Assignment # 5

Electrical Network Analysis (EE-211)

Submitted to: Ahsan Azhar

Class: BEE 12 C

Due: 16/06/2021

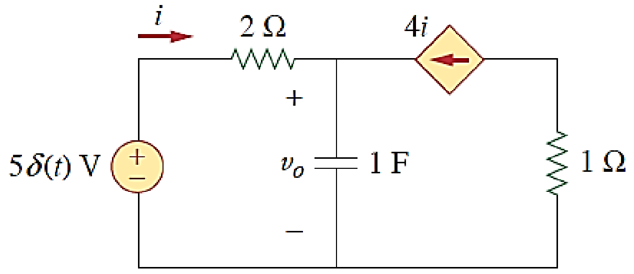
By

Muhammad Umer

CMS: 345834

Problem 16.16

The capacitor in the circuit of Fig. 16.39 is initially uncharged. Find v0(t) for t > 0.



Solution

The circuit in s-domain transforms to:

$5\,\delta(t) \longrightarrow 5$

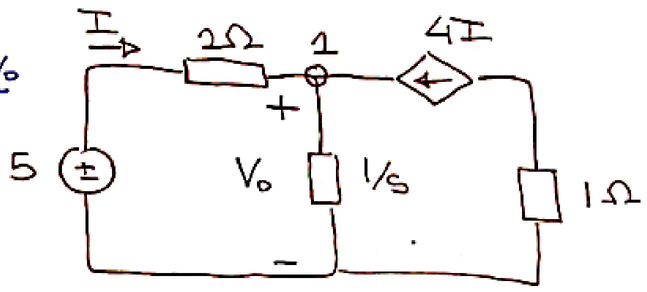
$i \longrightarrow I$

$2\Omega \longrightarrow 2\Omega$

$1F \longrightarrow \dfrac{1}{s}$

$1\Omega \longrightarrow 1\Omega$

$I = \dfrac{5 - V_o}{2}$



Applying Nodal Analysis at 1:

$\Rightarrow I + 4I = V_o(s)$

$\Rightarrow 5I = sV_o$

However, $I = \dfrac{5 - V_o}{2}$; Substituting

$\Rightarrow 5\left(\dfrac{5 - V_o}{2}\right) = sV_o$

$$\Rightarrow \quad 5(5 - V_o) = 2sV_o$$

$$\Rightarrow \quad 25 - 10V_o = 2sV_o$$

$$\Rightarrow \quad V_o(10 + 2s) = 25$$

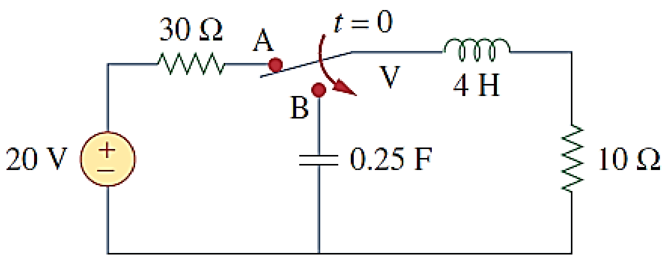
$$\Rightarrow \quad V_o = \frac{25}{10 + 2s} = \frac{12.5}{s + 5/2}$$

• $v_o(t) = L^{-1}(V_o(s))$

$$\boxed{v_o(t) = (12.5\, e^{-2.5t})\, u(t)\ V} \qquad \because e^{-at} \leftrightarrow \frac{1}{s+a}$$

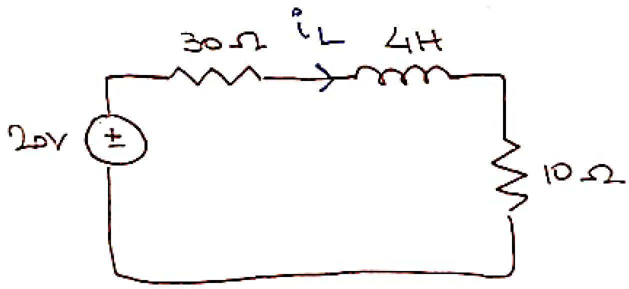
Problem 16.19

The switch in Fig. 16.42 moves from position A to position B at t = 0 (please note that the switch must connect to point B before it breaks the connection at A, a make before break switch). Find $v(t)$ for $t > 0$.



Solution

For $t < 0$



We can find $i_L(0)$ by shorting the inductor. Hence,

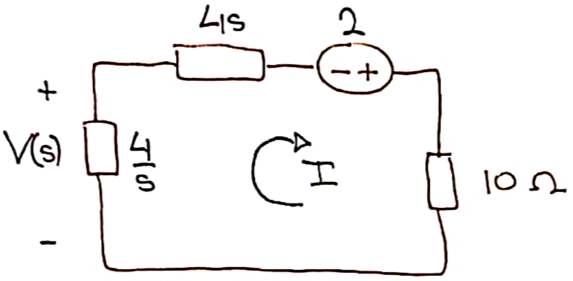
$$i_L(0) = \frac{20}{30 + 10} = \boxed{0.5\ A}$$

For $t > 0$

$\frac{1}{4}F \rightarrow \frac{4}{s}$

$4H \rightarrow 4s$ $\qquad\qquad$ $L\,i_L(0) \rightarrow 2V$

• $V(s) = -I\left(\frac{4}{s}\right)$



$\Rightarrow \frac{4}{s} I + 4sI - 2 + 10I = 0$

$\Rightarrow I\left(\frac{4}{s} + 4s + 10\right) = 2$

$\Rightarrow I = \dfrac{2}{\frac{4}{s} + 4s + 10} \Rightarrow \dfrac{2s}{4 + 4s^2 + 10s}$

• Using ohm's law for V required

$V = -I\left(\frac{4}{s}\right)$

$= \dfrac{-2\cancel{s}}{4 + 4s^2 + 10s}\left(\dfrac{4}{\cancel{s}}\right) = \dfrac{-8}{4s^2 + 10s + 4}$

$$\boxed{V(s) = \frac{-2}{s^2 + 2.5s + 1}}$$

$\Rightarrow \dfrac{-2}{s^2 + 2.5s + 1} = \dfrac{A}{s+2} + \dfrac{B}{s + 1/2}$ $\quad\therefore$ Poles

For A:

$$\left.\frac{-2}{s+1/2}\right|_{s=-2} \Rightarrow A = 4/3$$

For B:

$$\left.\frac{-2}{s+2}\right|_{s=-1/2} \Rightarrow B = -4/3$$

$$\Rightarrow V(s) = \frac{4}{3(s+2)} - \frac{4}{3(s+1/2)}$$

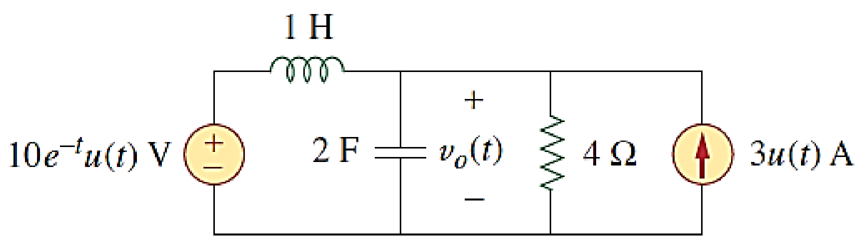
$$= \frac{4}{3}\left(\frac{1}{s+2} - \frac{1}{s+1/2}\right)$$

Taking $\mathcal{L}^{-1}$ of $V(s)$

$$\Rightarrow \boxed{v(t) = \frac{4}{3}\left(e^{-2t} - e^{-0.5t}\right) u(t)\ V}$$

Problem 16.35

Find $V_0(t)$ in the circuit of Fig. 16.58.



Solution

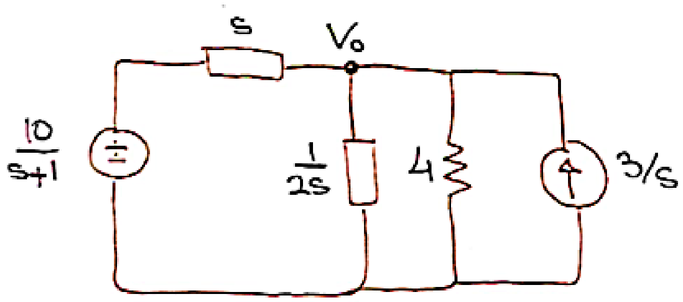
Converting this circuit into s-domain;

$$10e^{-t}u(t) \rightarrow \frac{10}{s+1}$$

$$1\ H \rightarrow s$$

$2F \rightarrow \frac{1}{2s}$

$3\,u(t) \rightarrow \frac{3}{s}$



Applying KCL to V_o,

$$\Rightarrow \frac{V_o}{1/2s} + \frac{V_o}{4} + \frac{V_o - 10/s+1}{s} = 3/s$$

Multiplying with s,

$$\Rightarrow 2V_o s^2 + \frac{V_o}{4}s + V_o = 3 + \frac{10}{s+1}$$

$$\Rightarrow V_o\left(2s^2 + \frac{s}{4} + 1\right) = 3 + \frac{10}{s+1}$$

$$\Rightarrow V_o = \frac{\frac{3s+3+10}{s+1}}{(2s^2+s/4+1)}$$

$$\Rightarrow V_o = \frac{3s+13}{(s+1)(2s^2+s/4+1)} = \frac{1.5s+6.5}{(s+1)(s^2+s/8+1/2)}$$

Using Heaviside Expansion ;

$$\Rightarrow \frac{1.5s+6.5}{(s+1)(s^2+s/8+1/2)} = \frac{A}{s+1} + \frac{B}{\left(s+\frac{1}{16}-\frac{\sqrt{127}}{16}j\right)} + \frac{B^*}{\left(s+\frac{1}{16}+\frac{\sqrt{127}}{16}j\right)}$$

- For A

$$\left.\frac{1.5s+6.5}{s^2+s/8+1/2}\right|_{s=-1} \Rightarrow A = \frac{40}{11}$$

- For B

$$\left.\frac{1.5s+6.5}{(s+1)\left(s+\frac{1}{16}+\frac{\sqrt{127}}{16}j\right)}\right|_{s=-\frac{1}{16}+\frac{\sqrt{127}}{16}j}$$

$$\Rightarrow B = \frac{1.5\left(-1/16+\sqrt{127}/16\,j\right)+6.5}{\left(15/16+\sqrt{127}/16\,j\right)\left(\frac{\sqrt{127}}{8}j\right)} = 3.93\angle -117.55^\circ$$

Substituting in old equation;

$$\Rightarrow \frac{1.5s+6.5}{(s+1)(s^2+\frac{s}{4}+1)} = \frac{40/11}{s+1} + \boxed{\frac{3.93\angle -117.55}{s+1/16-\sqrt{127}/16\,j} + \frac{3.93\angle 117.55}{s+1/16+\sqrt{127}/16\,j}}$$

Taking $\mathcal{L}^{-1}$

$$V_o(t) = \frac{40}{11}e^{-t} + \boxed{2Re^{\sigma t}\left(\cos(\omega t+\phi)\right)}$$

- Dotted Part:

$R = 3.93$, $\sigma = -1/16$, $\omega = \frac{\sqrt{127}}{16}$, $\phi = 117.55$

$$\boxed{V_o(t) = \frac{40}{11}e^{-t} + 7.86\,e^{-t/16}\cos\left(\sqrt{127}/16\,t - 117.55\right)u(t)\ V}$$