Electric Current-I

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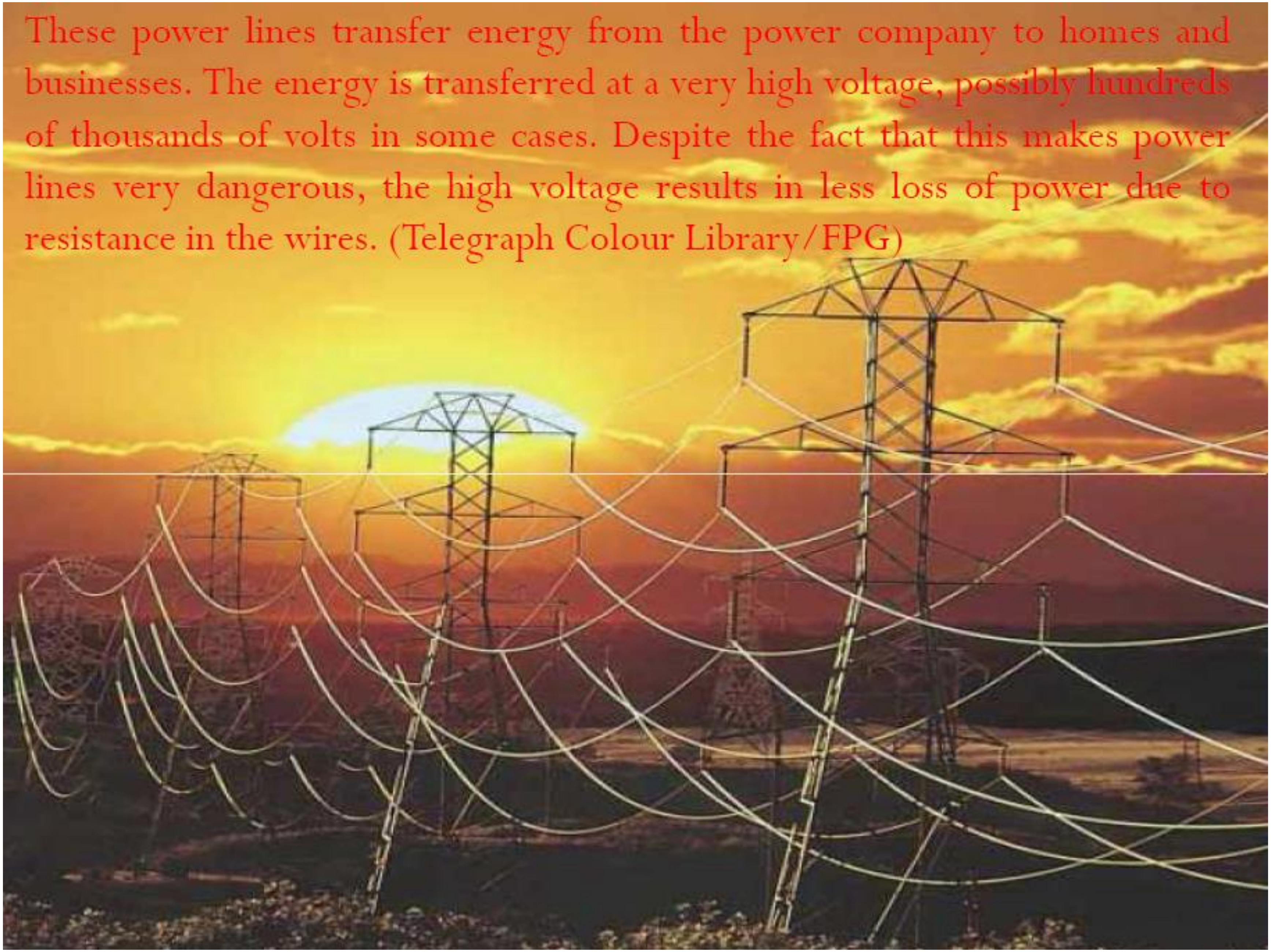
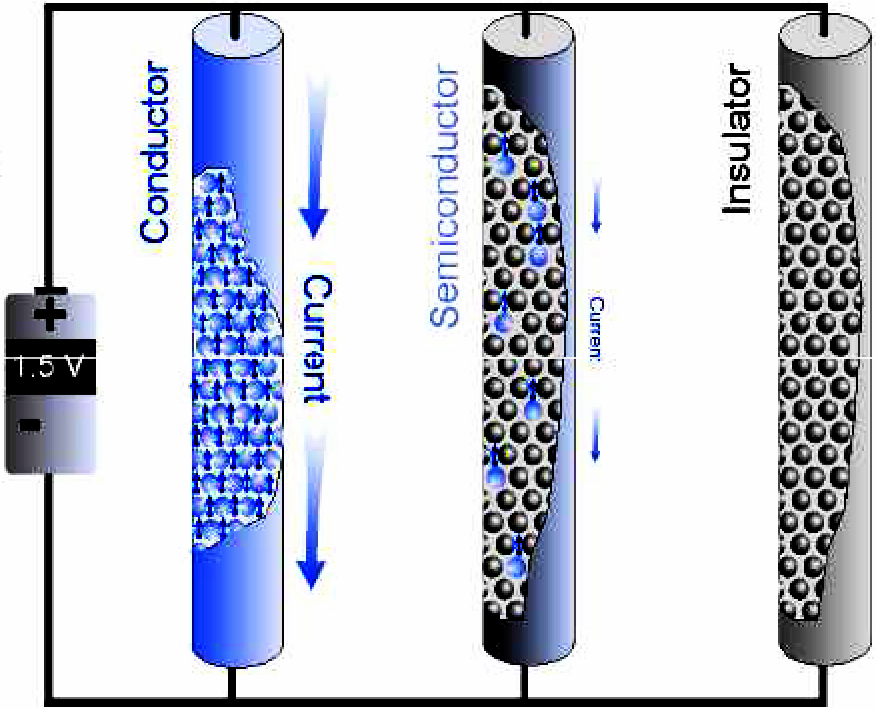


These power lines transfer energy from the power company to homes and businesses. The energy is transferred at a very high voltage, possibly hundreds of thousands of volts in some cases. Despite the fact that this makes power lines very dangerous, the high voltage results in less loss of power due to resistance in the wires. (Telegraph Colour Library/FPG)

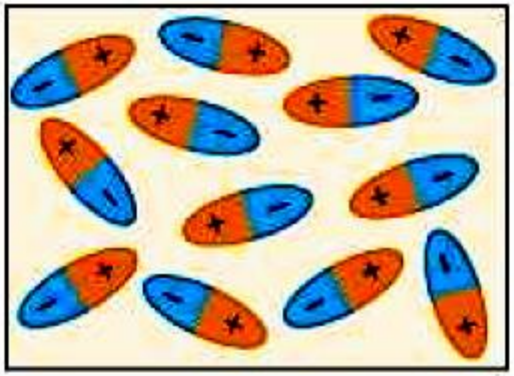
Types of materials

- All materials contain electrons.
- The electrons in conductor are free to move.
- The electrons in insulators are not free to move—they are tightly bound inside atoms.
- A semiconductor has a few free electrons and atoms with bound electrons that act as insulators.

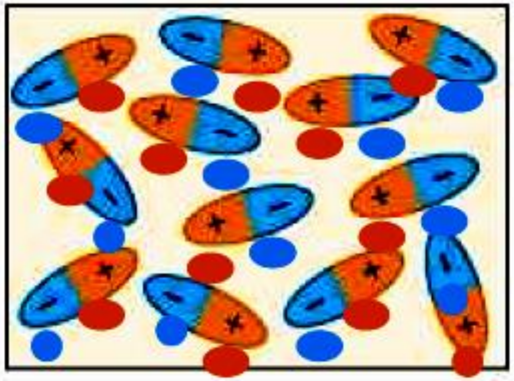


Insulator in an Electric Field

- If the molecules of the dielectric are polar, the dipoles are randomly oriented in the absence of an electric field the external electric field . When external electric field is applied, *these* dipole moments tend to align with the external field, and the dielectric is polarized.
- If the molecules of the dielectric are non polar, then the external electric field produces some charge separation and an *induced dipole moment. These* induced dipole moments tend to align with the external field, and the dielectric is polarized.

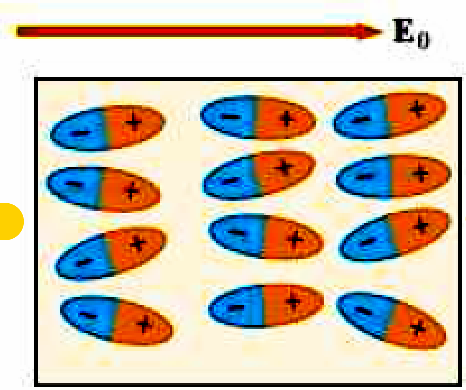


Polar



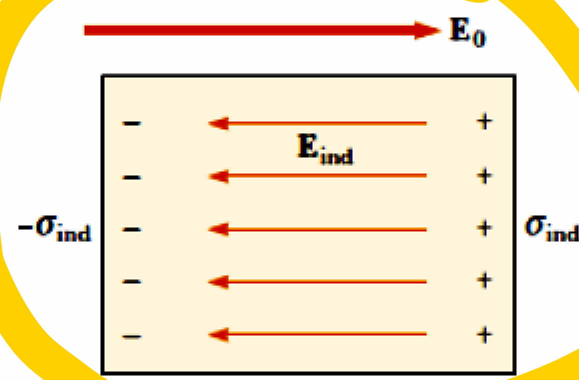
Non polar

- Thus, we can polarize a dielectric with an external field regardless of whether the molecules are polar or non polar.
- The degree of alignment of the molecules with the electric field depends on temperature and on the magnitude of the field.
- In general, the alignment increases with decreasing temperature and with increasing electric field.
- This polarization causes an induced negative surface charge on one side of the dielectric and an equal induced positive surface charge on the opposite side. This separation of charge results in a reduction in the net electric field within the dielectric.

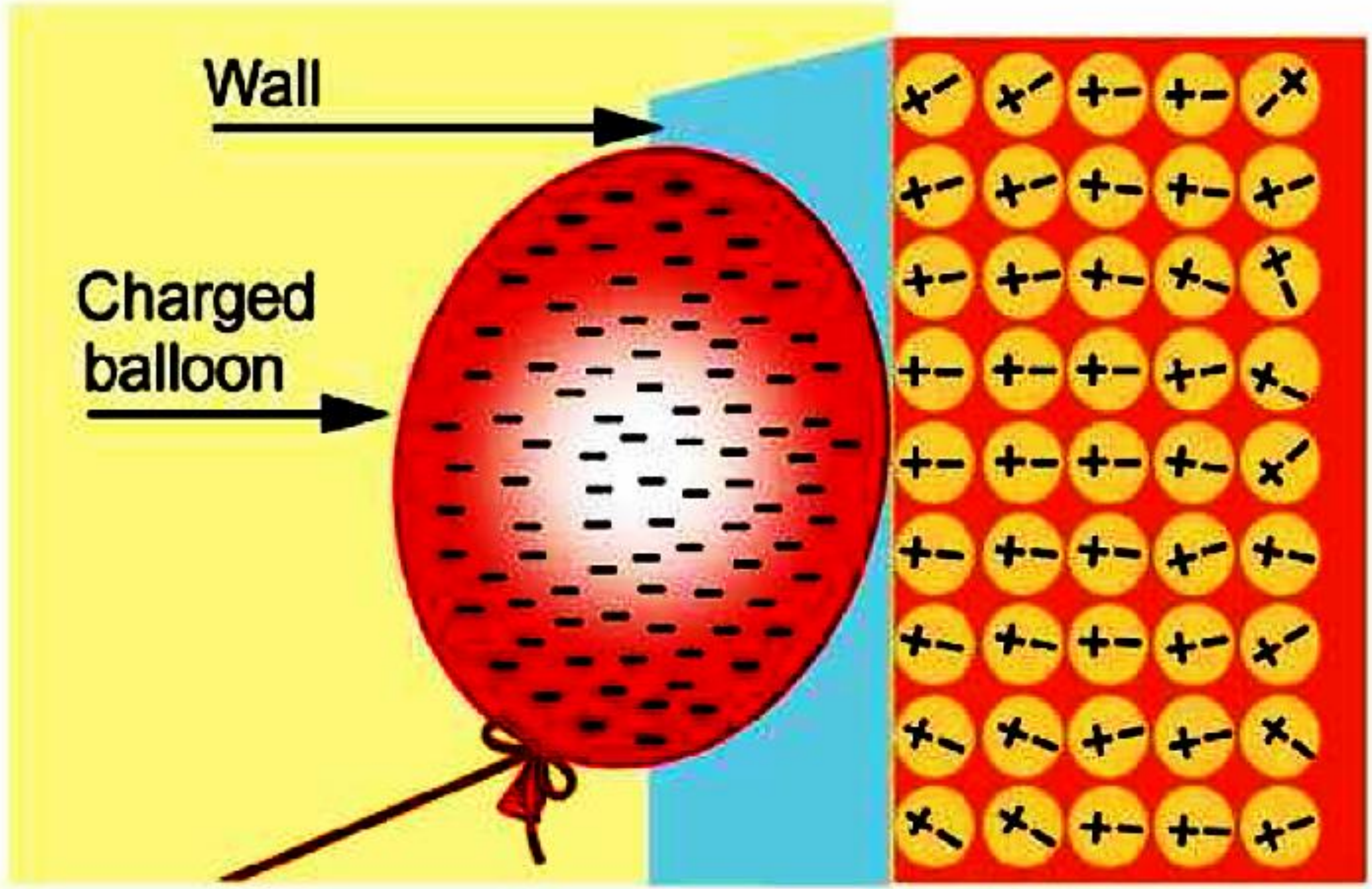


$$\vec{E}_{in} = \vec{E}_0 - \vec{E}_{ind}$$

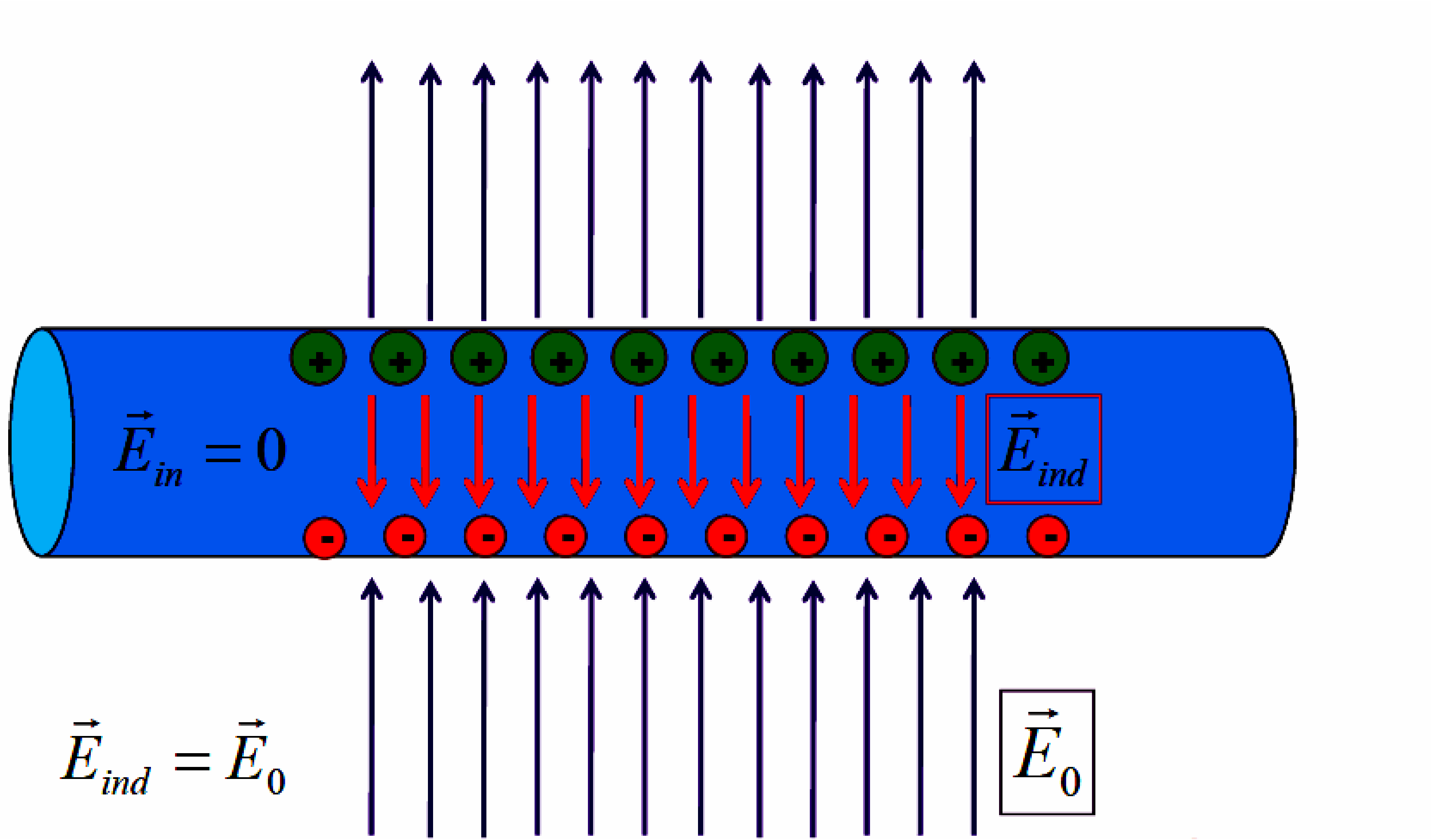
$$\vec{E}_{ind} = \frac{\vec{E}_0}{\kappa}$$



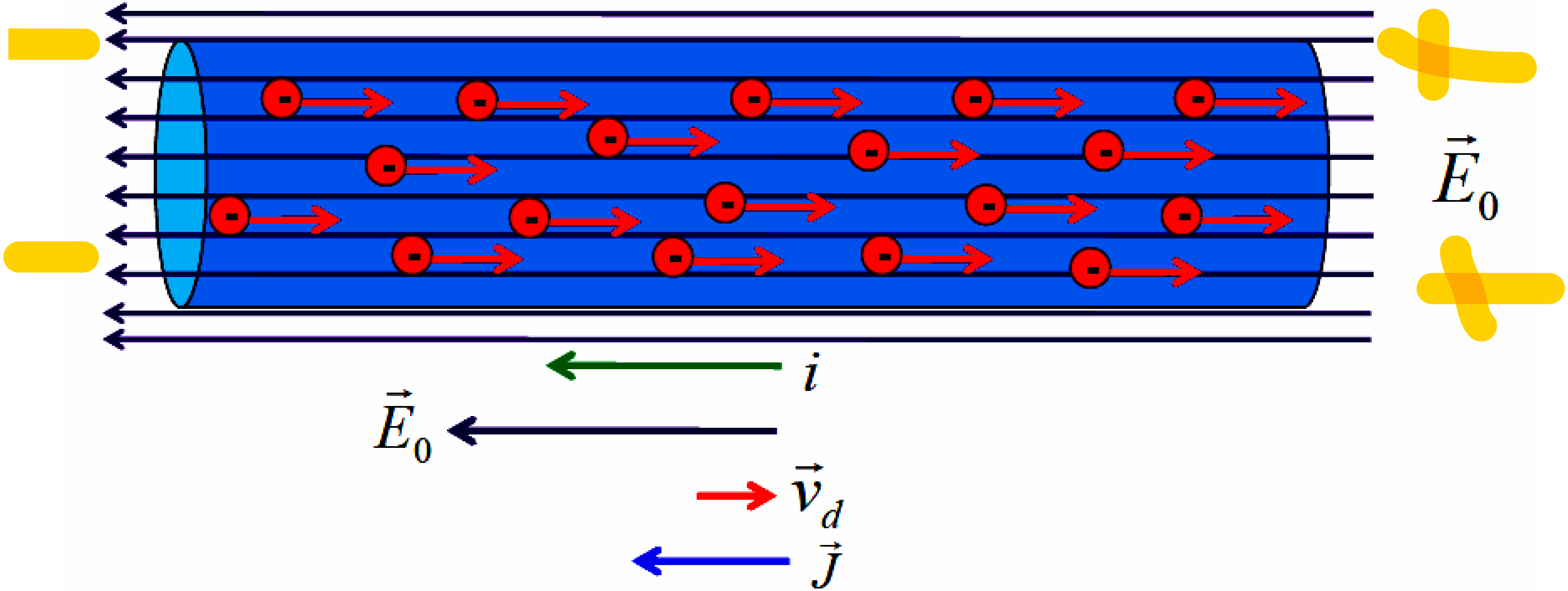
Polarization



Conductor in an Electric Field; Electrostatic Case



Conductor in an Electric Field; Electrodynamics Case



$$\vec{E}_{in} = \vec{E}_{out} = \vec{E}_0$$

Electric Current

The current is the rate at which charge flows through the surface.

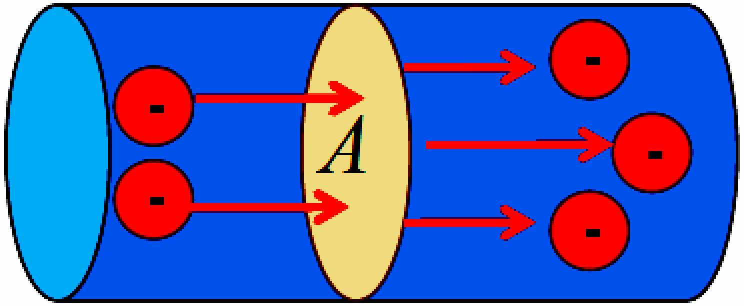
If dq is the amount of charge that passes through an area in a time interval d*t*, then the current *i* through that area is defined as:

$$\boxed{i = \frac{dq}{dt}}$$

Instantaneous Current

Charge that passes through that area in a time interval extending from 0 to t will be

$$q = \int dq = \int_0^t i\,dt$$



The unit of current is ampere (A), which is define as:

$$1 \ \ ampere = \frac{1 \ \ coulomb}{second}$$

If current is uniform

$$q = i\int_0^t dt = it$$

$$i = \frac{q}{t}$$

Uniform Current (DC)

A car's battery supplies, in general, up to 50 amperes when starting the car, but often we need to deal with smaller values:

$$1 \text{ milliampere} = 1 \text{ ma} = 10^{-3} A$$

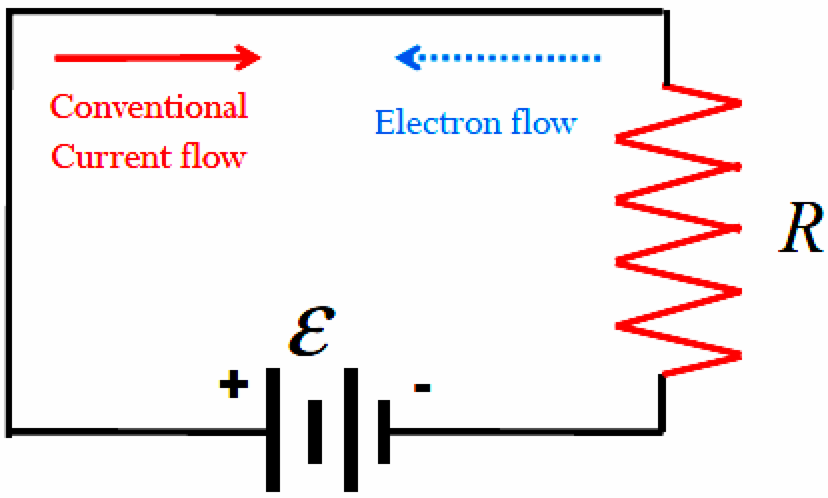
$$1 \text{ microampere} = 1\ \mu A = 10^{-6} A$$

$$1 \text{ nanoampere} = 1 \text{ n}A = 10^{-9} A$$

$$1 \text{ picoampere} = 1 \text{ p}A = 10^{-12} A$$

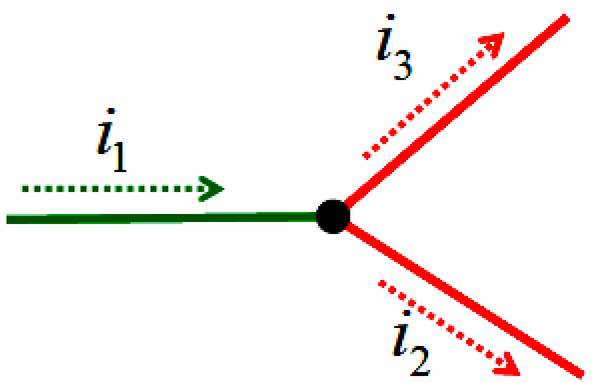
The direction of current flow is the direction in which positive charges move (Conventional current).

However, in a typical wire, the positive charges are fixed to the atoms and it is really the negative charges (electrons) that move. In that case the direction of current flow is reversed.



Charge is always conserved, and therefore current is conserved as well.

$$i_1 = i_2 + i_3$$



It shows that current is a scalar quantity.

Current Density

Consider a conductor of cross-sectional area A carrying a current i. The current density J in the conductor is defined as the current per unit area.

$$J = i / A$$

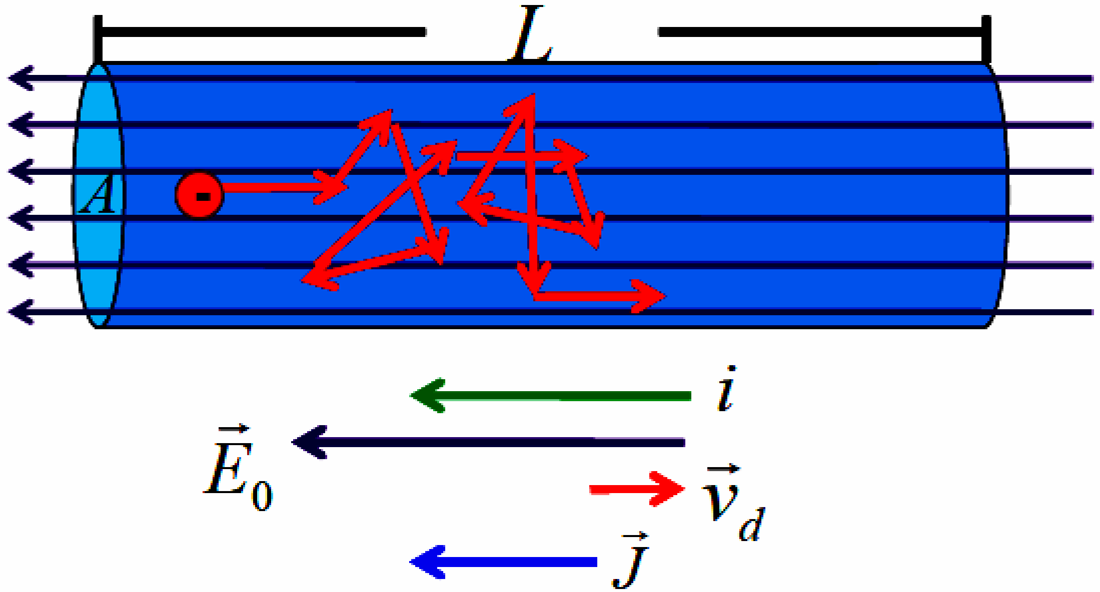
The direction of current density $\vec{J}$ is defined to be the direction of conventional current

The current passing through any surface can be determined by

$$i = \int \vec{J} \bullet d\vec{A}$$

Drift Speed

An electron inside a wire moves under the influence of the applied electric field and suffers many collisions with other electrons and ions that cause it to move on a highly irregular, jagged path as shown below.



$$\boxed{\vec{J} = -en\vec{v}_d}$$

The net effect is the drift of electron in the direction opposite to the electric field that causes the current.

Consider the motion of electrons in a portion of the conductor of length L and cross sectional area A. The electrons are moving with the drift speed v_d, so they will travel the length L in the time

$$t = L / v_d \qquad \therefore s = vt$$

If n is the number density of electrons in the conductor then total number of electrons travelling through the length L (from left face to right one) is

$$number\ of\ electrons = nAL$$

So the total charge travelling through the conductor will be

$$q = e(number\ of\ electronn)$$

$$q = enAL$$

So the current density is

$$J = \frac{i}{A} = \frac{q}{At} \qquad \therefore i = q / t$$

$$= \frac{enLA}{AL / v_d}$$

$$= env_d \qquad \therefore t = L / v_d$$

$$\boxed{\vec{J} = -en\vec{v}_d}$$

Vector form

$$\boxed{i = JA = env_d A}$$

The copper wire in a typical residential building has a cross-sectional area of $3.3\times10^{-6}\,m^2$ If it carries a current of 10.0 A, what is the drift speed of the electrons? Assume that free electron density of copper is

$n = 8.49\times10^{28}$ electrons/m^3 .

As we know

$$i = env_d A$$

$$v_d = \frac{i}{enA}$$

$$v_d = \frac{10A}{(1.6\times10^{-19}C)(8.49\times10^{28}/m^3)(3.3\times10^{-6}m^2)}$$

$$v_d = 2.22\times10^{-4}\,m/s$$

Suppose that the current through a conductor decreases exponentially with time according to the equation $I(t) = I_0 \exp(-t/\tau)$ where I_0 is the initial current (at $t = 0$), and τ is a constant having dimensions of time. Consider a fixed observation point within the conductor.

(a) How much charge passes this point between $t = 0$ and $t = \tau$?

(b) How much charge passes this point between $t = 0$ and $t = 10\,\tau$?

(c) How much charge passes this point between $t = 0$ and $t = \infty$?

(a)

$$q = \int_0^t i\,dt = \int_0^\tau I_\circ e^{-t/\tau}\,dt$$

$$= (-I_\circ \tau e^{-t/\tau})_0^\tau = I_\circ \tau (1 - e^{-1})$$

$$= (0.63) I_\circ \tau$$

(b)

$$q = \int_0^t i\,dt = \int_0^{10\tau} I_\circ e^{-t/\tau}\,dt$$

$$= (-I_\circ \tau e^{-t/\tau})_0^{10\tau} = I_\circ \tau (1 - e^{-10})$$

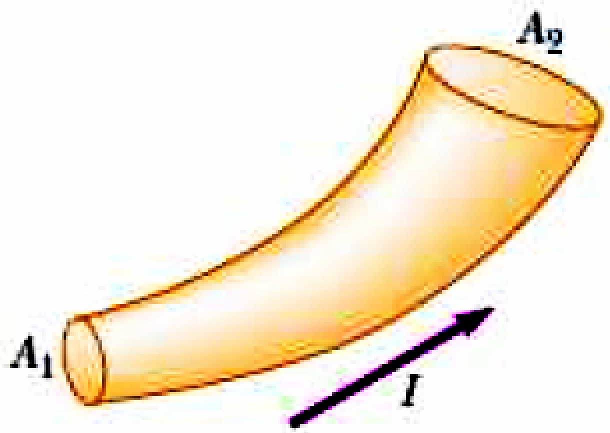
$$= (0.999) I_\circ \tau$$

(c)

$$q = \int_0^t i\,dt = \int_0^{\infty} I_\circ e^{-t/\tau}\,dt$$

$$= (-I_\circ \tau e^{-t/\tau})_0^{\infty} = I_\circ \tau$$

Figure represents a section of a circular conductor of non uniform diameter carrying a current of 5.00 A. The radius of cross section A1 is 0.4 cm. (a) What is the magnitude of the current density across A1? (b) If the current density across A2 is one-fourth the value across A1, what is the radius of the conductor at A2?



(a) $$J_1 = \frac{i}{A_1} = \frac{i}{\pi r_1^{\ 2}} = 99.5kA/m^2$$

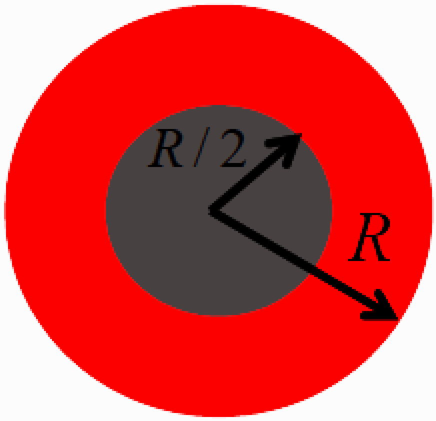
(b) $$J_2 = \frac{1}{4} J_1$$

$$\frac{i}{A_2} = \frac{1}{4}\frac{i}{A_1} \Rightarrow \frac{1}{\pi r_2^2} = \frac{1}{4\pi r_1^2}$$

$$r_2 = 2r_1 = 0.8cm$$

The current density in the cylindrical wire of radius R = 2mm is uniform across the cross sectional area of the wire and is $J = 2\times10^5\ A/m^2$.

What is the current through the outer portion of the wire between the radial distance R/2 and R.



$$i = JA$$

$$i = J\left[\pi R^2 - \pi\left(\frac{R}{2}\right)^2\right]$$

$$i = J\frac{3}{4}\pi R^2$$

$$i = 1.9\,A$$