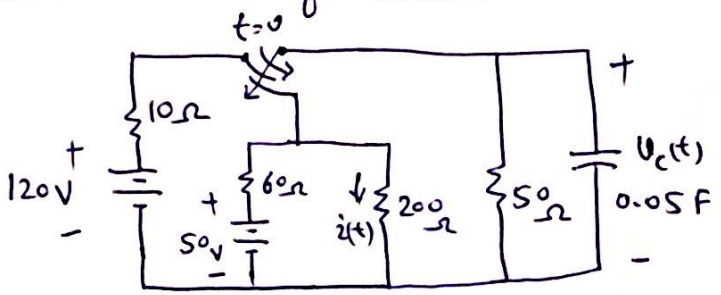
8.8 Driven RC Circuits

(PP 289 7th Ed HKD) (PP295 8th Ed)

Example 8.10 Driven RC Circuits

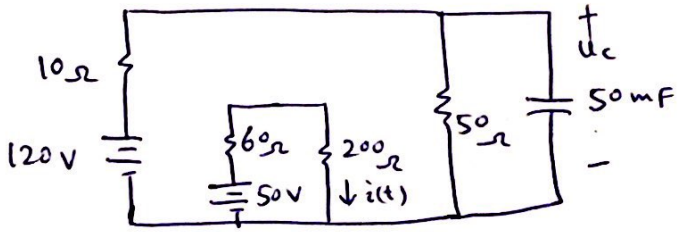
(PP 296 8th Ed HKD) (PP 296 8th Ed)

Find $v_c(t)$ and $i(t)$ for all time.



Solution:

At $t \leq 0$

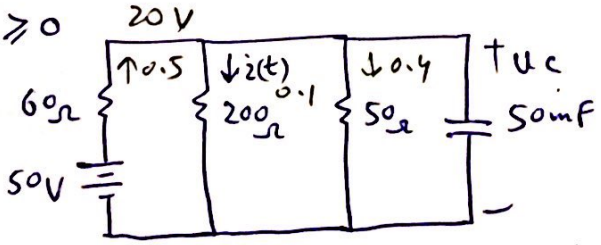


Now $v_c(0) = \left(\frac{50}{50+10}\right) 120 = 100\ V$

and $v_c(0^-) = v_c(0^+) = 100\ V$ Also $i(0^-) = \frac{50}{260}$

$= 0.192\ A$

At $t \geq 0$



So $R_{Th} = 50 // 200 // 60 = 24\ \Omega$ — eliminating independent sources

$\tau = R_{eq} C = 24 \times 50 \times 10^{-3}$

$\tau = 1.2\ s$

——— contd

— contd (297)

Now $v_c(t) = v_c(f) + v_c(n)$

And $v_c(n) = Ae^{-t/\tau} = Ae^{-t/1.2}$, V

Also $v_c(f) = \left[\frac{50//200}{(50//200)+60}\right] \times 50 = 20 \text{ V} = v_c(\infty)$

Hence $v_c = 20 + Ae^{-t/1.2}$

From initial condition $v_c(0) = 100$ V

so $v_c = 20 + A \times 1 = 100$

$A = 80$

Hence $v_c = 20 + 80e^{-t/1.2}$ V, $t \geq 0$ (Note: Here the author uses $t \geq 0$)

and $v_c = 100$ V, $t < 0$

Alternately

$$v_c(t) = v_c(\infty) + \left[v_c(0^+) - v_c(\infty)\right] e^{-t/\tau}$$

$$v_c(t) = 20 + \left[100 - 20\right] e^{-t/1.2}, \text{V}$$

or $v_c(t) = 20 + 80e^{-t/1.2}$ V, $t \geq 0$

Graphically

Now to find $i(t)$

At $t = 0^-$

$$i(0^-) = \frac{50}{60+200} = 192.3 \text{ mA}$$

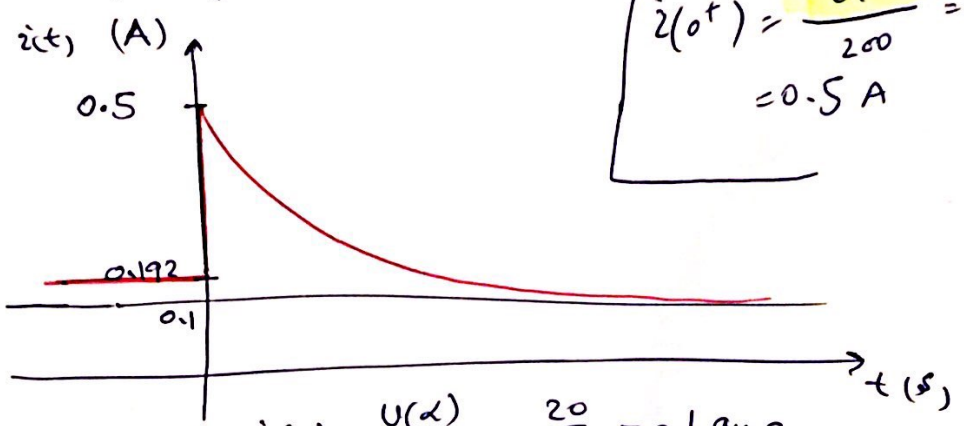
— contd

—contd (298)

because 200Ω and Capacitor in parallel.

$$i(0^+) = \frac{U_c(0^+)}{200} = \frac{100}{200}$$
$$= 0.5 \text{ A}$$

Graphically:



Note: $i(\infty) = \frac{U(\infty)}{200} = \frac{20}{200} = 0.1$ amp

Alternately

$$U_c(t) = 20 + 80\, e^{-t/1.2} \qquad t \geq 0$$

So $$i(t) = \frac{20 + 80\, e^{-t/1.2}}{200} = \frac{U_c(t)}{200} \qquad t > 0$$

$$i(t) = 0.1 + 0.4\, e^{-t/1.2} \text{ A} \qquad t > 0$$

Already $t < 0$

$$i(0^-) = 192.3 \text{ mA} \qquad t < 0$$

OR $$\left(0.1923\, u(-t) + \left(0.1 + 0.4\, e^{-\frac{t}{1.2}}\right) u(t) \right)$$

for all times