Engineering Mechanics

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Contents (Section 5.3 & 5.4)

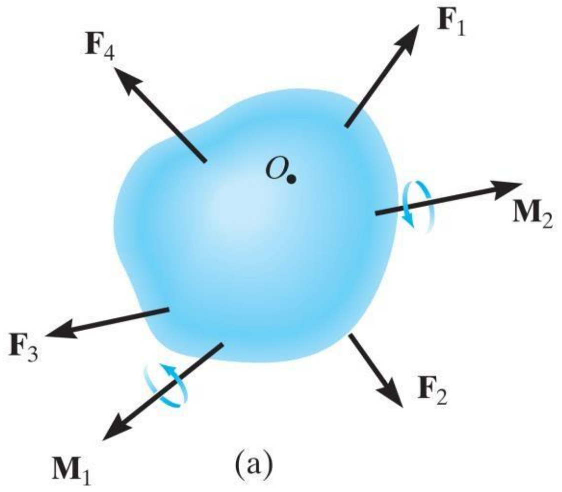
- Recap
- Equations of Equilibrium
- Two- and Three-Force Members

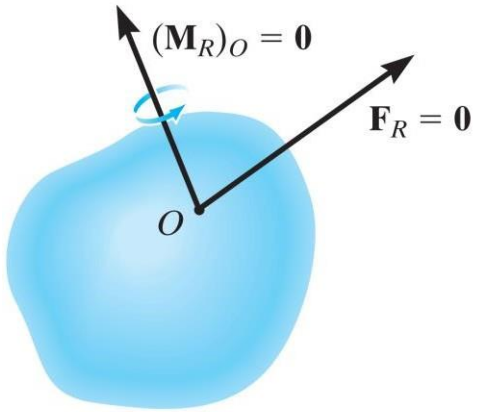
Conditions for Rigid-Body Equilibrium

$$\mathbf{F}_R = \Sigma \mathbf{F} = \mathbf{0}$$

$$(\mathbf{M}_R)_O = \Sigma \mathbf{M}_O = \mathbf{0}$$

Necessary



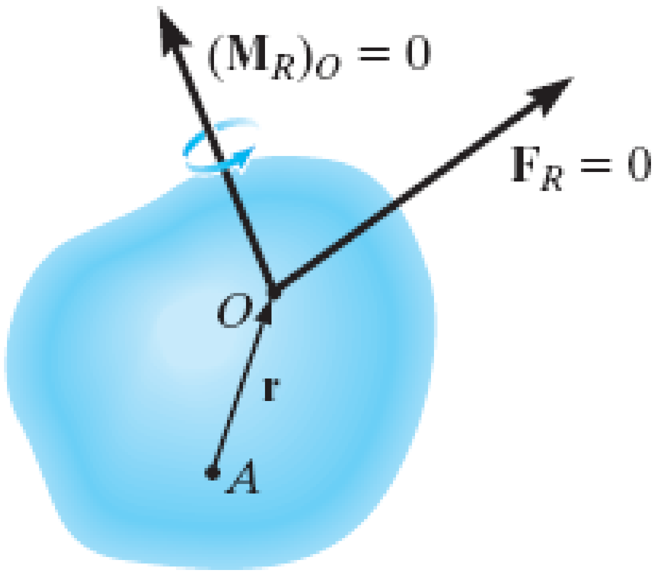


Conditions for Rigid-Body Equilibrium

$$\sum M_A = r \times F_R + (M_R)_O = 0$$

Since $\mathbf{r} \neq \mathbf{0}$, this equation is satisfied only if

$$F_R = \sum F = 0$$
$$(M_R)_O = \sum M_O = 0$$

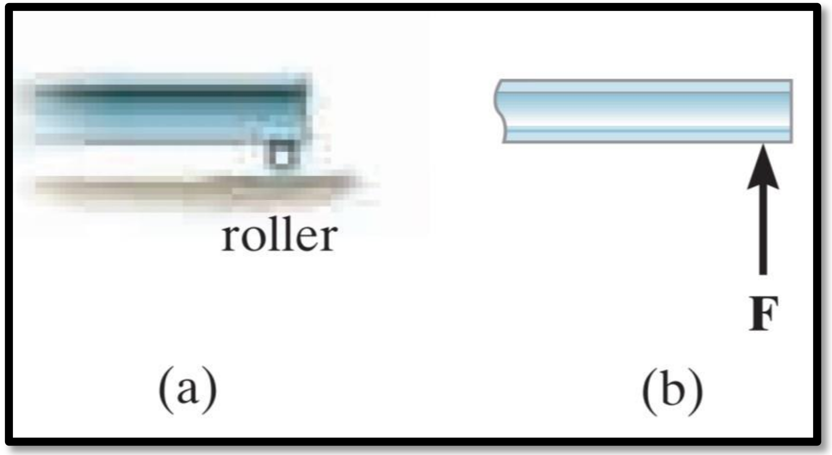


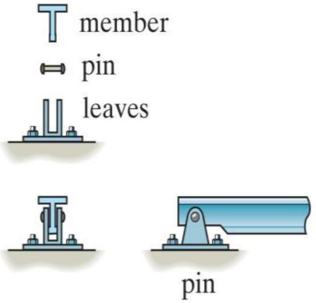
Free-Body Diagrams

Support Reactions.

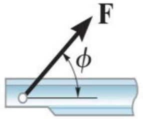
- A support prevents the translation of a body in a given direction by exerting a force on the body in the opposite direction.

- A support prevents the rotation of a body in a given direction by exerting a couple moment on the body in the opposite direction.



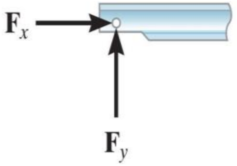


(c)



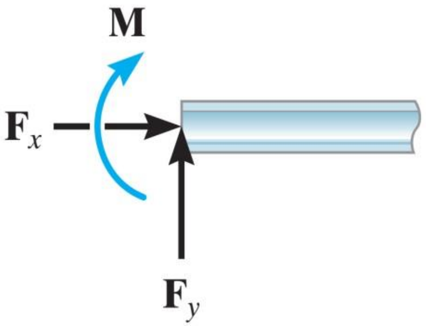
(d)

or



(e)

(f)

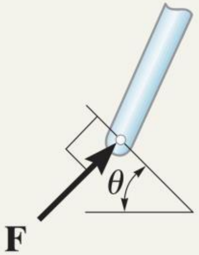


(g)

Support Reactions.

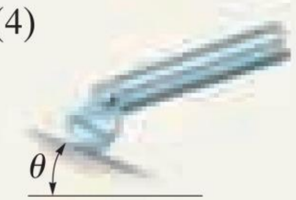
(3)

roller

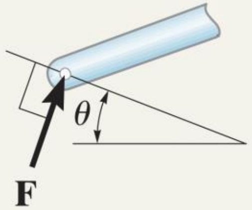


One unknown. The reaction is a force which acts perpendicular to the surface at the point of contact.

(4)



rocker



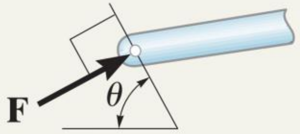
One unknown. The reaction is a force which acts perpendicular to the surface at the point of contact.

Free-Body Diagrams

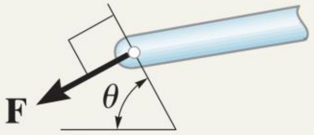
Support Reactions.

(6)

roller or pin in
confined smooth slot



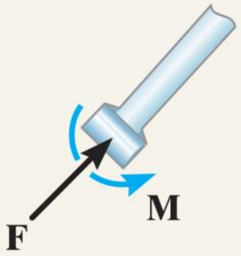
or



One unknown. The reaction is a force which acts perpendicular to the slot.

(9)

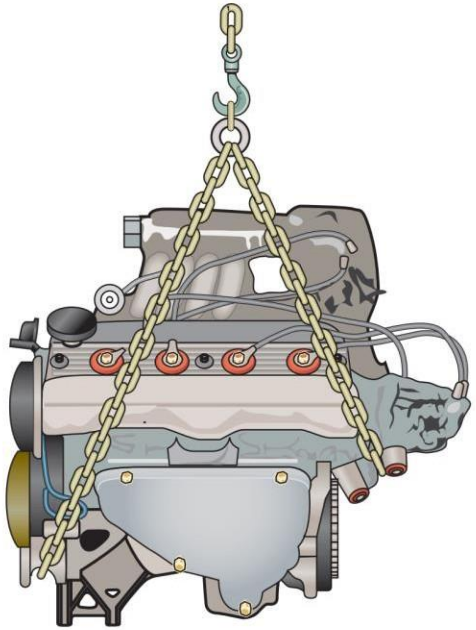
member fixed connected
to collar on smooth rod

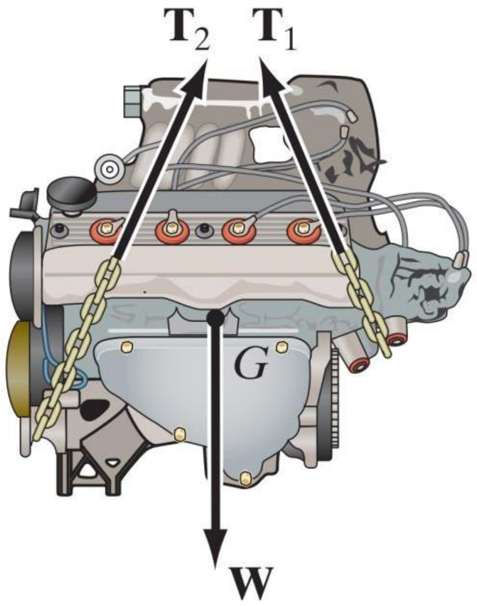


Two unknowns. The reactions are the couple moment and the force which acts perpendicular to the rod.

Free-Body Diagrams

Internal Forces.

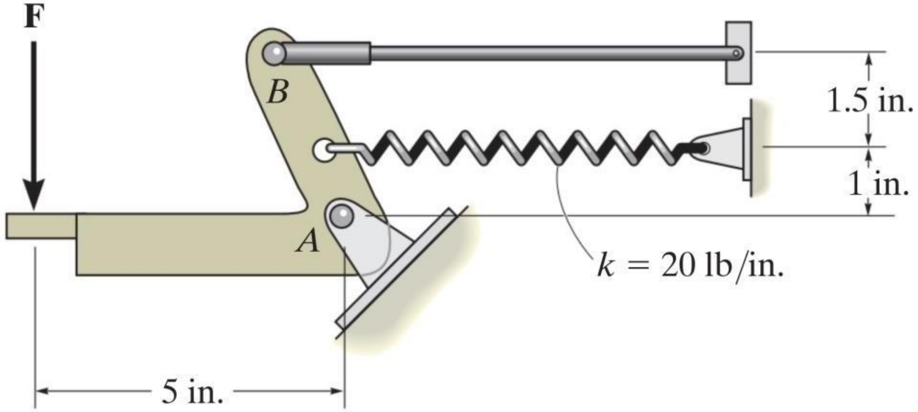




Example

Draw the free-body diagram of the foot lever shown in Fig. 5–8*a*. The operator applies a vertical force to the pedal so that the spring is stretched 1.5 in. and the force on the link at *B* is 20 lb.

(a)

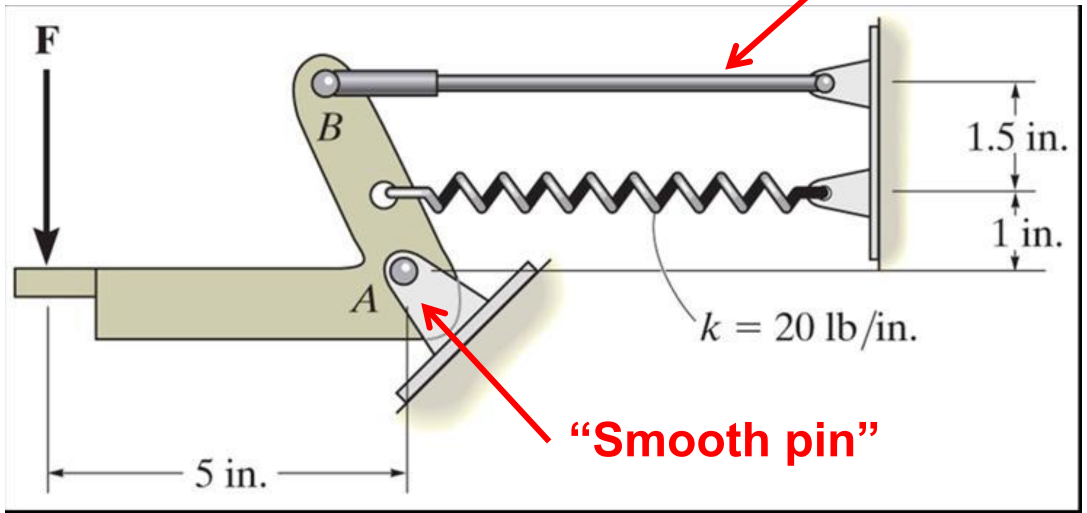


EXAMPLE: 2

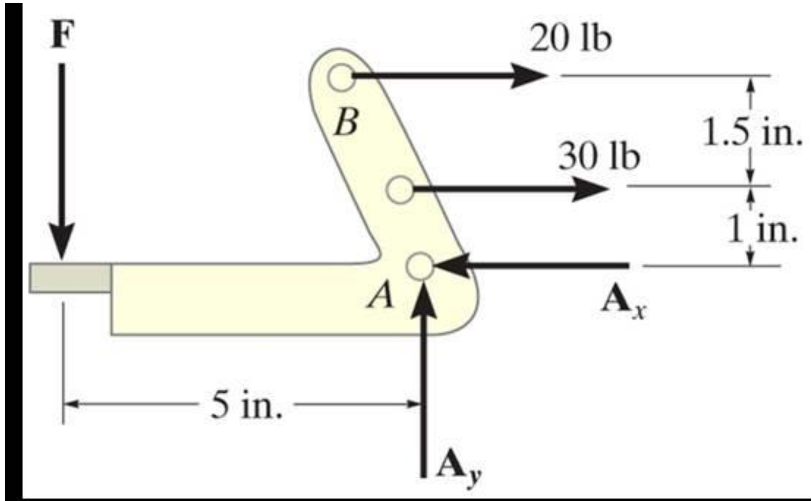
Given: The operator applies a vertical force to the pedal so that the spring is stretched 1.5 in. and the force in the short link at *B is 20 lb*.

Draw: A an idealized model and free-body diagram of the foot pedal.

"Weightless link" (see Table 5-1)



The idealized model



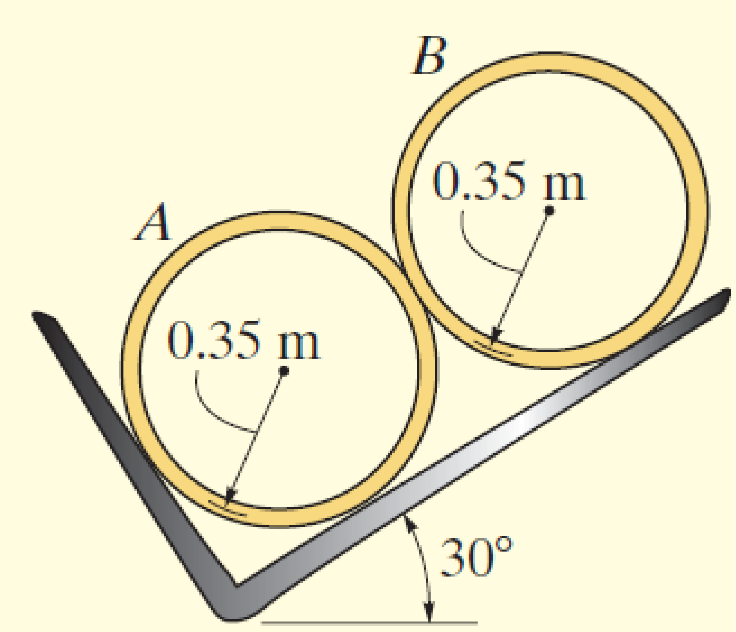
The free-body diagram

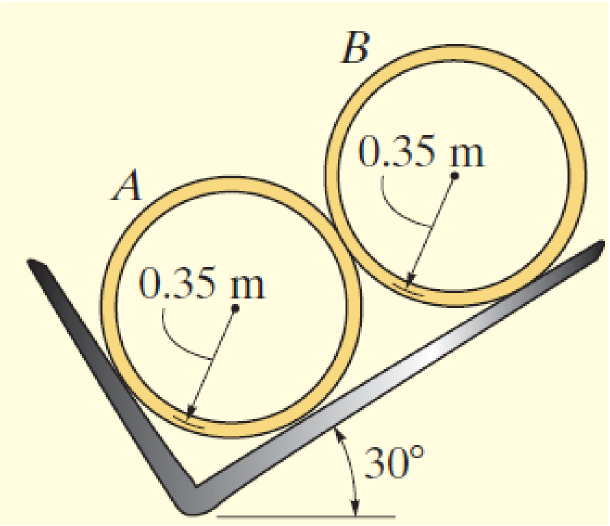
Free-Body Diagram's Description

By inspection of the photo the lever is loosely bolted to the frame at A. The rod at B is pinned at its ends and acts as a "short link." After making the proper measurements, the idealized model of the lever is shown in Fig. 5–8*b*. From this, the free-body diagram is shown in Fig. 5–8*c*. The pin support at A exerts force components $\mathbf{A}_x$ and $\mathbf{A}_y$ on the lever. The link at B exerts a force of 20 lb, acting in the direction of the link. In addition the spring also exerts a horizontal force on the lever. If the stiffness is measured and found to be $k = 20$ lb/in., then since the stretch $s = 1.5$ in., using Eq. 3–2, $F_s = ks = 20$ lb/in. (1.5 in.) $= 30$ lb. Finally, the operator's shoe applies a vertical force of $\mathbf{F}$ on the pedal. The dimensions of the lever are also shown on the free-body diagram, since this information will be useful when computing the moments of the forces. As usual, the senses of the unknown forces at A have been assumed. The correct senses will become apparent after solving the equilibrium equations.

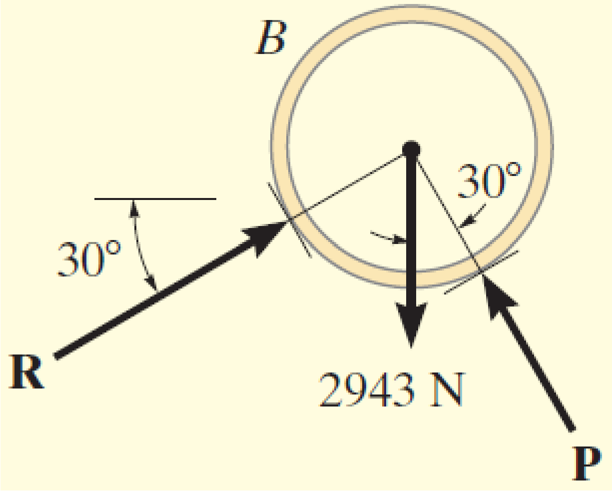
Example

Two smooth pipes, each having a mass of 300 kg, are supported by the forked tines of the tractor in Fig. 5–9*a*. Draw the free-body diagrams for each pipe and both pipes together.

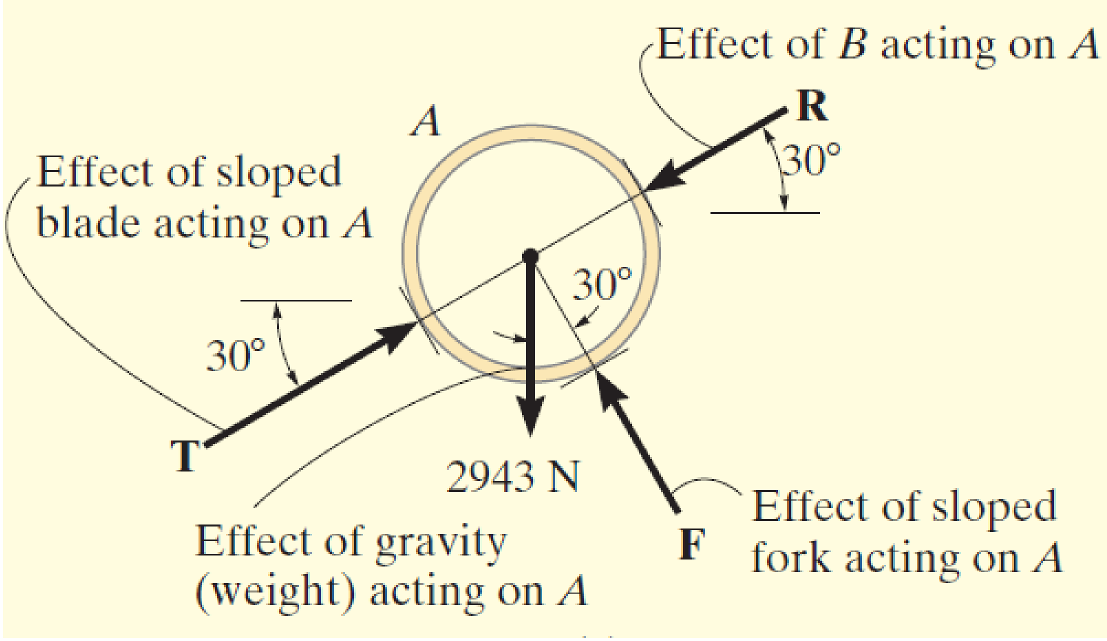




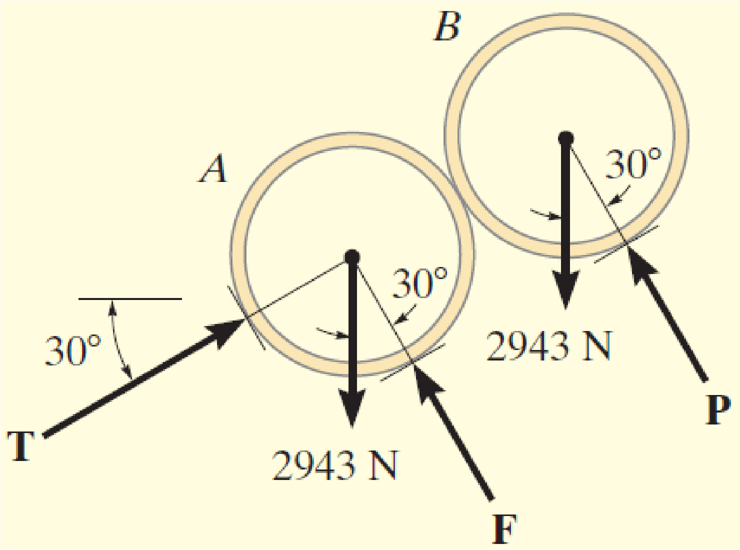
B
A
0.35 m
0.35 m
30°



B
30°
30°
R
2943 N
P



Effect of B acting on A
R
30°
A
Effect of sloped
blade acting on A
30°
30°
T
2943 N
Effect of sloped
fork acting on A
Effect of gravity
(weight) acting on A
F



B
A
30°
30°
30°
2943 N
T
P
2943 N
F

5.3 Equation of Equilibrium

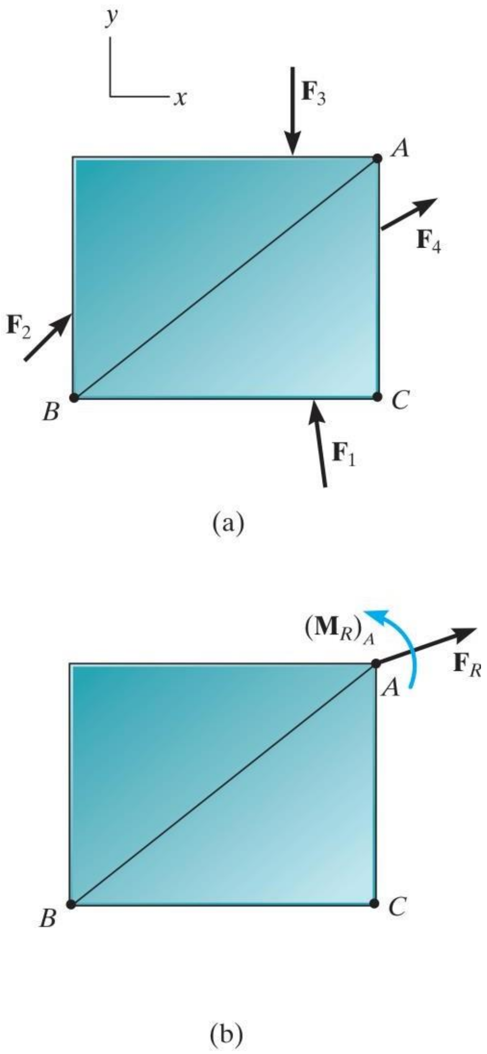
- For equilibrium of a rigid body in 2D,

$$\sum F_x = 0; \sum F_y = 0; \sum M_O = 0$$

- $\boldsymbol{\sum F_x}$ and $\boldsymbol{\sum F_y}$ represent sums of x and y components of all the forces
- $\boldsymbol{\sum M_O}$ represents the sum of the couple moments and moments of the force components

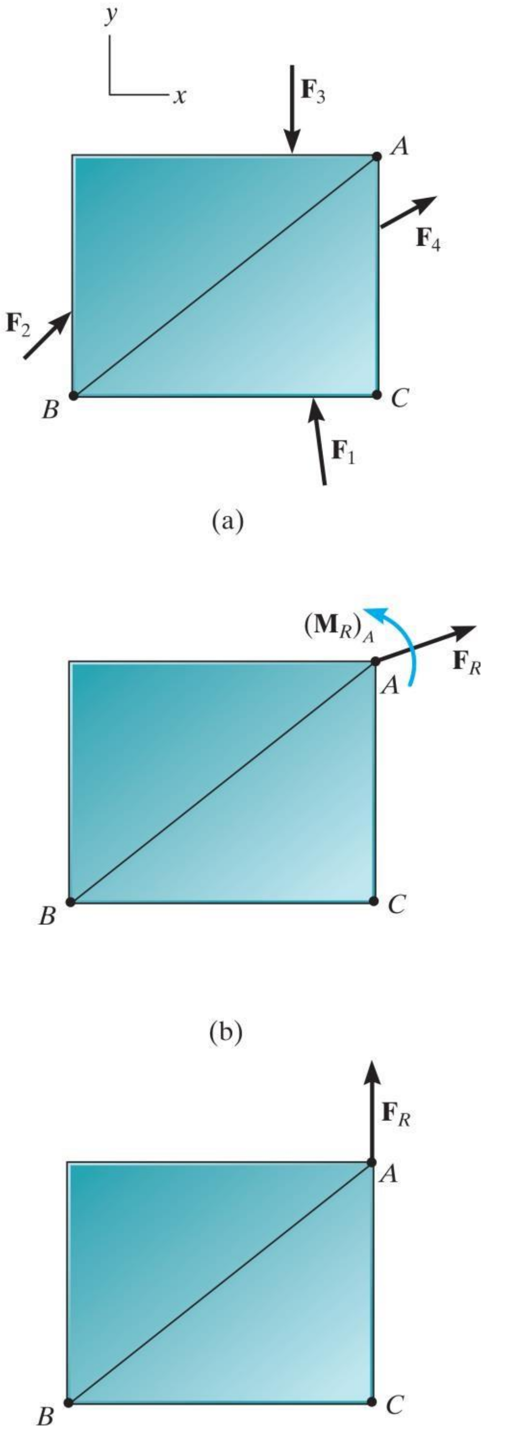
Equation of Equilibrium

$$\Sigma F_x = 0$$
$$\Sigma F_y = 0$$
$$\Sigma M_O = 0$$



Equation of Equilibrium

$$\Sigma F_x = 0$$
$$\Sigma F_y = 0$$
$$\Sigma M_O = 0$$



Alternative Sets of Equilibrium Equations.

$$\Sigma F_x = 0 \qquad \Sigma M_A = 0$$
$$\Sigma M_A = 0 \qquad \Sigma M_B = 0$$
$$\Sigma M_B = 0 \qquad \Sigma M_C = 0$$

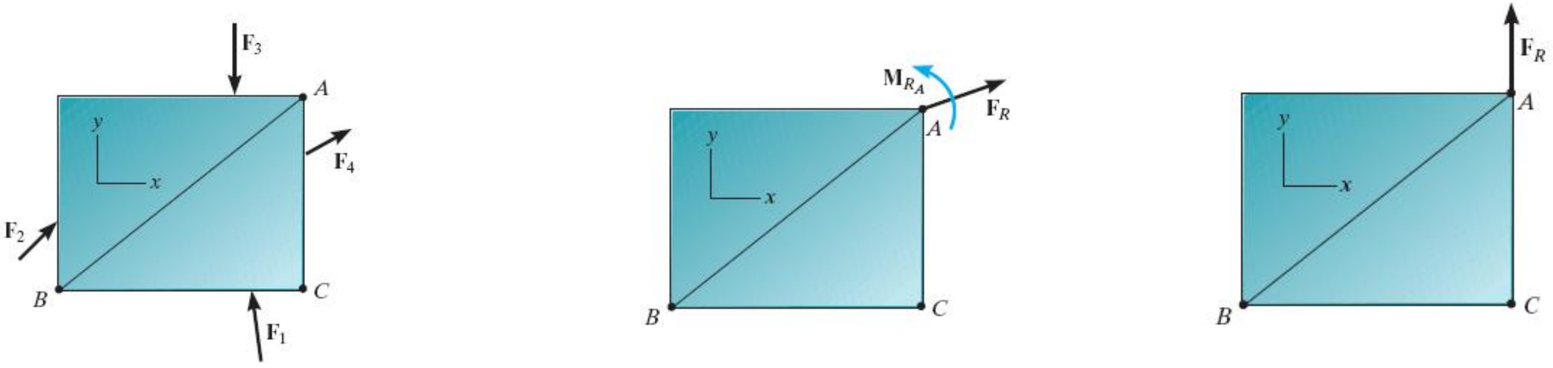
Equation of Equilibrium

Alternative Sets of Equilibrium Equations

- For coplanar equilibrium problems,
  $\sum F_x = 0$; $\sum F_y = 0$; $\sum M_O = 0$

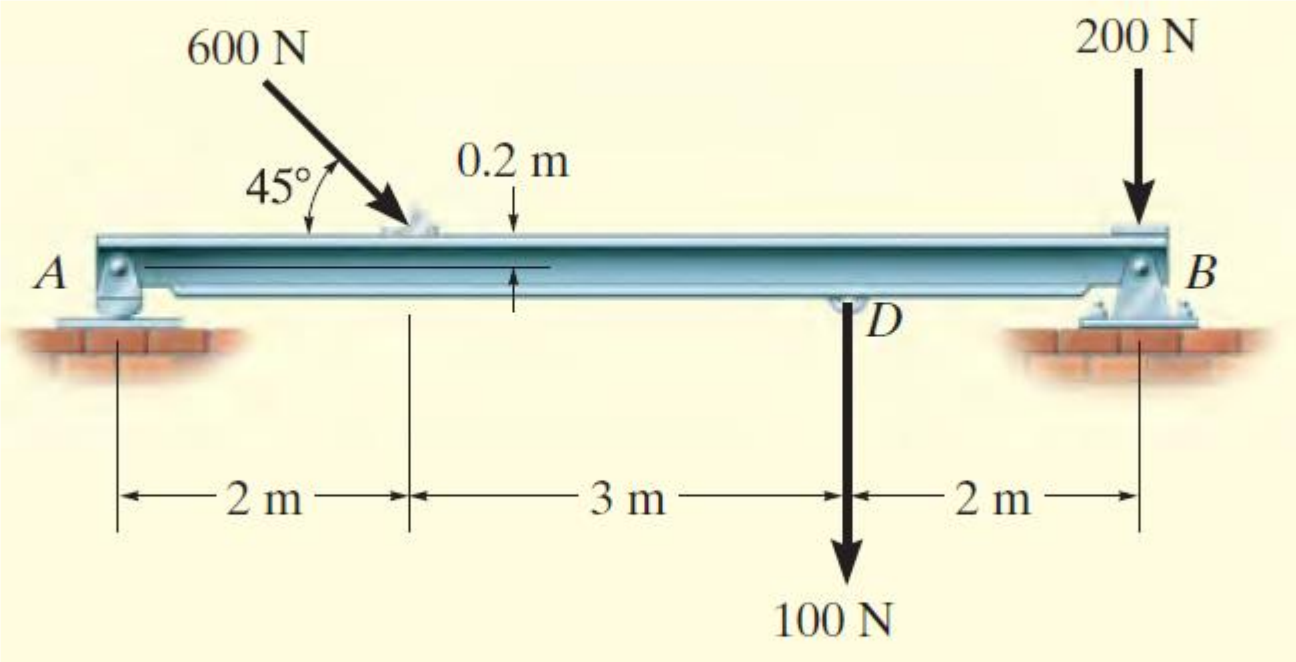
- 2 alternative sets of 3 independent equilibrium equations,

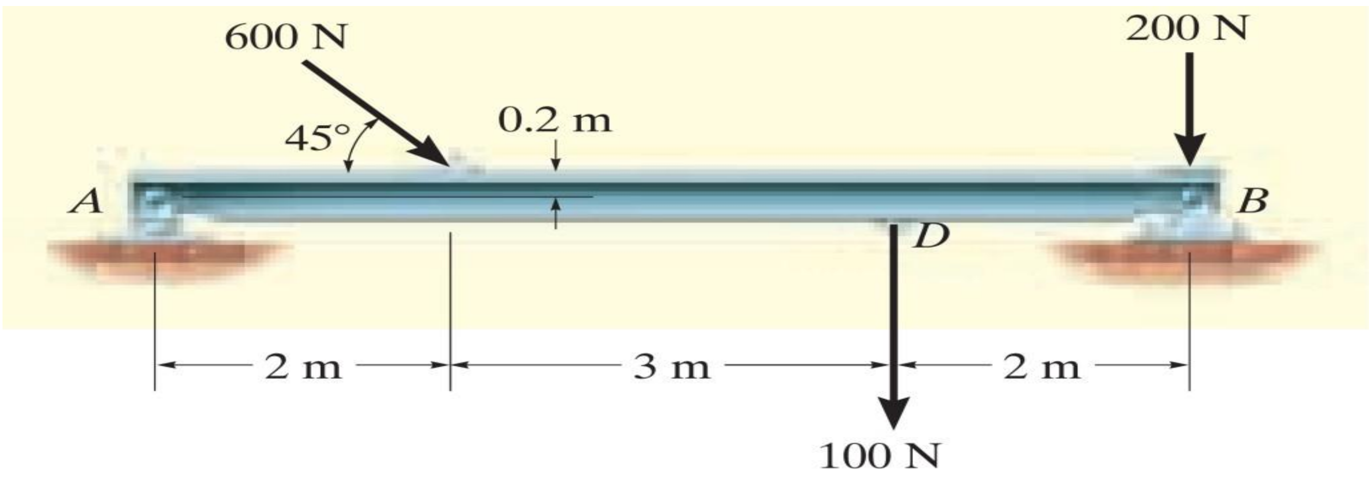
  $\sum F_a = 0$; $\sum M_A = 0$; $\sum M_B = 0$

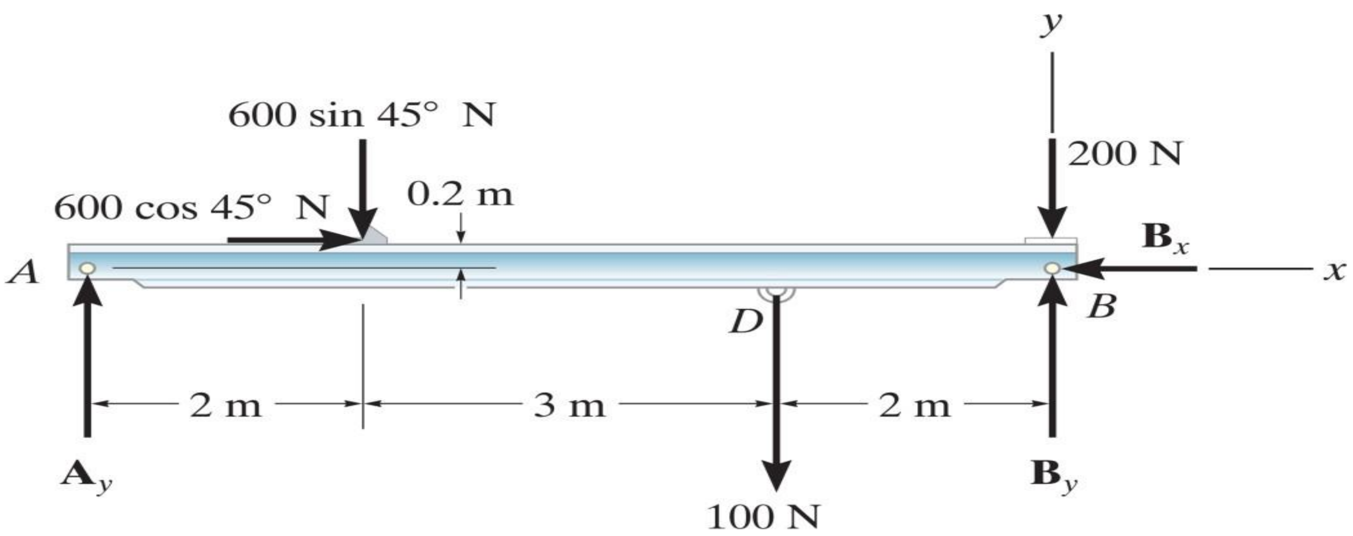


Example

Determine the horizontal and vertical components of reaction on the beam caused by the pin at *B* and the rocker at *A* as shown in Fig. 5–12*a*. Neglect the weight of the beam.



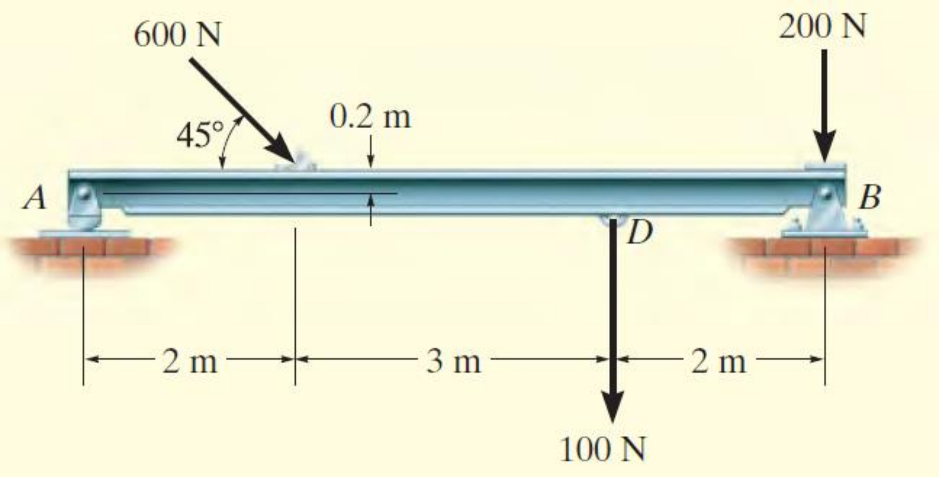

600 N
200 N
45°
0.2 m
A
B
D
2 m
3 m
2 m
100 N


y
600 sin 45° N
200 N
600 cos 45° N
0.2 m
B_x
A
x
B
D
2 m
3 m
2 m
A_y
B_y
100 N

$\xrightarrow{+} \Sigma F_x = 0;$

$600 \cos 45° \text{ N} - B_x = 0$

$B_x = 424 \text{ N}$



$\circlearrowleft + \Sigma M_B = 0;$

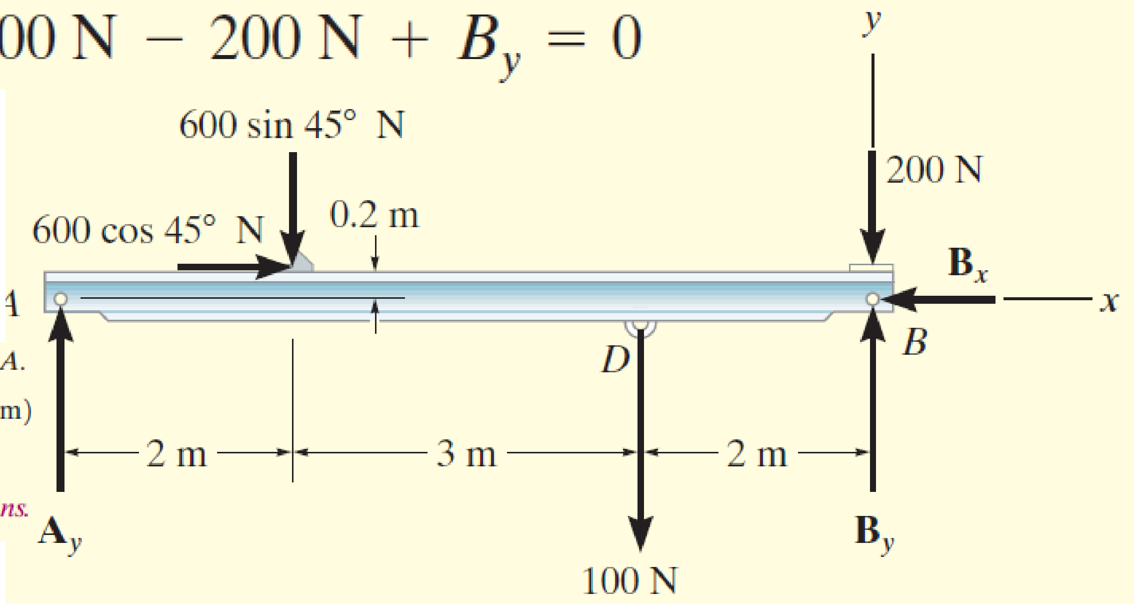
$100 \text{ N}(2 \text{ m}) + (600 \sin 45° \text{ N})(5 \text{ m}) - (600 \cos 45° \text{ N})(0.2 \text{ m}) - A_y(7 \text{ m}) = 0$

$A_y = 319 \text{ N}$

$+\uparrow \Sigma F_y = 0;$

$319 \text{ N} - 600 \sin 45° \text{ N} - 100 \text{ N} - 200 \text{ N} + B_y = 0$

$B_y = 405 \text{ N}$



NOTE: We can check this result by summing moments about point A.

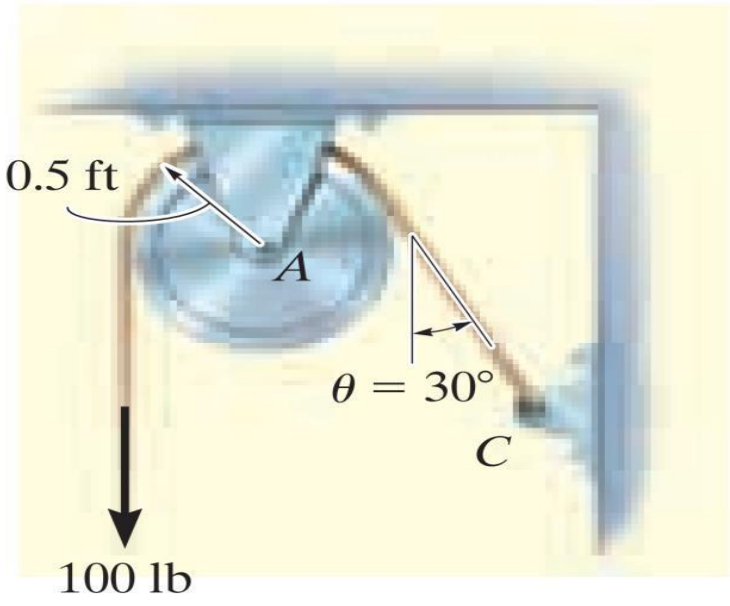
$\circlearrowleft + \Sigma M_A = 0; \quad -(600 \sin 45° \text{ N})(2 \text{ m}) - (600 \cos 45° \text{ N})(0.2 \text{ m})$

$-(100 \text{ N})(5 \text{ m}) - (200 \text{ N})(7 \text{ m}) + B_y(7 \text{ m}) = 0$

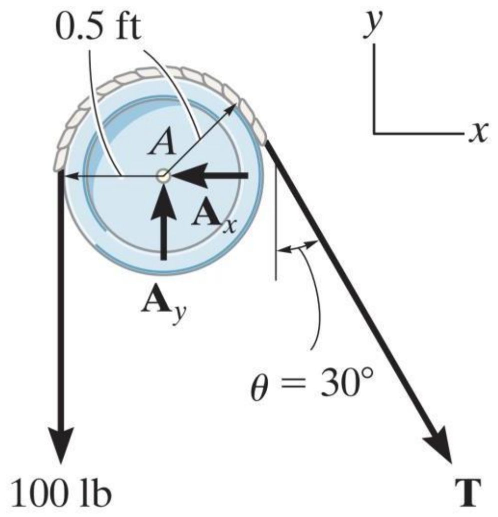
$B_y = 405 \text{ N}$ *Ans.*

Example

The cord supports a force of 100 lb and wraps over the frictionless pulley. Determine the tension in the cord at C and the horizontal and vertical components of reaction at pin A.



Equations of Equilibrium. Summing moments about point A to eliminate $\mathbf{A}_x$ and $\mathbf{A}_y$, Fig. 5–13c, we have



$$\circlearrowleft + \Sigma M_A = 0; \qquad 100 \text{ lb } (0.5 \text{ ft}) - T(0.5 \text{ ft}) = 0$$

$$T = 100 \text{ lb} \qquad \textit{Ans.}$$

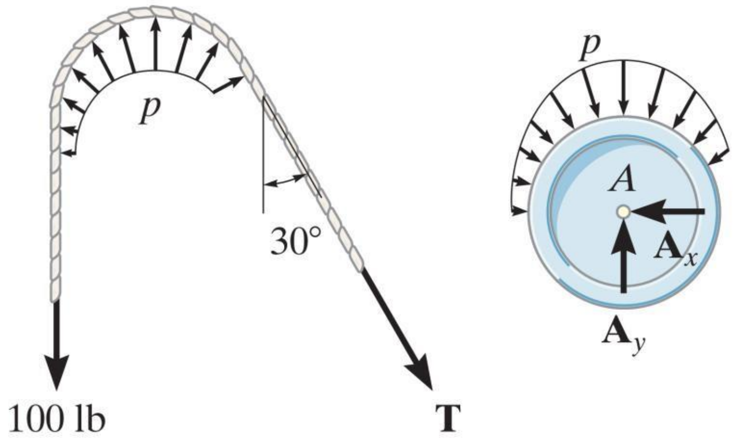
Using this result,

$$\xrightarrow{+} \Sigma F_x = 0; \qquad -A_x + 100 \sin 30° \text{ lb} = 0$$

$$A_x = 50.0 \text{ lb} \qquad \textit{Ans.}$$

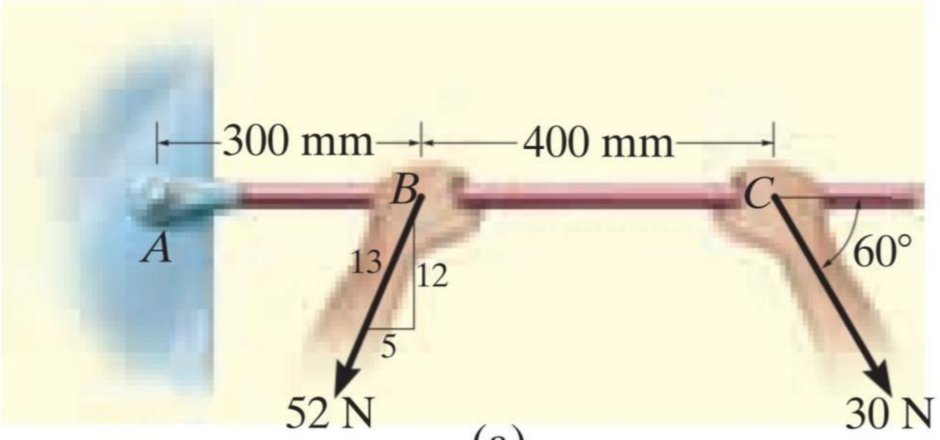
$$+\uparrow \Sigma F_y = 0; \qquad A_y - 100 \text{ lb} - 100 \cos 30° \text{ lb} = 0$$

$$A_y = 187 \text{ lb} \qquad \textit{Ans.}$$



Example

The box wrench is used to tighten the bolt at A. If the wrench does not turn when the load is applied to the handle, determine the torque or moment applied to the bolt and the force of the wrench on the bolt.



(a)

Equations of Equilibrium.

$\xrightarrow{+} \Sigma F_x = 0; \qquad A_x - 52\left(\frac{5}{13}\right) \text{N} + 30 \cos 60° \text{ N} = 0$

$$A_x = 5.00 \text{ N} \qquad \textit{Ans.}$$

$+\uparrow \Sigma F_y = 0; \qquad A_y - 52\left(\frac{12}{13}\right) \text{N} - 30 \sin 60° \text{ N} = 0$

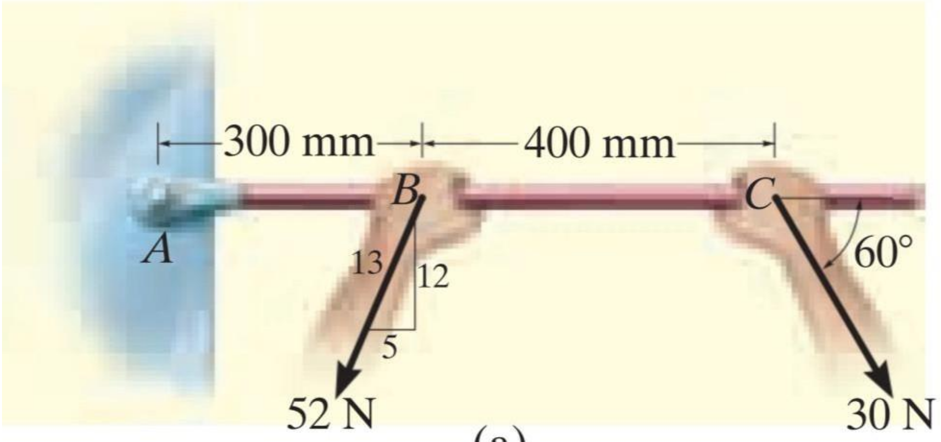
$$A_y = 74.0 \text{ N} \qquad \textit{Ans.}$$

$\circlearrowleft + \Sigma M_A = 0; \quad M_A - \left[52\left(\frac{12}{13}\right)\text{N}\right](0.3 \text{ m}) - (30 \sin 60° \text{ N})(0.7 \text{ m}) = 0$

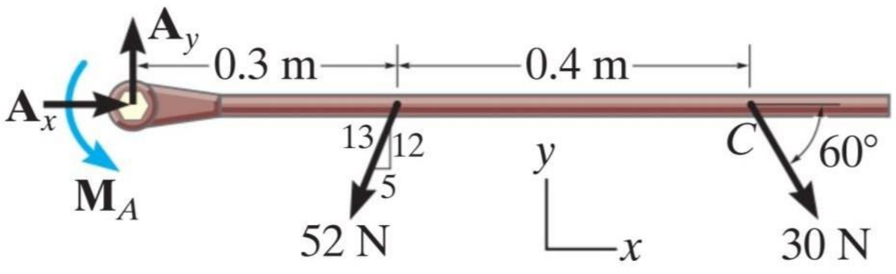
$$M_A = 32.6 \text{ N} \cdot \text{m} \qquad \textit{Ans.}$$

Note that $\mathbf{M}_A$ must be *included* in this moment summation. This couple moment is a free vector and represents the twisting resistance of the bolt on the wrench. By Newton's third law, the wrench exerts an equal but opposite moment or torque on the bolt. Furthermore, the resultant force on the wrench is

$$F_A = \sqrt{(5.00)^2 + (74.0)^2} = 74.1 \text{ N} \qquad \textit{Ans.}$$



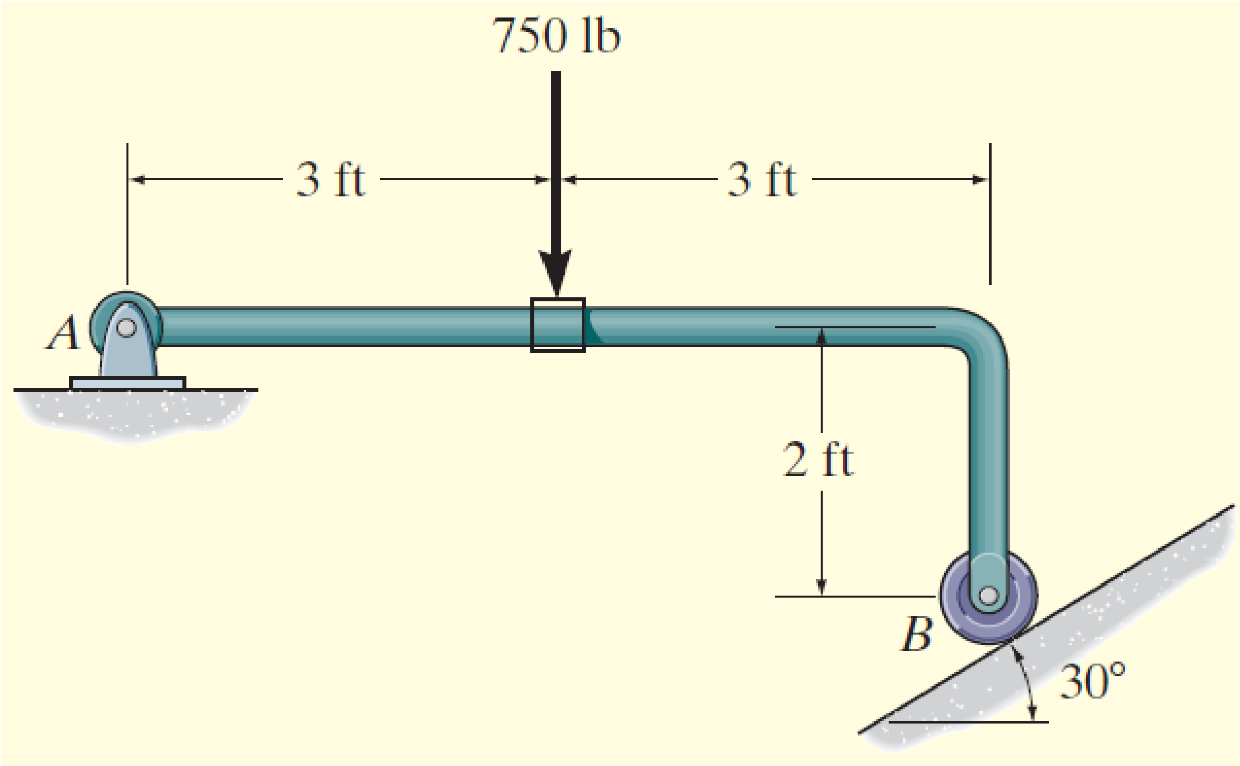
(a)



(b)

Example

Determine the horizontal and vertical components of reaction on the member at the pin *A*, and the normal reaction at the roller *B* in Fig. 5–16*a*.



$\circlearrowleft + \Sigma M_A = 0;$

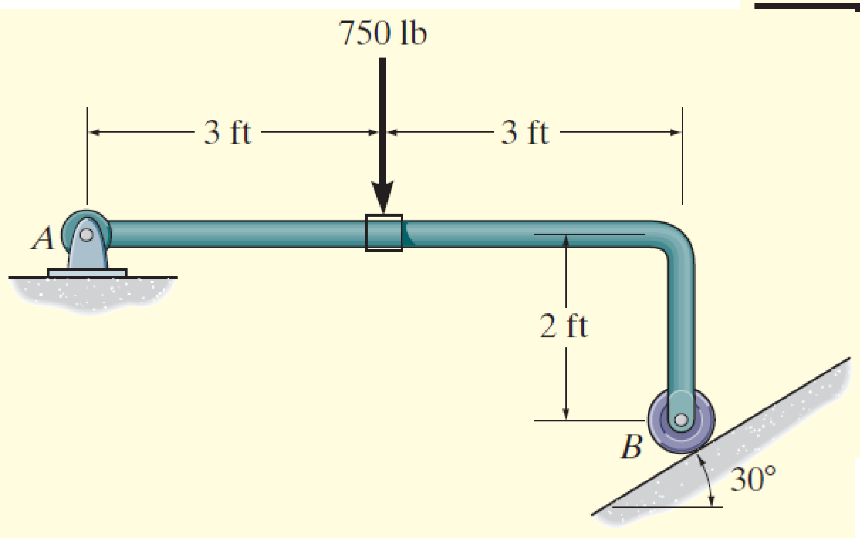
$$[N_B \cos 30°](6 \text{ ft}) - [N_B \sin 30°](2 \text{ ft}) - 750 \text{ lb}(3 \text{ ft}) = 0$$

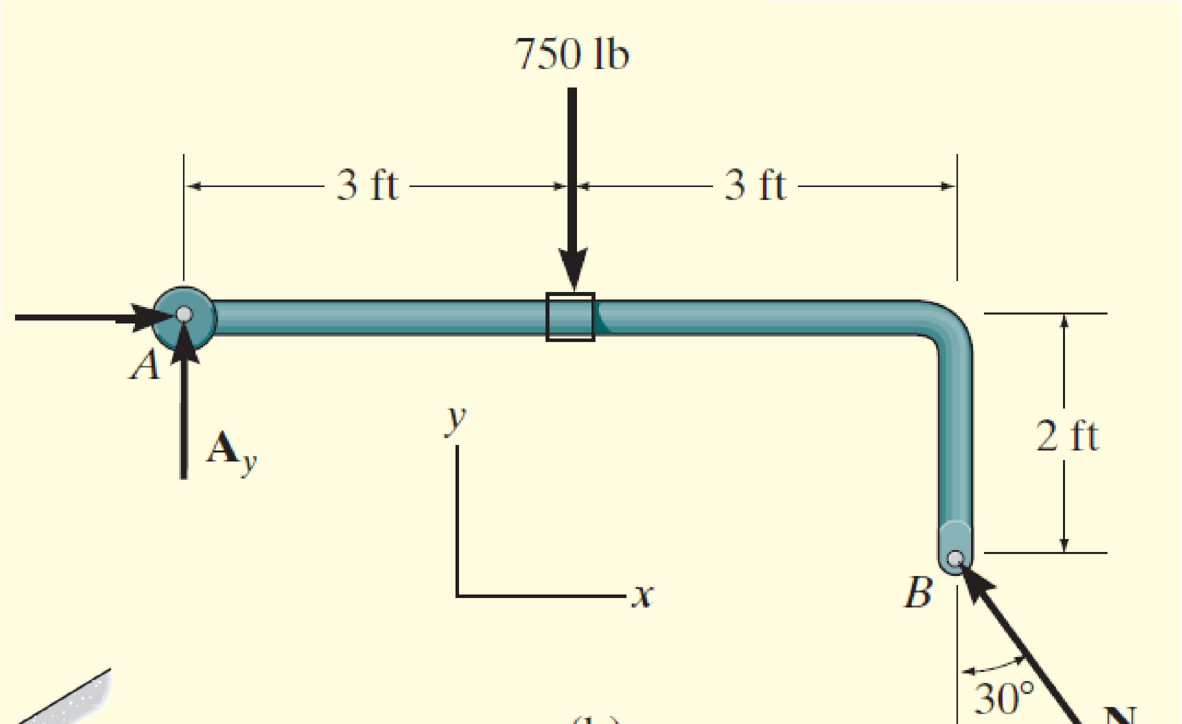
$$N_B = 536.2 \text{ lb} = 536 \text{ lb}$$

$\xrightarrow{+} \Sigma F_x = 0;$ $\quad A_x - (536.2 \text{ lb}) \sin 30° = 0$ $\quad A_x = 268 \text{ lb}$

$+\uparrow \Sigma F_y = 0;$ $\quad A_y + (536.2 \text{ lb}) \cos 30° - 750 \text{ lb} = 0$

$$A_y = 286 \text{ lb}$$

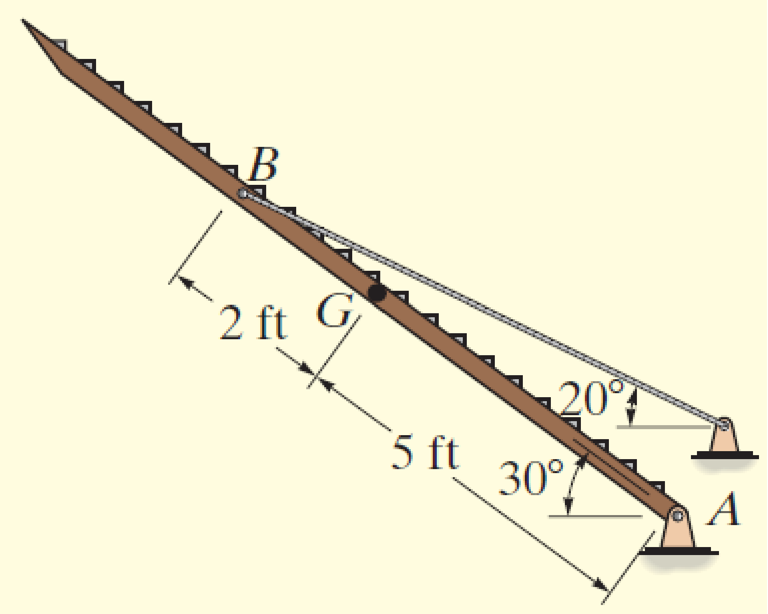


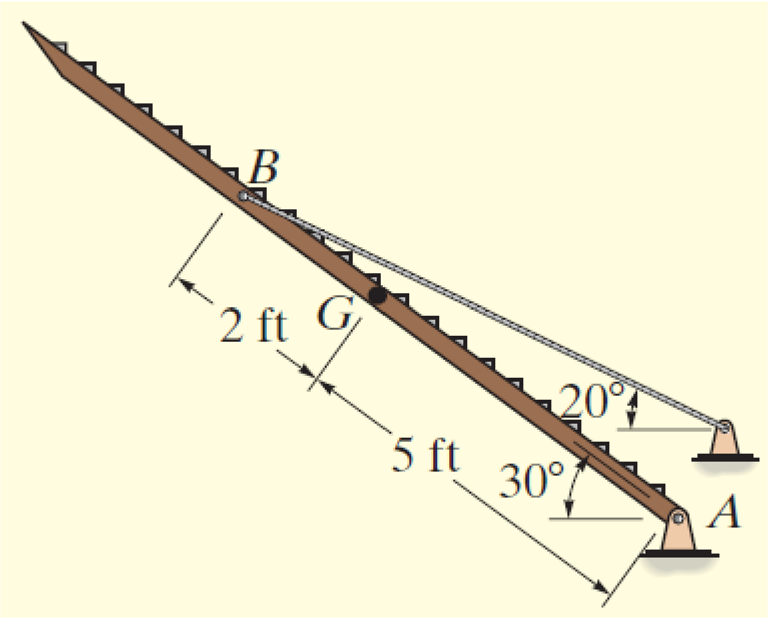


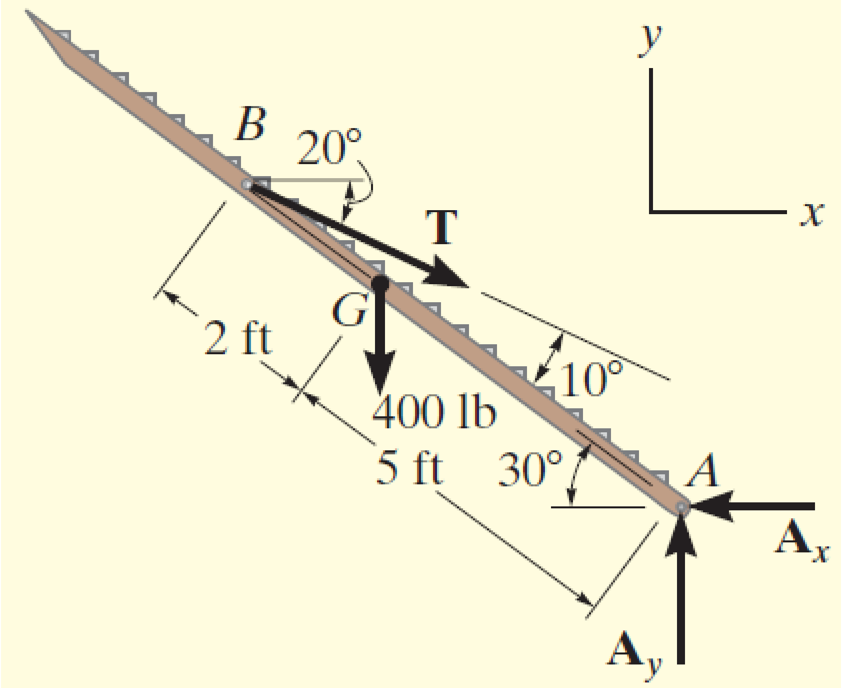
(b)

Example

The uniform truck ramp shown in Fig. 5–18*a* has a weight of 400 lb and is pinned to the body of the truck at each side and held in the position shown by the two side cables. Determine the tension in the cables.







$$\circlearrowleft + \Sigma M_A = 0;$$

$$-T \cos 20^\circ(7 \sin 30^\circ \text{ ft}) + T \sin 20^\circ(7 \cos 30^\circ \text{ ft})$$

$$+ 400 \text{ lb } (5 \cos 30^\circ \text{ ft}) = 0 \qquad T = 1425 \text{ lb}$$

The simplest way to determine the moment of **T** about A is to resolve it into components along and perpendicular to the ramp at B. Then the moment of the component along the ramp will be zero about A, so that

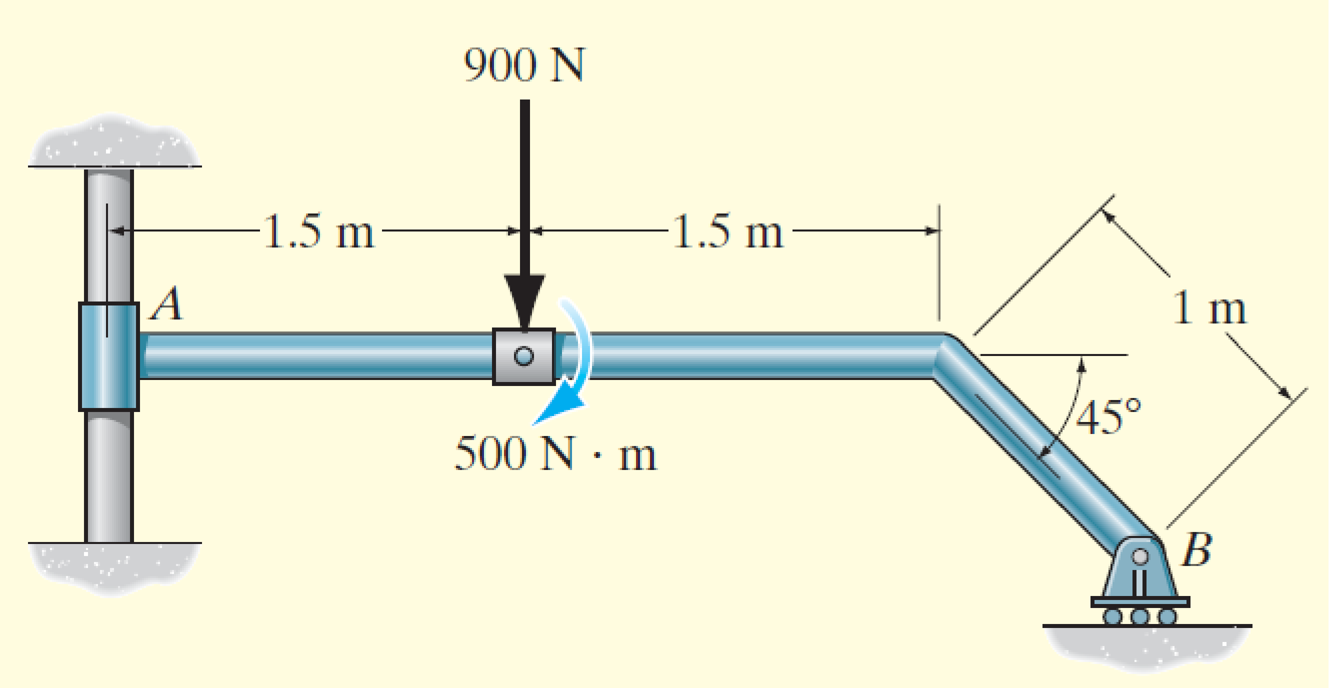
$$-T \sin 10^\circ(7 \text{ ft}) + 400 \text{ lb } (5 \cos 30^\circ \text{ ft}) = 0$$

$$T = 1425 \text{ lb} \qquad T' = \frac{T}{2} = 712 \text{ lb}$$

NOTE: As an exercise, show that $A_x = 1339$ lb and $A_y = 887.4$ lb.

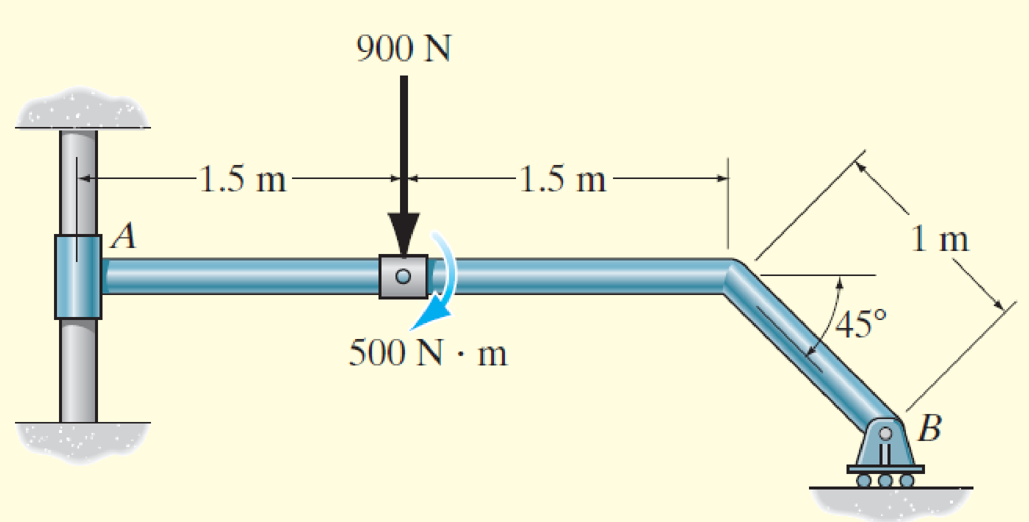
Example

Determine the support reactions on the member in Fig. 5–19*a*. The collar at *A* is fixed to the member and can slide vertically along the vertical shaft.



$\xrightarrow{+} \Sigma F_x = 0;$ $A_x = 0$

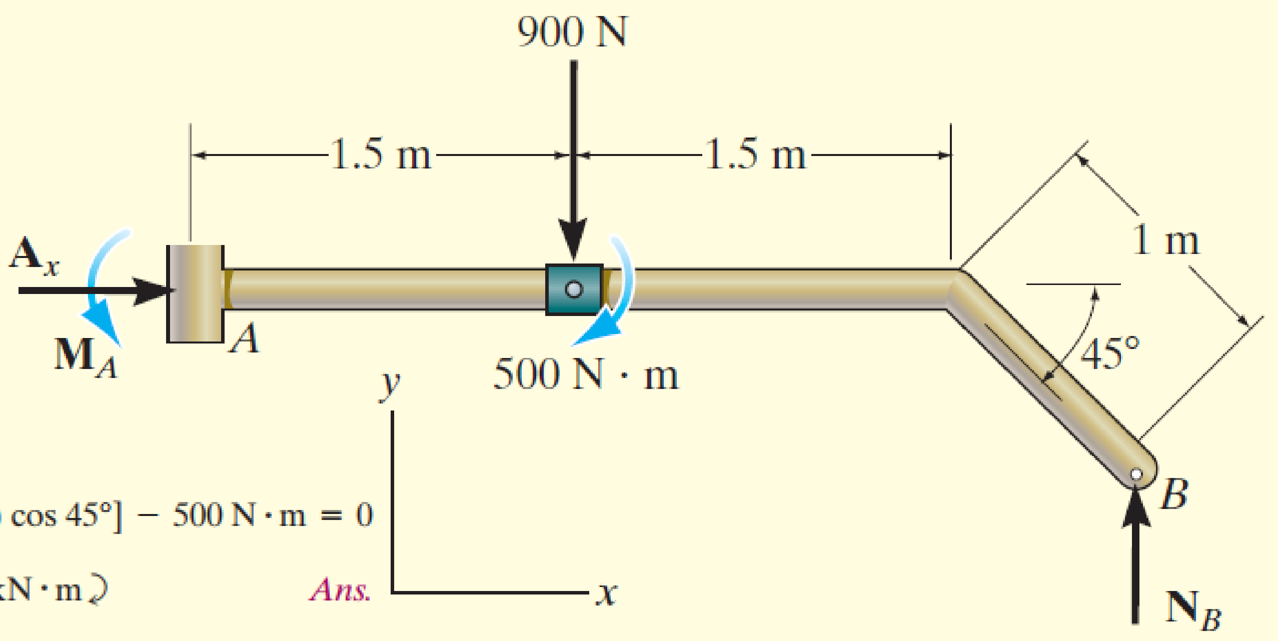
$+\uparrow \Sigma F_y = 0;$ $N_B - 900 \text{ N} = 0$



$\circlearrowleft + \Sigma M_A = 0;$

$$M_A - 900 \text{ N}(1.5 \text{ m}) - 500 \text{ N}\cdot\text{m} + 900 \text{ N}\,[3 \text{ m} + (1 \text{ m}) \cos 45°] = 0$$

$$M_A = -1486 \text{ N}\cdot\text{m} = 1.49 \text{ kN}\cdot\text{m} \circlearrowright$$



or

$\circlearrowleft + \Sigma M_B = 0;$ $M_A + 900 \text{ N}\,[1.5 \text{ m} + (1 \text{ m}) \cos 45°] - 500 \text{ N}\cdot\text{m} = 0$

$M_A = -1486 \text{ N}\cdot\text{m} = 1.49 \text{ kN}\cdot\text{m} \circlearrowright$ *Ans.*

Home Assignment

- Example 5.7, 5.9 & 5.10 .