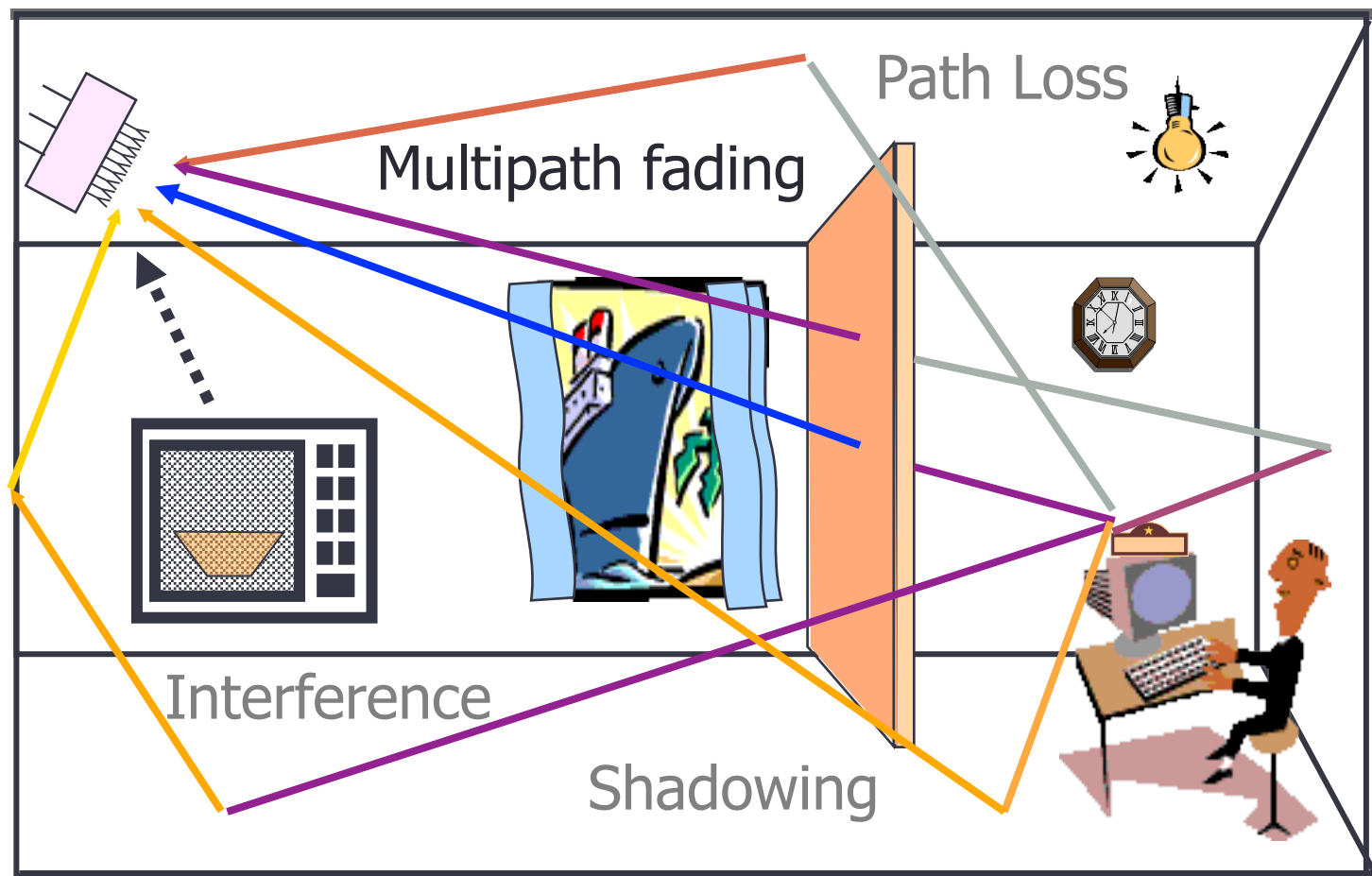




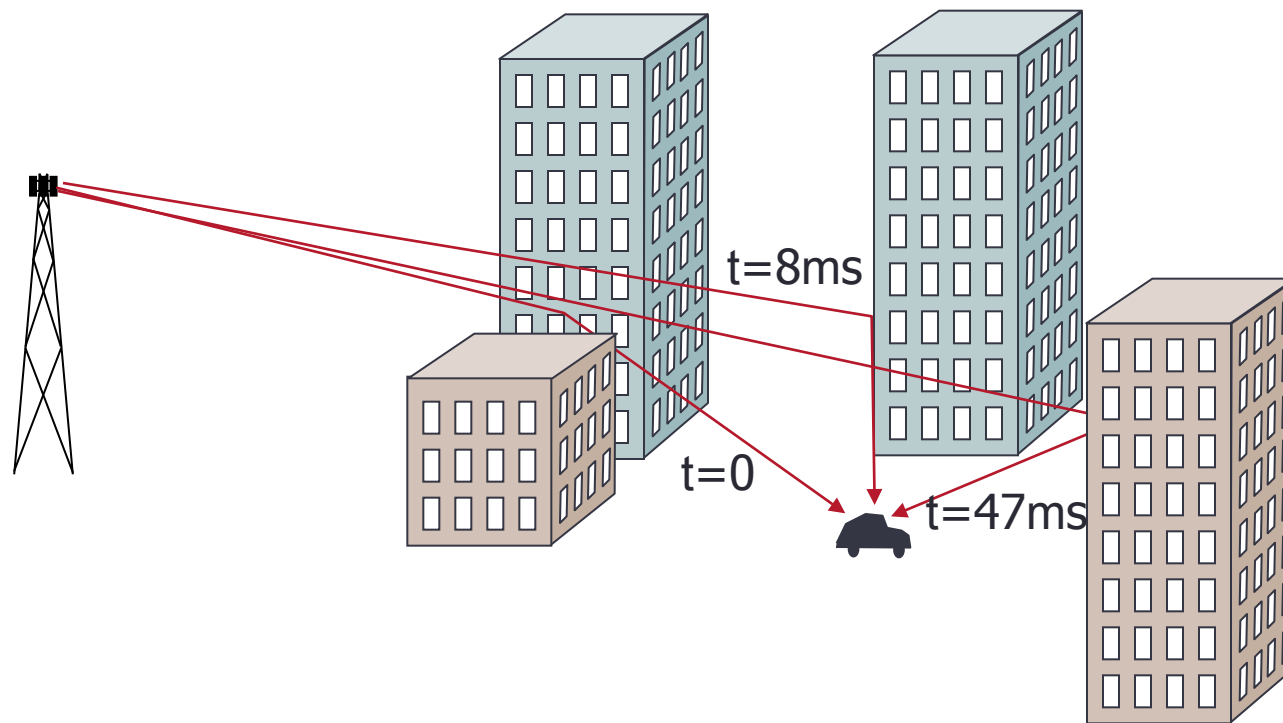
MULTIPATH FADING

Various Features in a Wireless Channel



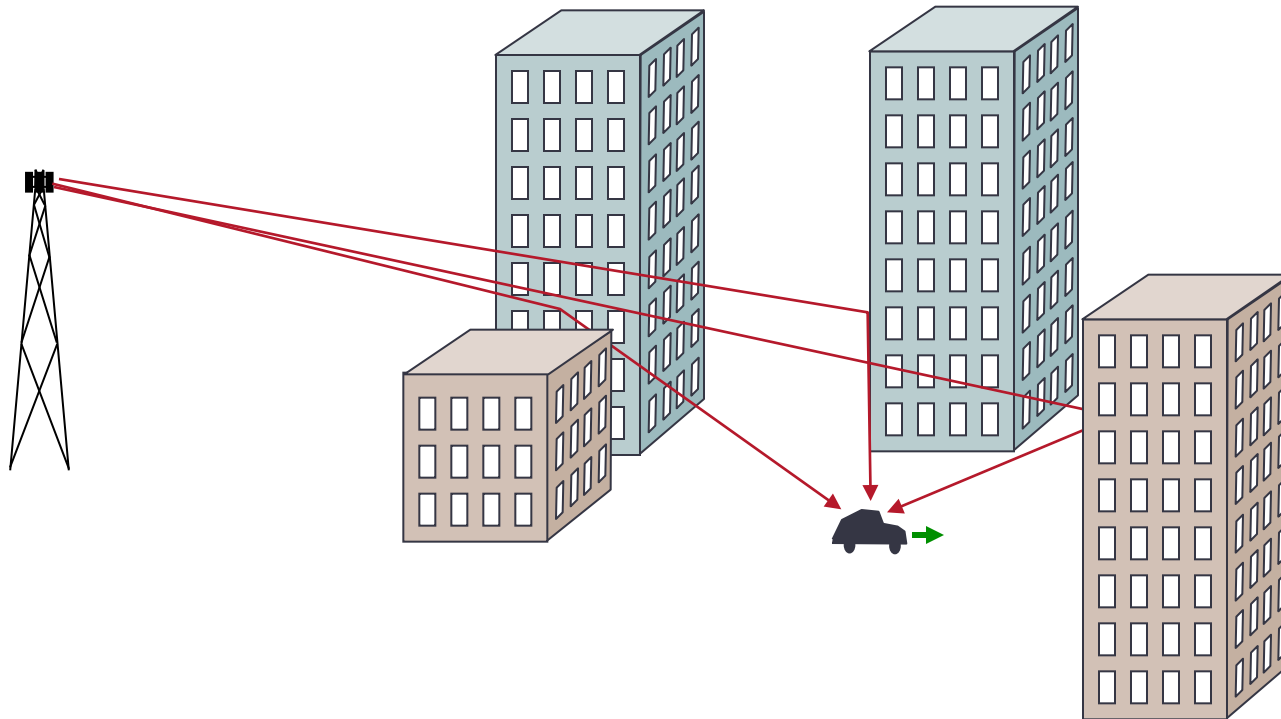
Excess Delay

- The propagation delay relative to that of the shortest path



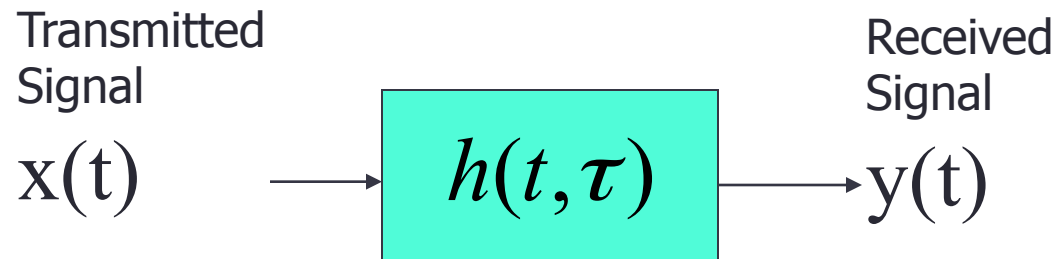
Strength Variation

- As the vehicle moves, the strength of each path varies because the surfaces are complex



The Channel is a Filter

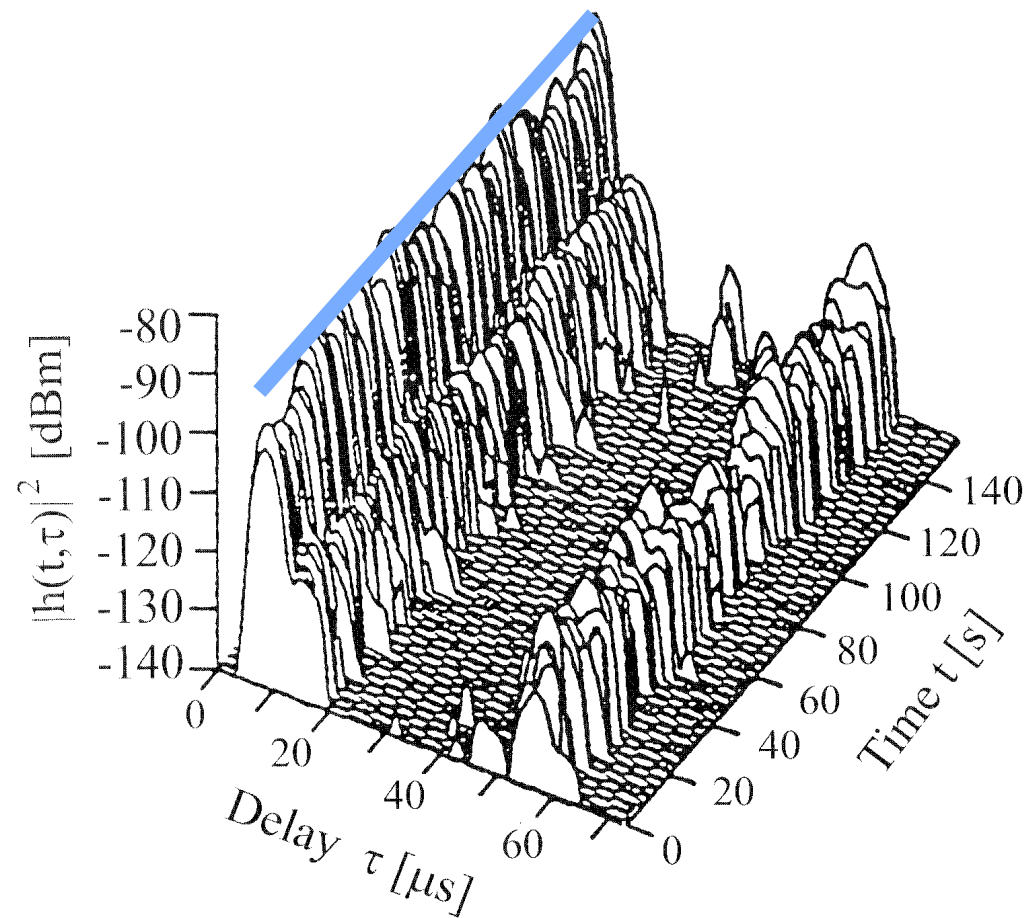
- The multipath channel can be represented as linear, time-varying bandpass filter



$$y(t) = \int_{-\infty}^{+\infty} x(t - \tau) h(t, \tau) d\tau$$

Measured Data from Darmstadt, Germany

[Molisch, '01]



Baseband Impulse Response

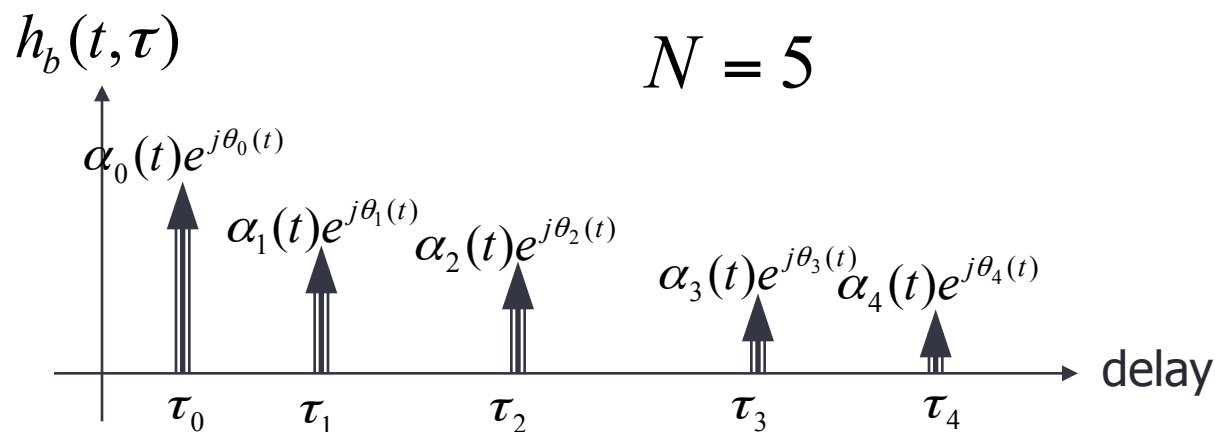
- More convenient to work with baseband signals

$$\begin{aligned}h(t, \tau) &= \text{Re} \left\{ h_b(t, \tau) e^{j\omega_c t} \right\} \\x(t) &= \text{Re} \left\{ c(t) e^{j\omega_c t} \right\} \\y(t) &= \text{Re} \left\{ r(t) e^{j\omega_c t} \right\} \\r(t) &= \frac{1}{2} \int_{-\infty}^{+\infty} c(t - \tau) h_b(t, \tau) d\tau\end{aligned}$$

The factor of $\frac{1}{2}$ ensures that baseband average power equals passband average power

Path Model

- The channel is assumed to comprise N discrete paths of propagation (rays)
- Each path has an amplitude $\alpha(t)$, a phase $\theta(t)$ and a propagation delay τ



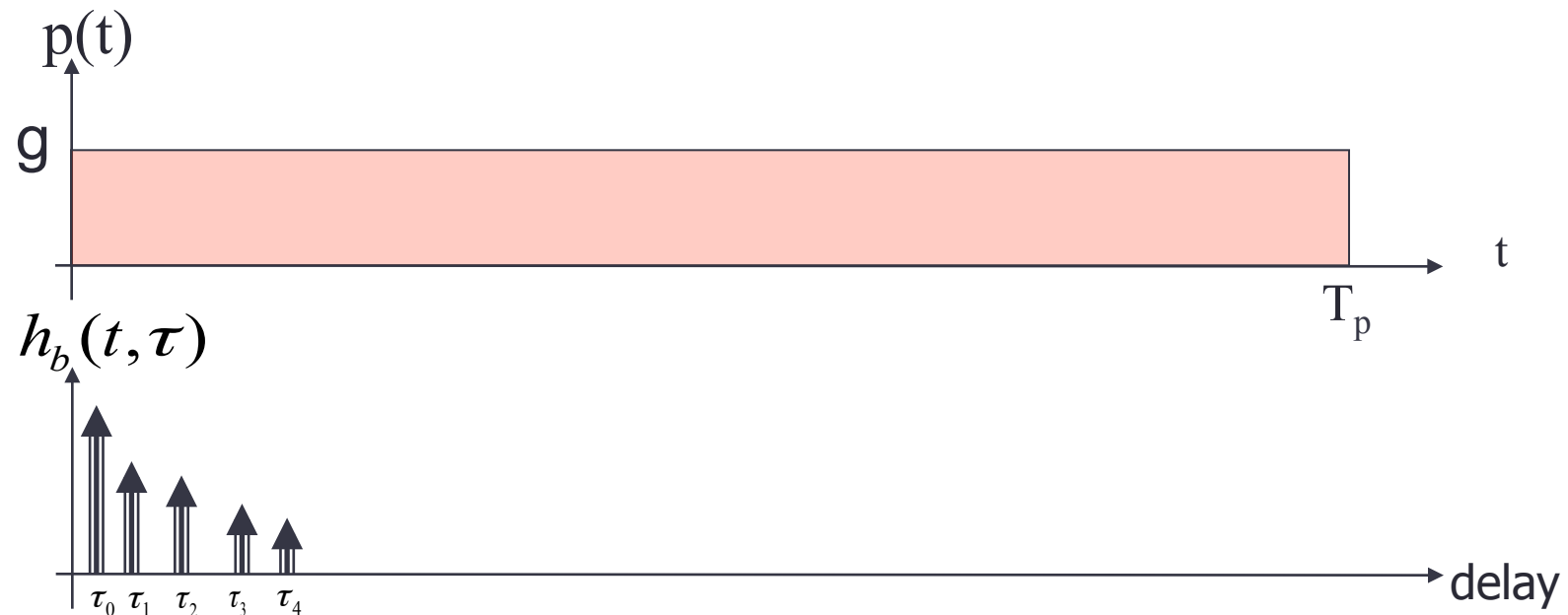
Probing the Channel

- The channel may be probed or “sounded” by transmitting a pulse $p(t)$ and recording the response at the receiver
- The response is the convolution of $p(t)$ with the channel impulse response

$$r(t) = \frac{1}{2} \sum_{i=0}^{N-1} \alpha_i(t) e^{j\theta_i(t)} p(t - \tau_i)$$

Pulse Width $\gg \tau_{\max}$

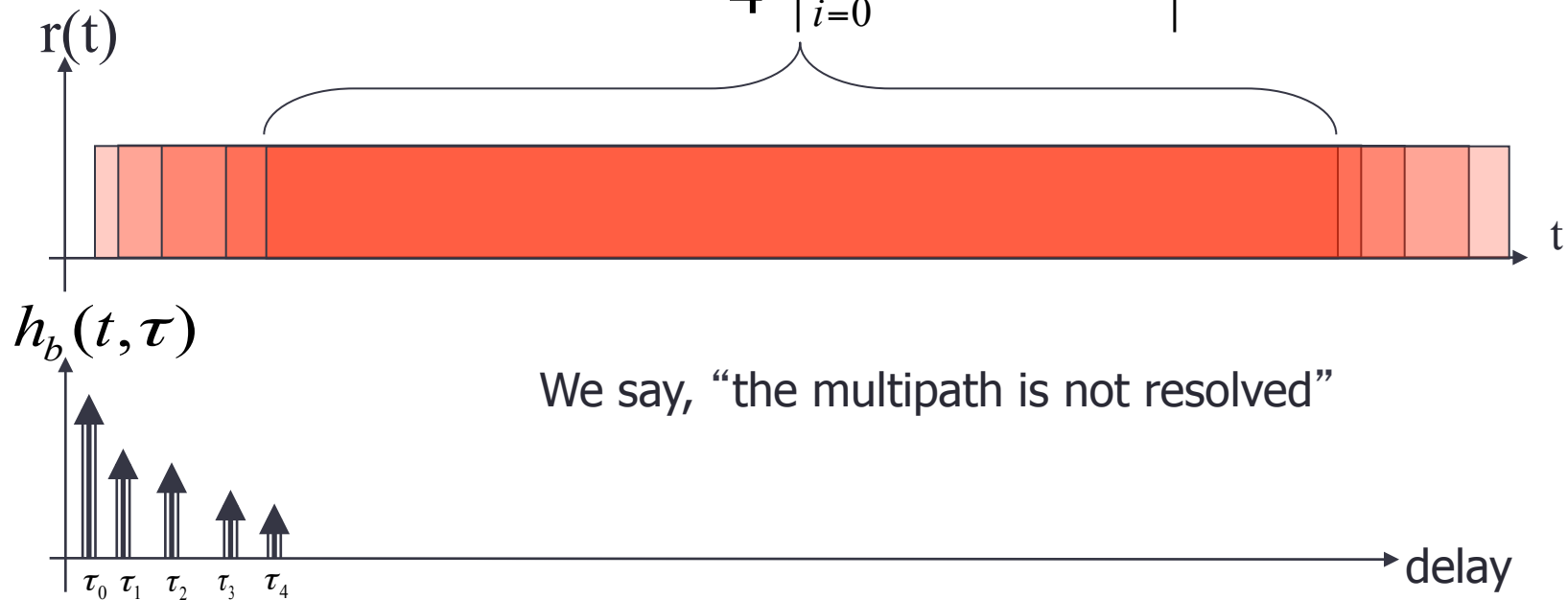
- Suppose a pulse much wider than the length of the impulse response is transmitted at time $t=0$



Instantaneous Power

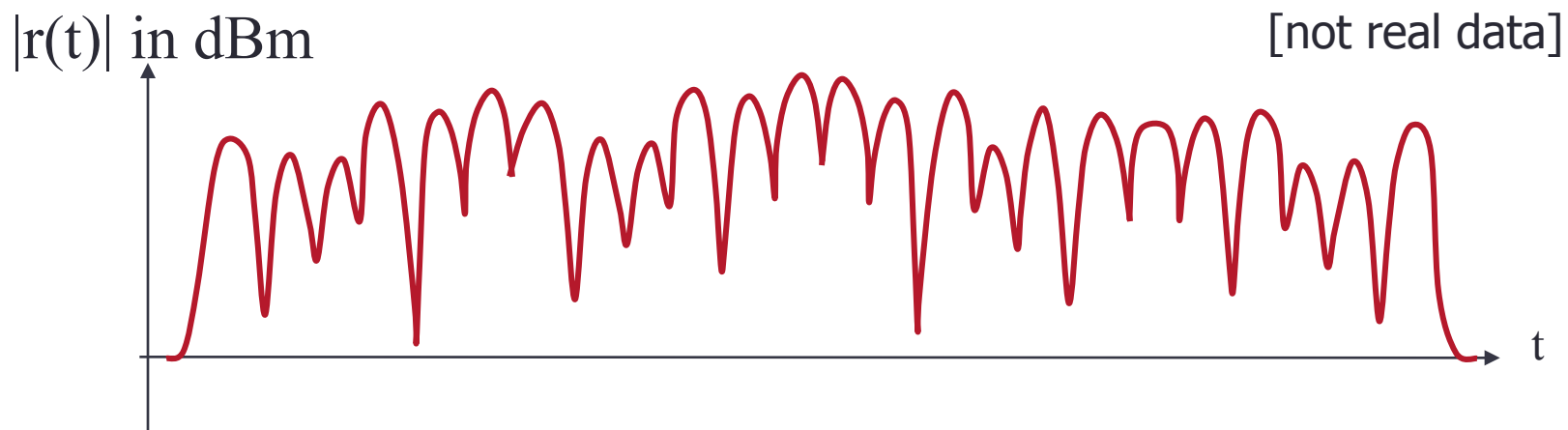
The **magnitude squared** of any sample in the interval t_4 and $t_0 + T_p$ will equal

$$|r(t)|^2 = \frac{\gamma^2}{4} \left| \sum_{i=0}^{N-1} \alpha_i(t) e^{j\theta_i(t)} \right|^2$$



Time Variation of the Probe Response

- If one or both of the terminals moves, the path phases change because the path lengths change
- The path amplitudes do not change much
- These changes yield large changes in the magnitude of the received waveform



Narrowband Fading

- This same type of fading happens to a digital waveform if the symbol period is much larger than (>10 times) the channel “length”
- Such long symbol periods correspond to “narrowband” signals

Average Power for Narrowband Signals

- Assuming the channel is ergodic, the ensemble average may be approximated by a time average:

$$\Omega = E_{\alpha\theta} \left\{ |r(t)|^2 \right\} \approx \frac{1}{T} \int_{t-\frac{T}{2}}^{t+\frac{T}{2}} \frac{\gamma^2}{4} \left| \sum_{i=0}^N \alpha_i(s) e^{j\theta_i(s)} \right|^2 ds$$

where the interval $[t-T/2, t+T/2]$ corresponds to a local area

Uncorrelated Scattering

- Assume that the phases of different paths are uncorrelated and that the energy of the pulse is one
- Then the time average simplifies to

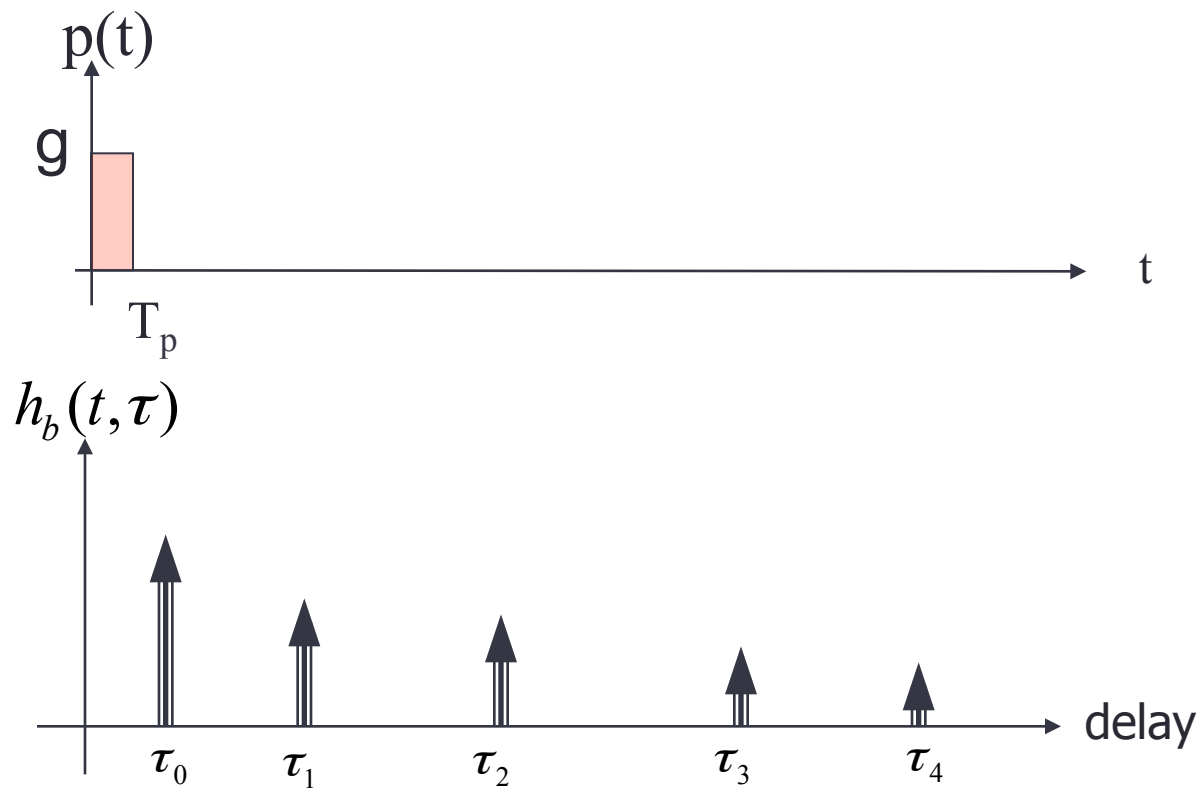
$$\Omega = E_{\alpha\theta} \left\{ |r(t)|^2 \right\} \approx \sum_{i=0}^N \overline{\alpha_i^2}$$

where

$$\overline{\alpha_i^2} \approx \frac{1}{T} \int_{t-\frac{T}{2}}^{t+\frac{T}{2}} \alpha_i^2(s) ds$$

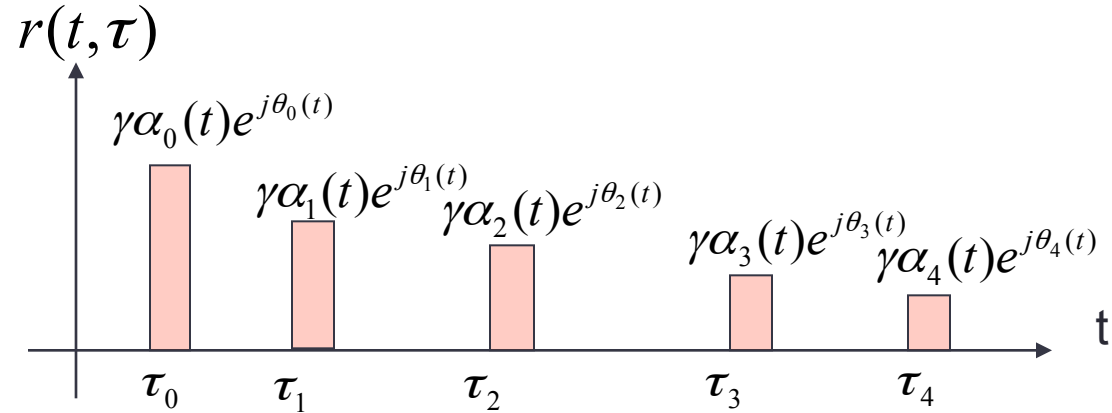
Pulse Width $\ll \tau_{\max}$

- Now consider a small pulse width



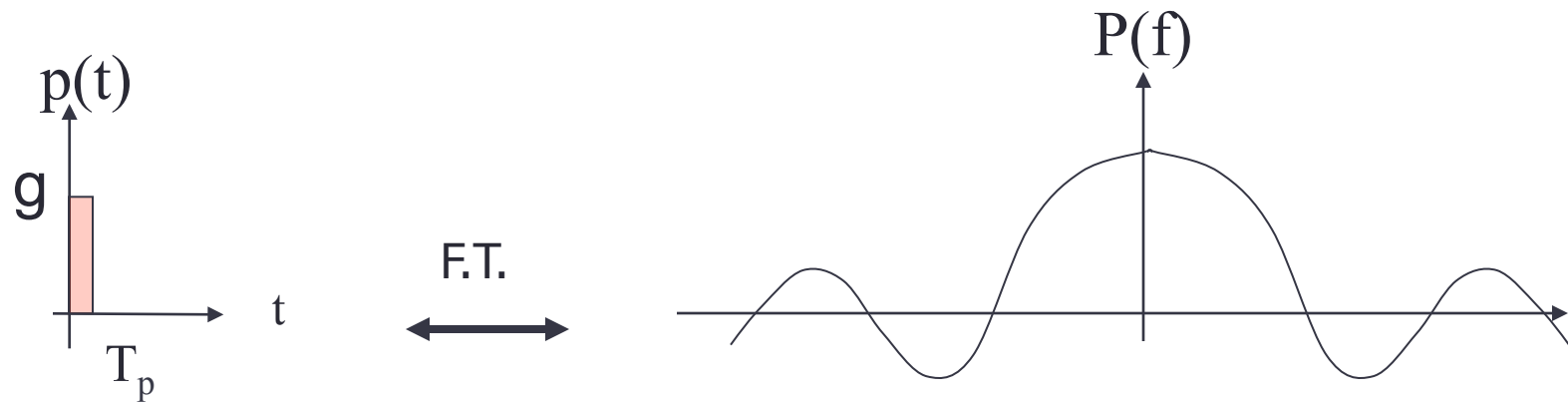
Multipath Resolved

- Pulses do not overlap



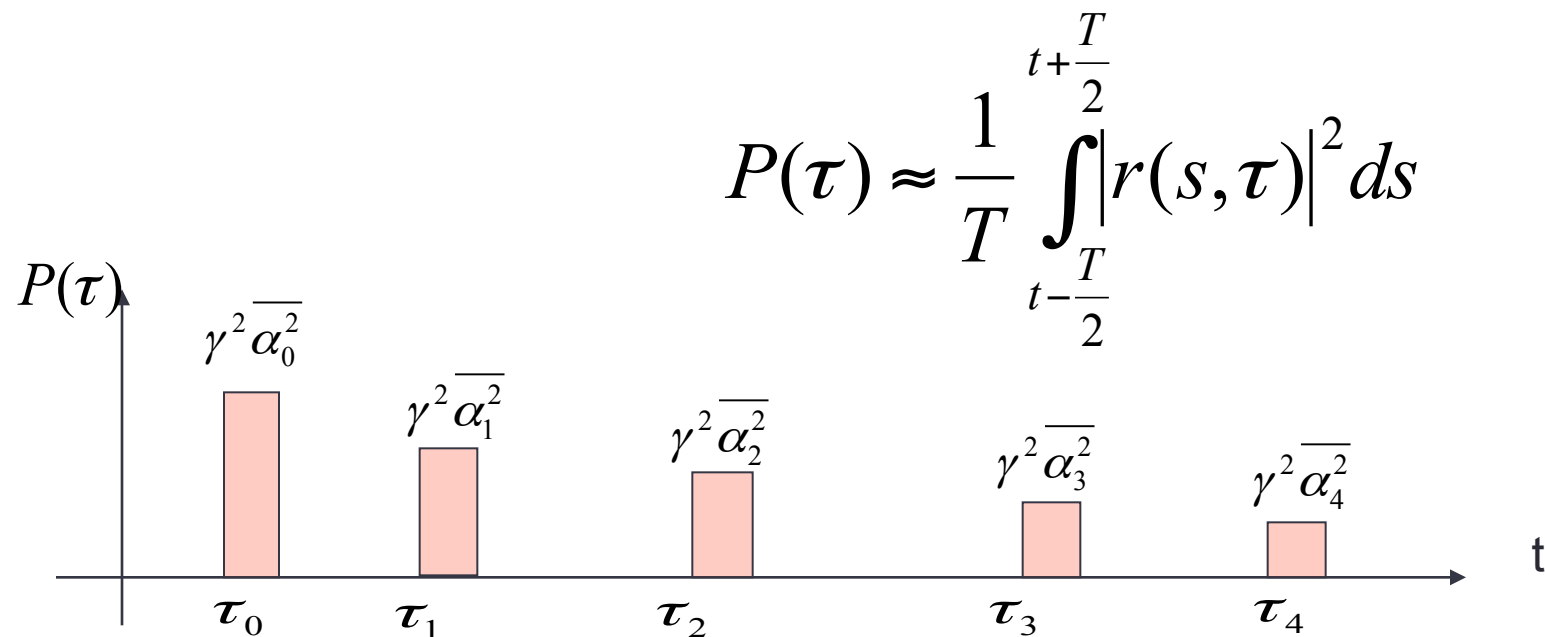
Wideband Signal

- The Fourier Transform of such a narrow pulse has a wide spectrum



Power Delay Profile (PDP)

- The PDP is a time-average of $|r(t, \tau)|^2$ over a small interval (assuming the terminal is moving)



W. Mohr, "Modeling of wideband mobile radio channels based on propagation measurements,"
in Proc. 16th Int. Symp. Personal, Indoor, Mobile Radio Communications, vol. 2, pp. 397-401, 1995

Average Power for Wideband Signals

- The average power is the integral of the PDP

$$\begin{aligned} P_{AVG} &= \int_0^{+\infty} P(\tau) d\tau \\ &= \sum_{i=0}^{N-1} \alpha_i^2 = \Omega \end{aligned}$$

Local Average Powers Are The Same

- Narrowband and Wideband averaged powers are equal

Moments of the PDP

- Channels are often described by their rms delay spread
- To compute rms delay spread, normalize the PDP to make it like a PDF for a random variable (unit area) and then find its standard deviation
- Must you use excess delay to compute rms delay spread?

Mean Delay

- Must first compute the mean delay

$$\bar{\tau} = \frac{\int_0^{+\infty} \tau P(\tau) d\tau}{\int_0^{+\infty} P(\tau) d\tau}$$

For this to be mean excess delay, the origin of the τ axis needs to be the time of the first arriving path

Second Moment

- Next need the second moment of this “PDF”

$$\overline{\tau^2} = \frac{\int_0^{+\infty} \tau^2 P(\tau) d\tau}{\int_0^{+\infty} P(\tau) d\tau}$$

RMS Delay Spread

- Recall that standard deviation is the square root of variance and variance is the second moment minus the first moment squared

Variance

$$\sigma_{\tau}^2 = \overline{\tau^2} - \left(\overline{\tau}\right)^2$$

rms delay spread

$$\sigma_{\tau} = \sqrt{\overline{\tau^2} - \left(\overline{\tau}\right)^2}$$

Example Data

[Rappaport, '02]

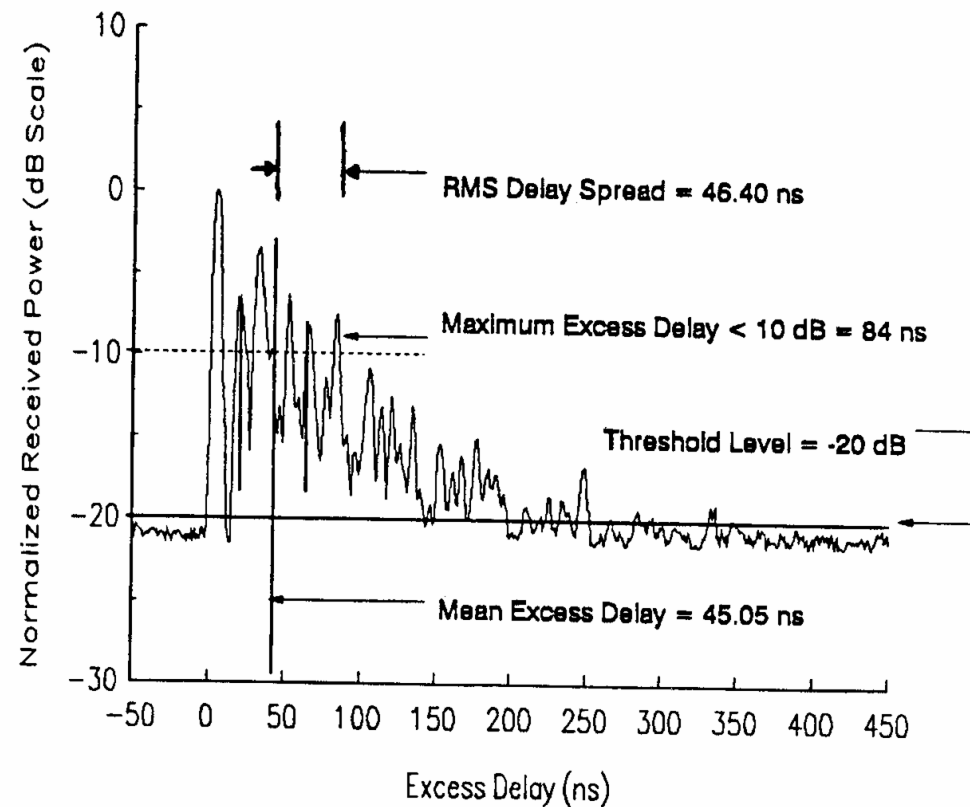


Figure 5.10 Example of an indoor power delay profile; rms delay spread, mean excess delay, maximum excess delay (10 dB), and threshold level are shown.

How RMS Delay Spread Can Be Used

- If $\sigma_\tau \ll$ symbol period, assume “narrowband” fading effects
- If $\sigma_\tau \gg$ symbol period, assume “wideband” fading effects (will need an equalizer, CDMA or OFDM)

The Frequency Domain View

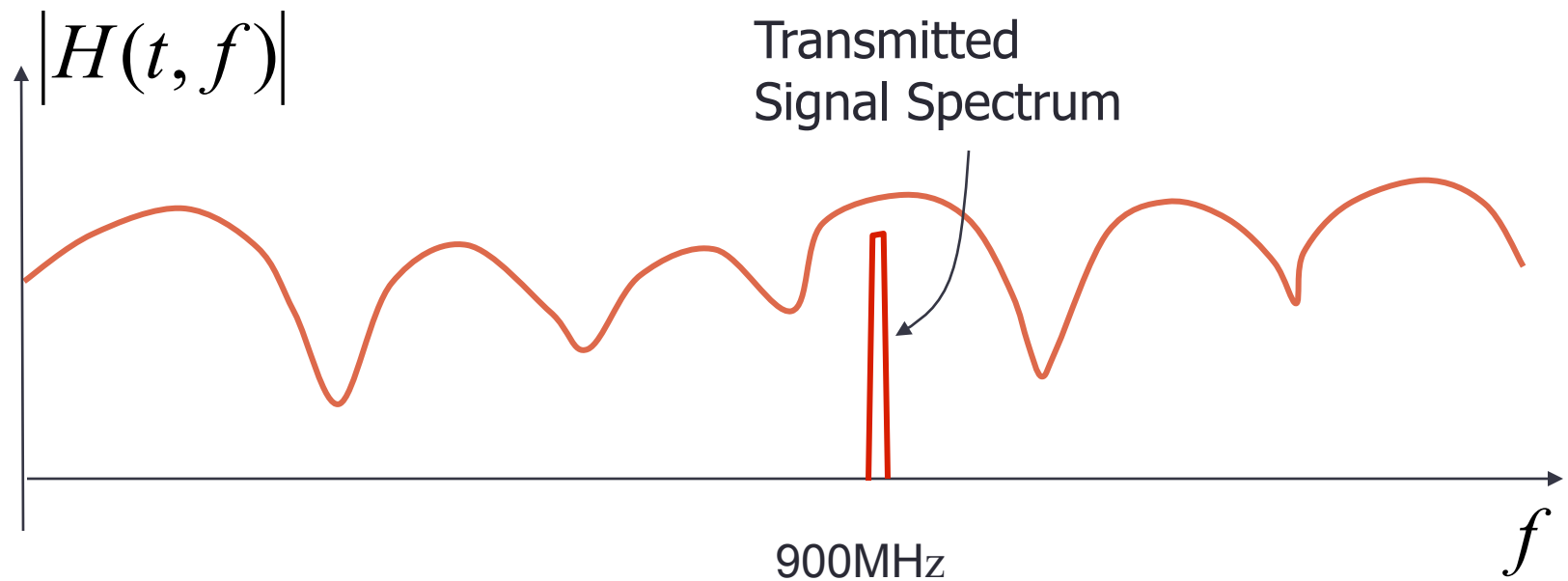
- $\sigma_\tau \ll \text{symbol period}$ implies that the frequency response of the channel,

$$H(t, f) = \int_{-\infty}^{+\infty} h(t, \tau) \exp(-j2\pi f\tau) d\tau,$$

doesn't vary much with frequency over the bandwidth of the transmitted signal

Narrowband Case

- The channel “appears flat to the signal”

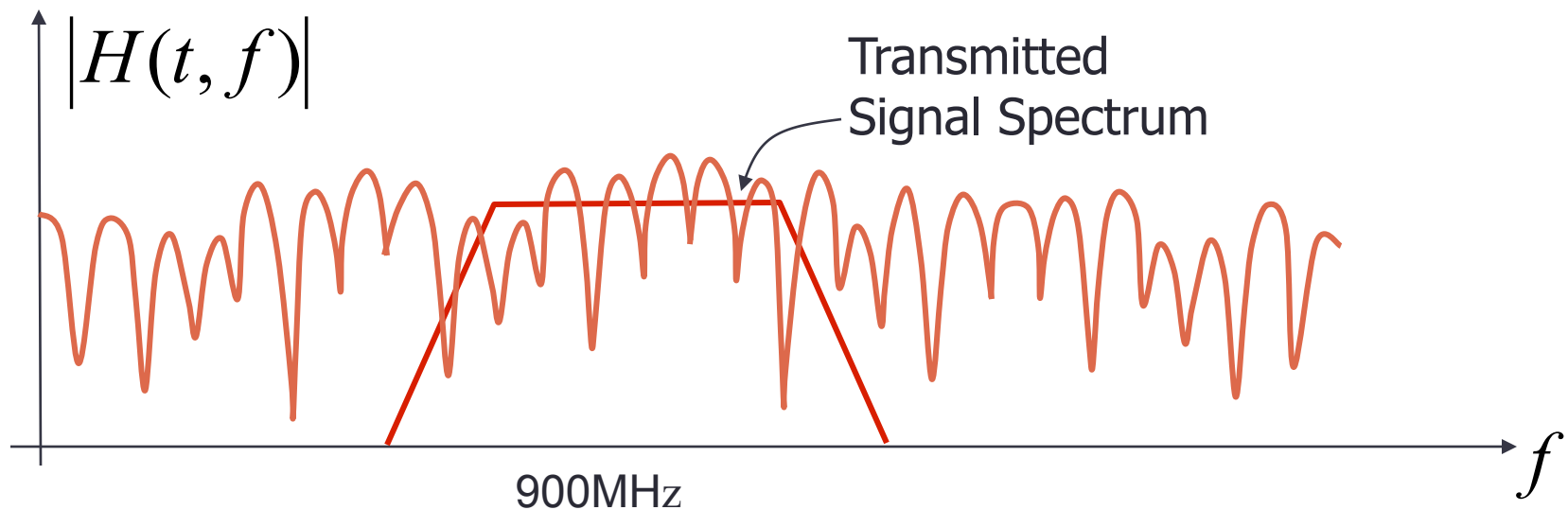


Flat Fading

- When $\sigma_\tau \ll \text{symbol period}$, we say the signal undergoes ***flat fading***
- The channel frequency response is approximately flat over the signal bandwidth

Wideband Case

- $\sigma_\tau \gg$ symbol period implies that the frequency response of the channel varies significantly with frequency over the bandwidth of the transmitted signal



Frequency Selective Fading

- When $\sigma_\tau \gg$ symbol period, we say the signal undergoes ***frequency selective fading***
- The channel frequency response is strong for some frequencies and not for others within the signal bandwidth

Complete Narrowband Statistical Fading Model

- $\alpha = |r(t)| = xy$ is the signal envelope. Recall $E\{\alpha^2\} = \Omega = \sum_{i=0}^{N-1} \overline{\alpha_i^2}$
- \mathcal{X} represents the small-scale or multipath fading, and has either the Rayleigh or Rician distribution with unit mean square value, i.e. Therefore,

$$E\{x^2\} = 1 \quad E\{\alpha^2\} = E\{y^2\}$$
- y represents the large-scale or shadow fading, and has a lognormal distribution. Recall $E\{y^2\} = \Omega$ is local average power
- Let $z = 10 \log_{10}(y^2)$. Then z is $N(\hat{p}(d), \sigma^2)$ (the z 's are the points on the scatter plot)
- $\hat{p}(d)$ is the path loss, with the model

$$\hat{p}(d) = p(d_o) - n 10 \log_{10} \left(\frac{d}{d_o} \right)$$

Complete Wideband Statistical Fading Model

- Recall $\Omega = \sum_{i=0}^{N-1} \overline{\alpha_i^2}$. Each path of the wideband model has independent small-scale fading: $\alpha_i = x_i y$

- x_i is the small-scale or multipath fading on each path, and has either the Rayleigh or Rician distribution with mean square value

$$E\{x_i^2\} = E\{\alpha_i^2\} / \Omega$$

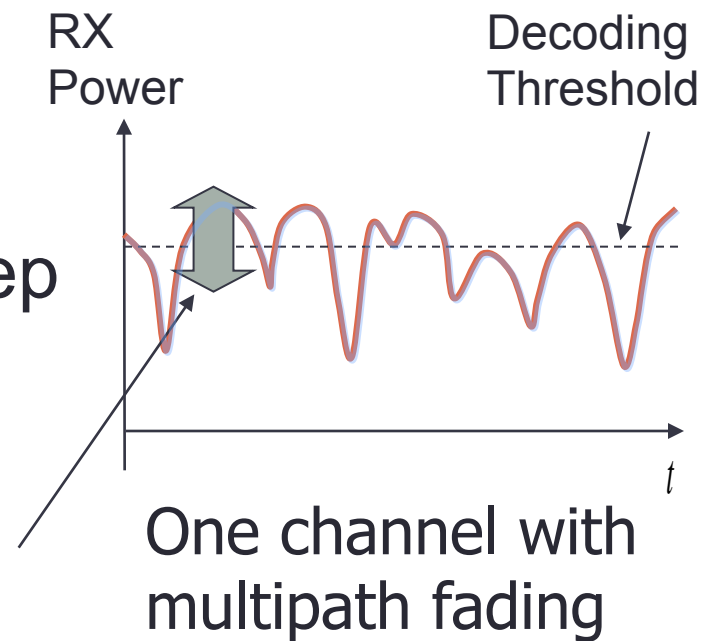
- y represents the large-scale or shadow fading, as before, and has a lognormal distribution.
- Let $z = 10 \log_{10}(y^2)$. Then z is $N(\hat{p}(d), \sigma^2)$ (the z 's are the points on the scatter plot)

- $\hat{p}(d)$ is the path loss, with the model
$$\hat{p}(d) = p(d_o) - n 10 \log_{10} \left(\frac{d}{d_o} \right)$$

Fade Margin

- Extra power (i.e. a fade margin) is required, to keep above the decoding threshold

Fade Margin



Summary

- The multipath channel model has a discrete number of propagation paths
- Each path has amplitude, phase and delay
- The PDP is the local average of the magnitude squared of the impulse response of the channel
- Average power of the channel is the integral of the PDP
- Average power is same for narrowband and wideband channels
- The fading is “flat” or “frequency selective” depending on the comparison between rms delay spread and the symbol period

References

- [Rapp, '02] T.S. Rappaport, *Wireless Communications*, Prentice Hall, 2002
- [Molisch, '01] Andreas F. Molisch (ed), *Wideband Wireless Digital Communications*, Prentice Hall PTR, 2001.
- W. Mohr, “Modeling of wideband mobile radio channels based on propagation measurements,” in *Proc. 16th Int. Symp. Personal, Indoor, Mobile Radio Communications*, vol. 2, pp. 397-401, 1995