Bayes Rule

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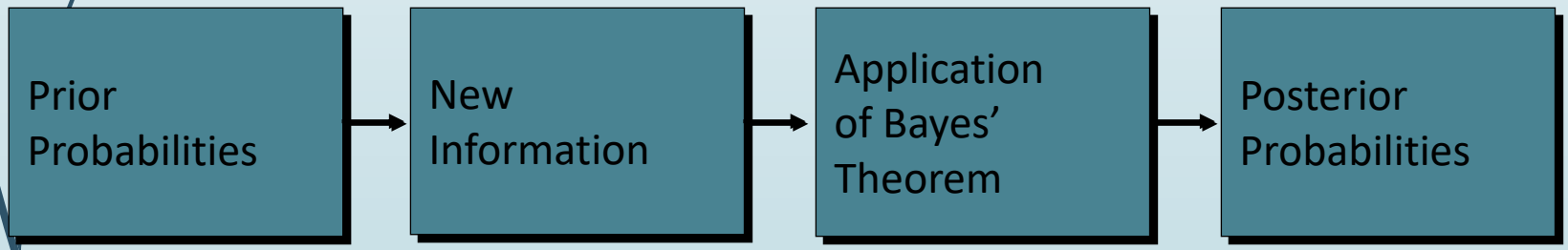
Bayes' Theorem

- Often, we begin probability analysis with initial or prior probabilities.
- Then, from a sample, special report, or a product test we obtain some additional information.
- Given this information, we calculate revised or posterior probabilities.
- Bayes' theorem provides the means for revising the prior probabilities.

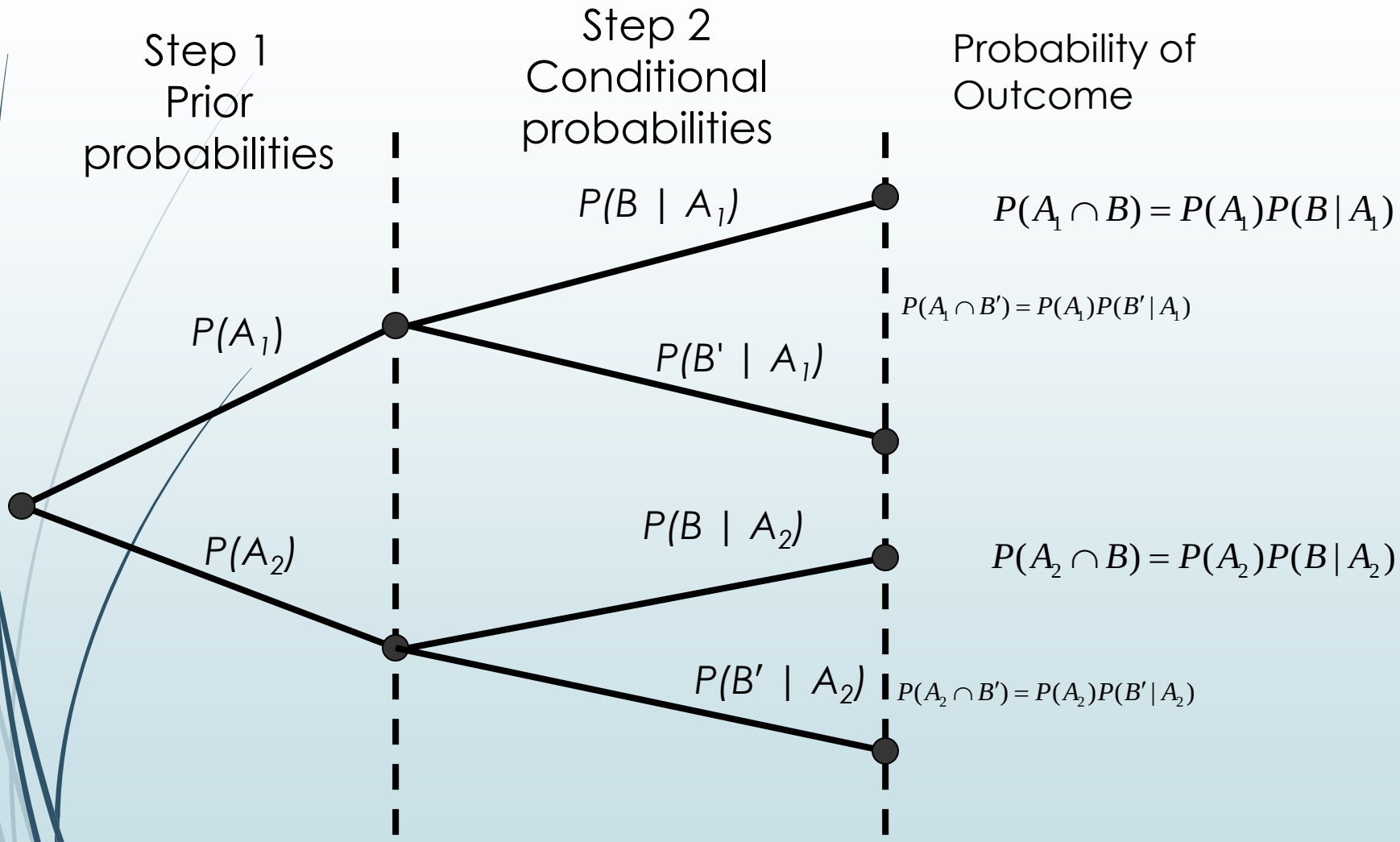


To find the posterior probability that event A_i will occur given that event B has occurred, we apply Bayes' theorem

$$P(A_i \mid B) = \frac{P(A_i)P(B \mid A_i)}{P(A_1)P(B \mid A_1) + P(A_2)P(B \mid A_2) + \ldots + P(A_n)P(B \mid A_n)}$$

Bayes' theorem is applicable when the events for which we want to compute posterior probabilities are mutually exclusive and their union is the entire sample space.

Probability Tree



Question 1

A problem on a multiple-choice quiz is answered correctly with probability 0.9 if a student is prepared. An unprepared student guesses between 4 possible answers, so the probability of choosing the right answer is one by four. Seventy five percent of students prepare for the quiz. If Mr. X gives a correct answer to this problem, what is the chance that he did not prepare for the quiz?

Question 2

There are three coins identical in appearance, one of which is ideal and other two biased with probabilities 1/3 and 2/3 respectively for a head. One coin is taken at random and tossed twice, if the head appears both times, what is the probability that the ideal coin is selected.

Question 3

A commercial system broadcast on three channels $C_i(1, 2, 3)$ with chances 4:2:3. These channels broadcast educational programs with respective probabilities 0.3,0.1,and 0.2. What is the probability that no educational program broadcast.

Question 4

In a certain region of country, it is known from experience that the probability of selecting an adult over 40 years of age with cancer is 0.05. If the probability of a doctor correctly diagnosing a person with cancer as having the disease is 0.78 and the probability of incorrectly diagnosing a person without cancer as having the disease is 0.06, what is the probability that a person is diagnosed as having cancer?

Question 5

A ternary communication channel is shown in figure. Suppose that the input symbols 0, 1, and 2 occur with probability ½, ¼, and ¼, respectively.

a) Find the probabilities of the output symbols.

b) Suppose that 1 is observed as an output. What is the probability that the input was 1?