Faraday's Law-II

Dr. M. Imran Malik

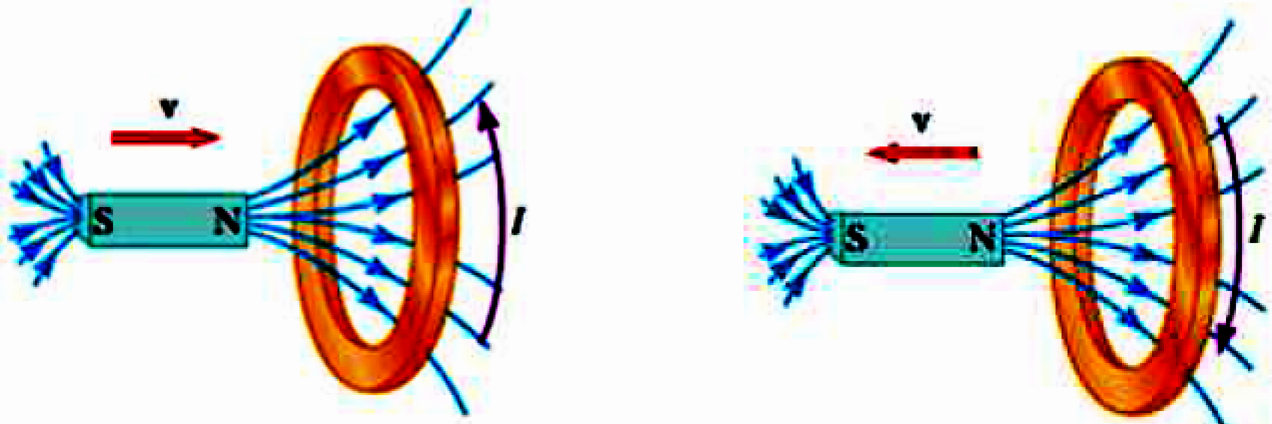
School of Electrical Engineering & Computer Science
National University of Sciences & Technology (NUST), Pakistan

In a commercial electric power plant, large generators produce energy that is transferred out of the plant by electrical transmission. These generators use magnetic induction to generate a potential difference when coils of wire in the generator are rotated in a magnetic field. The source of energy to rotate the coils might be falling water, burning fossil fuels, or a nuclear reaction. (Michael Melford/Getty Images)

Induced Electric Field

- As magnetic flux through the loop varies, induced emf and hence induced current flows through the loop.
- Before the magnetic flux began changing, there was no current in the loop; while the flux is changing, charges flow in the loop (induced current).
- For charges to begin moving, they must be accelerated by an electric field. Thus Faraday's law says that

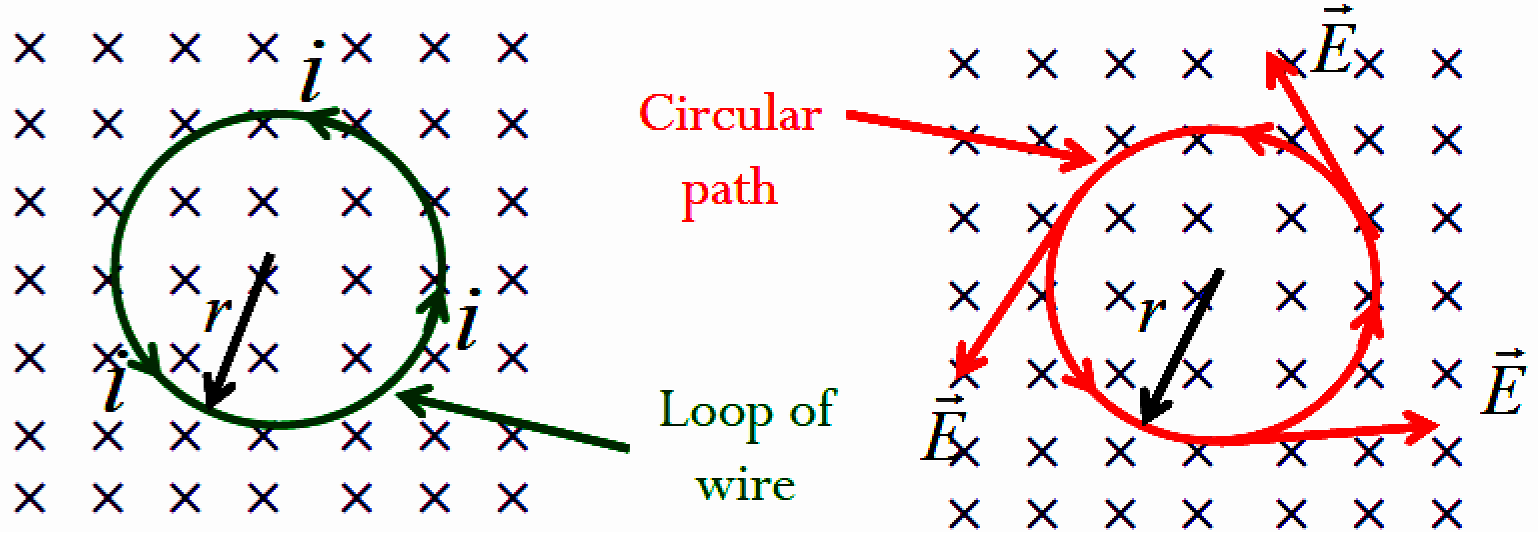
A changing magnetic flux induces an electric field.



Induced electric field is *non-conservative*.

Consider a steadily increasing magnetic field directed into the plan. If we place a circular loop of radius r there, an induced current starts flowing. Its mean circular electric field is induced in the region of changing magnetic flux. The induced electric field is as real as any that might be set by electric charges; for instance, it exerts a force q_0E on a test charge. Moreover, presences of electric field has nothing to do with the presence of loop of wire; if we were to remove the loop completely, electric field would still be present.

Increasing B field



Consider a test charge q_0 moving along the circular path of radius r.

The work done on the charge by induced electric field in one revolution is

$$W = \xi q_{\circ} \qquad (1)$$

Equivalently we can express this work as the electric force times displacement covered in one revolution

$$W = Fd = q_{\circ} E(2\pi r) \qquad (2)$$

From (1) and (2)

$$\xi = E(2\pi r)$$

The right side of above equation can be expressed as the line integral of E around the circle, which can be written in more general cases (for instance when E is not constant or when the chosen closed path is not circle) as

$$\xi = \oint_C \vec{E} \bullet \vec{dl}$$

Thus Faraday's law of induction can be written as

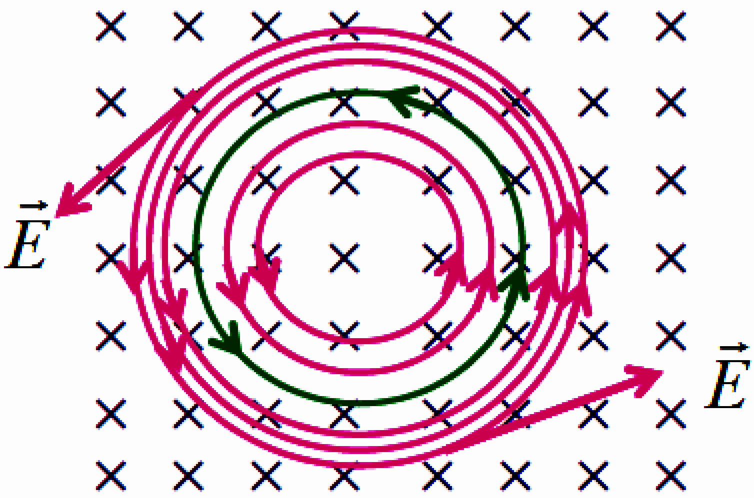
$$\boxed{\oint_C \vec{E} \bullet \vec{dl} = -N \frac{d\Phi}{dt}}$$

$$\therefore \xi = -N \frac{d\Phi}{dt}$$

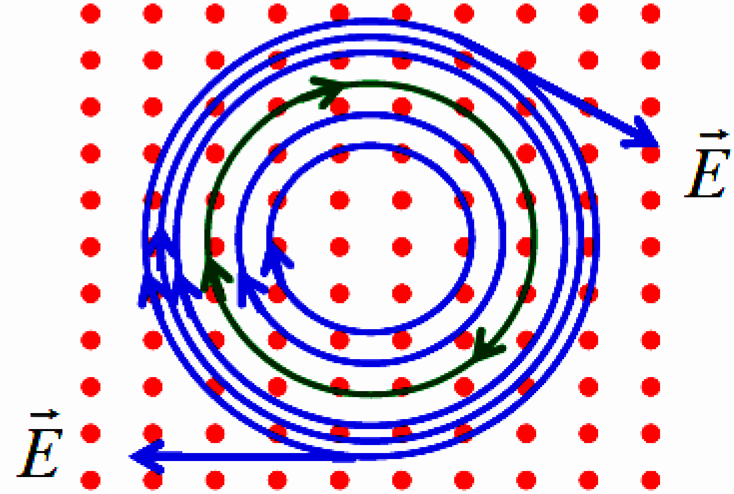
Thus Faraday's law of induction can be written as

A changing magnetic flux induces an electric field.

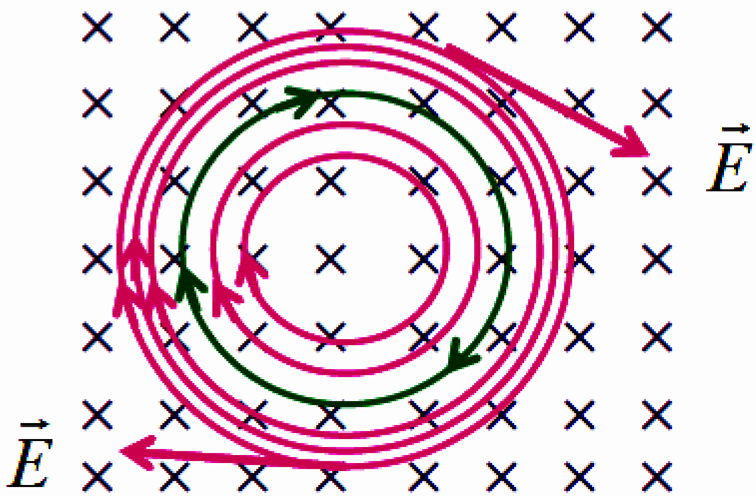
Increasing **B** field



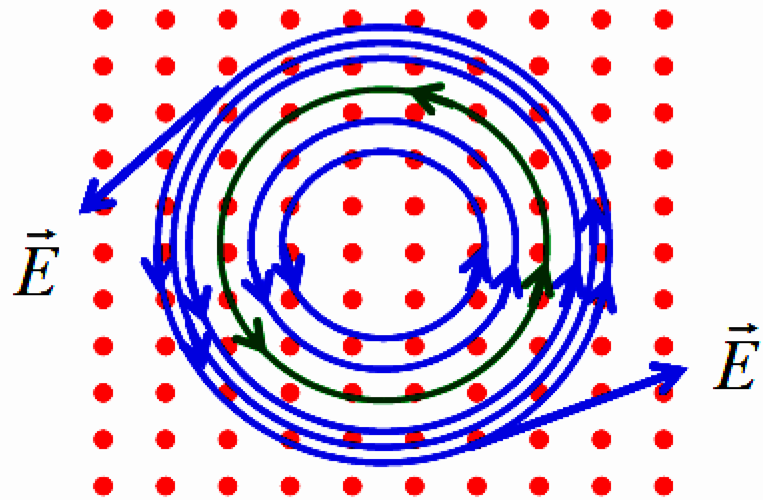
Increasing **B** field



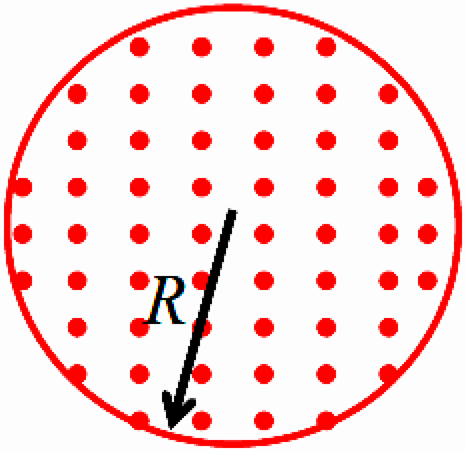
Decreasing **B** field



Decreasing **B** field



In the figure shown below R = 8.5cm and magnetic field is increasing uniformly at the rate 0.13T/s . What will be the magnitude of electric field at the distance (a) r = 5.2cm (b) r=12.5cm



(a) r = 5.2cm

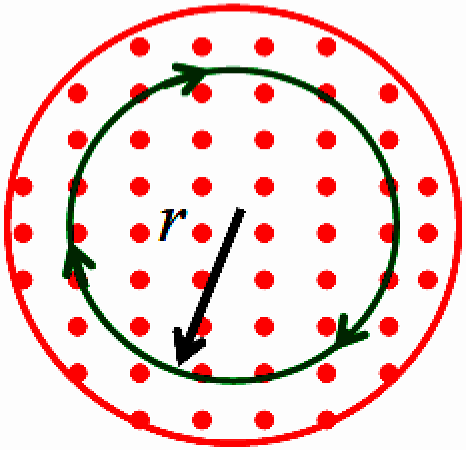
Consider an imaginary circular path of radius r.

$$\oint_C \vec{E} \bullet \vec{dl} = -\frac{d\Phi_B}{dt}$$

$$E(2\pi r) = -\frac{d(BA)}{dt}$$

$$E(2\pi r) = -(\pi r^2)\frac{dB}{dt}$$

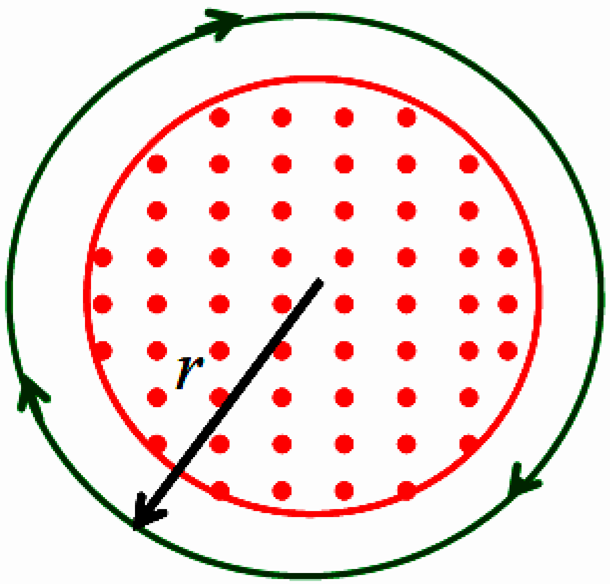
$$|E| = \frac{1}{2}\frac{dB}{dt}r = 3.4mV/m$$



(b) r = 12.5cm

Consider an imaginary circular path of radius r.

$$\oint_C \vec{E} \bullet \vec{dl} = -\frac{d\Phi_B}{dt}$$



$$E(2\pi r) = -\frac{d(BA)}{dt}$$

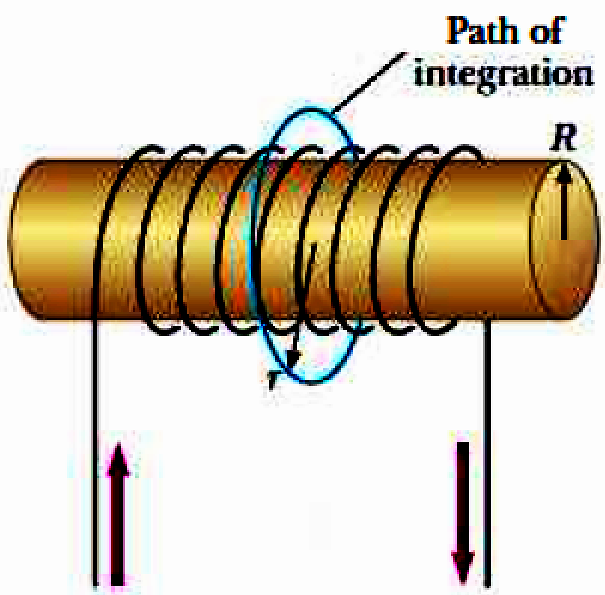
$$E(2\pi r) = -(\pi R^2)\frac{dB}{dt}$$

$$|E| = \frac{1}{2}\frac{dB}{dt}\frac{R^2}{r} = 3.8mV/m$$

A long solenoid of radius R *has n turns of wire per unit* length and carries a time-varying current that varies sinusoidally as $I = I_0 \cos\omega t$, *where* I_0 *is the maximum current* and ω is the angular frequency of the alternating current source . Determine the magnitude of the induced electric field;

(a) Outside the solenoid, a distance $r > R$ from its long central axis.

(b) Inside the solenoid, a distance $r < R$ from its long central axis.



(a) Outside the solenoid: Here magnetic field inside the solenoid is

$$B = \mu_{\circ} n i = \mu_{\circ} n i_{\circ} \cos \omega t$$

Let's consider an imaginary loop of radius r > R. The magnetic flux through this loop will be

$$\Phi_B = \int \vec{B} \bullet d\vec{A} = B \int dA = \mu_{\circ} n i_{\circ} \cos \omega t \, \pi R^2$$

So the induced electric field is

$$\oint_C \vec{E} \bullet \vec{dl} = -\frac{d\Phi_B}{dt}$$

$$E(2\pi r) = \mu_{\circ} n i_{\circ} \omega \sin(\omega t) \pi R^2$$

$$E = \frac{\mu_{\circ} n i_{\circ} \omega R^2}{2r} \sin(\omega t)$$

(b) Inside the solenoid: Here magnetic field inside the solenoid is

$$B = \mu_o n i = \mu_o n i_o \cos \omega t$$

Let's consider an imaginary loop of radius r < R. The magnetic flux through this loop will be

$$\Phi_B = \int \vec{B} \bullet d\vec{A} = B\int dA = \mu_o n i_o \cos(\omega t)\pi r^2$$

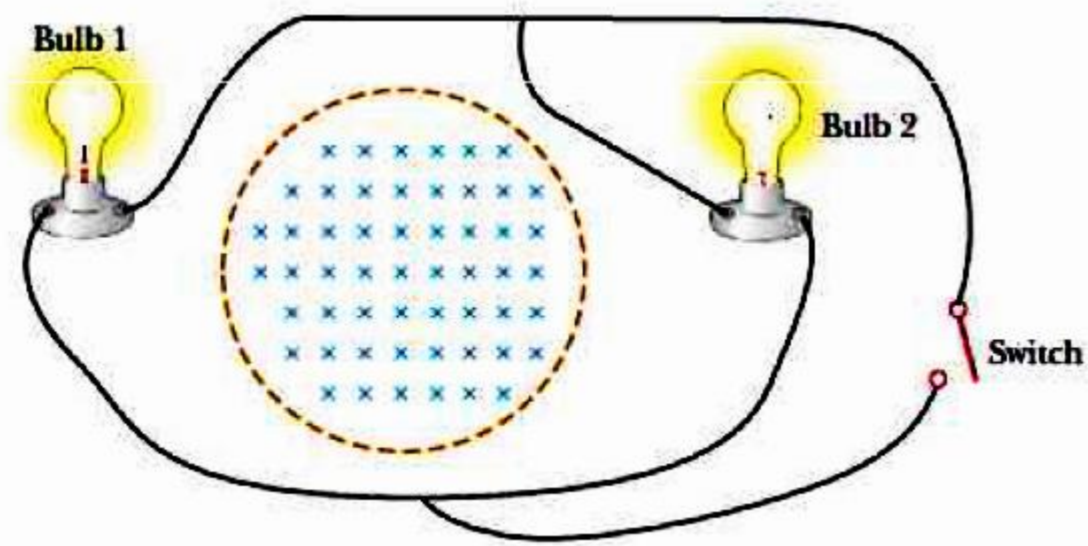
So the induced electric field is

$$\oint_C \vec{E} \bullet \vec{dl} = -\frac{d\Phi_B}{dt}$$

$$E(2\pi r) = \mu_o n i_o \omega \sin(\omega t)\pi r^2$$

$$E = \frac{\mu_o n i_o \omega}{2} r \sin(\omega t)$$

Two bulbs are connected to opposite sides of a loop of wire, as shown in Figure. A decreasing magnetic field (confined to the circular area shown in the figure) induces an emf in the loop that causes the two bulbs to light. What happens to the brightness of the bulbs when the switch is closed?



Bulb 1 glows brighter, and bulb 2 goes out.

Once the switch is closed, bulb 1 is in the large loop consisting of the wire to which it is attached and the wire connected to the switch. Because the changing magnetic flux is completely enclosed within this loop, a current exists in bulb 1. Bulb 1 now glows brighter than before the switch was closed because it is now the only resistance in the loop. As a result, the current in bulb 1 is greater than when bulb 2 was also in the loop.

Once the switch is closed, bulb 2 is in the loop consisting of the wires attached to it and those connected to the switch. There is no changing magnetic flux through this loop and hence no induced emf.