Engineering Mechanics : Statics

Chapter 6: Structural analysis

Chapter outline

- Simple Trusses
- The Method of Joints
- The Method of Sections
- Problems

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- Simple Trusses/Trusses
- The Method of Joints
- The Method of Sections

Chapter 6: Analysis of Structures

Some of the most common structures we see around us are buildings & bridges. In addition to these, one can also classify a lot of other objects as "structures."
For instance:

- The space station
- Chassis of your car
- Your chair, table, bookshelf etc. etc.

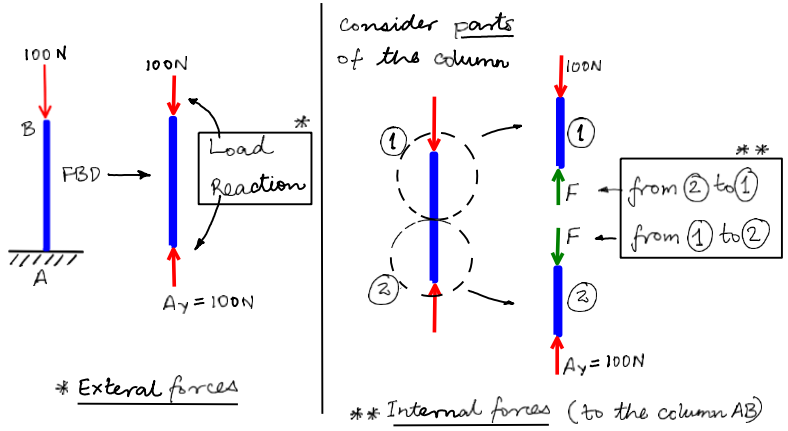
Almost everything has an internal structure and can be thought of as a "structure".

The objective of this chapter is to figure out the forces being carried by these structures so that as an engineer, you can decide whether the structure can sustain these forces or not.

Recall:

- External forces: "Loads" acting **on** your structure.
  Note: this includes "reaction" forces from the supports as well.

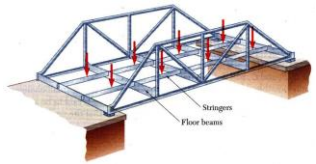
- Internal forces: Forces that develop within every structure that keep the different parts of the structure together.



In this chapter, we will find the internal forces in the following types of structures :

- Trusses
- Frames
- Machines

6.1 Trusses



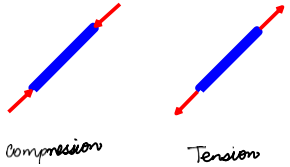
Trusses are used commonly in Steel buildings and bridges.

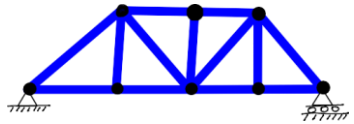
Definition: A truss is a structure that consists of

- All straight members
- connected together with pin joints
- connected only at the ends of the members
- and all external forces (loads & reactions) must be applied only at the joints.

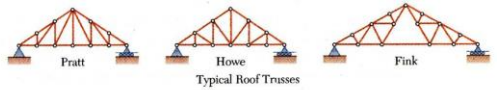
Note:

- Every member of a truss is a 2 force member.
- Trusses are assumed to be of negligible weight (compared to the loads they carry)

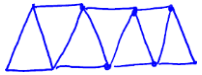




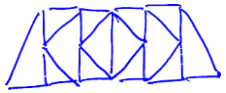
Types of Trusses



Simple Trusses: constructed from a "base" triangle by adding two members at a time.

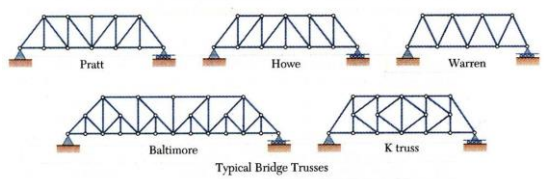


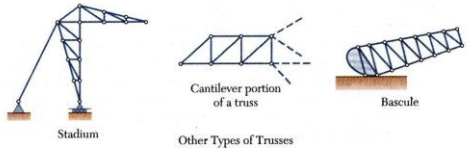
simple



simple

NOT simple





Note: For Simple Trusses (and in general statically determinate trusses)

$2n = m+r$

m: members, r: reactions, n: joints } $(m+r)$ unknowns

$2n$ equations

Note: This is a necessary condition for statical determinancy

This is not sufficient condition. So even if a truss satisfies the above relation it may not be determinate.

But if it is determinate then it satisfies the above relation

Chapter outline

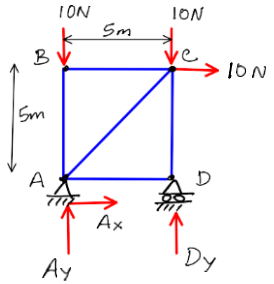
- Simple Trusses/Trusses
- The Method of Joints
- The Method of Sections

Analysis of Trusses: 6.2 **Method of Joints**

Problem

Consider the truss shown. Truss analysis involves:

(i) Determining the EXTERNAL reactions.
(ii) Determining the INTERNAL forces in each of the members (tension or compression).



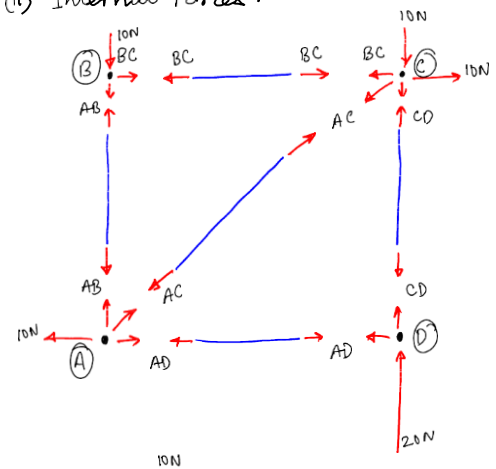
(i) External Reactions :

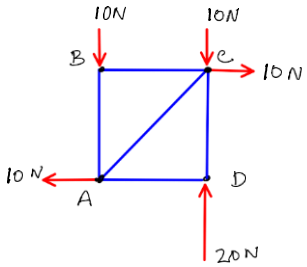
$\sum F_x = 0 \Rightarrow A_x + 10 = 0 \Rightarrow \boxed{A_x = -10N}$

$\sum F_y = 0 \Rightarrow A_y + D_y - 10 - 10 = 0 \Rightarrow \boxed{A_y = 0}$

$\sum M_A = 0 \Rightarrow -(10 \times 5) - (10 \times 5) + D_y \times 5 = 0 \Rightarrow \boxed{D_y = 20N}$

(ii) Internal Forces :

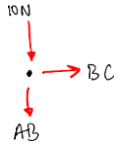




Joint D:

$\sum F_x = 0 \Rightarrow \boxed{AD = 0}$

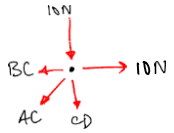
Joint B:



$\Rightarrow \sum F_x = 0 \Rightarrow \boxed{BC = 0}$ (10N Compressive)

$\sum F_y = 0 \Rightarrow -AB - 10 = 0 \Rightarrow \boxed{AB = -10}$

Joint C



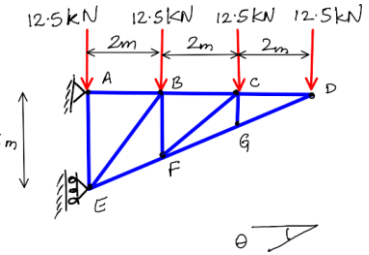
$\sum F_x = 0 \Rightarrow -BC - AC\cos 45° + 10 = 0$

$\Rightarrow AC = 10\sqrt{2} = \boxed{14.14N}$ T

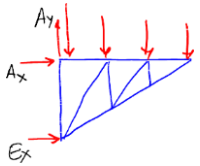
$\sum F_y = 0 \Rightarrow -10 - AC\sin 45° - CD = 0$

$\Rightarrow CD = -10N - 10 = \boxed{-20N}$ C

Problem



(i) External Reactions



$\sum f_x = 0$

$\Rightarrow A_x + E_x = 0$

$\sum f_y = 0$

$\Rightarrow A_y - 4 \times 12.5 = 0 \Rightarrow \boxed{A_y = 50\ kN}$

$\sin\theta = 5/13$

$\cos\theta = 12/13$

$\sum M_A = 0$

$\Rightarrow E_x(2.5) - 12.5(2+4+6) = 0 \Rightarrow \boxed{E_x = 60\ kN} \Rightarrow \boxed{A_x = -60\ kN}$

Joint D:

$\sum f_x = 0 \Rightarrow -CD - GD\cos\theta = 0 \Rightarrow CD = -32.5 \times \frac{12}{13} = \boxed{30\ kN}$ (T)

$\sum f_y = 0 \Rightarrow -12.5 - GD\sin\theta = 0 \Rightarrow GD = \frac{-12.5}{5} \times 13$

$= \boxed{-32.5\ kN}$ (comp)

Joint G:

$\sum f_y = 0 \Rightarrow \boxed{CG = 0}$

$\boxed{FG = GD = -32.5\ kN}$

Similarly, solve joints C, F and B in that order and calculate the rest of the unknowns.

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- Simple Trusses/Trusses
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Analysis of Trusses: 6.4 **Method of Sections**

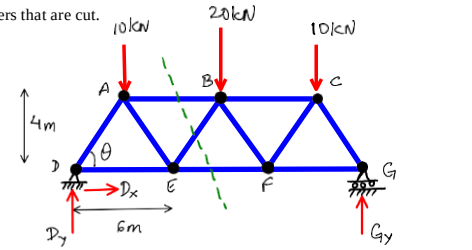
The method of joints is good if we have to find the internal forces in all the truss members.
In situations where we need to find the internal forces only in a few specific members of a truss, the method of sections is more appropriate.

Method of sections:

- Imagine a cut through the members of interest
- Try to cut the least number of members (preferably 3).
- Draw FBD of the 2 different parts of the truss
- Enforce Equilibrium to find the forces in the 3 members that are cut.

Problem

Find the force in member EF:



EXTERNAL (Entire Truss)

$\Sigma F_x = 0 \Rightarrow D_x = 0$

$\Sigma F_y = 0 \Rightarrow D_y + G_y = 0$

Symmetry $\Rightarrow D_y = G_y = 20$ kN

INTERNAL (Cut ----)

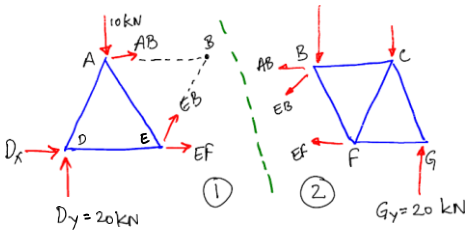
Body ① :-

$\Sigma F_x = 0 \Rightarrow EF + EB\cos\theta + AB = 0$

$\Sigma F_y = 0 \Rightarrow EB\sin\theta + 20\text{kN} - 10\text{kN} = 0$

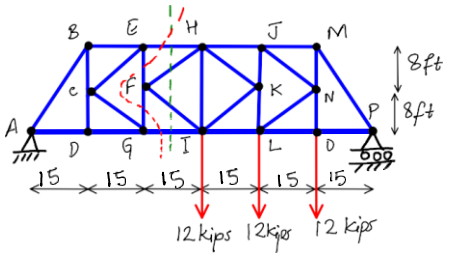
$\Sigma M_B = 0 \Rightarrow EF \times 4 - 20 \times 9 + 10 \times 6 = 0$

$\Rightarrow$ **EF = 30 kN** (T)



Problem

Find forces in the members EH and GI.



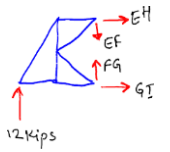
$\Sigma M_A = 0$ (for entire truss)

$\Rightarrow -36 \times 60 + P_y \times 90 = 0$

$\Rightarrow P_y = \frac{36 \times 60}{90} = 24$ kips

$\Rightarrow$ **A_y = 12 kips**

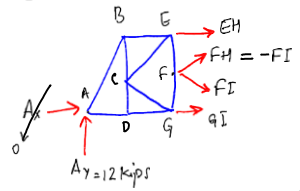
Cut ----



$\Sigma M_E = 0 \Rightarrow -12 \times 30 + GI \times 16 = 0 \Rightarrow GI = \frac{360}{16} =$ **22.5 kips** (T)

$\Sigma F_x = 0 \Rightarrow EH + GI = 0 \Rightarrow$ **EH = −22.5 kips** (C)

OR Cut - - - - -



$\Sigma F_x = 0 \Rightarrow EH + GI - FI\cos\theta + FI\cos\theta = 0$

$\Sigma F_y = 0 \Rightarrow 12 - FI\sin\theta - FI\sin\theta = 0$

$\Sigma M_F = 0 \Rightarrow -12\times30 - EH\times8 + GI\times8 = 0$

$\Rightarrow GI = \frac{12\times30}{16} = \boxed{22.5 \text{ kips}}$ (T) $\Rightarrow \boxed{EH = -22.5 \text{ kips}}$ (C)

$\Sigma F_x = 0 \Rightarrow \boxed{FH = -FI}$

$\Sigma F_y = 0$

$\Rightarrow EF - FG + FH\sin\theta - FI\sin\theta = 0$

$\Rightarrow EF - FG + 2FH\sin\theta = 0$