2. MOMENT OF INERTIA OF A FLYWHEEL

Theory:

A flywheel is comparatively big size wheel with its mass concentrated mostly in the rim. In order to determine its moment of inertia, the wheel is either set up against a wal where it moves round a horizontal axis or is fixed in a frame so that it may rotate aroun a vertical axis. A string, whose one end is fixed to small peg on its axis, is wrapped roun the axle and carries a weight at its other end. The weight Mg, falls through vertical heigh h and loses potential energy Mgh. This energy is spent

(i) in producing K.E. in the flywheel

(ii) in overcoming friction at the axis

(iii) in generating K.E. in the falling weight.

If the wheel makes n revolutions (after the height has been detached) before comin to rest and takes a time t for the purpose, the average angular velocity is $\frac{2\pi n}{t}$. Now th motion of the wheel is uniformly retarded by the frictional force at the axle and as th final velocity is zero. It's initial angular velocity should be $\omega = \frac{4\pi n}{t}$.

The kinetic energy of the wheel is $\frac{1}{2} I\omega^2$ and is dissipated in n rotations of the whee The energy lost per rotation in overcoming friction is $\frac{1}{2}\frac{I\omega^2}{n}$. If at the start of motion, tl string is wrapped n_1 times round the axle, the potential energy of the falling weight us up in overcoming friction is $n_1 \frac{\left(\frac{1}{2} I\omega^2\right)}{n}$. Also if v is the velocity of the falling weight at tl moment it leaves the peg, its kinetic energy is $\frac{1}{2} Mv^2$.

Hence, $$Mgh = \frac{1}{2} I\,\omega^2 + \frac{n_1}{n}\frac{1}{2} I\,\omega^2 + \frac{1}{2} Mv^2$$

Since $v = r\omega$, where r = radius of axle

$$Mgh = \frac{1}{2} I\,\omega^2 \left(1 + \frac{n_1}{n}\right) + \frac{1}{2} Mr^2\omega^2 .$$

or $$I = \frac{2\,Mgh}{\omega^2\left(1 + \frac{n_1}{n}\right)} - \frac{Mr^2}{\left(1 + \frac{n_1}{n}\right)}$$

EXPERIMENT 2

To determine the moment of inertia of a flywheel.

Apparatus:

Flywheel, two weights of about 300 and 200 gm. Cotton string, stop watch, set square, meter rod and vernier calipers'.

Method:

(1) Make a small loop at one end of the string and put it round the peg P, on the scale of the flywheel. (Fig.) Tie a weight Mg at its other end. Rotate the wheel with the hand and wrap the string round the axle.

(2) When the weight is at A (a little below the rim) put a set-square under it and make a mark A′ on the wall.

(3) Now, let the weight descend, the string will get unwrapped as the wheel turns. The length of the string is so adjusted that when the weight rests on a wooden block W, resting on the ground, the string is just tight and is on the verge of slipping off the peg P. A mark B′ is made at the level of B on the wall. Thus we know that when the weight will be let fall from A, it will fall through a height h = A′B′ before getting detached from the peg. Also count the rotation n_1, made by the wheel while the weight falls from A to B. This is facilitated by observing H, made on the rim, as the wheel rotates.

(4) The thread is wound up again so that the weight is at A, the block of the wood is removed and the weight is allowed to fall. As soon as the weight goes off the peg, start a stop watch. Count the number n, of rotations made by the wheel before coming to rest (starting from the moment M was detached from P) and note the time taken for this purpose.

(5) Measure the diameter of the axle at two mutually perpendicular directions and determine the mean radius.

(6) Put the weight of 200 gm. on the first weight M (= 300 gm) and repeat the experiment.

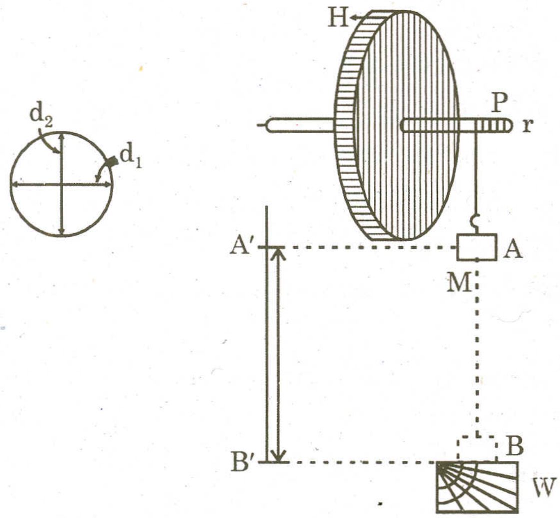


Fig. 1

Observations and Calculations:

(1) Vernier constant of the calipers = cm

(2) Diameter of the axle $= \dfrac{d_1 + d_2}{2} = d$ cm

(3) Mean Radius of the axis $= \dfrac{d}{2} = r$ cm

Rotation	No	Mass gm	Height cm	n_1	n	t	I gm cm^2
	1	300					
	2	300					
				Average	n	, t	= sec
	3	500					
	4	500		Average	n	= t	= sec

Mean = I =

Estimation of Error:

It is quite evident from the measurements of different quantities the main contributor of error is n_1 which can only be measured 1 part in 10 i.e., it introduces 10% error. On applying the theory of errors, total error introduced in the measurement of I is given by the following formula.

$$dI = \sqrt{\frac{2\,M^2g^2h^2}{w^4}\left(-\frac{1}{n}\right)^2 + M^2r^4\left(-\frac{1}{n}\right)^2}$$

$$= \frac{1}{n}\sqrt{\frac{2\,M^2g^2h^2}{w^4} + M^2r^4}$$

$\dfrac{dI}{I}$ % can be calculated easily.

Sources of Errors & Precautions:

(1) In determining the height, h, the position of the bottom of the weight to be marked on the wall in the two cases i.e., where the weight is at A and when at B.

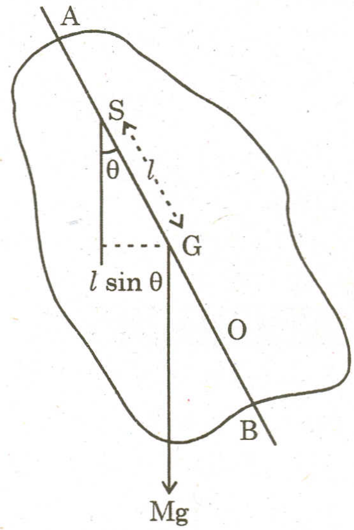
(2) While adjusting the length of the string, see that when the weight is resting on the block W, the string is just tight and is on the point of slipping of the peg.

(3) The timing and counting of rotations should commence from the instant the weight goes off the peg.

(4) If the wheel makes less than 100 rotations before stopping, there is considerable friction and the axel should be oiled.

(5) The diameter of the axle should be measured in two mutually perpendicular directions.

3. STUDY OF COMPOUND PENDULUM



Theory:

A rigid body, capable of vibrating about a horizontal axis passing through it, is called a compound pendulum. The mass of simple pendulum is concentrated in its bob but the mass of a compound pendulum is distributed over its whole body.

Consider a rigid body AB of mass M free to vibrate about a horizontal axis through S. S is called the centre of suspension. G is the position of center of gravity of the body such that SG = l. Let the body be displaced through a small angle 'θ'. The torque acting upon the body de to its weight Mg is

$$\tau = Mg \times l \sin\theta$$

Since 'θ' is very small, Sin θ = θ and $\tau = Mgl\theta$

This torque produces an angular acceleration 'α' in the body. If I is the moment of inertia of the body about S, then

$$\tau = I\alpha$$

$$\therefore \quad I\alpha = -Mgl\theta \quad \text{or} \quad \alpha = -\frac{Mgl}{I}\theta \qquad \text{...... (1)}$$

Negative sign indicates that α acts in such a way as to diminish θ.

If k is the radius of gyration of the body, its moment of inertia about G is Mk^2.

Thus the moment of inertia of the body S is given by

$$I = Mk^2 + Ml^2 = M(k^2 + l^2)$$

Substituting this value of I in (i) we get

$$\alpha = \frac{Mgl}{M(k^2 + l^2)}\theta = -\frac{gl}{(k^2 + l^2)}\theta$$

i.e., the angular acceleration is proportional to the angular displacement (θ). The motion of the body is therefore simple harmonic and its time-period is

$$T = \frac{2\pi}{\sqrt{\frac{gl}{k^2 + l^2}}} = 2\pi\sqrt{\frac{k^2 + l^2}{gl}} = 2\pi\sqrt{\frac{\frac{k^2}{l} + l}{g}}$$

If we put $\frac{k^2}{l} + l = L$, then $T = 2\pi\sqrt{\frac{L}{g}}$ which shows that the time-period of a rigid body is the same as that of a simple pendulum of length L. This is known as the length of the equivalent simple pendulum.

If we measure a length $GO = \frac{k^2}{l}$ along SG produced, then O is called the centre of oscillation. The points S and O are interchangeable, i.e., the time period is the same whether the body oscillates about S or O.

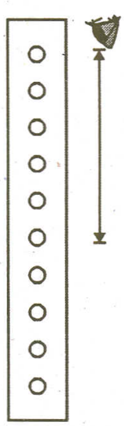
The expression $L = \frac{k^2}{l} + l$ may be written as a quadratic in l, thus: $l^2 + lL + k^2 = 0$. This gives two values of l, say l_1 and l_2, for which the body has equal times of vibration. From the theory of quadratic equations, $l_1 + l_2 = L$ and $l_1 l_2 = k^2$. In Fig. (ii) given below $l_1 = AC$ and $l_2 = CD$ therefore the radius of gyration $k = \sqrt{l_1 l_2}$ and moment of inertia of the body about its centre of gravity $I = Mk^2$ can be determined.

EXPERIMENT 3

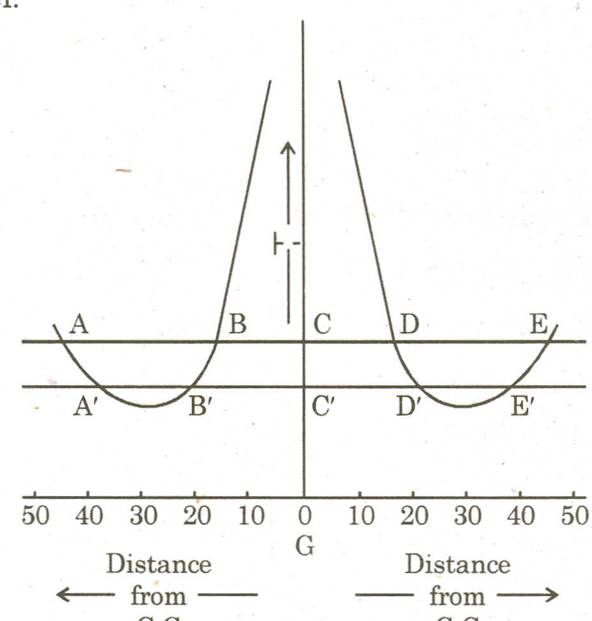
To determine the value of 'g' (acceleration due to gravity) by a compound pendulum (bar with holes) and calculate its radius of gyration and moment of inertia.

Apparatus:

Compound pendulum with two knife edges, support, sharp wedge, beam compass, telescope, stop watch, spirit level.



(i)



(ii)

Method:

(1) Balance the pendulum on the wedge, with knife-edges at the extreme holes on both sides, and mark the position of its centre of gravity at G.

(2) Level the support and suspend the pendulum on it. Draw a vertical line with a chalk along the holes of the pendulum or paste a thin white strip of paper but not covering its holes.

(3) Focus the eye-piece of the telescope till the cross-wires are distinctly visible. Now focus the telescope on the pendulum from a distance of about 3 metres such that the vertical cross-wire coincides with the line on the pendulum.

(4) Displace the pendulum slightly (not more than 5°) and release it to vibrate.

(5) Note the time for 50 vibrations and also measure the length from G up to the bottom of the first hole i.e., upto the axis about which the pendulum vibrates.

(6) Now shift the upper and lower knife-edges to the 2nd hole on both sides to keep the C.F. in its initial position and find out the time for 50 vibrations. Measure the distance form G to the bottom of the second hole.

(7) Repeat the process to note the time of 50 vibrations for each hole on both sides leaving one or two holes near G. Also measure the corresponding distances form G in every case.

(8) Calculate the time period T in each case.

(9) Take T along y-axis drawn in the middle of the graph paper and distance form G along x-axis on a large and suitable scale and plot a graph getting two smooth and mean curves on both sides of G (Fig. ii).

(10) Draw a line ABCDE parallel to the x- axis but not near the bends of the curve. The length of the equivalent simple pendulum, $L = l + \frac{k^2}{l}$ = AD or BE, i.e., AC = l and CD $= \frac{k^2}{l}$, D being the centre of oscillation. Similarly CE = l and BC $= \frac{k^2}{l}$, B being the centre of oscillation. Calculate 'g' using the relation $g = 4\pi^2 \frac{L}{T}$, where T is the value of time period at C.

(11) Draw another line A′B′C′D′E′ and get another value of 'g'.

(12) Weigh the pendulum and find out its mass M. From above readings, calculate $k^2 = AC$.

$$CD = BC.CE \quad \text{or} \quad k^2 = A'C'.\,C'D' = B'C'.C'E'$$

and
$$k = \frac{\sqrt{AC.CD} + \sqrt{BC.CE}}{2} \quad \text{or} \quad k = \frac{\sqrt{A'C'.C'D'} + \sqrt{B'C'.C'E'}}{2}.$$

Find out mean k.

(13) Find out moment of inertia of the pendulum using the relation $I = Mk^2$.

Observations and Calculations:

For the holes above G						For the holes below G					
Hole No.	Time for 50 vibrations			Time period	Distance from G	Hole No.	Time for 50 vibrations			Time period	Distance from G
	1	2	Mean t	$T = \frac{t}{50}$			1	2	Mean t	$T = \frac{t}{50}$	
1.	Sec.	Sec.	Sec.	Sec.	cm.	1.	Sec.	Sec.	Sec.	Sec.	cm.
2.						2.					
3.						3.					
4.						4.					

Value of g:

(i) Length of equivalent simple pendulum = $L = \frac{AD + DE}{2}$ = ... cm

From Graph:

Value of Time period at C = T = ... Sec.

$$g = 2\pi^2 \frac{L}{T^2} = \text{......} \ \text{cm/Sec}^2$$

(ii) Length of equivalent simple pendulum = $L = \frac{A'D' + B'E'}{2}$

value of time period at C' = T = Sec.

$$\therefore \quad g = 4\pi^2 \frac{L}{T^2} = \text{......} \ \text{cm/Sec}^2$$

Mean value of g = ... cm/Sec^2

(OR)

No.	L = AD + DE	T	T^2
1.			
2.			
3.			
4.			

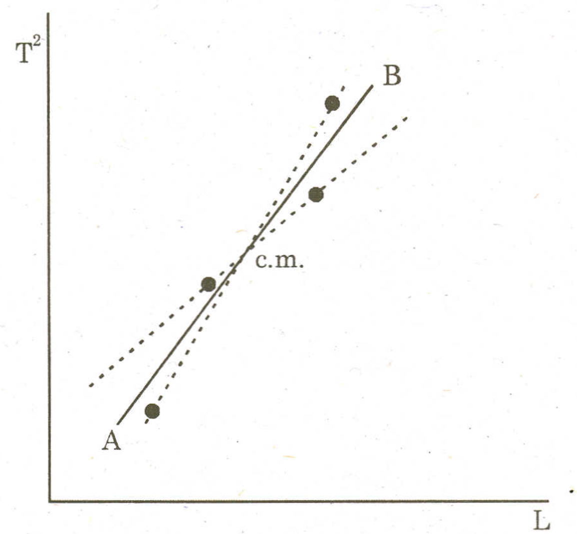


Fig. (iii).

A graph between L and T^2 may be plotted as shown in fig (iii), and the slope m of line AB = $\frac{4\pi^2}{g}$ and the error in m can be estimated from the other two lines.

Radius of Gyration k and Moment of Inertia I:

Mass of the pendulum = M = ... gm

From graph:

(i) $$k = \frac{\sqrt{AC.CD} + \sqrt{BC.CE}}{2} = \text{......} \ \text{cm}$$

$$I = Mk^2 = \text{......} \ \text{gm.cm}^2$$

(ii) $$k = \frac{\sqrt{A'C'.C'D'} + \sqrt{B'C'.C'E'}}{2} = \ldots\ldots \text{ cm}$$

$$I = Mk^2 = \ldots\ldots\ldots \text{ gm.cm}^2$$

Mean value of k = ... cm

Mean value of I = ... gm cm^2 = ... kg m^2

Precautions:

(1) The support should be horizontal and knife-edges should be sharp (Not broken or damaged).

(2) Amplitude of vibration should be kept small.

(3) Both knife-edges should be shifted to the corresponding holes for each observation to keep the position of C.G unchanged.

(4) Distances should always be measured from the C.G. to the lower edge of each hole.

(5) Times should be noted with the help of a good stop watch.

(6) A vibration should be counted wherever the reference line on the pendulum crosses the vertical cross-wire of the telescope.

(7) Take suitable and large scales and draw smooth and symmetrical curves on the graph.

Viva Voce:

Q.1 Define a compound pendulum.

Ans. It is a rigid body capable of vibrating abut a horizontal axis passing through it.

Q.2 Why is a compound pendulum preferred for the determination of 'g'?

Ans. Because it moves as one rigid body and we take into account the moment of inertia of the whole body.

Q.3 Define a simple pendulum.

Ans. It consists of a point mass suspended by a weightless, inextensible and flexible string from a fixed point about which the pendulum oscillates without friction. In practice, it is not possible to find a pendulum which may meet such ideal conditions.

Q.4 What is a second's pendulum?

Ans. A simple pendulum having a time period of two seconds.

Q.5 Define an equivalent simple pendulum.

Ans. A simple pendulum having the same time period as that of a compound pendulum is called equivalent simple pendulum.

Q.6 Why do you place the knife edges symmetrically?

Ans. The knife edges are kept symmetrical so that the centre of gravity does not shift.