Engineering Mechanics

Dr. Hina Gohar Ali

Hina.gohar@seecs.edu.pk

Office : IAEC building

Office Hours: Appointment through emails

Research Interest: Photovoltaic systems, Power & Control, Sliding mode control (SMC)

Contents (Section 3.4)

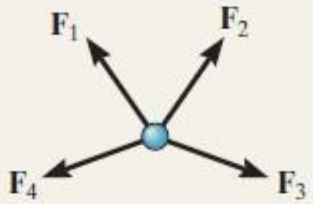
- Recap
- 3D Force System
- Examples

CHAPTER
Recap

Particle Equilibrium

When a particle is at rest or moves with constant velocity, it is in equilibrium. This requires that all the forces acting on the particle form a zero resultant force.

$$\mathbf{F}_R = \Sigma \mathbf{F} = \mathbf{0}$$



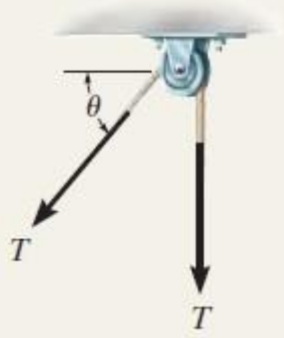
In order to account for all the forces that act on a particle, it is necessary to draw its free-body diagram. This diagram is an outlined shape of the particle that shows all the forces listed with their known or unknown magnitudes and directions.

Two Dimensions

The two scalar equations of force equilibrium can be applied with reference to an established x, y coordinate system.

$$\Sigma F_x = 0$$
$$\Sigma F_y = 0$$

The tensile force developed in a *continuous cable* that passes over a frictionless pulley must have a *constant* magnitude throughout the cable to keep the cable in equilibrium.



Cable is in tension

If the problem involves a linearly elastic spring, then the stretch or compression s of the spring can be related to the force applied to it.

$$F = ks$$

3.4 Three-Dimensional Force Systems

- For particle equilibrium

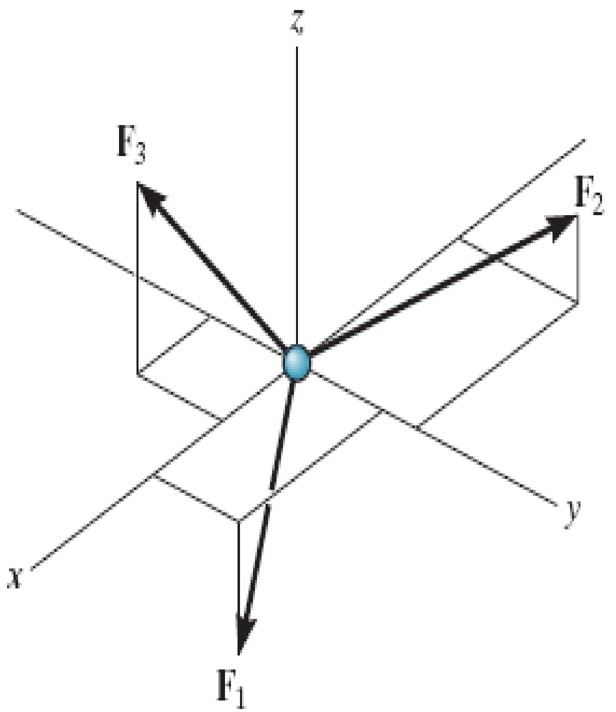
$$\sum \mathbf{F} = 0$$

- Resolving into **i**, **j**, **k** components

$$\sum F_x \mathbf{i} + \sum F_y \mathbf{j} + \sum F_z \mathbf{k} = 0$$

- Three scalar equations representing algebraic sums of the x, y, z forces

$$\sum F_x \mathbf{i} = 0$$
$$\sum F_y \mathbf{j} = 0$$
$$\sum F_z \mathbf{k} = 0$$



Using them **we can solve for at most three unknowns**, generally represented as coordinate direction angles or magnitudes of forces shown on the particle's free-body diagram.

Procedure for Analysis

Free-body Diagram

- Establish the z, y, z axes
- Label all known and unknown force

Equations of Equilibrium

- Apply $\sum F_x = 0$, $\sum F_y = 0$ and $\sum F_z = 0$

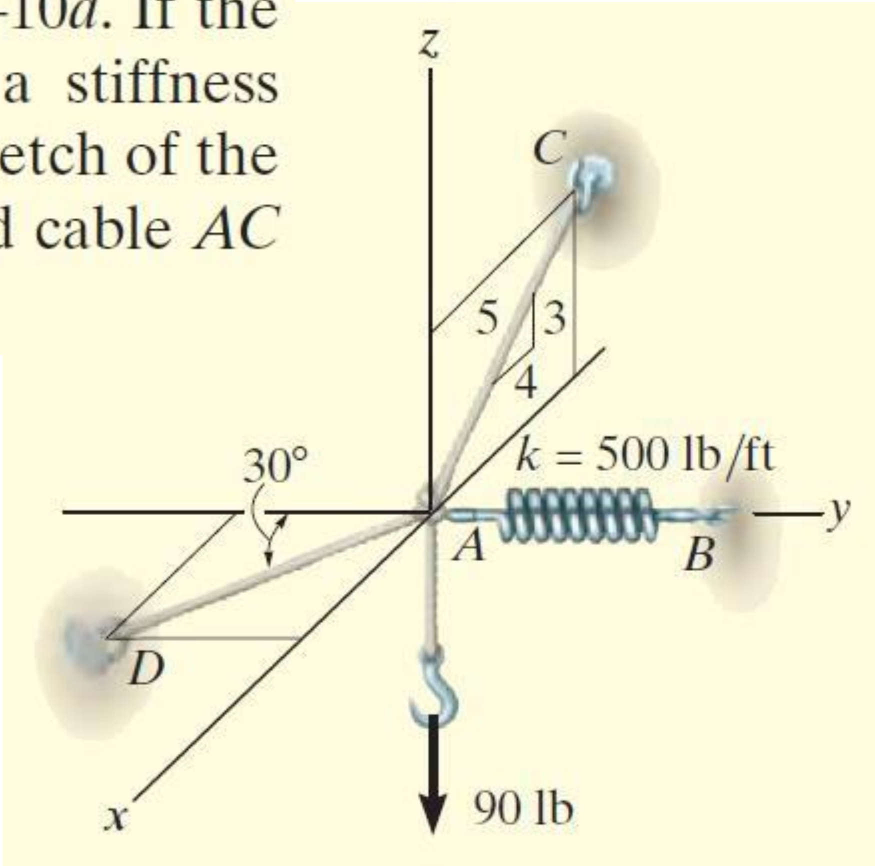
- If the three-dimensional geometry appears difficult, **then first express each force on the free-body diagram as a Cartesian vector**, substitute these vectors into $\sum F_x$ =0 and then set the **i**, **j**, **k** components equal to zero

- Negative results indicate that the sense of the force is opposite to that shown in the FBD.

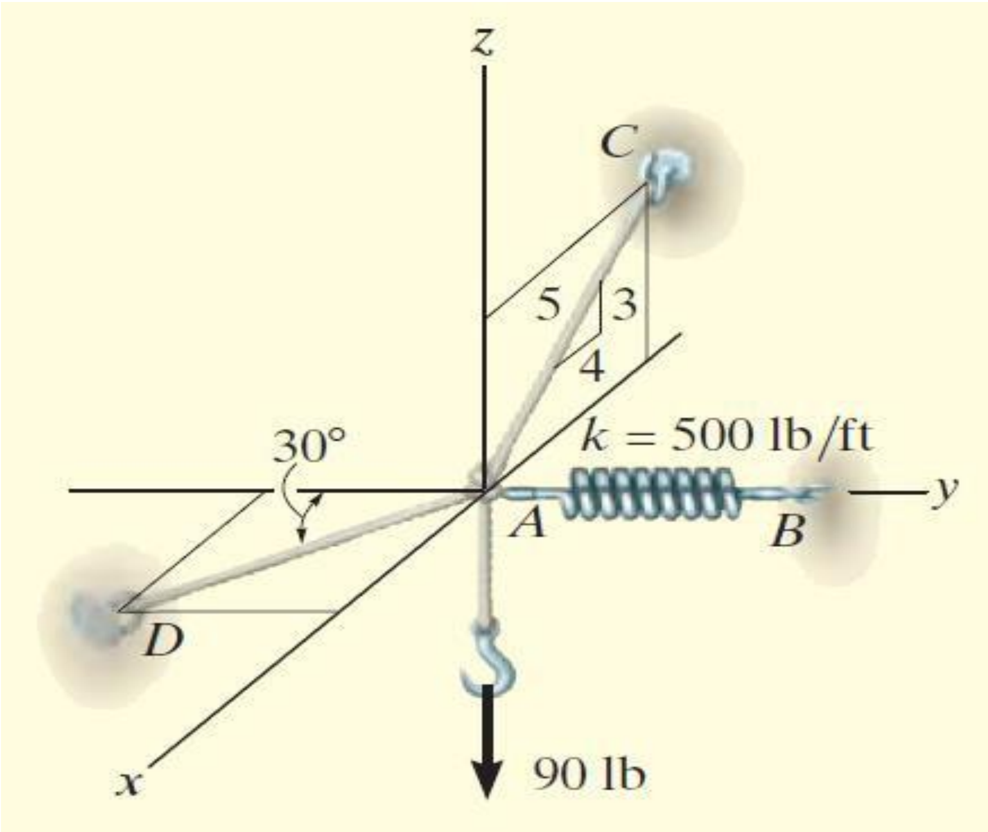
The ring at A is subjected to the force from the hook as well as forces from each of the three chains. If the electromagnet and its load have a weight W, then the force at the hook will be $\mathbf{W}$, and the three scalar equations of equilibrium can be applied to the free-body diagram of the ring in order to determine the chain forces, $\mathbf{F}_B$, $\mathbf{F}_C$, and $\mathbf{F}_D$.

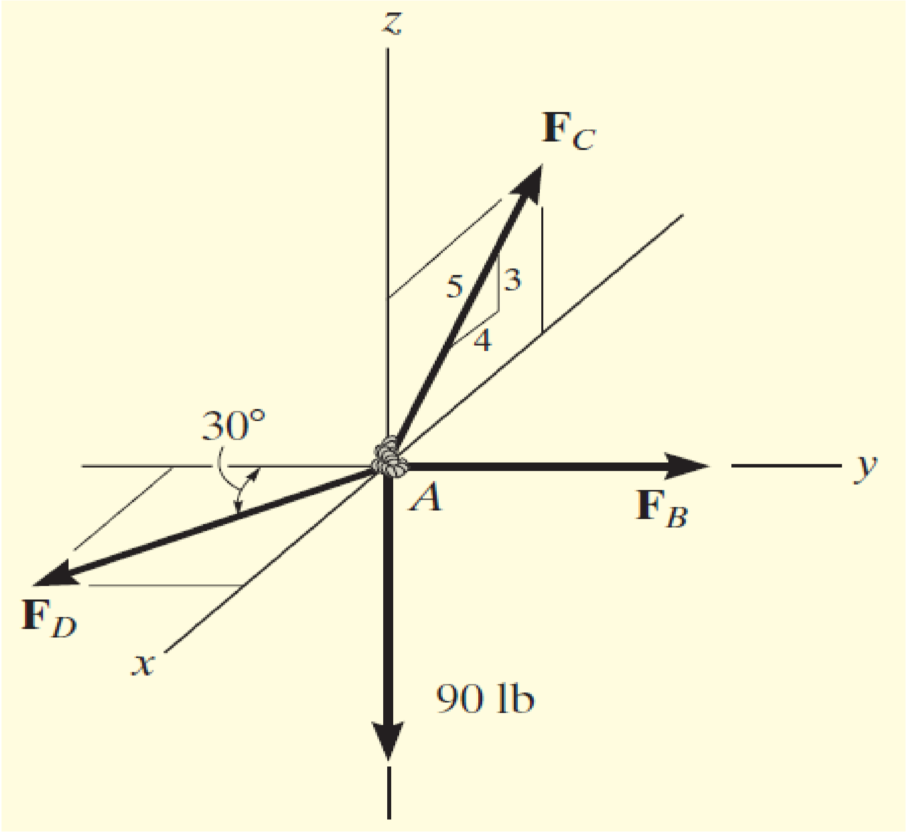
Example

A 90-lb load is suspended from the hook shown in Fig. 3–10*a*. If the load is supported by two cables and a spring having a stiffness $k = 500$ lb/ft, determine the force in the cables and the stretch of the spring for equilibrium. Cable *AD* lies in the *x*–*y* plane and cable *AC* lies in the *x*–*z* plane.



Free-Body Diagram. The connection at *A* is chosen for the equilibrium analysis since the cable forces are concurrent at this point. The free-body diagram is shown in Fig. 3–10*b*





$\Sigma F_x = 0;$ $\quad F_D \sin 30^\circ - \left(\frac{4}{5}\right)F_C = 0$

$\Sigma F_y = 0;$ $\quad -F_D \cos 30^\circ + F_B = 0$

$\Sigma F_z = 0;$ $\quad \left(\frac{3}{5}\right)F_C - 90 \text{ lb} = 0$

$F_C = 150 \text{ lb}$

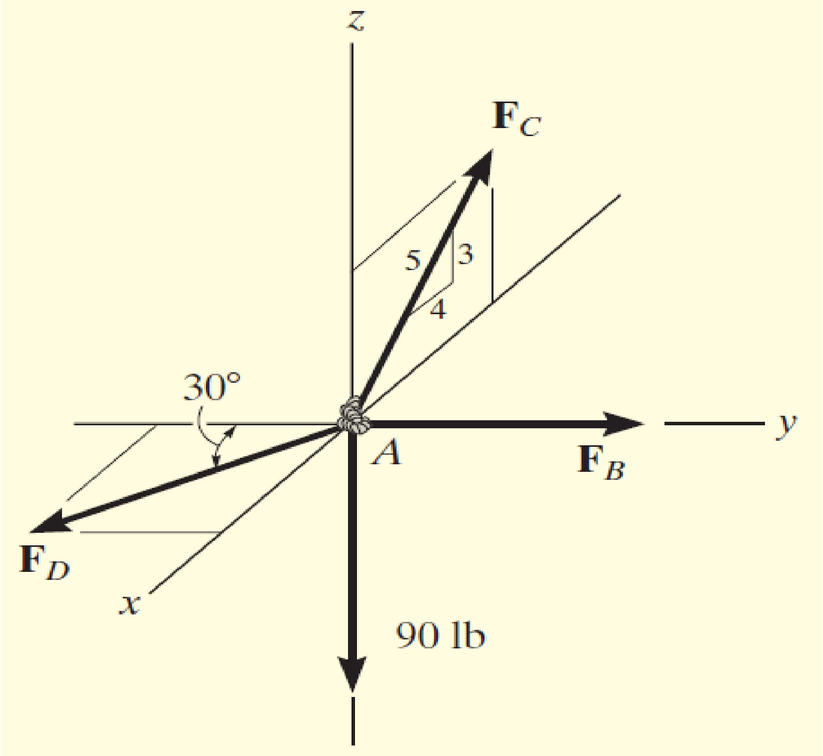
$F_D = 240 \text{ lb}$

$F_B = 207.8 \text{ lb}$

$F_B = k s_{AB}$

$207.8 \text{ lb} = (500 \text{ lb/ft})(s_{AB})$

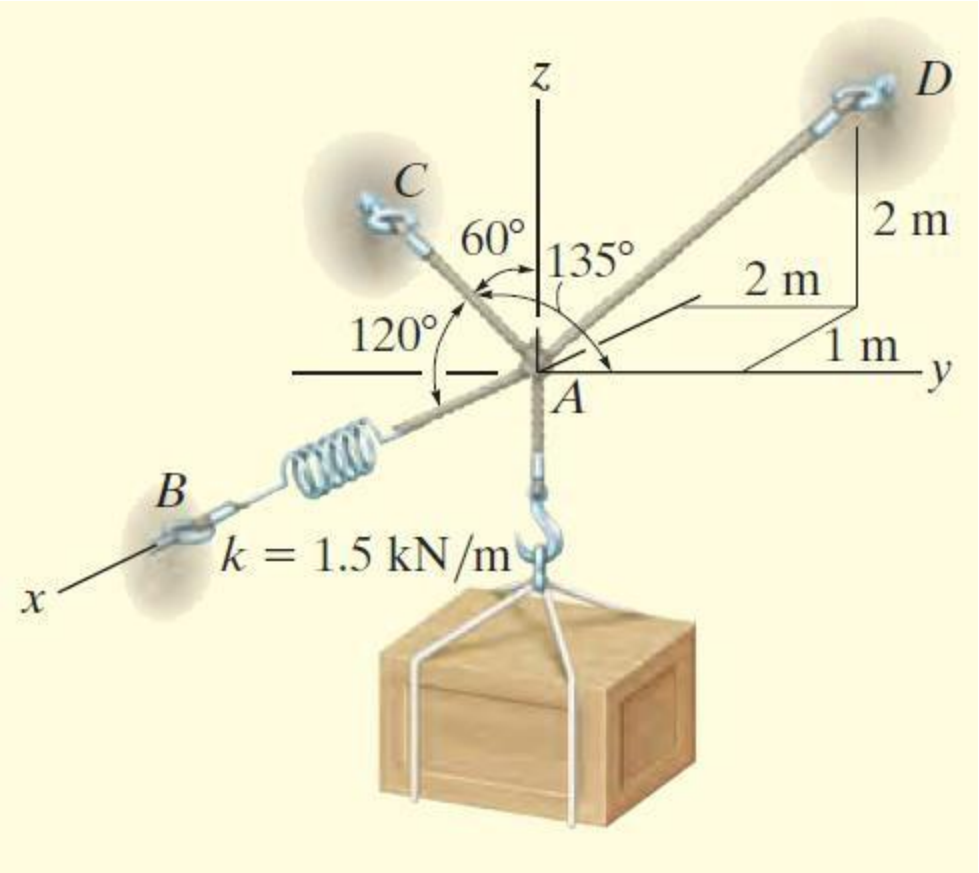
$s_{AB} = 0.416 \text{ ft}$



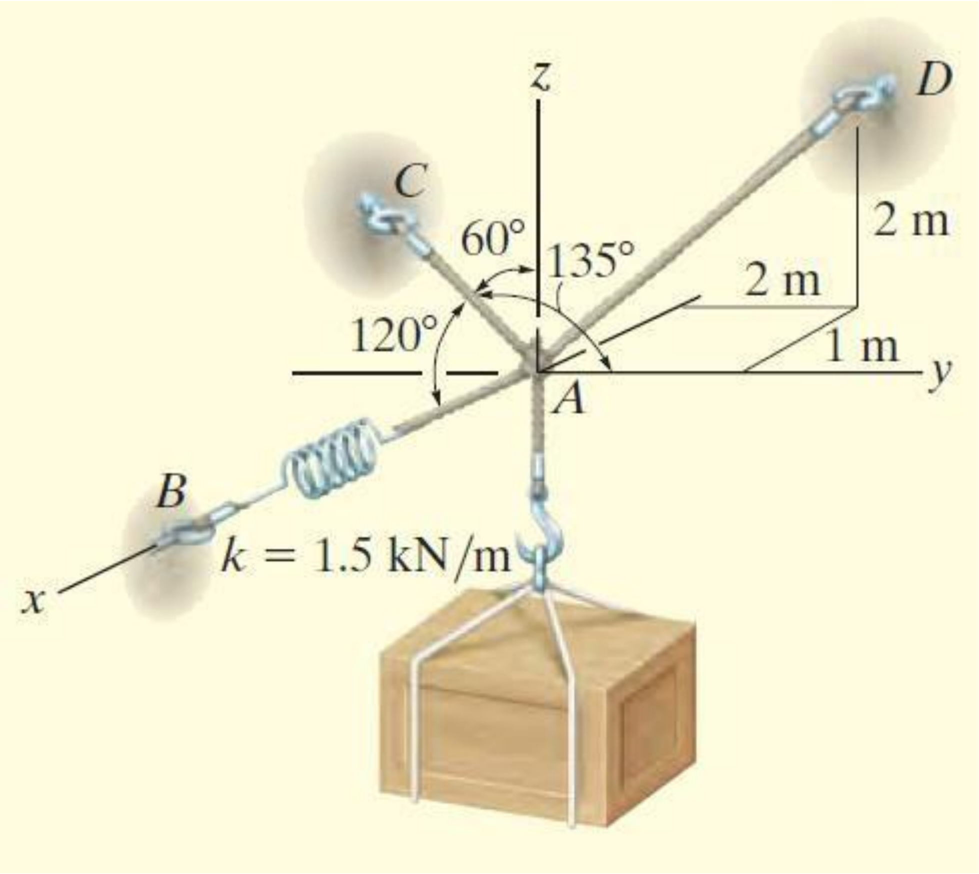
NOTE: Since the results for all the cable forces are positive, each cable is in tension; that is, it pulls on point *A* as expected, Fig. 3–10*b*

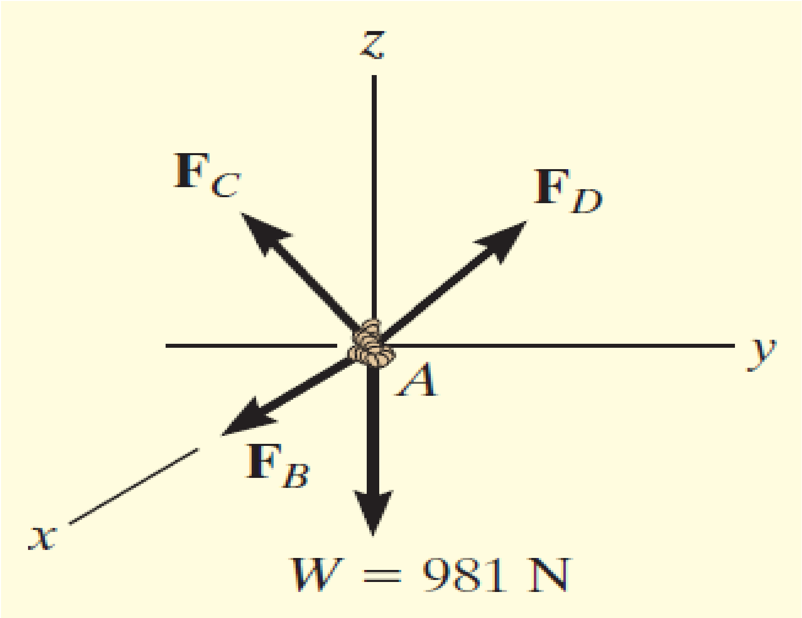
Example

Determine the tension in each cord used to support the 100-kg crate shown in Fig. 3–13*a*.

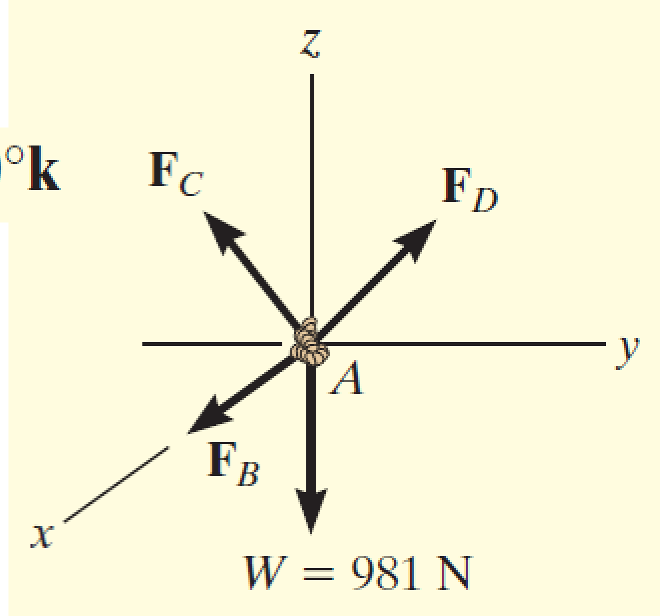


Free-Body Diagram. The force in each of the cords can be determined by investigating the equilibrium of point *A*. The free-body diagram is shown in Fig. 3–13*b*. The weight of the crate is W = 100(9.812) = 981 N





$\mathbf{F}_B = F_B\mathbf{i}$ D(-1 m, 2 m, 2 m)

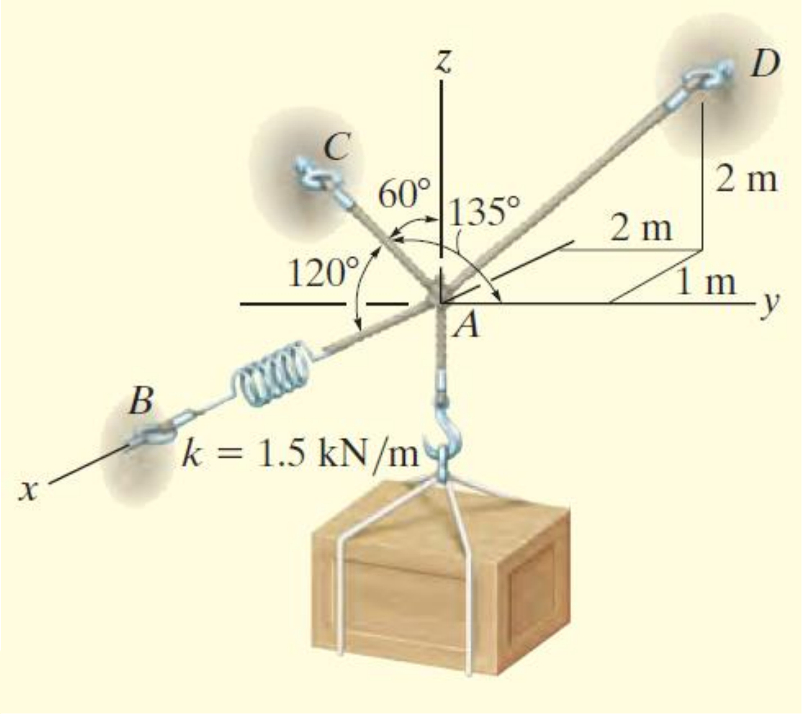


$$\mathbf{F}_C = F_C \cos 120^\circ \mathbf{i} + F_C \cos 135^\circ \mathbf{j} + F_C \cos 60^\circ \mathbf{k}$$
$$= -0.5F_C\mathbf{i} - 0.707F_C\mathbf{j} + 0.5F_C\mathbf{k}$$

$$\mathbf{F}_D = F_D\left[\frac{-1\mathbf{i} + 2\mathbf{j} + 2\mathbf{k}}{2\,\sqrt{(-1)^2 + (2)^2 + (2)^2}}\right]$$

$$= -0.333F_D\mathbf{i} + 0.667F_D\mathbf{j} + 0.667F_D\mathbf{k}$$

$$\mathbf{W} = \{-981\mathbf{k}\}\ \text{N}$$



$$\Sigma\mathbf{F} = \mathbf{0};$$

$$\mathbf{F}_B + \mathbf{F}_C + \mathbf{F}_D + \mathbf{W} = \mathbf{0}$$

$$F_B\mathbf{i} - 0.5F_C\mathbf{i} - 0.707F_C\mathbf{j} + 0.5F_C\mathbf{k}$$

$$-\ 0.333F_D\,\mathbf{i} + 0.667F_D\,\mathbf{j} + 0.667F_D\mathbf{k} - 981\mathbf{k} = \mathbf{0}$$

$$\Sigma F_x = 0; \qquad F_B - 0.5F_C - 0.333F_D = 0$$

$$\Sigma F_y = 0; \qquad -0.707F_C + 0.667F_D = 0$$

$$\Sigma F_z = 0; \qquad 0.5F_C + 0.667F_D - 981 = 0$$

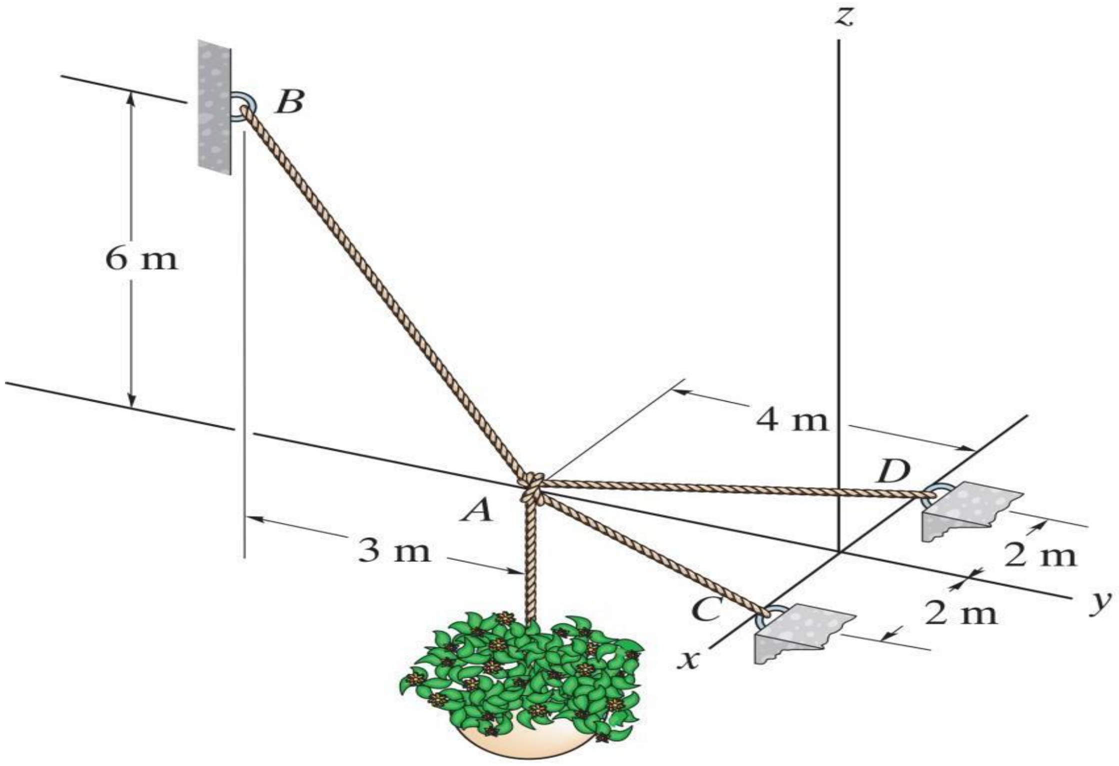
$$F_C = 813\ \text{N}$$

$$F_D = 862\ \text{N}$$

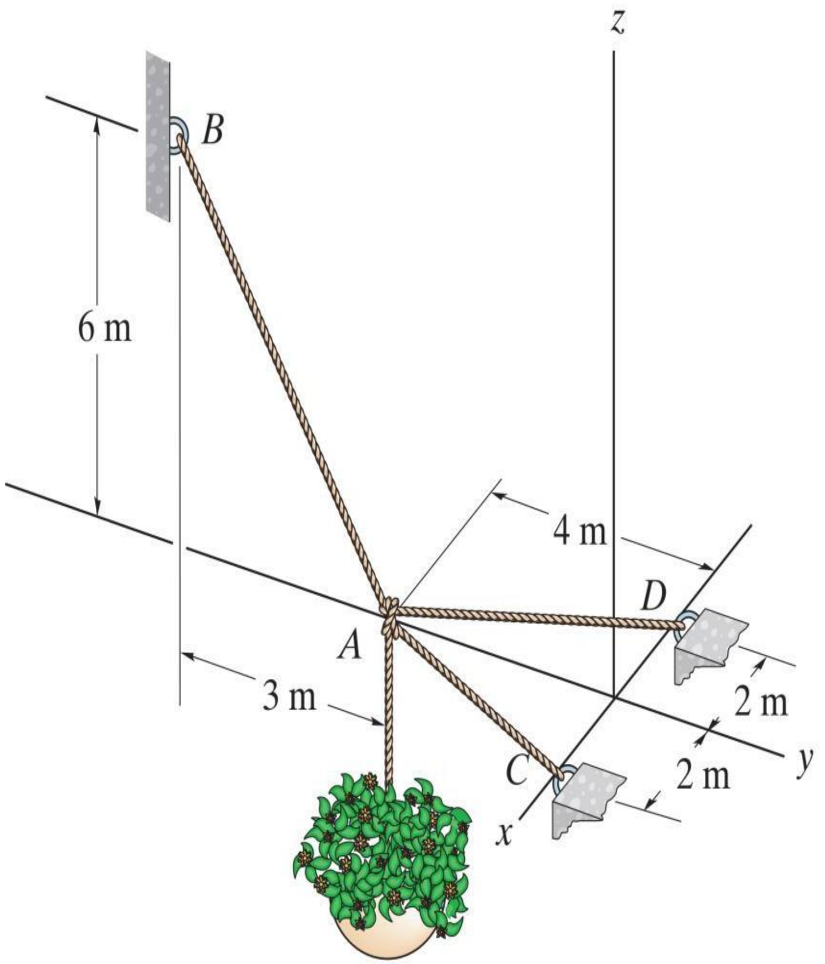
$$F_B = 694\ \text{N}$$

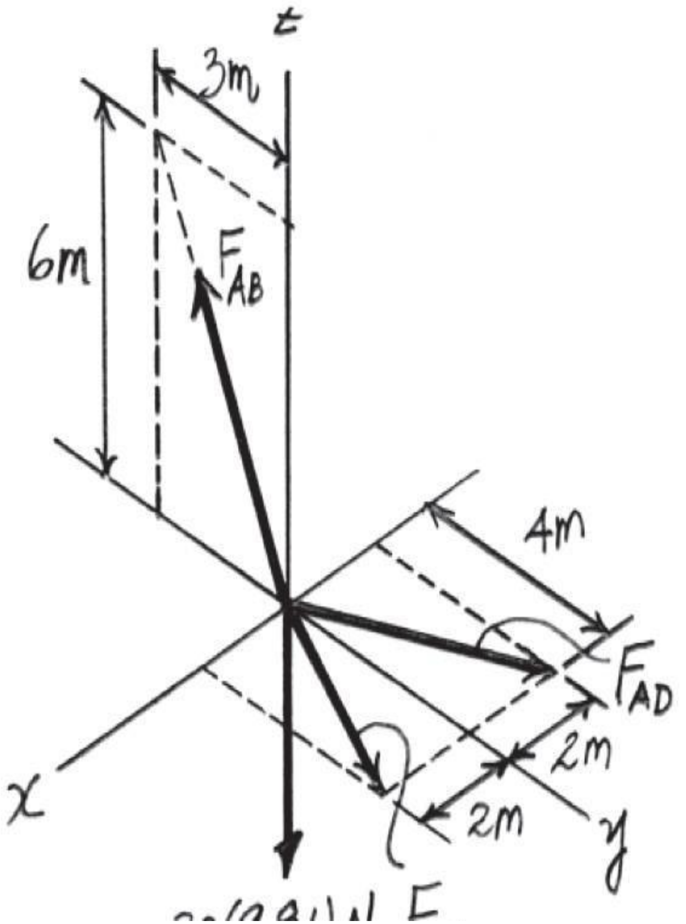
Example

Determine the force in each cable needed to support the 20-kg flowerpot



Free-Body Diagram. The connection at *A* is chosen for the equilibrium analysis since the cable forces are concurrent at this point.





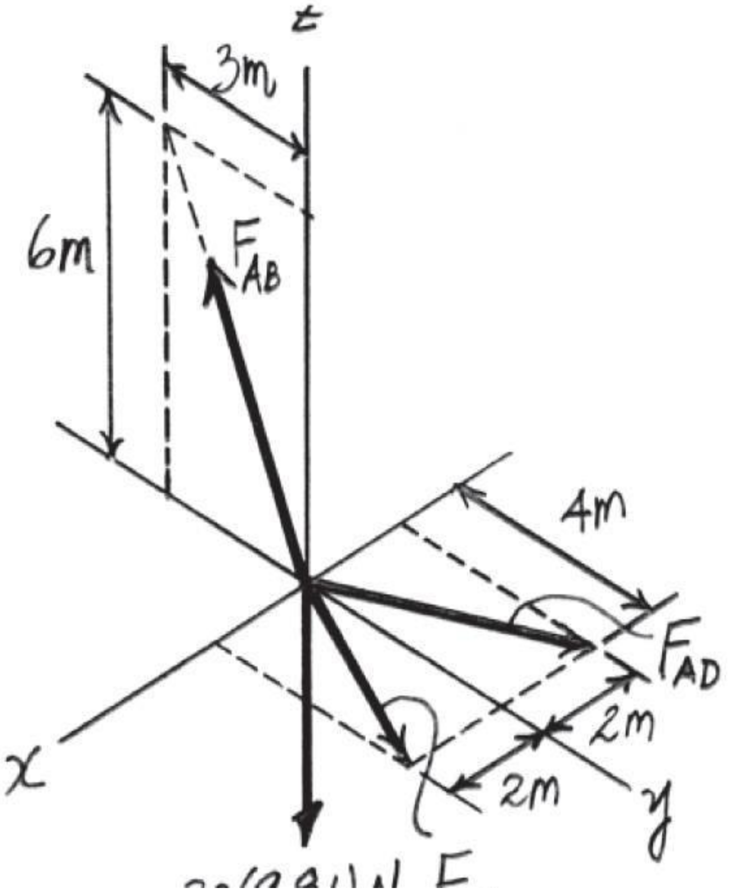
$$\Sigma F_z = 0; \qquad F_{AB}\left(\frac{6}{\sqrt{45}}\right) - 20(9.81) = 0 \qquad F_{AB} = 219.36\text{ N} = 219\text{ N}$$

$$\Sigma F_x = 0; \qquad F_{AC}\left(\frac{2}{\sqrt{20}}\right) - F_{AD}\left(\frac{2}{\sqrt{20}}\right) = 0 \qquad F_{AC} = F_{AD} = F$$

Using the results of $F_{AB} = 219.36\text{ N}$ and $F_{AC} = F_{AD} = F$,

$$\Sigma F_y = 0; \qquad 2\left[F\left(\frac{4}{\sqrt{20}}\right)\right] - 219.36\left(\frac{3}{\sqrt{45}}\right) = 0$$

$$F_{AC} = F_{AD} = F = 54.84\text{ N} = 54.8\text{ N}$$



Home Assignment

- Example 3.6, 3.7 & F3-8.