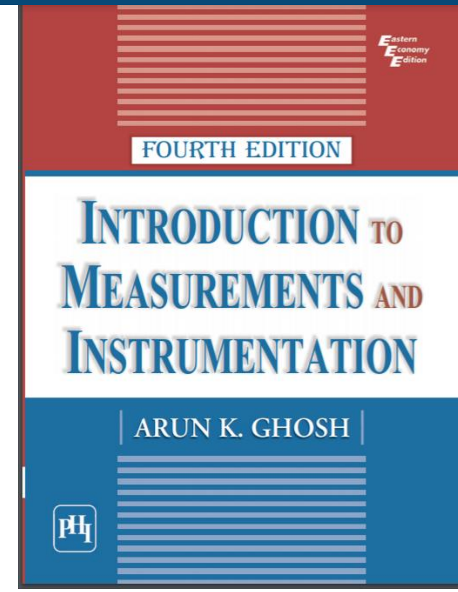
NUST
SCHOOL OF ELECTRICAL ENGINEERING
& COMPUTER SCIENCE

- Course: **EE383 Instrumentation and Measurements**
- Class: BEE12 (C)
- **Lectures: Week 11**
- Course Instructor:

INSTRUMENTATION
& MEASUREMENT



Week 11

- **Chapter 7**

  Strain Measurement

Book: Introduction to Measurements and Instrumentation, Arun K. Ghosh

Strain Measurement

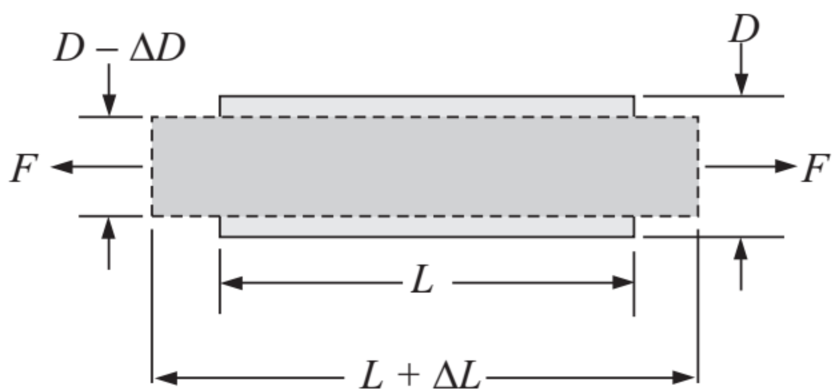
- **Strain?**

Strain

- ***Strain*** is the amount of deformation caused to the metals when a ***stress*** is applied to them.

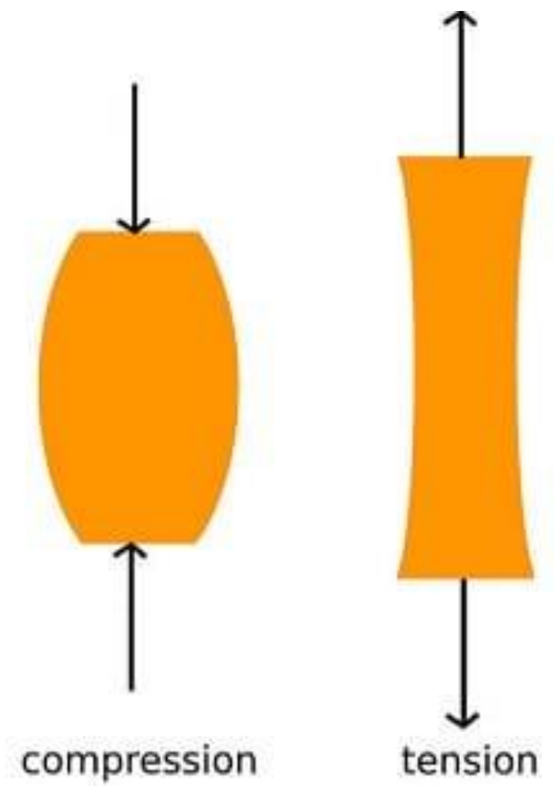
Strain Measurement

- **Strain**
  - Applied force mechanically deforms a solid to a certain extent



Strain Measurement

- **Strain**
  - For a tensile force, the length of the solid increases
  - For a compressive force, the length decreases



Strain Measurement

- **Longitudinal (axial) strain:**
  - When a body of length L is elongated by ΔL owing to the application of a force F
  - Ratio of the change in length ΔL to its original length L

$$\varepsilon = \frac{\Delta L}{L}$$

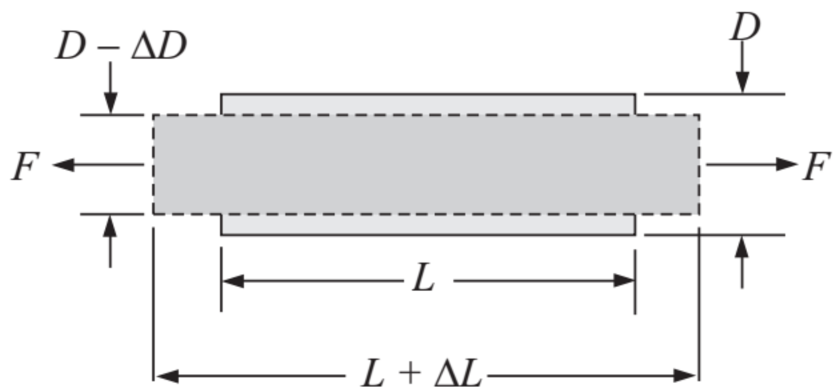


Fig. 7.1 Longitudinal and lateral strains of a solid.

Strain Measurement

- **Lateral strain**
  - When a body of length L is elongated by ΔL owing to the application of a force F, its perpendicular dimension D contracts by ΔD
  - The strain generated in the perpendicular direction is called the lateral strain

$$Lateral\ strain = \frac{\Delta D}{D}$$

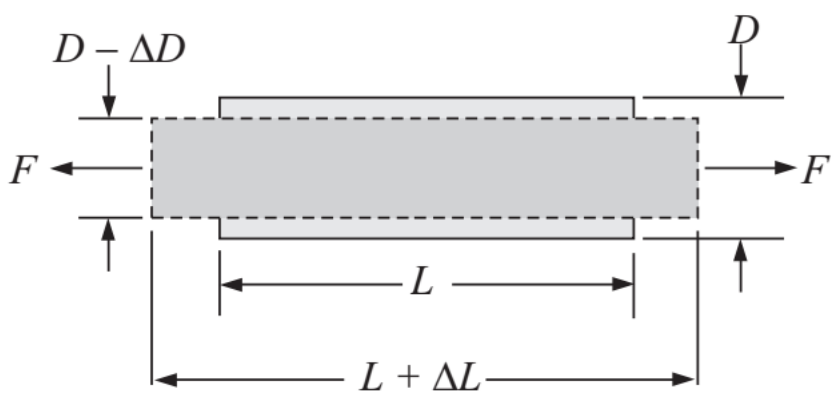
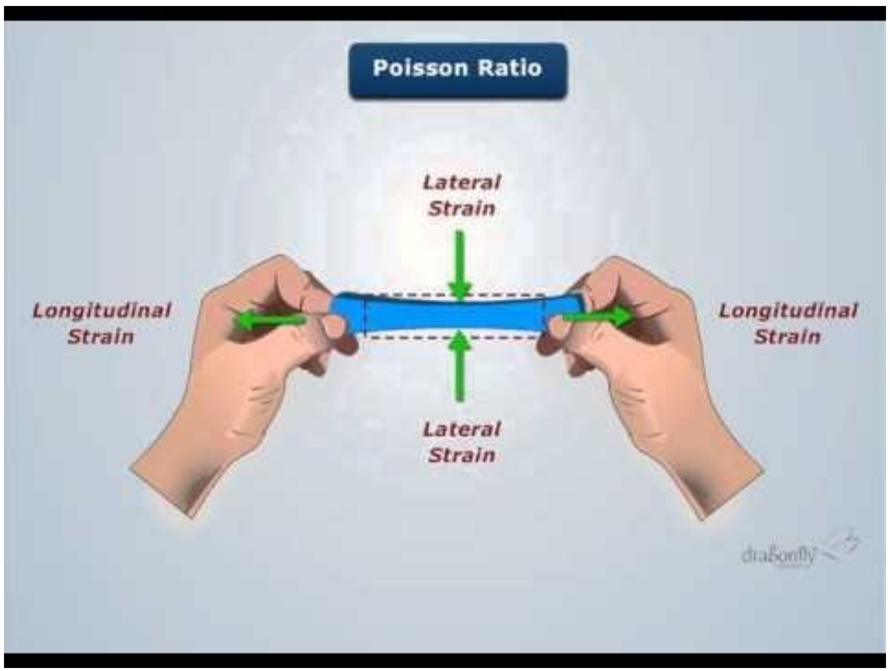


Fig. 7.1 Longitudinal and lateral strains of a solid.

Strain Measurement

- **Poisson ratio:** Poisson showed that the ratio between lateral strain and longitudinal strain is constant for a material

$$\nu = -\frac{\Delta D/D}{\Delta L/L}$$



Strain Measurement

- **Poisson ratio:** Poisson showed that the ratio between lateral strain and longitudinal strain is constant for a material

$$\nu = -\frac{\Delta D/D}{\Delta L/L}$$

- Poisson's ratio for all solids lie between 0 and 0.5: i.e. $0 < \nu < 0.5$
- For most of the materials, $\nu \approx 0.3$

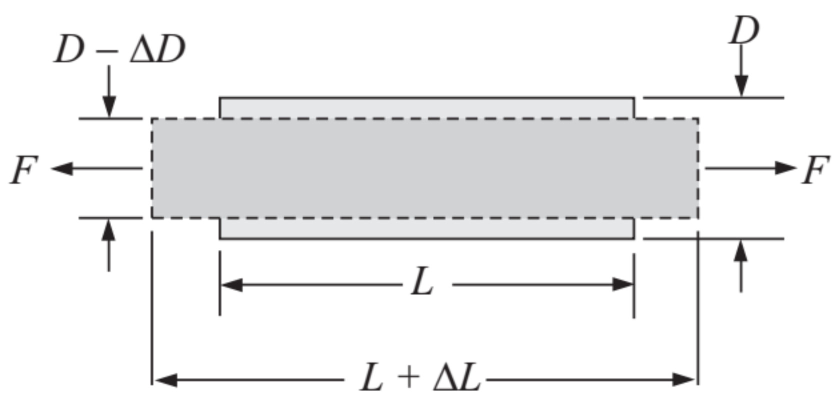


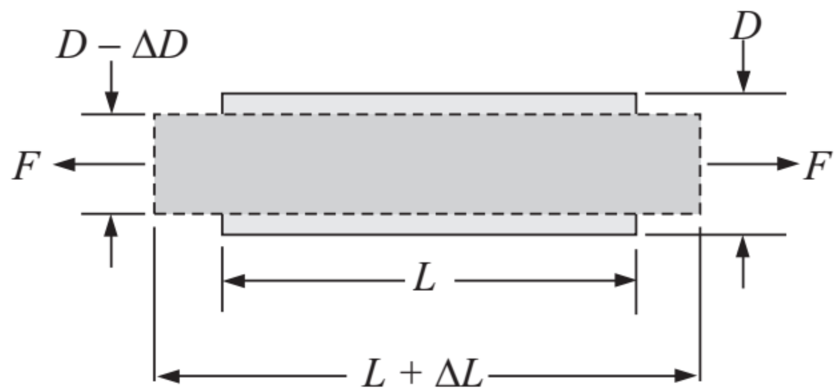
Fig. 7.1 Longitudinal and lateral strains of a solid.

Strain Measurement

- **Stress**

  - What is stress?
  - What is its origin?

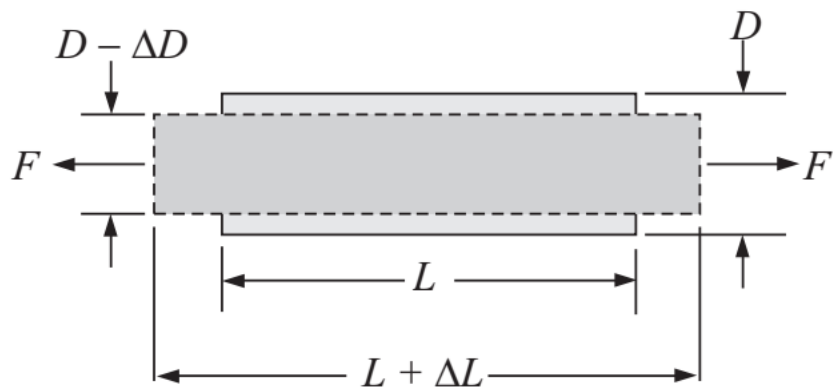
$$\sigma = \frac{F}{A}$$



Strain Measurement

- **Stress**
  - Interatomic forces – interatomic distances or bond lengths
  - Applied force increase or decrease the bond length
  - Forces of restitution come into play to restore the atoms to their original positions
  - Forces of restitution per unit area constitute the stress of the solid
  - Newton's third law of motion: the stress, which is a reaction, is equated to the applied force per unit area, which is the action.
  - Longitudinal stress *σ*,

$$\sigma = \frac{F}{A}$$

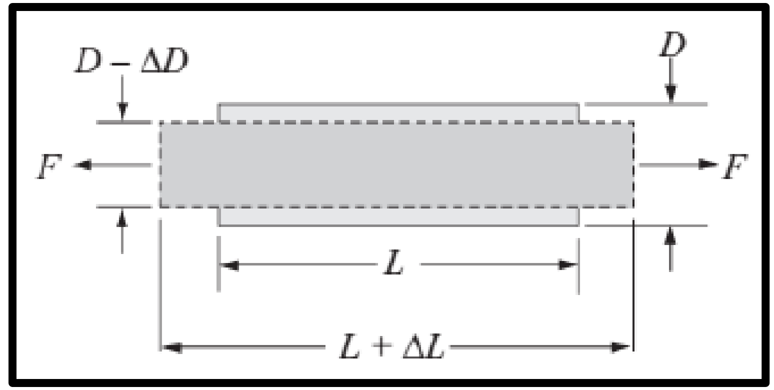


Basic Definitions Related to Strain Measurements

- **Axial Stress (σ)**
  - Axial or longitudinal stress is the force applied per unit area in the longitudinal direction

$$\sigma = \frac{F}{A}$$

  - ***A*** is the cross sectional area of the material.



- **Axial Strain (ϵ_A)**
  - Is the ratio of change in length to original length.

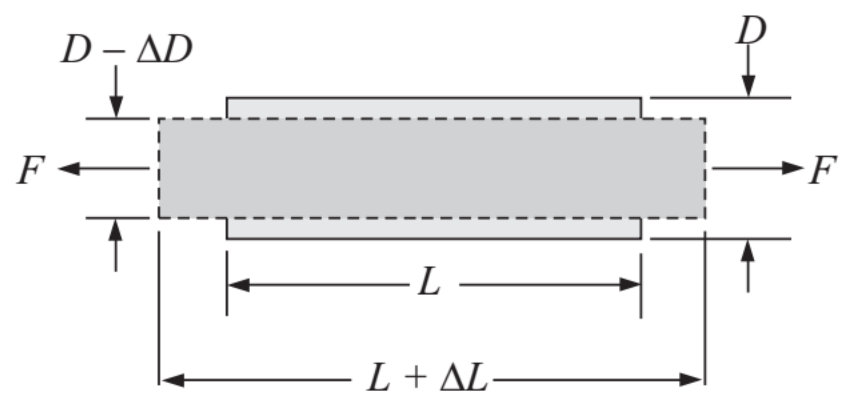
$$\epsilon_A = \frac{\Delta l}{l}$$

Strain Measurement

- **Stress-strain relations**
  - Within the elastic limit, the stress-strain relation is given by Hooke's law

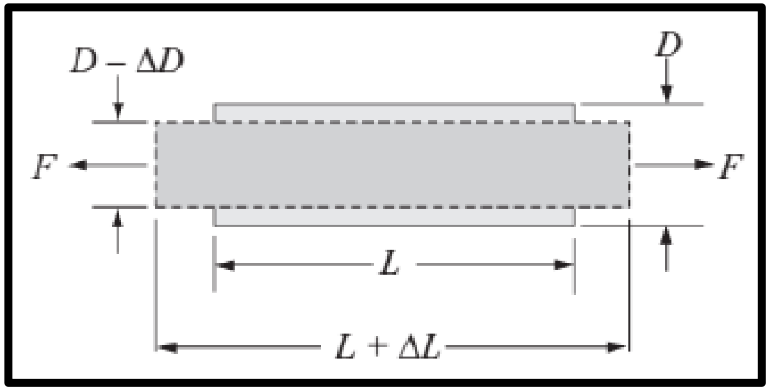
$$E = \frac{\sigma}{\varepsilon}$$

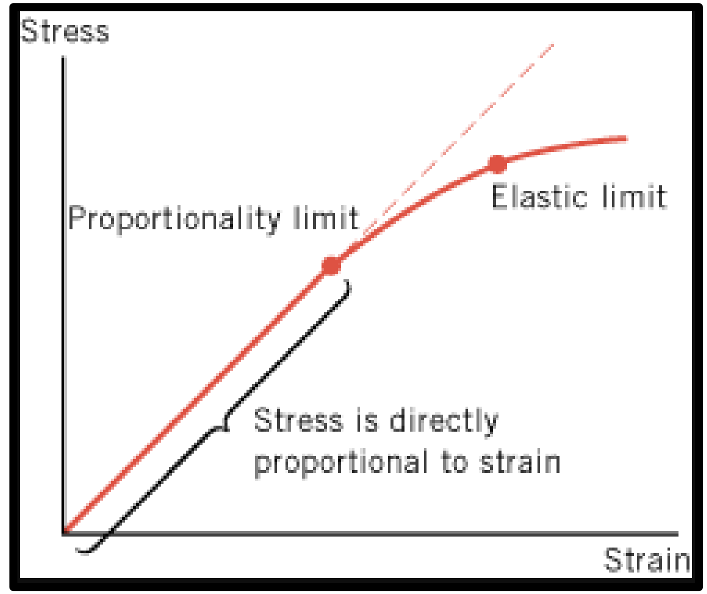
E is the Young's modulus
σ is the longitudinal stress,
ε is the longitudinal strain



Strain Measurement

- **Young's Modulus (E)**





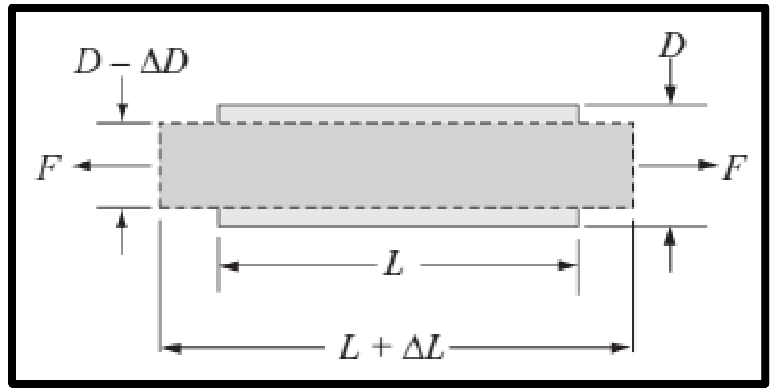
- Within the elastic limit the stress strain relation is given by Hooke's law

$$E = \frac{\sigma}{\epsilon_A} = \frac{\frac{F}{A}}{\frac{\Delta l}{l}}$$

Strain Measurement

- **Transverse Strain (ϵ_T)**
  - Is the ratio of change in width to original width.

$$\epsilon_T = \frac{\Delta D}{D}$$



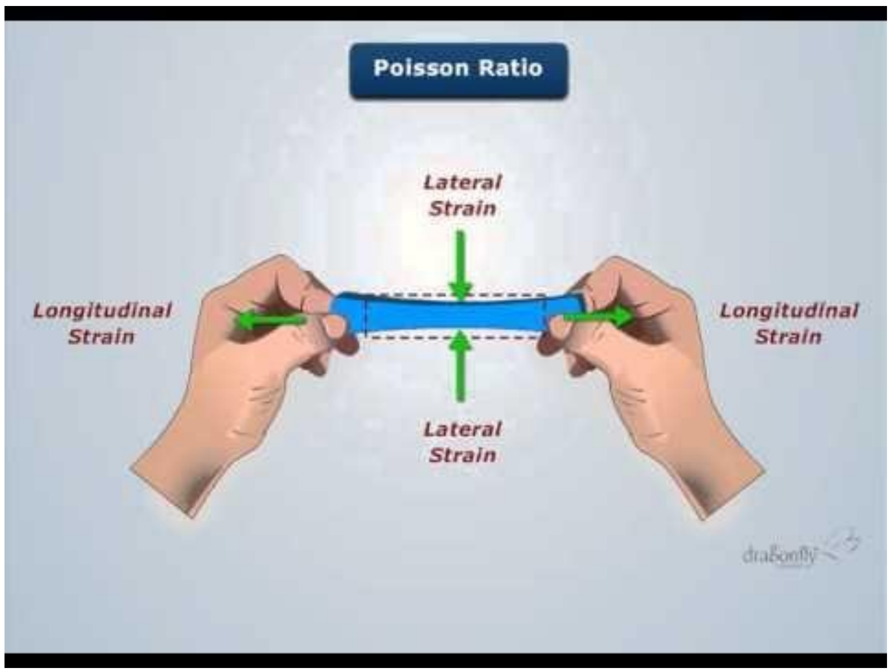
- **Poissons Ratio (ν)**
  - Is the ratio of transverse strain and axial strain.
  - The value of poisson ratio for any material is constant

$$\nu = \frac{\epsilon_T}{\epsilon_A} = -\frac{\frac{\Delta D}{D}}{\frac{\Delta l}{l}}$$

Strain Measurement

- **Poisson ratio:** Poisson showed that the ratio between lateral strain and longitudinal strain is constant for a material

$$\nu = -\frac{\Delta D / D}{\Delta L / L}$$

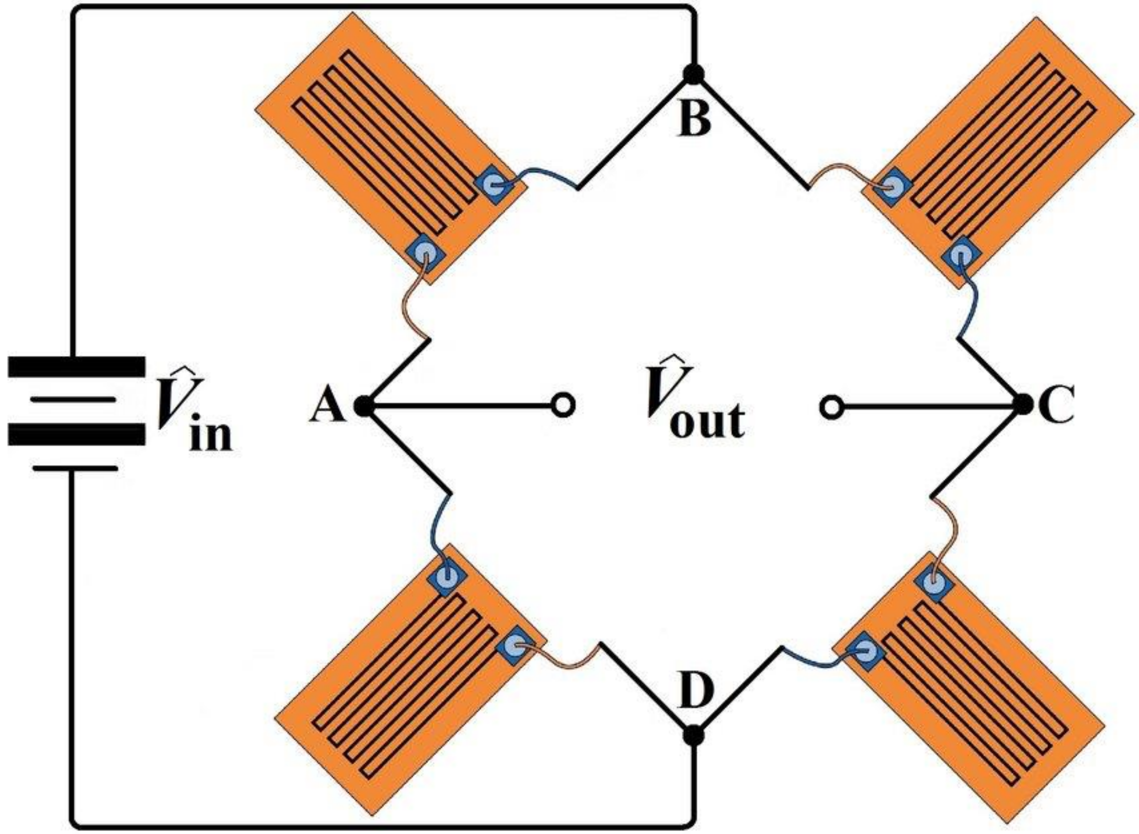


Strain Gage & Its Principle

- Strain gage is device used for strain measurement.
- Electrical strain gages are composed of a resistive or semiconducting material.
- When the material is strained its resistance changes.
- The change in resistance can be measured to determine the strain.

Strain Measurement

- Strain measurement considerations
    - Strain gauge to measure strain



Strain Measurement

- **Strain measurement considerations**
  - **Microstrain is a unit which is frequently used in strain measurements**

$$\text{Microstrain} = \frac{\Delta L \text{ (in } \mu\text{m)}}{L \text{ (in m)}} = \text{actual strain magnified } 10^6 \text{ times}$$

Strain Measurement

- **Gage factor (G_F)**
  - Is the **sensitivity factor of the strain gage.** i.e., the relative change in resistance (output factor) per unit strain.

$$G_F = \frac{\frac{dR}{R}}{\epsilon_A} = \frac{\frac{dR}{R}}{\frac{dl}{l}}$$

Strain Measurement

- Strain measurement considerations
  - Strains are likely to vary from point to point
    - This necessitates that the strain gauge should be as small as possible in size. Usually, the gauge length is around 5 mm

Strain Measurement

- **Various methods**

  1. **Mechanical**
  2. **Electrical**
  3. **Optical**

Strain Measurement

- **Various methods**

   1. **Mechanical**
      - **ΔL is measured, after magnification with the help of levers and gears, an*d* compared to the original length of the object.**
   2. Electrical
   3. Optical

Strain Measurement

- **Various methods**

  1. Mechanical
  2. **Electrical**
     - **Changes in resistance (simple or piezo) or inductance or capacitance**
     - **Capacitance- and inductance-based strain gauges: their sensitivity to vibration, mounting requirements, and circuit complexity limit their application**
  3. Optical

Strain Measurement

- **Various methods**

  1. Mechanical
  2. Electrical
  3. **Optical**
     - **The phenomena of interference, diffraction and scattering of light waves are utilized to measure strain**

Strain Measurement

- **Various methods**

1. **Mechanical**
   - ΔL is measured, after magnification with the help of levers and gears, an*d* comparing to the original length of the object.
2. **Electrical**
   - Changes in resistance (simple or piezo) or inductance or capacitance
   - Capacitance- and inductance-based strain gauges: their sensitivity to vibration, mounting requirements, and circuit complexity limit their application
3. **Optical**
   - The phenomena of interference, diffraction and scattering of light waves are utilized to measure strain

Applications of Strain Gage

- **Strain gage is used in many measurement applications.**
  - Pressure Measurement
  - Load Measurement
  - Torque Measurement
  - Monitoring masonry
  - Monitoring pipelines

Applications of Strain Gage

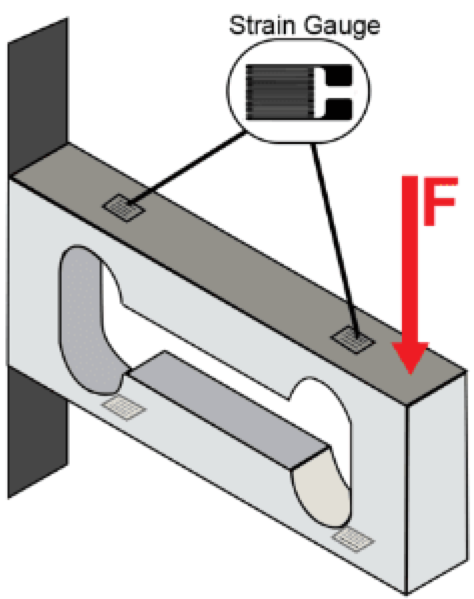
- **For structural health monitoring.**

Applications of Strain Gage

- **For strain monitoring in steel structures/ pipelines.**

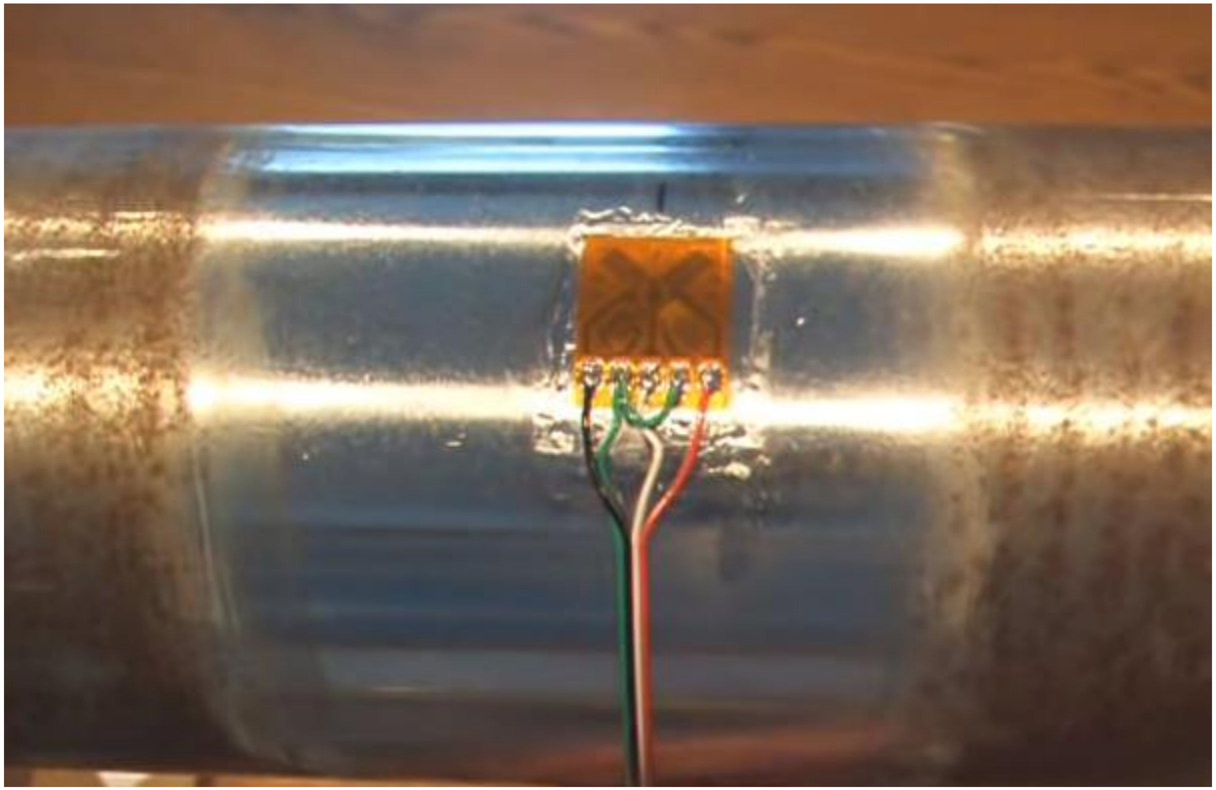
Applications of Strain Gage

- **In load cell for force measurements.**



Applications of Strain Gage

- **For torque measurement in a shaft.**

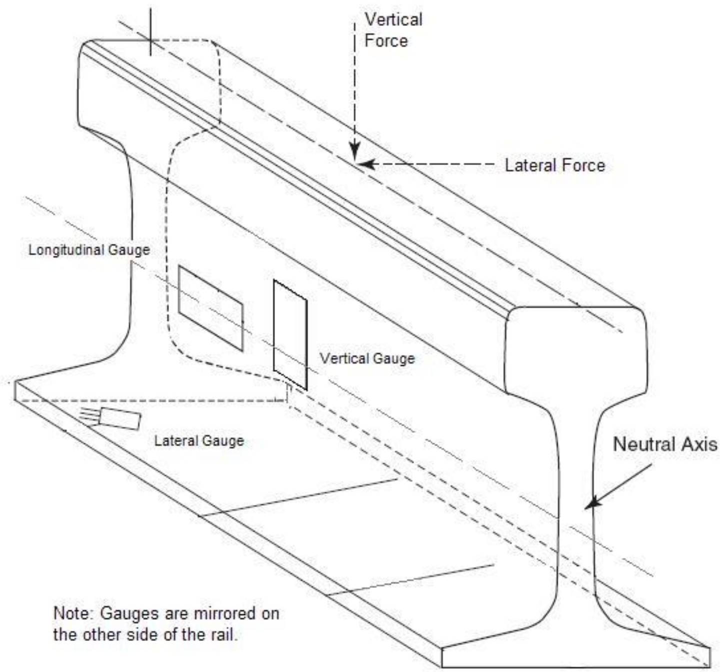


Applications of Strain Gage

- They are used in bridges for monitoring.
- The Tsing Ma Bridge in Hong Kong has 350 strain sensors.

Applications of Strain Gage

- **For railway line monitoring**



Applications of Strain Gage

Mechanical strain gauge used to measure the growth of a crack in a masonry foundation. This one is installed on the Hudson-Athens Lighthouse

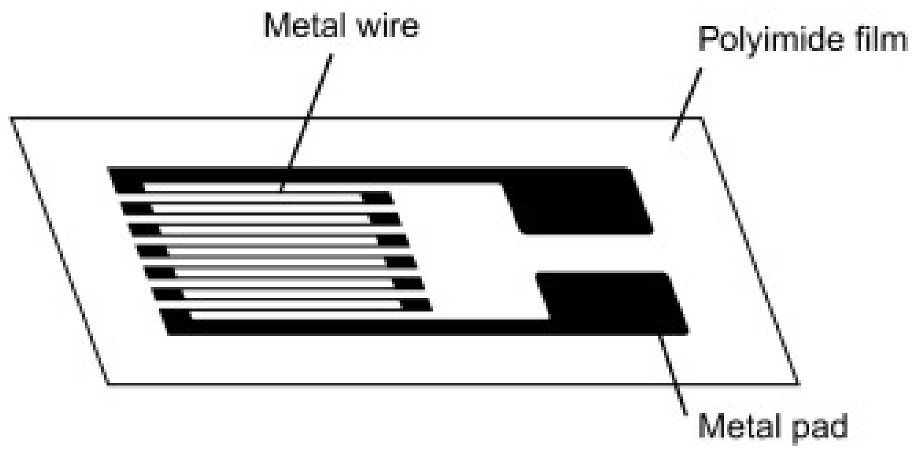
Strain Measurement

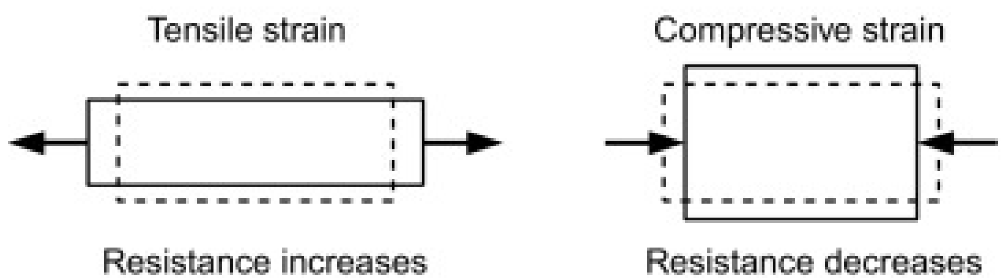
- **Various methods**

1. **Mechanical**
   - ΔL is measured, after magnification with the help of levers and gears, an*d* comparing to the original length of the object.
2. **Electrical (<u>Mostly used</u>)**
   - Changes in resistance (simple or piezo) or inductance or capacitance
   - Capacitance- and inductance-based strain gauges: their sensitivity to vibration, mounting requirements, and circuit complexity limit their application
3. **Optical (<u>that demand for accurate measurements</u>)**
   - The phenomena of interference, diffraction and scattering of light waves are utilized to measure strain

Resistance Strain Gauges

- **Principle**
  - Change in resistance due to strain
  - If a conducting wire is held under tension, its length increases slightly with a consequent reduction of its area of cross-section.





Resistance Strain Gauges

- **Principle**
  - Let us consider a conductor of length L, cross-sectional area A and resistivity ρ. Its resistance R is given by

$$R = \rho \frac{L}{A}$$

Resistance Strain Gauges

- **Principle**
  - Let us consider a conductor of length L, cross-sectional area A and resistivity ρ. Its resistance R is given by

$$R = \rho \frac{L}{A}$$

- Logarithmic differentiation technique

$$\Rightarrow \quad \ln R = \ln \rho + \ln L - \ln A$$

$$\frac{dR}{R} = \frac{d\rho}{\rho} + \frac{dL}{L} - \frac{dA}{A} \qquad \text{......................} \quad 1$$

Resistance Strain Gauges

$$\frac{dR}{R} = \frac{d\rho}{\rho} + \frac{dL}{L} - \frac{dA}{A} \quad \text{..........................} \quad 1$$

- If D is the cross-section dimension and C is a constant, area A can be written as

$$A = CD^2$$

Resistance Strain Gauges

$$\frac{dR}{R} = \frac{d\rho}{\rho} + \frac{dL}{L} - \frac{dA}{A} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad 1$$

- If D is the cross-section dimension and C is a constant, area A can be written as

$$A = CD^2$$

- Applying Log and differentiation

$$\ln A = 2 \ln D$$

$$\frac{dA}{A} = 2\frac{dD}{D}$$

Resistance Strain Gauges

$$\frac{dR}{R} = \frac{d\rho}{\rho} + \frac{dL}{L} - \frac{dA}{A} \qquad \cdots\cdots\cdots\cdots\cdots\cdots \quad 1$$

$$\frac{dA}{A} = 2\frac{dD}{D} \qquad \cdots\cdots\cdots\cdots\cdots\cdots \quad 2$$

- **Poisson ratio**

$$\nu = -\frac{lateral\ strain}{longitudinal\ strain} = -\frac{dD/_D}{\varepsilon}$$

Resistance Strain Gauges

$$\frac{dR}{R} = \frac{d\rho}{\rho} + \frac{dL}{L} - \frac{dA}{A} \qquad \text{.......................} \quad 1$$

$$\frac{dA}{A} = 2\frac{dD}{D} \qquad \text{.......................} \quad 2$$

- **Poisson ratio**

$$\nu = -\frac{lateral\ strain}{lngitudinal\ strain} = -\frac{dD/_{D}}{dL/_{L}} = -\frac{dD/_{D}}{\varepsilon}$$

$$\Rightarrow \quad \frac{dD}{D} = -\nu\varepsilon$$

Resistance Strain Gauges

$$\frac{dR}{R} = \frac{d\rho}{\rho} + \frac{dL}{L} - \frac{dA}{A} \quad \text{.........................} \quad 1$$

$$\frac{dA}{A} = 2\frac{dD}{D} \quad \text{.........................} \quad 2$$

$$\frac{dD}{D} = -\nu\varepsilon \quad \text{.........................} \quad 3$$

- **Putting eq. 3 into 2**

$$\Rightarrow \quad \frac{dA}{A} = -2\nu\varepsilon = -2\nu\frac{dL}{L}$$

Resistance Strain Gauges

$$\frac{dR}{R} = \frac{d\rho}{\rho} + \frac{dL}{L} - \frac{dA}{A} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad 1$$

$$\frac{dA}{A} = -2\nu\varepsilon = -2\nu\frac{dL}{L}$$

$$\Rightarrow \boxed{\frac{dR}{R} = \underbrace{\frac{d\rho}{\rho}}_{\substack{\text{resistivity}\\\text{change}}} + \underbrace{\frac{dL}{L}}_{\substack{\text{length}\\\text{change}}} + \underbrace{2\nu\frac{dL}{L}}_{\substack{\text{cross-section}\\\text{change}}}}$$

Resistance Strain Gauges

$$\boxed{\underbrace{\frac{dR}{R}}_{} = \underbrace{\frac{d\rho}{\rho}}_{\substack{\text{resistivity}\\\text{change}}} + \underbrace{\frac{dL}{L}}_{\substack{\text{length}\\\text{change}}} + \underbrace{2\nu\frac{dL}{L}}_{\substack{\text{cross-section}\\\text{change}}}}$$

1. **Piezoresistive change: change in resistivity of the material**
2. **Change in length**
3. **Change in cross-sectional area of the gauge**

Resistance Strain Gauges

- **Gauge Factor**
  - The sensitivity factor of the gauge
  - change in resistance of the gauge per unit strain

Resistance Strain Gauges

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  - The sensitivity factor of the gauge
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$$\boxed{\frac{dR}{R} = \underbrace{\frac{d\rho}{\rho}}_{\text{resistivity change}} + \underbrace{\frac{dL}{L}}_{\text{length change}} + \underbrace{2\nu\frac{dL}{L}}_{\text{cross-section change}}}$$

$$\Rightarrow \quad \boxed{G_f \equiv \frac{dR/R}{\varepsilon} = \frac{dR/R}{dL/L} = 1 + 2\nu + \frac{d\rho/\rho}{\varepsilon}}$$

Resistance Strain Gauges

- **Gauge Factor**
  - The sensitivity factor of the gauge
  - change in resistance of the gauge per unit strain

$$\boxed{G_f \equiv \frac{dR/R}{\varepsilon} = \frac{dR/R}{dL/L} = 1 + 2\nu + \frac{d\rho/\rho}{\varepsilon}}$$

Resistance Strain Gauges

- **Gauge Factor**

$$\boxed{G_f \equiv \frac{dR/R}{\varepsilon} = \frac{dR/R}{dL/L} = 1 + 2\nu + \frac{d\rho/\rho}{\varepsilon}}$$

for ${}^{d\rho}\!/_{\rho} = 0$ **and knowing** $0 \leq \nu \leq 1/2.$

$$\Rightarrow \qquad 1 \leq G_f \leq 2$$

Resistance Strain Gauges

- ❑ **Gauge Factor**

$$\boxed{G_f \equiv \frac{dR/R}{\varepsilon} = \frac{dR/R}{dL/L} = 1 + 2\nu + \frac{d\rho/\rho}{\varepsilon}}$$

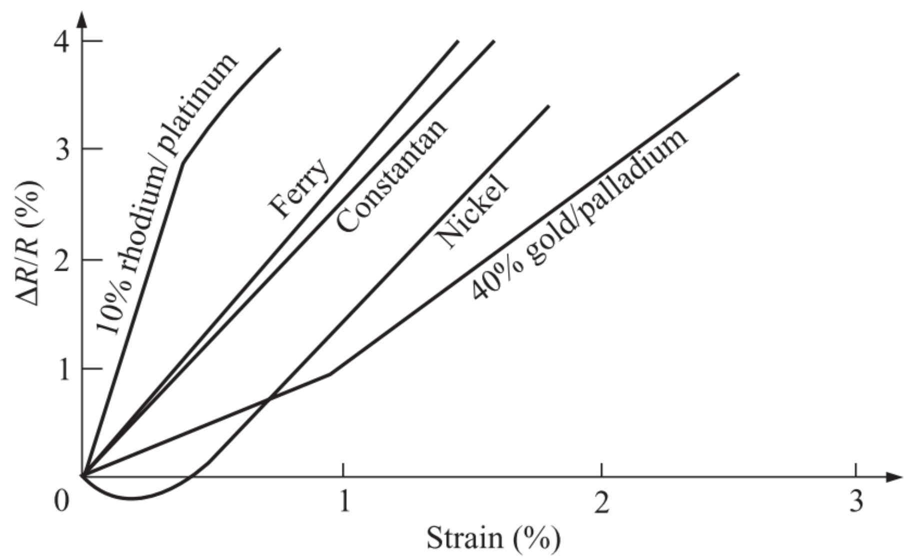

Fig. 7.2 Plot of fractional resistance change vs. fractional strain for a few materials.

Resistance Strain Gauges

- **Gauge Factor**

$$\boxed{G_f \equiv \frac{dR/R}{\varepsilon} = \frac{dR/R}{dL/L} = 1 + 2\nu + \frac{d\rho/\rho}{\varepsilon}}$$

Table 7.1 Gauge factors for strain gauge materials

Material	*Low-strain* G_f	*High-strain* G_f	*Elongation* (%)
Copper	2.6	2.2	0.5
Constantan/ferry	2.1	1.9	1.0
40% gold/palladium	0.9	1.9	0.8
Nickel	−12	2.7	–
Platinum	6.1	2.4	0.4
Silver	2.9	2.4	0.8
Semiconductor	~ -100	~ -600	–

Piezoresistivity

- **Origin of piezoresistivity**
  - Change in resistivity of the semiconductor when strained

?

Piezoresistivity

❑ Origin of piezoresistivity

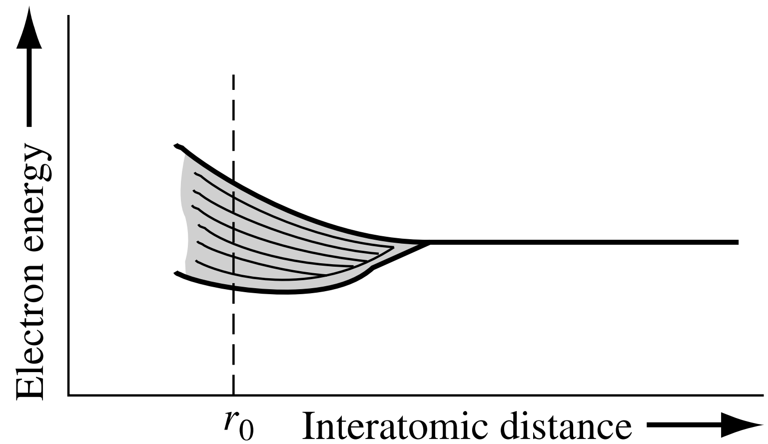

Figure 3.2 | The splitting of an energy state into a band of allowed energies.

Piezoresistivity

- **Origin of piezoresistivity**
  - Change in resistivity of the semiconductor when strained
    - When strained, the interatomic spacings within the material change
    - The change in the interatomic spacings eventually changes the bandgaps in each atom

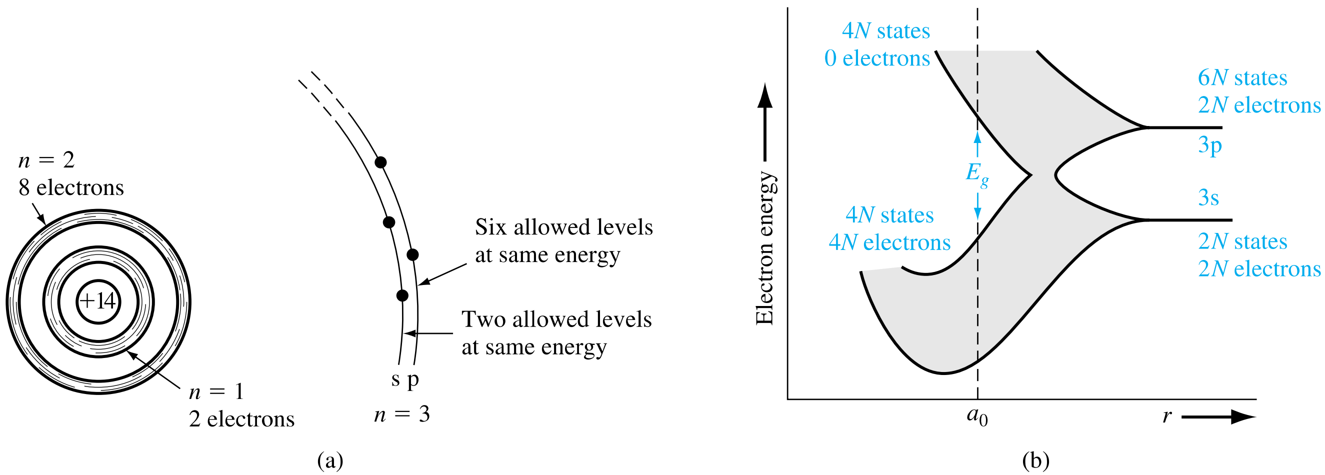

Figure 3.4 | (a) Schematic of an isolated silicon atom. (b) The splitting of the 3s and 3p states of silicon into the allowed and forbidden energy bands.
(From Shockley [6].)

Ref: Semiconductor Physics and Devices: Basic Principles by Donald A. Neamen

Piezoresistivity

- **Origin of piezoresistivity**
  - Change in resistivity of the semiconductor when strained
    - Change in the interatomic spacings changes the atomic bandgaps
    - Band gaps change makes it easier (or harder depending on the material and strain) for electrons to be raised into the conduction band

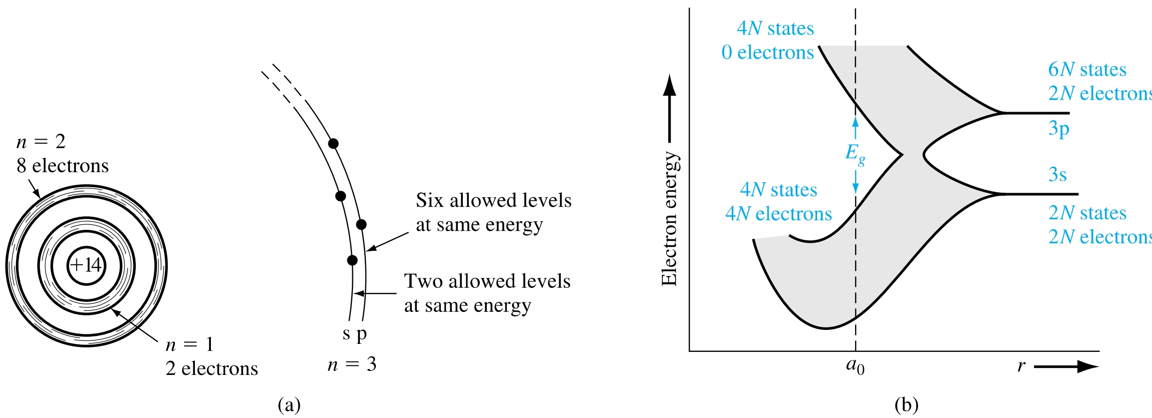

Figure 3.4 | (a) Schematic of an isolated silicon atom. (b) The splitting of the 3s and 3p states of silicon into the allowed and forbidden energy bands.
(From Shockley [6].)

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Resistance Strain Gauges

- **Types**

  1. **Wire-wound**
  2. **Foil**
  3. **Semiconductor**

Resistance Strain Gauges

- **Wire-wound gauges**

  1. **Bonded**

  2. **Unbonded**

Resistance Strain Gauges

- **Bonded wire-wound gauges**
  - bonded to the surface of the specimen being tested
    - wire diameter ~ 25 μm
    - adhesive cement transmits the strain and acts as an insulator

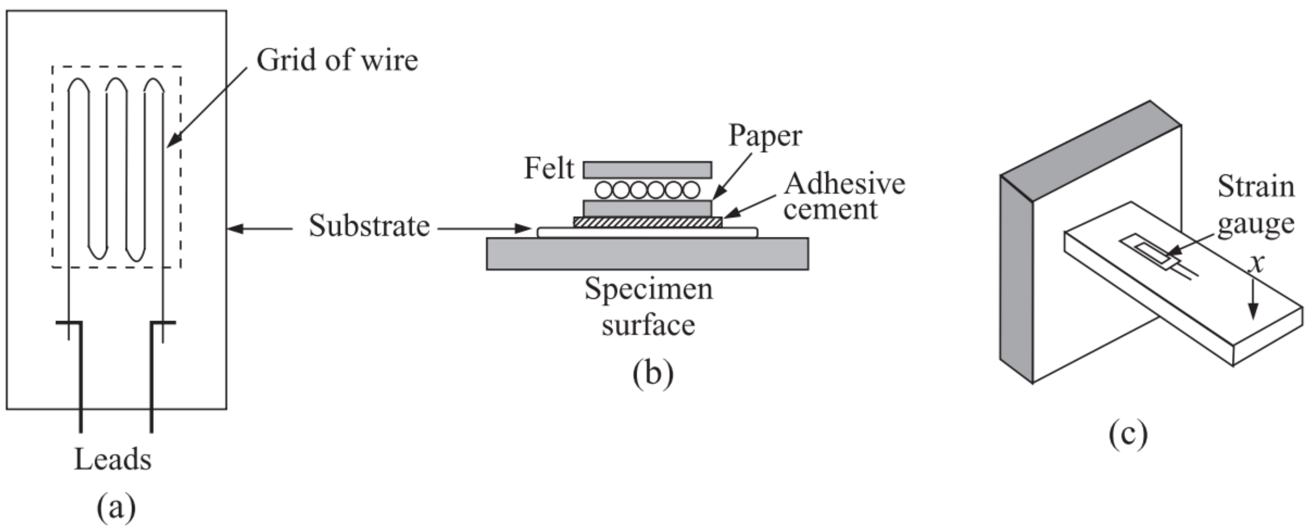


Fig. 7.3 Schematic view of a bonded strain gauge: (a) construction, (b) bonding on the surface, and (c) actual placement.

Resistance Strain Gauges

- **Bonded wire-wound gauges**

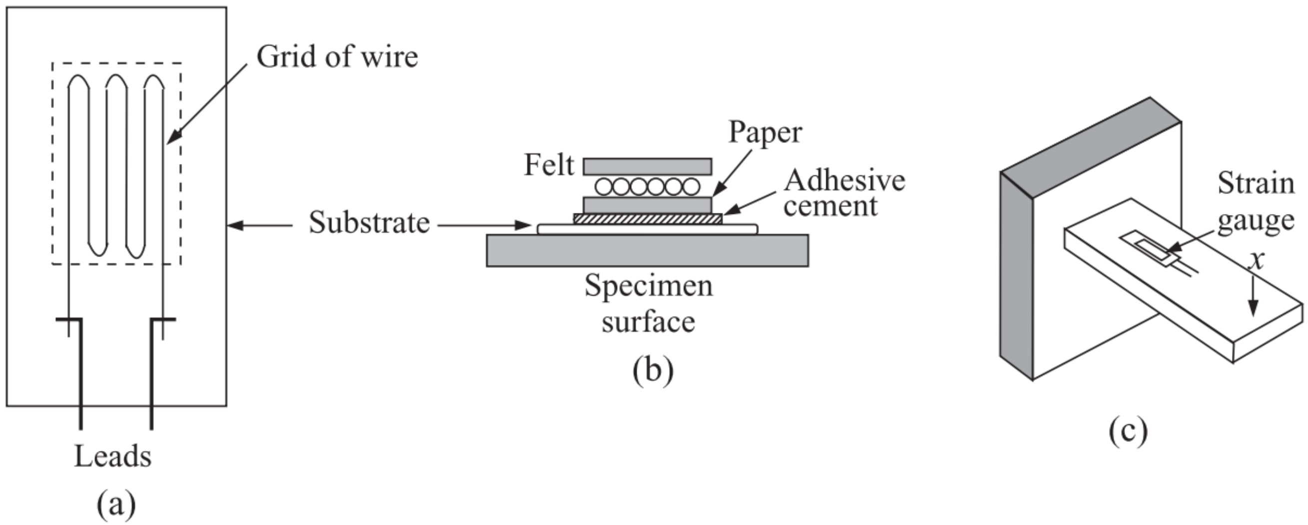


Fig. 7.3 Schematic view of a bonded strain gauge: (a) construction, (b) bonding on the surface, and (c) actual placement.

Table 7.2 General specifications of bonded strain gauges

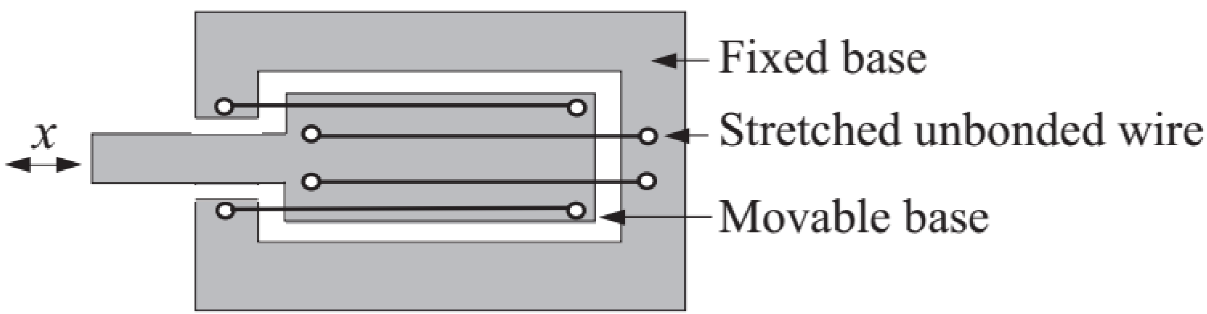
Size	Typically 3 mm × 3 mm, but seldom bigger than 2.5 mm × 12.5 mm
Resistance value	120 – 1000 Ω
Maximum excitation voltage	5 – 10 V
Material	Ni-Cu, Ni-Cr or Ni-Fe alloys

Resistance Strain Gauges

- **Unbonded metal wire gauge**

  - The resistance wires are connected in the form of a Wheatstone bridge

    A Wheatstone bridge is a divided bridge circuit used for the measurement of static or dynamic electrical resistance



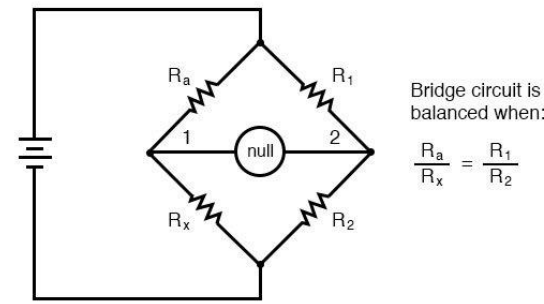


Fig. 7.4 Schematic view of unbonded strain gauge.

- A small motion increases tension in two wires while decreasing it in two others
  - bridge unbalance because of resistance changes
  - output voltage is proportional to the input displacement which can be calibrated in terms of strain

Resistance Strain Gauges

- **Foil type Gauges**
  - resistor/sensor comprises a thin sheet or foil with thickness of less than 5 μm
  - fabrication: photo-etching or masked vacuum deposition

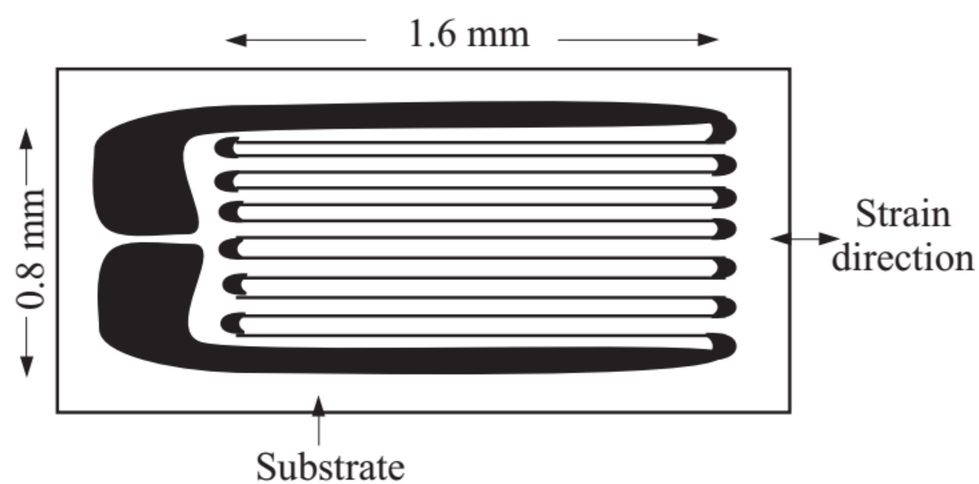


Fig. 7.5 Foil type strain gauge.

Resistance Strain Gauges

- **Semiconductor Type Gauges**
  - Piezoresistivity or piezoresistive effect
    - The resistivity of doped silicon and germanium changes when stressed
  - Gauge factor is well around 100

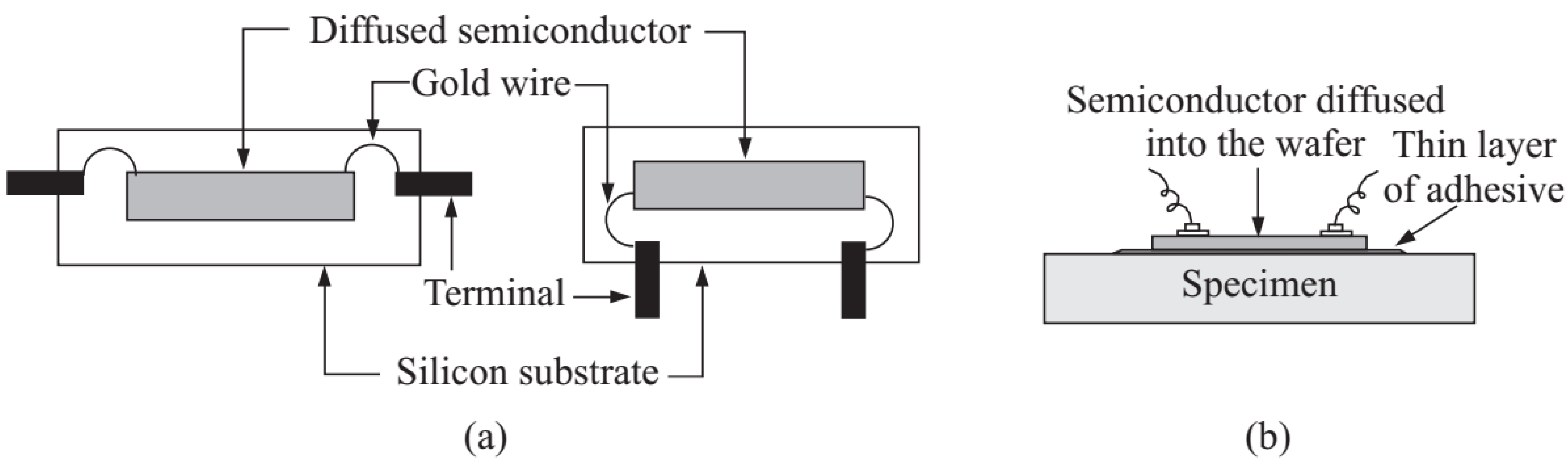


Fig. 7.6 Bonded-type semiconductor strain gauge: (a) construction, and (b) bonding.

Resistance Strain Gauges

- ❑ **Semiconductor Type Gauges**

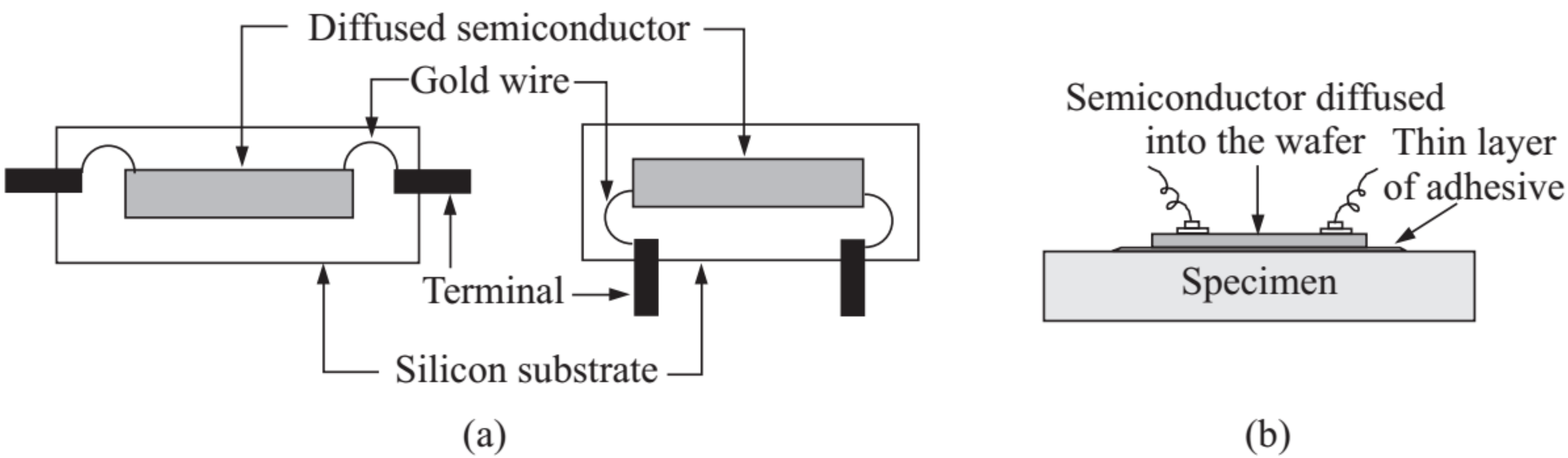
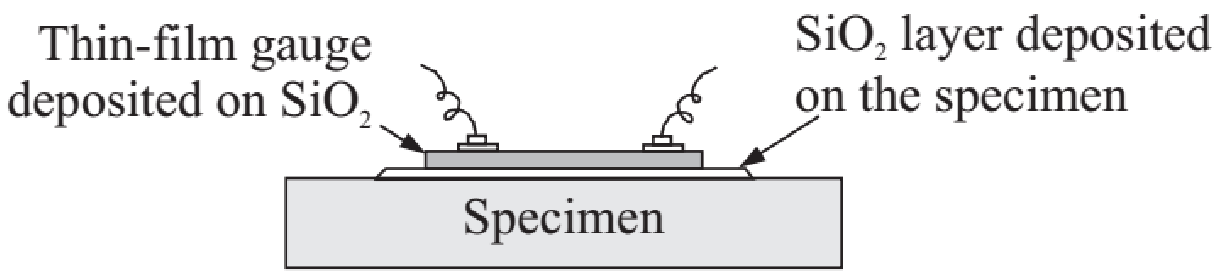


Fig. 7.6 Bonded-type semiconductor strain gauge: (a) construction, and (b) bonding.

- ❑ Ohmic contacts
  - ❑ Au wires as fermi level of gold matches that of the semiconductors
- ❑ Adhesion: epoxy

Resistance Strain Gauges

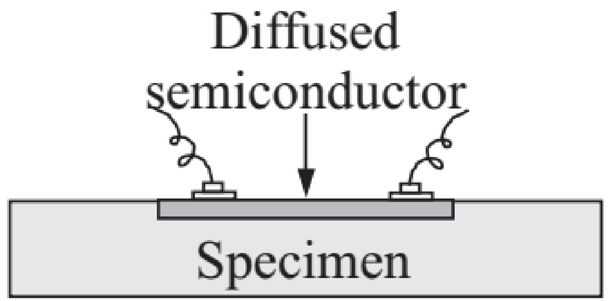
- **Semiconductor Type Gauges: Improvements**



- Inclusion of SiO_2 layer, adhesive is not required
- much more stable and the resistance values experience less drift
- the specimen can either be a thin diaphragm or a thick beam

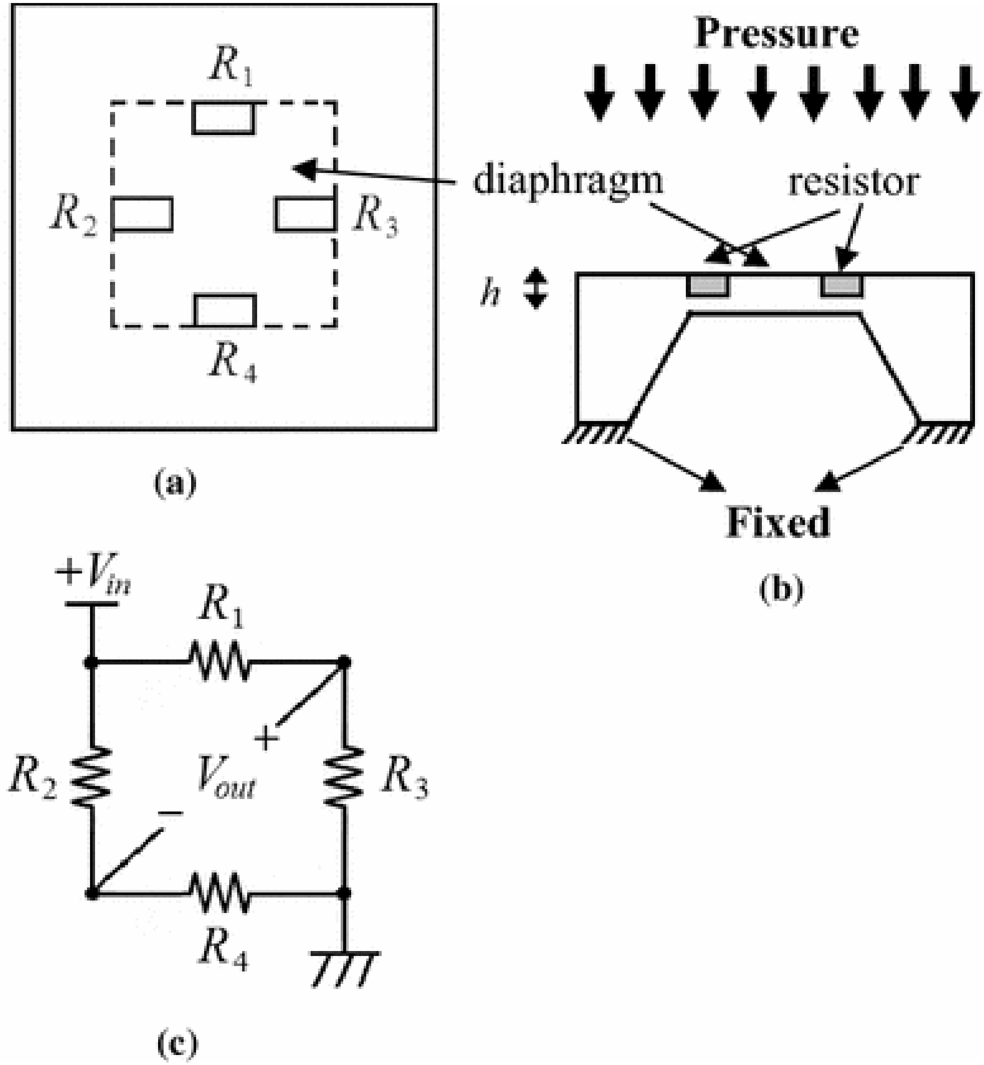
Resistance Strain Gauges

- **Semiconductor Type Gauges: Improvements**



- By eliminating bonding agents, errors due to creep and hysteresis are eliminated
- limited to moderate-temperature applications
  - temperature compensation
- small, inexpensive, accurate and generate a strong output signal
- used as sensing elements in pressure transducers

Strain Measurement Method



Strain Measurement: Resistance Strain Gauges

Advantages and disadvantages. Table 7.3 lists the advantages and disadvantages of semiconductor strain gauges.

Table 7.3 Advantages and disadvantages of semiconductor strain gauges

Advantages	*Disadvantages*
1. High unit resistance and high G_f	1. High temperature sensitivity.
2. Low hysteresis.	2. Tendency to drift.
3. Good frequency response which makes them amenable to ac measurements.	3. Nonlinear characteristic curve.
4. Very small size (0.7 to 7 mm).	

But the disadvantages can easily be tackled by employing a second gauge for temperature compensation, and software compensation in computer-controlled instrumentation to make the calibration curve linear.

Strain Measurement Method

- Strain measurement: challenging
- The displacements associated with strains are very small and therefore, corresponding changes in resistance are small

Strain Measurement Method

- Strain measurement: challenging
- The displacements associated with strains are very small and therefore, corresponding changes in resistance are small

Example 7.1

A strain gauge, having $G_f = 2.0$ and $R = 120\,\Omega$, is used to measure strains generated by pressures of 50 psi and 50000 psi in aluminium. The corresponding strains are 5 and 5000 microstrains. Calculate the per cent changes of resistance of the strain gauge.

Solution

For 5 microstrain: $\Delta R = G_f \varepsilon R = 2(5 \times 10^{-6})(120) = 0.0012\,\Omega = 0.0012\,\Omega$

$$\therefore \quad \frac{\Delta R}{R} = \frac{0.0012}{120} \times 100 = 0.001$$

For 5000 microstrain: $\Delta R = 2(5000 \times 10^{-6})(120) = 1.2\,\Omega$

$$\therefore \quad \frac{\Delta R}{R} = 1\%$$

$$G_F = \frac{\frac{dR}{R}}{\epsilon_A} = \frac{\frac{dR}{R}}{\frac{dl}{l}}$$

Strain Measurement Method

Conventional methods

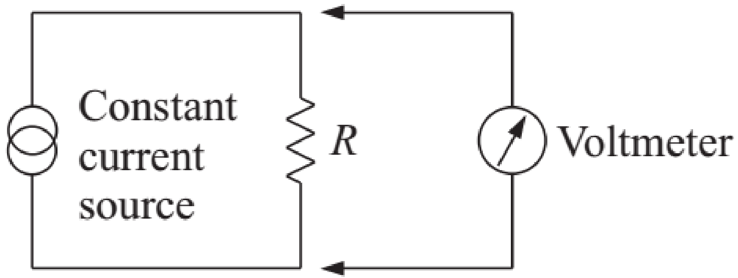
1. **Current injection**
2. **Ballast circuit**

Strain Measurement Method

Conventional methods:

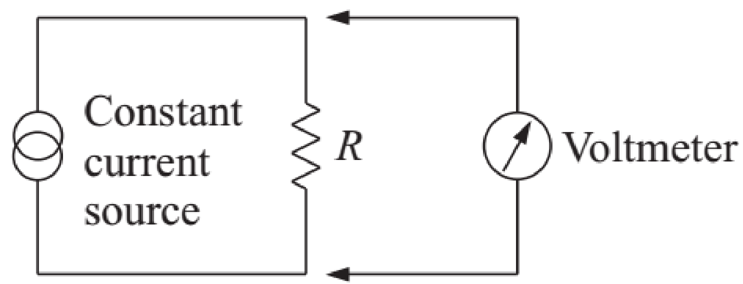
1. Current injection

- ❑ The resistance can be calculated from the Ohm's law.



Strain Measurement Method

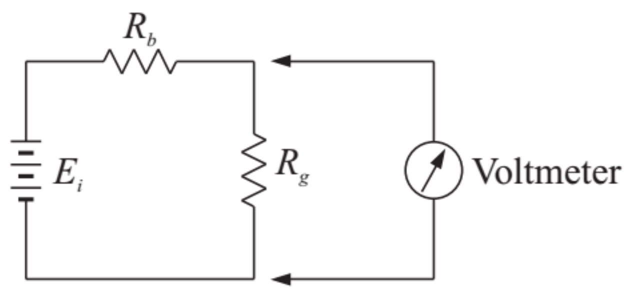
- **Conventional methods: 1. Current injection**
    - The resistance can be calculated from the Ohm’s law.



- Drawbacks
- a very low current is injected to avoid Joule heating of the resistor
- Since the resistance change is very small, the change in voltage is often on the order of the thermal noise

Strain Measurement Method

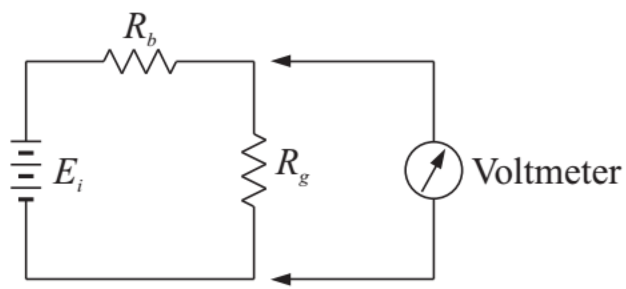
- **Conventional methods: 2. Ballast Circuit**
    - A voltage source in series with a ballast (high resistance) R_b is used to produce a low current to be passed though the gauge R_g



$$E_o = \frac{R_g}{R_b + R_g} E_i$$

Strain Measurement Method

- **Conventional methods: 2. Ballast Circuit**
  - A voltage source in series with a ballast (high resistance) R_b is used to produce a low current to be passed though the gauge R_g



$$E_o = \frac{R_g}{R_b + R_g} E_i$$

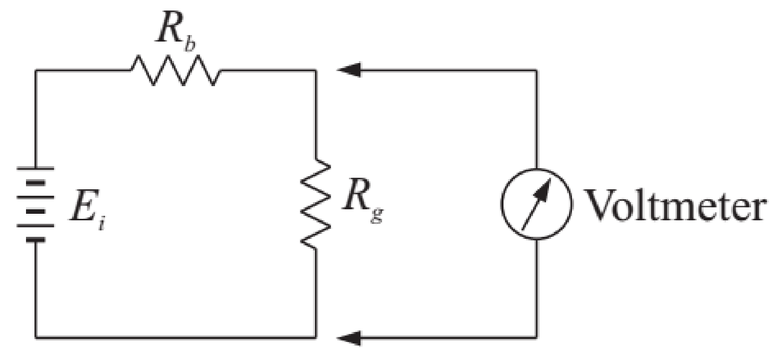
- **Applying differential**

$$dE_o = \left[\frac{dR_g}{R_b + R_g} - \frac{R_g \, dR_g}{(R_b + R_g)^2} \right] E_i = \frac{R_b R_g}{(R_b + R_g)^2} E_i \cdot \frac{dR_g}{R_g} = \frac{R_b R_g}{(R_b + R_g)^2} E_i \cdot G_f \varepsilon \qquad (7.17)$$

$$G_f \equiv \frac{dR/R}{\varepsilon}$$

Strain Measurement Method

- **Conventional methods: 2. Ballast Circuit**



$$E_o = \frac{R_g}{R_b + R_g} E_i$$

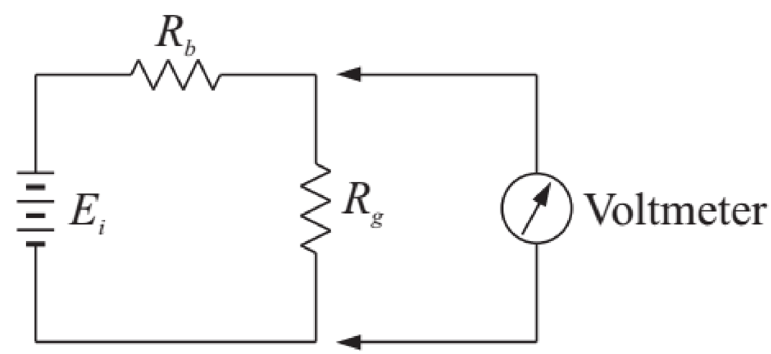
$$dE_o = \left[\frac{dR_g}{R_b + R_g} - \frac{R_g\, dR_g}{(R_b + R_g)^2} \right] E_i = \frac{R_b R_g}{(R_b + R_g)^2} E_i \cdot \frac{dR_g}{R_g} = \frac{R_b R_g}{(R_b + R_g)^2} E_i \cdot G_f \varepsilon \qquad (7.17)$$

- **Sensitivity of the circuit is given by**

$$S = \frac{dE_o}{\varepsilon} = \frac{R_b R_g}{(R_b + R_g)^2} E_i \cdot G_f \qquad \text{[From Eq. (7.17)]}$$

Strain Measurement Method

❑ Conventional methods: 2. Ballast Circuit



$$E_o = \frac{R_g}{R_b + R_g} E_i$$

$$dE_o = \left[\frac{dR_g}{R_b + R_g} - \frac{R_g\, dR_g}{(R_b + R_g)^2} \right] E_i = \frac{R_b R_g}{(R_b + R_g)^2} E_i \cdot \frac{dR_g}{R_g} = \frac{R_b R_g}{(R_b + R_g)^2} E_i \cdot G_f \varepsilon \qquad (7.17)$$

$$S = \frac{dE_o}{\varepsilon} = \frac{R_b R_g}{(R_b + R_g)^2} E_i \cdot G_f \qquad \text{[From Eq. (7.17)]}$$

For maximum S w.r.t. R_b

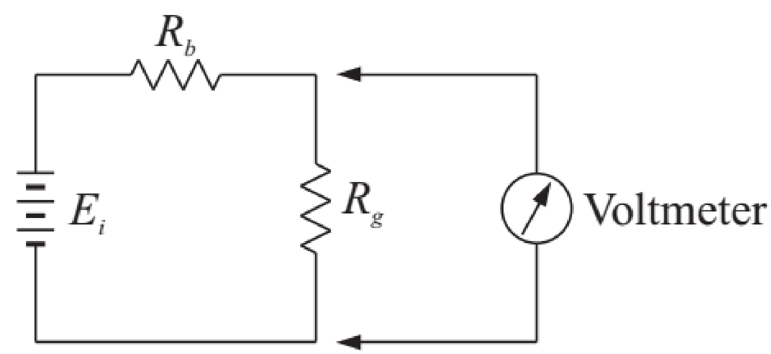
$$0 = \frac{dS}{dR_b} = \frac{(R_g - R_b)}{(R_b + R_g)^3} R_g E_i G_f \qquad (7.18)$$

Equation (7.18) yields

$$R_b = R_g$$

Strain Measurement Method

❑ Conventional methods: 2. Ballast Circuit



$$E_o = \frac{R_g}{R_b + R_g} E_i$$

$$dE_o = \left[\frac{dR_g}{R_b + R_g} - \frac{R_g\, dR_g}{(R_b + R_g)^2} \right] E_i = \frac{R_b R_g}{(R_b + R_g)^2} E_i \cdot \frac{dR_g}{R_g} = \frac{R_b R_g}{(R_b + R_g)^2} E_i \cdot G_f \varepsilon \qquad (7.17)$$

$$S = \frac{dE_o}{\varepsilon} = \frac{R_b R_g}{(R_b + R_g)^2} E_i \cdot G_f \qquad \text{[From Eq. (7.17)]}$$

$$0 = \frac{dS}{dR_b} = \frac{(R_g - R_b)}{(R_b + R_g)^3} R_g E_i G_f \qquad (7.18)$$

Equation (7.18) yields

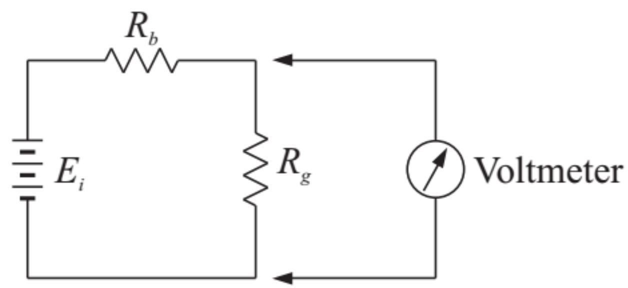
$$R_b = R_g$$

Then from Eq. (7.17), we get

$$dE_o = \frac{G_f}{4} \varepsilon E_i \qquad (7.19)$$

Strain Measurement Method

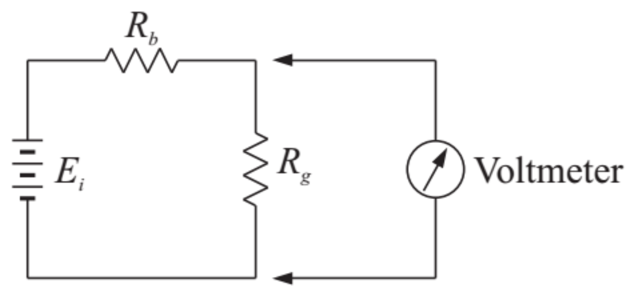
- ❑ **Conventional methods: 2. Ballast Circuit**



$$dE_o = \frac{G_f}{4} \varepsilon E_i$$

Strain Measurement Method

❑ Conventional methods: 2. Ballast Circuit



$$dE_o = \frac{G_f}{4}\varepsilon E_i$$

From Eq. (7.19), we observe that the change in voltage is indeed small. let us consider a typical case of $G_f = 2$, and $\varepsilon = 5$ microstrain. Then Eq. (7.19) yields a voltage change of $0.0000025E_i$ V which requires measurement by a digital voltmeter of 6 decades of precision!

Strain Measurement Method

Conventional methods

1. Current injection

2. Ballast circuit

❑

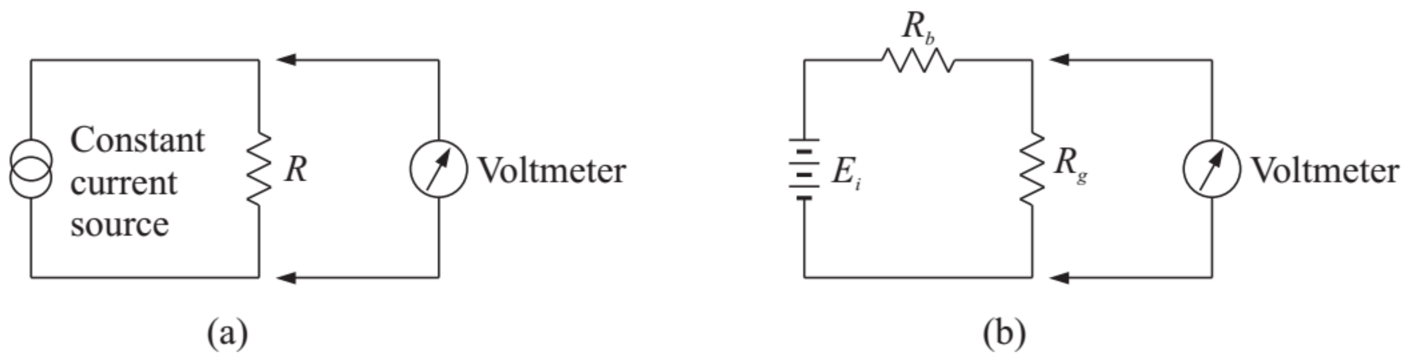


Fig. 7.8 Resistance measurement: (a) by the current injection method, and (b) by the ballast circuit method.

❑ Conclusion

❑ both the conventional current injection and ballast circuit methods are not suitable for strain measurements

Strain Measurement Method

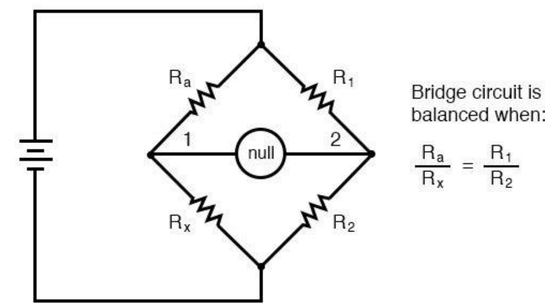
- **Bridge circuit method**
    - **Static measurement: null-type**
        - no current flows through the measuring instrument which is equivalent to having a measuring instrument of infinite input impedance.
        - Thus the loading of the measured medium is almost nil here
    - **Dynamic measurement:**
        - Voltage sensitive and current sensitive
        - very small current flows through measuring instrument thus loading the measured medium minimally
- **Crucial advantage**
    - it is very easy to eliminate stray inputs, like temperature effects, by incorporating compensatory devices in suitable arms of the bridge

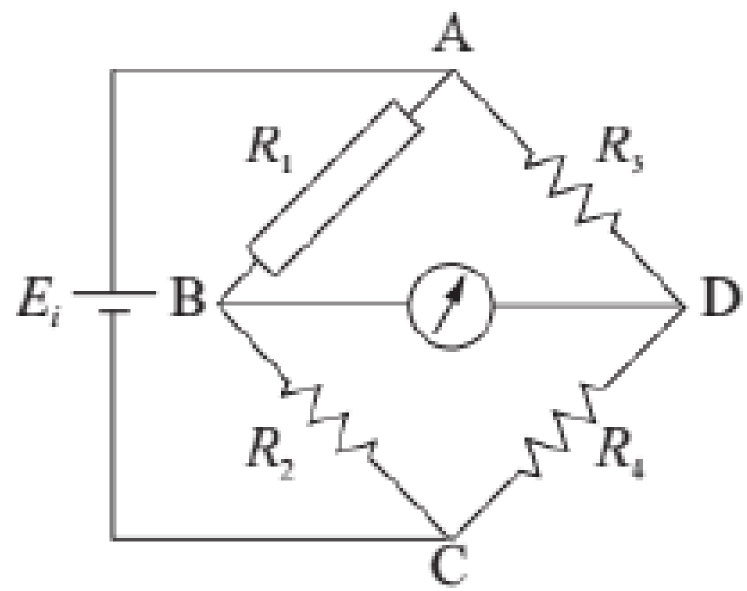
Strain Measurement Method

- Strain gage is used in a Wheatstone bridge.
- Under no strain condition, the resistance of all the arms of bridge is equal i.e.,

$$R_1 = R_2 = R_3 = R_4$$

- When a strain is applied the resistance of strain gage changes. Resultantly a potential difference appears across nodes **'BD'** of the bridge.





Strain Measurement Method

- **Bridge circuit method: Static measurement**

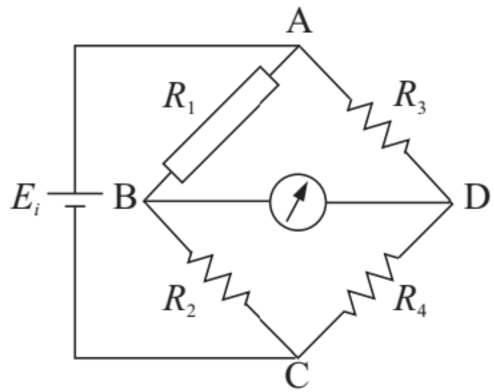


Fig. 7.9 Wheatstone bridge arrangement for strain measurement. The resistance R_1 represents the strain gauge.

When the bridge is balanced, no current flows through the galvanometer. Then

$$\frac{R_1}{R_2} = \frac{R_3}{R_4}$$

Strain Measurement Method

- **Bridge circuit method: Static measurement**

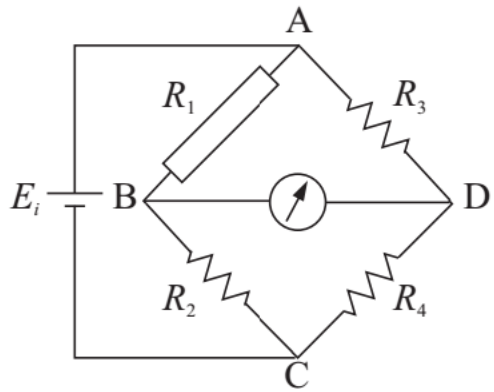


Fig. 7.9 Wheatstone bridge arrangement for strain measurement. The resistance R_1 represents the strain gauge.

When the bridge is balanced, no current flows through the galvanometer. Then

$$\frac{R_1}{R_2} = \frac{R_3}{R_4}$$

Now, if R_1 changes to $R_1 + \Delta R_1$ owing to strain, R_2 has to change to $R_2 + \Delta R_2$ to balance the bridge.

$$\Delta R_2 = \Delta R_1 \frac{R_4}{R_3}$$

Strain Measurement Method

- **Bridge circuit method: Static measurement**

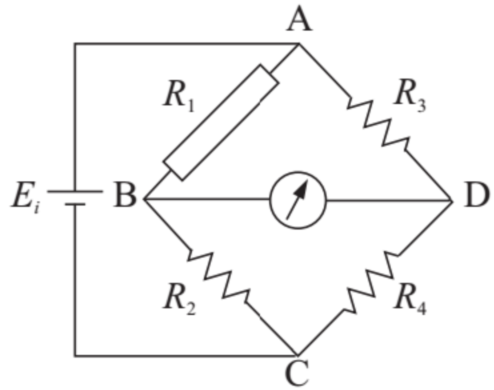


Fig. 7.9 Wheatstone bridge arrangement for strain measurement. The resistance R_1 represents the strain gauge.

When the bridge is balanced, no current flows through the galvanometer. Then

$$\frac{R_1}{R_2} = \frac{R_3}{R_4}$$

$$\Delta R_2 = \Delta R_1 \frac{R_4}{R_3}$$

And if $R_1 = R_2 = R_3 = R_4 = R_g$, which normally is, then $\Delta R_2 = \Delta R_1 \equiv \Delta R_g = G_f R_g \varepsilon$

$$G_f \equiv \frac{dR/R}{\varepsilon}$$

Strain Measurement Method

- **Bridge circuit method: Static measurement**

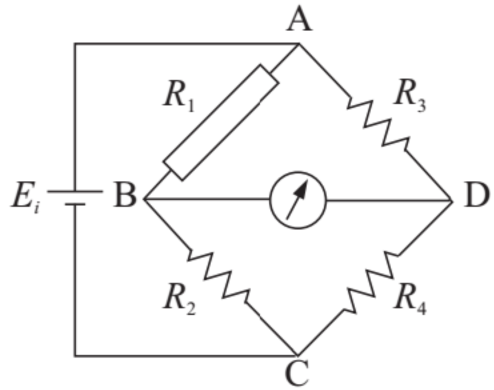


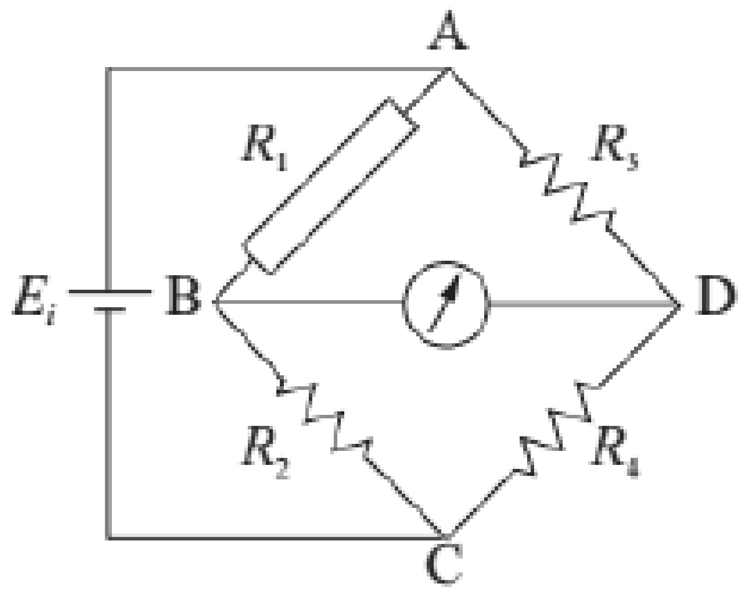
Fig. 7.9 Wheatstone bridge arrangement for strain measurement. The resistance R_1 represents the strain gauge.

$$\Delta R_2 = \Delta R_1 \equiv \Delta R_g = G_f R_g \varepsilon$$

- **Change in resistance R_2 is a direct measure of the strain**

Strain Measurement Method

- Two types of measurement are used.
    - **Static Measurements**
        - No potential difference between nodes **BD.**
        - No deflection of galvanometer.
    - **Dynamic Measurements**
        - Voltage Sensitive Measurement (uses voltmeter)
        - Current Sensitive Measurement (uses ammeter)



Strain Measurement Method

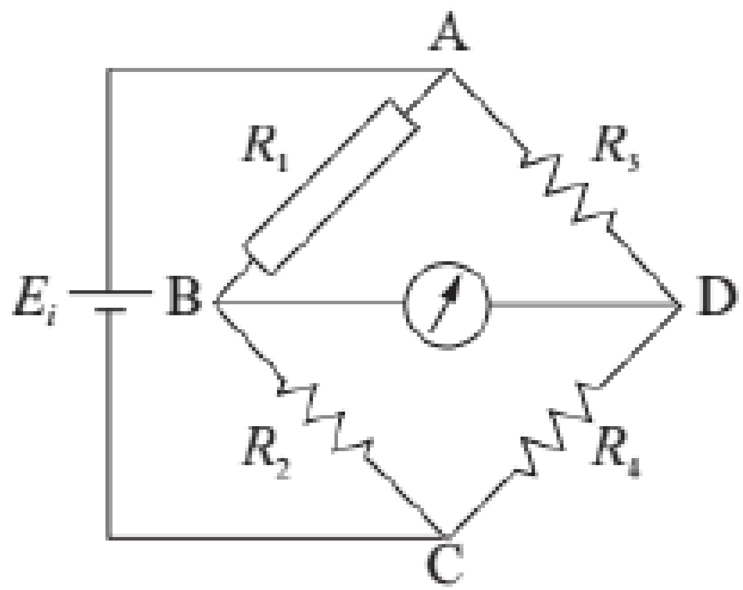
- **Bridge circuit method: Dynamic measurements**
  - Voltage sensitive bridge
  - Current sensitive bridge

Strain Measurement Method

Voltage Sensitive Measurements (Quarter-Bridge)

- **Potential Difference across nodes 'BD'**

$$\Delta E_o = \frac{G_f \epsilon}{4} E_i$$



- **Sensitivity of the bridge**

$$S = \frac{Chnage\ in\ Output\ Voltage}{Strain} = \frac{\Delta E_o}{\epsilon}$$

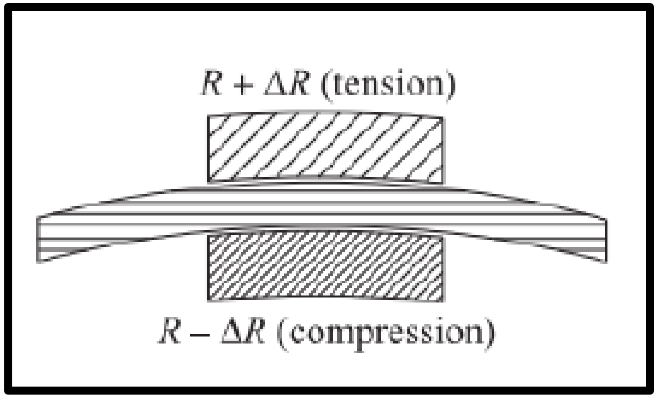
$$S = \frac{G_f}{4} E_i$$

Strain Measurement Method

Voltage Sensitive Measurements (Half-Bridge)

- Potential Difference across nodes 'AC'

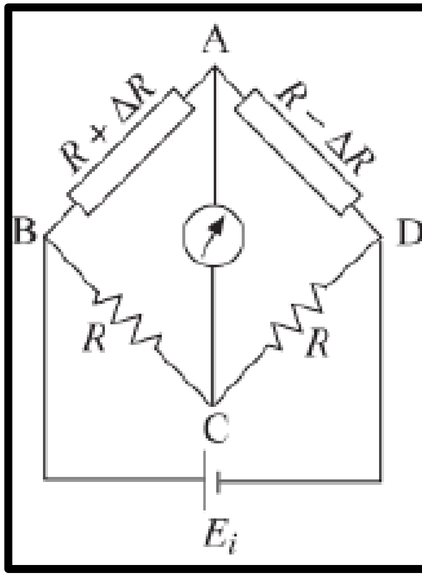
$$\Delta E_o = \frac{G_f \epsilon}{2} E_i$$



- Sensitivity of the bridge

$$S = \frac{Change\ in\ Output\ Voltage}{Strain} = \frac{\Delta E_o}{\epsilon}$$

$$S = \frac{G_f}{2} E_i$$

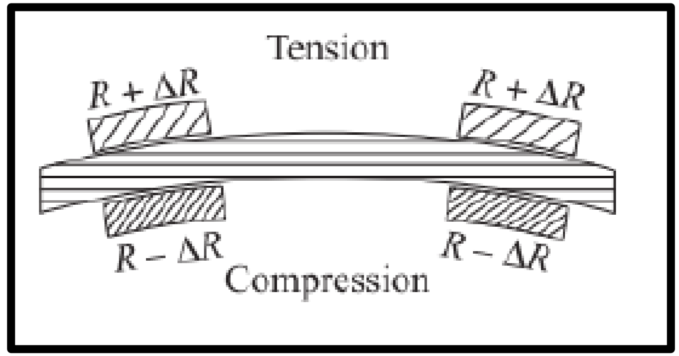


Strain Measurement Method

Voltage Sensitive Measurements (Full-Bridge)

- **Potential Difference across nodes 'AC'**

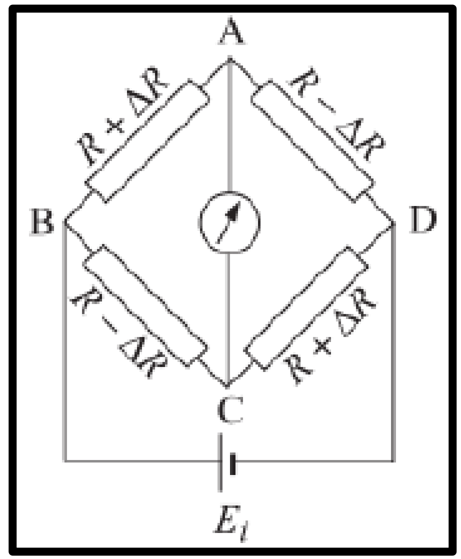
$$\Delta E_o = G_f \epsilon E_i$$



- **Sensitivity of the bridge**

$$S = \frac{Chnage\ in\ Output\ Voltage}{Strain} = \frac{\Delta E_o}{\epsilon}$$

$$S = G_f E_i$$

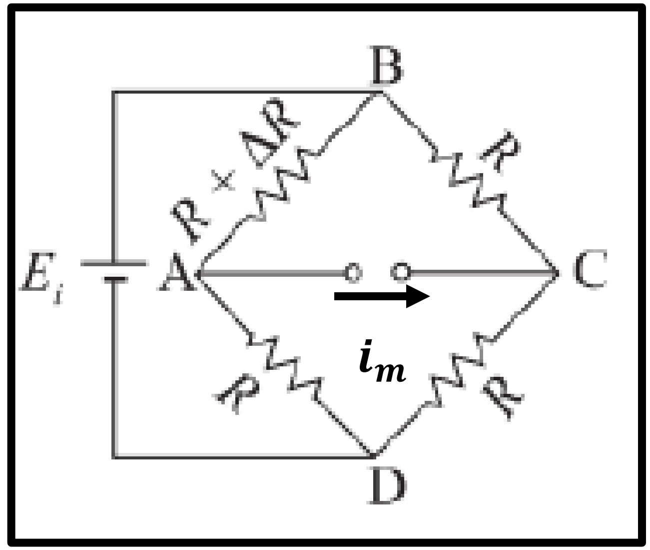


Strain Measurement Method

Current Sensitive Measurements

- **The current i_m flowing through the ammeter is proportional to strain.**

$$i_m = G_f \epsilon K \text{ where } K = \frac{E_i}{4(R+R_m)}$$



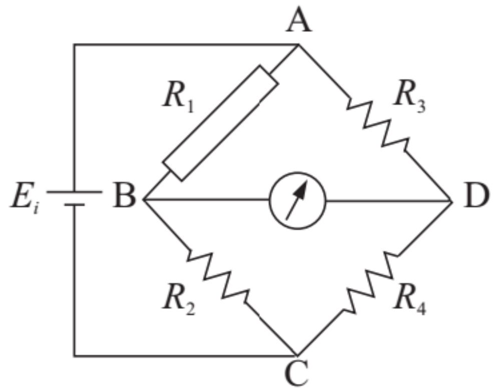
- **Sensitivity of the bridge**

$$S = \frac{ammeter\ current}{Strain} = \frac{i_m}{\epsilon}$$

$$S = G_f K$$

Strain Measurement Method

- **Voltage Sensitive Bridge: Quarter bridge**



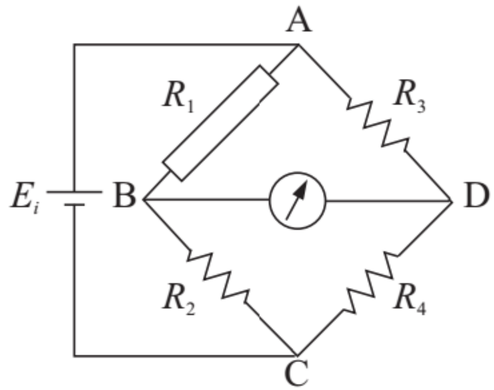
- for equal initial resistance of all the arms

- output voltage caused by a change in resistance in the strain gauge

$$\Delta E_o = \left(\frac{R + \Delta R}{2R + \Delta R} - \frac{1}{2} \right) E_i = \frac{\Delta R / R}{4 + 2\Delta R / R} E_i$$

Strain Measurement Method

- ❑ **Voltage Sensitive Bridge: Quarter bridge**



- ❑ **output voltage caused by a change in resistance in the strain gauge**

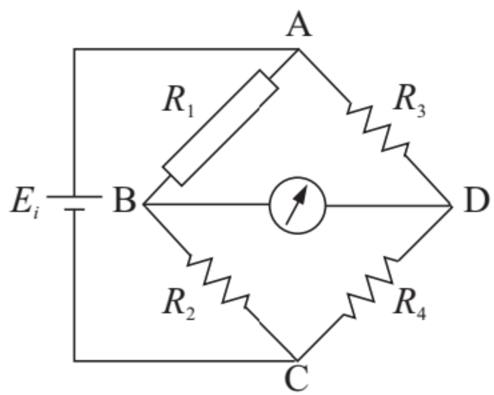
$$\Delta E_o = \left(\frac{R+\Delta R}{2R+\Delta R} - \frac{1}{2}\right) E_i = \frac{\Delta R/R}{4+2\Delta R/R} E_i$$

$$\cong \frac{\Delta R/R}{4} E_i \qquad \left[\because \quad 4 \gg \frac{2\Delta R}{R}\right]$$

$$= \frac{G_f \varepsilon}{4} E_i$$

Strain Measurement Method

- ❑ **Voltage Sensitive Bridge: Quarter bridge**



- ❑ **output voltage caused by a change in resistance in the strain gauge**

$$\Delta E_o = \left(\frac{R + \Delta R}{2R + \Delta R} - \frac{1}{2} \right) E_i = \frac{\Delta R/R}{4 + 2\Delta R/R} E_i$$

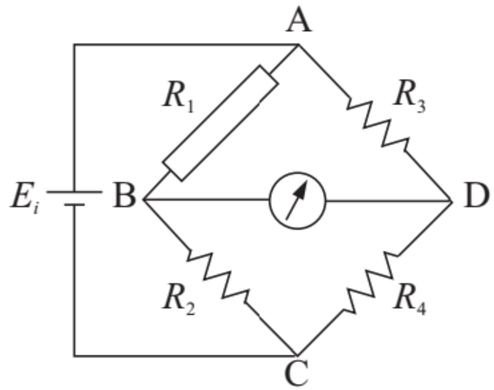
$$\cong \frac{\Delta R/R}{4} E_i \qquad \left[\because \quad 4 \gg \frac{2\Delta R}{R} \right]$$

$$= \frac{G_f \varepsilon}{4} E_i$$

$$S = \frac{\Delta E_o}{\varepsilon} = \frac{G_f E_i}{4}$$

Strain Measurement Method

- Voltage Sensitive Bridge: Quarter bridge



- Sensitivity

$$S = \frac{\Delta E_o}{\varepsilon} = \frac{G_f E_i}{4}$$

Strain Measurement Method

- **Voltage Sensitive Bridge: Half bridge**
  - strain gauges are bonded on top and bottom of the stressed member

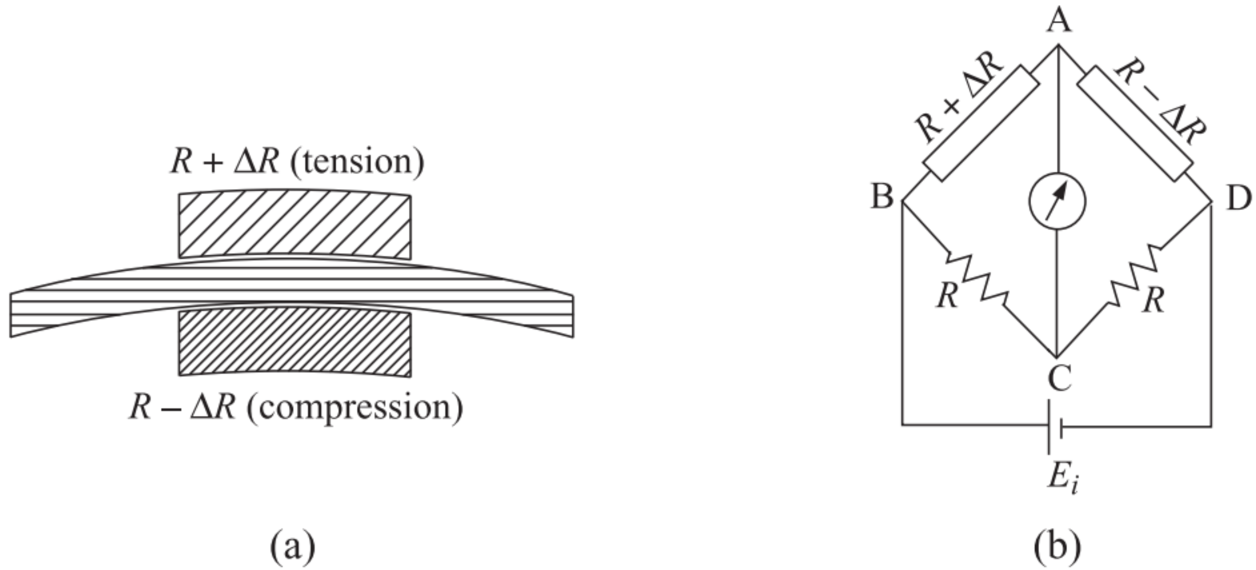
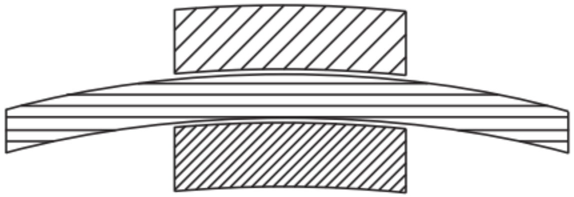
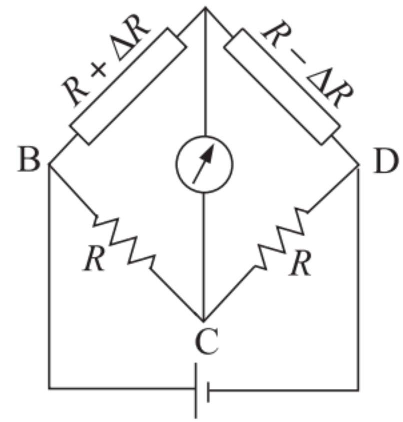


Fig. 7.10 Half-bridge arrangement for measurement of strain: (a) fixing of gauges on a cantilever, and (b) bridge configuration.

Strain Measurement Method

- **Voltage Sensitive Bridge: Half bridge**





- **output voltage**

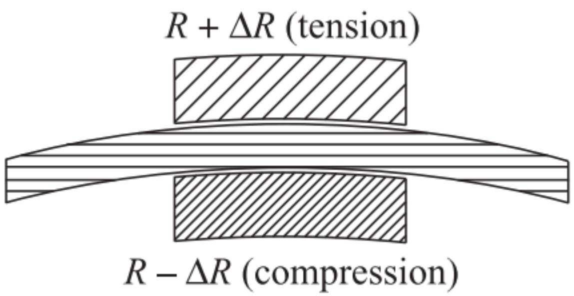
$$\Delta E_o = \left[\frac{R+\Delta R}{(R+\Delta R)+(R-\Delta R)} - \frac{1}{2}\right] E_i$$

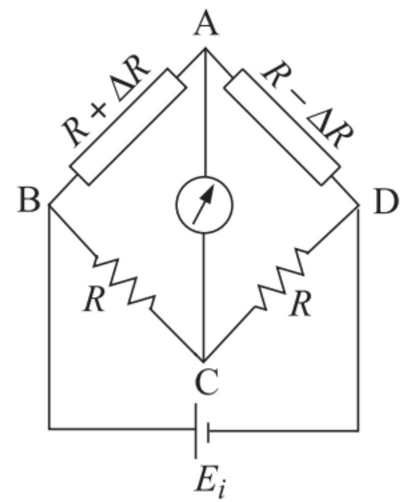
$$= \frac{\Delta R}{2R} E_i$$

$$= \frac{G_f \varepsilon}{2} E_i$$

Strain Measurement Method

- Voltage Sensitive Bridge: Half bridge





- output voltage

$$\Delta E_o = \left[\frac{R+\Delta R}{(R+\Delta R)+(R-\Delta R)} - \frac{1}{2}\right] E_i$$

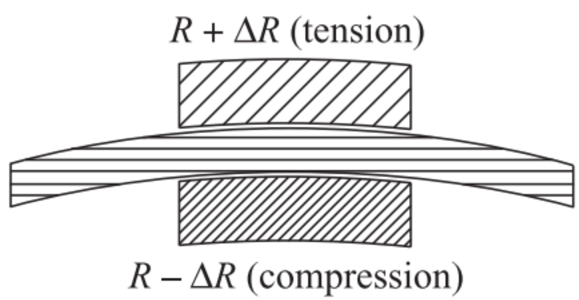
$$= \frac{\Delta R}{2R} E_i$$

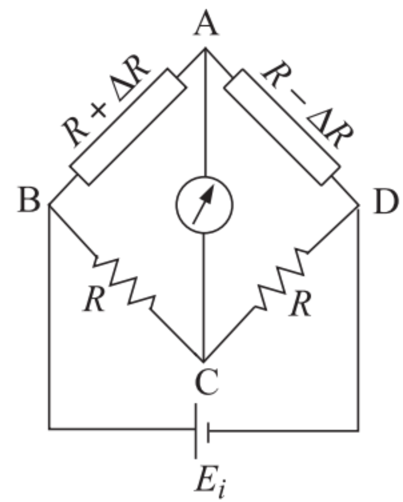
$$= \frac{G_f \varepsilon}{2} E_i$$

$$S = \frac{G_f E_i}{2}$$

Strain Measurement Method

- ❑ **Voltage Sensitive Bridge: Half bridge**





$$S = \frac{G_f E_i}{2}$$

- ❑ Advantages of a half-bridge over a quarter-bridge

1. The sensitivity is doubled
2. Unlike quarter-bridge it is not susceptible to errors arising out of change in the ambient temperature

Strain Measurement Method

- **Voltage Sensitive Bridge: Full bridge**

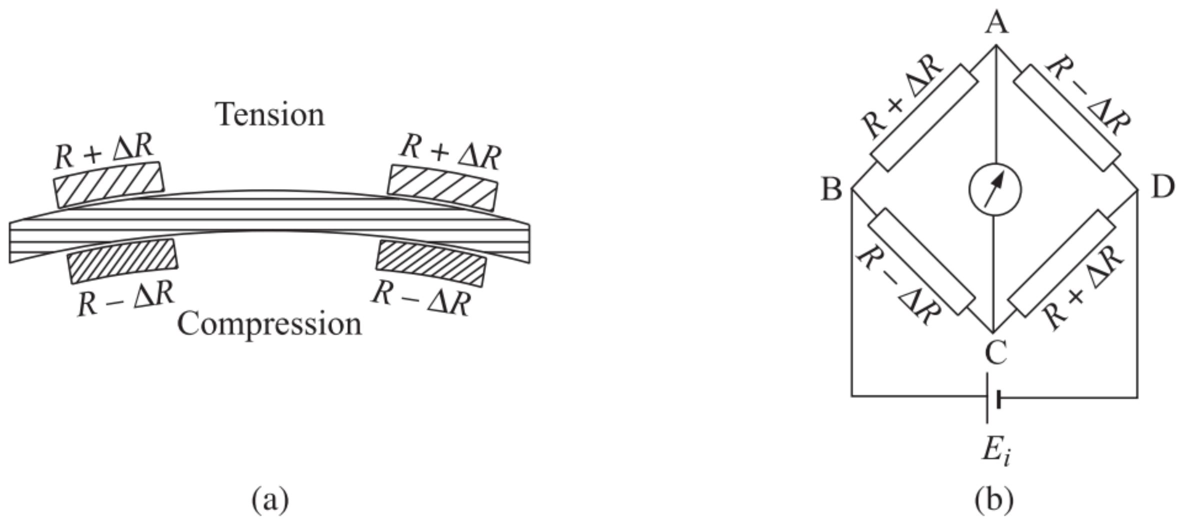
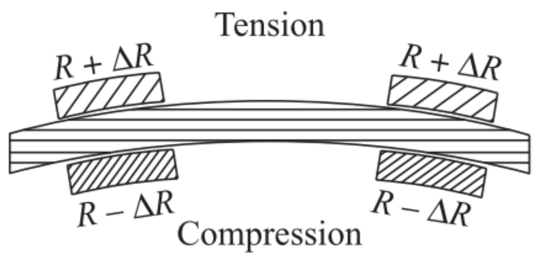
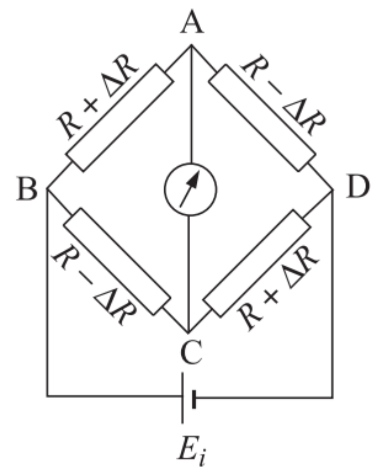


Fig. 7.11 Full-bridge arrangement for measurement of strain: (a) fixing of gauges on the specimen, and (b) bridge configuration.

Strain Measurement Method

- **Voltage Sensitive Bridge: Full bridge**





$$\Delta E_o = G_f \varepsilon E_i$$

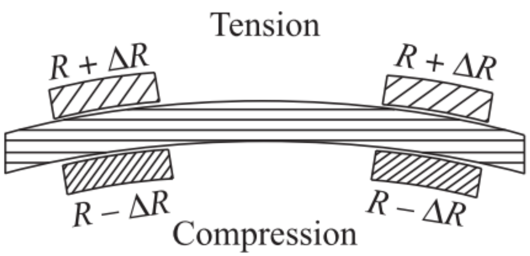
$$S = G_f E_i$$

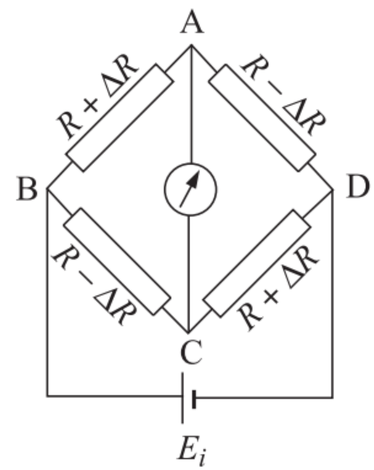
- **Advantages**

1. Higher sensitivity than Quarter and Half bridges
2. Immune to temperature effects

Strain Measurement Method

- **Voltage Sensitive Bridge: Full bridge**





$$\Delta E_o = G_f \varepsilon E_i$$

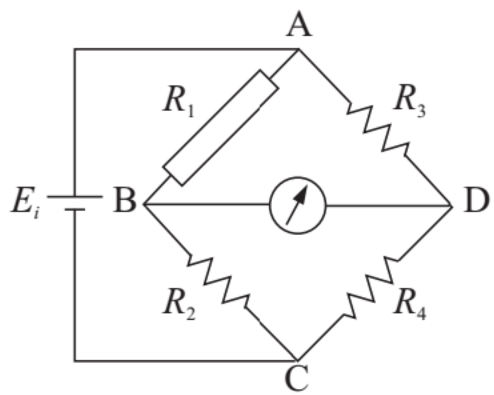
$$S = G_f E_i$$

- **Drawback**
- gauges occupy a considerable space in this arrangement
  - the measurement is an average strain value over a rather large area

Strain Measurement Method

- **Current Sensitive Bridge:**

  Output current is measured by an ammeter



Strain Measurement Method

- ❑ **Current Sensitive Bridge: output current is measured by an ammeter**

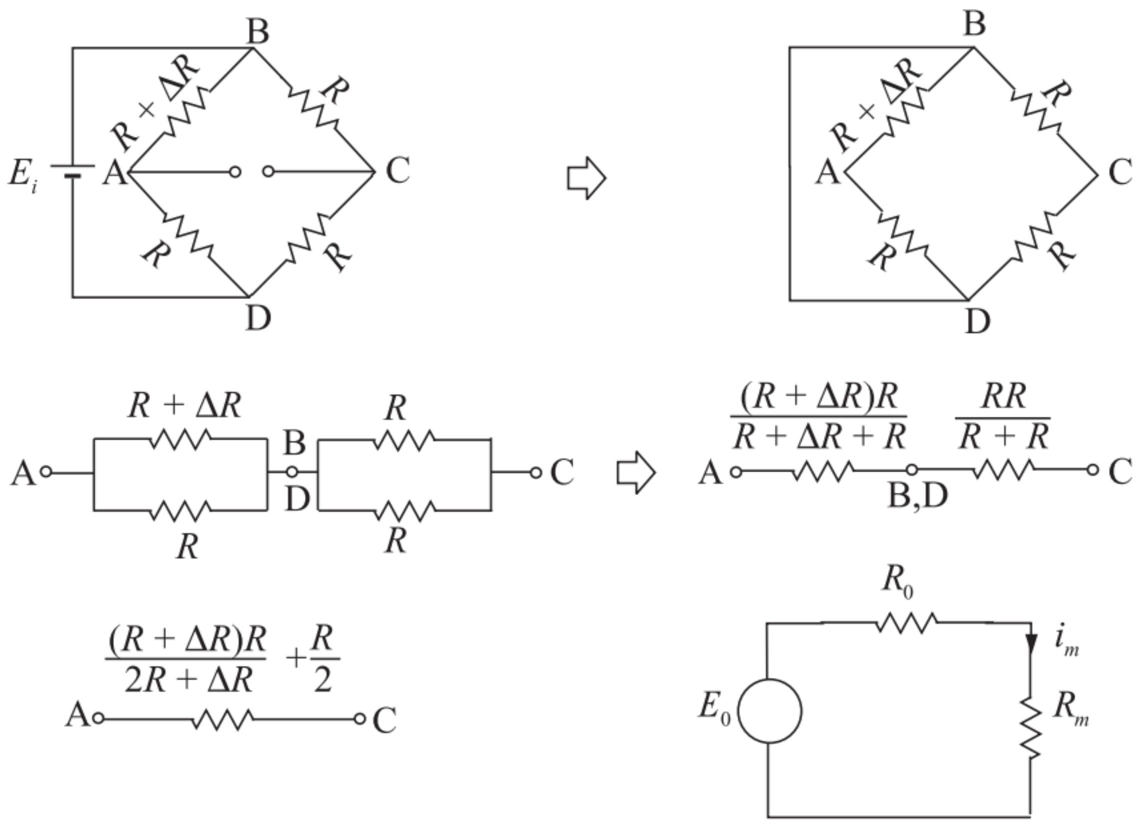


Fig. 7.12 Reduction of the bridge to its Thevenin-equivalent resistance.

Strain Measurement Method

- Current Sensitive Bridge: output current is measured by an ammeter

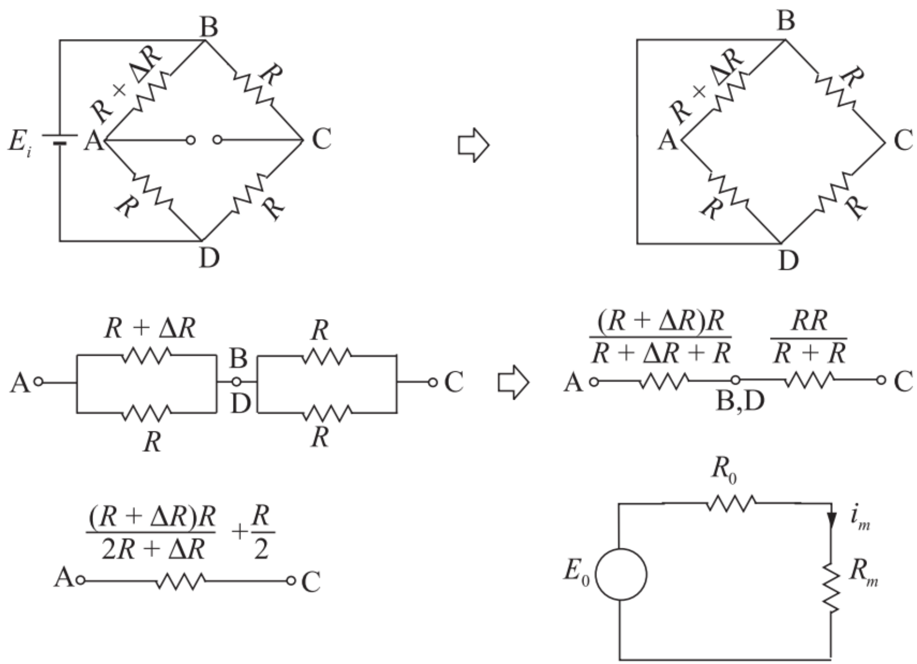


Fig. 7.12 Reduction of the bridge to its Thevenin-equivalent resistance.

$$R_o = \frac{(R + \Delta R)R}{2R + \Delta R} + \frac{R}{2} = \frac{4 + (3\Delta R/R)}{4 + (2\Delta R/R)} R$$

$$E_o = E_A - E_C = \left(\frac{R + \Delta R}{2R + \Delta R} - \frac{R}{2R} \right) E_i$$

$$= \frac{\Delta R/R}{4 + (2\Delta R/R)} E_i$$

The current through the measuring ammeter of resistance R_m is

$$i_m = \frac{E_m}{R_m} = \frac{E_o}{R_o + R_m}$$

Strain Measurement Method

- Current Sensitive Bridge: output current is measured by an ammeter

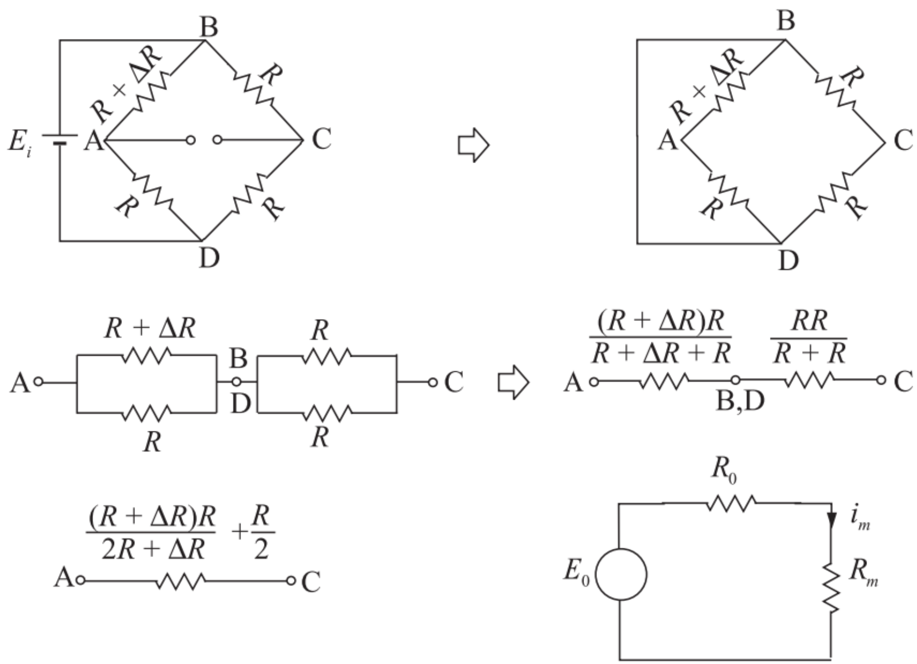


Fig. 7.12 Reduction of the bridge to its Thevenin-equivalent resistance.

$$R_o = \frac{(R+\Delta R)R}{2R+\Delta R} + \frac{R}{2} = \frac{4+(3\Delta R/R)}{4+(2\Delta R/R)}R$$

$$E_o = E_A - E_C = \left(\frac{R+\Delta R}{2R+\Delta R} - \frac{R}{2R}\right)E_i$$

$$= \frac{\Delta R/R}{4+(2\Delta R/R)}E_i$$

The current through the measuring ammeter of resistance R_m is

$$i_m = \frac{E_m}{R_m} = \frac{E_o}{R_o+R_m}$$

$$i_m = \frac{\Delta R}{R}\frac{1}{R_m[4+(2\Delta R/R)]+R[4+(3\Delta R/R)]}E_i$$

$$\cong \frac{\Delta R}{R}\frac{1}{4(R+R_m)}E_i$$

$$= G_f \varepsilon K \qquad \left[K \equiv \frac{E_i}{4(R+R_m)}\right]$$

Strain Measurement Method

Example 7.2

A 100 Ω strain gauge of gauge factor 2 is connected to the first arm of a Wheatstone bridge. Under no strain condition, all the arms have equal resistance. When the gauge is subjected to a strain, the second arm resistance has to be changed to 100.56 Ω to obtain a balance. Find the value of the strain.

Solution

Substituting the respective values in Eq. (7.2), we get

$$\varepsilon = \frac{\Delta R}{G_f R_g} = \frac{0.56}{2 \times 100} = 0.0028$$

Strain Measurement Method

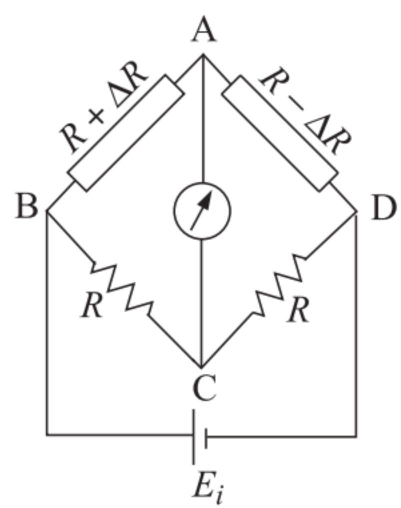
Example 7.3

A bridge circuit has two fixed resistors and two strain gauges all of which have a value of 120 Ω. The gauge factor is 2.04 and the strain applied to twin strain gauges, one in tension and the other in compression, is 0.000165. If the battery current in the initial balanced condition of the bridge is 50 mA, determine

(a) The voltage output of the bridge, and

(b) The sensitivity in volt per unit strain.

If the galvanometer connected to output terminals reads 100 μV per scale division and if 1/10th of a division can be read, determine the resolution.



Example 7.3

A bridge circuit has two fixed resistors and two strain gauges all of which have a value of 120 Ω. The gauge factor is 2.04 and the strain applied to twin strain gauges, one in tension and the other in compression, is 0.000165. If the battery current in the initial balanced condition of the bridge is 50 mA, determine

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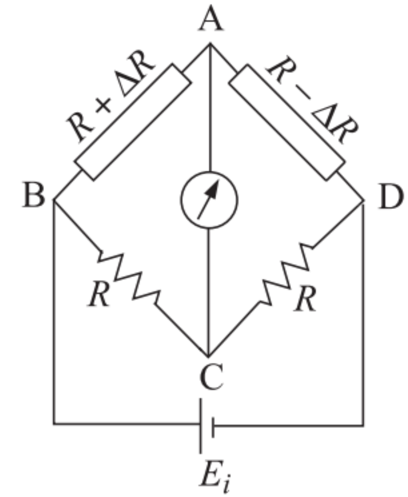


Solution

In the initial balanced condition of the bridge, the equivalent bridge resistance is $R = 120\ \Omega$. Hence, the battery voltage is

$$E_i = 50 \times 10^{-3} \times 120 = 6\ \text{V}$$

(a) A strain of 0.000165 produces an output of

$$\Delta E_o = \frac{G_f \varepsilon}{2} E_i = \frac{2.04 \times 0.000165 \times 6}{6} = 1.01\ \text{mV}$$

(b) Therefore,

$$\text{Sensitivity} = \frac{1.01 \times 10^{-3}}{0.000165} = 6.12\ \ \text{V/strain} = 6.12\ \mu\text{V}/\mu\text{-strain}.$$

The instrument has 100 μV graduation and 1/10th of a division can be read. That means, 10 μV can be read. This corresponds to $10 \div 6.12 \cong 1.63$ μ-strain. Therefore, the resolution is 1.63 μ-strain.

Solution 7.3 in Detail

$R = 120\,\Omega$

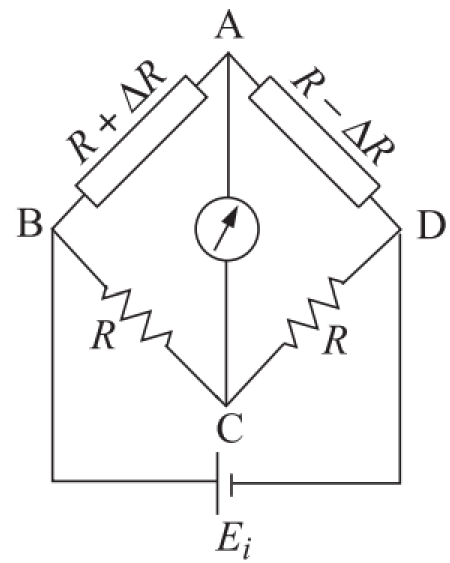
$G_f = 2.04$

$\varepsilon = 0.000165$

$E_i = 10\,V$

$\Delta E_0 = ?$

$S = \dfrac{\Delta E_0}{\varepsilon} = ?$



$R = 120\,\Omega$

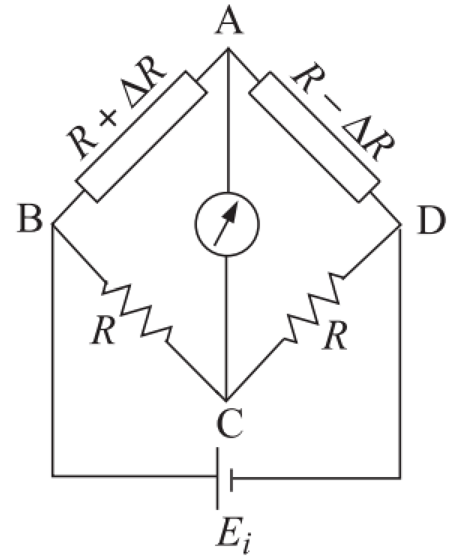
$G_f = 2.04$

$\varepsilon = 0.000165$

$E_i = 10\,V$

$\Delta E_0 = ?$

$S = \dfrac{\Delta E_0}{\varepsilon} = ?$



$$\Delta E_0 = \frac{G_f\,\varepsilon}{2} E_i$$

$R = 120\,\Omega$

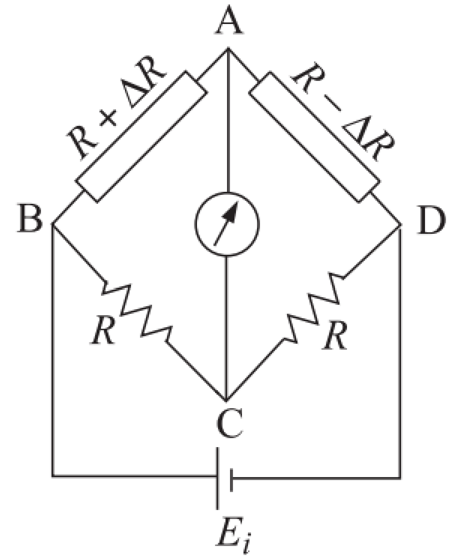
$G_f = 2.04$

$\varepsilon = 0.000165$

$E_i = 10\,V$

$\Delta E_o = ?$

$S = \dfrac{\Delta E_o}{\varepsilon} = ?$



$$\Delta E_o = \frac{G_f \varepsilon}{2} E_i$$

$$= \frac{2.04(0.000165)(10)}{2}$$

$$\Delta E_o = 1.7\,mV$$

$R = 120\,\Omega$

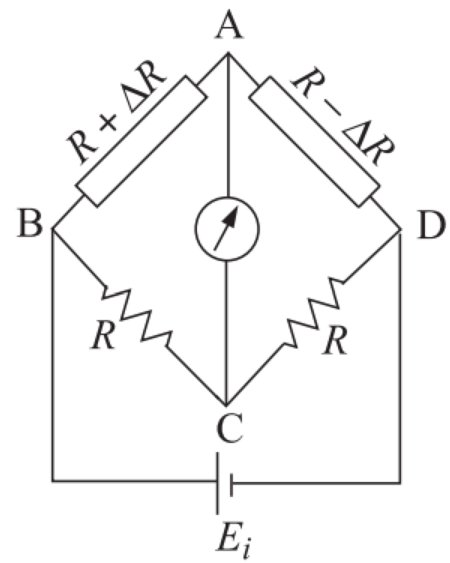
$G_f = 2.04$

$\varepsilon = 0.000165$

$E_i = 10\,V$

$\Delta E_o = ?$

$S = \dfrac{\Delta E_o}{\varepsilon} = ?$



$$\Delta E_o = \frac{G_f \varepsilon}{2} E_i = 1.7\,mV$$

$$S = \frac{\Delta E_o}{\varepsilon} = \frac{0.0017}{0.000165} = 10.2\,V/\text{strain}$$

$R = 120\,\Omega$

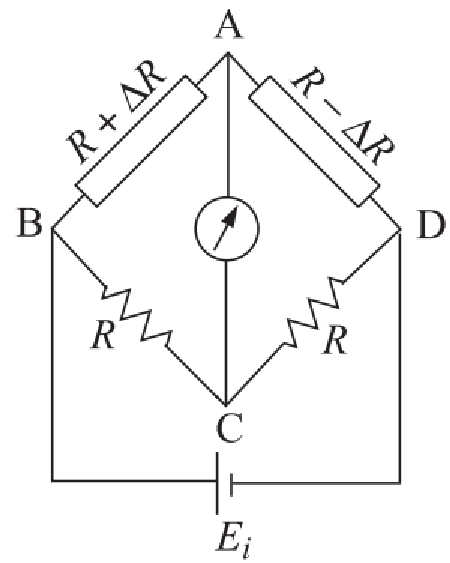
$G_f = 2.04$

$\varepsilon = 0.000165$

$E_i = 10\,V$

$\Delta E_o = ?$

$S = \dfrac{\Delta E_o}{\varepsilon} = ?$



$\Delta E_o = \dfrac{G_f\,\varepsilon}{2} E_i = 1.7\,mV$

$S = \dfrac{\Delta E_o}{\varepsilon} = \dfrac{0.0017}{0.000165} = 10.2\,V/\text{strain} = 10.2\,\mu V/\mu\text{-strain}$

$R = 120\,\Omega$

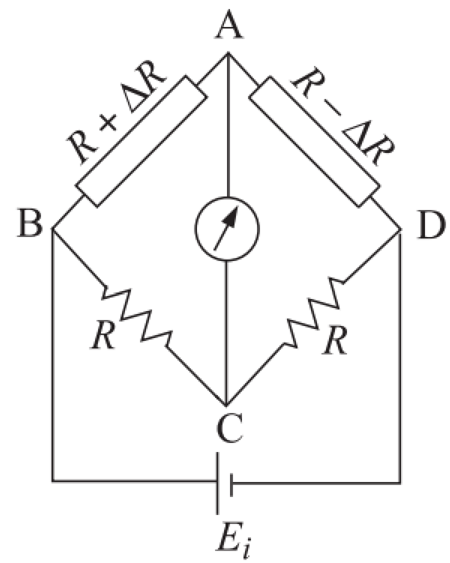
$G_f = 2.04$

$\varepsilon = 0.000165$

$E_i = 10\,V$

$\Delta E_o = ?$

$S = \dfrac{\Delta E_o}{\varepsilon} = ?$



$\Delta E_o = \dfrac{G_f\,\varepsilon}{2} E_i = 1.7\,mV$

$S = \dfrac{\Delta E_o}{\varepsilon} = 10.2\,\mu V \,|\, \mu\text{-strain}$

Example 7.3

A bridge circuit has two fixed resistors and two strain gauges all of which have a value of 120 Ω. The gauge factor is 2.04 and the strain applied to twin strain gauges, one in tension and the other in compression, is 0.000165. If the battery current in the initial balanced condition of the bridge is 50 mA, determine

(a) The voltage output of the bridge, and

(b) The sensitivity in volt per unit strain.

If the galvanometer connected to output terminals reads 100 μV per scale division and if 1/10th of a division can be read, determine the resolution.

Solution

In the initial balanced condition of the bridge, the equivalent bridge resistance is $R = 120\,\Omega$. Hence, the battery voltage is

$$E_i = 50 \times 10^{-3} \times 120 = 6\text{ V}$$

(a) A strain of 0.000165 produces an output of

$$\Delta E_o = \frac{G_f \varepsilon}{2} E_i = \frac{2.04 \times 0.000165 \times 6}{6} = 1.01\text{ mV}$$

(b) Therefore,

$$\text{Sensitivity} = \frac{1.01 \times 10^{-3}}{0.000165} = 6.12\ \text{ V/strain} = 6.12\,\mu\text{V}/\mu\text{-strain}.$$

The instrument has 100 μV graduation and 1/10th of a division can be read. That means, 10 μV can be read. This corresponds to $10 \div 6.12 \cong 1.63\,\mu$-strain. Therefore, the resolution is 1.63 μ-strain.

Strain Measurement Method

Temperature Effects and Compensation

Variations in the ambient temperatures affect the strain measurements by Wheatstone bridges in the following three ways:

1. Change in the gauge factor of the strain gauge
2. Temperature-induced strain in the gauge element
3. Temperature-induced resistance changes in long lead wires.

Effect of Temperature on Strain Measurement

Change in gage factor

The gage factor changes slightly with change in temperature as shown in the figure.

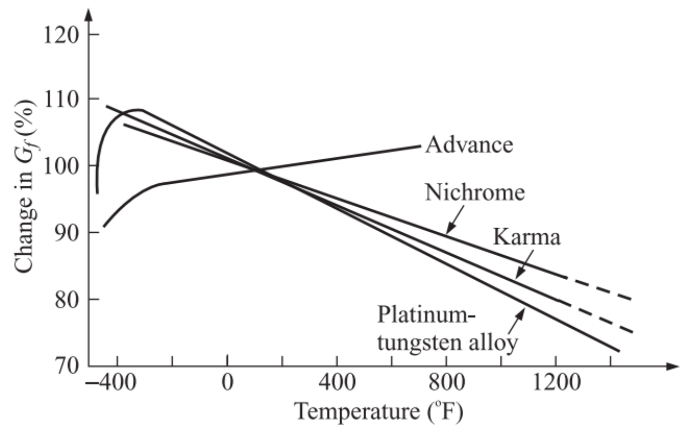


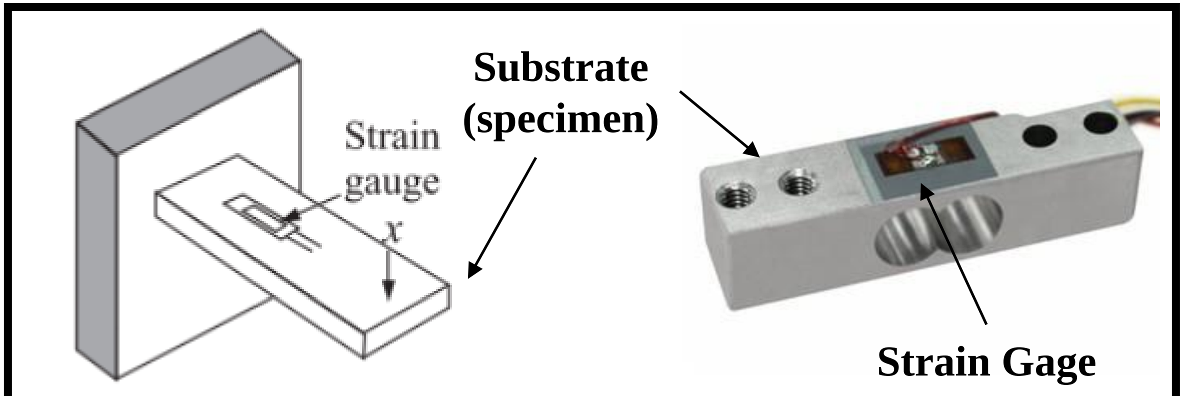
Fig. 7.14 Variation in gauge factors of the various strain gauge materials as a function of operating temperature.

The materials such as advance (copper-nickel alloys) are relatively insensitive to temperature variation are most popular choice for strain gage materials.

Effect of Temperature on Strain Measurement

Temperature-induced strain in gage element

Thermal expansion coefficient of **strain gage** and the **substrate (specimen)** on which the strain gage is applied is different.



$$\epsilon_{thermal} = \alpha_x \, \Delta t$$

- Where α_x is the thermal expansion coefficient.
- Temperature-induced strain in the gage element and substrate will be different.

Although the total surface area of gage element transverse to its measuring axis is very small compared to the similar area of substrate material, actually all the substrate thermal strain is transferred to the strain gage. → **because strain is a relative quantity.**

Strain Measurement Method

❑ Temperature Effects

❑ Change in the Gauge Factor

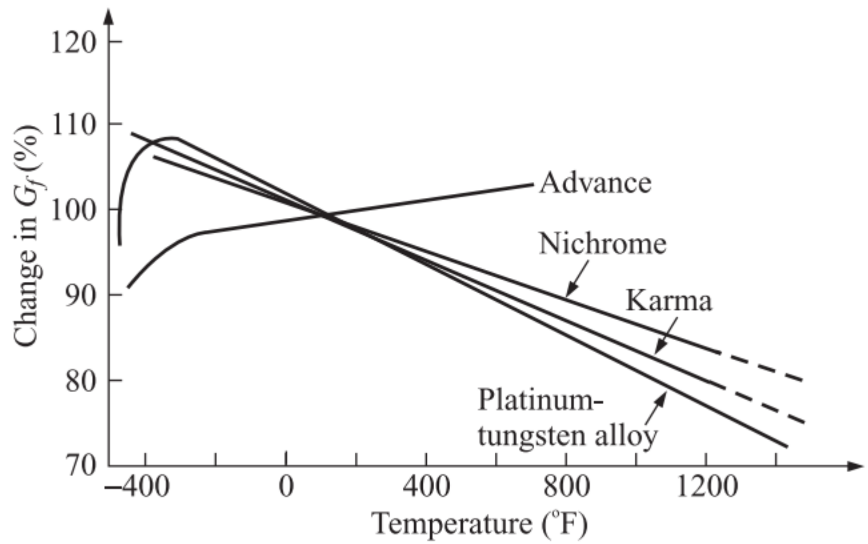


Fig. 7.14 Variation in gauge factors of the various strain gauge materials as a function of operating temperature.

Example 7.6

The TCR for isoelastic[9] is 260 ppm/°F. Its G_f is 3.5. Calculate the apparent strain produced by a variation of 1°F of temperature.

*Temperature coefficient of resistance (**TCR**) is the calculation of a relative change of resistance per degree of temperature change*

Solution

The generated apparent strain is

$$\varepsilon = \frac{dR/R}{G_f} = \frac{260 \times 10^{-6}}{3.5} = 74 \text{ microstrain}$$

[9] An alloy of 36% Ni, 8% Cr, 0.5% Mo and 55.5% Fe.

Strain Measurement Method

❑ Temperature Effects and Compensation

Example 7.7

A gauge, made of a material having a temperature coefficient of resistance of 12×10^{-4} per °C, has a resistance of 120 Ω and a gauge factor of 2. It is connected to a bridge having resistances of 120 Ω each. The bridge is balanced at ambient temperature. If the temperature changes by 20°C, find

(a) the output voltage of the bridge if the input voltage is 10 V, and

(b) the equivalent strain represented by the change in temperature.

Strain Measurement Method

❑ Temperature Effects

Example 7.7

A gauge, made of a material having a temperature coefficient of resistance of 12×10^{-4} per °C, has a resistance of 120 Ω and a gauge factor of 2. It is connected to a bridge having resistances of 120 Ω each. The bridge is balanced at ambient temperature. If the temperature changes by 20°C, find

(a) the output voltage of the bridge if the input voltage is 10 V, and

(b) the equivalent strain represented by the change in temperature.

Solution

(a) Change in resistance of the gauge due to the change in temperature is

$$\Delta R = \alpha R \Delta T = (120 \times 12 \times 10^{-4} \times 20)\,\Omega = 2.88\,\Omega$$

Since this is a quarter bridge, from Eq. (7.22), we get

$$\Delta E_o = \frac{\Delta R}{4R} E_i = \frac{2.88}{4 \times 120} \times 10\,\text{V} = 0.06 \;\; \text{V}$$

(b) The strain corresponding to a resistance change of 2.88 Ω is

$$\frac{\Delta R}{G_f \varepsilon} = \frac{2.88}{120 \times 2} = 0.012 = 12000\,\text{microstrain}$$

Temperature Effects and Compensation

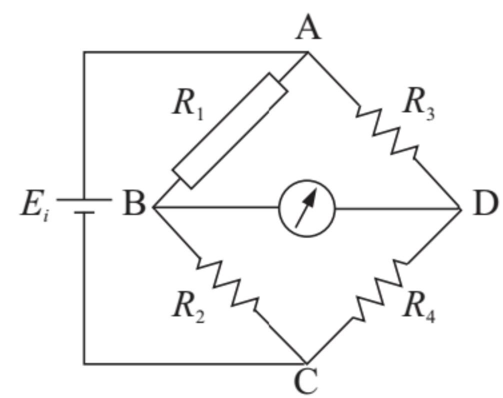
$R = 120\,\Omega$

$G_f = 2$

$\alpha = 12 \times 10^{-4}$ per °C

$E_i = 10V$

$\Delta T = 20°C$

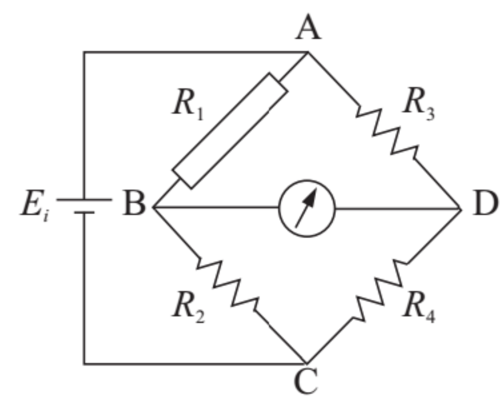


❑ Temperature Effects and Compensation

$R = 120\,\Omega$ $\quad G_f = 2$

$\alpha = 12 \times 10^{-4}$ per °C $\quad E_i = 10V$

$\Delta T = 20$ °C



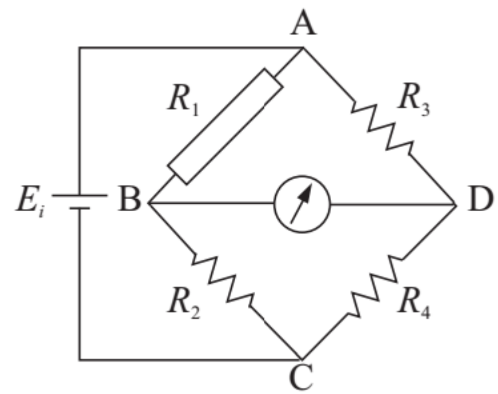
$$\Delta R = \alpha R \Delta T = 2.88\,\Omega$$

❑ Temperature Effects and Compensation

$R = 120\,\Omega$ $G_f = 2$

$\alpha = 12 \times 10^{-4}$ per °C $E_i = 10V$

$\Delta T = 20$ °C



$$\Delta R = \alpha R \Delta T = 2.88\,\Omega$$

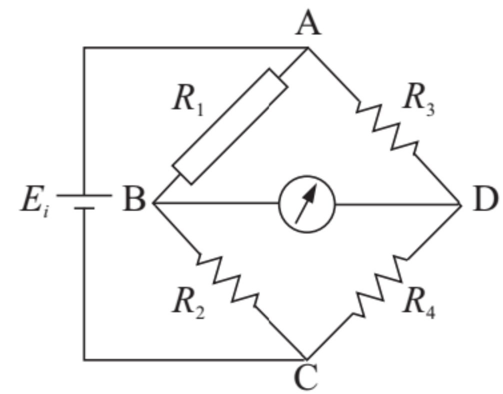
$$\Delta E_o = \frac{\Delta R}{4R} E_i = 0.06V$$

❑ Temperature Effects and Compensation

$R = 120\,\Omega$ $G_f = 2$

$\alpha = 12\times10^{-4}$ per °C $E_i = 10V$

$\Delta T = 20°C$



$$\Delta R = \alpha R \Delta T = 2.88\,\Omega$$

$$\Delta E_o = \frac{\Delta R}{4R} E_i = 0.06V$$

$$\varepsilon = \frac{\Delta R}{R G_f} = 0.0012 = 12000\ \mu\text{-strain}$$

Strain Measurement Method

❑ Temperature Effects and Compensation

Example 7.7

A gauge, made of a material having a temperature coefficient of resistance of 12×10^{-4} per °C, has a resistance of 120 Ω and a gauge factor of 2. It is connected to a bridge having resistances of 120 Ω each. The bridge is balanced at ambient temperature. If the temperature changes by 20°C, find

(a) the output voltage of the bridge if the input voltage is 10 V, and

(b) the equivalent strain represented by the change in temperature.

Strain Measurement Method

❑ Temperature Effects

Example 7.7

A gauge, made of a material having a temperature coefficient of resistance of 12×10^{-4} per °C, has a resistance of 120 Ω and a gauge factor of 2. It is connected to a bridge having resistances of 120 Ω each. The bridge is balanced at ambient temperature. If the temperature changes by 20°C, find

(a) the output voltage of the bridge if the input voltage is 10 V, and

(b) the equivalent strain represented by the change in temperature.

Solution

(a) Change in resistance of the gauge due to the change in temperature is

$$\Delta R = \alpha R \Delta T = (120 \times 12 \times 10^{-4} \times 20)\,\Omega = 2.88\,\Omega$$

Since this is a quarter bridge, from Eq. (7.22), we get

$$\Delta E_o = \frac{\Delta R}{4R} E_i = \frac{2.88}{4 \times 120} \times 10\,\text{V} = 0.06\ \text{V}$$

(b) The strain corresponding to a resistance change of 2.88 Ω is

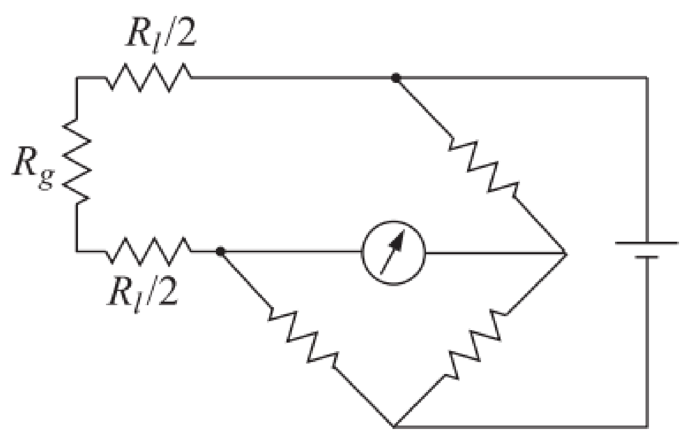
$$\frac{\Delta R}{G_f \varepsilon} = \frac{2.88}{120 \times 2} = 0.012 = 12000\ \text{microstrain}$$

Strain Measurement Method

- **Temperature Effects**
  - Temperature-induced strain in the gauge element
  - A difference in the coefficients of thermal expansion between the gauge and the substrate material may also generate spurious strain readings

Strain Measurement Method

- **Temperature Effects**
  - Temperature-induced resistance changes in the lead wires
  - Strain gauges are sometimes mounted at a distance from the measuring equipment
  - Any change in the lead wire resistance (R_l) cannot be distinguished from the changes in the resistance R_g of the strain gauge

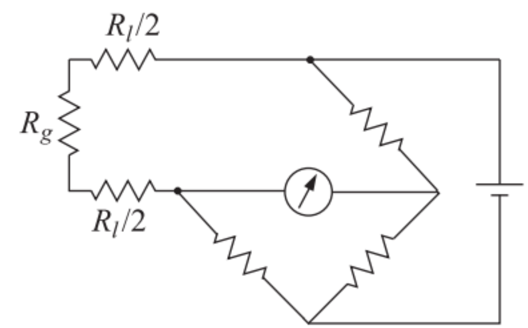


Strain Measurement Method

- **Temperature Effects**
  - Temperature-induced resistance changes in the lead wires
  - Strain gauges are sometimes mounted at a distance from the measuring equipment
  - Any change in the lead wire resistance (R_l) cannot be distinguished from the changes in the resistance R_g of the strain gauge

$$\Delta E_o = \frac{1}{4}\frac{\Delta R_g}{R_g + R_l}E_i = \frac{1}{4}\frac{\Delta R_g}{R_g(1+\alpha)}E_i$$

$$\alpha = \frac{R_l}{R_g} = \frac{\text{Total leadwire resistance}}{\text{Gauge resistance}}$$



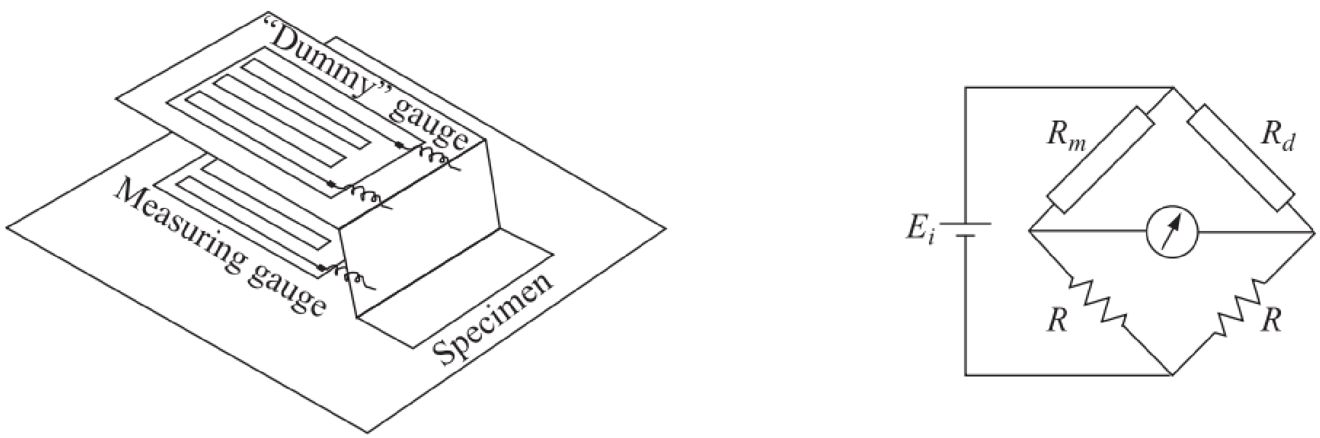
$$\Delta E_o = \frac{1}{4}\frac{G_f}{1+\alpha}\varepsilon E_i \equiv \frac{1}{4}(G_f)_{\text{eff}}\,\varepsilon E_i$$

$$(G_f)_{\text{eff}} = \frac{G_f}{1+\alpha} \approx G_f(1-\alpha) \quad \text{for } \alpha \ll 1$$

- Error introduced by the lead-wire resistance becomes significant if the ratio *α* exceeds 0.1%. It effectively lowers the sensitivity of the bridge circuit by reducing the gauge factor.

Strain Measurement Method

- **Temperature Compensation**
  - half-bridge and full-bridge configurations for measurement of strain are automatically compensated for temperature effects
  - For the quarter-bridge arrangement, the temperature compensation can be made here by incorporating a dummy gauge in one of the arms of the bridge



(a) (b)

Fig. 7.15 Temperature compensation for quarter-bridge: (a) Placement of the *dummy* gauge on the specimen, and (b) incorporation of the *dummy* (R_d) in the bridge.

Strain Measurement Method

- **Temperature Compensation**
  - Lead-wire effects: for correction, an additional third lead can be incorporated
  - The third wire is merely a sense lead with almost no current flowing through it
  - Theoretically, if the lead wires to the strain gauge have the same nominal resistance, the same TCR, and are maintained at the same temperature, full compensation is obtained.

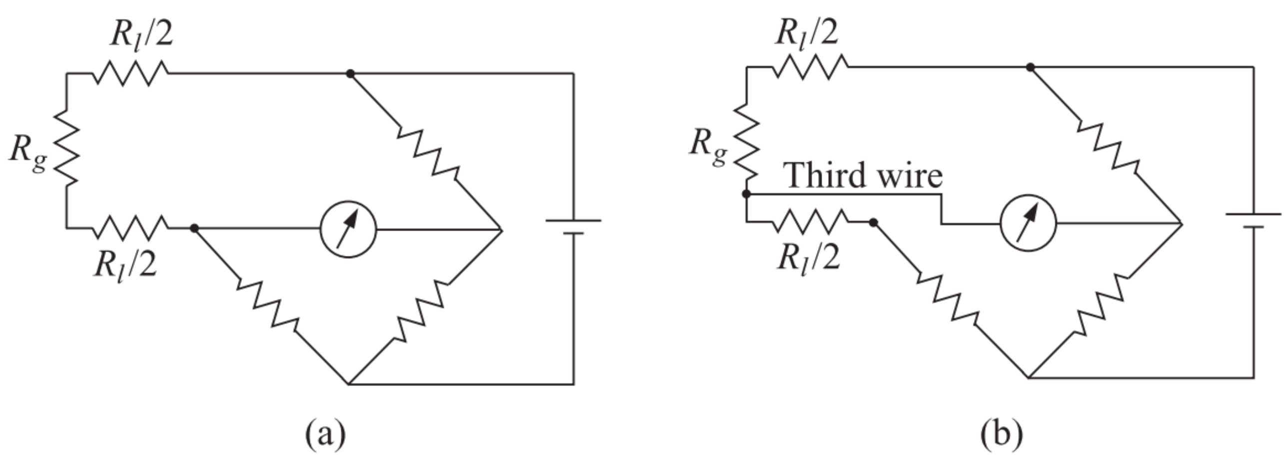


Fig. 7.16 (a) Resistance of long lead-wires interfering in the bridge measurement, and (b) three-wire

Strain Measurement Method

- **Bridge Excitation Voltage**
  - Joule heating, i^2R, can produce temperature change
  - Temperature change is also influenced by heat dissipation
    - Heat dissipation depend upon the thermal conductivity and thickness of the substrate
    - **What should be the power density?**

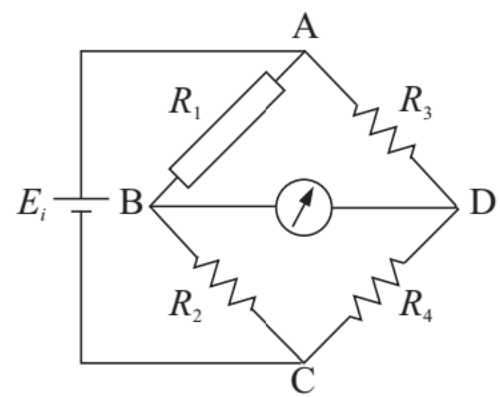
Table 7.4 Power density requirement with respect to accuracy of measurement and substrate

Required accuracy	*Substrate*		*Power density* (ρ_E)
	Thermal conductivity	*Thickness*	(mW/cm^2)
High	Good	Thick	300–750
		Thin	150–300
	Poor	Thick	75–150
		Thin	7.5–30
Average	Good	Thick	750–1500
		Thin	150–750
	Poor	Thick	150–300
		Thin	15–75

Strain Measurement Method

- Bridge Excitation Voltage
  - In a Wheatstone with excitation voltage E_i, power generated in the strain gauge of cross sectional area A_g,

$$P_g = \frac{E_i^2}{4R_g}$$



the required power density is given by

$$\rho_E = \frac{P_g}{A_g}$$

$$E_i|_{\max} = 2\sqrt{\rho_E R_g A_g}$$

Strain Measurement Method

- ❑ **Fibre-optic Strain Gauges**

* ***Self-study***

* ***We briefly discussed in "sensors and transducers"***

Queries

Thanks!