Ex:- A Periodic voltage $v(t)$ (in volts) of period 5ms and specified by

7 harmonics 356.9

94.96%

$$v(t) = \begin{cases} 60 & (0 < t < \frac{5}{4}\text{ms}) \\ 0 & (\frac{5}{4}\text{ms} < t < 5\text{ms}) \end{cases}$$

$\frac{1-j}{j} = \frac{1}{j} - \frac{j}{j} = -j-1.$

$c_0 = \frac{1}{8}(60)(\frac{8}{4})$

$c_6 = -\frac{j}{\pi}(10),\ P_6 = 1.34$

Power % 94.12%

$v(t+5\text{ms}) = v(t)$. is applied across the terminals of a $15\,\Omega$ resistor.

(a) obtain expressions for the coefficients c_n of the complex Fourier series representation of $v(t)$, and write down the values of the first five non-zero terms.

(b) Calculate the power associated with each of the first five non-zero terms of the Fourier series

(c) Calculate the total power delivered to the $15\,\Omega$ resistor.

(d) What is the percentage of the total power delivered to the resistor by the first five non-zero terms of the Fourier Series?

Sol:

$$C_n = \frac{1}{5}\int_0^{5/4} 60\, e^{-j\frac{2n\pi}{5}t}\, dt = 12\left[\frac{-5}{j2n\pi} e^{-j\frac{2n\pi}{5}t}\right]_0^{5/4} = \frac{30}{jn\pi}\left[1 - e^{-\frac{jn\pi}{2}}\right], n \neq 0,\quad C_0 = \frac{1}{5}(60)(\frac{5}{4}) = 15.$$

$c_1 = \frac{30}{\pi}(1-j).$

First five non-zero terms are $C_0 = 15$, $C_1 = \frac{30}{\pi j}(1+j)$, $C_2 = \frac{30}{j\pi} = -\frac{30}{\pi}j$, $C_3 = \frac{10}{j\pi}(1-j) = \frac{10}{\pi}(-1-j)$

$C_4 = 0$, $C_5 = \frac{6}{j\pi}(1+j) = \frac{6}{\pi}(1-j)$. (b). Power associated with the first five non-zero terms are

$P_0 = \frac{15^2}{15} = 15\text{w}$, $P_1 = \frac{1}{15}[2|c_1|^2] = \frac{2}{15}(13.50)^2 = 24.30\text{W}$, $P_2 = \frac{1}{15}[2|c_2|^2] = \frac{2}{15}(9.55)^2 = 12.16\text{W}$

$P_3 = \frac{1}{15}[2|c_3|^2] = \frac{2}{15}(4.50)^2 = 2.70\text{W}$, $P_4 = 0$, $P_5 = \frac{1}{15}[2|c_5|^2] = \frac{2}{15}(2.70)^2 = 0.97\text{W}$

The total power delivered by first five terms is $P = P_0 + P_1 + P_2 + P_3 + P_4 + P_5 = 55.13\text{W}$.

(c) Total power delivered by $15\,\Omega$ resistor is $P = \frac{1}{15}[\frac{1}{5}\int_0^{5/4} 60^2 dt] = \frac{1}{15}(\frac{1}{5})(60^2)(\frac{5}{4}) = 60\text{W}$

(d) % of total power delivered by the first five non-zero terms is $\frac{55.13}{60} \times 100 = 91.9\%$.

$= \frac{1}{2}(a_n + j b_n)$ (4.56)

In general the coefficients c_n ($n = 0, \pm 1, \pm 2, \dots$) are complex, and may be expressed in the form

$$c_n = |c_n| e^{j\phi_n}$$

where $|c_n|$, the magnitude of c_n, is given from the definitions (4.56) by

$$|c_n| = \sqrt{[(\tfrac{1}{2}a_n)^2 + (\tfrac{1}{2}b_n)^2]} = \tfrac{1}{2}\sqrt{(a_n^2 + b_n^2)} \Rightarrow \sqrt{a_n^2 + b_n^2} = 2|c_n|.$$

so that $2|c_n|$ is the amplitude of the nth harmonic. The argument ϕ_n of c_n is related to the phase of the nth harmonic.

Section 4.2.2. page 282–283: The amplitude and phase of the nth harmonic is $A_n = \sqrt{a_n^2 + b_n^2}$, $\phi_n = \tan^{-1}\left(\frac{a_n}{b_n}\right)$ with care being taken over choice of quadrant.

EXAMPLE 4.18 Find the complex form of the Fourier series expansion of the periodic function $f(t)$ defined by

$$f(t) = \cos \tfrac{1}{2}t \quad (-\pi < t < \pi), \qquad f(t + 2\pi) = f(t)$$

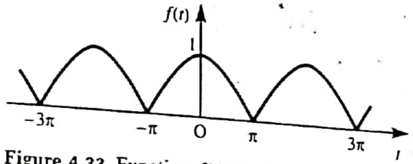

Figure 4.33 Function $f(t)$ of Example 4.18.

Solution A graph of the function $f(t)$ over the interval $-3\pi \leqslant t \leqslant 3\pi$ is shown in Figure 4.33. Here the period T is 2π, so from (4.61) the complex coefficients c_n are given by

$$c_n = \frac{1}{2\pi}\int_{-\pi}^{\pi} \cos \tfrac{1}{2}t\, e^{-jnt}\, dt = \frac{1}{4\pi}\int_{-\pi}^{\pi} (e^{jt/2} + e^{-jt/2})\, e^{-jnt}\, dt$$

$$= \frac{1}{4\pi}\int_{-\pi}^{\pi} (e^{-j(n-1/2)t} + e^{-j(n+1/2)t})\, dt$$

$$= \frac{1}{4\pi}\left[\frac{-2\,e^{-j(2n-1)t/2}}{j(2n-1)} - \frac{2\,e^{-j(2n+1)t/2}}{j(2n+1)}\right]_{-\pi}^{\pi}$$

$$= \frac{j}{2\pi}\left[\left(\frac{e^{-jn\pi}e^{j\pi/2}}{2n-1} + \frac{e^{-jn\pi}e^{-j\pi/2}}{2n+1}\right) - \left(\frac{e^{jn\pi}e^{-j\pi/2}}{2n-1} + \frac{e^{jn\pi}e^{j\pi/2}}{2n+1}\right)\right]$$

Now $e^{j\pi/2} = \cos \frac{1}{2}\pi + j \sin \frac{1}{2}\pi = j$, $e^{-j\pi/2} = -j$ and $e^{jn\pi} = e^{-jn\pi} = \cos n\pi = (-1)^n$, so that

$$c_n = \frac{j}{2\pi}\left(\frac{j}{2n-1} - \frac{j}{2n+1} + \frac{j}{2n-1} - \frac{j}{2n+1}\right)(-1)^n$$

$$= \frac{(-1)^n}{\pi}\left(\frac{1}{2n+1} - \frac{1}{2n-1}\right) = \frac{-2(-1)^n}{(4n^2-1)\pi}$$

Note that in this case c_n is real, which is as expected, since the function $f(t)$ is an even function of t.

From (4.57), the complex Fourier series expansion for $f(t)$ is

$$f(t) = \sum_{n=-\infty}^{\infty} \frac{2(-1)^{n+1}}{(4n^2-1)\pi} e^{jnt}$$

This may readily be converted back to the trigonometric form, since, from the definitions (4.56),

$$a_0 = 2c_0, \qquad a_n = c_n + c_n^*, \qquad b_n = j(c_n - c_n^*)$$

so that in this particular case

$$a_0 = \frac{4}{\pi}, \qquad a_n = 2\left[\frac{2}{\pi}\frac{(-2)^{n+1}}{4n^2+1}\right] = \frac{4}{\pi}\frac{(-1)^{n+1}}{4n^2-1}, \qquad b_n = 0$$

Thus the trigonometric form of the Fourier series is

$$f(t) = \frac{2}{\pi} + \frac{4}{\pi}\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{4n^2-1} \cos nt$$

which corresponds to the solution to Exercise 1(e).

EXAMPLE 4.19 Obtain the complex form of the Fourier series of the sawtooth function $f(t)$ defined by

$$f(t) = \frac{2t}{T} \quad (0 < t < 2T), \qquad f(t + 2T) = f(t)$$

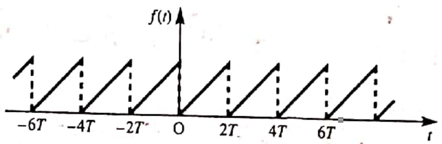

Figure 4.34 Function $f(t)$ of Example 4.19.

Solution A graph of the function $f(t)$ over the interval $-6T < t < 6T$ is shown in Figure 4.34. Here the period is $2T$, that is, $\omega = \pi/T$, so from (4.61) the complex coefficients c_n are given by

$$c_n = \frac{1}{2T}\int_0^{2T} f(t)\, e^{-jn\pi t/T}\, dt = \frac{1}{2T}\int_0^{2T} \frac{2}{T}t\, e^{-jn\pi t/T}\, dt$$

$$= \frac{1}{T^2}\left[\frac{Tt}{-jn\pi}e^{-jn\pi t/T} - \frac{T^2}{(jn\pi)^2}e^{-jn\pi t/T}\right]_0^{2T} \qquad (n \neq 0)$$

Half-wave rectification in which only the positive portions of the signal remain. In full-wave rectification, either positive or negative portions are inverted so that the entire signal has the same polarity. (Converts AC into DC).