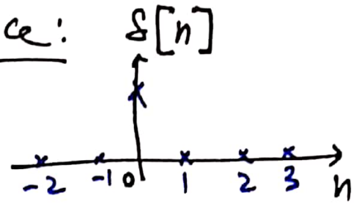
Z-transform of unit pulse / impulse sequence:



$$\delta[n] = [1, 0, 0, \ldots, \ldots]$$

$$Z[\delta(n)] = \sum_{n=-\infty}^{\infty} \delta[n] z^{-n} = z^0 = 1$$

Similarly, $\delta[n-m] = z^{-m}$. All z except 0 (if $m > 0$) or ∞ (if $m < 0$).

Inverse Z-transform: Inverse Z-transform is used to derive $x[n]$ from $X(z)$, and is formally defined as

$$x[n] = \frac{1}{2\pi j} \oint X(z) z^{n-1} dz \quad \text{——— (1)}$$

Here the symbol $\oint$ indicates an integration in counter-clockwise direction around a closed path within the complex z-plane (known as contour integral).
Such contour integral could be evaluated using Cauchy's residue theorem. We often use Z-transform pairs table with partial fraction expansion to calculate inverse inverse Z-transform.

Ex: Find the inverse Z-transform of $X(z) = \frac{8z-19}{(z-2)(z-3)}$.

We consider,

$$\boxed{Z[a^n] = \frac{z}{z-a}}$$

$$\frac{X(z)}{z} = \frac{8z-19}{z(z-2)(z-3)} = \frac{\left(-\frac{19}{6}\right)}{z} + \frac{3/2}{z-2} + \frac{5/3}{z-3}.$$

$$X(z) = -\frac{19}{6} + \frac{3}{2}\left(\frac{z}{z-2}\right) + \frac{5}{3}\left(\frac{z}{z-3}\right).$$

By using the Z-transform table, we have,

$$x[n] = -\frac{19}{6}\delta[n] + \frac{3}{2} 2^n u(n) + \frac{5}{3} 3^n u(n)$$

$$= -\frac{19}{6}\delta[n] + \left[\frac{3}{2} 2^n + \frac{5}{3} 3^n\right] u[n].$$

[Z transform (1]

EX:- For each of the following parts, determine the inverse z-transform $\frac{1}{az^{-1}+1} = \frac{z}{z+a}$ and specify the associated ROC. Assume that $a \in \mathbb{R}$.

a) Determine the inverse that is right-sided.

We know that $a^n u[n] \longleftrightarrow \frac{1}{1-az^{-1}} = \frac{z}{z-a}$, $|z|>|a|$.

Now, $X(z) = \frac{1}{az^{-1}+1} = \frac{z}{z+a} = \frac{z}{z-(-a)}$

$$x[n] = (-a)^n u[n] \quad , (ROC: |z|>|a|)$$

b) Determine the inverse that is left sided.

We know that $a^{-n} u[-n-1] \longleftrightarrow \frac{az}{1-az} = \frac{z}{z-\frac{1}{a}}$

$(|z|<|\frac{1}{a}|)$

'or' $X(z) = \frac{z}{z+a} = \frac{z}{z-(-a)}$

giving $x[n] = (\frac{1}{a})^n u[-n-1]$ Roc: $|z|< \frac{1}{|a|}$).

Practice Problem: Consider the z-transform

$$H(z) = \frac{1}{(2z^{-1}-1)((\frac{1}{2})z^{-1}-1)}$$

a) Find the poles and zeros of the z-transform.

b) Sketch the poles, zeros and the possible ROCs.

c) For each ROC, compute the corresponding signal $h(n)$.

Hints: (i) $|z|<\frac{1}{2}$, (ii) $|z|>2$, (iii) $\frac{1}{2}<|z|<2$

$$H(z) = \frac{1}{3}\frac{1}{\frac{1}{2}z^{-1}-1} - \frac{4}{3}\frac{1}{2z^{-1}-1}$$

(i) $h[n] = \frac{1}{3}(\frac{1}{2})^n u[-n-1] - \frac{4}{3}(2)^n u[-n-1]$

(ii) $h[n] = -\frac{1}{3}(\frac{1}{2})^n u[n] + \frac{4}{3}(2)^n u[n]$

(iii) $h[n] = -\frac{1}{3}(\frac{1}{2})^n u[n] - \frac{4}{3}(2)^n u[-n-1]$.

[z transform 12]