DISPLACEMENT CURRENT

Displacement Current

- Maxwell's curl equation for static EM fields is:

$$\nabla \times \mathbf{H} = \mathbf{J}$$

- But the divergence of the curl of any vector field is identically zero, hence:

$$\nabla \cdot (\nabla \times \mathbf{H}) = 0 = \nabla \cdot \mathbf{J}$$

- The continuity of current equation, however, requires that:

$$\nabla \cdot \mathbf{J} = -\frac{\partial \rho_v}{\partial t} \neq 0$$

- Thus the above equations are obviously incompatible for time-varying conditions

- We must modify Maxwell's curl equation to agree with the continuity equation

Displacement Current

- To do this, we add a term to Maxwell's curl equation so that it becomes:

$$\nabla \times \mathbf{H} = \mathbf{J} + \mathbf{J}_d$$

- where $\mathbf{J}_d$ is to be determined and defined

- Again, the divergence of the curl of any vector is zero, hence:

$$\nabla \cdot (\nabla \times \mathbf{H}) = 0 = \nabla \cdot \mathbf{J} + \nabla \cdot \mathbf{J}_d$$

- In order for the above equation to agree with the continuity equation:

$$\nabla \cdot \mathbf{J}_d = -\nabla \cdot \mathbf{J} = \frac{\partial \rho_v}{\partial t} = \frac{\partial}{\partial t}(\nabla \cdot \mathbf{D}) = \nabla \cdot \frac{\partial \mathbf{D}}{\partial t}$$

- Or:

$$\mathbf{J}_d = \frac{\partial \mathbf{D}}{dt}$$

Displacement Current

- Substituting $\mathbf{J}_d$ into Maxwell's curl equation, we get:

$$\nabla \times \mathbf{H} = \mathbf{J} + \frac{\partial \mathbf{D}}{\partial t}$$

- This is Maxwell's equation (based on Ampere's circuit law) for a time-varying field

- The term $\mathbf{J}_d$ is known as *displacement current density* and **J** is the conduction current density

- The insertion of $\mathbf{J}_d$ into Maxwell's curl equations was one of the major contributions of Maxwell

- Without the term $\mathbf{J}_d$, electromagnetic wave propagation (radio or TV waves, for example) would be impossible

Displacement Current

- At low frequencies, $\mathbf{J}_d$ is usually neglected compared with $\mathbf{J}$, however, at radio frequencies, the two terms are comparable

- At the time of Maxwell, high-frequency sources were not available and the curl equation could not be verified experimentally

- It was years later that Hertz succeeded in generating and detecting radio waves thereby verifying the curl equation

- This is one of the rare situations where mathematical argument paved the way for experimental investigation

Displacement Current

- Based on the displacement current density, we define the *displacement current* as:

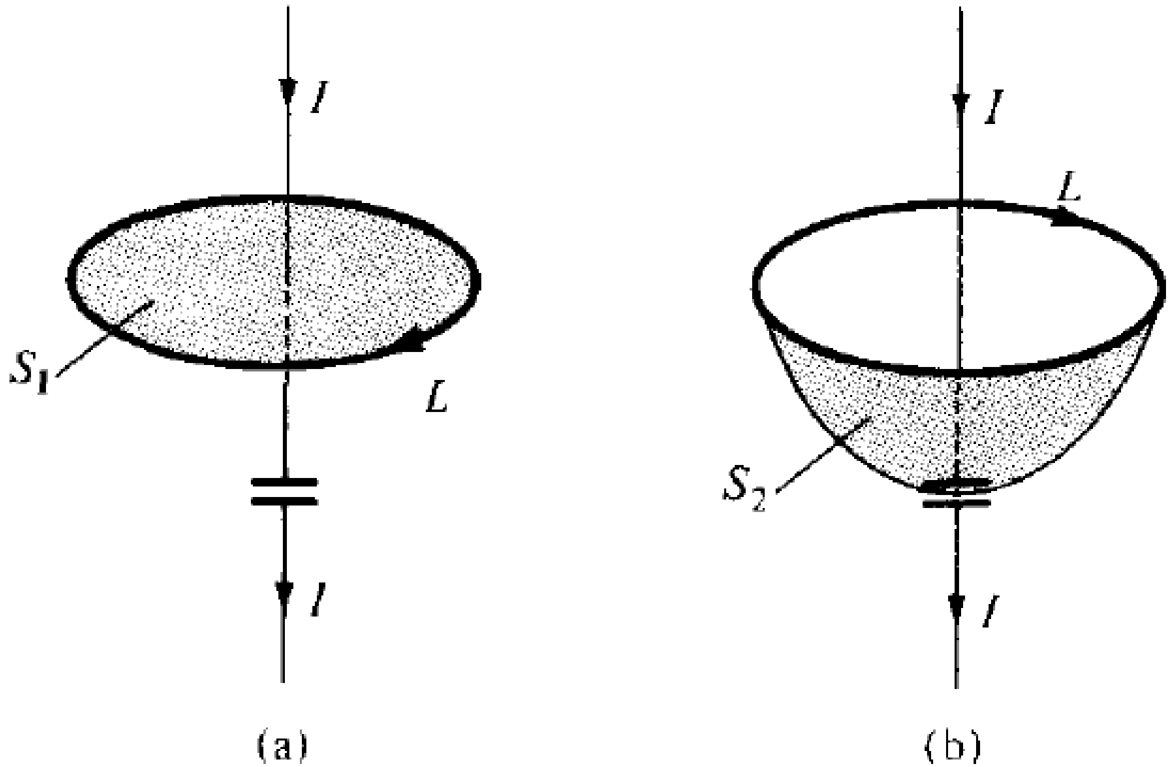
$$I_d = \int \mathbf{J}_d \cdot d\mathbf{S} = \int \frac{\partial \mathbf{D}}{\partial t} \cdot d\mathbf{S}$$

- It must be kept in mind that displacement current is a result of time-varying electric field

- A typical example of such current is the current through a capacitor when an alternating voltage source is applied to its plates

Displacement Current

- Consider the example of current through a capacitor illustrated in figure:



- We apply Ampere's law to the two different surfaces shown in the figure

Displacement Current

- Applying an unmodified form of Ampere's circuit law to a closed path *L* shown in figure (a) gives:

$$\oint_L \mathbf{H} \cdot d\mathbf{l} = \int_{S_1} \mathbf{J} \cdot d\mathbf{S} = I_{enc} = I$$

- If we use the balloon-shaped surface S_2 that passes between the capacitor plates, as in figure (b):

$$\oint_L \mathbf{H} \cdot d\mathbf{l} = \int_{S_2} \mathbf{J} \cdot d\mathbf{S} = I_{enc} = 0$$

- Because no conduction current (J = 0) flows through S_2

- This is contradictory because the same closed path *L* is used

Displacement Current

- To resolve the conflict, we need to include the displacement current in Ampere's circuit law

- The total current density is $\mathbf{J} + \mathbf{J}_d$

- In the first case, $\mathbf{J}_d = 0$ so that the equation remains valid

- While in the second case, $\mathbf{J} = 0$ so that:

$$\oint_L \mathbf{H} \cdot d\mathbf{l} = \int_{S_2} \mathbf{J}_d \cdot d\mathbf{S} = \frac{d}{dt} \int_{S_2} \mathbf{D} \cdot d\mathbf{S} = \frac{dQ}{dt} = I$$

- So we obtain the same current for either surface, though it is conduction current in S_1 and displacement current in S_2

Maxwell's Equations

- For a field to be "qualified" as an electromagnetic field, it must satisfy all four Maxwell's equations

Differential Form	Integral Form	Remarks
$\nabla \cdot \mathbf{D} = \rho_v$	$\oint_S \mathbf{D} \cdot d\mathbf{S} = \int_v \rho_v \, dv$	Gauss's law
$\nabla \cdot \mathbf{B} = 0$	$\oint_S \mathbf{B} \cdot d\mathbf{S} = 0$	Nonexistence of isolated magnetic charge*
$\nabla \times \mathbf{E} = -\dfrac{\partial \mathbf{B}}{\partial t}$	$\oint_L \mathbf{E} \cdot d\mathbf{l} = -\dfrac{\partial}{\partial t} \int_S \mathbf{B} \cdot d\mathbf{S}$	Faraday's law
$\nabla \times \mathbf{H} = \mathbf{J} + \dfrac{\partial \mathbf{D}}{\partial t}$	$\oint_L \mathbf{H} \cdot d\mathbf{l} = \int_S \left(\mathbf{J} + \dfrac{\partial \mathbf{D}}{\partial t} \right) \cdot d\mathbf{S}$	Ampere's circuit law

*This is also referred to as Gauss's law for magnetic fields.

Problem-1

- In free space, $\mathbf{E} = 20\cos(\omega t - 50x)\mathbf{a_y}$ V/m. Calculate
- (a) $\mathbf{J}_d$
- (b) $\mathbf{H}$
- (c) ω