

National University of Sciences & Technology
School of Electrical Engineering and Computer Science
Department of Basic Sciences

MATH-101: Calculus and Analytical Geometry (3+0): BEE2k20-ABC Fall 2020

Assignment 4	
CLO-2 (Understand integration and use it to compute areas, volumes, and arc length.)	
Maximum Marks: 10	Instructor: Dr. Naila Amir
Announcement Date: 18 th January 2021	Due Date: 24 th January 2020

Instructions:

- Understanding the question is part of the assignment and copying is not allowed.
- Express your answer in the most simplified form. Direct calculations using calculator are not allowed, you need to show the detail of your work to get the maximum marks.
- This is an individual assignment.
- Assignment must be handwritten and properly scanned in a single pdf file. This page must be part of every assignment.
- Assignment must be properly tagged and is required to be submitted on MS teams.
- Assignment is not acceptable after deadline.

Tasks: Attempt all questions.

Students Name	CMS Id.	Section
Muhammad Umer	345834	BEE 12C

Total Marks	Marks Obtained
10 Marks	

Question # 1: [10 marks]

- a) Sketch the region bounded by the graphs of the functions
 $y^2 = x + 1$, and $y = x^2 + 4x + 1$.
- b) Determine the points of intersection of the curves given in part (a).
- c) Decide the limits of integration by using parts (a) and (b) and compute the area of the region that is bounded inside the graphs of both the curves.

Q 1

a) Graph (General)

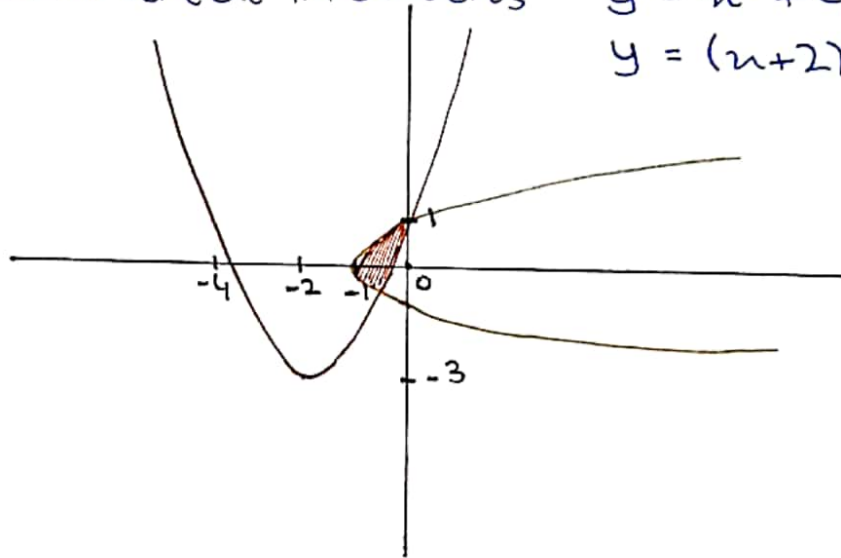
No Asymptotes

Horizontal & Vertical Translations

$$x = y^2 - 1$$

$$y = x^2 + 4x + 1$$

$$y = (x+2)^2 - 3$$



b) Points of Intersection

$$x = y^2 - 1 \quad \text{--- (i)}$$

$$y = (x+2)^2 - 3 \quad \text{--- (ii)}$$

Writing (ii) with y as a function of x

$$y + 3 = (x+2)^2$$

$$x = \sqrt{y+3} - 2$$

We are only concerned with the +ve solution / curve of the radical.

$$y^2 - 1 = \sqrt{y+3} - 2$$

$$y^2 + 1 = \sqrt{y+3}$$

$$y^4 + 1 + 2y^2 = y + 3$$

$$y^4 + 2y^2 - y - 2 = 0$$

$$(y-1)(y^3 + y^2 + 3y + 2) = 0$$

Hence,

$$y = 1 \quad \text{or} \quad y = -0.7152$$

which are the limits of integration.

c) Area b/w curves

$$A = \int_a^b |f(y) - g(y)| dy$$

$$= \int_{-0.7152}^1 \sqrt{y+3} - 2 - (y^2 - 1) dy$$

$$= \int_{-0.7152}^1 \sqrt{y+3} - 2 - y^2 + 1 dy$$

$$= \int_{-0.7152}^1 -y^2 + \sqrt{y+3} - 1 dy$$

$$= - \int_{-0.7152}^1 y^2 - \sqrt{y+3} + 1 dy$$

$$= - \left(\frac{y^3}{3} - \frac{2}{3} (y+3)^{3/2} + y \right) \Big|_{-0.7152}^1$$

$$= - \left[\left(\frac{1}{3} - \frac{2}{3} (4)^{3/2} + 1 \right) - \left(-\frac{0.3658}{3} - \frac{2}{3} (2.284)^{3/2} - 0.7152 \right) \right]$$

$$= - \left[(-4) - (-3.1383) \right]$$

$$= - \left[-4 + 3.1383 \right]$$

$$= - \left[-0.8617 \right]$$

$$A = 0.8617 \text{ sq. unit}$$