CONVECTION AND CONDUCTION CURRENTS

Introduction

- Until now, we considered electrostatic fields in free space or a space that has no materials in it

- Thus what we have developed so far under electrostatics may be regarded as the "vacuum" field theory

- Similarly, we shall develop the theory of electric phenomena in material space

- Most of the formulas derived for Electrostatic fields in free space are still applicable, though some may require modification

Electric Current

- Electric voltage (or potential difference) and current are two fundamental quantities in electrical engineering

- Before examining how electric field behaves in a conductor or dielectric, it is appropriate to consider electric current

- Electric current is generally caused by the motion of electric charges

- **The current (in amperes) through a given area is the electric charge passing through the area per unit time**

$$I = \frac{dQ}{dt}$$

Current Density

- Thus in a current of one ampere, charge is being transferred at a rate of one coulomb per second

- We now introduce the concept of current density **J**

- If current ΔI flows through a surface ΔS, the current density is:

$$J_n = \frac{\Delta I}{\Delta S}$$

- OR

$$\Delta I = J_n \Delta S$$

- The equations above assume that the current density is perpendicular to the surface

Current Density

- If the current density is not normal to the surface, then:

$$\Delta I = \mathbf{J} \cdot \Delta S$$

- Thus, the total current flowing through a surface *S* is:

$$I = \int_S \mathbf{J} \cdot d\mathbf{S}$$

- The current density at a given point is the current through a unit normal area at that point

Convection Current

- Convection current, as distinct from conduction current, does not involve conductors and consequently does not satisfy Ohm's law

- It occurs when current flows through mediums such as liquid, rarefied gas, or a vacuum (Rarefied gas: A gas whose pressure is much less than atmospheric pressure)

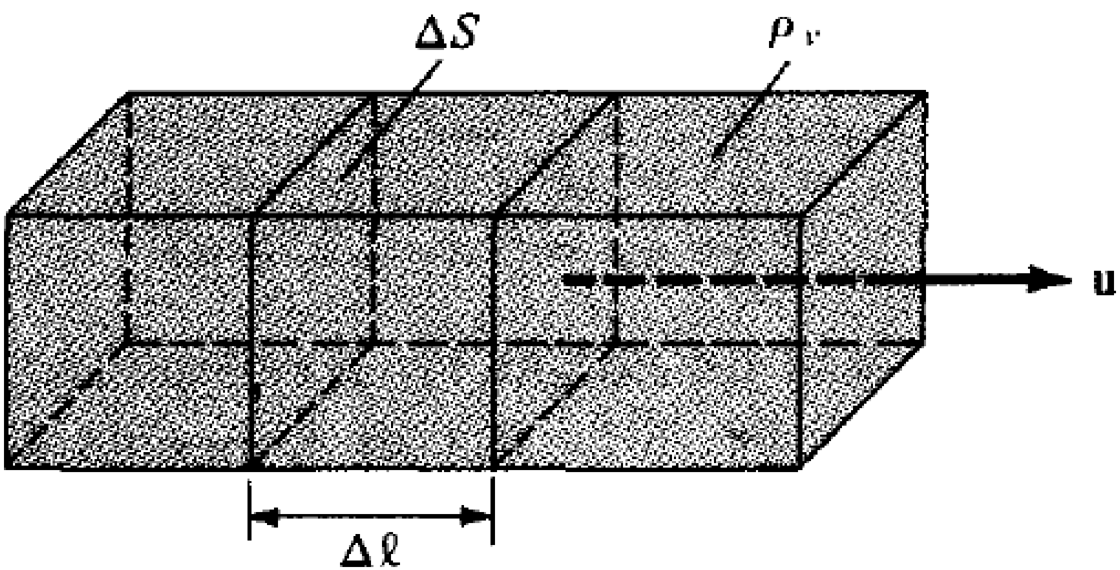
- A beam of electrons in a vacuum tube, for example, is a convection current

Convection Current

- Consider a filament shown in Figure below

- If there is a flow of charge of volume density ρ_v, at velocity $\mathbf{u} = u_y \mathbf{a_y}$, the current through the filament is:

$$\Delta I = \frac{\Delta Q}{\Delta t} = \rho_v \, \Delta S \frac{\Delta \ell}{\Delta t} = \rho_v \, \Delta S \, u_y$$



Convection Current

- The y-directed current density J_y is given by:

$$J_y = \frac{\Delta I}{\Delta S} = \rho_v u_y$$

- Hence in general:

$$\mathbf{J} = \rho_v \mathbf{u}$$

- The current I is the convection current and J is the convection current density in amperes/square meter (A/m^2)

Conduction Current

- Conduction current requires a conductor

- A conductor is characterized by a large amount of free electrons that provide conduction current due to an impressed electric field

- When an electric field **E** is applied, the force on an electron with charge $-e$ is:

$$\mathbf{F} = -e\mathbf{E}$$

- Since the electron is not in free space, it will not be accelerated under the influence of the electric field only

- Rather, it suffers constant collision with the atomic lattice and drifts from one atom to another

Conduction Current

- Consider an electron with mass m is moving in an electric field **E** with an average drift velocity **u**

- According to Newton's law, the average change in momentum equals the applied force, thus:

- $$\frac{m\mathbf{u}}{\tau} = -e\mathbf{E} \qquad \text{OR} \qquad \mathbf{u} = -\frac{e\tau}{m}\mathbf{E}$$

- where τ is the average time interval between collisions

- This indicates that the drift velocity of the electron is directly proportional to the applied field

Conduction Current Density

- If there are *n* electrons per unit volume, the electronic charge density is given by:

$$\rho_v = -ne$$

- Thus the *conduction current density* is:

$$\mathbf{J} = \rho_v \mathbf{u} = \frac{ne^2\tau}{m}\mathbf{E} = \sigma\mathbf{E}$$

- OR

$$J = \sigma E$$

- where $\boldsymbol{\sigma = ne^2\tau/m}$ is the conductivity of the conductor

- The above relationship is known as the point form of Ohm's law

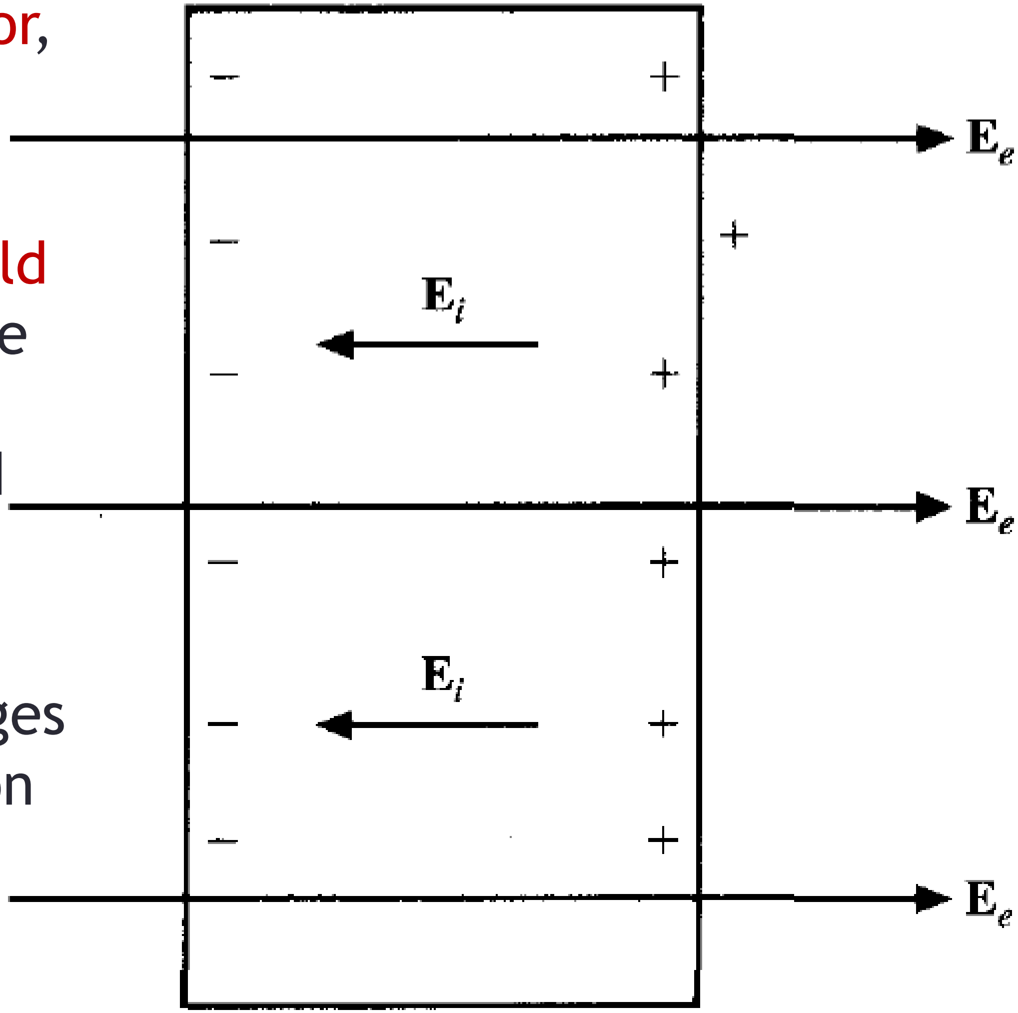
Conductors

- A conductor has abundance of charge that is free to move

- Consider an isolated conductor, such as shown in Figure

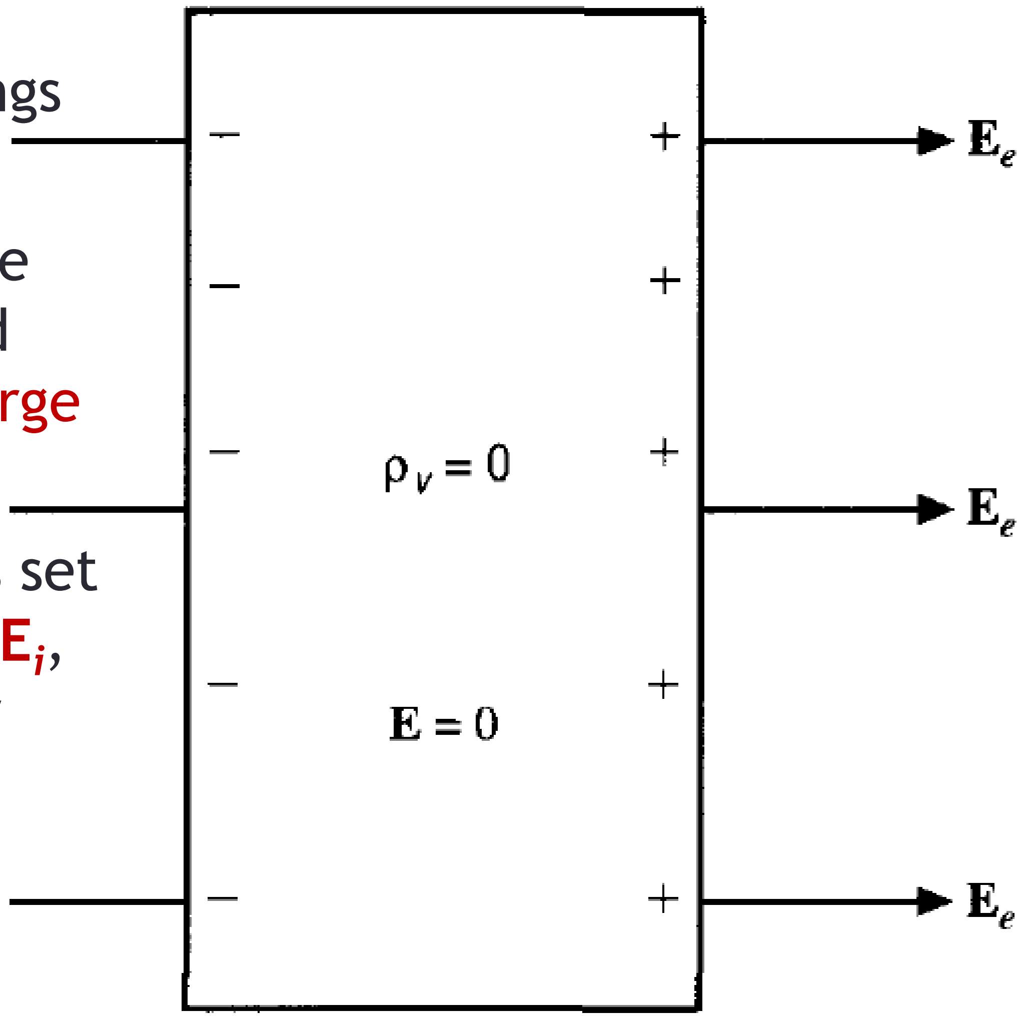
- When an external electric field E_e is applied, the positive free charges are pushed along the same direction as the applied field

- While the negative free charges move in the opposite direction



Conductors

- The charge migration takes place very quickly
- The free charges do two things
- First, they accumulate on the surface of the conductor and form an induced surface charge
- Second, the induced charges set up an internal induced field $\mathbf{E}_i$, which cancels the externally applied field $\mathbf{E}_e$



Conductors

- The previous discussion leads to an important property of a conductor:

- *A **perfect conductor** cannot contain an electrostatic field within it*

- A conductor is called an equipotential body, implying that the potential is the same everywhere in the conductor

- This is based on the fact that $\mathbf{E} = -\nabla V = 0$

Conductors

- Another way of looking at this is to consider Ohm's law, $\mathbf{J} = \sigma\mathbf{E}$
- To maintain a finite current density **J**, in a perfect conductor $(\sigma \to \infty)$, requires that the electric field inside the conductor must vanish
- In other words, $\mathbf{E} \to 0$ because $\sigma \to \infty$ in a perfect conductor
- According to Gauss's law, if **E** = 0, the charge density ρ_v must be zero
- Thus under static conditions:

$$\mathbf{E} = 0, \qquad \rho_v = 0, \qquad V_{ab} = 0 \qquad \text{inside a conductor}$$

Problem-1

- For the current density $\mathbf{J} = 10zsin^2\emptyset\ \mathbf{a_\rho}$ A/m^2, find the current through the cylindrical surface $\rho = 2m, 1m \leq z \leq 5m.$