Engineering Mechanics

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Contents (Section 2.4-2.6)

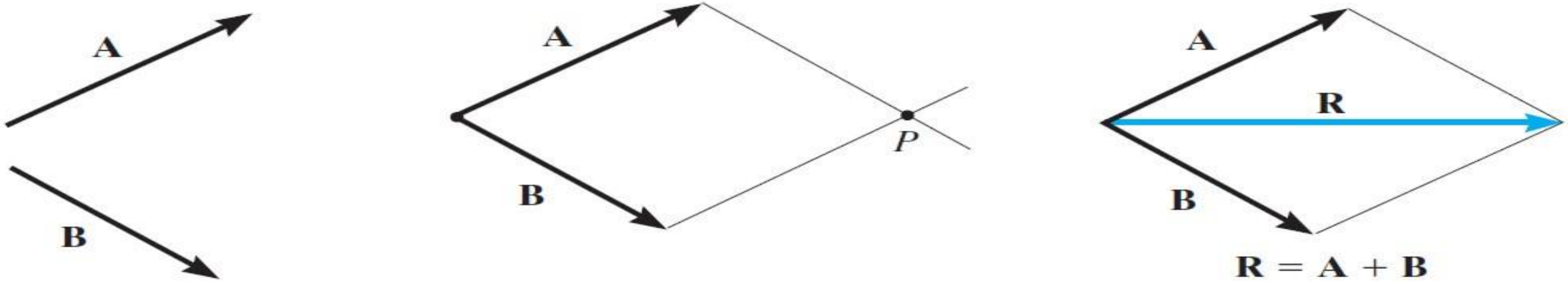
- Recap
- Cartesian Vectors
- Addition of Cartesian Vectors

RECAP
Engineering Mechanics

Force Vectors

Vector Addition and subtraction:

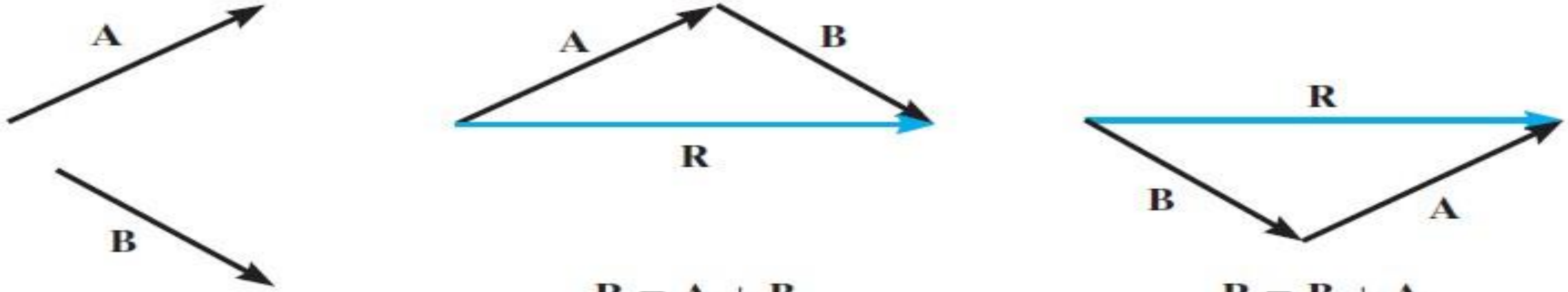
1. Parallelogram Method:



$\mathbf{R} = \mathbf{A} + \mathbf{B}$

Parallelogram law

2. 2. Head to Tail Method (Triangle rule):



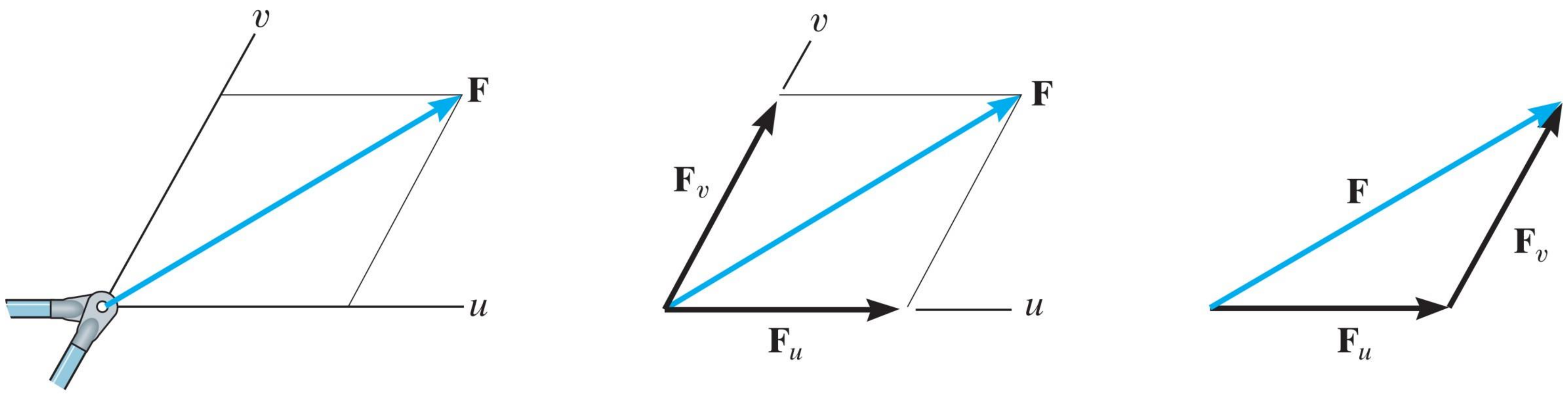
$\mathbf{R} = \mathbf{A} + \mathbf{B}$

Triangle rule

$\mathbf{R} = \mathbf{B} + \mathbf{A}$

Triangle rule

Components of Force Vectors (splitting force vector)

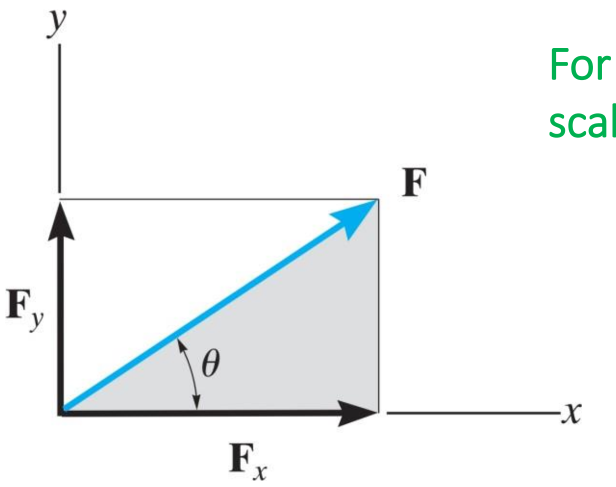


(a) (b) (c)

Addition of a System of Coplanar Forces

- Representation of vector in Rectangular components

When a force is resolved into two components along the x and y axes, the components are then called rectangular components.

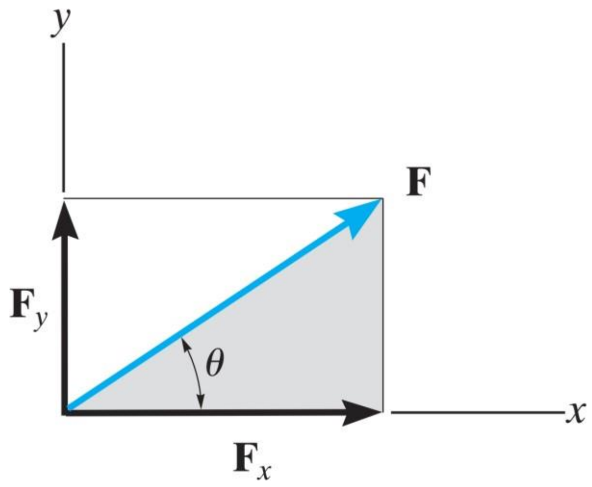


For analytical work we can represent these components in one of two ways, using either scalar notation or Cartesian vector notation.

$$F_x = F\cos\theta \qquad \text{and} \qquad F_y = F\sin\theta$$

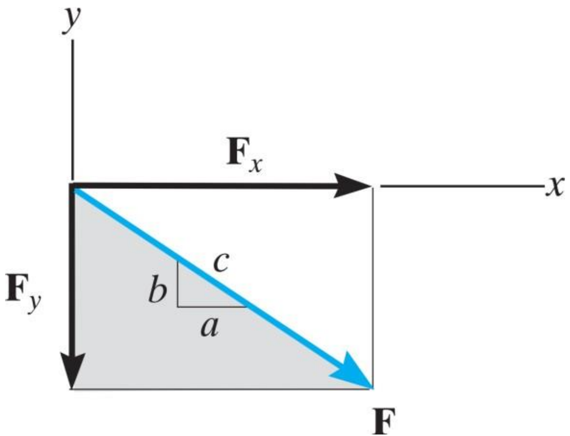
Addition of a System of Coplanar Forces

- Representation of vector in Rectangular components



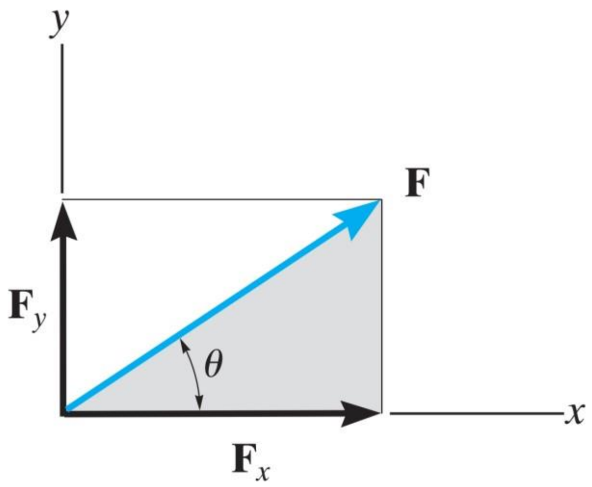
$$F_x = F\cos\theta \qquad \text{and} \qquad F_y = F\sin\theta$$

The direction of **F** can also be defined using a small "slope" triangle

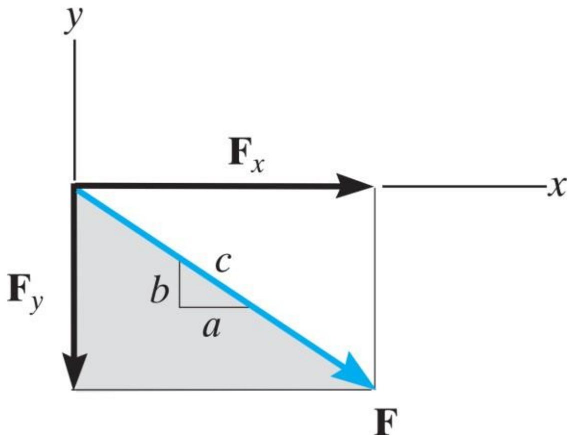


Addition of a System of Coplanar Forces

- Representation of vector in Rectangular components



$$F_x = F\cos\theta \qquad \text{and} \qquad F_y = F\sin\theta$$



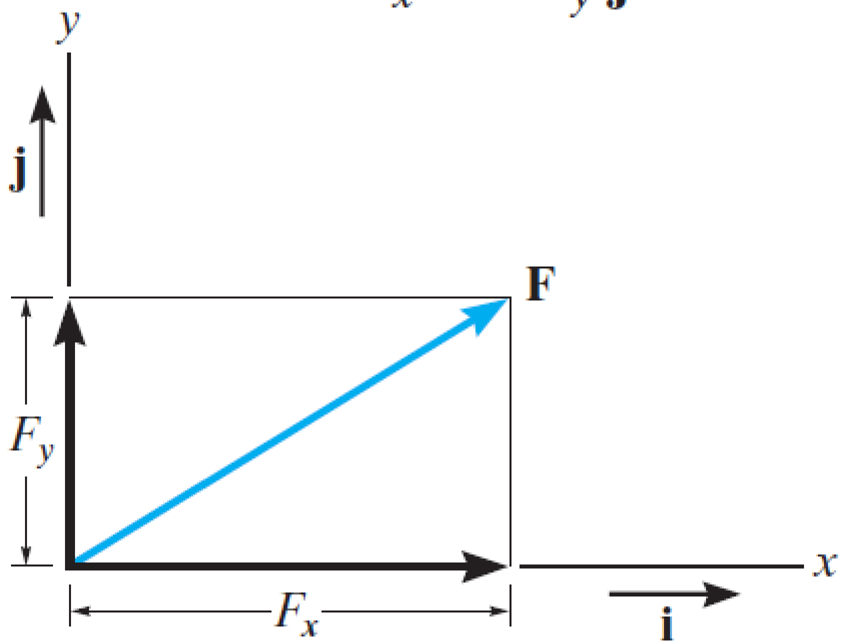
$$F_x = F\left(\frac{a}{c}\right)$$

$$\frac{F_y}{F} = \frac{b}{c}$$

Cartesian Vector Notation. It is also possible to represent the x and y components of a force in terms of Cartesian unit vectors $\mathbf{i}$ and $\mathbf{j}$.

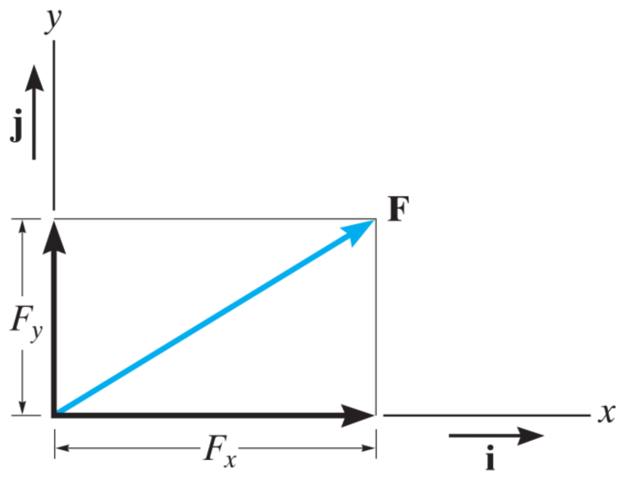
Each of these unit vectors has a **dimensionless magnitude of one**, and so they can be used to designate the *directions* of the *x* and *y* axes, respectively

$$\mathbf{F} = F_x \mathbf{i} + F_y \mathbf{j}$$

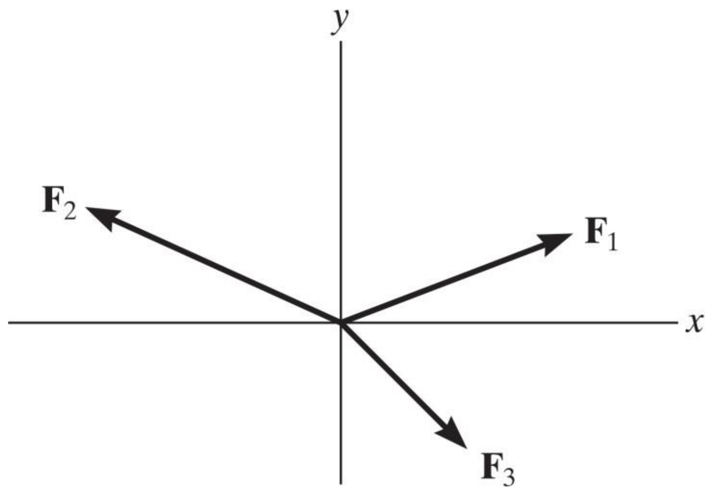


Coplanar force system refers to the number of **forces** which remain in same plane. It is also stated as the number of **forces** in a system which remains in single plane.

Cartesian Vector Notation

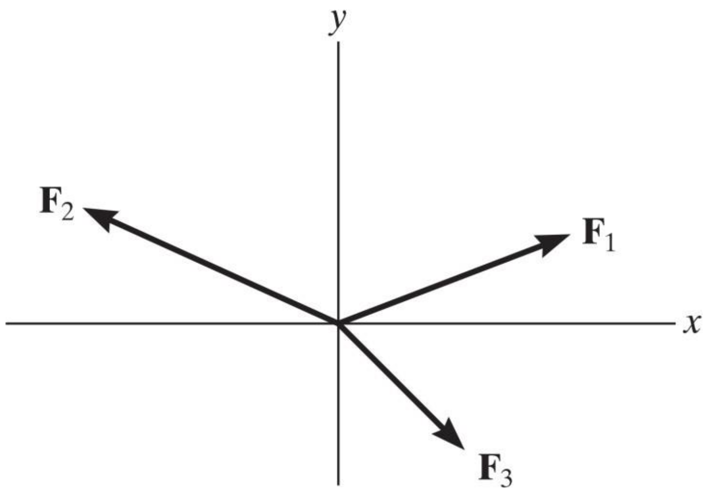


$$\mathbf{F} = F_x\,\mathbf{i} + F_y\,\mathbf{j}$$

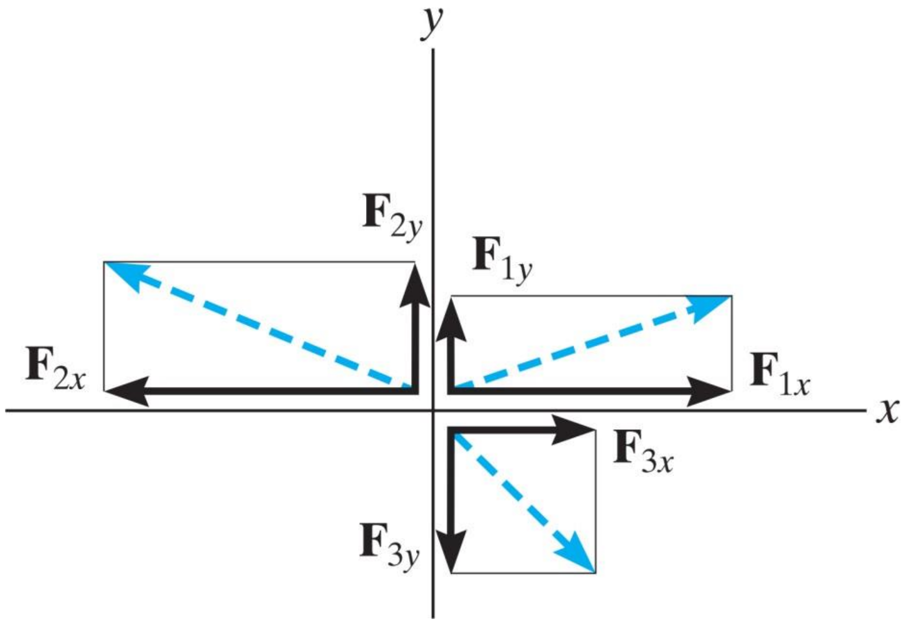


$$\mathbf{F}_1 = F_{1x}\,\mathbf{i} + F_{1y}\,\mathbf{j}$$
$$\mathbf{F}_2 = -F_{2x}\,\mathbf{i} + F_{2y}\,\mathbf{j}$$
$$\mathbf{F}_3 = F_{3x}\,\mathbf{i} - F_{3y}\,\mathbf{j}$$

Cartesian Vector Notation



$$
\begin{aligned}
\mathbf{F}_1 &= F_{1x}\,\mathbf{i} + F_{1y}\,\mathbf{j} \\
\mathbf{F}_2 &= -F_{2x}\,\mathbf{i} + F_{2y}\,\mathbf{j} \\
\mathbf{F}_3 &= F_{3x}\,\mathbf{i} - F_{3y}\,\mathbf{j}
\end{aligned}
$$

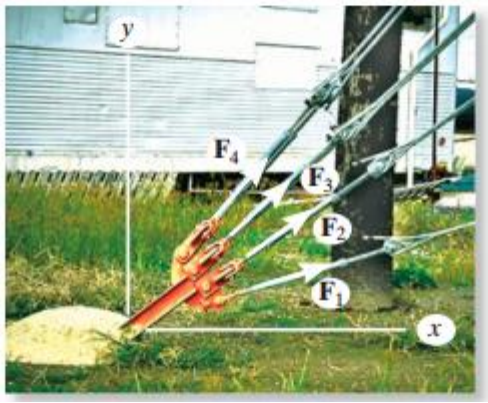


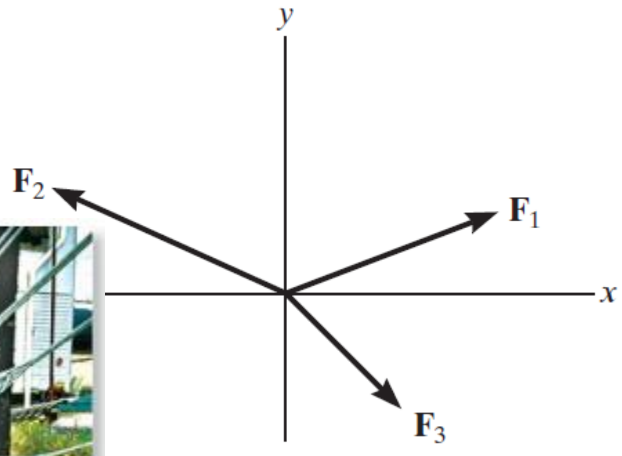
$$
\begin{aligned}
\mathbf{F}_R &= \mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3 \\
&= F_{1x}\,\mathbf{i} + F_{1y}\,\mathbf{j} - F_{2x}\,\mathbf{i} + F_{2y}\,\mathbf{j} + F_{3x}\,\mathbf{i} - F_{3y}\,\mathbf{j} \\
&= (F_{1x} - F_{2x} + F_{3x})\mathbf{i} + (F_{1y} + F_{2y} - F_{3y})\mathbf{j} \\
&= (F_{Rx})\mathbf{i} + (F_{Ry})\mathbf{j}
\end{aligned}
$$

Coplanar Force Resultants.

$$\mathbf{F}_1 = F_{1x}\,\mathbf{i} + F_{1y}\,\mathbf{j}$$
$$\mathbf{F}_2 = -F_{2x}\,\mathbf{i} + F_{2y}\,\mathbf{j}$$
$$\mathbf{F}_3 = F_{3x}\,\mathbf{i} - F_{3y}\,\mathbf{j}$$

The vector resultant is therefore

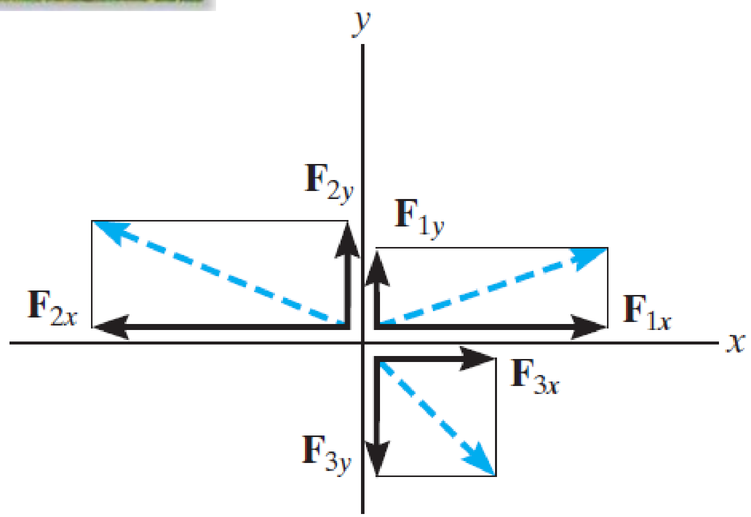




(a)

$$\mathbf{F}_R = \mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3$$
$$= F_{1x}\mathbf{i} + F_{1y}\,\mathbf{j} - F_{2x}\,\mathbf{i} + F_{2y}\,\mathbf{j} + F_{3x}\,\mathbf{i} - F_{3y}\,\mathbf{j}$$
$$= (F_{1x} - F_{2x} + F_{3x})\mathbf{i} + (F_{1y} + F_{2y} - F_{3y})\mathbf{j}$$
$$= (F_{Rx})\mathbf{i} + (F_{Ry})\mathbf{j}$$

If *scalar notation* is used, then we have

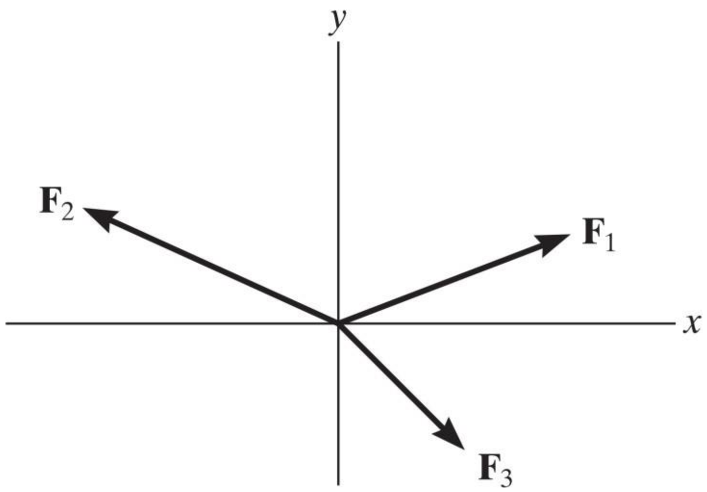


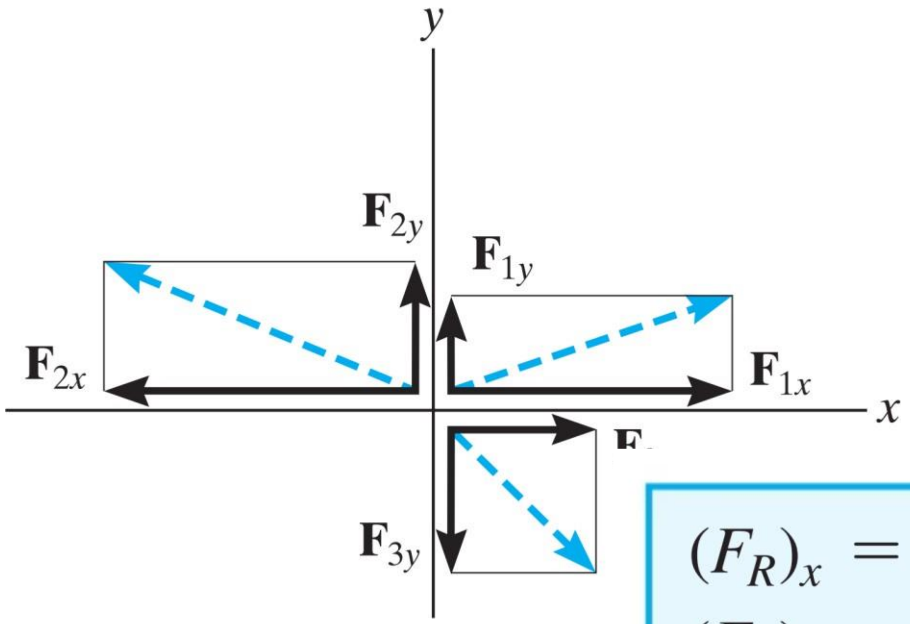
(b)

$$(\overset{+}{\rightarrow}) \qquad F_{Rx} = F_{1x} - F_{2x} + F_{3x}$$
$$(+\uparrow) \qquad F_{Ry} = F_{1y} + F_{2y} - F_{3y}$$

Fig. 2–17

Cartesian Vector Notation



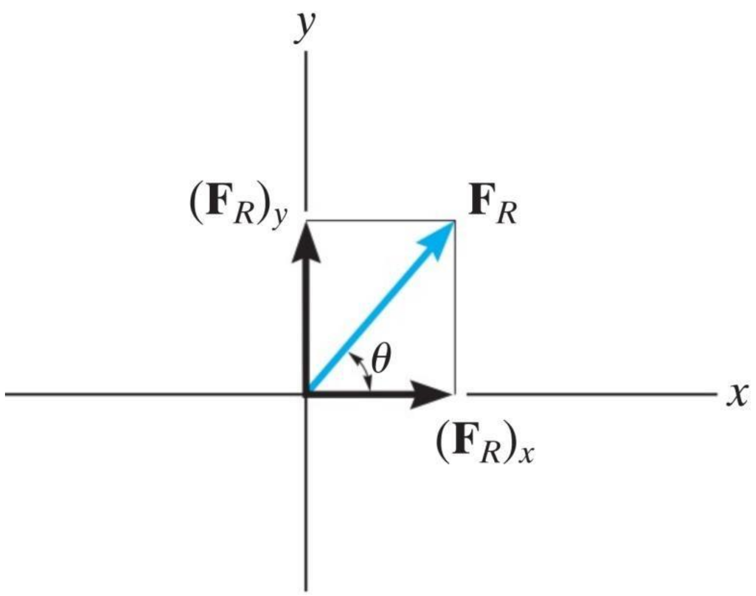


$$(F_R)_x = \Sigma F_x$$
$$(F_R)_y = \Sigma F_y$$

$$\mathbf{F}_1 = F_{1x}\,\mathbf{i} + F_{1y}\,\mathbf{j}$$
$$\mathbf{F}_2 = -F_{2x}\,\mathbf{i} + F_{2y}\,\mathbf{j}$$
$$\mathbf{F}_3 = F_{3x}\,\mathbf{i} - F_{3y}\,\mathbf{j}$$

$$\begin{aligned}\mathbf{F}_R &= \mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3\\ &= F_{1x}\,\mathbf{i} + F_{1y}\,\mathbf{j} - F_{2x}\,\mathbf{i} + F_{2y}\,\mathbf{j} + F_{3x}\,\mathbf{i} - F_{3y}\,\mathbf{j}\\ &= (F_{1x} - F_{2x} + F_{3x})\mathbf{i} + (F_{1y} + F_{2y} - F_{3y})\mathbf{j}\\ &= (F_{Rx})\mathbf{i} + (F_{Ry})\mathbf{j}\end{aligned}$$

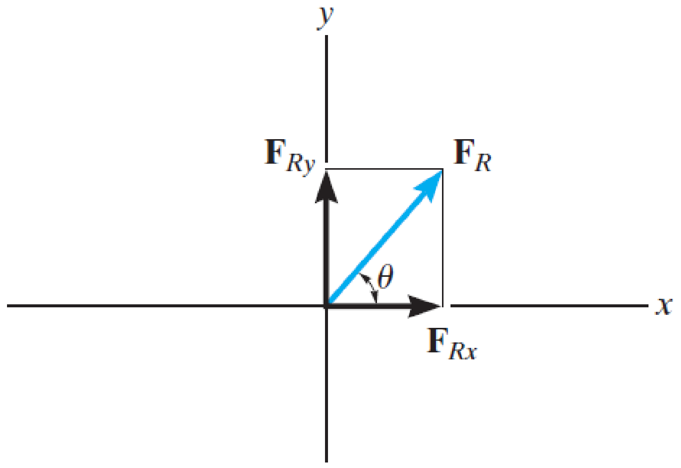
Cartesian Vector Notation



$|F_R|$=????

θ =????

Resultant Force: Magnitude & Orientation



(c)

$$\boxed{\begin{aligned} F_{Rx} &= \Sigma F_x \\ F_{Ry} &= \Sigma F_y \end{aligned}}$$

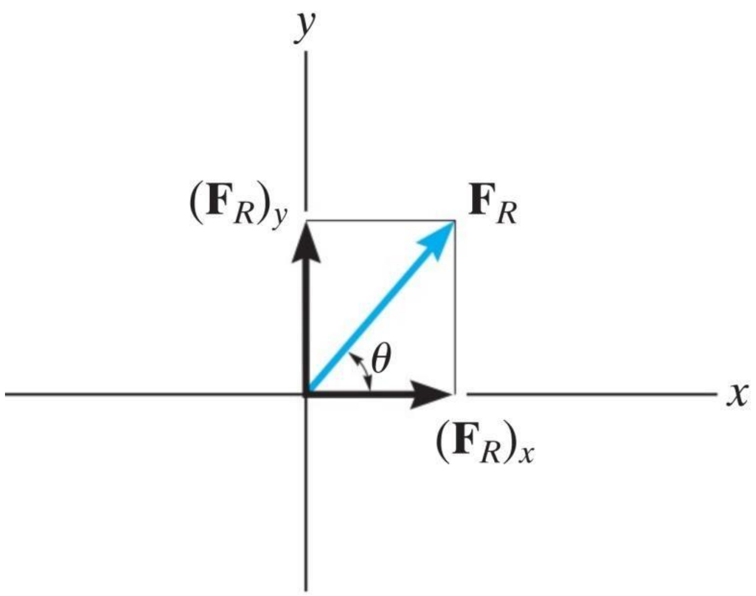
From this sketch, the magnitude of $\mathbf{F}_R$ is then found from the Pythagorean theorem; that is,

$$F_R = \sqrt{F_{Rx}^2 + F_{Ry}^2}$$

Also, the angle θ, which specifies the direction of the resultant force, is determined from trigonometry:

$$\theta = \tan^{-1}\left|\frac{F_{Ry}}{F_{Rx}}\right|$$

Cartesian Vector Notation

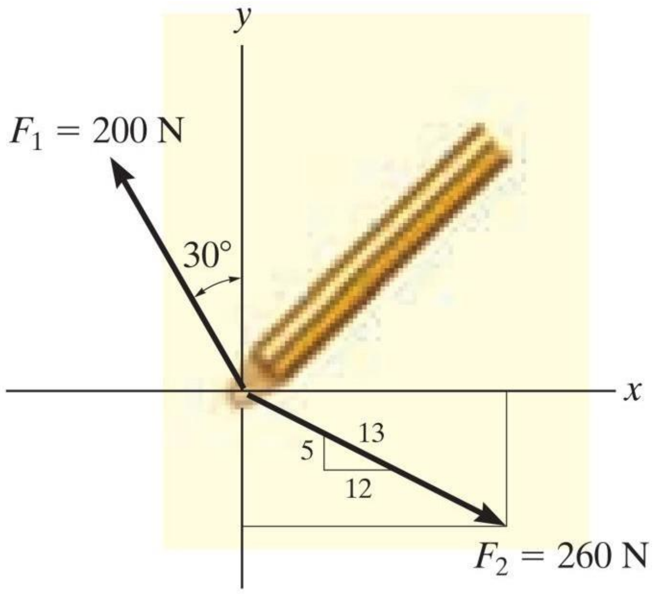


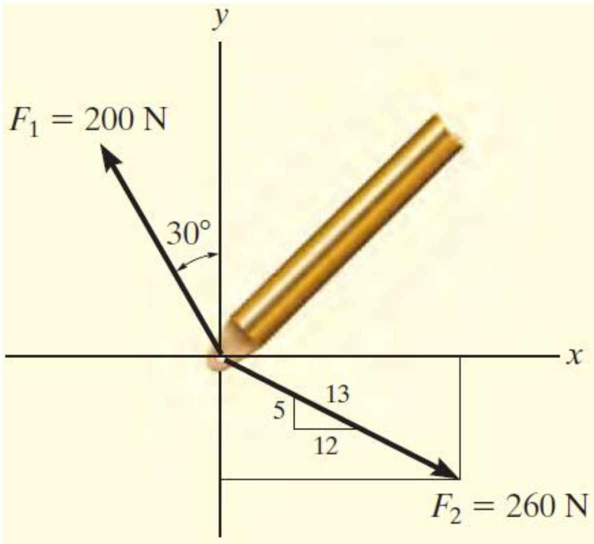
$$|FR| = \sqrt{(F_R)_x^2 + (F_R)_y^2}$$

$$\theta = \tan^{-1}\left|\frac{(F_R)_y}{(F_R)_x}\right|$$

Example

- Determine the *x* and *y* components of **F**1 and **F**2 acting on the boom. Express each force as a Cartesian vector.





$$F_{1x} = -200 \sin 30^\circ \text{ N} = -100 \text{ N}$$

$$F_{1y} = 200 \cos 30^\circ \text{ N} = 173 \text{ N}$$

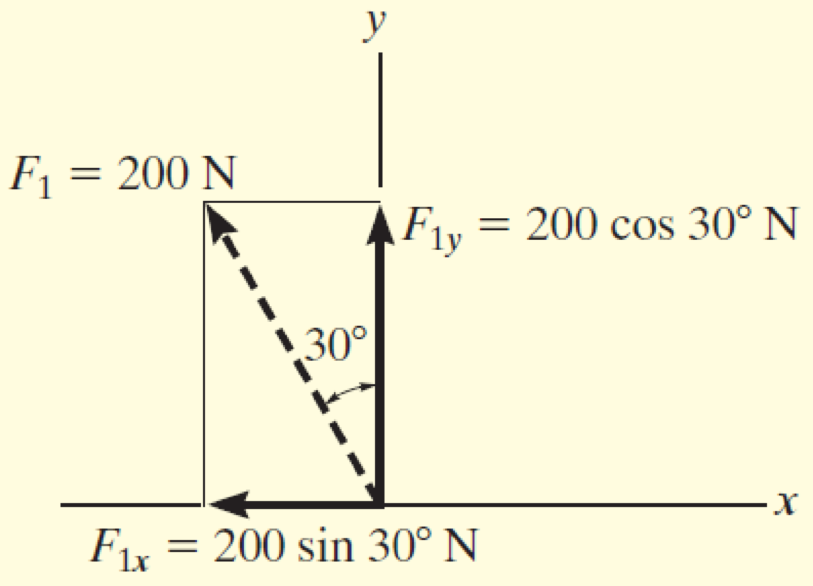
$$\frac{F_{2x}}{260 \text{ N}} = \frac{12}{13} \qquad F_{2x} = 260 \text{ N}\left(\frac{12}{13}\right) = 240 \text{ N}$$

$$F_{2y} = 260 \text{ N}\left(\frac{5}{13}\right) = 100 \text{ N}$$

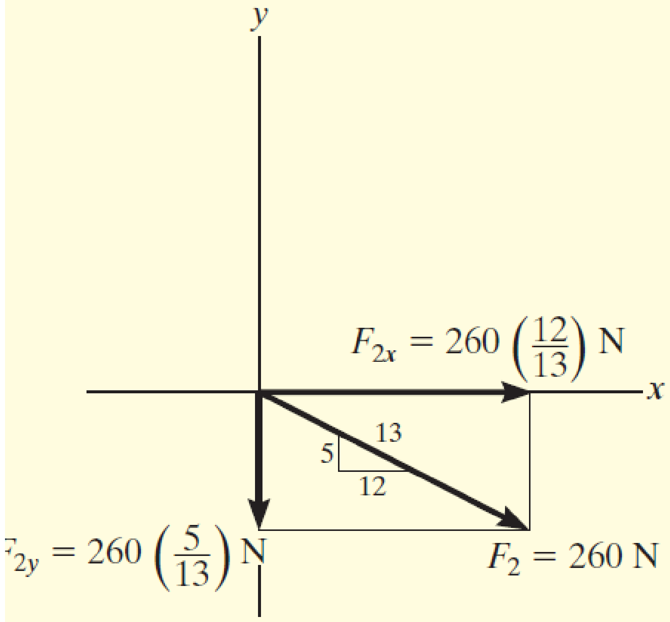
Cartesian Vector Notation.

$$\mathbf{F}_1 = \{-100\mathbf{i} + 173\mathbf{j}\} \text{ N} \qquad \textit{Ans.}$$

$$\mathbf{F}_2 = \{240\mathbf{i} - 100\mathbf{j}\} \text{ N} \qquad \textit{Ans.}$$



(b)

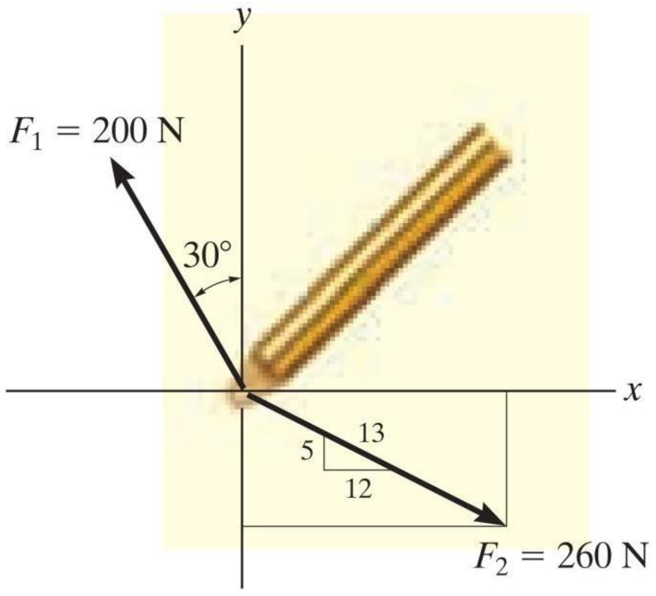


(c)

Fig. 2–18

Example

- Determine the *x* and *y* components of **F**1 and **F**2 acting on the boom. Express each force as a Cartesian vector.



$$F_{1x} = -200 \sin 30° \text{ N} = -100 \text{ N} = 100 \text{ N} \leftarrow$$

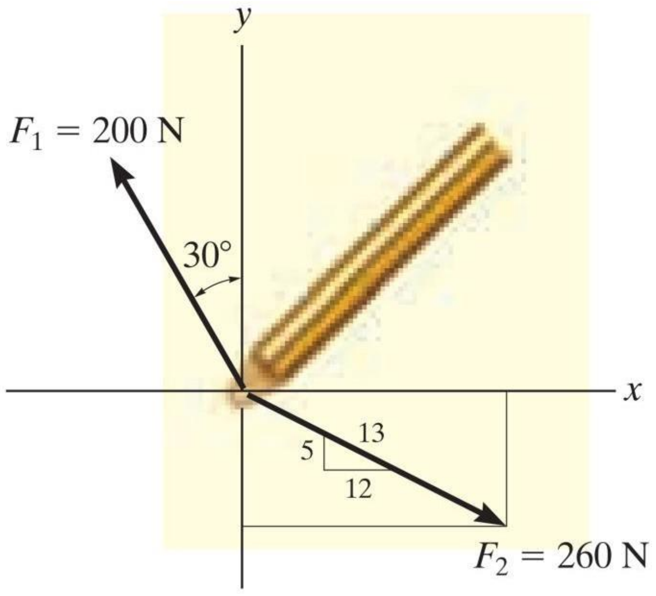
$$F_{1y} = 200 \cos 30° \text{ N} = 173 \text{ N} = 173 \text{ N} \uparrow$$

$$\frac{F_{2x}}{260 \text{ N}} = \frac{12}{13} \qquad F_{2x} = 260 \text{ N}\left(\frac{12}{13}\right) = 240 \text{ N}$$

$$F_{2y} = 260 \text{ N}\left(\frac{5}{13}\right) = 100 \text{ N}$$

Example

- Determine the *x* and *y* components of **F**1 and **F**2 acting on the boom. Express each force as a Cartesian vector.



$$\mathbf{F}_1 = \{-100\mathbf{i} + 173\mathbf{j}\}\,\mathrm{N}$$

$$\mathbf{F}_2 = \{240\mathbf{i} - 100\mathbf{j}\}\,\mathrm{N}$$

Cartesian Vectors

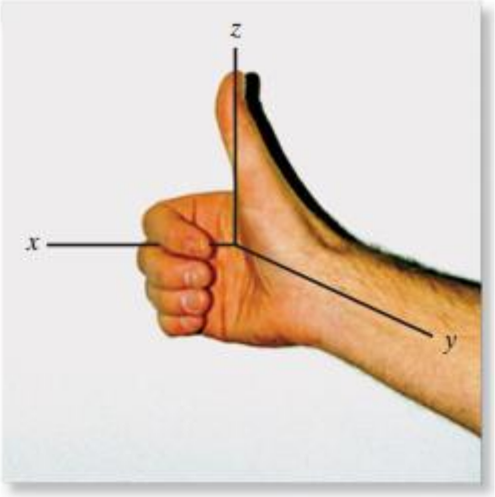
The operations of vector algebra, when applied to solving problems in *three dimensions*, are greatly simplified if the vectors are first represented in Cartesian vector form.

In this section we will present a general method for doing this; then in the next section we will use this method for finding the resultant force of a system of concurrent forces.

Right-Handed Coordinate System.

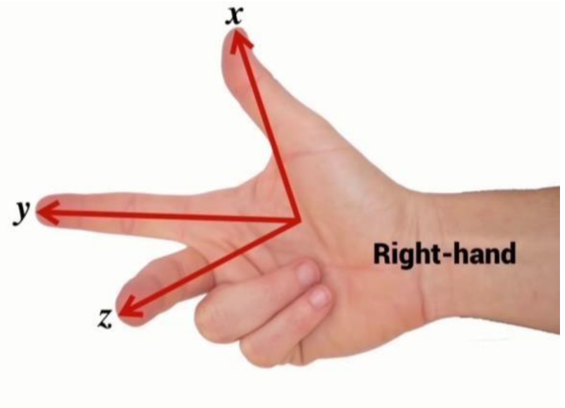
We will use a righthanded coordinate system to develop the theory of vector algebra that follows.

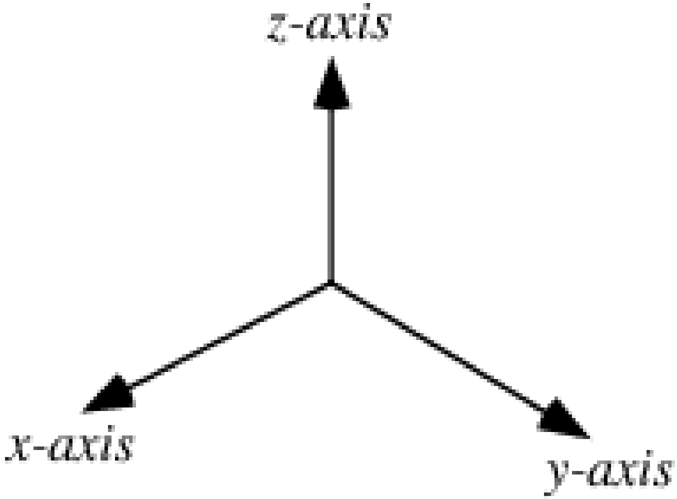
A rectangular coordinate system is said to be *right-handed* if the thumb of the right hand points in the direction of the positive z axis when the right-hand fingers are curled about this axis and directed from the positive x towards the positive y axis, Fig. 2–21



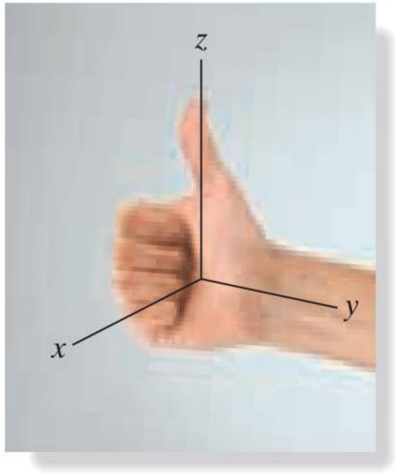
Cartesian Representation in 3D (RIGHT HAND RULE)

- 3 Finger Method





- Curling Method



Rectangular Components of a Vector.

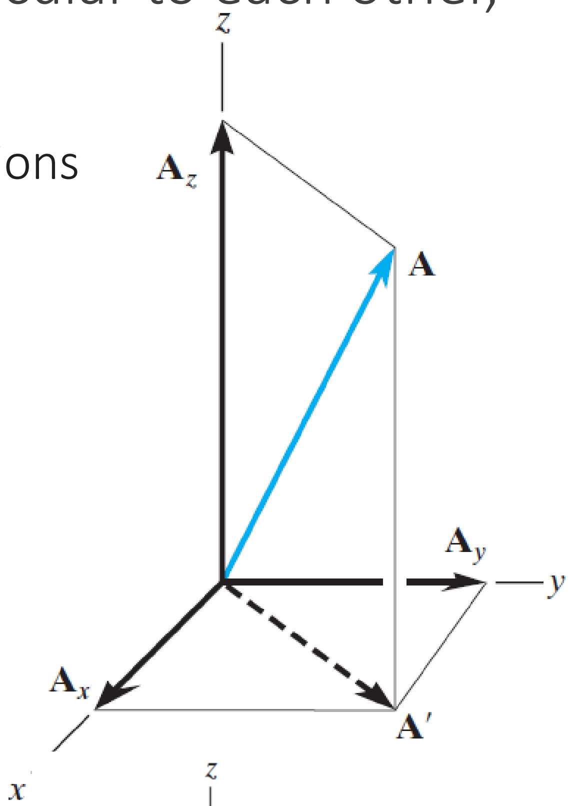
If the components of a given vector are perpendicular to each other, they are called as **Rectangular components**.

$$\mathbf{A} = \mathbf{A}' + \mathbf{A}_z$$

by two successive applications of the parallelogram law

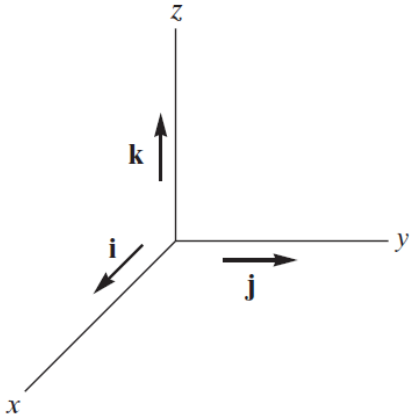
$$\mathbf{A}' = \mathbf{A}_x + \mathbf{A}_y$$

$$\mathbf{A} = \mathbf{A}_x + \mathbf{A}_y + \mathbf{A}_z$$



Cartesian Unit Vectors.

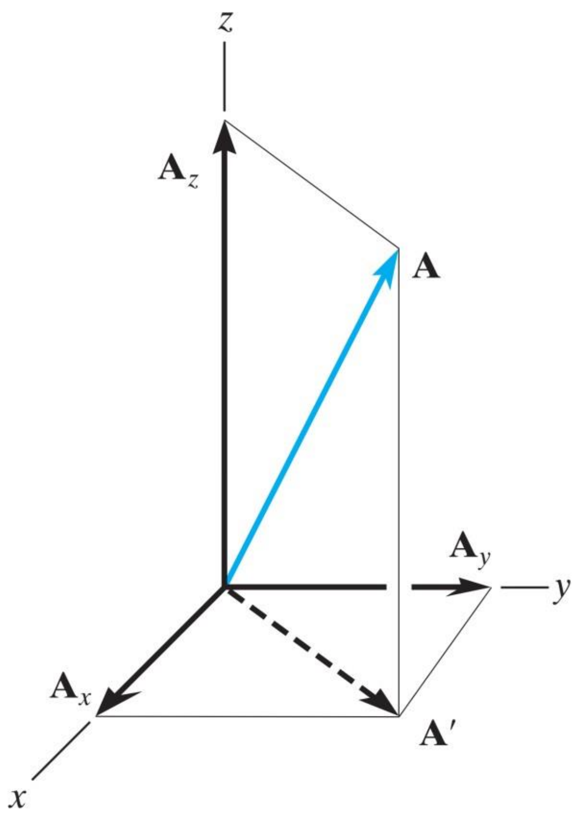
In three dimensions, the set of Cartesian unit vectors, **i**, **j**, **k**, is used to designate the directions of the *x, y, z* axes, respectively



Cartesian Representation in 3D

- Rectangular components

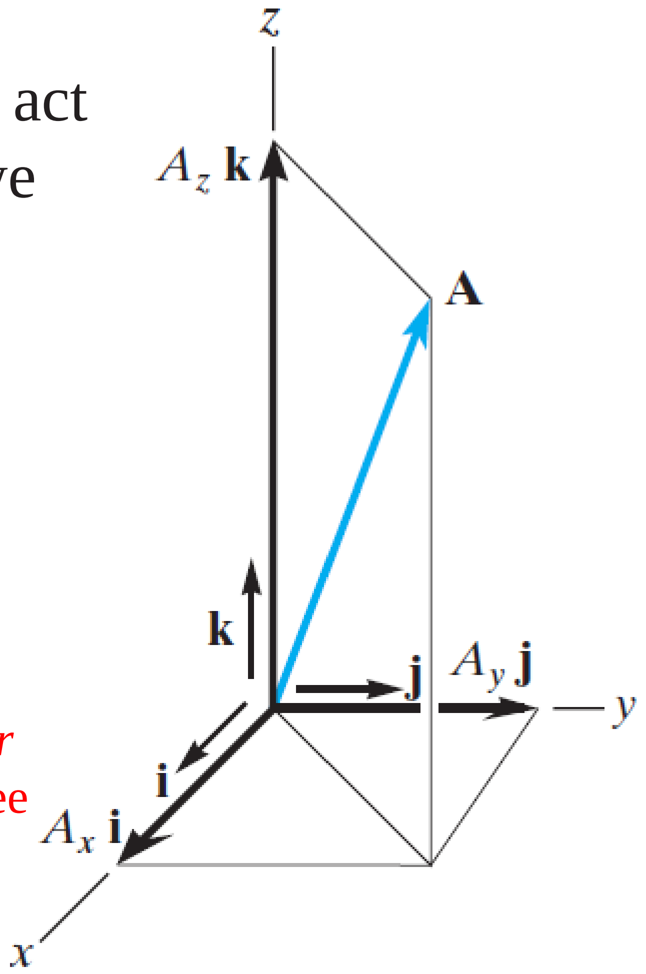
$$\mathbf{A} = \mathbf{A}_x + \mathbf{A}_y + \mathbf{A}_z$$



Cartesian Vector Representation.

Since the three components of **A** in previous Eq. act in the positive **i**, **j**, and **k** directions, Fig. 2–24, we can write **A** in Cartesian vector form as

$$\boxed{\mathbf{A} = A_x\mathbf{i} + A_y\mathbf{j} + A_z\mathbf{k}}$$



separating the *magnitude* and *direction* of each *component vector* will simplify the operations of vector algebra, particularly in three dimensions.

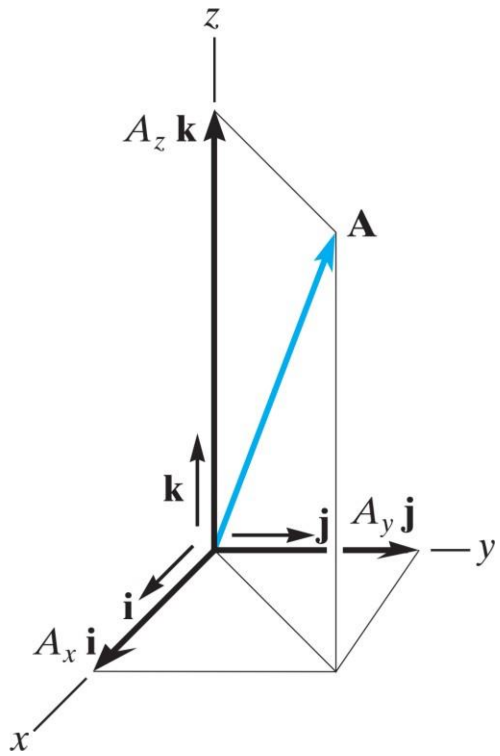
Cartesian Representation in 3D

- Rectangular components

$$\mathbf{A} = \mathbf{A}_x + \mathbf{A}_y + \mathbf{A}_z$$

- Cartesian Unit Vectors

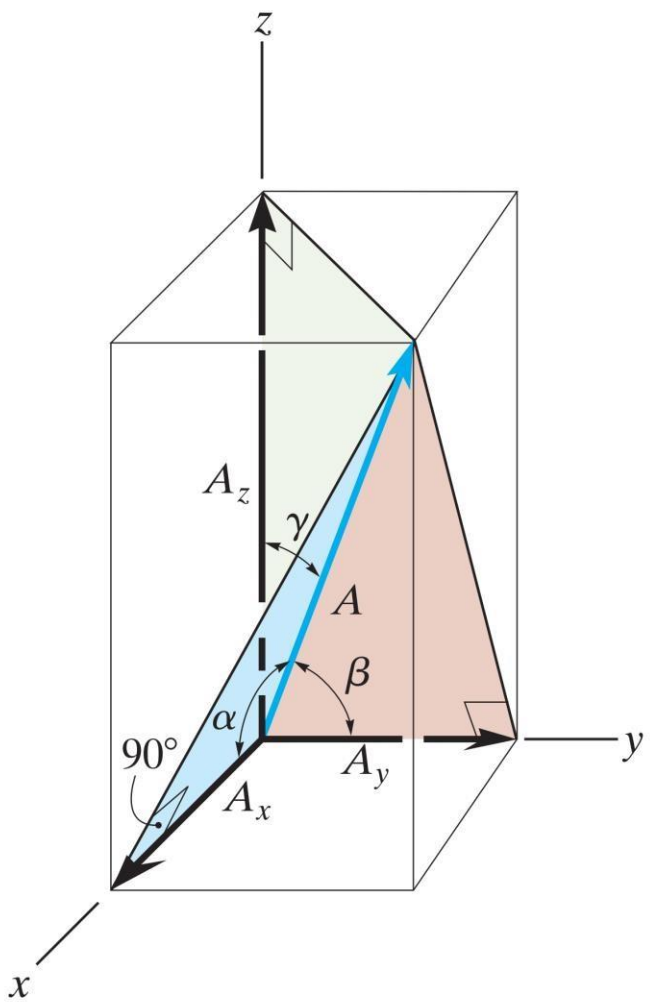
$$\boxed{\mathbf{A} = A_x\mathbf{i} + A_y\mathbf{j} + A_z\mathbf{k}}$$



Cartesian Representation in 3D

- Magnitude

- Angles

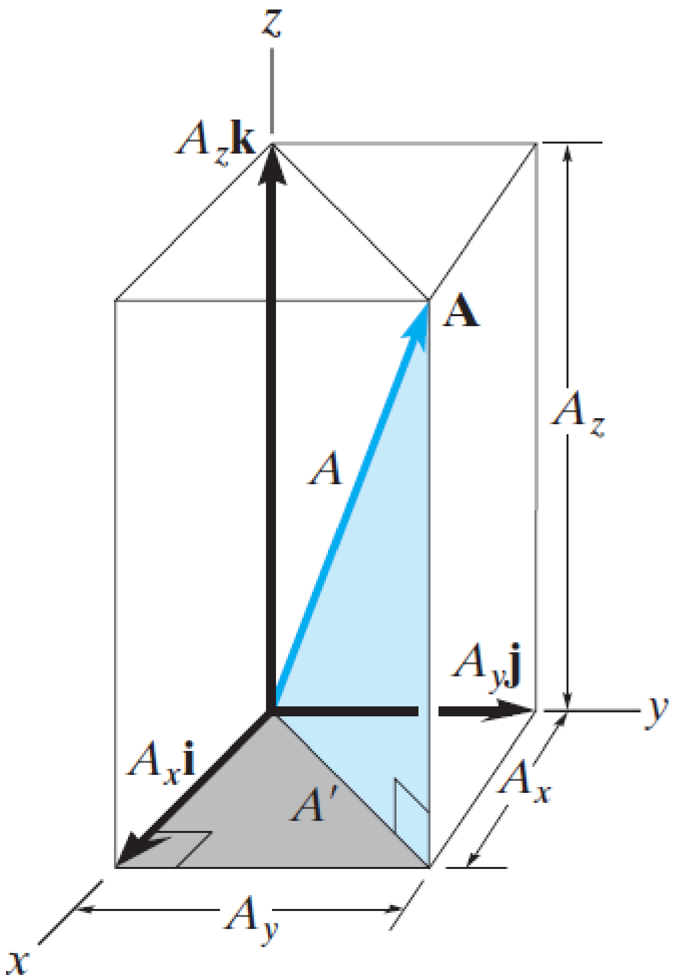


Magnitude of a Cartesian Vector.

$$A = \sqrt{A'^2 + A_z^2}$$ Blue right triangle,

$$A' = \sqrt{A_x^2 + A_y^2}$$ Gray right triangle

$$\boxed{A = \sqrt{A_x^2 + A_y^2 + A_z^2}}$$

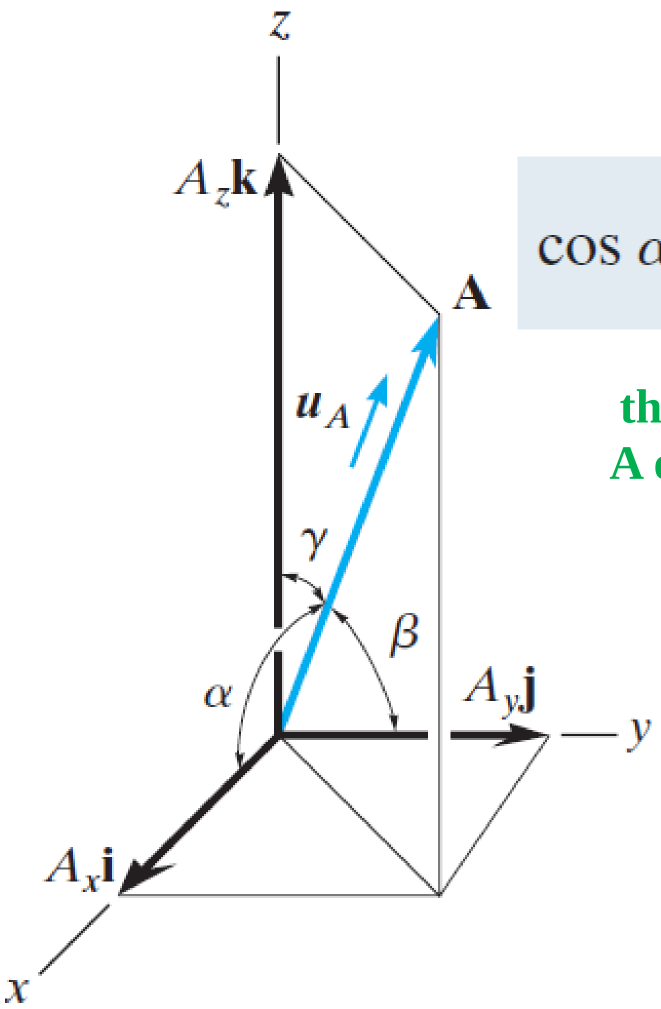


Hence, the magnitude of **A** *is equal to the positive square root of the sum of the squares of its components.*

Direction of a Cartesian Vector.

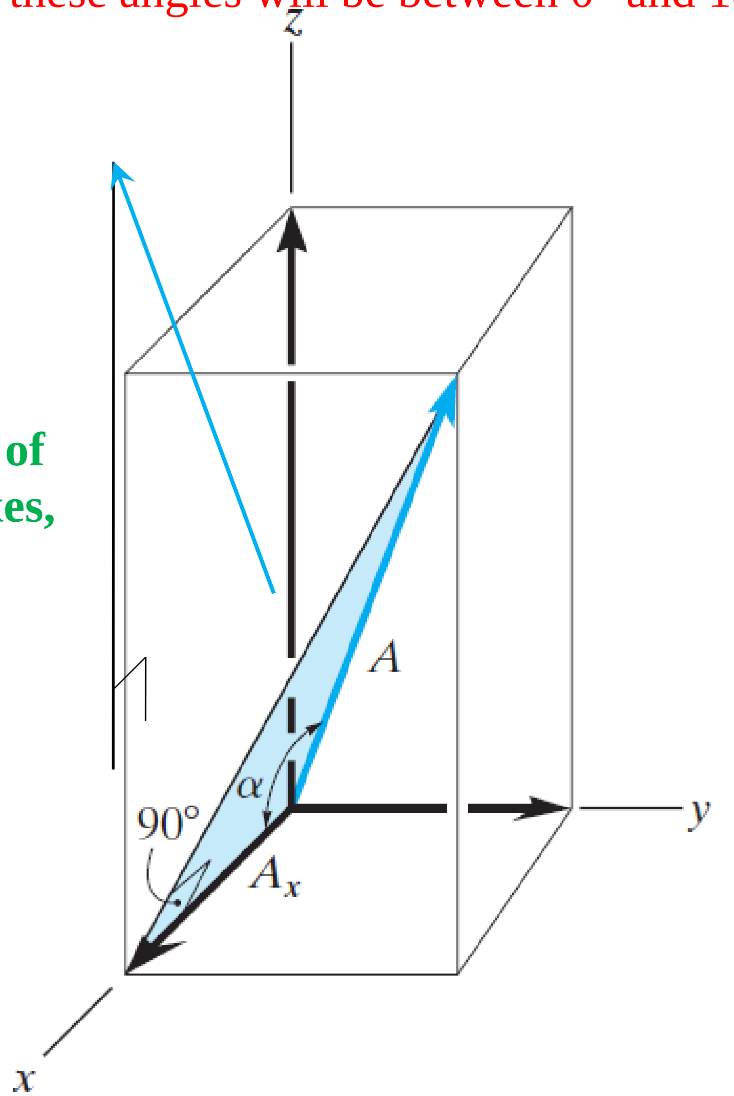
We will define the *direction* of **A** by the *coordinate direction angles* α (alpha), β (beta), and γ (gamma), measured between the *tail* of **A** and the *positive x, y, z* axes provided they are located at the tail of **A**

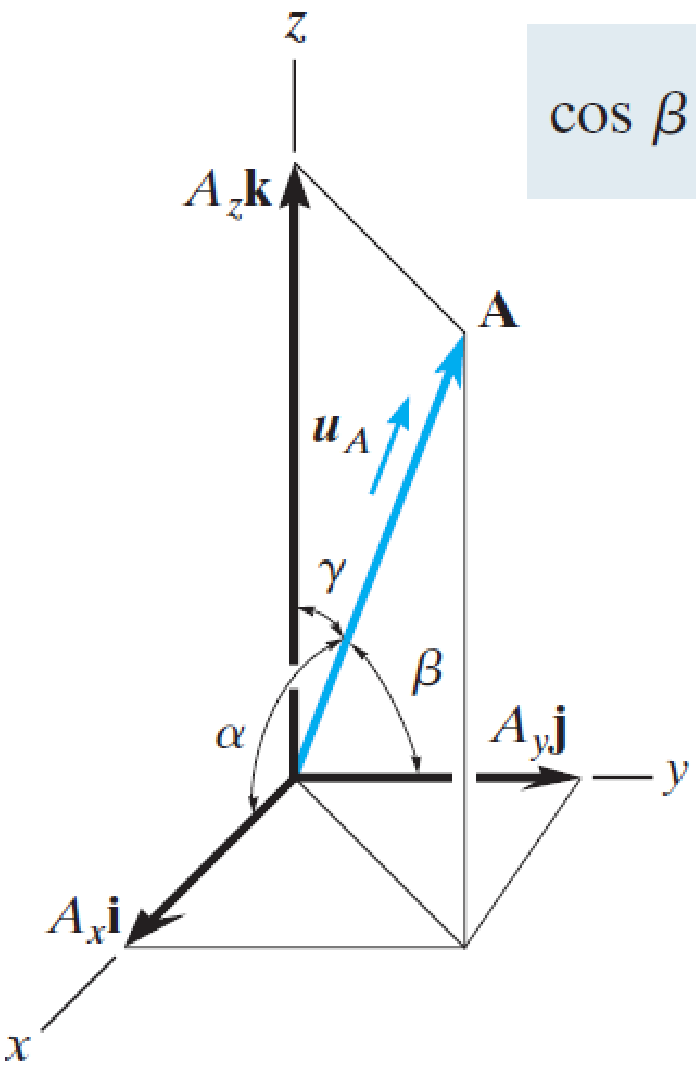
Note that regardless of where **A** is directed, each of these angles will be between 0° and 180°.



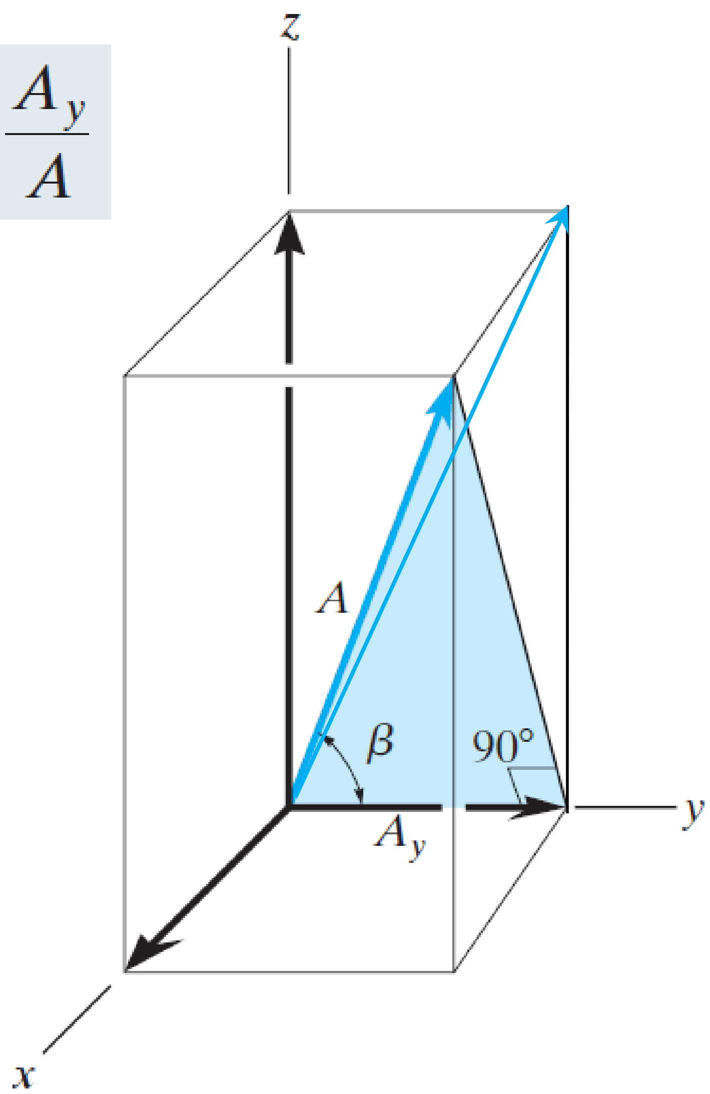
$$\cos\alpha = \frac{A_x}{A}$$

the projection of A onto the *x* axes,

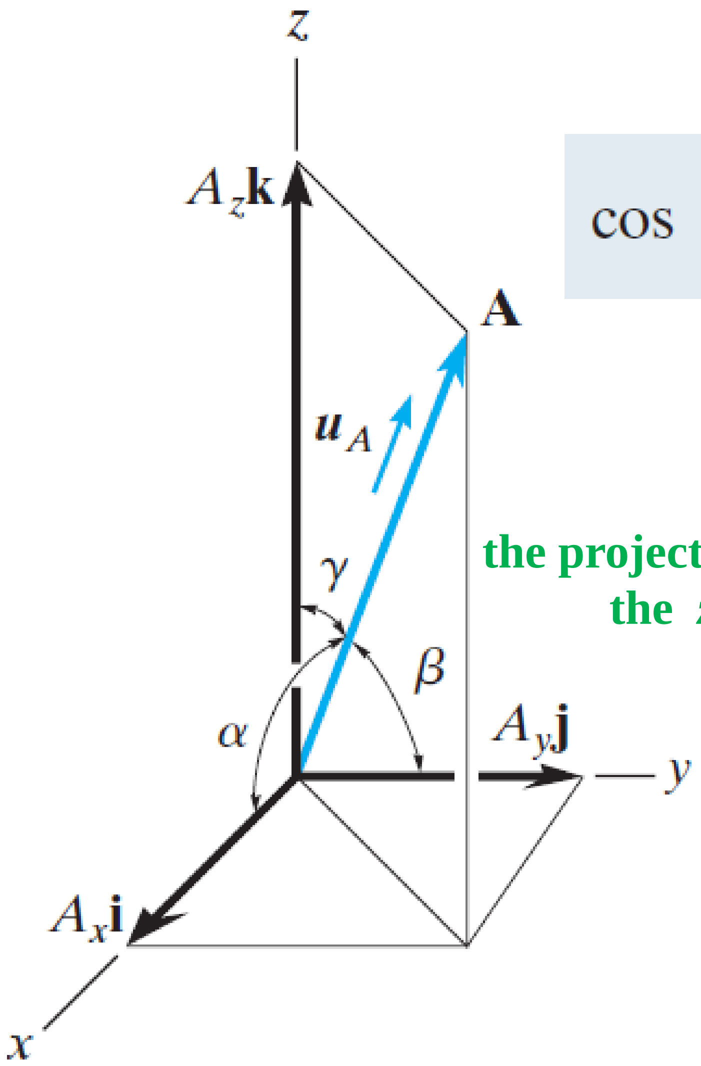




$$\cos\beta = \frac{A_y}{A}$$

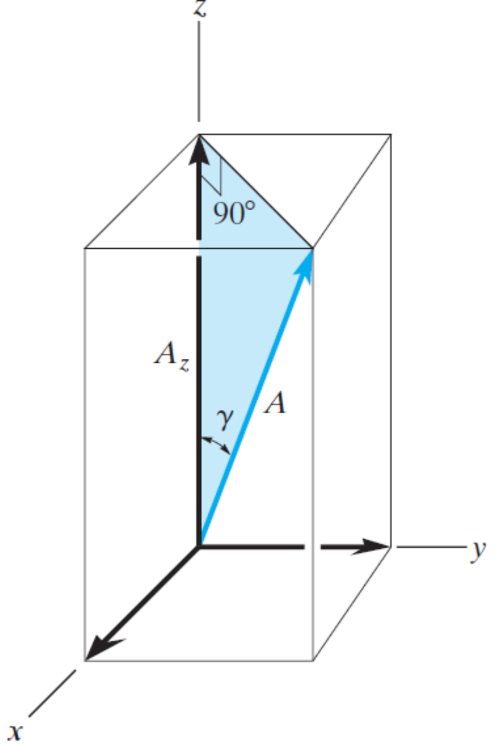


the projection of A onto the *y* axes,



$$\cos\ \gamma = \frac{A_z}{A}$$

the projection of A onto the z axes,



$$\cos \alpha = \frac{A_x}{A} \qquad \cos \beta = \frac{A_y}{A} \qquad \cos\ \gamma = \frac{A_z}{A}$$

These numbers are known as the *direction cosines* of A.

$$\mathbf{u}_A = \frac{\mathbf{A}}{A} = \frac{A_x}{A}\mathbf{i} + \frac{A_y}{A}\mathbf{j} + \frac{A_z}{A}\mathbf{k}$$

An easy way of obtaining these above direction cosines is to form a unit vector $\mathbf{u}_A$ in the direction of A,

$$\mathbf{u}_A = \cos\alpha\,\mathbf{i} + \cos\beta\,\mathbf{j} + \cos\gamma\,\mathbf{k}$$

$$\boxed{\mathbf{u}_A = \frac{\mathbf{A}}{A} = \frac{A_x}{A}\mathbf{i} + \frac{A_y}{A}\mathbf{j} + \frac{A_z}{A}\mathbf{k}}$$

$$\mathbf{u}_A = \cos\alpha\,\mathbf{i} + \cos\beta\,\mathbf{j} + \cos\gamma\,\mathbf{k}$$

$$\boxed{\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1}$$

Here we can see that if only *two* of the coordinate angles are known, the third angle can be found using this equation

Finally, if the magnitude and coordinate direction angles of $\mathbf{A}$ are known, then $\mathbf{A}$ may be expressed in Cartesian vector form as

$$\boxed{\begin{aligned}\mathbf{A} &= A\mathbf{u}_A \\ &= A\cos\alpha\,\mathbf{i} + A\cos\beta\,\mathbf{j} + A\cos\gamma\,\mathbf{k} \\ &= A_x\mathbf{i} + A_y\mathbf{j} + A_z\mathbf{k}\end{aligned}} \tag{2–9}$$

Cartesian Representation in 3D

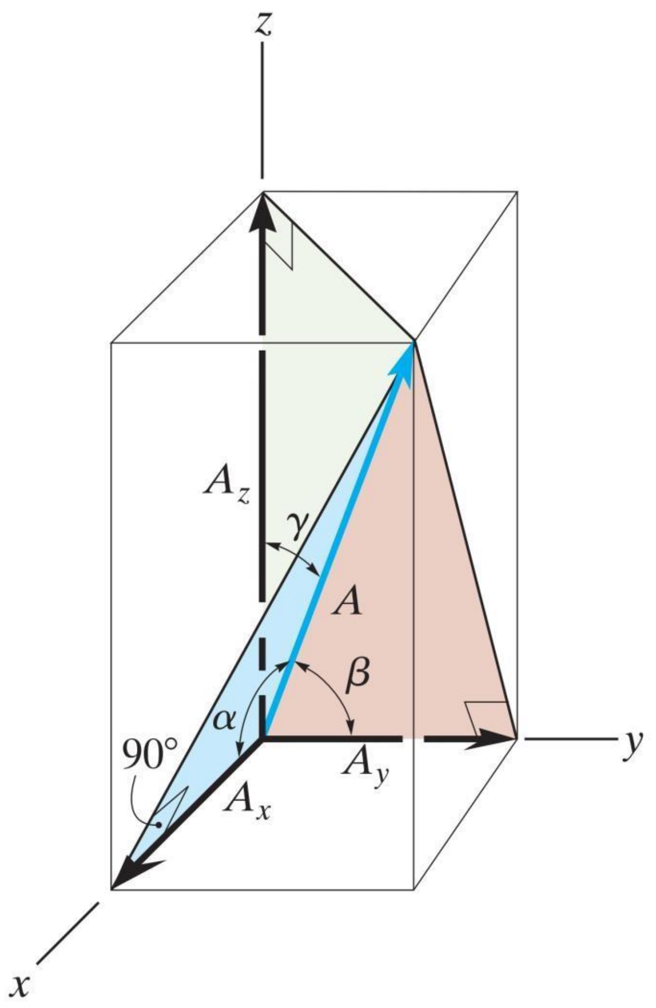
- Magnitude

$$A = \sqrt{A_x^2 + A_y^2 + A_z^2}$$

- Angles

$$\cos\alpha = \frac{A_x}{A} \quad \cos\beta = \frac{A_y}{A} \quad \cos\gamma = \frac{A_z}{A}$$

$$\begin{aligned}\mathbf{A} &= A\,\mathbf{u}_A \\ &= A\cos\alpha\,\mathbf{i} + A\cos\beta\,\mathbf{j} + A\cos\gamma\,\mathbf{k} \\ &= A_x\mathbf{i} + A_y\mathbf{j} + A_z\mathbf{k}\end{aligned}$$



Transverse and Azimuth Angle Representation

- Sometimes

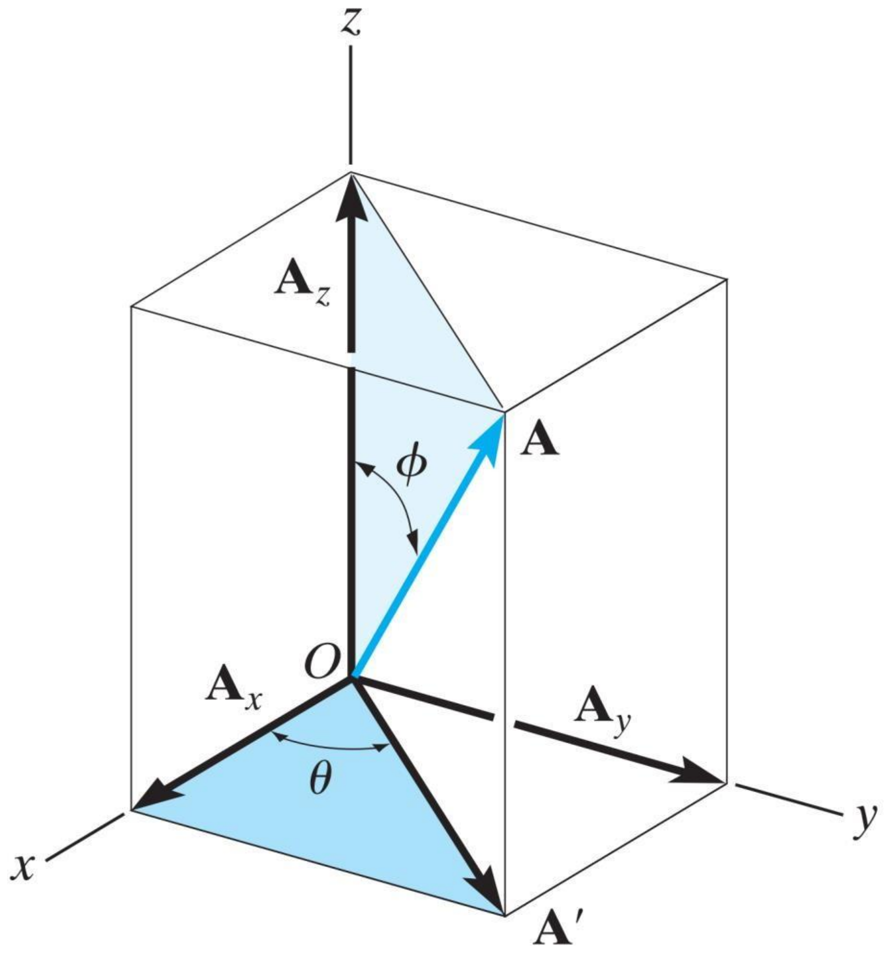
$$A_z =$$

$$A' =$$

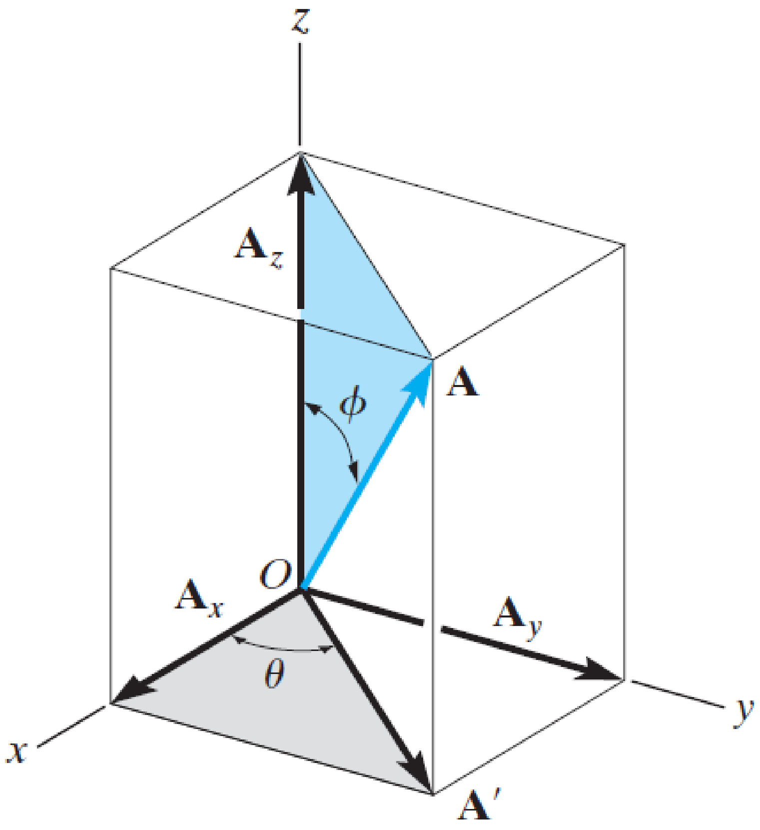
$$A_x =$$

$$A_y =$$

$$\mathbf{A} =$$



Sometimes, the direction of **A** can be specified using **two angles theta, and phi**, such as shown in Fig. 2–28. The components of **A** can then be determined by applying trigonometry **first to the blue right triangle**, which yields



$$A_z = A\cos\phi$$

$$A' = A\sin\phi$$

$$A_x = A' \cos\theta = A\sin\phi\cos\theta$$

$$A_y = A' \sin\theta = A\sin\phi\sin\theta$$

$$\mathbf{A} = A\sin\phi\cos\theta\,\mathbf{i} + A\sin\phi\sin\theta\,\mathbf{j} + A\cos\phi\,\mathbf{k}$$

You should not memorize this equation, rather it is important to understand how the components were determined using trigonometry.

Transverse and Azimuth Angle Representation

- Sometimes

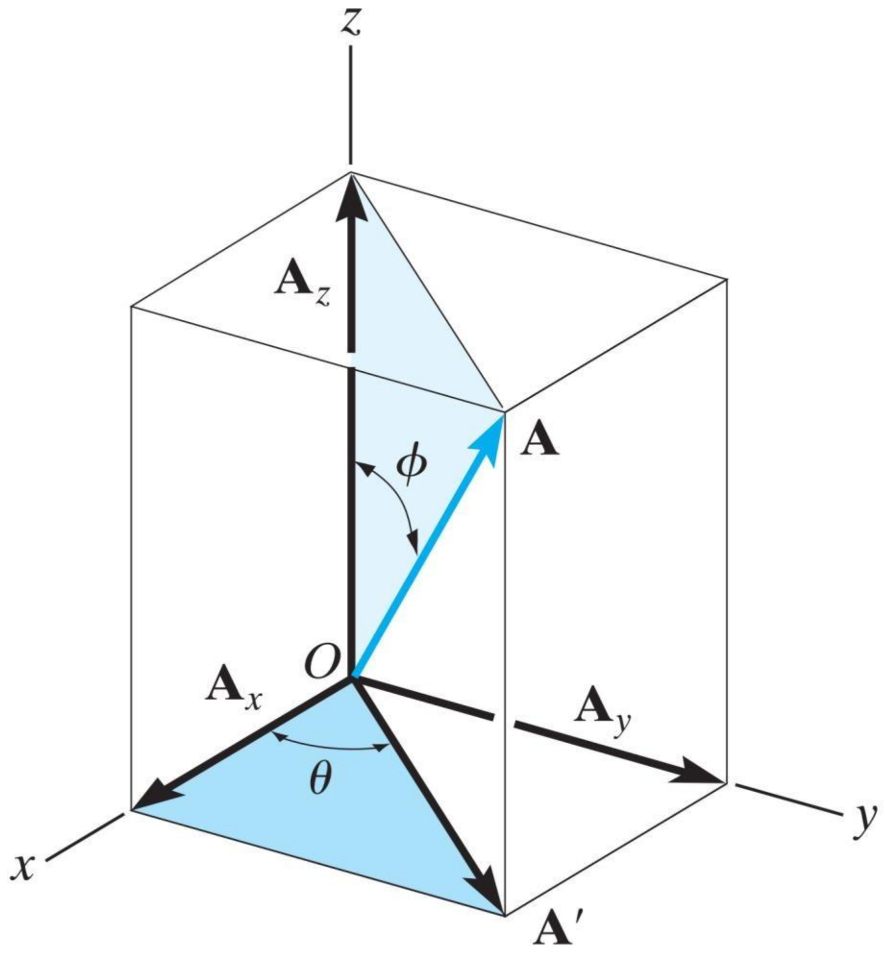
$$A_z = A \cos\phi$$

$$A' = A \sin\phi$$

$$A_x = A' \cos\theta = A \sin\phi \cos\theta$$

$$A_y = A' \sin\theta = A \sin\phi \sin\theta$$

$$\mathbf{A} = A \sin\phi \cos\theta\,\mathbf{i} + A \sin\phi \sin\theta\,\mathbf{j} + A \cos\phi\,\mathbf{k}$$

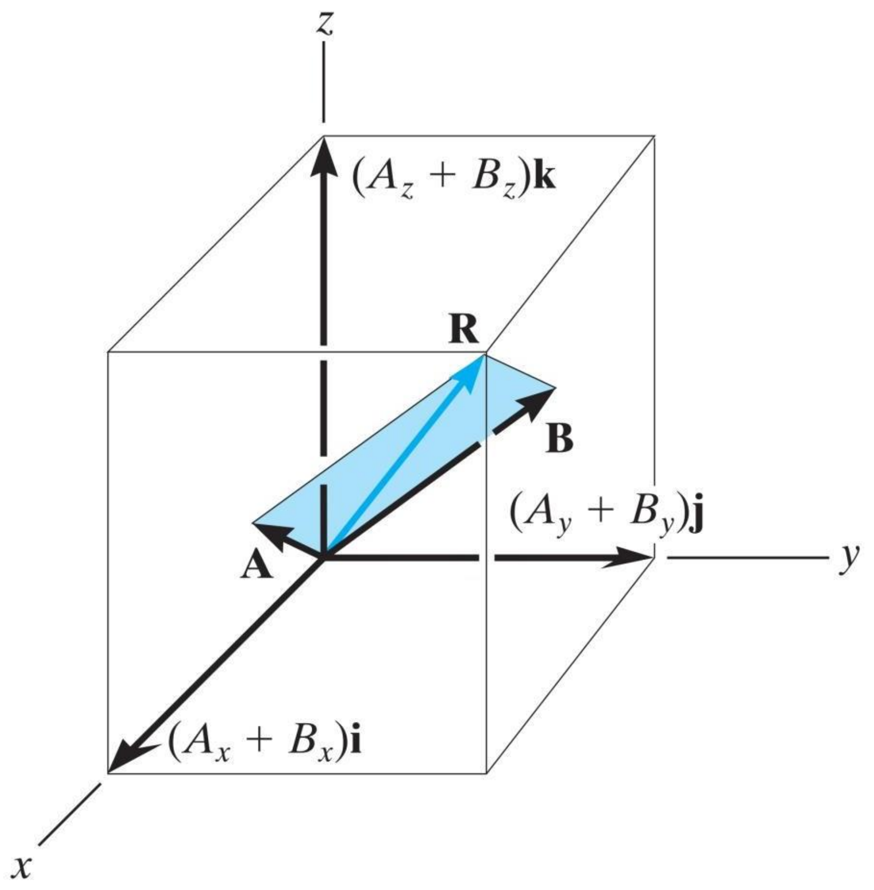


Addition of Cartesian Vectors

$$\mathbf{R} = \mathbf{A} + \mathbf{B} = (A_x + B_x)\mathbf{i} + (A_y + B_y)\mathbf{j} + (A_z + B_z)\mathbf{k}$$

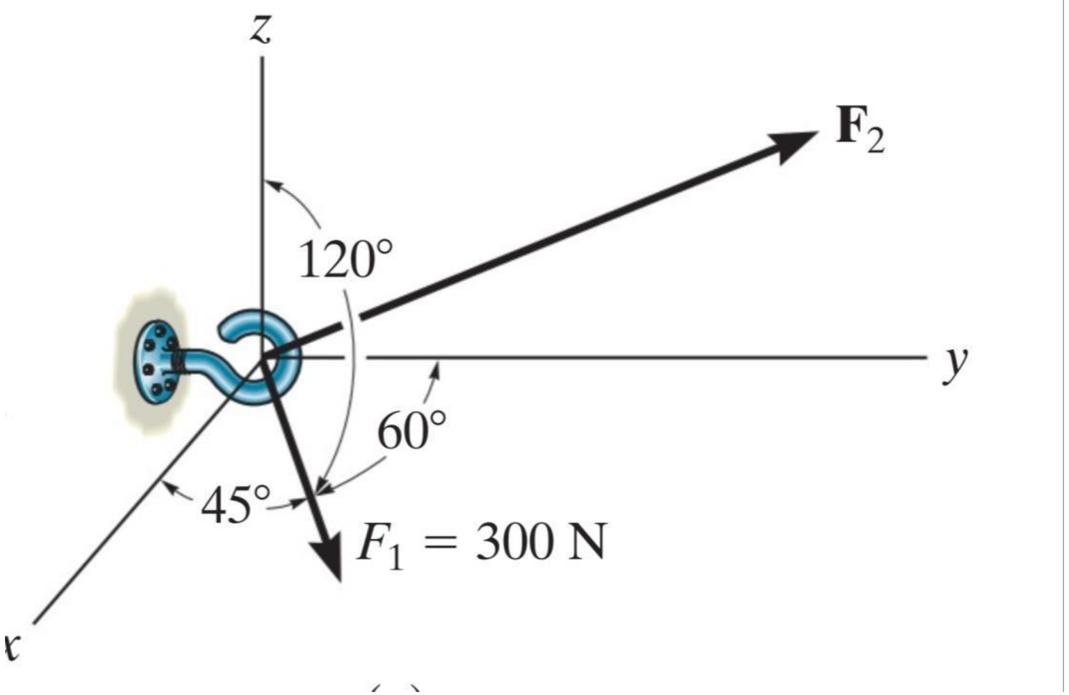
- In general

$$\boxed{\mathbf{F}_R = \Sigma\mathbf{F} = \Sigma F_x\mathbf{i} + \Sigma F_y\mathbf{j} + \Sigma F_z\mathbf{k}}$$



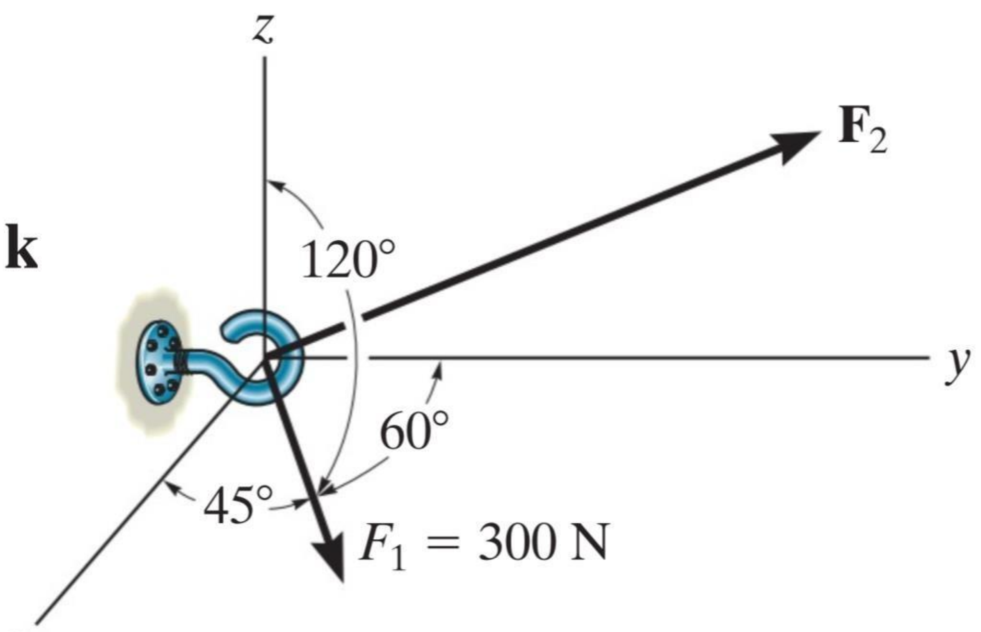
Example

- Two forces act on the hook. Specify the magnitude of **F**2 and its coordinate direction angles so that the resultant force **F**$_R$ acts along the positive *y* axis and has a magnitude of 800 N.



Example

$$\mathbf{F}_1 = F_1 \cos \alpha_1 \mathbf{i} + F_1 \cos \beta_1 \mathbf{j} + F_1 \cos \gamma_1 \mathbf{k}$$
$$= 300 \cos 45^\circ\, \mathbf{i} + 300 \cos 60^\circ\, \mathbf{j} + 300 \cos 120^\circ\, \mathbf{k}$$
$$= \{212.1\mathbf{i} + 150\mathbf{j} - 150\mathbf{k}\}\,\mathrm{N}$$
$$\mathbf{F}_2 = F_{2x}\mathbf{i} + F_{2y}\mathbf{j} + F_{2z}\mathbf{k}$$



$$\mathbf{F}_R = \mathbf{F}_1 + \mathbf{F}_2$$
$$800\mathbf{j} = 212.1\mathbf{i} + 150\mathbf{j} - 150\mathbf{k} + F_{2x}\mathbf{i} + F_{2y}\mathbf{j} + F_{2z}\mathbf{k}$$
$$800\mathbf{j} = (212.1 + F_{2x})\mathbf{i} + (150 + F_{2y})\mathbf{j} + (-150 + F_{2z})\mathbf{k}$$

Example

$$0 = 212.1 + F_{2x} \qquad F_{2x} = -212.1 \text{ N}$$

$$800 = 150 + F_{2y} \qquad F_{2y} = 650 \text{ N}$$

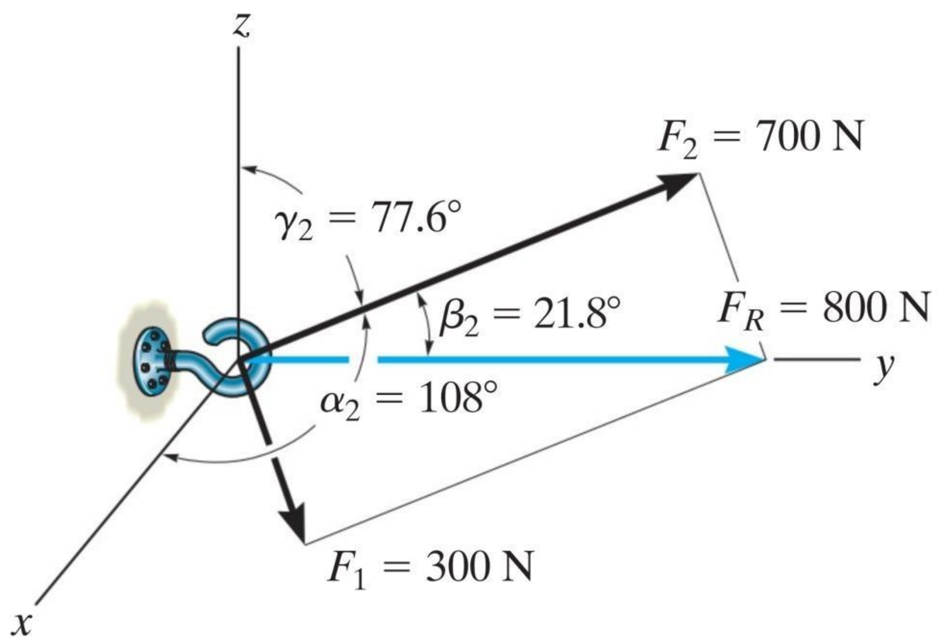
$$0 = -150 + F_{2z} \qquad F_{2z} = 150 \text{ N}$$

$$F_2 = \sqrt{(-212.1 \text{ N})^2 + (650 \text{ N})^2 + (150 \text{ N})^2}$$
$$= 700 \text{ N}$$

$$\cos \alpha_2 = \frac{-212.1}{700}; \qquad \alpha_2 = 108^\circ$$

$$\cos \beta_2 = \frac{650}{700}; \qquad \beta_2 = 21.8^\circ$$

$$\cos \gamma_2 = \frac{150}{700}; \qquad \gamma_2 = 77.6^\circ$$



Home Assignment

- Example 2.6