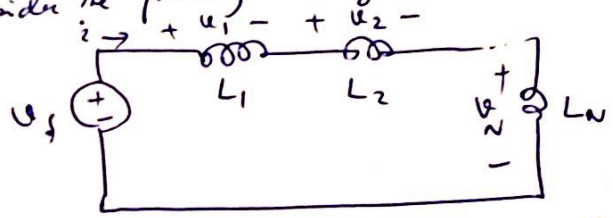
7.3 Inductance and Capacitance Combinations

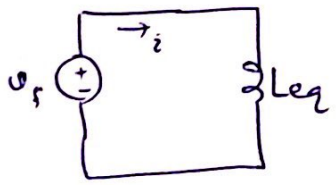
(pp 235 8th Ed HKD)

Inductors in Series (pp 235)

Consider the following arrangement:



The desired equivalent circuit (with v_s and $i(t)$ remaining same) is:



Now $v_s = v_1 + v_2 + \cdots v_N$

$$= L_1 \frac{di}{dt} + L_2 \frac{di}{dt} + \cdots L_N \frac{di}{dt}$$

$$= (L_1 + L_2 + \cdots L_N) \frac{di}{dt}$$

For the equivalent circuit

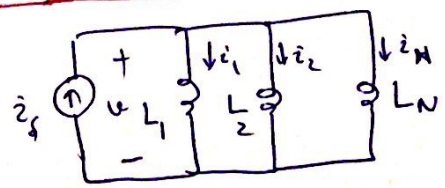
$$v_s = L_{eq} \frac{di}{dt}$$

Thus $$L_{eq} = L_1 + L_2 + \cdots L_N$$

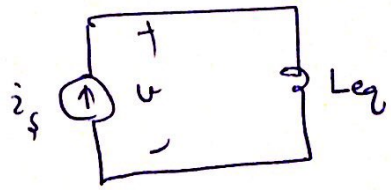
or $$L_{eq} = \sum_{n=1}^{N} L_n$$

Inductors in Parallel (PP236)

Consider



and equivalent cct:



Now $v = L_1 \frac{di_1}{dt}$ or $i_1 = \frac{1}{L_1}\int v\,dt$

and $i_2 = \frac{1}{L_2}\int v\,dt$ and $i_3 = \frac{1}{L_3}\int v\,dt$

For the equivalent cct:-

$$i_s = \frac{1}{L_{eq}}\int v\,dt$$

So $$i_s = i_1 + i_2 + \cdots i_N = \left(\frac{1}{L_1} + \frac{1}{L_2} + \cdots \frac{1}{L_N}\right)\int v\,dt$$

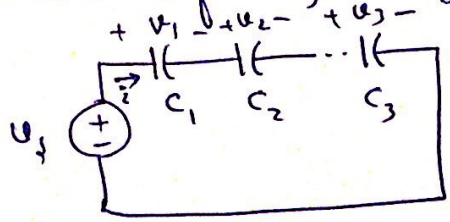
Hence $$\frac{1}{L_{eq}} = \frac{1}{L_1} + \frac{1}{L_2} + \cdots \frac{1}{L_N}$$

For the special case of two inductors in parallel,

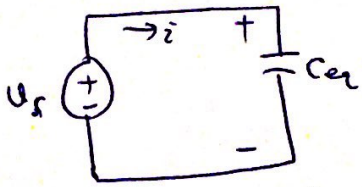
$$L_{eq} = \frac{L_1 L_2}{L_1 + L_2}$$

Capacitors in Series (PP 236)

Consider the following arrangement:



and equivalent circuit



Because same charge appears on each plate

$$Q = C_1 v_1 \quad , \quad Q = C_2 v_2 \quad \text{and} \quad Q = C_3 v_3$$

also $\quad Q = C_{eq} v_s$

Now $\quad v_s = v_1 + v_2 + v_3 = \dfrac{Q}{C_1} + \dfrac{Q}{C_2} + \dfrac{Q}{C_3}$

and $\quad v_s = \dfrac{Q}{C_{eq}}$

So $\quad \dfrac{Q}{C_{eq}} = \dfrac{Q}{C_1} + \dfrac{Q}{C_2} + \dfrac{Q}{C_3}.$

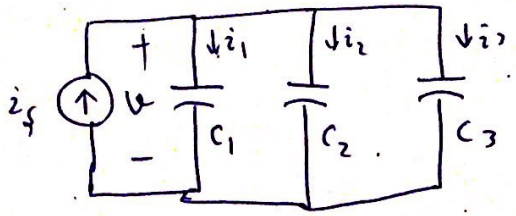
or $\quad \dfrac{1}{C_{eq}} = \dfrac{1}{C_1} + \dfrac{1}{C_2} + \dfrac{1}{C_3}$

For the special case of two capacitors in parallel

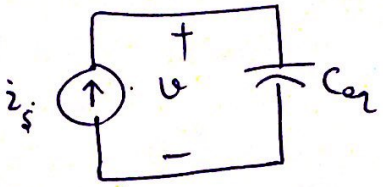
$$C_{eq} = \frac{C_1 C_2}{C_1 + C_2}$$

Capacitors in Parallel (PP 237)

Consider the following arrangement:-



and the equivalent circuit



Now $Q_1 = C_1 v$ and $Q_2 = C_2 v$, $Q_3 = C_3 v$

Also $Q_{eq} = C_{eq} v$

also $i_1 = C_1 \frac{dv}{dt}$.
$i_2 = C_2 \frac{dv}{dt}$ etc etc

Now $i_s = i_1 + i_2 + i_3$

and $i = \frac{dQ}{dt}$

So $\frac{dQ_1}{dt} + \frac{dQ_2}{dt} + \frac{dQ_3}{dt} = \frac{dQ_{eq}}{dt}$

or $Q_1 + Q_2 + Q_3 = Q_{eq}$ ——— (important).

Hence $C_1 v + C_2 v + C_3 v = C_{eq} v$

Therefore $C_{eq} = C_1 + C_2 + C_3$

———— (M2)