National University of Sciences & Technology
School of Electrical Engineering and Computer Science
Department of Basic Sciences

MATH-243: Vector Calculus (3+0): BEE-2k20-C Fall 2021

Quiz - 2: Partial and Directional derivatives	
CLO-1: Interpret the consequences of del (nabla) operator on scalar and vector fields.	
Maximum Marks: **10**	Instructor: Dr. Naila Amir
Date: **15 − 10 − 2021**	Duration: 10 Minutes
Name: Master Solution	CMS ID:

Question: Multiple choice questions. Circle the correct option. Overwriting, cutting and multiple selections will not be considered.

1) Let $\mathbf{r} = \langle x, y, z\rangle$ and $r = |\mathbf{r}|$, then $\nabla e^r =$ _______?
   a) $\frac{e^r}{r}$
   b) $e^r\mathbf{r}$
   c) $\frac{e^r}{r}\mathbf{r}$
   d) $e^{-r}r\mathbf{r}$
   a) None of these.

$r = |\vec{r}| = \sqrt{x^2+y^2+z^2} \Rightarrow e^r = e^{\sqrt{x^2+y^2+z^2}}$

$\vec{\nabla} e^r = \left\langle \frac{e^{\sqrt{x^2+y^2+z^2}}(2x)}{2\sqrt{x^2+y^2+z^2}}, \frac{e^{\sqrt{x^2+y^2+z^2}}(2y)}{2\sqrt{x^2+y^2+z^2}}, \frac{e^{\sqrt{x^2+y^2+z^2}}(2z)}{2\sqrt{x^2+y^2+z^2}} \right\rangle$

$= \left\langle \frac{xe^r}{r}, \frac{ye^r}{r}, \frac{ze^r}{r} \right\rangle$

$= \frac{e^r}{r}\langle x, y, z\rangle = \frac{e^r}{r}\vec{r}$

2) The equation of a tangent plane of surface $z = -x$ at $(1,0,-1)$ is _______.
   a) $z - x = 0$
   b) $x + y + z = 2$
   c) $x + z = 0$
   d) $y + z = 2$
   e) None of these.

$F(x,y,z) = z + x$

$F_x = 1$, $F_y = 0$, $F_z = 1$

$1(x-1) + 0(y-0) + 1(z+1) = 0$

$\Rightarrow x - 1 + z + 1 = 0$

$\Rightarrow x + z = 0$

Alternative: Consider $f(x,y) = -x$ and proceed as discussed in class

3) The equation of the tangent line to the curve: $x^2 - xy + y^2 = 1$, at $(-1,1)$ is:
   a) $4x - y = 14$
   b) $4x - 5y = -14$
   c) $x + 3y = 5$
   d) $2x - y = -4$
   e) None of these.

$f(x,y) = x^2 - xy + y^2$

$f_x = 2x - y \Rightarrow f_x(-1,1) = -2 - 1 = -3$

$f_y = -x + 2y \Rightarrow f_y(-1,1) = 1 + 2 = 3$

$-3(x+1) + 3(y-1) = 0 \Rightarrow -3[x+1-(y-1)] = 0$

$\Rightarrow x + 1 - y + 1 = 0 \Rightarrow x - y + 2 = 0$

4) The directional derivative of $f(x,y) = x^2 + y^2$ at $(1,1)$ in the direction of the line segment directed from $(0,0)$ to $(1,1)$ is:
   P Q
   a) $\sqrt{2}$
   b) 2
   c) $\frac{1}{\sqrt{2}}$
   d) $2\sqrt{2}$
   e) None of these.

$\vec{\nabla} f = \langle 2x, 2y\rangle \Rightarrow \vec{\nabla} f(1,1) = \langle 2, 2\rangle$

$\vec{u} = \langle 1, 1\rangle / \sqrt{2}$

$\left[\because \vec{PQ} = \langle 1-0, 1-0\rangle = \langle 1, 1\rangle\right.$

$\vec{\nabla} f \cdot \vec{u} = \frac{2+2}{\sqrt{2}} = \frac{4}{\sqrt{2}} = \frac{2\times 2}{\sqrt{2}} = 2\sqrt{2}$

5) If $u = e^{-x^2+yz}$, then $\frac{\partial u}{\partial x} =$ ________.

a) e^{-x^2+yz}
b) ze^{-x^2+yz}
c) $-2xyze^{-x^2+yz}$
d) yze^{-x^2+yz}
e) None of these.

$\frac{\partial u}{\partial x} = e^{-x^2+yz}[-2x+0]$

$\Rightarrow \frac{\partial u}{\partial x} = -2x\, e^{-x^2+yz}$

6) The maximum value of the directional derivative of $u = 2x^2 + 3xy^2 + 5z^2$ at $(1,0,-1)$ is:

a) $\langle 4,0,-10\rangle$
b) $\langle 4,0,-10\rangle/\sqrt{160}$
c) $\sqrt{160}$
d) $1/\sqrt{160}$
e) None of these.

$\vec{\nabla} u = \langle 4x + 3y^2, 6xy, 10z\rangle$

$\vec{\nabla} u(1,0,-1) = \langle 4, 0, -10\rangle$

$|\vec{\nabla} u| = \sqrt{16+0+100} = \sqrt{116}$

7) If $z = x^2 + y^3$; $x = t^2 + t^3$ & $y = t^3 + t^9$,then $\frac{dz}{dt}$ at $t = 1$ is:

a) 0
b) 1
c) -1
d) 164
e) None of these.

$x = t^2+t^3 \Rightarrow x = 2$ [for $t=1$]
$y = t^3+t^9 \Rightarrow y = 2$ [for $t=1$]

$\frac{dz}{dt} = z_x \frac{dx}{dt} + z_y \frac{dy}{dt}$

$= (2x)(2t+3t^2) + (3y^2)(3t^2+9t^8)$

$\left.\frac{dz}{dt}\right|_{t=1} = 2(2)(2+3) + 3(2)^2(3+9) = 164$

8) If $f(x, y) = \sin(xy) + x^2 \ln y$ then $f_y\left(0,\frac{\pi}{2}\right)$ is

a) 0
b) 1
c) -1
d) π
e) None of these.

$f_y = x\cos(xy) + \frac{x^2}{y}$

$\Rightarrow f_y(0,\pi/2) = 0 + 0 = 0$

9) If $f(x, y) = c$ is an implicit function, then $\frac{dy}{dx} =$ ________.

a) $\frac{1}{f_y}$
b) $-\frac{1}{f_x}$
c) $-\frac{f_y}{f_x}$
d) $-\frac{f_x}{f_y}$
e) None of these.

$f(x,y) = c$

$\Rightarrow f_x + f_y \frac{dy}{dx} = 0$

$\Rightarrow \frac{dy}{dx} = -\frac{f_x}{f_y}$

10) The vector normal to the surface: $x^2y + \ln y - 2xz = 0$, at $(2, 1,0)$ is:

a) $\langle 4,5,-2\rangle$
b) $\langle 4,5,2\rangle$
c) $\langle 4,0,-2\rangle$
d) $\langle -4,5,-2\rangle$
e) None of these.

$f(x,y,z) = x^2y + \ln y - 2xz$

$\vec{\nabla} f = \langle 2xy - 2z, x^2 + \frac{1}{y}, -2x\rangle$

$\vec{\nabla} f(2,1,0) = \langle 4-0, 4+1, -2(2)\rangle$

$= \langle 4, 5, -4\rangle$