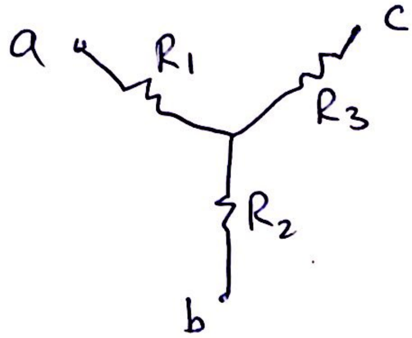
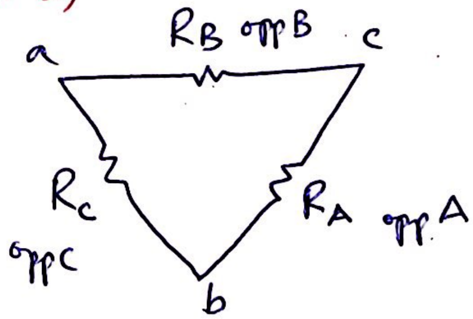
$\Delta - Y$ Conversion : The Proof

(PP155 8th Ed HKD)





Now $R_{ab} = R_1 + R_2$ and $R_{ab} = R_C // (R_A + R_B)$

Hence $$R_1 + R_2 = \frac{R_C R_A + R_C R_B}{R_A + R_B + R_C} \quad \text{———} \; (1)$$

Between $R_{bc} = R_2 + R_3$ and $R_{bc} = R_A // (R_B + R_C)$

So $$R_2 + R_3 = \frac{R_A R_B + R_A R_C}{R_A + R_B + R_C} \quad \text{———} \; (2)$$

and $$R_1 + R_3 = \frac{R_A R_B + R_B R_C}{R_A + R_B + R_C} \quad \text{———} \; (3)$$

Now (1) − (2) is:

$$R_1 + R_2 - R_2 - R_3 = \frac{(R_A R_C + R_B R_C) - (R_A R_B + R_A R_C)}{R_A + R_B + R_C}$$

or $$R_1 - R_3 = \frac{R_B R_C - R_A R_B}{R_A + R_B + R_C} \quad \text{———} \; (4)$$

Adding (3) + (4) is:

$$2R_1 = \frac{2 R_B R_C}{R_A + R_B + R_C}$$

$$R_1 = \frac{R_B R_C}{R_A + R_B + R_C} \quad \text{and} \quad R_2 = \frac{R_A R_C}{R_A + R_B + R_C}$$

and $$R_3 = \frac{R_A R_B}{R_A + R_B + R_C}$$

———— contd

— contd (155) We proved :-

$$R_1 = \frac{R_B R_C}{R_A + R_B + R_C} \text{ ——— } (C)$$

Therefore $R_A + R_B + R_C = \dfrac{R_B R_C}{R_1}$

and $\quad = \dfrac{R_A R_B}{R_3}$

and $\quad = \dfrac{R_A R_C}{2}$

Hence $\dfrac{R_B R_C}{R_1} = \dfrac{R_A R_B}{R_3} = \dfrac{R_A R_C}{R_2}$

Consider

$$\frac{R_B \cancel{R_C}}{R_1} = \frac{R_A \cancel{R_C}}{R_2}$$

So $R_B = \dfrac{R_1 R_A}{R_2}$ ——— (A) $\quad$ {determining R_B and R_C in terms of R_A}

and $\dfrac{R_B R_C}{R_1} = \dfrac{R_A R_B}{R_3}$

so $R_C = \dfrac{R_1 R_A}{R_3}$ ——— (B)

Putting (A) and (B) in (C) we get :-

$$R_1 = \frac{\left(\frac{R_1 R_A}{R_2}\right)\left(\frac{R_1 R_A}{R_3}\right)}{R_A + \frac{R_1 R_A}{R_2} + \frac{R_1 R_A}{R_3}} = \frac{(R_1 R_A)(R_1 R_A)}{R_A R_2 R_3 + R_1 R_A R_3 + R_1 R_A R_2}$$

$$1 = \frac{R_1 R_A}{R_2 R_3 + R_1 R_3 + R_1 R_2} \quad \text{or } R_A = \frac{R_1 R_3 + R_1 R_2 + R_2 R_3}{R_1}$$

and $R_B = \dfrac{R_1 R_3 + R_1 R_2 + R_2 R_3}{R_2}$ and

$$R_C = \frac{R_1 R_3 + R_1 R_2 + R_2 R_3}{R_3}$$