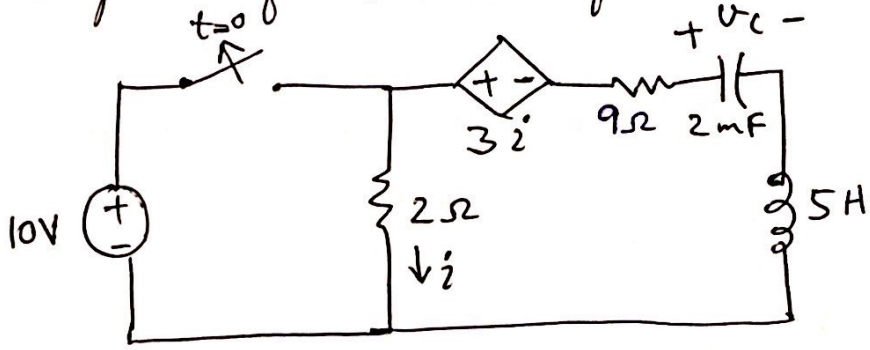
Series

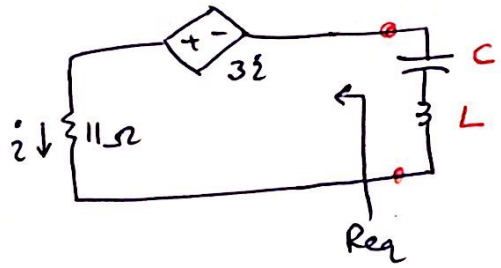
Example: 9.8 The Source-Free RLC Circuit: underdamped

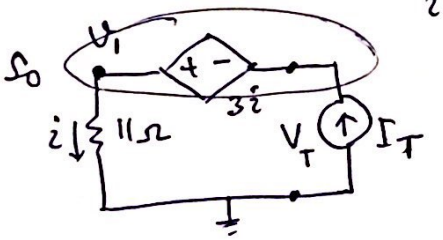
(pp 349 8th Ed Hkd)

Find an expression for $v_c(t)$ valid for $t > 0$.



Solution: At $t > 0$





$$\frac{v_1}{11} - I_T = 0$$

So $v_1 = 11 I_T$

$v_1 - v_T = 3i = +3I_T$

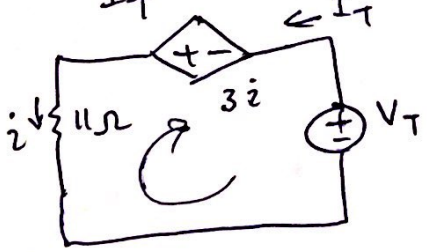
$11 I_T - V_T = 3 I_T$

$V_T = 8 I_T$

$\frac{V_T}{I_T} = 8\ \Omega$

Hence $R_{eq} = 8\ \Omega$

or



$-11i + 3i + V_T = 0$

$-11 I_T + 3 I_T + V_T = 0$

$V_T = 8 I_T$

$\frac{V_T}{I_T} = 8\ \Omega$

contd

— contd (349) — "Example 9.8"

So $\alpha = \frac{8}{2\times 5} = 0.8$ (exponential damping coefficient)

and $\omega_0 = \frac{1}{\sqrt{5\times 2\times 10^{-3}}} = 10$ (resonant frequency)

— As $\alpha < \omega_0$ so it is an underdamped case.

Hence $v_c(t) = e^{-\alpha t}(A_1 \cos \omega_d t + A_2 \sin \omega_d t)$

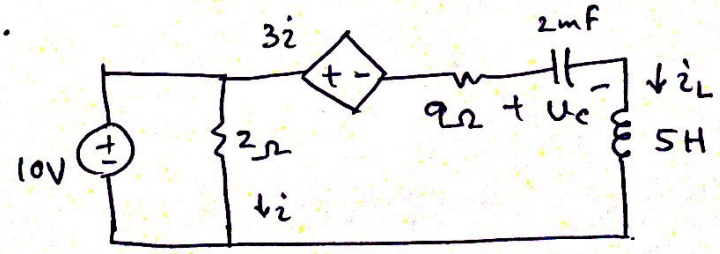
where $\alpha = 0.8$

and $\omega_d = \sqrt{\omega_0^2 - \alpha^2} = \sqrt{100 - 0.64}$

$\omega_d = 9.968$

— So $v_c(t) = e^{-0.8t}(A_1 \cos 9.968t + A_2 \sin 9.968t)$ — (A)

— To determine A_1 and A_2, we find $v_c(t)$ at $t = 0^-$.



So $i(0^-) = \frac{10}{2} = 5$ A and $i_L(0^-) = 0$ (because of capacitor)

Therefore $v_c(0^-) = -3i + 10 = -3\times 5 + 10$

$v_c(0^-) = -5$ V

So $v_c(0^+) = -5$ V

contd.

— contd (349)

Substituting in (A)

$$v_c(0) = -5 = 1(A_1 \times 1 + A_2 \times 0)$$

$$\text{So } A_1 = -5$$

— Hence (A) becomes

$$v_c(t) = e^{-0.8t}(-5\cos 9.968t + A_2 \sin 9.968t)$$

— Taking the derivative & evaluating at $t=0$ yields.

$$\left.\frac{dv_c}{dt}\right|_{t=0} = 4 + 9.968 A_2 \qquad \text{(B)}$$

— Now $i = -C\frac{dv_c}{dt} = 0$ because $i_L(0^-) = 0$

Putting in (B)

$$0 = 4 + 9.968 A_2$$

$$\text{So } A_2 = -\frac{4}{9.968} = -0.4013 \text{ V}$$

— Therefore $v_c(t) = -e^{-0.8t}(5\cos 9.968t + 0.4013 \sin 9.968t)$, V

for $t > 0$.

— Pspice yields