Assignment # 4

Linear Algebra & ODE (MATH-121)

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Class: BEE 12C

Due: June 6th, 2021

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Question: Solve the following differential equations with all possible methods.

1. $(3xy - 2ay^2)dx + (x^2 - 2axy)dy = 0$

Method 1: Homogeneous Substitution

$$\Rightarrow \frac{dy}{dx} = - \frac{3xy - 2ay^2}{x^2 - 2axy}$$

- $y = vx \qquad \Rightarrow \quad y' = v + xv'$

$$\Rightarrow v + xv' = -\left(\frac{3vx^2 - 2av^2x^2}{x^2 - 2avx^2}\right)$$

$$\Rightarrow x v' = -\left(\frac{3v - 2v^2a}{1 - 2va}\right) - v$$

$$\Rightarrow v' = \left(\frac{2v^2a - 3v}{1 - 2va} - v\right)\frac{1}{x}$$

$$\Rightarrow v' = \left(\frac{2v^2a - 3v - v + 2v^2a}{1 - 2va}\right)\frac{1}{x}$$

$$\Rightarrow v' = \frac{1}{x}\left(\frac{4v^2a - 4v}{1 - 2va}\right)$$

$$\frac{dx}{x} = dv\left(\frac{4v(av-1)}{1-2va}\right)^{-1}$$

$$C + \ln(x) = \int\left(-\frac{1}{4v} - \frac{a}{4(av-1)}\right)dv$$

$$C + \ln(x) = -\frac{1}{4}\int \frac{1}{v} + \frac{a}{av-1}\, dv$$

$$C + \ln(x) = -\frac{1}{4}\left(\ln(v) + \ln(av-1)\right)$$

$$C - \ln(x)^{-4} = \ln(v(av-1))$$

$$\ln(v(av-1)) + \ln(x)^4 = C$$

$$v(av-1)x^4 = C$$

$$\rightarrow \quad v = \frac{y}{x}$$

$$\Rightarrow \quad \frac{y}{x}\left(a\frac{y}{x} - 1\right) x^{\cancel{4}3} = C$$

$$y\left(\frac{ay-x}{\cancel{x}}\right) x^{\cancel{3}2} = C$$

$$\boxed{x^3y - ax^2y^2 = C}$$

Method 2: Exact Method

$$\underset{M}{(3xy - 2ay^2)}dx + \underset{N}{(x^2 - 2axy)}dy = 0$$

$$\frac{\partial M}{\partial y} = 3x - 4ay = M_y$$

$$\frac{\partial N}{\partial x} = 2x - 2ay = N_x$$

Not exact,

$$u = \frac{M_y - N_x}{N} = \frac{3x - 4ay - 2x + 2ay}{x(x-2ay)}$$

$$u = \frac{\cancel{x-2ay}}{x\cancel{(x-2ay)}}$$

$$u = \frac{1}{x}$$

$I.F = e^{\int u(x)dx} = \exp(\ln(x)) = x$

Multiplying in Old Equation:

$(3x^2y - 2axy^2)dx + (x^3 - 2ax^2y)dy = 0$

Which is exact

Taking M, and integrating wrt x

$f(x,y) = x^3y - 2a\frac{x^2}{2}y^2 + h(y)$

$\quad " \quad = x^3y - ax^2y^2 + h(y)$

Differentiating wrt y

$\frac{\partial f}{\partial y} = \cancel{x^3} - 2a\cancel{x^2}y + h'y = \cancel{x^3} - 2a\cancel{x^2}y$

$h'(y) = 0$

$h(y) = c$

$f(x,y) = x^3y - ax^2y^2$

$f(x,y) = c$

$\boxed{x^3y - ax^2y^2 = c}$

Method 3: Homogeneous IF

$\Rightarrow \quad xM + yN = 3x^2y - 2axy^2 + x^2y - 2axy^2$

$1/u = 4x^2y - 4axy^2 \neq 0$

$u(x,y) = (4x^2y - 4axy^2)^{-1}$

Multiplying with this IF the original equation;

$$\Rightarrow \quad \frac{3xy-2ay^2}{4x^2y-4axy^2}\,dx + \frac{x^2-2axy}{4x^2y-4axy^2}\,dy = 0$$

$\Rightarrow$ Which is now exact;

- $\dfrac{\partial f}{\partial x} = \dfrac{3xy-2ay^2}{4x^2y-4axy^2} = \dfrac{3x-2ay}{4x^2-4axy}$
- $\dfrac{\partial f}{\partial y} = \dfrac{x^2-2axy}{4x^2y-4axy^2} = \dfrac{x-2ay}{4xy-4ay^2}$

Integrating wrt x (the first part),

$$\Rightarrow \frac{3x-2ay}{(x)(-ay+x)} \Rightarrow \frac{A}{x} + \frac{B}{x-ay}$$

$$A: \quad \left.\frac{3x-2ay}{(x-ay)}\right|_{x=0} \Rightarrow A = 2$$

$$B: \quad \left.\frac{3x-2ay}{x}\right|_{x=ay} \Rightarrow B = 1$$

$$f(x,y) = \frac{1}{4}\int \frac{2}{x} + \frac{1}{x-ay}\,dx$$

$$\text{〃} \quad = \frac{1}{4}\left[2\ln(x) + \ln(x-ay)\right]$$

$$\text{〃} \quad = \frac{1}{2}\ln(x) + \frac{1}{4}\ln(x-ay) + h(y)$$

Differentiating wrt y

$$\frac{\partial f}{\partial y} = \frac{1(-a)}{4(x-ay)} + h'(y) = \frac{x-2ay}{4xy-4ay^2}$$

$$\Rightarrow h'(y) = \frac{a}{4(x-ay)} + \boxed{\frac{x-2ay}{4y(x-ay)}}^{\star}$$

$$\star \quad \frac{x-2ay}{y(x-ay)} = \frac{A}{y} + \frac{B}{x-ay}$$

$$A: \quad \left.\frac{x-2ay}{x-ay}\right|_{y=0} \Rightarrow A = 1$$

$$B: \quad \left.\frac{x-2ay}{y}\right|_{y=\frac{x}{a}} \Rightarrow B = -a$$

$$\Rightarrow h'(y) = \cancel{\frac{a}{4(x-ay)}} + \frac{1}{4y} - \cancel{\frac{a}{4(x-ay)}}$$

$$\Rightarrow h'(y) = \frac{1}{4y}$$

$$h(y) = \frac{1}{4}\ln(y)$$

Standard form

$$f(x,y) = c$$

$$\Rightarrow \frac{1}{2}\ln(x) + \frac{1}{4}\ln(x-ay) + \frac{1}{4}\ln(y) = c_1$$

$$\Rightarrow 2\ln(x) + \ln(x-ay) + \ln(y) = 4c_1$$

$$\Rightarrow \ln(x)^2 + \ln(x-ay) + \ln(y) = 4c_1$$

$$\Rightarrow \ln((x^2)(x-ay)(y)) = 4c_1$$

$$\Rightarrow (x^2)(x-ay)(y) = e^{4c_1}$$

$$\Rightarrow x^2(xy - ay^2) = c_1 \qquad \therefore c_1 = e^{4c_1}$$

$$\Rightarrow \boxed{x^3y - ax^2y^2 = c_1} \qquad \therefore c_1 = e^{4c_1}$$

2. $(x^2y^2 + xy + 1)ydx + (x^2y^2 - xy + 1)xdy = 0$

Method 1: Integration Factor

$$y(x^2y^2+xy+1)\,dx + x(x^2y^2-xy+1)\,dy = 0$$

$$y \quad f(x,y) \quad dx \quad x \quad g(x,y) \quad dy$$

$$U = \frac{1}{xM - yN}$$

$$U = \frac{1}{\cancel{x^3y^3} + x^2y^2 + \cancel{xy} - \cancel{x^3y^3} + x^2y^2 - \cancel{xy}}$$

$$U = \frac{1}{2x^2y^2}$$

$$\Rightarrow \frac{\cancel{y}(x^2y^2+xy+1)}{2x^2y^{\cancel{2}}}\,dx + \frac{x(x^2y^2-xy+1)dy}{2x^2y^2} = 0$$

$$\Rightarrow \underbrace{\left(\frac{y}{2} + \frac{1}{2x} + \frac{1}{2x^2y}\right)}_{M} dx + \underbrace{\left(\frac{x}{2} - \frac{1}{2y} + \frac{1}{2xy^2}\right)}_{N} dy = 0$$

$$\frac{\partial f}{\partial x} = \frac{y}{2} + \frac{1}{2x} + \frac{1}{2x^2y}$$

$$\frac{\partial f}{\partial y} = \frac{x}{2} - \frac{1}{2y} + \frac{1}{2xy^2}$$

Integrating first with x

$$f(x,y) = \frac{yx}{2} + \frac{1}{2}\ln(x) - \frac{1}{2yx} + h(y)$$

Differentiating wrt y

$$\Rightarrow \frac{\partial f}{\partial y} = \cancel{\frac{x}{2}} + \cancel{\frac{1}{2y^2x}} + h'(y)$$

$$= \cancel{\frac{x}{2}} - \frac{1}{2y} + \cancel{\frac{1}{2y^2x}}$$

$$h'(y) = \int h'(y)\,dy = -\int \frac{1}{2y}\,dy$$

$$h(y) = -\frac{1}{2}\ln(y)$$

- $f(x,y) = c$

$$\Rightarrow \frac{xy}{2} + \frac{1}{2}\ln(x) - \frac{1}{2xy} - \frac{1}{2}\ln(y) = c$$

$$\Rightarrow xy + \ln(x) - \frac{1}{xy} - \ln(y) = 2c$$

$$\Rightarrow \boxed{xy + \ln(x) - \frac{1}{xy} - \ln(y) = c_1} \quad \therefore c_1 = 2c$$

Method 2: Substitution Method

Let $u = xy$

$y = \frac{u}{x} \Rightarrow dy = \frac{x\,du - u\,dx}{x^2}$

Substituting in main eq. (Only $u = xy$)

$x(u^2 - u + 1)dy + y(u^2 + u + 1)dx = 0$

Now using y and dy;

$$\Rightarrow \left[(u/x)(u^2+u+1) - (u/x)(u^2-u+1)\right]dx =$$

$$(-u^2 + u - 1)\,du$$

$$\Rightarrow \left(2\frac{u^2}{x}\right)dx = \left(-u^2 + u - 1\right)du$$

$$\Rightarrow \frac{1}{x}\,dx = \left[-\frac{1}{2} + \frac{1}{2u} - \frac{1}{2u^2}\right]du$$

$$\Rightarrow \ln(x) = -\frac{1}{2}u + \frac{1}{2}\ln(u) + \frac{1}{2u} + c$$

$\rightarrow u = xy$ Substitution

$$\Rightarrow \boxed{\ln(x) = \frac{1}{2xy} + \frac{1}{2}\ln(xy) - \frac{1}{2}xy + c}$$

3. $\left(1+e^{\frac{x}{y}}\right)dx+e^{\frac{x}{y}}\left(1-\frac{x}{y}\right)dy=0$

Method 1: Homogeneous Substitution

$$\frac{dy}{dx} = -\frac{(1+e^{x/y})}{e^{x/y}(1-x/y)}$$

Which is a homogeneous equation of 0^{th} order

$\because$ Since x is in the nominator in powers, fractions, etc. It would be better to use dx/dy.

$$\Rightarrow \frac{dx}{dy} = -\frac{e^{x/y}(1-x/y)}{(1+e^{x/y})}$$

Using $x = vy$

$\cdot\ x' = v'y + v$

$$\Rightarrow v'y + v = -\frac{e^v(1-v)}{1+e^v}$$

$$\Rightarrow v'y = \frac{-e^v(1-v)}{1+e^v} - v$$

$$\Rightarrow v'y = \frac{-e^v + \cancel{e^v(v)} - v - \cancel{e^v(v)}}{1+e^v}$$

$$\Rightarrow \frac{dv}{dy}y = \frac{-(v+e^v)}{1+e^v}$$

$$\Rightarrow \frac{1+e^v}{v+e^v}dv = -\frac{dy}{y}$$

$$\Rightarrow \int \frac{1+e^v}{v+e^v}\,dv = -\int \frac{1}{y}\,dy$$

$$\Rightarrow \ln(v+e^v) = -\ln(y) + c_1$$

$$\Rightarrow \ln(v+e^v) + \ln(y) = c_1$$

$$\Rightarrow \ln|(v+e^v)(y)| = c_1$$

$$\Rightarrow yv + ye^v = c \qquad \therefore e^{c_1} = c$$

Replacing with $v = y^{-1}x$

$$\Rightarrow \boxed{x + ye^{x/y} = c}$$

Method 2: Exact Method

$$\underbrace{(1+e^{x/y})}_{M}dx + \underbrace{e^{x/y}(1-x/y)}_{N}dy = 0$$

$$\frac{\partial M}{\partial y} = -e^{x/y}\frac{x}{y^2}$$

$$\frac{\partial N}{\partial x} = e^{x/y}(-1/y) + (1-x/y)e^{x/y}\frac{1}{y}$$

$$= \cancel{-\frac{1}{y}e^{x/y}} + \cancel{\frac{e^{x/y}}{y}} - e^{x/y}\frac{x}{y^2}$$

$$= -e^{x/y}\frac{x}{y^2}$$

$$\therefore \frac{\partial M}{\partial x} = \frac{\partial N}{\partial x} \quad (\text{Exact})$$

$$\frac{\partial f}{\partial x} = 1 + e^{x/y}$$

$$\frac{\partial f}{\partial y} = e^{x/y}(1 - x/y)$$

Integrating wrt x (1st)

$$f(x,y) = x + ye^{x/y} + h(y)$$

Differentiating wrt y

$$\frac{\partial f}{\partial y} = -e^{x/y}\frac{x}{y} + h'(y) + e^{x/y}$$

$$\Rightarrow -e^{x/y}\frac{x}{y} + h'(y) = e^{x/y}(1 - x/y) - e^{x/y}$$

$$\Rightarrow \cancel{-e^{x/y}\frac{x}{y}} + h'(y) = \cancel{e^{x/y}} \cancel{-e^{x/y}\frac{x}{y}} \cancel{-e^{x/y}}$$

$$\Rightarrow h'(y) = 0$$

$$\int h'(y)\,dy = \int 0 \; dy$$

$$h(y) = C_1$$

$$\Rightarrow x + ye^{x/y} + C_1 = C_2$$

$$\Rightarrow x + ye^{x/y} = C \qquad \therefore C_2 - C_1 = C$$

4. $(1+\theta^2)\frac{dy}{d\theta} = \tan^{-1}\theta - y$

Method 1: Linear Equation Method

$$(1+\theta^2)\, dy/d\theta = \tan^{-1}(\theta) - y$$

$$\frac{dy}{d\theta} + \frac{y}{(1+\theta^2)} = \frac{\tan^{-1}(\theta)}{(1+\theta^2)}$$

$$y' \qquad p(\theta)\,y \qquad q(\theta)$$

$$u = e^{\int p(\theta)\,d\theta}$$

$$u = \exp\left(\int \frac{1}{1+\theta^2}\,d\theta\right)$$

$$u = \exp\left(\tan^{-1}(\theta)\right)$$

Multiplying with old eq.

$$\Rightarrow \quad y\,e^{\tan^{-1}(\theta)} = \int e^{\tan^{-1}(\theta)} \tan^{-1}(\theta)/1+\theta^2\,d\theta$$

$$\Rightarrow \quad y\,e^{\tan^{-1}(\theta)} = \tan^{-1}(\theta)\int \frac{e^{\tan^{-1}\theta}}{\theta^2+1}\,d\theta - \int \frac{d}{d\theta}\tan^{-1}(\theta)\int \frac{e^{\tan^{-1}\theta}}{\theta^2+1}\,d\theta$$

$$\Rightarrow \quad y\,e^{\tan^{-1}\theta} = \tan^{-1}(\theta)\,e^{\tan^{-1}\theta} - \int \frac{1}{\theta^2+1}\left(e^{\tan^{-1}(\theta)}\right)d\theta$$

$$\Rightarrow \quad y\,e^{\tan^{-1}\theta} = \tan^{-1}(\theta)\,e^{\tan^{-1}\theta} - e^{\tan^{-1}\theta} + C$$

$$\Rightarrow \quad \boxed{y = \tan^{-1}(\theta) - 1 + \frac{C}{e^{\tan^{-1}(\theta)}}}$$

Method 2: Exact Method

$\Rightarrow (1+\theta^2)\,dy = (\tan^{-1}(\theta) - y)\,d\theta$

$\Rightarrow \underbrace{-(1+\theta^2)}_{N}\,dy + \underbrace{(\tan^{-1}(\theta) - y)}_{M}\,d\theta = 0$

$\Rightarrow \dfrac{\partial M}{\partial y} = -1$

$\Rightarrow \dfrac{\partial N}{\partial \theta} = -2\theta$

Hence, this isn't exact. We first find the IF to make it exact.

$\Rightarrow \dfrac{M_y - N_x}{N}$

$\Rightarrow \dfrac{-1+2\theta}{-(1+\theta^2)} \Rightarrow \dfrac{-(1-2\theta)}{-(1+\theta^2)}$

$\Rightarrow \dfrac{1-2\theta}{1+\theta^2} = 4$

- $IF = e^{\int 4\,d\theta}$

$\Rightarrow IF = \exp\left(\int \dfrac{1-2\theta}{1+\theta^2}\,d\theta\right)$

$= \exp\left(\int \dfrac{1}{1+\theta^2} - \dfrac{2\theta}{1+\theta^2}\,d\theta\right)$

$$= \exp\left(\tan^{-1}(\theta) - \ln(1+\theta^2)\right)$$

$$= \left(e^{\tan^{-1}(\theta)}\right) \cdot \left(e^{\ln(1+\theta^2)^{-1}}\right)$$

$$\boxed{IF = \frac{e^{\tan^{-1}(\theta)}}{1+\theta^2}}$$

Multiplying with old eq.

$$\Rightarrow \frac{(\tan^{-1}(\theta) - y)}{1+\theta^2}\left(e^{\tan^{-1}(\theta)}\right) d\theta - e^{\tan^{-1}(\theta)}\, dy = 0$$

- $$\frac{\partial f}{\partial \theta} = \frac{\tan^{-1}(\theta) - y}{1+\theta^2}\left(e^{\tan^{-1}\theta}\right) = M$$
- $$\frac{\partial f}{\partial y} = -e^{\tan^{-1}(\theta)} = N$$

Integrating N wrt y

$$f(x,y) = -e^{\tan^{-1}(\theta)} y + h(\theta)$$

Now, differentiating wrt θ

$$\frac{\partial f}{\partial \theta} = -\frac{e^{\tan^{-1}(\theta)}}{1+\theta^2} y + h'(\theta) = M$$

$$\Rightarrow h'(\theta) = \frac{e^{\tan^{-1}(\theta)}}{1+\theta^2}\left(\tan^{-1}(\theta) - \cancel{y} + \cancel{y}\right)$$

$$\Rightarrow h'(\theta) = \frac{e^{\tan^{-1}(\theta)}(\tan^{-1}(\theta)}{1+\theta^2}$$

Integrating wrt θ

$$\Rightarrow \quad h(\theta) = e^{\tan^{-1}(\theta)}\left(\tan^{-1}(\theta) - 1\right)$$

Substituting back;

$$F(x,y) = c$$

$$\Rightarrow \quad \boxed{e^{\tan^{-1}(\theta)}\left(\tan^{-1}(\theta) - y - 1\right) = c}$$

5. $(2xy + y - \tan y)dx + (x^2 - x\tan^2 y + \sec^2 y)dy = 0$

$$\underbrace{(2xy + y - \tan(y))}_{M}dx + \underbrace{(x^2 - x\tan^2(y) + \sec^2(y))}_{N}dy = 0$$

- $$\frac{\partial M}{\partial y} = 2x + 1 - \sec^2(y)$$
  $$= 2x - \tan^2(y)$$
- $$\frac{\partial N}{\partial x} = 2x - \tan^2(y)$$

Hence, the equation is exact

$$\therefore \frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$$

- $$\frac{\partial f}{\partial x} = 2xy + y - \tan(y)$$
- $$\frac{\partial f}{\partial y} = x^2 - x\tan^2(y) + \sec^2(y)$$

Integrating 1st wrt x

$$f(x,y) = \frac{\cancel{2}x^2}{\cancel{2}}y + yx - x\tan(y) + h(y)$$

Differentiating wrt y

$$\Rightarrow \frac{\partial f}{\partial y} = x^2 + x - x\sec^2(y) + h'(y)$$

Comparing with N

$$\Rightarrow \cancel{x^2} + x(1-\sec^2 y) + h'(y) = \cancel{x^2} - x\tan^2(y) + \sec^2(y)$$

$$\Rightarrow -x\cancel{\tan^2}(y) + h'(y) = -x\cancel{\tan^2}(y) + \sec^2(y)$$

$$\Rightarrow h'(y) = \sec^2(y)$$

Integrating wrt y

$$\Rightarrow h(y) = \tan(y)$$

Substituting it back;

$$\Rightarrow f(x,y) = c$$

$$\Rightarrow \boxed{x^2y + yx - x\tan(y) + \tan(y) = c}$$

6. $x\log x\frac{dy}{dx} + y = \log x^2$

Method 1: Linear Equation Method

$$x\log(x)\frac{dy}{dx} + y = \log(x)^2$$

$$\Rightarrow \frac{dy}{dx} + \frac{y}{x\log(x)} = \frac{2\cancel{\log(x)}}{x\cancel{\log(x)}}$$

$$\Rightarrow \frac{dy}{dx} + \frac{y}{x\log(x)} = \frac{2}{x}$$

y' $\quad p(x)\,y \quad q(x)$

$$\mu = \exp\left(\int p(x)\,dx\right)$$

$$= \exp\left(\int \frac{1}{x\log(x)}\,dx\right)$$

$$= \exp\left(\int \frac{1}{u}\,du\right)$$

$$= \exp(\ln(u))$$

$$= u$$

$\therefore \bar{u} = \ln(x)$

$d\bar{u} = \frac{1}{x}\,dx$

$$\mu = \ln(x)$$

Multiplying with old eq.

and writing linear equation solution

$$\Rightarrow y\ln(x) = \int \frac{2}{x}\ln(x)\,dx + C$$

$$y \ln(x) = 2\int \frac{1}{x} \ln(x)\, dx + C$$

$$y \ln(x) = \cancel{2} \frac{[\ln(x)]^2}{\cancel{2}} + C$$

$$\boxed{y = \ln(x) + C \operatorname{Log}^{-1}(x)}$$

Method 2: Exact Method

$$(x \log x)\, dy = (\operatorname{Log} x^2 - y)\, dx$$

$$\underbrace{x \ln(x)}_{N}\, dy - \underbrace{(\operatorname{Log} x^2 - y)}_{M}\, dx = 0$$

$$\frac{\partial M}{\partial y} = 1$$

$$\frac{\partial N}{\partial x} = 1 + \ln(x)$$

Turning into Exact from IF

$$u = \exp\left(\int \frac{M_y - N_x}{N}\, dx\right)$$

$$\Rightarrow \frac{M_y - N_x}{N} = \frac{\cancel{1} - \cancel{1} - \ln(x)}{x \ln(x)}$$

$$= -\frac{1}{x}$$

$$u = \exp\left(\int -\frac{1}{x}\, dx\right)$$

$u = \frac{1}{x}$, Multiplying with old eq.

$$\Rightarrow \underbrace{\ln(x)}_{N}\, dy - \underbrace{\frac{(\log x^2 - y)}{x}}_{M}\, dx = 0$$

$$\frac{\partial M}{\partial y} = \frac{1}{x}$$

$$\frac{\partial N}{\partial x} = \frac{1}{x}$$

Hence, they are now Exact.

$$\frac{\partial f}{\partial x} = \frac{y - 2\ln(x)}{x}$$

$$\frac{\partial f}{\partial y} = \ln(x)$$

Integrating 1st w.r.t x

$$f(x,y) = \int \frac{y}{x} - \frac{2\ln(x)}{x}\,dx$$

$$'' = y\ln(x) - \cancel{2}\,\frac{\ln(x^2)}{\cancel{2}} + h(y)$$

Differentiating w.r.t y,

$$\frac{\partial F}{\partial y} = \ln(x) + h'(y) = N$$

$$\Rightarrow \cancel{\ln(x)} + h'(y) = \cancel{\ln(x)}$$

$$\Rightarrow h'(y) = 0$$

$$\Rightarrow h(y) = c_1 \quad , \quad f(x,y) = c_2$$

$$\Rightarrow y\ln(x) - \ln(x^2) + c_1 = c_2$$

$$\Rightarrow \boxed{y\ln(x) - \ln(x^2) = c} \qquad \therefore c_2 - c_1 = c$$

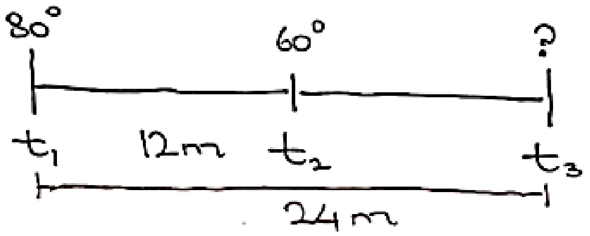
Question: If the air is maintained at 30°C and the temperature of the body cools from 80°C to 60°C in 12 minutes, find the temperature of the body after 24 minutes. (Clearly state all the conditions and solve it after modeling the problem)

$T_m = 30°C$

$T_1 = 80°C$

$T_2 = 60°C$

$t_1 - t_2 = 12$ minutes



• For k:

$$\Rightarrow \frac{T(t_1) - T_m}{T(t_2) - T_m} = e^{k(t_1 - t_2)}$$

$$\Rightarrow \frac{80 - 30}{60 - 30} = e^{k(12)}$$

$$\Rightarrow 5/3 = e^{12k}$$

$$\Rightarrow \ln(5/3) = 12k$$

$$\Rightarrow \boxed{k = 0.0425}$$

We can now find $T(t_3)$;

$$\Rightarrow \frac{T(t_1) - T_m}{T(t_3) - T_m} = e^{k(t_1 - t_3)}$$

$$\Rightarrow \quad \frac{80-30}{T(t_3)-30} = e^{k(24)}$$

$$\Rightarrow \quad \frac{50}{e^{(0.0425)(24)}} = T(t_3) - 30 \qquad \therefore k = 0.0425$$

$$\Rightarrow \quad 18.03 = T(t_3) - 30$$

$$\Rightarrow \quad \boxed{T(t_3) = 48.03^\circ C}$$