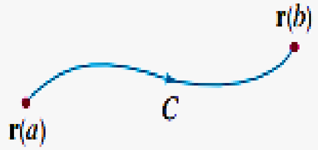
line- and surface integrals

Vector Calculus(MATH-243)

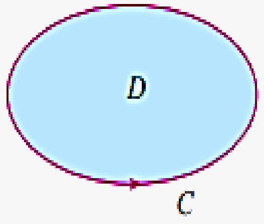
Instructor: Dr. Naila Amir

Fundamental Theorem of Calculus $\quad \int_a^b F'(x)\,dx = F(b) - F(a)$

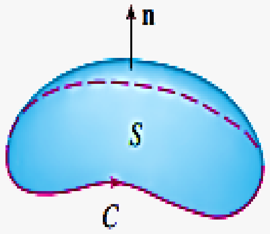
Fundamental Theorem for Line Integrals $\quad \int_C \nabla f \cdot d\mathbf{r} = f(\mathbf{r}(b)) - f(\mathbf{r}(a))$



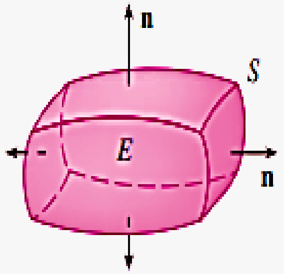
Green's Theorem (Circulation form) $\quad \iint_D \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y} \right) dA = \int_C P\,dx + Q\,dy$



Stokes' Theorem $\quad \iint_S \operatorname{curl} \mathbf{F} \cdot d\mathbf{S} = \int_C \mathbf{F} \cdot d\mathbf{r}$

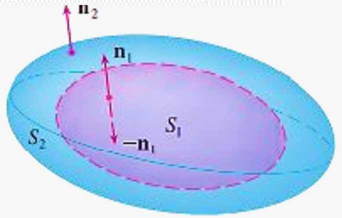


Divergence Theorem $\quad \iiint_E \operatorname{div} \mathbf{F}\,dV = \iint_S \mathbf{F} \cdot d\mathbf{S}$



Divergence Theorem for regions that are finite unions of simple solid regions

$$\iiint_E \operatorname{div} \mathbf{F}\,dV = -\iint_{S_1} \mathbf{F} \cdot d\mathbf{S} + \iint_{S_2} \mathbf{F} \cdot d\mathbf{S}$$



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Vector Calculus

Book: Calculus Early Transcendentals (6th Edition) By James Stewart.

- **Chapter: 16**

Book: Thomas' Calculus Early Transcendentals (14th Edition) By George B. Thomas, Jr., Joel Hass, Christopher Heil, Maurice D. Weir.

- **Chapter: 16**

Question # 1: Green's Theorem

Determine the moments of inertia about the coordinate axes of a thin wire lying along the curve:

$$\mathbf{r}(t) = t\,\mathbf{i} + \frac{2\sqrt{2}}{3}t^{3/2}\,\mathbf{j} + \frac{t^2}{2}\,\mathbf{k};\ \ 0 \leq t \leq 2,$$

if the density function is $\rho(x, y, z) = \frac{1}{x+1}$.

Question # 2: Surface Area

Calculate the surface area of the surface $4x^2 + 4y^2 + z^2 - 6z + 5 = 0$ oriented inward.

Question # 3: Tangent Plane to a Surface

Determine an equation for the tangent plane to the circular cylinder:

$$x^2 + (y-3)^2 = 9; \qquad 0 \leq z \leq 5,$$

at the point $\left(\frac{3\sqrt{3}}{2}, \frac{9}{2}, 0\right)$.

(Hint: Parametrize the surface first.)

Question # 4: Line & Surface Integrals of Vector Fields

1) For constants a, b, c, and e consider the vector field:

$$\mathbf{F} = \langle ax + by + 5z, x + cz, 3y + e\,x \rangle .$$

(a) Suppose that the flux of $\mathbf{F}$ through any closed surface is 0. What does this tell us about the value of the constants a, b, c, and e?

(b) Suppose instead that the line integral of $\mathbf{F}$ around any closed curve is 0. What does this tell us about the values of the constants a, b, c, and e?

2) Let S be the boundary surface of the solid given by $0 \leq z \leq \sqrt{4 - y^2}$ and $0 \leq x \leq \pi/2$. Determine the outward unit normal vector field on each of the four sides of S.

Question # 5: Surface Integral of Scalar Field

Use the divergence theorem to calculate the outward flux of the field:

$$\mathbf{F}(x, y, z) = \langle z^2x, y^3/3 \ + \tan z\,, x^2z + y^2 \rangle,$$

through the surface S where S is surface $z = \sqrt{1 - x^2 - y^2}; z > 0$ oriented upward.

Paper Pattern & Syllabus for OHT - II

Paper Pattern

Total Marks: 30		
Q - 1: CLO-2 [$5 \times 2 = 10$ marks]	True/False with Justification
Q - 2: CLO-2 [11 marks]	1 Long question
Q – 3: CLO-2 [9 marks]	1 Long question

Syllabus

Lectures: 23, 24, 25, 26, 27, 28 (till Stokes’ theorem)