Chapter2: Boolean Algebra and Logic Gates

Lecture3- Canonical Forms

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Objectives

- Study Canonical Forms
- Standard and non-standard Forms
- Conversion of Canonical Forms

Canonical Forms

- A canonical form is a standard method for representing Boolean functions.
- The two canonical forms that are used are:
  - Sum of Minterms
  - Product of Maxterms
- These forms are sometimes considered the "brute force" method of representing functions as they seldom represent a function in a minimized form.
- Examples of these forms are:

  $F_1 = xyz' + xy'z + x'y'z'$

  $F_2 = (x+y+z')(x+y'+z)(x'+y'+z')$

Minterms

- Any given binary variable can be represented in two forms:
  - x, its normal form, and
  - X', its complement
- If we consider two binary variables and the AND operation, there are four combinations of the variables:
  - xy
  - Xy'
  - X'y
  - X'y'
- Each of the above four AND terms is called a minterm or a standard product.
- n variables can be combined to form 2^n minterms.

Minterms Expressed

Minterms for Three Variables

X	Y	Z	Product Term	Symbol
0	0	0	$\overline{X}\,\overline{Y}\,\overline{Z}$	m_0
0	0	1	$\overline{X}\,\overline{Y}Z$	m_1
0	1	0	$\overline{X}Y\overline{Z}$	m_2
0	1	1	$\overline{X}YZ$	m_3
1	0	0	$X\overline{Y}\,\overline{Z}$	m_4
1	0	1	$X\overline{Y}Z$	m_5
1	1	0	$XY\overline{Z}$	m_6
1	1	1	XYZ	m_7

Maxterms Expressed

Maxterms for Three Variables

X	Y	Z	Sum Term	Symbol
0	0	0	$X + Y + Z$	M_0
0	0	1	$X + Y + \overline{Z}$	M_1
0	1	0	$X + \overline{Y} + Z$	M_2
0	1	1	$X + \overline{Y} + \overline{Z}$	M_3
1	0	0	$\overline{X} + Y + Z$	M_4
1	0	1	$\overline{X} + Y + \overline{Z}$	M_5
1	1	0	$\overline{X} + \overline{Y} + Z$	M_6
1	1	1	$\overline{X} + \overline{Y} + \overline{Z}$	M_7

Truth Table to Expression (Sum of Minterms)

- Any Boolean function can be expressed as a sum of minterms or sum of products (i.e. the ORing of terms).
  - You can form the function algebraically by forming a minterm for each combination of the variables that produces a 1 in the function. (Each row with output of 1 becomes a product term)
  - Sum (OR) product terms together.

x	y	z	G
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	0
1	1	0	1
1	1	1	1

xyz + xyz’ + x’yz

Minterms and Maxterms Expressed

Table 2.3
Minterms and Maxterms for Three Binary Variables

			Minterms		Maxterms	
x	y	z	Term	Designation	Term	Designation
0	0	0	$x'y'z'$	m_0	$x + y + z$	M_0
0	0	1	$x'y'z$	m_1	$x + y + z'$	M_1
0	1	0	$x'yz'$	m_2	$x + y' + z$	M_2
0	1	1	$x'yz$	m_3	$x + y' + z'$	M_3
1	0	0	$xy'z'$	m_4	$x' + y + z$	M_4
1	0	1	$xy'z$	m_5	$x' + y + z'$	M_5
1	1	0	xyz'	m_6	$x' + y' + z$	M_6
1	1	1	xyz	m_7	$x' + y' + z'$	M_7

Sum of Minterms Example

x	y	z		Function F_1	Required Minterms
0	0	0		1	$x'y'z'$
0	0	1		0	
0	1	0		0	
0	1	1		1	$x'yz$
1	0	0		1	$xy'z'$
1	0	1		0	
1	1	0		0	
1	1	1		0	

$$F_1 = x'y'z' + x'yz + xy'z'$$

$$= m_0+m_3+m_4$$

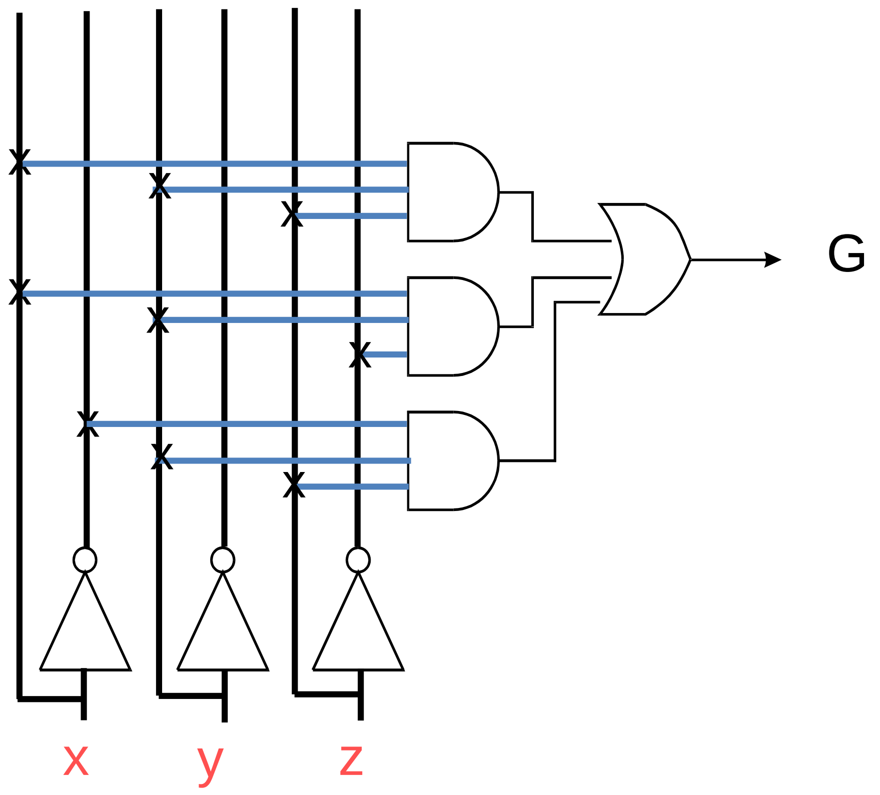
$$= \sum(0,3,4)$$

Equivalent Representations of Circuits

- All three formats are equivalent
- Number of 1's in truth table output column equals AND terms for Sum-of-Products (SOP)

x	y	z	G
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	0
1	1	0	1
1	1	1	1

G = xyz + xyz' + x'yz



Truth Table to Expression (Product of Maxterms)

- Any Boolean function can be expressed as a product of maxterms or product of sums (i.e. the ANDing of terms).
  - You can form the function algebraically by forming a maxterm for each combination of the variables that produces a 0 in the function. (Each row with output of 0 becomes a standard sums)
  - AND these maxterms together.

Product of Maxterms Example

x	y	z		Function F_1	Required Maxterms
0	0	0		1	
0	0	1		0	$x + y + z'$
0	1	0		0	$x + y' + z$
0	1	1		1	
1	0	0		1	
1	0	1		0	$x' + y + z'$
1	1	0		0	$x' + y' + z$
1	1	1		0	$x' + y' + z'$

$F_1 = (x + y + z')(x + y' + z)(x' + y + z')(x' + y' + z)(x' + y' + z')$

$= M_1M_2M_5M_6M_7$

$= \pi(1,2, 5, 6, 7)$

Minterms and Maxterms

- Each variable in a Boolean expression is a literal
- Boolean variables can appear in normal (x) or complement form (x')
- Each AND combination of terms is a minterm
- Each OR combination of terms is a maxterm
- Example:

Minterms

x	y	z	Minterm	
0	0	0	x'y'z'	m_0
0	0	1	x'y'z	m_1
			…	
1	0	0	xy'z'	m_4
			…	
1	1	1	xyz	m_7

Maxterms

x	y	z	Maxterm	
0	0	0	x+y+z	M_0
0	0	1	x+y+z'	M_1
			…	
1	0	0	x'+y+z	M_4
			…	
1	1	1	x'+y'+z'	M_7

Obtaining Sum of Minterms Form

A	B	C	F = A'B + B' + C	Required Minterms	Required Designations
0	0	0	1	A'B'C'	m_0
0	0	1	1	A'B'C	m_1
0	1	0	1	A'BC'	m_2
0	1	1	1	A'BC	m_3
1	0	0	1	AB'C'	m_4
1	0	1	1	AB'C	m_5
1	1	0	0		
1	1	1	1	ABC	m_7

F = A´B´C´ + A´B´C + A´BC´ + A´BC + AB´C´ + AB´C+ ABC

$= m_0 + m_1 + m_2 + m_3 + m_4 + m_5 + m_7$

F(A, B, C) = ∑(0, 1, 2, 3, 4, 5, 7)

Obtaining Product of Maxterms

A	B	C		F = A'B + B'C	Required Maxterms	Required Designations
0	0	0		0	A + B + C	M_0
0	0	1		1		
0	1	0		1		
0	1	1		1		
1	0	0		0	A' + B + C	M_4
1	0	1		1		
1	1	0		0	A' + B' + C	M_6
1	1	1		0	A' + B' + C'	M_7

F = (A+B+C)(A´+B+C)(A´+B´+C)(A´+B´+C´)

= $M_0 \, . \, M_4 \, . \, M_6 \, . \, M_7$

F(A, B, C) = π(0, 4, 6, 7)

Canonical Form Conversion

- A function represented as Sum of minterms can be represented as the Product of maxterms of the remaining terms.
- The complement of a function expressed in sum of minterms equals the sum of minterms missing from the original function
  - ➢ $F(A, B, C) = \sum(0, 3,4) = m_0+m_3+m_4$
  - ➢ $F'(A, B, C) = \sum(1,2,5,6,7) = m_1+m_2+m_5+m_6+m_7$
- Now if we take the complement of F' using DeMorgan's theorem, we obtain F in the product of maxterms form:
  - $(F')' = (m_1+m_2+m_5+m_6+m_7)'$
  - $F = m_1' . m_2' . m_5' . m_6' . m_7'$ [Complement of minterms]
  - $= M_1M_2M_5M_6M_7$ [maxterms]
  - $= \pi(1,2, 5, 6, 7)$
- This implies the following relation:

  $m_i' = Mj$
- So sum of minterms: $\sum(0,3,4)$ = product of maxterms: $\pi(1,2, 5, 6, 7)$

Table A: Conversion of Forms

		Desired Form			
Given Form		Minterm Expansion of F	Maxterm Expansion of F	Minterm Expansion of F’	Maxterm Expansion of F’
	Minterm Expansion of F	-	maxterm nos are those nos, not on the minterm list of F	List minterms not present in F	Maxterm nos are same as minterm nos of F
	Maxterm Expansion of F	minterm nos are those nos, not on the maxterm list of F	-	minterm nos are same as maxterm nos of F	List maxterms not present in F

Table B: Application of Table A

	Desired Form				
Given Form		Minterm Expansion of F	Maxterm Expansion of F	Minterm Expansion of F’	Maxterm Expansion of F’
	F=∑(3,4,5,6,7)	-	F=π(0,1,2)	∑(0,1,2)	π(3,4,5,6,7)
	F=π(0,1,2)	∑(3,4,5,6,7)	-	∑(0,1,2)	π(3,4,5,6,7)

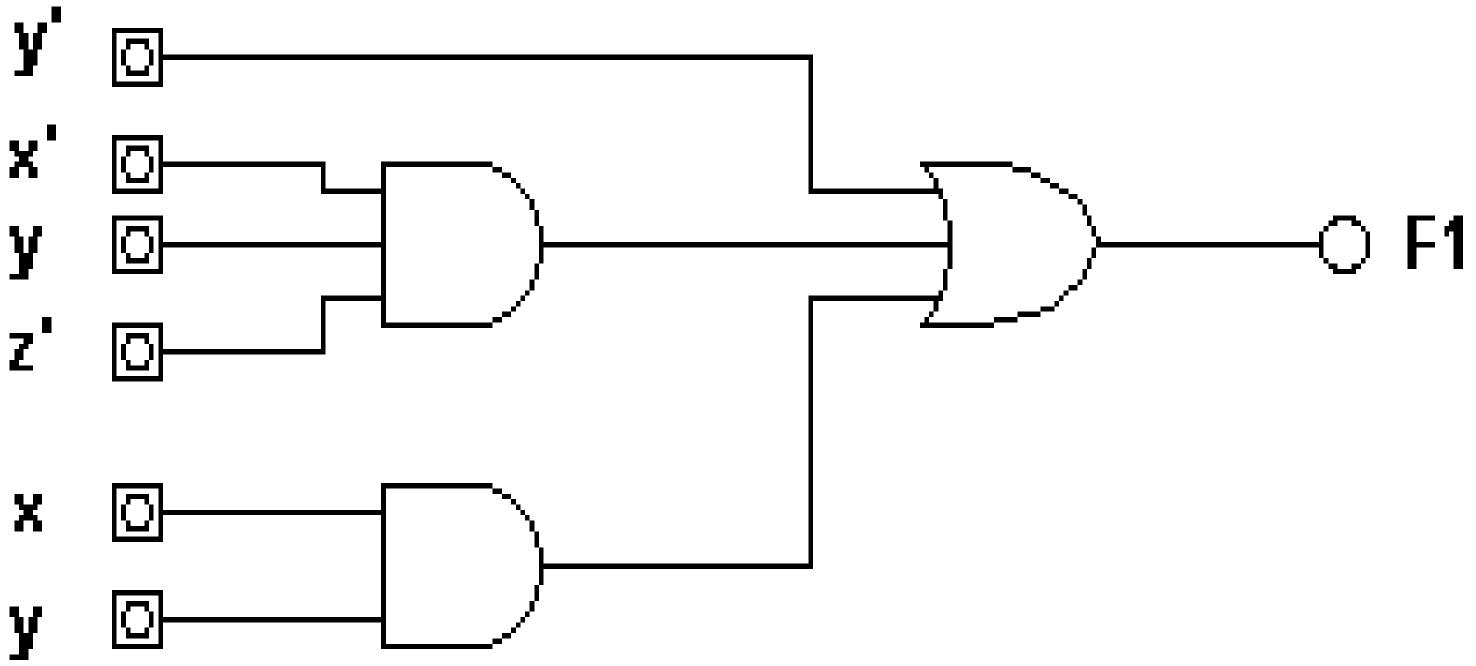
Standard Forms

- Standard forms are those forms that allow the terms forming the function to consist of any number of the variables.
- There are two standard forms:
  - sum of products (SOP)
  - product of sums (POS)
- Examples of these two forms are:

$F=xy'+x'yz$	SOP
$G=(X+Y')(X'+Y+Z)$	POS

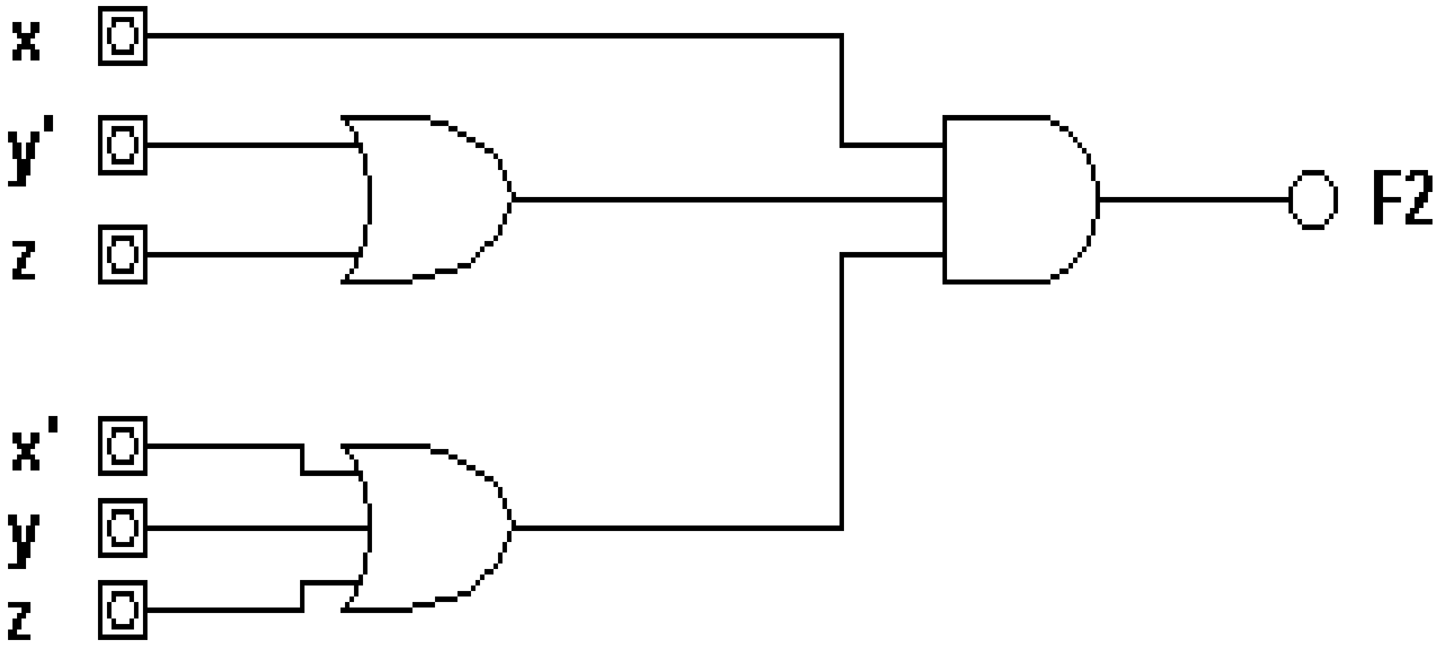
Sum of Products

- The Sum of Products (SOP) is a Boolean expression containing AND terms, called product terms, of one or more literals each.
  - $F_1 = y' + xy + x'yz'$



Product of Sums

- The Product of Sums (POS) is a Boolean expression containing OR terms, called sum terms, of one or more literals each.
  - ➢ $F_2 = x(y' + z)(x' + y + z')$



Two Level Implementations

- The standard type of expression results in a two-level gating structure

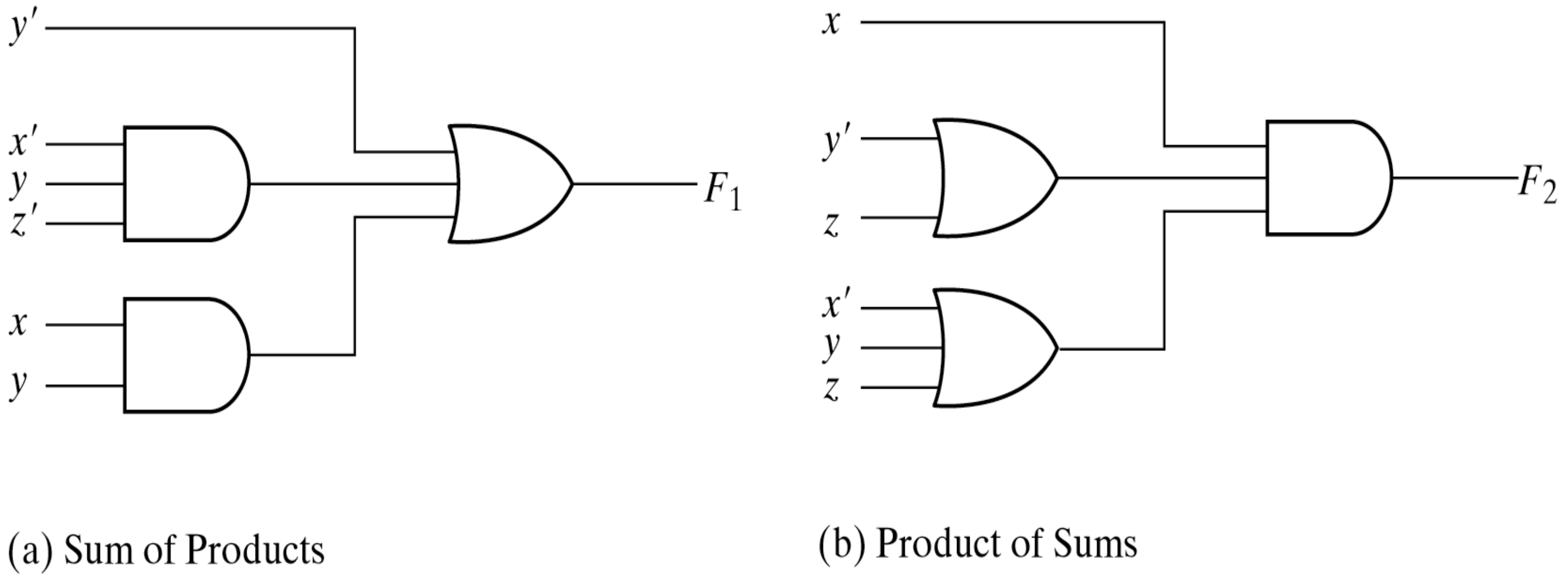
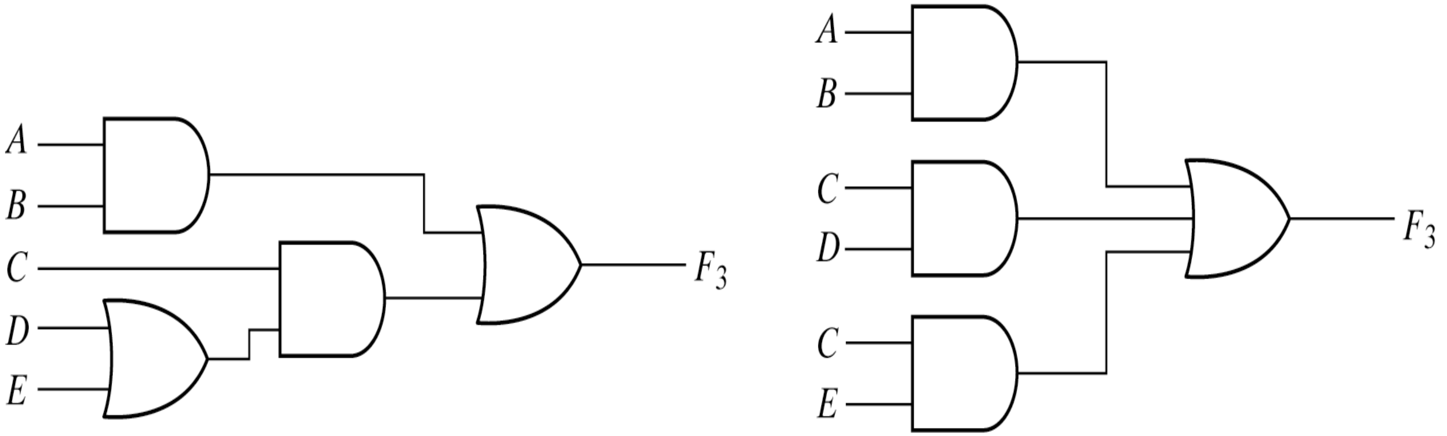


Fig. 2-3 Two-level implementation

Conversion from Nonstandard to Standard Form

- A Boolean function may be expressed in a nonstandard form (fig 2.4a shows a function that is neither in sum of products nor in product of sums). It has three levels of gating
- It can be converted to a standard form (Sum of product) by using distributive law to remove parenthesis
- Two-level implementation is preferred as it produces the least amount of delay



(a) $AB + C(D + E)$ (b) $AB + CD + CE$

Fig. 2-4 Three- and Two-Level implementation

The End