Chapter1: Digital Systems and Binary Numbers

Lecture3- Study Complements, Perform Subtraction using Complements

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Objectives

- Study Complements
- Perform Subtraction of Unsigned Numbers using Complements

Complements

- Complements are used in digital computers to simplify the subtraction operation and for logical manipulation. We can perform subtraction by adder circuits i.e

  $$A - B = A+(-B)$$

- There are two types of complements for each base-r system:
  - The radix complement, called the r's complement.
  - The diminished radix complement, called the (r-1)'s complement.
- When the value of the base r is substituted in the name, the two types are referred as the 2's complement and 1's complement for binary numbers and the 10's complement and 9's complement for decimal numbers.

Diminished Radix Complement (DRC)

- Given a number *N* in base *r* having *n* digits, the (r-1)’s complement of N is defined as:

  $(r^n – 1) – N$; where

  r: radix or base of the given number

  n: number of digits of integer part

  N: Given number
- Decimal numbers are in base-10.

  (r-1) = (10-1) = 9.
- The 9’s complement would be defined as:

  $(10^n – 1) – N$
- So, to determine the 9’s complement of 52:

  $(10^2 – 1) – 52 = 47$
- Another example is to determine the 9’s complement of 3124:

  $(10^4 – 1) – 3124 = 6875$

Finding Diminished Radix Complement (DRC)

- The DRC or (r-1)’s complement of decimal number is obtained by subtracting each digit from 9
- The (r-1)’s complement of octal or hexadecimal number is obtained by subtracting each digit from 7 or F, respectively
- The DRC (1’s complement) of a binary number is obtained by subtracting each digit from 1. It can also be formed by changing 1’s to 0’s and 0’s to 1’s

Diminished Radix Complement for Binary Numbers

- For binary numbers r = 2 and (r-1) = 1. So, the 1's complement would be defined as:

  $(2^n – 1) – N$

- To determine the 1's complement of 1000101:

  $(2^7 – 1) – 1000101 = 0111010$

- To determine the 1's complement of 11110111101:

  $(2^{11} – 1) - 11110111101 = 00001000010$

Note: 1's complement can be done by switching all 0's to 1's and 1's to 0's.

Complements

- Radix Complement (r's Complement)

The r's complement of an n-digit number N in base-r is defined as:

$r^n - N$; for $N \neq 0$

0 ; for $N = 0$

- We may obtain r's complement by adding 1 to (r-1)'s complement. Since $r^n - N = [(r^n - 1) - N]+1$
- 10's complement of 3229 is:

  $10^4 - 3229 = 6771$
- 2's complement of 101101 is:

  $2^6 - 101101 = 010011$

Note that to determine 2's complement, leave the least significant 0's and the first 1 unchanged and then switch the remaining 1's to 0' and 0's to 1's.

2’s Complement

- Another method to find 2’s complement is
  - Complement (reverse) each bit
  - Add 1
- Example:

Starting value	00000001
Step 1: reverse the bits	11111110
Step 2: add 1 to the value from Step 1	11111110 +00000001
Sum: two’s complement representation	11111111

Note that 00000001 + 11111111 = 00000000

Notes on Complements

- A couple of points on complements to keep in mind:
  - ➢ If you are trying to determine the complement of a value that contains a radix point:
    - o Remove the radix point.
    - o Determine the complement.
    - o Replace the radix point in the same relative position.
  - ➢ The complement of a complement will restore the original number i.e

    $N = 2^n - 1 - [(2^n - 1) - N]$ 1,s complement

    $N = r^n - (r^n - N)$ 2,s complement

Your Turn

- Find 9’s and 10’s complement of the following:
  - N=972.85
  - N=0.975
  - 7256
- Find 1’s and 2’s complement of the following:
  - 1011.101
  - 0.10110
  - 1101101

Subtraction with Complements

- In digital computers the use of borrows to complete subtraction is inefficient. Complements are used to overcome this inefficiency.
- The subtraction of two *n*-digit unsigned numbers *M* – *N* in base *r* can be done as follows:
  - ➢ Add the minuend, M, to the r's complement of the subtrahend, N:
    - o $M - N = M + (r^n - N) = M - N + r^n$
  - ➢ If $M \geq N$, the sum will produce an end carry, r^n, which can be discarded by $-r^n$; what is left is the result of M – N. This gives us correct answer
  - o If $M < N$, the sum does not produce an end carry and is equal to $r^n - (N - M)$, which is the r's complement of (N – M). This shows –ve answer expressed in r's complement form. To obtain the answer in a familiar form, take the r's complement of the sum and place a negative sign in front i.e –(N-M).

10’s Complement Subtraction

- Using 10’s complement, subtract 62513 – 2140

M =	62513
10’s complement of N =	97860
Sum	160373
Discard end carry	-100000
Answer	60373

- Note that the extra 9 in the 10’s complement of N is to fill the space holder 0.

10’ Complement Subtraction

- Using 10’s complement, subtract 2140 - 62513

M =	02140
10’s complement of N =	37487
Sum	39627
There is no end carry.	
10’s complement of 39627	60373
(Add - sign) Answer	-60373

2’s Complement Subtraction

- Using 2’s complement, subtract 1001001 - 1000110

$$
\begin{array}{rr}
M = & 1001001 \\
\text{2's complement of } N = & \underline{0111010} \\
\text{Sum} & 10000011 \\
\text{Discard end carry } 2^7 & \underline{-10000000} \\
\text{Answer} & 0000011
\end{array}
$$

2’s Complement Subtraction

- Using 2’s complement, subtract 1000110 - 1001001

M =	1000110
2’s complement of N =	0110111
Sum	1111101
There is no end carry.	
2’s complement of 1111101	0000011
(Add – sign) Answer	-0000011

Subtraction with r-1's Complement

- The subtraction of two *n*-digit unsigned numbers $M - N$ in base r using r – 1's complement can be done as follows:
  - Add the minuend, M, to the r – 1's complement of the subtrahend, N:
    - $M - N = M + (-N) = M + (r - 1\text{'s complement of } N) = M + [(r^n - 1) - N] = M - N + (r^n - 1)$
  - If M > N, M – N , a +ve value after added to $r^n - 1$ will produce an end carry, r^n, which can be discarded by $-r^n$ and 1 added to the least significant digit (LSD) of SUM i.e end around carry.; what is left is the result of M – N. This gives us correct +ve answer. Examples are 72532 – 3250 and 1010100 – 1000011. Mathematically,
    - $M - N = M - N + r^n - 1 - r^n + 1$
  - If $M \leq N$, $M - N = (r^n - 1) - (N - M)$. Here (N – M) is a +ve value and after subtracted from $r^n - 1$, doesn't produce an end carry . The result $(r^n - 1) - (N - M)$ shows r – 1's complement of (N – M). This shows – ve answer. To obtain the answer in a familiar form, take the r – 1's complement of the SUM and place a – ve sign in front i.e – (N – M). Examples are 3250 – 72532 and 1000011 – 1010100. Mathematically,
    - $M - N = (r^n - 1) - (N - M) = -[(r^n - 1) - \{(r^n - 1) - (N - M)\}] = -[r^n - 1 - r^n + 1 + (N - M)] = -(N - M)$

Subtraction using 9's Complement

- You can use the 9's complement for performing subtraction.
- You can add the minuend M to the 9's complement i.e (r-1)'s complement of subtrahend N. Then inspect the result.
  - ➢ If an end carry occurs discard end carry by – r^n , and add 1 to the least significant digit i.e end around carry
  - ➢ If there is no end carry take 9's complement i.e (r-1)'s complement of the result obtained and place a negative sign
  - ➢ Note: Remember that 9's complement is 1 less than 10's complement. This means we must compensate by adding 1 when an end carry occurs. Removing an end-carry and adding one is called an *end-around carry*.

9'S Complement Subtraction Example

- Using 9's complement, subtract 62513– 2140

M=	62513
9'S complement of N=	+ 97859
SUM	160372
$-r^n$	-100000
	60372
end around carry	+1
Answer=	60373

9’S Complement Subtraction Example

- Using 9’s complement, subtract 2140 – 62513

M= 02140

9’S complement of N= + 37486

SUM 39626

No end around carry;

9’s complement 60373

(Add – Sign) Answer= -60373

Subtraction using 1’s Complement

- You can also use the 1’s complement for performing subtraction.
- You can add the minuend M to the 1’s complement i.e (r-1)’s complement of subtrahend N. Then inspect the result.
  - If an end carry occurs discard the end carry by $-r^n$ and add 1 to the least significant digit i.e end around carry.
  - If there is no end carry take 1’s complement i.e (r-1)’s complement of the result obtained and place a negative sign
  - Note: Remember that 1’s complement is 1 less than 2’s complement. This means we must compensate by adding 1 when an end carry occurs. Removing an end-carry and adding one is called an *end-around carry*.

1's Complement Subtraction

- Using 1's complement, subtract 1001001 - 1000110

M =	1001001
1's complement of N =	0111001
Sum	10000010
Discard end carry 2^7	-10000000
	0000010
Add 1 to compensate	+0000001
Answer	0000011

1’s Complement Subtraction

- Using 1’s complement, subtract 1000110 - 1001001

M =	1000110
1’s complement of N =	0110110
Sum	1111100
There is no end carry.	
1’s complement of 1111100	0000011
(Add – sign) Answer	-0000011

Your Turn

- Perform subtraction 110110.10-10101.01 using 2's complement. Redo it using 1's complement.

The End