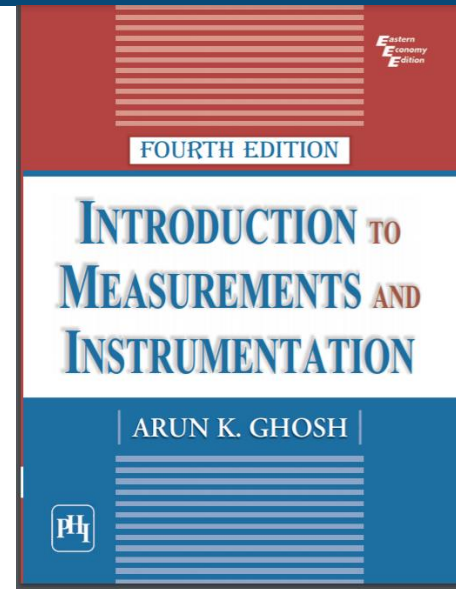
NUST

SCHOOL OF ELECTRICAL ENGINEERING & COMPUTER SCIENCE

- Course: **EE383 Instrumentation and Measurements**
- Session: Fall 2022
- Class: BEE12
- **Lectures: Week 9**
- Course Instructor: Dr. Shahzad Younis

INSTRUMENTATION
& MEASUREMENT



Week 9

- **Chapter 6**

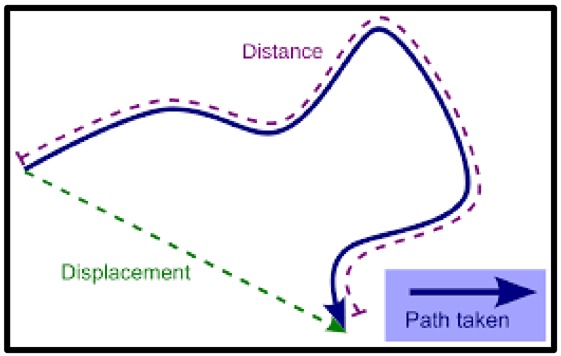
  Displacement Measurement

Book: Introduction to Measurements and Instrumentation, Arun K. Ghosh

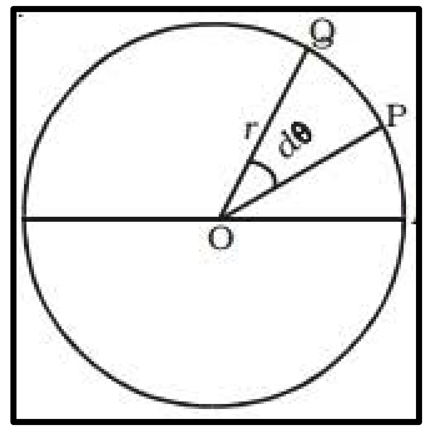
Displacement Measurement

Displacement can be of two types.

Linear Displacement



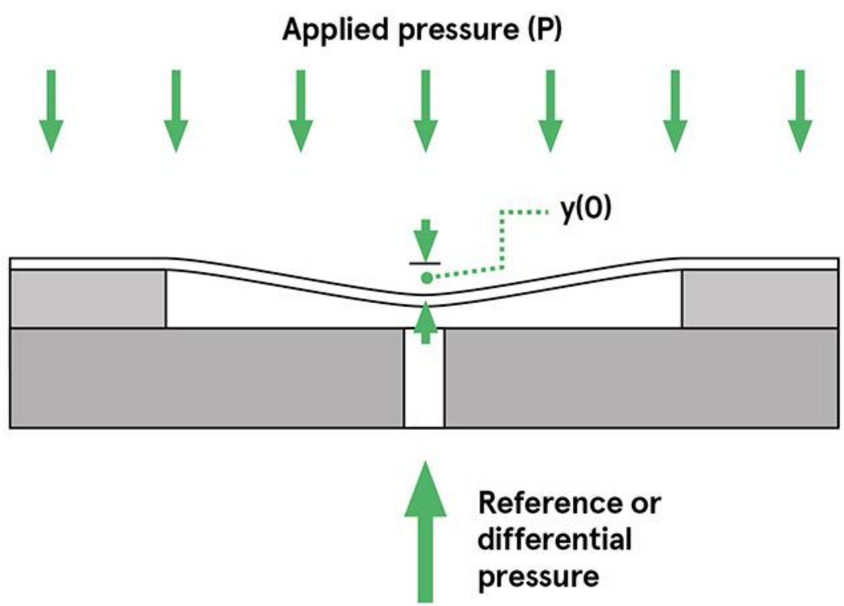
Angular Displacement



Measurement of displacement is fundamental to many measurements.

Displacement Measurement

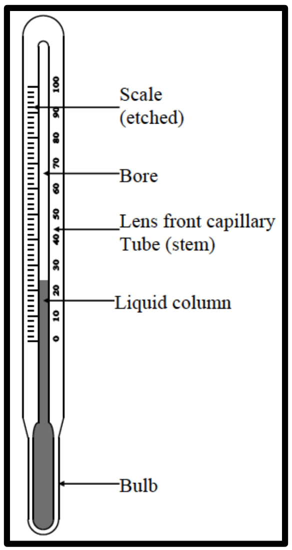
- Many environmental parameters, such as force, strain, pressure, temperature, level etc. ultimately result into displacement
- Measurement of displacement, linear or angular, is fundamental to many other measurements (force, strain, …)

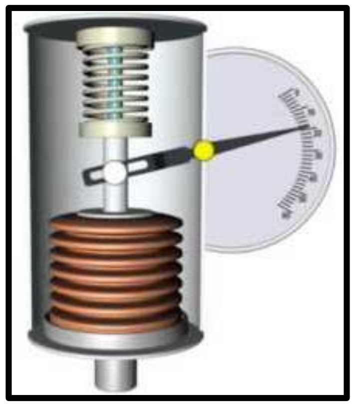


Displacement Measurement

Many measurements are dependent on displacement measurement.

Temperature Measurement	Pressure Measurement	Level Measurement





Displacement Measurement

Displacements are commonly measured using,

- Measuring tapes
- Vernier Scale
- Micrometers
- Screw gauge

All these devices are self sufficient and very commonly used.

Displacement Measurement

In this chapter, few transducers have been discussed which are used as component in an instrumentation system.

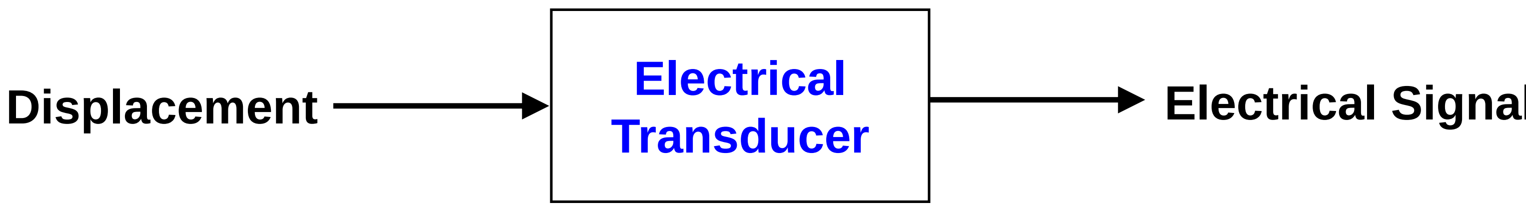
- Electrical
- Pneumatic
- Optical
- Ultrasonic
- Magnetostrictive
- Digital

Displacement Measurement

- Classification
  - Electrical
  - Pneumatic
  - Optical
  - Ultrasonic
  - Magnetostrictive
  - Digital

Displacement Measurement

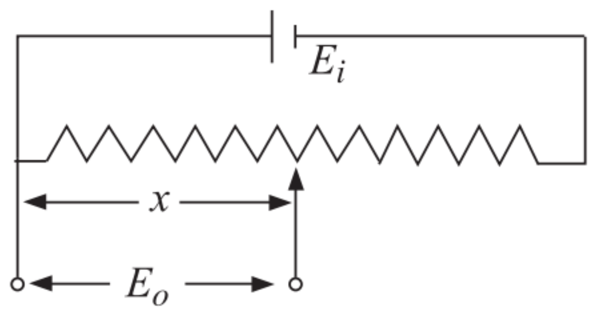
- **Electrical Transducers**
  - **Convert displacement to an electrical signal**



- Passive electrical components**: resistance, inductance and capacitance**
  - Resistive, inductive and capacitive transduction of displacement

Displacement Measurement

- **Resistive Transducer: Potentiometer**
  - A resistance element provided with a movable contact
  - Potentiometer or pot



Displacement Measurement

❑ **Resistive Transducer: Potentiometer**

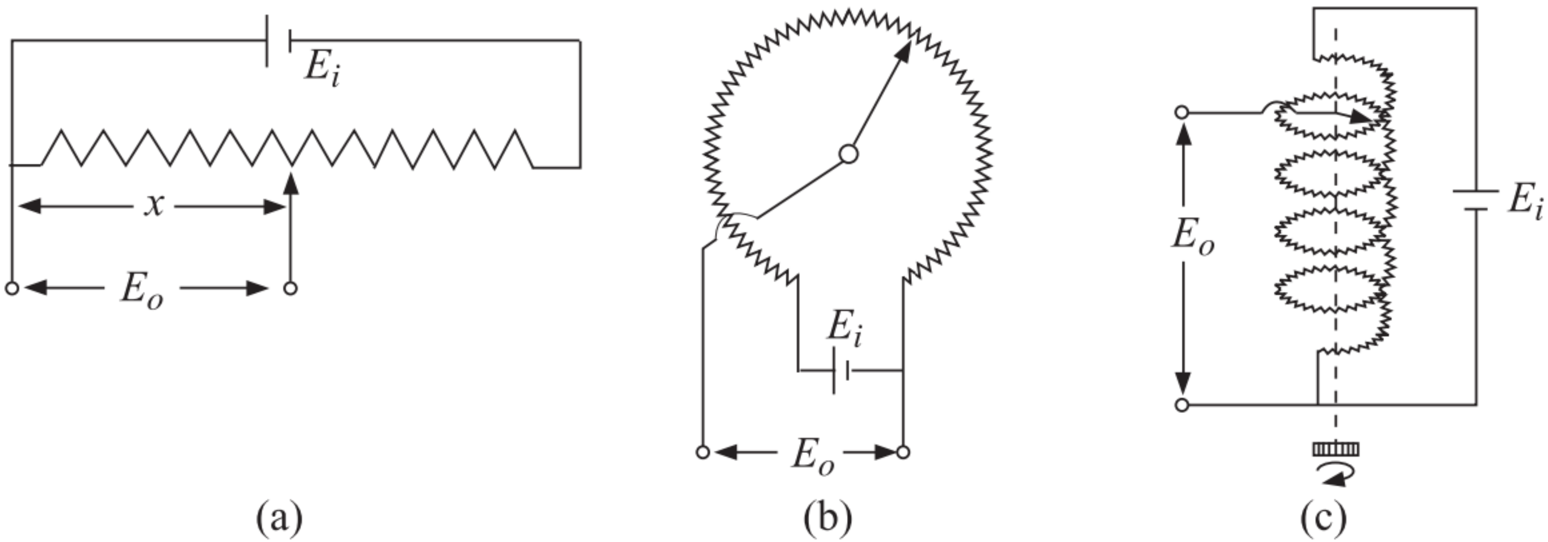


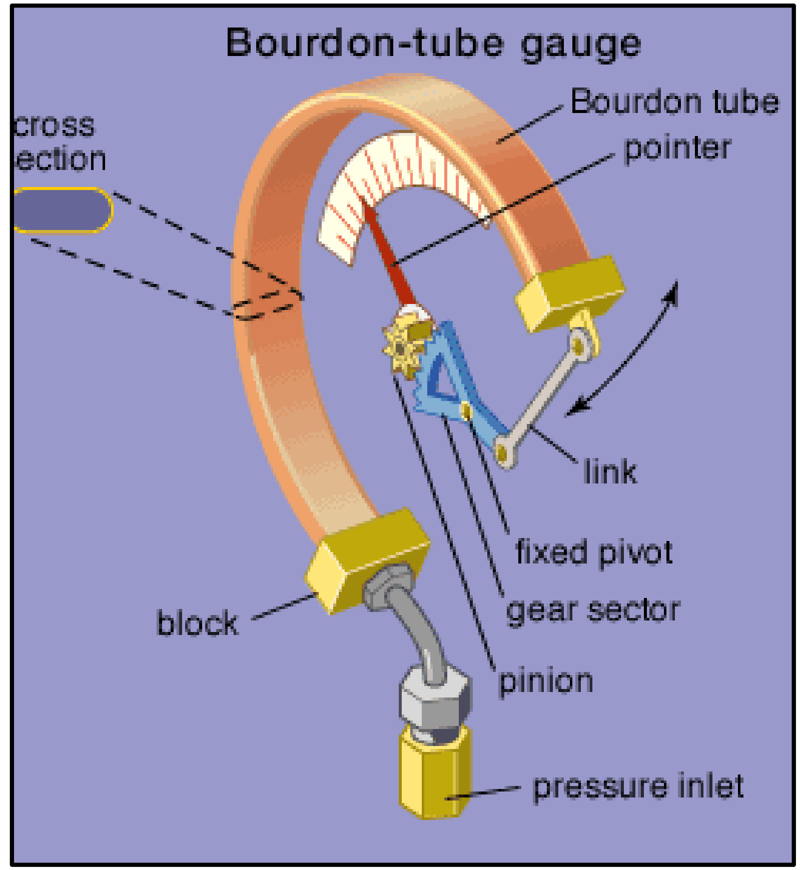
Fig. 6.4 Schematic representations of potentiometers: (a) translational, (b) rotary, and (c) helical.

❑ The motion of the contact can be translational, rotational, or helical (a combination of the two motions)

Resistive Transducer

Application of translational displacement sensor

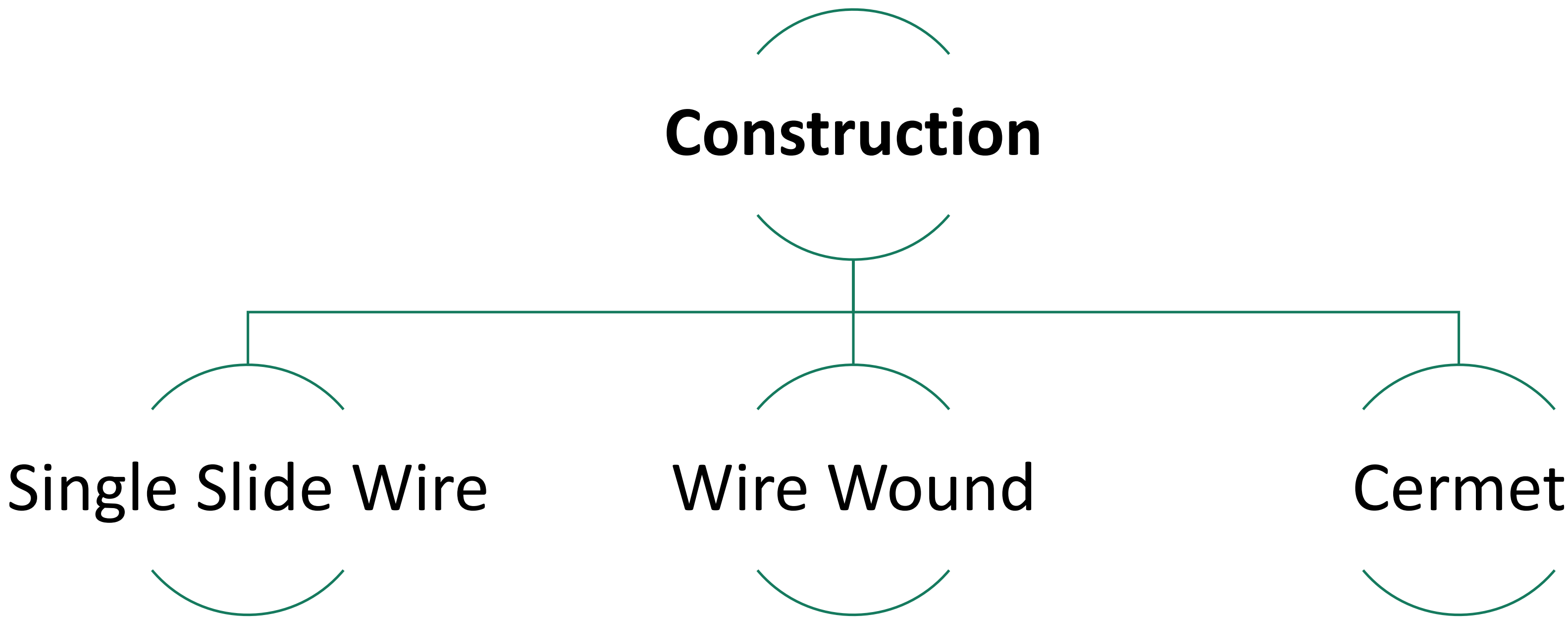
Application of rotary displacement sensor



By using the potentiometer, analogue gauges can be converted to digital gauges

Construction of Potentiometer

Resistive transducers are constructed in one of the three forms.



Resistive Transducer: Potentiometer

- **Construction: single slide wire**
  - Stepless variation of resistance
  - Small values of resistance
    - length is limited by the desired stroke in a translational device and by the diameter in a rotational one
    - resistance per unit length can be increased by reducing the wire cross-sectional area: at the expense of the strength and wear resistance

Construction of Potentiometer

Single Slide Wire

Advantage: Single slide wire offers step less variation of resistance as slider moves over it.

Disadvantage: The values of resistance are quite low.

How to increase resistance?

- By increasing the length of transducer.
- By reducing the area of cross section of wire → This may reduce the strength of wire and potentiometer may wear out quickly.

Resistive Transducer: Potentiometer

- **Construction: wire-wound**
  - wire is wound on a straight or circular card depending on the type of motion: translational or rotational

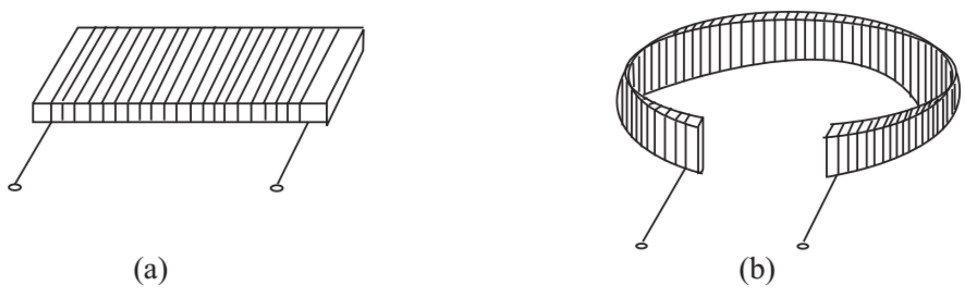


Fig. 6.5 Wire-wound resistance elements on: (a) straight insulating card and (b) circular insulating card.

Construction of Potentiometer

Wire Wound: The wire is wound on a straight (for translational transducer) or circular (for rotational transducer) insulating card.

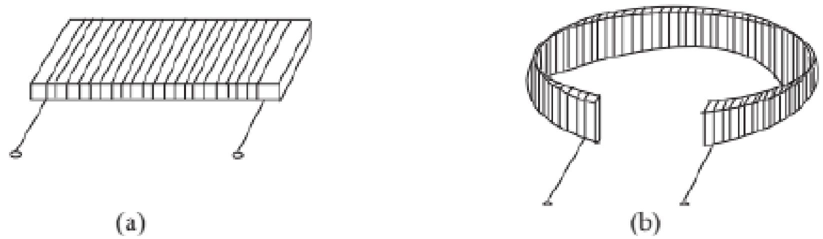


Fig. 6.5 Wire-wound resistance elements on: (a) straight insulating card and (b) circular insulating card.

The wire wound construction produces a step wise increase in resistance as the wiper moves from one turn of wire to another.

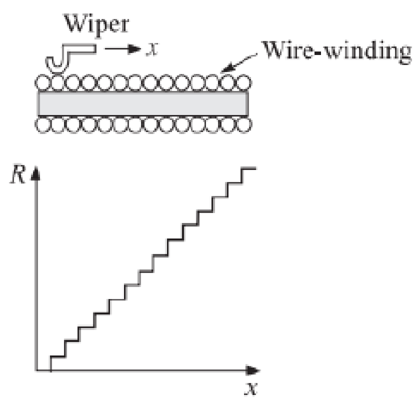


Fig. 6.6 Stepwise increase of resistance of wire-wound potentiometers.

Resistive Transducer: Potentiometer

- **Construction: wire-wound**
  - a stepwise increase in resistance

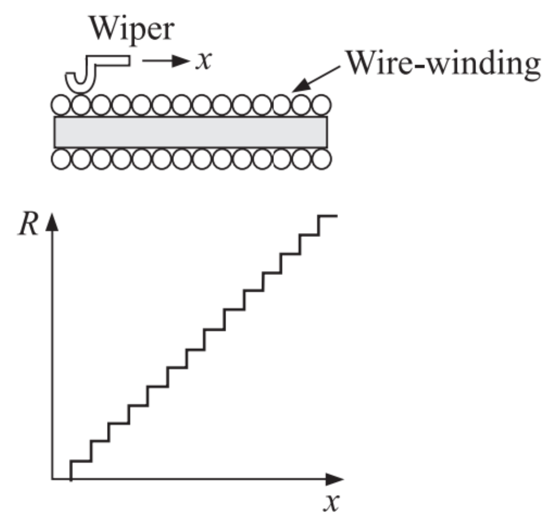


Fig. 6.6 Stepwise increase of resistance of wire-wound potentiometers.

Resistive Transducer: Potentiometer

- **Construction: wire-wound**
  - a stepwise increase in resistance
  - This limits the resolution, R
  - R=? For 400 turns on 20 mm long card

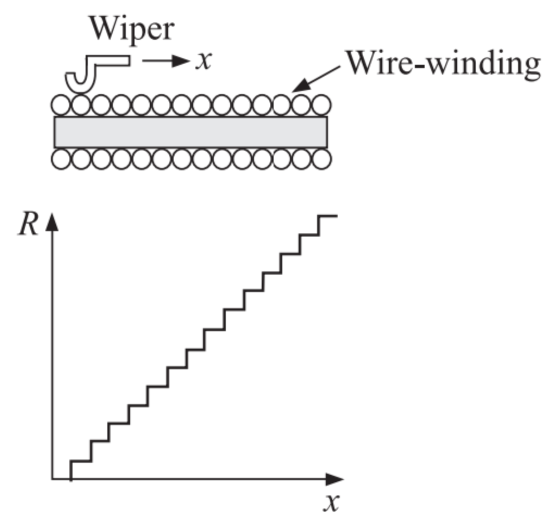


Fig. 6.6 Stepwise increase of resistance of wire-wound potentiometers.

Resistive Transducer: Potentiometer

- **Construction: wire-wound**
  - a stepwise increase in resistance
  - This limits the resolution, R
  - R=? For 400 turns on 20 mm long card

$$R = \frac{360 \times 10^{-3}}{\pi n D}$$

D is diameter in m and *n* is number of turns per mm

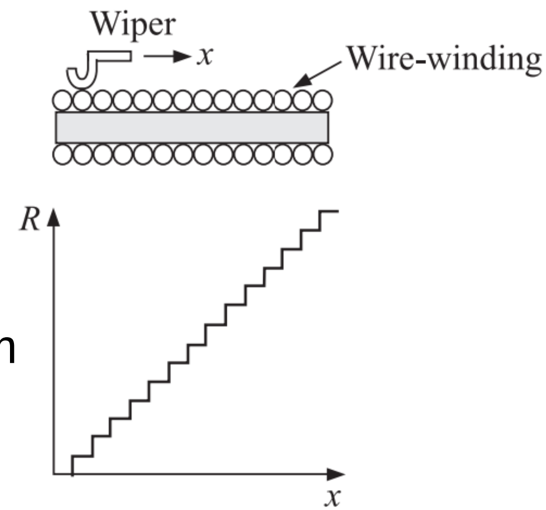
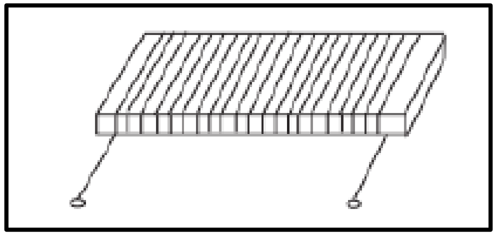


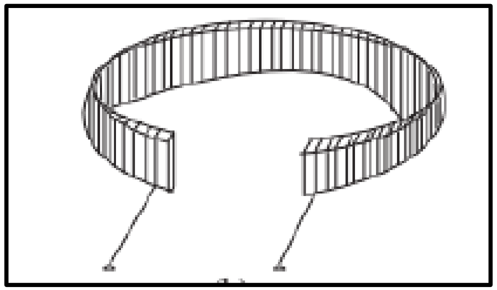
Fig. 6.6 Stepwise increase of resistance of wire-wound potentiometers.

Resistive Transducer: Potentiometer

Wire Wound:



If there are 400 turns on a 20 mm long card. What will be the resolution?



Resolution of a rotational resistive transducer is.

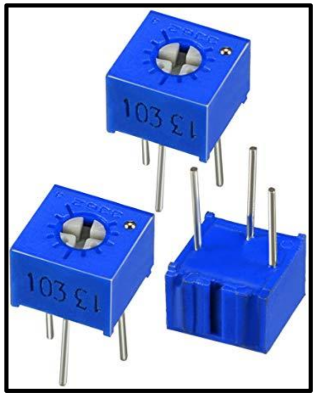
$$R = \frac{360 \times 10^{-3}}{\pi n D}$$

No of turns

where D is the diameter of the potentiometer in metres and n is the number of turns/mm.

Resistive Transducer: Potentiometer

Cermet: Is composed of precious metal particles fused into a ceramic base.



Advantages:

→ Stepless variation of resistance offering a very high resolution.

→ Large power ratings.

→ Low cost.

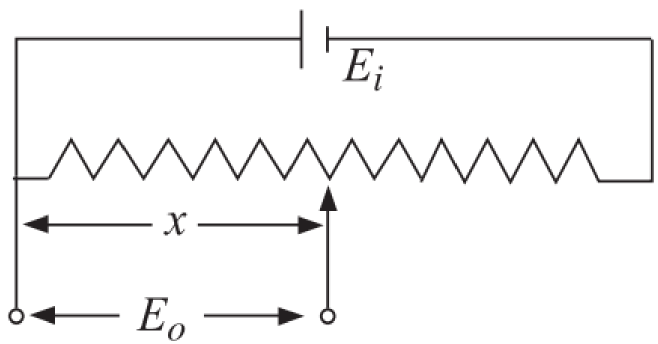
Resistive Transducer: Potentiometer

- **Construction: cermet**
  - **metal particles fused into a ceramic base constitute cermet**

1. Stepless variation of resistance offering a very high resolution
2. Large power ratings because it is not easily fusible
3. Low cost
4. Moderate temperature coefficients
5. Utility in ac applications

Resistive Transducer: Potentiometer

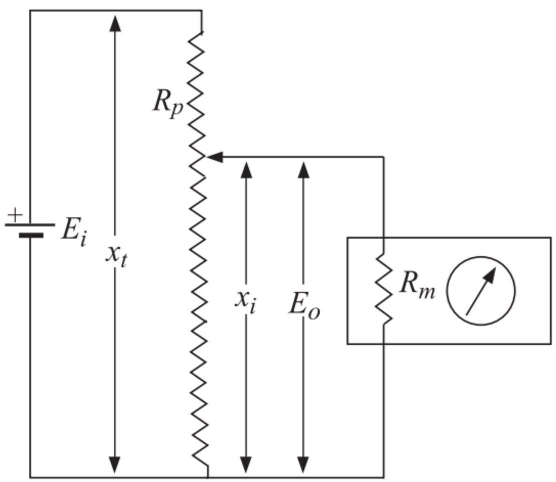
- Input-output relation is ideally linear for a resistor



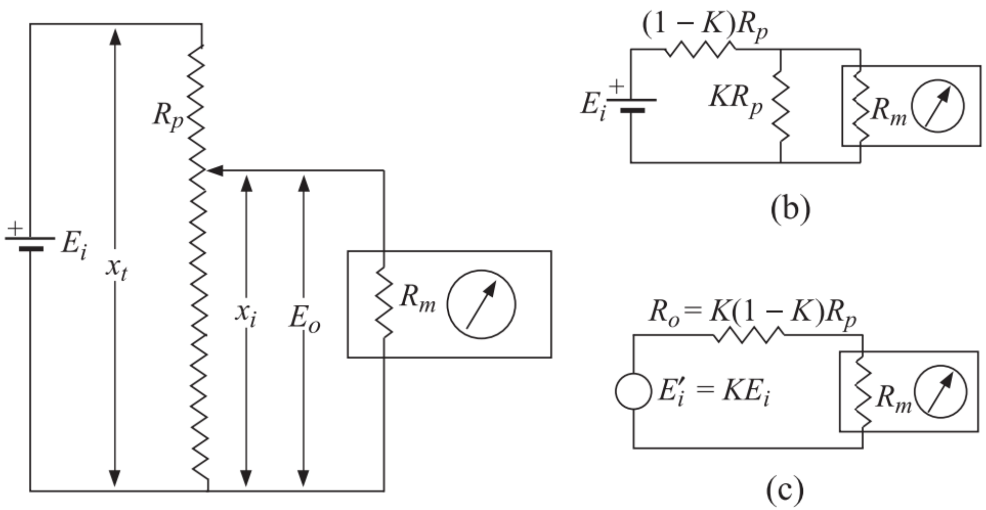
$$E_o = \frac{x}{L} E_i \equiv Kx$$

Resistive Transducer: Potentiometer

- **Characteristics: loading effects**
  - voltage measuring arrangement loads the output
    - Input-output relation becomes far from linear



Resistive Transducer: Potentiometer



- $K = \frac{x_i}{x_t}$

Resistive Transducer: Potentiometer

- **Characteristics: loading effects**
  - voltage measuring arrangement loads the output
    - Input-output relation becomes far from linear

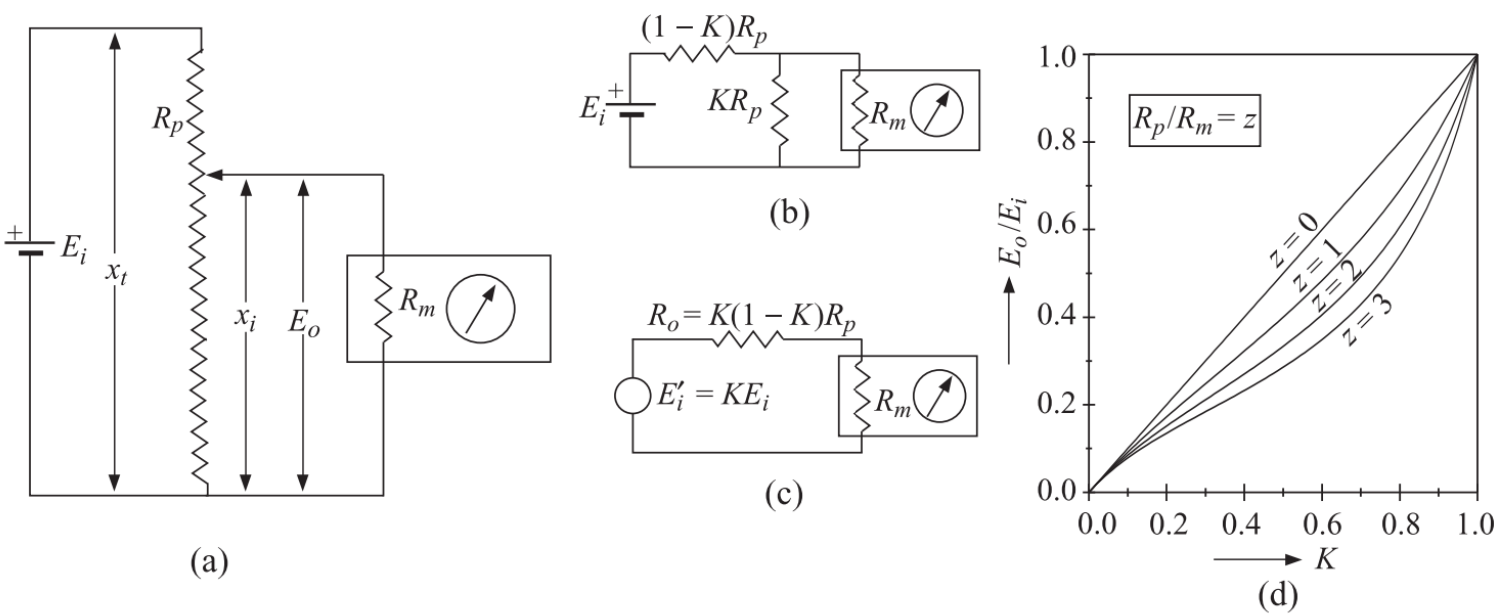


Fig. 6.7 Potentiometer loading effect: (a) circuit arrangement, (b) re-drawn circuit, (c) Thevenin equivalent circuit, and (d) characteristic curves.

Resistive Transducer: Potentiometer

❑ Characteristics: loading effects

➢ $K = \dfrac{x_i}{x_t}$

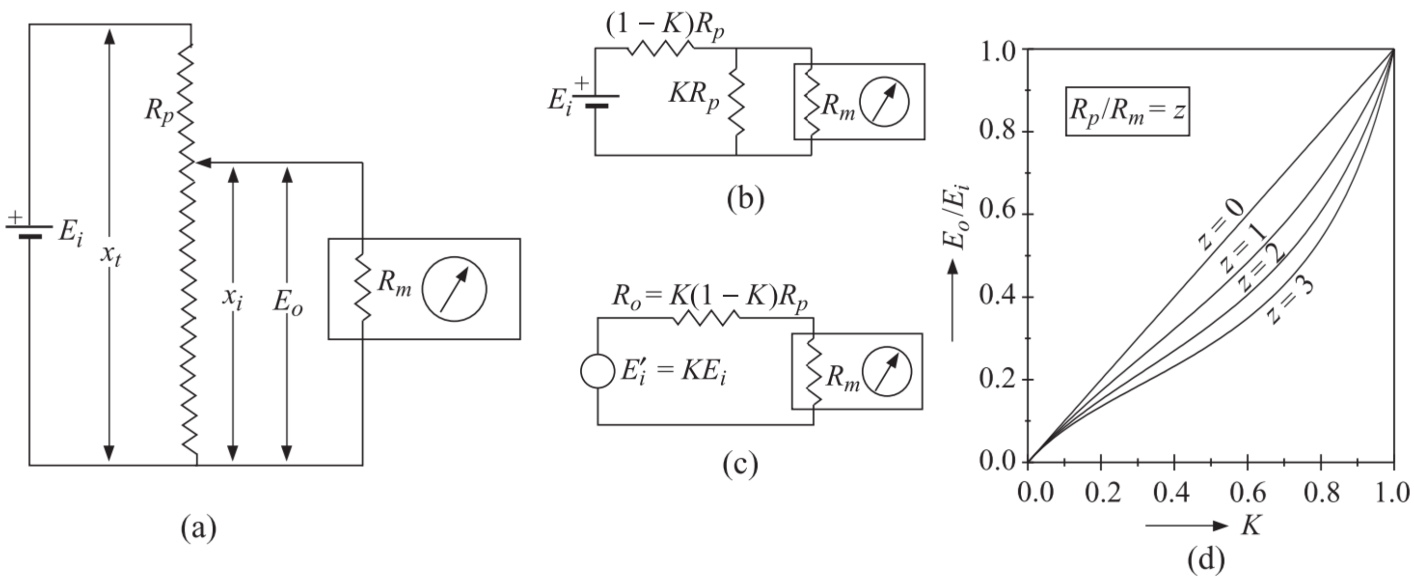


Fig. 6.7 Potentiometer loading effect: (a) circuit arrangement, (b) re-drawn circuit, (c) Thevenin equivalent circuit, and (d) characteristic curves.

$$\frac{E_o}{E_i} = \frac{K}{1 + K(1-K)\dfrac{R_p}{R_m}}$$

❑ In actual practice, $R_m \neq \infty$ and, therefore, the characteristics curve is nonlinear

Resistive Transducer: Potentiometer

❑ Characteristics: loading effects

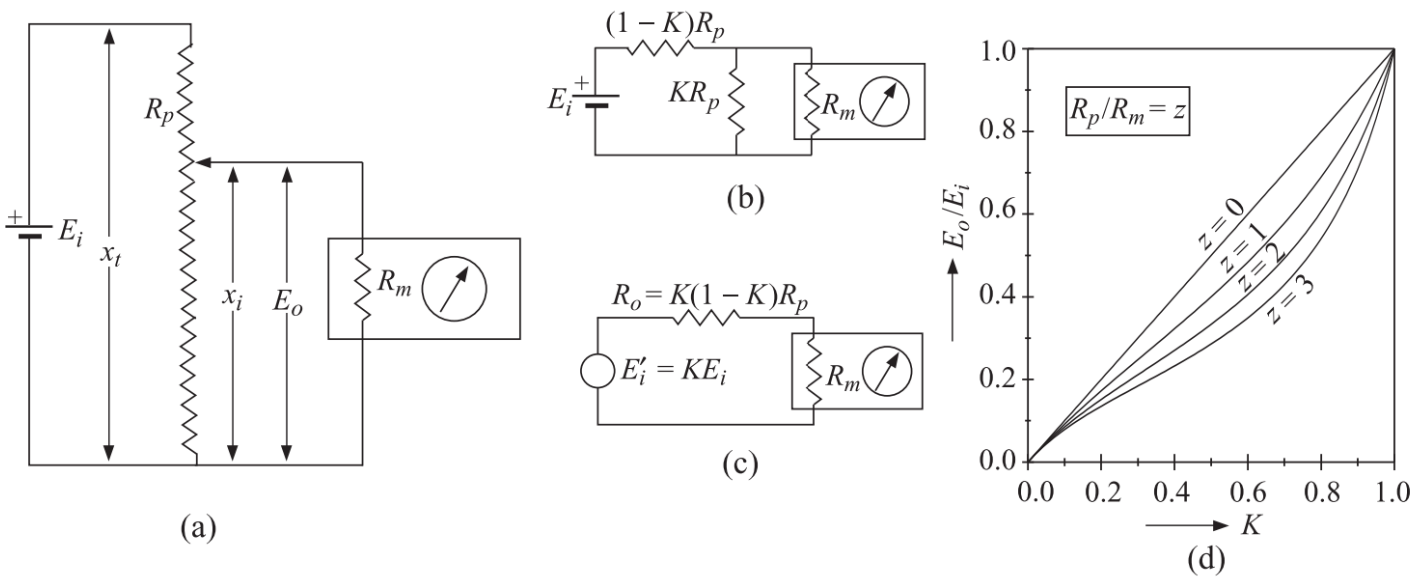


Fig. 6.7 Potentiometer loading effect: (a) circuit arrangement, (b) re-drawn circuit, (c) Thevenin equivalent circuit, and (d) characteristic curves.

- $K = \dfrac{x_i}{x_t}$

$$\frac{E_o}{E_i} = \frac{1}{1 + \dfrac{R_o}{R_m}} = \frac{K}{1 + K(1-K)\dfrac{R_p}{R_m}}$$

Table 6.1 Error caused by loading of the potentiometer

R_p/R_m	1.0	0.1	< 0.1
Maximum error (%)	12	1.5	$15(R_p/R_m)$

Resistive Transducer: Potentiometer

- **Characteristics: power ratings**
  - The typical available power rating is 5 W at room temperature

$$(E_i)_{\max} = \sqrt{PR_p} \text{ volt}$$

where P is the rated power in watts.

Resistive Transducer: Potentiometer

- **Characteristics: Linearity and sensitivity**
  - Typical values of sensitivity are 200 mV/mm for translational or 200 mV/deg for rotational devices

$$(E_i)_{\max} = \sqrt{PR_p} \text{ volt}$$

where P is the rated power in watts.

- For high sensitivity, the output voltage E_o and, so the input voltage E_i should be high
  - *Max* E_i is limited by R_p and power rating
  - For linearity: R_p has to be kept low in comparison to the resistance of the measuring instrument R_m for linearity

Resistive Transducer: Potentiometer

Example 6.3

The output of a potentiometer is to be read by a 10 kΩ voltmeter, holding non-linearity to 1%. A family of potentiometers having a thermal rating of 5W and resistances ranging from 100 Ω to 10 kΩ in 100 Ω steps are available. Choose from this family the pot that has the greatest possible sensitivity and meets other requirements. What is the sensitivity if pots are single-turn (360°) units?

Solution

To hold linearity to 1%, $R_p = R_m/15 = 666.7\,\Omega$. Pots available in this range are 600 Ω and 700 Ω. To ensure a high sensitivity we should choose 700 Ω, but then the nonlinearity goes above 1%. So, we have no alternative but to choose the 600 Ω pot. With this pot, the maximum excitation voltage is $\sqrt{5 \times 600} \cong 54.8$ V, and, therefore, the required sensitivity is $54.8/360 \cong 152$ mV/degree.

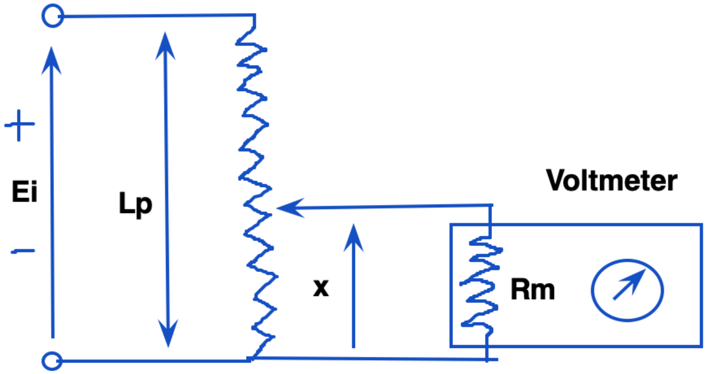
Resistive Transducer: Potentiometer

Table 6.2 Advantages and disadvantages of potentiometers

Advantages	*Disadvantages*
1. Inexpensive and simple to set up.	1. Mechanical loading owing to wiper friction.
2. Rather large displacements can be measured.	2. Electrical noise from the sliding contact.
3. Sufficient output to drive control circuits.	3. Wear and misalignment owing to friction.
4. Frequency response and resolution limited for the wire-wound, but unlimited for others.	4. Quick manipulation generates heat and associated problems.

❑ Numerical problem

Potentiometer: loading effect



Ei = 100V **Rm = 10 k ohm**

Lp = 100 mm

x = 10 mm

Potentiometer resistance, Rp = 10 k Ohm

$$R_x = \frac{x}{L_p} R_p = 1\,k\Omega$$

$$R_{xm} = R_x \| R_m = 0.91\,k\Omega$$

$$V_m = E_i \frac{R_{xm}}{R_{xm} + (R_p - R_x)} = 9.17V$$ **Measured voltage**

$$V_x = \frac{R_x}{R_p} E_i = 10\,V$$ **actual voltage**

loading error = ?

$$error = \frac{|V_x - V_m|}{V_x}$$

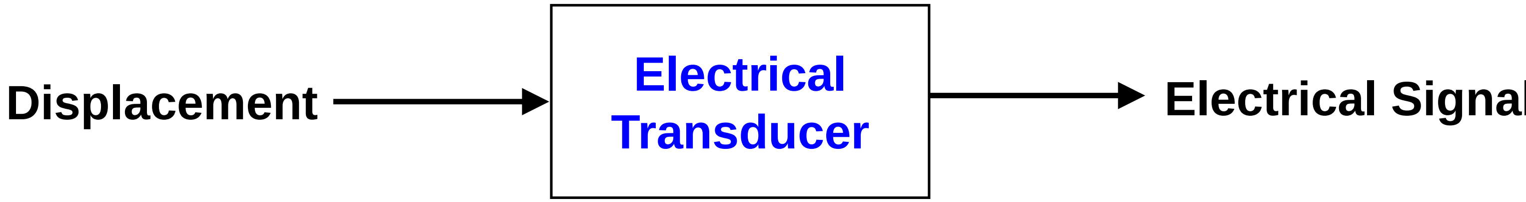
$$= \frac{|10 - 9.17|}{10} = 8.26\%$$

Displacement Measurement

- Classification
  - Electrical
  - Pneumatic
  - Optical
  - Ultrasonic
  - Magnetostrictive
  - Digital

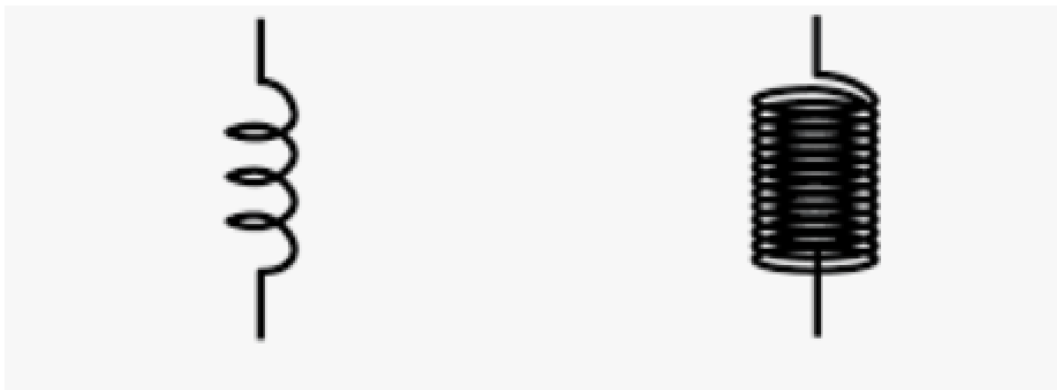
Displacement Measurement

- **Electrical Transducers**
  - **Convert displacement to an electrical signal**

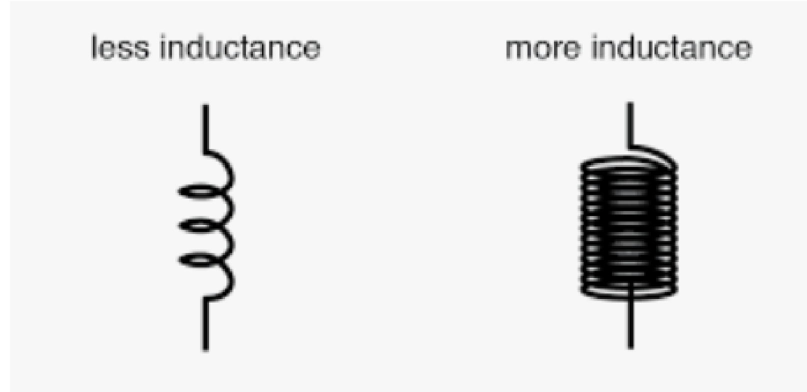


- Passive electrical components**: resistance, inductance and capacitance**
  - Resistive, inductive and capacitive transduction of displacement

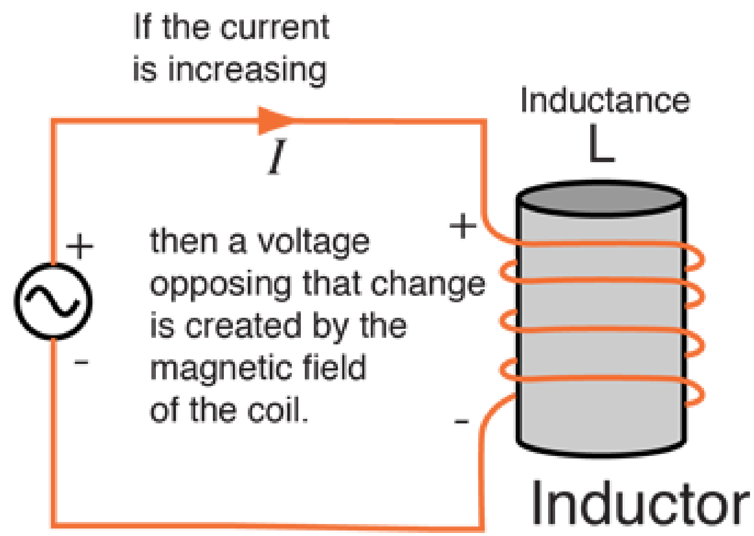
About Inductance?



About Inductance?



In electromagnetism and electronics, **inductance** is the tendency of an electrical conductor to oppose a change in the electric current flowing through it. The flow of electric current creates a magnetic field around the conductor.

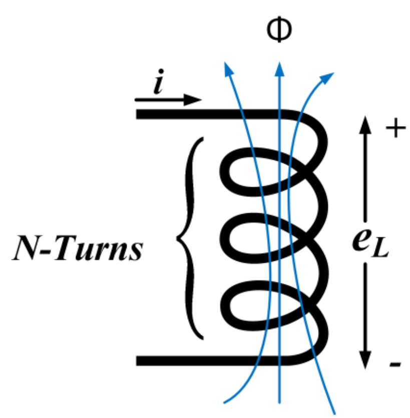


$$L = \frac{\Phi(i)}{i}$$

L = inductance

$\Phi(i)$ = magnetic flux of current i

i = current



$$\boxed{e_L = N\frac{\Delta\phi}{\Delta t}}$$

$$\boxed{L = N\frac{\Delta\phi}{\Delta i}}$$

Where L is the inductance in Henry, e_L is the induced counter-emf in volts and is the rate of change of current in A/s.

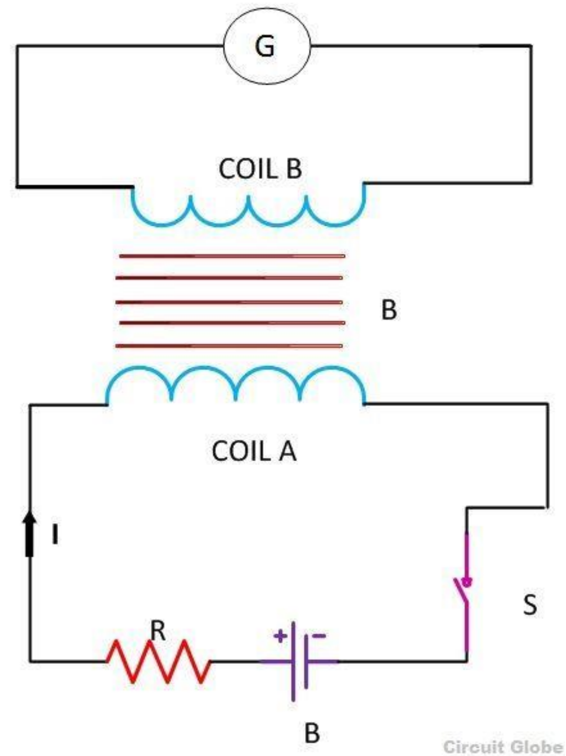
About Mutual Inductance?

- **Mutual Inductance** between the two coils is defined as the property of the coil due to which it opposes the change of current in the other coil, or you can say in the neighbouring coil.

- When the current in the neighbouring coil changes, the flux sets up in the coil and because of this, changing flux emf is induced in the coil called Mutually Induced emf and the phenomenon is known as **Mutual Inductance**.

Example: *Two coils namely coil A and coil B are placed nearer to each other.*
When the switch S is closed, and the current flows in the coil, it sets up the flux φ in the coil A and emf is induced in the coil and if the value of the current is changed by varying the value of the resistance (R), the flux linking with the coil B also changes because of this changing current.

Thus this phenomenon of the linking flux of the coil A with the other coil, B is called ***Mutual Inductance****.*



Inductive Transducer

- **Inductive transducers can be of various types; below are few:**

  1. Linear variable differential transformer (LVDT)
  2. Rotary variable differential transformer (RVDT)
  3. Synchro(s)

Linear variable differential transformer (LVDT)

- ❑ Electromechanical device that produce an ac output voltage proportional to the relative displacement of a transformer and an iron core

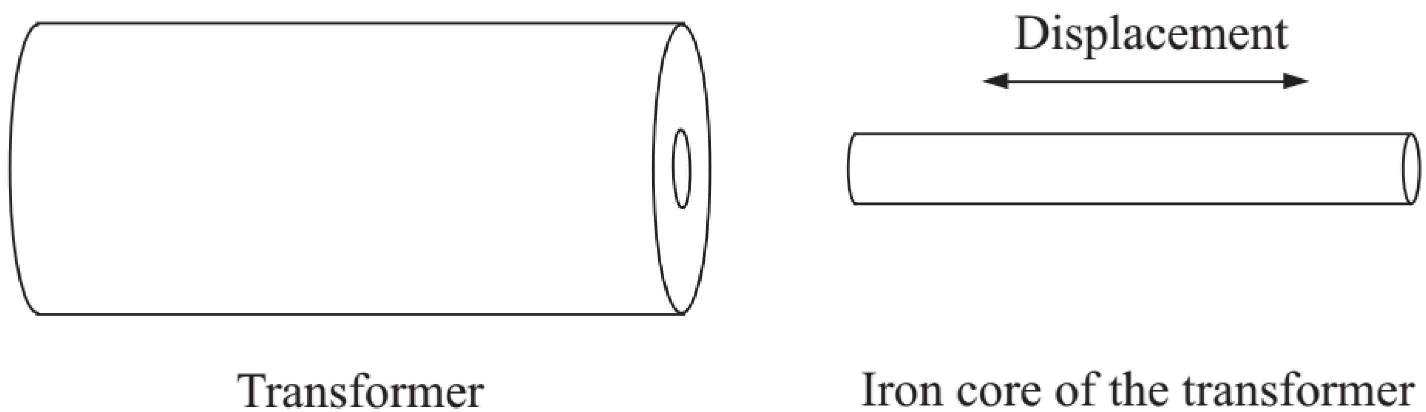
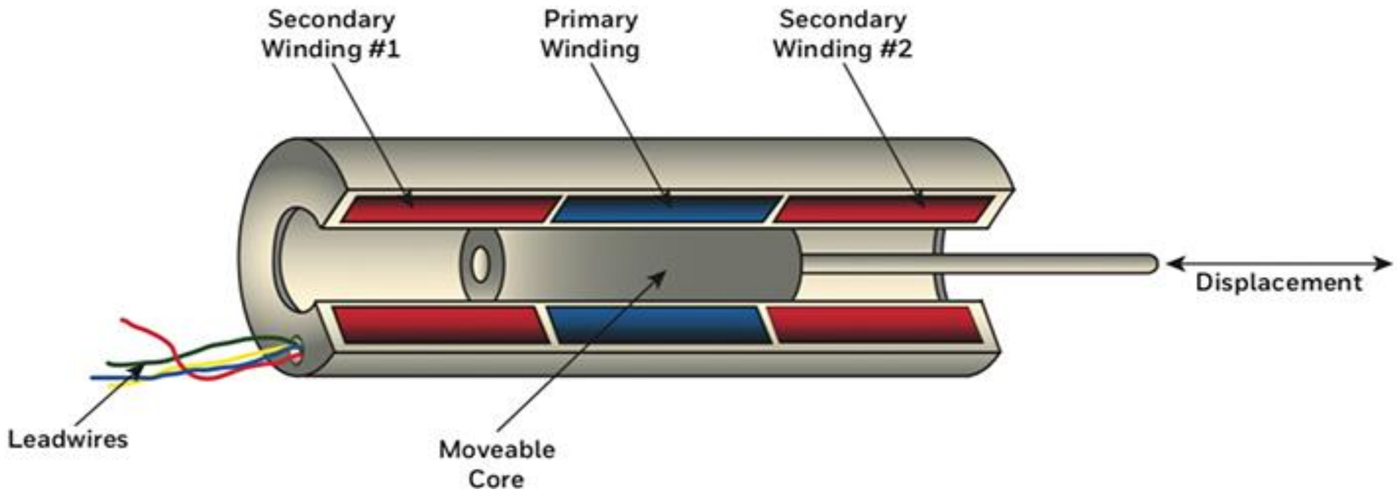


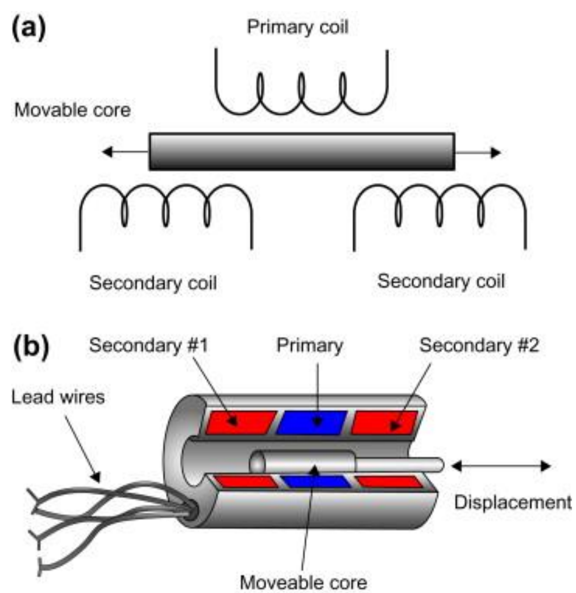
Fig. 6.8 Basics of LVDT.

- ❑ **Most commonly used inductive transducer in the industry**

Linear Variable Differential Transformer (LVDT)

Cross sectional view of LVDT.

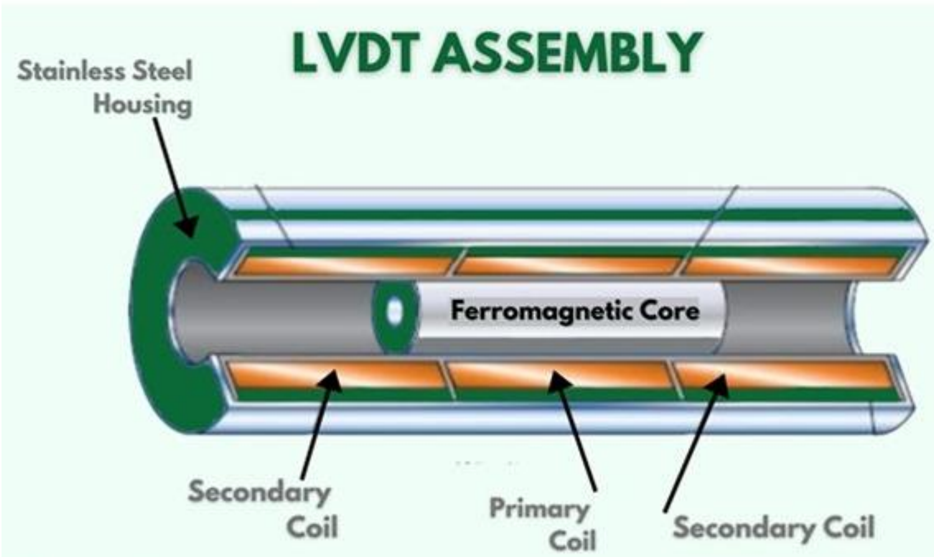


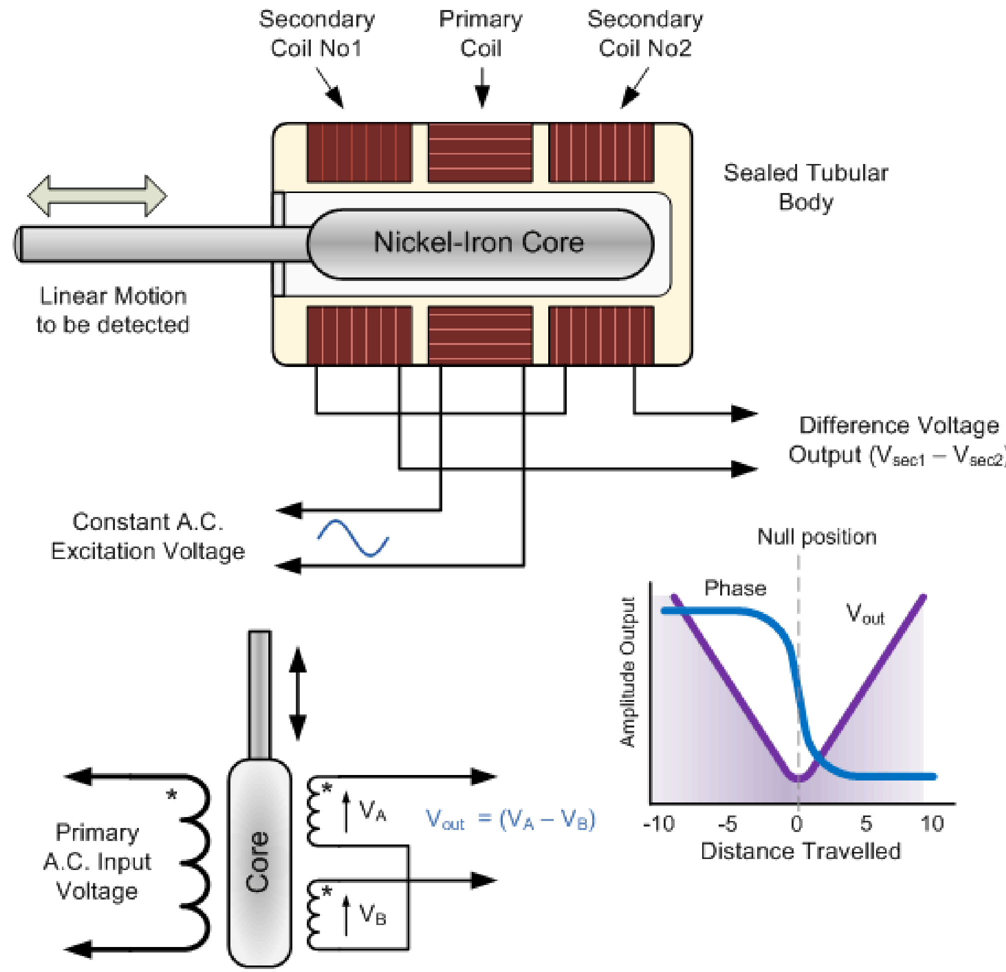


The term LVDT stands for the Linear Variable Differential Transformer. It is the most widely used inductive transducer that converts the linear motion into the electrical signal.

The output across secondary of this transformer is the differential thus it is called so.

Linear Variable Differential Transformer (LVDT)





Linear Variable Differential Transformer (LVDT)

Commercial Examples

Long Stroke LVDT

0.5 to 18.5 in range

Short Stroke LVDT

0.1 to 0.4 in range

Linear variable differential transformer (LVDT)

- **Structure**
  - Based on transformer principle (Faradays Law: Any change in the magnetic environment of a coil of wire will cause a voltage (emf) to be "induced" in the coil.
  - One primary and two secondary windings and a high permeability μ movable core
  - Secondary windings are identical in terms of number of turns and placement on both sides of the primary winding

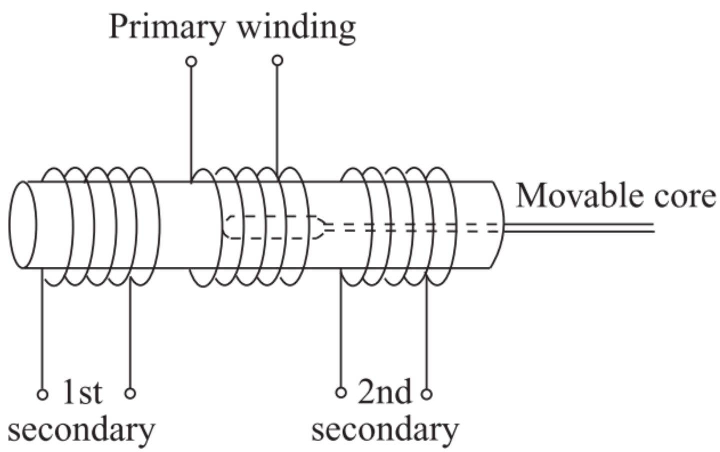


Fig. 6.9 Construction of LVDT.

Linear variable differential transformer (LVDT)

- ❑ **Structure**
  - ❑ One primary and two secondary windings and a high permeability μ movable core
  - ❑ Secondary windings are identical in terms of number of turns and placement on both sides of the primary winding

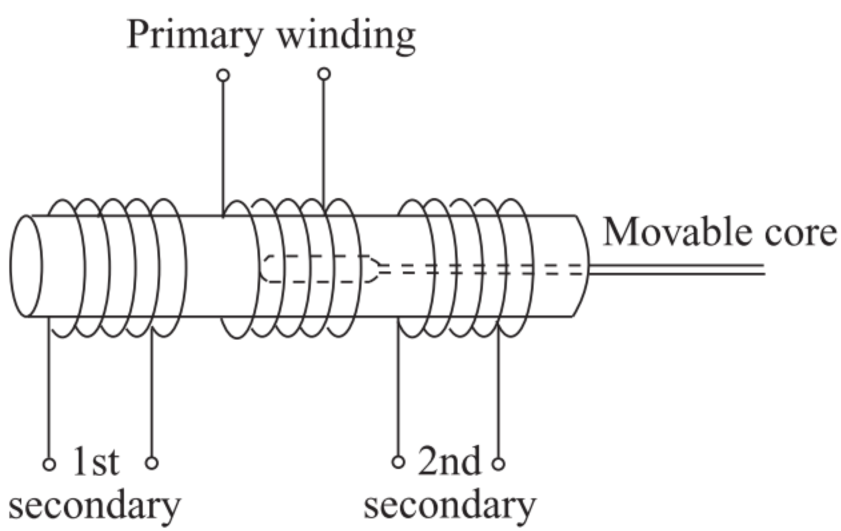
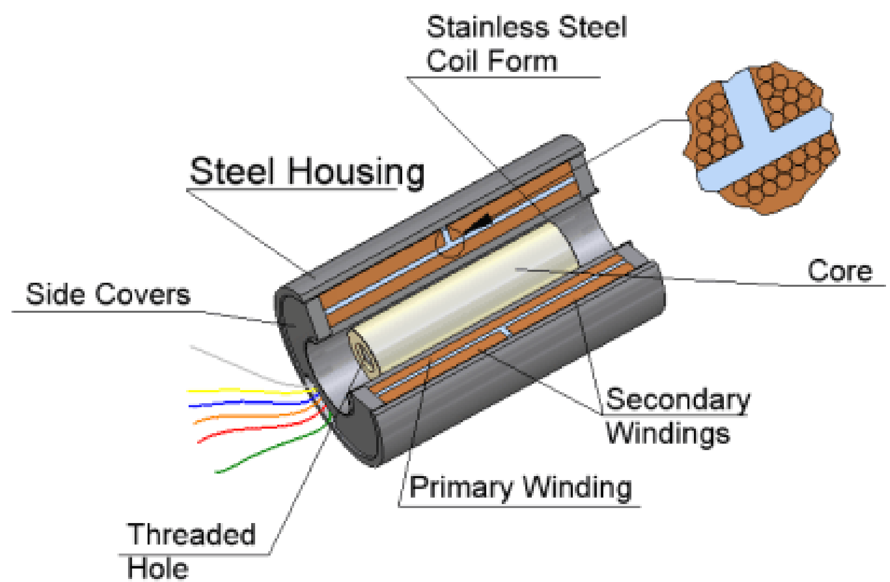


Fig. 6.9 Construction of LVDT.



Linear variable differential transformer (LVDT)

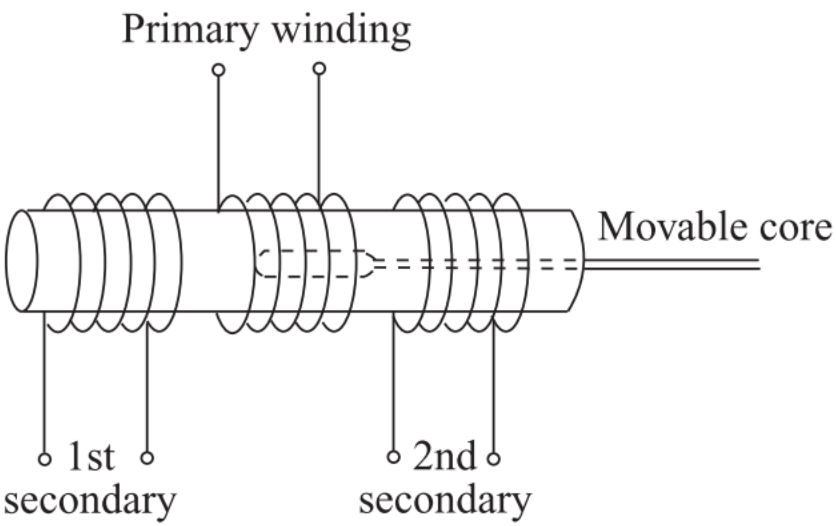
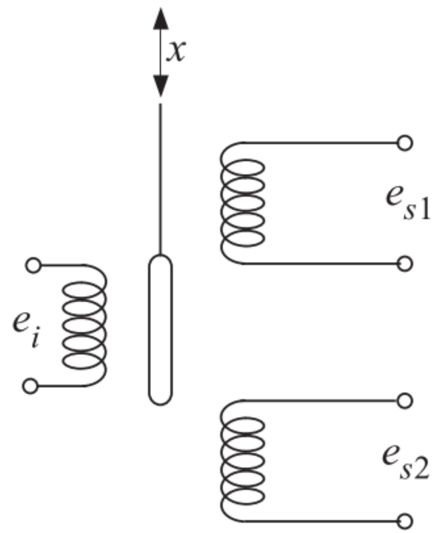


Fig. 6.9 Construction of LVDT.

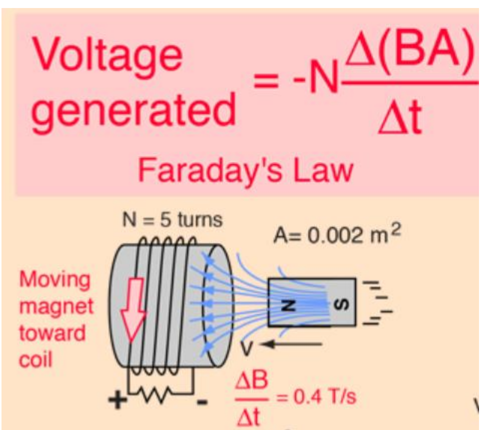
Real LVDT

Linear variable differential transformer (LVDT)

- **Working mechanism**
  - **Core movement causes a change in the mutual inductance between the coils**



Ref.



Any change in the magnetic environment of a coil of wire will cause a voltage (emf) to be "induced" in the coil.

Linear variable differential transformer

- ❑ **Working mechanism**
  - ❑ **Core movement causes a change in the mutual inductance between the coils**

A sinusoidal voltage e_i

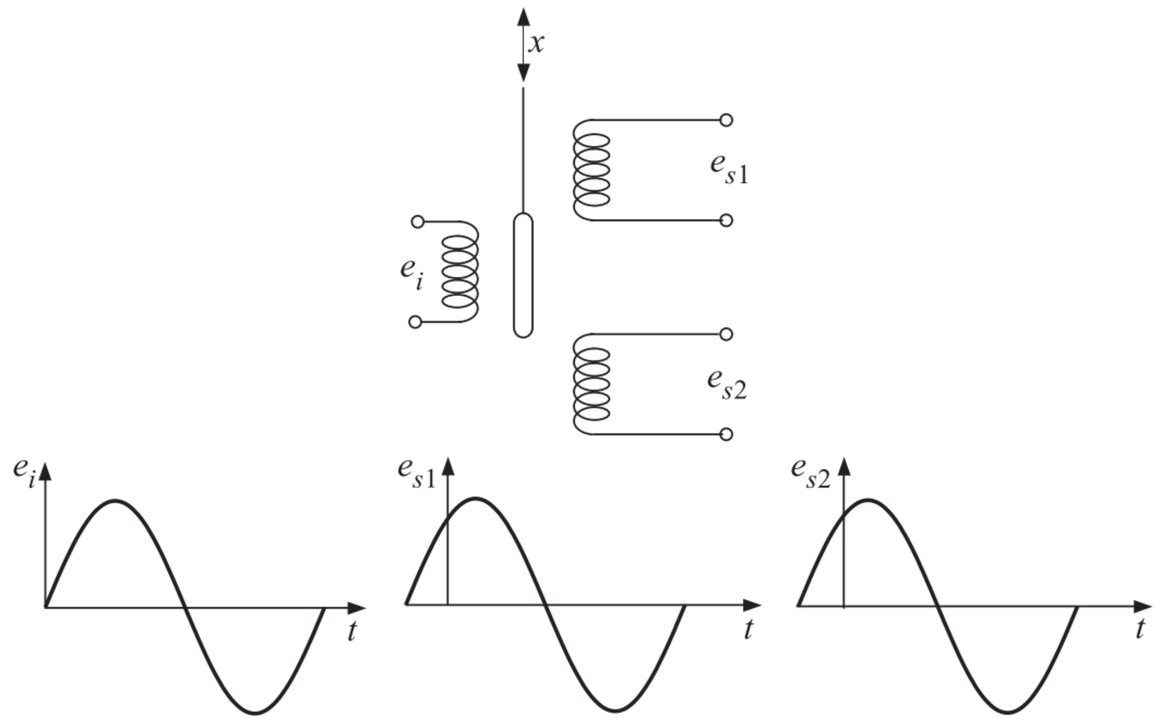
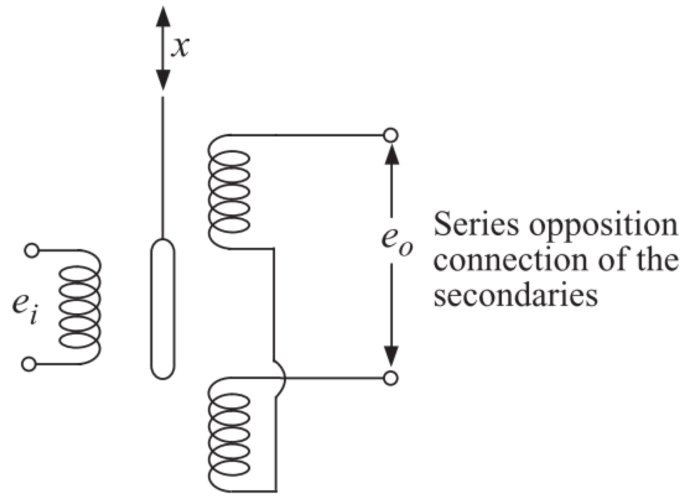


Fig. 6.10 Core in null position and the corresponding voltages.

- ❑ When the core is in the middle position, a sinusoidal voltage of equal amplitude appears across the two secondary windings

Linear variable differential transformer

- **Working mechanism**
  - Secondary windings are connected in series opposition



Linear variable differential transformer

- ❑ **Working mechanism**
- ❑ Secondary windings are connected in series opposition: their voltages cancel each other to produce a null voltage

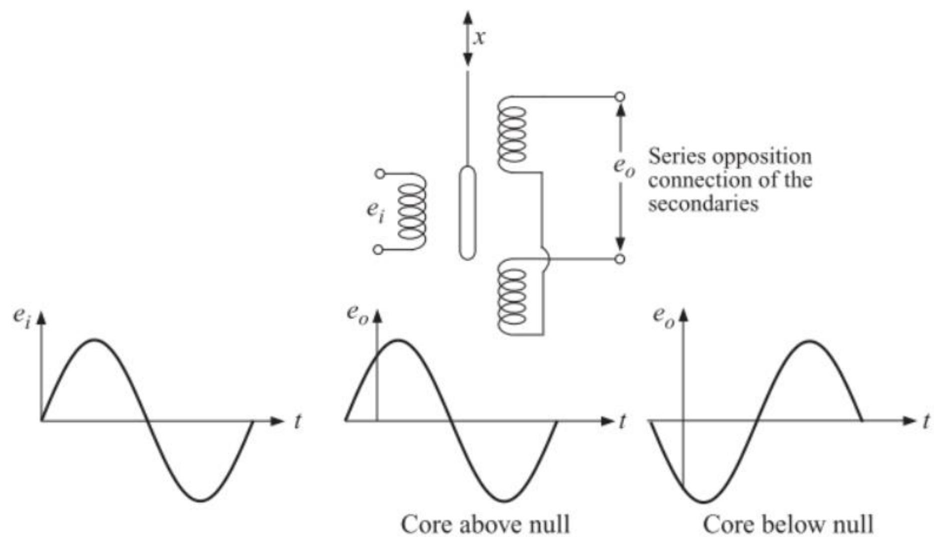


Fig. 6.11 Series opposing connection of secondaries and voltages for different core positions.

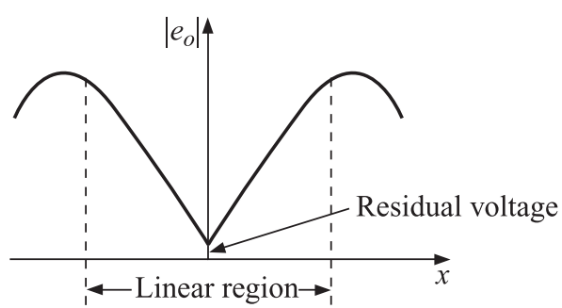
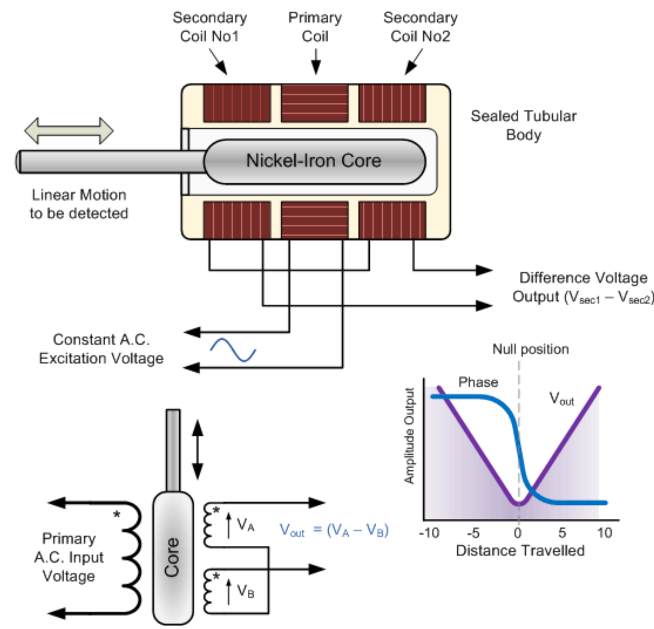
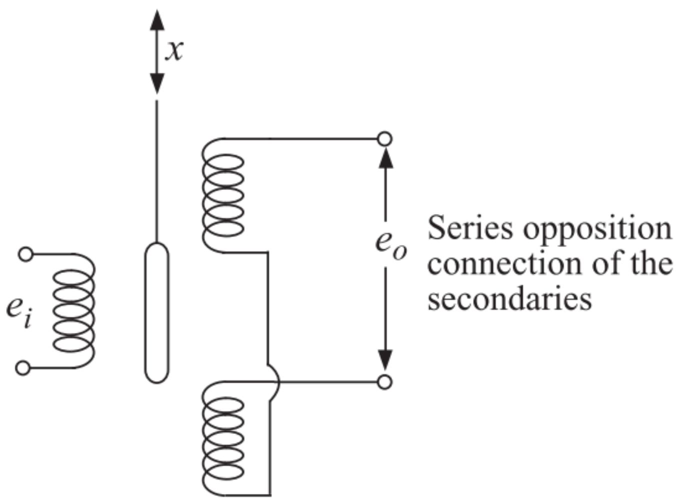


Fig. 6.12 Magnitude of output voltage for core displacement of an LVD



Linear variable differential transformer

- ❑ **Working mechanism**
- ❑ Secondary windings are connected in series opposition: their voltages cancel each other to produce a null voltage



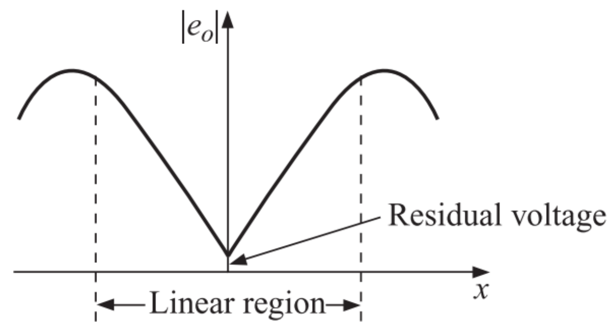


Fig. 6.12 Magnitude of output voltage for core displacement of an LVD

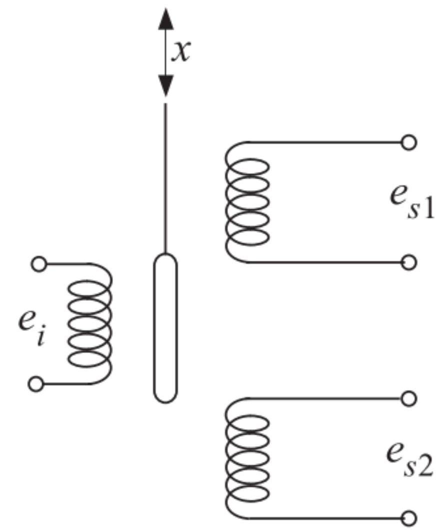
- ❑ **With the core's displacement, the output voltage increases linearly, undergoing a 180° phase-shift while passing through the null**

(Fig. 6.12). The output loses its linear relationship with displacement beyond some limits and this property restricts the range of the LVDT. The normal range is from ±10 μm to ±10 mm.

Linear variable differential transformer

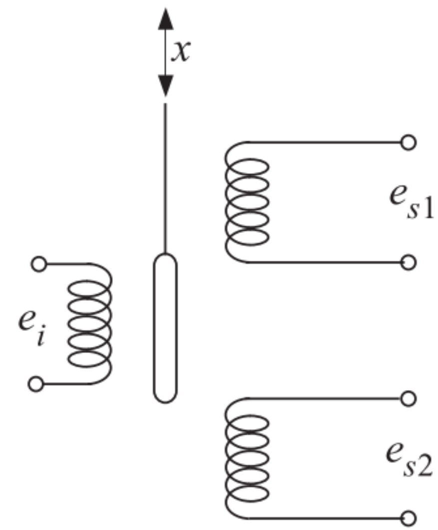
- **Circuit Analysis**
  - When the secondary is open circuited
  - applying KVL to the primary gives

$$i_p R_p + L_p \frac{di_p}{dt} = e_i$$



Linear variable differential transformer

- ❑ **Circuit Analysis**
  - ❑ When the secondary is open circuited
  - ❑ applying KVL to the primary gives



$$i_p R_p + L_p \frac{di_p}{dt} = e_i$$

taking Laplace transform,

$$(sL_p + R_p)I_p = E_i$$

$$I_p = \frac{E_i}{sL_p + R_p} \equiv \frac{E_i/R_p}{\tau_p s + 1} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad 1$$

Where τ_p = L_p / R_p

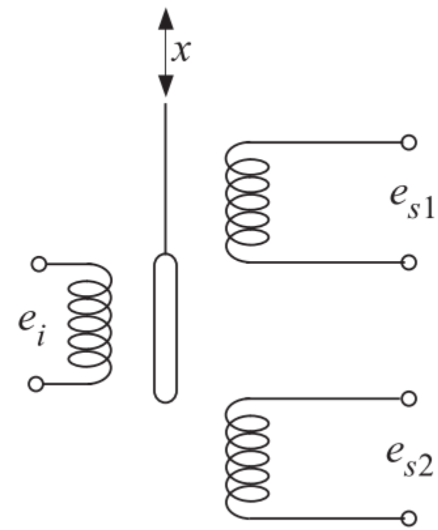
Linear variable differential transformer

- **Circuit Analysis**
  - Voltages induced in the secondary windings

$$e_{s1} = M_1 \frac{di_p}{dt}$$

$$e_{s2} = M_2 \frac{di_p}{dt}$$

mutual inductances of coefficients M_1 and M_2

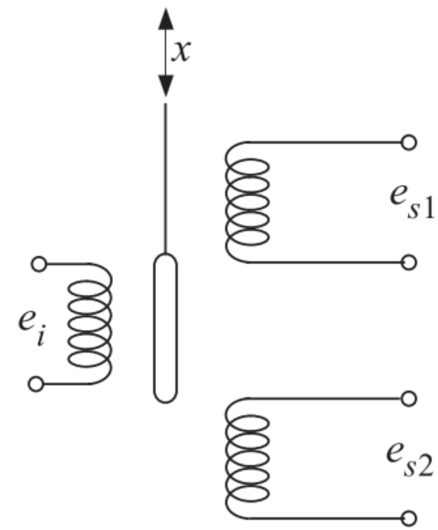


Linear variable differential transformer

- **Circuit Analysis**
  - Voltages induced in the secondary windings

$$e_{s1} = M_1 \frac{di_p}{dt}$$

$$e_{s2} = M_2 \frac{di_p}{dt}$$



taking Laplace transform,

$$E_{s1} = sM_1 I_p \qquad \text{.....................} \quad 2$$

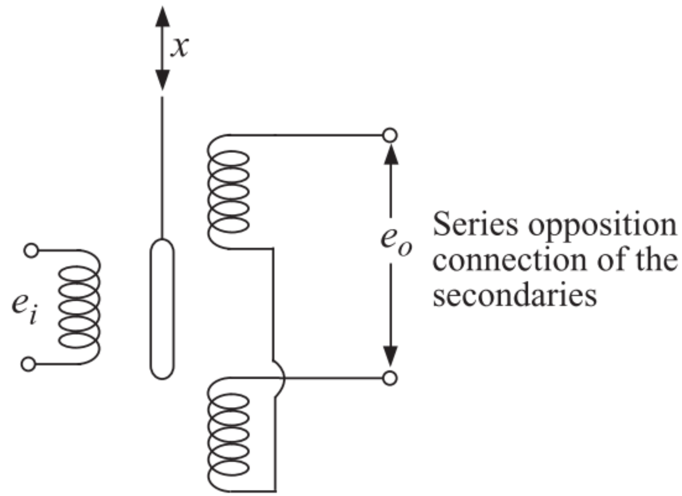
$$E_{s2} = sM_2 I_p \qquad \text{.....................} \quad 3$$

Linear variable differential transformer

$$I_p = \frac{E_i}{sL_p + R_p} \equiv \frac{E_i/R_p}{\tau_p s + 1} \qquad \text{.....................} \quad 1$$

$$E_{s1} = sM_1 I_p \qquad \text{.....................} \quad 2$$

$$E_{s2} = sM_2 I_p \qquad \text{.....................} \quad 3$$

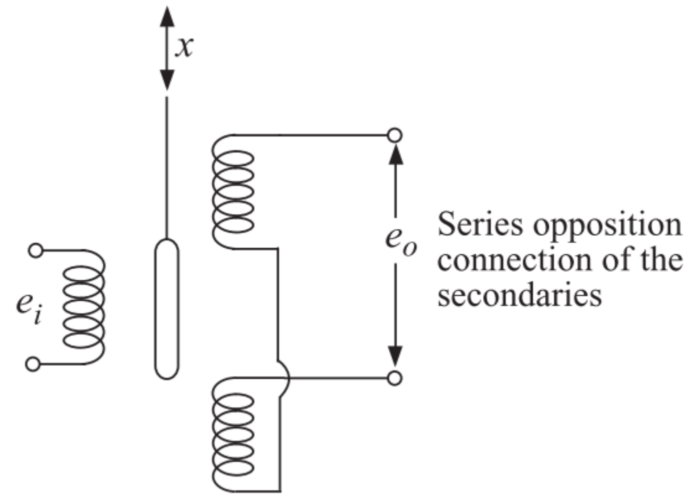


Linear variable differential transformer

$$I_p = \frac{E_i}{sL_p + R_p} \equiv \frac{E_i/R_p}{\tau_p s + 1} \quad \text{.....................} \quad (1)$$

$$E_{s1} = sM_1 I_p \quad \text{.....................} \quad (2)$$

$$E_{s2} = sM_2 I_p \quad \text{.....................} \quad (3)$$



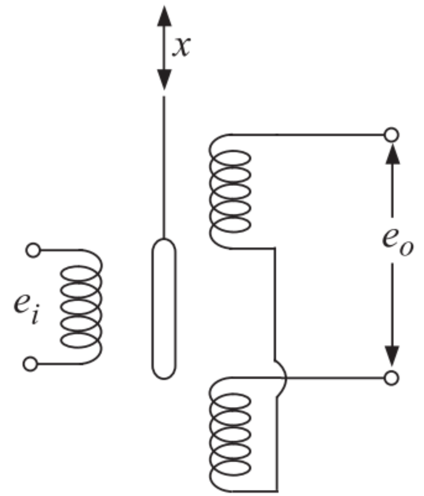
- ❑ Due to the series opposition connection of the secondary windings,
  - ❑ Taking (eq. 2 – eq. 3) and substituting I_p from eq. 1

$$E_o \equiv E_{s1} - E_{s2} = (M_1 - M_2)sI_p = \frac{(M_1 - M_2)s/R_p}{\tau_p s + 1} E_i \quad \text{.....................} \quad (4)$$

Linear variable differential transformer

$$E_o \equiv E_{s1} - E_{s2} = (M_1 - M_2)sI_p = \frac{(M_1 - M_2)s/R_p}{\tau_p s + 1} E_i \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad 4$$

- **Simplifying and replacing *jω* with *s***



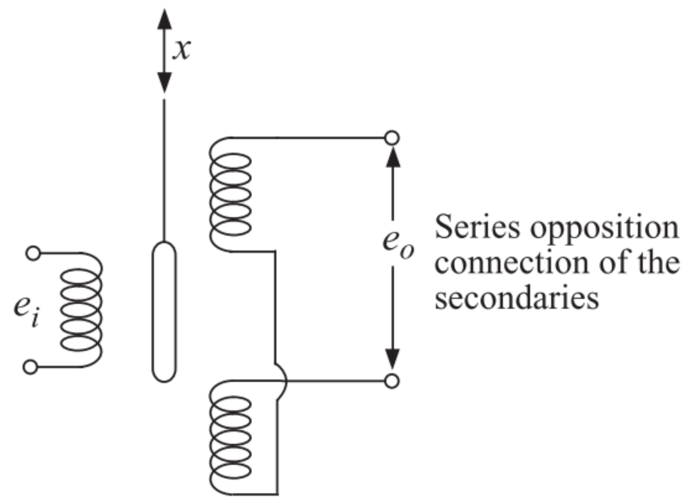
$$\frac{E_o(s)}{E_i(s)} = \frac{s(M_1 - M_2)/R_p}{\tau_p s + 1}$$

$$\frac{E_o(j\omega)}{E_i(j\omega)} = \frac{j\omega(M_1 - M_2)/R_p}{j\omega\tau_p + 1} \equiv \frac{\omega(M_1 - M_2)/R_p}{\sqrt{(\omega\tau_p)^2 + 1}} \angle\phi$$

$$\phi = \frac{\pi}{2} - \tan^{-1} \omega\tau_p$$

Linear variable differential transformer

$$\frac{E_o(s)}{E_i(s)} = \frac{s(M_1 - M_2)/R_p}{\tau_p s + 1}$$



$$\frac{E_o(j\omega)}{E_i(j\omega)} = \frac{j\omega(M_1 - M_2)/R_p}{j\omega\tau_p + 1} \equiv \frac{\omega(M_1 - M_2)/R_p}{\sqrt{(\omega\tau_p)^2 + 1}} \angle\phi$$

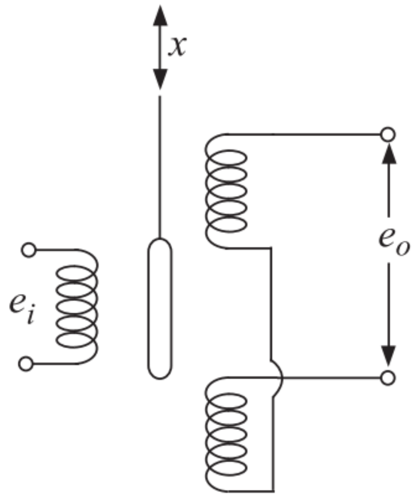
$$\phi = \frac{\pi}{2} - \tan^{-1}\omega\tau_p$$

- If A_o is the amplitude of the output and A_i is the input's amplitude
  - Ratio of the amplitudes

$$\frac{A_o}{A_i} = \frac{\omega(M_1 - M_2)/R_p}{\sqrt{(\omega\tau_p)^2 + 1}} \qquad \text{.....................} \quad (5)$$

Linear variable differential transformer

$$\frac{A_o}{A_i} = \frac{\omega(M_1 - M_2)/R_p}{\sqrt{(\omega\tau_p)^2 + 1}} \quad \text{....................} \quad (5)$$



- Amplitude of the output A_o
  - K and K' are constants and x is the displacement

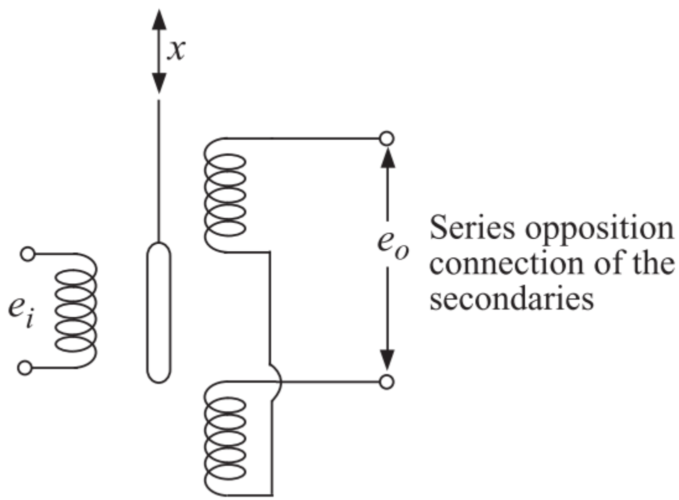
$$\boxed{A_o = K(M_1 - M_2) \equiv K'x}$$

$$K = A_i \frac{\omega/R_p}{\sqrt{(\omega\tau_p)^2 + 1}}$$

A_i is the amplitude of the input excitation

Linear variable differential transformer

- **Circuit Analysis**
  - **amplitude linearly varies with the difference in mutual inductances**



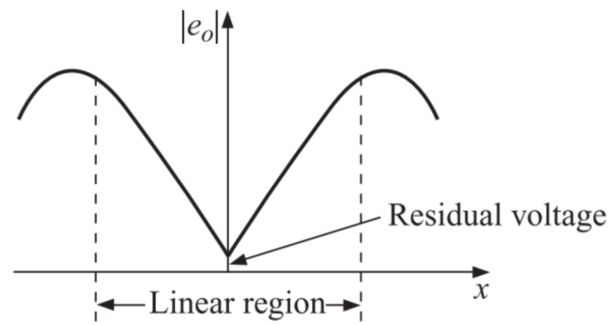


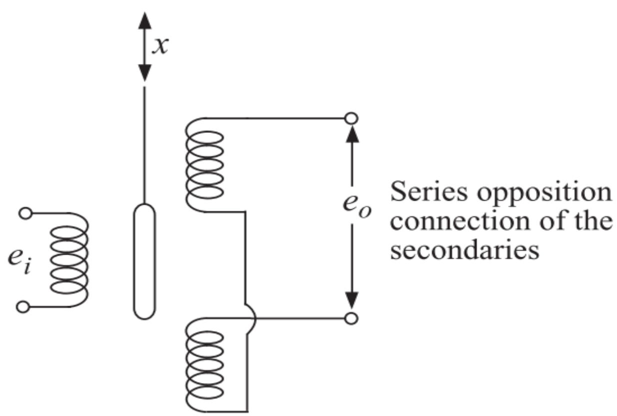
Fig. 6.12 Magnitude of output voltage for core displacement of an LVD

$$\boxed{A_o = K(M_1 - M_2) \equiv K'x}$$

- **Value of (M_1−M_2) increase with the core's displacement; after a certain point, it starts falling as the core moves past one of the secondaries**

Linear variable differential transformer

- ❑ **Excitation frequency**
  - ❑ Excitation frequency must be much higher than the core-movement frequency
    - ❑ For good dynamic response
    - ❑ Necessary to distinguishing them in amplitude modulated signal.



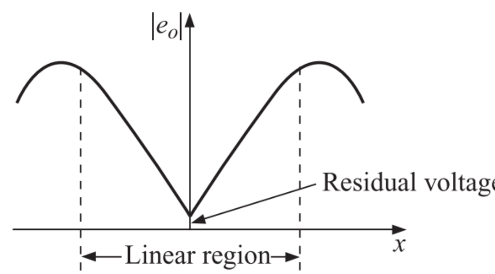


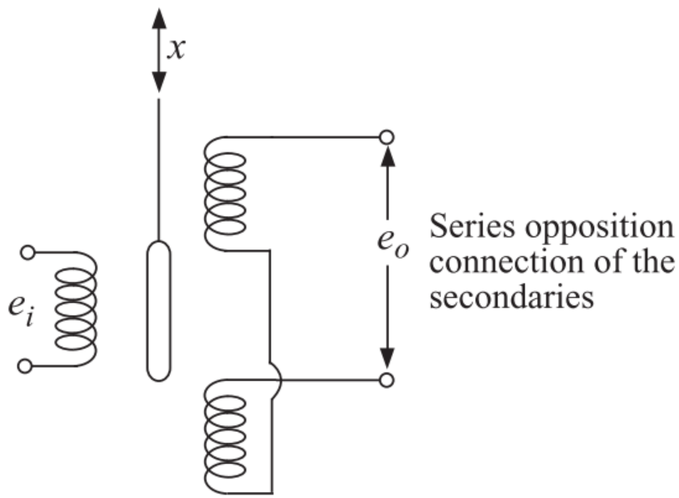
Fig. 6.12 Magnitude of output voltage for core displacement of an LVD

- ❑ **Rule of thumb**

$$\boxed{\frac{\text{Maximum core-movement frequency}}{\text{Excitation frequency}} = \frac{1}{10}}$$

Linear variable differential transformer

- **Residual voltage**
  - Ideally, output voltage at null position is zero
  - Stray capacitance coupling between primary and secondary result in a small but non-zero voltage



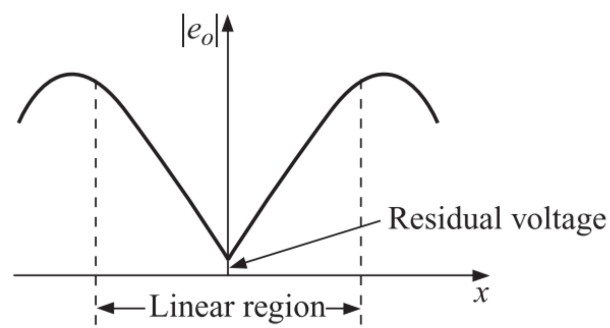


Fig. 6.12 Magnitude of output voltage for core displacement of an LVD

Linear variable differential transformer

Typical specifications. Typical specifications of an LVDT are given in Table 6.3.

Table 6.3 Typical specifications of an LVDT

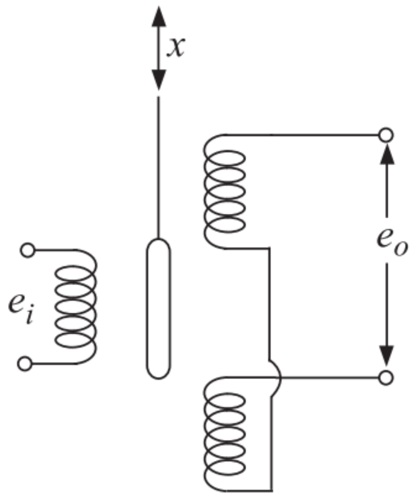
Normal excitation voltage	*Normal range*	*Operating temperature*
1.0 V at 2 to 10 kHz	±10µm to ±10 mm	–40 to +100 °C

Linear variable differential transformer

Table 6.4 Advantages and disadvantages of LVDT

Advantages	*Disadvantages*
1. Linearity is good up to 5 mm.	1. Rather large threshold.
2. Output voltage is stepless and hence the resolution is good (~ 1µm).	2. Affected by stray electromagnetic fields. Hence, proper shielding of the device is necessary.
3. Output is rather high. Therefore, intermediate amplification is not necessary.	3. AC input generates noises.
4. Sensitivity is high (~ 40 V/mm).	4. Sensitivity is lower at higher temperatures.
5. Low power and low hysteresis device.	
6. Short response time, only limited by the inertia of the iron core and the rise time of the amplifiers.	
7. Does not load the measurand mechanically.	
8. Solid and robust, capable of working in a wide variety of environments. No permanent damage to the LVDT if measurements exceed the designed range.	
9. Relatively low cost owing to its popularity.	

Linear variable differential transformer



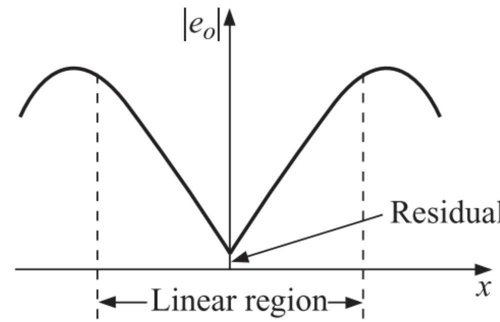


Fig. 6.12 Magnitude of output voltage for core displacement of an LVD

- ❑ **Principle of operation**
- ❑ Displacement, to be measured, displaces the moveable core causing a change in the mutual inductance of the transformer thereby generating a voltage at the output of the two secondary windings connected in a series opposition.

Rotary variable differential transformer

- To measure rotational angles

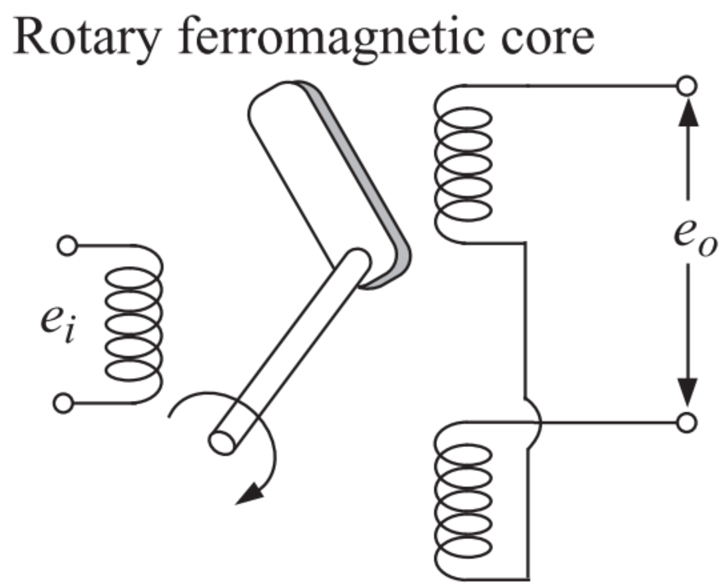


Fig. 6.15 Schematic diagram of an RVDT.

Rotary variable differential transformer

- ❑ **Principle of operation**
  - ❑ Same as LVDT, but it uses a rotary ferromagnetic core

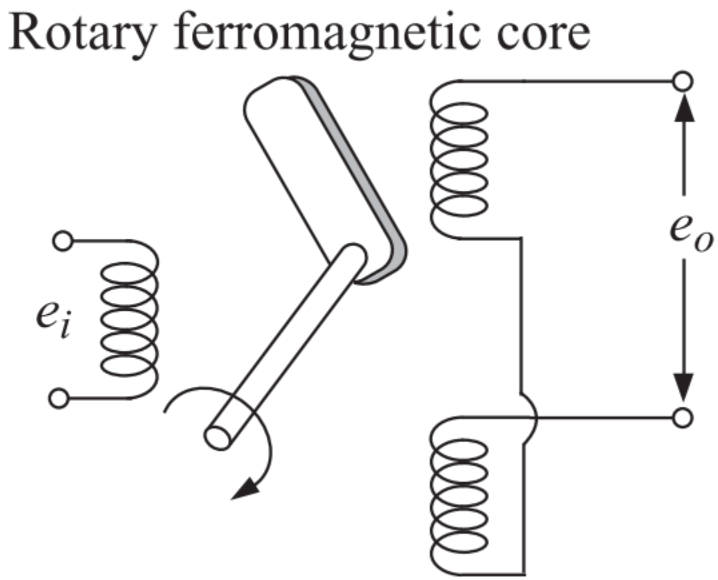


Fig. 6.15 Schematic diagram of an RVDT.

Synchro

❑ Based on Variations of Reluctance

- ❑ Magnetic reluctance, or magnetic resistance, is a concept used in the analysis of magnetic circuits. It is defined as the ratio of magnetomotive force to magnetic flux.
- ❑ It represents the opposition to magnetic flux, and depends on the geometry and composition of an object

$$\mathcal{R} = \frac{\mathcal{F}}{\phi}$$

$\mathcal{R}$ = reluctance in ampere-turns per weber

$\mathcal{F}$ = magnetomotive force

ϕ = magnetic flux in webers

❑ To measure angles

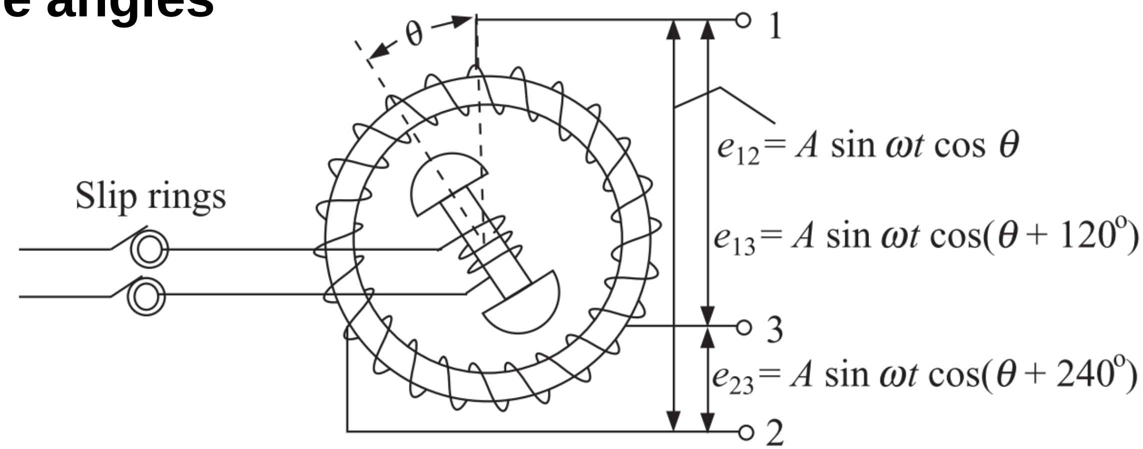


Fig. 6.16 Schematic presentation of a synchro.

❑ Essentially a transformer

- ❑ Rotor: single phase winding
- ❑ Stator: three phase winding (the phases being displaced by 120 degree)

Synchro

- **To measure angles**
  - Motion of the rotor produces a variable mutual inductance between the two windings

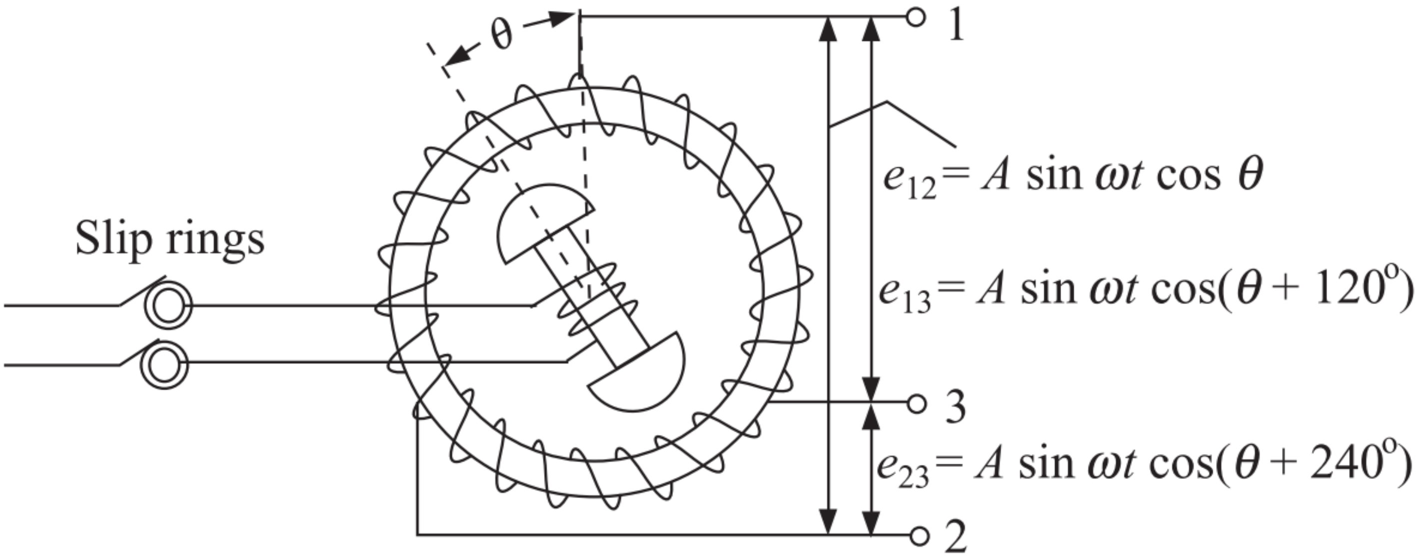


Fig. 6.16 Schematic presentation of a synchro.

Synchro

- **To measure angles**
  - Motion of the rotor produces a variable mutual inductance between the two windings

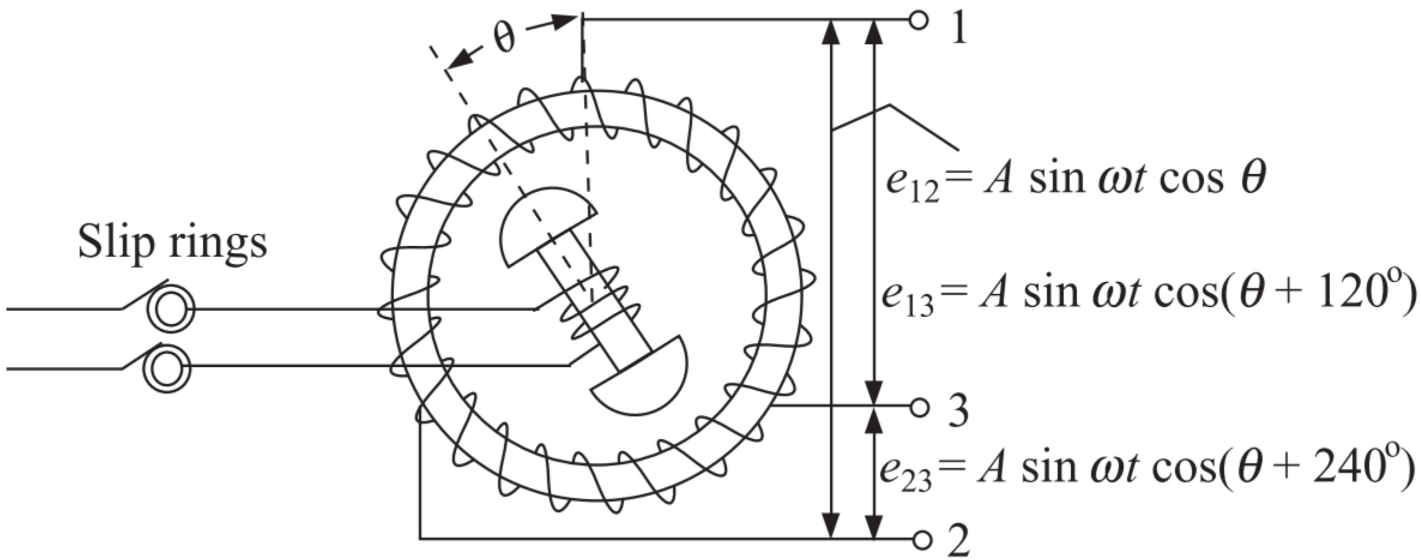
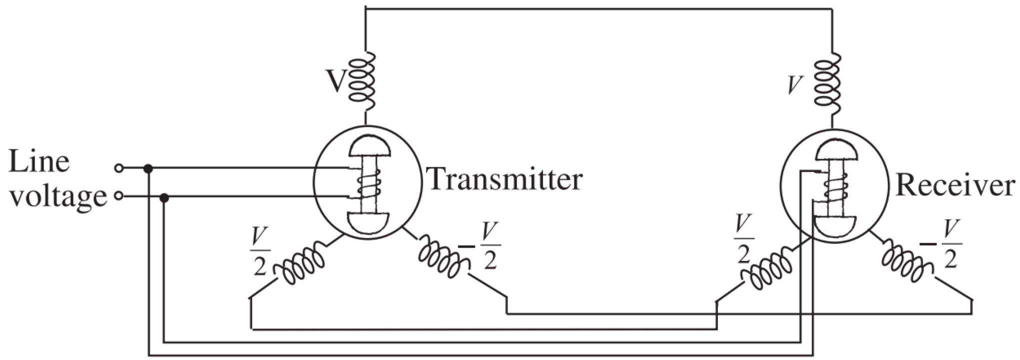


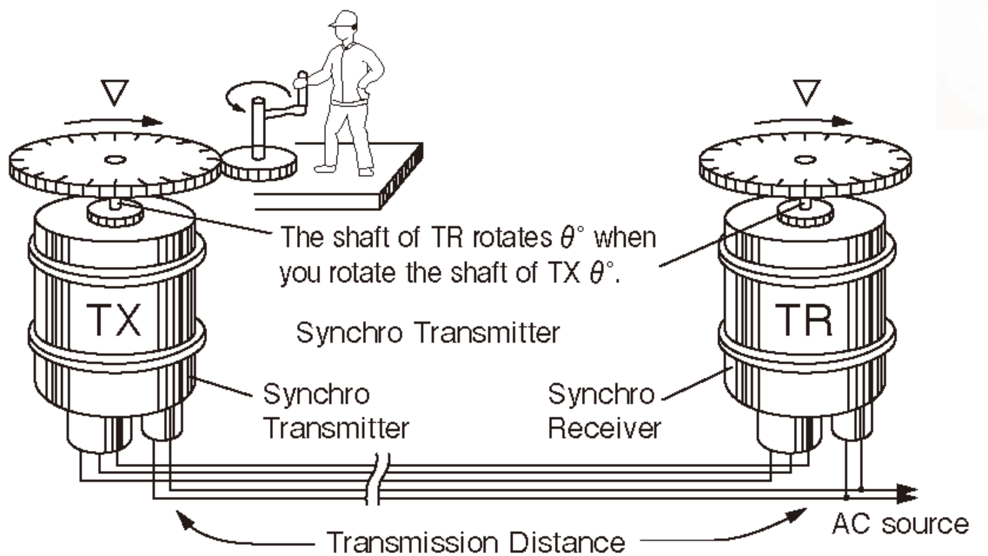
Fig. 6.16 Schematic presentation of a synchro.

- The angle can be measured using the output voltages of the stator's windings

Synchros

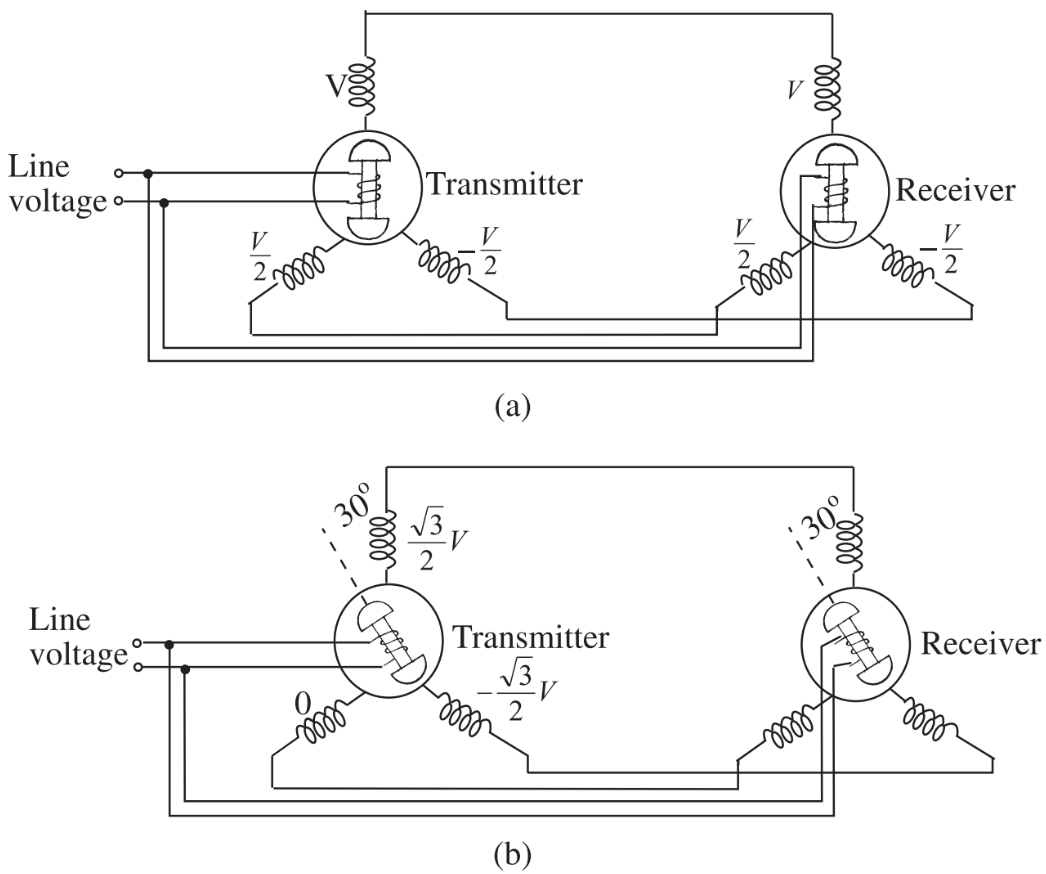
- Servomechanism: automatic motion control feedback system





Synchros

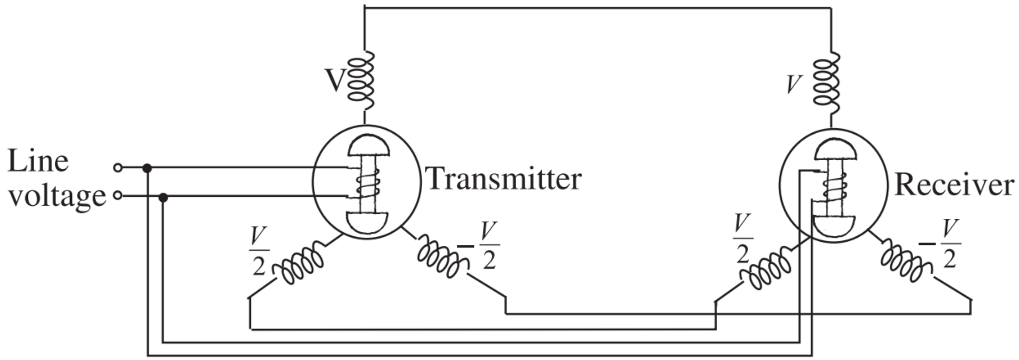
- ❑ **Servomechanism: automatic motion control feedback system**



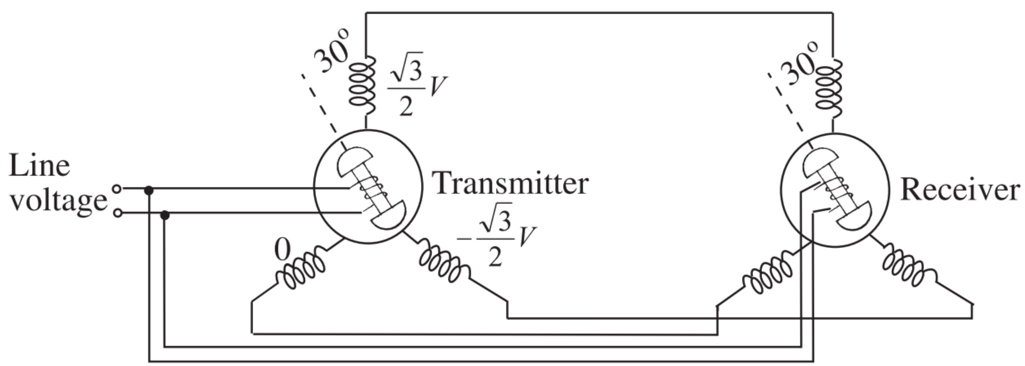
- ❑ When the transmitter’s rotor rotates, due to voltage unbalance, a current flow causes a rotation of the receiver’s rotor through an identical angle

Synchros

- ❑ **Servomechanism: automatic motion control feedback system**
- ❑ Compare Angular position of the load with Commanded Position



(a)



(b)

- ❑ Any application ?

to convert an angular position of a shaft into an electric signal.

Synchros

autopilot system

Table 6.5 Typical specifications of a synchro

Normal excitation voltage	*Sensitivity*	*Nonlinearity*
1.0 V at 50 Hz to 400 kHz	1 V/deg	about 0.25 %

Displacement Measurement

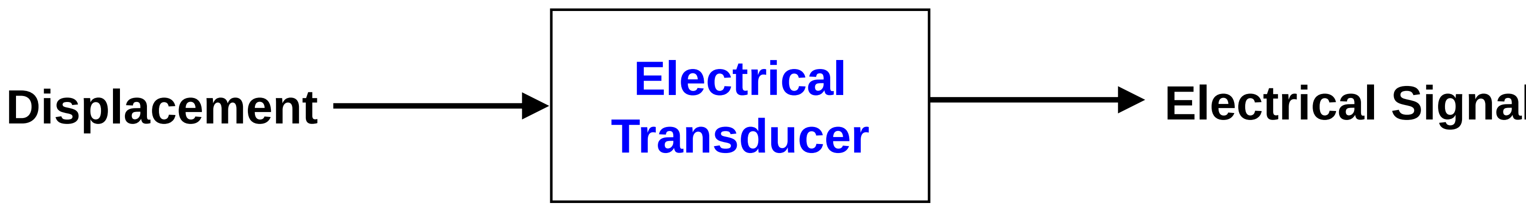
displacement, motion, …

Concept Check?
Is LVDT active or passive transducer?

- The **active transducer** is also called as self generating type **transducer**.
- The **passive transducer** is also called as externally powered **transducer**.
- The **active transducer** does not require any auxiliary (external) power supply.
- Example of **passive transducer is LVDT** (linear variable differential transformer).

Displacement Measurement

- **Electrical Transducers**
  - **Convert displacement to an electrical signal**

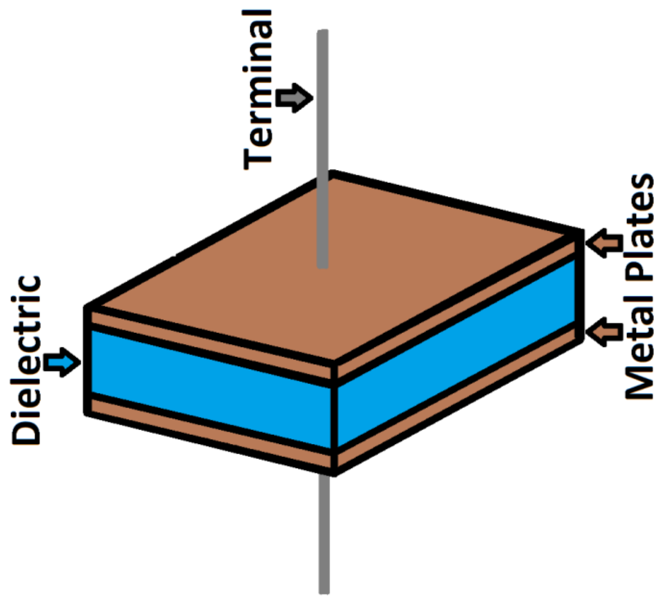


- Passive electrical components**:** resistance, inductance and **capacitance**
  - Resistive, inductive and capacitive transduction of displacement

Capacitive Displacement Transducer

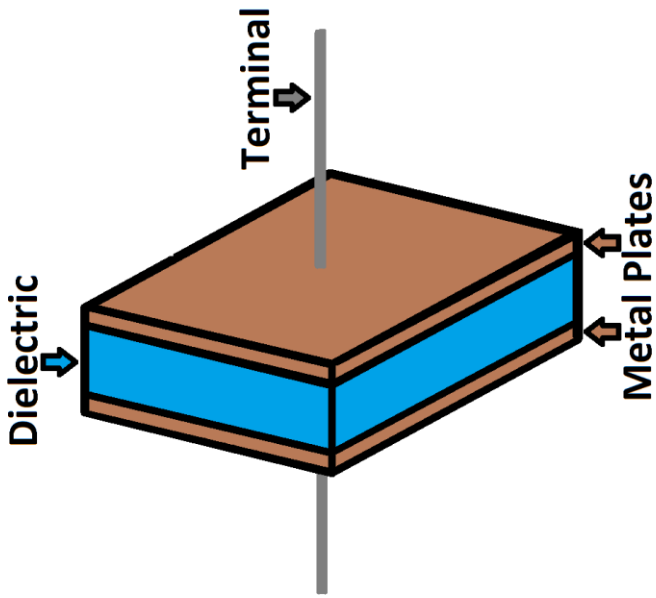
Capacitive Transducers

- Capacitance change due to the environmental stimulus



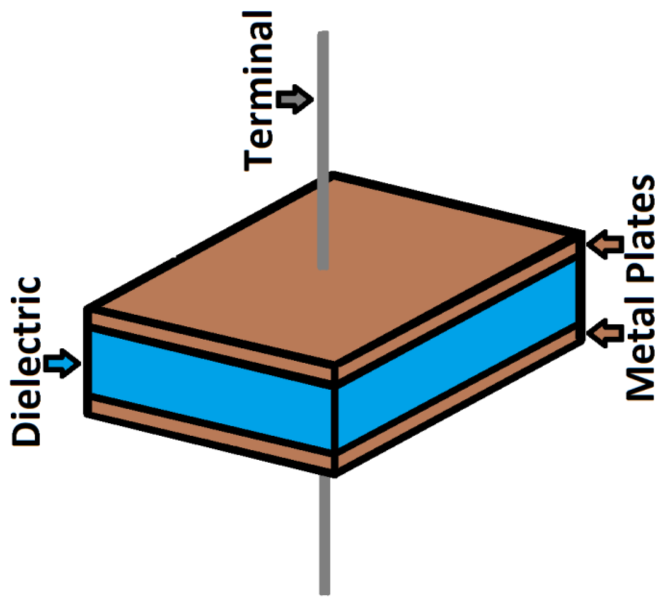
Capacitive Transducers

- Sensing a variety of stimuli:
  - Directly: motion, chemical composition, electric field
  - Indirectly: pressure, acceleration, fluid level, and fluid composition



motion, displacement,…

Capacitive Transducers



- **Features**
  - Low cost and stable and uses simple conditioning circuits
  - Can detect 10^{-8} m displacements with good stability and high speed under wide environmental variations

Capacitive Transducers

❑ **Utilization: Widely researched for micro-nano technologies**

LETTERS NATURE NANOTECHNOLOGY DOI: 10.1038/NNANO.2011.184

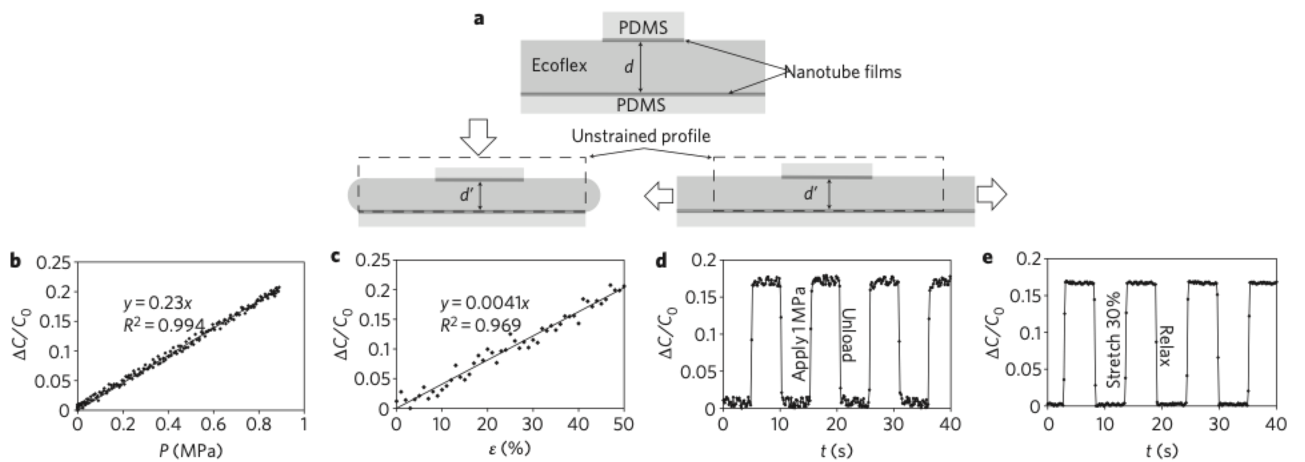
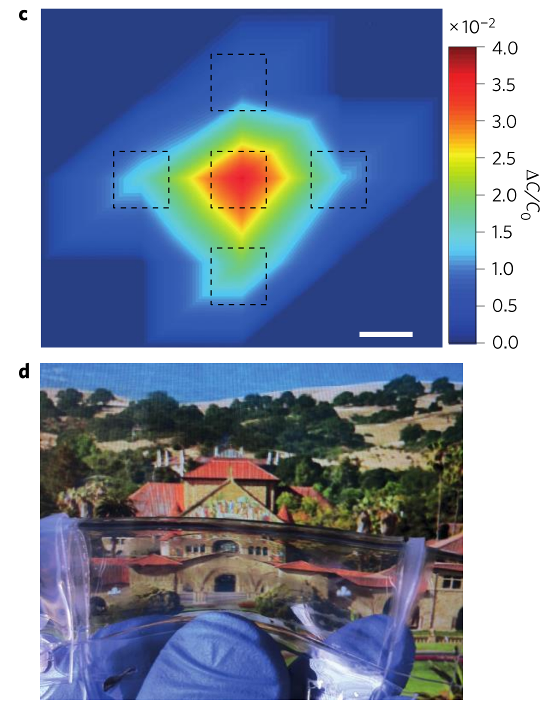


Figure 3 | Use of stretchable nanotube films in compressible capacitors that can sense pressure and strain. a, Schematic showing a stretchable capacitor with transparent electrode (top), and the same capacitor after being placed under pressure (left) and being stretched (right). **b**,**c**, Change in capacitance $\Delta C/C_0$ versus pressure P (**b**) and strain ε (**e**). **d**,**e**, $\Delta C/C_0$ versus time t over four cycles of applied pressure (**d**) and stretching (**e**).



Skin-like pressure and strain sensors based on transparent elastic films of carbon nanotubes

Darren J. Lipomi[1†], Michael Vosgueritchian[1†], Benjamin C-K. Tee[2†], Sondra L. Hellstrom[3], Jennifer A. Lee[1], Courtney H. Fox[1] and Zhenan Bao[1*]

Capacitive Transducers

Single-Probe Capacitive Position Gauge for Nanometrology Applications

The D-510 family of PISeca™ single-electrode capacitive displacement gauges performs high-precision, non-contact measurements of geometric quantities representing displacement, separation, position, length or other linear dimension against any kind of electrically conductive target. These single-probe nanometrology sensors combine superior resolution and linearity with very high bandwidth for dynamic measurements.

>> D-510 Single-Probe Capacitive Position Gauge for Nanometrology Applications

Features & Advantages

- Sub-Nanometer Resolution, Measuring Ranges to 500 μm
- Absolute, Non-Contact Measurement of Distance / Motion / Vibration
- Multi-Axis Measurements Possible
- Excellent Measuring Linearity to 0.1 %
- Plug & Play: Easy Setup and Integration
- Very Temperature Stable
- Bandwidth to 10 kHz
- Guard-Ring Electrode Design for Better Sensor Linearity
- ILS Linearization System in the Signal Conditioner Electronics Improves Output Signal Linearity
- All Systems Factory Calibrated for Highest Possible Linearity / Accuracy

- They are in the market

Capacitive Transducers

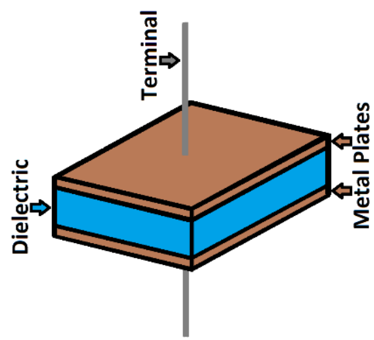
- **Capacitance of a parallel plate capacitor**

$$\boxed{C = \frac{\varepsilon A}{x} \text{ farad}}$$

$\varepsilon = \varepsilon_0 \varepsilon_r$ is the permittivity of the intervening medium (farad/metre)

x is the distance between the plates (metre)

A is the overlapping area of the plates (metre^2)



Capacitive Transducers

Journals & Magazines > IEEE Transactions on Nanotech... > Volume: 15 Issue: 6

Nanostructural Analysis of CMOS-MEMS-Based Digital Microphone for Performance Optimization

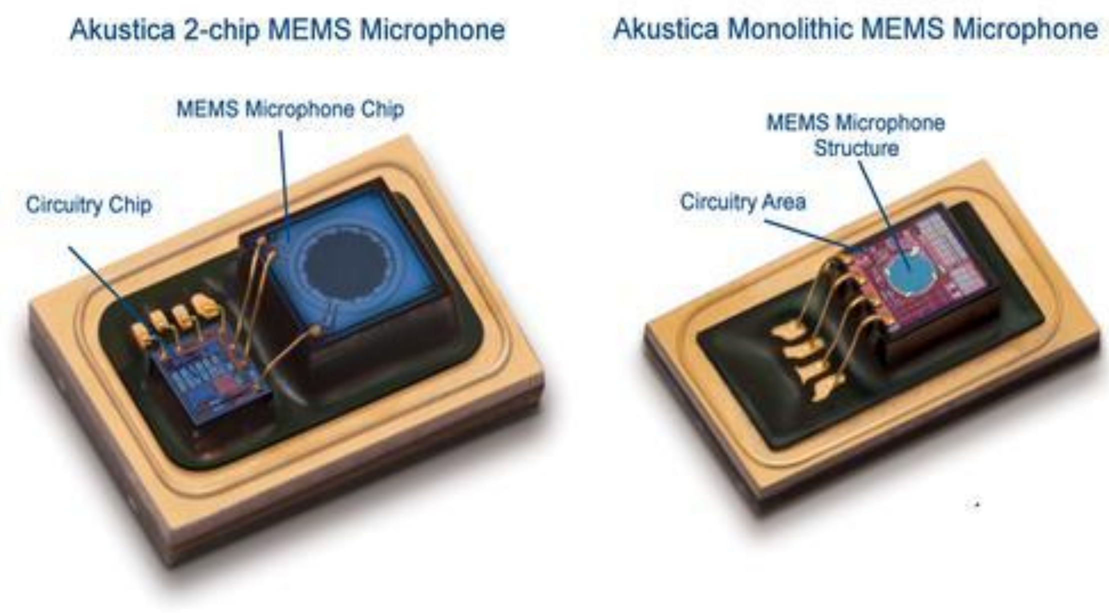
Publisher: IEEE

Cite This

PDF

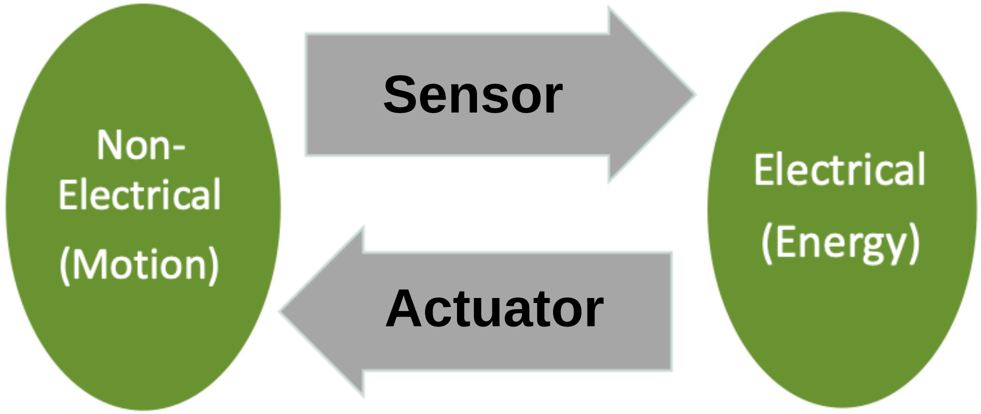
Mansoor Ali Khan ; Rongkun Zheng

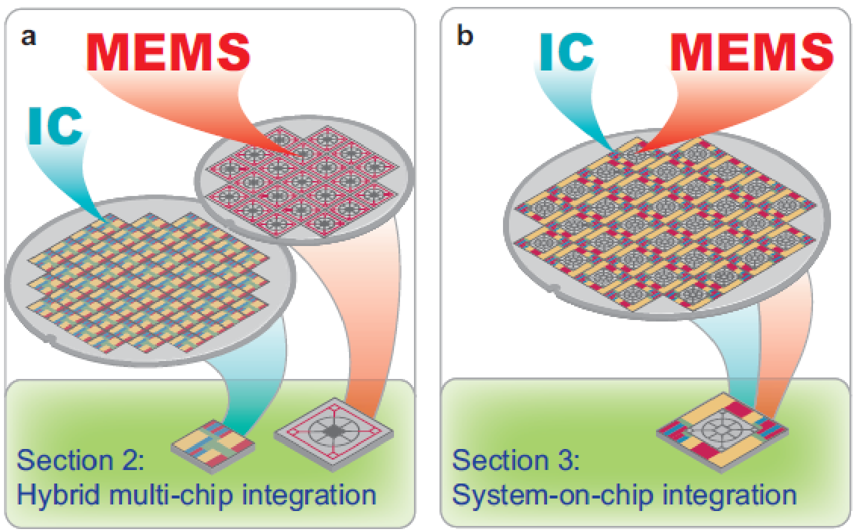
All Authors

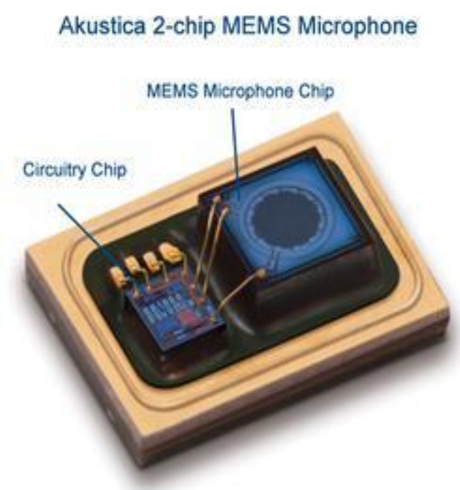


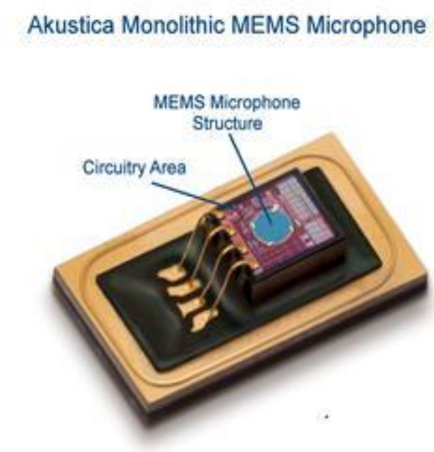
Capacitive Transducers

Example: CMOS-MEMS Integration







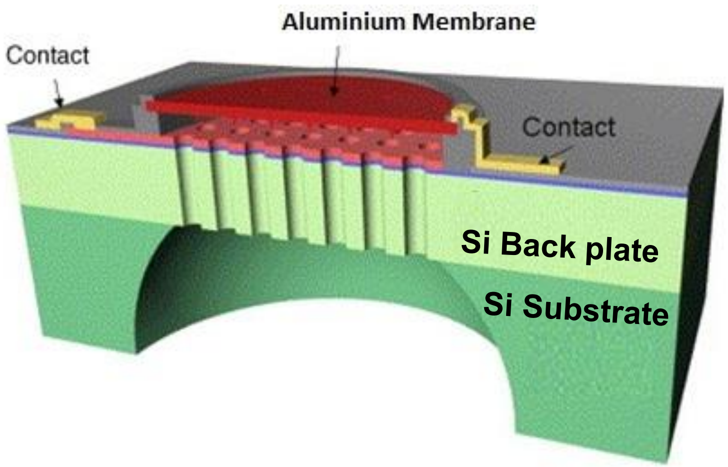


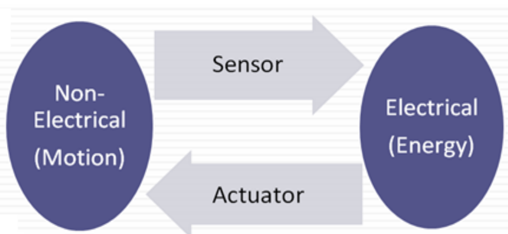
AKUSTICA

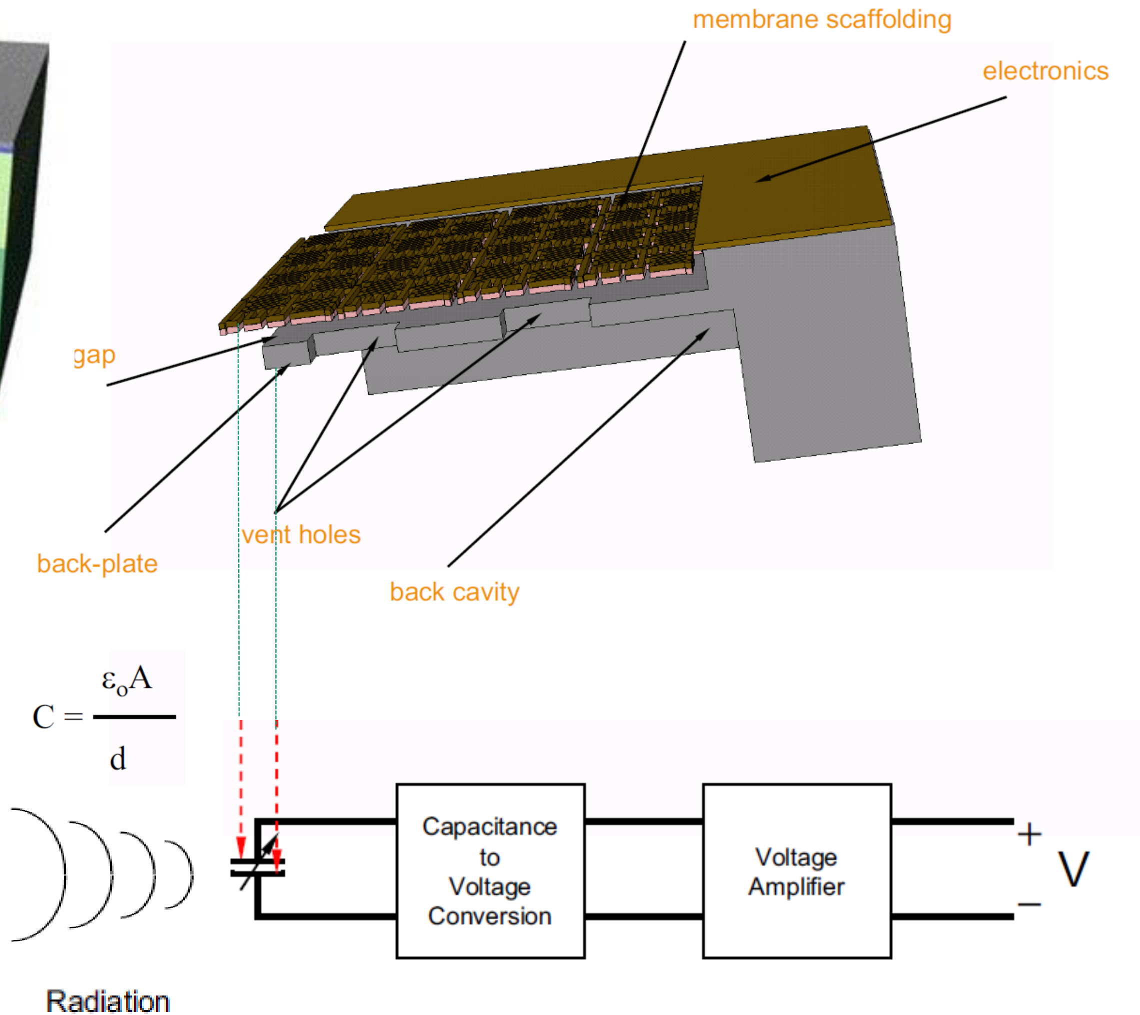
Source: IEEE, 2016

Capacitive Transducers

Example: Working Principle of CMOS-MEMS Microphone







$$C = \frac{\varepsilon_o A}{d}$$

Source: IEEE, 2016

Capacitive Transducers

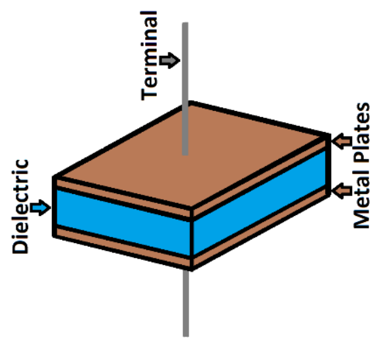
- **Capacitance of a parallel plate capacitor**

$$\boxed{C = \frac{\varepsilon A}{x} \text{ farad}}$$

$\varepsilon = \varepsilon_0 \varepsilon_r$ is the permittivity of the intervening medium (farad/metre)

x is the distance between the plates (metre)

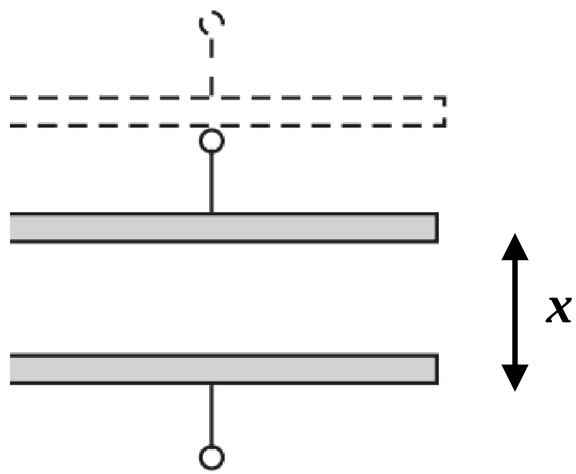
A is the overlapping area of the plates (metre^2)



Capacitive Transducers

- **Variable Capacitance**

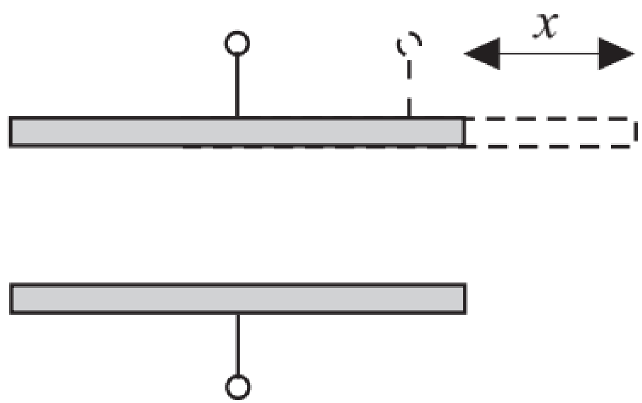
1. Variation in the distance x between the plates



Capacitive Transducers

- **Variable Capacitance**

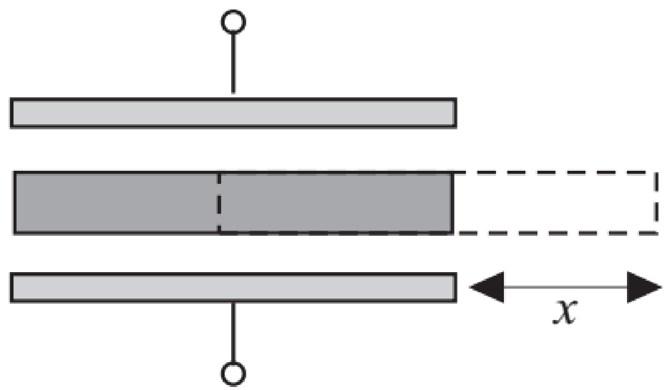
  2. Variation in the effective overlapping area A between the plates



Capacitive Transducers

- **Variable Capacitance**

3. Variation in the relative permittivity ε_r of the intervening medium between the plates



Capacitive Transducers

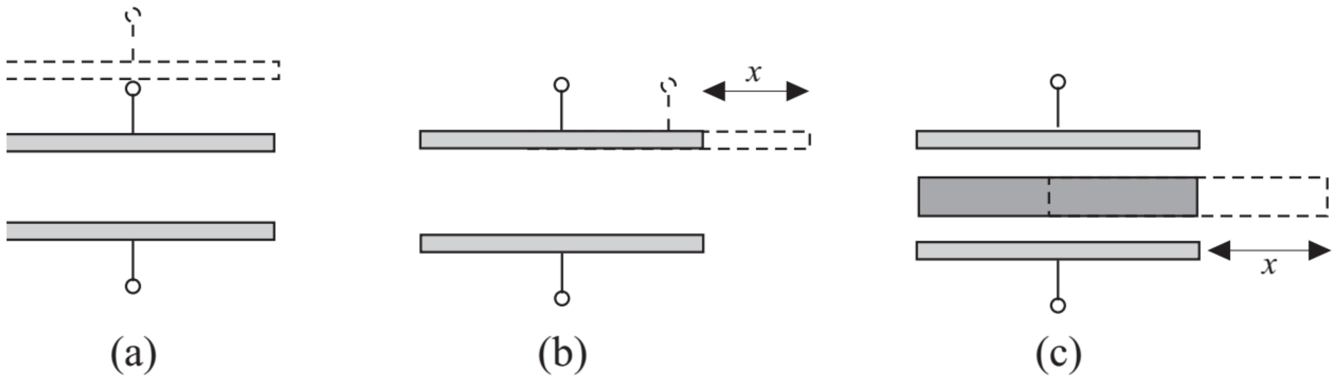


Fig. 6.18 Three kinds of variation in capacitative transducers: (a) change in the gap, (b) change in the area and (c) change in the permittivity.

- **Variable Capacitance**

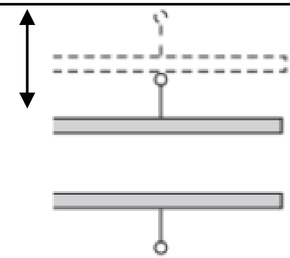
1. Variation in the distance x between the plates
2. Variation in the effective overlapping area A between the plates
3. Variation in the relative permittivity ε_r of the intervening medium b/w plates

Capacitive Transducers

Change in capacitance is used to measure displacement.

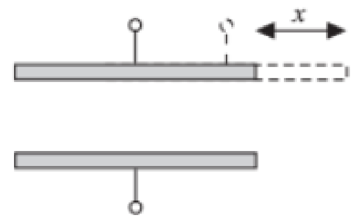
$$\boxed{C = \frac{\varepsilon A}{x} \text{ farad}}$$

- $\epsilon = \epsilon_o \epsilon_r$ is the permittivity of intervening medium (farad/meters)
  - *The absolute permittivity, often simply called permittivity and denoted by the Greek letter ε (epsilon), is a measure of the electric polarizability. he ability of a substance to store electrical energy in an electric field.*
- x is the distance between plates (meters)
- A is the overlapping area of plates (meters^2)



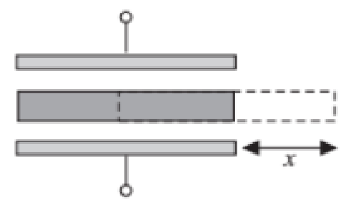
(a)

<change in the gap>



(b)

<change in the area>

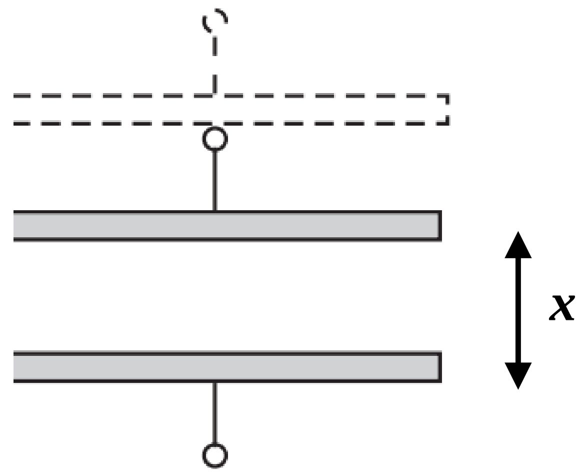


(c)

<change in the permittivity>

1. Change in the gap x between the plates

$$\boxed{C = \frac{\varepsilon A}{x} \text{ farad}}$$



- Nonlinear
  - Capacitance varies inversely as x

1. Change in the gap *x* between the plates

$$C = \frac{\varepsilon A}{x}$$

$$\Rightarrow \frac{dC}{dx} = \frac{d}{dx}\left(\frac{\varepsilon A}{x}\right) = \varepsilon A \frac{d(x^{-1})}{dx}$$

- The **derivative** is a **function** -- a rule -- that assigns to each value of x the slope of the tangent line at the point (x, f(x)) on the graph of f(x).
- It is the rate of change of f(x) at that point.

1. Change in the gap *x* between the plates

$$C = \frac{\varepsilon A}{x}$$

$$\frac{dC}{dx} = \frac{d}{dx}\left(\frac{\varepsilon A}{x}\right) = \varepsilon A \frac{d(x^{-1})}{dx}$$

$$\Rightarrow \quad \frac{dC}{dx} = \frac{-\varepsilon A}{x^2}$$

1. Change in the gap x between the plates

$$C = \frac{\varepsilon A}{x}$$

$$\frac{dC}{dx} = \frac{d}{dx}\left(\frac{\varepsilon A}{x}\right) = \varepsilon A \frac{d(x^{-1})}{dx}$$

$$\Rightarrow \quad \frac{dC}{dx} = \frac{-\varepsilon A}{x^2} = \frac{-\text{Constant}}{x^2}$$

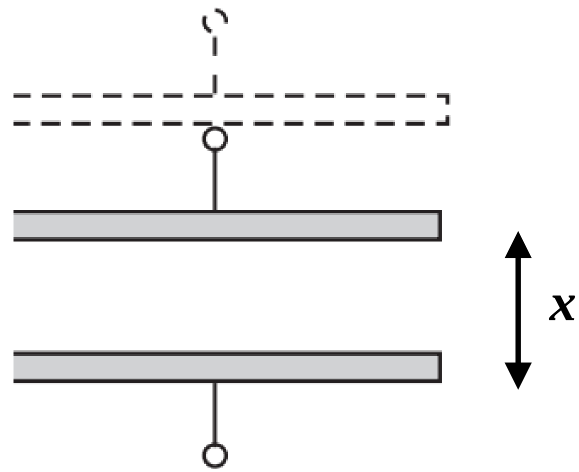
1. Change in the gap x between the plates

$$\text{Sensitivity} = \frac{\Delta q_o}{\Delta q_i} = \frac{dC}{dx} = \frac{-\text{Constant}}{x^2}$$

1. Change in the gap x between the plates

$$\boxed{C = \frac{\varepsilon A}{x} \text{ farad}}$$

$$\Rightarrow \boxed{\text{Sensitivity} = S = \frac{dC}{dx} = -\frac{\text{constant}}{x^2}}$$

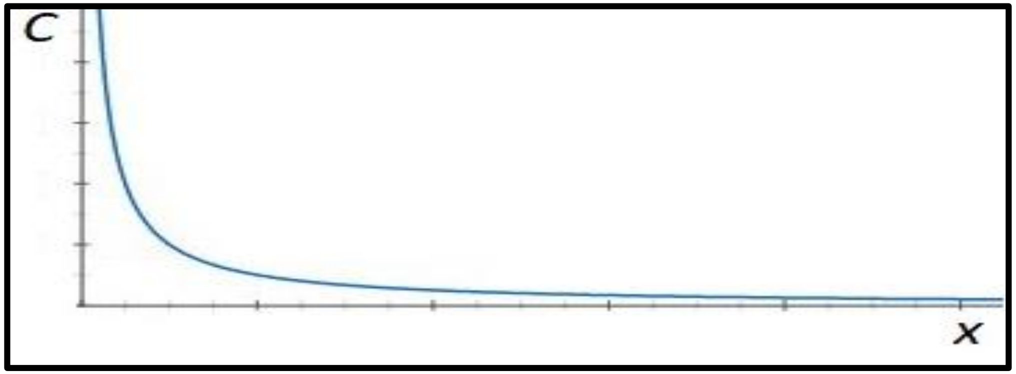


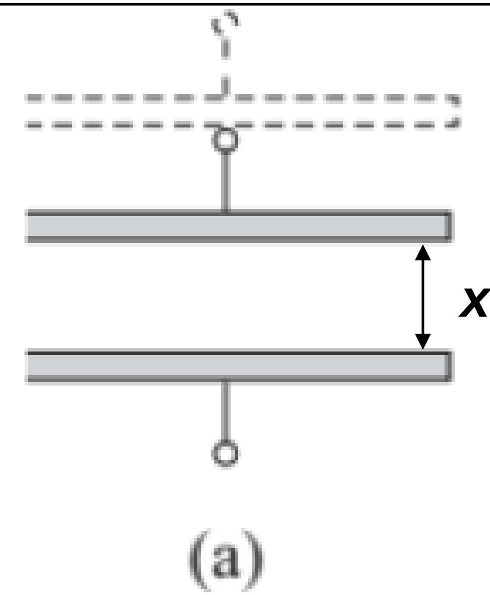
- Nonlinear
  - Capacitance varies inversely as x
  - Sensitivity S is not constant
    - Decreases as x increases

1. Change in the gap x between the plates

$$\boxed{C \propto \frac{1}{x}}$$

→Plot of 'C' vs 'x' is a rectangular hyperbola.





<change in the gap>

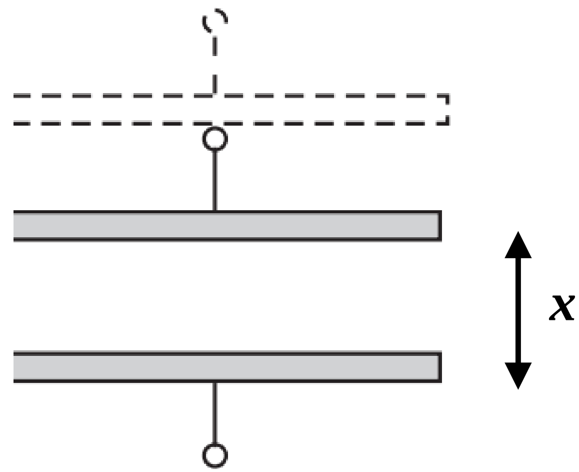
→It is not convenient to measure the non linear change in capacitance.

→Therefore, the change in output **should be linearized.**

1. Change in the gap *x* between the plates

$$\boxed{C = \frac{\varepsilon A}{x}\ \text{farad}}$$

$$\Rightarrow \quad \boxed{\text{Sensitivity} = S = \frac{dC}{dx} = -\frac{\text{constant}}{x^2}}$$



- **Linearization Techniques:**

  1. By measuring the per cent change in capacitance
  2. Using a charge amplifier
  3. Measuring impedance
  4. Differential arrangement

Linearization of input/output relationship

Measuring percent change in capacitance

$$\boxed{C = \frac{\varepsilon A}{x}}$$

$$\frac{dC}{dx} = -\frac{\epsilon A}{x^2} = -\frac{1}{x} * \left(\frac{\epsilon A}{x}\right)$$

$$\frac{dC}{dx} = -\frac{1}{x} * C$$

$$\boxed{\frac{dC}{C} = -\frac{dx}{x}}$$

Percent change in 'C' is linearly related to percent change in 'x'.

Measuring Impedance

$$\boxed{C = \frac{\varepsilon A}{x}}$$

$$X_c = \frac{1}{2\pi f C}$$

$$X_c = \frac{1}{2\pi f} * \left(\frac{x}{\epsilon A}\right)$$

$$\boxed{X_c \propto x}$$

Where 'f' is the frequency of exciting voltage.

Linearization of input/output relationship

By using a charge amplifier

$$e_i = \frac{\int i\,dt}{C}$$

$$e_o = \frac{\int i_x\,dt}{C_x}$$

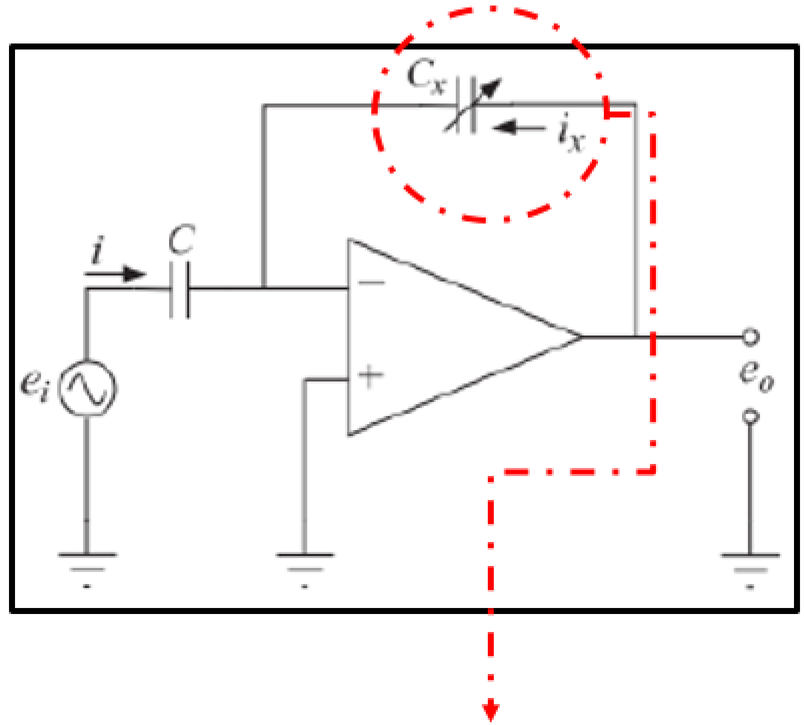
$$i = -i_x$$

$$e_o = \frac{\int i_x\,dt}{C_x} = -\frac{\int i\,dt}{C_x} \leftarrow \boxed{\int i\,dt = C\,e_i}$$

$$e_o = -\frac{C e_i}{C_x} \leftarrow \boxed{C_x = \frac{\epsilon A}{x}}$$

$$e_o = -\frac{C e_i}{\epsilon A}x$$

$$e_o \propto x$$



Displacement measuring capacitor

The output voltage changes linearly with displacement

Linearization of input/output relationship

By using a differential arrangement of capacitors

$$\boxed{Q = EC_{LN}}$$

$$E_{LM} = \frac{Q}{C_{LM}} = \frac{EC_{LN}}{C_{LM}} \qquad (1)$$

$$E_{MN} = \frac{Q}{C_{MN}} = \frac{EC_{LN}}{C_{MN}} \qquad (2)$$

$$C_{LN} = \frac{\epsilon A}{2d} \qquad (3)$$

'd' is the distance between two adjacent plates

When 'M' is midway, $C_{LM} = C_{MN}$

If 'M' is displaced upwards by a distance x

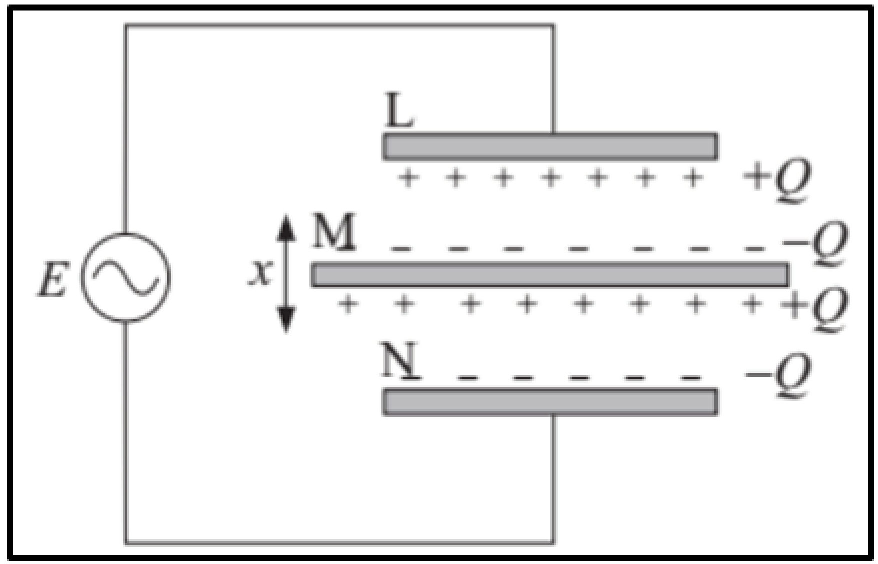
$$C_{LM} = \frac{\epsilon A}{d - x}, C_{MN} = \frac{\epsilon A}{d + x} \qquad (4)$$

Substituting (3) & (4) in (1) & (2)

$$E_{LM} = \frac{E(d - x)}{2d}, E_{MN} = \frac{E(d + x)}{2d}$$

$$\Delta E = E_{LM} - E_{MN} = \frac{E}{d}x$$

$$\boxed{\Delta E \propto x}$$

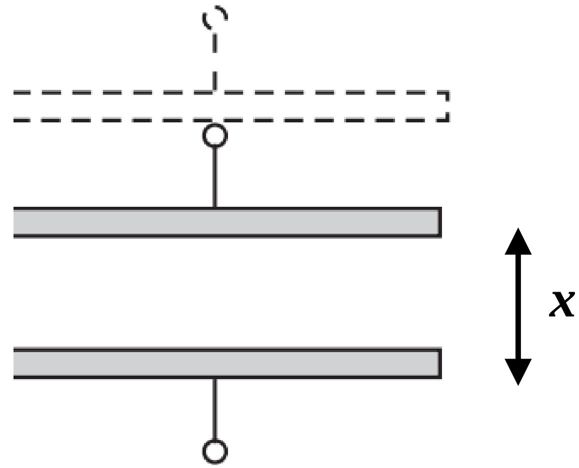


The voltage difference changes linearly with displacement

Steps/Techniques in Detail..

1. Change in the gap *x* between the plates

$$\boxed{C = \frac{\varepsilon A}{x} \text{ farad}}$$



$$\Rightarrow \boxed{\text{Sensitivity} = S = \frac{dC}{dx} = -\frac{\text{constant}}{x^2}}$$

- **Linearization Techniques:**

  1. By measuring the per cent change in capacitance
  2. Using a charge amplifier
  3. Measuring impedance
  4. Differential arrangement

1. Change in the gap *x* between the plates

- **Linearization: 1. By measuring the per cent change in capacitance**

$$C = \frac{\varepsilon A}{x}$$

1. Change in the gap *x* between the plates

- **Linearization: 1. By measuring the per cent change in capacitance**

$$C = \frac{\varepsilon A}{x}$$

$$\Rightarrow \frac{dC}{dx} = \frac{-\varepsilon A}{x^2}$$

1. Change in the gap x between the plates

- **Linearization: 1. By measuring the per cent change in capacitance**

$$C = \frac{\varepsilon A}{x}$$

$$\Rightarrow \frac{dC}{dx} = \frac{-\varepsilon A}{x^2} = -\frac{1}{x}\left(\frac{\varepsilon A}{x}\right)$$

1. Change in the gap x between the plates

- **Linearization: 1. By measuring the per cent change in capacitance**

$$C = \frac{\varepsilon A}{x}$$

$$\frac{dC}{dx} = \frac{-\varepsilon A}{x^2} = -\frac{1}{x}\left(\frac{\varepsilon A}{x}\right) = -\frac{1}{x}(C)$$

1. Change in the gap x between the plates

- **Linearization: 1. By measuring the per cent change in capacitance**

$$C = \frac{\varepsilon A}{x}$$

$$\frac{dC}{dx} = \frac{-\varepsilon A}{x^2} = -\frac{1}{x}\left(\frac{\varepsilon A}{x}\right) = -\frac{1}{x}(C)$$

$$\frac{dC}{dx} = -\frac{C}{x}$$

1. Change in the gap x between the plates

- **Linearization: 1. By measuring the per cent change in capacitance**

$$C = \frac{\varepsilon A}{x}$$

$$\frac{dC}{dx} = \frac{-\varepsilon A}{x^2} = -\frac{1}{x}\left(\frac{\varepsilon A}{x}\right) = -\frac{1}{x}(C)$$

$$\frac{dC}{dx} = -\frac{C}{x}$$

$$\Rightarrow \frac{dC}{C} = -\frac{dx}{x}$$

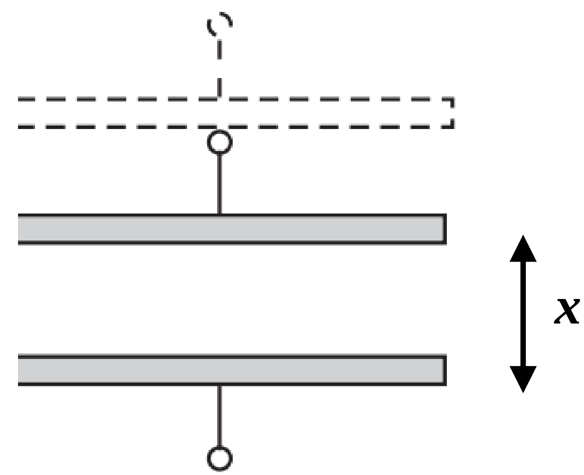
1. Change in the gap x between the plates

- **Linearization: 1. By measuring the per cent change in capacitance**

$$C = \frac{\varepsilon A}{x} \text{ farad}$$

$$\Rightarrow \quad \frac{dC}{dx} = -\frac{\varepsilon A}{x^2} = -\frac{C}{x}$$

$$\frac{dC}{C} = -\frac{dx}{x}$$



- **Percent changes of C and x are linearly related**

1. Change in the gap x between the plates

LETTERS NATURE NANOTECHNOLOGY DOI: 10.1038/NNANO.2011.184

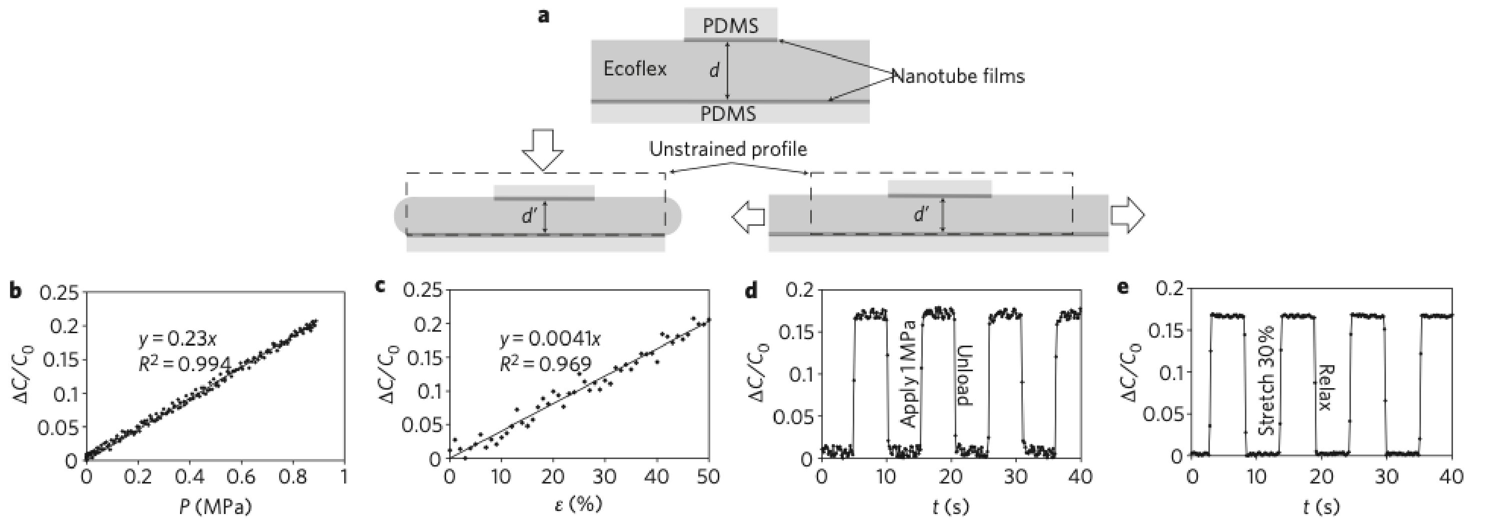


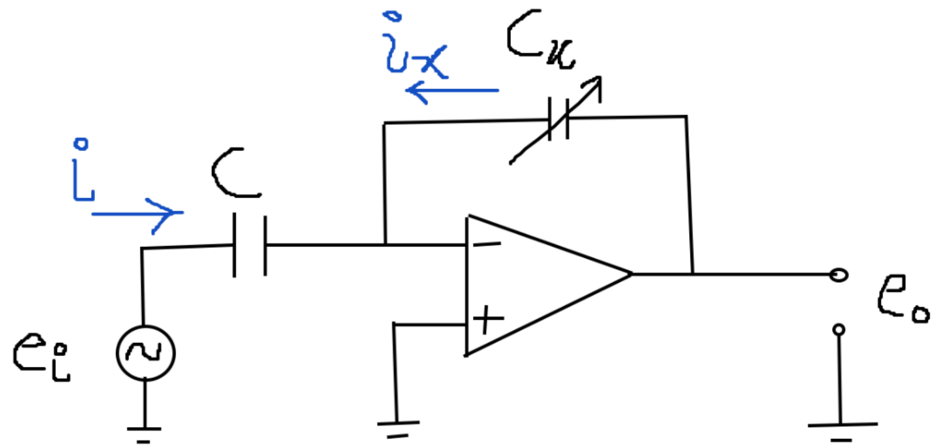
Figure 3 | Use of stretchable nanotube films in compressible capacitors that can sense pressure and strain. a, Schematic showing a stretchable capacitor with transparent electrode (top), and the same capacitor after being placed under pressure (left) and being stretched (right). **b**,**c**, Change in capacitance $\Delta C/C_0$ versus pressure P (**b**) and strain ε (**e**). **d**,**e**, $\Delta C/C_0$ versus time t over four cycles of applied pressure (**d**) and stretching (**e**).

- **Linearization: 1. By measuring the per cent change in capacitance**

$$C = \frac{\varepsilon A}{x} \text{ farad} \qquad \boxed{\frac{dC}{C} = -\frac{dx}{x}}$$

1. Change in the gap x between the plates

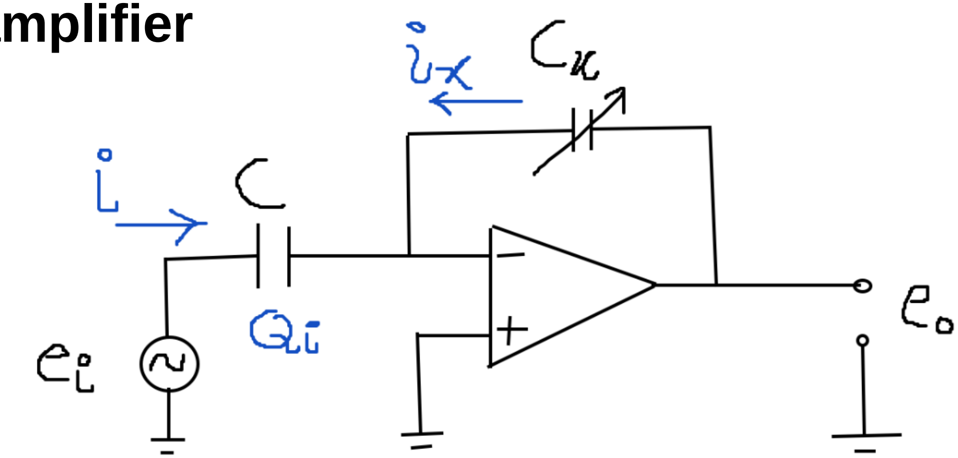
- **Linearization: 2. By using a charge amplifier**



1. Change in the gap *x* between the plates

- **Linearization: 2. By using a charge amplifier**

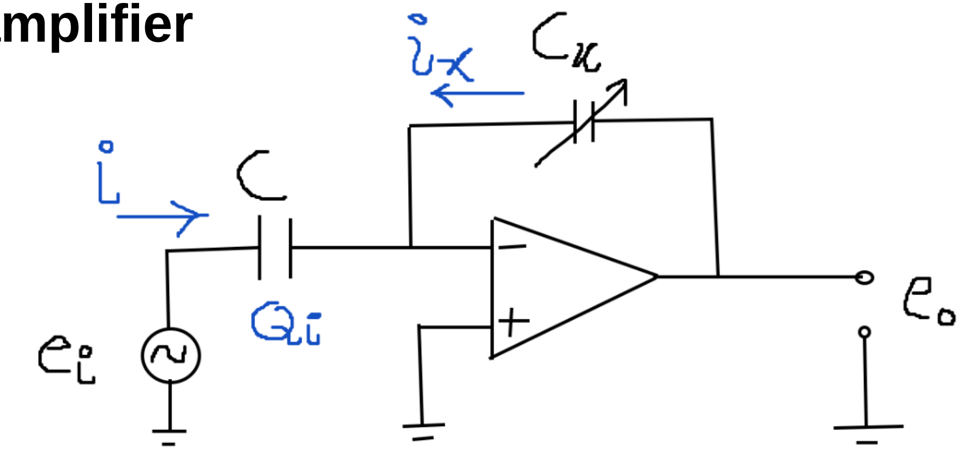
$$e_i = \frac{Q_i}{C}$$



1. Change in the gap *x* between the plates

- **Linearization: 2. By using a charge amplifier**

$$e_i = \frac{Q_i}{C} = \frac{\int i\,dt}{C}$$

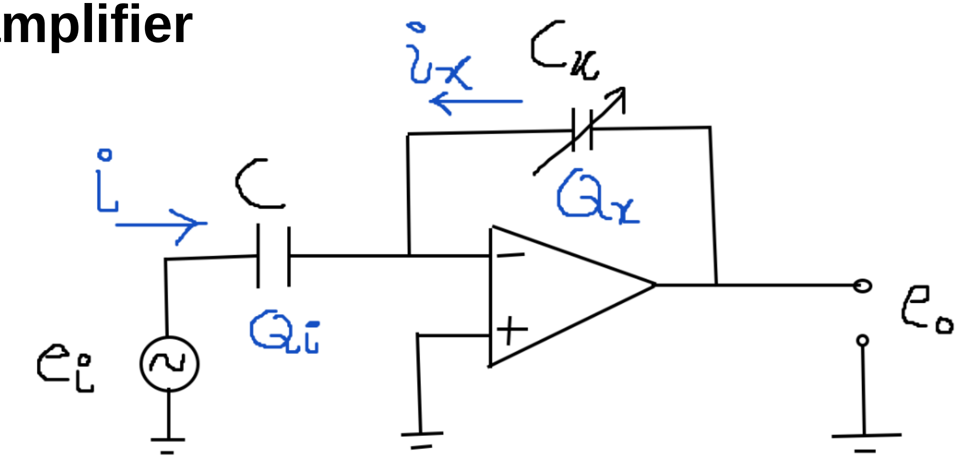


1. Change in the gap *x* between the plates

- **Linearization: 2. By using a charge amplifier**

$$e_i = \frac{Q_i}{C} = \frac{\int i\,dt}{C}$$

$$e_o = \frac{Q_o}{C_x} = \frac{\int i_x\,dt}{C_x}$$



1. Change in the gap x between the plates

❑ Linearization: 2. By using a charge amplifier

$$e_i = \frac{\int i dt}{C}$$

$$e_o = \frac{\int i_x dt}{C_x}$$

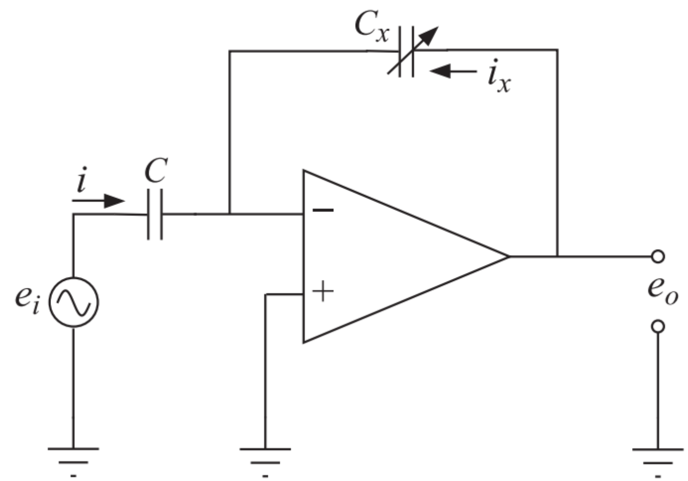


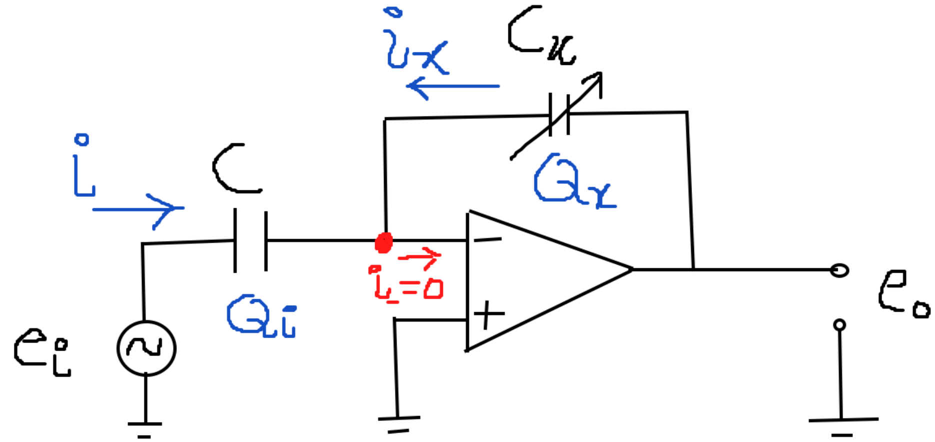
Fig. 6.19 Use of the op-amp to linearise the input-output relation.

1. Change in the gap *x* between the plates

- **Linearization: 2. By using a charge amplifier**

$$e_i = \frac{Q_i}{C} = \frac{\int i\,dt}{C}$$

$$e_o = \frac{Q_o}{C_x} = \frac{\int i_x\,dt}{C_x}$$



for the OPAMPS,

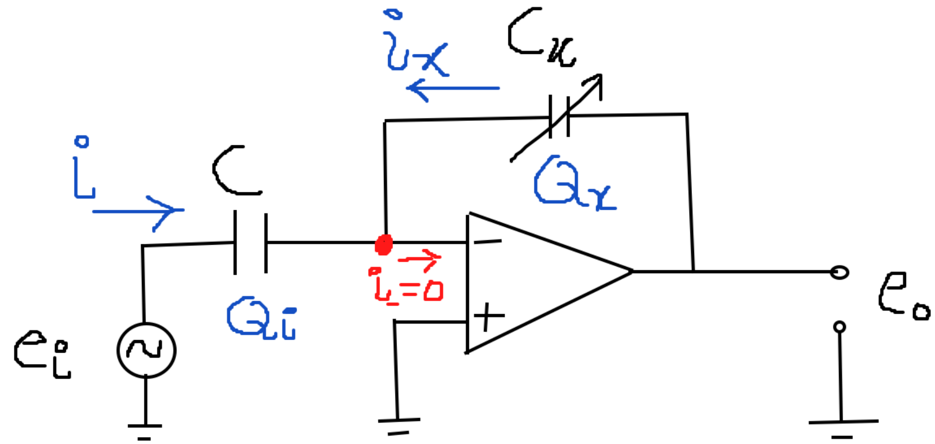
$$i + i_x = 0$$

1. Change in the gap *x* between the plates

- **Linearization: 2. By using a charge amplifier**

$$e_i = \frac{Q_i}{C} = \frac{\int i\,dt}{C}$$

$$e_o = \frac{Q_o}{C_x} = \frac{\int i_x\,dt}{C_x}$$



for the OPAMPS,

$$i + i_x = 0$$

$$\Longrightarrow i_x = -i$$

1. Change in the gap *x* between the plates

- **Linearization: 2. By using a charge amplifier**

$$e_i = \frac{\int i dt}{C} \quad \text{.....................} \quad (1)$$

$$e_o = \frac{\int i_x dt}{C_x} \quad \text{.....................} \quad (2)$$

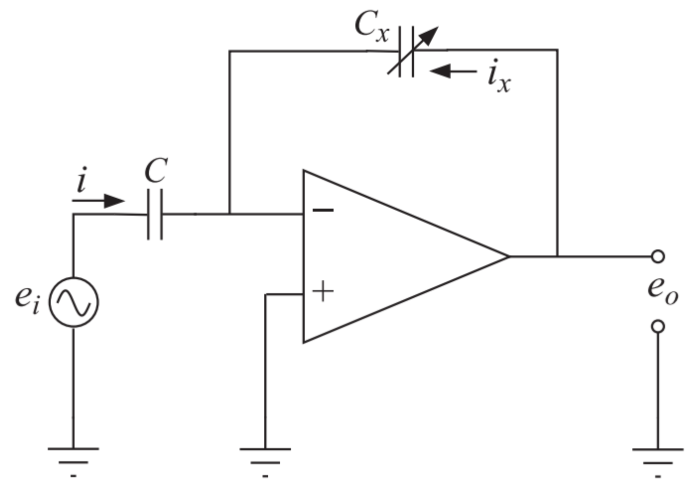


Fig. 6.19 Use of the op-amp to linearise the input-output relation.

Since for the opamp, $i_x = -i$

$$(2) \Rightarrow \quad e_o = \frac{\int i_x dt}{C_x} = -\frac{\int i dt}{C_x}$$

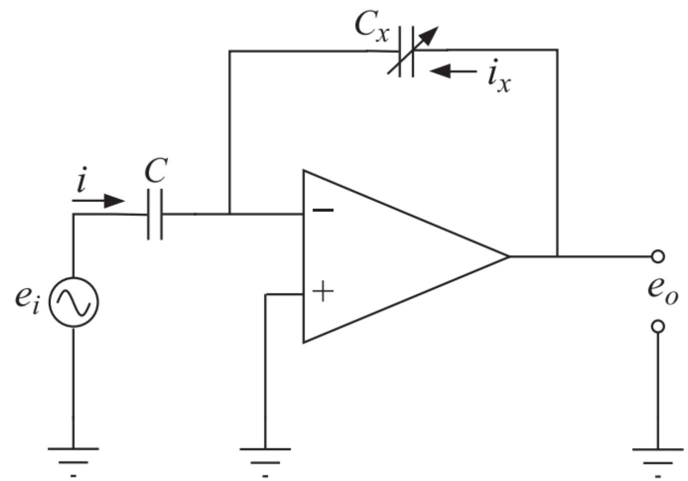
1. Change in the gap x between the plates

- **Linearization: 2. By using a charge amplifier**

$$e_i = \frac{\int i dt}{C} \quad \text{......................} \quad (1)$$

$$e_o = \frac{\int i_x dt}{C_x} \quad \text{......................} \quad (2)$$

$$e_o = \frac{\int i_x dt}{C_x} = -\frac{\int i dt}{C_x} \quad \text{......................} \quad (3)$$



$$(1) \Rightarrow \int i\, dt = e_i\, C$$

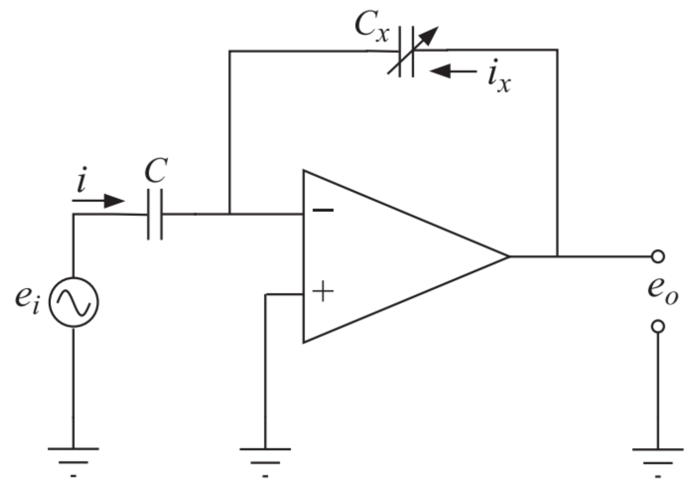
1. Change in the gap x between the plates

- **Linearization: 2. By using a charge amplifier**

$$e_i = \frac{\int i dt}{C} \quad \text{..................... } (1)$$

$$e_o = \frac{\int i_x dt}{C_x} \quad \text{..................... } (2)$$

$$e_o = \frac{\int i_x dt}{C_x} = -\frac{\int i dt}{C_x} \quad \text{..................... } (3)$$



$$\int i\, dt = e_i\, C$$

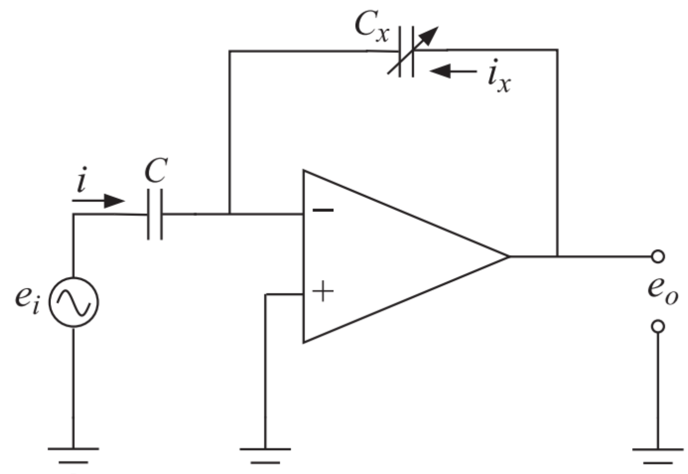
$$(3) \Rightarrow \quad e_o = \frac{\int i_x dt}{C_x} = -\frac{\int i dt}{C_x} = -\frac{C}{C_x} e_i$$

1. Change in the gap *x* between the plates

- **Linearization: 2. By using a charge amplifier**

$$e_i = \frac{\int i dt}{C} \quad \text{.....................} \quad 1$$

$$e_o = \frac{\int i_x dt}{C_x} \quad \text{.....................} \quad 2$$



$$e_o = \frac{\int i_x dt}{C_x} = -\frac{\int i dt}{C_x} = -\frac{C}{C_x} e_i$$

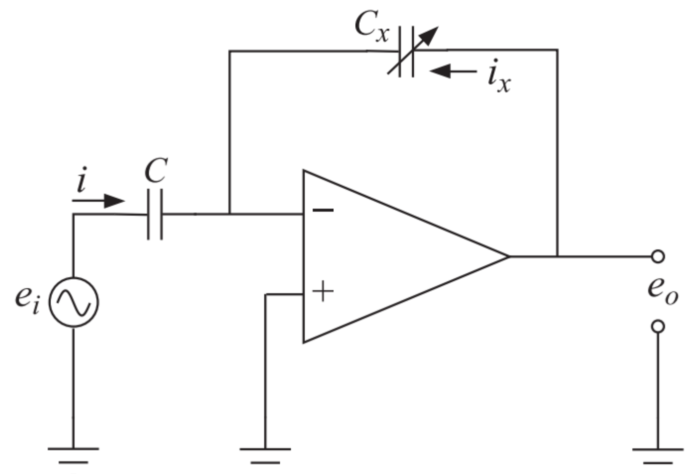
$$C_x = \frac{\varepsilon A}{x}$$

1. Change in the gap x between the plates

- **Linearization: 2. By using a charge amplifier**

$$e_i = \frac{\int i dt}{C} \quad \text{..................... } 1$$

$$e_o = \frac{\int i_x dt}{C_x} \quad \text{..................... } 2$$



$$e_o = \frac{\int i_x dt}{C_x} = -\frac{\int i dt}{C_x} = -\frac{C}{C_x} e_i$$

$$C_x = \frac{\varepsilon A}{x} \Rightarrow e_o = -\frac{Ce_i}{\varepsilon A} x$$

In Eq. (6.13), since all other factors are constant, the output voltage varies linearly with the displacement.

1. Change in the gap x between the plates

❑ Linearization: 2. By using a charge amplifier

$$e_i = \frac{\int i dt}{C}$$

$$e_o = \frac{\int i_x dt}{C_x}$$

Since for the opamp, $i_x = -i$

$$\Rightarrow \quad e_o = \frac{\int i_x dt}{C_x} = -\frac{\int i dt}{C_x}$$

$$= -\frac{C}{C_x} e_i$$

$$= -\frac{C e_i}{\varepsilon A} x$$

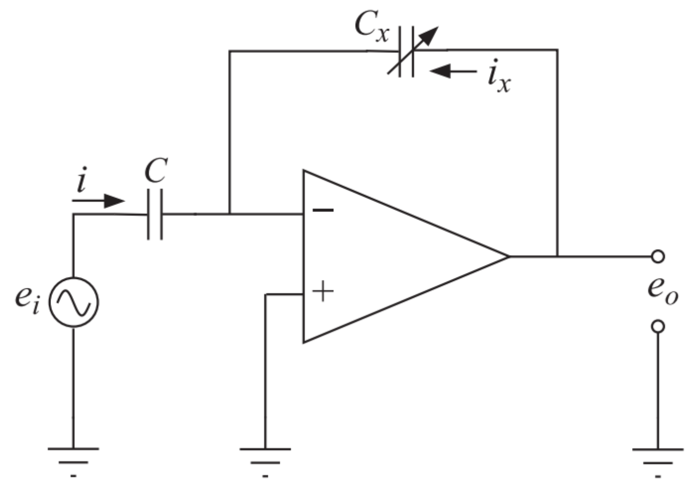


Fig. 6.19 Use of the op-amp to linearise the input-output relation.

1. Change in the gap x between the plates

- **Linearization: 2. By using a charge amplifier**

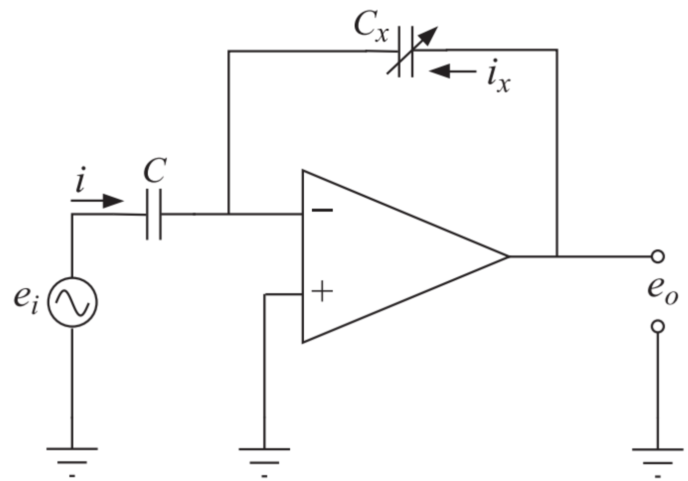


Fig. 6.19 Use of the op-amp to linearise the input-output relation.

$$e_o = -\frac{Ce_i}{\varepsilon A}x$$

- **Output voltage varies linearly with the displacement**

OPAMPS Home assignment

- What are they?
- What is their gain?
- Which one is the inverting input?
- Which one is the non-inverting input?
- What is the differential input?
- What is virtual ground?
- ..
- .

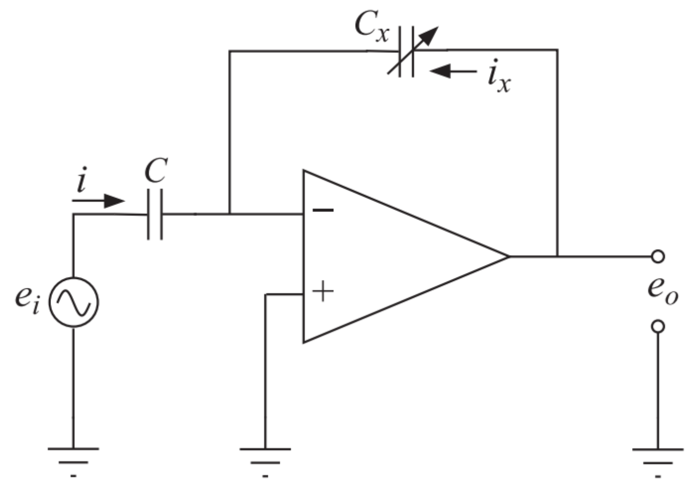
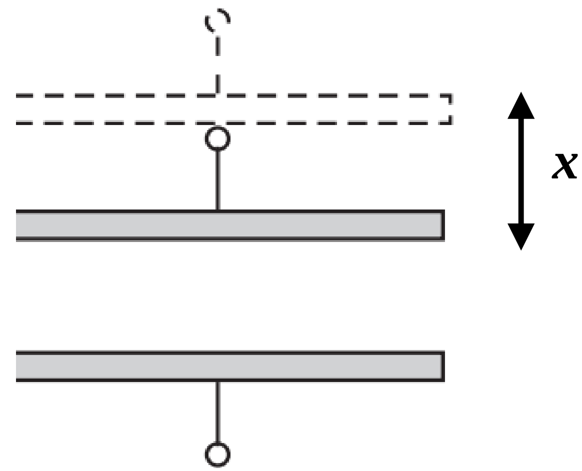


Fig. 6.19 Use of the op-amp to linearise the input-output relation.

1. Change in the gap x between the plates

- **Linearization: 3. By measuring Impedance**
  - Measuring impedance rather than the capacitance is another way of linearization

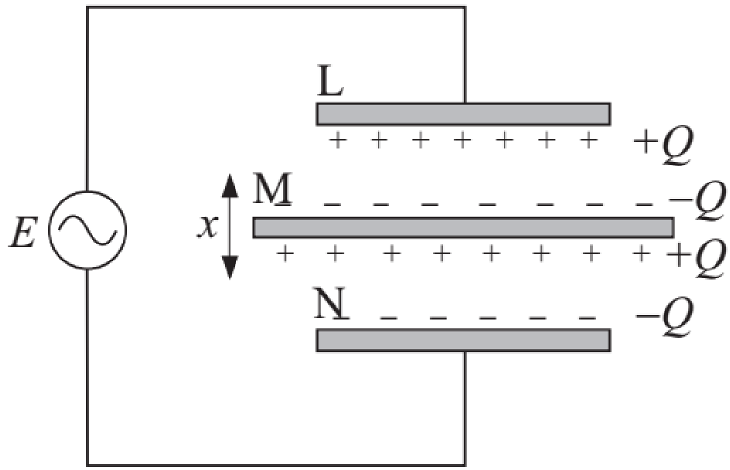
$$X_C = \frac{1}{2\pi f C} = \frac{x}{2\pi f \varepsilon_r \varepsilon_0 A}$$



- **Output voltage varies linearly with the displacement**

1. Change in the gap x between the plates

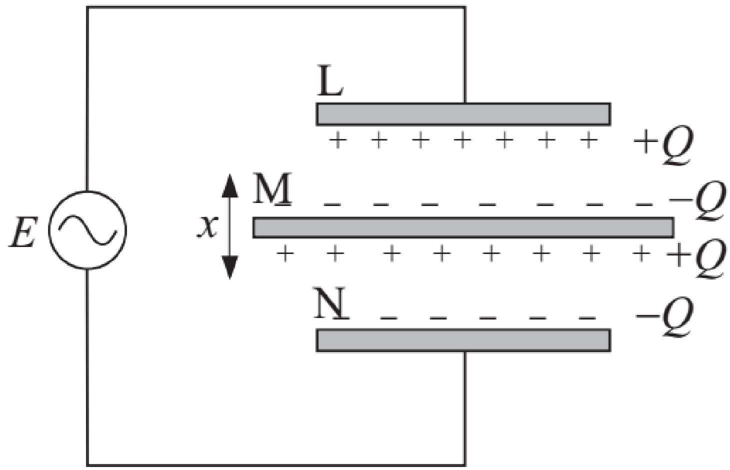
- **Linearization: 4. By using differential arrangement of capacitors**



- Three plate capacitor
  - Plate M is movable and plates L and N are fixed

1. Change in the gap x between the plates

- **Linearization: 4. By using differential arrangement of capacitors**



- Three plate capacitor
  - Plate M is movable and plates L and N are fixed

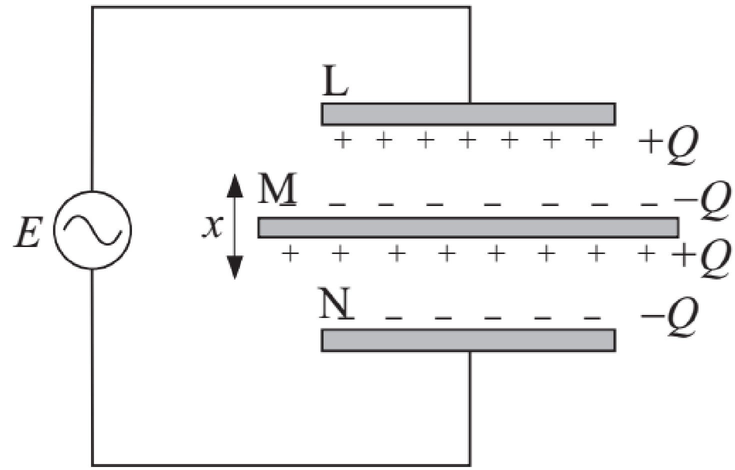
1. Change in the gap *x* between the plates

- **Linearization: 4. By using differential arrangement of capacitors**

$$E_{\mathrm{LM}} = \frac{Q}{C_{\mathrm{LM}}} = \frac{EC_{\mathrm{LN}}}{C_{\mathrm{LM}}} \quad \text{......................} \quad (1)$$

$$E_{\mathrm{MN}} = \frac{Q}{C_{\mathrm{MN}}} = \frac{EC_{\mathrm{LN}}}{C_{\mathrm{MN}}} \quad \text{......................} \quad (2)$$

$$C_{\mathrm{LN}} = \frac{\varepsilon A}{2d}$$



- ***Q*** **is the amount of charge on any plate and** ***d*** **is the distance between two adjacent plates**

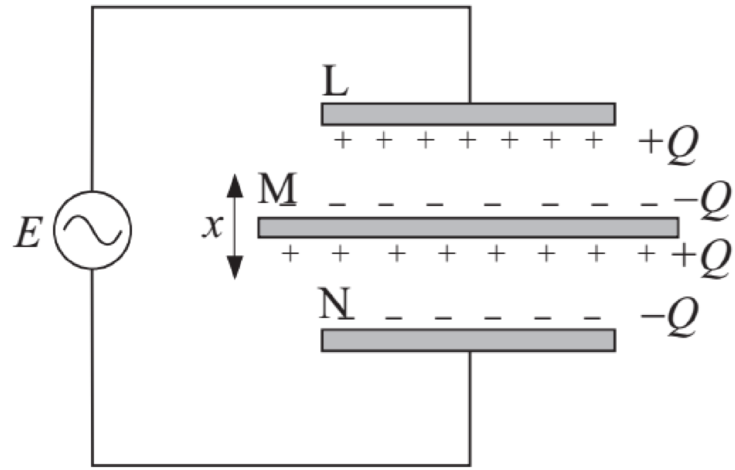
1. Change in the gap x between the plates

- **Linearization: 4. By using differential arrangement of capacitors**

$$E_{\mathrm{LM}} = \frac{Q}{C_{\mathrm{LM}}} = \frac{EC_{\mathrm{LN}}}{C_{\mathrm{LM}}} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad 1$$

$$E_{\mathrm{MN}} = \frac{Q}{C_{\mathrm{MN}}} = \frac{EC_{\mathrm{LN}}}{C_{\mathrm{MN}}} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad 2$$

$$C_{\mathrm{LN}} = \frac{\varepsilon A}{2d}$$



- **When M is right at the midway,**

$$C_{\mathrm{LM}} = C_{\mathrm{MN}}$$

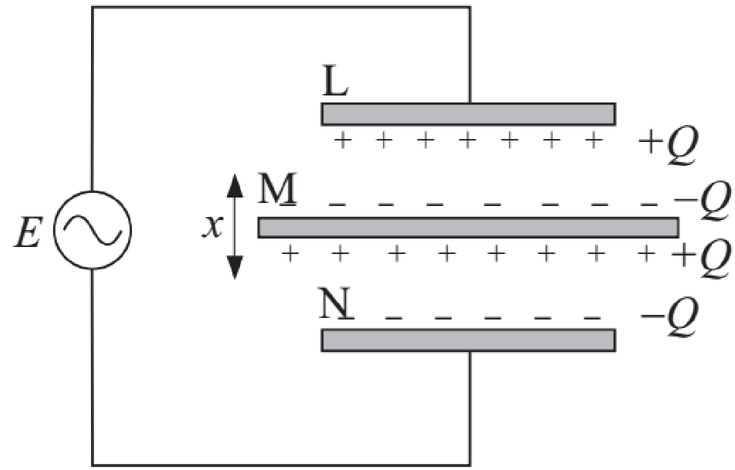
1. Change in the gap x between the plates

- **Linearization: 4. By using differential arrangement of capacitors**

$$E_{\mathrm{LM}} = \frac{Q}{C_{\mathrm{LM}}} = \frac{EC_{\mathrm{LN}}}{C_{\mathrm{LM}}} \quad \text{......................} \quad (1)$$

$$E_{\mathrm{MN}} = \frac{Q}{C_{\mathrm{MN}}} = \frac{EC_{\mathrm{LN}}}{C_{\mathrm{MN}}} \quad \text{......................} \quad (2)$$

$$C_{\mathrm{LN}} = \frac{\varepsilon A}{2d}$$



- **If *M* is displaced upwards by a distance x,**

$$C_{\mathrm{LM}} = \frac{\varepsilon A}{d - x}$$

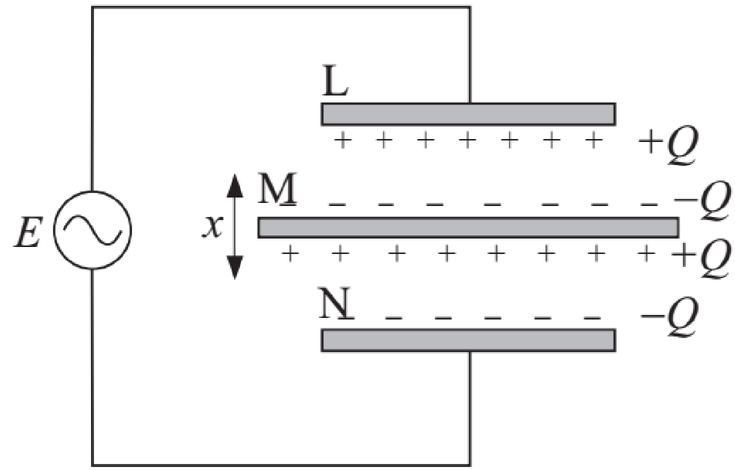
$$C_{\mathrm{MN}} = \frac{\varepsilon A}{d + x}$$

1. Change in the gap x between the plates

- **Linearization: 4. By using differential arrangement of capacitors**

$$E_{\mathrm{LM}} = \frac{Q}{C_{\mathrm{LM}}} = \frac{EC_{\mathrm{LN}}}{C_{\mathrm{LM}}} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad 1$$

$$E_{\mathrm{MN}} = \frac{Q}{C_{\mathrm{MN}}} = \frac{EC_{\mathrm{LN}}}{C_{\mathrm{MN}}} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad 2$$



$$C_{\mathrm{LN}} = \frac{\varepsilon A}{2d}$$

$$C_{\mathrm{LM}} = \frac{\varepsilon A}{d - x}$$

$$C_{\mathrm{MN}} = \frac{\varepsilon A}{d + x}$$

- **Putting C_{LN} , C_{LM} and C_{MN} in eq. 1 and 2, respectively.**

$$\Rightarrow \quad E_{\mathrm{LM}} = E \cdot \frac{d - x}{2d}$$

$$E_{\mathrm{MN}} = E \cdot \frac{d + x}{2d}$$

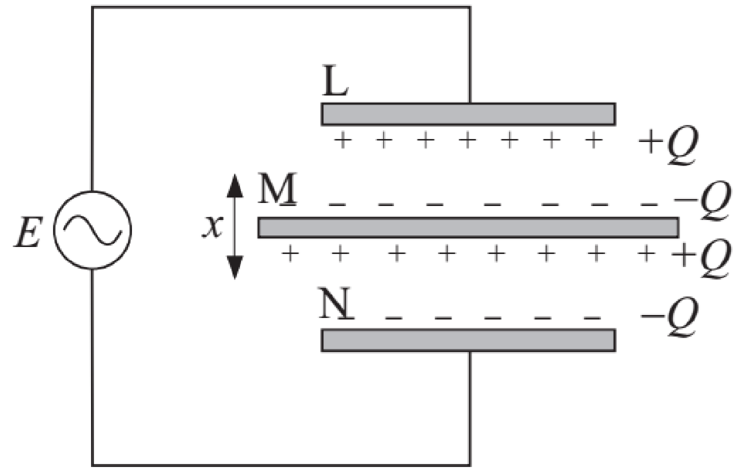
1. Change in the gap x between the plates

- **Linearization: 4. By using differential arrangement of capacitors**

$$E_{\mathrm{LM}} = E \cdot \frac{d-x}{2d}$$

$$E_{\mathrm{MN}} = E \cdot \frac{d+x}{2d}$$

$$\Rightarrow \quad \boxed{\Delta E = E_{\mathrm{LM}} - E_{\mathrm{MN}} = \frac{E}{d}x}$$



- **ΔE has a linear relation with the displacement x of the movable plate M**

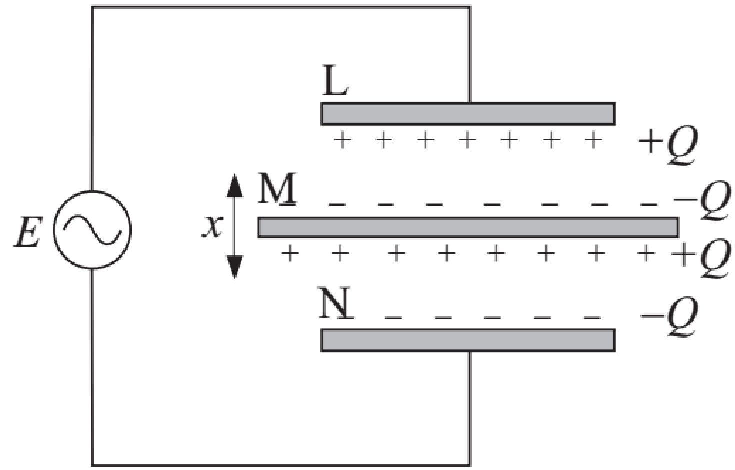
1. Change in the gap *x* between the plates

- **Linearization: 4. By using differential arrangement of capacitors**

$$E_{\mathrm{LM}} = E \cdot \frac{d-x}{2d}$$

$$E_{\mathrm{MN}} = E \cdot \frac{d+x}{2d}$$

$$\Rightarrow \quad \boxed{\Delta E = E_{\mathrm{LM}} - E_{\mathrm{MN}} = \frac{E}{d}x}$$



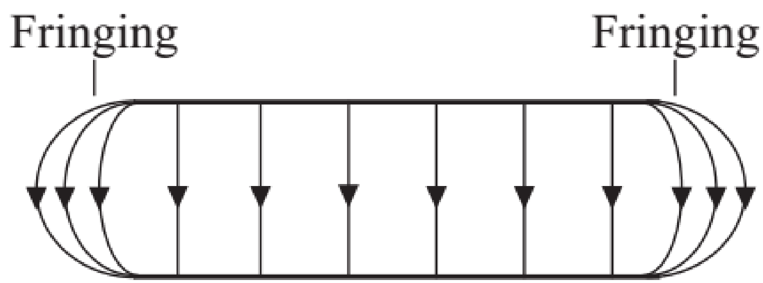
- **Features**
  - This arrangement, with appropriate instrumentation, can measure displacements between 10^{-8} mm and 10 mm with an accuracy of about 0.1%

1. Change in the gap x between the plates

- **fringing flux**

1. Change in the gap *x* between the plates

- **Effect of fringing flux**
  - Increases with plate spacing with respect to plate dimensions
  - Measured capacitance can be much larger than calculated
    - When plate spacing increases relative to the plates length and width



1. Change in the gap x between the plates

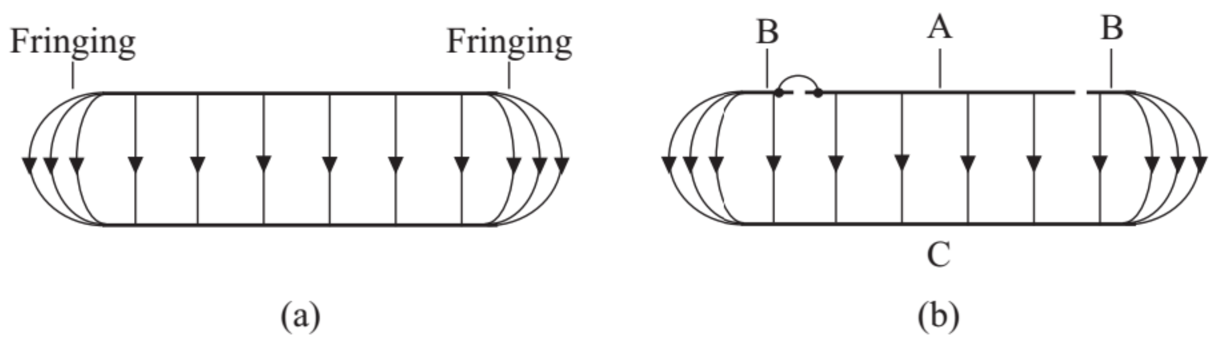


Fig. 6.21 (a) Fringing of flux at the ends of plates of a parallel-plate capacitor. (b) Guard ring (B) to eliminate the effect of fringing in a capacitor. B is at the same electrical potential with A.

- **Guard ring to eliminate the fringing effect**
  - Circular plates A of the capacitor is surrounded (but, electrically connected) by a concentric annular plate B in the same plane
  - Fringing effect is at the plate B

Example 6.7

Figure 6.22 shows a circuit with a variable air gap parallel plate capacitor as the sensing element. Show that the circuit acts as a velocity sensor for very small displacements, and find the proportionality constant between the voltage e_o and the input velocity v. Nominal (zero displacement) capacitance C is 50 pF and the nominal (zero displacement) distance between the capacitor plates x_0 is 5 mm.

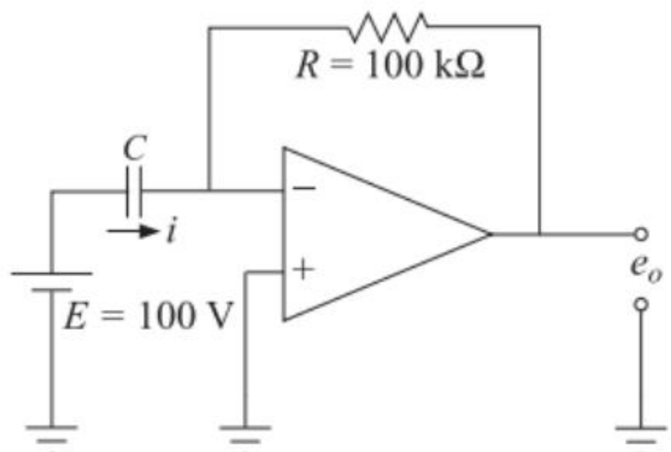
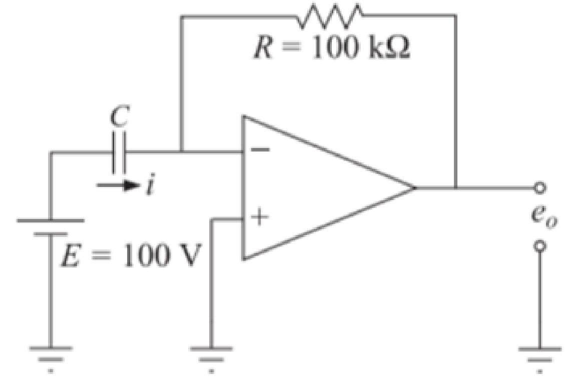


Fig. 6.22 Variable air gap parallel plate capacitor (Example 6.7).

Example 6.7



Solution

We know, $C = \dfrac{\varepsilon A}{x}$, where terms have their usual meaning. Therefore,

$$\frac{dx}{dt} = \frac{d}{dt}\left(\frac{\varepsilon A}{C}\right) = -\frac{\varepsilon A}{C^2}\frac{dC}{dt} = -\frac{Cx}{C^2}\frac{dC}{dt}$$

$$= -\frac{x}{C}\frac{dC}{dt} \qquad \text{(i)}$$

We also know, $C = \dfrac{Q}{E}$. Therefore,

$$\frac{dC}{dt} = \frac{1}{E}\frac{dQ}{dt} = \frac{i}{E}$$

where i denotes current. Substituting the value of $\dfrac{dC}{dt}$ in Eq. (i), we get on rearranging

$$i = -\frac{EC}{x}\frac{dx}{dt}$$

Therefore,

$$e_o = -iR = \frac{ECR}{x}\frac{dx}{dt} = \frac{(100)(50\times10^{-12})(100\times10^{3})}{0.5}\frac{dx}{dt} = 1.0\times10^{-3}v$$

Queries

Thanks!