POISSON’S AND LAPLACE’S EQUATIONS

Introduction

➢ The procedure for determining the electric field **E** in the preceding lectures has generally been using either:

1. Coulomb's law or Gauss's law when the charge distribution is known
2. Using $\mathbf{E} = -\nabla V$ when the potential V is known throughout the region

➢ In most practical situations, however, neither the charge distribution nor the potential distribution is known

Boundary-value Problems

- We shall consider practical electrostatic problems where only electrostatic conditions (charge and potential) at some boundaries are known and it is desired to find **E** and V throughout the region

- Such problems are usually tackled using:

1. Poisson's equation
2. Or Laplace's equation
3. Or the Method of Images

- These problems are usually referred to as boundary value problems

Poisson's Equations

- Poisson's and Laplace's equations are easily derived from Gauss's law (for a linear material medium)

$$\nabla \cdot \mathbf{D} = \nabla \cdot \varepsilon \mathbf{E} = \rho_v \qquad \text{AND} \qquad \mathbf{E} = -\nabla V$$

- Using the two equations above, we get for an in-homogenous medium:

$$\nabla \cdot (-\varepsilon \nabla V) = \rho_v$$

- While for a homogenous medium:

$$\nabla^2 V = -\frac{\rho_v}{\varepsilon}$$

- This is known as Poisson's equation

Laplace's Equations

- A special case of this equation occurs when $\rho_v = 0$ (i.e., for a charge-free region)

$$\nabla^2 V = 0$$

- This is known as Laplace's equation

- Laplace's equation is of primary importance in solving electrostatic problems involving a set of conductors maintained at different potentials (capacitors and vacuum tube diodes)

Problem-1

- In a one-dimensional device, the charge density is given by $\rho_v = \rho_{vo}x/a$. If **E** = 0 at *x* = 0 and *V* = 0 at *x* = *a*, find *V* and **E**.

Problem-2

- The two plates of a parallel-plate capacitor are separated by a distance d and maintained at potentials 0 and V_o. The medium between the plates have no charge density. Determine:

a) Potential at any point between the plates

b) The surface charge densities at the plates

Problem-3

- Conducting spherical shells with radii a = 10 cm and b = 30 cm are maintained at a potential difference of 100 V such that V(r = b) = 0 and V(r = a) = 100 V. Determine V and **E** in the region between the shells. If ε_r = 2.5 in the region, determine the total charge induced on the shells and the capacitance of the capacitor.