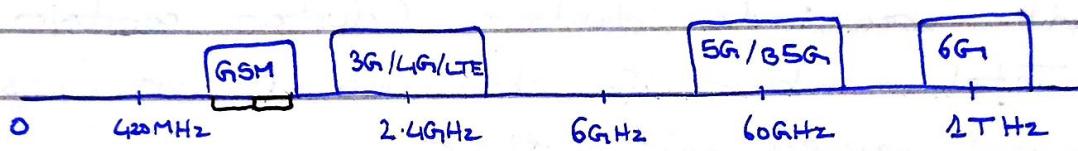
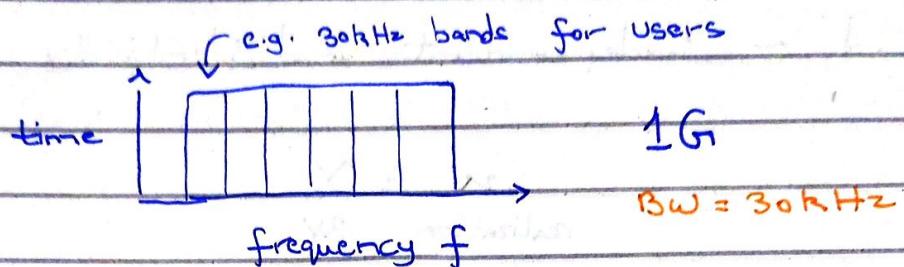


## Mobile Communication Systems

### FDMA

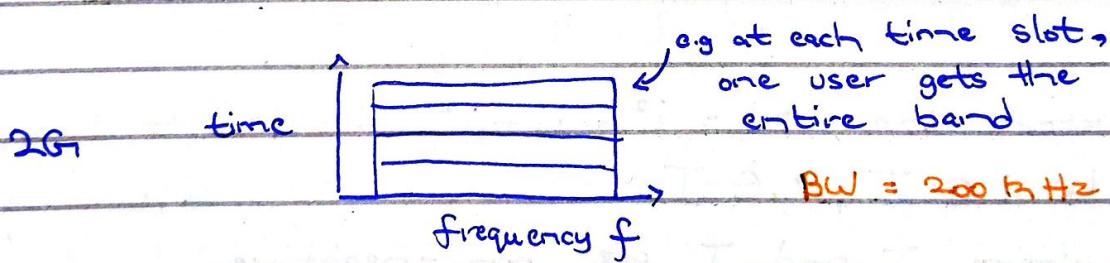


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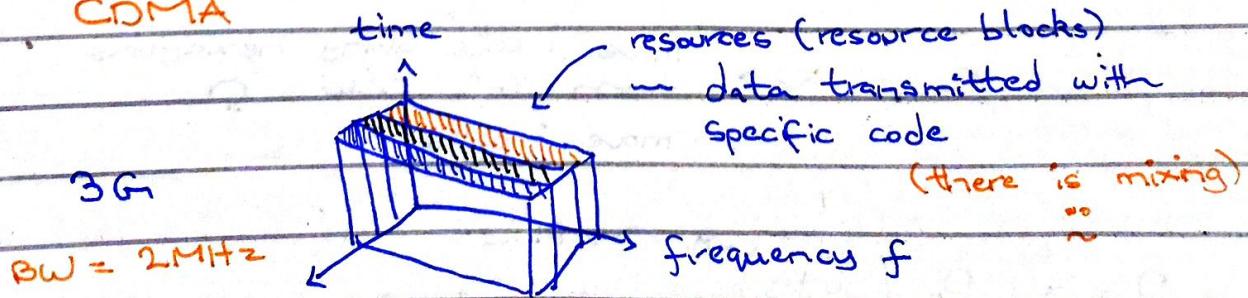
7GHz ~  
Bandwidth

$$R = B \log_2 (1 + SNR) \text{ bps}$$

### TDMA



### CDMA



## Cellular Systems

i.i.d ~ independent and identically distributed

$$\underline{n \in X}$$

realization      RV

Divide area into clusters (clusters contain cell(s))

- ~ Total number of channels C are assigned and reused per cluster.
- ~ Added co-channel interference.

[ Small cells serve more users but more BS, higher cost & more interference  
Always a tradeoff

Cluster size  $N = i^2 + ij + j^2$  Related to hexagonal  
 $\sqcup i \in I, j \in J$  geometry

where I and J are non-negative integers

Location Rule  $\Rightarrow$  move i cells along hexagons  
turn  $60^\circ$  ccw  $\sqcup$   
move j cells

$$Pr = P_0 \left( \frac{d_{des}}{d_0} \right)^{-n} \rightsquigarrow \text{PLE}$$

$\{P_0, d_0\}$  reference values

## Mobile Communication Systems

### Reuse Ratio

$$d = \frac{D}{R} = \sqrt{3N}$$

↓ distance  
 ↓ radius

### Adjacent Cell Interference

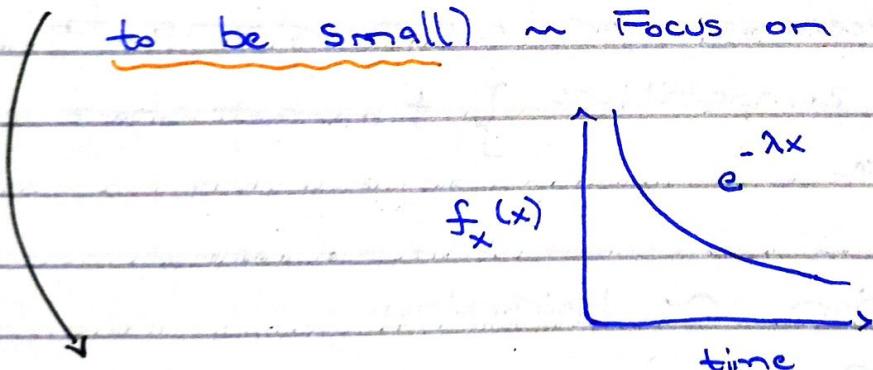
Ideal BP filters don't exist

↳ Adjacent channels interfere

### Trunking Theory

Inter-arrival times are independent

exponential random variables (as calls tend to be small) ~ Focus on RVs



### Server Model

Likewise, durations of service (holding times) are also independent exponential RVs with  $E[X] = H$

## Mobile Communication Systems

### Traffic Intensity

Erlang  $\triangleq$  channel occupied on average  
 e.g. one erlang can be attributed to one continuous call

### Channels

Each call  $\sim$  requires a channel

$\hookrightarrow$  call  $\sim$  requires a channel Ignore

- In trunking, channels are "pooled"
- When the pool is exhausted,  
 new call requests are simply denied  
 $\hookrightarrow$  "Blocks Call Cleared"  $\sim$  Erlang B

### Probability of Blocking

$$p = B(A, c)$$

$\hookrightarrow$ , where  $A = A_u U$ ,  $A_u = \lambda H$

$$p = B(A, c) = \frac{A^c / c!}{\sum_{k=0}^c A^k / k!}$$

Erlang B  
Formula

A is the total offered traffic

Problem

$$C = 5, 10, 50, 100$$

$$A_U = 0.05 \text{ Erlangs}$$

$$A = A_U U \rightarrow U = \frac{A}{A_U} = ?$$

$$P_B = 0.01$$

$$C = 5 \gg U = \frac{1.5}{0.05} = 30 \xrightarrow{\text{= } A \text{ (found from Erlang B chart)}}$$

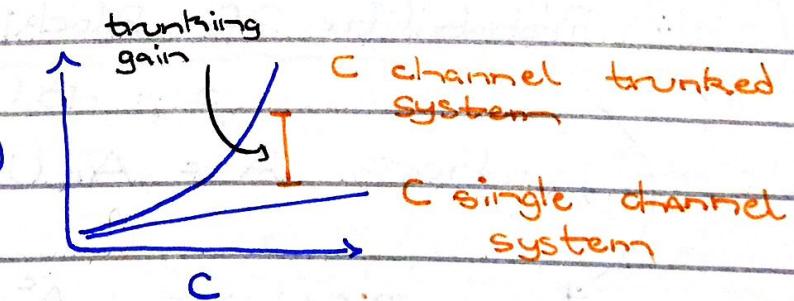
↳ Do the same for  $C = 10, 50, 100$

For reference ;  $\underline{C = 100 \gg U = 1900}$

100 channels can serve

1900 users with

$$P_B = 0.01$$



Blocked Calls Delayed ~ Erlang C

→ Blocked calls (users) go into queue rather than getting dropped

## Mobile Communication Systems

Capacity  $\rightarrow$   $B \log_2 (1 + SNR)$

is bandwidth of channel  
# of users in the system

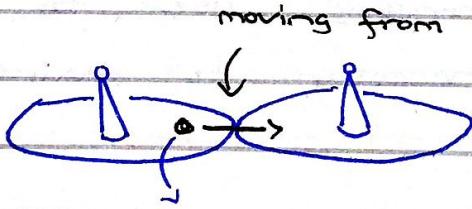
definition depends upon context

- We increased  $B$  by frequency reuse ; etc  
 → How do we improve SNR / SIR / SINR ?

Channels  $\left[ \begin{array}{l} \text{control} \rightarrow \text{always on (beacon signals)} \\ \text{data} \qquad \qquad \qquad \text{at low data rates} \end{array} \right]$

RSSI : Received signal strength indicator

### Hand-offs



the subscriber compares received powers and connects to BS with higher power (frequency hand-off)

- To avoid excessive switching ;

$$\text{Let } |P_a - P_b| \geq H(\theta)$$

be the condition

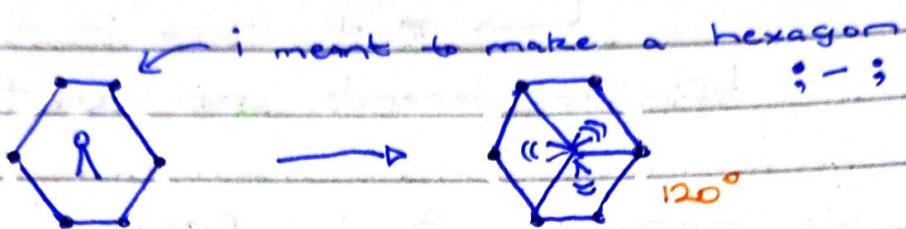
after which we switch

- Hand-offs have higher priority

but call will be dropped if existing channels in the new cell are exhausted

## Sectoring (Increase SINR by decreasing interference)

Omnidirectional antennas are replaced with 3 or 6 directional antennas



- trade off** {
- Now, significant interference comes from 2 sources instead of 6 ( $T_1$  interferers)
  - However, channels are divided into pools and thus, a loss in trunking gain is observed
  - BS commands user to utilize lowest power necessary to maintain GOS  
(To reduce inter user interference)

~ forward  $\triangleq$  downlink channel  
~ reverse  $\triangleq$  uplink channel

## Outage

$P\{SINR < \bar{C}\}$  SINR drops below operating level

26/03/2023

## Mobile Communication Systems

### Path Loss

$$\begin{aligned} \text{Powers} &\triangleq P_n ; n \in \{t, r\} \\ \text{Feeder Losses} &\triangleq L_n \\ \text{Antenna Gain} &\triangleq G_n \end{aligned} \quad \left. \begin{array}{l} \text{PL Integrated} \\ \hline \end{array} \right.$$

$$P_r = \frac{P_t G_t G_r}{L_t L_r} \quad \text{Friis Eqn}$$

(all in Watts)

dBm and dBW

$$1 \text{ dBm} = 10 \log_{10} \left( \frac{1 \text{ W}}{1 \text{ mW}} \right)$$

→ radiated

$$\text{EIRP} \quad P_{ti} = \frac{P_t G_t}{L_t} \quad \Rightarrow \quad P_{ri} = \frac{P_{ti}}{L_{PL}} \quad \begin{array}{l} \rightarrow \text{EIRP} \\ \rightarrow \text{path loss} \end{array}$$

$$L_{PL} = \frac{P_{ti}}{P_{ri}} = \frac{P_{ti}}{\frac{P_r G_r}{L_r}}$$

### FSPL

$$L_{PL} = \left( \frac{4\pi d}{\lambda} \right)^2 ; d \text{ is far field distance}$$

$d \gg 2D^2 ; d \gg D \text{ and } d \gg \lambda$

D is largest dimension of antenna

PLF of FSPL is always 2.

~ Power Flux Density

$$P_d = \frac{E I B P}{4\pi d^2} = \frac{|E|^2}{n}$$

$E$  ≈ envelope of electric field in V/m

~ Effective Aperture

$$G_t = \frac{4\pi A_e}{\lambda^2}; A_e = A \eta \xrightarrow{\text{efficiency}}$$

$\downarrow$   
area

$$P_{ri} = P_d A_e$$

- Antennas should be spaced by at least  $\lambda/2$  for uncorrelated channels. Reason why cellphones have a single antenna.

~ Field Near Transmitter

Assuming transmitter is high enough;

$$\frac{P_t G_t}{4\pi d_0^2} = \frac{E_0^2}{120\pi}; E_0 \text{ is the envelope}$$

$$E(d, t) = \frac{E_0 d_0}{d} \cos(\omega_c(t - d/c))$$

28/09/23

## Mobile Communication System

Flat Earth PL

$$P_{ri} = P_t G_t G_r \frac{h_t^2 h_r^2}{L^4}$$

$$L = \frac{d^4}{h_t^2 h_r^2} \quad \text{now depends on heights}$$

$\Rightarrow 10 \text{ dB SNR}$  (conservative system)

$y = h_x + n$   
goal is to use signal processing to extract  $x$  from  $y$        $\hookrightarrow$  stochastic process

$\Rightarrow n \lambda/2$  must hold for antennas in order for channels to be uncorrelated ;  $n \in \mathbb{R}$

Analytical : { Free Space ~ Flat Earth }

Empirical : { All the rest ; }

$\rightarrow$  Flat earth model was derived using horizontal polarization

$\rightarrow$  slow fading  
Log Normal Shadowing [ Large Scale Fading ]

Fade Margin should be present in the system

### shadowing effect

- Sample outage by moving on the locus of radius of base station;

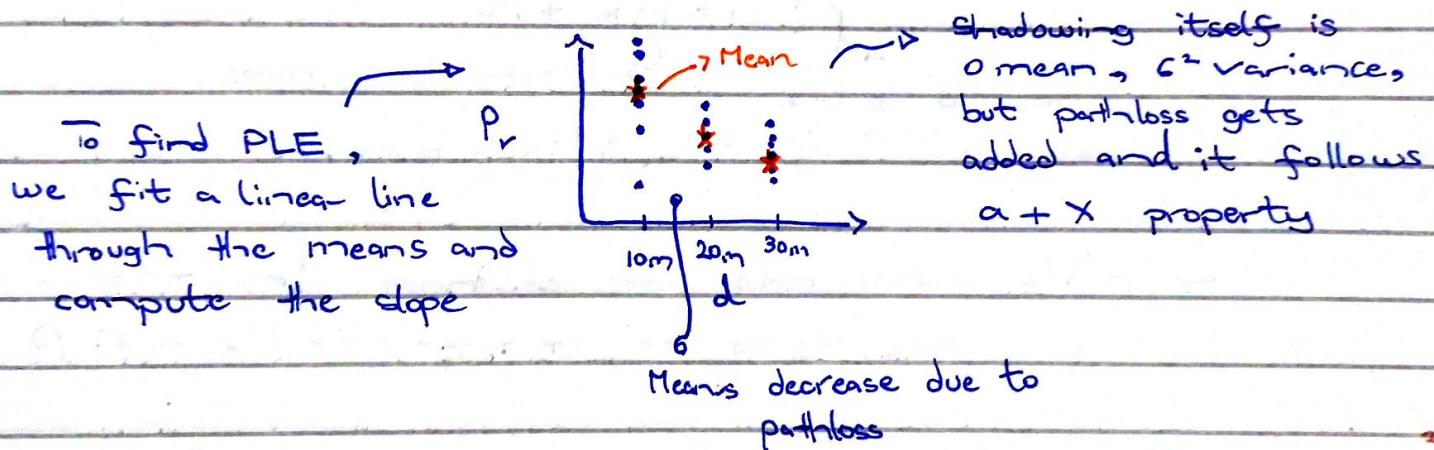
### Likelihood of Coverage

$$P(P_r(d) > Y) = P(X_0 > \beta)$$

If  $X$  is an RV  $\sim N(\mu, \sigma^2)$

$$\alpha X \sim N(\mu, \alpha^2 \sigma^2)$$

$$\alpha + X \sim N(\mu + \alpha, \sigma^2)$$



$$L(d) = \bar{L}(d_0) + 10n \log_{10} \left( \frac{d}{d_0} \right) + X_0$$

Zero mean,  $\sigma$  Gaussian RV

$\Rightarrow Q$ -Function:

$$Q(z) = \frac{1}{\sqrt{2\pi}} \int_z^{+\infty} \exp\left(-\frac{x^2}{2}\right) dx$$

$$P(X > b) = Q\left(\frac{b - \mu}{\sigma}\right); \mu, \sigma \text{ are params of RV } X$$

## Mobile Communication System

Inverse Q Problems |  $\alpha = -30 \text{ dBm}$ ,  $b = 9 \text{ dB}$   
 $P_r = 0.95$

$$P(P_r > b) = Q\left(\frac{b - (-30)}{9}\right) = 0.95$$

From table and ;  $Q(z) = 1 - Q(-z)$

$$\frac{-b + 30}{9} = 1.65$$

$$\Rightarrow b = \underline{-44.85 \text{ dBm}}$$

## Likelihood of Coverage or Fraction of Time

$P(P_r(R) > \gamma)$   $\rightarrow R \triangleq \text{radius of cell}$

$\gamma \triangleq \text{threshold}$

Useful Service Area  $\rightarrow$  polar integral &  
error functions

$\therefore$  should use the chart instead ; slides

## Mobile Communication System

### Example

$d$ (m)	$P_r$ (dBm)
100	0
200	-20
1000	-35
3000	-70

a) Find  $n$

$$d_0 = 100 \quad P_r(d_0) = 0 \text{ dBm}$$

Let  $\hat{p}_i$  be  $P_r$  at  $d_i$ :

$$\rightarrow \hat{p}_1 = P(d_0) - 10n \log(d_1/d_0) = 0$$

$$\hat{p}_2 = P(d_0) - 10n \log(d_2/d_0) = -3n$$

$$\hat{p}_3 = P(d_0) - 10n \log(d_3/d_0) = -10n$$

$$\hat{p}_4 = P(d_0) - 10n \log(d_4/d_0) = -14.77n$$

$$J(n) = \sum_{i=1}^4 (p_i - \hat{p}_i)^2$$

$$= (0-0)^2 + (-3n - (-3n))^2 + (-35 - (-10n))^2 \\ + (-70 - (-14.77n))^2$$

$$= 327.15n^2 - 2887.6n + 6525$$

$$\frac{\partial J(n)}{\partial n} = 0 \Rightarrow n = 4.1 \cdot 4$$

b) Find  $\sigma^2 / G$

$$\sigma^2 = \frac{I(4.4)}{4} = 38 \Rightarrow G = 6.17 \text{ dB}$$

c) Find  $P_r$  at 2km

$$P_r(2\text{km}) = P(d_0) - 10n \log \left( \frac{2\text{km}}{d_0} \right)$$
$$= -57.24 \text{ dBm}$$

d) Predict likelihood of coverage @ 2km

$$> -60 \text{ dBm}$$

$$\Phi \left( \frac{-60 - (-57.24)}{6.17} \right) = P(P_r(2\text{km}) > -60 \text{ dBm})$$
$$\Rightarrow 67.4 \%$$

e) % age Useful area

From graph

↪ Find the useful area;

$$U = 67.4 \%$$

## Mobile Communications

### Excess Delay

↳ Propagation Delay relative to the shortest path ; Suppose three signals are transmitted and receiver gets it with  $t_i = i$  ;

$$y(n) = \alpha_0 x(n) + \alpha_1 x(n-1) + \alpha_2 x(n-2)$$

↳ LCCD - Linear but not Time Invariant (LTI)

Linearity ~ Superposition Property

$$y(z) = \alpha_0 X(z) + \alpha_1 X(z) z^{-1} + \alpha_2 X(z) z^{-2}$$

- $\alpha_i$  are random in nature as well
- ↳ taps

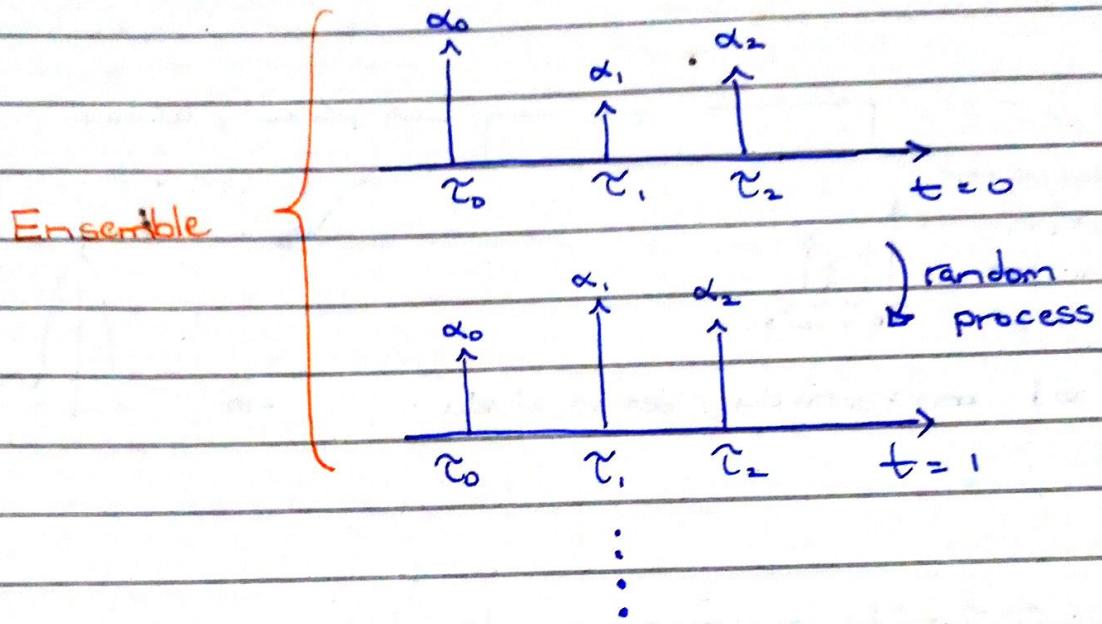
Channel is a Filter

$$x(t) \rightarrow h(t, \tau) \rightarrow y(t)$$

↳ convolution

$$y(t) = \int_{-\infty}^{\infty} x(t-\tau) h(t, \tau) d\tau$$

↳ Impulse is important as an input as it contains all frequency components



### Baseband Impulse Response

↳ original  $\rightarrow$  passband : transmitted (modulated)

$$h(t, \tilde{\tau}) = \operatorname{Re} \{ h_b(t, \tilde{\tau}) e^{j\omega_b t} \}$$

$$x(t) = \operatorname{Re} \{ c(t) e^{j\omega_b t} \}$$

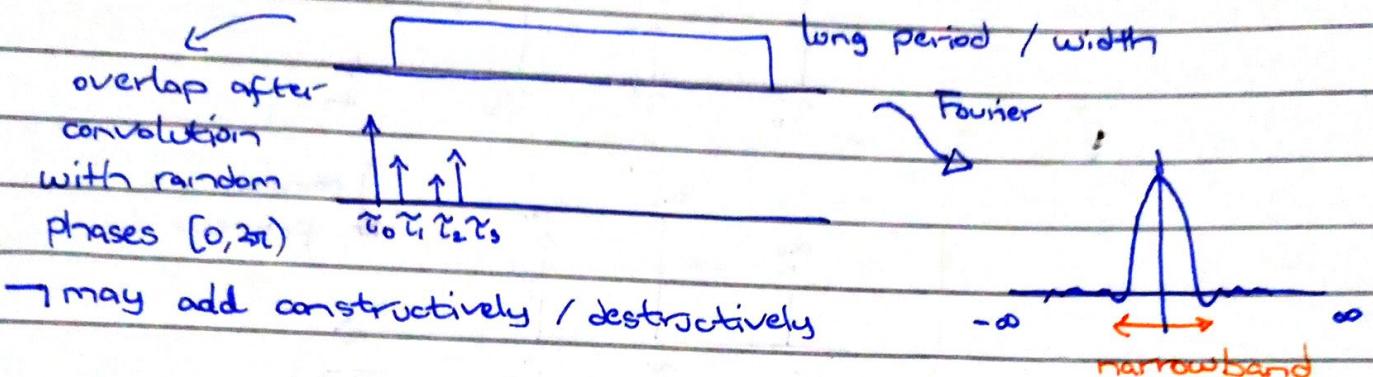
$$y(t) = \operatorname{Re} \{ r(t) e^{j\omega_b t} \}$$

$$r(t) = \frac{1}{2} \int_{-\infty}^{\infty} c(t - \tilde{\tau}) h_b(t, \tilde{\tau}) d\tilde{\tau}$$

↳ ensures passband average power equals baseband average power

- Each path has amplitude  $\alpha(t)$  and phase  $\theta(t)$ ; propagation delay  $\tilde{\tau}$

## Narrowband Fading



## Ergodicity

↳ ensemble average equals time average

mean of means

$$\theta \in [0, 2\pi)$$

↳ uniform distribution

↳ incorrect but good enough (isotropic scattering model)

## ISI

↳ inter symbol interference

↗ in wideband

↳ delayed copies of same signal  $x_n$  could come when  $x_{n+1}$  is being resolved

↳ equalizers mitigate ISI

↳ OFDM ~ solution

↗ diversity gain

Solutions : Narrowband : MIMO

Wideband : OFDM

## Mobile Communication System

- Wideband and narrowband average powers are equal
- Channels are often described by their RMS delay spread

### Moments of the PDP

$$\mu = E[x] \text{ Mean}$$

$$\rightarrow E[x^2] \text{ Variance if } \mu = 0$$

$$\rightarrow E[x^3] \text{ Skewness}$$

$$\rightarrow E[x^4] \text{ Kurtosis}$$

$$\sqrt{\sigma_x^2} = \sqrt{\bar{x}_2 - \bar{x}_1^2}$$

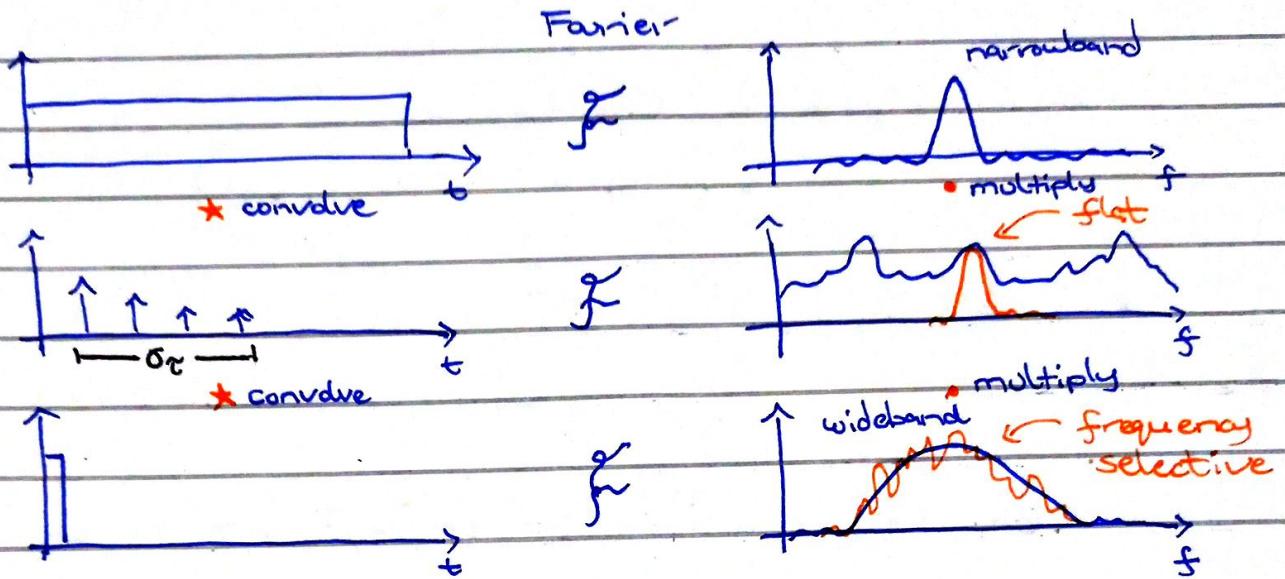
↗ RMS      ↘ mean      ↘ 2nd moment  
 Delay Spread

$\sigma_x \ll$  Symbol Period  $\rightarrow$  Narrowband

$\sigma_x \gg$  Symbol Period  $\rightarrow$  Wideband

Recap	CTFT	: In (Cont.)	out (Cont.)
	DTFT	: In (Discrete)	out (Cont.)
	DFT	: In (Gen.)	out ( <u>Cont. * ↑</u> ) or Discrete

↳



Coherence Bandwidth  $\sim \Delta f$  of the Channel  
( Bandwidth of Channel alone  
does not mean much )

Correlation  
We want to find two points on our channel random process with  $\max(\text{correlation})$  ;  
given  $a(t)$  { message width }  $\gg \sigma_a$  , to minimize fading effects { experiences flat fading }

Normalized Covariance  $\rho_{xy} = \frac{\mathbb{E}[(x - m_x)(y - m_y)^*]}{\sigma_x \sigma_y}$  ↳ STD's

## Mobile Communication Systems

### Wide Sense Stationary Uncorrelated Scattering (WSSUS)

- └ Assume path gains at different delays are uncorrelated
- └ Assume f response depends only on  $\Delta f$
- └ Correlation, now, is of the entire process

### Coherence Bandwidth

$P_{\text{af}} = X/100 \gg$  It quantifies the corelationship b/w two points

- └ If 30 kHz is 90% ; CB ; then, all points separated by 30 kHz are 90% correlated

- If  $BW_{\text{transmit}} > 50\% B$ , channel is frequency selective

- Equalizer → mitigates frequency distortions

Adaptive Tapped Delay Filter

↳ reverses effect of channel taps dynamically

↳ requires CSI - CSI is determined by using pilot signals

pilot symbols are at the start of each message

Correlation in Time ~ Same / Similar to  $f$   
→ Channel response changes as the vehicle moves

- Doppler Shift  $f_d = f_0 \frac{v}{c} \cos \theta$

### Fast Fading

if  $T_s > T_c$

$\downarrow$        $\downarrow$   
symbol time   coherence time

or, in terms of Doppler spread :

$$\underline{B_s < B_d = \frac{k}{T_c}}$$

## Mobile Communication System

### Problem

→ Binary BPSK  $R_b = 250 \text{ kbps}$

- a) Range of values for  $\sigma_2$  such that Rx signal is flat?

$$T_s = k / R_b ; \text{ where } k \text{ is number of modulated bits}$$

$$= 4 \mu\text{s}$$

→ Rx signal is flat for  $\sigma_2 < 0.4 \mu\text{s}$

- b) Coherence Time

$$f_0 = 5.8 \text{ GHz} ; f_m = f_0 \sqrt{\frac{k}{c}} = 259.3 \text{ Hz}$$

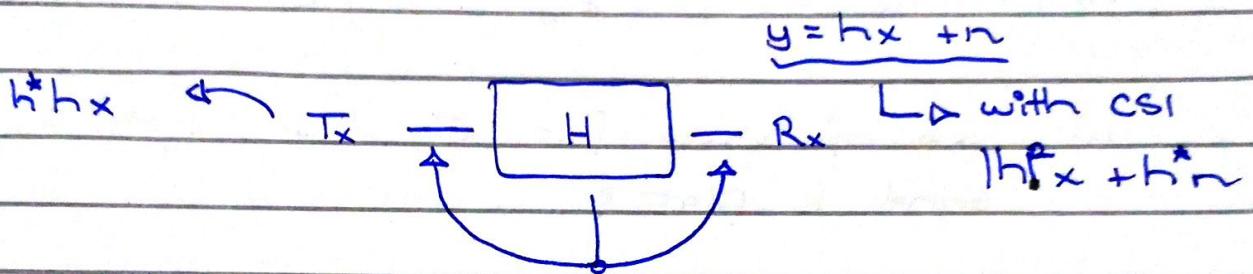
$$T_c = \frac{0.423}{f_m} = 1.63 \text{ ms}$$

As  $T_s \ll T_c$ , slow fading

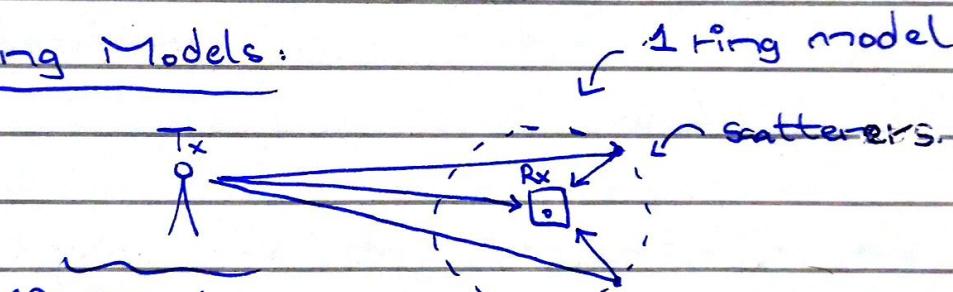
- c) How many bits can be sent such that the channel appears static?

$$\frac{T_c}{T_s} \approx 407 \text{ bits}$$

## Small Scale Fading Distributions



### {Ring Models:



if Tx also  
has scatterers

### 2 ring model

CLT → when number of samples  $\rightarrow \infty$ , the addition yields {underlying distribution approaches gaussian}

$$\rightarrow Z = X + jY$$

Complex Gaussian

## Mobile Communication Systems

### Small Scale Fading Distribution

- $R = \sqrt{X + jY} \rightsquigarrow$  Gaussian as per CLT  
where  $X \sim N(0, \sigma^2)$   
 $Y \sim N(0, \sigma^2)$

then,  $R$  has a Rayleigh distribution

$$\underbrace{X+Y}_{\text{RV}} \xrightarrow{\text{convolved}} f_X * f_Y$$

- Now, if there's a LOS component

then,

$$R = \sqrt{(X + \text{Re}(A))^2 + (Y + \text{Im}(A))^2}; A \text{ is the power}$$

of LOS component

If  $|Z| \sim \text{Rayleigh}(\sigma^2)$

$|Z|^2 \sim \text{Exponential}(\sigma^2)$

elif  $|Z| \sim \text{Rice}(\dots)$

$|Z|^2 \sim \text{Non central chi square}(\dots)$

### Level Crossing Rate

Expected rate at which normalized fading crosses the threshold  $\rho$  with either +ive or -ive slope.

Rician shifts mean of Rayleigh;

- Hence, LCR at same threshold is less for Rician

L LCR can be used to estimate the velocity of the vehicle.

### Average Fade Duration

duration fade is below the threshold used / impacts interleaving depth

#### - Example

$[1101101101110110]$  →  
in stream

1	1	0	1
1	0	1	1
0	1	1	1
0	1	1	0

→ Read data  
out as columns

err  
 $1100 \boxed{1011} 0111 1110$  → after  
interleaving  
burst errors

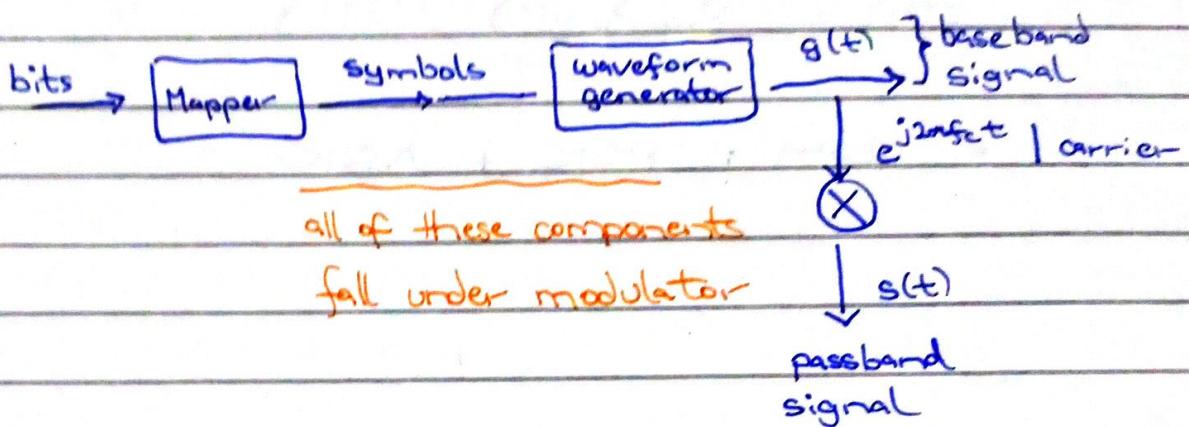
→ deep fade → removes the bits (err)

Followed by de-interleaving

- bursts can't be corrected; hence, we spread out the errors.  $[n, m, k]$   
 $12 \quad 4 \quad 2$

## Mobile Communication Systems

### Modulation



#### - Properties of a Good Modulator

- [-] Compact PSD
- [-] Low BER (Bit-error rate)
- [-] Friendly to Non-Linear Amplifiers

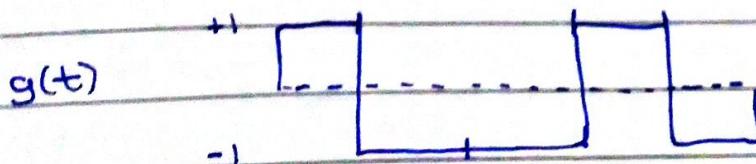
- o Mapper : N-coded bits →  $M = 2^N$  possible waveforms

Let  $p(t) = \begin{cases} 1 & \text{at } t \\ 0 & \text{elsewhere} \end{cases}$  → localized in time

trade-off

bits: 1 0 0 1 0  
 bpsk  $x_s$ : 1 -1 -1 1 -1

does not remain  
 localized in frequency  
 PSD should be  
 compact



$$g(t) = x_0 p(t) + x_1 p(t - T_s) + \dots$$

$$= \sum_{n=0}^N x_n p(t - nT_s)$$

$$s(t) = \operatorname{Re} \{ g(t) e^{j2\pi f_c t} \}$$

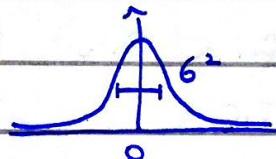
- QAM : PSK + ASK

→ Receiver Side

AUGIN  
↑

$$r(t) = a(t) A \sum_n x_n p(t - nT_s) \cos(2\pi f_c t + \theta_n) + n(t)$$

↓ Adds  
A W GRN → Gaussian  
↳ affects all frequencies  
never repeats



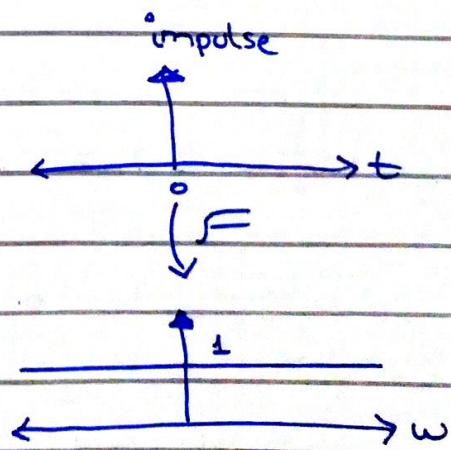
- PSD Analysis  $X \rightarrow \Phi_{xx} \xrightarrow{\mathcal{F}} S_{xx}(\omega)$

Auto correlation

$$\operatorname{conv}(x, y)^{(t)} = \int x(t) \cdot y(-t)$$

$$\operatorname{corr}(x, y)^{(t)} = \int x(t) \cdot y(t)$$

<sup>auto corr</sup>  
 $\operatorname{corr}(n(t), n(t)) = S(t)$



- Next lecture

## Mobile Communication Systems

## Demodulator

$$r(t) \Rightarrow \frac{\sqrt{2}}{T_s} \cos(2\pi f_c t) dt$$

channel gains and TX power

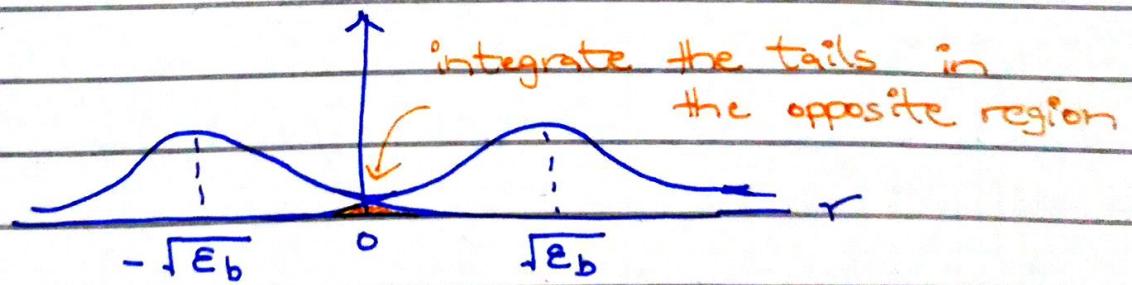
noise

$$R = \int_{(n-1)T_s}^{T_s} (\cdot) dt = x_n \sqrt{E_b} + v_n$$

For BPSK  $x_n \in \{1, -1\}$

if  $x_n = 1 \gg R = \sqrt{E_b} + v_n \sim f_{R|x_n=1}$

else if  $x_n = -1 \gg R = -\sqrt{E_b} + v_n \sim f_{R|x_n=-1}$

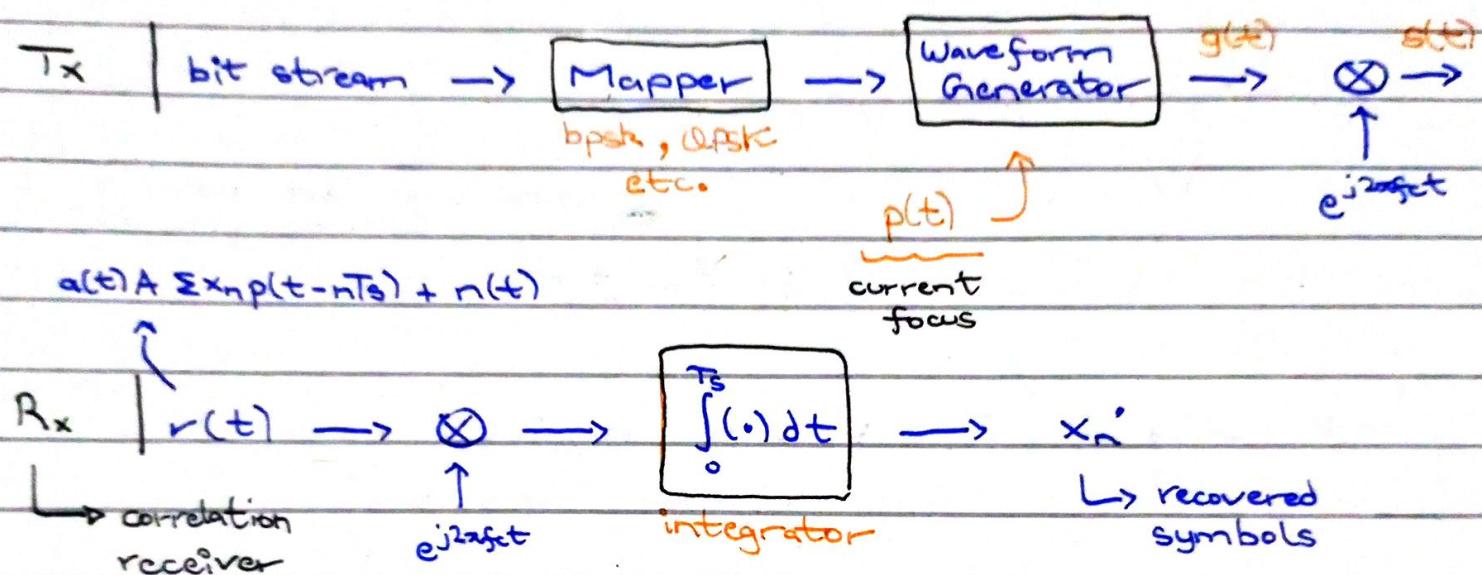


$$P(\text{error}) = Q\left(\frac{\sqrt{2E_b}}{\sqrt{N_0/2}}\right) = Q\left(\sqrt{\frac{E_b}{N_0/2}}\right)$$

$N_0/2$ : noise power (variance)

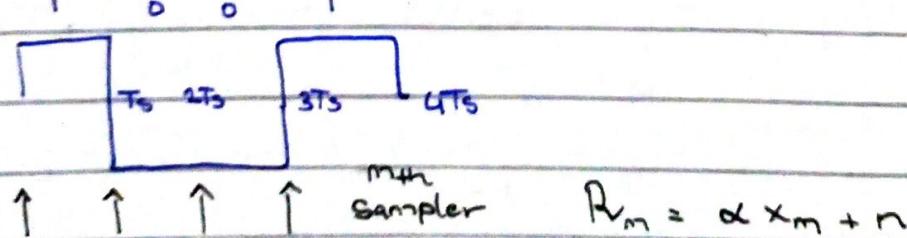
## Mobile Communication Systems

### Nyquist Pulses

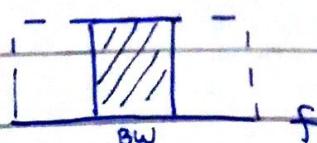


- Digital - "over" of correlation receiver  $\sim$  matched receiver / filter

Let  $p(t) = \begin{cases} 1 & 0 \leq t < T_s \\ 0 & T_s \leq t < 2T_s \\ 0 & 2T_s \leq t < 3T_s \\ 1 & 3T_s \leq t < 4T_s \\ 0 & \text{otherwise} \end{cases}$   $\rightarrow$  but it is not localized in frequency

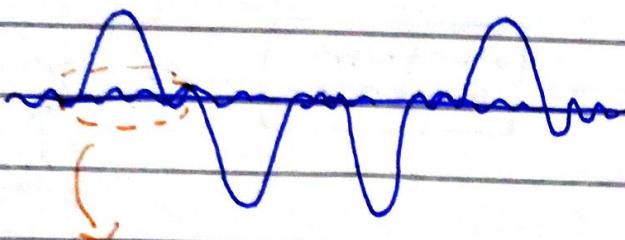


- Sinc  $\rightarrow$  for localization in frequency



Let  $p(t) = \text{sinc}(t)$

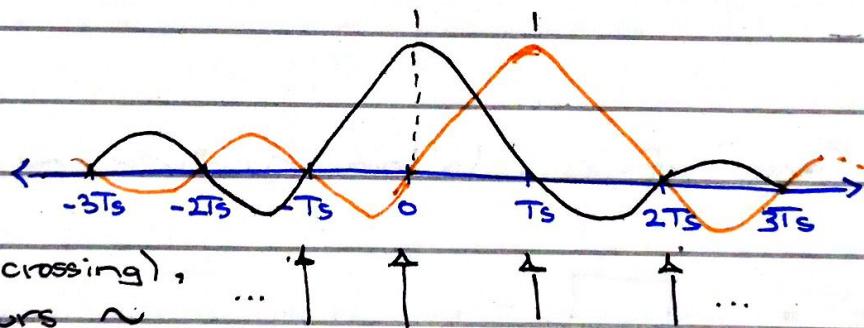
1 0 0 1



now, ISI is present ;  $R_m \neq \alpha x_m + n$

now there are other symbols present

↳ beauty of sinc ; zero crossings occur at regular intervals



at  $T_s$  (zero crossing), ... ↑ ↑ ↑ ↑ ...  
no ISI occurs ~

### Nyquist Condition

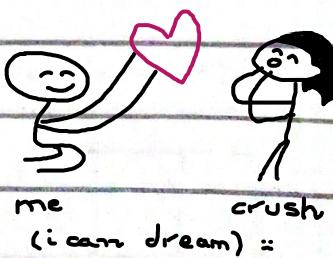
ignore  
 $\tilde{p}(t) = p(t) \sum_n \delta(t - nT_s) = \delta(t)$

or

$$\tilde{p}(t) \sum_{n=-\infty}^{+\infty} \delta(t - nT_s) = \delta(t)$$

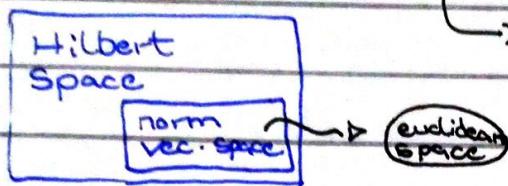
### ISI conditions

- Non-nyquist Pulses
- Non-ideal sampler
- Multipath fading



## Mobile Communication Systems

### Geometry of Signals



→ geometrically compute the BER

$$L_2 \rightarrow \|x_2 - x_1\|^2 = \|x_2 - x_1\|$$

$$L_1 \rightarrow \|x_2 - x_1\|$$

$\mathbb{R}$   $\Rightarrow$  basis function  $\{1\}$

number  
line



$\mathbb{R}^2 \Rightarrow$  basis  $\{\begin{bmatrix} 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \end{bmatrix}\}$

and so on

$L_2$  sq. integrable func

$$\int_{-\infty}^{\infty} |f(x)|^2 dx < \infty$$

From Fourier, sines and cosines linearly combined act as the basis of our  $L_2$  sq. integrable space.

inner product

$$\langle x, y \rangle = \int_{-\infty}^{\infty} x(t) y(t) dt$$

→ receiver just computes the inner product.

→ Draw a threshold line b/w two noiseless points for optimal decision boundary.

$$P(\text{bit error}) = Q\left(\frac{d_{xy}}{\sqrt{2N_0}}\right) \rightarrow \text{distance b/w two noiseless points}$$

└ DPSK       $M = 2^k$ ;  $k$  is bits per symbol  
when possible bits  $> 2$ ; use union bound

$$\underbrace{P(\text{symbol error})}_{\text{upper bound}} = \frac{1}{M} \sum_i^M \sum_{j \neq i}^M \left[ \frac{d_{ij}}{\sqrt{2N_0}} \right]$$

$\left. \begin{array}{l} \triangleq \text{cosine - in-phase} \\ \triangleq \text{sine - quadrature} \end{array} \right\}$

- PAPR : Peak to average Power Ratio
- OFFSET DPSK To counter pulse shaping effects
  - └ shift IP or I channel & Q channel to harbor a difference of half a time period
- higher the bits per symbol ; higher the data rate but lower the inter symbol distance { ergo, higher the BER for the same SNR }
- higher power increases the constellation circle's radius ; higher distance b/w symbols

## Mobile Communication Systems

### Gray Coding

- Adjacent bits / symbol differ by only one bit

### Matched Filter

- $r(t) = s(t) + n(t)$ ;  $n(t)$  is WGN with spectral height  $N_0/2$

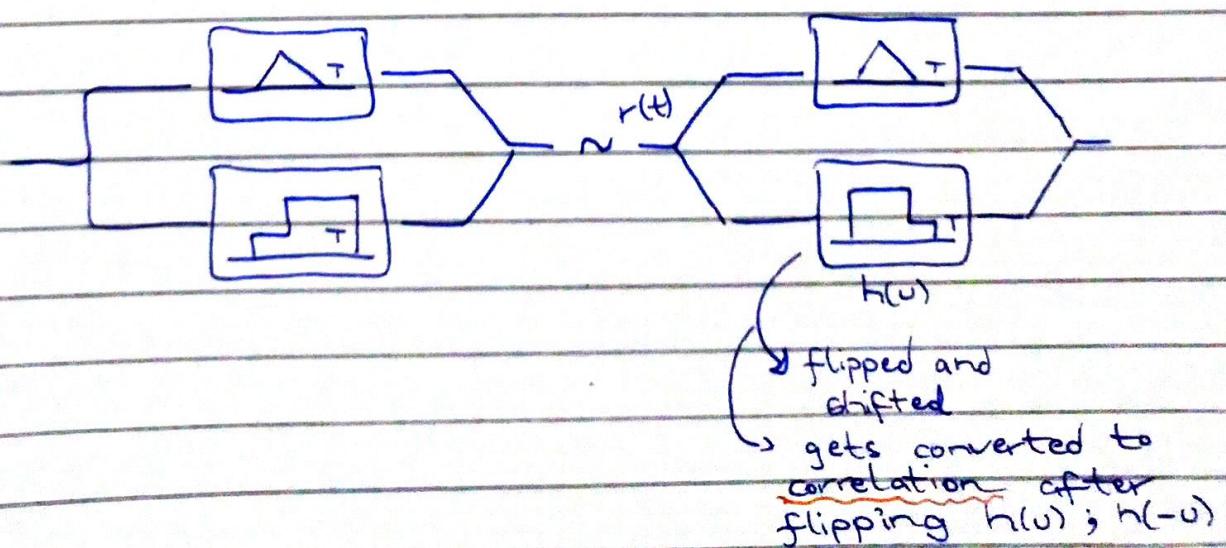
with  $n(u)$  &  $n(v)$  as noise {WGN}

$$\mathbb{E}(n(u)n(v)) = \frac{N_0}{2} \delta(u-v)$$

### Cauchy-Schwarz Inequality

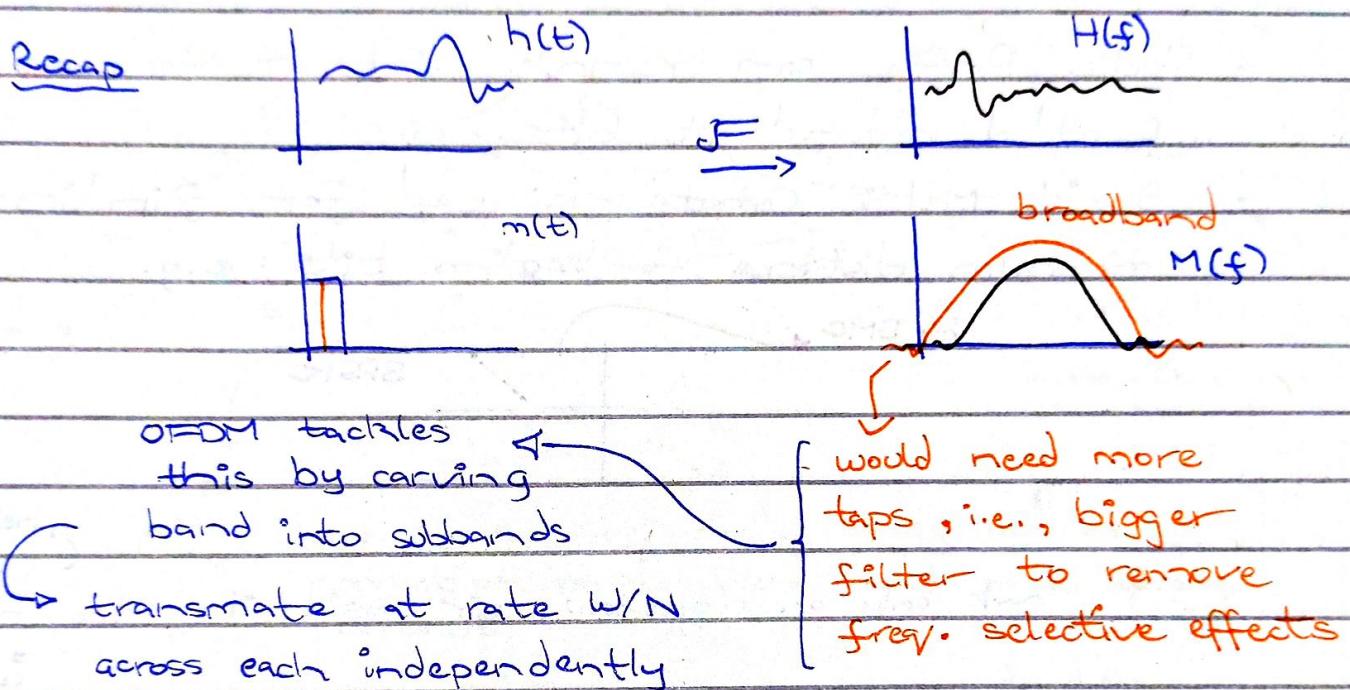
$$\langle s, a \rangle^2 \leq \|s\|^2 \|a\|^2$$

Receiver



# Mobile Communication Systems

OFDM - { multiplexing  
modulation } - both



## Naïve Implementation

$$r(t) = \alpha \left[ a_n^{(0)} e^{j2\pi f_0 t} + a_k^{(1)} e^{j2\pi f_1 t} + \dots + a_k^{(N-1)} e^{j2\pi f_{N-1} t} \right] p(t) + n(t)$$

...  $\underbrace{\dots}_{\text{block modulation}}$  ...

$\downarrow$  fading  $\downarrow$  noise

$\downarrow$  receiver block

$$\rightarrow a_n^{(0)} \int_0^T e^{j2\pi f_0 t} e^{-j2\pi f_0 t} dt + a_k^{(1)} \int_0^T e^{j2\pi f_1 t} e^{-j2\pi f_0 t} dt + \dots$$

$\underbrace{\qquad\qquad\qquad}_{100\% \text{ correlation}}$   $\underbrace{\qquad\qquad\qquad}_{\text{orthogonality property}} \dashrightarrow = 0$

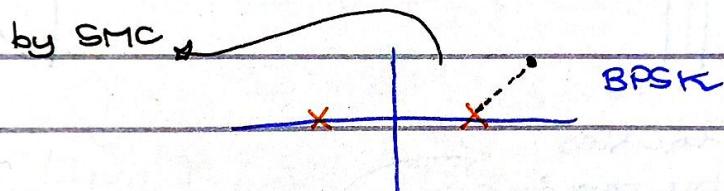
$\rightarrow \alpha a_n^{(0)} + \checkmark \rightarrow \text{noise}$

FFT is only applicable for powers of 2.

↳ Cooley Tukey

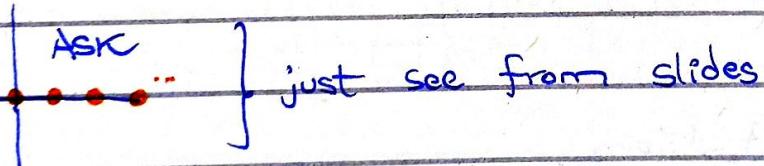
↳ can be used for modulation / demodulation

- Cyclic Prefix can remove ISI at the expense of added redundant bits.
- Serial Metric Compute is used for finding minimum distance for decoding bits; e.g.



PSD ↗ only an overview, sir said its too hard

:(



30 / 11 / 23

## Mobile Communication Systems

### Multiple Access

Recap :

1G - FDMA	(30 kHz)
2G - TDMA + FDMA	(200 kHz)
{ TDMA only possible if digital scheme}	

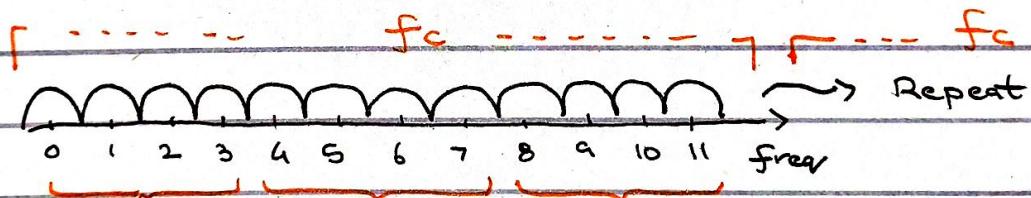
3G - CDMA (UMTS) (2 MHz)

4G - [ uplink SC-FDMA (20 - 40 MHz)  
downlink OFDMA ]

$M = 12$  sub-carriers

$N = M/\alpha = 4$  sub-carrier/user

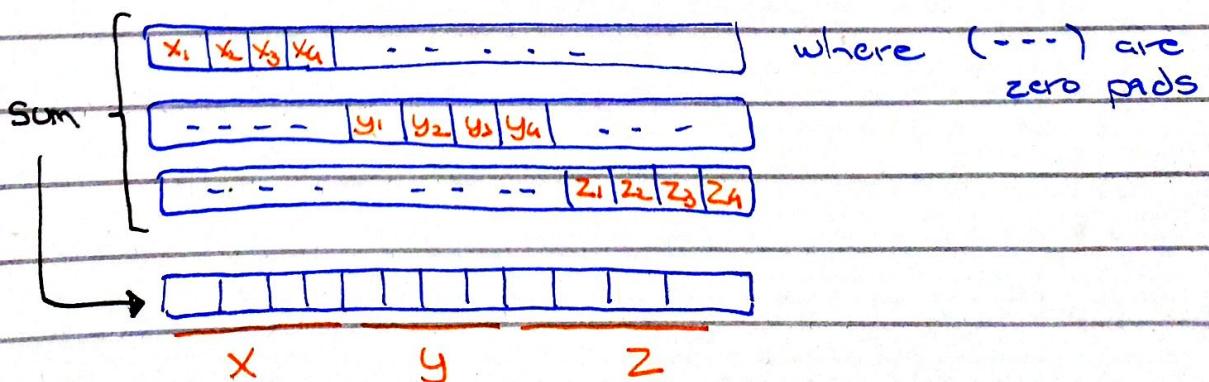
$\alpha = 3$  users



$\gamma$  disjoint set (no overlap)

$\gamma_j$   $j=1$   $\gamma_2$   $\gamma_3$

### Data blocks



Read em up

[unicast | multicast | broadcast]

OFDMA  
is like broadcast

Single Carrier ~ No PAPR issues

SC - FDMA

↳ multiplexing in the frequency domain