EE-381 Robotics-1
UG ELECTIVE

Lecture 7

Dr. Hafsa Iqbal
Department of Electrical Engineering,
School of Electrical Engineering and Computer Science,
National University of Sciences and Technology,
Pakistan

Tangent Functions

- Inverse tangent vs arc tangent (atan2)

$$\operatorname{atan2}(y, x) = \begin{cases} \arctan\left(\frac{y}{x}\right) & \text{if } x > 0, \\ \arctan\left(\frac{y}{x}\right) + \pi & \text{if } x < 0 \text{ and } y \geq 0, \\ \arctan\left(\frac{y}{x}\right) - \pi & \text{if } x < 0 \text{ and } y < 0, \\ +\frac{\pi}{2} & \text{if } x = 0 \text{ and } y > 0, \\ -\frac{\pi}{2} & \text{if } x = 0 \text{ and } y < 0, \\ \text{undefined} & \text{if } x = 0 \text{ and } y = 0. \end{cases}$$

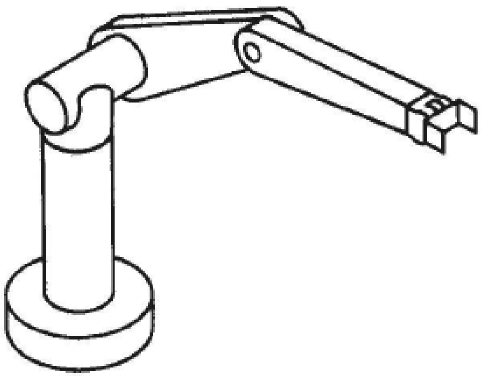
IK for General Robot Arm in 3D

- 6-DOF puma robot

https://www.youtube.com/watch?v=tjOhGqOHfhg

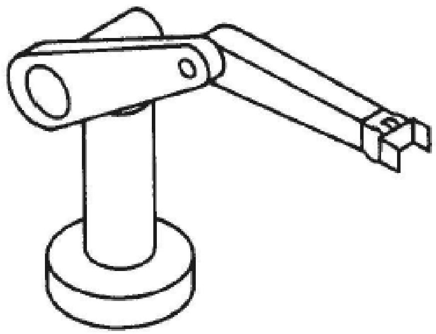
IK for General Robot Arm in 3D

- Multiple possible configurations
  - Left/right handed
  - Elbow (up/down)
  - Wrist position



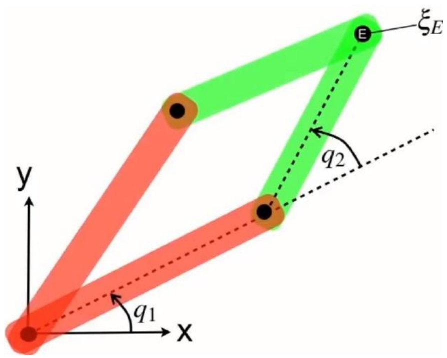
LEFT

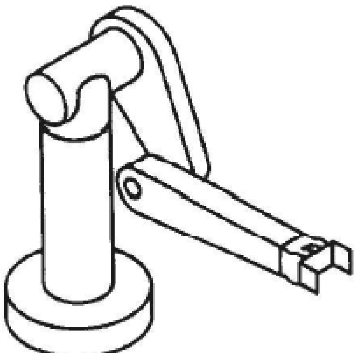
clockwise rotation



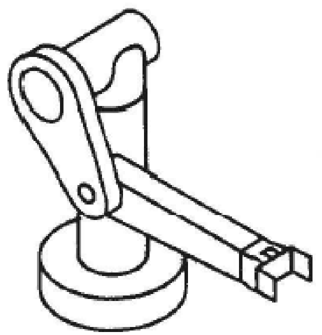
RIGHT

(flip/not flip)





left and elbow down



right and elbow down

Robot Arm Configuration Change

- A characteristic of inverse kinematics is that there is often more than one solution, that is, more than one set of joint angles gives exactly the same end-effector pose
- The end point remains the same whereas joint angles need to adjust themselves
- How robot move from one configuration to other? Or more specifically can we compute the trajectory for configuration change?
- Example case from left handed configuration to right hand configuration or vice versa.

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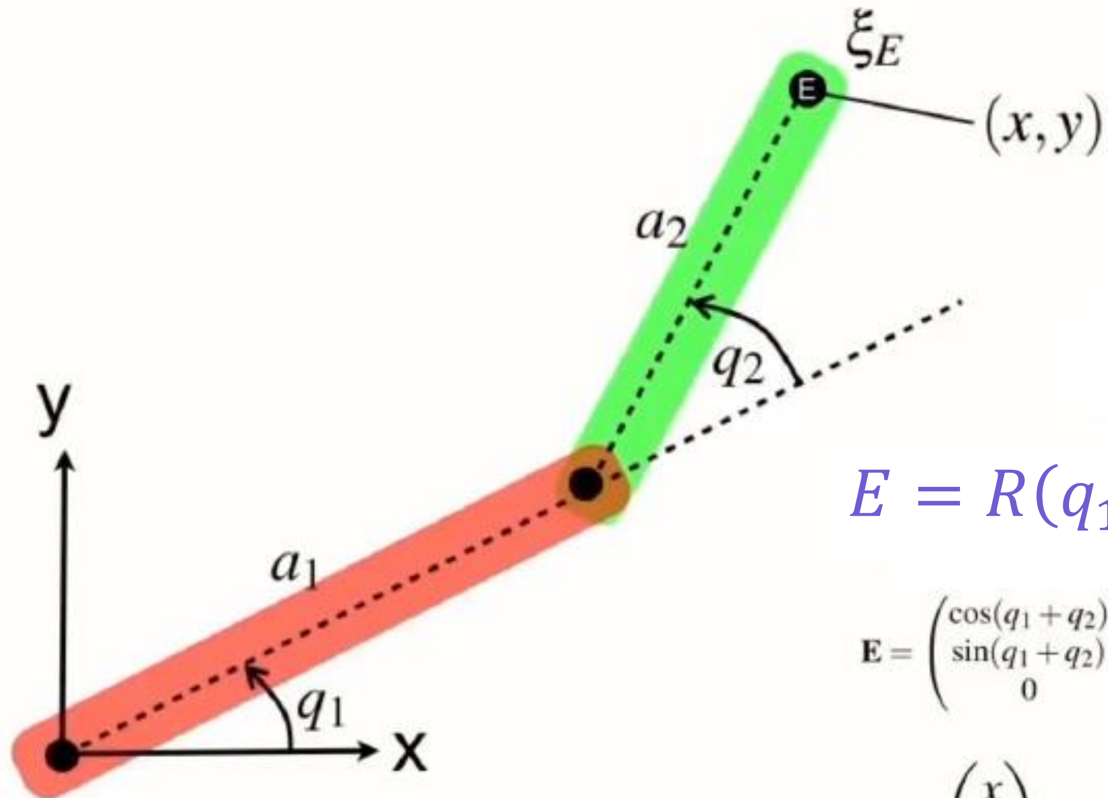
Solving for IK of a Manipulator

- Analytical Solution
  - Pros?
  - Cons?
- What is the alternative and when we need it?

IK through **Numerical Analysis**



$$E = R(q_1)T_x(a_1)R(q_2)T_x(a_2)$$

$$\mathbf{E} = \begin{pmatrix} \cos(q_1+q_2) & -\sin(q_1+q_2) & a_2\cos(q_1+q_2)+a_1\cos q_1 \\ \sin(q_1+q_2) & \cos(q_1+q_2) & a_2\sin(q_1+q_2)+a_1\sin q_1 \\ 0 & 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} a_2\cos(q_1+q_2)+a_1\cos q_1 \\ a_2\sin(q_1+q_2)+a_1\sin q_1 \end{pmatrix}$$

Solving for IK of a Manipulator

- Numerical Solution
- We can derive the forward kinematics of a Robot

$$\xi = \mathcal{K}(\boldsymbol{q})$$

- In IK problem, we know the desired pose (ξ^*)
- To find **q**, we adjust **q** until
- This is formally achieved through optimization ξ^*

$$\xi \rightarrow \xi^*$$

$$q^* = arg\min_{q} |\mathcal{K}(\boldsymbol{q}) \ominus \xi^*|$$

IK through Numerical Analysis

Serial and parallel link manipulators

- Problems
  - Initial joint coordinates is critical
  - No knowledge about a particular robot configuration
  - Computationally expensive

Reachability and Singularity

- Reach-ability?
  - End-Effector is not capable of adopting particular orientations
- Singularity:
  - Loss of degree of freedom

Redundant Robots

- Robots with high degree of freedom

$$\mathcal{C} \subset \mathbb{R}^N$$

$$\mathcal{T} \subset \mathbb{R}^3 \times \mathbb{S}(3)$$

*A highly redundant robot with N joints has large configuration space

$$\xi_E \sim (x, y, z, \theta_r, \theta_p, \theta_y)$$

We will prefer to use Numerical Solution for IK because of large #(joints).