Introduction to Probability

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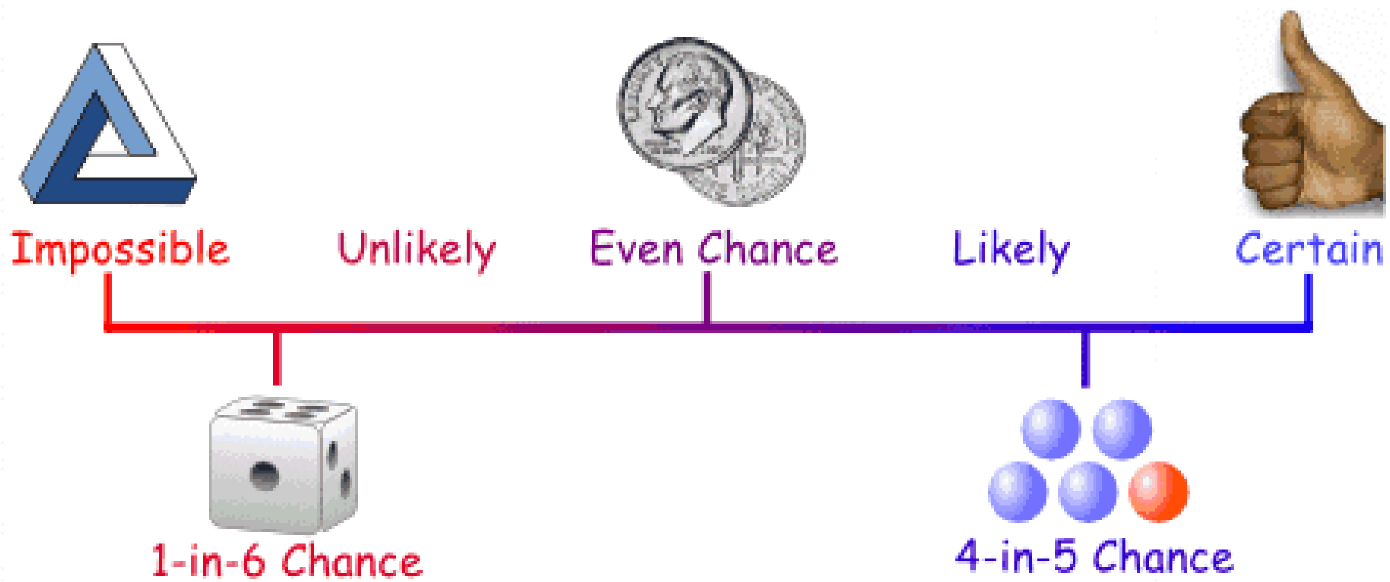
Introduction to Probability

- Events
- Experiments
- Sample Space
- Counting Rules
- Combinations
- Permutations
- Assigning Probabilities

Probability

Probability is a measure of chance.



Experiments

These are processes that generate well-defined outcomes

Experiment	Experimental Outcomes
Toss a coin	head, tail
Select a part for inspection	defective, non defective
Conduct a sales call	Purchase, no purchase
Roll a die	1, 2, 3, 4, 5, 6
Play a Hockey game	Win, lose, tie

The result of an experiment is called outcome.

An experimental outcome is also called a sample point.

Collection of all possible sample points is called sample space.

Subset of sample space is called event.

A single performance of an experiment is called trial.

Random Experiment

An experiment which produce different results even though , it is repeated a large number of times under similar conditions, and we cannot predict the experiment result in advance.

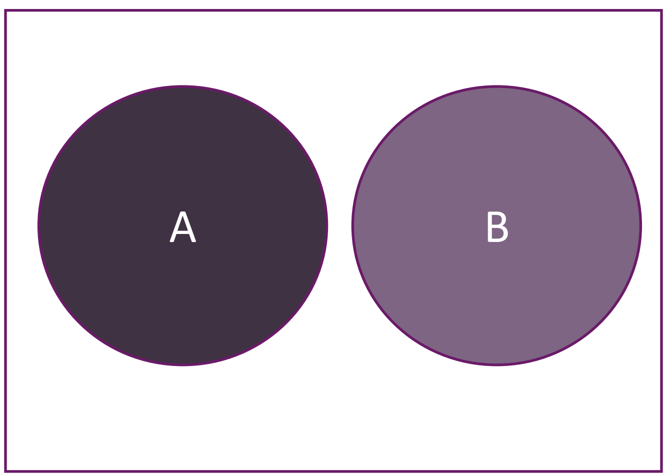
A random experiments has following properties

- All the outcome of experiment is known in advance.
- Random experiment can be repeated any no of times under similar conditions.
- Random experiments always has more than one outcome.
- Outcome of random experiments cannot be predicted in advance; it has some degree of uncertainty.

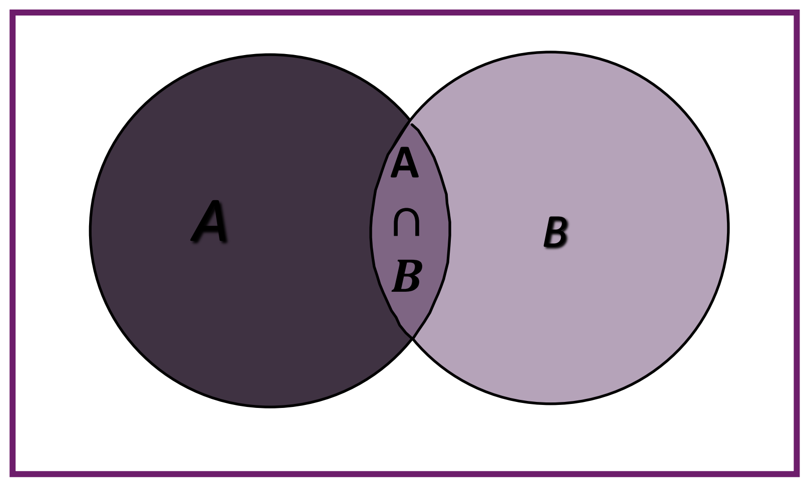
Events

Mutually Exclusive Events… two events are mutually exclusive, if they cannot both occur at the same tome

Mutually Exclusive Events



Not Mutually Exclusive Events



Events

Equally Likely Events.....The two events A and B are said to be equally likely, when one event is as likely to occur as the other.

Events

Exhaustive Event….. Events A, B, C, …….. are exhaustive if their union equals the whole sample space, i.e.,

$$A \cup B \cup C \cup \cdots \ldots = S$$

Sample Space

The **sample space** for an experiment is the set of all experimental outcomes

- For a coin toss:

$$S = \{\text{Head, Tail}\}$$

- Selecting a part for inspection:

$$S = \{\text{Defective, Nondefective}\}$$

- Rolling a die:

$$S = \{1, 2, 3, 4, 5, 6\}$$

Sample Space

Ways to display the sample space:

- List all possible outcomes
- Table
- Tree diagram

Counting Rules

To assign probabilities, we must first count experimental outcomes. We have 3 useful counting rules for **multiple-step** experiments.

a) Counting rule for multiplication

b) Counting rule for combinations

c) Counting rule for permutations

Counting rule for multiplication

If an experiment can be described as a sequence of k steps with n_1 possible outcomes on the fist step, n_2 possible outcomes on the second step, then the total number of experimental outcomes is given by:

$$(n_1)(n_2)\ldots(n_k)$$

With or Without Replacement

- **With Replacement** means the same item can be chosen more than once.

- **Without Replacement** means the same item cannot be selected more than once. In other words, you don't replace the first item you choose before you choose a second.

Class Assignment

The three-letter words can be formed using A, B, C, and D. List all possible three-letter words using

- With replacement
- Without replacement
  - When order is important
  - When order is not important

Counting Rule for Permutations

Sometimes the order of selection matters. This rule allows us to count the number of experimental outcomes when ***n*** objects are to be selected from a set of ***N*** objects and the *order* of selection matters

$$P_n^N = n!\binom{N}{N} = \frac{N!}{(N-n)!}$$

Counting Rule for Combinations

This rule allows us to count the number of experimental outcomes when we select ***n*** objects from a (usually larger) set of ***N*** objects

The number of N objects taken n at a time is

$$C_n^N = \binom{N}{n} = \frac{N!}{n!(N-n)!}$$

where

$$N! = N(N-1)(N-2)\ldots(2)(1)$$
$$n! = n(n-1)(n-2)\ldots(2)(1)$$

And by definition

$$0! = 1$$

Question 1

A product is assembled at three stages at first stage there are 5 assembly line at 2^{nd} there are 4, and 3^{rd} there are 6 in how many different ways the product may be pass through the assembly process.

Question 2

A computer rating service is commissioned to rank the top three brands of intelligent terminals. A total of 10 brands are to be included in the study. In how many different ways can the computer rating service arrive at the final ranking?

Question 3

How many 7 digits telephone numbers are possible if the first number is not allowed to be 0 or 1?

Question 4

Ten coins are tossed simultaneously. In how many of the outcomes will the third coin turn up a head?

Question 5

A deck of cards contains 10 red cards numbered 1 to 10 and 10 black cards numbered 1 to 10.how many ways are there arranging the 20 cards in a row? Suppose we draw a card at random and lay them in a row. What is the probability red and black card alternate?

Question 6

Certain automobile license plates consist of a sequence of three letters followed by three digits.

a) If no repetitions of letters are permitted, how many possible license plates are there?

b) If no letters and no digits are repeated, how many license plates are possible?

Question 7

Out of six Chips, two are defective. If three chips are randomly chosen for testing (without replacement), compute the probability that two of them are defective? List all the outcomes in the sample space.

Assigning Probabilities

- **Objective**

Assigning probabilities based on rule or formula

  - The Classical OR A Priori Definition of Probability
  - The Relative frequency OR A Posteriori Method Definition of Probability
  - The Axiomatic Definition of Probability

- **Subjective**

Assigning probabilities based on judgment

Classical Method

This method of assigning probabilities is indicated if each experimental outcome is *equally likely*

$$P(E_i) = \frac{1}{n}$$

Relative Frequency Method

- This method is indicated when the data are available to estimate the proportion of the time the experimental outcome will occur if the experiment is repeated a large number of times.
- If a random experiment is repeated a large number of times, say n, under identical conditions and if an event A is observed to occur m times, then
- $P(A) = \lim_{n \to \infty} \frac{m}{n}$

The Axiomatic Definition of Probability

- Let A_i denote the *i*th experimental outcome and $P(A_i)$ is its probability of occurring. Then:

$$0 \leq P(A_i) \leq 1 \text{ for all } i$$

- The sum of the probabilities for all experimental outcomes must be must equal 1. For n experimental outcomes:

$$P(A_1) + P(A_2) + \ldots + P(A_n) = 1$$

- If A_1 and A_2 is mutually exclusive events in sample space, then

$$\mathrm{P}(A_1 \cup A_2) = P(A_1) + P(A_2)$$

Subjective Method

- When economic conditions and a company's circumstances change rapidly it might be inappropriate to assign probabilities based solely on historical data.
- We can use any data available as well as our experience and intuition, but ultimately a probability value should express our <u>degree of belief</u> that the experimental outcome will occur.
- The best probability estimates often are obtained by combining the estimates from the classical or relative frequency approach with the subjective estimate.

Question 1

A coin is tossed four times. what is the probability that

a) At least one head appears?
b) Exactly one head appear?
c) At most two tails appears?

Question 2

If two dice are thrown. what is the probability of getting

a) A double six?
b) A Sum of at least 8?
c) $P(x>y)$?

Question 3

Six balls drawn from a bag contains 4 white and 6 black balls. What is the probability that

a) Both colors are equally represented in the sample?

b) At least two white balls drawn?

Question 4

An urn contains five red, five black, and five white balls. If three balls are chosen without replacement at random, what is the probability that

a) All are different colors?
b) All are same colors?
c) Exactly two different colors?

Question 5

Three distinct integers are chosen at random from the first 20 positive integers. What is the probability that?

a) their product is even

b) their sum is even