PROPERTIES OF SIGNALS

Periodic Signals

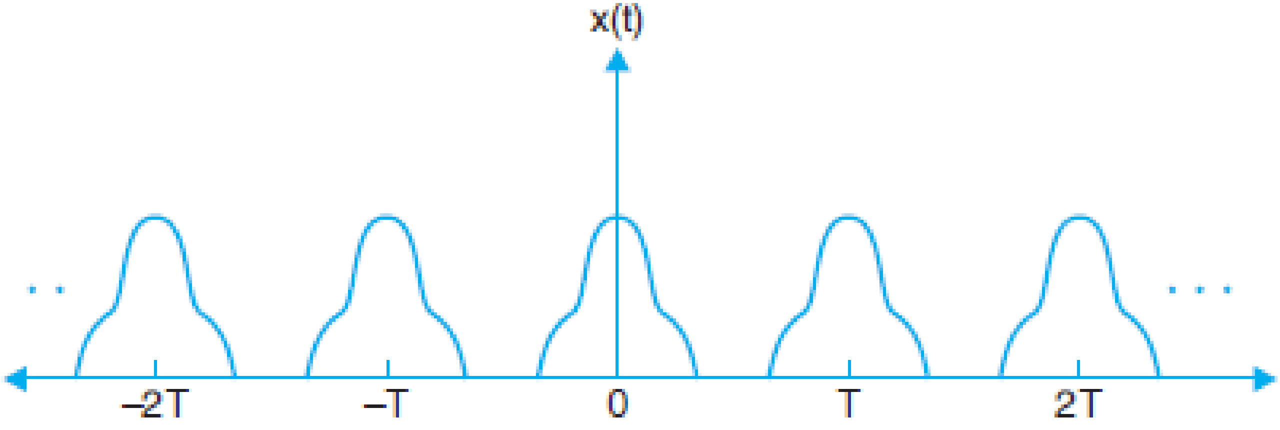
- A periodic continuous-time signal x(t) has the property that for a positive value of time T,

$$x(t) = x(t + T), \text{ for all value of } t$$

- Then x(t) is periodic with time period T

- The fundamental period T_0 is the smallest positive value of T for which the above equation holds.

- Thus x(t) is also periodic with period 2T, 3T, ...



Periodic Signals

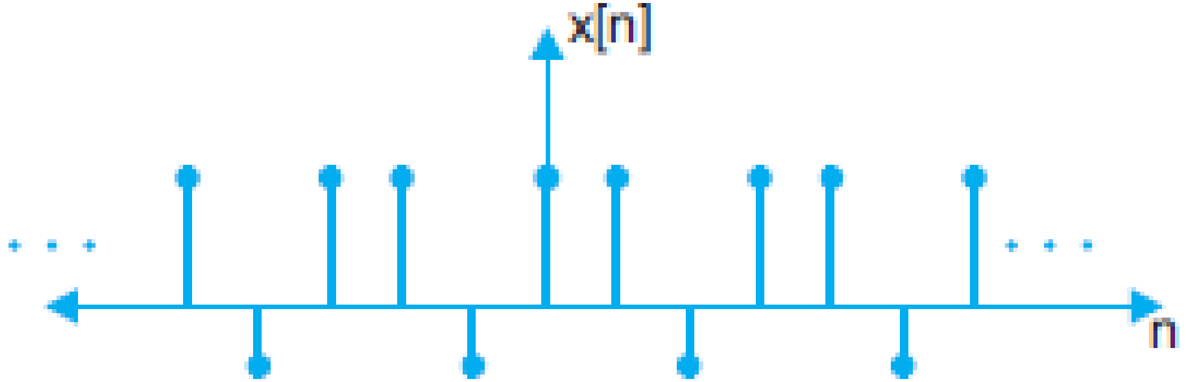
- A periodic discrete-time signal x[n] has the property that for a positive integer N,

$$x[n] = x[n + N], \text{ for all values of } n.$$

- The discrete time signal x[n] is periodic with period N if it is unchanged by a time shift of N

- The fundamental period N_0 is the smallest positive value of N for which the above equation holds

- Thus x[n] is periodic with period 2N, 3N, ...



Periodic Signals – Sine Wave

- An important periodic signal is the sinusoidal signal shown in figure below:

$$x(t) = A\cos(\omega_0 t + \phi),$$

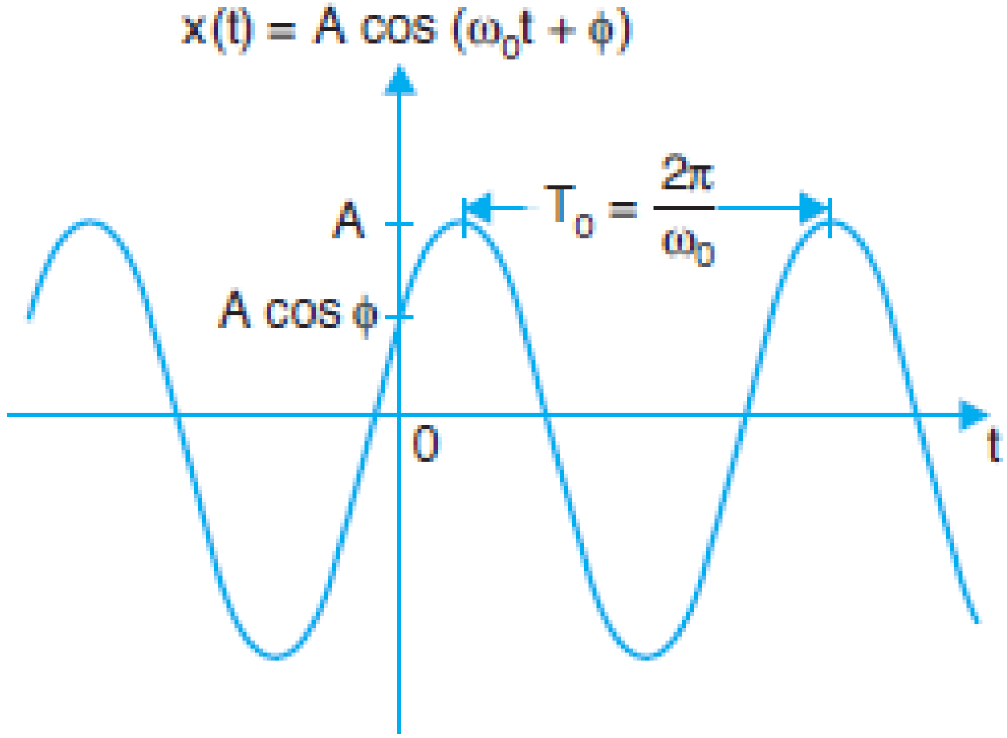
- Unit of t is seconds, unit of ϕ is radians and that of ω_0 radians/second, respectively

- $\omega_0 = 2\pi f_0 = 2\pi/T_0$, where f_0 is in cycles/second or hertz(Hz)

- The sinusoidal signal is periodic with fundamental period $T_0 = 1/f_0$

- Signals that are not periodic are said to be aperiodic

Periodic Signals – Sine Wave



Energy of a Signal

- We may consider the area under a signal x(t) as a possible measure of its size, because it takes account not only of the amplitude but also of the duration

- However, this will be a defective measure because even for a large signal x(t), its positive and negative areas could cancel each other, indicating a signal of small size

- This difficulty can be corrected by defining the signal size as the area under $x^2(t)$, which is always positive

$$E_x = \int_{-\infty}^{\infty} x^2(t)\, dt$$

Energy of a Signal

- This definition can be generalized to a complex valued signal x(t) as:

$$E_x = \int_{-\infty}^{\infty} |x(t)|^2 \, dt$$

- The signal energy must be finite for it to be a meaningful measure of the signal size

- A necessary condition for the energy to be finite is that the signal amplitude → 0 as |t| → ∞, otherwise the integral will not converge

- When the amplitude of x(t) does not → 0 as |t| → ∞, the signal energy is infinite

- A more meaningful measure of the signal size in such a case would be the time average of the energy, if it exists

Power of a Signal

- Time average of the energy is called the power of the signal:

$$P_x = \lim_{T \to \infty} \frac{1}{T} \int_{-T/2}^{T/2} x^2(t)\, dt$$

- We can generalize this definition for a complex signal x(t) as:

$$P_x = \lim_{T \to \infty} \frac{1}{T} \int_{-T/2}^{T/2} |x(t)|^2\, dt$$

- Generally, the mean of an entity averaged over a large time interval approaching infinity exists if the entity either is periodic or has a statistical regularity

- If such a condition is not satisfied, the average may not exist, for instance, a ramp signal $x(t) = t$ increases indefinitely as $|t| \to \infty$, and neither the energy nor the power exists for this signal

Energy & Power of a DT Signal

- The energy and power for a CT signal are given as:

$$E_\infty = \int_{-\infty}^{\infty} |x(t)|^2 \, dt$$

$$P_\infty = \lim_{T \to \infty} \frac{1}{2T} \int_{-T}^{T} |x(t)|^2 \, dt$$

- The energy and power for a DT signal are given as:

$$E_\infty = \sum_{n=-\infty}^{\infty} |x[n]|^2$$

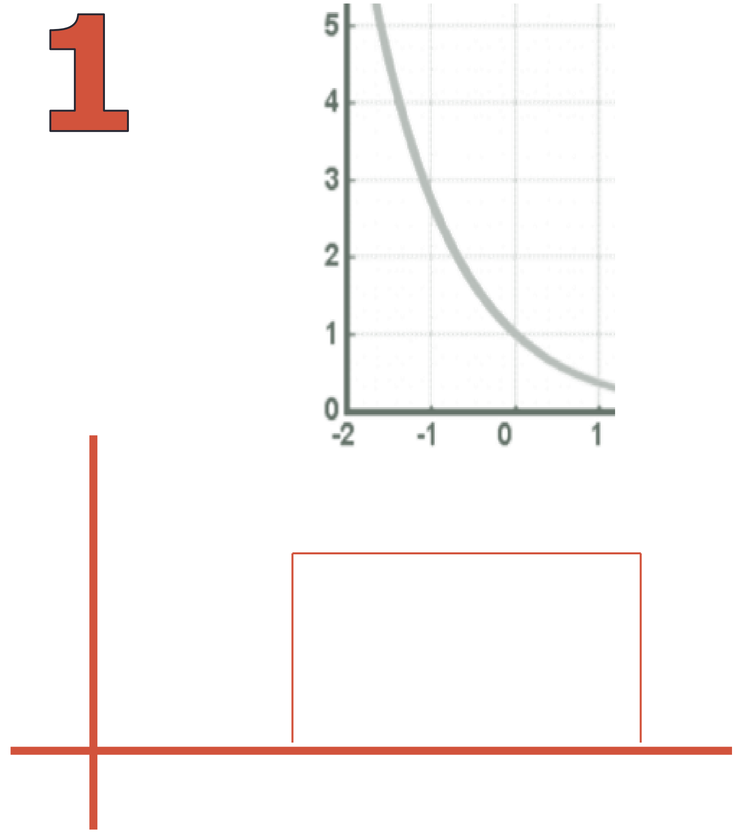
$$P_\infty = \lim_{N \to \infty} \frac{1}{2N+1} \sum_{n=-N}^{N} |x[n]|^2$$

Classification of Signals based upon Energy and Power

1- Signals with finite total energy and with zero average power:

These are signals for which $E < \infty$

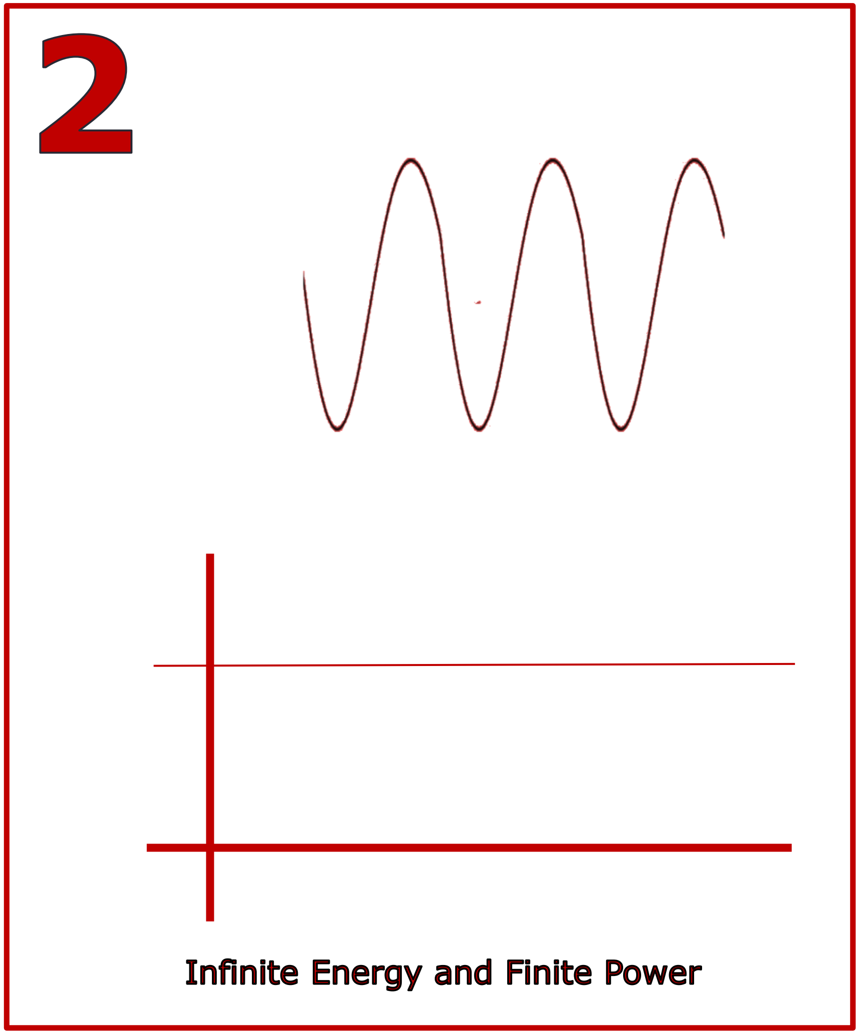
Such a signal must have zero average power since in the CT case, to calculate P, $T \to \infty$; any finite value of E divided by infinite T would give zero power



Finite Energy and Zero Power

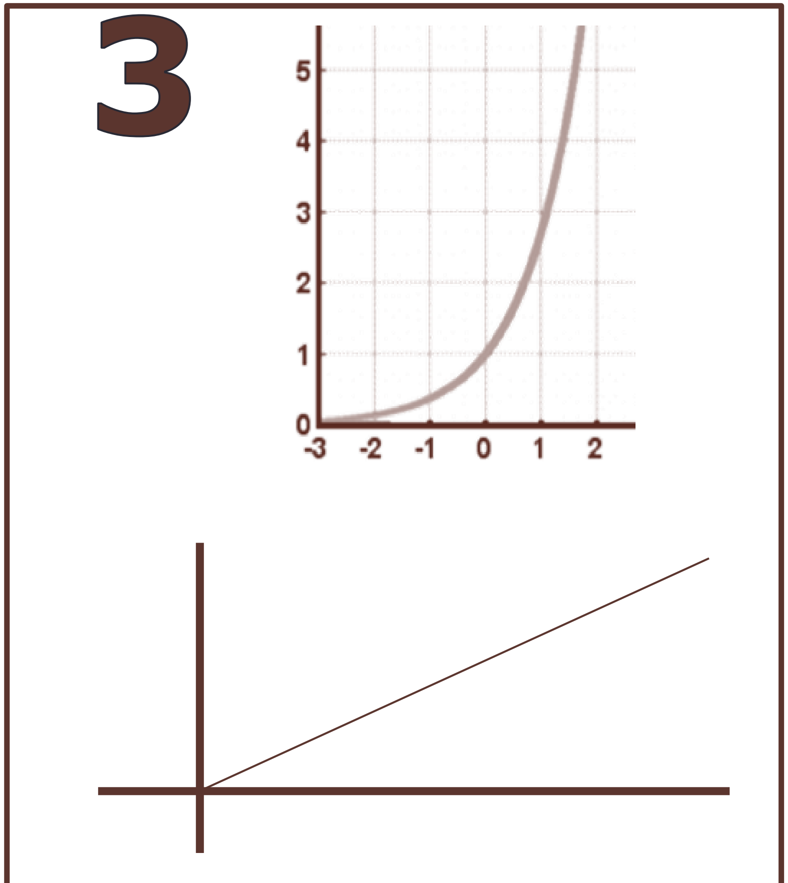
Classification of Signals based upon Energy and Power

2- Signals with infinite total energy and finite average power:



Classification of Signals based upon Energy and Power

3- Signals with infinite total energy and infinite average power



Infinite Energy and Infinite Power

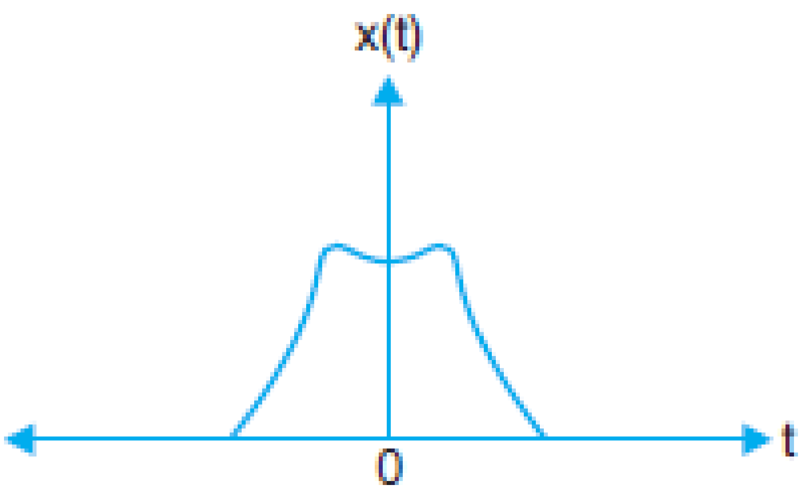
Even Signals

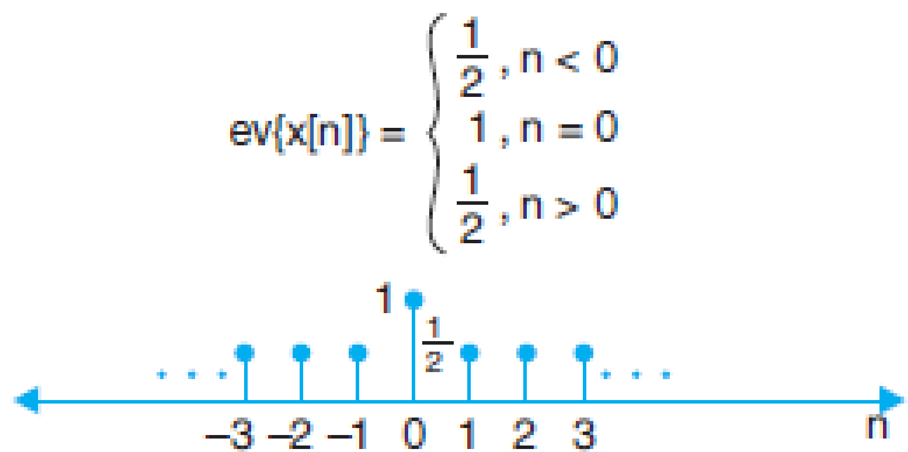
- A continuous signal f(t) is referred to as an even signal if it is identical to its time-reversed counterpart, i.e.

$$f(t) = f(-t); \quad \text{for all } t$$

- The discrete signal f[n] is said to be even if

$$f[n] = f[-n]; \quad \text{for all } n$$





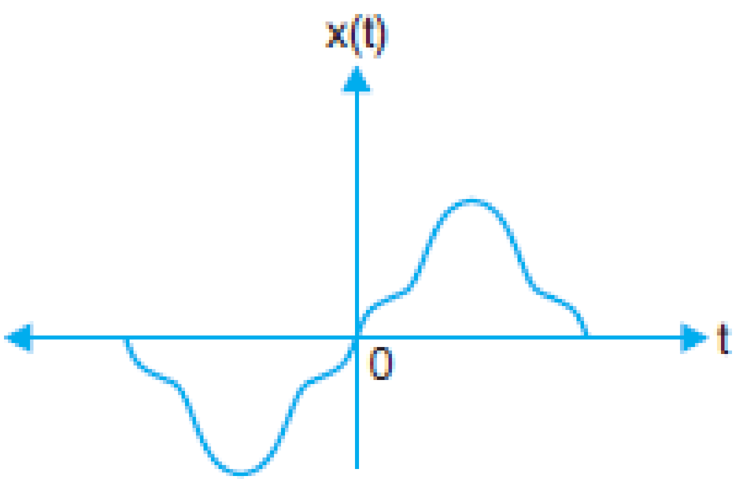
Odd Signals

- A continuous signal f(t) is referred to as an odd signal if it is not identical to its time-reversed counterpart, as shown below:

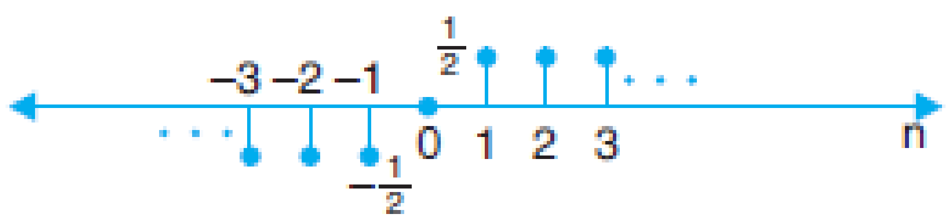
$$f(-t) = -f(t); \quad \text{for all } t$$

- It may be noted that an odd continuous time signal will be zero at origin, i.e., f(0) = 0 at t = 0

- The signal f[n] is said to be odd if: $f[-n] = -f[n]; \quad \text{for all } n$



$$\mathrm{od}\{x[n]\} = \begin{cases} -\frac{1}{2}, & n < 0 \\ 0, & n = 0 \\ \frac{1}{2}, & n > 0 \end{cases}$$



Even and Odd Signal Decomposition

- A signal can be decomposed into its even and odd components

- Decomposition of continuous signal f(t) can be done as:

$$f(t) = f_e(t) + f_o(t)$$

- Here, $f_e(t)$ is the even and $f_o(t)$ is the odd component of continuous signal f(t)

- Obviously, the even function has the property:

$$f_e(-t) = f_e(t)$$

- And the odd function has the property:

$$f_o(-t) = -f_o(t)$$

Even and Odd Signal Decomposition

- Replacing t by $-t$ in the expression of $f(t)$, we get:

$$f(-t) = f_e(-t) + f_o(-t) = f_e(t) - f_o(t)$$

- Solving from the expression of f(t) and f(–t), we get:

$$f_e(t) = \frac{1}{2}[f(t) + f(-t)]$$

AND

$$f_o(t) = \frac{1}{2}[f(t) - f(-t)]$$

DT Even and Odd Signal Decomposition

- Similarly, for DT signal $f[n]$, we have:

$$f_e[n] = \frac{1}{2}\left[[f[n] + f[-n] \right]$$

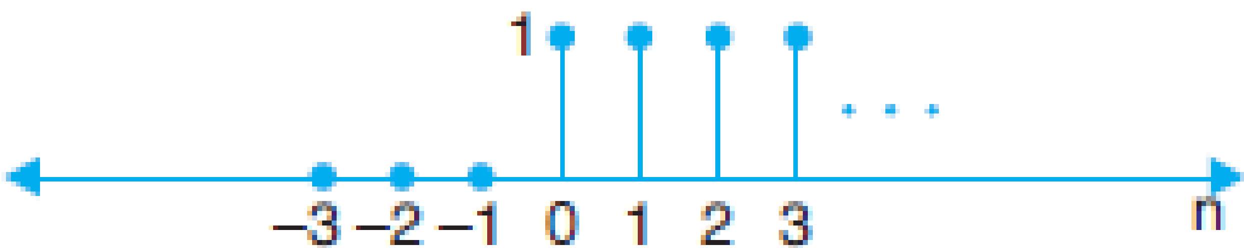
AND

$$f_o[n] = \frac{1}{2}\left[[f[n] - f[-n] \right]$$

Problem-1 / Signal Decomposition

- Find the Even and Odd signal components of the signal below:

$$x[n] = \begin{cases} 1, & n > 0 \\ 0, & n < 0 \end{cases}$$



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