Sampling Distributions

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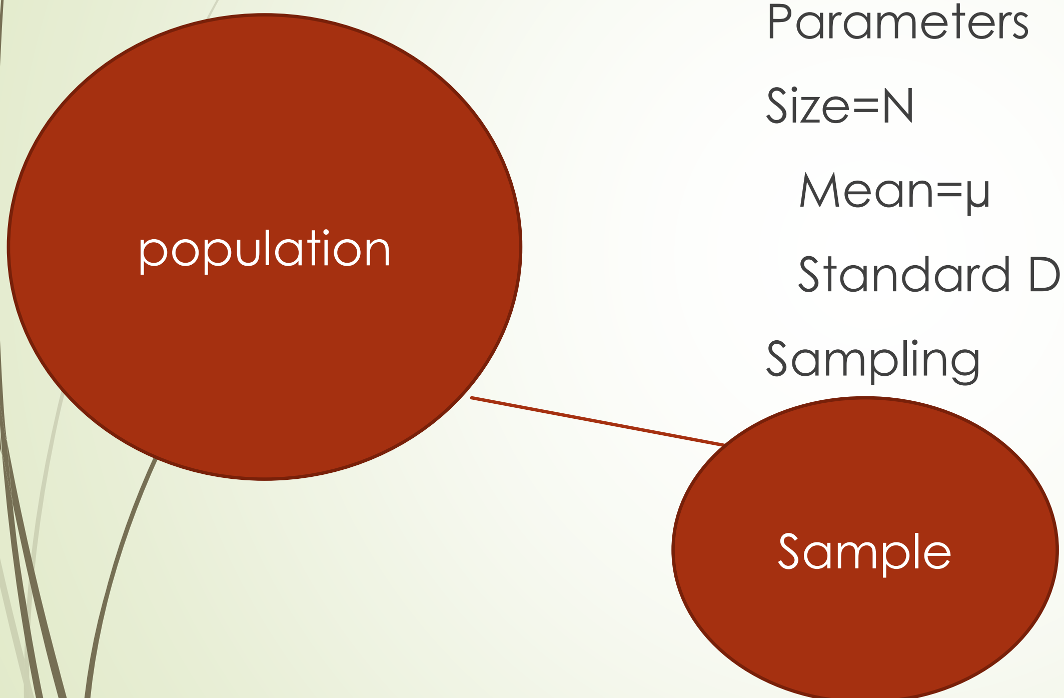
Definitions

- **Population:** The entire group of individuals is called the population
- **Parameter:** A numerical measurement describing some characteristic of a Population
- **Sample:** Representative part of population
- **Statistic:** A numerical measurement describing some characteristic of a sample
- **Sampling:** The process by which researchers select a representative subset or part of the total population that could be studied for their topic so that they will be able to draw conclusions about the entire population.

Sampling



Parameters

Size=N

Mean=μ

Standard Deviation=σ

Sampling

Statistic

Size=n

Mean=$\bar{X}$

Standard Deviation=S

Sampling Distribution

A sampling distribution is defined as a probability distribution of the value such as mean, standard deviation, a proportion, etc computed from all possible samples of the same size, which might be selected with or without replacement from a population.

$\bar{X}$	f($\bar{X}$)
2	1/9
3	2/9
4	3/9
5	2/9
6	1/9
Sum	1

$\hat{P}$	f($\hat{P}$)
0	1/20
1/3	9/20
2/3	9/20
1	1/20
Sum	1

Sampling Distribution of the Mean

The sampling distribution of the mean is the probability distribution of the means $\bar{X}$ of all possible random samples of same size that could be selected from the given population. The sampling distribution has the following properties.

- The mean of the sampling distribution of sample mean $\overline{\mathrm{X}}$ is equal to the population mean

$$\mu_{\overline{\mathrm{X}}} = \mu \,,$$

regardless the sampling is done with or without replacement.

Sampling Distribution of the Mean

- The standard deviation of sampling distribution is given by,

$$\sigma_{\bar{X}} = \frac{\sigma}{\sqrt{n}},$$

when sampling is performed with replacement from a finite or infinite population, or

$$\sigma_{\bar{X}} = \frac{\sigma}{\sqrt{n}}\sqrt{\frac{N-n}{N-1}}$$

When sampling is performed without replacement from a finite population of size N

Sampling Distribution of the Mean

- **Shape of the distribution**

a) If the population sampled is normally distributed, then the sampling distribution of the mean $\bar{X}$, will also be normal regardless of sample size.

b) If the population sampled is non-normal, then for sufficiently large sample size, the sampling distribution of $\bar{X}$ will approximate the normal distribution.

Sampling distribution of Mean

Example 1:

Population: 2, 4, 6, 8 and 10

N=5

n=2

With replacement

Total Sample points=N^n=5×5=25

Sample Space

	2	4	6	8	10
2	(2,2) (2+2)/2=2	(2,4) (2+4)/2=3	(2,6) 4	(2,8) 5	(2,10) 6
4	(4,2) 3	(4,4) 4	(4,6) 5	(4,8) 6	(4,10) 7
6	(6,2) 4	(6,4) 5	(6,6) 6	(6,8) 7	(6,10) 8
8	(8,2) 5	(8,4) 6	(8,6) 7	(8,8) 8	(8,10) 9
10	(10,2) 6	(10,4) 7	(10,6) 8	(10,8) 9	(10,10) 10

Sampling Distribution of $\bar{X}$

$\bar{X}$	f	f($\bar{X}$)
2	1	1/25
3	2	2/25
4	3	3/25
5	4	4/25
6	5	5/25
7	4	4/25
8	3	3/25
9	2	2/25
10	1	1/25
Sum	25	1

The Mean and standard deviation of sampling distribution of $\bar{X}$

$\bar{X}$	f($\bar{X}$)	$\bar{X}$f($\bar{X}$)	$\bar{X}^2$	$\bar{X}^2$f($\bar{X}$)
2	1/25	2/25	4	4/25
3	2/25	6/25	9	18/25
4	3/25	12/25	16	48/25
5	4/25	20/25	25	100/25
6	5/25	30/25	36	180/25
7	4/25	28/25	49	196/25
8	3/25	24/25	64	192/25
9	2/25	18/25	81	162/25
10	1/25	10/25	100	100/25
Sum	1	150/25		1000/25

The Mean and standard deviation of sampling distribution of $\bar{X}$

The mean of sampling distribution is

$\mu_{\bar{X}}$ =E($\bar{X}$)=150/25=6

The standard deviation of sampling distribution is

$\sigma_{\bar{X}}=\sqrt{E(\bar{X}^2)-(\mathrm{E}(\bar{X}))^2}$

$\sigma_{\bar{X}}=\sqrt{\sum \bar{X}^2 \mathrm{f}(\bar{X})-(\sum \bar{X}\mathrm{f}(\bar{X}))^2}$

$\sigma_{\bar{X}}=\sqrt{1000/25-(6)^2}$

$\sigma_{\bar{X}}=\sqrt{40-36}=\sqrt{4}=2$

The Mean and standard deviation of Population

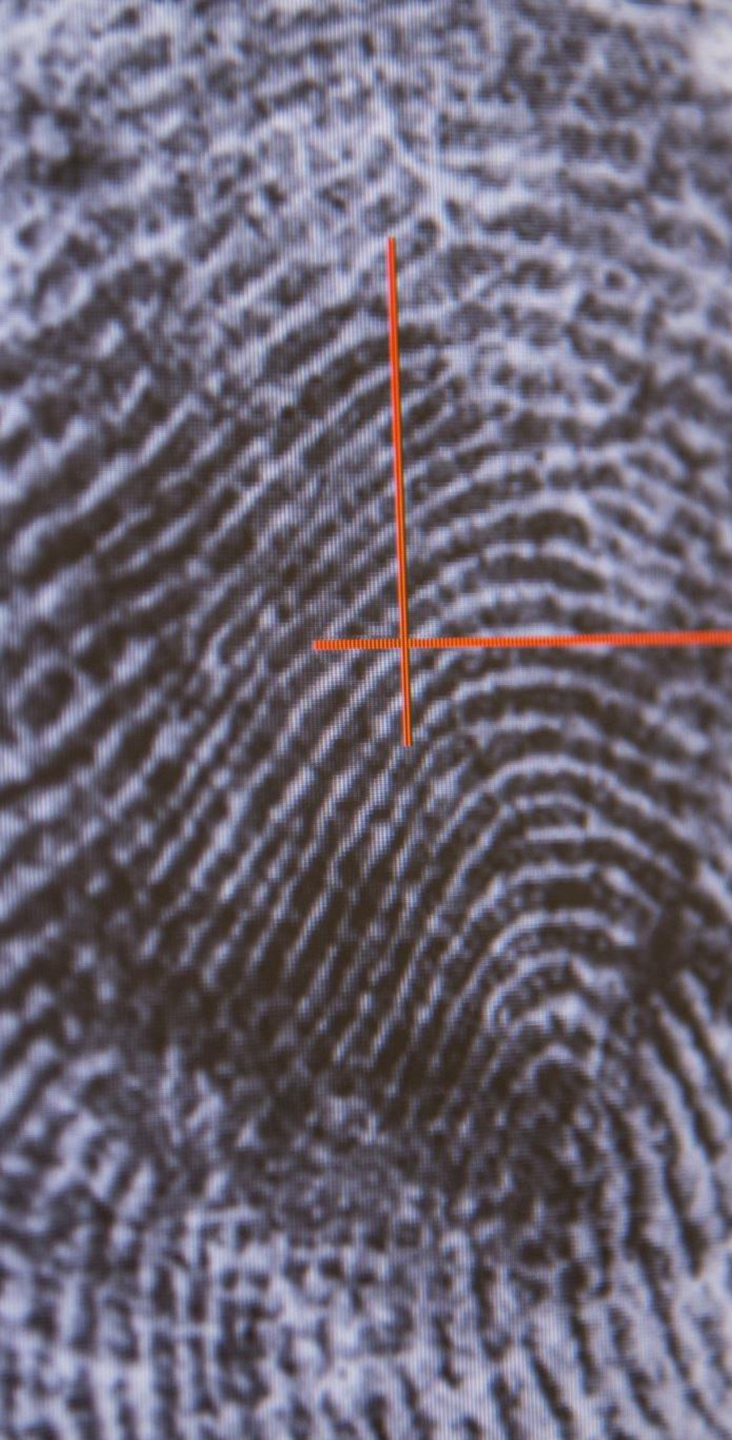
Population: 2, 4, 6, 8 and 10

$$\mu=\frac{2+4+6+8+10}{5}=\frac{30}{5}=6$$

$$\sigma=\sqrt{\frac{(2-6)^2+(4-6)^2+(6-6)^2+(8-6)^2+(10-6)^2}{5}}$$

$$\sigma=\sqrt{\frac{(-4)^2+(-2)^2+(0)^2+(2)^2+(4)^2}{5}}$$

$$\sigma=\sqrt{\frac{40}{5}}=\sqrt{8}=2\sqrt{2}$$



Verifications

- The mean of the sampling distribution of sample mean $\overline{X}$ is equal to the population mean.

$$\mu_{\overline{X}} = \mu = 6$$

- The standard deviation of sampling distribution is given by, $\sigma_{\overline{X}} = \frac{\sigma}{\sqrt{n}}$

$$2 = \frac{2\sqrt{2}}{\sqrt{2}}$$

$$2 = 2$$

Question 1

Compute the sampling distribution of mean for samples of size 3 without replacement for the following population distribution:

X	2	4	6	8
f(x)	1/6	2/6	2/6	1/6

And also verify that

$$\mu_{\bar{X}} = \mu \text{ and } \sigma_{\bar{X}} = \frac{\sigma}{\sqrt{n}}\sqrt{\frac{N-n}{N-1}}$$