POLARIZATION IN DIELECTRICS, DIELECTRIC CONSTANT AND STRENGTH

Flux Density in Dielectrics

- We now consider the case in which the dielectric region contains free charge

- If ρ_v is the free charge volume density, the total volume charge density ρ_t, is given by:

$$\rho_t = \rho_v + \rho_{pv} = \nabla \cdot \varepsilon_o \mathbf{E}$$

- Hence:

$$\begin{aligned} \rho_v &= \nabla \cdot \varepsilon_o \mathbf{E} - \rho_{pv} \\ &= \nabla \cdot (\varepsilon_o \mathbf{E} + \mathbf{P}) \\ &= \nabla \cdot \mathbf{D}' \end{aligned}$$

- Where:

$$\mathbf{D}' = \varepsilon_o \mathbf{E} + \mathbf{P}$$

- Therefore, the net effect of the dielectric on the electric field **E** is to increase **D** inside it by an amount of **P**

Flux Density in Dielectrics

- Therefore, due to the application of **E** to the dielectric material, the flux density is greater than it would be in free space

- It should be noted that the definition of **D** for free space is a special case of that in the previous equation because **P** = 0 in free space

- The polarization **P** varies directly as the applied electric field **E** for some dielectrics:

$$\mathbf{P} = \chi_e \varepsilon_o \mathbf{E}$$

- where χ_e, known as the electric susceptibility of the material - measure of how susceptible (or sensitive) a given dielectric is to electric fields

Dielectric Constant

- Substituting $\mathbf{P} = \chi_e \varepsilon_o \mathbf{E}$ into $\mathbf{D}' = \varepsilon_o \mathbf{E} + \mathbf{P}$, we get:

$$\mathbf{D}' = \varepsilon_o (1 + \chi_e)\,\mathbf{E} = \varepsilon_o \varepsilon_r \mathbf{E}$$

Or $\mathbf{D}' = \varepsilon \mathbf{E}$ where $\varepsilon = \varepsilon_o \varepsilon_r$

And $\varepsilon_r = 1 + \chi_e = \dfrac{\varepsilon}{\varepsilon_o}$

- In above equations, ε is called the permittivity of the dielectric, ε_o is the permittivity of free space and ε_r is called the dielectric constant or relative permittivity

Dielectric Strength

- The theory of dielectrics we have discussed so far assumes ideal dielectrics

- Practically, no dielectric is ideal

- When the electric field in a dielectric is sufficiently large, it begins to pull electrons completely out of the molecules, and the dielectric becomes conducting

- Dielectric breakdown is said to have occurred when a dielectric becomes conducting

- *The **dielectric strength** is the maximum electric field that a dielectric can tolerate or withstand without breakdown*

Dielectric Properties

- A material is said to be ***linear*** if **D** varies linearly with **E** and nonlinear otherwise

- Materials for which ε (or σ) does not vary in the region being considered and is therefore the same at all points (i.e., independent of x, y, z) are said to be ***homogeneous***

- They are said to be ***inhomogeneous*** (or nonhomogeneous) when ε is dependent on the space coordinates

- ***Isotropic*** dielectrics are those which have the same properties in all directions

Problem-1

- A parallel-plate capacitor with plate separation of 2 mm has a 1-kV voltage applied to its plates. If the space between its plates is filled with polystyrene (ε_r = 2.55), find E, P, and ρ_{ps}.

Problem-1

$$E = \frac{V}{d} = \frac{1000}{2 \times 10^{-3}} = \boxed{500\ \mathrm{KV/m}}$$

$$\vec{P} = \chi_e \varepsilon_0 E = (\varepsilon_r - 1)(\varepsilon_0)(E)$$

$$\Rightarrow P = (2.25 - 1)\left(\frac{10^{-9}}{36\pi}\right)(500{,}000)$$

$$\Rightarrow \boxed{P = 6.85 \times 10^{-6}\ \mathrm{C/m^2}}$$

$$\rho_{ps} = \vec{P} \cdot \vec{a}_n = \boxed{6.85 \times 10^{-6}\ \mathrm{C/m^2}}$$

$\vec{a}_n$ depends upon the direction of the distance vector $\vec{d}$ of the plates.

Problem-2

- In a dielectric material, E_x = 5 V/m and $\mathbf{P} = (1/10\pi)(3\mathbf{a}_x - \mathbf{a}_y + 4\mathbf{a}_z)$ nC/m^2. Calculate:

(a) χ_e

(b) **E**

(c) **D**

Problem-2

(a) $\vec{P} = \varepsilon_0 \chi_e \vec{E}$

Since $\vec{E}$ is along $\vec{a}_x$

$\Rightarrow P_x = \varepsilon_0 \chi_e E_x$

$$\frac{3}{10\cancel{\pi}} \times \cancel{10}^{-9} = \frac{\cancel{10}^{-9}}{36\cancel{\pi}} \chi_e (5)$$

$$\Rightarrow \boxed{\chi_e = 2.16}$$

Problem-2

(b) $$\vec{E} = \frac{\vec{P}}{\varepsilon_0 \chi_e} = \frac{1}{10\pi} \frac{(3\vec{a}_x - a_y + 4a_z)\times 10^{-9}(36\pi)}{10^{-9}(2.16)}$$

$$\Rightarrow \boxed{\vec{E} = 5\vec{a}_x - 1.667\vec{a}_y + 6.667\vec{a}_z \ V/m}$$

(c) $$\vec{D} = \varepsilon\vec{E} = \varepsilon_0\varepsilon_r\vec{E} = \varepsilon_0\varepsilon_r\frac{\vec{P}}{\varepsilon_0\chi_e} = \frac{\varepsilon_r\vec{P}}{\chi_e}$$

$$\varepsilon_r = 1+\chi_e$$

$$\Rightarrow \vec{D} = \frac{(1+\chi_e)}{\chi_e}\vec{P} = \frac{(3.16)}{2.16}(\vec{P})$$

$$\Rightarrow \boxed{\vec{D} = 139.7\vec{a}_x - 46.6\vec{a}_y + 186.3\vec{a}_z \ pC/m^2}$$