *B*.
- c) the area of the parallelogram.
- d) an equation for the plane of the parallelogram.

Q - 3: [5+5=10 marks]

Each of the following equations in parts (a) & (b) describes $\mathbf{r}(t)$ which represents the position of a particle in space at time t. Find the particle's velocity and acceleration vectors. Then find the particle's speed and direction of motion at the given value of t. Write the particle's velocity at that time as the product of its speed and direction.

a)
$$\mathbf{r}(t) = \langle \sec t, \tan t, \frac{4}{3}t \rangle$$
; $t = \frac{\pi}{6}$.

b)
$$\mathbf{r}(t) = \langle e^{-t}, 2\cos(3t), 2\sin(3t) \rangle$$
; $t = \frac{\pi}{6}$.

Q - 4: [5+5=10 marks]

a) Find a vector function that represents the curve of intersection of the surface $z = x^2 + y^2$ and the plane 5x - 6y + z - 8 = 0.

[Hint: Parametric equations for a shifted circle:
$$(x-x_0)^2+(y-y_0)^2=r^2$$
 are given as: $x=x_0+r\cos t$, $y=y_0+r\sin t$; $0\leq t\leq 2\pi$.]

b) Find a vector function that represents the curve of intersection of the cylinder $x^2 + y^2 = 9$ and the hyperbolic paraboloid z = xy.