Gauss' Law- III

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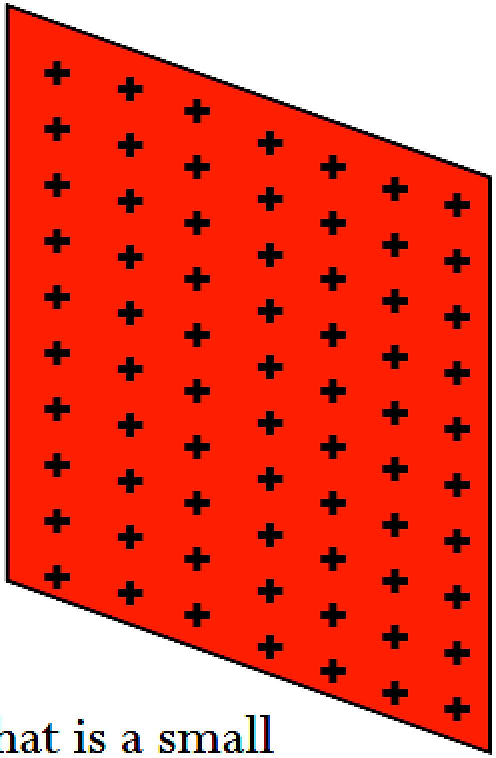
E due to a Nonconducting Sheet of Charges

Consider a nonconducting, infinite sheet. Let positive charge q is sprayed on one side of the sheet. The charges stick where they land. The uniform surface charge density will be

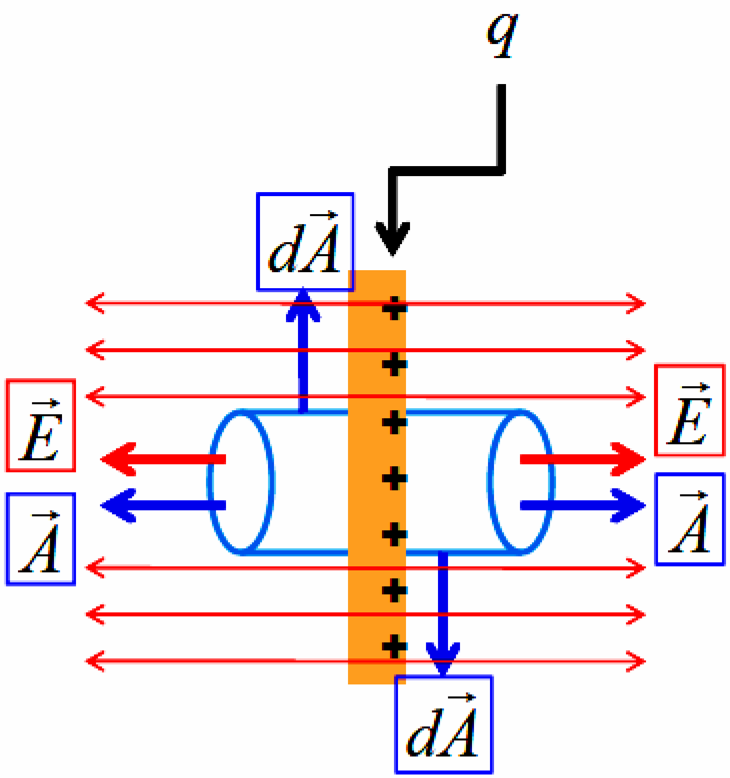
$$\sigma = q / A$$



- By symmetry, E must be perpendicular to the plane and must have the same magnitude at all points equidistant from the plane.
- The fact that the direction of E is away from positive charges indicates that the direction of E on one side of the plane must be opposite its direction on the other side.

To find electric field, let's consider a Gaussian surface that is a small cylinder whose axis is perpendicular to the plane and whose ends each have an area *A and* are equidistant from the plane.

- For curved part of Gaussian surface —the electric field is perpendicular to vector area everywhere on the surface—there is no contribution to the surface integral from this surface.
- For flat ends, electric field is constant at all points and is parallel to vector area. we will restrict our attention to only the flat ends of the cylinder..
- The enclosed charge will be σA.

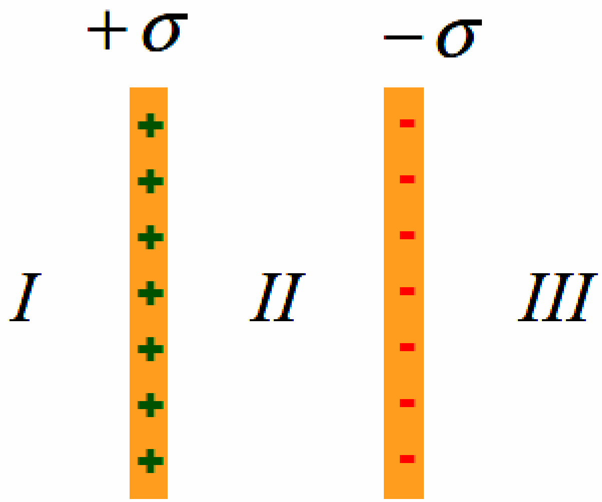


According to Gauss' law

$$\oint \vec{E}.d\vec{A} = \frac{q_{enc}}{\varepsilon_o}$$

$$\underbrace{EA}_{End1} + \underbrace{EA}_{End2} = \frac{\sigma A}{\varepsilon_o} \quad \Rightarrow \quad \boxed{E = \frac{\sigma}{2\varepsilon_o}}$$

Two thin, large and flat nonconducting sheets are separated by a distance that is very small compared to their height and width. The sheets are given equal but opposite uniform surface charge densities $\pm\sigma$ where $\sigma = q/A$. Ignore edge effects and for points far from the edges, find (*a*) the electric field between the plates (*b*) the electric field outside the plates on either side.



Each sheet of charge sets up an electric field

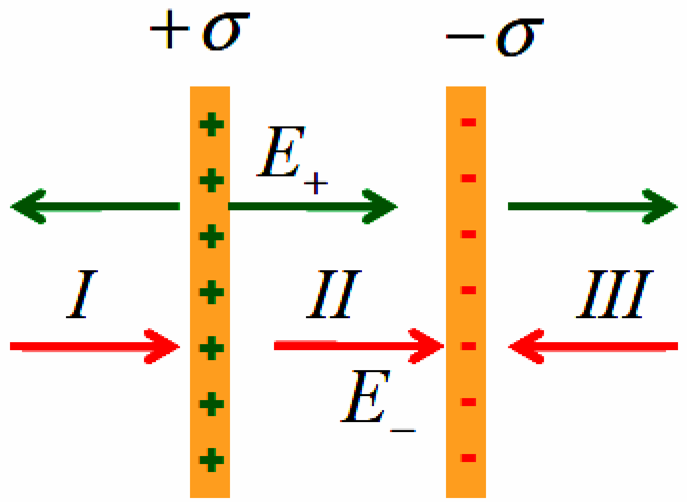
$$E_+ = E_- = \frac{\sigma}{2\varepsilon_o}$$

Net electric field between the plate (region II)will be

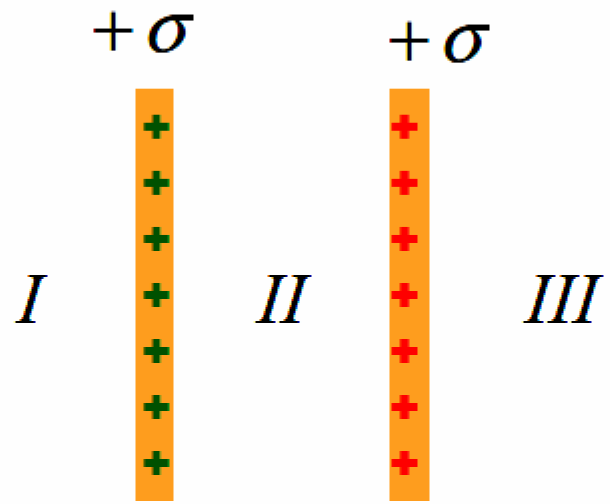
$$E = E_+ + E_- = \frac{\sigma}{\varepsilon_o}$$

Net electric field outside the plates (region I &III)will be

$$E = 0$$



Two thin, large and flat nonconducting sheets are separated by a distance that is very small compared to their height and width. The sheets are given equal and uniform surface charge densities $+\sigma$ where $\sigma = q/A$. Ignore edge effects and for points far from the edges, find (*a*) the electric field between the plates (*b*) the electric field outside the plates on either side.



Each sheet of charge sets up an electric field

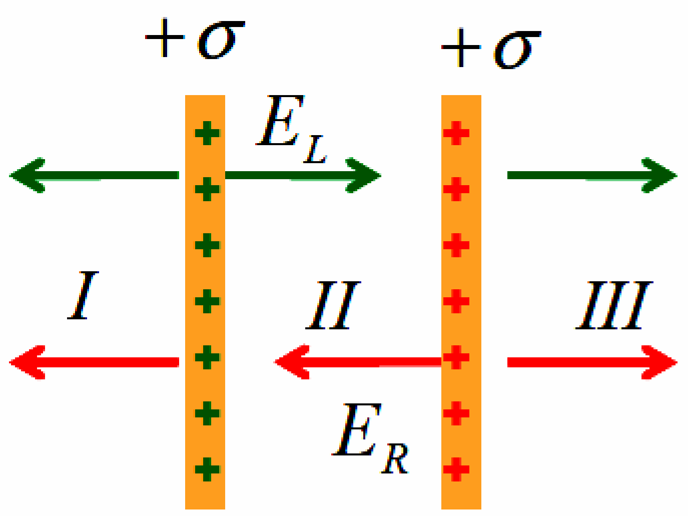
$$E_R = E_L = \frac{\sigma}{2\varepsilon_o}$$

Net electric field between the plate (region II)will be

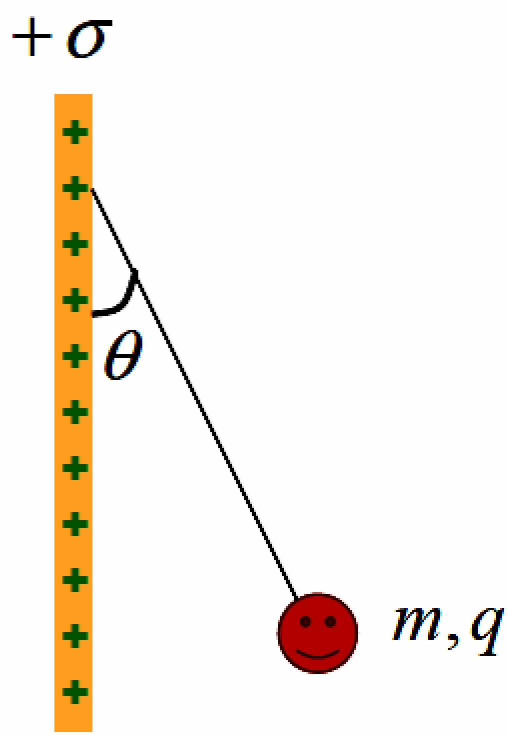
$$E = 0$$

Net electric field outside the plates (region I &III)will be

$$E = E_R + E_L = \frac{\sigma}{\varepsilon_o}$$



A small sphere whose mass is **1.12mg** carries a charge **q = 19.7μC**. It hangs in the earth gravitational field from a silk thread that makes an angle **θ = 27.4** with a large uniformly charged nonconducting sheet as shown in fig. Calculate uniform charge density **σ** for the sheet



The net force on the small sphere is zero; this force is the vector sum of the force of gravity W, the electric force F, and the tension T.

$$T\cos\theta = mg \qquad (I)$$

$$T\sin\theta = F_E \qquad (II)$$

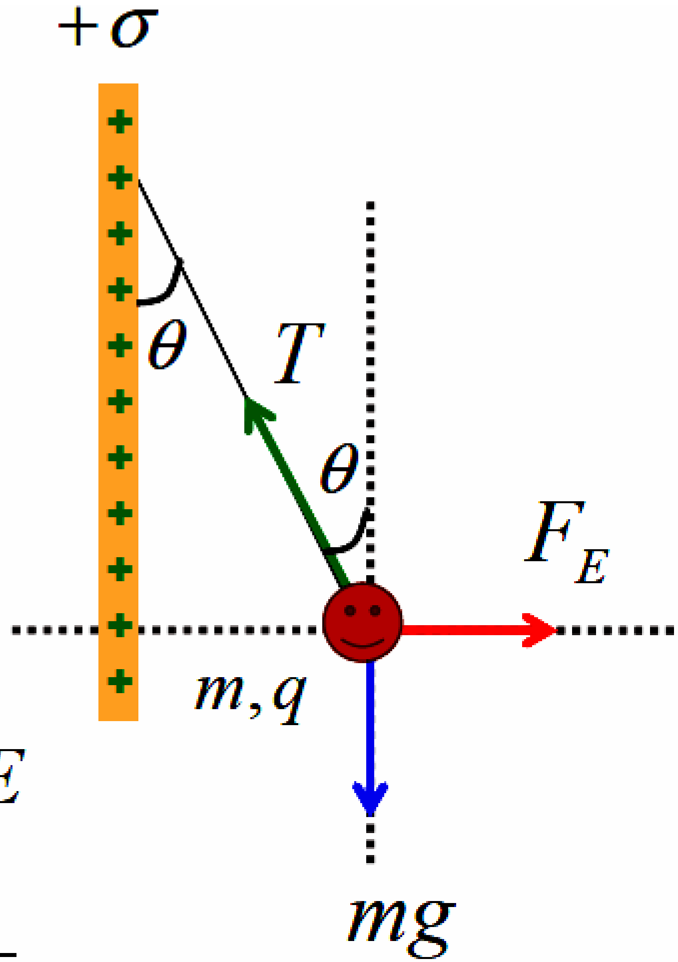
Form eq.(I) $T = mg/\cos\theta$

So eq.(II) $F_E = mg\tan\theta$

$$qE = mg\tan\theta \qquad \therefore F_E = qE$$

$$q\frac{\sigma}{2\varepsilon_\circ} = mg\tan\theta \qquad \therefore E = \frac{\sigma}{2\varepsilon_\circ}$$

$$\boxed{\sigma = \frac{2\varepsilon_\circ mg\tan\theta}{q}}$$



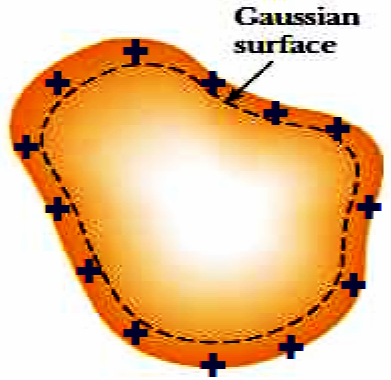
A Charged Isolated Conductor

A good electrical conductor contains charges (electrons) that are free to move about within the material. When there is no net motion of charge within a conductor, the conductor is in electrostatic equilibrium.

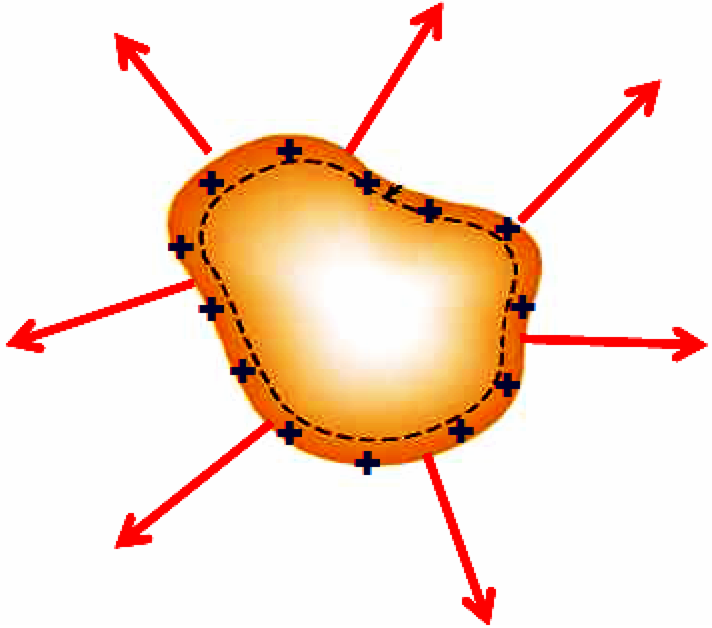
The electric field inside a conductor in static equilibrium must be zero. Otherwise, the field would exert forces on the conduction electrons producing motion or currents which is not a static equilibrium state.

If we place a Gaussian surface just inside the surface of charged conductor, the E field is zero for all points on the Gaussian surface. Therefore the flux through the Gaussian surface is zero and according to Gauss' Law the net charge enclosed by the Gaussian surface must also be zero.

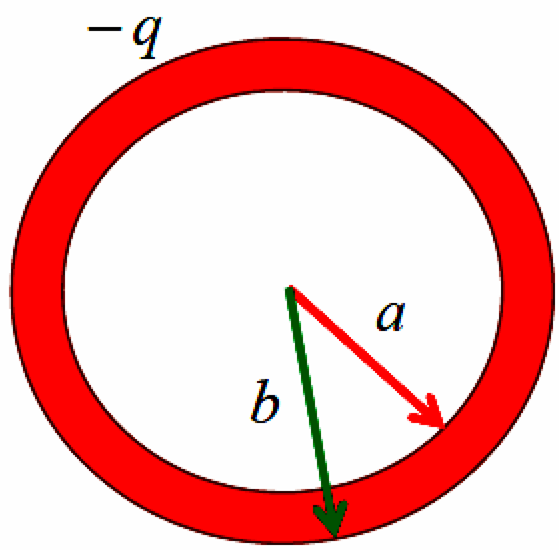
If excess charge is placed on an isolated conductor, the charge will move to the surface of the conductor. None of the excess charge will be found within the body of the conductor.



The electric field just outside a charged conductor is perpendicular to the surface of the conductor from the condition of electrostatic equilibrium. (If E had a component parallel to the conductor's surface, the free charges would move along the surface; in such a case, the conductor would not be in equilibrium.)



A conducting spherical shell of inner radius **a** and outer radius **b** carries a net charge **-q**. (a) what charges appear on the inner and outer surface of the shell? Using Gauss's law, find the electric field in the regions (a) inside the shell, (b) within the shell, (c) outside the shell



(a) All the charges will reside on the outer surface of shell

(b). Inside the shell

As charge enclosed is zero, so $E = 0$

(c). Within the shell

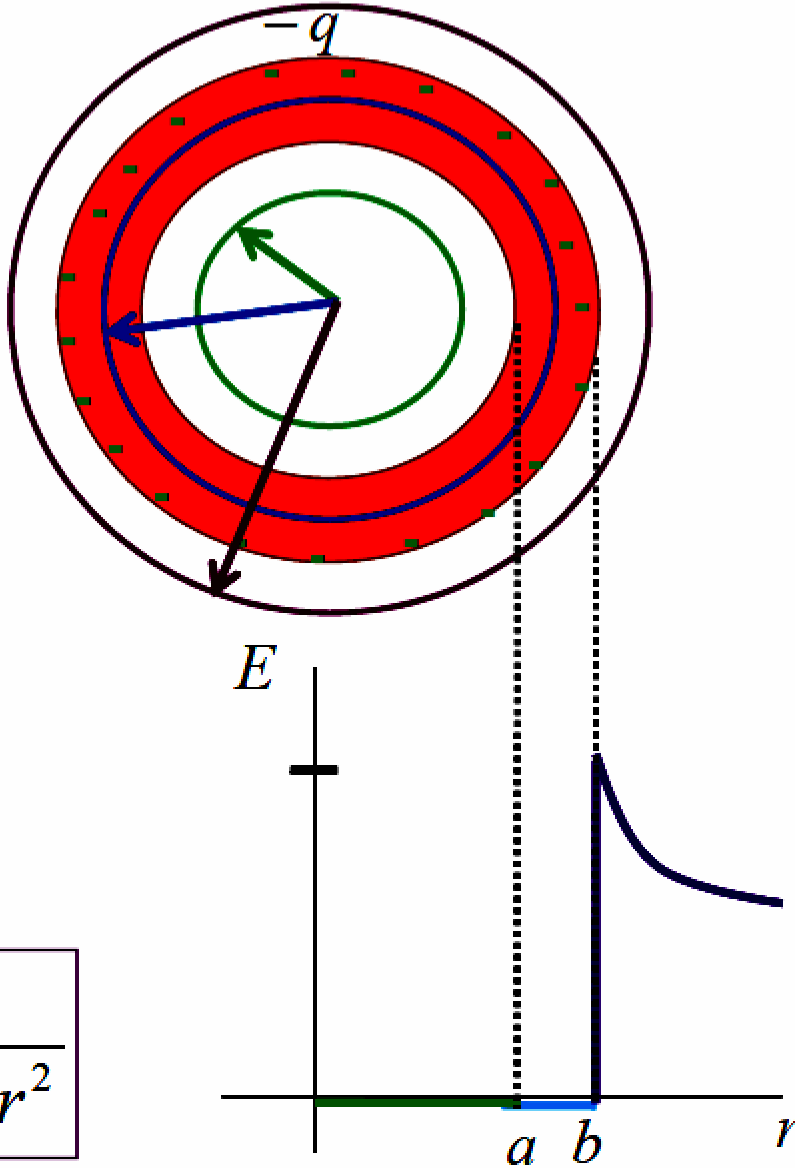
As charge enclosed is zero, so $E = 0$

(d). Outside the shell

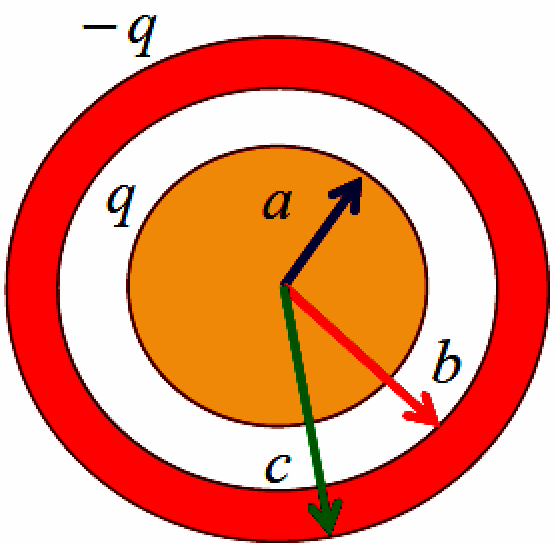
$$\oint \vec{E}.d\vec{A} = \frac{q_{enc}}{\varepsilon_o}$$

$$E\oint dA = \frac{q}{\varepsilon_o} \quad \therefore q_{enc} = q$$

$$E \times 4\pi r^2 = \frac{q}{\varepsilon_o} \Rightarrow \boxed{E = \frac{q}{4\pi\varepsilon_o r^2}}$$

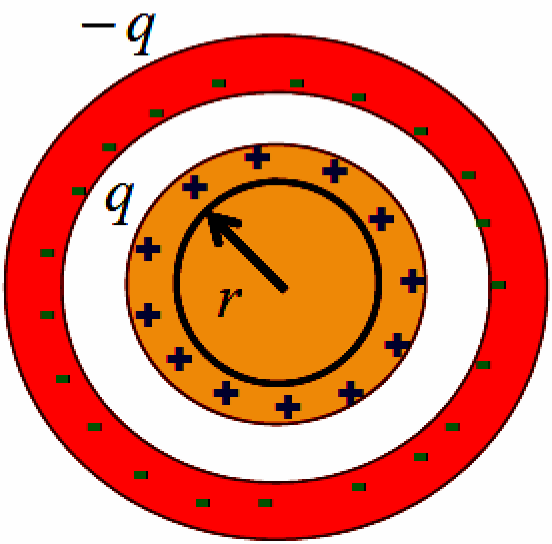


A solid conducting sphere of radius **a** carries a net positive charge **q**. A conducting spherical shell of inner radius **b** and outer radius **c** is concentric with the solid sphere and carries a net charge **'-q**. Using Gauss's law, find the electric field in the regions (a) within the solid sphere, (b) Between the sphere and the shell, (c) inside the shell (d) outside the shell (d) what charges appear on the inner and outer surface of the shell?



The sphere and shell are both conducting, so all charge will reside on the surfaces. Due to attraction of charges, -q will reside on the inner surface of shell.

(a). Within the solid sphere



Consider a spherical Gaussian surface of radius r such that r < a. According to Gauss's law

$$\oint \vec{E}.d\vec{A} = q_{enc} / \varepsilon_o$$

As charge enclosed is zero, so

$$E = 0$$

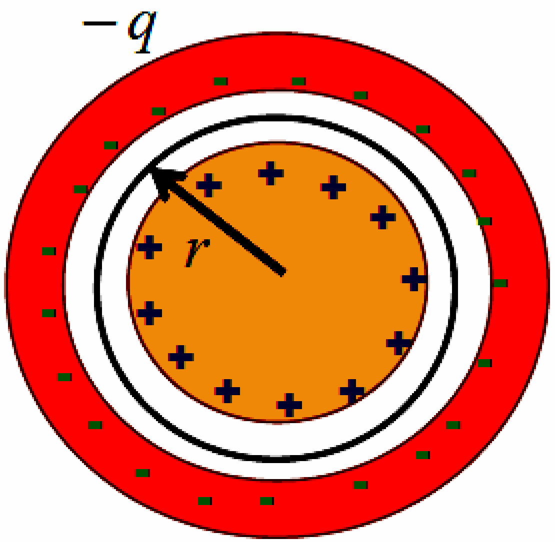
Note: You can say, without using Gauss's law, that electric field is zero inside the conductor.

(b). Between the sphere and shell

Consider a spherical Gaussian surface of radius r such that a < r < b.

According to Gauss's law

$$\oint \vec{E}.d\vec{A} = q_{enc} / \varepsilon_o$$



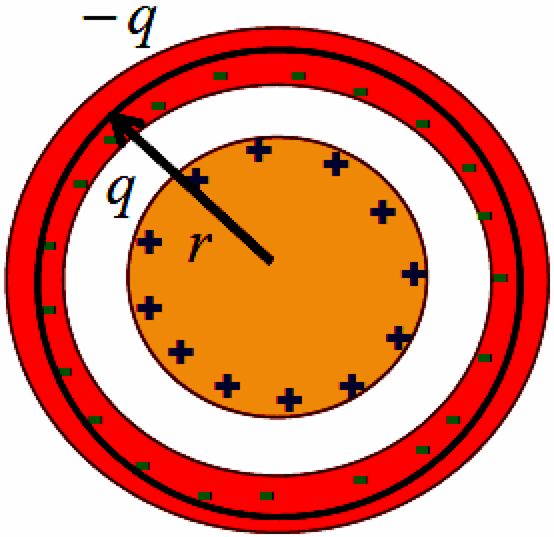
As electric field always remains perpendicular to the surface of conductor and enclosed charge is q, so

$$E \oint dA = q / \varepsilon_o$$

$$E \times 4\pi r^2 = q / \varepsilon_o$$

$$E = \frac{q}{4\pi r^2 \varepsilon_o}$$

(c). Inside the shell



Consider a spherical Gaussian surface of radius r such that b < r < c. According to Gauss's law

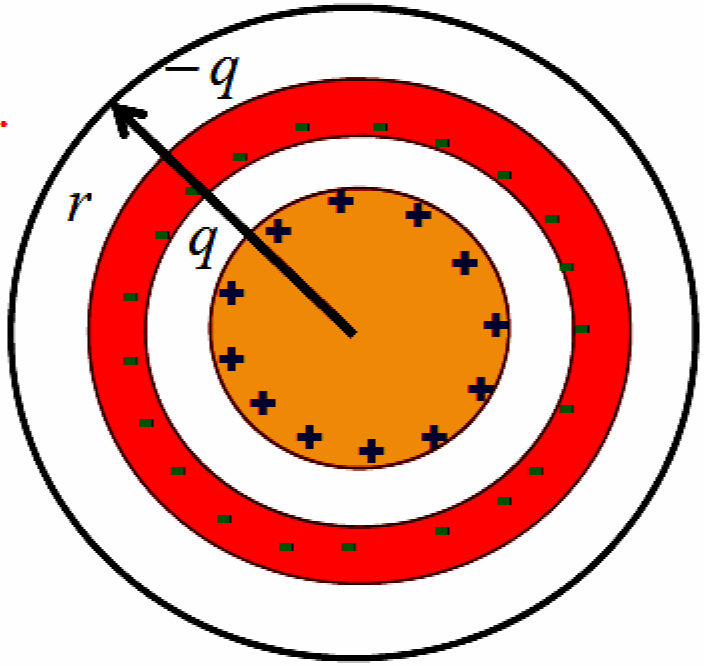
$$\oint \vec{E}.d\vec{A} = q_{enc} / \varepsilon_o$$

As charge enclosed is zero, so

$$E = 0$$

Note: You can say, without using Gauss's law, that electric field is zero inside the conductor.

(d). Outside the shell

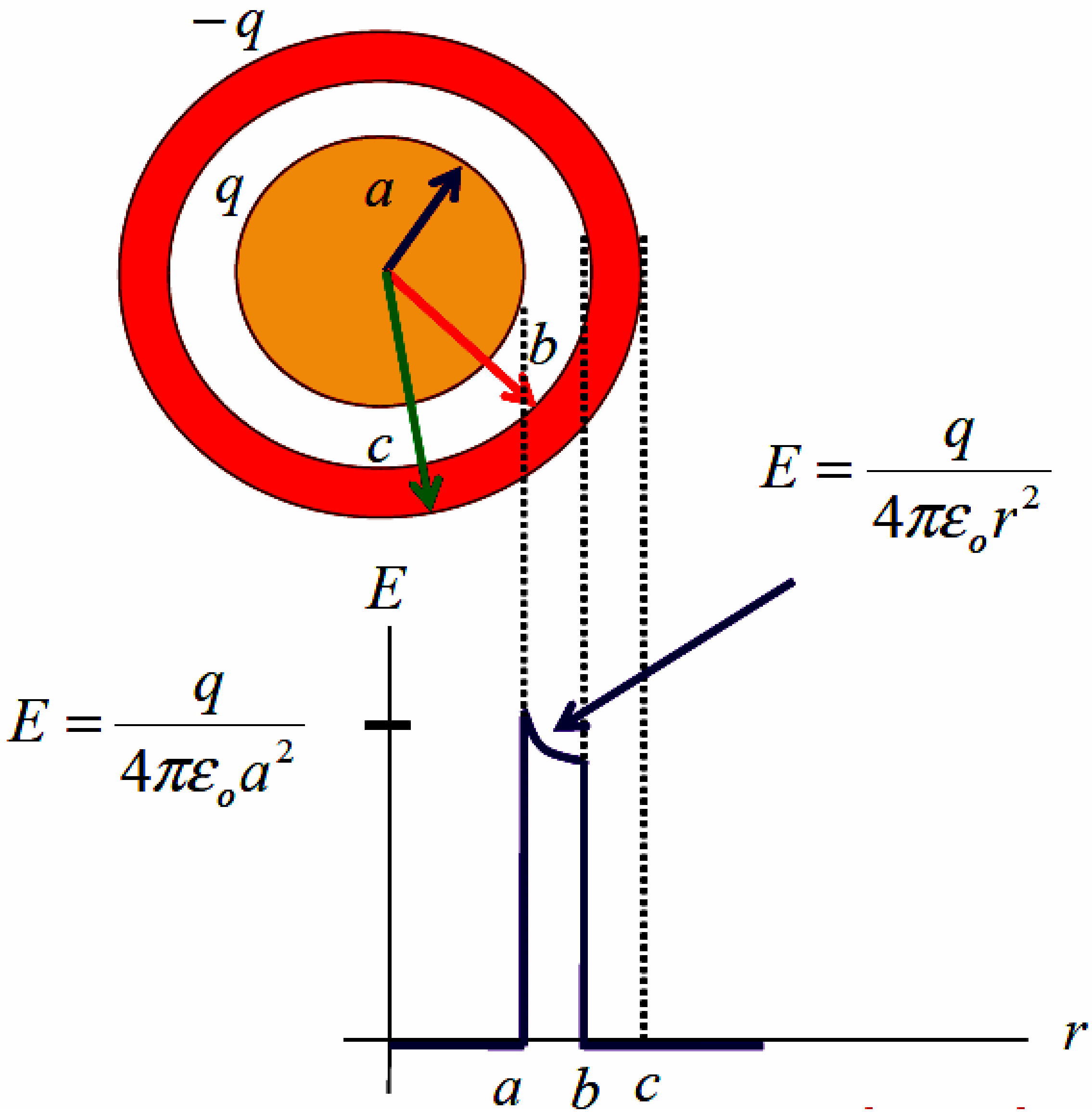


Consider a spherical Gaussian surface of radius r such that r > c.

$$\oint \vec{E}.d\vec{A} = q_{enc} / \varepsilon_o$$

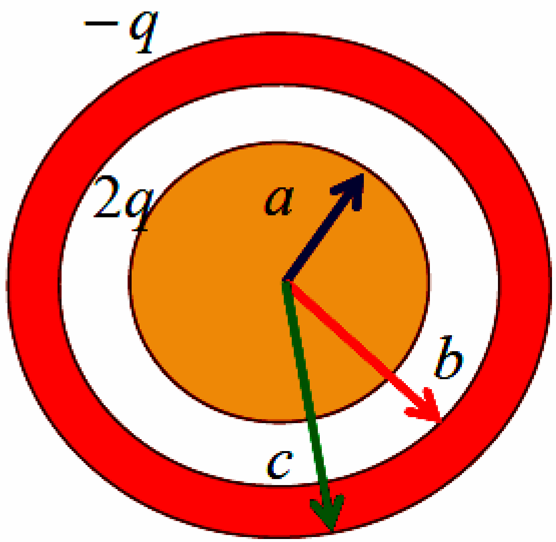
As charge enclosed is zero, so

$$E = 0$$

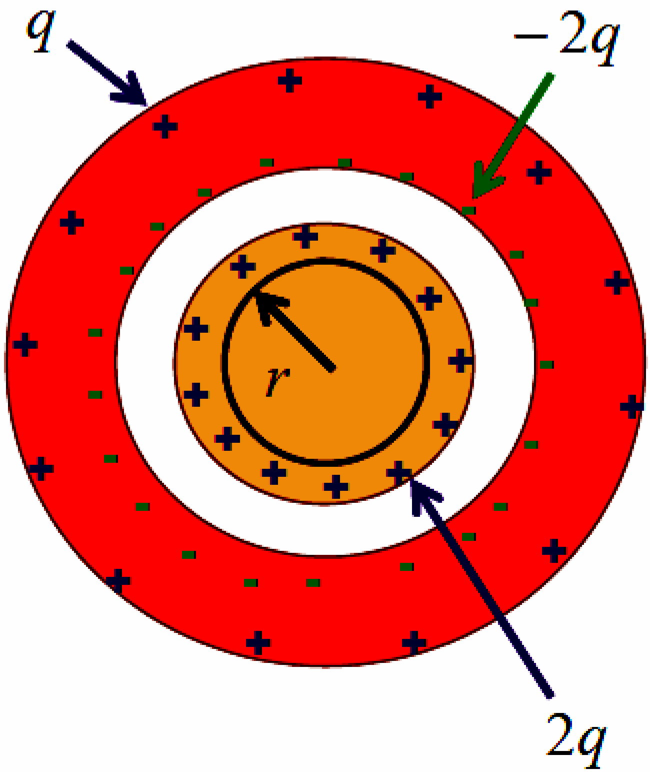


$-q$
q
a
b
c
$E = \frac{q}{4\pi\varepsilon_o r^2}$
E
$E = \frac{q}{4\pi\varepsilon_o a^2}$
r
a b c

A solid conducting sphere of radius **a** carries a net positive charge 2**q**. A conducting spherical shell of inner radius **b** and outer radius **c** is concentric with the solid sphere and carries a net charge **'-q**. Using Gauss's law, find the electric field in the regions (a) within the solid sphere, (b) Between the sphere and the shell, (c) inside the shell (d) outside the shell (d) what charges appear on the inner and outer surface of the shell?



The sphere and shell are both conducting, so all charge will reside on the surfaces. Due to attraction of charges, -2q will reside on the inner surface of shell and +q will remain on the outer one.



(a). Within the solid sphere

Consider a spherical Gaussian surface of radius r such that r < a. According to Gauss's law

$$\oint \vec{E}.d\vec{A} = q_{enc} / \varepsilon_o$$

As charge enclosed is zero, so

$$E = 0$$

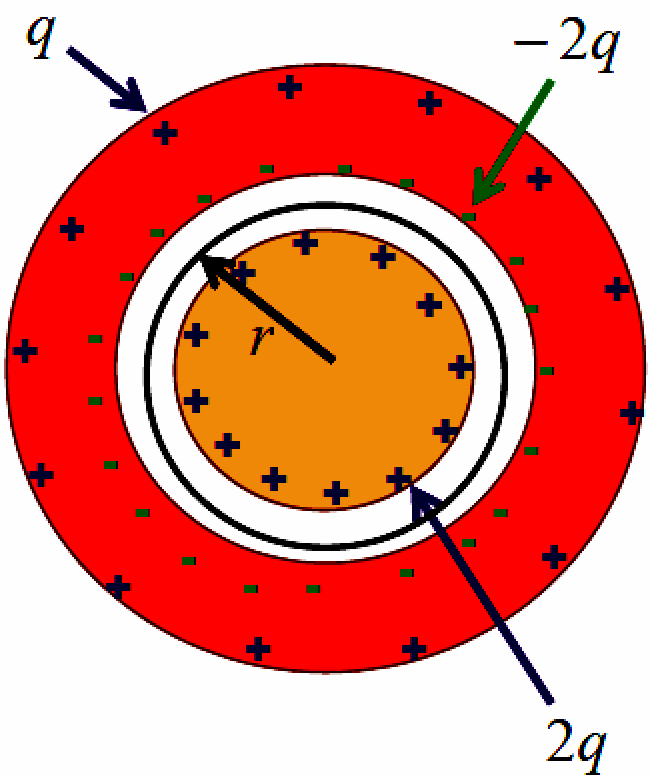
Note: You can say, without using Gauss's law, that electric field is zero inside the conductor.

(b). Between the sphere and shell

Consider a spherical Gaussian surface of radius r such that a < r < b.

According to Gauss's law

$$\oint \vec{E}.d\vec{A} = q_{enc} / \varepsilon_o$$



As electric field always remains perpendicular to the surface of conductor and enclosed charge is 2q, so

$$E\oint dA = 2q / \varepsilon_o$$

$$E \times 4\pi r^2 = 2q / \varepsilon_o$$

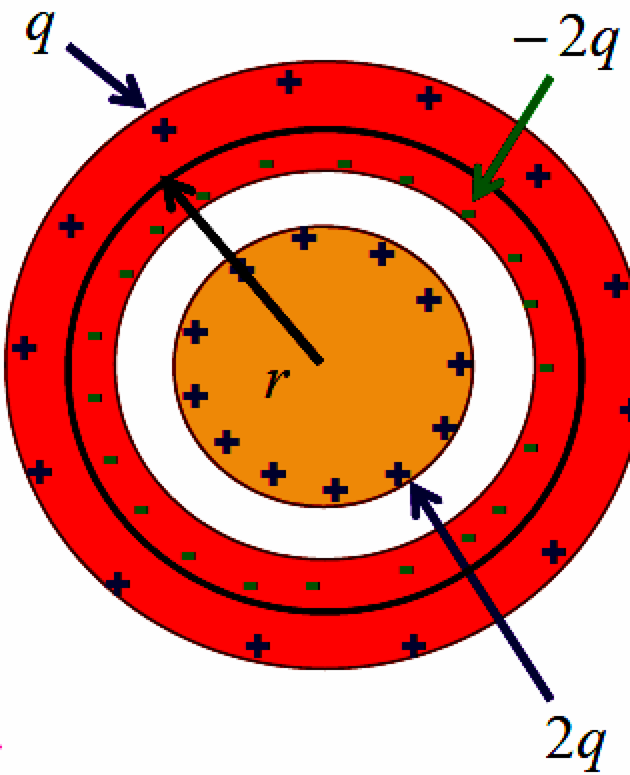
$$E = \frac{2q}{4\pi r^2 \varepsilon_o} \quad \Rightarrow \quad E = \frac{2kq}{r^2}$$

(c). Inside the shell

Consider a spherical Gaussian surface of radius r such that b < r < c.

According to Gauss's law

$$\oint \vec{E}.d\vec{A} = q_{enc} / \varepsilon_o$$



As charge enclosed is zero, so

$$E = 0$$

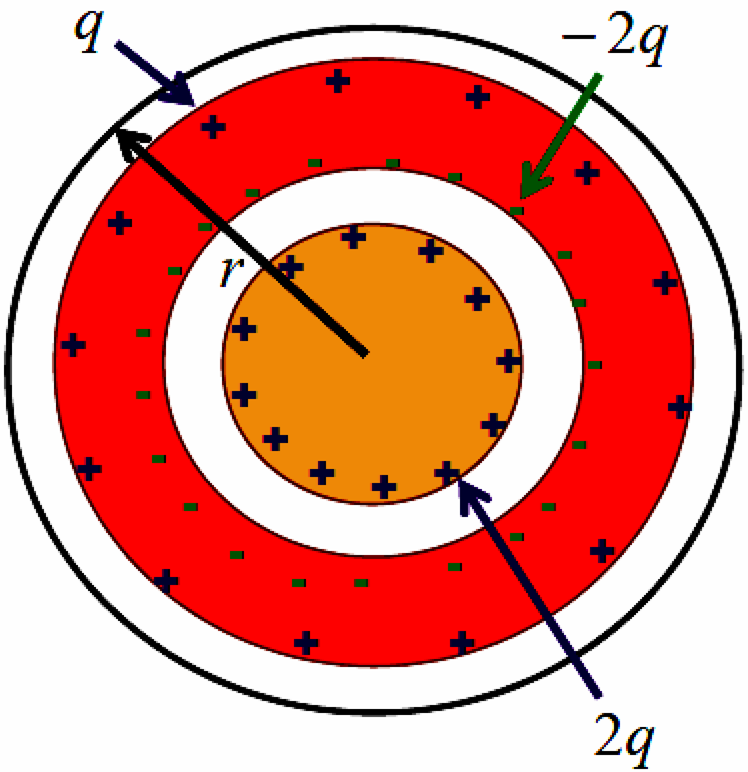
Note: You can say, without using Gauss's law, that electric field is zero inside the conductor.

(d). Outside the shell

Consider a spherical Gaussian surface of radius r such that r > c.

$$\oint \vec{E}.d\vec{A} = q_{enc} / \varepsilon_o$$

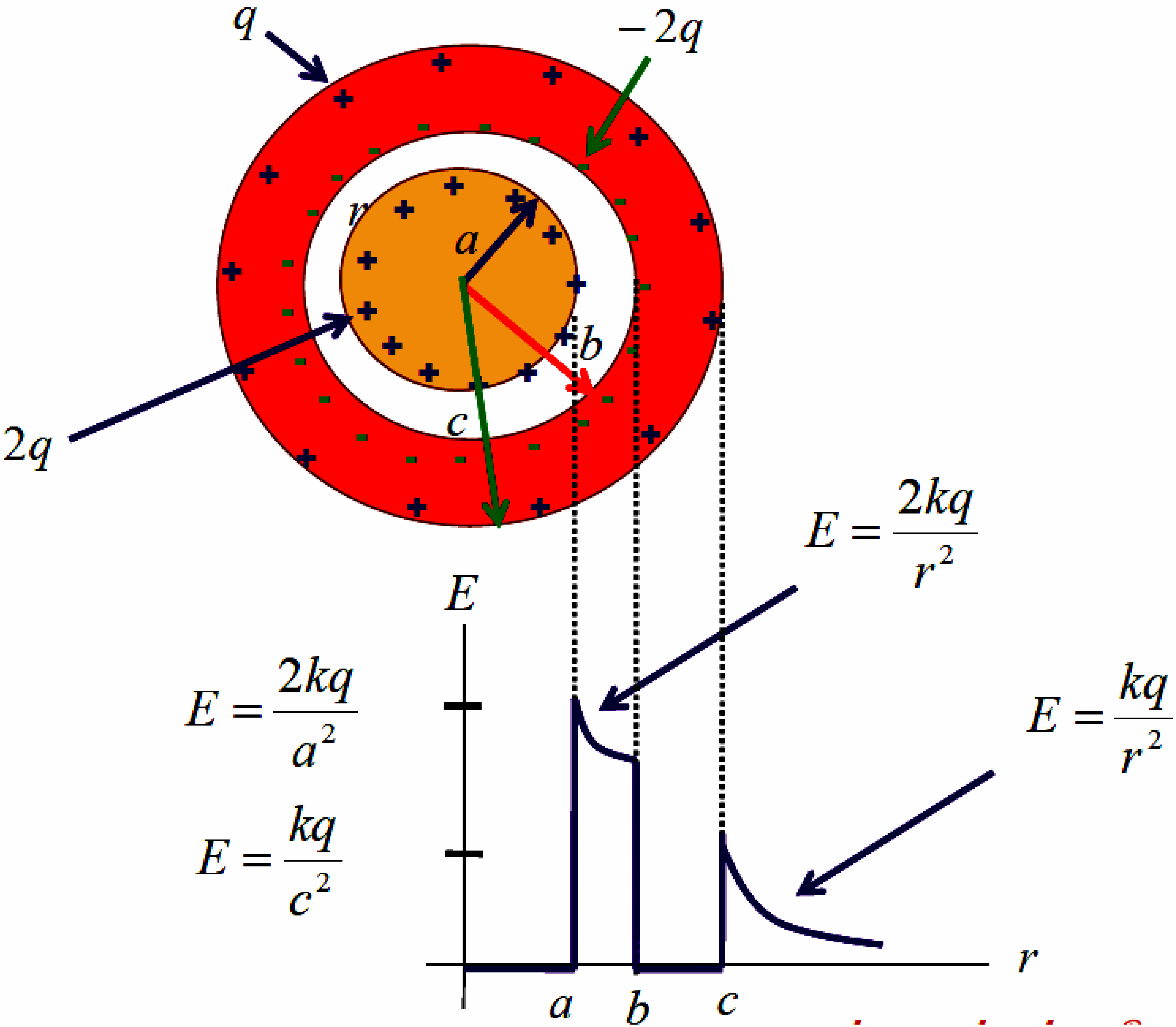
As electric field always remains perpendicular to the surface of conductor and enclosed charge is q, so



$$E\oint dA = q / \varepsilon_o$$

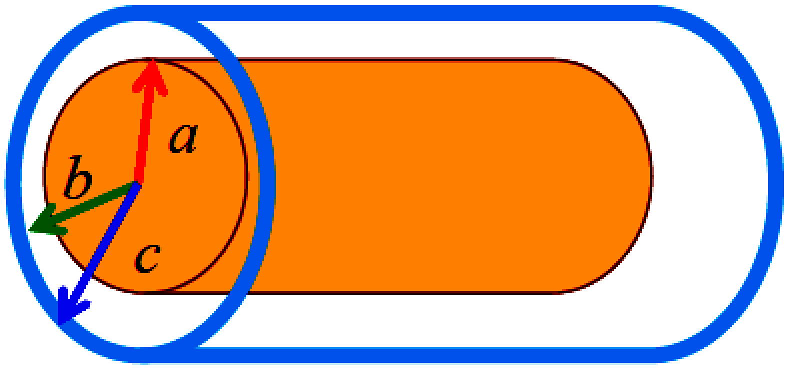
$$E \times 4\pi r^2 = q / \varepsilon_o$$

$$E = \frac{q}{4\pi r^2 \varepsilon_o} \quad \Rightarrow \quad E = \frac{kq}{r^2}$$



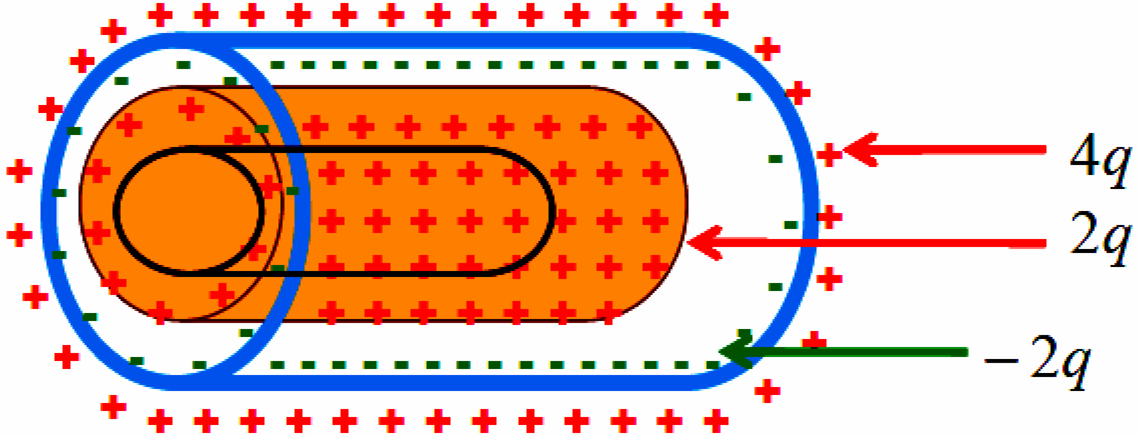
q
−2q
2q
r
a
b
c
E
$E = \frac{2kq}{r^2}$
$E = \frac{2kq}{a^2}$
$E = \frac{kq}{c^2}$
$E = \frac{kq}{r^2}$
r
a
b
c

A solid conducting very long cylinder of length ***l*** and radius **a** carries a net positive charge 2**q**. A conducting long cylindrical shell having length ***l*** and of inner radius **b** and outer radius **c** is concentric with the solid cylinder and carries a net charge **2q**. What charges appear on the inner and outer surface of the shell? Using Gauss's law, find the electric field in the regions (a) within the solid cylinder, (b) Between the cylinder and the shell, (c) inside the shell (d) outside the shell.



The solid cylinder and shell are both conducting, so all charge will reside on the surfaces. Due to repulsion of charges, -2q will reside on the inner surface of shell and +4q will be on the outer one.

(a). Within the solid cylinder



Consider a long cylindrical Gaussian surface of length l and radius r such that r < a. According to Gauss's law

$$\oint \vec{E}.d\vec{A} = q_{enc} / \varepsilon_o$$

As charge enclosed is zero, so

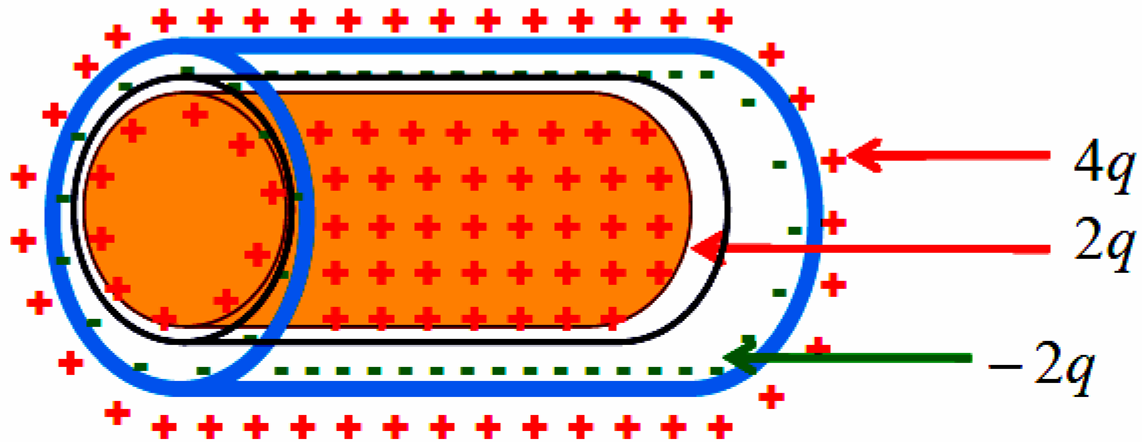
$$E = 0$$

Note: You can say, without using Gauss's law, that electric field is zero inside the conductor.

(b). Between the cylinder and shell

Consider a long cylindrical Gaussian surface of length l and radius r such that a < r < b. According to Gauss's law

$$\oint \vec{E}.d\vec{A} = q_{enc} / \varepsilon_o$$



As electric field always remains perpendicular to the surface of conductor and enclosed charge is 2q, so

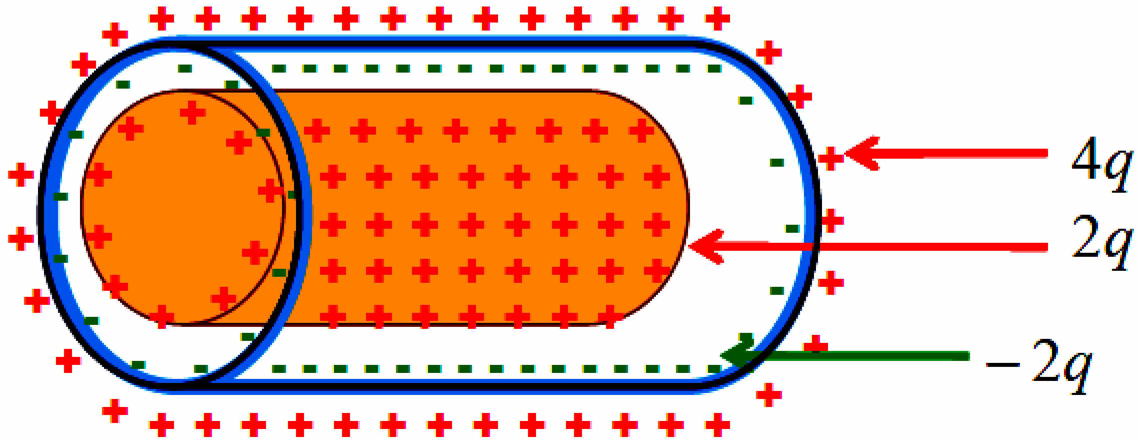
$$E\oint dA = 2q / \varepsilon_o$$

$$E \times 2\pi r l = 2q / \varepsilon_o \quad \Rightarrow \quad E = \frac{q}{\pi r l \varepsilon_o}$$

(c). Inside the shell

Consider a long cylindrical Gaussian surface of length l and radius r such that b < r < c. According to Gauss's law

$$\oint \vec{E}.d\vec{A} = q_{enc} / \varepsilon_o$$



As charge enclosed is zero, so

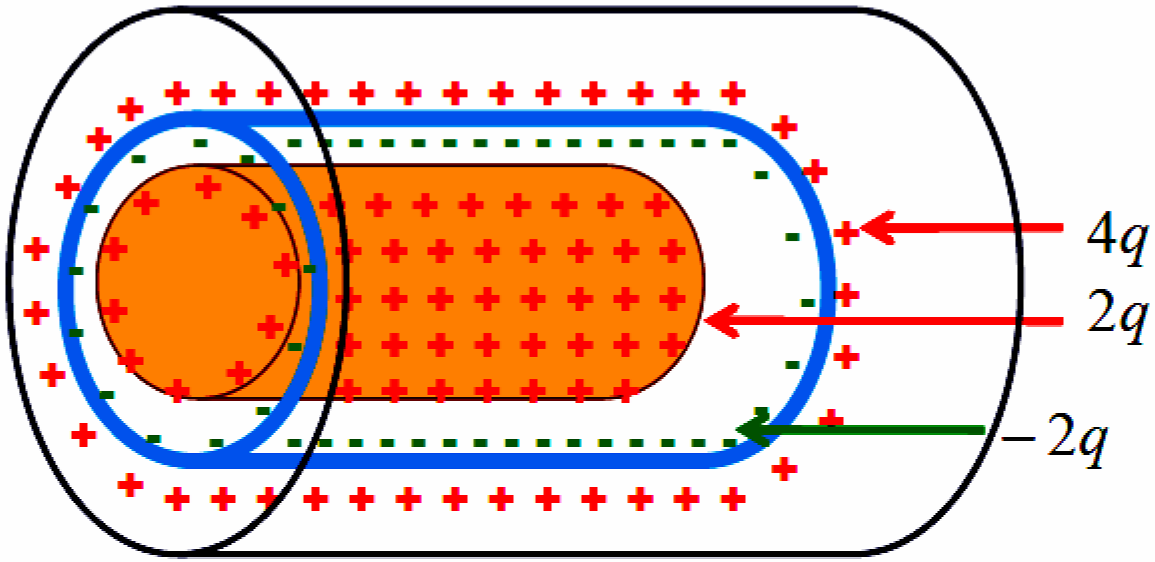
$$E = 0$$

Note: You can say, without using Gauss's law, that electric field is zero inside the conductor.

(d). Outside the shell

Consider a long cylindrical Gaussian surface of length l and radius r such that r > c.

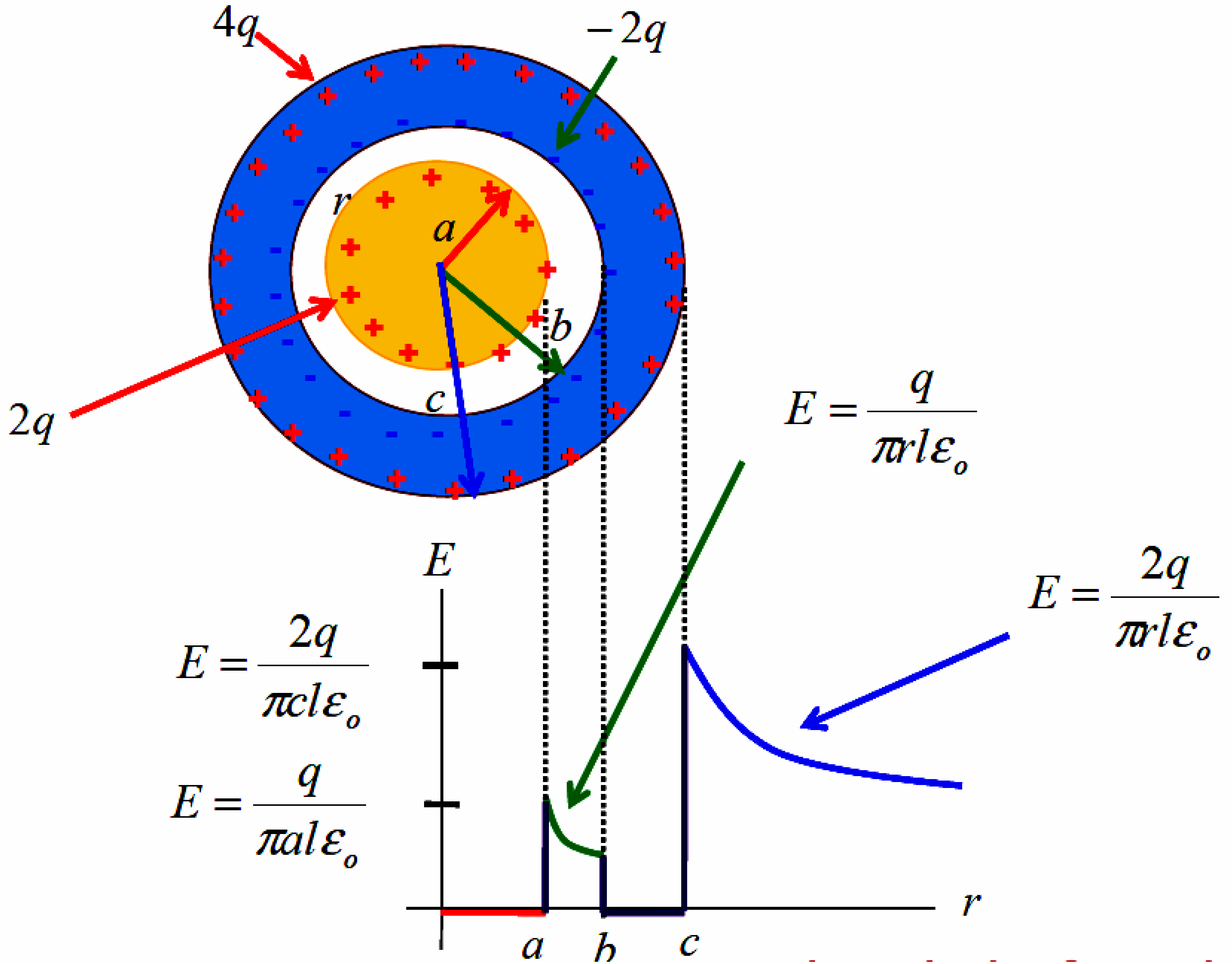
$$\oint \vec{E}.d\vec{A} = q_{enc} / \varepsilon_o$$



As electric field always remains perpendicular to the surface of conductor and enclosed charge is 4q, so

$$E\oint dA = 4q / \varepsilon_o$$

$$E \times 2\pi r l = 4q / \varepsilon_o \quad \Rightarrow \quad E = \frac{2q}{\pi r l \varepsilon_o}$$

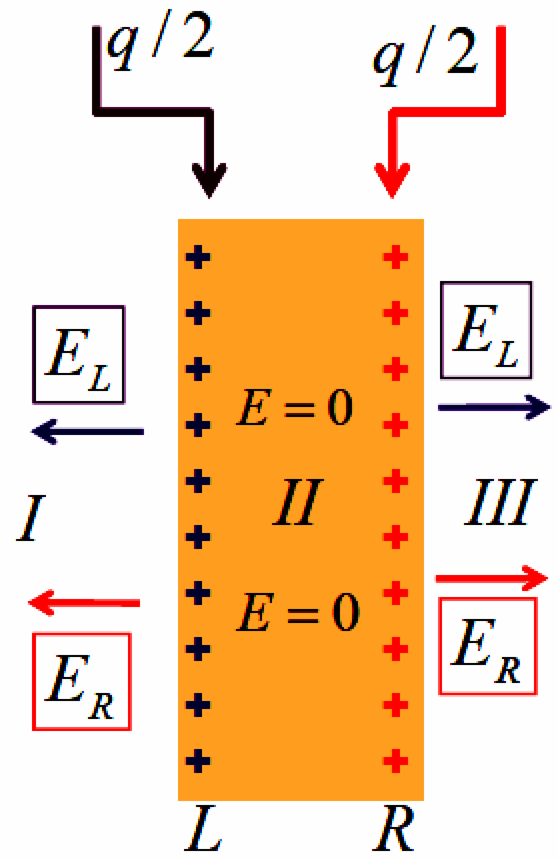


$4q$
$-2q$
r
a
b
c
$2q$
$E = \frac{q}{\pi r l \varepsilon_o}$
E
$E = \frac{2q}{\pi r l \varepsilon_o}$
$E = \frac{2q}{\pi c l \varepsilon_o}$
$E = \frac{q}{\pi a l \varepsilon_o}$
r
a
b
c

E due to a Conducting Plate of Charges

Consider an isolated conducting, infinite plate. Let positive charge q is sprayed on it. The charges will move to the surface setting q/2 on each face as shown. The uniform surface charge density of each face will be

$$\sigma = q/2A$$



We can consider each face of the conducting plate as a sheet of charge which establishes an electric field

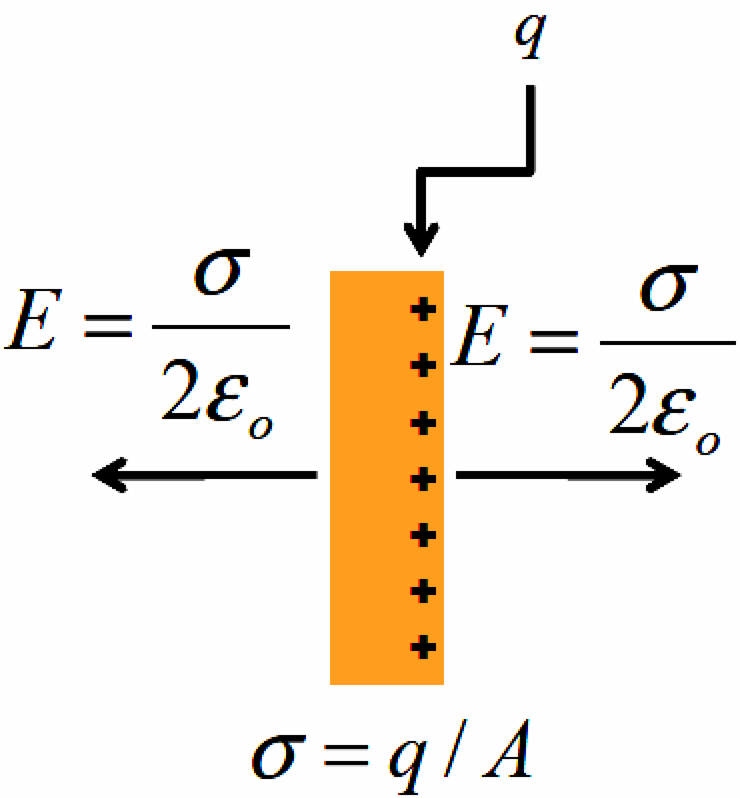
$$E_R = E_L = \frac{\sigma}{2\varepsilon_\circ}$$

So net electric field in region I and III will be

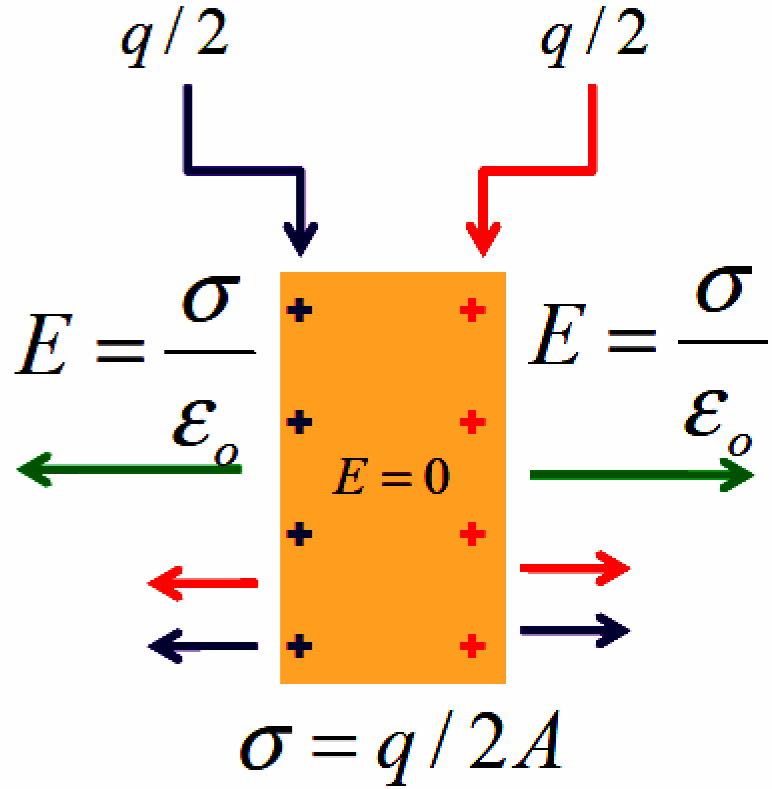
$$E = E_R + E_L = \frac{\sigma}{\varepsilon_\circ}$$

Note: In region II $E = 0$

Nonconducting Sheet

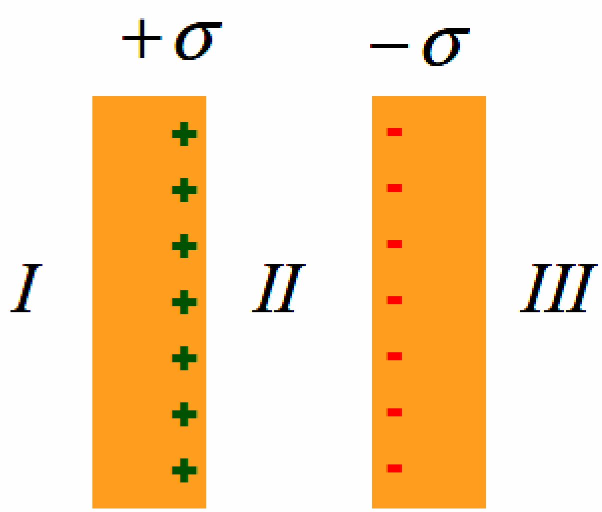


Isolated Conducting Plate



Electric field due to conducting plate is twice that of due to nonconducting sheet if same amount of charge is sprayed on them.

Two large, flat metal plates are separated by a distance that is very small compared to their height and width. The conductors are given equal but opposite uniform surface charge densities $\pm\sigma$ where $\sigma = q/A$. Ignore edge effects and for points far from the edges, find (*a*) the electric field between the plates (*b*) the electric field outside the plates on either side. (*c*) How would your results be altered if the two plates were nonconductors?



Here, conductors are not isolated. There is an attraction between the positive charges on one plate and negative charges on the other that draws the charges to the inner surfaces of the plates. Each inner surface has charge q (instead of q/2) and surface charge density $\sigma = q/A$. Regarded as a sheet of charge, each surface sets up an electric field

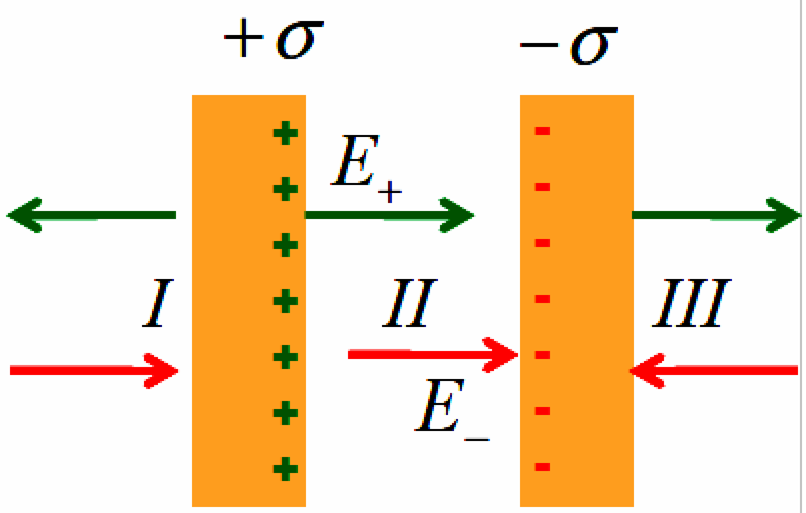
$$E_+ = E_- = \frac{\sigma}{2\varepsilon_o}$$

(a) Net electric field between the plate (region II) will be

$$E = E_+ + E_- = \frac{\sigma}{\varepsilon_o}$$

(b) Net electric field outside the plates (region I&III)will be

$$E = 0$$



(c) In this situation, there will be no difference whether these plates are conductors or insulators.

Typical Electric Field Calculations Using Gauss's Law

Charge Distribution	Electric Field	Location
Insulating sphere of radius R, uniform charge density, and total charge Q	$k_e \frac{Q}{r^2}$	$r > R$
	$k_e \frac{Q}{R^3} r$	$r < R$
Thin spherical shell of radius R and total charge Q	$k_e \frac{Q}{r^2}$	$r > R$
	0	$r < R$
Line charge of infinite length and charge per unit length λ	$2k_e \frac{\lambda}{r}$	Outside the line
Nonconducting, infinite charged plane having surface charge density σ	$\frac{\sigma}{2\epsilon_0}$	Everywhere outside the plane
Conductor having surface charge density σ	$\frac{\sigma}{\epsilon_0}$	Just outside the conductor
	0	Inside the conductor