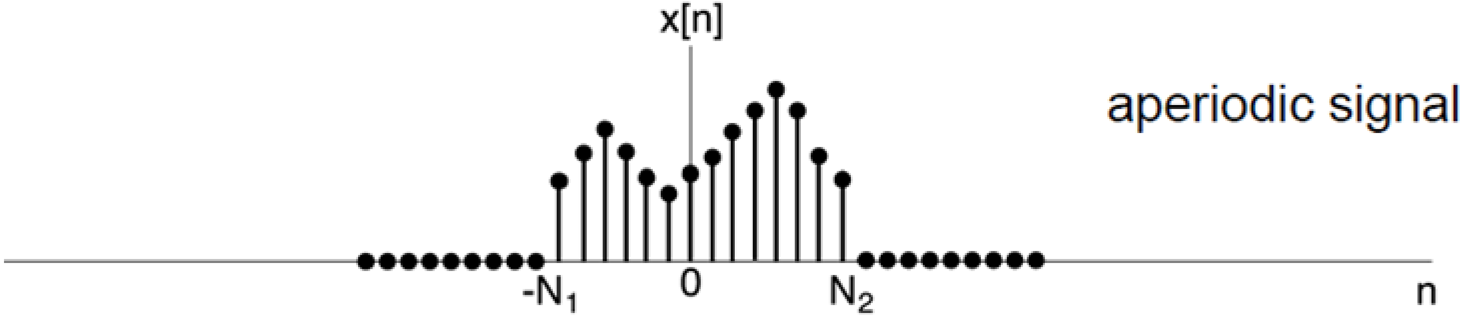
DISCRETE TIME FOURIER TRANSFORM (DTFT)

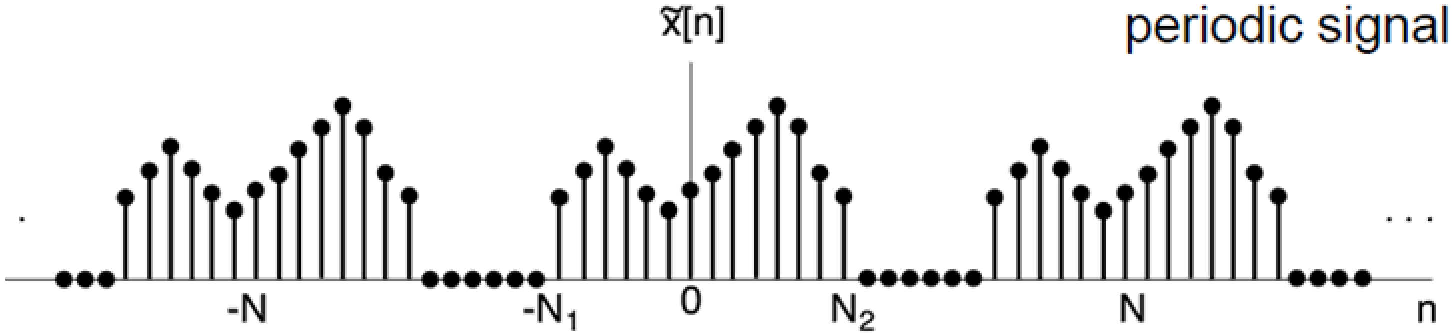
Discrete Time Fourier Transform

Derivation: (Analogous to CTFT except $e^{j\omega n} = e^{j(\omega+2\pi)n}$)

- $x[n]$ - aperiodic and (for simplicity) of finite duration
- N is large enough so that $x[n] = 0$ if $|n| \geq N/2$
- $\tilde{x}[n] = x[n]$ for $|n| \leq N/2$ and periodic with period N

Discrete Time Fourier Transform





$$\tilde{x}[n] = x[n] \text{ for } any\ n \text{ as } N \to \infty$$

DTFT - Derivation

$$\tilde{x}[n] = \sum_{k=<N>} a_k e^{jk\omega_0 n}, \quad \omega_0 = \frac{2\pi}{N}$$

synthesis eq.

$$a_k = \frac{1}{N} \sum_{n=<N>} \tilde{x}[n] e^{-jk\omega_0 n}$$

analysis eq.

$$= \frac{1}{N} \sum_{n=-N_1}^{N_2} \tilde{x}[n] e^{-jk\omega_0 n} = \frac{1}{N} \sum_{n=-\infty}^{\infty} x[n] e^{-jk\omega_0 n}$$

DTFT - Derivation

Define

$$X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x[n]e^{-j\omega n} \quad \boxed{-\text{ periodic in } \omega \text{ with period } 2\pi}$$

$$\Downarrow$$

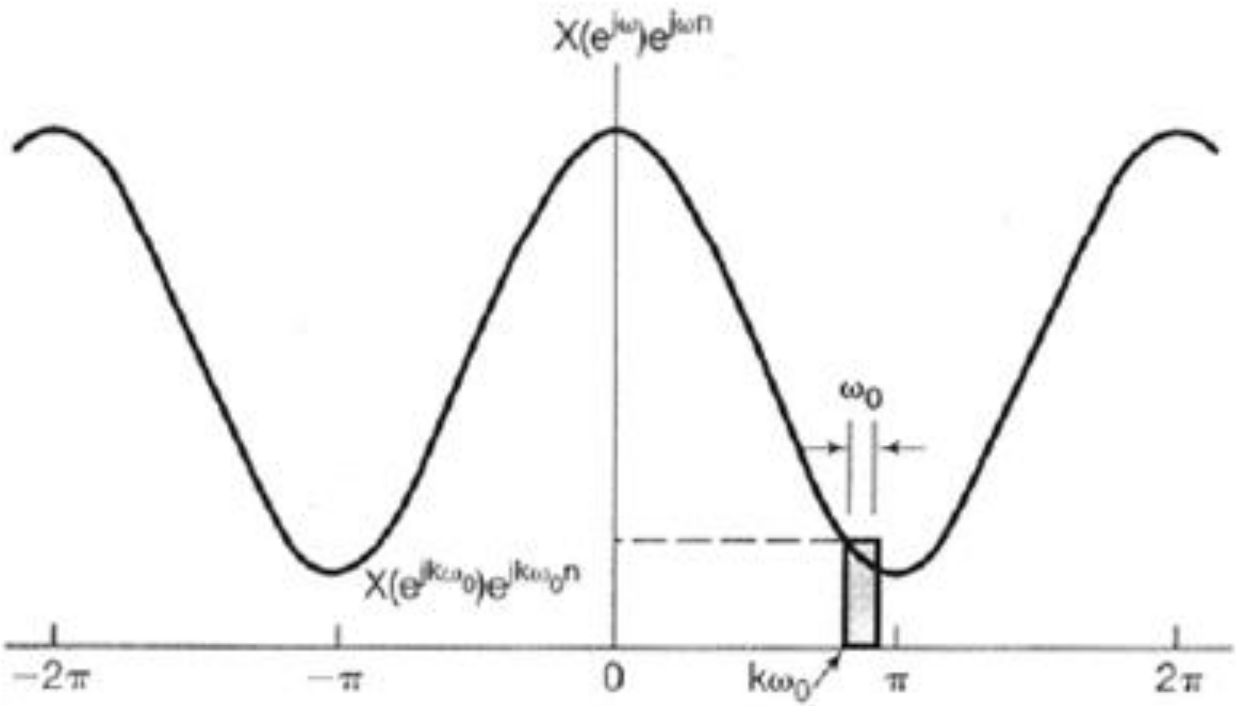
$$a_k = \frac{1}{N} X(e^{jk\omega_0})$$

We see that the coefficients a_k are proportional to samples of $X(e^{j\omega})$ where $\omega_0 = 2\pi / N$ is the spacing of the samples in the frequency domain

DTFT - Derivation

$$\tilde{x}[n] = \sum_{k=<N>} \underbrace{\frac{1}{N} X(e^{jk\omega_0})}_{a_k} e^{jk\omega_0 n} = \frac{1}{2\pi} \sum_{k=<N>} X(e^{jk\omega_0}) e^{jk\omega_0 n} \omega_0 \quad (*)$$

$$\text{As } N \to \infty : \qquad \tilde{x}[n] \to x[n] \text{ for every } n$$



DTFT - Derivation

$$\omega_0 \to 0, \sum \omega_0 \to \int d\omega$$

The sum in $(*) \to$ an integral

$\Downarrow$ The DTFT Pair

$$x[n] = \frac{1}{2\pi} \int_{2\pi} X(e^{j\omega}) e^{j\omega n} d\omega \qquad \text{Synthesis equation}$$

$$X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x[n] e^{-j\omega n} \qquad \text{Analysis equation}$$

Any 2π
interval in ω

DTFT – Example-1

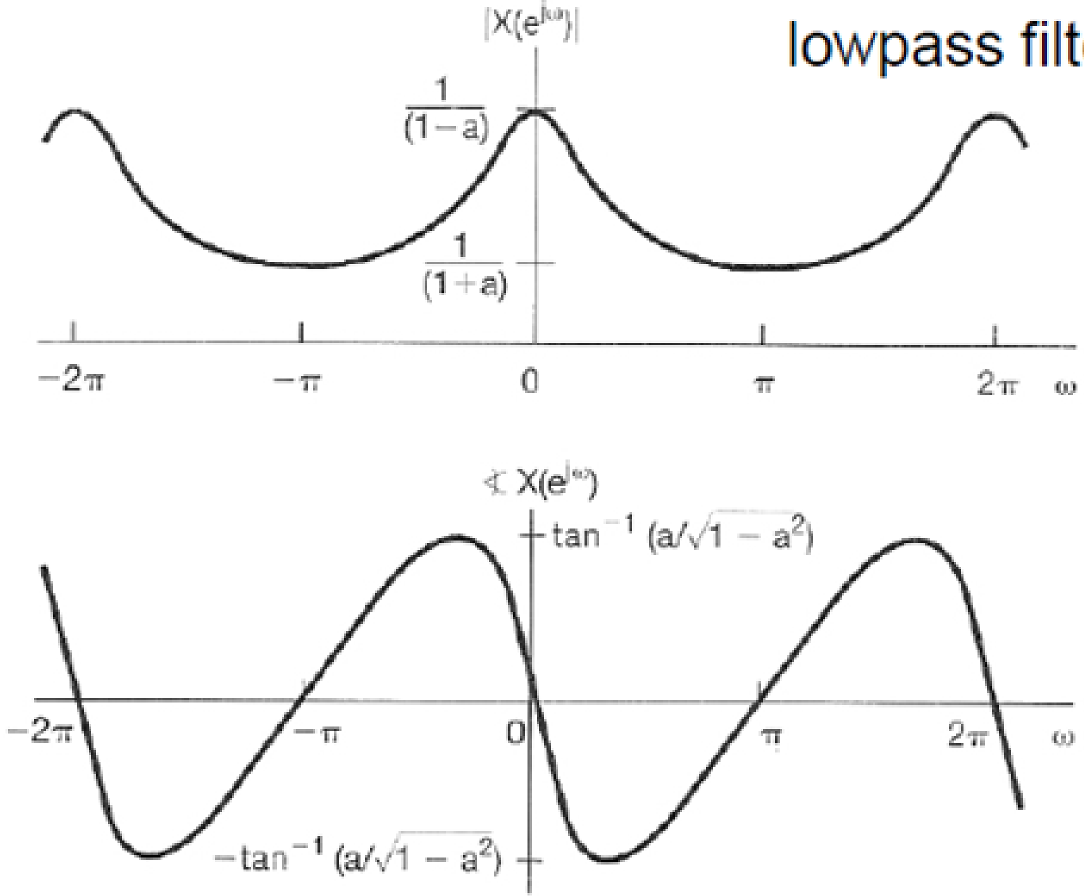
$$x[n] = a^n u[n], \quad |a| < 1$$

DTFT – Example-1

$$x[n] = a^n u[n], \quad |a| < 1$$

a>0

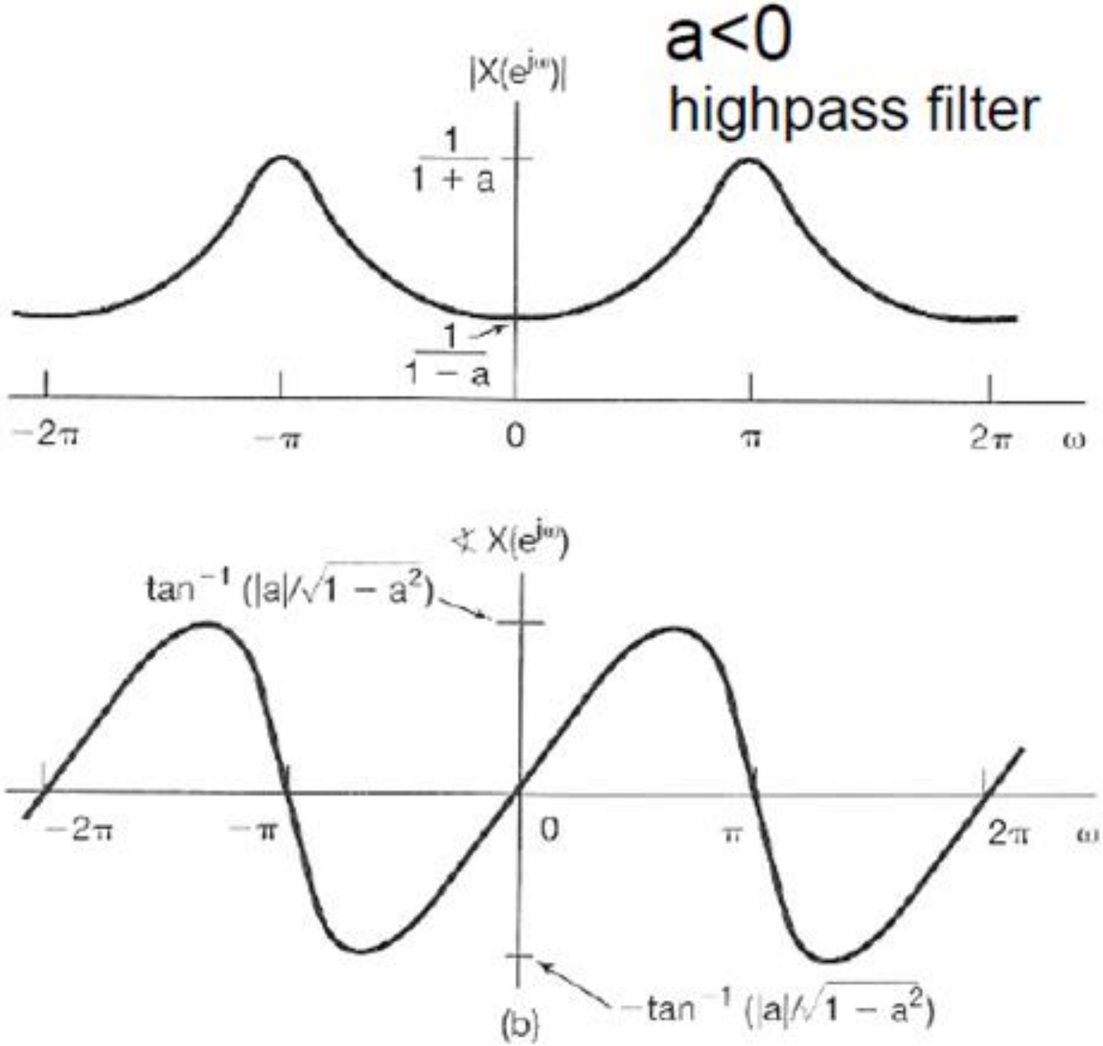
lowpass filter



(a)

DTFT – Example-1

$$x[n] = a^n u[n], \quad |a| < 1$$



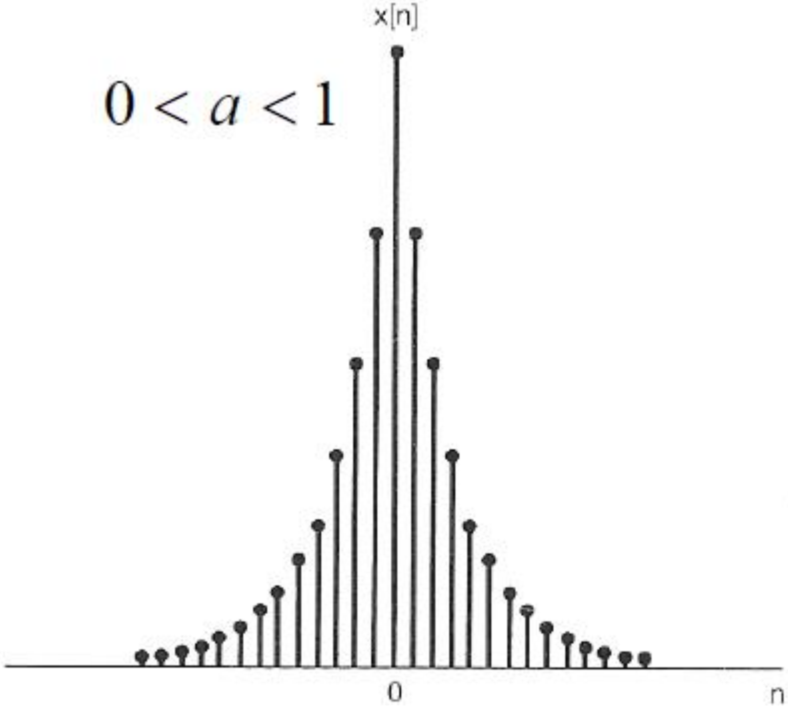
DTFT – Example-2

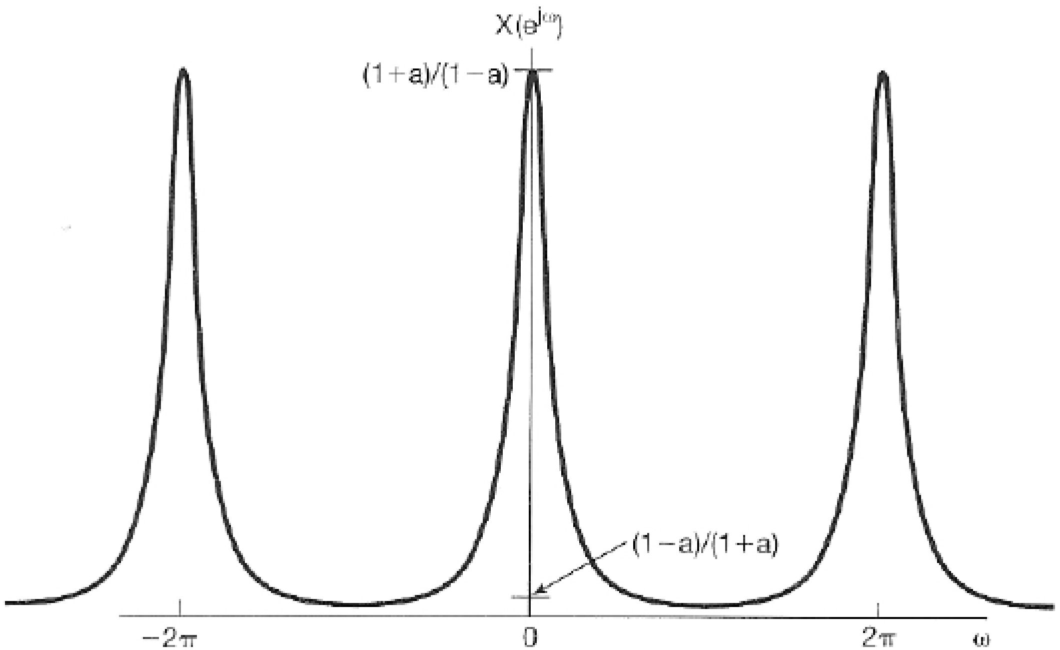
$$x[n] = a^{|n|}, \quad |a| < 1$$

DTFT – Example-2

$$x[n] = a^{|n|}, \quad |a| < 1$$

$0 < a < 1$





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