MAGNETIC BOUNDARY CONDITIONS

Magnetic Boundary Conditions

- The conditions that **H** (or **B**) field must satisfy at the boundary between two different media

- Two laws, Gauss law for magnetic fields and Ampere circuit law are used for derivations

- Gauss law for magnetic fields is:

$$\oint \mathbf{B} \cdot d\mathbf{S} = 0$$

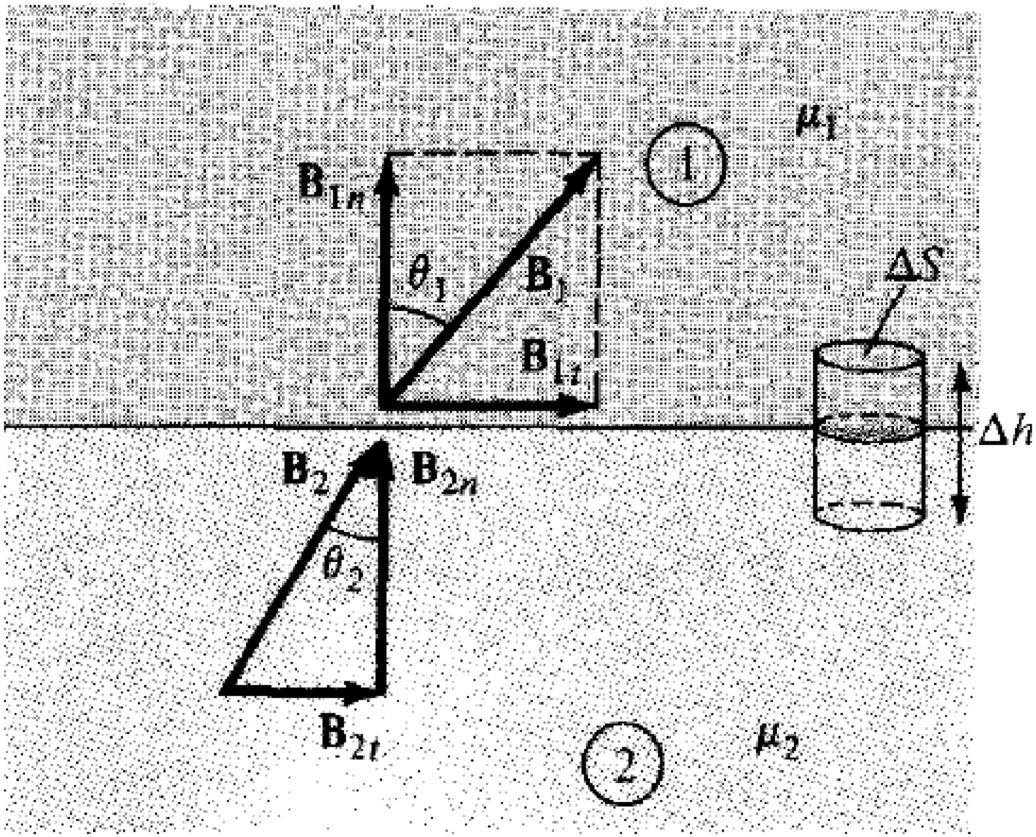
- Ampere's circuital law is:

$$\oint \mathbf{H} \cdot d\mathbf{l} = I$$

Magnetic Boundary Conditions

- Consider the boundary between two magnetic media 1 and 2, characterized, respectively, by μ_1 and μ_2 as in figure below:

- To apply the Gauss law, we choose the Gaussian surface shown:



Magnetic Boundary Conditions

- Applying the above equation to the Gaussian surface in the figure and allowing *Δh→ 0*, we get:

$$B_{1n}\,\Delta S - B_{2n}\,\Delta S = 0$$

- Therefore:

$$\boxed{\mathbf{B}_{1n} = \mathbf{B}_{2n}} \qquad \text{or} \qquad \mu_1\mathbf{H}_{1n} = \mu_2\mathbf{H}_{2n}$$

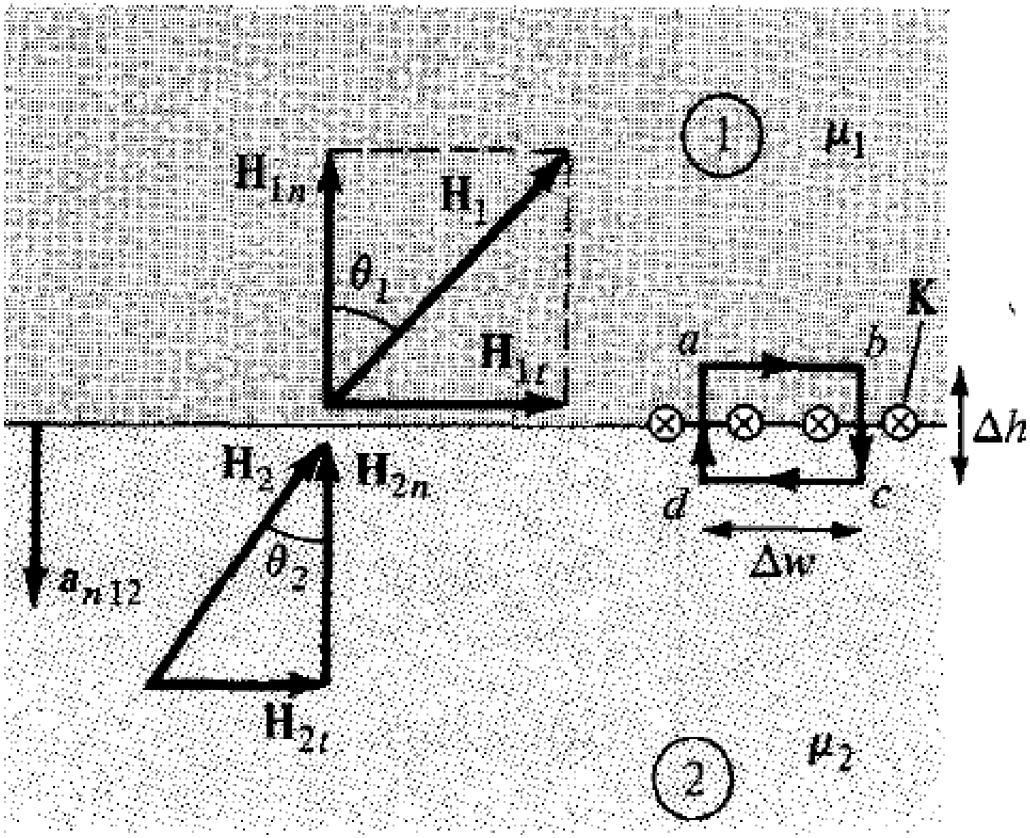
- So the normal component of **B** is continuous at the boundary

- Whereas the normal component of **H** is discontinuous at the boundary; **H** undergoes some change at the interface

Magnetic Boundary Conditions

- Consider the closed path abcda in figure below where surface current K on the boundary is assumed normal to the path

- We apply Ampere's law to determine the tangential components:



Magnetic Boundary Conditions

- For the closed path abcda in the figure, we get:

$$K \cdot \Delta w = H_{1t} \cdot \Delta w + H_{1n} \cdot \frac{\Delta h}{2} + H_{2n} \cdot \frac{\Delta h}{2} - H_{2t} \cdot \Delta w - H_{2n} \cdot \frac{\Delta h}{2} - H_{1n} \cdot \frac{\Delta h}{2}$$

- As *Δh→ 0*, we get:

$$H_{1t} - H_{2t} = K$$

- This shows that the tangential component of **H** is also discontinuous if there is current flow at the boundary

- The above equation maybe written in terms of **B** as:

$$\frac{B_{1t}}{\mu_1} - \frac{B_{2t}}{\mu_2} = K$$

Magnetic Boundary Conditions

- For a general case:

$$(\mathbf{H}_1 - \mathbf{H}_2) \times \mathbf{a}_{n12} = \mathbf{K}$$

- where $\mathbf{a}_{n12}$ is a unit vector normal to the interface and is directed from medium 1 to medium 2

- If the boundary is free of current or the media are not conductors (for K is free current density), K = 0 and we get:

$$\boxed{\mathbf{H}_{1t} = \mathbf{H}_{2t}} \quad \text{or} \quad \frac{\mathbf{B}_{1t}}{\mu_1} = \frac{\mathbf{B}_{2t}}{\mu_2}$$

- Thus the tangential component of **H** is continuous while that of **B** is discontinuous at the boundary

Magnetic Boundary Conditions

- If the fields make an angle θ with the normal to the interface, for the normal components, we get:

$$B_1 \cos\theta_1 = B_{1n} = B_{2n} = B_2 \cos\theta_2$$

- While for the tangential components, we get:

$$\frac{B_1}{\mu_1}\sin\theta_1 = H_{1t} = H_{2t} = \frac{B_2}{\mu_2}\sin\theta_2$$

- Dividing the above two equations, we get:

$$\frac{\tan\theta_1}{\tan\theta_2} = \frac{\mu_1}{\mu_2}$$

- This is the law of refraction for magnetic flux lines at a boundary with no surface current (Similar to that for electric flux lines)

Problem-1

- A unit normal vector from region 2 ($\mu = 2\mu_o$) to region 1 ($\mu = \mu_o$) is $\mathbf{a}_{n21} = (6\mathbf{a}_x + 2ay - 3\mathbf{a}_z)/7$. If $\mathbf{H}_1 = 10\mathbf{a}_x + \mathbf{a}_y + 12\mathbf{a}_z$ A/m and $\mathbf{H}_2 = H_{2x}\mathbf{a}_x - 5\mathbf{a}_y + 4\mathbf{a}_z$ A/m, determine:

(a) H_{2x}

(b) The surface current density **K** on the interface

(c) The angles $\mathbf{B}_1$ and $\mathbf{B}_2$ make with the normal to the interface.

Problem-2

- A current of 6A flows from $M(2, 0, 5)$ to $N(5, 0, 5)$ in a straight solid conductor in free space. An infinite current filament lies along the z axis and carries 50A in the $\mathbf{a}_z$ direction. Compute the vector torque on the wire segment with respect to an origin at (0, 0, 5).