Electric Field-II

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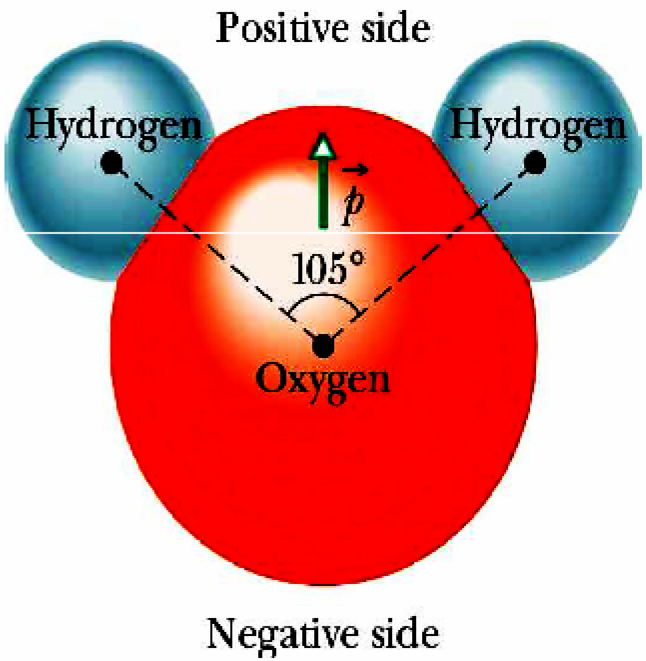
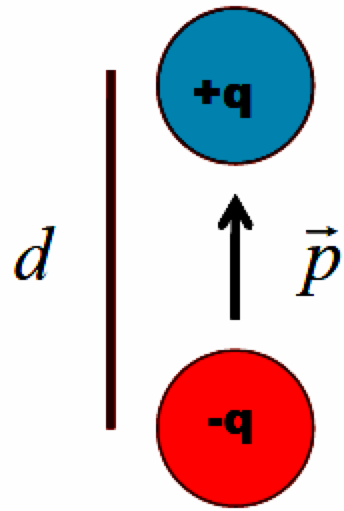
Electric Dipole

- A pair of equal and opposite charges q separated by displacement d is called an electric dipole.
- It has an electric dipole moment $p = qd$





The electric dipole is a good model of many molecules, such as H2O

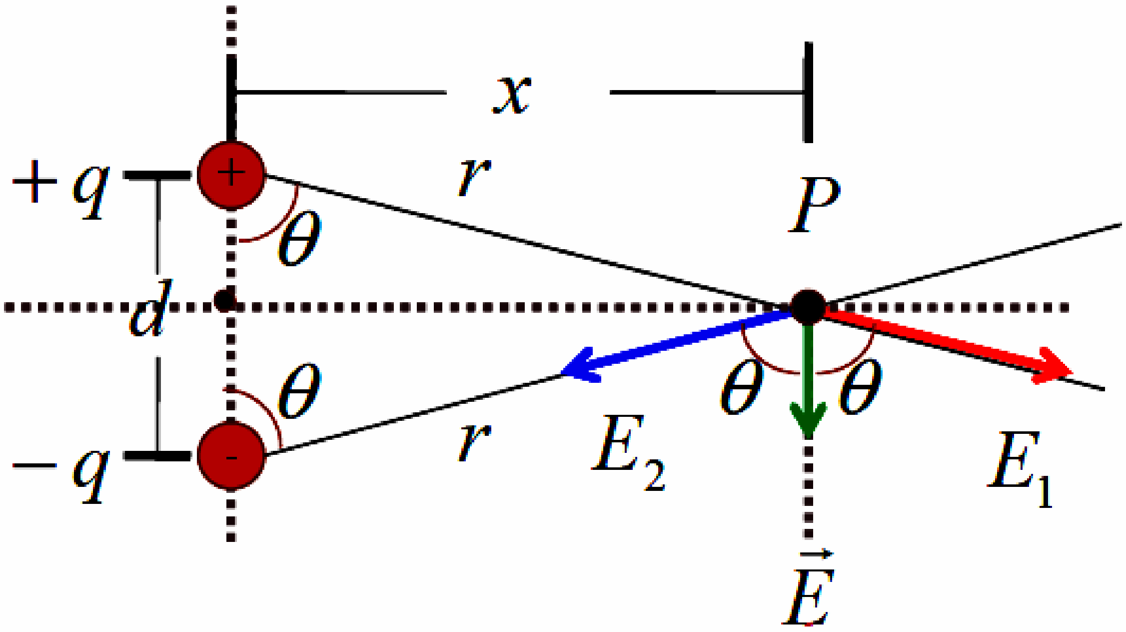
Neutral atoms and molecules behave as dipoles when placed in an external electric field (Polarization).

Electric Field of Dipole

Electric field at the perpendicular bisector

At P, the fields E1 and E2 due to the two charges +q and –q are equal in magnitude because P is equidistant from the charges.

$$E_1 = E_2 = \frac{kq}{r^2} = \frac{kq}{x^2 + (d/2)^2}$$



The x components of E1 and E2 cancel each other, and the y components add up because they are both in the negative y direction. Therefore, E is along negative y-axis and has a magnitude

$$E = 2E_1 \cos\theta$$

$$\cos\theta = \frac{d/2}{r} = \frac{d}{2\sqrt{x^2 + (d/2)^2}}$$

$$E = 2\frac{kq}{x^2 + (d/2)^2}\frac{d}{2\sqrt{x^2 + (d/2)^2}}$$

$$\boxed{E = \frac{kp}{\left(x^2 + (d/2)^2\right)^{3/2}}} \qquad \therefore p = qd$$

Binomial Expansion: if x<1 then,

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \dots\dots\dots$$

If x>>d, then,

$$E = kp\left(x^2 + (d/2)^2\right)^{-3/2}$$

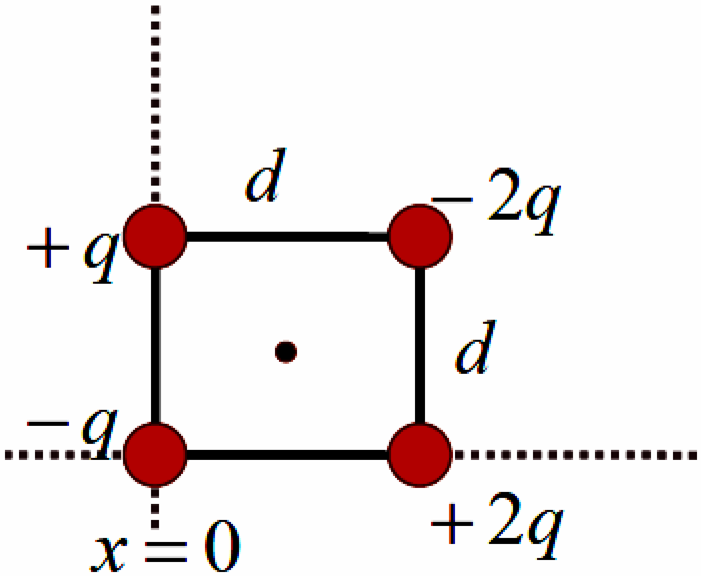
$$E = kpx^{-3}\left(1 + (d/2x)^2\right)^{-3/2}$$

$$= \frac{kp}{x^3}\left(1 - \frac{3}{2}\left(\frac{d}{2x}\right)^2 + \ldots\ldots\right) = \frac{kp}{x^3}$$

$$\boxed{\vec{E} = -\frac{kp}{x^3}\hat{j}}$$

Thus, we see that, at distances far from a dipole but along the perpendicular bisector of the line joining the two charges, the magnitude of the electric field created by the dipole varies as $1/r^3$, whereas the more slowly varying field of a point charge varies as $1/r^2$.

Find the electric field at the center of the square shown below, where q=11.8nC and d=5.2cm.



Electric field due to dipole 1; +q & -q, is

$$\vec{E}_1 = -\frac{kp}{(x^2+(d/2)^2)^{3/2}}\hat{j}$$

$$= -\frac{kqd}{((d/2)^2+(d/2)^2)^{3/2}}\hat{j}$$

$$\therefore x = d/2$$

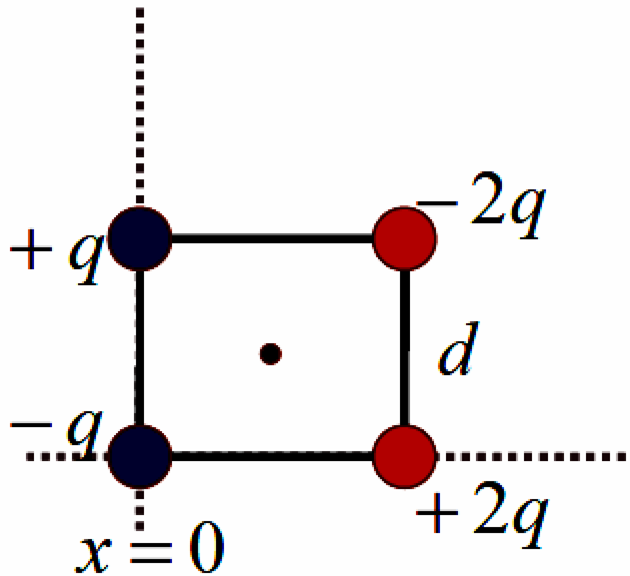
$$= -\frac{kqd}{(2(d/2)^2)^{3/2}}\hat{j}$$

Electric field due to dipole 2; +2q & -2q, is

$$\vec{E}_2 = \frac{kp}{(x^2+(d/2)^2)^{3/2}}\hat{j}$$

$$= \frac{2kqd}{((d/2)^2+(d/2)^2)^{3/2}}\hat{j}$$

$$= \frac{2kqd}{(2(d/2)^2)^{3/2}}\hat{j}$$



Net electric field at the center of square will be

$$\vec{E} = \vec{E}_1 + \vec{E}_2$$

$$= -\frac{kqd}{(2(d/2)^2)^{3/2}}\hat{j} + \frac{2kqd}{(2(d/2)^2)^{3/2}}\hat{j}$$

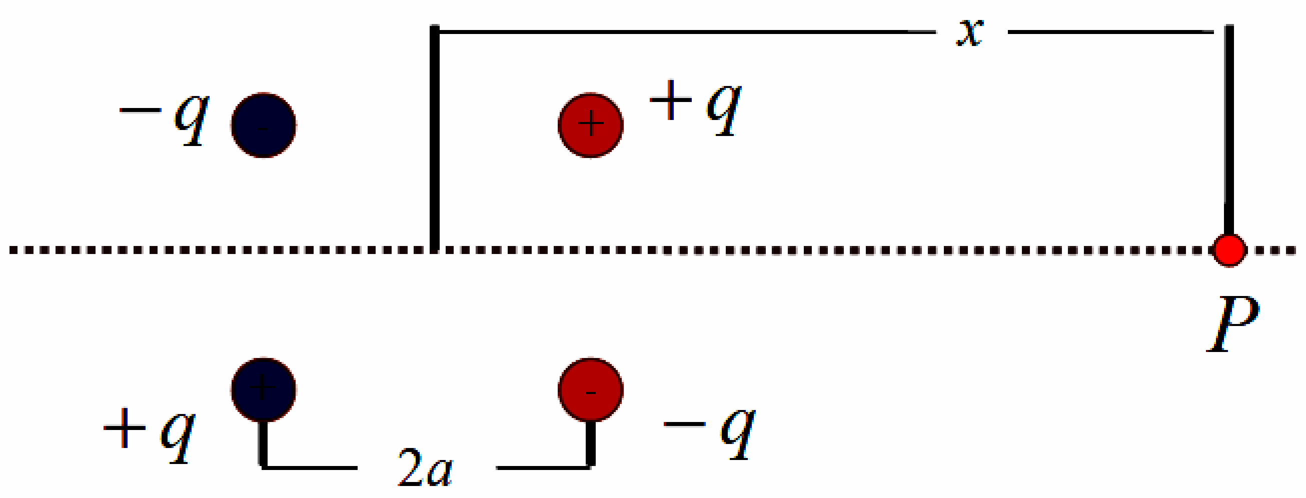
$$= \frac{kqd}{(2(d/2)^2)^{3/2}}\hat{j}$$

Substituting values for k, q and d;

$$\vec{E} = 1.1 \times 10^5\,\hat{j} N/C$$

If x >> a for an electrical quadrupole shown below, show that electric field at point P is given by

$$E = \frac{3(2qa^2)}{2\pi\varepsilon_\circ x^4}$$



Electric field due to dipole 1 is

$$\vec{E}_1 = \frac{kp}{(x+a)^3}\hat{j} = \frac{2kqa}{(x+a)^3}\hat{j}$$

Electric field due to dipole 2 is

$$\vec{E}_2 = -\frac{kp}{(x-a)^3}\hat{j} = -\frac{2kqa}{(x-a)^3}\hat{j}$$

Net electric field at point P will be $\vec{E} = \vec{E}_1 + \vec{E}_2$

$$\vec{E} = \frac{2kqa}{(x+a)^3}\hat{j} - \frac{2kqa}{(x-a)^3}\hat{j}$$

$$= 2kqa\left[(x+a)^{-3} - (x-a)^{-3}\right]\hat{j}$$

$$= \frac{2kqa}{x^3}\left[(1+a/x)^{-3} - (1-a/x)^{-3}\right]\hat{j}$$

$$= \frac{2kqa}{x^3}\left[(1-3a/x) - (1+3a/x)\right]\hat{j}$$

$$= -\frac{3(2qa^2)}{2\pi\varepsilon_o x^4}\hat{j}$$

Electric Field of Dipole

Electric field at the axis of dipole for y>>d.

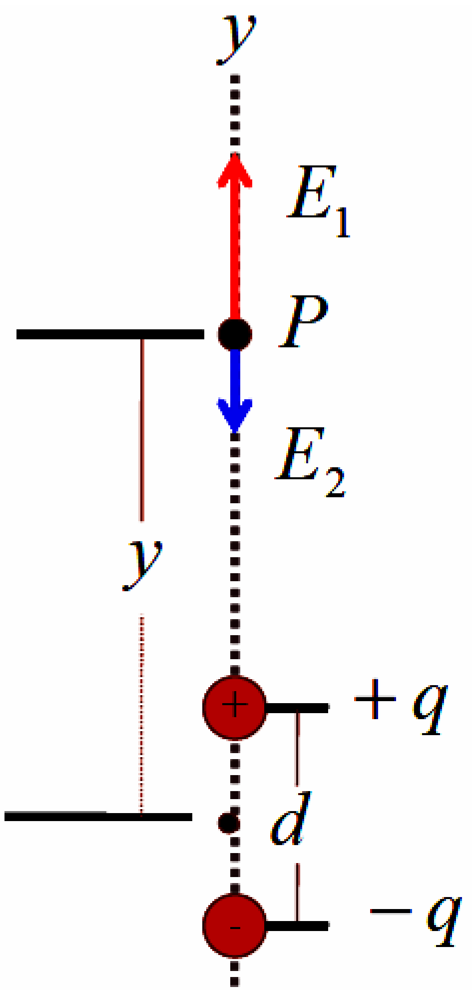
Let E1 is the field at P due to +q and E2 is due to charge –q. The net field will be

$$\vec{E} = \vec{E}_1 + \vec{E}_2$$

$$= \frac{kq}{(y-d/2)^2}\hat{j} - \frac{kq}{(y+d/2)^2}\hat{j}$$

$$= \frac{kq}{y^2(1-d/2y)^2}\hat{j} - \frac{kq}{y^2(1+d/2y)^2}\hat{j}$$

$$= \frac{kq}{y^2}\left[\left(1-\frac{d}{2y}\right)^{-2} - \left(1+\frac{d}{2y}\right)^{-2}\right]\hat{j}$$



If y>>d, then,

Neglecting square and higher terms

$$\vec{E} = \frac{kq}{y^2}\left[\left(1+\frac{d}{y}+\ldots\right)-\left(1-\frac{d}{y}+\ldots\right)\right]\hat{j}$$

$$= \frac{kq}{y^2}\frac{2d}{y}\hat{j}$$

$$= \frac{2kp}{y^3}\hat{j}$$

For dipole, with dipole moment along y axis and r >> d

$$\boxed{\vec{E} = -(kp/r^3)\hat{j}}$$ At perpendicular Bisector

$$\boxed{\vec{E} = (2kp/r^3)\hat{j}}$$ At axis of dipole

Motion of charged Particles in a Uniform Electric Field

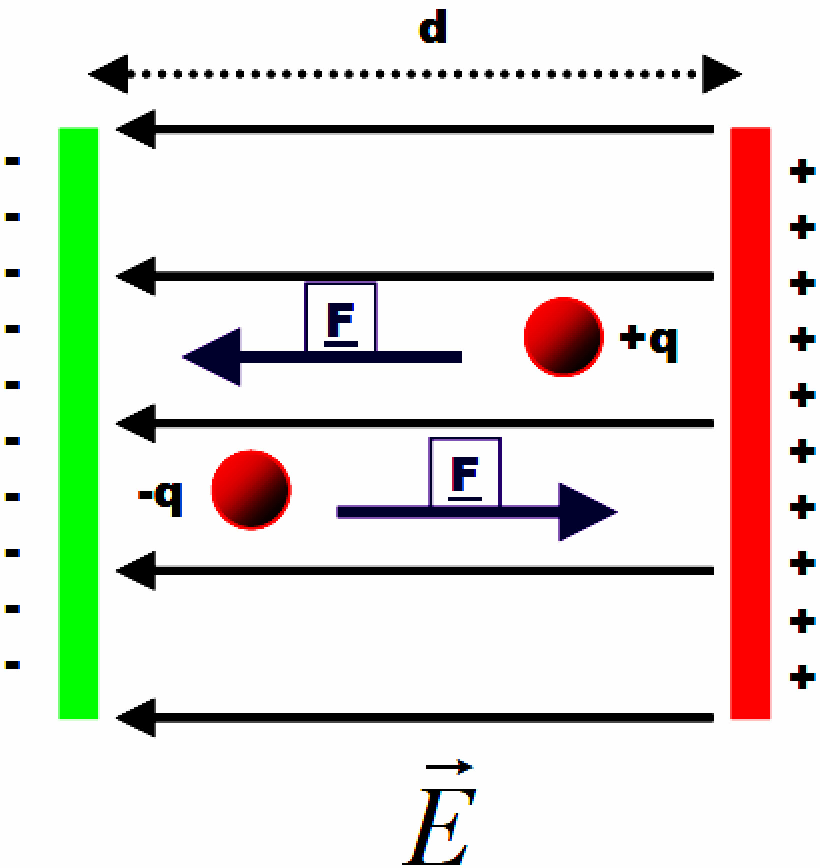
- When a particle of charge q and mass m is placed in an electric field E, the electric force exerted on the charge is qE.
- By Newton's second law, this electric force cause the particle to accelerate.

$$F = ma = qE \qquad \Rightarrow \qquad a = \frac{qE}{m}$$

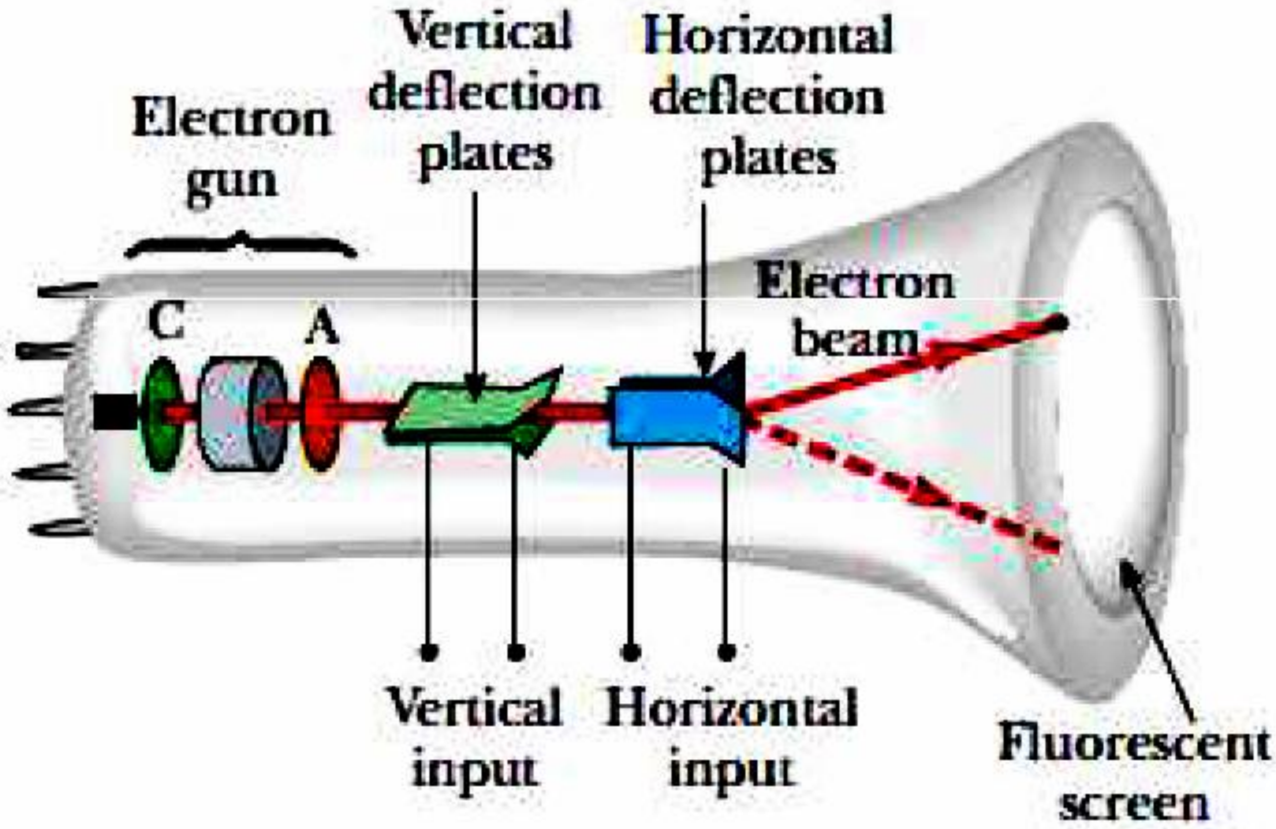
- If E is uniform (that is, constant in magnitude and direction), then the acceleration is constant.
- If the particle has a positive charge, then its acceleration is in the direction of the electric field.
- If the particle has a negative charge, then its acceleration is in the direction opposite the electric field.

Electric Field as Accelarator

- A uniform electric field can be produced by maintaining a voltage difference across any insulating space, such as air or a vacuum.
- Electric fields are used to create beams of high-speed electrons by accelerating them.
- Electron beams are used in x-ray machines, televisions, computer displays, and many other technologies.



The Cathode Ray Tube



A Dipole in an Electric Field

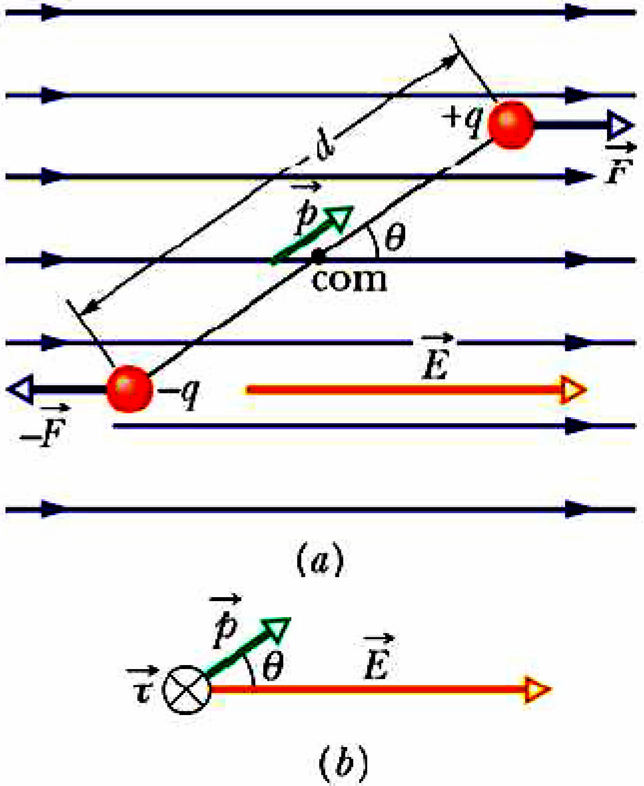
When we place an electric dipole in an external electric field, the force on positive charge will be in one direction and the force on negative charge will be in other direction. This will result in net torque about center of mass of dipole that tends to align dipole moment p along the electric field **E**.

The net torque due to these two forces will have magnitude

$$\tau = F_1 r_\perp + F_2 r_\perp$$

$$\tau = F\frac{d}{2}\sin\theta + F\frac{d}{2}\sin\theta$$

$$= Fd\sin\theta$$



As force on charge q placed in electric field **E** is; $F = qE$

$$\tau = qEd\sin\theta$$

Moreover electric dipole moment is defined as; $p = qd$

$$\tau = pE\sin\theta$$

$$\boxed{\vec{\tau} = \vec{p} \times \vec{E}}$$

In dynamics, if forces acting on a system are conservative, we can represent the system equally well using either force equations or energy equations

The work done by the external field in turning the dipole from initial angle $\theta_\circ$ to some final angle θ is

$$W = \int_{\theta_\circ}^{\theta} \vec{\tau} \bullet d\vec{\theta} = \int_{\theta_\circ}^{\theta} -\tau d\theta$$

Torque tends to decrease θ

$$= -\int_{\theta_\circ}^{\theta} pE \sin\theta d\theta = -pE \int_{\theta_\circ}^{\theta} \sin\theta d\theta$$

$$= pE[\cos\theta]_{\theta_\circ}^{\theta}$$

$$= pE(\cos\theta - \cos\theta_\circ)$$

Work-Energy theorem,

$$\Delta U = -W$$

$$U - U_\circ = -pE(\cos\theta - \cos\theta_\circ)$$

Let's choose reference point for this system to be at $\boxed{\theta_\circ = 90^\circ}$

So potential energy at that point will be zero $U_\circ = 0$

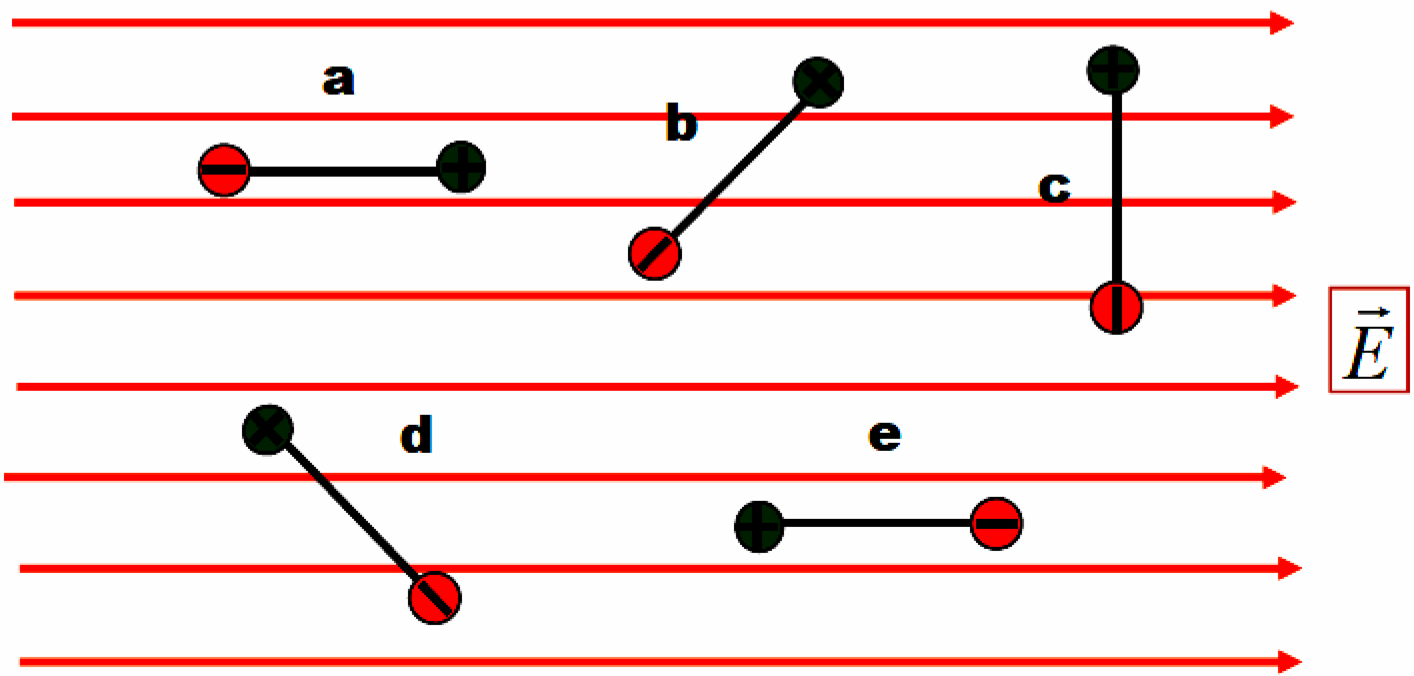
$$U = -pE\cos\theta$$

$$\boxed{U = -\vec{p} \bullet \vec{E}}$$

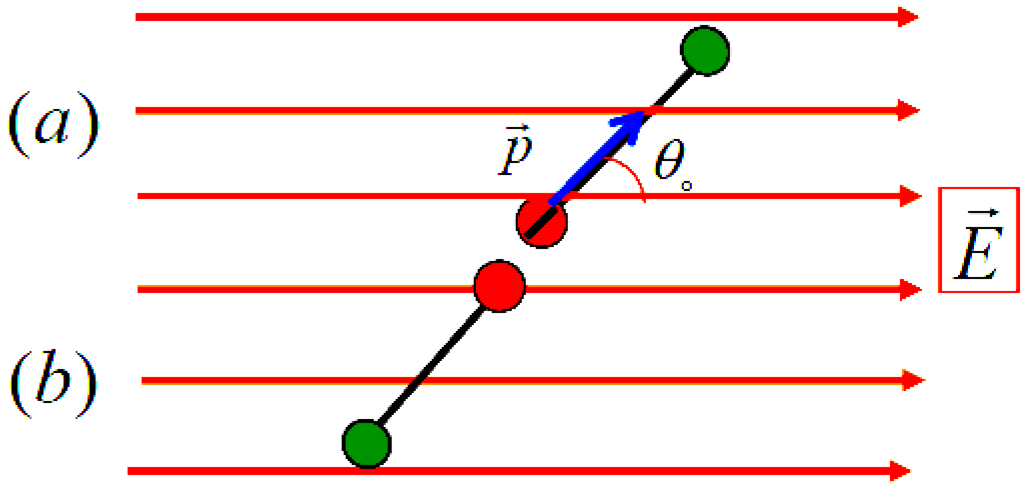
Potential energy is minimum when **p** and **E** are parallel.

Thus we can interpret the motion of a dipole in an external electric field either on the basis of torque that rotates the dipole into alignment with the field or a potential energy that becomes minimum when the dipole is aligned with the field.

In which configuration, the potential energy of the dipole system is the greatest, zero and smallest?



Find the work required to turn an electric dipole end far end in a uniform electric field **E**, in terms of electric dipole moment **p** and the initial angle θ_0 between **p** and **E**.



If the initial angle between **p** and **E** is θ_0, then potential energy will be

$$U_i = -pE\cos\theta_\circ$$

Thus final potential energy is

$$U_f = -pE\cos(\theta_\circ + \pi)$$

So the work required to turn dipole end far end is

$$\begin{aligned} W &= -(U_f - U_i) \\ &= -\{-pE\cos(\theta_\circ + \pi) + pE\cos\theta_\circ\} \\ &= pE\{\cos(\theta_\circ + \pi) - \cos\theta_\circ\} \\ &= pE\{-\cos\theta_\circ - \cos\theta_\circ\} \\ &= -2pE\cos\theta_\circ \end{aligned}$$