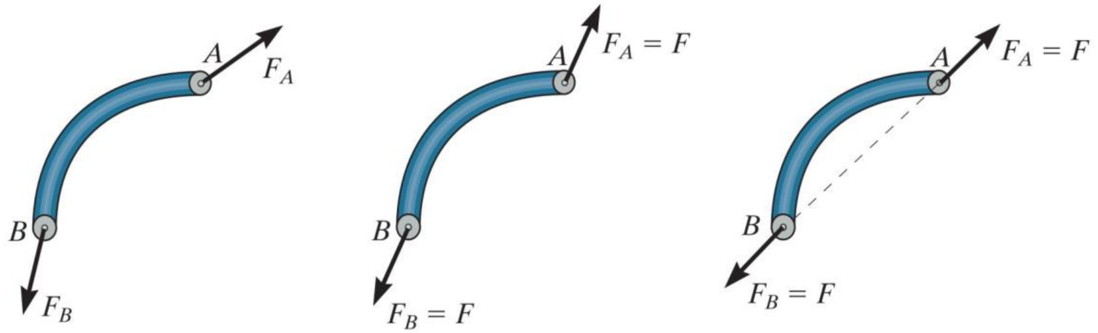
Engineering Mechanics: Statics

Chapter 5: Equilibrium of a Rigid Body

5.4 Two- and Three-Force Members

Two-Force Members.

- When forces are applied at only two points on a member, the member is called a two-force member
- Only force magnitude must be determined
- For any two-force member to be in equilibrium, the two forces acting on the member *must have the same magnitude, act in opposite directions, and have the same line of action, directed along the line joining the two points where these forces act.*



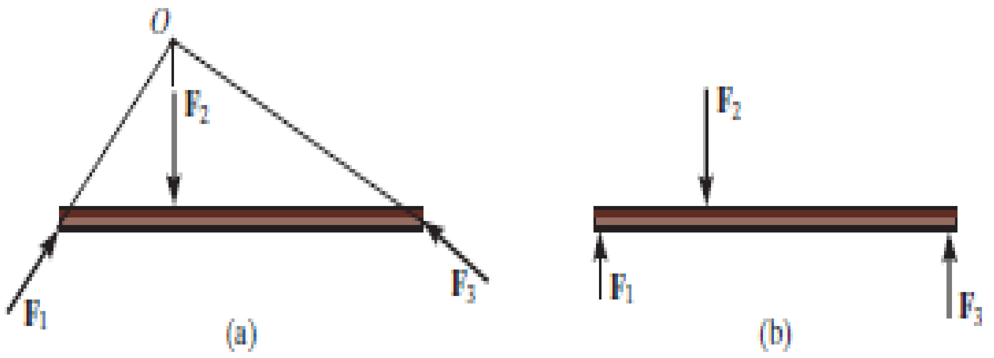
(a) (b) (c)

Two- and Three-Force Members

Three-Force Members.

- If a member is subjected to only *three forces*, it is called a ***three-force member***.
- Moment equilibrium can be satisfied only if the three forces form a *concurrent* or *parallel* force system.
- Consider the member subjected to the three forces

$\mathbf{F}_1$, $\mathbf{F}_2$, and $\mathbf{F}_3$, shown in Fig. 5–21*a*. If the lines of action of $\mathbf{F}_1$ and $\mathbf{F}_2$ intersect at point *O*, then the line of action of $\mathbf{F}_3$ must *also* pass through point *O* so that the forces satisfy $\Sigma \mathbf{M}_O = \mathbf{0}$. As a special case, if the three forces are all parallel, Fig. 5–21*b*, the location of the point of intersection, *O*, will approach infinity.



Three-force member

EQUILIBRIUM IN THREE-DIMENSIONS.

5.5 Free-Body Diagrams

- The first step in solving three-dimensional equilibrium problems, as in the case of two dimensions, is to draw a free-body diagram.

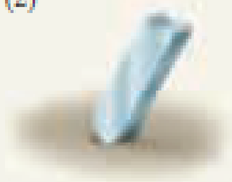
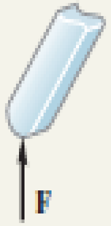
Support Reactions.

As in the two-dimensional case:

- If a **support prevents translation** of a body in a given direction, then **a force is developed** on the body in the opposite direction.
- A couple moment is developed when rotation of the attached member is prevented
- The force's orientation is defined by the coordinate angles α, β and γ

EQUILIBRIUM IN THREE-DIMENSIONS.

Free-Body Diagrams

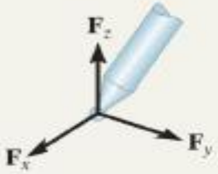
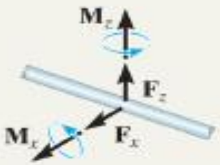
Types of Connection	Reaction	Number of Unknowns
(1)  cable		One unknown. The reaction is a force which acts away from the member in the known direction of the cable.
(2)  smooth surface support		One unknown. The reaction is a force which acts perpendicular to the surface at the point of contact.

EQUILIBRIUM IN THREE-DIMENSIONS.

Free-Body Diagrams

Free-Body Diagrams

Support Reactions.

(3)  roller		One unknown. The reaction is a force which acts perpendicular to the surface at the point of contact.
(4)  ball and socket		Three unknowns. The reactions are three rectangular force components.
(5)  single journal bearing		Four unknowns. The reactions are two force and two couple-moment components which act perpendicular to the shaft. Note: The couple moments are *generally not applied* if the body is supported elsewhere. See the examples.

EQUILIBRIUM IN THREE-DIMENSIONS.

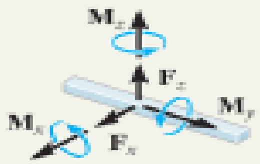
Free-Body Diagrams

Free-Body Diagrams

Support Reactions.

(6)

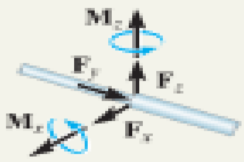
single journal bearing with square shaft



Five unknowns. The reactions are two force and three couple-moment components. *Note*: The couple moments *are generally not applied* if the body is supported elsewhere. See the examples.

(7)

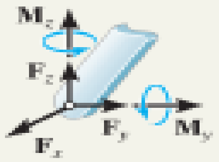
single thrust bearing



Five unknowns. The reactions are three force and two couple-moment components. *Note*: The couple moments *are generally not applied* if the body is supported elsewhere. See the examples.

(8)

single smooth pin



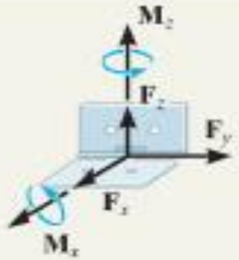
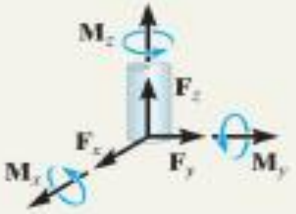
Five unknowns. The reactions are three force and two couple-moment components. *Note*: The couple moments *are generally not applied* if the body is supported elsewhere. See the examples.

EQUILIBRIUM IN THREE-DIMENSIONS.

Free-Body Diagrams

Free-Body Diagrams

Support Reactions.

(9)	 single hinge		Five unknowns. The reactions are three force and two couple-moment components. *Note*: The couple moments *are generally not applied* if the body is supported elsewhere. See the examples.
(10)	 fixed support		Six unknowns. The reactions are three force and three couple-moment components.

Applications.

This ball-and-socket joint provides a connection for the housing of an earth grader to its frame. (4) (© Russell C. Hibbeler)

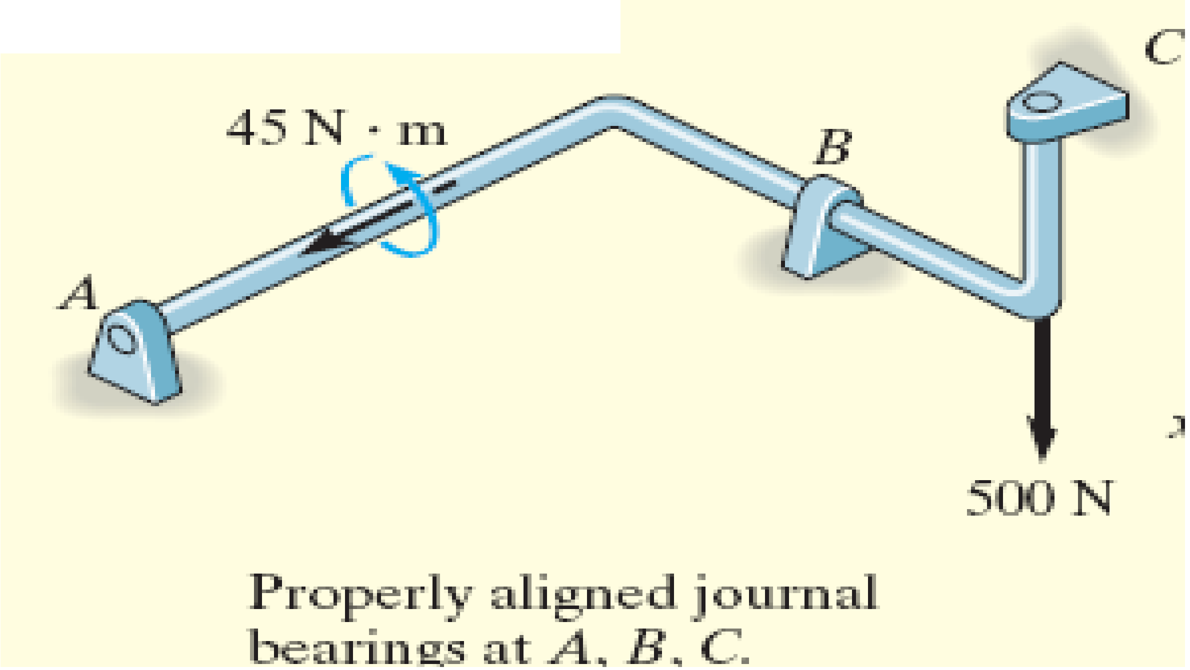
The journal bearings support the ends of the shaft. (5) (© Russell C. Hibbeler)

This thrust bearing is used to support the drive shaft on a machine. (7) (© Russell C. Hibbeler)

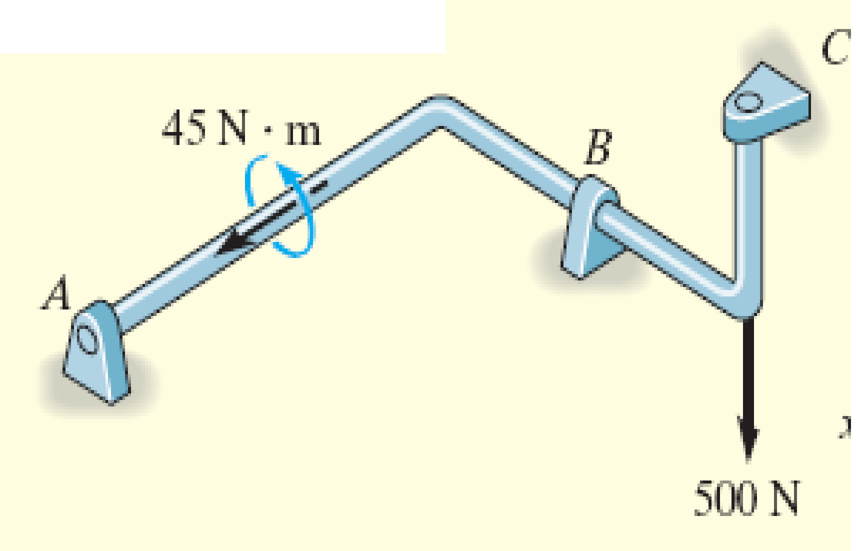
This pin is used to support the end of the strut used on a tractor. (8) (© Russell C. Hibbeler)

Example-1a

Consider the two rods and plate, along with their associated free-body diagrams shown in Fig. 5–23. The *x, y, z* axes are established on the diagram and the **unknown reaction** components are indicated in the *positive sense*. The weight is neglected.

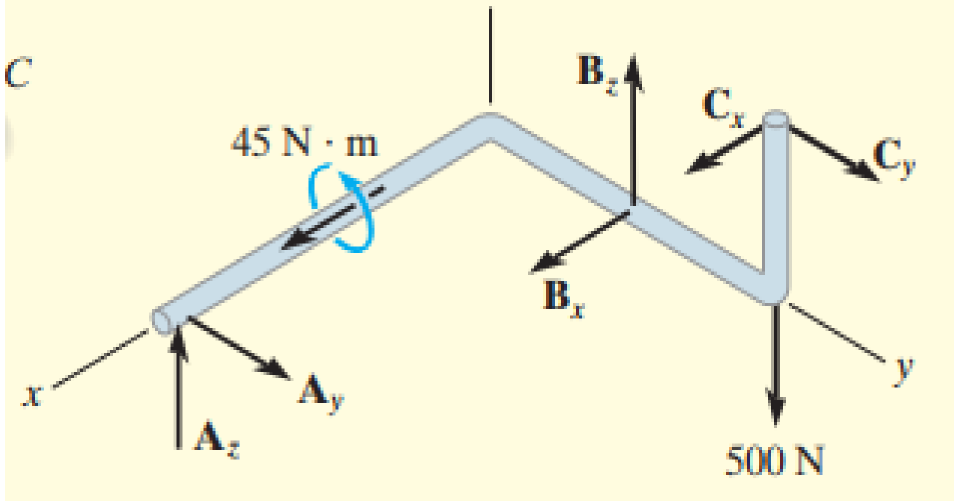


Actual Figure.



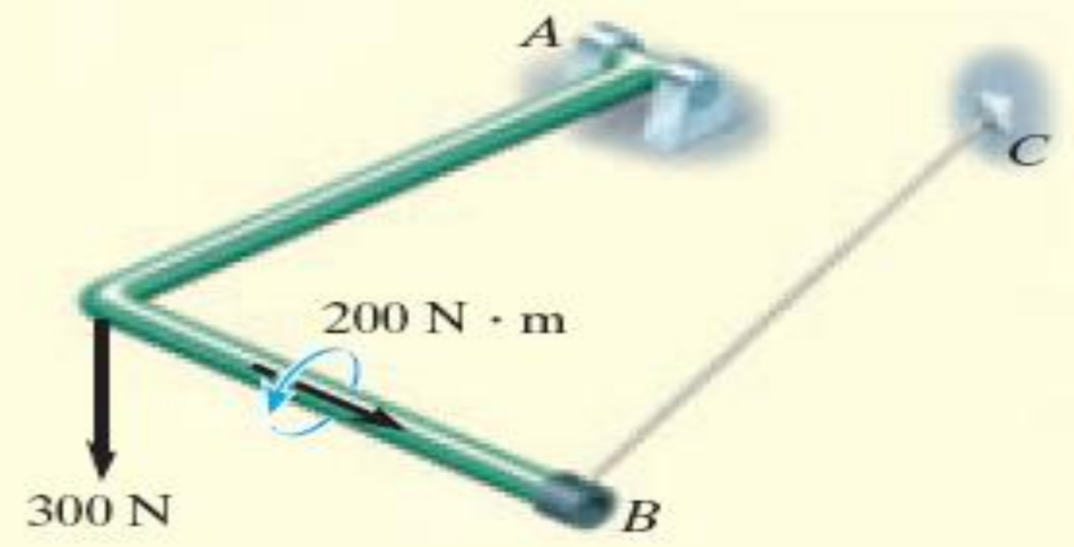
Properly aligned journal bearings at A, B, C.

FBD.



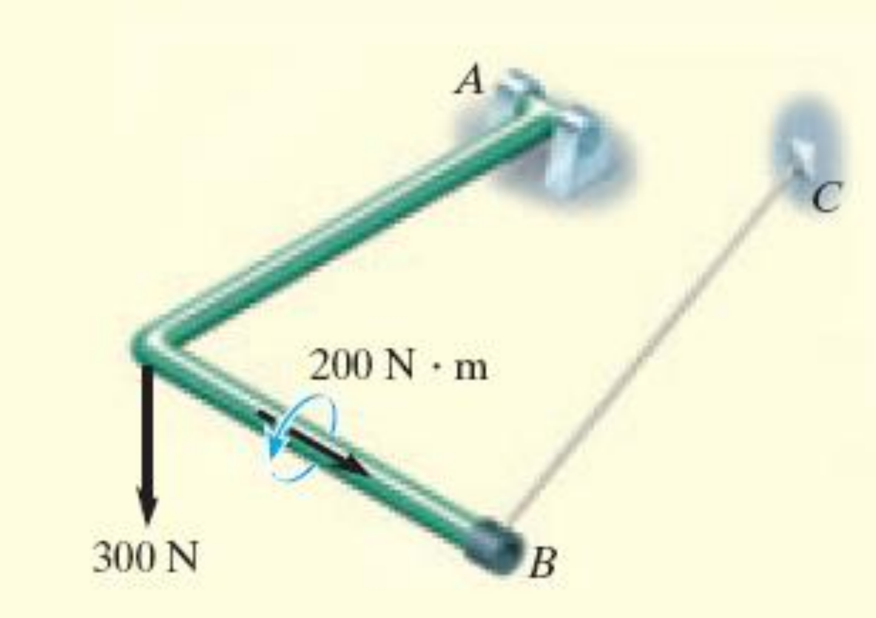
The force reactions developed by the bearings are sufficient for equilibrium since they prevent the shaft from rotating about each of the coordinate axes.

Example-1b



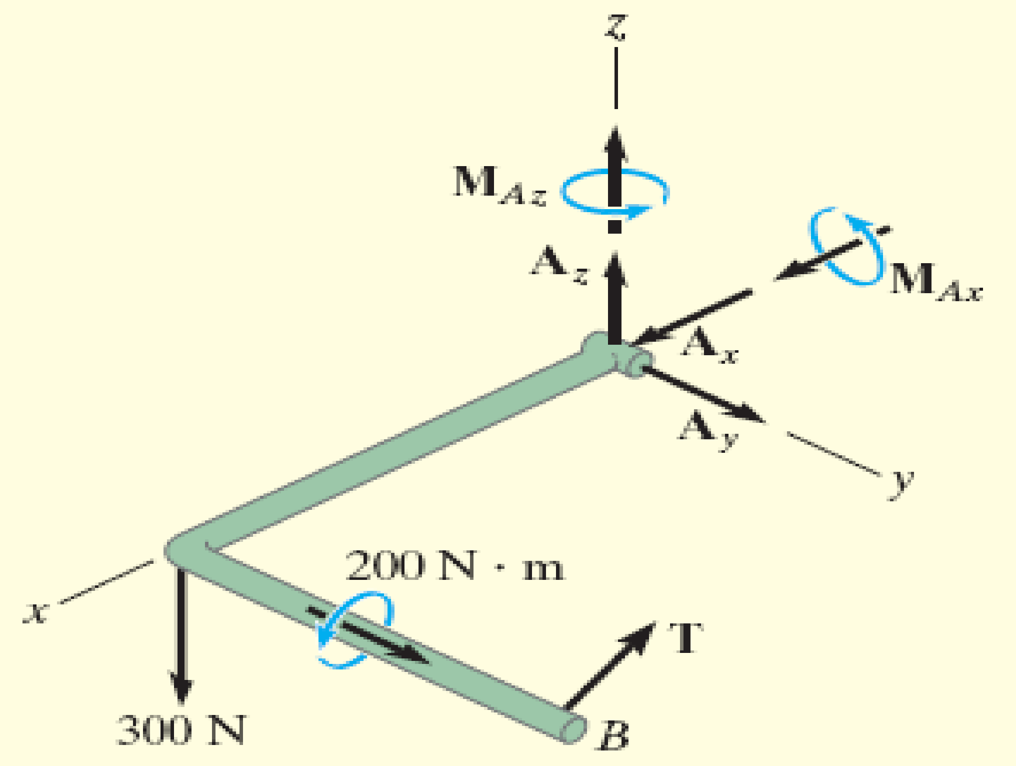
Pin at *A* and cable *BC*.

Actual Figure.



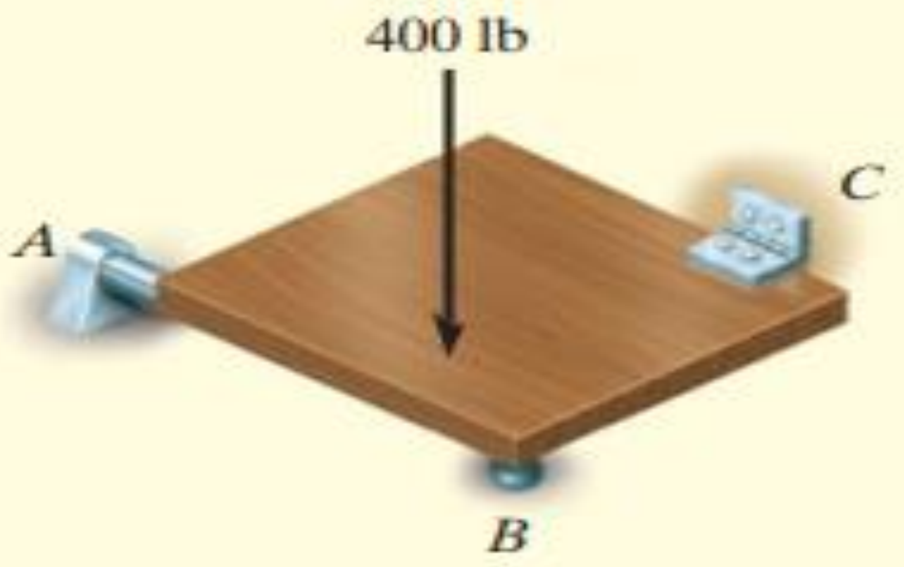
Pin at A and cable BC.

FBD.



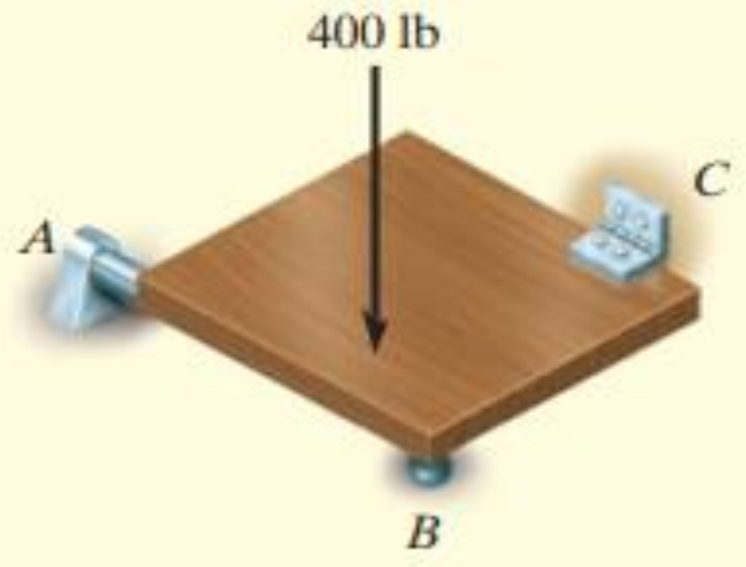
Moment components are developed by the pin on the rod to prevent rotation about the x and z axes.

Example-1c



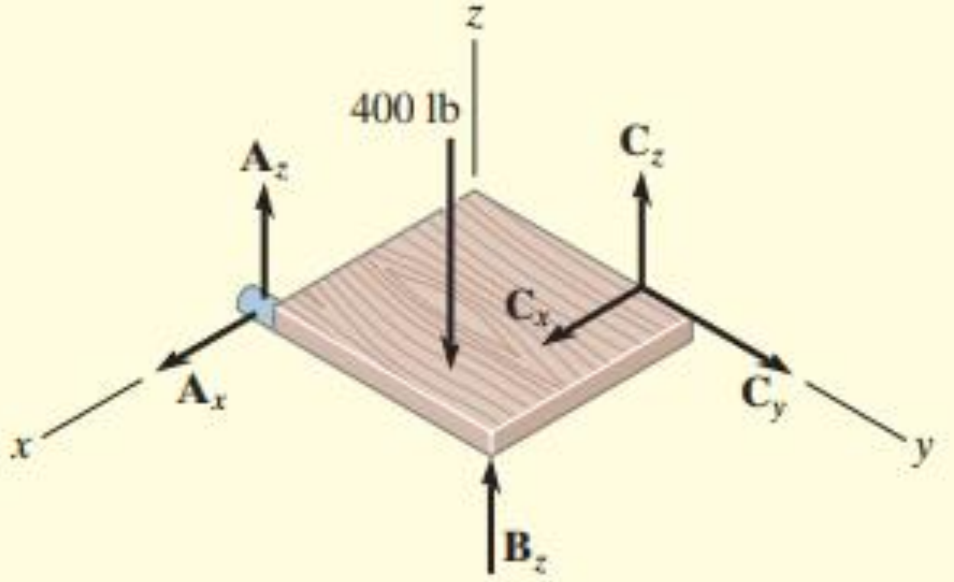
Properly aligned journal bearing at A and hinge at C. Roller at B.

Actual Figure.



Properly aligned journal bearing at A and hinge at C. Roller at B.

FBD.



Only force reactions are developed by the bearing and hinge on the plate to prevent rotation about each coordinate axis. No moments at the hinge are developed.

5.6 Equations of Equilibrium

- The conditions for equilibrium of a rigid body subjected to a three-dimensional force system require that both the *resultant* force and *resultant* couple moment acting on the body be equal to *zero*.

Vector Equations of Equilibrium.

The two conditions for equilibrium of a rigid body may be expressed mathematically in vector form as

$$\Sigma \mathbf{F} = \mathbf{0}$$
$$\Sigma \mathbf{M}_O = \mathbf{0}$$

Scalar Equations of Equilibrium. If all the external forces and couple moments are expressed in Cartesian vector form; we have

$$\Sigma \mathbf{F} = \Sigma F_x \mathbf{i} + \Sigma F_y \mathbf{j} + \Sigma F_z \mathbf{k} = \mathbf{0}$$

$$\Sigma \mathbf{M}_O = \Sigma M_x \mathbf{i} + \Sigma M_y \mathbf{j} + \Sigma M_z \mathbf{k} = \mathbf{0}$$

$$\Sigma F_x = 0 \qquad \Sigma M_x = 0$$
$$\Sigma F_y = 0 \qquad \Sigma M_y = 0$$
$$\Sigma F_z = 0 \qquad \Sigma M_z = 0$$

Equations of Equilibrium

- **Six scalar equations** are required to express the conditions for the equilibrium of a rigid body in the general three dimensional case.

$$\sum F_x = 0 \quad \sum F_y = 0 \quad \sum F_z = 0$$
$$\sum M_x = 0 \quad \sum M_y = 0 \quad \sum M_z = 0$$

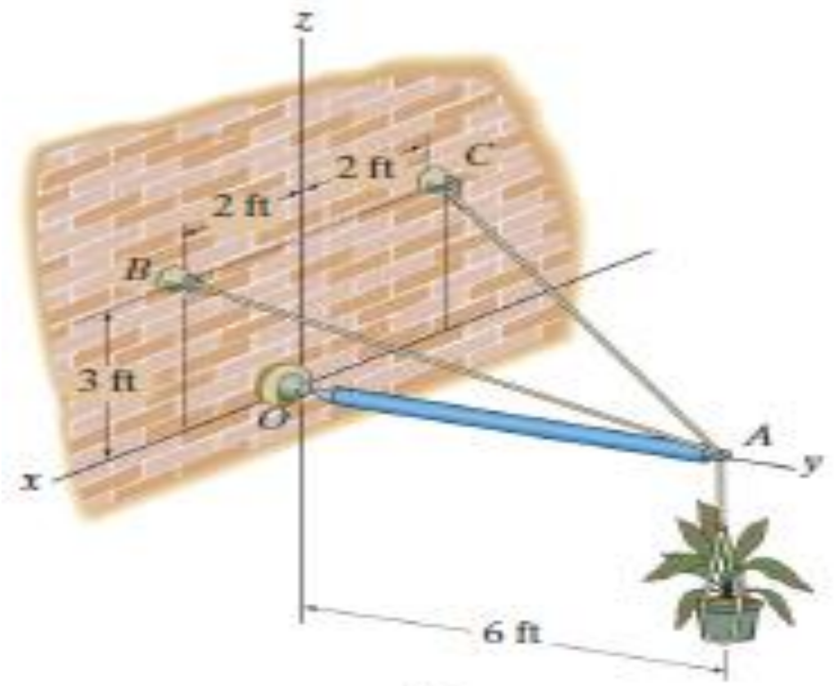
- These equations can be solved for no more than 6 unknowns which generally represent reactions at supports or connections.

- The scalar equations are conveniently obtained by applying the vector forms of the conditions for equilibrium,

$$\sum \vec{F} = 0 \qquad \sum \vec{M}_O = \sum \left(\vec{r} \times \vec{F} \right) = 0$$

Example-2

The boom is used to support the 75-lb flowerpot in Fig. 5–30*a*. Determine the tension developed in wires *AB* and *AC*.

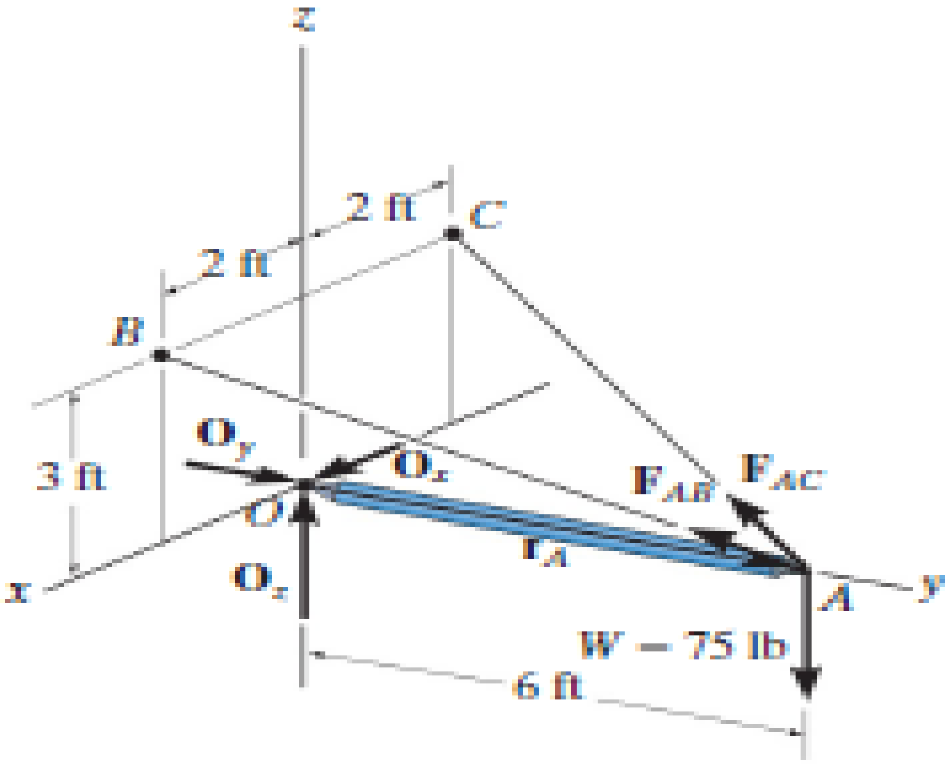


(a)

Fig. 5–30

SOLUTION (*VECTOR ANALYSIS*)

Free-Body Diagram. The free-body diagram of the boom is shown in Fig. 5–30*b*.



(b)

SOLUTION

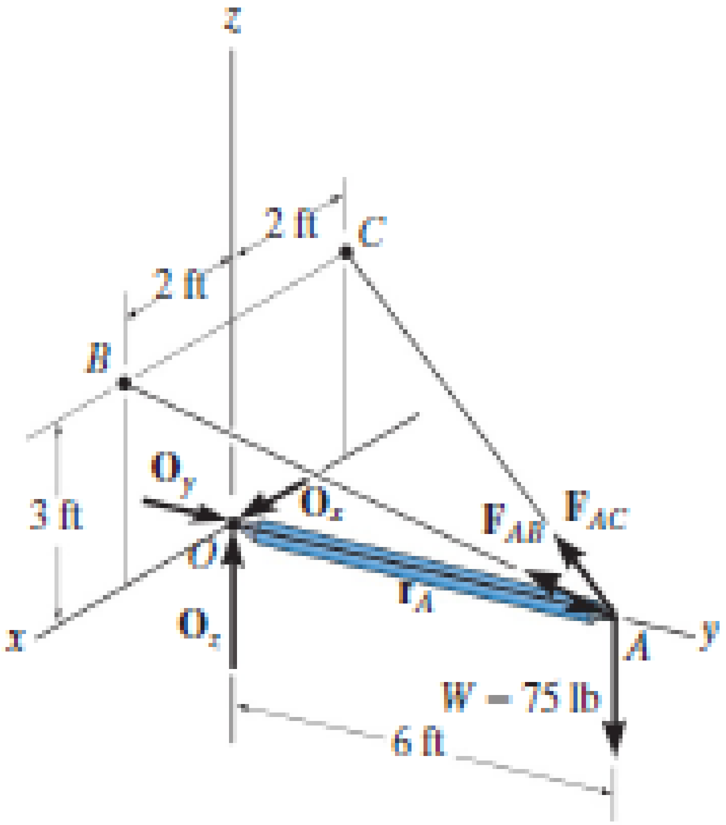
Equations of Equilibrium. Here the cable forces are directec angles with the coordinate axes, so we will use a vector analysis

$$\mathbf{F}_{AB} = F_{AB}\left(\frac{\mathbf{r}_{AB}}{r_{AB}}\right) = F_{AB}\left(\frac{\{2\mathbf{i} - 6\mathbf{j} + 3\mathbf{k}\}\ \text{ft}}{\sqrt{(2\ \text{ft})^2 + (-6\ \text{ft})^2 + (3\ \text{ft})^2}}\right)$$

$$= \tfrac{2}{7}F_{AB}\mathbf{i} - \tfrac{6}{7}F_{AB}\mathbf{j} + \tfrac{3}{7}F_{AB}\mathbf{k}$$

$$\mathbf{F}_{AC} = F_{AC}\left(\frac{\mathbf{r}_{AC}}{r_{AC}}\right) = F_{AC}\left(\frac{\{-2\mathbf{i} - 6\mathbf{j} + 3\mathbf{k}\}\ \text{ft}}{\sqrt{(-2\ \text{ft})^2 + (-6\ \text{ft})^2 + (3\ \text{ft})^2}}\right)$$

$$= -\tfrac{2}{7}F_{AC}\mathbf{i} - \tfrac{6}{7}F_{AC}\mathbf{j} + \tfrac{3}{7}F_{AC}\mathbf{k}$$



(b)

SOLUTION

We can eliminate the force reaction at O by writing the moment equation of equilibrium about point O.

$$\Sigma \mathbf{M}_O = 0; \qquad \mathbf{r}_A \times (\mathbf{F}_{AB} + \mathbf{F}_{AC} + \mathbf{W}) = \mathbf{0}$$

$$(6\mathbf{j}) \times \left[\left(\tfrac{2}{7}F_{AB}\mathbf{i} - \tfrac{6}{7}F_{AB}\mathbf{j} + \tfrac{3}{7}F_{AB}\mathbf{k}\right) + \left(-\tfrac{2}{7}F_{AC}\mathbf{i} - \tfrac{6}{7}F_{AC}\mathbf{j} + \tfrac{3}{7}F_{AC}\mathbf{k}\right) + (-75\mathbf{k})\right] = \mathbf{0}$$

$$\left(\tfrac{18}{7}F_{AB} + \tfrac{18}{7}F_{AC} - 450\right)\mathbf{i} + \left(-\tfrac{12}{7}F_{AB} + \tfrac{12}{7}F_{AC}\right)\mathbf{k} = \mathbf{0}$$

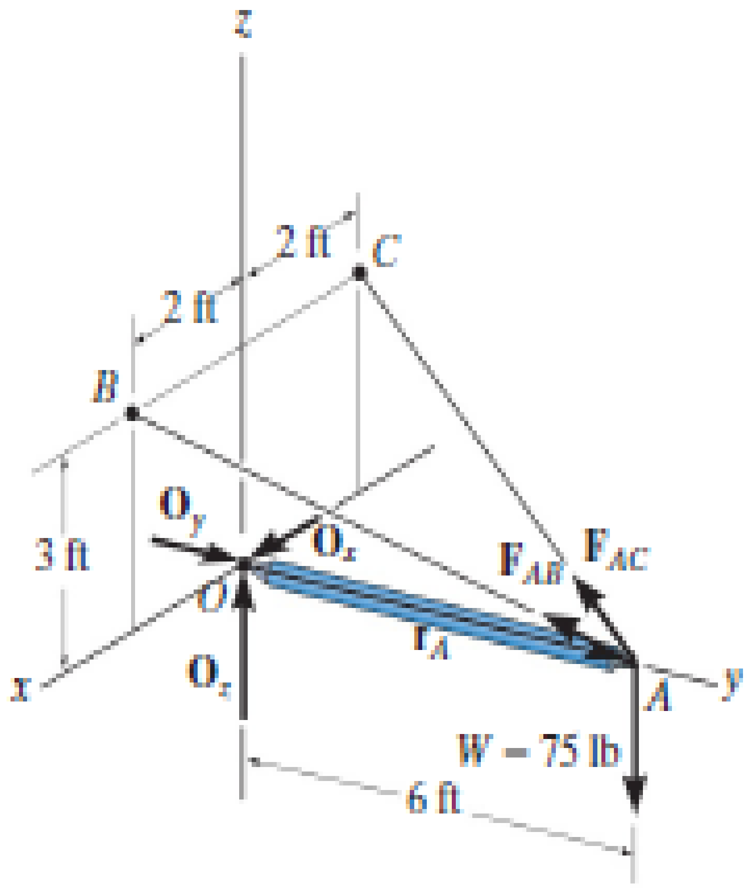
$$\Sigma M_x = 0; \qquad \tfrac{18}{7}F_{AB} + \tfrac{18}{7}F_{AC} - 450 = 0 \qquad (1)$$

$$\Sigma M_y = 0; \qquad 0 = 0$$

$$\Sigma M_z = 0; \qquad -\tfrac{12}{7}F_{AB} + \tfrac{12}{7}F_{AC} = 0 \qquad (2)$$

Solving Eqs. (1) and (2) simultaneously,

$$F_{AB} = F_{AC} = 87.5 \text{ lb} \qquad \textit{Ans.}$$



(b)

Home Assignment

- Example 5-18 & 5-19.