National University of Sciences & Technology
School of Electrical Engineering and Computer Science
Department of Basic Sciences

MATH-243: Vector Calculus (3+0): BEE-2k20-C Fall 2021

| Quiz - 3: Scalar and Vector Fields | |
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| **CLO-1:** Interpret the consequences of del (nabla) operator on scalar and vector fields. | |
| Maximum Marks: **10** | Instructor: Dr. Naila Amir |
| Date: $3-11-2021$ | Duration: 10 Minutes |
| Name: Master Solution | CMS ID: |

**Question:** For the vector field, $\mathbf{F}(x,y,z)=\langle z,y,x\rangle$ and the scalar field $g(x,y,z)=x+y+z$, verify the identity: $\nabla\times(g\mathbf{F})=g(\nabla\times\mathbf{F})+\nabla g\times\mathbf{F}$.

**Note:** You are required to show details of your work to get maximum marks.

Solution:

$$\vec{F}(x,y,z)=\langle z,y,x\rangle$$

$$g(x,y,z)=x+y+z$$

$$g\vec{F}=(x+y+z)\langle z,y,x\rangle$$

$$=\langle xz+yz+z^2,\ xy+y^2+yz,\ x^2+xy+xz\rangle$$

$$\vec{\nabla}\times(g\vec{F})=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ \partial/\partial x & \partial/\partial y & \partial/\partial z\\ xz+yz+z^2 & xy+y^2+yz & x^2+xy+xz\end{vmatrix}$$

$$=\hat{i}\left[x-y\right]-\hat{j}\left[2x+\cancel{y}+z-x-\cancel{y}-2z\right]+\hat{k}\left[y-z\right]$$

$$=\langle x-y,\ z-x,\ y-z\rangle \longrightarrow (1)$$

$$\vec{\nabla}\times\vec{F}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ \partial/\partial x & \partial/\partial y & \partial/\partial z\\ z & y & x\end{vmatrix}$$

$$=\langle 0,-(1-1),0\rangle$$

$$=\langle 0,0,0\rangle=\vec{0} \rightarrow (i)$$

$g(\vec{\nabla} \times \vec{F}) = (x+y+z)\langle 0,0,0\rangle$ [using (i)]

$= \langle 0,0,0\rangle \rightarrow$ (ii)

$\vec{\nabla} g = \langle g_x, g_y, g_z\rangle$

$= \langle 1, 1, 1\rangle$

$$\vec{\nabla} g \times \vec{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & 1 \\ z & y & x \end{vmatrix}$$

$= \langle x-y, z-x, y-z\rangle \rightarrow$ (iii)

Adding (ii) & (iii)

$g(\vec{\nabla} \times \vec{F}) + \vec{\nabla} g \times \vec{F} = \langle 0,0,0\rangle + \langle x-y, z-x, y-z\rangle$

$= \langle x-y, z-x, y-z\rangle \rightarrow$ ②

From ① & ②

$\vec{\nabla} \times (g\vec{F}) = g(\vec{\nabla} \times \vec{F}) + \vec{\nabla} g \times \vec{F}$.

Hence proved.