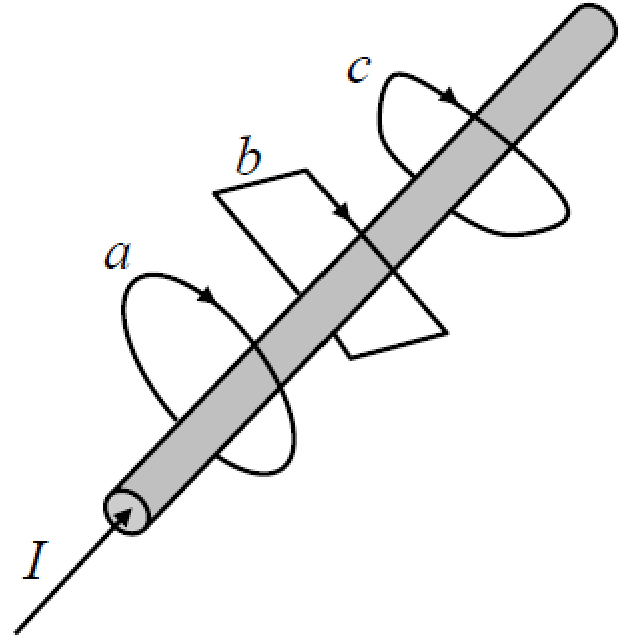
AMPERE'S CIRCUIT LAW - MAXWELL'S EQUATIONS

Ampere's Circuit Law

- Ampere's circuit law states that "the line integral of the tangential component of **H** around a closed path is the same as the net current I_{enc} enclosed by the path"

$$\oint \boldsymbol{H}.d\boldsymbol{l} = I_{enc}$$

- The closed integral in the above expression can be performed on any closed path "a" or "b" or "c"



Maxwell's Third Equation

- We have the following equation from Ampere's law:

$$\oint \boldsymbol{H}.\,d\boldsymbol{l} = I_{enc}$$

- Applying Stoke's Theorem to the left-hand side, we get:

$$I_{enc} = \oint \boldsymbol{H}.\,d\boldsymbol{l} = \int (\nabla \times \boldsymbol{H}) \cdot d\boldsymbol{S}$$

But

$$I_{enc} = \int \boldsymbol{J} \cdot d\boldsymbol{S}$$

- Comparing the two equations above, we get

$$\nabla \times \boldsymbol{H} = \boldsymbol{J}$$

- Maxwell's Third Equation also called Ampere's Law in point or differential form

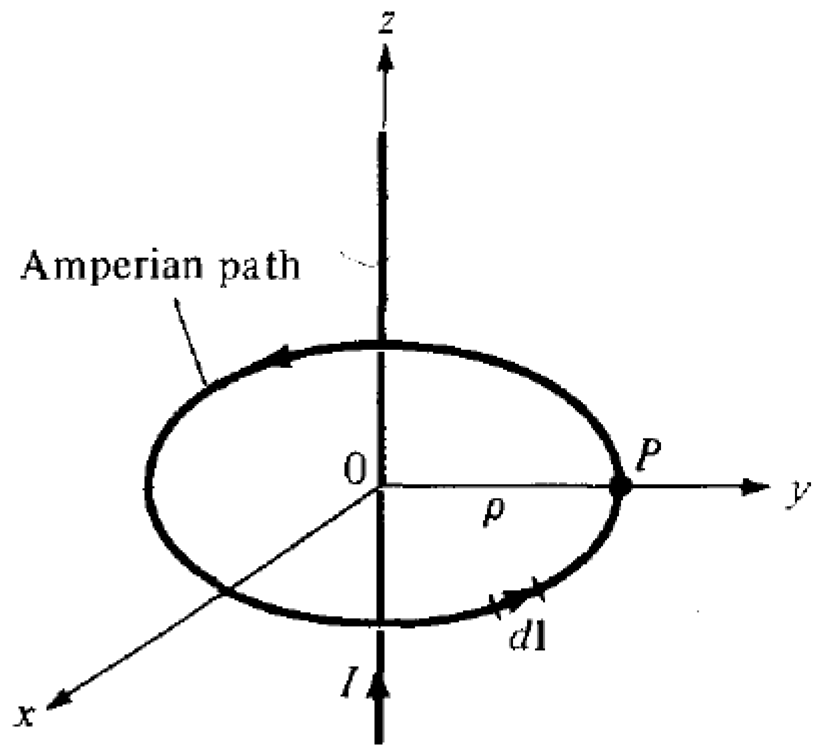
Application of Ampere's Law – Infinite Line Current

- Useful in calculating the magnetic field in problems that involve symmetrical geometries and symmetrical current distribution

- Consider an infinitely long filamentary current *I* along the z-axis as in Figure

- To determine **H** at an observation point *P*, we allow a closed path pass through *P*

- This path, on which Ampere's law is to be applied, is known as an Amperian path (analogous to the term Gaussian surface)



Application of Ampere's Law – Infinite Line Current

- We choose a concentric circle as the Amperian path
- Since this path encloses the whole current *I*, according to Ampere's law:

$$I = \int H_\phi \mathbf{a}_\phi \cdot \rho \, d\phi \, \mathbf{a}_\phi = H_\phi \int \rho \, d\phi = H_\phi \cdot 2\pi\rho$$

OR

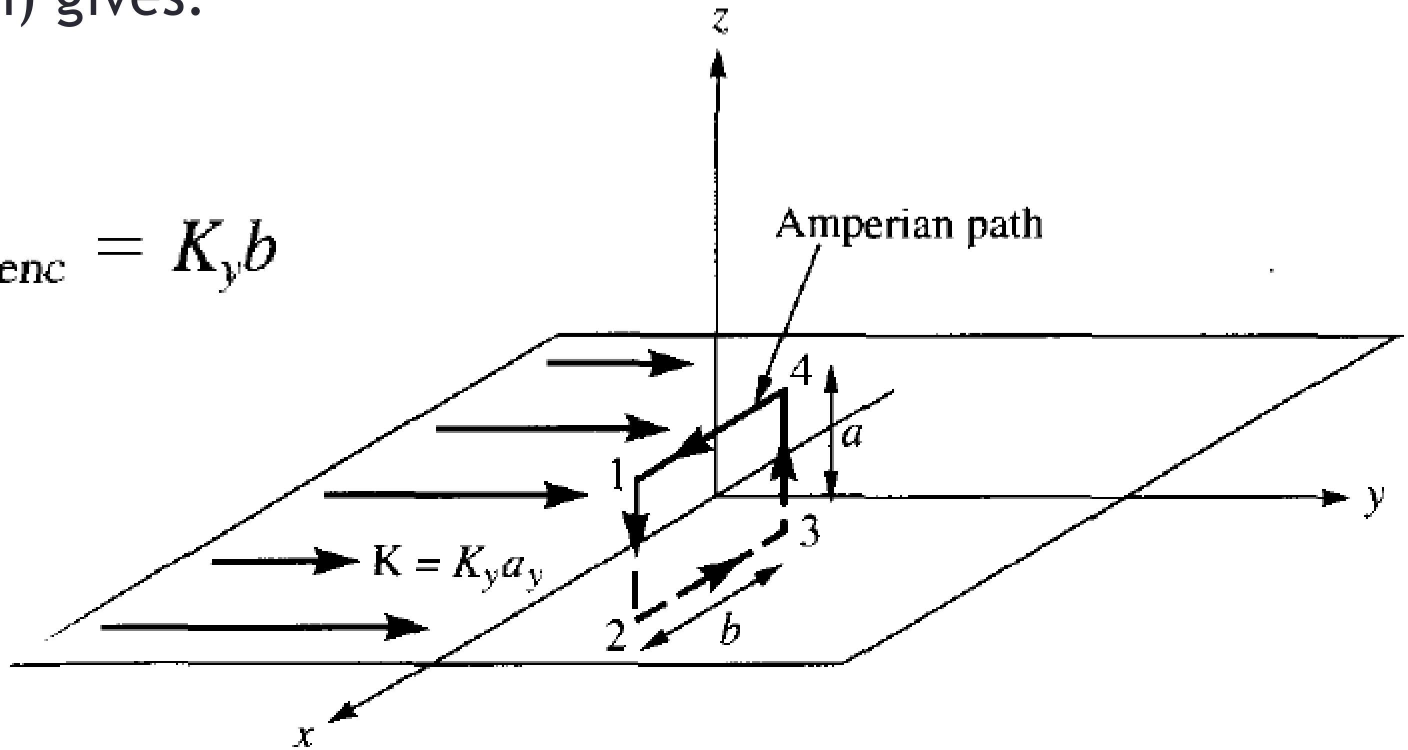
$$\mathbf{H} = \frac{I}{2\pi\rho} \mathbf{a}_\phi$$

Application of Ampere's Law – Infinite Current Sheet

- Consider an infinite current sheet in the *z = 0* plane

- If the sheet has a uniform current density $\mathbf{K} = K_y \mathbf{a}_y$ A/m as shown in Figure, applying Ampere's law to the rectangular closed path (Amperian path) gives:

$$\oint \mathbf{H} \cdot d\mathbf{l} = I_{enc} = K_y b$$



Application of Ampere's Law – Infinite Current Sheet

- We regard the infinite sheet as comprising of filaments or line currents

- Therefore, from the right-hand rule **H** will be cancelled along z-axis due to symmetrical pair of lines

- So the resultant **H** has only x-component, that is:

$$\mathbf{H} = \begin{cases} H_o \mathbf{a}_x & z > 0 \\ -H_o \mathbf{a}_x & z < 0 \end{cases}$$

- Here H_o is yet to be determined

Application of Ampere's Law – Infinite Current Sheet

- Evaluating the line integral of **H** along the closed path gives:

$$\oint \mathbf{H} \cdot d\mathbf{l} = \left(\int_1^2 + \int_2^3 + \int_3^4 + \int_4^1 \right) \mathbf{H} \cdot d\mathbf{l}$$
$$= 0(-a) + (-H_o)(-b) + 0(a) + H_o(b)$$
$$= 2H_o b$$

- Therefore, by comparison we get:

$$H_o = \frac{1}{2} K_y$$

Application of Ampere’s Law – Infinite Current Sheet

- By substituting H_o, we get:

$$\mathbf{H} = \begin{cases} \frac{1}{2} K_y \mathbf{a}_x, & z > 0 \\ -\frac{1}{2} K_y \mathbf{a}_x, & z < 0 \end{cases}$$

- In general, for an infinite sheet of current density **K** A/m,

$$\mathbf{H} = \frac{1}{2} \mathbf{K} \times \mathbf{a}_n$$

- where $\mathbf{a}_n$ is a unit normal vector directed from the current sheet to the point of interest

Magnetic Flux and Flux Density

- The magnetic flux density **B** is similar to the electric flux density **D**

- As **D** = ε_o**E** in free space, the magnetic flux density **B** is related to the magnetic field intensity **H** as:

$$\mathbf{B} = \mu_o \mathbf{H}$$

- μ_o = constant known as the permeability of free space

- The constant is in henrys/meter (H/m) and has the value of:

$$\mu_o = 4\pi \times 10^{-7} \text{ H/m}$$

Magnetic Flux and Flux Density

- The Magnetic Flux through a surface S is given by:

$$\Psi = \int_S \mathbf{B} \cdot d\mathbf{S}$$

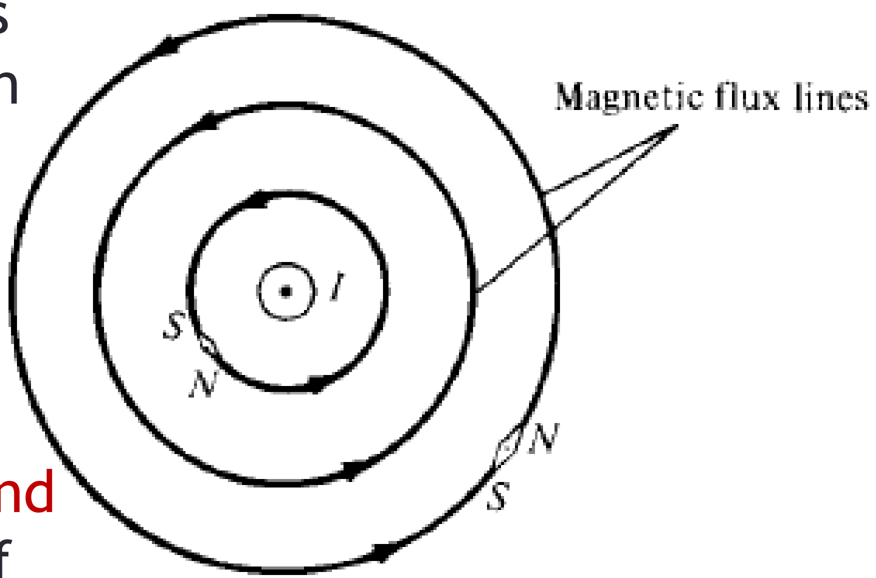
- The magnetic flux Ψ is in webers (Wb) and the magnetic flux density is in webers/square meter (Wb/m^2) or teslas

Magnetic Flux Lines

- The magnetic flux line is the path to which **B** is tangential at every point in a magnetic field

- It is the line along which the needle of a magnetic compass will orient itself if placed in the magnetic field

- For example, the magnetic flux lines due to a straight long wire are shown in Figure



- The magnetic flux lines are closed and do not cross each other regardless of the current distribution

Magnetic Flux Lines

- In an electrostatic field, the flux passing through a closed surface is the same as the charge enclosed; that is,

$$\Psi = \oint \boldsymbol{D} . d\boldsymbol{S} = Q$$

- Thus it is possible to have an isolated electric charge

- On the other hand, it is **not possible** to have isolated magnetic poles (or magnetic charges)

- For example, if we desire to have an isolated magnetic pole by dividing a magnetic bar successively into two, we end up with pieces each having north and south poles

Magnetic Flux Lines

- Thus the total flux through a closed surface in a magnetic field must be zero; that is,

$$\oint \mathbf{B} \cdot d\mathbf{S} = 0$$

- This equation is referred to as the **law of conservation of magnetic flux or Gauss's law for magnetostatic fields**

Maxwell's 4th Equation

- From law of conservation of magnetic flux, we have:

$$\oint \mathbf{B} \cdot d\mathbf{S} = 0$$

- By applying Divergence Theorem to the above equation, we get:

$$\oint_S \mathbf{B} \cdot d\mathbf{S} = \int_v \nabla \cdot \mathbf{B} \, dv = 0$$

Or

$$\nabla \cdot \mathbf{B} = 0$$

- This is Maxwell's 4th Equation

Problem-1

- Consider an infinitely long transmission line consisting of two concentric cylinders having their axes along the z-axis, where the z-axis is out of the page. The inner conductor has radius a and carries current I while the outer conductor has inner radius b and thickness t and carries return current $-I$. Using Ampere's law determine **H** at different regions around the conductors assuming that current is uniformly distributed in both conductors.