NATIONAL UNIVERSITY OF SCIENCES & TECHNOLOGY

Linear Control Systems (EE-371)

Assignment # 2

(CLO-2)

Submission Details

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In this case study, we investigate the time response of the vehicle dynamics that relate the pitch angle output to the elevator deflection input.

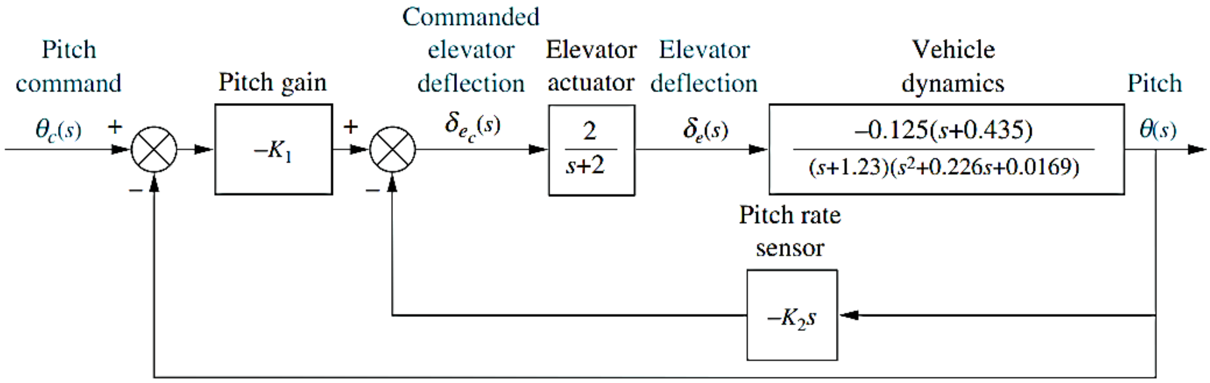


Figure 1: Pitch control loop for the UFSS vehicle

The transfer function relating pitch angle, *$\theta(s)$*, to elevator surface angle, *$\delta_e(s)$*, for the UFSS vehicle is:

$$\frac{\theta(s)}{\delta_e(s)} = \frac{-0.125(s+0.435)}{(s+1.23)(s^2+0.226s+0.0169)}$$

Requirements:

1. Using only the second-order poles shown in the transfer function, predict percent overshoot, rise time, peak time, and settling time? [do it by hand]

In MATLAB: [submit proper MATLAB codes and response plots]

2. Using Laplace transforms, find the analytical expression for the response of the pitch angle to a step input in elevator surface deflection. [Use MATLAB to find the time domain expression by taking inverse Laplace transform].
3. Plot the step response of the system having just two poles [Let's say there is pole zero cancellation. Maintain the steady state gain of system].
4. Plot the step response of the system having just two poles and a zero [Let's say additional pole is cancelled by 5 times rule of thumb but maintain the steady state gain of your system]
5. Plot the step response of the complete system [having 3 poles and a zero].
6. Compare the responses in just 2 to 3 lines.

1 Rise Time

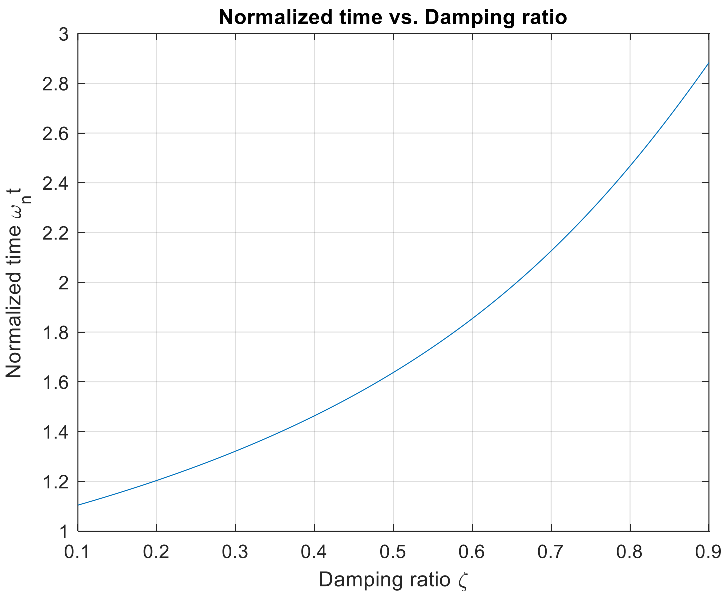
```
%% Rise Time
syms wn_t zeta;
zeta_fit = linspace(0.1, 0.9, 5000);
c_t = 1 - (1/sqrt(1-zeta^2))*exp(-zeta*wn_t)*cos(wn_t*sqrt(1-zeta^2)- ...
    atan(zeta/sqrt(1-zeta^2)));
c_fit = subs(c_t, zeta, zeta_fit);

norm_time = zeros(1, 5000);
for i = 1:5000
    t1 = vpasolve(c_fit(i) == 0.9, wn_t, [0 5]);
    t2 = vpasolve(c_fit(i) == 0.1, wn_t, [0 5]);
    norm_time(i) = t1 - t2;
end

plot(zeta_fit, norm_time)
grid
title('Normalized time vs. Damping ratio')
xlabel('Damping ratio \zeta')
ylabel('Normalized time \omega_{n}t')

% polyfit returns coefficients of nth order polynomial
P = polyfit(zeta_fit, norm_time, 4);
poly2sym(P, zeta)

                                Output
P =
    0.5089    0.7837    0.2144    0.8848    1.0114
```



Resultant polynomial can be written in terms of zeta as,

$$\omega_n T_r = 0.5089\zeta^4 + 0.7837\zeta^3 + 0.2144\zeta^2 + 0.8848\zeta + 1.0114$$

, which is used to find the rise time by substituting $\zeta = 0.8692$.

2 Definitions

```
z1 = -0.435; p1 = [-1.23 -0.113-1j*0.0642 -0.113+1j*0.0642]; k1 = -0.125;
z2 = -0.435; p2 = [-0.113-1j*0.0642 -0.113+1j*0.0642]; k2 = -0.1016;
z3 = []; p3 = [-0.113-1j*0.0642 -0.113+1j*0.0642]; k3 = -0.0442;

G1 = zpk(z1, p1, k1); % Represents original tf
G2 = zpk(z2, p2, k2); % Represents G1 + 5 times rule reduction
G3 = zpk(z3, p3, k3); % Represents G1 + pole-zero cancellation reduction

G1 =
          -0.125 (s+0.435)
  ---------------------------------
  (s+1.23) (s^2 + 0.226s + 0.01689)

Continuous-time zero/pole/gain model.

G2 =
     -0.1016 (s+0.435)
  ------------------------
  (s^2 + 0.226s + 0.01689)

Continuous-time zero/pole/gain model.

G3 =
          -0.0442
  ------------------------
  (s^2 + 0.226s + 0.01689)

Continuous-time zero/pole/gain model.
```

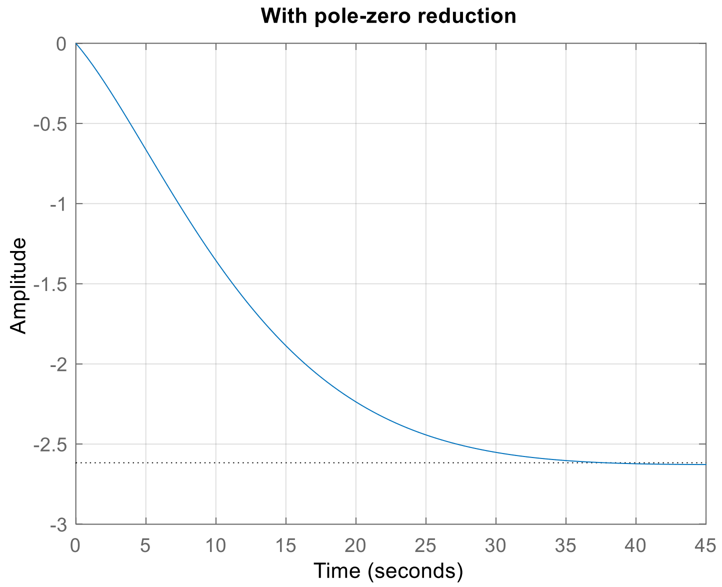
3 Time Domain Expression

```
syms s;
G1_s = (-0.125*(s+0.435))/((s+1.23)*(s^2 + 0.226*s + 0.0169))
g1_t = ilaplace(1/s*G1_s)
display(g1_t)

                                        Output
g1_t =
(28352500*exp(-(113*t)/1000)*(cos((9*51^(1/2)*t)/1000) +
(1891291*51^(1/2)*sin((9*51^(1/2)*t)/1000))/10411038))/10577879 - (165625*exp(-
(123*t)/100))/2566231 - 18125/6929
```

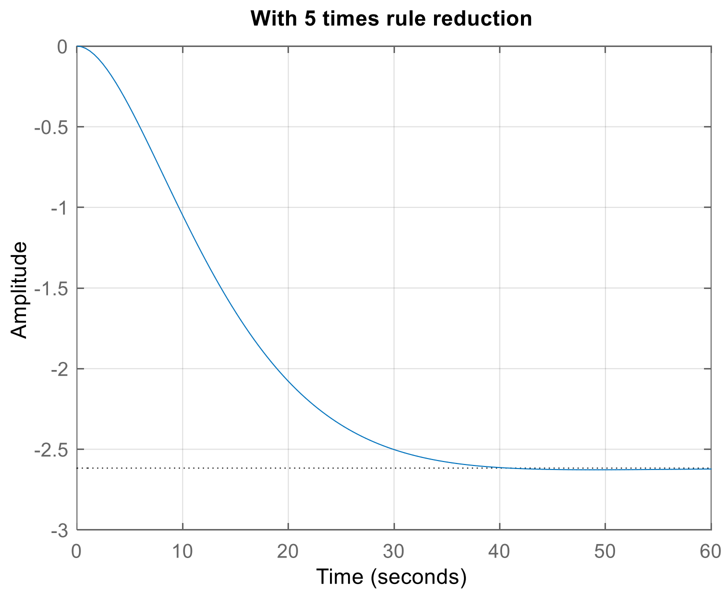
4 Step Response (With Pole-Zero Reduction)

```
figure
step(G2)
grid
title('With pole-zero reduction')
```



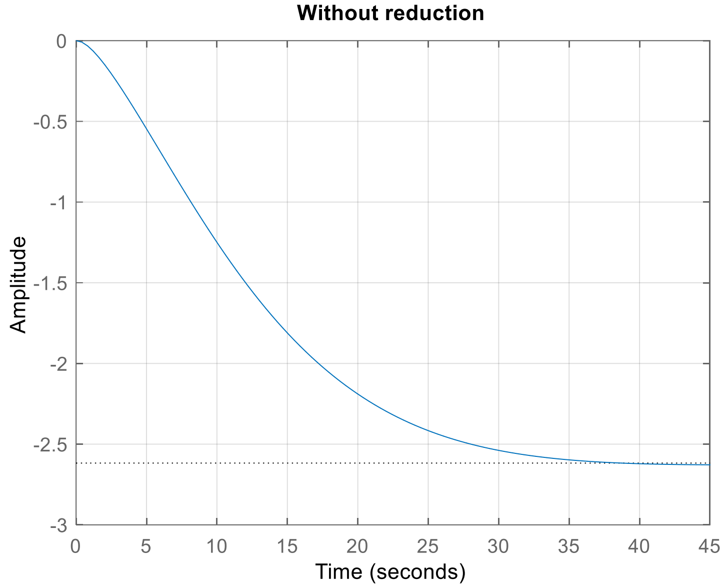
5 Step Response (With 5-Times Rule Reduction)

```
figure
step(G3)
grid
title('With 5 times rule reduction')
```



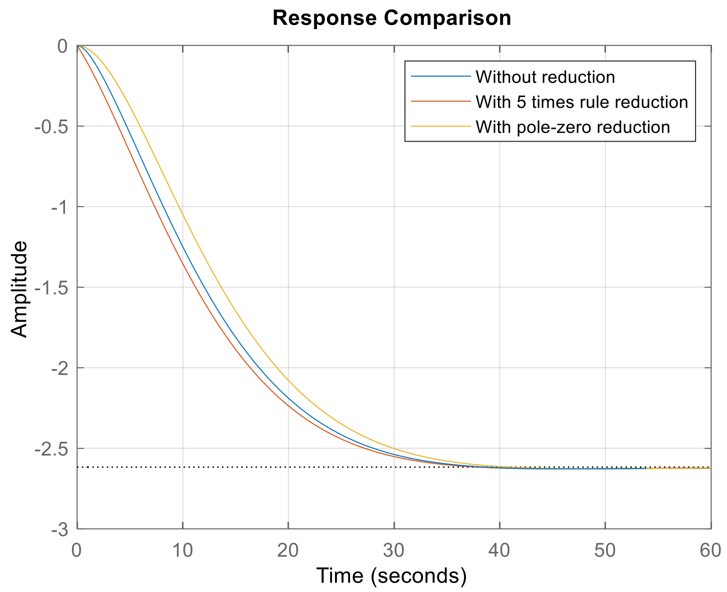
6 Step Response (Without Reduction)

```
figure
step(G1)
grid
title('Without reduction')
```



7 Comparison

```
hold on
step(G1); step(G2); step(G3);
legend('Without reduction', 'With pole-zero reduction', 'With 5 times rule reduction')
grid on
title('Response Comparison')
```



Comments: The transfer function reduced using 5-times rule has the fastest response as it can be inferred as a second order system with an additional zero, and zeros decrease the response time albeit increasing the overshoot. The complete system also has a faster response than the system reduced to a simple second order system using pole-zero cancellation as the additional zero at -0.435, which is closer to the imaginary axis than the pole at -1.23, has a greater impact on the transient response and thus the system's overall speed.