

Uniform Distribution & Exponential Distribution

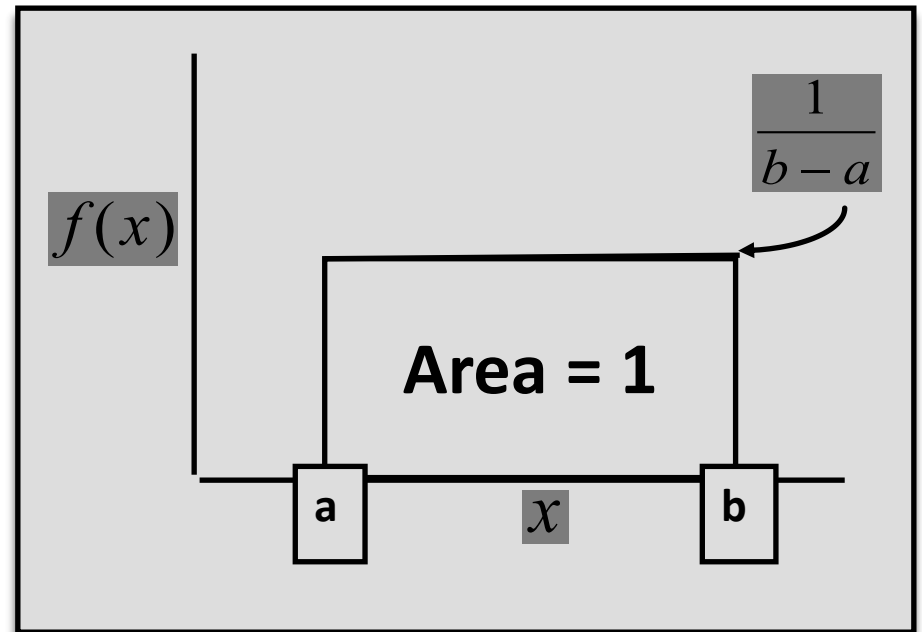
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Uniform Distribution

The uniform distribution is a continuous distribution in which the same height, of $f(X)$, is obtained over a range of values.

$$f(x) = \begin{cases} \frac{1}{b-a} & \text{for } a \leq x \leq b \\ 0 & \text{for all other values} \end{cases}$$



Uniform Distribution

The Parameters of uniform distribution are a and b.

The distribution function of the uniform random variable X is given by

$$F(x) = \begin{cases} 0 & \text{for } x < a \\ \frac{x-a}{b-a} & \text{for } a \leq x \leq b \\ 1 & \text{for } x > b \end{cases}$$

Mean

$$\text{Mean} = \frac{a+b}{2}$$

Variance

$$\text{Variance} = \frac{(b-a)^2}{12}$$

Uniform Distribution

The thickness x of a protective coating applied to a conductor designed to work in corrosive conditions follows a uniform distribution over the interval $[20, 40]$ microns. Find the mean, standard deviation and cumulative distribution function of the thickness of the protective coating. Find also the probability that the coating is less than 35 microns thick.

Uniform Distribution

The arrival time of an engineer to her office is uniformly distributed in the interval between 8 and 9 A.M. Find the probability that the engineer will arrive during the next minute given that she has not arrived by 8.30.

Exponential Distribution

A random variable X follows the exponential distribution with rate $\mu > 0$ if its p.d.f is :

$$f(x) = \frac{1}{\mu} e^{-x/\mu} \text{ for } x \geq 0, \mu > 0$$

$\mu = \text{mean}$

$e = 2.71828$

Exponential Distribution

Examples

- The life time of an electronic device.
- The time between arrivals of two successive buses.
- The duration time of a car service.
- Time until next earthquake occur
- Time required to complete a questionnaire

Exponential Distribution

The exponential distribution has only one parameter μ .

The distribution Function of the exponential random variable X is given by

$$P(x \leq x_0) = 1 - e^{-x_0 / \mu}$$

The Mean and standard deviation of exponential distribution are equal.

$$\text{Mean} = \mu$$

Exponential Distribution

The duration of the long distant calls is found to be exponentially distributed with a mean of 3 minutes. What is the probability that a call will last

- more than three minutes
- less than 5 minutes

Exponential Distribution

Jobs are sent to a printer at an average rate of 3 jobs per hour.

- What is the expected time between jobs?
- What is the probability that the next job is sent within 5 minutes?

Exponential Distribution

The time between arrivals of taxies at a busy intersection is exponentially distributed with mean of 10 minutes.

- Determine X such that the probability that you wait more than x minutes is 0.10.
- What is the probability that you wait longer than an hour for a taxi?
- Suppose you have already been waiting for one hour for a taxi, what is the probability that one arrive within the next 10 minutes.