Engineering Mechanics : Statics

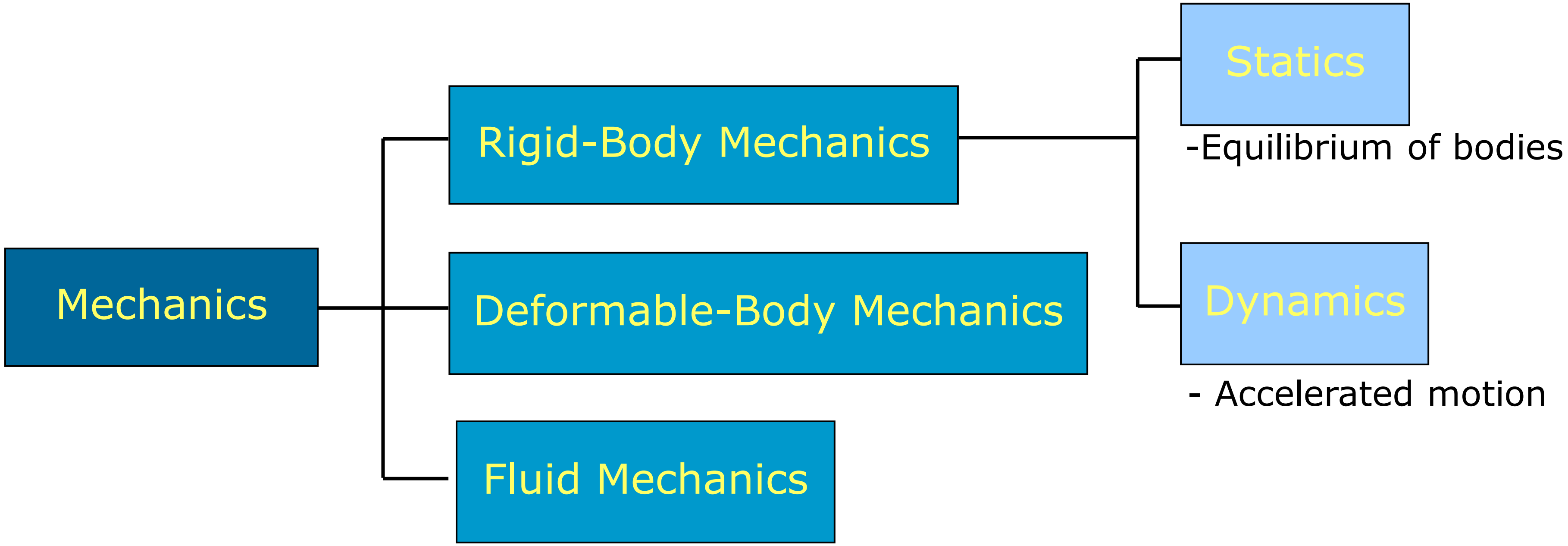
Chapter 13: kinetics of a particle

Chapter outline

- Newton's Second Law of Motion
- The Equation of Motion
- Equation of Motion for a System of Particles
- Equations of Motion: Rectangular Coordinates

12.1 INTRODUCTION

Engineering Mechanics



Dynamics: Deals with the accelerated motion of a body

kinematics, which treats only the geometric aspects of the motion,

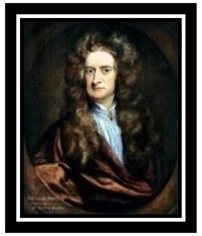
kinetics, which is the analysis of the forces causing the motion.

INTRODUCTION

- *Kinetics*: study of the relations existing between the forces acting on a body, the mass of the body, and the motion of the body. Kinetics is used to predict the motion caused by given forces or to determine the forces required to produce a given motion.

Kinetics is a branch of dynamics that deals with the relationship between the change in motion of a body and the forces that cause this change.

Newton's Laws

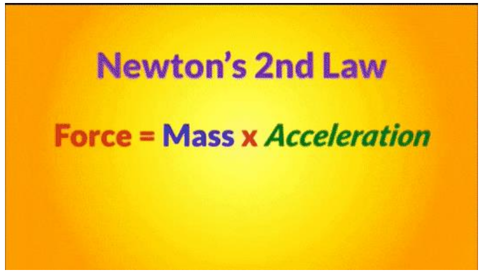


- **Law I:** If the resultant force on a particle is zero, the particle will remain at rest (if originally at rest) or will move with constant speed in a straight line (if originally in motion).

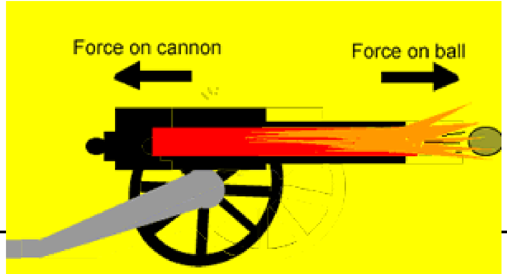
Law II: The acceleration of a particle is proportional to the resultant force acting on it in the direction of this force.

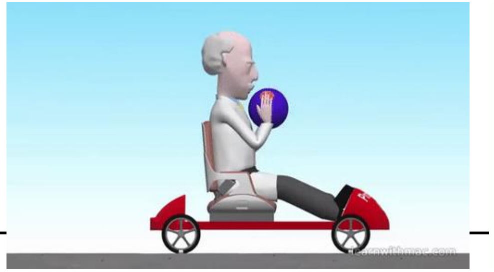
If above Law (II law) is applied to a particle of mass *m*, it may be stated as

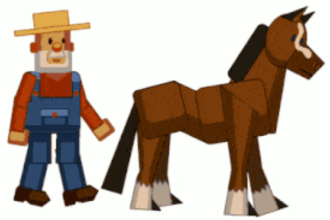
$\mathbf{F} = m\mathbf{a}$ where $\mathbf{F} =$ resultant force; $\mathbf{a} =$ resulting acceleration.



Law III: The forces of action and reaction between interacting bodies are equal in magnitude, opposite in direction, and collinear.







13.1 Newton's Second Law

13..2 The Equation of Motion

13.3 The Equation of Motion for a System of Particles

13.4 Equations of Motion: Rectangular Coordinates

Newton's Law of Gravitational Attraction.

$$\boxed{F = G\frac{m_1 m_2}{r^2}}$$

$G =$ universal constant of gravitation; according to experimental evidence $G = 66.73(10^{-12})\ \mathrm{m^3/(kg \cdot s^2)}$

Newton's Law of Gravitational Attraction.

$$\boxed{F = G\frac{m_1 m_2}{r^2}}$$

$$g = GM_e/r^2,$$

$$W = mg$$

$G =$ universal constant of gravitation; according to experimental evidence $G = 66.73(10^{-12})\ \mathrm{m}^3/(\mathrm{kg}\cdot\mathrm{s}^2)$

Newton's Second Law of Motion

- Newton's Law of Gravitational Attraction

$$\boxed{F = G\frac{m_1 m_2}{r^2}}$$

F: force of attraction between the two particles, N

G: universal constant of gravitation, m^3/kgs^2

$$G = 66.73 \times 10^{-12}$$

m_1, m_2: mass of each of the two particles, kg

r: distance between the centers of the two particles, m

Newton's Second Law of Motion

- For an object "near" the earth surface

the force of attraction = the weight (W) of the object

$$F = G\frac{M_e m}{r^2} = m\frac{GM_e}{r^2}$$

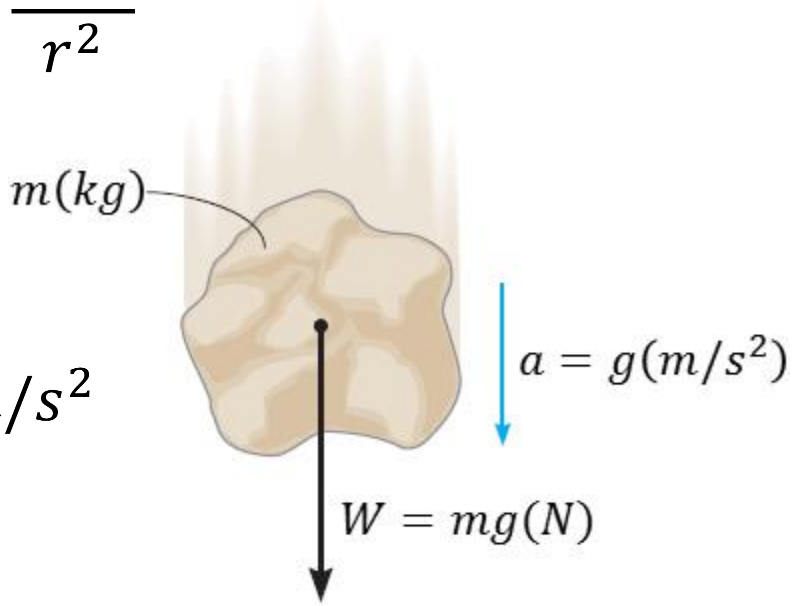
$$W = mg$$

M_e: the mass of the earth, kg

m: mass of the object, kg

g: gravitational acceleration, m/s^2

$$g = \frac{GM_e}{r^2} \approx 9.81$$



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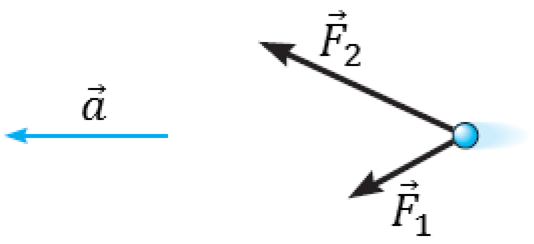
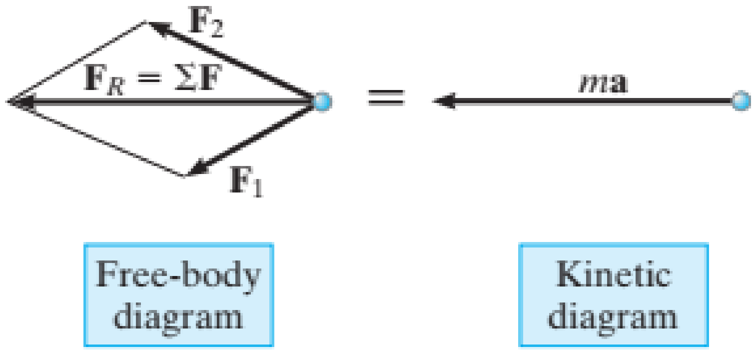
When more than one force acts on an object, the resultant force is vector summation of all the forces

$$\vec{F}_R = \sum \vec{F}_i = m\vec{a}$$





(b) (c)

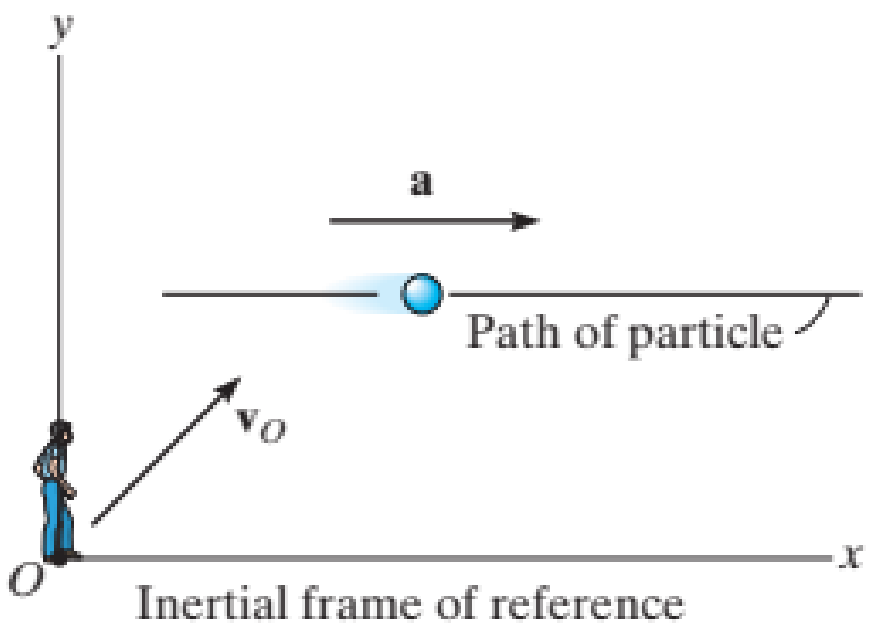
Fig. 13–2

$$\boxed{\Sigma \mathbf{F} = m\mathbf{a}}$$

Inertial Reference Frame.

- The acceleration of the particle should be measured with respect to a reference frame that is either fixed or translates with a constant velocity
- In this way, the observer will not accelerate and measurements of the particle's acceleration will be the same from any reference of this type

⇒Such a frame of reference is commonly known as a Newtonian or inertial reference frame



When studying the motions of rockets and satellites, it is justifiable to consider the inertial reference frame as fixed to the stars, whereas dynamics problems concerned with motions on or near the surface of the earth may be solved by using an inertial frame which is assumed fixed to the earth.

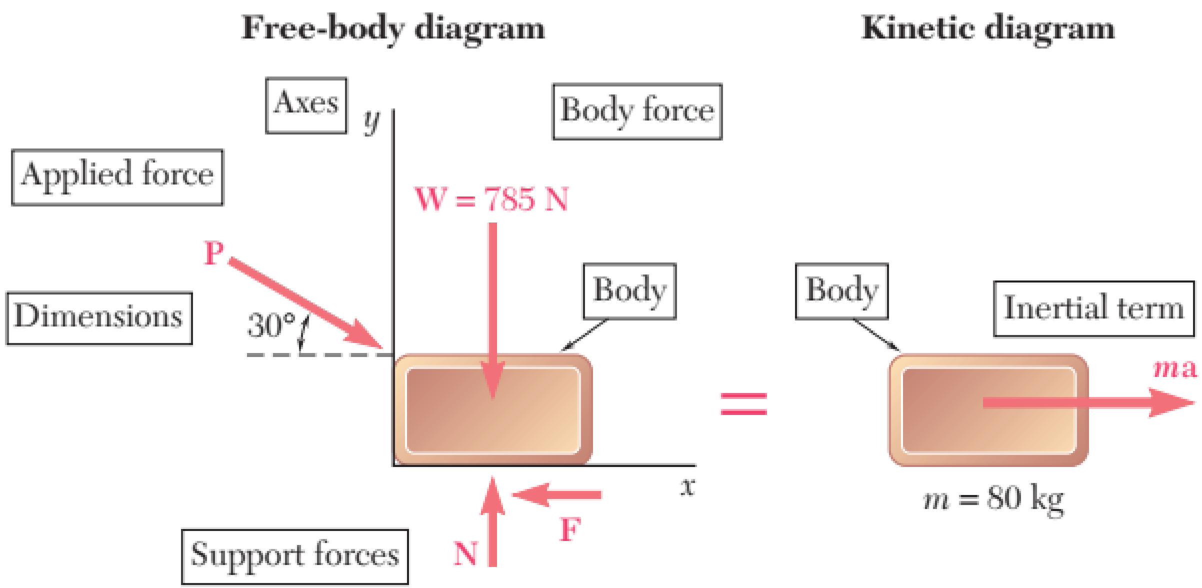
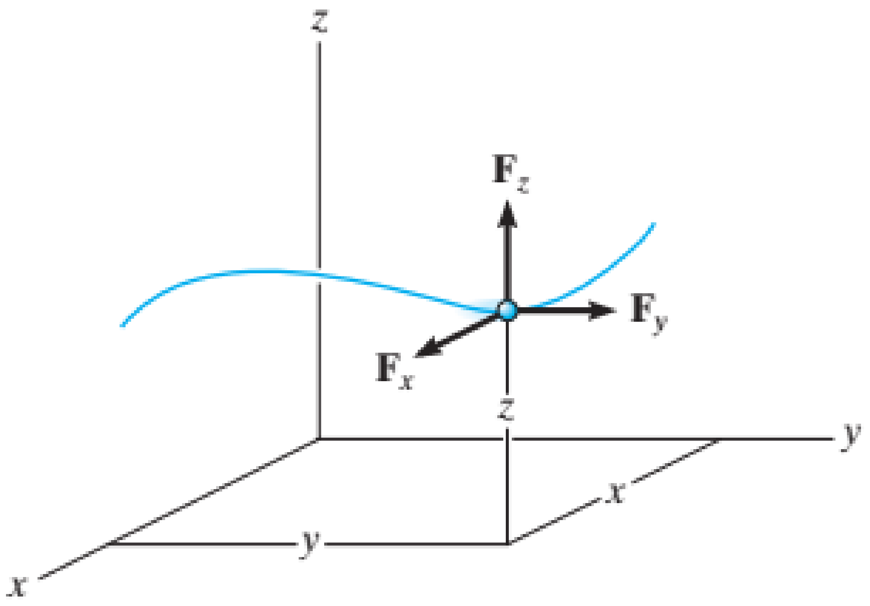


Fig. 12.9 Steps in drawing a free-body diagram and a kinetic diagram for solving dynamics problems.

$$\boxed{\Sigma \mathbf{F} = m\mathbf{a}_G}$$

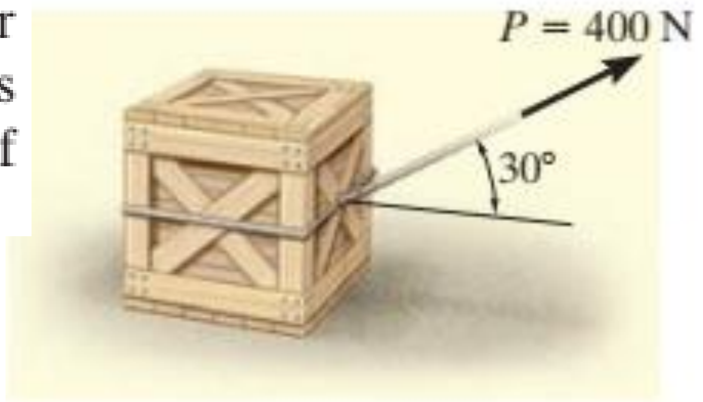
When a particle is moving relative to an inertial x,y,z frame of reference, the forces acting on the particle, as well as its acceleration, may be expressed in terms of their $\vec{i}, \vec{j}, \vec{k}$ components



$$\Sigma \mathbf{F} = m\mathbf{a}; \qquad \Sigma F_x\mathbf{i} + \Sigma F_y\mathbf{j} + \Sigma F_z\mathbf{k} = m(a_x\mathbf{i} + a_y\mathbf{j} + a_z\mathbf{k})$$

$$\boxed{\begin{aligned} \Sigma F_x &= ma_x \\ \Sigma F_y &= ma_y \\ \Sigma F_z &= ma_z \end{aligned}}$$

The 50-kg crate shown in Fig. 13–6*a* rests on a horizontal surface for which the coefficient of kinetic friction is $\mu_k = 0.3$. If the crate is subjected to a 400-N towing force as shown, determine the velocity of the crate in 3 s starting from rest.



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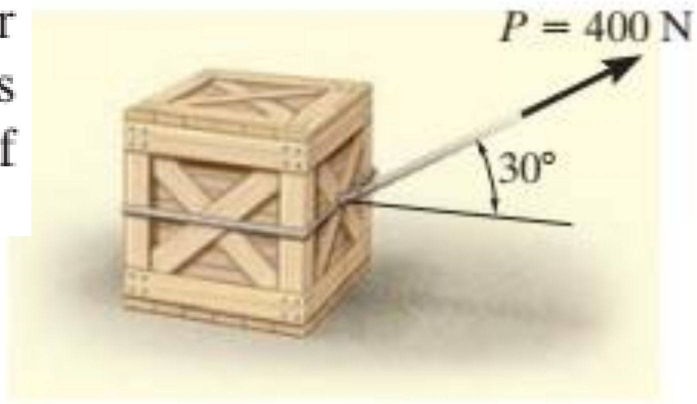


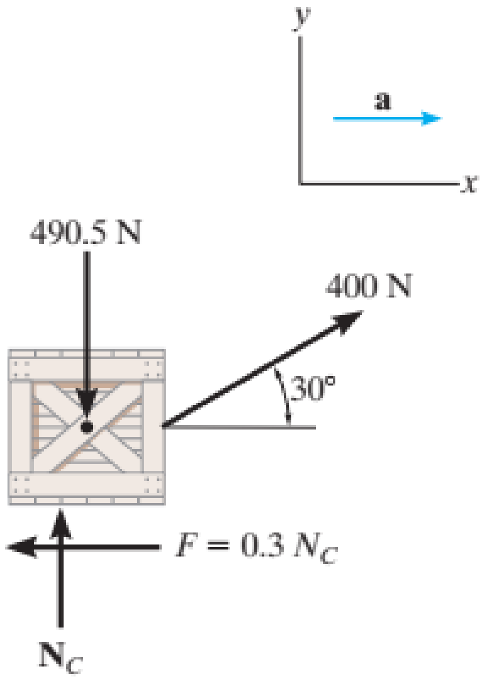
$$\overset{+}{\rightarrow} \Sigma F_x = ma_x; \qquad 400 \cos 30^\circ - 0.3N_C = 50a \qquad (1)$$

$$+\uparrow \Sigma F_y = ma_y; \qquad N_C - 490.5 + 400 \sin 30^\circ = 0 \qquad (2)$$

Solving Eq. 2 for N_C, substituting the result into Eq. 1, and solving for a yields

$$N_C = 290.5 \text{ N}$$

$$a = 5.185 \text{ m/s}^2$$



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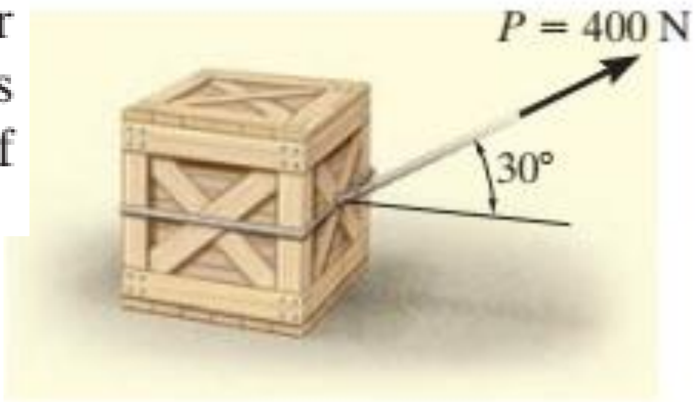
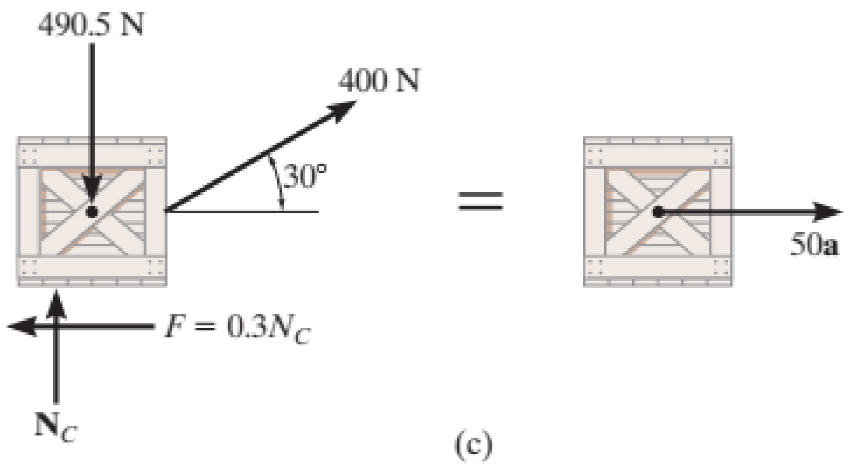


Kinematics. Notice that the acceleration is *constant*, since the applied force **P** is constant. Since the initial velocity is zero, the velocity of the crate in 3 s is

$$(\xrightarrow{+}) \qquad v = v_0 + a_c t = 0 + 5.185(3)$$
$$= 15.6 \text{ m/s} \rightarrow \qquad \textit{Ans.}$$



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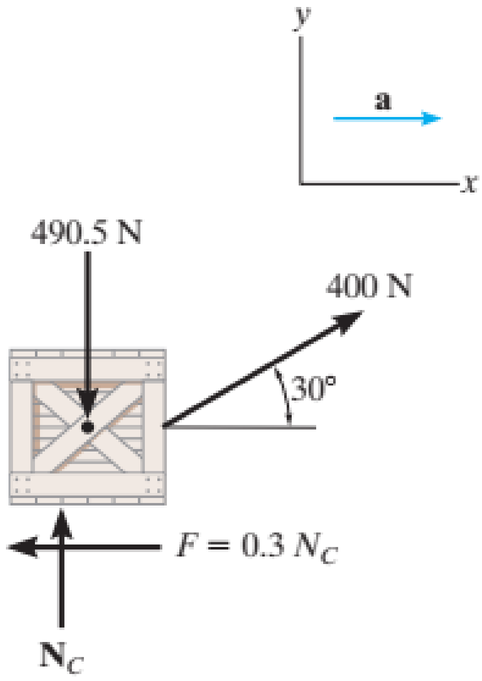
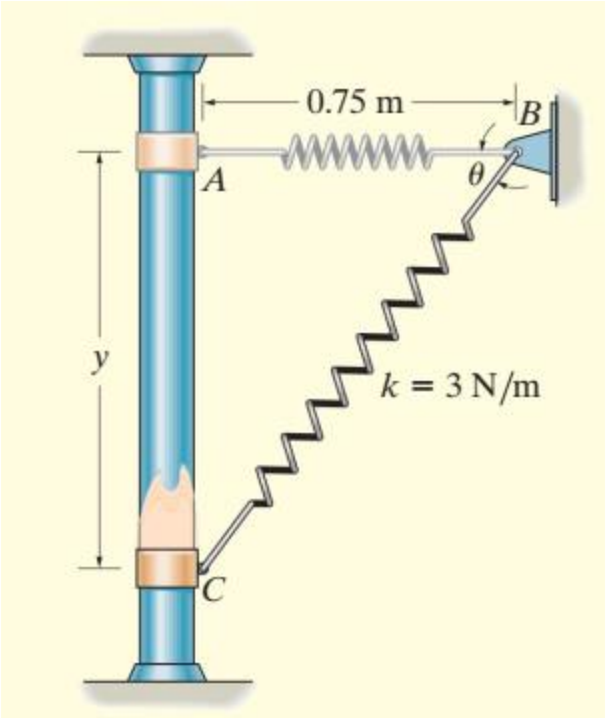


A smooth 2-kg collar, shown in Fig. 13–9*a*, is attached to a spring having a stiffness $k = 3\ \text{N/m}$ and an unstretched length of 0.75 m. If the collar is released from rest at A, determine its acceleration and the normal force of the rod on the collar at the instant $y = 1$ m.



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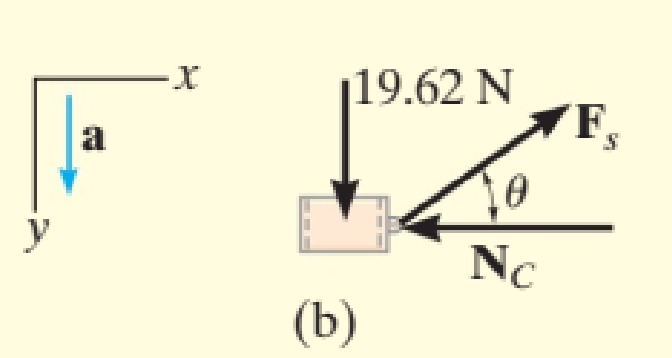

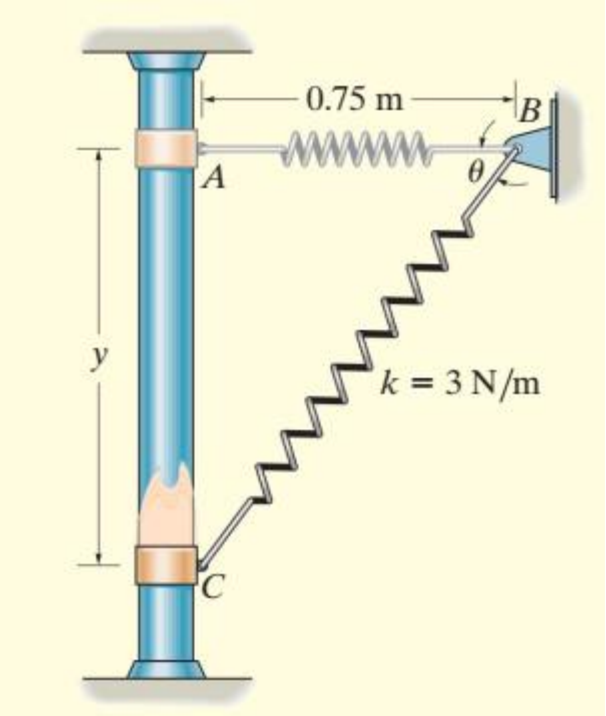

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13.4 Equations of Motion: Rectangular Coordinates

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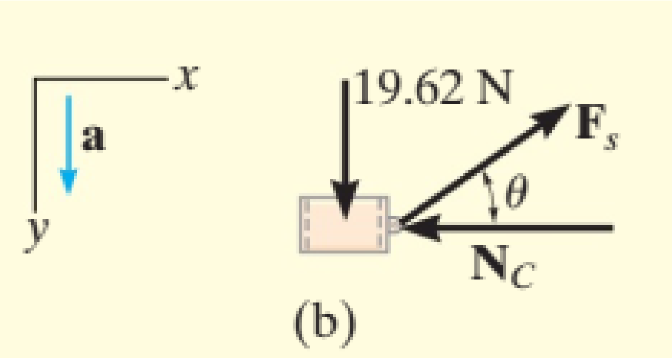
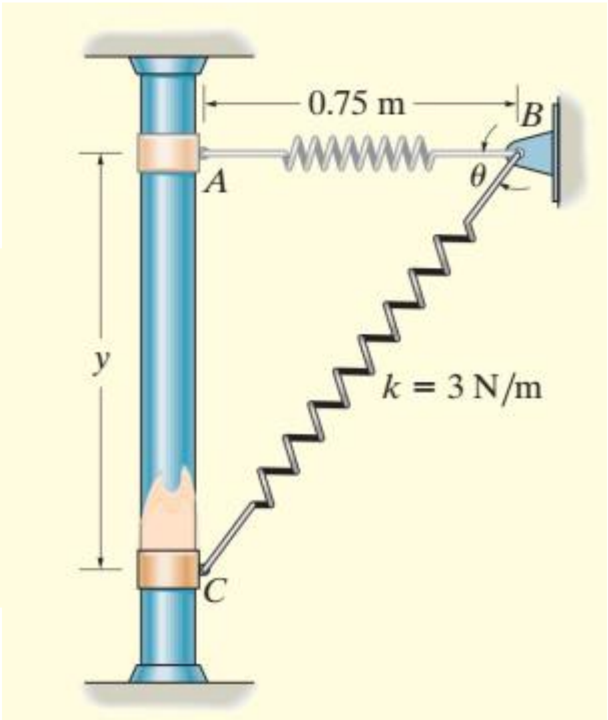




Equations of Motion.

$\xrightarrow{+} \Sigma F_x = ma_x; \qquad -N_C + F_s \cos\theta = 0$

$+\downarrow \Sigma F_y = ma_y; \qquad 19.62 - F_s \sin\theta = 2a$

$$F_s = ks.$$

$$F_s = ks = 3\left(\sqrt{y^2 + (0.75)^2} - 0.75\right)$$

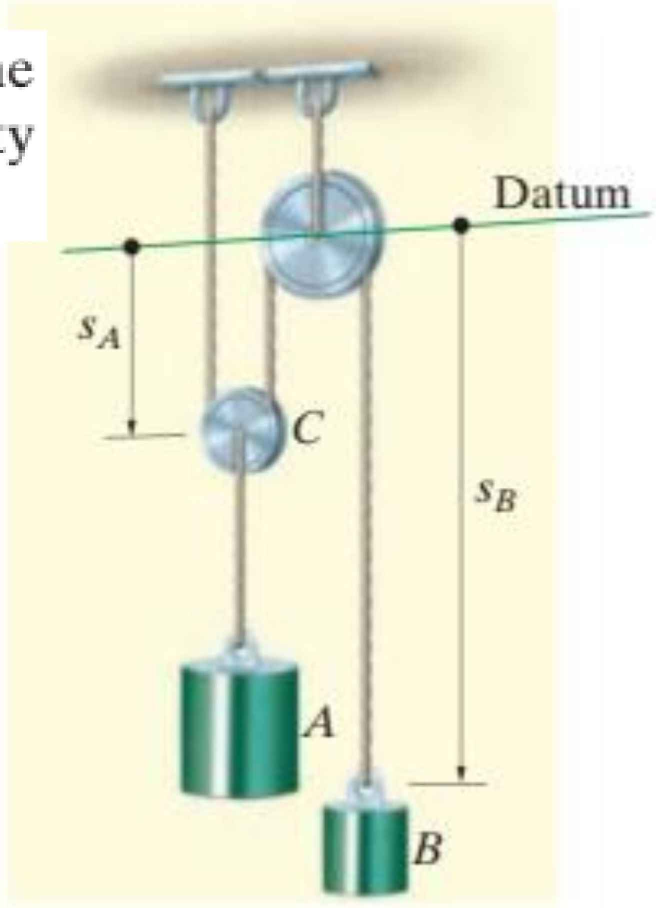
$$\tan\theta = \frac{y}{0.75}$$

$$y = 1 \text{ m}$$

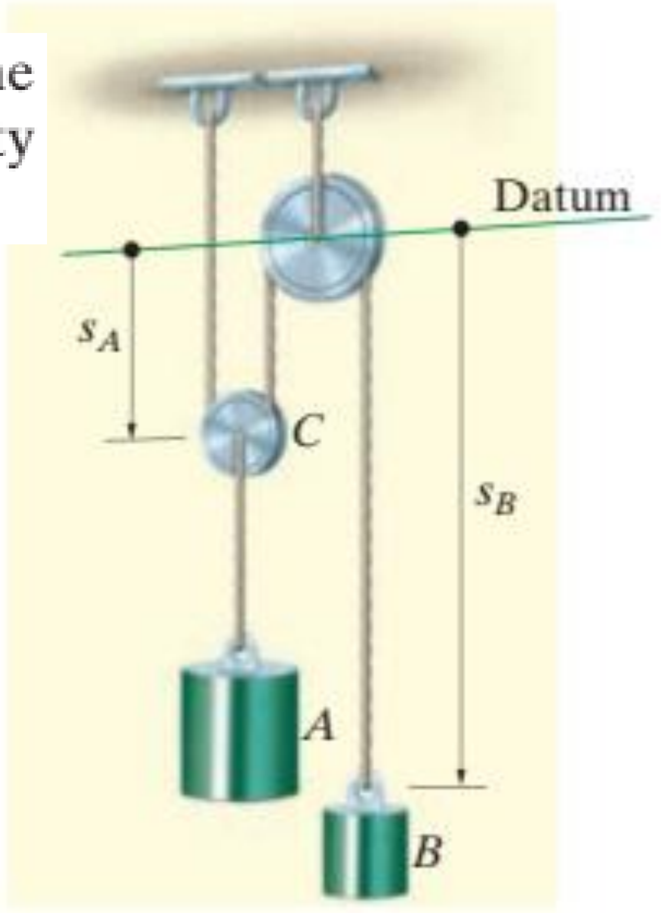
$$N_C = 0.900 \text{ N}$$

$$a = 9.21 \text{ m/s}^2 \downarrow$$

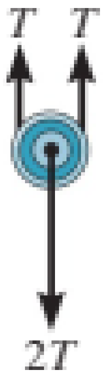
The 100-kg block A shown in Fig. 13–10a is released from rest. If the masses of the pulleys and the cord are neglected, determine the velocity of the 20-kg block B in 2 s.



The 100-kg block A shown in Fig. 13–10a is released from rest. If the masses of the pulleys and the cord are neglected, determine the velocity of the 20-kg block B in 2 s.



Free-Body Diagrams. Since the mass of the pulleys is *neglected*, then for pulley C, $ma = 0$ and we can apply $\Sigma F_y = 0$, as shown in Fig. 13–10b. The free-body diagrams for blocks A and B are shown in Fig. 13–10c and d, respectively. Notice that for A to remain stationary $T = 490.5$ N, whereas for B to remain static $T = 196.2$ N. Hence A will move down while B moves up. Although this is the case, we will assume both blocks accelerate downward, in the direction of $+s_A$ and $+s_B$. The three unknowns are T, a_A, and a_B.



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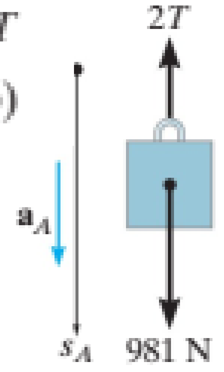


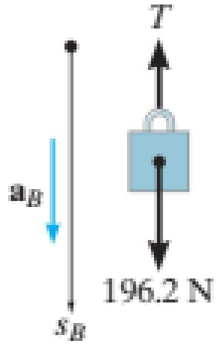
(c)

Equations of Motion. Block A,

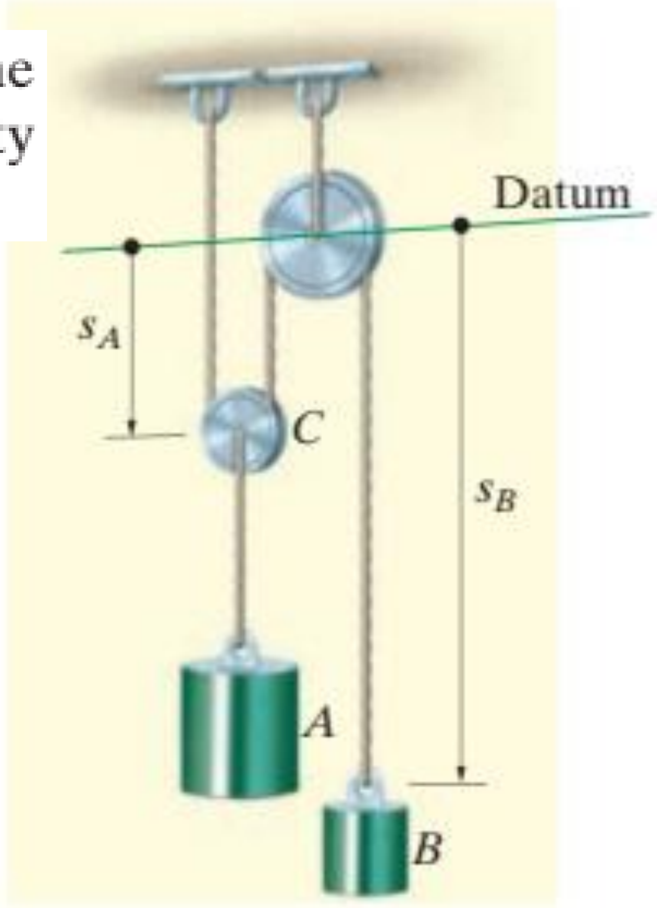
$$+\downarrow \Sigma F_y = ma_y; \qquad 981 - 2T = 100a_A$$

Block B,

$$+\downarrow \Sigma F_y = ma_y; \qquad 196.2 - T = 20a_B$$



The 100-kg block A shown in Fig. 13–10a is released from rest. If the masses of the pulleys and the cord are neglected, determine the velocity of the 20-kg block B in 2 s.



Kinematics. The necessary third equation is obtained by relating a_A to a_B using a dependent motion analysis, discussed in Sec. 12.9. The coordinates s_A and s_B in Fig. 13–10a measure the positions of A and B from the fixed datum. It is seen that

$$2s_A + s_B = l$$

where l is constant and represents the total vertical length of cord. Differentiating this expression twice with respect to time yields

$$2a_A = -a_B \qquad (3)$$

Notice that when writing Eqs. 1 to 3, the *positive direction was always assumed downward*. It is very important to be *consistent* in this assumption since we are seeking a simultaneous solution of equations. The results are

$$T = 327.0 \text{ N}$$
$$a_A = 3.27 \text{ m/s}^2$$
$$a_B = -6.54 \text{ m/s}^2$$

Hence when block A accelerates *downward*, block B accelerates *upward* as expected. Since a_B is constant, the velocity of block B in 2 s is thus

$(+\downarrow)$
$$\begin{aligned} v &= v_0 + a_B t \\ &= 0 + (-6.54)(2) \\ &= -13.1 \text{ m/s} \end{aligned} \qquad \textit{Ans.}$$

The negative sign indicates that block B is moving upward.

13.1 Newton's Second Law

13..2 The Equation of Motion

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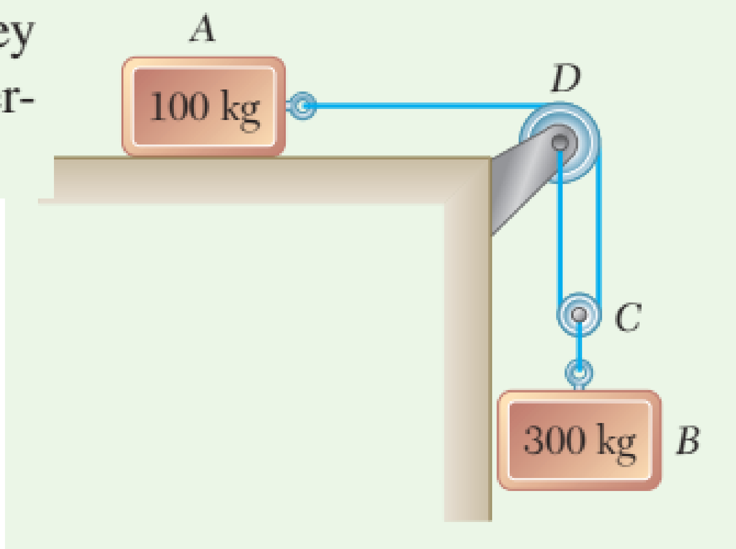
13.4 Equations of Motion: Rectangular Coordinates

The two blocks shown start from rest. The horizontal plane and the pulley are frictionless, and the pulley is assumed to be of negligible mass. Determine the acceleration of each block and the tension in each cord.



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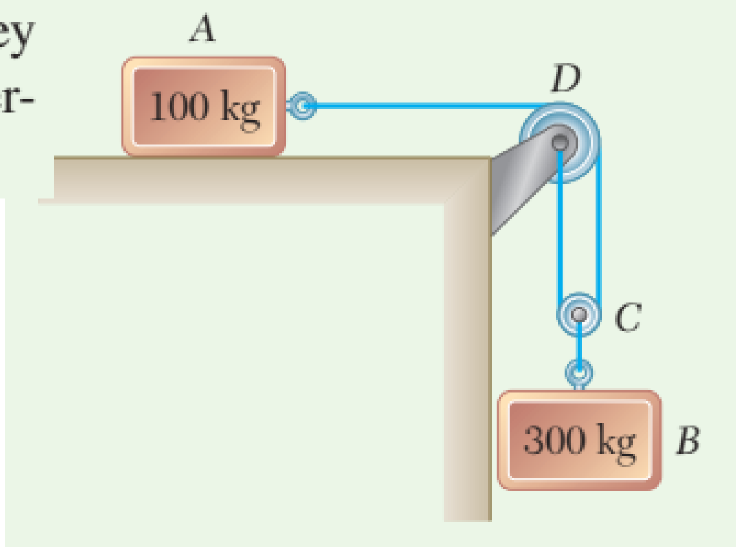


STRATEGY: You are interested in finding the tension in the rope and the acceleration of the two blocks, so use Newton's second law. The two blocks are connected by a cable, indicating that you need to relate their accelerations using the techniques discussed in Chapter 11 for objects with dependent motion.

MODELING: Treat both blocks as particles and assume that the pulley is massless and frictionless. Since there are two masses, you need two systems: block *A* by itself and block *B* by itself. The free-body and kinetic diagrams for these objects are shown in Figs. 1 and 2. To help determine the forces acting on block *B*, you can also isolate the massless pulley *C* as a system (Fig. 3).

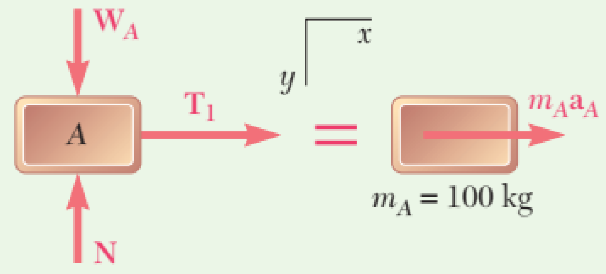


Fig. 1 Free-body diagram and kinetic diagram for *A*.

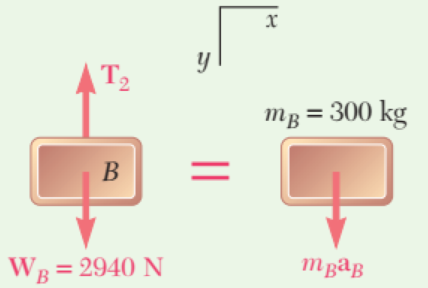
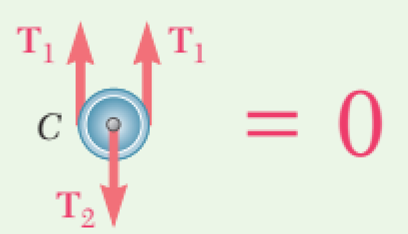


Fig. 2 Free-body diagram and kinetic diagram for *B*.



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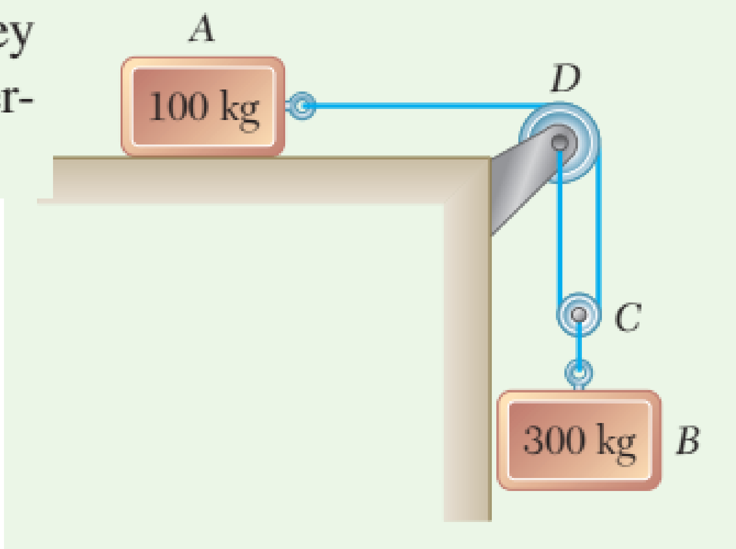


Kinetics. Apply Newton's second law successively to block A, block B, and pulley C.

Block A. Denote the tension in cord ACD by T_1 (Fig. 1). Then you have

$$\xrightarrow{+}\Sigma F_x = m_A a_A: \qquad T_1 = 100a_A \qquad (1)$$

Block B. Observe that the weight of block B is

$$W_B = m_B g = (300\text{ kg})(9.81\text{ m/s}^2) = 2940\text{ N}$$

Denote the tension in cord BC by T_2 (Fig. 2). Then

$$+\downarrow\Sigma F_y = m_B a_B: \qquad 2940 - T_2 = 300a_B \qquad (2)$$

Pulley C. Assuming m_C is zero, you have (Fig. 3)

$$+\downarrow\Sigma F_y = m_C a_C = 0: \qquad T_2 - 2T_1 = 0 \qquad (3)$$

At this point, you have three equations, (1), (2), and (3), and four unknowns, T_1, T_2, a_B, and a_A. Therefore, you need one more equation, which you can get from kinematics.

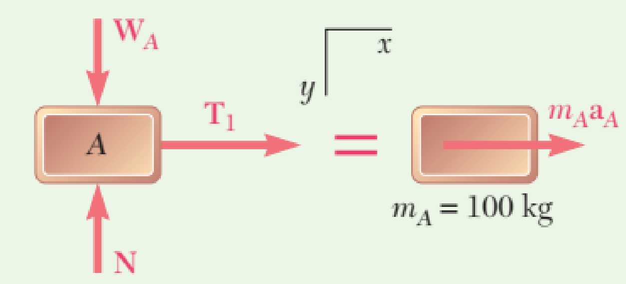


Fig. 1 Free-body diagram and kinetic diagram for A.

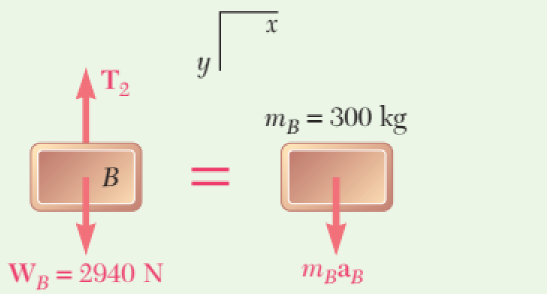
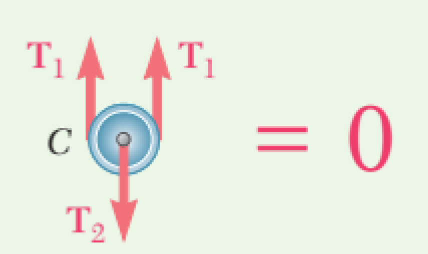


Fig. 2 Free-body diagram and kinetic diagram for B.



13.1 Newton's Second Law

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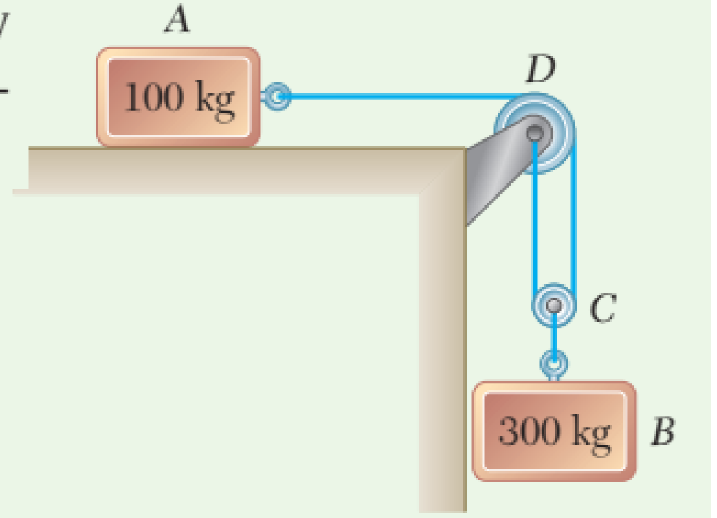
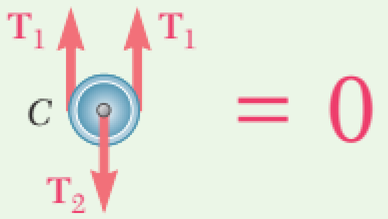


Kinematics. It is important to make sure that the directions you assumed for the kinetic diagrams are consistent with the kinematic analysis. Note that if block A moves through a distance x_A to the right, block B moves down through a distance

$$x_B = \frac{1}{2}x_A$$

Differentiating twice with respect to t, you have

$$a_B = \frac{1}{2}a_A$$



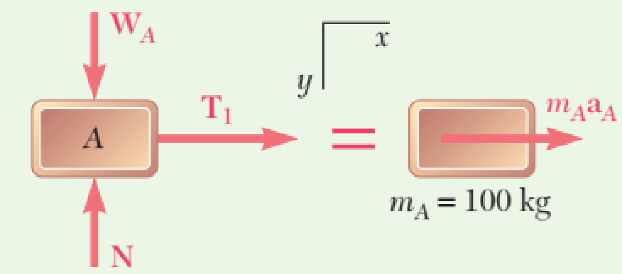


Fig. 1 Free-body diagram and kinetic diagram for A.

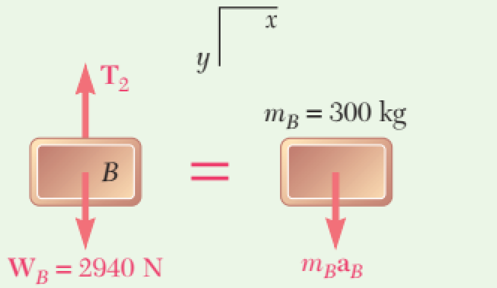


Fig. 2 Free-body diagram and kin diagram for B.

$$2940 - T_2 = 300(\tfrac{1}{2}a_A)$$

$$T_2 = 2940 - 150a_A \qquad \textbf{(5)}$$

Now substitute for T_1 and T_2 from Eqs. (1) and (5), respectively, into Eq. (3).

$$2940 - 150a_A - 2(100a_A) = 0$$

$$2940 - 350a_A = 0 \qquad a_A = 8.40\ \text{m/s}^2 \blacktriangleleft$$

Then substitute the value obtained for a_A into Eqs. (4) and (1).

$$a_B = \tfrac{1}{2}a_A = \tfrac{1}{2}(8.40\ \text{m/s}^2) \qquad a_B = 4.20\ \text{m/s}^2 \blacktriangleleft$$

$$T_1 = 100a_A = (100\ \text{kg})(8.40\ \text{m/s}^2) \qquad T_1 = 840\ \text{N} \blacktriangleleft$$

Recalling Eq. (3), you have

$$T_2 = 2T_1 \qquad T_2 = 2(840\ \text{N}) \qquad T_2 = 1680\ \text{N} \blacktriangleleft$$

13.1 Newton’s Second Law

13..2 The Equation of Motion

13.3 The Equation of Motion for a System of Particles

13.4 Equations of Motion: Rectangular Coordinates

Home assignment

Reading Assignment & Examples