

# Communication Systems

## EE-351

### Lecture 4

# Fourier Transform (FT):

Consider an aperiodic continuous time signal  $x(t)$ , its FT is given as:

$$X(F) = \int_{-\infty}^{\infty} x(t) e^{-j2\pi Ft} dt$$
$$x(t) \xleftrightarrow{\text{Fourier Transform pair}} X(F)$$

$x(t)$  might be real but,  $e^{-j2\pi Ft}$  is a complex quantity in  $X(F) = \int_{-\infty}^{\infty} x(t) e^{-j2\pi Ft} dt$ , therefore,  $X(F)$  in general is complex.

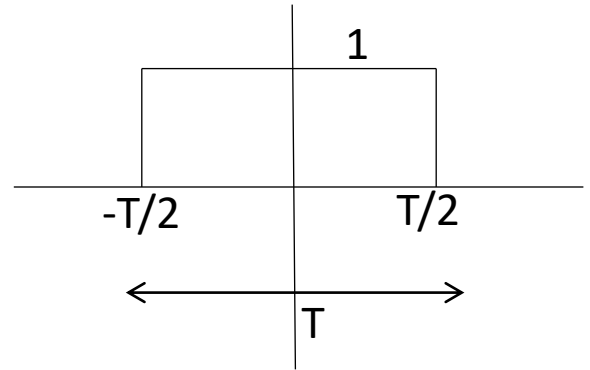
Two components of  $X(F)$  are:

- Magnitude spectrum
- Phase spectrum

Given  $X(F)$ , the corresponding time domain signal  $x(t)$  is given by the inverse Fourier Transform as:

$$x(t) = \int_{-\infty}^{\infty} X(F) e^{j2\pi Ft} dF$$

# Fourier Transform (example)



Consider the pulse  $x(t)$  defined as:

$$x(t) = P_T(t) = \begin{cases} 1, & |t| \leq T/2 \\ 0, & \text{otherwise} \end{cases}$$

$$X(F) = \int_{-\infty}^{\infty} x(t) e^{-j2\pi Ft} dt = \int_{-\infty}^{\infty} P_T(t) e^{-j2\pi Ft} dt$$

$$P_T(F) = \int_{-T/2}^{T/2} 1 \cdot e^{-j2\pi Ft} dt = \frac{e^{-j2\pi FT/2} - e^{-j2\pi F(-T/2)}}{-j2\pi F} = \frac{-2j \sin(2\pi FT/2)}{-2j\pi F}$$

$$P_T(F) = \frac{\sin(\pi FT)}{\pi F}$$

# Fourier Transform (example)

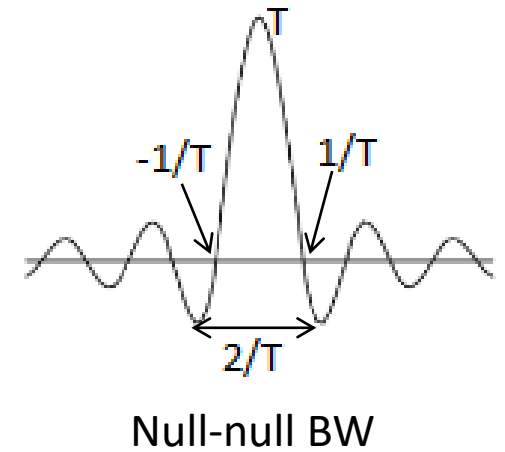
$$P_T(F) = \frac{T \sin(\pi FT)}{\pi FT} = T \text{sinc}(FT)$$

At  $F = 0, T \text{sinc}(0) = T$

At  $F = \frac{k}{T}, T \text{sinc}\left(\frac{kT}{T}\right) = T \text{sinc}(k) = 0$

$$P_T(t) \xleftrightarrow{\text{Fourier Transform pair}} P_T(F) = T \text{sinc}(FT)$$

$$P_T(t) = \int_{-\infty}^{\infty} P_T(F) e^{j2\pi Ft} dF = \int_{-\infty}^{\infty} T \text{sinc}(FT) e^{j2\pi Ft} dF$$



# Fourier Transform (modulation property)

Modulation simply refers to:

$$\text{Modulated signal} \rightarrow x_m(t) = x(t)e^{j2\pi F_c t} \leftarrow \text{Carrier freq.}$$

$x(t)$  times a complex sinusoid with a very high freq.

If  $x(t) \xleftrightarrow{\text{Fourier Transform pair}} X(F)$

$x_m(t) \longleftrightarrow ??$

$$X_m(F) = \int_{-\infty}^{\infty} x_m(t) e^{-j2\pi Ft} dt = \int_{-\infty}^{\infty} x(t) e^{j2\pi F_c t} e^{-j2\pi Ft} dt = \int_{-\infty}^{\infty} x(t) e^{-j2\pi(F-F_c)t} dt$$

# Fourier Transform (modulation property)

$$X_m(F) = X(F - F_c)$$

Fourier Transform  
of modulated  
signal

Fourier Transform  
 $X(F)$  shifted by  $F_c$

Modulation in time  $\equiv$  shift in frequency by  $F_c$

# Linear time Invariant (LTI) system

Another principle of communication is the transmission of a signal through a linear system

Consider a signal  $x(t)$  given as an input to a LTI system

If system is linear

$$\underbrace{ax_1(t) + bx_2(t)}_{\substack{\text{Linear combination} \\ \text{of inputs}}} \rightarrow \underbrace{ay_1(t) + by_2(t)}_{\substack{\text{Linear combination} \\ \text{of corresponding} \\ \text{outputs}}}$$

# Linear time Invariant (LTI) system

If  $x(t) \rightarrow y(t)$

$$\underbrace{x(t - t_o)}_{\text{Time shifted input}} \rightarrow \underbrace{y(t - t_o)}_{\text{Time shifted output}}$$

Therefore, linearity + time invariance = linear time invariant (LTI) system

An LTI system is characterized by impulse response  $h(t)$

$$y(t) = x(t) * h(t)$$

Input signal

Convolution  
operator

This convolution operation is represented as:

$$\int_{-\infty}^{\infty} x(\tau)h(t - \tau)d\tau = \int_{-\infty}^{\infty} x(t - \tau)h(\tau)d\tau$$

$$Y(F) = X(F) \cdot H(F) \quad (\text{multiplication in freq. domain})$$



# Amplitude Modulation (AM)

Any communication system involves a carrier signal as:

$$c(t) = A_c \cos(2\pi F_c t)$$

$A_c$  is Amplitude of the carrier and  $F_c$  is carrier frequency

Hence, AM signal is defined as:

$$x(t) = (1 + k_a m(t)) \times A_c \cos(2\pi F_c t)$$

$$x(t) = A_c (1 + k_a m(t)) \cos(2\pi F_c t)$$

$x(t)$  is amplitude modulated signal,  $k_a$  sensitivity of this AM signal,  $m(t)$  is message signal or baseband signal

⇒ Amplitude of the carrier varies according to the message or Amplitude of the carrier is modulated according to the message