Assignment 2

Q.

t	v(t)
0	0
10	227.04
15	362.78
20	517.35
22.5	602.92
30	901.67

Compute:
- v at $t = 16$ s
- a at $t = 21$ s
- s between $t = 23 - 29$

n = 6 (data points) − 1 = 5

Splines

$$S_0^{(t)} = a_0 + b_0(t-0) + c_0(t-0)^2 + d_0(t-0)^3$$
$$S_1^{(t)} = a_1 + b_1(t-10) + c_1(t-10)^2 + d_1(t-10)^3$$
$$S_2^{(t)} = a_2 + b_2(t-15) + c_2(t-15)^2 + d_2(t-15)^3$$
$$S_3^{(t)} = a_3 + b_3(t-20) + c_3(t-20)^2 + d_3(t-20)^3$$
$$S_4^{(t)} = a_4 + b_4(t-22.5) + c_4(t-22.5)^2 + d_4(t-22.5)^3$$

$4n$ = 20 unknowns

Constraint 1 (Passing through 2 points)

$S_0(0) = a_0 \Rightarrow \boxed{a_0 = 0}$ — i

$S_0(10) = \boxed{10b_0 + 100c_0 + 1000d_0 = 227.04}$ — ii

$S_1(10) = a_1 \Rightarrow \boxed{a_1 = 227.04}$ — iii

$S_1(15) = \boxed{5b_1 + 25c_1 + 125d_1 = 135.74}$ — iv

$S_2(15) = a_2 \Rightarrow \boxed{a_2 = 362.78}$ — v

$S_2(20) = \boxed{5b_2 + 25c_2 + 125d_2 = 154.57}$ — vi

$S_3(20) = a_3 \Rightarrow \boxed{a_3 = 517.35}$ — vii

$S_3(22.5) = \boxed{2.5b_3 + 6.25c_3 + 15.625d_3 = 85.57}$ — viii

$$S_4(22.5) = a_4 \Rightarrow \boxed{a_4 = 602.92} \quad - ix$$

$$S_4(30) = \boxed{7.5b_4 + 56.25c_4 + 421.875d_4 = 298.75} \quad - x$$

Constraint 2 (d/dt equal at nodes)

- $\left.\frac{d}{dt} S_0(t)\right|_{t=t_1} = \left.\frac{d}{dt} S_1(t)\right|_{t=t_1}$

$$\Rightarrow \left(b_0 + 2c_0 t + 3d_0 t^2\right)_{t=10} = \left(b_1 + 2c_1(t-10) + 3d_1(t-10)^2\right)_{t=10}$$

$$\Rightarrow b_0 + 20c_0 + 300d_0 = b_1$$

$$\Rightarrow \boxed{b_0 - b_1 + 20c_0 + 300d_0 = 0} \quad - xi$$

- $\left.\frac{d}{dt} S_1(t)\right|_{t=t_2} = \left.\frac{d}{dt} S_2(t)\right|_{t=t_2}$

$$\Rightarrow \left(b_1 + 2c_1(t-10) + 3d_1(t-10)^2\right)_{t=15} = \left(b_2 + 2c_2(t-15) + 3d(t-15)^2\right)_{t=15}$$

$$\Rightarrow b_1 + 10c_1 + 75d_1 = b_2$$

$$\Rightarrow \boxed{b_1 - b_2 + 10c_1 + 75d_1 = 0} \quad - xii$$

- $\left.\frac{d}{dt} S_2(t)\right|_{t=t_3} = \left.\frac{d}{dt} S_3(t)\right|_{t=t_3}$

$$\Rightarrow \left(b_2 + 2c_2(t-15) + 3d_2(t-15)^2\right)_{t=20} = \left(b_3 + 2c_3(t-20) + 3d_3(t-20)^2\right)_{t=20}$$

$$\Rightarrow b_2 + 10c_2 + 75d_2 = b_3$$

$$\Rightarrow \boxed{b_2 - b_3 + 10c_2 + 75d_2 = 0} \quad - xiii$$

- $\left.\frac{d}{dt} S_3(t)\right|_{t=t_4} = \left.\frac{d}{dt} S_4(t)\right|_{t=t_4}$

$$\Rightarrow \left(b_3 + 2c_3(t-20) + 3d_3(t-20)^2\right)_{t=22.5} = \left(b_4 + 2c_4(t-22.5) + 3d(t-22.5)^2\right)_{t=22.5}$$

$$\Rightarrow b_3 + 5c_3 + 18.75d_3 = b_4$$

$$\Rightarrow \boxed{b_3 - b_4 + 5c_3 + 18.75d_3 = 0} \quad - xiv$$

Constraint 3 (d^2/dt^2 equal at nodes)

- $\left.\frac{d^2}{dt^2} S_0(t)\right|_{t=t_1} = \left.\frac{d^2}{dt^2} S_1(t)\right|_{t=t_1}$

$\Rightarrow (2c_0 + 6d_0 t)_{t=10} = (2c_1 + 6d_1(t-10))_{t=10}$

$\Rightarrow 2c_0 + 60d_0 = 2c_1$

$\Rightarrow \boxed{2c_0 - 2c_1 + 60d_0 = 0}$ — xv

- $\left.\frac{d^2}{dt^2} S_1(t)\right|_{t=t_2} = \left.\frac{d^2}{dt^2} S_2(t)\right|_{t=t_2}$

$\Rightarrow (2c_1 + 6d_1(t-10))_{t=15} = (2c_2 + 6d_2(t-15))_{t=15}$

$\Rightarrow 2c_1 + 30d_1 = 2c_2$

$\Rightarrow \boxed{2c_1 - 2c_2 + 30d_1 = 0}$ — xvi

- $\left.\frac{d^2}{dt^2} S_2(t)\right|_{t=t_3} = \left.\frac{d^2}{dt^2} S_3(t)\right|_{t=t_3}$

$\Rightarrow (2c_2 + 6d_2(t-15))_{t=20} = (2c_3 + 6d_3(t-20))_{t=20}$

$\Rightarrow 2c_2 + 30d_2 = 2c_3$

$\Rightarrow \boxed{2c_2 - 2c_3 + 30d_2 = 0}$ — xvii

- $\left.\frac{d^2}{dt^2} S_3(t)\right|_{t=t_4} = \left.\frac{d^2}{dt^2} S_4(t)\right|_{t=t_4}$

$\Rightarrow (2c_3 + 6d_3(t-20))_{t=22.5} = (2c_4 + 6d_4(t-22.5))_{t=22.5}$

$\Rightarrow 2c_3 + 15d_3 = 2c_4$

$\Rightarrow \boxed{2c_3 - 2c_4 + 15d_3 = 0}$ — xviii

Constraint 4 (Natural Splines)

- $\left.\frac{d^2}{dt^2} S_0(t)\right|_{t=t_0} = 0$

$\Rightarrow \quad (2c_0 + 6d_0 t)_{t=t_0} = 0$

$\Rightarrow \quad 2c_0 + 6d_0(0) = 0$

$\Rightarrow \quad \boxed{c_0 = 0} \quad - xix$

- $\left.\frac{d^2}{dt^2} S_4(t)\right|_{t=t_5} = 0$

$\Rightarrow \quad (2c_4 + 6d_4(t - 22.5))_{t=30} = 0$

$\Rightarrow \quad \boxed{2c_4 + 45d_4 = 0} \quad - xx$

System Matrix

$$
\begin{bmatrix}
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 10 & 100 & 1000 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 5 & 25 & 125 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 5 & 25 & 125 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2.5 & 6.25 & 15.6 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 7.5 & 56.2 & 421.8 & 0 \\
0 & 1 & 20 & 300 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 10 & 75 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 10 & 75 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 5 & 18.7 & 0 & -1 & 0 & 0 & 0 \\
0 & 0 & 2 & 60 & 0 & 0 & -2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 2 & 30 & 0 & 0 & -2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2 & 30 & 0 & 0 & -2 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2 & 15 & 0 & 0 & -2 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2 & 45
\end{bmatrix}
\begin{bmatrix} a_0 \\ b_0 \\ c_0 \\ d_0 \\ a_1 \\ b_1 \\ c_1 \\ d_1 \\ a_2 \\ b_2 \\ c_2 \\ d_2 \\ a_3 \\ b_3 \\ c_3 \\ d_3 \\ a_4 \\ b_4 \\ c_4 \\ d_4 \end{bmatrix}
=
\begin{bmatrix} 0 \\ 227.04 \\ 227.04 \\ 135.74 \\ 362.78 \\ 154.57 \\ 547.35 \\ 85.57 \\ 602.92 \\ 298.75 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}
$$

A X B

Solved through MATLAB

$a_0 = 0$	$b_0 = 21.4265$	$c_0 = 0$	$d_0 = 0.0128$
$a_1 = 227.04$	$b_1 = 25.2590$	$c_1 = 0.3833$	$d_1 = -0.0011$
$a_2 = 362.78$	$b_2 = 29.0096$	$c_2 = 0.3669$	$d_2 = 0.0028$
$a_3 = 517.35$	$b_3 = 32.8884$	$c_3 = 0.4089$	$d_3 = 0.0508$
$a_4 = 602.92$	$b_4 = 35.8849$	$c_4 = 0.7897$	$d_4 = -0.0351$

Solution for
X

- Velocity at $t = 16$ s

$$S_2(t)\Big|_{t=16} = \left(362.78 + 29.0096(t-15) + 0.3669(t-15)^2 \cdots + 0.0028(t-15)^3\right)_{t=16}$$

$$S_2(16) = \boxed{392.1593 \text{ unit/s}} = V$$

- Acceleration at $t = 21$ s

$$\frac{dS_3(t)}{dt}\Big|_{t=21} = \left(b_3 + 2c_3(t-20) + 3d_3(t-20)^2\right)_{t=21}$$

$$= 32.8884 + 2(0.4089)(1) + 3(0.0508)(1)^2$$

$$a = \boxed{33.8586 \text{ unit}/s^2}$$

- Distance b/w $t = 23$ and $t = 29$ s

$$\int_{23}^{29} S_4(t)\,dt = \left[a_4 t + b_4\left(\frac{t^2}{2} - 22.5t\right) + c_4\frac{(t-22.5)^3}{3} \cdots d_4\left(\frac{(t-22.5)^4}{4}\right)\right]_{23}^{29}$$

$$S = \boxed{4427.697 \text{ unit}}$$