Testing of Hypothesis

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Testing of Hypothesis

Testing of hypothesis is a procedure which enables us to decide on the basis of information obtained by sampling whether to accept or reject any specified statement or hypothesis about the value of the population parameter.

We accept the hypothesis as being true, when it supported by sample data. We reject the hypothesis when the sample data fail to support it.

Null Hypothesis & Alternative Hypothesis

Null Hypothesis

- A statement about the population parameter
- Its denoted by H_o
- Must contain condition of equality

Alternative Hypothesis

- The statement that directly contradict the null hypothesis.
- Its denoted by H_1
- In a mathematical formulation of the alternative hypothesis, there will typically be an inequality, or not equal to symbol.

Simple hypothesis & Composite Hypothesis

Simple hypothesis

A simple hypothesis specifies the population distribution completely.

Example: $H_0 : \mu = 65$ is a simple hypothesis

Composite Hypothesis

A composite hypothesis does not specify the population distribution completely.

Example: $H_1 : \mu > 65$, $H_1 : \mu < 65$, and $H_1 : \mu \# 65$ are composite hypotheses.

Test Statistic

- The test statistic is a value computed from the sample data, and it is used in making the decision about the rejection of the null hypothesis.
- It is important to remember that a test statistic does not prove the hypothesis to be correct but it furnishes the evidence against the hypothesis
- The sampling distribution of most commonly used test statistics are normal, t, chi-square and f.

Acceptance and Rejection Regions

Region of acceptance: The region of acceptance is a range of values. If the test statistic falls within the region of acceptance, the null hypothesis is not rejected. The region of acceptance is defined so that the chance of making a Type I error is equal to the significance level.

Region of rejection: The set of values outside the region of acceptance is called the region of rejection. If the test statistic falls within the region of rejection, the null hypothesis is rejected. In such cases, we say that the hypothesis has been rejected at the α level of significance.

Level of Significance

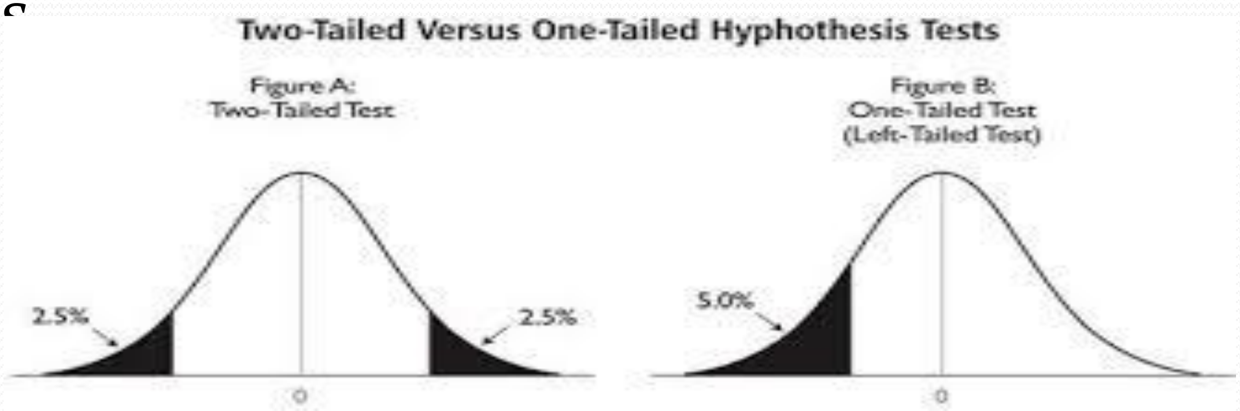
The level of significance is the probability of a Type I error (reject the null hypothesis when it is true) and is usually denoted by α.

Common level of significance are 0.05, 0.01, and 0.1.

One Tailed Test & Two Tailed Test

One Tailed Test: A test for which the entire rejection region is located in only one of the two tails either in the right tail or the left tail of the sampling distribution of the test statistics, is called one tailed or one sided test.

Two Tailed Test: The test is referred to the two tail test, when the rejection region is divided equally between the two tails of the sampling distribution of the test statistics



General procedure of testing of hypothesis

- State the null and alternative hypothesis
- Decide upon Significance level
- Choose an appropriate test statistics
- Determine the rejection region which is truly based in alternative hypothesis
- Compute the value of test statistics
- Conclusion: Accept or Reject the Null Hypothesis

Testing of Hypothesis for Population Mean

- State the **Null hypothesis** and **Alternative hypothesis**. Three possible forms are
  - $H_o : \boldsymbol{\mu = \mu_0}$ **and** $H_1 : \boldsymbol{\mu \# \mu_0}$
  - $H_o : \boldsymbol{\mu \geq \mu_0}$ **and** $H_1 : \boldsymbol{\mu < \mu_0}$
  - $H_o : \boldsymbol{\mu \leq \mu_0}$ **and** $H_1 : \boldsymbol{\mu > \mu_0}$
- **Level of Significance:**

Define the value of α.

- **Test Statistics:**

Choose an appropriate test statistics from the following table

Testing of Hypothesis for Population Mean

	$n\ large (n > 30)$	$n\ small (n \leq 30)$
σ known	$\boldsymbol{z} = \dfrac{\overline{\boldsymbol{X}} - \mu}{\frac{\sigma}{\sqrt{n}}}$	$\boldsymbol{z} = \dfrac{\overline{\boldsymbol{X}} - \mu}{\frac{\sigma}{\sqrt{n}}}$
σ unknown	$\boldsymbol{z} = \dfrac{\overline{\boldsymbol{X}} - \mu}{\frac{S}{\sqrt{n}}}$	$\boldsymbol{t} = \dfrac{\overline{\boldsymbol{X}} - \mu}{\frac{s}{\sqrt{n}}}$

Testing of Hypothesis for Population Mean

- **Rejection Region**

Find the rejection region, which actually depends on the alternative hypothesis, from the table of areas under the normal curve by finding areas exactly equal to α.

When the alternative hypothesis is	The rejection region value will be
$H_1 : \boldsymbol{\mu} \# \boldsymbol{\mu_0}$(two sided)	$z < -z_{\alpha/2}$ and $z > z_{\alpha/2}$
$H_1 : \boldsymbol{\mu} < \boldsymbol{\mu_0}$ (one sided)	$z < -z_{\alpha}$
$H_1 : \boldsymbol{\mu} > \boldsymbol{\mu_0}$ (one sided)	$z > z_{\alpha}$

Testing of Hypothesis for Population Mean

- **Computations:**

Calculate the value of z or t from the sample data.

- **Conclusion:**

Reject $H_{o,}$ when the calculated value of z or t falls in the rejection region, otherwise, accept it.

In the case of rejection, the decision would be that $\boldsymbol{\mu}$ differ from $\boldsymbol{\mu_0}$.

Question 1

A process is in control when the average amount of instant coffee that is packed in a jar, is 6 oz. The standard deviation is 0.2 oz. A sample of 100 jars is selected at random and the sample average is found to be 6.1 oz. Is the process out of control? [$\alpha = 5\%$]

Solution

- Hypothesis
  - H_o : $\boldsymbol{\mu}$ **=6**
  - H_1 : $\boldsymbol{\mu}$**#6**
- **Level of Significance:**

  α=0.05.
- **Test Statistics:**

Here n=100, σ=0.2, $\overline{X}$=6.1, so n is large and σ is known, then we use

$$\boldsymbol{z} = \frac{\overline{\boldsymbol{X}} - \mu}{\frac{\sigma}{\sqrt{n}}}$$

Solution

- The critical region is z < $-z_{0.025}$=-1.96 and z > $z_{0.025}$=1.96
- **Computations:**

Here n=100, σ=0.2, $\bar{X}$=6.1, and μ_0=6

$$z = \frac{\bar{X}-\mu}{\frac{\sigma}{\sqrt{n}}} = \frac{6.1-6}{\frac{0.2}{\sqrt{100}}} = 5$$

- **Conclusion:**

the calculated value of z falls in the rejection region ,so we reject H_o. Here we conclude that the process is out of control.

Question 2

In order to ensure efficient usage of a server, it is necessary to estimate the mean number of concurrent users. According to records, the average number of concurrent users at 100 randomly selected times is 37.7, with standard deviation 9.2.

- Construct a 90 % one sided confidence interval that provides an upper limit on average number of concurrent users.
- At the 1% significance level, do these data provide significance evidence that the mean number of concurrent users is greater than 35?

Question 3

A company is planning a large telephone survey and is interested in assessing how long it will take. In a short pilot study, 21 people are contacted by telephone and asked the specified set of questions. The times of these telephone surveys have a sample mean of 9.39 minutes with standard deviation of 1.041 minutes. Can the company safely conclude that the telephone surveys will last on average no more than 10 minutes each?[$\alpha = 5\%$]

Question 4

A fertilizer mixing machine is set to give 12 kg of nitrate for every 100kg bag of fertilizer. Six 100kg bags are examined. The percentages of nitrate are as follows: 11, 12, 13, 14, 11, and 12. Is there reason to believe that the machine is defective at 5% level of significance? {Tabulated value=2.571}

Table T Critical Values of the *t* Distribution

df	One-Tail = .4 Two-Tail = .8	.25 .5	.1 .2	.05 .1	.025 .05	.01 .02	.005 .01	.0025 .005	.001 .002	.0005 .001
1	0.325	1.000	3.078	6.314	12.706	31.821	63.657	127.32	318.31	636.62
2	0.289	0.816	1.886	2.920	4.303	6.965	9.925	14.089	22.327	31.598
3	0.277	0.765	1.638	2.353	3.182	4.541	5.841	7.453	10.214	12.924
4	0.271	0.741	1.533	2.132	2.776	3.747	4.604	5.598	7.173	8.610
5	0.267	0.727	1.476	2.015	2.571	3.365	4.032	4.773	5.893	6.869
6	0.265	0.718	1.440	1.943	2.447	3.143	3.707	4.317	5.208	5.959
7	0.263	0.711	1.415	1.895	2.365	2.998	3.499	4.029	4.785	5.408
8	0.262	0.706	1.397	1.860	2.306	2.896	3.355	3.833	4.501	5.041
9	0.261	0.703	1.383	1.833	2.262	2.821	3.250	3.690	4.297	4.781
10	0.260	0.700	1.372	1.812	2.228	2.764	3.169	3.581	4.144	4.587
11	0.260	0.697	1.363	1.796	2.201	2.718	3.106	3.497	4.025	4.437
12	0.259	0.695	1.356	1.782	2.179	2.681	3.055	3.428	3.930	4.318
13	0.259	0.694	1.350	1.771	2.160	2.650	3.012	3.372	3.852	4.221
14	0.258	0.692	1.345	1.761	2.145	2.624	2.977	3.326	3.787	4.140
15	0.258	0.691	1.341	1.753	2.131	2.602	2.947	3.286	3.733	4.073
16	0.258	0.690	1.337	1.746	2.120	2.583	2.921	3.252	3.686	4.015
17	0.257	0.689	1.333	1.740	2.110	2.567	2.898	3.222	3.646	3.965
18	0.257	0.688	1.330	1.734	2.101	2.552	2.878	3.197	3.610	3.922
19	0.257	0.688	1.328	1.729	2.093	2.539	2.861	3.174	3.579	3.883
20	0.257	0.687	1.325	1.725	2.086	2.528	2.845	3.153	3.552	3.850
21	0.257	0.686	1.323	1.721	2.080	2.518	2.831	3.135	3.527	3.819
22	0.256	0.686	1.321	1.717	2.074	2.508	2.819	3.119	3.505	3.792
23	0.256	0.685	1.319	1.714	2.069	2.500	2.807	3.104	3.485	3.767
24	0.256	0.685	1.318	1.711	2.064	2.492	2.797	3.091	3.467	3.745
25	0.256	0.684	1.316	1.708	2.060	2.485	2.787	3.078	3.450	3.725
26	0.256	0.684	1.315	1.706	2.056	2.479	2.779	3.067	3.435	3.707
27	0.256	0.684	1.314	1.703	2.052	2.473	2.771	3.057	3.421	3.690
28	0.256	0.683	1.313	1.701	2.048	2.467	2.763	3.047	3.408	3.674
29	0.256	0.683	1.311	1.699	2.045	2.462	2.756	3.038	3.396	3.659
30	0.256	0.683	1.310	1.697	2.042	2.457	2.750	3.030	3.385	3.646
40	0.255	0.681	1.303	1.684	2.021	2.423	2.704	2.971	3.307	3.551
60	0.254	0.679	1.296	1.671	2.000	2.390	2.660	2.915	3.232	3.460
120	0.254	0.677	1.289	1.658	1.980	2.358	2.617	2.860	3.160	3.373
∞	0.253	0.674	1.282	1.645	1.960	2.326	2.576	2.807	3.090	3.291

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