ELECTRIC POTENTIAL AND POTENTIAL ENERGY

Introduction

- The electric field intensity **E** due to a charge distribution can be obtained from Coulomb's law in general or from Gauss's law when the charge distribution is symmetric

- It would be desirable if we could find some as yet undefined scalar function with a single integration and then determine the electric field from this scalar by some simple straightforward procedure, such as differentiation

- This scalar function does exist and is known as the *potential* or *scalar potential field*

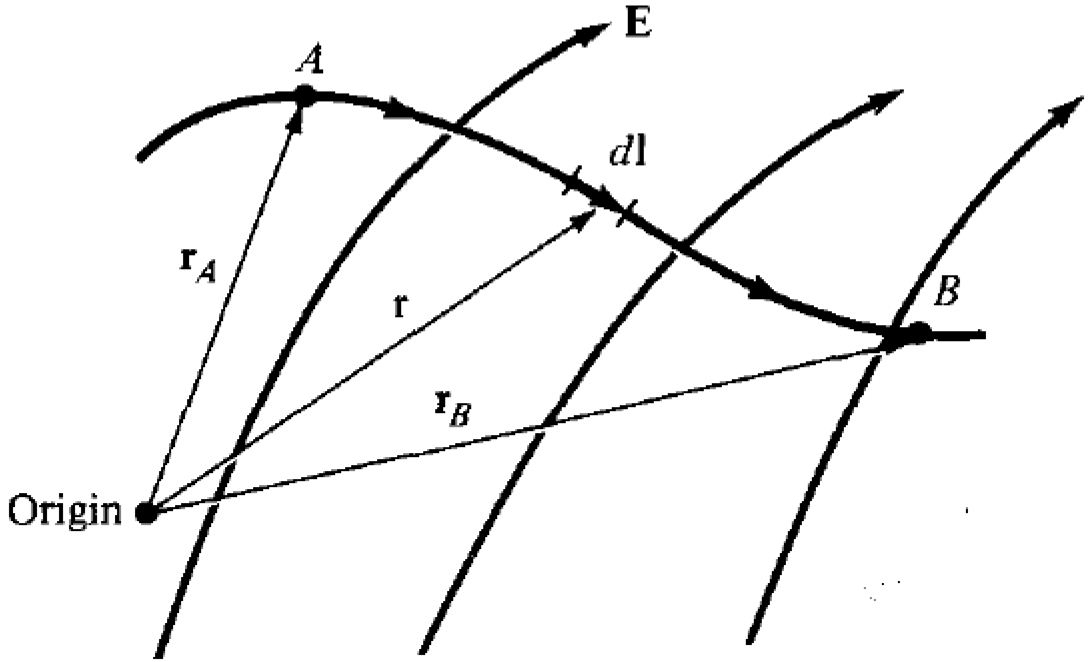
- The method of obtaining **E** from the *electric scalar potential* V is discussed in this lecture

Electric Potential

- Suppose we wish to move a point charge Q from point A to point B in an electric field **E**

- From Coulomb's law, the force on Q is **F** = Q**E** so that the work done in displacing the charge by $d\mathbf{l}$ is:

$$dW = -\mathbf{F} \cdot d\mathbf{l} = -Q\mathbf{E} \cdot d\mathbf{l}$$



Electric Potential

- The negative sign indicates that the work is being done by an external agent

- Thus, the total work done, or the potential energy required, in moving *Q* from A to B is:

$$W = -Q \int_A^B \mathbf{E} \cdot d\mathbf{l}$$

- Dividing *W* by *Q* in the above equation gives the potential energy per unit charge:

$$V_{AB} = \frac{W}{Q} = -\int_A^B \mathbf{E} \cdot d\mathbf{l}$$

- This quantity, denoted by V_{AB}, is known as the potential difference between points A and B

Electric Potential – Important Points

1. In determining V_{AB}, A is the initial point while B is the final point

2. If V_{AB} is negative, there is a loss in potential energy in moving Q from A to B; this implies that the work is being done by the field

3. If V_{AB} is positive, there is a gain in potential energy in the movement; an external agent performs the work

4. V_{AB} is independent of the path taken (to be proved later)

5. V_{AB} is measured in joules per coulomb, commonly referred to as volts (V)

Electric Potential - Point Charge

- As an example, if the **E** field in the figure is due to a point charge *Q* located at the origin, then:

$$\mathbf{E} = \frac{Q}{4\pi\varepsilon_o r^2}\,\mathbf{a}_r$$

- Then the equation for potential difference between point A and B becomes:

$$V_{AB} = -\int_{r_A}^{r_B} \frac{Q}{4\pi\varepsilon_o r^2}\,\mathbf{a}_r \cdot dr\,\mathbf{a}_r$$

$$= \frac{Q}{4\pi\varepsilon_o}\left[\frac{1}{r_B} - \frac{1}{r_A}\right]$$

- Or:

$$V_{AB} = V_B - V_A$$

- Here V_A and V_B are potentials or absolute potentials at A and B

Electric Potential - Point Charge

- The potential difference V_{AB} may be regarded as the potential at B with reference to A

- In problems involving point charges, it is customary to choose infinity as reference where the potential is zero

- Thus, if $V_A = 0$ as $r_A \rightarrow \infty$, the potential at any point $(r_B \rightarrow r)$ due to a point charge Q located at the origin is:

$$V = \frac{Q}{4\pi\varepsilon_o r}$$

- If the point charge Q is not located at the origin but at a point whose position vector is **r'**, the potential $V(x, y, z)$ or simply $V(r)$ at r becomes:

$$V(\mathbf{r}) = \frac{Q}{4\pi\varepsilon_o|\mathbf{r} - \mathbf{r}'|}$$

Electric Potential - Point Charge

- Note that because **E** points in the radial direction, any contribution from a displacement in the θ or $\emptyset$ direction is wiped out by the dot product, i.e.

$$\mathbf{E}.d\mathbf{l} = E\,cos\theta\;dl = Edr$$

- Hence the potential difference V_{AB} is independent of the path as mentioned earlier (work done depends upon displacement)

- The potential at any point is the potential difference between that point and a chosen point at which the potential is zero

- By assuming zero potential at infinity, the potential at a distance r from the point charge is the work done per unit charge by an external agent in transferring a test charge from infinity to that point

Electric Potential - Multiple Charges

- The superposition principle, which we applied to electric fields, applies to potentials

- For *n* point charges Q_1, Q_2,….,Q_n located at points with position vectors $\mathbf{r}_1$, $\mathbf{r}_2$,…., $\mathbf{r}_n$, the potential at $\mathbf{r}$ is:

$$V(\mathbf{r}) = \frac{Q_1}{4\pi\varepsilon_o|\mathbf{r} - \mathbf{r}_1|} + \frac{Q_2}{4\pi\varepsilon_o|\mathbf{r} - \mathbf{r}_2|} + \cdots + \frac{Q_n}{4\pi\varepsilon_o|\mathbf{r} - \mathbf{r}_n|}$$

- Or:

$$V(\mathbf{r}) = \frac{1}{4\pi\varepsilon_o} \sum_{k=1}^{n} \frac{Q_k}{|\mathbf{r} - \mathbf{r}_k|} \qquad \text{(point charges)}$$

Electric Potential - Charge Distributions

- For continuous charge distributions, we replace Q_k with charge element $\rho_L\, d$l, $\rho_s\, dS$, or $\rho_v\, dv$ and the summation becomes an integration, so the potential at *r* becomes:

$$V(\mathbf{r}) = \frac{1}{4\pi\varepsilon_o} \int_L \frac{\rho_L(\mathbf{r}')dl'}{|\mathbf{r} - \mathbf{r}'|} \qquad \text{(line charge)}$$

$$V(\mathbf{r}) = \frac{1}{4\pi\varepsilon_o} \int_S \frac{\rho_S(\mathbf{r}')dS'}{|\mathbf{r} - \mathbf{r}'|} \qquad \text{(surface charge)}$$

$$V(\mathbf{r}) = \frac{1}{4\pi\varepsilon_o} \int_v \frac{\rho_v(\mathbf{r}')dv'}{|\mathbf{r} - \mathbf{r}'|} \qquad \text{(volume charge)}$$

- Where the primed coordinates are used to denote source point location and the unprimed coordinates refer to field point

Relationship Between E and V

- As discussed, the potential difference between points A and B is independent of the path taken, hence:

$$V_{BA} = -V_{AB}$$

- Therefore: $$V_{BA} + V_{AB} = \oint \mathbf{E} \cdot d\mathbf{l} = 0$$

- Or: $$\oint \mathbf{E} \cdot d\mathbf{l} = 0$$

- This shows that the line integral of **E** along a closed path must be zero

- Physically, this implies that no net work is done in moving a charge along a closed path in an electrostatic field

Relationship Between E and V

- Applying Stokes's theorem, we get:

$$\oint \mathbf{E} \cdot d\mathbf{l} = \int (\nabla \times \mathbf{E}) \cdot d\mathbf{S} = 0$$

- Or:

$$\nabla \times \mathbf{E} = 0$$

- Any vector field that satisfies the above equations is said to be conservative

- Thus, an electrostatic field is a conservative field

- The above equation is referred to as Maxwell's second equation for static electric fields

Relationship Between E and V

- From the way we defined potential, $V = -\int \mathbf{E}.d\mathbf{l}$, it follows that:

$$dV = -\mathbf{E} \cdot d\mathbf{l} = -E_x\,dx - E_y\,dy - E_z\,dz$$

- The differential of a scalar quantity may be written as:

$$dV = \frac{\partial V}{\partial x}dx + \frac{\partial V}{\partial y}dy + \frac{\partial V}{\partial z}dz$$

- By comparing we get:

$$E_x = -\frac{\partial V}{\partial x}, \qquad E_y = -\frac{\partial V}{\partial y}, \qquad E_z = -\frac{\partial V}{\partial z}$$

- As:

$$\mathbf{E} = E_x \boldsymbol{a_x} + E_y \boldsymbol{a_y} + E_z\, \boldsymbol{a_z}$$

- So we get:

$$\mathbf{E} = -\nabla V$$

Relationship Between E and V

- Therefore, the electric field intensity is the gradient of *V*

- The negative sign shows that the direction of **E** is opposite to the direction in which *V* increases or **E** is directed from higher to lower levels of *V*

- Since the curl of the gradient of a scalar function is always zero ($\nabla \times \nabla V = 0$), this implies that **E** must be a gradient of some scalar function

- So by using the potential difference *V*, a vector problem is reduced to a scalar problem

Problem-1

- An infinite line charge having charge density ρ_L C/m^2 is placed along the z-axis. Find the work done in moving a point charge Q:

a) From point b to a on the y-axis, where a is closer to the line charge compared to b.

b) In a circle around the line charge.