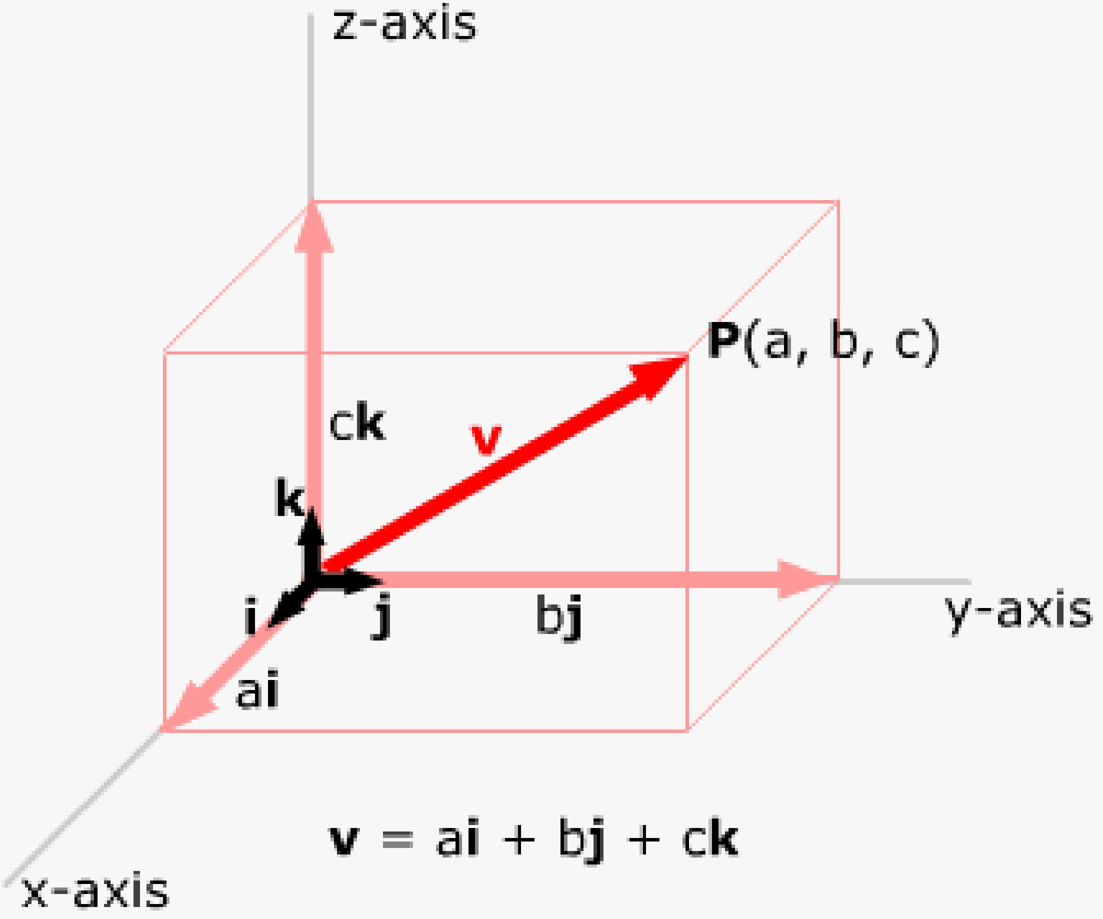
Vectors in Coordinate Systems



Vector Calculus(MATH-243)
Instructor: Dr. Naila Amir

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Vectors And
The Geometry Of Space

Book: Thomas' Calculus Early Transcendentals (14th Edition) By George B. Thomas, Jr., Joel Hass, Christopher Heil, Maurice D. Weir.

Section: 12.2, 12.3, 12.4

Book: Calculus Early Transcendentals (6^{th} Edition) By James Stewart.

Section: 12.2, 12.3, 12.4

Components of a Vector

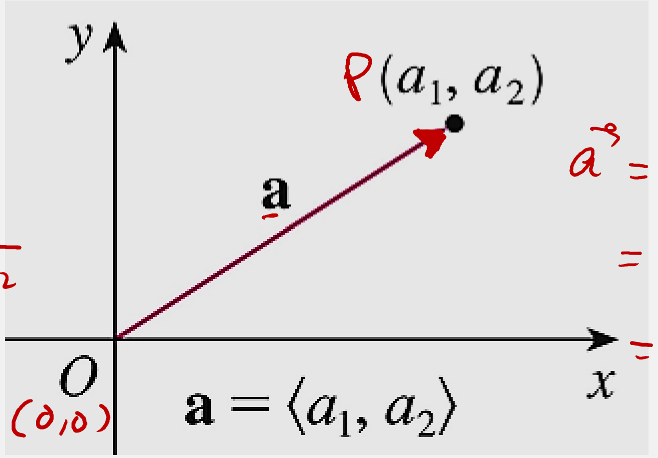
- For some purposes, it's best to introduce a coordinate system and treat vectors algebraically.
- Let's place the initial point of a vector **a** at the origin of a rectangular coordinate system.
- Then, the terminal point of **a** has coordinates of the form (a_1, a_2) or (a_1, a_2, a_3).
- This depends on whether our coordinate system is two- or three-dimensional.

$\vec{a} = \vec{OP}$

$|\vec{a}| = |\vec{OP}|$

$= \sqrt{(a_1-0)^2 + (a_2-0)^2}$

$= \sqrt{a_1^2 + a_2^2}$

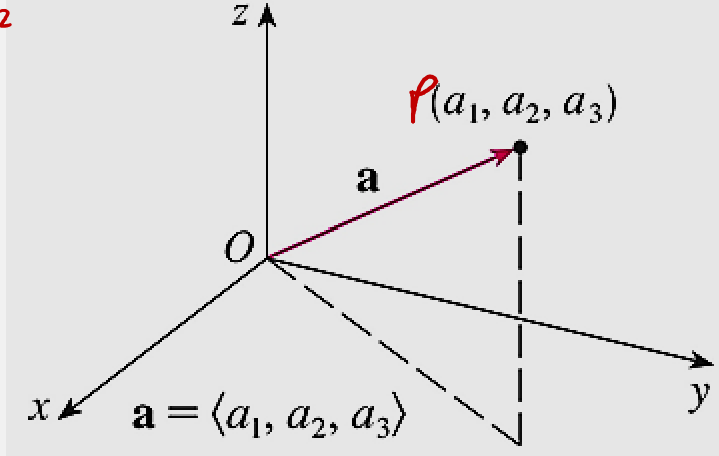


$\vec{a} = \vec{OP} \in \mathbb{R}^2$

$\vec{a} = \langle a_1, a_2 \rangle$

$= a_1 \hat{i} + a_2 \hat{j}$

$= a_1 \langle 1, 0 \rangle + a_2 \langle 0, 1 \rangle$



Components of a Vector

These coordinates are called the components of vector $\mathbf{a}$ and we write:

$$\mathbf{a} = \langle a_1, a_2 \rangle \quad \text{or} \quad \mathbf{a} = \langle a_1, a_2, a_3 \rangle.$$

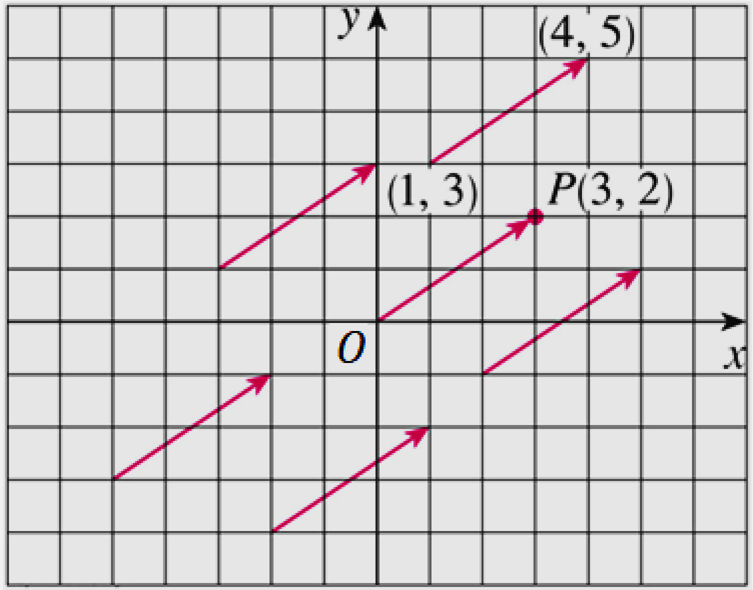
We use the notation $\langle a_1, a_2 \rangle$ for the ordered pair that refers to a vector so as not to confuse it with the ordered pair (a_1, a_2) that refers to a point in the plane.

Components of a Vector

For instance, the vectors shown are all equivalent to the vector $\overrightarrow{OP} = \langle 3,2 \rangle$ whose terminal point is $P(3, 2)$. What they have in common is that the terminal point is reached from the initial point by a displacement of three units to the right and two upward.

We can think of all these geometric vectors as representations of the algebraic vector $\overrightarrow{OP} = \langle 3,2 \rangle$.

The particular representation $\overrightarrow{OP}$ from the origin to the point $P(3,2)$ is called the position vector of the point P.

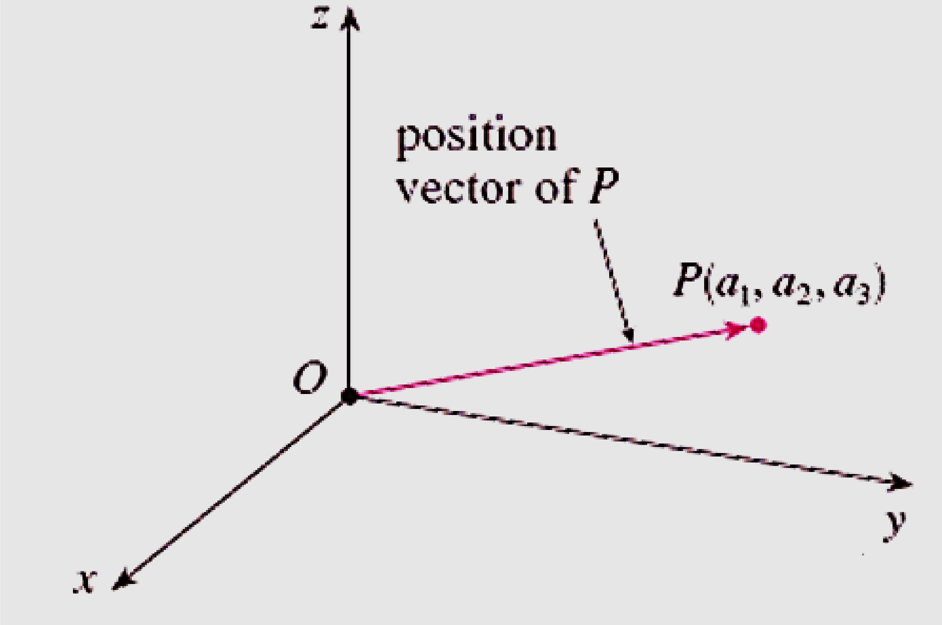


Position Vector

In three dimensions, the vector

$$\mathbf{a} = \overrightarrow{OP} = \langle a_1, a_2, a_3 \rangle,$$

is the position vector of the point $P(a_1, a_2, a_3)$.



Another Representation of a Vector

Given the points $A(x_1, y_1, z_1)$ and $B(x_2, y_2, z_2)$, the vector $\mathbf{a}$ with representation $\overrightarrow{AB}$ is given as:

$$\mathbf{a} = \overrightarrow{AB} = \langle x_2 - x_1, y_2 - y_1, z_2 - z_1 \rangle. \qquad (1)$$

$\overrightarrow{AB} \neq \overrightarrow{BA}$
I F I F

$= (x_2 - x_1)\hat{i} + (y_2 - y_1)\hat{j} + (z_2 - z_1)\hat{k}.$

$\overrightarrow{AB} = -(\overrightarrow{BA})$

Example:

Find the vector represented by the directed line segment with initial point $A(2, -3, 4)$ and terminal point $B(-2, 1, 1)$.

Solution:

By using equation (1), the vector corresponding to $\overrightarrow{AB}$ is given as:

$$\mathbf{a} = \langle -2 - 2, 1 - (-3), 1 - 4 \rangle = \langle -4, 4, -3 \rangle. = -4\hat{i} + 4\hat{j} - 3\hat{k}.$$

Length Of Vector

The magnitude, length or norm of the vector **v** is the length of any of its representations. It is denoted by the symbol $|\mathbf{v}|$ or $\|\mathbf{v}\|$.

$P(a_1, a_2)$
$O(0, 0)$

- The length of the two-dimensional (2-D) vector $\mathbf{a} = \langle a_1, a_2 \rangle$ is given as:

$\vec{AB} = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2}$

where $A(x_1, y_1)$
$B(x_2, y_2)$

$$|\mathbf{a}| = \sqrt{a_1^2 + a_2^2}.$$

$\vec{a} = \vec{OP}$

$|\vec{a}| = \sqrt{(a_1-0)^2 + (a_2-0)^2}$

$= \sqrt{a_1^2 + a_2^2}$

- The length of the three-dimensional (3-D) vector $\mathbf{a} = \langle a_1, a_2, a_3 \rangle$ is:

$$|\mathbf{a}| = \sqrt{a_1^2 + a_2^2 + a_3^2}.$$

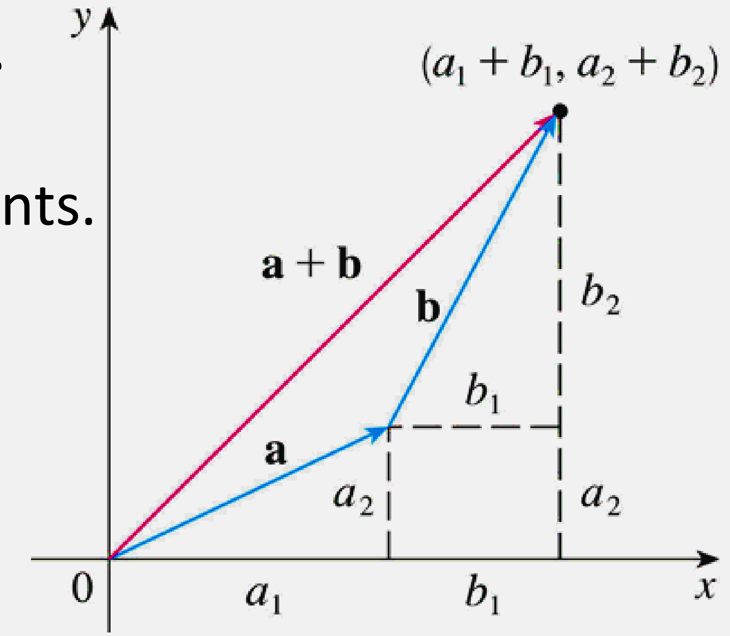
Algebraic Vectors (Addition & Subtraction of Algebraic Vectors)

How do we add vectors algebraically?
The figure shows that, if $\mathbf{a} = \langle a_1, a_2 \rangle$ and $\mathbf{b} = \langle b_1, b_2 \rangle$, then the sum is:

$$\mathbf{a} + \mathbf{b} = \langle a_1 + b_1, a_2 + b_2 \rangle,$$

at least for the case where the components are positive.

- In order to add algebraic vectors, we add their components.
- Similarly, to subtract vectors, we subtract components.



2-D Algebraic Vectors

$\vec{a}, \vec{b} \in \mathbb{R}^2$

If $\mathbf{a} = \langle a_1, a_2 \rangle$ and $\mathbf{b} = \langle b_1, b_2 \rangle$, then

$(-1)\,\vec{b} = \langle -b_1, -b_2 \rangle$

✓ **1.** $\mathbf{a} + \mathbf{b} = \langle a_1 + b_1, a_2 + b_2 \rangle$, closure law w.r.t addition

$\vec{a} + [-\vec{b}]$

$= \langle a_1 + (-b_1), a_2 + (-b_2) \rangle$

2. $\mathbf{a} - \mathbf{b} = \langle a_1 - b_1, a_2 - b_2 \rangle$,

$\vec{a} - \vec{b} = \langle a_1 - b_1, a_2 - b_2 \rangle$

✓ 3. $c\mathbf{a} = \langle ca_1, ca_2 \rangle$. closure law w.r.t scalar multiplication

3-D Algebraic Vectors

If $\mathbf{a} = \langle a_1, a_2, a_3 \rangle$ and $\mathbf{b} = \langle b_1, b_2, b_3 \rangle$, then

$\vec{a}, \vec{b} \in \mathbb{R}^3$

✓ 1. $\mathbf{a} + \mathbf{b} = \langle a_1 + b_1, a_2 + b_2, a_3 + b_3 \rangle$,

2. $\mathbf{a} - \mathbf{b} = \langle a_1 - b_1, a_2 - b_2, a_3 - b_3 \rangle$,

✓ 3. $c\mathbf{a} = \langle ca_1, ca_2, ca_3 \rangle$.

Properties of Vectors

If $\mathbf{a}$, $\mathbf{b}$, and $\mathbf{c}$ are vectors in a vector space V and c & d are scalars, then

Vector Space

1. $\mathbf{a}+\mathbf{b}=\mathbf{b}+\mathbf{a}$
2. $\mathbf{a}+(\mathbf{b}+\mathbf{c})=(\mathbf{a}+\mathbf{b})+\mathbf{c}$
3. $\mathbf{a}+\mathbf{0}=\mathbf{a}$,
4. $\mathbf{a}+(-\mathbf{a})=\mathbf{0}$,

+ closure law w.r.t addition $\Rightarrow$ Additive Abelian Group

5. $c(\mathbf{a}+\mathbf{b})=c\mathbf{a}+c\mathbf{b}$,
6. $(c+d)\mathbf{a}=c\mathbf{a}+d\mathbf{a}$,
7. $(cd)\mathbf{a}=c(d\mathbf{a})$,
8. $1\mathbf{a}=\mathbf{a}$.

+ closure law w.r.t scalar multiplication

Vectors in $V_3 = \mathbb{R}^3$

Three vectors in $V_3 = \mathbb{R}^3$ play a special role. Let

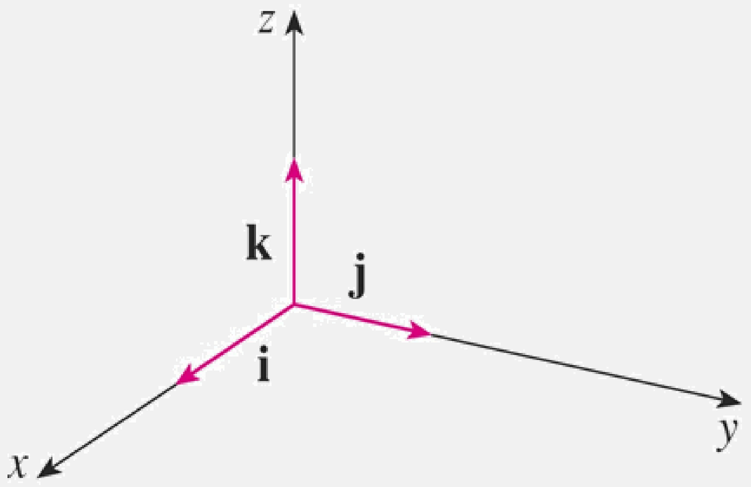
$$\mathbf{i} = \langle 1,0,0 \rangle,\ \ \mathbf{j} = \langle 0,1,0 \rangle \text{ and } \mathbf{k} = \langle 0,0,1 \rangle.$$

These vectors $\mathbf{i}$, $\mathbf{j}$, and $\mathbf{k}$ are called the standard basis vectors. They have length 1 (unit vectors) and point in the directions of the positive $x-$, $y-$, and z $-$axes.

$$\left.\begin{array}{l}\hat{i}\cdot\hat{i}\\ \hat{j}\cdot\hat{j}\\ \hat{k}\cdot\hat{k}\end{array}\right\} = 1$$

$$\Rightarrow |\hat{i}| = 1 = |\hat{j}| = |\hat{k}|$$

$$\hat{i}\cdot\hat{j} = 0 = \hat{j}\cdot\hat{k} = \hat{k}\cdot\hat{i}$$

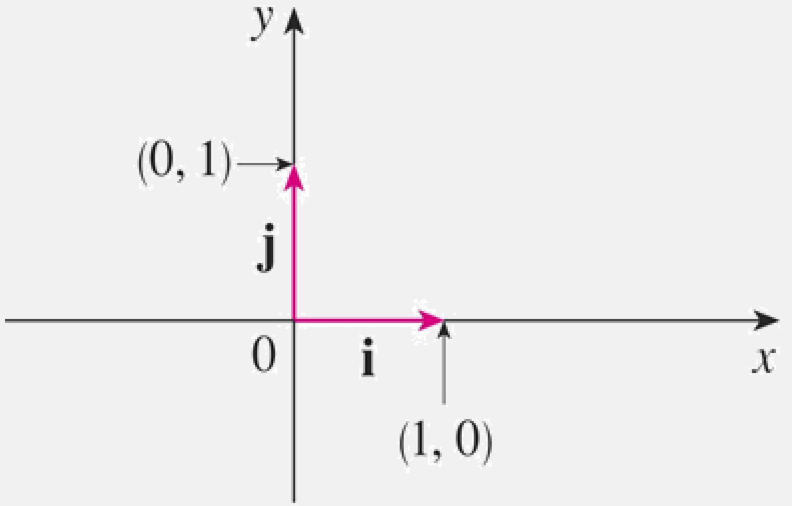


Vectors in $V_2 = \mathbb{R}^2$

Similarly, in two dimensions, we define:

$$\mathbf{i} = \langle 1,0 \rangle \text{ and } \mathbf{j} = \langle 0,1 \rangle.$$

These vectors $\mathbf{i}$ and $\mathbf{j}$ are called the standard basis vectors. They have length 1 and point in the directions of the positive $x-$ and $y-$axes.



Product of Vectors

The question arises:

Is it possible to multiply two vectors so that their product is a useful quantity?

And the answer is……… **Yes**

- One such product is the dot product.
- Another is the cross product.

Product

Dot Product → Scalar
2.3

Cross Product → Vector
2.4

The Dot Product

If $\mathbf{a} = \langle a_1, a_2, a_3 \rangle$ and $\mathbf{b} = \langle b_1, b_2, b_3 \rangle$, then the dot product of **a** and **b** is the number $\mathbf{a} \bullet \mathbf{b}$ given by:

$$\mathbf{a} \bullet \mathbf{b} = a_1 b_1 + a_2 b_2 + a_3 b_3.$$

The result is not a vector. It is a real number, that is, a scalar. For this reason, the dot product is sometimes called the **scalar product** or **inner product**.

Examples:

$\hat{i} \cdot \hat{i} = \hat{j} \cdot \hat{j} = \hat{k} \cdot \hat{k} = 1$

$\hat{i} \cdot \hat{j} = \hat{j} \cdot \hat{k} = \hat{k} \cdot \hat{i} = 0$

1. $\langle 2, 4 \rangle \cdot \langle 3, -1 \rangle = 2(3) + 4(-1) = 2.$

2. $\langle -1, 7, 4 \rangle \cdot \langle 6, 2, -½ \rangle = (-1)(6) + 7(2) + 4(-½) = 6.$

3. $(\mathbf{i} + 2\mathbf{j} - 3\mathbf{k}) \cdot (2\mathbf{j} - \mathbf{k}) = \boxed{7.}$

Properties of Dot Product

The dot product obeys many of the laws that hold for ordinary products of real numbers. These are stated in the following theorem.

Theorem: If $\mathbf{a}$, $\mathbf{b}$, and $\mathbf{c}$ are vectors in a vector space V and c is a scalar, then

1. $\mathbf{a} \cdot \mathbf{a} = |\mathbf{a}|^2$
2. $\mathbf{a} \cdot \mathbf{b} = \mathbf{b} \cdot \mathbf{a}$
3. $\mathbf{a} \cdot (\mathbf{b} + \mathbf{c}) = \mathbf{a} \cdot \mathbf{b} + \mathbf{a} \cdot \mathbf{c}$
4. $(c\mathbf{a}) \cdot \mathbf{b} = c(\mathbf{a} \cdot \mathbf{b}) = \mathbf{a} \cdot (c\mathbf{b})$
5. $\mathbf{0} \cdot \mathbf{a} = 0$

$\vec{0} \in \mathbb{R}^3$

$\vec{a} \in \mathbb{R}^3$

$\vec{0} \cdot \vec{a} = \langle 0, 0, 0 \rangle \cdot \langle a_1, a_2, a_3 \rangle$

$= (0)(a_1) + (0)(a_2) + (0)(a_3)$

$= 0 + 0 + 0$

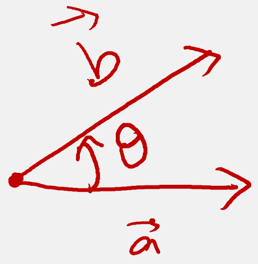
$= 0$

Geometric Interpretation of Dot Product

Algebraically, the dot product is the **sum of the products** of the corresponding entries of the two vectors. Geometrically, it is the product of the Euclidean magnitudes of the two vectors and the cosine of the angle between them. This means that the dot product $\mathbf{a} \bullet \mathbf{b}$ can be given a geometric interpretation in terms of the angle θ between $\mathbf{a}$ and $\mathbf{b}$. If θ is the angle between the vectors $\mathbf{a}$ and $\mathbf{b}$, then

α: angle between $\vec{a}$ and +ve x-axis

$$\mathbf{a} \bullet \mathbf{b} = |\mathbf{a}||\mathbf{b}| \cos\theta\,,$$



or

$$\cos\alpha = \frac{\vec{a}\cdot\hat{i}}{|\vec{a}|} \Rightarrow \alpha = \cos^{-1}\left(\frac{\vec{a}\cdot\vec{i}}{|\vec{a}|}\right)$$

$$\cos\theta = \frac{\mathbf{a} \bullet \mathbf{b}}{|\mathbf{a}||\mathbf{b}|}.$$

β: angle between $\vec{a}$ and +ve y-axis

$$\cos\beta = \frac{\vec{a}\cdot\hat{j}}{|\vec{a}|} \Rightarrow \beta = \cos^{-1}\left(\frac{\vec{a}\cdot\vec{j}}{|\vec{a}|}\right)$$

γ: angle between $\vec{a}$ and +ve z-axis

$$\cos\gamma = \frac{\vec{a}\cdot\hat{k}}{|\vec{a}|}\ ; \quad \gamma = \cos^{-1}\left(\frac{\vec{a}\cdot\hat{k}}{|\vec{a}|}\right)$$

Orthogonal Vectors

Two nonzero vectors $\mathbf{a}$ and $\mathbf{b}$ are called perpendicular or orthogonal if the angle between them is $\theta = \pi/2$, i.e., two vectors $\mathbf{a}$ and $\mathbf{b}$ are orthogonal if and only if:

$$\mathbf{a} \bullet \mathbf{b} = |\mathbf{a}||\mathbf{b}| \cos\left(\frac{\pi}{2}\right) = 0.$$

Example:

Show that $2\mathbf{i} + 2\mathbf{j} - \mathbf{k}$ is perpendicular to $5\mathbf{i} - 4\mathbf{j} + 2\mathbf{k}$.

Solution:

$$(2\mathbf{i} + 2\mathbf{j} - \mathbf{k}) \bullet (5\mathbf{i} - 4\mathbf{j} + 2\mathbf{k}) = 2(5) + 2(-4) + (-1)(2) = 0.$$

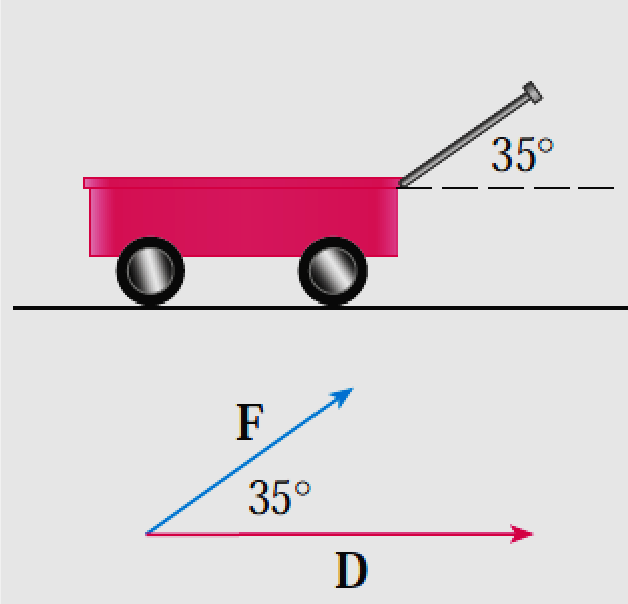
Thus, we conclude that the given vectors are perpendicular.

An Application of Dot Product: Work Done

The work done by a constant force $\mathbf{F}$ is the dot product $\mathbf{F}.\mathbf{D}$, where $\mathbf{D}$ is the displacement vector.

Example:

A wagon is pulled a distance of $100\ \mathrm{m}$ along a horizontal path by a constant force of $70\ \mathrm{N}$. The handle of the wagon is held at an angle of 35° above the horizontal. Find the work done by the force.



Solution:

If $\mathbf{F}$ and $\mathbf{D}$ are the force and displacement vectors respectively, then the work done is:

$$\mathbf{F} \bullet \mathbf{D} = |\mathbf{F}||\mathbf{D}|\cos(35^\circ) = (70)(100)\cos(35^\circ)$$
$$\approx 5734\ \mathrm{N.m} = 5734\ \mathrm{J}.$$

Thus, the work done by the force is $5734\ \mathrm{J}$.

The Cross Product

The cross product $\mathbf{a} \times \mathbf{b}$ of two vectors $\mathbf{a}$ and $\mathbf{b}$, unlike the dot product, is a vector quantity. For this reason, the cross product is also called the **vector product**. Note that $\mathbf{a} \times \mathbf{b}$ is defined only when $\mathbf{a}$ and $\mathbf{b}$ are three-dimensional (3-D) vectors. If $\mathbf{a} = \langle a_1, a_2, a_3 \rangle$ and $\mathbf{b} = \langle b_1, b_2, b_3 \rangle$, then the cross product of $\mathbf{a}$ and $\mathbf{b}$ is the vector:

$\vec{a} \times \vec{b} \neq \vec{b} \times \vec{a}$

$$\mathbf{a} \times \mathbf{b} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix} = \langle a_2 b_3 - a_3 b_2, a_3 b_1 - a_1 b_3, a_1 b_2 - a_2 b_1 \rangle.$$

$$\vec{b} \times \vec{a} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ b_1 & b_2 & b_3 \\ a_1 & a_2 & a_3 \end{vmatrix}$$

Example:

If $\mathbf{a} = \langle 1, 3, 4 \rangle$ and $\mathbf{b} = \langle 2, 7, -5 \rangle$, then determine $\mathbf{a} \times \mathbf{b}$.

Solution:

$$\mathbf{a} \times \mathbf{b} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & 3 & 4 \\ 2 & 7 & -5 \end{vmatrix} = \hat{i}(-15 - 28) - \hat{j}(-5 - 8) + \hat{k}(7 - 6)$$

$$= \langle -15 - 28, 8 + 5, 7 - 6 \rangle$$

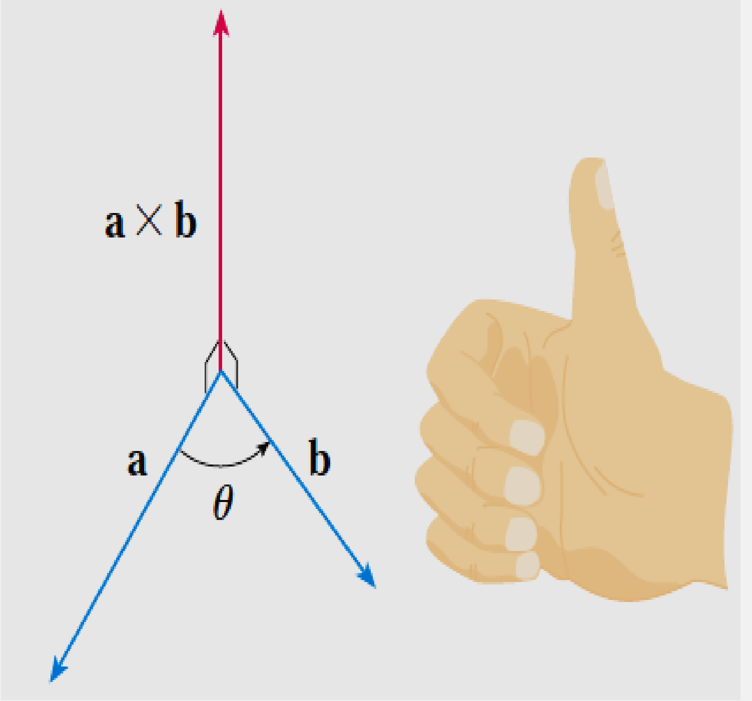
$$= \langle -43, 13, 1 \rangle$$

$$= -43\mathbf{i} + 13\mathbf{j} + \mathbf{k}.$$

Some Properties of Cross Product

- The cross product $\mathbf{a} \times \mathbf{b}$ is orthogonal to both $\mathbf{a}$ and $\mathbf{b}$. This means:
$$(\mathbf{a} \times \mathbf{b}) \cdot \mathbf{a} = 0 \quad \text{and} \quad (\mathbf{a} \times \mathbf{b}) \cdot \mathbf{b} = 0.$$

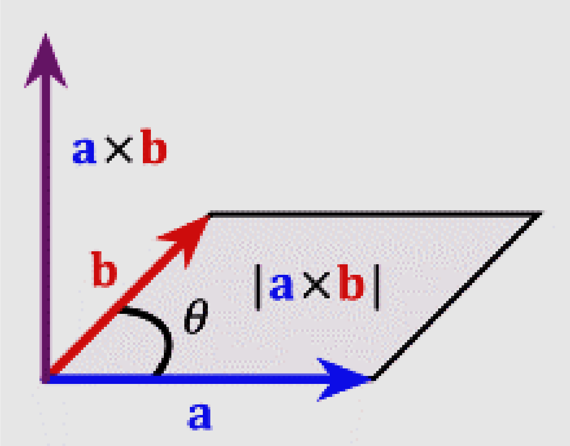
- If $\mathbf{a}$ and $\mathbf{b}$ are represented by directed line segments with the same initial point, then by using above property, $\mathbf{a} \times \mathbf{b}$ points in a direction perpendicular to the plane through $\mathbf{a}$ and $\mathbf{b}$. It turns out that the direction of $\mathbf{a} \times \mathbf{b}$ is given by the *right-hand rule:* If the fingers of our right-hand curl in the direction of a rotation (through an angle less than π) from $\mathbf{a}$ to $\mathbf{b}$, then the thumb points in the direction of $\mathbf{a} \times \mathbf{b}$.



Some Properties of Cross Product

- Now that we know the direction of the vector $\mathbf{a} \times \mathbf{b}$, the remaining thing we need to complete its geometric description is its length $|\mathbf{a} \times \mathbf{b}|$. If θ is the angle between $\mathbf{a}$ and $\mathbf{b}$ (so $0 \leq \theta \leq \pi$), then:

$$|\mathbf{a} \times \mathbf{b}| = |\mathbf{a}||\mathbf{b}| \sin\theta\,.$$



- Two nonzero vectors $\mathbf{a}$ and $\mathbf{b}$ are parallel if and only if:

$$\mathbf{a} \times \mathbf{b} = \mathbf{0}.$$

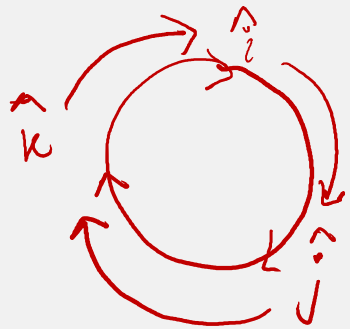
- The length of the cross product $\mathbf{a} \times \mathbf{b}$ is equal to the area of the parallelogram determined by $\mathbf{a}$ and $\mathbf{b}$.

Some Properties of Cross Product

- For the standard basis vectors **i**, **j**, and **k** we have:

$$\checkmark\ \mathbf{i}\times\mathbf{j}=\mathbf{k}, \qquad \checkmark\ \mathbf{j}\times\mathbf{k}=\mathbf{i}, \qquad \checkmark\ \mathbf{k}\times\mathbf{i}=\mathbf{j},$$

$$\checkmark\ \mathbf{j}\times\mathbf{i}=-\mathbf{k}, \qquad \checkmark\ \mathbf{k}\times\mathbf{j}=-\mathbf{i}, \qquad \checkmark\ \mathbf{i}\times\mathbf{k}=-\mathbf{j},$$



  Observe that:

$$\mathbf{i}\times\mathbf{j}\neq\mathbf{j}\times\mathbf{i}.$$

$$\vec{a}\times\vec{b}\neq\vec{b}\times\vec{a}$$

  Thus, the cross product is not commutative.

- The associative law for multiplication does not usually hold; that is, in general,

$$(\mathbf{a}\times\mathbf{b})\times\mathbf{c}\neq\mathbf{a}\times(\mathbf{b}\times\mathbf{c}).$$

Some Properties of Cross Product

If $\mathbf{a}$, $\mathbf{b}$, and $\mathbf{c}$ are vectors and c is a scalar, then

1. $\mathbf{a} \times \mathbf{b} = -\mathbf{b} \times \mathbf{a}$

2. $(c\mathbf{a}) \times \mathbf{b} = c(\mathbf{a} \times \mathbf{b}) = \mathbf{a} \times (c\mathbf{b})$

3. $\mathbf{a} \times (\mathbf{b} + \mathbf{c}) = \mathbf{a} \times \mathbf{b} + \mathbf{a} \times \mathbf{c}$

4. $(\mathbf{a} + \mathbf{b}) \times \mathbf{c} = \mathbf{a} \times \mathbf{c} + \mathbf{b} \times \mathbf{c}$

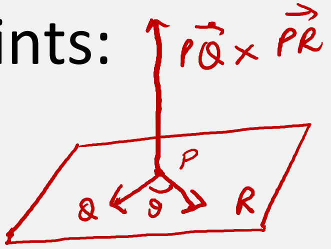
5. $\mathbf{a} \cdot (\mathbf{b} \times \mathbf{c}) = (\mathbf{a} \times \mathbf{b}) \cdot \mathbf{c}$

6. $\mathbf{a} \times (\mathbf{b} \times \mathbf{c}) = (\mathbf{a} \cdot \mathbf{c})\mathbf{b} - (\mathbf{a} \cdot \mathbf{b})\mathbf{c}$

Example:

Find a vector perpendicular to the plane that passes through the points:

$$P(1,4,6), Q(-2,5,-1), R(1,-1,1).$$



Solution:

The vector $\overrightarrow{PQ} \times \overrightarrow{PR}$ is perpendicular to both $\overrightarrow{PQ}$ and $\overrightarrow{PR}$. Therefore, it is perpendicular to the plane through P, Q, and R. Now

$$\overrightarrow{PQ} = -3\mathbf{i} + \mathbf{j} - 7\mathbf{k} \quad \text{and} \quad \overrightarrow{PR} = -5\mathbf{j} - 5\mathbf{k}.$$

Thus,

$$\overrightarrow{PQ} \times \overrightarrow{PR} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -3 & 1 & -7 \\ 0 & -5 & -5 \end{vmatrix} = -40\mathbf{i} - 15\mathbf{j} + 15\mathbf{k} = \langle -40, -15, 15 \rangle.$$

Example:

Find the area of the triangle with vertices $P(1,4,6), Q(-2,5,-1), R(1,-1,1)$.

Solution:

From previous example:

$$\overrightarrow{PQ} \times \overrightarrow{PR} = \langle -40, -15, 15 \rangle.$$

The area of the parallelogram with adjacent sides PQ and PR is the length of this cross product, i.e.,

$$\left|\overrightarrow{PQ} \times \overrightarrow{PR}\right| = \sqrt{(-40)^2 + (-15)^2 + (15)^2} = 5\sqrt{82}.$$

The area A of the triangle PQR is half the area of this parallelogram, that is:

$$A = \frac{1}{2}\left|\overrightarrow{PQ} \times \overrightarrow{PR}\right| = \frac{5}{2}\sqrt{82}.$$

An Application of Cross Product: Torque

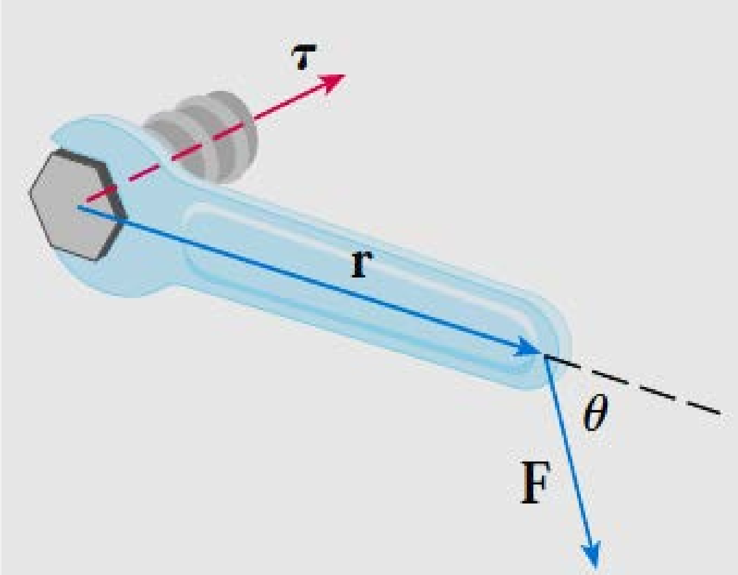
Consider a force $\mathbf{F}$ acting on a rigid body at a point given by a position vector $\mathbf{r}$. For instance, if we tighten a bolt by applying a force to a wrench, we produce a turning effect. The **torque** $\boldsymbol{\tau}$ (relative to the origin) is defined to be the cross product of the position and force vectors, i.e.,

$$\boldsymbol{\tau} = \mathbf{r} \times \mathbf{F},$$

and measures the tendency of the body to rotate about the origin. The direction of the torque vector indicates the axis of rotation, and the magnitude of the torque vector is:

$$|\boldsymbol{\tau}| = |\mathbf{r} \times \mathbf{F}| = |\mathbf{r}|\,|\mathbf{F}| \sin\theta\,,$$

where θ is the angle between the position and force vectors.



Example:

A bolt is tightened by applying a 40-N force to a 0.25-m wrench as shown in figure. Find the magnitude of the torque about the center of the bolt.

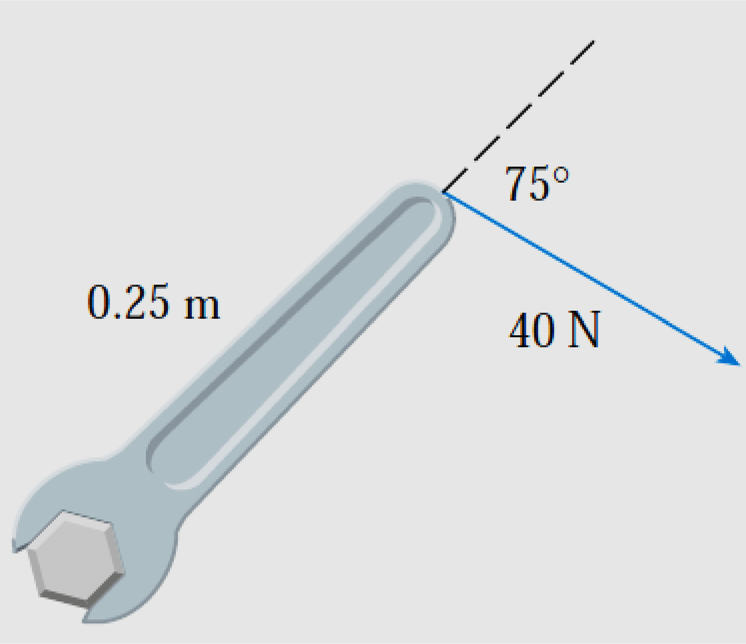
Solution:

The magnitude of the torque vector is given as:

$$|\boldsymbol{\tau}| = |\mathbf{r} \times \mathbf{F}| = |\mathbf{r}|\,|\mathbf{F}| \sin(75^{\circ})$$

$$= (0.25)(40) \sin\left(75^{\circ}\right)$$

$$\approx 9.66 \text{ N.m}$$



Practice Questions

Book: Thomas' Calculus Early Transcendentals (14th Edition) By George B. Thomas, Jr., Joel Hass, Christopher Heil, Maurice D. Weir.

Chapter: 12

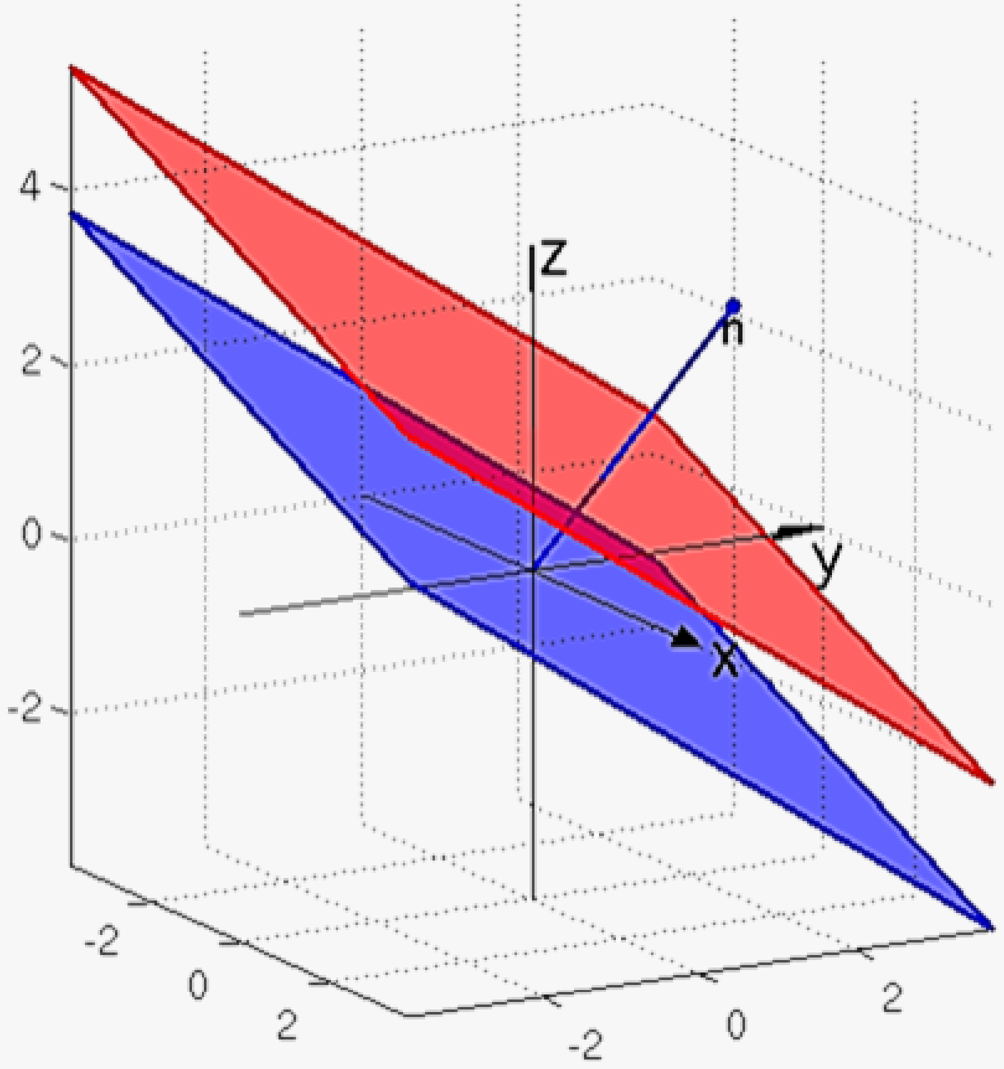
Exercise-12.1: Q – 1 to 4, Q – 6, Q – 11 to 18.

Exercise-12.2: Q – 1, Q – 6, Q – 7 to 20, Q – 26&27.

Exercise-12.3: Q – 2 to 10, Q – 13 to 20, Q – 23 to 25, Q – 45 to 48.

Exercise-12.4: Q – 1 to 7, Q – 14 to 18, Q – 21 to 32, Q – 39 to 42.

Equations of Lines and Planes



12

Vectors And
The Geometry Of Space

Book: Thomas' Calculus Early Transcendentals (14th Edition) By George B. Thomas, Jr., Joel Hass, Christopher Heil, Maurice D. Weir.

Section: 12.5

Book: Calculus Early Transcendentals (6th Edition) By James Stewart.

Section: 12.5

Lines in 2-D & 3-D

- A line in the xy −plane is determined when a point on the line and the direction of the line (its slope or angle of inclination) are given. The equation of the line can then be written using the point-slope form. Otherwise, we can determine equation of a line in 2-D if information about two points on the line is known.

2-D $\leftarrow \begin{cases} y = mx + c & \text{or} \\ y - y_0 = m(x - x_0) \end{cases}$

- A line L in 3-D space is determined when we have information about a point $P_0(x_0, y_0, z_0)$ on L and the direction of L, that can be determined with the help of a vector $\mathbf{v} = a\mathbf{i} + b\mathbf{j} + c\mathbf{k}$, that is parallel to L.