National University of Sciences and Technology (NUST)
School of Electrical Engineering and Computer Science

Assignment 1a

Submitted to: Dr. HINA GOHAR | Dated: 8/04/2021

Section: BEE 12C | Semester: 2nd

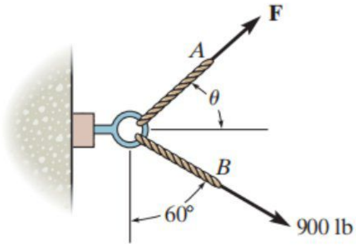
Group C

Chapter	Participants	ID
Force Vectors	**Amina Bashir**	**343489**
	• Problem 2-09 • Problem 2-54 • Problem 2-68	
Force Vectors and Equilibrium of a Particle	**Muhammad Adil**	**343869**
	• Problem 2-87 • Problem 2-99 • Problem 2-113 • Problem 3-15	
Equilibrium of a Particle	**Saad Bakhtiar**	**341150**
	• Problem 3-44 • Problem 3-51 • Problem 3-60 • Problem 3-64	
Force System Resultants	**Muhammad Adil Azeem**	**342459**
	• Problem 4-05 • Problem 4-53 • Problem 4-81	
Force System Resultants	**Muhammad Umer**	**345834**
	• Problem 4-101 • Problem 4-116 • Problem 4-138 • Problem 4-157	

PROBLEM: 2-9

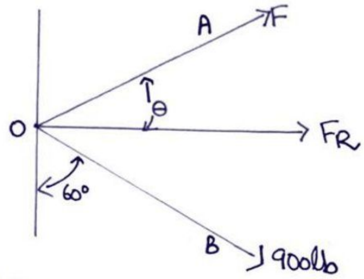
If the resultant force acting on the support is to be 1200 lb, directed horizontally to the right, determine the force **F** in rope *A* and the corresponding angle u.

Diagram



SOLUTION

2-9 $F_{Resultant} = 1200 lb$ directed horizontally to right



Given Data:-

Resultant force $= |\vec{F_R}| = 1200 lb$

Direction $= \theta_R = 0°$

Say force along $\vec{OB}$ be F_B.

$|\vec{F_B}| = 900 lb$

If F_B has direction θ_2

$\theta_2 = 90° - 60° = 30°$ with positive x-axis

To find:-

force $\vec{F}$ along rope $OA = ?$

Solution:-

Let us apply parallelogram law of addition of Two Vectors

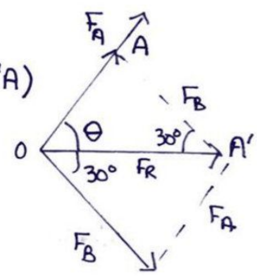
By law of cosine in $\Delta OAA'$

$(OA')^2 = (OA')^2 + (AA')^2 + 2(OA')(AA')\cos(\angle OA'A)$

$(F_{OA})^2 = (F_2)^2 + (F_R)^2 + 2(F_2)(F_R)\cos 30°$

$(F_{OA})^2 = 900^2 + 1200^2 - 2(900)(1200)\cos 30°$

$(F_{OA})^2 = 379385.13 lb^2$



Taking square root on both sides

$$\boxed{F_{OA} = 615.942 lb}$$

Using law of sines for $\Delta OAA'$:-

$$\frac{\sin\theta}{F_B} = \frac{\sin \angle OA'A}{F_A}$$

$$\frac{\sin\theta}{900} = \frac{\sin 30^\circ}{615.942}$$

$$\sin\theta = 0.730588$$

$$\theta = \sin^{-1}(0.73058)$$

Since θ lies in I quadrant

$$\boxed{\theta = 49.9357^\circ}$$

$\therefore$ acute angle of $\Delta OAA'$

Conclusion:-

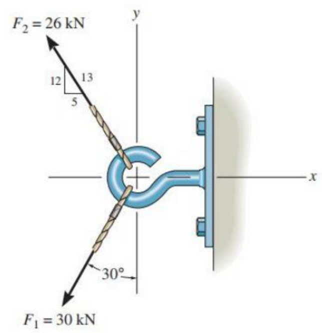
Force acting along rope A (say F_{OA}) has

- magnitude = 615.94 lb
- direction = 49.9357° with positive x-axis

PROBLEM: 2-54

Determine the magnitude of the resultant force and its direction measured counter clockwise from the positive x axis.

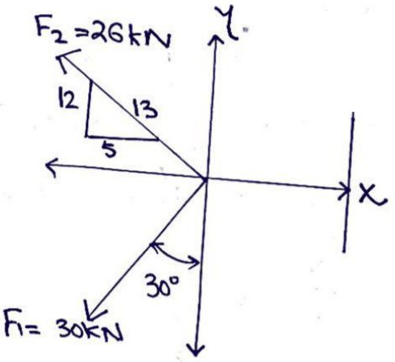
DIAGRAM



SOLUTION

Problem 54:-

Determine magnitude of resultant force and its direction measured counter clockwise from positive x-axis.



Given Data:-

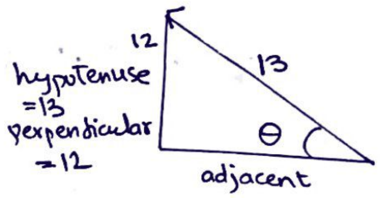
force $|\vec{F_1}| = 30kN$

angle θ_1 with horizontal $= 90° - 30° = 60°$

force $|\vec{F_2}| = 26kN$

For angle $\vec{F_2}$ (say θ_2)

$$\cos\theta_2 = \frac{\text{adjacent}}{\text{hypotenuse}}$$



$$\cos\theta_2 = \frac{5}{13}$$

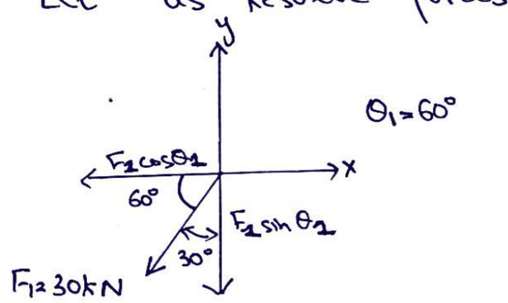
Similarly $\sin\theta_2 = \frac{12}{13}$

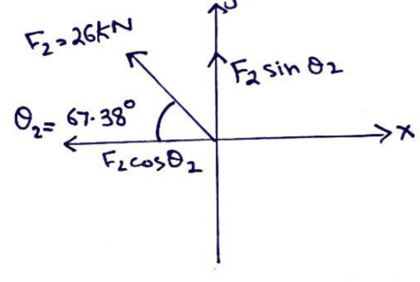
Since $\vec{F_R}$ is resultant force of $\vec{F_1}$ and $\vec{F_2}$ we conclude

$$\vec{F_R} = \vec{F_1} + \vec{F_2}$$

where $\vec{F_{Rx}} = \vec{F_{1x}} + \vec{F_{2x}}$, $\vec{F_{Ry}} = \vec{F_{1y}} + \vec{F_{2y}}$

Let us Resolve forces $\vec{F_1}$ and $\vec{F_2}$





$$\vec{F_{1x}} = F_1 \cos 60^\circ (-\hat{i})\ kN$$
$$= 30 \cos 60^\circ (-\hat{i})\ kN$$
$$= 15 kN\ (-\hat{i})$$

$$\vec{F_{1y}} = F_1 \sin 60^\circ (-\hat{j})\ kN$$
$$= 30 \sin 60^\circ (-j)\ kN$$
$$= -25.9808\ kN (+j)$$

$$\vec{F_{2x}} = F_2 \cos\theta_2 (-\hat{i})$$
$$= \left(26 \times \frac{5}{13}\right) kN\ (-\hat{i})$$
$$= 10 kN (-\hat{i})$$

$$\vec{F_{2}y} = F_2 \sin\theta_2 (+\hat{j})$$
$$= 26 \times \frac{12}{13}\ (+j)\ kN$$
$$= 24\ kN\ (+\hat{j})$$

$$\vec{F_{R_x}} = \vec{F_{1x}} + \vec{F_{2x}} = 25(-\hat{i})\ kN$$
$$\vec{F_{Ry}} = \vec{F_{1y}} + \vec{F_{2}y} = +1.9808\ (\hat{j})\ kN$$

Magnitude of $\vec{F_R}$:-

$$F_R = \sqrt{(F_{Rx})^2 + (F_{Ry})^2}$$

$$F_R = \sqrt{(-25)^2 + (-1.9808)^2}$$

$$\boxed{F_R = 25.0783\ kN}$$

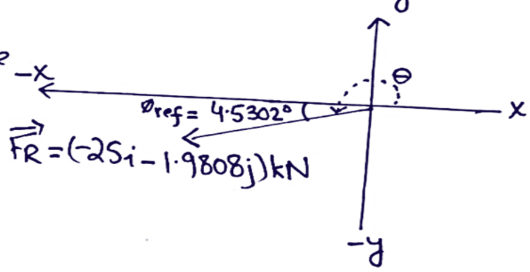
Direction of $\vec{F_R}$:-

Let angle with nearest x-axis be ϕ_{ref}.

$$\phi_{ref} = \tan^{-1} \frac{y}{x}$$
$$\phi_{ref} = \tan^{-1}\left(\frac{1.9808}{25}\right)$$
$$\phi_{ref} = 4.5302^\circ$$

Since $F_{Rx} < 0$ and $F_{Ry} < 0$ $\vec{F_R}$ lies in III quadrant.

Therefore if θ_R is angle measured counterclockwise from +X-axis



$$\theta_R = 360^\circ + \phi_{ref} - 180^\circ$$
$$\theta_R = 360^\circ + 4.5302^\circ - 180^\circ$$
$$\theta_R = 180^\circ + 4.5302^\circ$$

$$\boxed{\theta_R = 184.5302^\circ}$$

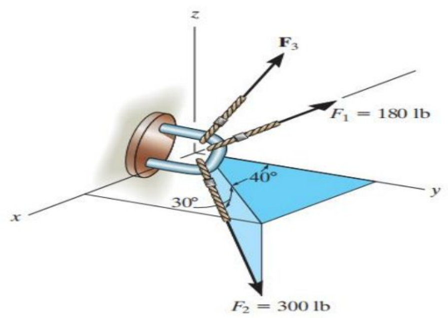
Conclusion :-

The resultant force F_R has
Magnitude = 25.0783 kN
direction = 184.5302° measured counter clockwise from +ve x-axis.

PROBLEM:2-68

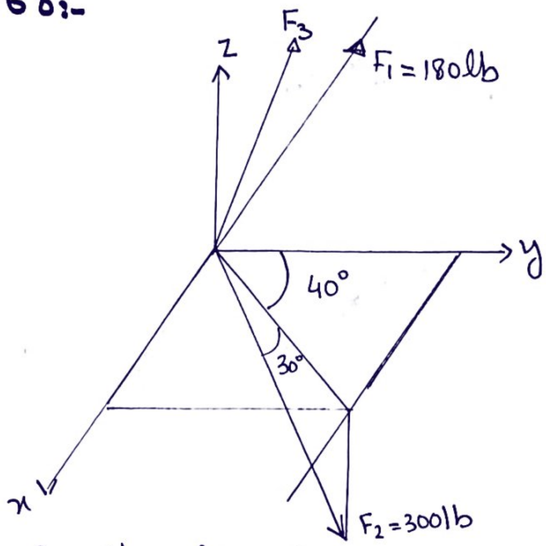
Determine the magnitude and coordinate direction angles of $\mathbf{F}_3$ so that the resultant of the three forces is zero.

DIAGRAM



SOLUTION

Problem 68:-



Resultant of three forces is zero.

Given Data:-

Force F_1 is along -x-axis

$$F_1 = (180(-\hat{i}) + 0\hat{j} + 0\hat{k})\ lb$$

Force F_2 has magnitude = 300lb

F_R (say resultant force) has magnitude = 0lb.

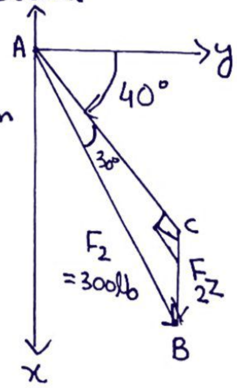
To find:-

i- Magnitude of F_3 = ?

ii- Direction angles of F_3 = ?

Solution:-

Let us resolve F_2 in ints perpendicular components.



The component of F_2 along $(-\hat{k})$ direction is:- In ΔABC

$$\sin 30^\circ = \frac{\text{Perpendicular}}{\text{Hypotenuse}}$$

$$\sin 30^\circ = F_{2(z)} \div (300)$$

$$300 \sin 30^\circ = F_{2z}(-\hat{k})$$

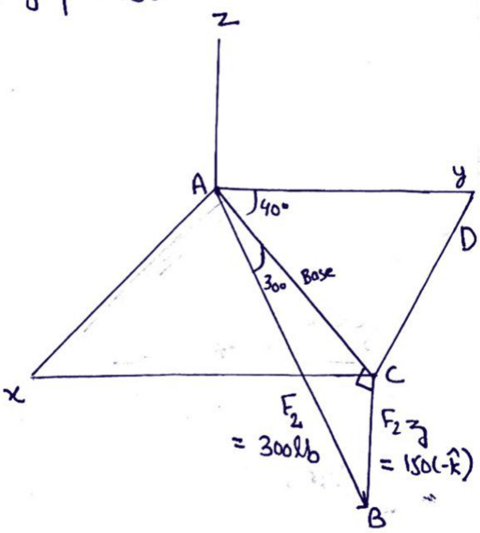
$$\vec{F}_{2z} = 150(-\hat{k})$$

Let us consider F_2 in x-y plane:-

In ΔABC :-

$$\cos 30^\circ = \frac{\text{Base}}{\text{Hypotenuse}}$$

$$\text{Base} = 300(\cos 30^\circ)\,lb$$



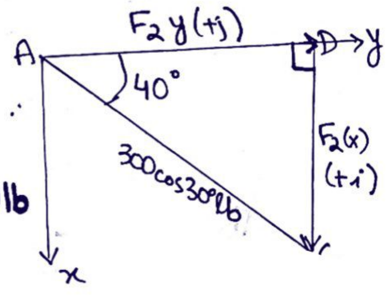
In ΔADC :-

$\vec{F_2}y$ = Base

$\vec{F_2}x$ = Perpendicular

$$\sin 40^\circ = \frac{\text{Perpendicular}}{\text{Hypotenuse}}$$

$$\sin 40^\circ = \frac{\vec{F_2 x}\,(+\hat{i})}{300\cos 30^\circ}$$



$$\vec{F}_{2x} = 300\cos 30^\circ \sin 40^\circ\,(+\hat{i})\,lb$$

$$\vec{F}_{2x} = 167.00112\,lb\,(+\hat{i})$$

$$\cos 40^\circ = \frac{\text{Base}}{\text{Hypotenuse}}$$

$$\cos 40^\circ = \frac{F_2 y\,(+j)}{300\cos 30^\circ}$$

$\Longrightarrow$

300cos30°

$$\overrightarrow{F_2y} = 300\cos 30^\circ \cos 40^\circ (+\hat{j})\,lb$$

$$\overrightarrow{F_2y} = 199.0242lb\,(+\hat{j})$$

$$\overrightarrow{F_2} = [150(-\hat{k}) + 199.024(+\hat{j}) + 167.00112(+\hat{i})]\,lb$$

Let $\overrightarrow{F_3}$ be

$$\overrightarrow{F_3} = (F_3(x) + \hat{i} + F_3(y)(+\hat{j}) + F_3(z) + \hat{k})\,lb$$

$$180(-\hat{i}) + 150(-\hat{k}) + 199.024(+\hat{j}) + 167.0011(+\hat{i}) + \overrightarrow{F_3} = \vec{0}$$

$$+12.99(-\hat{i}) + 199.024(+\hat{j}) + (150)(-\hat{k}) + \overrightarrow{F_3} = \vec{0}$$

$$\overrightarrow{F_3} = 12.99(+\hat{i}) + 199.024(-\hat{j}) + 150(+\hat{k})\,lb$$

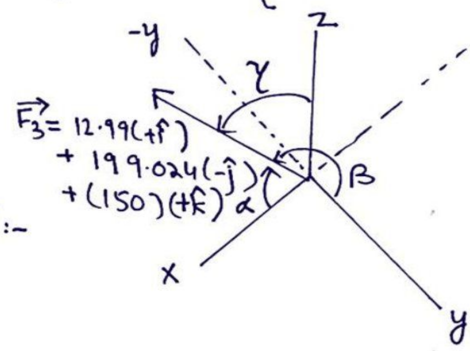
The magnitude of F_3 is

$$|\overrightarrow{F_3}| = \sqrt{(F_{3x})^2 + (F_{3y})^2 + (F_{3z})^2}$$

$$|\overrightarrow{F_3}| = \sqrt{(12.99)^2 + (-199.024)^2 + (150)^2}$$

$$|\overrightarrow{F_3}| = 249.588lb$$

Co-ordinate direction angles α, β, γ will be



- $\cos\alpha = \dfrac{+F_{3x}}{F}$

Since $\cos\alpha > 0$, α is in I quadrant $0 \leq \alpha \leq 90^\circ$:-

$$\cos\alpha = \frac{12.99}{249.558}$$

$$\alpha = \cos^{-1}\left(\frac{12.99}{249.558}\right)$$

$$\boxed{\alpha = 87.0163^\circ}$$

- $\cos\beta = -\dfrac{F_{3y}}{F_3}$

Since $\cos\beta < 0$, β is in II quadrant $90^\circ \leq \beta \leq 180^\circ$

$$\cos\beta = \left(-\frac{199.024}{249.558}\right)$$

$$\beta = \cos^{-1}(-0.7975)$$

$$\boxed{\beta = 142.8926^\circ}$$

- $\cos\gamma = +\frac{F_{3z}}{F_3}$

Since $\cos\gamma > 0$, we conclude $0 \leq \gamma \leq 90^\circ$:-

$$\cos\gamma = \frac{+150}{249.558}$$

$$\gamma = \cos^{-1}\left(\frac{150}{249.558}\right)$$

$$\boxed{\gamma = 53.054^\circ}$$

Conclusion :-

Given that $\vec{F_R} = \vec{0}$ we obtained result

- F_3 has magnitude

$$|\vec{F_3}| = 249.58\ lb$$

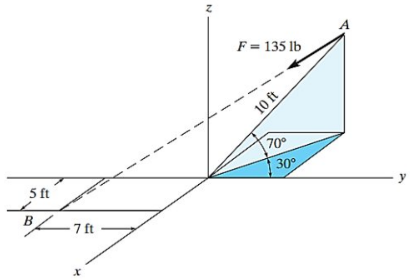
- F_3 has co-ordinate direction angles

$$\alpha = 87.016^\circ$$
$$\beta = 142.8926^\circ$$
$$\gamma = 53.054^\circ$$

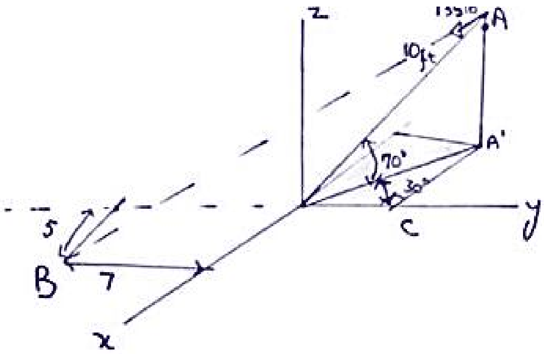
PROBLEM: 2-87

Express force **F** as a Cartesian vector; then determine its coordinate direction angles.

DIAGRAM



SOLUTION



Given Data:-

$|\vec{F}| = 135 lb$

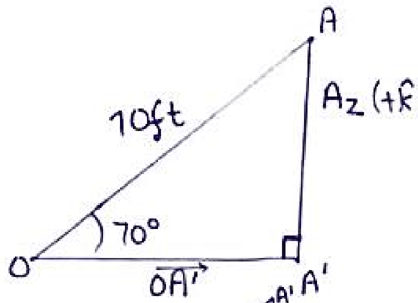
To find:-

i. $\vec{F}$ in cartesian form.

ii. Coordinate direction angles.

$\vec{OA} = ?$

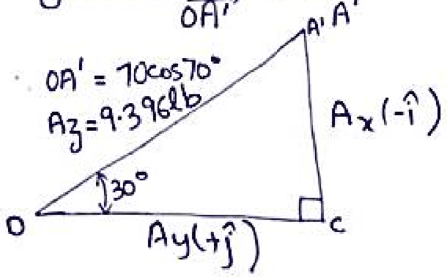
Using Δ OAA'



$A_z(+\hat{k}) = 10\sin 70°(\hat{k})$

$= 9.3969 lb (+\hat{k})$

for A_x & A_y we consider Δ OA'C :-



$\dfrac{A_y(+\hat{j})}{OA'} = \cos 30°$

$A_y(+\hat{j}) = OA'\cos 30°$

$A_y(+j) = 10\cos 70° \cos 30°$

Similarly for A_x :-

$\dfrac{A_x(-\hat{i})}{OA'} = \sin 30°$

$A_x(-\hat{i}) = \sin 30° (10)(\cos 70°)$

Hence

$\vec{OA} = 10\sin 30°\cos 70°(-\hat{i}) + 10\cos 70°\cos 30°(+\hat{j}) + 10\sin 70°(\hat{k})$

$\vec{OB} = (5, -7, 0)$

$r_{AB} = \vec{OB} - \vec{OA}$

$= (5 + 10\sin 30° \cos 70°)\hat{i} + (-7 - 10\cos 70°\cos 30°)+\hat{j}$

$- 10\sin 70° (+\hat{k})$

$r_{AB} = 6.7101(\hat{i}) + (9.9619)(-\hat{j}) + (-9.3969)(+\hat{k})$

Since $\vec{F}$ and $\vec{r}_{AB}$ are parallel vectors we conclude

$\boxed{\text{They can be represented by same unit vectors.}}$

$$\hat{r}_{AB} = \frac{\vec{r}_{AB}}{|\vec{r}_{AB}|} = \frac{6.7101\hat{i} + 9.9619(-\hat{j}) + (9.3969)(+\hat{k})}{\sqrt{(6.7101)^2 + (-9.9619)^2 + (9.3969)^2}}$$

$$\hat{r}_{AB} = \frac{6.7101\hat{i} + 9.9619(-\hat{j}) + 9.3969(+\hat{k})}{15.2501}$$

$$\hat{r}_{AB} = 0.4394\hat{i} + 0.6532(-\hat{j}) + 0.6162(-\hat{k})$$

Since vector $\vec{F}$ is expressed as

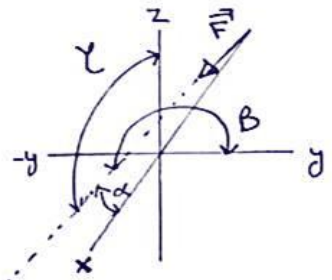
$$\vec{F} = |\vec{F}| \times (u\hat{F})$$

$$\vec{F} = 135(0.4394\hat{i} + (0.6532)-\hat{j} + 0.6162(-\hat{k}))$$

$$\vec{F} = 59.265\hat{i} + 88.182(-\hat{j}) + 83.187(-\hat{k})$$

(b) Co-ordinate direction angles:-

we observe $0 < \alpha < 90°$, $90° < \beta < 180°$ and $90° < \gamma < 180°$



$$|\vec{F}| = \sqrt{(59.265)^2 + (88.182)^2 + (83.187)^2}$$

$$|\vec{F}| = 135\,lb$$

$$\alpha = \cos^{-1}\left(\frac{F_x}{F}\right) = \cos^{-1}\left(\frac{59.265}{135}\right)$$

$$\boxed{\alpha = 63.789°}$$

$$\beta = \cos^{-1}\left(-\frac{F_y}{F}\right) = \cos^{-1}\left(-\frac{88.182}{135}\right)$$

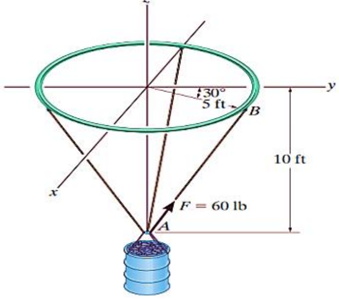
$$\boxed{\beta = 130.783°}, \quad \gamma = \cos^{-1}\left(-\frac{F_z}{F}\right) = \cos^{-1}\left(\frac{-83.18}{135}\right)$$

$$\boxed{\gamma = 128.039°}$$

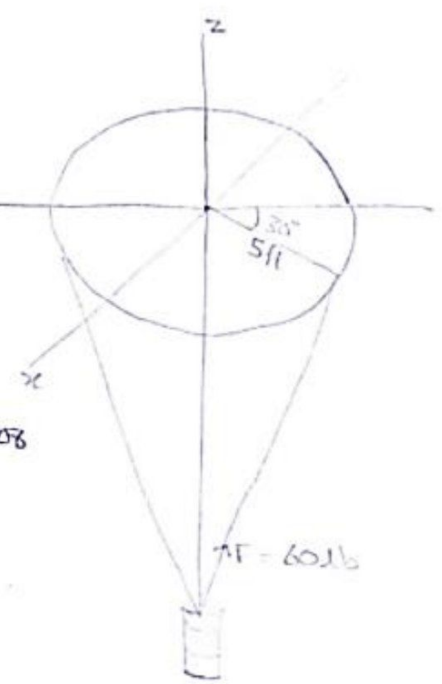
PROBLEM: 2-99

The load at creates a force of in wire AB. Express this force as a Cartesian vector acting on and directed toward as shown.

DIAGRAM



SOLUTION



Coordinates of point B are $B(5\sin 30°, 5\cos 30°, 0)$

$B = (2.50, 4.33, 0)$ ft

Now

Find the position vector r_{AB}

$$r_{AB} = \{(2.50-0)i + (4.33-0)j + (0-(-10))k\} \text{ ft}$$

$$= \{2.50i + 4.33j + 10k\} \text{ ft}$$

$$r_{AB} = \sqrt{(2.50)^2 + (4.33)^2 + (10)^2}$$

$$r_{AB} = 11.180 \text{ ft}$$

$$u_{AB} = \frac{\vec{r}_{AB}}{r_{AB}} = \frac{2.50i + 4.33j + 10k}{11.180}$$

$$u_{AB} = 0.2236i + 0.3873j + 0.8944k$$

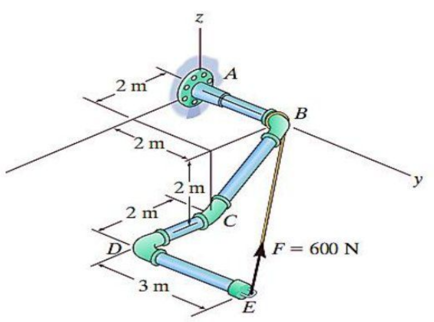
$$F = Fu_{AB} = 60\,[0.2236i + 0.3873j + 0.8944k]$$

$$F = \{13.4i + 23.2j + 53.7k\} \text{ lb}$$

PROBLEM: 2-113

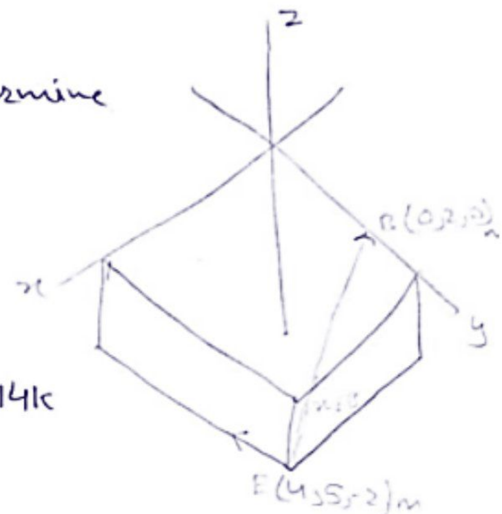
Determine the magnitudes of the components of F=600N acting along and perpendicular to segment *DE* of the pipe assembly.

DIAGRAM



SOLUTION

First of all we will determine u_{EB} and u_{ED}



$$u_{EB} = \frac{\vec{r}_{EB}}{r_{EB}}$$

$$= \frac{(0-4)i + (2-5)j + [0-(-2)]k}{\sqrt{(0-4)^2 + (2-5)^2 + (0+2)^2}}$$

$$= -0.7428i - 0.5571j + 0.3714k$$

$$u_{ED} = -j$$

Force Vector "F"

$$F = Fu_{EB} = 600(-0.7428i - 0.5571j + 0.3714k)$$
$$= [-445.66i - 334.25j + 222.83k]\,N$$

Magnitude of F parallel to segment DE

$$F_{ED} = F \cdot u_{ED}$$
$$= (-445.66i - 334.25j + 222.83k) \cdot (-j)$$
$$= (-445.66)(0) + (-334.25)(-1) + (222.83)(0)$$
$$= 334.25$$

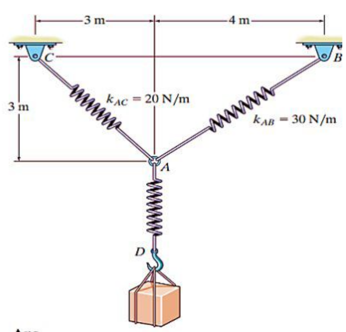
F is perpendicular to DE

$$(F_{ED})_{per} = \sqrt{F^2 - (F_{ED})^2}$$
$$= \sqrt{(600)^2 - (334.25)^2}$$
$$= 498\,N$$

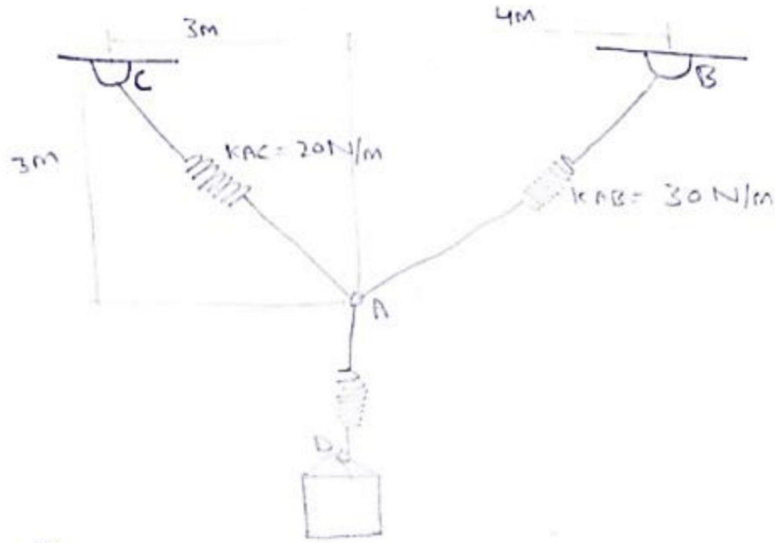
PROBLEM: 3-15

The un-stretched length of spring *AB* is 3 m. If the block is held in the equilibrium position shown, determine the mass of the block at *D*.

DIAGRAM



SOLUTION



$k = 30$

$x = 5 - 3 = 2$

$F = kx$

$F = (30)(2) = 60N$

$\Sigma F_x = 0$

$T \cos 45^\circ - 60\left(\frac{4}{5}\right) = 0$

$T = 67.88\ N$

$\Sigma F_y = 0$

$-W + 67.88 \sin 45^\circ + 60\left(\frac{3}{5}\right) = 0$

$W = 84\ N$

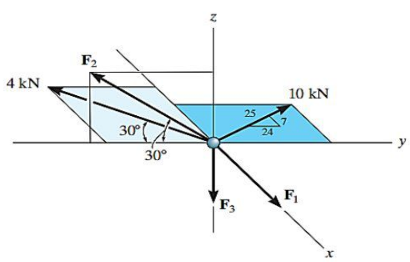
$M = \frac{84}{9.81}$

$M = 8.56kg$

PROBLEM: 3-44

Determine the magnitudes of **F**1, **F**2, and **F**3 for equilibrium of the particle.

DIAGRAM



SOLUTION

According to the conditions of Equilibrium:

$\sum F_x = 0.$

Now, according to the figure:

$$F_1 - 4000 \sin 30^\circ \overset{-10000}{\cdot} \left(\frac{7}{25}\right) = 0 \qquad \left\{ \begin{aligned} \sin\theta &= \frac{7}{25} \\ \cos\theta &= \frac{24}{25} \end{aligned} \right.$$

or $F_1 = 2000 + 2800$

$F_1 = 4800N$ or $4.8kN$

$\sum F_y = 0.$

$$10000\left(\frac{24}{25}\right) - 4000 \cos 30^\circ - F_2 \cos 30^\circ = 0$$

or $F_2 \cos 30^\circ = 9600 - 3464$

$F_2 = \dfrac{6136}{\cos 30^\circ}$

$F_2 = 7.085 kN$

Now again:

$\sum F_z = 0$

Using the value of F_2:

$F_2 \sin 30^\circ - F_3 = 0$

$F_3 = F_2 \sin 30^\circ$

$= 7.085 \times 10^3 \sin 30^\circ$

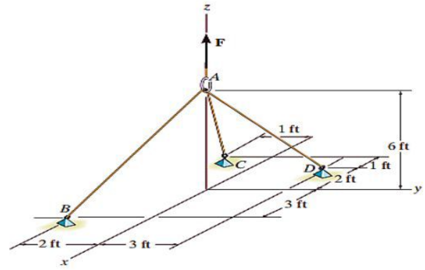
$F_3 = 3543 N$

or $F_3 = 3.543kN$

PROBLEM: 3-51

Determine the greatest force **F** that can be applied to the ring if each cable can support a maximum force of 800 lb.

DIAGRAM



SOLUTION

In the figure given.

The equations of equilibrium can be written as:

$\sum F_x = 0;$

Now,

$$F_{(AB)}\left(\frac{3}{7}\right) - F_{(AC)}\left(\frac{3}{\sqrt{46}}\right) - F_{(AD)}\left(\frac{2}{7}\right) = 0 \quad \text{—①}$$

$\sum F_y = 0;$

$$-F_{(AB)}\left(\frac{2}{7}\right) - F_{(AC)}\left(\frac{1}{\sqrt{46}}\right) + F_{(AD)}\left(\frac{3}{7}\right) = 0 \quad \text{—②}$$

$\sum F_z = 0;$

$$-F_{(AB)}\left(\frac{6}{7}\right) - F_{(AC)}\left(\frac{6}{\sqrt{46}}\right) - F_{(AD)}\left(\frac{6}{7}\right) + F = 0 \quad \text{—③}$$

By solving the above eq.s ①, ② & ③, we have:

$F_{(AC)} = 0.226\,F,$

$F_{(AD)} = 0.42\,F,$

$F_{(AB)} = 0.513\,F$

Now, from the figure given we see that the cable AB is subjected to the greatest tension. The tension of cable AB will reach the limit first and will be $F_{(AB)} = 800\,lb$.

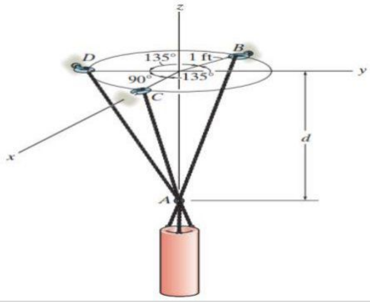
Thus,

$$F_{(AB)} = 0.513\,F$$
$$800 = 0.513\,F$$
$$F = 1559\ lb$$

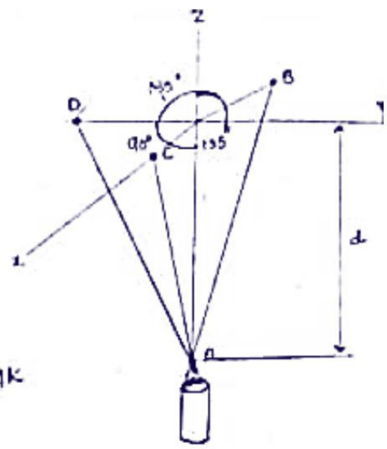
PROBLEM: 3-60

The 800-lb cylinder is supported by three chains as shown. Determine the force in each chain for equilibrium. Take d = 1 ft.

DIAGRAM



SOLUTION



From the figure, we can sort out as:

$$F_{(AD)} = F_{(AD)}\left(\frac{-1j+1k}{\sqrt{(-1)^2+1^2}}\right)$$

calculating: $F_{(AD)} = -0.7071\,F_{(AD)}\,j + 0.707\,F_{(AD)}\,k$

$$F_{(AC)} = F_{(AC)}\left(\frac{1i+1k}{\sqrt{1^2+1^2}}\right) = 0.7071\,F_{AC}\,i + 0.7071\,F_{(AC)}\,k$$

&

$$F_{(AB)} = F_{(AB)}\left(\frac{-0.7071i+0.7071j+1k}{\sqrt{(0.7071)^2+(0.7071)^2+1^2}}\right)$$

$$F_{(AB)} = -0.5\,F_{AB}\,i + 0.5\,F_{AB}\,j + 0.7071\,F_{AB}\,k$$

As we know that:

$$F = (-800k)\,lb$$

Since $\sum F = 0$;

Then, $F_{(AD)} + F_{(AC)} + F_{(AB)} + F = 0$

Putting the values from the above eqs.

$$(-0.7071\,F_{(AD)}\,j + 0.7071\,F_{(AD)}\,k) + (0.7071\,F_{(AC)}\,i + 0.7071\,F_{(AC)}\,k) + (-0.5\,F_{(AB)}\,i + 0.5\,F_{(AB)}\,j + 0.7071\,F_{(AB)}\,k) + (-800k) = 0$$

$$\Rightarrow (0.7071\,F_{(AC)} - 0.5\,F_{(AB)})\,i + (-0.7071\,F_{(AD)} + 0.5\,F_{(AB)})\,j + (0.7071\,F_{(AD)} + 0.7071\,F_{(AC)} + 0.7071\,F_{(AB)} - 800)\,k = 0$$

Now using the eqs. of equilibrium:

$\sum F_x = 0$;

$$0.7071\,F_{(AC)} - 0.5\,F_{(AB)} = 0 \quad -(1)$$

$\sum F_y = 0$: $-0.7071\,F_{(AD)} + 0.5\,F_{(AB)} = 0 \quad -(2)$

$\sum F_z = 0$;

$$0.7071\,F_{(AD)} + 0.7071\,F_{(AC)} + 0.7071\,F_{(AB)} - 800 = 0 \quad -(3)$$

Solving the above eqs. (1), (2) & (3) we have:

$F_{AB} = 469\,lb$

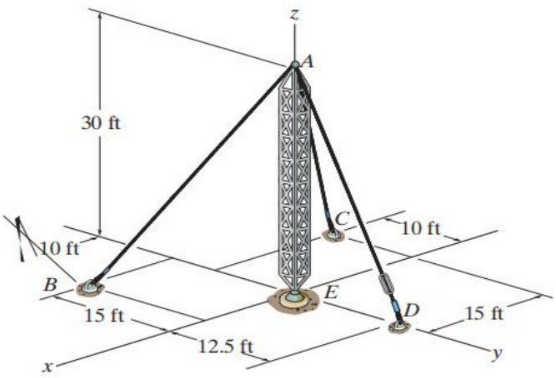
$F_{AC} = 331\,lb$

$F_{AD} = 331\,lb.$

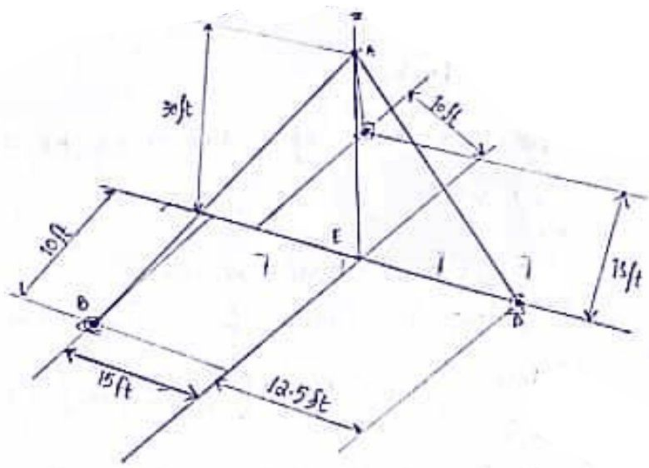
PROBLEM: 3-64

If cable *AD* is tightened by a turnbuckle and develops a tension of 1300 lb, determine the tension developed in cables *AB* and *AC* and the force developed along the antenna tower *AE* at point *A*.

DIAGRAM



SOLUTION



The forces on the free body diagram can be described in Cartesian Vector form. as:

$$F_{(AB)} = F_{(AB)} \left[\frac{(10-0)i + (-15-0)j + (-30-0)k}{\sqrt{(10-0)^2 + (-15-0)^2 + (-30-0)^2}} \right]$$

$$\vec{F}_{(AB)} = F_{(AB)}\left[\frac{2}{7}i - \frac{3}{7}j - \frac{6}{7}k\right]$$

$$= \frac{2}{7}F_{(AB)}i - \frac{3}{7}F_{(AB)}j - \frac{6}{7}F_{(AB)}k$$

$$F_{(AC)} = F_{(AC)}\left[\frac{(-15-0)i + (-10-0)j + (-30-0)k}{\sqrt{(-15-0)^2 + (-10-0)^2 + (-30-0)^2}}\right]$$

$$= F_{(AC)}\left[-\frac{3i}{7} - \frac{2}{7}j - \frac{6}{7}k\right]$$

$$F_{(AC)} = \frac{3}{7}F_{(AC)}i - \frac{2}{7}F_{(AC)}j - \frac{6}{7}F_{(AC)}k$$

$$F_{(AD)} = F_{(AD)}\left[\frac{(0-0)i + (12.5-0)j + (-30-0)k}{\sqrt{(0-0)^2 + (12.5-0)^2 + (-30-0)^2}}\right]$$

$$F_{(AD)} = \{500j - 1200k\}lb$$

And last:

$$F_{(AE)} = F_{AE}k$$

From the eqs. of equilibrium we know that:

$$\Sigma F = 0$$

so,

$$F_{(AB)} + F_{(AC)} + F_{(AD)} + F_{(AE)} = 0$$

Putting the values from the above eqs.

$$\left(\frac{2}{7}F_{AB}i - \frac{3}{7}F_{AB}j - \frac{6}{7}F_{AB}k\right) + \left(-\frac{3}{7}F_{AC}i - \frac{2}{7}F_{AC}j - \frac{6}{7}F_{AC}k\right) + (500j - 1200k) + F_{AE}k = 0$$

rearranging:

$$\left(\frac{2}{7}F_{AB} - \frac{3}{7}F_{AC}\right)i + \left(-\frac{3}{7}F_{AB} - \frac{2}{7}F_{AC} + 500\right)j + \left(-\frac{6}{7}F_{AB} - \frac{6}{7}F_{AC} - 1200 + F_{AE}\right) = 0$$

Now if we equate the i, j & k components to zero it gives:

$$\frac{2}{7}F_{AB} - \frac{3}{7}F_{AC} = 0 \quad \text{—— (1)}$$

$$-\frac{3}{7}F_{AB} - \frac{2}{7}F_{AC} + 500 = 0 \quad \text{—— (2)}$$

$$-\frac{6}{7}F_{AB} - \frac{6}{7}F_{AC} + F_{AE} - 1200 = 0 \quad \text{—— (3)}$$

Solving the above eqs. (1), (2) and (3); we have:

$F_{AB} = 808\,lb$

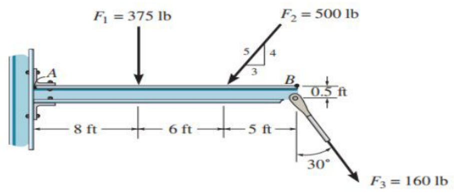
$F_{AC} = 538\,lb$

$F_{AC} = 2354\,lb$

PROBLEM: 4-5

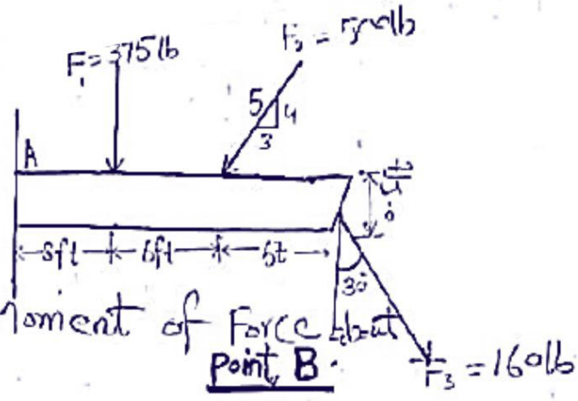
Determine the moment about point *B* of each of the three forces acting on the beam.

DIAGRAM



SOLUTION

P 4-5:



Determine Moment of Force about Point B.

Solution:

Using Anti-Clockwise Direction

For F_1

$$\circlearrowleft + (M_{F_1})_B = 375 \times 11 \Rightarrow 4125 \text{ lbft}$$

$$\boxed{= 4.125 \text{ k lbft}} \text{ (Counter-clockwise)}$$

For F_2

$$\circlearrowleft + (M_{F_2})_B = 500 \times \frac{4}{5} \times 5 \Rightarrow 2000 \text{ lbft}$$

$$\boxed{= 2 \text{ k lbft}}$$

For F_3

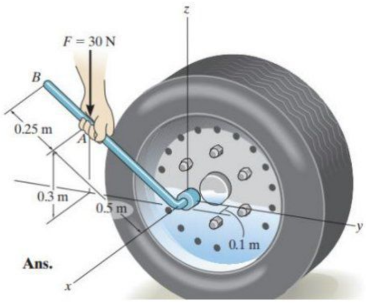
$$\circlearrowleft + (M_{F_3})_B = 160 \sin 30° \times 0.5 - 160 \cos 30° (0)$$

$$\boxed{= 40 \text{ lbft}}$$

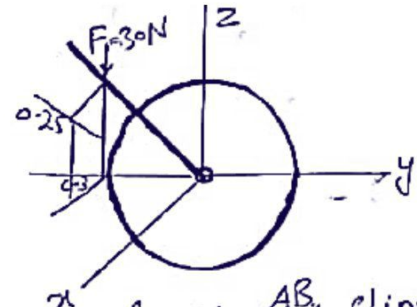
PROBLEM: 4-53

Solve Prob. 4–52 if the cheater pipe *AB* is slipped over the handle of the wrench and the 30-N force can be applied at any point and in any direction on the assembly.

DIAGRAM



SOLUTION



Solve p 4-52 if pipe AB is slipped over handle of wrench =?

Solution: if F applied at end of pipe:

if F was applied at end of pipe then moment would be following:

$$M_x = 30 \times \frac{4}{5} \times 0.75$$

$$\boxed{= 18\ Nm}$$

Since 18 Nm > ~~15~~ 14 Nm

So, this force would be enough to remove lug nut.

if F applied at perpendicular to arm:

then moment will be following:

$$M_x = 30 \times 0.75 \quad \boxed{= 22.5\ Nm}$$

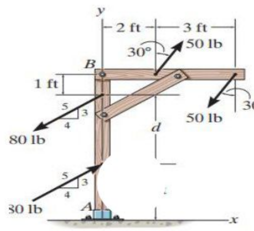
Since 22.5 Nm > 14 Nm

So, this would enough to remove lug nut.

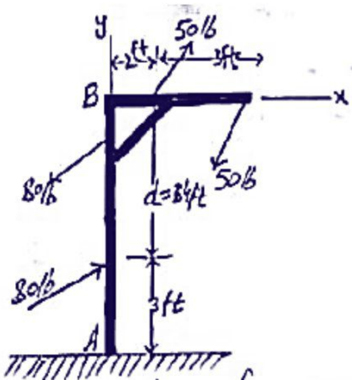
PROBLEM: 4-81

Two couples act on the frame. If determine the resultant couple moment. Compute the result by resolving each force into x and y components and (a) finding the moment of each couple (Eq. 4–13) and (b) summing the moments of all the force components about point B.

DIAGRAM



SOLUTION



ⓐ find moment of each force =?

ⓑ summing moments of all force components about point B =?

Solution:

(a)

sum of forces couples moments in vertical direction:

$$\curvearrowleft M_y = 50 \cos 30^\circ$$

$$\boxed{= 129.9 \text{ lb.ft}}$$

sum of forces couples moments in horizontal direction:

$$\curvearrowleft M_x = -80 \times \frac{4}{5} \times d$$

$$= -80 \times \frac{4}{5} \times 4$$

$$\boxed{= -256 \text{ lb.ft}}$$

Since both forces point in +z or -z direction So, resultant moment will be:

$$\curvearrowright M_R = \curvearrowright M_x + \curvearrowright M_y$$

$$= 129.9 + (-256)$$

$$\curvearrowright M_R = -126.1 \text{ lb}\cdot\text{ft}$$

$$\boxed{\circlearrowleft M_R = +126.1 \text{ lb}\cdot\text{ft}}$$

(b)

Sum of all moments about point B:

$$\curvearrowright M_B = 80 \times \frac{4}{5} - 80 \times \frac{4}{5} \times (1+4) - 50\cos 30^\circ \times 2 + 50\cos 30^\circ \times 5$$

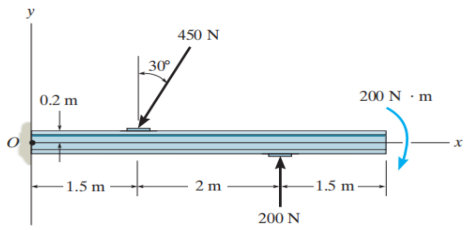
$$\curvearrowright M_B = -126.1 \text{ ft}\cdot\text{lb}$$

$$\boxed{\circlearrowleft M_B = +126.1 \text{ lb}\cdot\text{ft}}$$

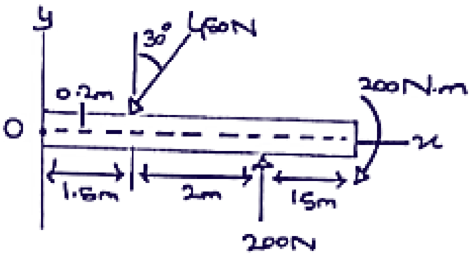
PROBLEM: 4-101

Replace the loading system acting on the beam by an equivalent resultant force and couple moment at point *O*.

DIAGRAM



SOLUTION



$$\overset{+}{\rightarrow} F_{Rx} = \Sigma F_x = -450 \sin(30^\circ)$$

$$\boxed{F_{Rx} = -225}$$

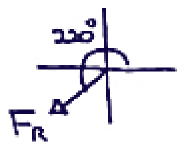
$$+\uparrow F_{Ry} = \Sigma F_y = 200 - 450 \cos(30)$$

$$\boxed{F_{Ry} = -189.7}$$

$$\boxed{F_R = \sqrt{(-225)^2 + (-189.7)^2} = 294.3\ N}$$

As both components are negtive,

$$\theta = \tan^{-1}\left(\frac{|F_y|}{|F_x|}\right) + 180^\circ = \underline{220.13^\circ}$$



For moment,

$$\circlearrowleft^{+} M_{RO} = \Sigma M_O + \Sigma M$$

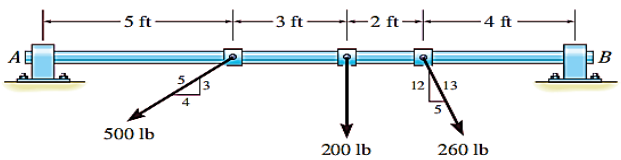
$$= 450 \cos(30)(1.5) - 450 \sin(30)(0.2) - 200(3.5) + 200$$

$$\boxed{M_{RO} = 39.6\ Nm \circlearrowright}$$

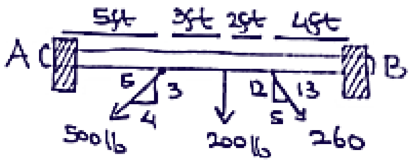
PROBLEM: 4-116

Replace the three forces acting on the shaft by a single resultant force. Specify where the force acts, measured from end *B*.

DIAGRAM



SOLUTION



$$\overset{+}{\rightarrow} \Sigma F_x = 260\left(\frac{5}{13}\right) - 500\left(\frac{4}{5}\right)$$

$$\boxed{F_{Rx} = -300 \text{ lb}}$$

$$+\uparrow \Sigma F_y = -200 - 500\left(\frac{3}{5}\right) - 260\left(\frac{12}{13}\right)$$

$$\boxed{F_{Ry} = -740 \text{ lb}}$$

$$F_R = \sqrt{(F_{Rx})^2 + (F_{Ry})^2} = \sqrt{(-300)^2 + (-740)^2}$$

$$\boxed{F_R = 798 \text{ lb}}$$

$$\theta = \tan^{-1}\left(\frac{|F_y|}{|F_x|}\right) + 180^\circ \qquad \therefore \text{ (in 3}^{rd}\text{ quadrant)}$$

$$\boxed{\theta = 247.9^\circ}$$

$$\circlearrowleft + M_B = F_{Ry}(x) = 500\left(\frac{3}{5}\right)(9) + 260\left(\frac{12}{13}\right)(4) + 200(6)$$

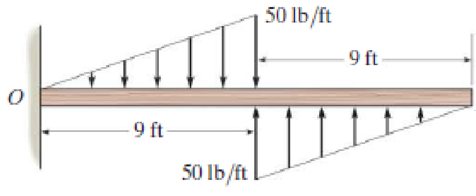
$$740(x) = 4860$$

$$\boxed{x = 6.567 \text{ ft}}$$

PROBLEM: 4-138

Replace the loading by an equivalent resultant force and couple moment acting at point O.

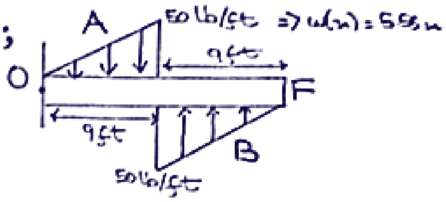
DIAGRAM



SOLUTION

4-138

At A, the loading function is;



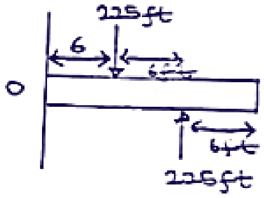
$$F_A = \int_0^9 w(x)\,dx = \int_0^9 5.55x\,dx$$

$$= \frac{5.55x^2}{2}\Big|_0^9$$

$$\boxed{F_A = 225 \text{ lb} \downarrow}$$

$$\bar{X}_A = \frac{\int_0^9 x\,w(x)\,dx}{F_A} = \frac{\int_0^9 5.55x^2\,dx}{225}$$

• From O, $\boxed{\bar{X}_A = 6 \text{ ft}}$



As B is an inverted image of A;

$$\boxed{F_B = 225 \text{ lb} \uparrow}$$

• From F, $\boxed{\bar{X}_B = 6 \text{ft}}$
• From O, $\boxed{\bar{X}_B = 12 \text{ft}}$

$$\bar{X}_{AB} = 12 - 6 = 6 \text{ft}$$

The resultant force is;

$$F_R = \Sigma F; \quad F_A + F_B$$

$$\Rightarrow -225 + 225$$

$$\boxed{F_R \Rightarrow 0}$$

As it can be expressed as a couple moment,

$$\Sigma M_o \,; \quad \circlearrowleft^{+} M_{RO}$$

$$\Sigma M_o = Fd$$

$$= F \bar{x}_{AB}$$

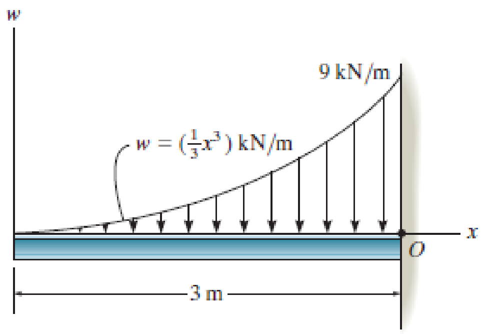
$$= (225)(6)$$

$$\boxed{M_{Ro} = 1350 \text{ lb.ft}}$$

PROBLEM: 4-157

Determine the equivalent resultant force and couple moment at point O.

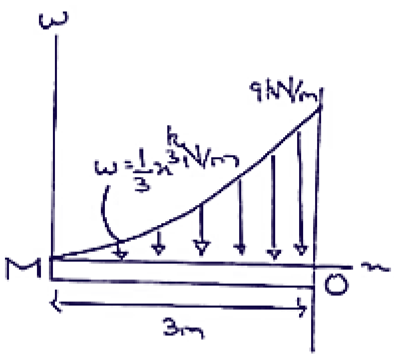
DIAGRAM



SOLUTION

4.157

The resultant force is;



$$+\uparrow F_R = -\int_0^3 w(x)\,dx$$

$$= -\int_0^3 \frac{1}{3} x^3 \, dx$$

$$= -\frac{1}{12} x^4 \Big|_0^3$$

$$\boxed{F_R = -6.75 \text{ kN}}$$

$$\bar{X} = \frac{-\int_0^3 x\,w(x)\,dx}{F_R} = \frac{-\int_0^3 \frac{1}{3}x^4\,dx}{F_R}$$

$$\bar{X} = 2.4\ m \quad \text{(from M)}$$

$$\boxed{\bar{X}_o = 3 - 2.4 = 0.6\ m} \quad \text{(from O)}$$

$$\underset{\circlearrowright +}{\Sigma M_o} = (6.75)(0.6) = \boxed{4050\ N.m}$$