FORCES DUE TO MAGNETIC FIELDS

Forces Due To Magnetic Fields

- We considered the basic laws and techniques commonly used in calculating magnetic field **B** due to current-carrying elements
- Now we will study the force a magnetic field exerts on charged particles, current elements, and loops
- Such a study is important to problems on electrical devices such as ammeters, voltmeters, galvanometers, motors, and magneto-hydrodynamic generators

Forces Due To Magnetic Fields

- There are at least three ways in which force due to magnetic fields can be experienced

- The force can be:

1. Due to a moving charged particle in a **B** field,
2. On a current element in an external **B** field,
3. Between two current elements

Force On a Charged Particle

- The electric force $\mathbf{F}_e$ on a stationary or moving electric charge Q in an electric field is given by Coulomb's experimental law and is related to the electric field intensity $\mathbf{E}$ as:

$$\mathbf{F}_e = Q\mathbf{E}$$

- A magnetic field can exert force only on a moving charge

- From experiments, it is found that the magnetic force $\mathbf{F}_m$ experienced by a charge Q moving with a velocity $\mathbf{u}$ in a magnetic field $\mathbf{B}$ is:

$$\mathbf{F}_m = Q\mathbf{u} \times \mathbf{B}$$

- $\mathbf{F}_m$ is perpendicular to both $\mathbf{u}$ and $\mathbf{B}$.

Force On a Charged Particle

- Electric force $\mathbf{F}_e$ is independent of the velocity of the charge and can perform work on the charge and change its kinetic energy

- On the other hand, $\mathbf{F}_m$ depends on the charge velocity and is normal to it

- $\mathbf{F}_m$ cannot perform work because it is at right angles to the direction of motion of the charge ($\mathbf{F}_m \bullet d\mathbf{l} = 0$), so $\mathbf{F}_m$ does not cause an increase in kinetic energy of the charge

- The magnitude of $\mathbf{F}_m$ is generally small compared to $\mathbf{F}_e$ except at high velocities

Force On a Charged Particle

- For a moving charge *Q* in the presence of both electric and magnetic fields, the total force on the charge is given by:

$$\mathbf{F} = \mathbf{F}_e + \mathbf{F}_m$$

- OR:

$$\mathbf{F} = Q(\mathbf{E} + \mathbf{u} \times \mathbf{B})$$

- This is known as the *Lorentz force equation*

- If the mass of the charged particle moving in **E** and **B** fields is *m*, by Newton's second law of motion

$$\mathbf{F} = m\frac{d\mathbf{u}}{dt} = Q\left(\mathbf{E} + \mathbf{u} \times \mathbf{B}\right)$$

Force On a Current Element

- Force on a conduction current element "$I d\mathbf{l}$" of a current-carrying conductor due to the magnetic field **B** can be determined from the equation of force on a moving charged particle

- We have:

$$I\, d\mathbf{l} = \frac{dQ}{dt}\, d\mathbf{l} = dQ\, \frac{d\mathbf{l}}{dt} = dQ\, \mathbf{u}$$

- Therefore, an elemental charge dQ moving with velocity **u** (thereby producing convection current element "dQ **u**") is equivalent to a conduction current element $I d\mathbf{l}$, *that* is:

$$I\, d\mathbf{l} = dQ\, \mathbf{u}$$

Force On a Current Element

- Thus the force on a current element $I d\mathbf{l}$ in a magnetic field **B** is found by merely replacing "Q**u**" by "I d**l**"; that is:

$$d\mathbf{F} = I\, d\mathbf{l} \times \mathbf{B}$$

- If the current I is through a closed path L or circuit, the force on the circuit is given by:

$$\mathbf{F} = \oint_L I\, d\mathbf{l} \times \mathbf{B}$$

- Keep in mind that the magnetic field produced by the current element $I d\mathbf{l}$ does not exert force on the element itself just as a point charge does not exert force on itself

Force On a Current Element

- If instead of the line current element $I d\mathbf{l}$, we have surface current element $\mathbf{K}\ dS$ and volume current element $\mathbf{J}\ dv$, the differential force is:

$$d\mathbf{F} = \mathbf{K}\, dS \times \mathbf{B} \qquad \text{or} \qquad d\mathbf{F} = \mathbf{J}\, dv \times \mathbf{B}$$

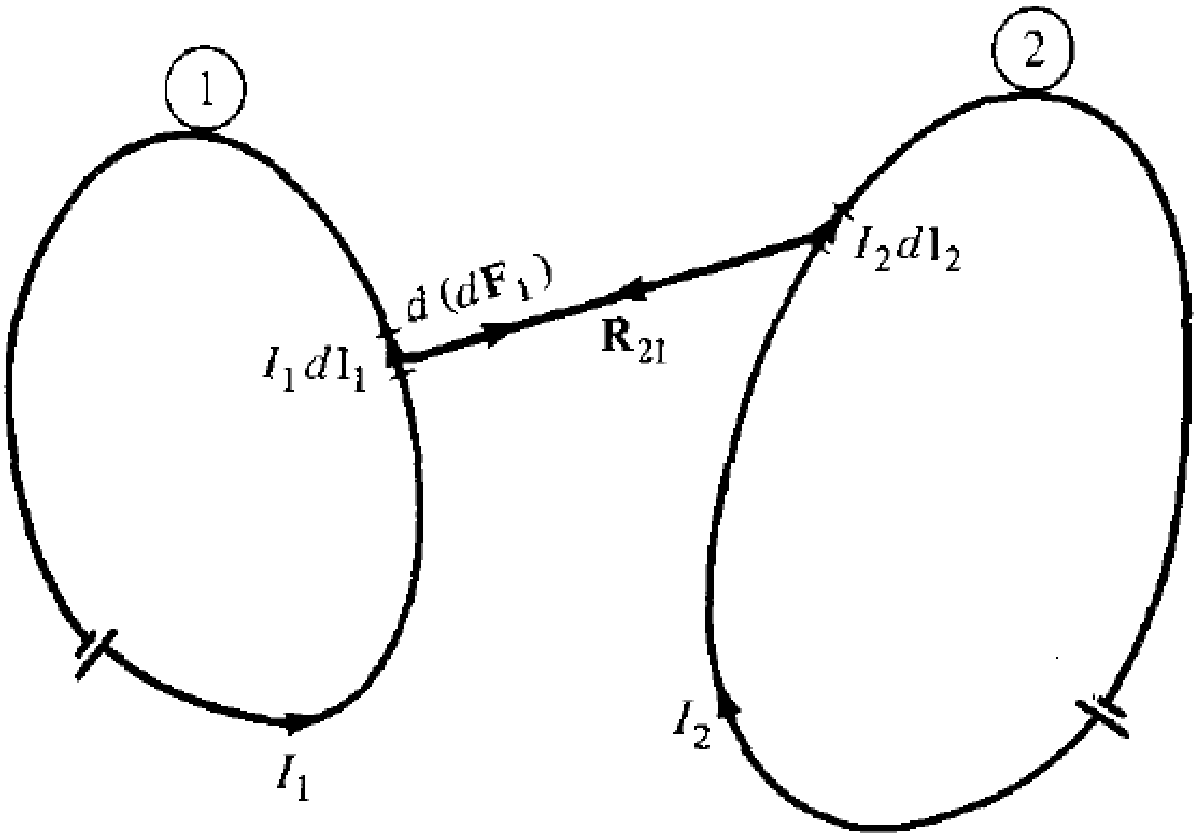
- For a closed surface S or a closed volume, we have:

$$\mathbf{F} = \int_S \mathbf{K}\, dS \times \mathbf{B} \qquad \text{or} \qquad \mathbf{F} = \int_v \mathbf{J}\, dv \times \mathbf{B}$$

- The **B** field that exerts force on the current elements must be due to another external element

Force Between Two Current Elements

- We now consider the force between two current elements $I_1 d\mathbf{l}_1$ and $I_2 d\mathbf{l}_2$, as shown in figure below



Force Between Two Current Elements

- The force $d(dF_1)$ on element $I_1 d\mathbf{l}_1$ due to the field $d\mathbf{B}_2$ produced by element $I_2 d\mathbf{l}_2$ is given as:

$$d(d\mathbf{F}_1) = I_1\, d\mathbf{l}_1 \times d\mathbf{B}_2$$

- But from Biot-Savart's law, we have:

$$d\mathbf{B}_2 = \frac{\mu_o I_2\, d\mathbf{l}_2 \times \mathbf{a}_{R_{21}}}{4\pi R_{21}^2}$$

- Hence:

$$d(d\mathbf{F}_1) = \frac{\mu_o I_1\, d\mathbf{l}_1 \times (I_2\, d\mathbf{l}_2 \times \mathbf{a}_{R_{21}})}{4\pi R_{21}^2}$$

Force Between Two Current Elements

- This equation is essentially the law of force between two current elements and is analogous to Coulomb's law, which expresses the force between two stationary charges

- The total force $\mathbf{F}_1$, on current loop 1 due to current loop 2 is:

$$\mathbf{F}_1 = \frac{\mu_o I_1 I_2}{4\pi} \oint_{L_1} \oint_{L_2} \frac{d\mathbf{l}_1 \times (d\mathbf{l}_2 \times \mathbf{a}_{R_{21}})}{R_{21}^2}$$

- The force $\mathbf{F}_2$ on loop 2 due to the magnetic field $\mathbf{B}_1$ from loop 1 is obtained from the above equation by interchanging subscripts 1 and 2

Problem-1

- In a velocity filter, uniform **E** and **B** fields are oriented at right angles to each other. An electron moves with a speed of 8 x 10^6 $\mathbf{a}_x$ m/s at right angles to both fields and passes un-deflected through the field.

(a) If the magnitude of **B** is 0.5 $\mathbf{a}_z$ mWb/m^2, find the value of E $\mathbf{a}_y$.

(b) Will this filter work for positive and negative charges and any value of mass?

Problem-2

- A conducting current strip carrying $\mathbf{K} = 12\mathbf{a}_z$ A/m lies in the x = 0 plane between y = 0.5 and y = 1.5 m. There is also a current filament of I = 5 A in the $\mathbf{a}_z$ direction on the z axis. Find the force per unit length exerted on the:

a) filament by the current strip:

b) strip by the filament:

$$\mathbf{F} = \oint_L I\, d\mathbf{l} \times \mathbf{B} \qquad\qquad \mathrm{B} = \frac{\mu_0 I}{2\pi d}\mathrm{a}_n$$