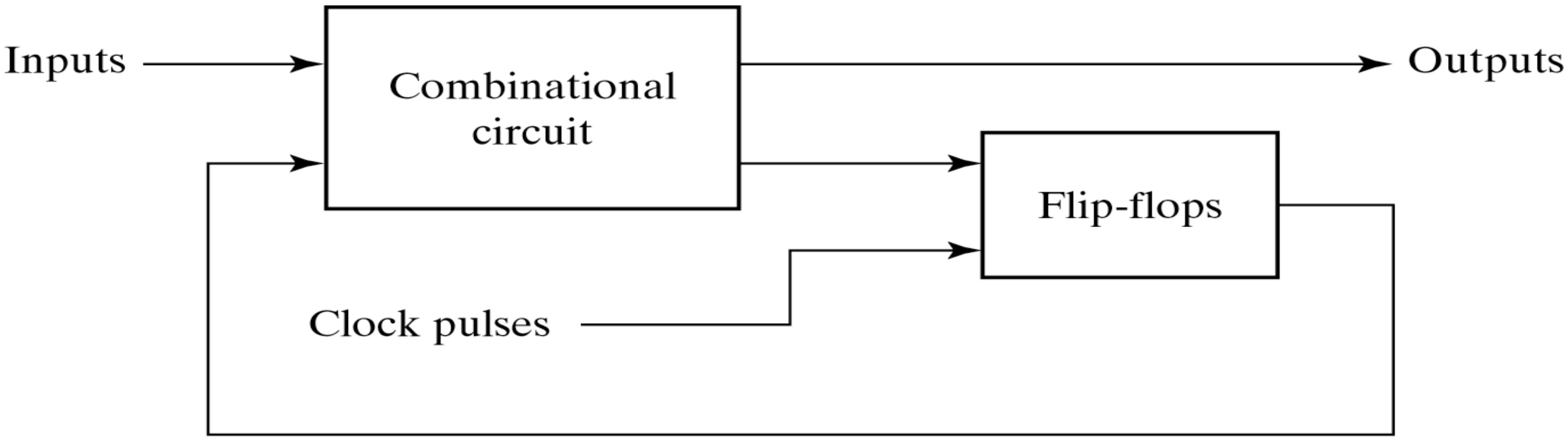
Chapter5-Synchronous Sequential Logic

Lecture5- Analysis of Clocked Sequential Circuits

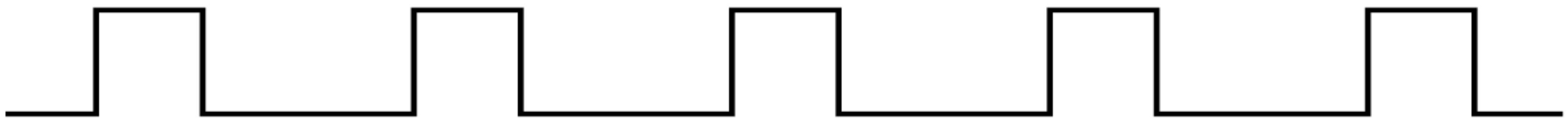
Objectives

- Analyze Clocked Sequential Circuits with D, JK, and T Flip-flops

Block Diagram of Synchronous Sequential Circuit



(a) Block diagram



(b) Timing diagram of clock pulses

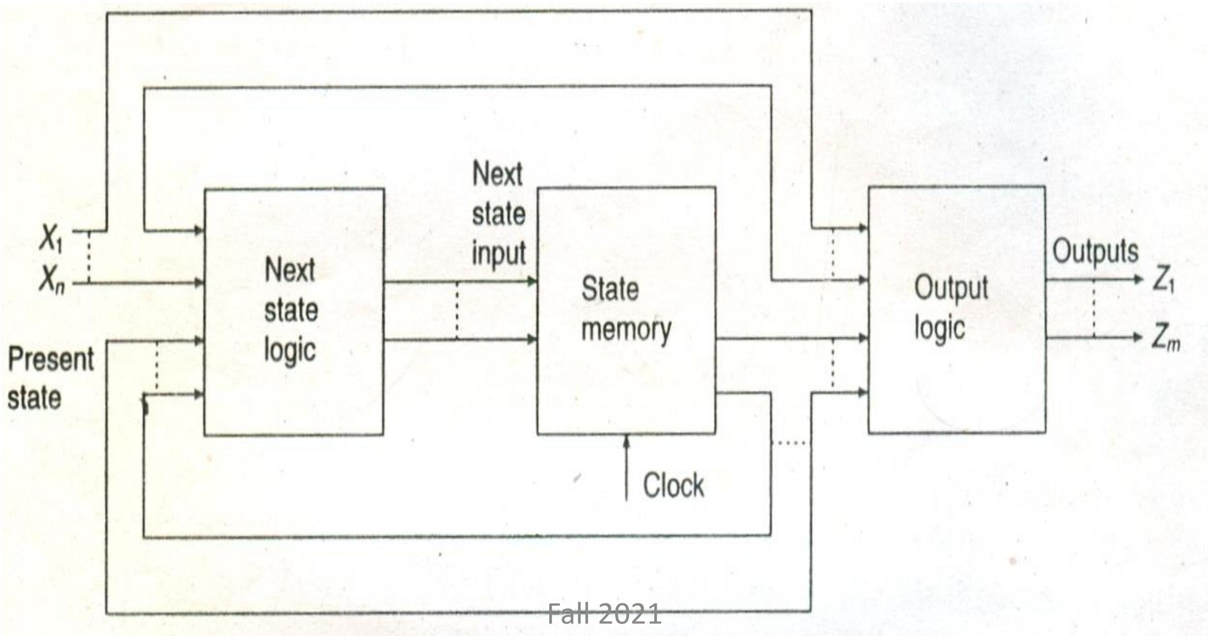
Fig. 5-2 Synchronous Clocked Sequential Circuit

Mealy and Moore Models

- There are two types of synchronous sequential machines namely Mealy and Moore. These machines differ in the way the output is generated.
  - In the first type of machine the output depends on both its present state and also its inputs. This type of machine is referred to as the Mealy machine and its behavior is defined by the following equations:

    Next State=f(Present State, inputs)

    Output=g(Present State, inputs)

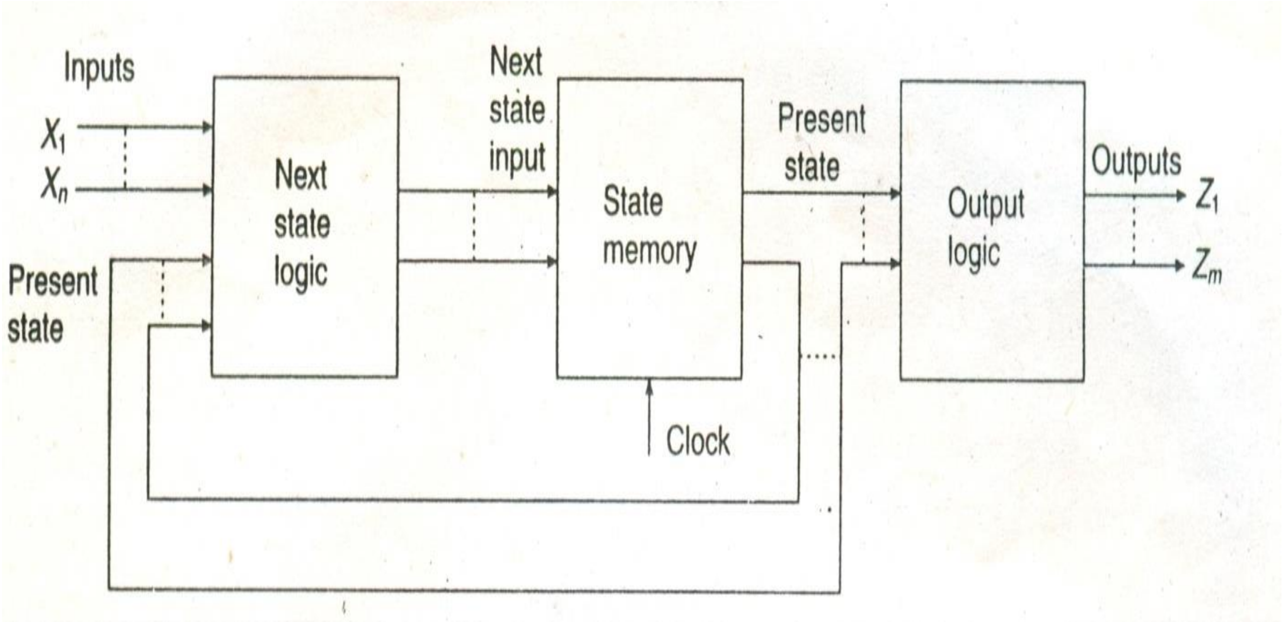


Mealy and Moore Models

The second of these machines has an output that depends only on its present state and is referred to as Moore machine. The behavior of the machine is defined by the following equations:

Next State=f(Present State, inputs)

Output=g(Present State)



Analysis of Clocked Sequential Circuits

- The behavior of a clocked sequential circuit is determined from the inputs, the outputs, and the state of its flip flops.
  - The outputs and the next state are both a function of the inputs and the present state.
- Analysis starts with the given logic diagram or a set of equations from which block diagram can be drawn and consists of obtaining a table or a diagram for the time sequence of inputs, outputs, and internal states.
- From the state table or diagram we can determine the function performed by the clocked sequential circuit.

State Equations

- The behavior of a clocked sequential circuit can be described algebraically by means of state equations.
- A state equation (also called transition equation) specifies the next state as a function of the present state and inputs.
  - It is an algebraic equation that specifies the condition for a flip flop state transition.
- Consider the circuit shown in next slide. It consists of two D flip-flops A and B and an output y.

Example of Sequential Circuit

Since the D input determines the next state:

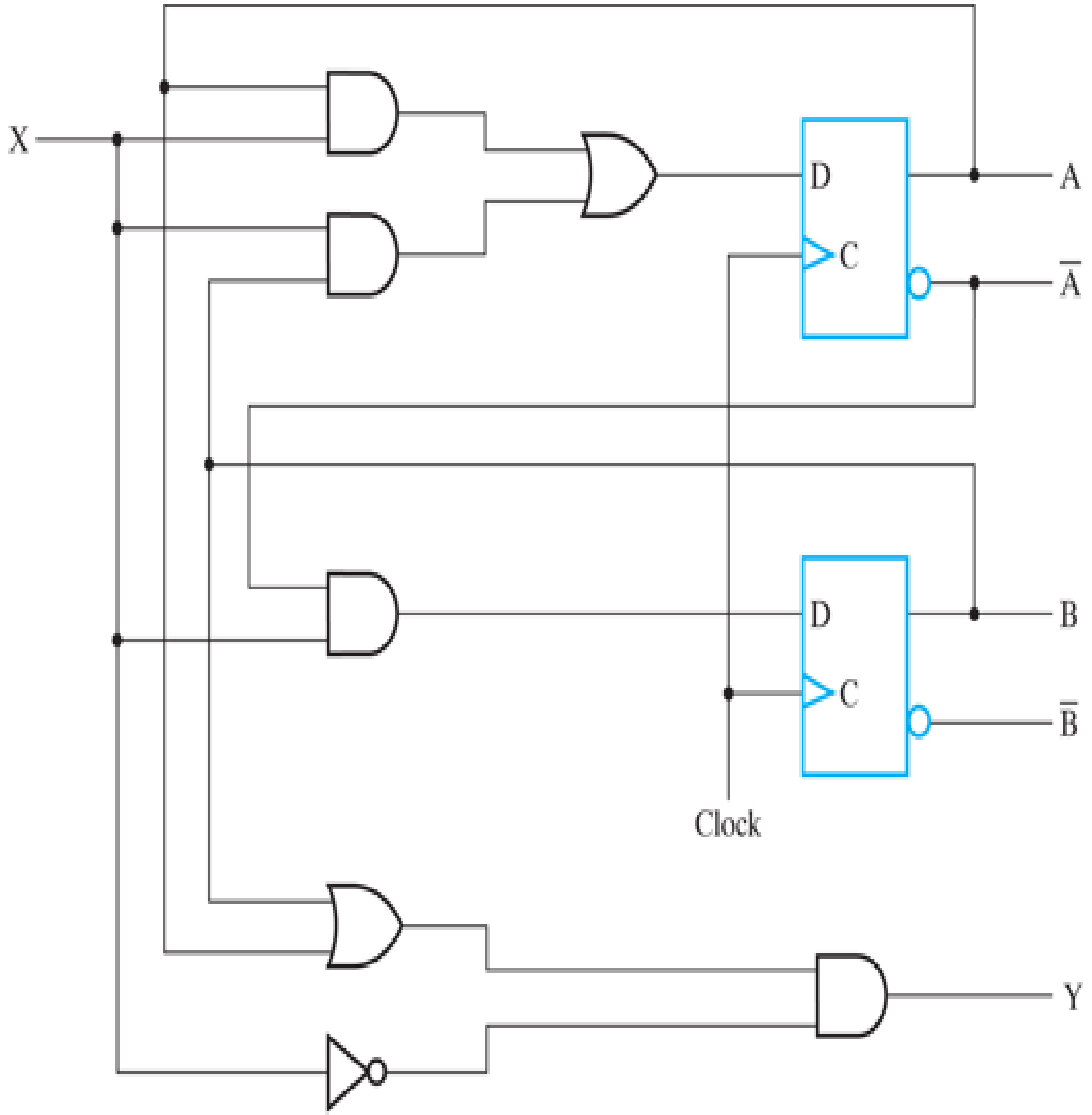
- $A(t + 1) = A(t)x(t) + B(t)x(t)$

  $= Ax + Bx;$

  State Equation of A
- $B(t + 1) = A'(t)x(t) = A'x$

  State Equation of B
- $y(t) = [A(t) + B(t)]x'(t) = (A + B)x'$

  Output Equation of Circuit



State Table- Our Example Cont…

- The time sequence of inputs, outputs, and flip flop states can be enumerated in a state table (sometimes called a transition table).
  - The table consists of four sections
    - Present state shows the states of the flip flops at time t
    - Input gives input values for each possible present state
    - Next state shows the states of the flip flops one cycle later at t + 1
    - Output gives the value of circuit outputs at time t for each present state and input condition
- The derivation of a state table requires listing all possible binary combinations of present state and inputs.
  - In our example, we have eight combinations from 000 to 111.
- The next state values are then determined from the logic diagram or from the state equations.

State Table- Our Example Cont…

Present State		Input	Next State		Output
A	B	X	A	B	Y
0	0	0	0	0	0
0	0	1	0	1	0
0	1	0	0	0	1
0	1	1	1	1	0
1	0	0	0	0	1
1	0	1	1	0	0
1	1	0	0	0	1
1	1	1	1	0	0

Generic Procedure for State Table

- A sequential circuit with m flip flops and n inputs needs 2^{m+n} rows in the state table.
- The numbers 0 through $2^{m+n} - 1$ are listed under the present state and input columns.
- The next state section has m columns, one for each flip flop.
  - Next state values are derived from the state equations.
- The output section has as many columns as there are output values
  - Output values are derived from the circuit or the Boolean function in the same matter as a truth table.

An alternate form of State Table

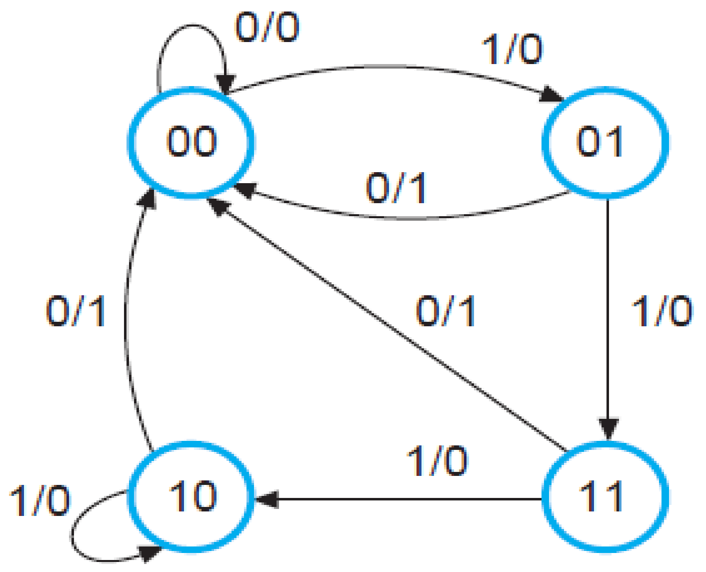
Present state		Next state				Output	
		X = 0		X = 1		X = 0	X = 1
A	B	A	B	A	B	Y	Y
0	0	0	0	0	1	0	0
0	1	0	0	1	1	1	0
1	0	0	0	1	0	1	0
1	1	0	0	1	0	1	0

Present State		Input	Next State		Output
A	B	X	A	B	Y
0	0	0	0	0	0
0	0	1	0	1	0
0	1	0	0	0	1
0	1	1	1	1	0
1	0	0	0	0	1
1	0	1	1	0	0
1	1	0	0	0	1
1	1	1	1	0	0

State Diagram

- Information in a state table can be represented graphically in the form of a state diagram.
- In a state diagram:
  - ➢ a state is represented by a circle
  - ➢ transitions between states are indicated by directed lines connecting the circles
  - ➢ Binary numbers inside the circles represent state of the flip flops
  - ➢ Directed lines are labeled with two binary numbers separated by a slash
    - o The input value during the present state is labeled first
    - o The second number gives the output after the present state with the given input

Example State Diagram



Present state		Next state				Output	
		X = 0		X = 1		X = 0	X = 1
A	B	A	B	A	B	Y	Y
0	0	0	0	0	1	0	0
0	1	0	0	1	1	1	0
1	0	0	0	1	0	1	0
1	1	0	0	1	0	1	0

Flip Flop Input Equations

- The part of the circuit that generates the inputs to flip flops is described algebraically by a set of Boolean functions called flip flop input equations (excitation equations).
- The notation of an input equation consists of the flip flop input symbol with a subscript to denote the name of the flip flop output
  - ➢ $D_Q = x + y$
- In our example the following input equations would be used:
  - ➢ $D_A = Ax + Bx$
  - ➢ $D_B = A'x$
  - ➢ $Y = (A + B)x'$

Analysis With D Flip Flops

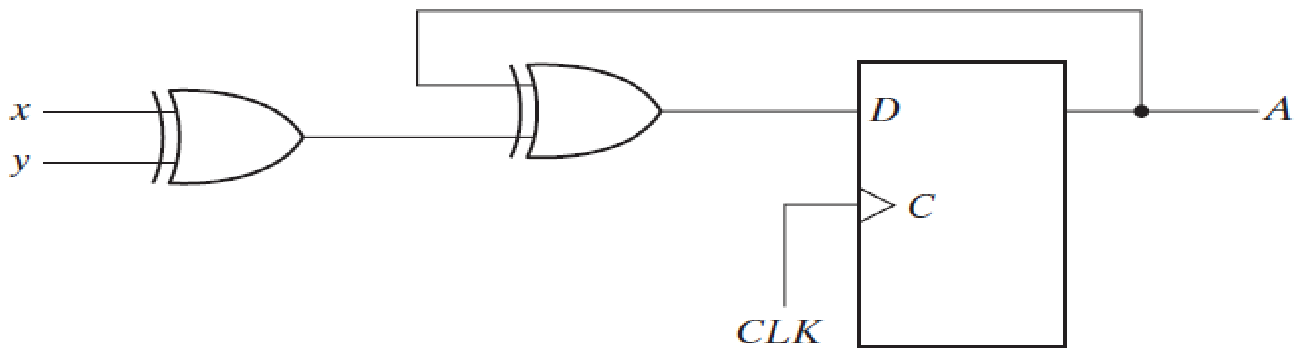
- We start analysis of a sequential circuit described by the following input equation:

  $D_A = A \oplus x \oplus y$

- The symbol D_A implies a D flip flop with output A.
- The x and y variables are the inputs to the circuit.
- No outputs are given so the output is implied to come from the output of the flip flop.
- The state table has one column for flip-flop A, two columns for the two inputs, and one column for the next state of A. The binary numbers under Axy are listed from 000 through 111.
- The next-state values are obtained from the state equation

  $A(t+1)=A \oplus x \oplus y$

Analysis With D Flip Flops Cont…

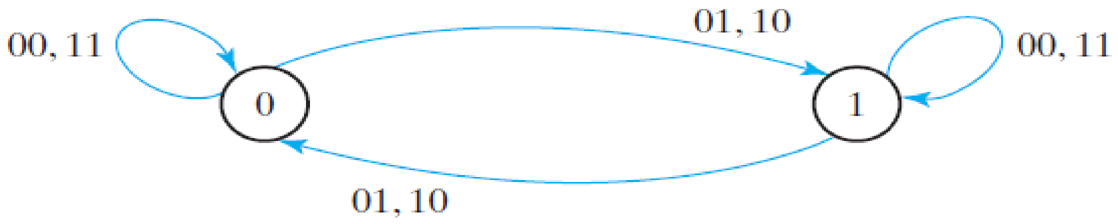


(a) Circuit diagram

Present state	Inputs		Next state
A	x	y	A
0	0	0	0
0	0	1	1
0	1	0	1
0	1	1	0
1	0	0	1
1	0	1	0
1	1	0	0
1	1	1	1

(b) State table

- **DA** = $A \oplus x \oplus y$



(c) State diagram

Fig. 5-17 Sequential Circuit with *D* Flip-Flop

Analysis Notes

- With D type flip flops, the state equation is the same as the input equation.
- With JK and T flip flops, it is necessary to refer to the corresponding characteristic table or characteristic equation to obtain the next state values.

JK and T Flip Flop Analysis

- For a D type flip-flop, the state equation is same as the input equation. When other than the D type flip-flop is used, such as JK or T, it is necessary to refer to the corresponding characteristic table or characteristic equation to obtain the next-state values.
- The next-state values of a sequential circuit that uses flip flops such as JK or T type can be derived using the following procedure:
  - ➢ Determine the flip flop input equations in terms of the present state and input variables
  - ➢ List the binary values of each input equation
  - ➢ Use the corresponding flip flop characteristic table to determine the next state values in the state table

JK Analysis Example

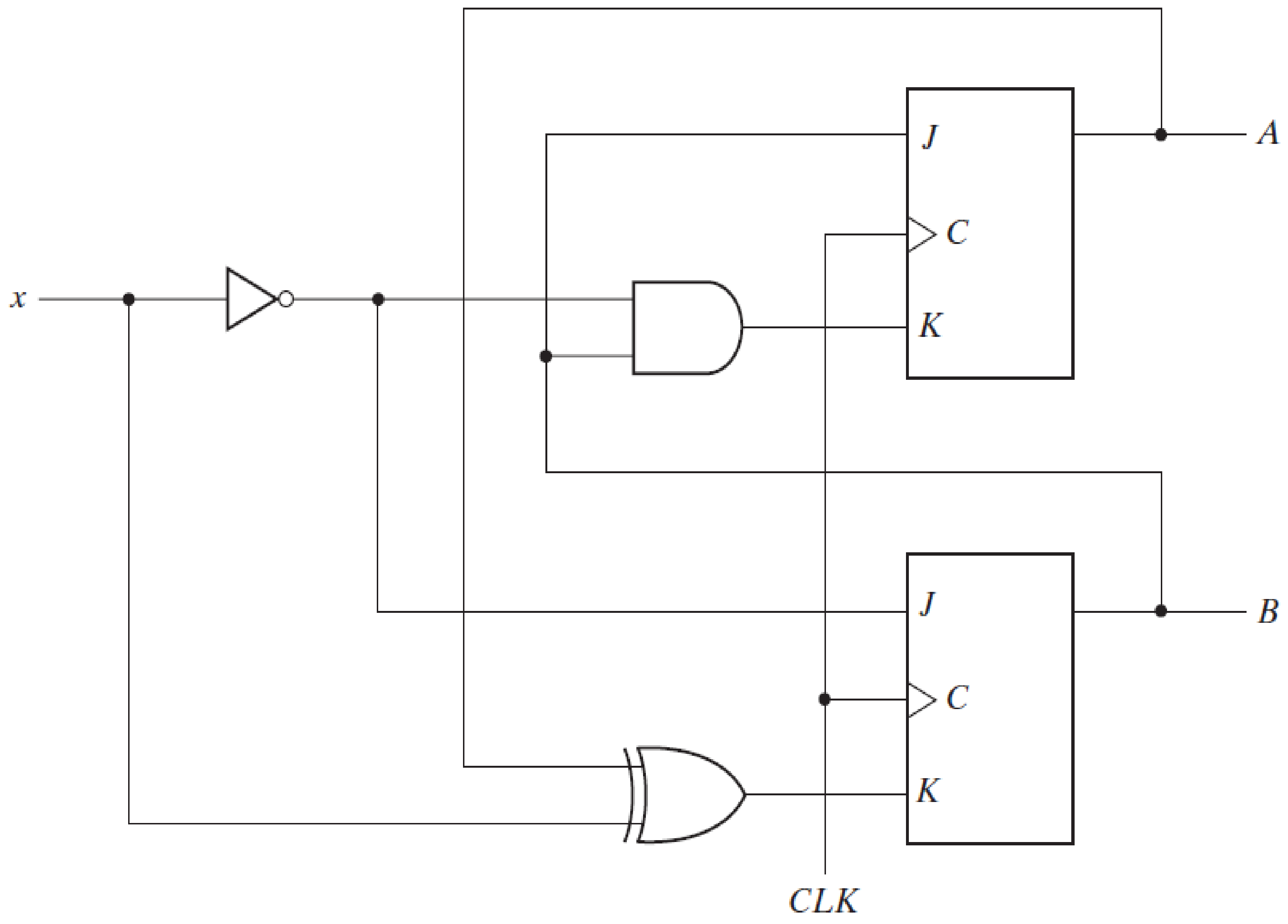


Fig. 5-18 Sequential Circuit with *JK* Flip-Flop

- $J_A = B$;
- $J_B = x'$
- $K_A = Bx'$;
- $K_B = A'x + Ax' = A \oplus x$

JK Analysis State Table

Present state		Input	Next state		Flip-flop inputs			
A	B	X	A	B	J_A	K_A	J_B	K_B
0	0	0	0	1	0	0	1	0
0	0	1	0	0	0	0	0	1
0	1	0	1	1	1	1	1	0
0	1	1	1	0	1	0	0	1
1	0	0	1	1	0	0	1	1
1	0	1	1	0	0	0	0	0
1	1	0	0	0	1	1	1	1
1	1	1	1	1	1	0	0	0

- $J_A = B$
- $J_B = x'$
- $K_A = Bx'$
- $K_B = A'x + Ax' = A \oplus x$

JK Flip Flop			
J	K	$Q(t+1)$	
0	0	$Q(t)$	No change
0	1	0	Reset
1	0	1	Set
1	1	$Q'(t)$	Complement

JK Analysis State Diagram

Present State			Input		Next State			Flip-Flop Inputs			
A	B		x		A	B		J_A	K_A	J_B	K_B
0	0		0		0	1		0	0	1	0
0	0		1		0	0		0	0	0	1
0	1		0		1	1		1	1	1	0
0	1		1		1	0		1	0	0	1
1	0		0		1	1		0	0	1	1
1	0		1		1	0		0	0	0	0
1	1		0		0	0		1	1	1	1
1	1		1		1	1		1	0	0	0

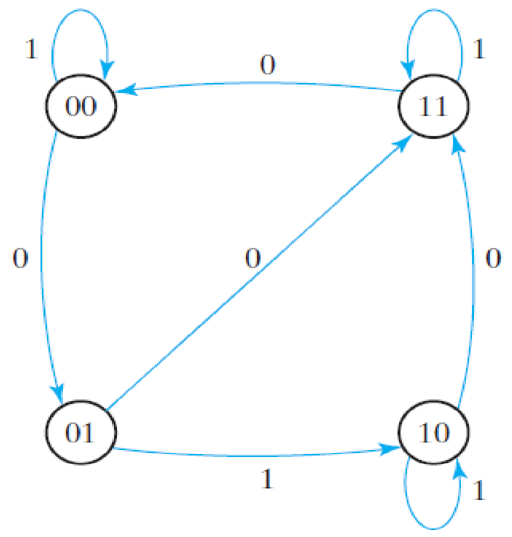


Fig. 5-19 State Diagram of the Circuit of Fig. 5-18

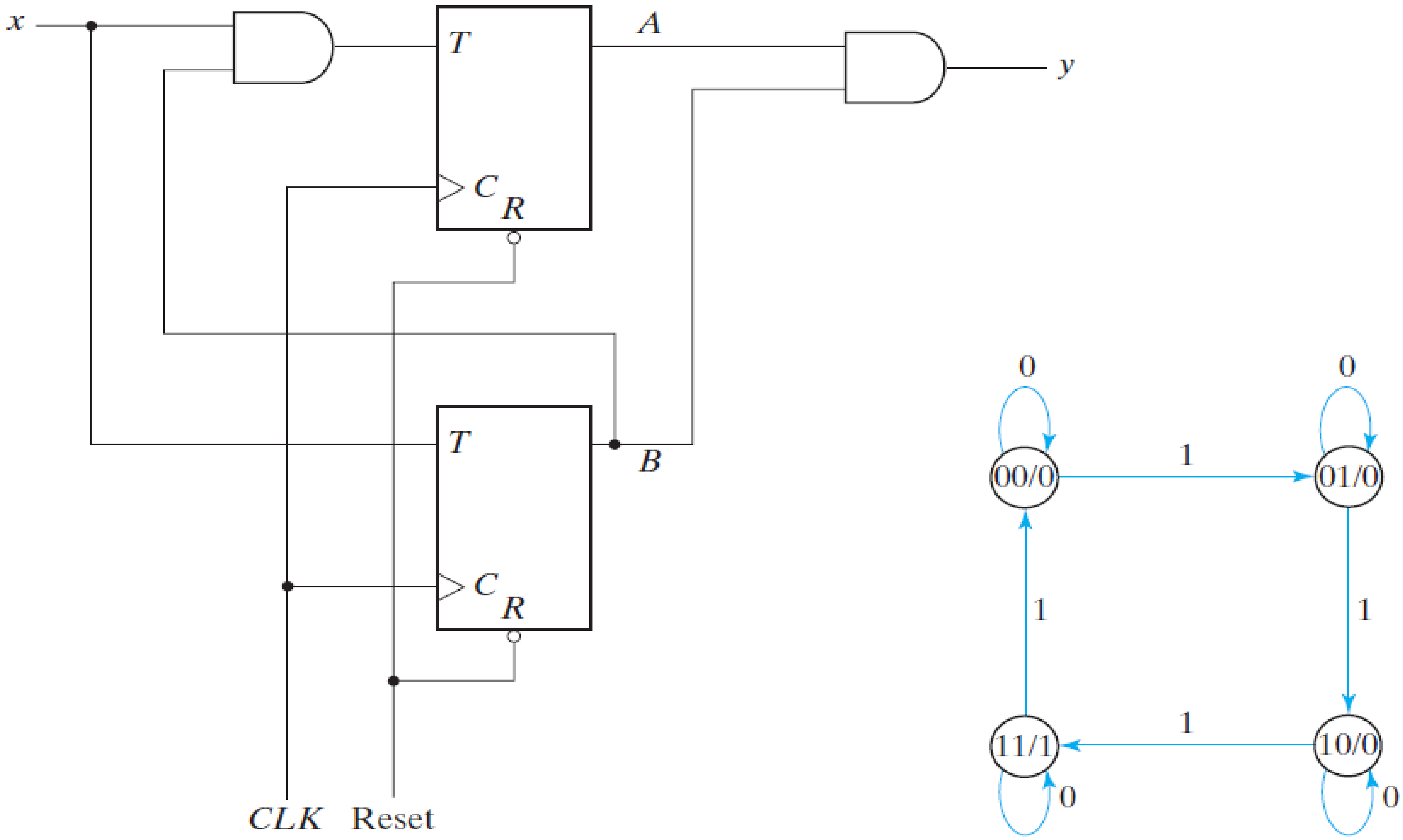
- $J_A = B$
- $J_B = x'$
- $K_A = Bx'$
- $K_B = A'x + Ax' = A \oplus x$

- $A(t+1) = J_A A' + K_A' A = BA' + (Bx')'A$
  $= A'B + AB' + Ax$
- $B(t+1) = J_B B' + K_B' B = x'B' + (A \oplus x)'B$
  $= B'x' + ABx + A'Bx'$

T Flip Flop Analysis

- Analysis of a sequential circuit with T flip flops follows the same procedure outlined for JK flip flops.
- The next state values in the state table can be obtained wither by using the characteristic table or the characteristic equation
  - $Q(t + 1) = T \oplus Q = T'Q + TQ'$

T Flip Flop Analysis Example



(a) Circuit diagram

(b) State diagram

Fig. 5-20 Sequential Circuit with T Flip-Flops

T Flip Flop Analysis State Table

Present State		Input	Next State		Output
A	B	x	A	B	y
0	0	0	0	0	0
0	0	1	0	1	0
0	1	0	0	1	0
0	1	1	1	0	0
1	0	0	1	0	0
1	0	1	1	1	0
1	1	0	1	1	1
1	1	1	0	0	1

- **$T_A = Bx; T_B = x$**
- **$Y = AB$**

T Flip Flop		
T	Q(t + 1)	
0	Q(t)	No change
1	Q'(t)	Complement

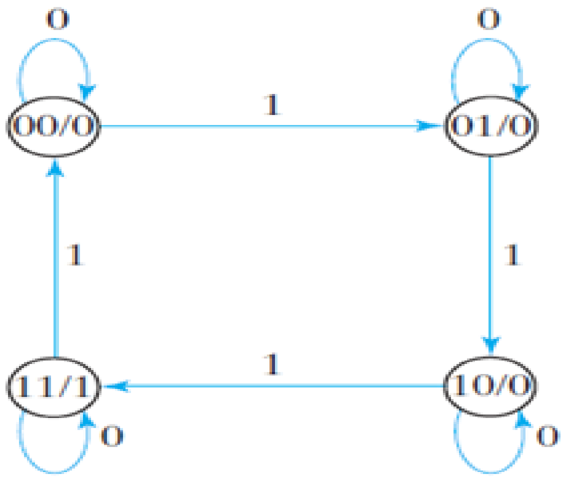
- **$A(t+1) = T_A \oplus A = Bx \oplus A$**

  $=(Bx)'A+(Bx)A'$

  $=AB'+Ax'+A'Bx$

- **$B(t+1) = T_B \oplus B = x \oplus B$**

  $=Bx'+B'x$



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