Chapter1: Digital Systems and Binary Numbers

Lecture1- Introduction to Digital Systems

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Digital Systems
Binary Numbers
Number Base Conversion
Octal and Hexadecimal Numbers
Complements
Signed Binary Numbers
Binary Codes
Binary Storage and Registers
Binary Logic and Logic Gates
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Objectives

- Introduction to Digital Systems
- History of Number Systems
- Study Commonly Used Number Systems

Analog and Digital Systems

- Real world is analog but digital circuits are found in an astonishingly wide range of electronic systems.
- Analog systems process information that varies continuously . Examples of analog represented variables are:
  - ➢ a mercury thermometer
  - ➢ needle speedometer of cars
  - ➢ sine wave voltages indicated on a galvanometer
  - ➢ audio amplifier
  - ➢ simple light dimmer switch
- Digital systems process discrete information. Discrete means distinct or separated as opposed to continuous or connected. The examples are:
  - ➢ telephone switching exchanges
  - ➢ Speedometer of cars with numerical readout
  - ➢ electronic calculators
  - ➢ ten position switch
  - ➢ digital computers

Analog and Digital Systems

Analog watch

Digital watch

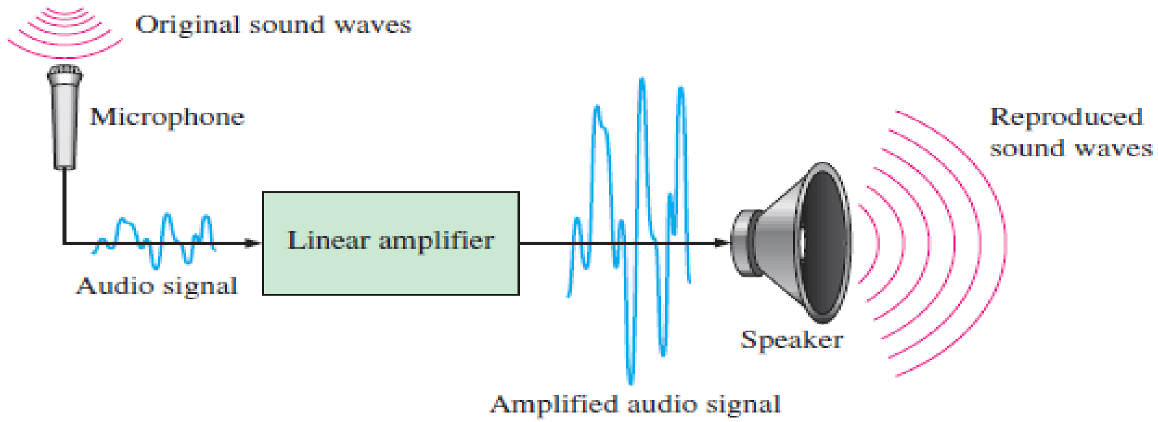


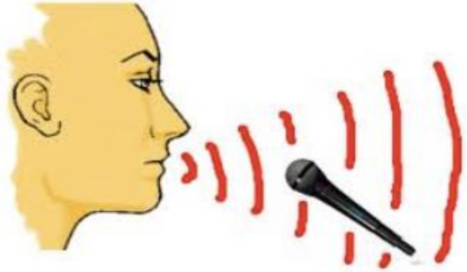
Figure: A basic audio public address system

Analog and Digital Systems

- Analog systems process signals that take on continuous range of values. Examples are

Automobile speedometer

Mercury thermometer



Sound through a microphone

- Digital system use discrete set of values that can be represented by 1’s and 0’s. Examples are

Digital Multimeter

Digital thermometer

Analog and Digital Systems

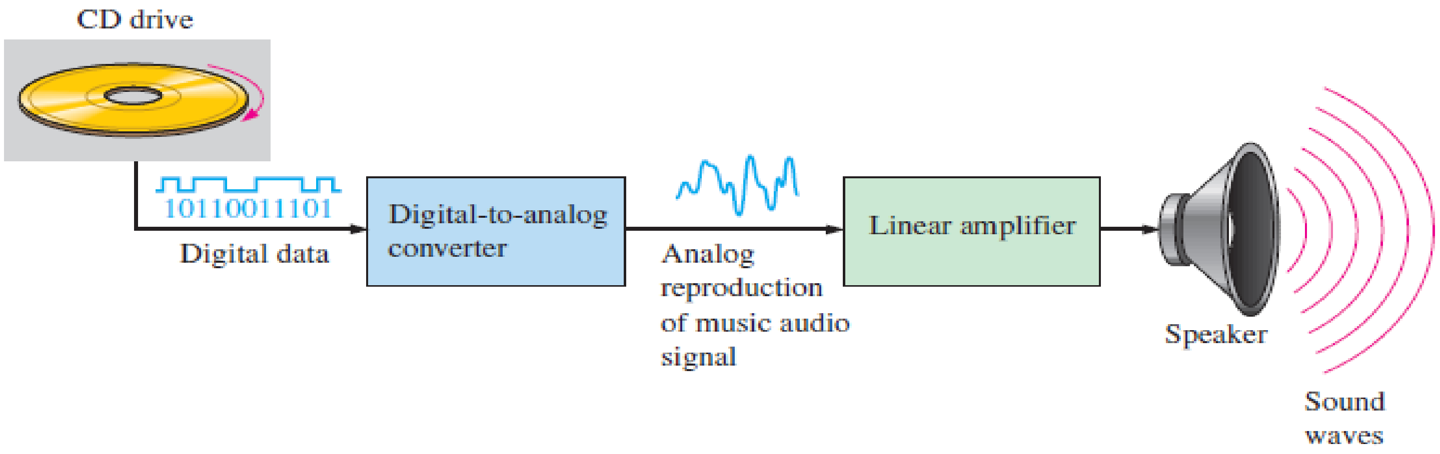


Figure: Block diagram of a CD player

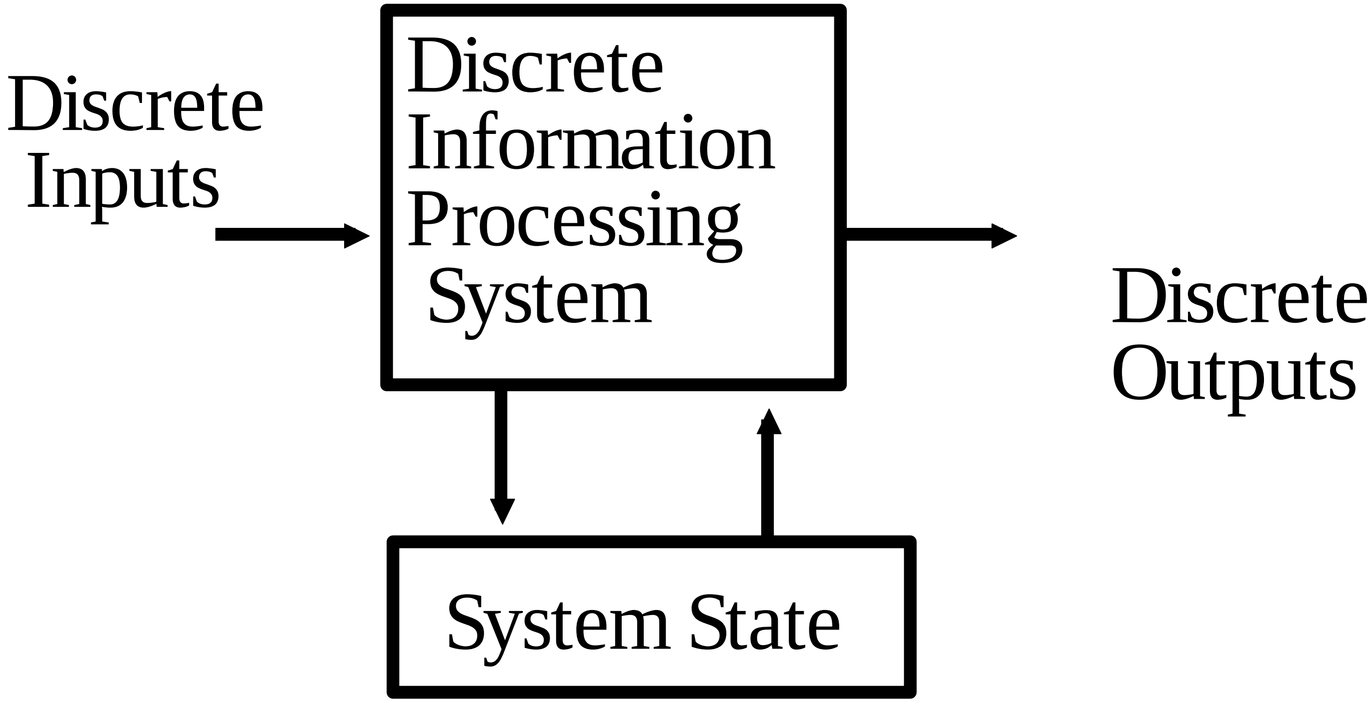
Digital Systems

- Digital Systems have such a prominent role in everyday life that we refer to the present technological period as the digital age.
- Digital systems manipulate discrete elements of information and have wide applications.
  - Digital systems are used in communication, business transactions, traffic control, space guidance, medical treatment, weather monitoring, the Internet, and many other commercial, industrial, and scientific enterprises.
  - We have digital telephones, digital television, digital versatile discs, digital cameras, and digital computers.
- The discrete elements of information are represented in a digital system by physical quantities called signals i.e voltage and current.
- The signals in present-day electronic digital systems use just two discrete values and are therefore said to be binary. A binary digit, called a bit, has two values: 0 and 1.
- Why binary?
  - reliability: a transistor circuit is either ON or OFF (two stable states)

Digital Systems

- Digital Systems represent systems that understand, represent and manipulate discrete elements.
  - ➢ A discrete element is any set that has a finite number of elements, for example 10 decimal digits, 26 letters of the alphabet, etc.
- Discrete elements are represented by signals, such as electrical signals (voltages and currents)
- The signals in most electronic digital systems use two discrete values, termed as binary.
- Digital Systems take a set of discrete information inputs and discrete internal information (system state) and generates a set of discrete information outputs.

Digital Systems



Why Digital Components?

- Why do we choose to use digital components?
  - ➢ The main reason for using digital components is that they can easily be programmed, allowing a single hardware unit to be used for many different purposes.
  - ➢ Advances in circuit technology decrease the price of technology dramatically.
  - ➢ Digital integrated circuits can perform at speeds of hundreds of millions of operations per second.
  - ➢ Error-checking and correction can be used to ensure the reliability of the machine.

Binary Digits

- A binary digit, called a bit, is represented by one of two values: 0 or 1.
  - Discrete elements can be represented by groups of bits called binary codes. For example, the decimal digits 0 to 9 are represented as follows:

Decimal	Binary Code
0	**0000**
1	**0001**
2	**0010**
3	**0011**
4	**0100**
5	**0101**
6	**0110**
7	**0111**
8	**1000**
9	**1001**

Different Bases

- In order to represent numbers of different bases, we surround a number in parenthesis and then place a subscript with the base of the number. Few examples of different number bases are:
  - A decimal number→ $(9233)_{10}$
  - A binary number → $(11011)_2$
  - A base 5 number →$(3024)_5$
- Decimal number digits are 0 through 9
- Binary number digits are 0 through 1
- Base 5 number digits are 0 through 4
- Base (radix) *r* number digits are 0 through *r* - 1

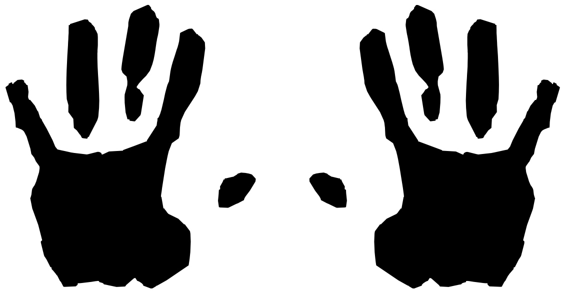
Commonly Used Bases

Name	Radix	Digits (0 through r-1)
Binary	2	0,1
Octal	8	0,1,2,3,4,5,6,7
Decimal	10	0,1,2,3,4,5,6,7,8,9
Hexadecimal	16	0,1,2,3,4,5,6,7,8,9,A,B,C,D,E,F

Decimal Numbers

- A decimal number such as 5723 represents a quantity equal to:
  - 5 thousands
  - 7 hundreds
  - 2 tens
  - 3 ones
- Or it can be written as:

  $5 \times 10^3 + 7 \times 10^2 + 2 \times 10^1 + 3 \times 10^0$



- The 5, 7, 2, and 3 represent coefficients.
- The decimal number system is said to be of base or radix 10 because it uses the 10 digits (0...9) and the coefficients are multiplied by powers of 10.

Binary Numbers

- The binary system contains only two values in the allowed coefficients (0 and 1).
- The binary system uses powers of 2 as the multipliers for the coefficients.
- For example, we can represent the binary number 10111.01_2 as:

$$1 \text{ X } 2^4 + 0 \text{ X } 2^3 + 1 \text{ X } 2^2 + 1 \text{ X } 2^1 + 1 \text{ X } 2^0 + 0 \text{ X } 2^{-1} + 1 \text{ X } 2^{-2} = 23.25_{10}$$

Understanding Binary Numbers

- Binary numbers are made of binary digits (bits):

  0 and 1
- How many items does a binary number represent?

  $(1011)_2 = 1x2^3 + 0x2^2 + 1x2^1 + 1x2^0 = (11)_{10}$
- What about fractions?

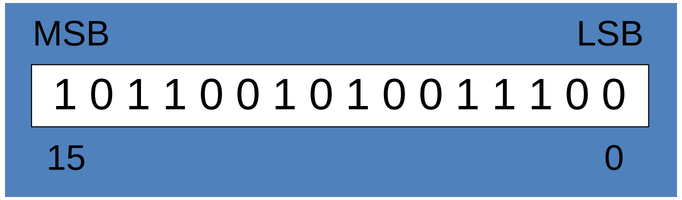
  $(110.10)_2 = 1x2^2 + 1x2^1 + 0x2^0 + 1x2^{-1} + 0x2^{-2}$
- Groups of eight bits are called a *byte*

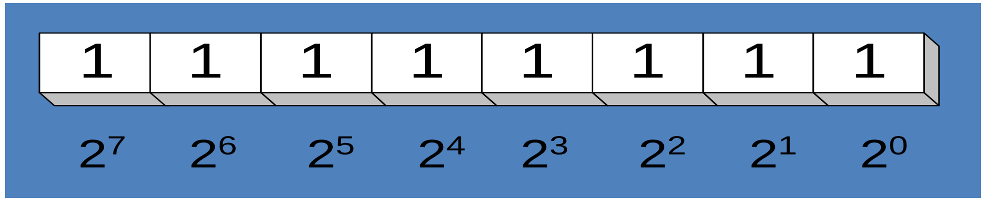
  $(11001001)_2$
- Groups of four bits are called a *nibble.*

  $(1101)_2$

Understanding Binary Numbers (Cont…)

- Each digit (bit) is either 1 or 0
- Each bit represents a power of 2
- Bit numbering
- MSB: most significant bit
- LSB: least significant bit





Why Binary Numbers?

- Easy to represent 0 and 1 using electrical values.
- Possible to tolerate noise.
- Easy to transmit data
- Easy to build binary circuits.

Powers of Two

n	2^n	n	2^n	n	2^n
0	1	8	256	16	65,536
1	2	9	512	17	131,072
2	4	10	1,024	18	262,144
3	8	11	2,048	19	524,288
4	16	12	4,096	20	1,048,576
5	32	13	8,192	21	2,097,152
6	64	14	16,384	22	4,194,304
7	128	15	32,768	23	8,388,608

Important Powers of Two are:

- 2^{10} is referred to as Kilo, called "K"
- 2^{20} is referred to as Mega, called "M"
- 2^{30} is referred to as Giga, called "G"
- 2^{40} is referred to as Tera, called "T"

Octal Numbers

- The octal number system is a base-8 system that contains the coefficient values of 0 to 7.
- The octal system uses powers of 8 as the multipliers for the coefficients.
- For example, we can represent the octal number 72032_8 as:

$$7 \times 8^4 + 2 \times 8^3 + 0 \times 8^2 + 3 \times 8^1 + 2 \times 8^0 = (29722)_{10}$$

Hexadecimal Numbers

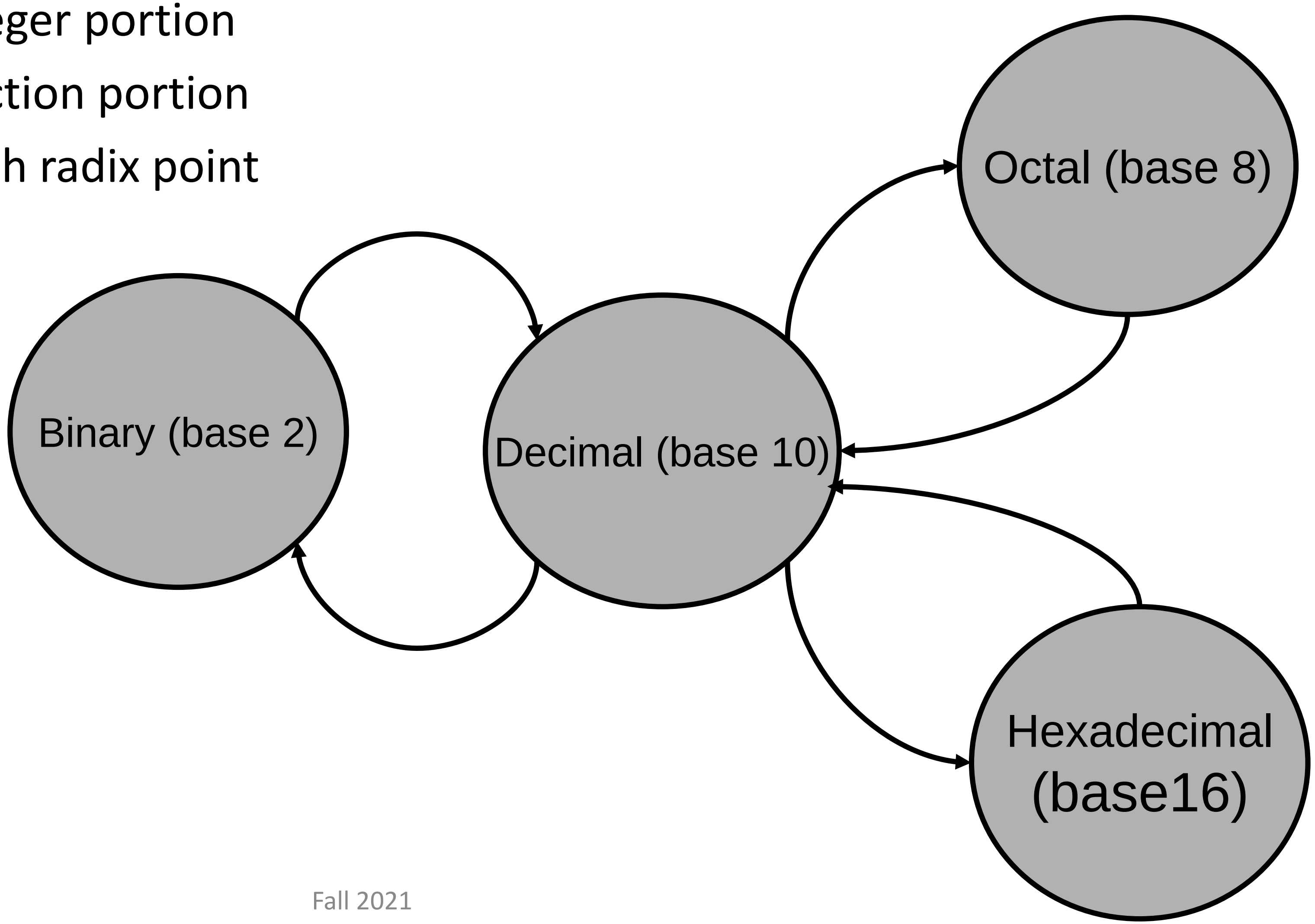
- The hexadecimal number system is a base-16 system that contains the coefficient values of 0 to 9 and A to F.
- The letters A, B, C, D, E, F represent the coefficient values of 10, 11, 12, 13, 14, and 15, respectively.
- The hexadecimal system uses powers of 16 as the multipliers for the coefficients.
- For example, we can represent the hexadecimal number $C34D_{16}$ as:
  - ➢ $12 \times 16^3 + 3 \times 16^2 + 4 \times 16^1 + 13 \times 16^0 = (49997)_{10}$

Numbers with Different Bases

Decimal (base 10)	Binary (base 2)	Octal (base 8)	Hexadecimal (base 16)
00	0000	00	0
01	0001	01	1
02	0010	02	2
03	0011	03	3
04	0100	04	4
05	0101	05	5
06	0110	06	6
07	0111	07	7
08	1000	10	8
09	1001	11	9
10	1010	12	A
11	1011	13	B
12	1100	14	C
13	1101	15	D
14	1110	16	E
15	1111	17	F

Conversion between bases

- To convert from one base to other:
  - Convert the integer portion
  - Convert the fraction portion
  - Join the two with radix point



Decimal-r Conversion

- Conversion of a number in base r to decimal is done by expanding the number in a power series and adding all the terms.
- For example, $(C34D)_{16}$ is converted to decimal:

  $12 \text{ X } 16^3 + 3 \text{ X } 16^2 + 4 \text{ X } 16^1 + 13 \text{ X } 16^0 = (49997)_{10}$
- $(11010.11)_2$ is converted to decimal:

  $1 \text{ X } 2^4 + 1 \text{ X } 2^3 + 0 \text{ X } 2^2 + 1 \text{ X } 2^1 + 0 \text{ X } 2^0 + 1 \text{ X } 2^{-1} + 1 \text{ X } 2^{-2} = 26.75$
- In general N=$(\text{Number})_r$ = $\left(\sum_{i=0}^{i=n-1} a_i \bullet r^i\right) + \left(\sum_{j=-m}^{j=-1} a_j \bullet r^j\right)$

  (Integer Portion) + (Fraction Portion)

Decimal-r Conversion

- If a decimal number has a radix point, it is necessary to separate the number into an integer part and a fraction part.
- The conversion of a decimal integer into a number in base-r is done by dividing the number and all successive quotients by r and accumulating the remainders in reverse order of computation.
- For example, to convert decimal 13 to binary:

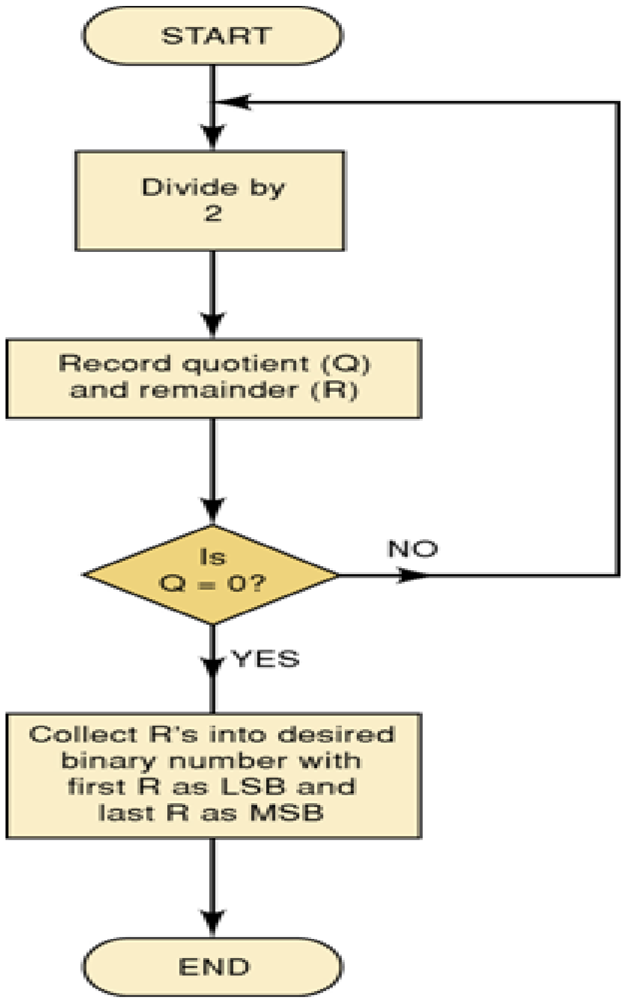
	Integer Quotient		Remainder	Coefficient
13/2 =	6	+	½	$a_0 = 1$
6/2 =	3	+	0	$a_1 = 0$
3/2 =	1	+	½	$a_2 = 1$
1/2 =	0	+	½	$a_3 = 1$

Answer $(13)_{10} = (a_3\, a_2\, a_1\, a_0)_2 = (1101)_2$

Decimal to Binary Conversion

Repeated Division

This flowchart describes the process and can be used to convert from decimal to any other number system.



Decimal to Binary Conversion Example

- Convert $(37)_{10}$ to binary

Division	Quotient	Remainder
37 / 2	18	1
18 / 2	9	0
9 / 2	4	1
4 / 2	2	0
2 / 2	1	0
1 / 2	0	1

$(37)_{10} = 100101_2$

Decimal-r Conversion (Converting Fractions)

- To convert the fraction portion repeatedly multiply the fraction by the radix and save the integer digits that result. The process continued until the fraction becomes 0 or the number of digits have sufficient accuracy. The new radix fraction digits are the integer digits in computed order.
- For example convert fraction $(0.6875)_{10}$ to base 2

0.6875 * 2 = 1.3750	integer = 1
0.3750 * 2 = 0.7500	integer = 0
0.7500 * 2 = 1.5000	integer = 1
0.5000 * 2 = 1.0000	integer = 1

Answer = $(0.1011)_2$

Converting Fractions Cont…

- When converting fractions, we must use multiplication rather than division. The new radix fraction digits are the integer digits in *computed order*.

		Integer		Fraction	Coefficent
0.8432×2	=	1	+	0.6864	$a_{-1} = 1$
0.6864×2	=	1	+	0.3728	$a_{-2} = 1$
0.3728×2	=	0	+	0.7456	$a_{-3} = 0$
0.7456×2	=	1	+	0.4912	$a_{-4} = 1$
0.4912×2	=	0	+	0.9824	$a_{-5} = 0$
0.9824×2	=	1	+	0.9648	$a_{-6} = 1$
0.9648×2	=	1	+	0.9296	$a_{-7} = 1$

Continue until fraction becomes 0 or until sufficient accuracy.

$$(0.8432)_{10} = (0.a_{-1}a_{-2}a_{-3}a_{-4}a_{-5}a_{-6}a_{-7})_2 = (0.1101011)_2$$

Another example:

- **Convert 0.8125 decimal to binary.**
  - To convert the decimal 0.8125 to binary, we multiply by the radix 2.
  - **$(0.1101)_2$**

$$\begin{array}{r} .8125 \\ \times \quad 2 \\ \hline 1.6250 \end{array}$$

$$\begin{array}{r} .6250 \\ \times \quad 2 \\ \hline 1.2500 \end{array}$$

$$\begin{array}{r} .2500 \\ \times \quad 2 \\ \hline 0.5000 \end{array}$$

$$\begin{array}{r} .5000 \\ \times \quad 2 \\ \hline 1.0000 \end{array}$$

Decimal to Octal Conversion

- In converting decimal to octal we must divide integer part by 8 till quotient becomes lesser than divisor.

	Integer Quotient		Remainder	Coefficent
35 / 8 =	4	+	3/8	$a_0 = 3$
4 / 8 =	0	+	4/8	$a_1 = 4$

$$(35)_{10} = (a_1a_0)_8 = (43)_8$$

Converting Fractions (Decimal to Octal)

- Decimal to Octal fraction conversion takes the same approach but it multiplies by the base 8.

	Integer		Fraction	Coefficent
$0.8432 \times 8 =$	6	+	0.7456	$a_{-1} = 6$
$0.7456 \times 8 =$	5	+	0.9648	$a_{-2} = 5$
$0.9648 \times 8 =$	7	+	0.7184	$a_{-3} = 7$
$0.7184 \times 8 =$	5	+	0.7472	$a_{-4} = 5$
$0.7472 \times 8 =$	5	+	0.9776	$a_{-5} = 5$
$0.9776 \times 8 =$	7	+	0.8208	$a_{-6} = 7$
$0.8208 \times 8 =$	6	+	0.5664	$a_{-7} = 6$

Continue until fraction becomes 0 or until sufficient accuracy.

$$(0.8432)_{10} = (0.a_{-1}a_{-2}a_{-3}a_{-4}a_{-5}a_{-6}a_{-7})_8 = (0.6575576)_8$$

The End