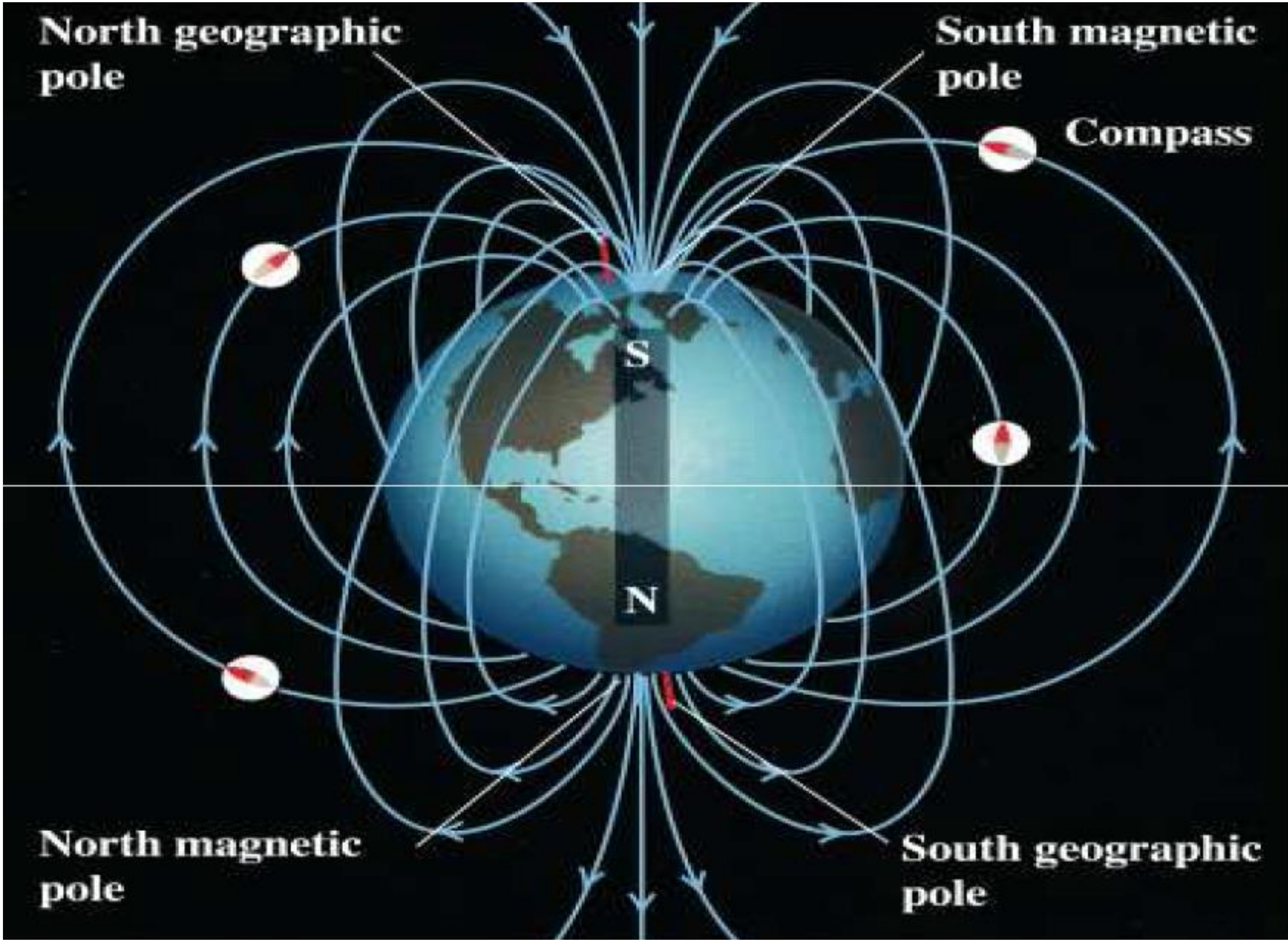
Magnetic Field-I

Dr. M. Imran Malik

School of Electrical Engineering & Computer Science
National University of Sciences & Technology (NUST), Pakistan


North geographic pole
South magnetic pole
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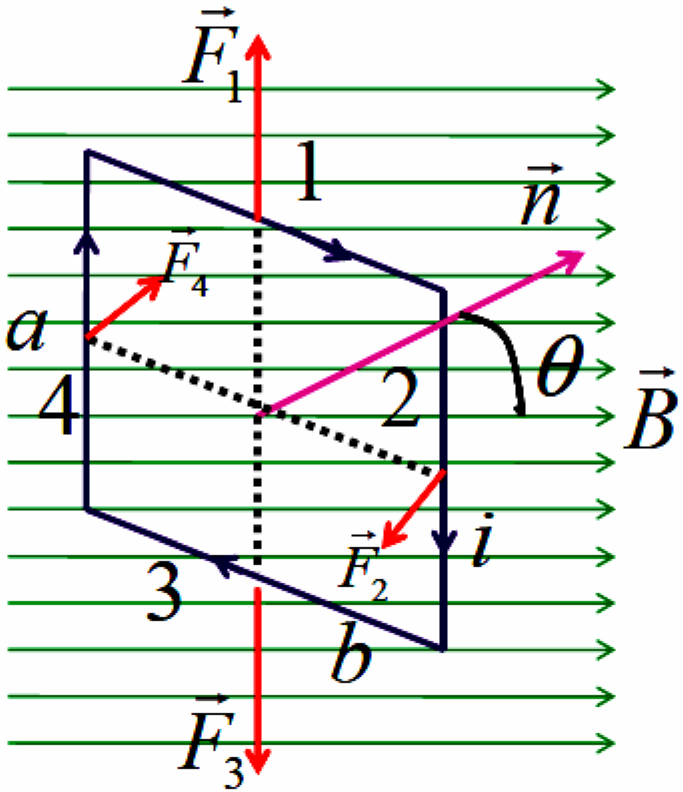
North magnetic pole
South geographic pole

Torque on a Current Carrying Loop

Consider a rectangular loop of length a and width b placed inside a uniform magnetic field B such that plane of loop makes an angle θ with B. If current *i* passes through the loop clockwise, magnetic forces on four sides 1,2,3 and 4 will be

$$F_1 = F_3 = ibB\sin(90^\circ - \theta) = ibB\sin\theta$$

$$F_2 = F_4 = iaB$$



Line of action for forces F1 and F3 is same, so they will cancel each other's effect. Forces F1 and F4 will generate a torque in the loop

$$\tau = F_2 r_\perp + F_4 r_\perp$$

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$$\tau = F_2 \frac{b}{2}\sin\theta + F_4 \frac{b}{2}\sin\theta$$

$$= iaB(\frac{b}{2}\sin\theta) + iaB(\frac{b}{2}\sin\theta)$$

$$= iabB\sin\theta$$

$$= iAB\sin\theta \qquad\qquad \therefore A = ab$$

$$\boxed{\vec{\tau} = iA\hat{n}\times\vec{B}}$$

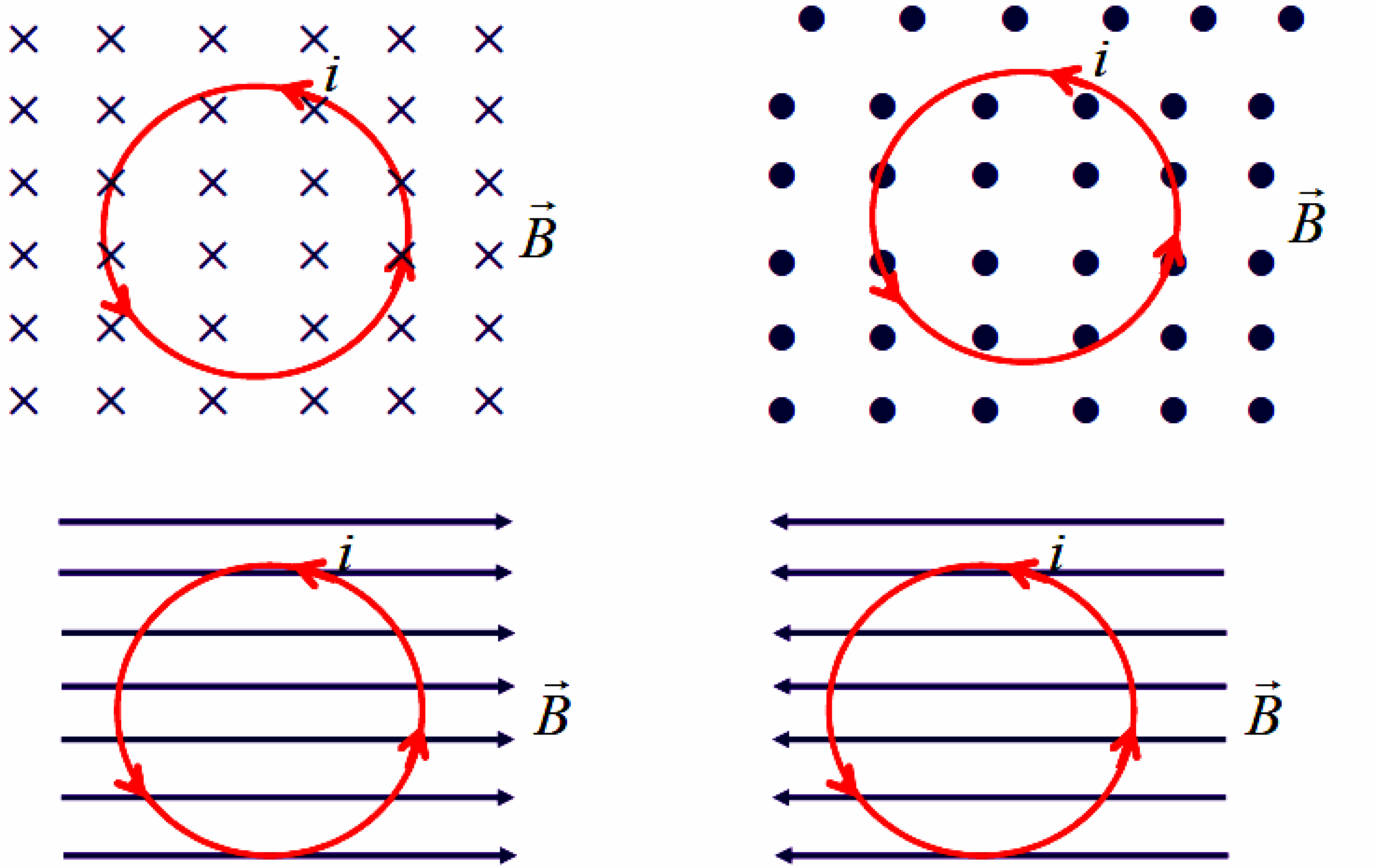
$$\boxed{\tau(\theta) = iAB\sin\theta}$$

Torque depends on current *i* through the loop, area A of the loop, strength of magnetic field B and angle θ between plane of loop and magnetic field

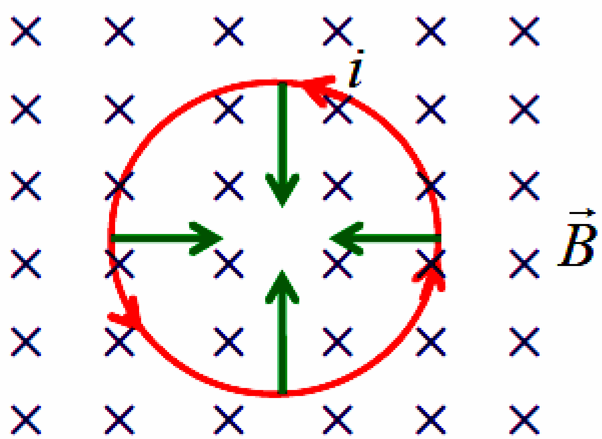
Note: Above expression for torque is valid for any shape of current carrying loop

Rotation, Contraction, and Expansion

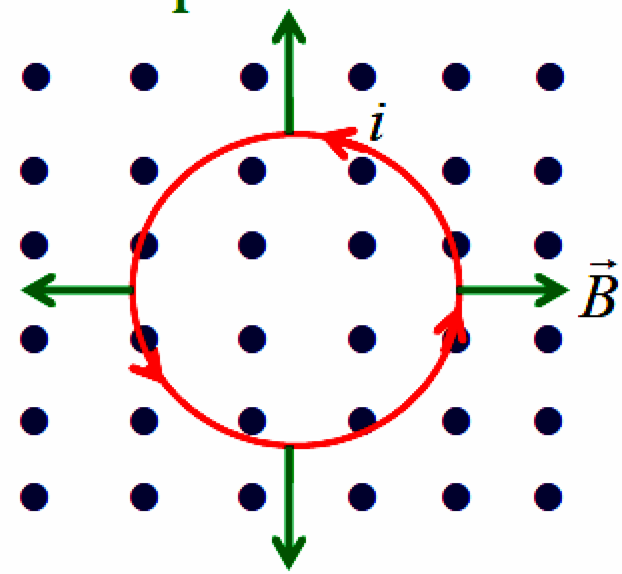
Consider a flexible current carrying loop placed in magnetic field



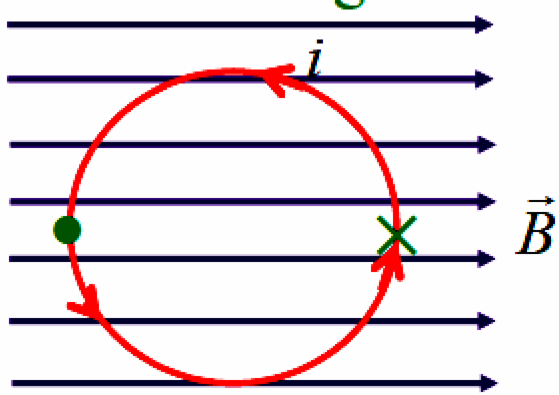
Compression



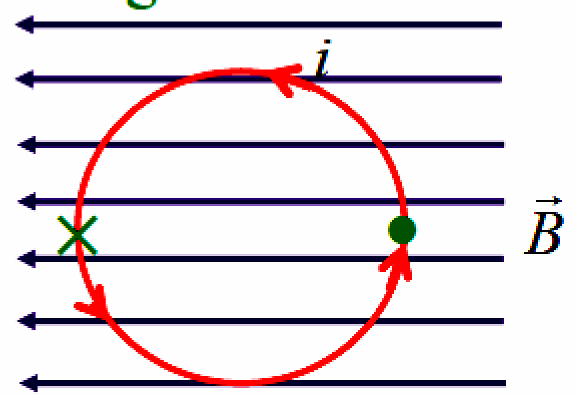
Expansion



Rotation Left to Right



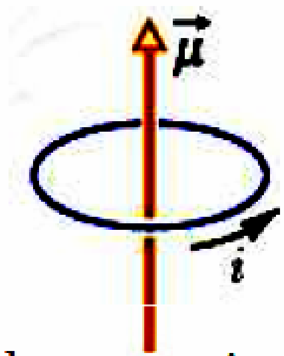
Rotation Right to Left

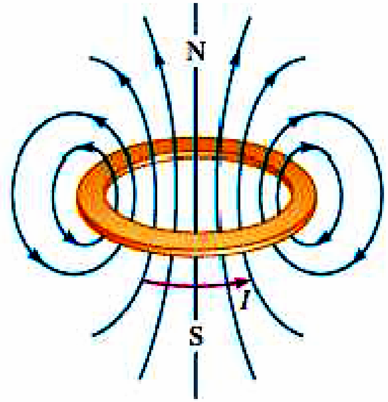


The Magnetic Dipole Moment

If current i passes through a loop of area A it behaves like a small magnetic dipole. Whose magnetic dipole moment is defined as

$$\vec{\mu} = i\vec{A}$$



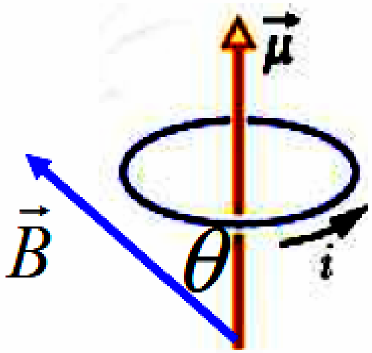


Its SI unit is $Am^2 = Nm/T = J/T$

As we have seen that when a loop of plane area A carrying current i is placed in a magnetic field B such that area makes an angle θ with B then magnetic field B exert torque on the loop

$$\vec{\tau} = iA\hat{n} \times \vec{B}$$

$$\boxed{\vec{\tau} = \vec{\mu} \times \vec{B}}$$



In dynamics, if forces acting on a system are conservative, we can represent the system equally well using either force equations or energy equations

The work done by the external field in turning the dipole from initial angle $\theta_\circ$ to some final angle θ is

$$W = \int_{\theta_\circ}^{\theta} \vec{\tau} \bullet d\vec{\theta} = \int_{\theta_\circ}^{\theta} -\tau d\theta$$

Torque tends to decrease θ

$$= -\int_{\theta_\circ}^{\theta} \mu B \sin\theta d\theta = -\mu B \int_{\theta_\circ}^{\theta} \sin\theta d\theta$$

$$= \mu B [\cos\theta]_{\theta_\circ}^{\theta}$$

$$= \mu B (\cos\theta - \cos\theta_\circ)$$

Work-Energy theorem,

$$\Delta U = -W$$

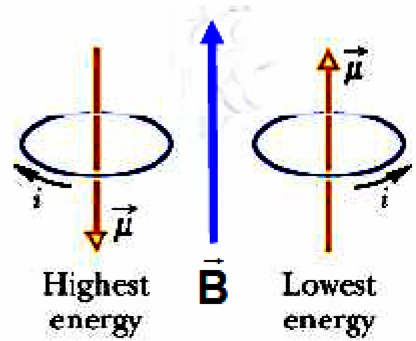
$$U - U_{\circ} = -\mu B(\cos\theta - \cos\theta_{\circ})$$

Let's choose reference point for this system to be at $\theta_{\circ} = 90^{\circ}$

So potential energy at that point will be zero $U_{\circ} = 0$

$$U = -\mu B\cos\theta$$

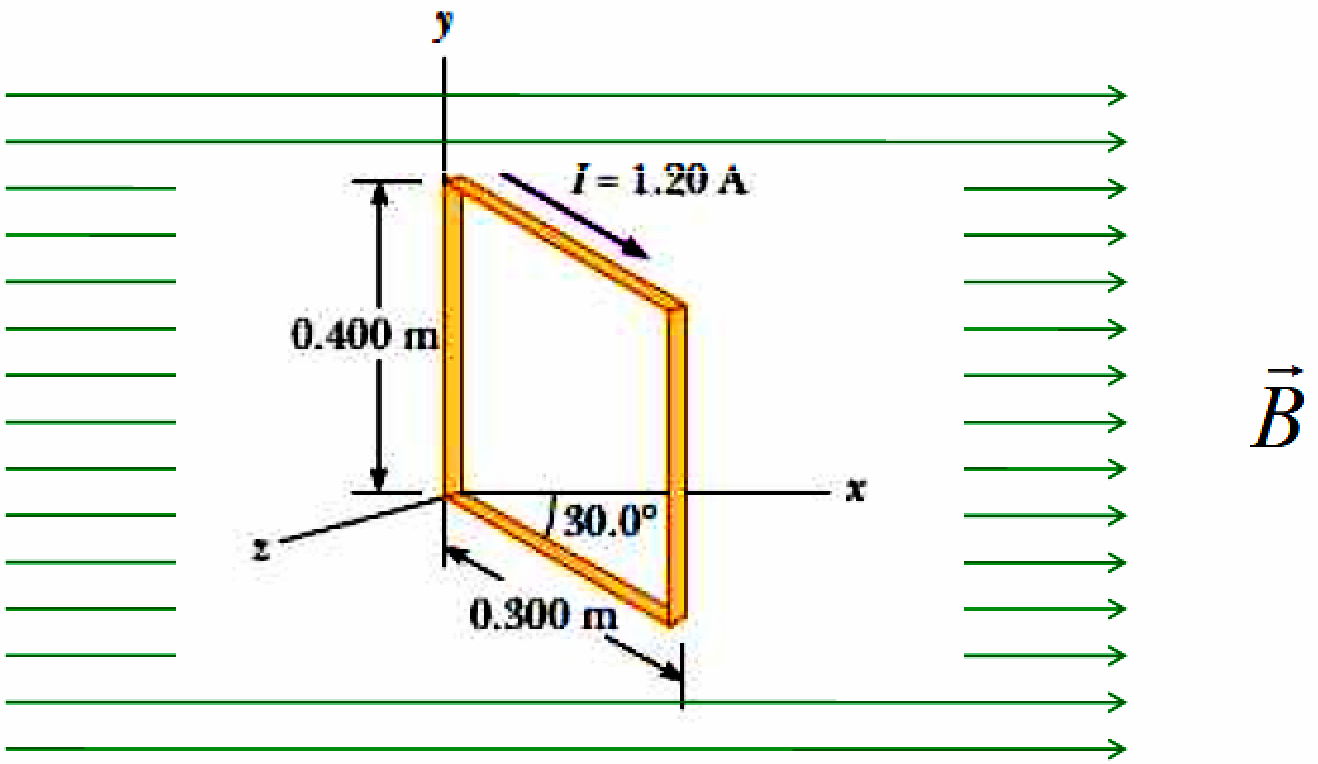
$$\boxed{U = -\vec{\mu} \bullet \vec{B}}$$



Potential energy is minimum when **μ** and **B** are parallel.

Thus we can interpret the motion of a magnetic dipole in an external magnetic field either on the basis of torque that rotates the dipole into alignment with the field or a potential energy that becomes minimum when the dipole is aligned with the field.

A rectangular current carrying loop shown in figure below consists of N=100 closely wrapped turns and is hinged along the y axis. If plane of loops makes an angle 30.0° with the x axis, what is the magnitude of the torque exerted on the loop by a uniform magnetic field B=0.8T directed along the x axis? What is the expected direction of rotation of the loop?



As $$\vec{\tau} = N\vec{\mu}\times\vec{B} = NiA\hat{n}\times\vec{B} \qquad \therefore \vec{\mu} = iA\hat{n}$$

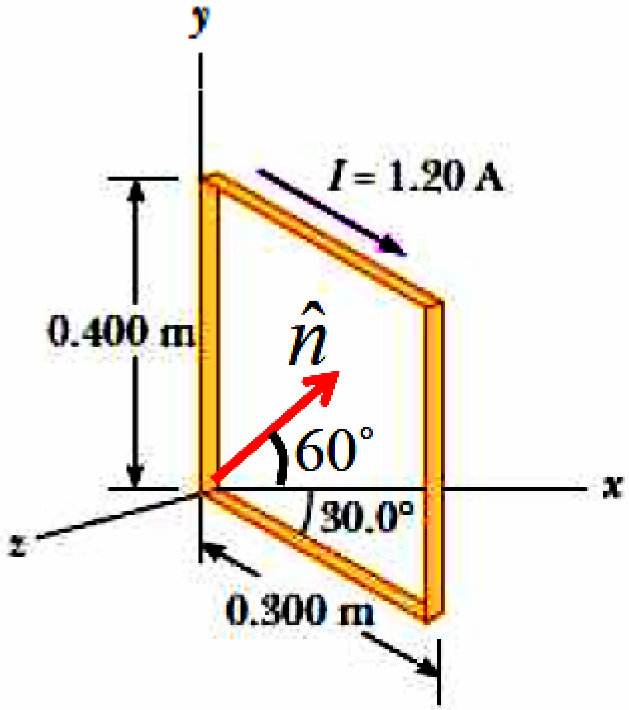
$$\tau = NiAB\sin\theta$$

$$\tau = 100(1.2)(0.4\times0.3)(0.8)\sin 60$$

$$\tau = 9.98\,Nm$$

Note that θ is the angle between the magnetic moment μ and the B field. The loop will rotate so as to align the magnetic moment with the B field.

Looking down along the y-axis, the loop will rotate in a clockwise direction.



You can see that from right hand rule.

Typical magnetic dipole moments

Small bar magnet	5 J/T
Earth	8.0×10^{22} J/T
Proton	1.4×10^{-26} J/T
Electron	9.3×10^{-24} J/T

Electric dipole and magnetic dipole

	Electric Dipole	Magnetic Dipole
Moment	$\vec{p}=q\,\vec{d}$	$\vec{\mu}=i\,\vec{A}$
Torque	$\vec{\tau}=\vec{p}\times\vec{E}$	$\vec{\tau}=\vec{\mu}\times\vec{B}$
Potential Energy	$U=-\vec{p}\cdot\vec{E}$	$U=-\vec{\mu}\cdot\vec{B}$