Engineering Mechanics

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- Objectives
- Equilibrium
- Free Body Diagram
- Coplanar Force System

CHAPTER
Objectives

Objectives

- Free-body diagram concept for a particle
- Particle equilibrium problems using the equations of equilibrium

3.1 Condition for the Equilibrium of a Particle

- Particle at *equilibrium* if
  - At rest
  - Moving at constant velocity

- Definition: ''A particle is said to be in ***equilibrium*** if it remains at rest if originally at rest, or has a constant velocity if originally in motion.''

To maintain equilibrium it is essential to satisfy

- Newton's first law of motion

$$\sum \mathbf{F} = 0$$

where $\sum \mathbf{F}$ is the vector sum of all the forces acting on the particle.

- Newton’s second law of motion can be written as

$$\sum \mathbf{F} = m\mathbf{a}$$

- When the force fulfill Newton's first law of motion,

$$m\mathbf{a} = 0$$

$$\mathbf{a} = 0$$

therefore, the particle is moving in constant velocity or at rest

3.2 The Free-Body Diagram

- A drawing that shows the particle with all the forces that act on it is called a free-body diagram (FBD)
- Best representation of all the unknown forces ($\sum \mathbf{F}$) which acts on a body
- A sketch showing the particle "free" from the surroundings with all the forces acting on it
- Consider two common connections in this subject –
  - Spring
  - Cables and Pulleys

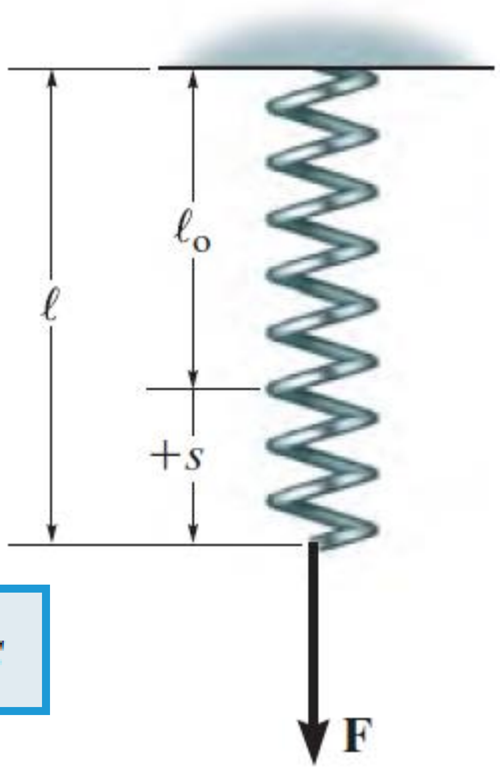
Springs.

Linear elastic spring: change in length is directly proportional to the force acting on it

spring constant or stiffness k: defines the elasticity of the spring

Magnitude of force when spring is elongated or compressed

$$\boxed{F = ks}$$

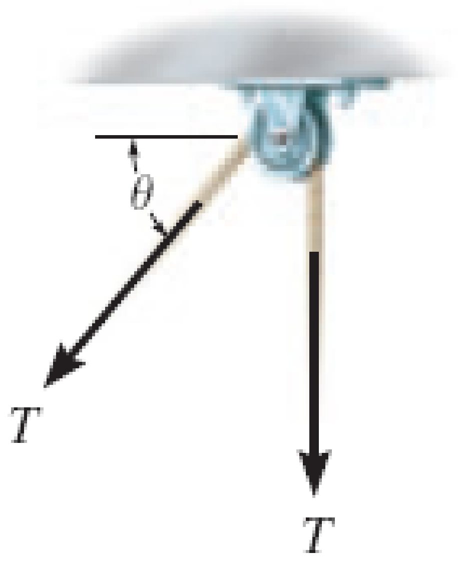


If s is positive, causing an elongation, then **F** must pull on the spring; whereas if s is negative, causing a shortening, then **F** must push on it.

example, if the spring in Fig. 3–1 has an unstretched length of 0.8 m and a stiffness $k = 500$ N/m and it is stretched to a length of 1 m, so that $s = l - l_o = 1\text{ m} - 0.8\text{ m} = 0.2\text{ m}$, then a force $F = ks = 500\text{ N/m}(0.2\text{ m}) = 100\text{ N}$ is needed.

Cables and Pulleys.

- Cables (or cords) are assumed of negligible weight and cannot stretch
- Also, a cable can support *only* a tension or “pulling” force, and this force always acts in the direction of the cable.
- Tension force must have a constant magnitude for equilibrium
- For any angle θ, the cable is subjected to a constant tension T



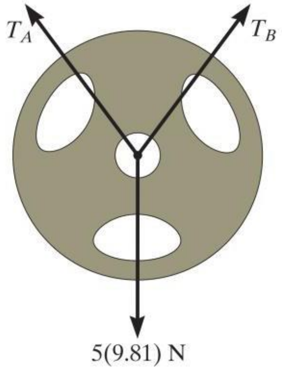
Cable is in tension

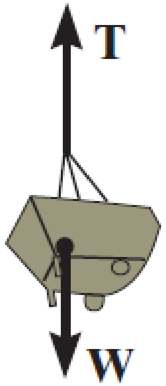
Procedure for Drawing a FBD

- Draw outlined shaped
- Show all forces (active + reactive)

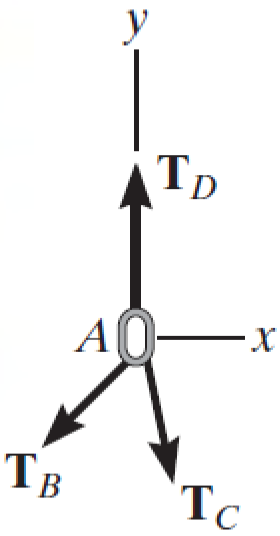
- Active forces: particle in motion
- Reactive forces: constraints that prevent motion

- Identify each force





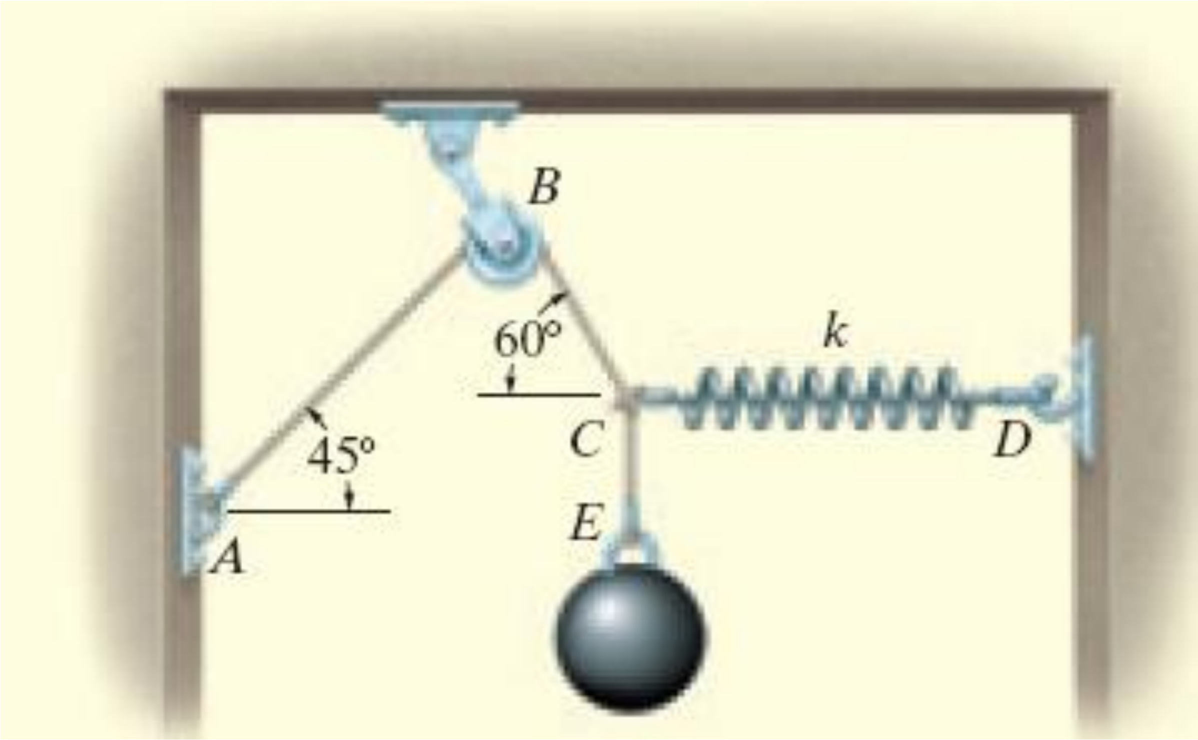
The bucket is held in equilibrium by the cable, and instinctively we know that the force in the cable must equal the weight of the bucket. By drawing a free-body diagram of the bucket we can understand why this is so. This diagram shows that there are only two forces *acting on the bucket*, namely, its weight **W** and the force **T** of the cable. For equilibrium, the resultant of these forces must be equal to zero, and so $T = W$.



The spool has a weight W and is suspended from the crane boom. If we wish to obtain the forces in cables AB and AC, then we should consider the free-body diagram of the ring at A. Here the cables AD exert a resultant force of **W** on the ring and the condition of equilibrium is used to obtain $\mathbf{T}_B$ and $\mathbf{T}_C$.

Example

The sphere has a mass of 6kg and is supported as shown. Draw a free-body diagram of the sphere, the cord CE and the knot at C.

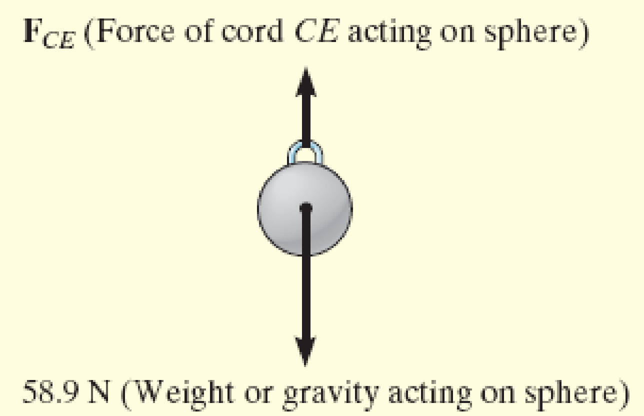


Example

FBD at Sphere

Two forces acting, weight and the force on cord CE.

Weight of 6kg (9.81m/s^2) = 58.9N



Cord CE

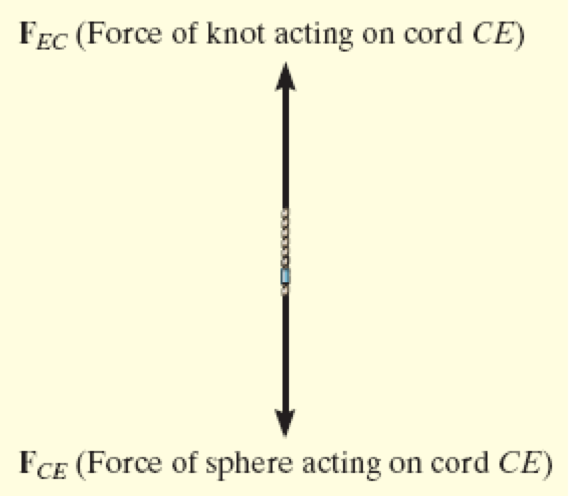
Two forces acting: sphere and knot

Newton's 3rd Law:

$\mathbf{F}_{CE}$ is equal but opposite

$\mathbf{F}_{CE}$ and $\mathbf{F}_{EC}$ pull the cord in tension

For equilibrium, $\mathbf{F}_{CE} = \mathbf{F}_{EC}$

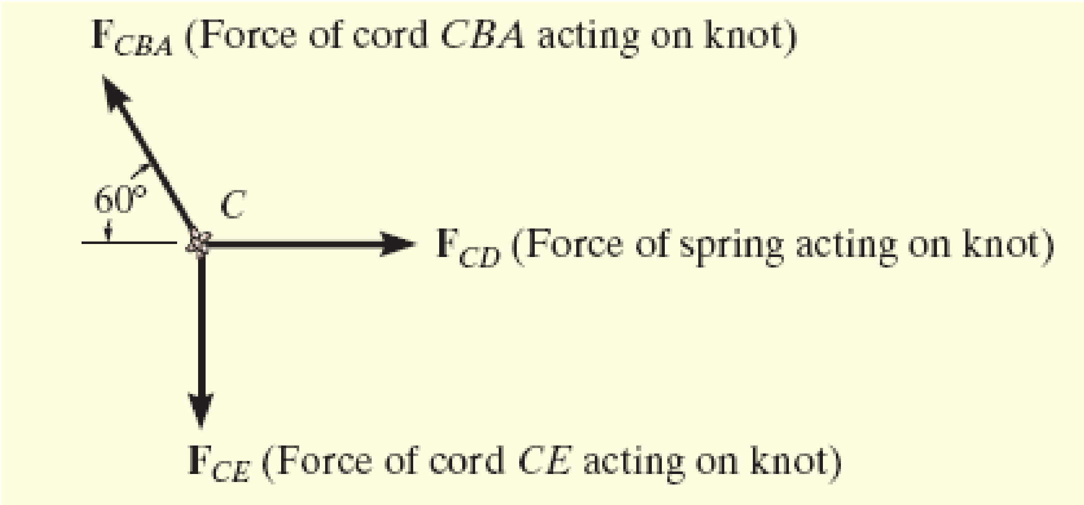


Example

FBD at Knot

3 forces acting: cord CBA, cord CE and spring CD

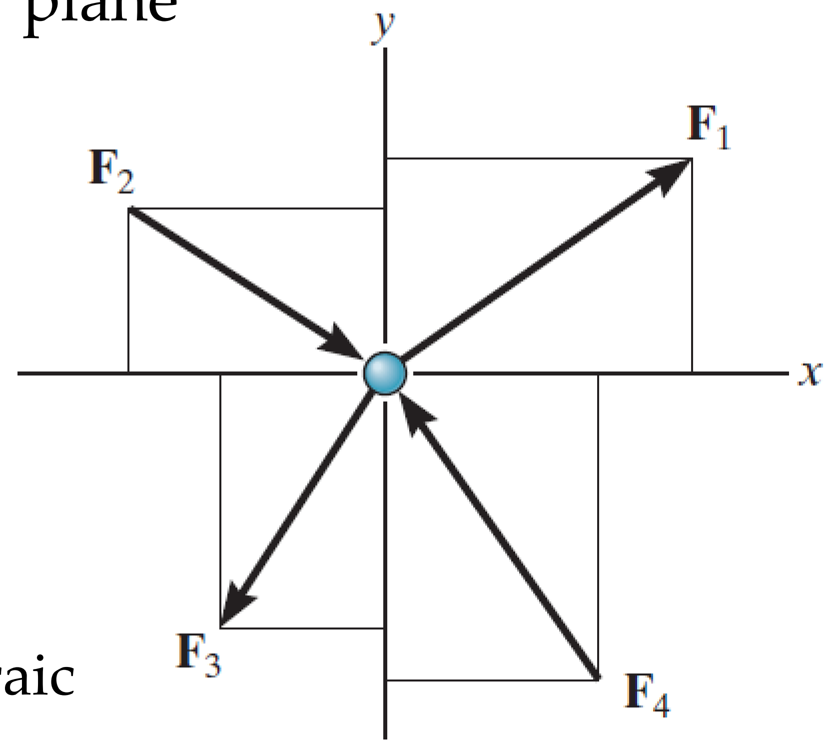
Important to know that the weight of the sphere does not act directly on the knot but subjected to by the cord CE



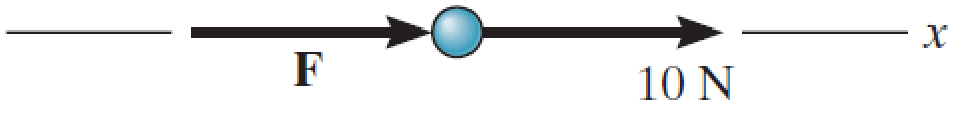
3.3 Coplanar Force Systems

- A particle is subjected to coplanar forces in the x-y plane
- Resolve into **i** and **j** components for equilibrium

$$\Sigma \mathbf{F} = \mathbf{0}$$
$$\Sigma F_x \mathbf{i} + \Sigma F_y \mathbf{j} = \mathbf{0}$$
$$\boxed{\begin{aligned} \Sigma F_x &= 0 \\ \Sigma F_y &= 0 \end{aligned}}$$



- Scalar equations of equilibrium require that the algebraic sum of the x and y components to equal to zero



$$\overset{+}{\rightarrow} \Sigma F_x = 0;$$
$$+F + 10 \text{ N} = 0$$
$$F = -10 \text{ N}$$

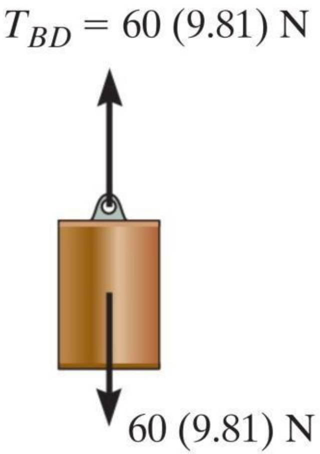
Here the *negative sign* indicates that **F** must act to the left to hold the particle in equilibrium.

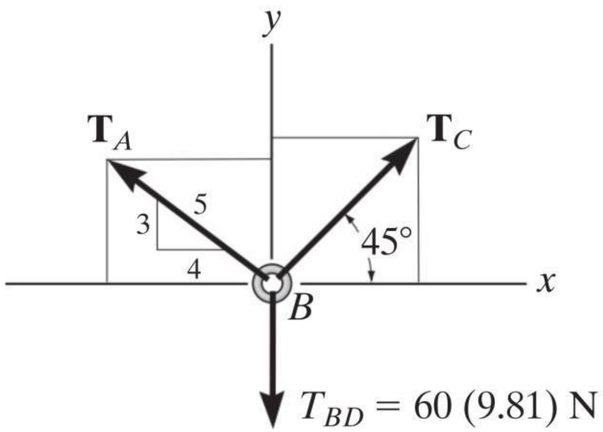
Example

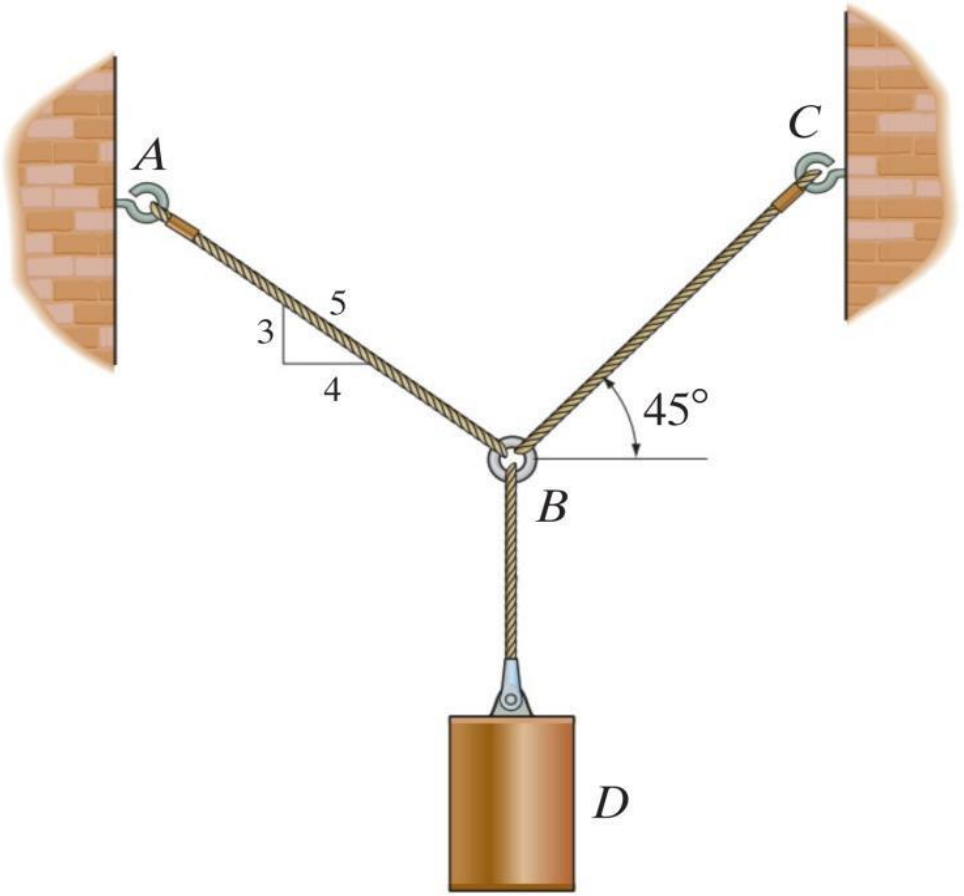
Determine the tension in cables BA and BC necessary to support (60kg) cylinder.

$$\xrightarrow{+} \Sigma F_x = 0;$$

$$+\uparrow \Sigma F_y = 0;$$







Equations of Equilibrium. Applying the equations of equilibrium along the x and y axes, we have

$$\xrightarrow{+} \Sigma F_x = 0; \qquad T_C \cos 45^\circ - \left(\tfrac{4}{5}\right)T_A = 0 \qquad (1)$$

$$+\uparrow \Sigma F_y = 0; \quad T_C \sin 45^\circ + \left(\tfrac{3}{5}\right)T_A - 60(9.81)\text{ N} = 0 \qquad (2)$$

Equation (1) can be written as $T_A = 0.8839T_C$. Substituting this into Eq. (2) yields

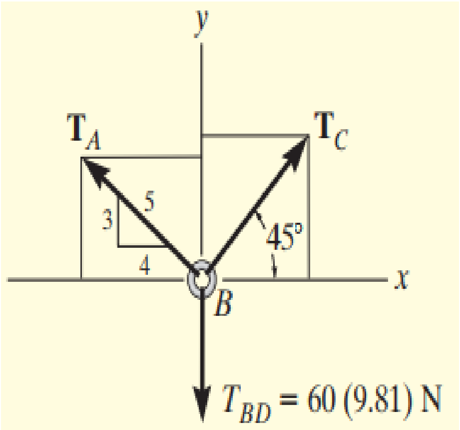
$$T_C \sin 45^\circ + \left(\tfrac{3}{5}\right)(0.8839T_C) - 60(9.81)\text{ N} = 0$$

So that

$$T_C = 475.66\text{ N} = 476\text{ N} \qquad \textit{Ans.}$$

Substituting this result into either Eq. (1) or Eq. (2), we get

$$T_A = 420\text{ N} \qquad \textit{Ans.}$$

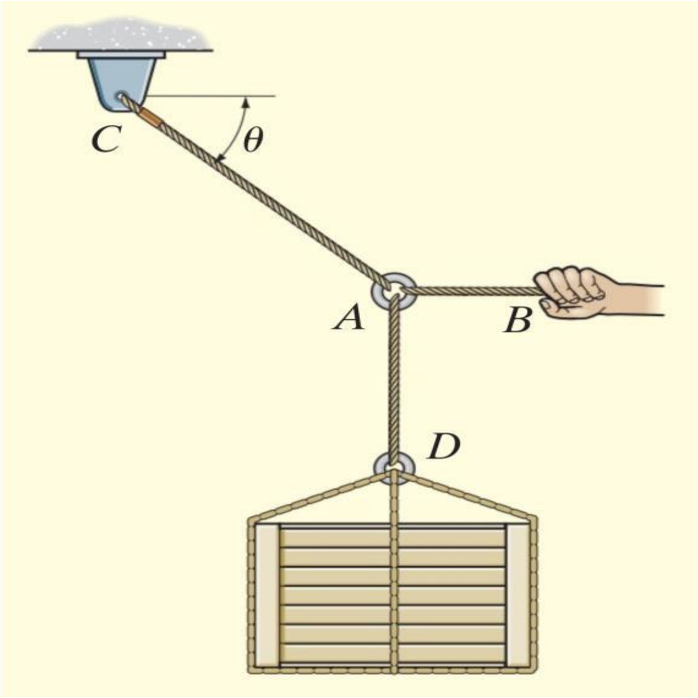


Example

The 200-kg crate is suspended using the ropes AB and AC. Each rope can withstand a maximum force of 10 kN before it breaks. If AB always remains horizontal, determine the smallest angle u to which the crate can be suspended before one of the ropes breaks.

$$\overset{+}{\rightarrow} \Sigma F_x = 0;$$

$$+\uparrow \Sigma F_y = 0;$$



Example

The 200-kg crate is suspended using the ropes *AB* and *AC*. Each rope can withstand a maximum force of 10 kN before it breaks. If *AB* always remains horizontal,
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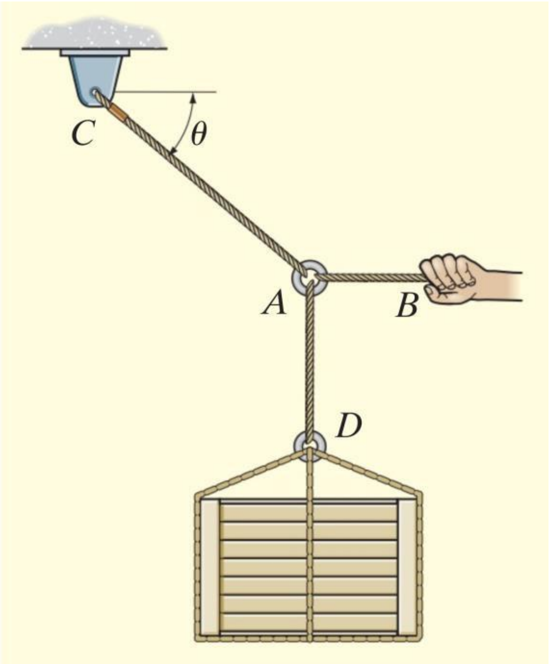
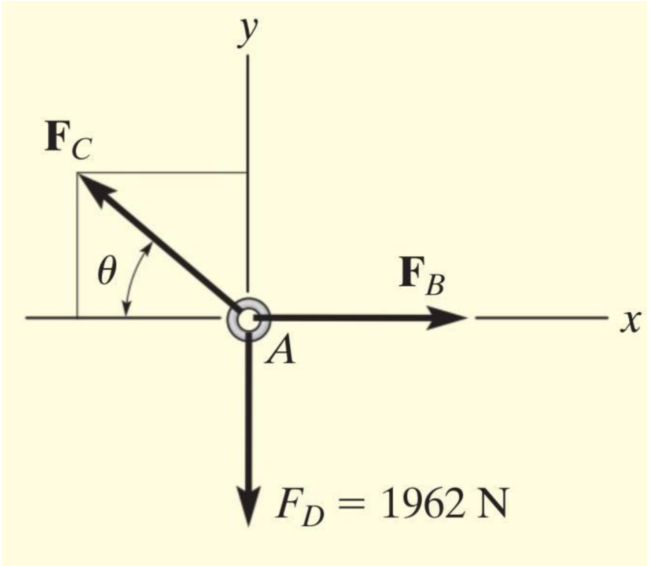


$$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad -F_C \cos\theta + F_B = 0; \quad F_C = \frac{F_B}{\cos\theta}$$

$$+\uparrow \Sigma F_y = 0; \qquad F_C \sin\theta - 1962 \text{ N} = 0$$



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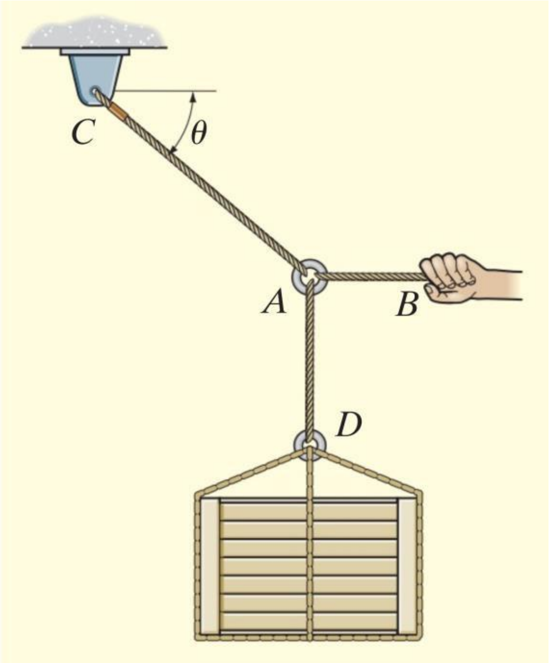
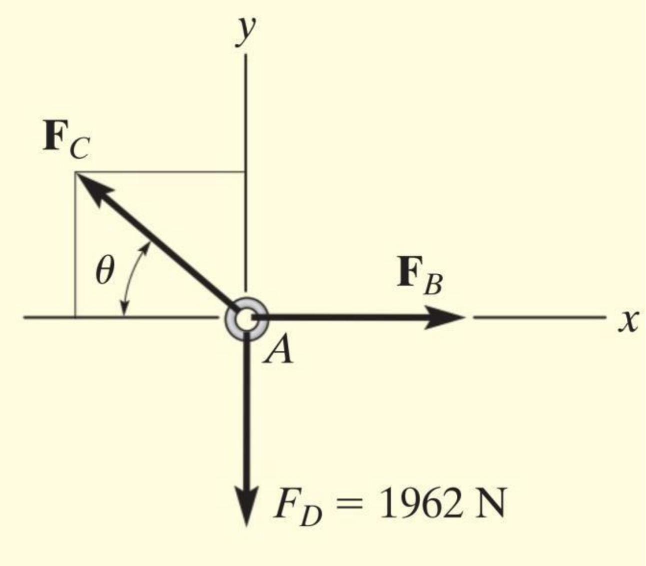


$$\xrightarrow{+} \Sigma F_x = 0; \qquad -F_C \cos\theta + F_B = 0; \quad F_C = \frac{F_B}{\cos\theta}$$

$$+\uparrow \Sigma F_y = 0; \qquad F_C \sin\theta - 1962\text{ N} = 0$$

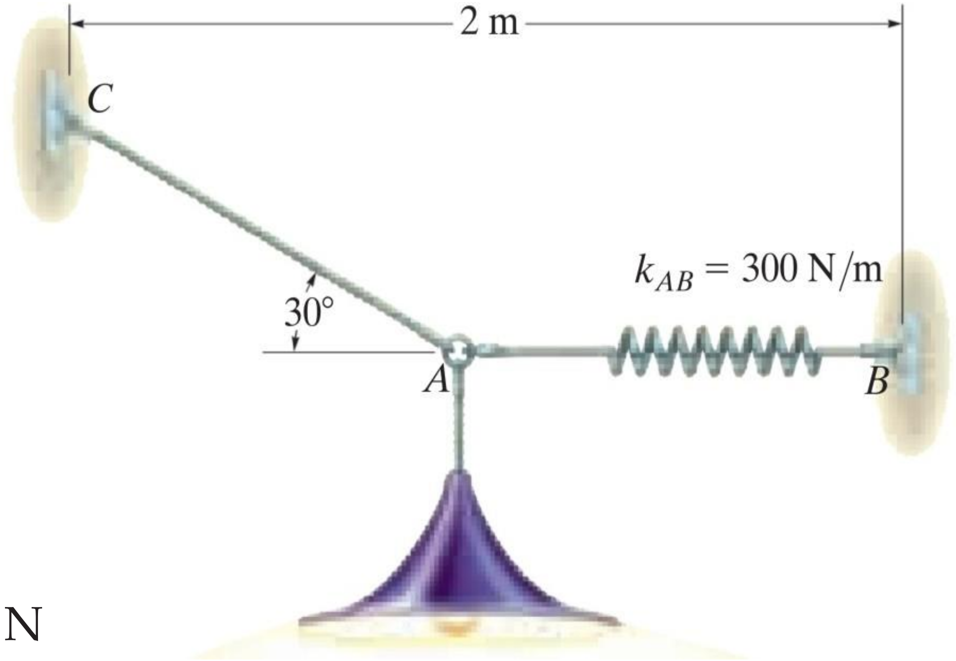
$$10(10^3)\text{ N} = \frac{F_B}{\cos 11.31^\circ}$$

$$F_B = 9.81\text{ kN}$$



Example

Determine the required length of cord AC in Fig. 3–8a so that the 8-kg lamp can be suspended in the position shown. The *undeformed* length of spring AB is $l'_{AB} = 0.4$ m, and the spring has a stiffness of $k_{AB} = 300$ N/m.



$$\overset{+}{\rightarrow} \Sigma F_x = 0;$$

$$+\uparrow \Sigma F_y = 0;$$

Free-Body Diagram. The lamp has a weight W = 819.812 = 78.5 N and so the free-body diagram of the ring at A is shown in Fig.

$$\Sigma F_x = 0;$$

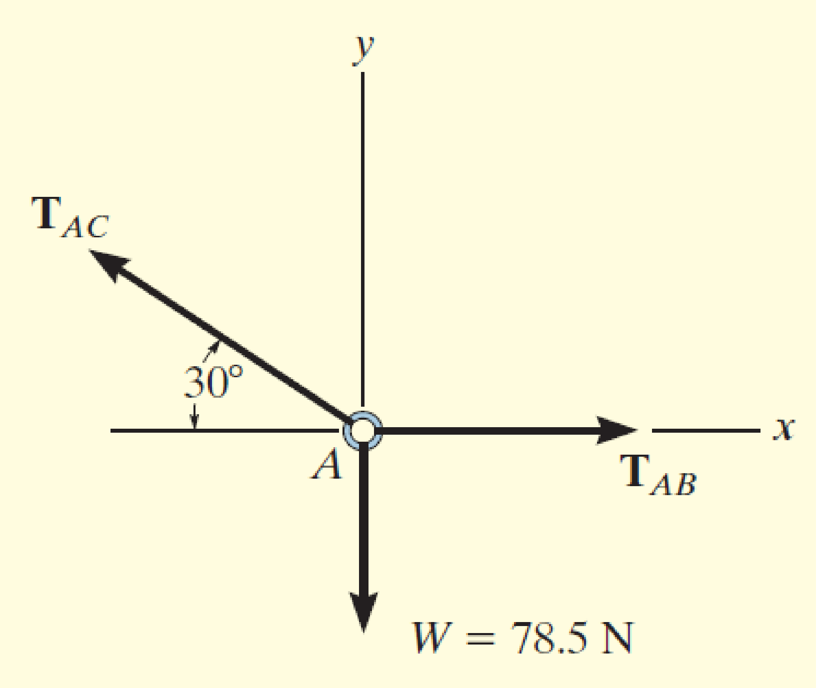
$$T_{AB} - T_{AC} \cos 30^\circ = 0$$

$$\Sigma F_y = 0;$$

$$T_{AC} \sin 30^\circ - 78.5 \text{ N} = 0$$

$$T_{AC} = 157.0 \text{ N}$$

$$T_{AB} = 135.9 \text{ N}$$



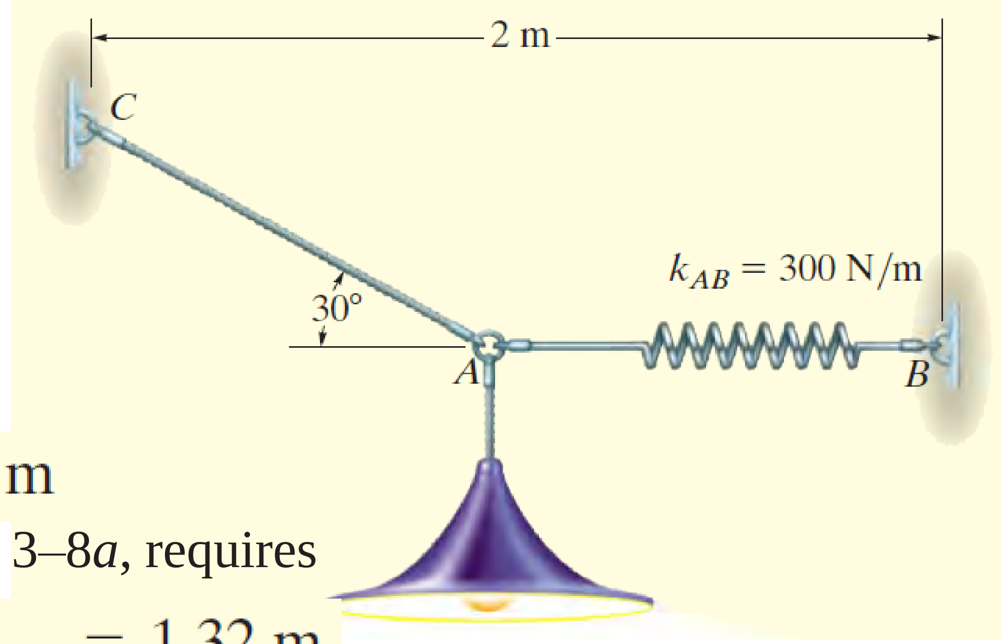
$$T_{AB} = k_{AB} s_{AB};$$

$$135.9 \text{ N} = 300 \text{ N/m}(s_{AB})$$

$$s_{AB} = 0.453 \text{ m}$$

$$l_{AB} = l'_{AB} + s_{AB}$$

$$l_{AB} = 0.4 \text{ m} + 0.453 \text{ m} = 0.853 \text{ m}$$



The horizontal distance from C to B, Fig. 3–8*a*, requires

$$2 \text{ m} = l_{AC} \cos 30^\circ + 0.853 \text{ m} \qquad l_{AC} = 1.32 \text{ m}$$

Home Assignment

- F3-1. & Problem 3-36 & 3-39.