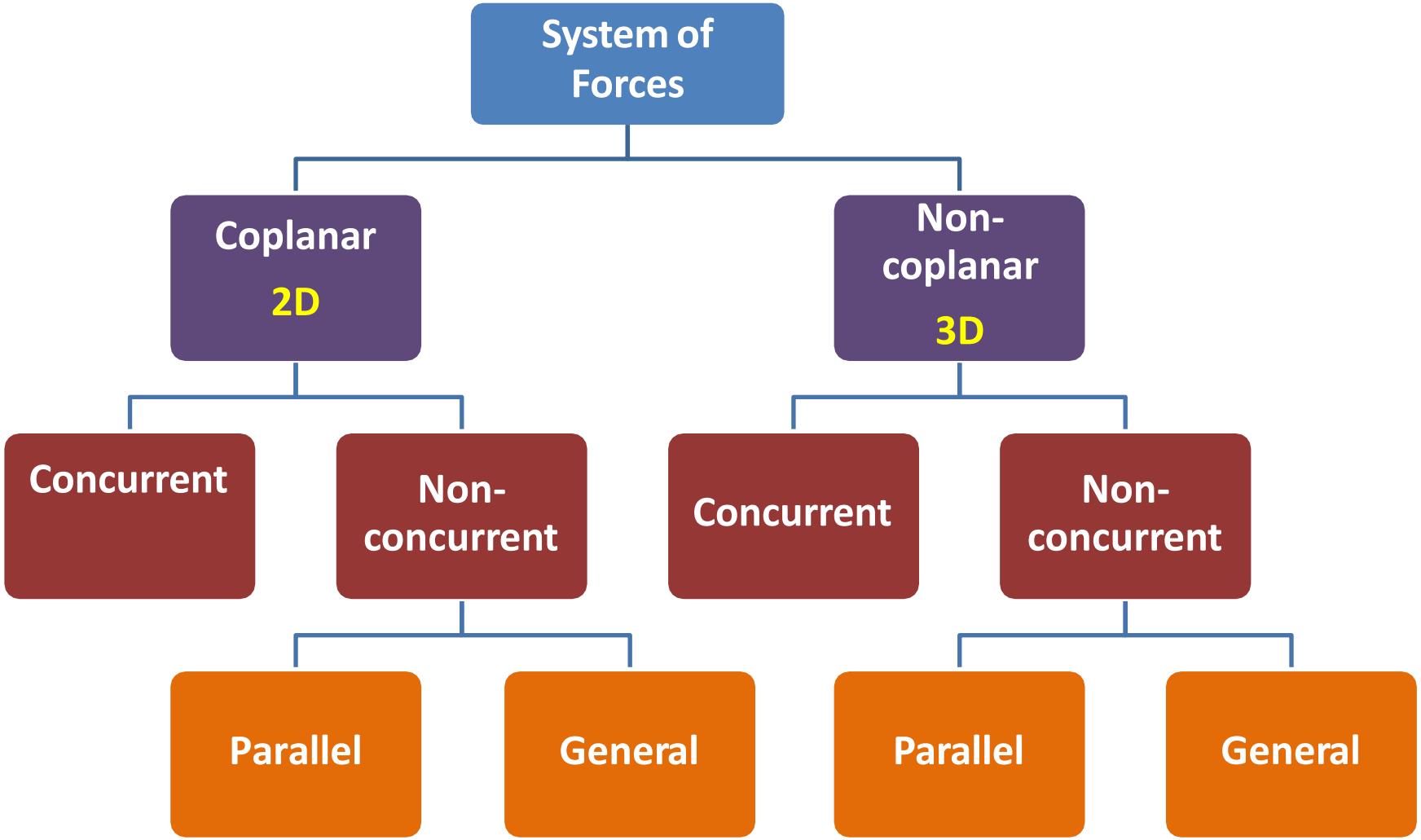
Engineering Mechanics: Statics

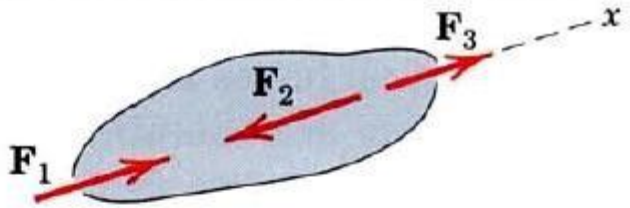
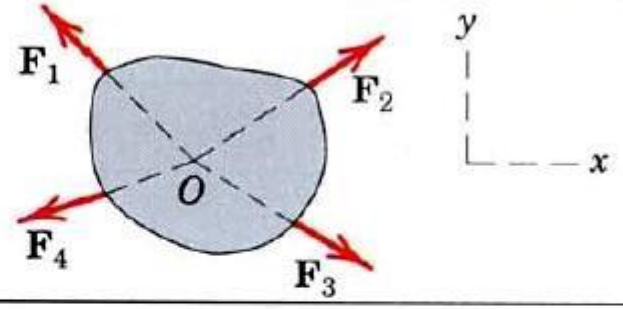
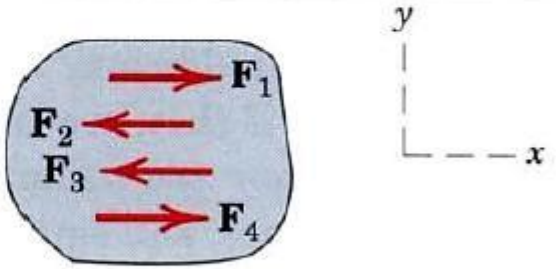
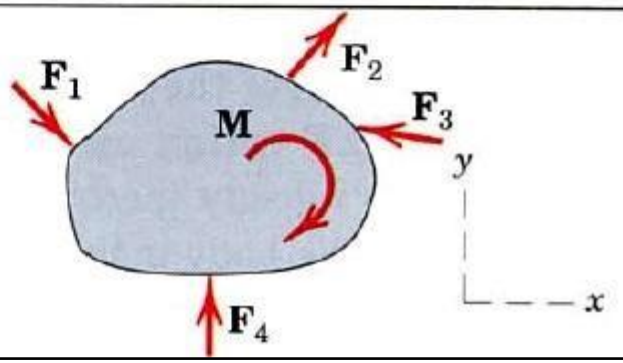
Chapter 5: Equilibrium of a Rigid Body

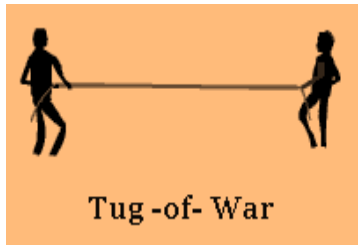
Revision

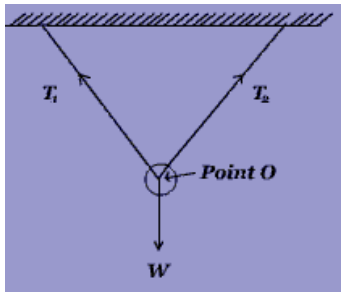
System of Forces: Several forces acting simultaneously upon a body

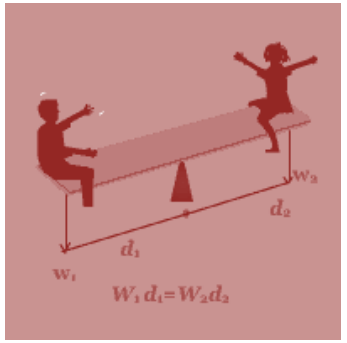


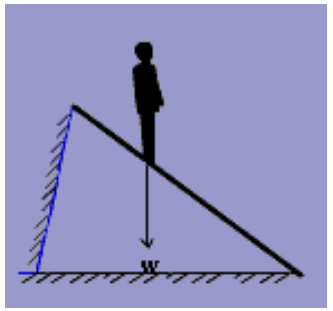
Coplanar System of Forces 2D

1. Collinear	
2. Concurrent at a point	
3. Parallel	
4. General	

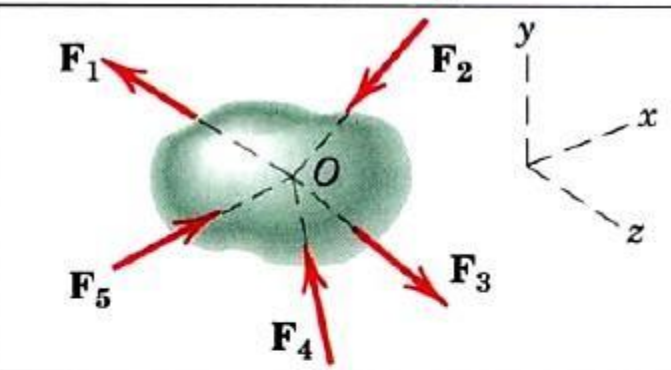
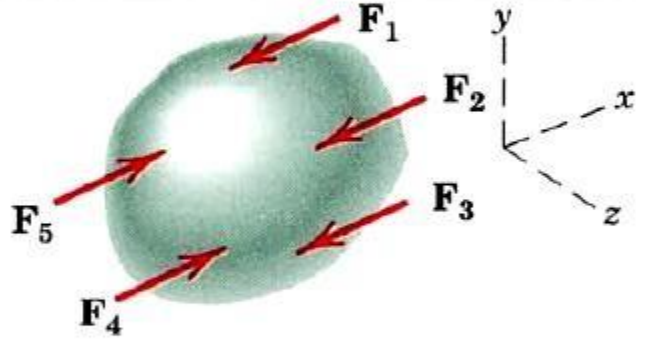
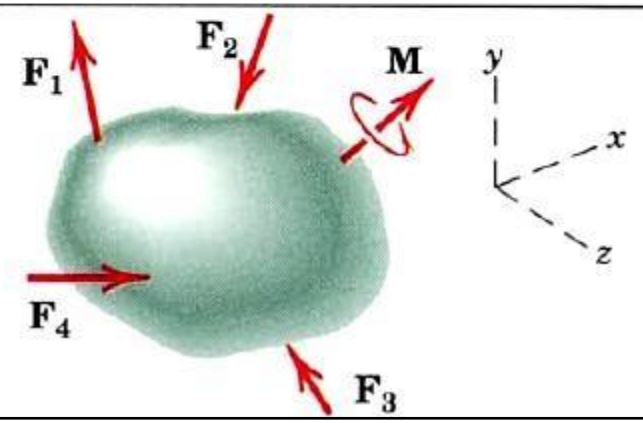


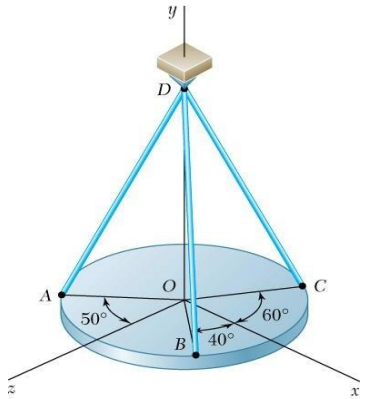


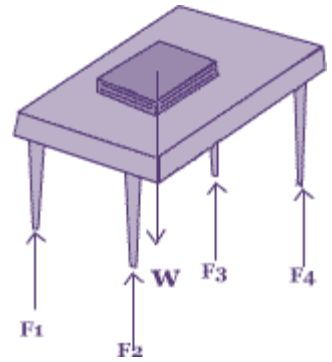




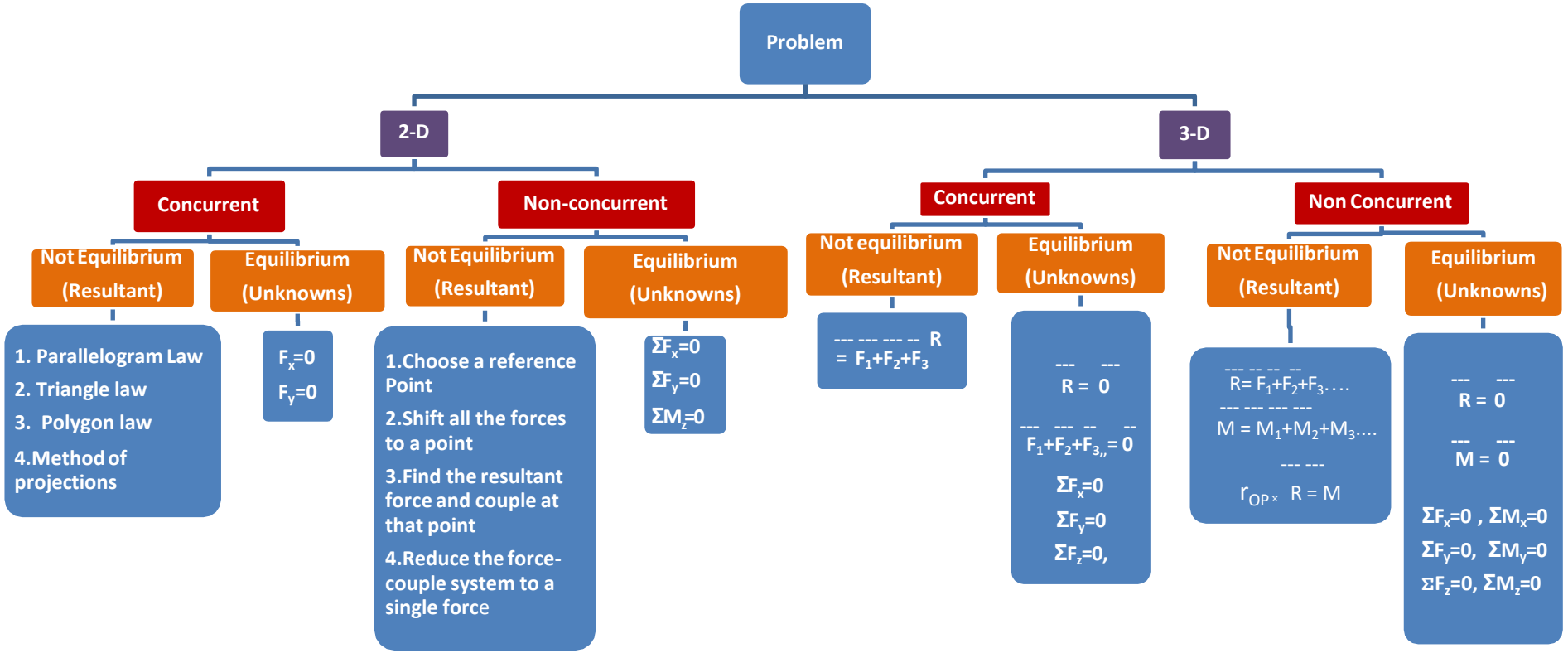
Non-Coplanar System of Forces 3D

Force System	Free-Body Diagram
Concurrent at a point	
Parallel	
General	

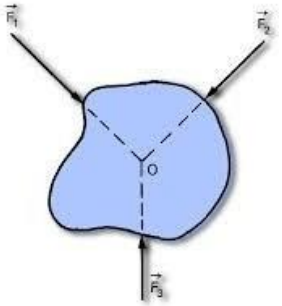


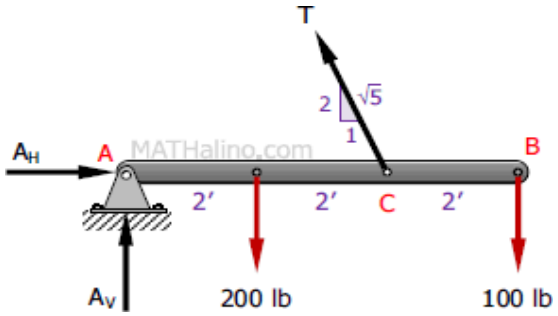


Method of approach to solve CONCURRENT and NON CONCURRENT FORCE Systems



Method of approach to solve Coplanar (2D) problems

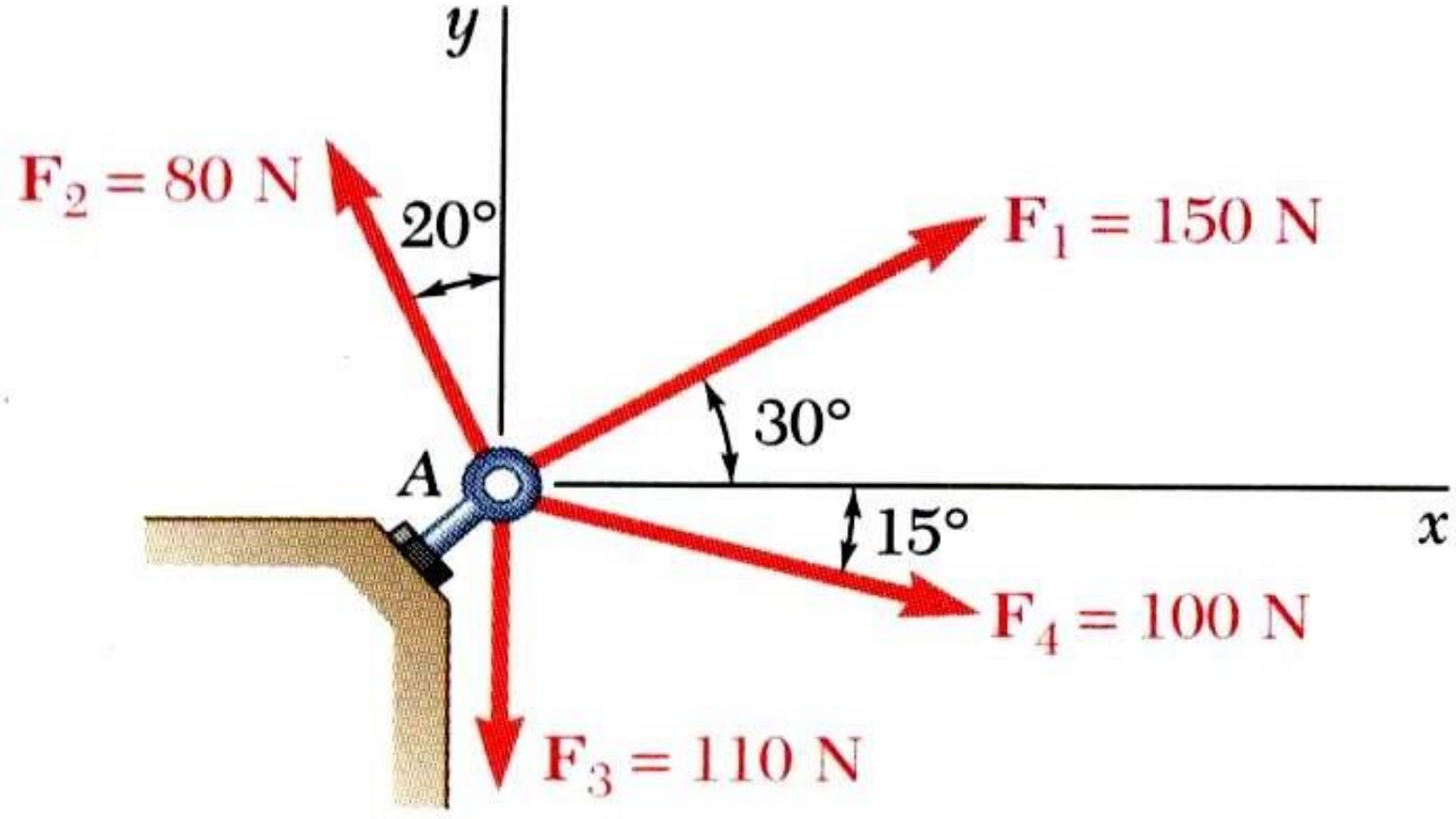




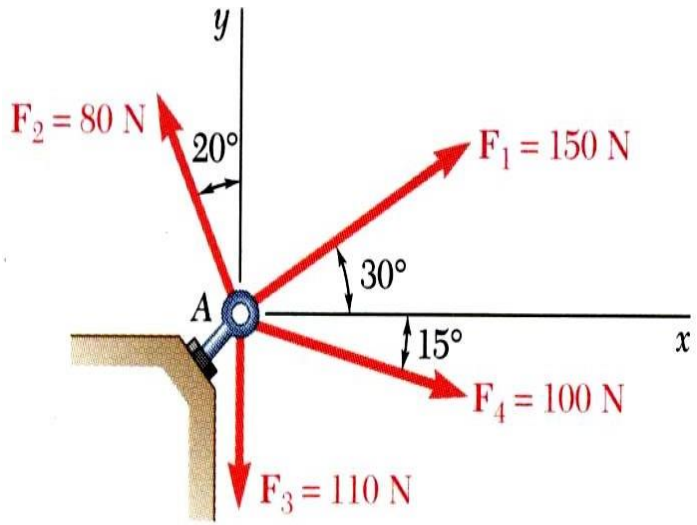
- **Problem**
  - **2-D**
    - **Concurrent**
      - **Not Equilibrium (Resultant)**
        - 1. Parallelogram Law
        - 2. Triangle law
        - 3. Polygon law
        - 4. Method of projections
      - **Equilibrium (Unknowns)**
        - $F_x=0$
        - $F_y=0$
    - **Non-concurrent**
      - **Not Equilibrium (Resultant)**
        - 1. Choose a reference Point
        - 2. Shift all the forces to a point
        - 3.Find the resultant force and couple at that point
        - 4.Reduce the force-couple system to a single force
      - **Equilibrium (Unknowns)**
        - $\Sigma F_x=0$
        - $\Sigma F_y=0$
        - $\Sigma M_z=0$

Determine the resultant of the following figure

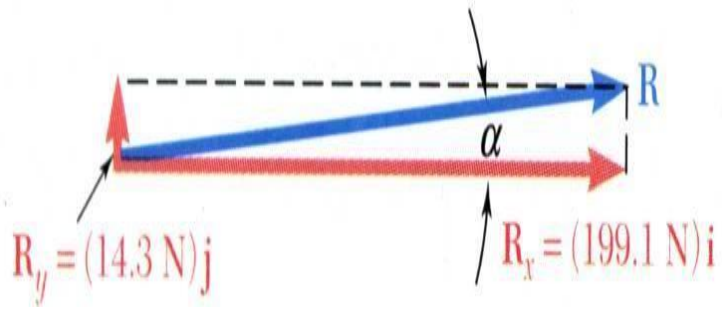
Problem



Problem solution



Force	Mag	x −comp	y −comp
F_1	150	$150\cos 30$	$+150Sin30$
F_2	80	$-80\sin 20$	$+80Cos20$
F_3	110	0	-110
F_4	100	$+100\cos 15$	$-100\sin 15$
		$\sum F_x = +199.1$	$\sum F_y = +14.3$



Resultant is $R = \sqrt{\sum F_{X_2} + \sum F_{\ 2}}$

^{Y}

$$R = \sqrt{199.1^2 + 14.3^2}$$

$$R = 199.6\text{N}$$

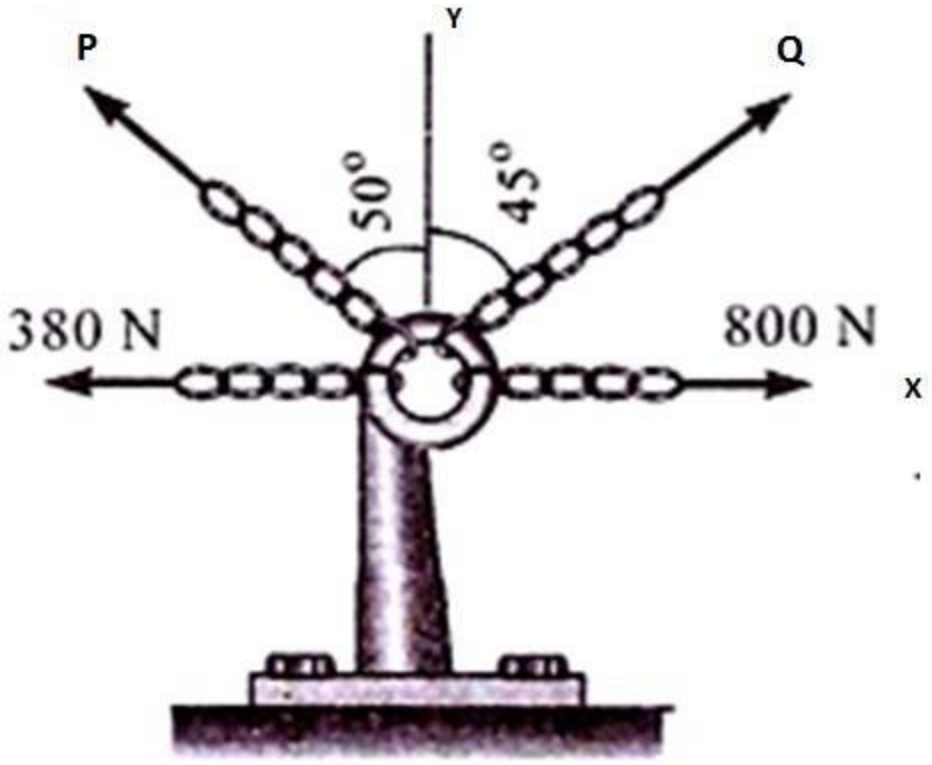
Direction is

$$\tan\alpha = \frac{14.3\ \text{N}}{199.1\text{N}}$$

$$\alpha = 4.1°$$

Problem

The resultant of the four concurrent forces as shown in Fig acts along Y-axis and is equal to 300N. Determine the forces P and Q.

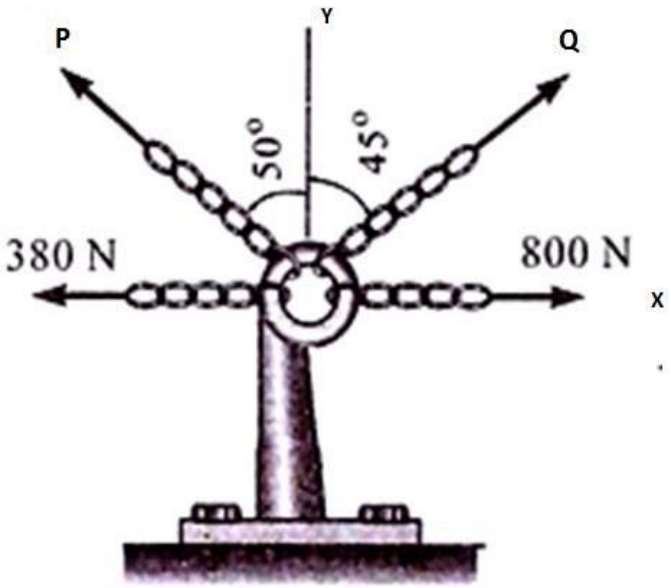


$$\sum F_x = 0$$

$$\sum F_y = R = 300N$$

Problem – 2D- Concurrent - Resultant

Solution



Force	Mag	$x-comp$	$y-comp$
F_1	800	800	0
F_2	380	-380	0
F_3	Q	$+QSin45$	$+QCos45$
F_4	P	$-PSin50$	$+PCos50$

$$\sum F_x = 0 \qquad \sum F_x = 800 - 380 + QSin45 - P\sin 50 = 0$$

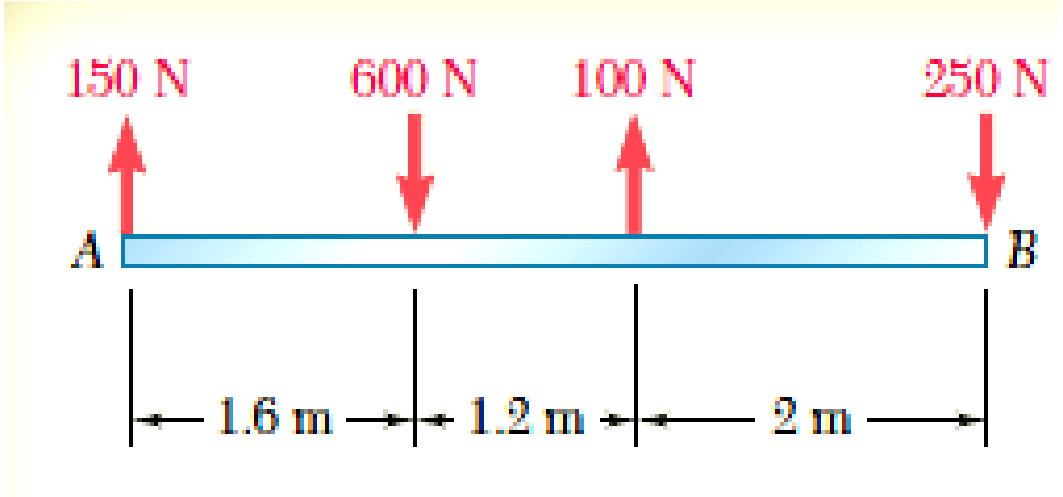
$$\sum F_y = R = 300N \qquad \sum F_y = QCos45 + PCos50 = R = 300$$

P = 511 N

Q =- 40.3N

Problem

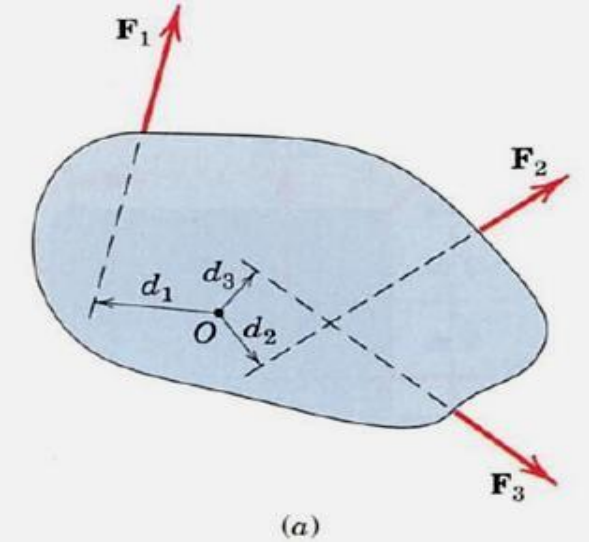
Determine the resultant of the following figure



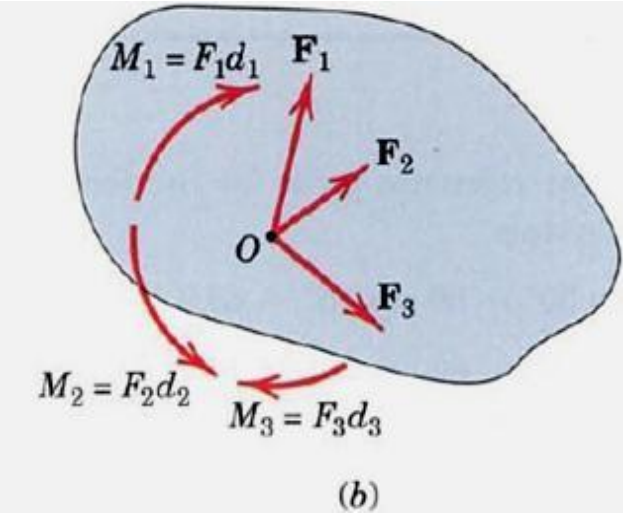
Problem – 2D- Non Concurrent - Resultant

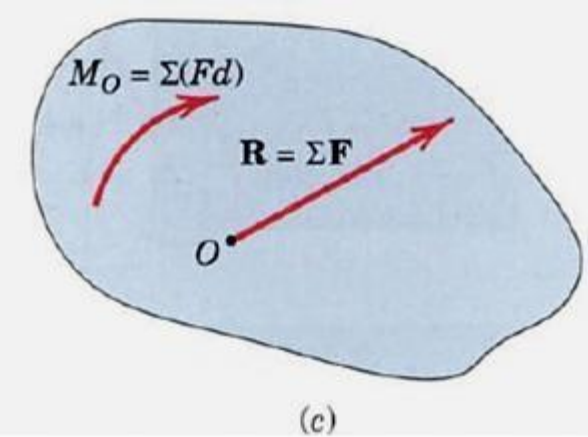
Resultant of General forces in a plane –
Coplanar non-concurrent

Step 1: Choose a reference point

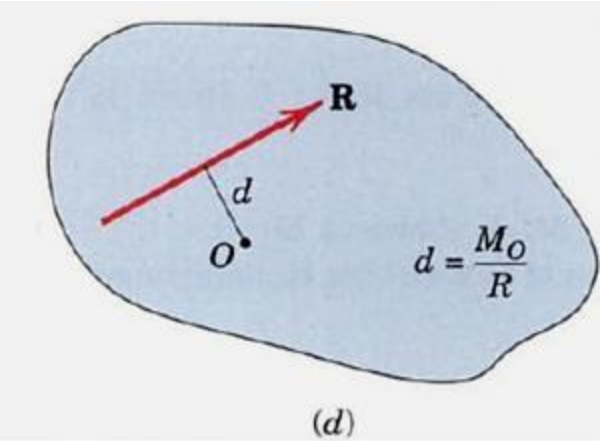


Step 2: Shift all the forces to a point





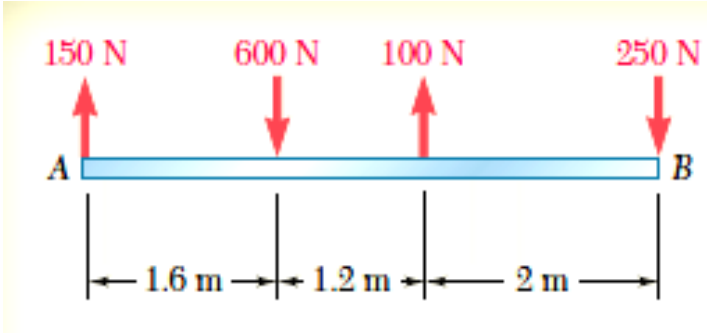
Step 3: Find the resultant force and moment of forces about O



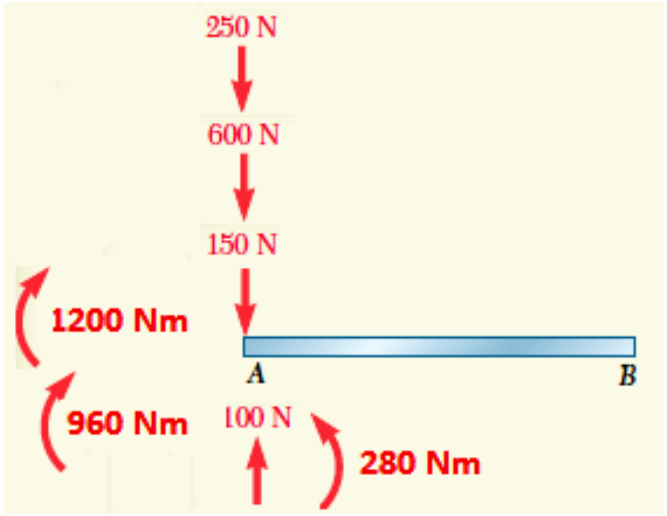
Step 4: Reduce resultant force and moment to a single force

Solution

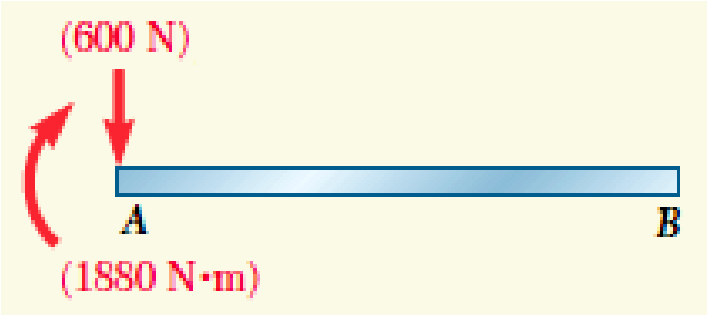
Step:1: Choose A as reference Point



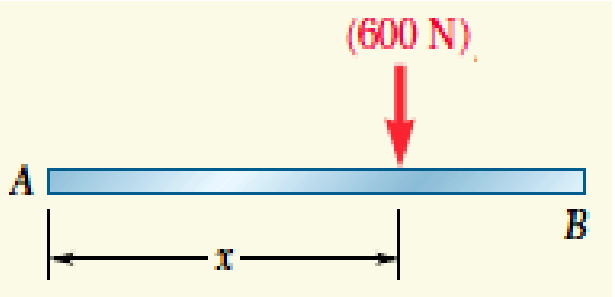
Step:2: Shift all forces to point A



Step 3: Find resultant force and couple



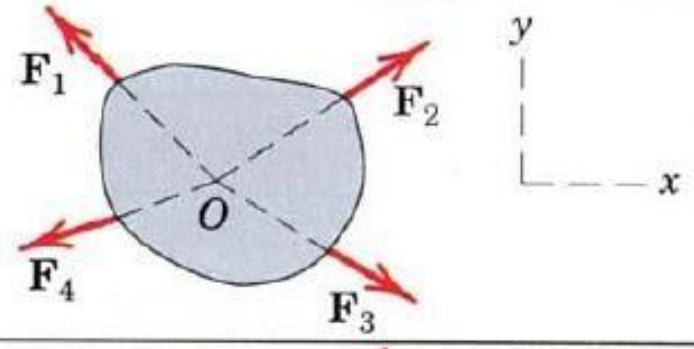
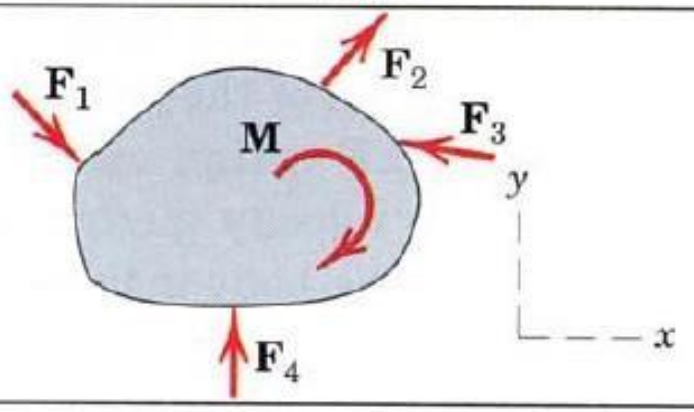
Step:4: Reduce it to a single force



x = 1880/600

x = 3.13m

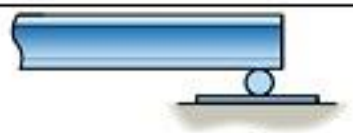
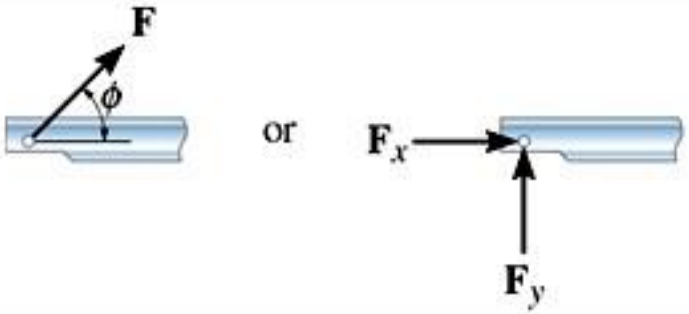
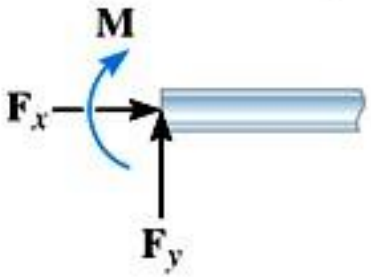
EQUATIONS OF EQUILIBRIUM

Force System	Free-Body Diagram	Independent Equations
Concurrent		$\Sigma F_x = 0$ $\Sigma F_y = 0$
Non Concurrent		$\Sigma F_x = 0$ $\Sigma M_z = 0$ $\Sigma F_y = 0$

Types of supports and reaction forces (2D)

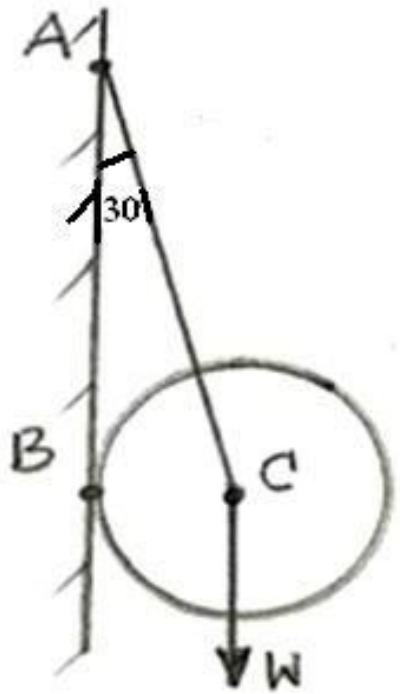
Types of Connection	Reaction	Number of Unknowns
(1)  cable		One unknown. The reaction is a force which acts away from the member in
(2)  smooth surface support		One unknown. The reaction is a force which acts perpendicular to the surface

Types of supports and reaction forces (2D)

Constraints	Type and direction of forces produced
 roller The connection point on the bar can not move downward.	
 pin The joint can not move in vertical and horizontal directions.	
 fixed support The support prevents translation in vertical and horizontal directions and also rotation, Hence a couple moment is developed on the body in that direction as well.	

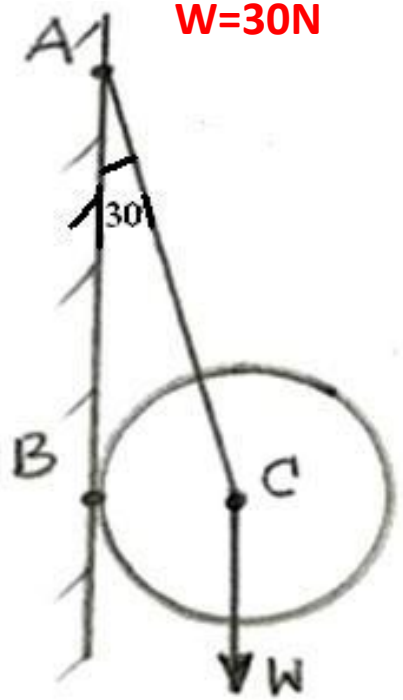
Problem

Find tension in the string and reaction at B



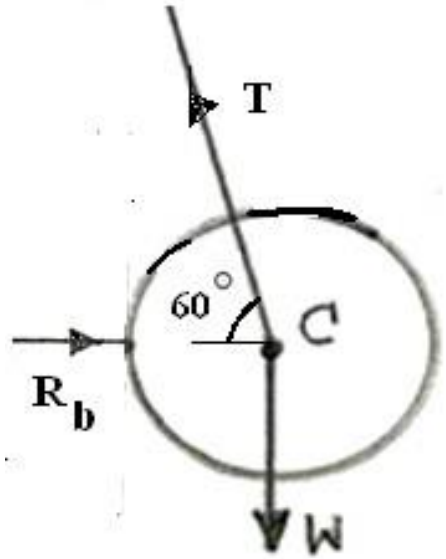
Problem – 2D - Concurrent - Equilibrium

Solution



W=30N

FBD of **C**



Since the body is in equilibrium and the forces are concurrent …..

$\sum F_X = 0$; $R_b - T\cos 60^0 = 0$ ……1

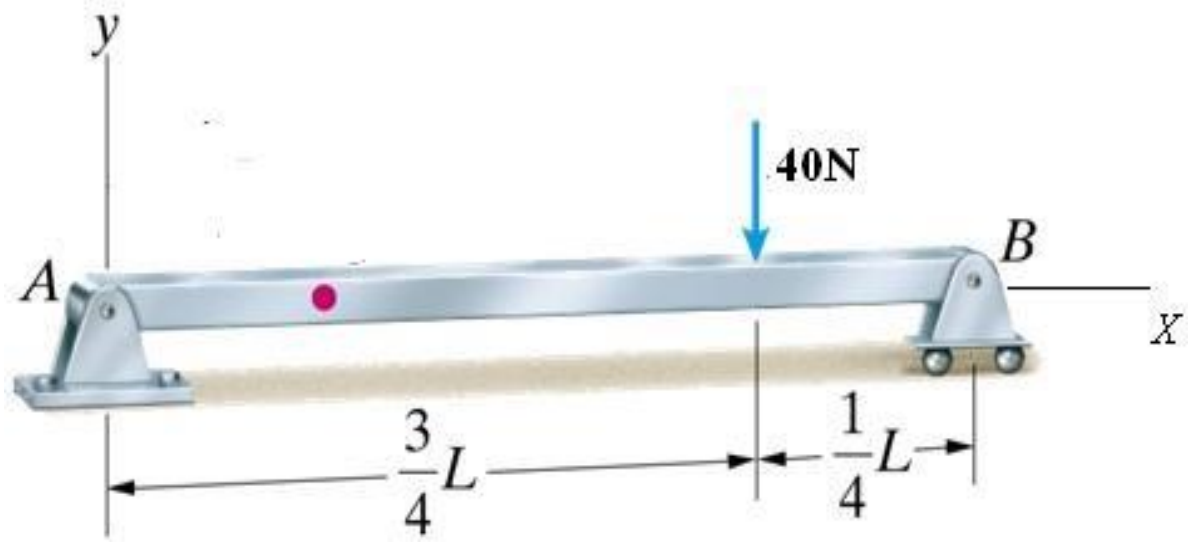
$\sum F_Y = 0$; $T\sin 60^0 - W = 0$ ……2

T= 34.64N

R_b = 17.32N

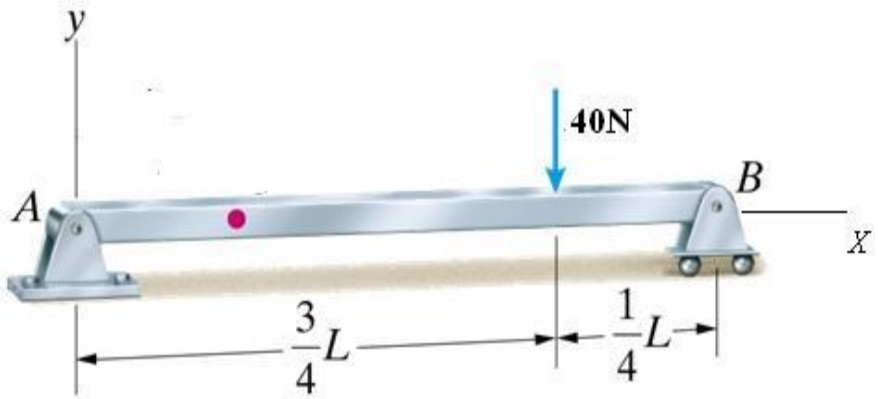
Problem

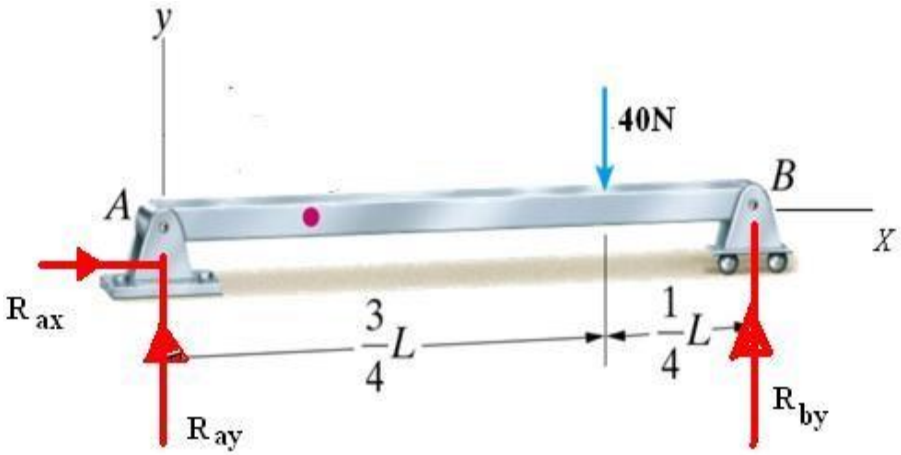
Determine the reactions at A and B



Problem – 2D - Non Concurrent - Equilibrium

Solution





Since the body is in equilibrium and the forces are general forces then……

$\sum F_X = 0$; $R_{ax} = 0$ 1

$\sum F_Y = 0$; $R_{ay} + R_{by} - 40 = 0$.....2

$\sum M_A = 0$; $(R_{by} * L) - 40*(3L/4) = 0$…..3

R_{by}=30N

R_{ay}=10N

R_{ax}=0N

Home Assignment

- Revise Chapter 2, 3 &4.