Solution Practice Problems
Lecture #34
Q1:- Solve the initial-value problem
$\frac{y^{2}u^{2} + x^{2}u^{2}y = (xyu)^{2}}{y^{2}};  u(x,0) = 3e^{(x^{2}/4)}$
$(J_{x}, J_{y})$
Solution: Consider
u(n,y) = f(x) g(y) = 0
Using this solution in given PDE we got
Using this colution in given PDE we got  y² [f'(n) gry)]² + x² [f(n) g'(y)]² = [nyf(n)gro)]²
· · · · · · · · · · · · · · · · · · ·
=> y2[f'(x)]2 g2 + x2 f2[g'(y)]2 = x2 y2 f2 g2
=> y'[f'(x)]2 gh + xx \$2 pg'(y)]2 = 1
x y p g x y g
Γ <sub>2</sub> , γ <sup>2</sup> , Γ <sub>2</sub> ,
$\Rightarrow \frac{1}{\chi^2} \left[ \frac{f'(\chi)}{f(\chi)} \right]^2 + \frac{1}{\chi^2} \left[ \frac{g'(\chi)}{g(\chi)} \right]^2 - 1$
J ( g(y) )
1 [ E(m) ] <sup>2</sup> 1 [ -(1) <sup>2</sup> 1 <sup>2</sup>
$\frac{1}{2} \left[ \frac{f'(x_1)}{f'(x_1)} \right]^2 = \frac{1}{2} \left[ \frac{g'(y_1)}{g'(y_1)} \right]^2 = \frac{1}{2}$
where d' is separation constant. Thus,
soft constant, thus,
$\frac{1}{1}\left[f'(x)\right] = \lambda  \text{and}  1 - \frac{1}{1}\left[g'(y)\right]^{2} = \lambda^{2}$
, x f(x)
=> f(x) = x d f(x) >0 and 1 - 22 = 1 [9(4)]2
y' ( g(y) ]
⇒ g'(y) = 11-22
<u>पु १५७)</u>
> 9(y) = y g(y) \[ 1 - \lambda^2 \]
From (1)
P(1/2) = 1 2 P(x)
=) df(x) = x 2 f(x) => df(x) = x 2 dx
ola fin)
=> ln/f(x)/= 4x2+ ln/Al.

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=> du 1fins 1 = 2 x2 lue + lu 1A1
- Later when the first and a said
=> lu   f(n)   = lu   A e dx2
= 3) P(n) = A e d2/2 3) (A is asloi trang constant)
From (2)
g'(y) = y g(y) N 1-12
-> olg (4) = y g (4) 11-42
dy
=> olg(q) = y 11-22 dy
- 1. 19 ml - 42 11-42 4 shulls
$\frac{1}{2} \int \ln  g(y)  = \frac{y^2 \int [-\lambda^2 + \ln  B ]}{2}$
> eln 19 cys1 = y2 [1-22 he + en/131
$\mathcal{G}$
$\frac{1}{2} \qquad q(y) = B e^{\frac{y(1-x)^2}{2}} \rightarrow (9)  (B \text{ is and otherway complete})$
From (3) 8 (4)
$u(x,y) = C \exp(dx^2/2) \exp(y^2 \sqrt{1-4^2}/2)$
=> umone C= AB => um,y) = C exp [dx2 + y2 NI-d2] -> (5)
2) ((1), 4) 2 ( (2) (1) 2 ( (2) ((2) ((2
Given that u(1,0) = 3 e(x2/4)
Using this initial carelition in 3 me get.
$\frac{3 \exp\left[\frac{x^2}{4}\right] = C \exp\left[\frac{dx^2}{2} + 0\right]}{2}$
Comparison of both sides yield
C=3 and d2 1/2.
Thus solution 5 takes the form
u(n,y) = 3 exp [ 2 + y2 [1-1]
=> u(x,y) = 3 exp[1 (x2+ y2 [3)]

Q2:- Use the separation of variables u(x,y) = fm)+y(y)
to solve it a constitution of variables u(x,y) = fm) +g(y)
to solve the equation:
4x + 4y = 1.
Sol- For the present case:
u(m,y) = f(n) + g(y)
Thus
Uz + Uz = [f'(x)] + [g'(y)] = 1
y = (-1-y) = 1-y
\ \( \text{Te} \) \ \( \text{7}^2 \) \ \( \text{12} \)
$= \sum_{\alpha} \left[ f'(\alpha) \right]^2 = \left[ 1 - \left[ g'(y) \right]^2 = \lambda^2$
where d' is a separation constant, Thus,
me obtain
F'(n) = 2 and 1- [g'(y)] = 2 - 2
2) F(n) = 2n + A and q(y) = y 1/2 + B
where A and B are a goitrary constants.
Thus, the solution of the given PDF
is!
u(m,y) = dx + y N1 - 12 + C,
where CzA+B is an arboitrary constant.
3.000 3
Q3:- Use the separation of variables $u(x,y) = f(x) + g(y)$
to some the equation.
<u> </u>
<u>Sol.</u> $U(x,y) = f(x) + g(y)$
=> Un = P'(n) and Uy = g'(y)
Thus
Ux + Uy + x2 = 0 takes the form
[ \( \text{P'(M)} \) + \( \text{P'(Y)} \) + \( \text{P'(M)} \) + \( \text{P'(M)} \) \( \t
where it is the separation constant. Thus
$[f'(\pi)]^2 + \chi^2 = \lambda^2  \text{and}  -g'(y) = \lambda^2.$
Now
[f'(n)]2 + x2 = 1/2
$\Rightarrow f'(x) = \sqrt{\lambda^2 - x^2} \Rightarrow f(x) = \sqrt{\lambda^2 - x^2} dx + A.$

Let x=dsint => dx2 d cost dt
Thus, P(x) = (N/2-x2 dx + A = (N/2-13 gint (2 cost) dx + A
$= \lambda^2 \left( \left[ \lambda \right] - \sin^2 t \right) dt_1 = \lambda^2 \left( \left[ \cos t \right] dt + A \right).$
$= \lambda^2 \left\{ \begin{array}{c} 1 + \cos 2t - \\ 2 \end{array} \right\} \text{ of } A$
$\frac{3^2}{2} \left( \begin{array}{c} t + 8 \text{ in } 2t + 2A \\ 2 & 3^2 \end{array} \right)$
$= \frac{\lambda^2}{2} \left[ \frac{\sin^{-1}(x)}{\lambda} + \frac{\cos t \cos t}{x} \right] + A$
2 ( ( ) ) × ]
$\frac{2}{3}\frac{\lambda^{2}}{\lambda^{2}}\left[\frac{8in^{-1}}{\lambda}+\frac{\lambda}{\lambda}\right]\frac{\lambda^{2}}{1-\frac{\lambda^{2}}{\lambda^{2}}}$
$\frac{2}{2} \left[ \frac{\lambda^2 - \lambda^2}{\lambda^2} + \frac{2}{3} \ln^{-1} \left( \frac{\lambda}{\lambda} \right) \right] \circ A$
12 x2 12 x2 12 x4
$\frac{2}{2} \frac{f(x)}{f(x)} = \frac{x}{x} \frac{1}{x^2 - x^2} + \frac{2}{x^2} \frac{2ix^{-1}}{x^2} \left(\frac{x}{x}\right) \neq \frac{1}{x^2}.$
and $g'(y) = -\lambda^2$
3) g(y) = -2 y 2 B.
Tuis
M(x,y) = x 1/2-x2 + 2 8in-1 (x) - 12y +C
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where c = A & B is an aroboitrary constant.
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