ELECTROSTATIC FIELDS- COULOMB’S LAW AND FIELD INTENSITY

Introduction

- An electrostatic field is produced by a static charge distribution
- Two fundamental laws governing electrostatic fields:
- (1) Coulomb's law, and (2) Gauss's law
- Both of these laws are based on experimental studies and they are interdependent
- Based on Coulomb's law, the concept of electric field intensity will be introduced and applied to cases involving point, line, surface, and volume charges

Coulomb’s Law

- Deals with the force a point charge exerts on another point charge
- By a point charge we mean a charge that is located on a body whose dimensions are much smaller than other relevant dimensions
- Coulomb's law states that the force ***F*** between two point charges Q1 and Q2 is:
- 1. Along the line joining them
- 2. Directly proportional to the product Q_1Q_2 of the charges
- 3. Inversely proportional to the square of the distance *R* between them

Coulomb's Law

- Mathematically:

$$F = \frac{k\,Q_1Q_2}{R^2}$$

- where $k = {}^1/_{4\pi\varepsilon_o}$ is the proportionality constant

- In SI units, charges Q1 and Q2 are in coulombs (C), the distance R is in meters (m), and the force F is in newton (N)

- The constant ε_o is known as the *permittivity of free space* (in farads per meter) and has the value:

$$\varepsilon_o = 8.854 \times 10^{-12} \simeq \frac{10^{-9}}{36\pi} \text{ F/m}$$

$$\text{or } k = \frac{1}{4\pi\varepsilon_o} \simeq 9 \times 10^9 \text{ m/F}$$

Coulomb's Law

- If point charges Q1 and Q2 are located at points having position vectors **r1** and **r2**, then the force $\mathbf{F}_{12}$ on Q2 due to Q1:

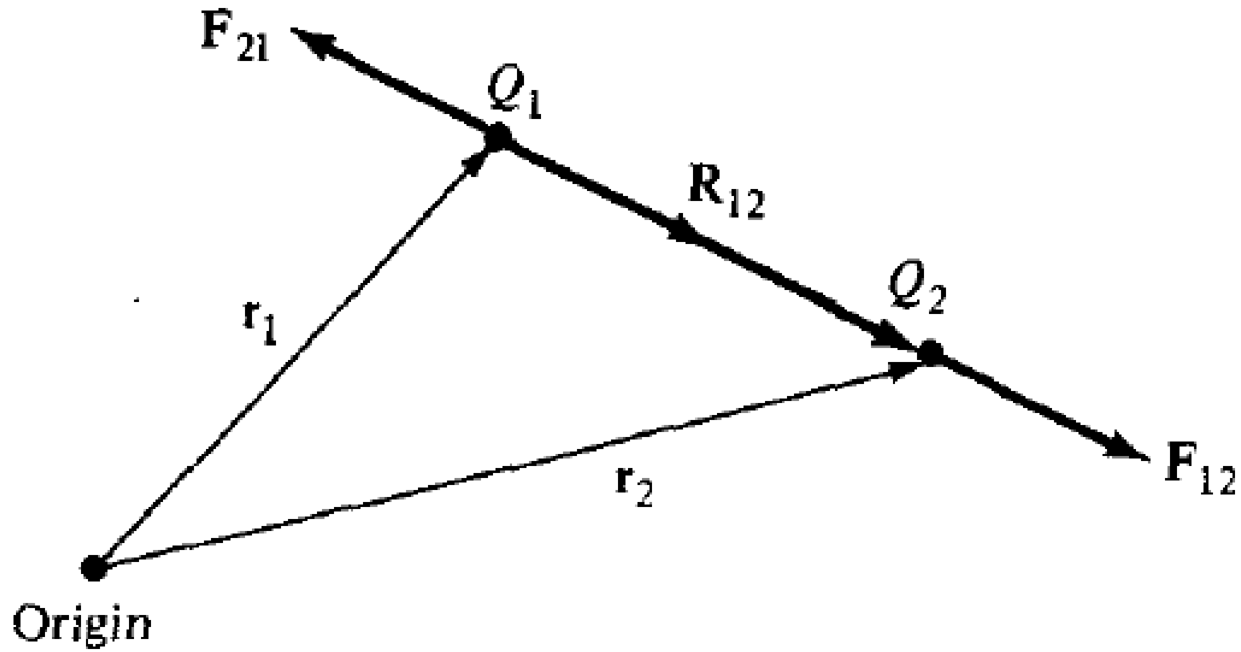
$$\mathbf{F}_{12} = \frac{Q_1 Q_2}{4\pi\varepsilon_o R^2}\,\mathbf{a}_{R_{12}}$$

Where

$$\mathbf{R}_{12} = \mathbf{r}_2 - \mathbf{r}_1$$

$$R = |\mathbf{R}_{12}|$$

$$\mathbf{a}_{R_{12}} = \frac{\mathbf{R}_{12}}{R}$$



Coulomb's Law

- Using the equations on the previous slide, the relation for force may also be written as:

$$\mathbf{F}_{12} = \frac{Q_1 Q_2}{4\pi\varepsilon_o R^3}\mathbf{R}_{12}$$

OR

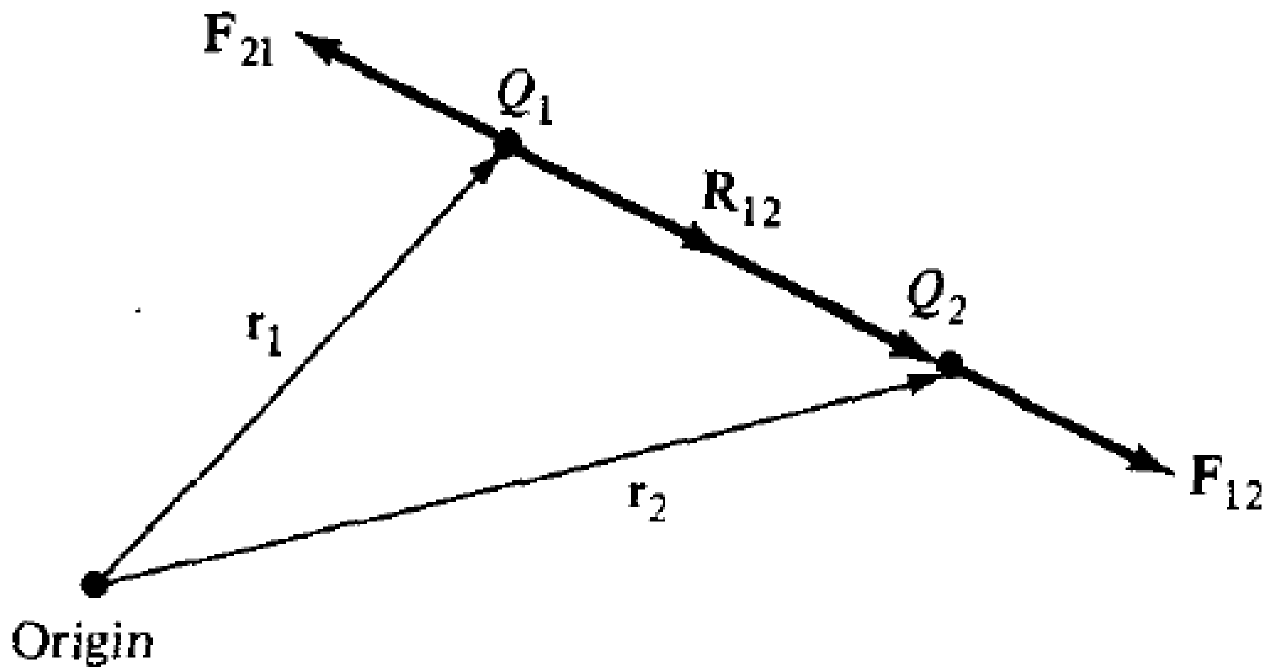
$$\mathbf{F}_{12} = \frac{Q_1 Q_2 (\mathbf{r}_2 - \mathbf{r}_1)}{4\pi\varepsilon_o |\mathbf{r}_2 - \mathbf{r}_1|^3}$$

Coulomb's Law – Important Points

- The force F_{21} on Q1 due to Q2 is given by:

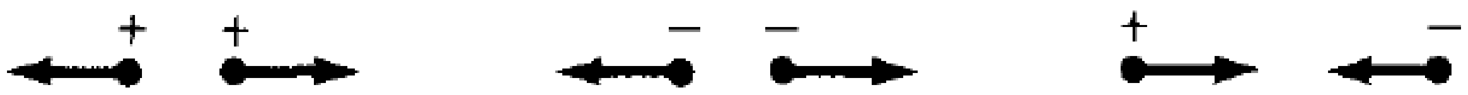
$$\mathbf{F}_{21} = |\mathbf{F}_{12}|\mathbf{a}_{R_{21}} = |\mathbf{F}_{12}|(-\mathbf{a}_{R_{12}})$$

OR $\mathbf{F}_{21} = -\mathbf{F}_{12}$, Since $\mathbf{a}_{R_{21}} = -\mathbf{a}_{R_{12}}$



Coulomb's Law – Important Points

- Like charges (charges of the same sign) repel each other while unlike charges attract, as illustrated below:



- The distance R between the charged bodies Q1 and Q2 must be large compared with the linear dimensions of the bodies; that is, Q1 and Q2 must be point charges

- Q1 and Q2 must be static (at rest)

- The signs of Q1 and Q2 must be taken into account while solving problems

Coulomb's Law – Principle of Superposition

- If we have more than two point charges, we can use the principle of superposition to determine the force on a particular charge

- The principle states that if there are N charges Q1, Q2, …… QN located, respectively, at points with position vectors $\mathbf{r}_1$, $\mathbf{r}_2$,….$\mathbf{r}_N$, the resultant force **F** on a charge Q located at point **r** is the vector sum of the forces exerted on Q by each of the charges Q1, Q2, …… QN, that is:

$$\mathbf{F} = \frac{QQ_1(\mathbf{r} - \mathbf{r}_1)}{4\pi\varepsilon_o|\mathbf{r} - \mathbf{r}_1|^3} + \frac{QQ_2(\mathbf{r} - \mathbf{r}_2)}{4\pi\varepsilon_o|\mathbf{r} - \mathbf{r}_2|^3} + \cdots + \frac{QQ_N(\mathbf{r} - \mathbf{r}_n)}{4\pi\varepsilon_o|\mathbf{r} - \mathbf{r}_N|^3}$$

OR

$$\mathbf{F} = \frac{Q}{4\pi\varepsilon_o} \sum_{k=1}^{N} \frac{Q_k(\mathbf{r} - \mathbf{r}_k)}{|\mathbf{r} - \mathbf{r}_k|^3}$$

Electric Field Intensity

- The electric field intensity (or electric field strength) **E** is the force per unit charge when placed in the electric field

$$\mathbf{E} = \frac{\mathbf{F}}{Q}$$

- The electric field intensity **E** is obviously in the direction of the force **F** and is measured in newton/coulomb or volts/meter

- The electric field intensity at point **r** due to a point charge located at **r'** is obtained from Coulomb's law as:

$$\mathbf{E} = \frac{Q}{4\pi\varepsilon_o R^2}\,\mathbf{a}_R = \frac{Q(\mathbf{r} - \mathbf{r}')}{4\pi\varepsilon_o |\mathbf{r} - \mathbf{r}'|^3}$$

Electric Field Intensity – Principle of Superposition

- For N point charges Q1, Q2,….., Q_N located at **r1, r2,….., r**$_N$, the electric field intensity at point **r** is obtained as:

$$\mathbf{E} = \frac{Q_1(\mathbf{r} - \mathbf{r}_1)}{4\pi\varepsilon_o|\mathbf{r} - \mathbf{r}_1|^3} + \frac{Q_2(\mathbf{r} - \mathbf{r})_2}{4\pi\varepsilon_o|\mathbf{r} - \mathbf{r}_2|^3} + \cdots + \frac{Q_N(\mathbf{r} - \mathbf{r}_N)}{4\pi\varepsilon_o|\mathbf{r} - \mathbf{r}_N|^3}$$

OR

$$\mathbf{E} = \frac{1}{4\pi\varepsilon_o}\sum_{k=1}^{N}\frac{Q_k(\mathbf{r} - \mathbf{r}_k)}{|\mathbf{r} - \mathbf{r}_k|^3}$$

Problem-1

- Point charges 5 nC and -2 nC are located at (2,0,4) and (-3,0,5), respectively.
  - (a) Determine the force on a 1 nC point charge located at (1,-3, 7)
  - (b) Find the electric field E at (1,-3,7)