Course : EE 451 Mobile Communication Systems

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- Problem 1 | $d = 10\,km$, $P_t = 50\,W$, $f_c = 6\,GHz$, $G_t = G_r = 1$

a) Power at receiver

As free-space propagation is assumed,

- $$P_r = \frac{P_t\, G_t\, G_r\, \lambda^2}{(4\pi)^2\, d^2}$$, where $\lambda = c/f$

Substituting,

$$P_r = \frac{(50)(1)(1)(3e8/6e9)^2}{(4\pi)^2\,(10e3)^2} = \boxed{7.91\,e-12\ W}$$

b) Magnitude of E at receiver

As $P_r = P_d\, A_e = \frac{|E|^2}{120\pi} \cdot \frac{G_r\, \lambda^2}{4\pi}$

Rearranging,

$$|E|^2 = \frac{P_r\,(4\pi)(120\pi)}{G_r\,\lambda^2} = \frac{(7.91e-12)(480\pi^2)}{(1)(3e8/6e9)^2}$$

$$= 1.49\,e-5\ V^2/m^2$$

$$|E| = \sqrt{|E|^2} = \boxed{3.87\,e-3\ V/m}$$

c) Receiver power in dBm

$$P_{r\,(dBm)} = 10\log_{10}\left(P_{r(W)}/1e-3\right) = \boxed{-81\ dBm}$$

d) Decoding $P_{r(sensitivity)} = -96\,dBm$

- As the received power, i.e., $-81\,dBm$, is well above the receiver sensitivity; the receiver would be able to decode the message.

  $\therefore \{-81\,dBm > -96\,dBm\}$

∘ Problem 2 | $L = 25\,dB + 10\log_{10} d^{2.8}$, $\beta = 2.8$

a) Required Transmitter Power

$P_{r(sensitivity)} = -95\,dBm$, $d = 10\,km$

$$\gg \quad P_r = P_t - (25 + 2.8(10\log_{10}(10e3)))$$

$$P_t = (-95) + (25 + 112)$$

$$= \boxed{42\,dBm}$$

b) With $\beta = 3.1$

$$\gg \quad P_r = P_{t_{new}} - (25 + 3.1(10\log_{10}(10e3)))$$

$$P_{t_{new}} = (-95) + (124 + 25)$$

$$= 54\,dBm$$

- Hence, an increase of $\boxed{12\,dB}$ is required in transmit power to meet the desired specification.

c) With Log-normal shadowing, $\sigma = 8\,dB$

└ Assuming c) is independent of b), and $\beta = 2.8$

$$P_R(\overline{P_r} > -95) = 0.9$$

or, $$P_R(\overline{P_r} < -95) = 0.1$$

$$\Rightarrow \quad Q\left(-\left\{\frac{-95 - x}{8}\right\}\right) = 0.1$$

From Q-table $\gg$ $\dfrac{95 + x}{8} = 1.3$, where $x \triangleq \overline{P_r}$

$\overline{P_r} = -84.6$, which is the new sensitivity catering for thermal outage

$$\gg \overline{P_r} = P_{t_{new}} - (25 + 2.8(10\log_{10}(10e3)))$$

$$P_{t_{new}} = (-84.6) + (25 + 112)$$

$$= 52.4$$

- Hence, an increase of $\boxed{10.4 \text{ dB}}$ is needed in transmit power to account for 10% thermal noise outage.

d) Fraction of Useful Area

$\beta = 2.8$; $\sigma/\beta = 2.857$

With $P_R[\overline{P_r} > \gamma] = 0.9$; Using the graph of $U(\gamma)$:

Useful Area Fraction $= \boxed{98.3\ \%}$

- Problem 3

a) Estimate of Power Delay Profile

As $d = 2\lambda$ and PDP of the channel is found by taking the spatial average of $|h(t;\tau)|^2$ over a local area.

$$P(\tau) = \overline{|h(t;\tau)|^2}$$

$$\gg P(0\mu) = \frac{|h(0;0\mu)|^2 + |h(1;0\mu)|^2 + |h(2;0\mu)|^2}{3}$$

Assumption of Watts

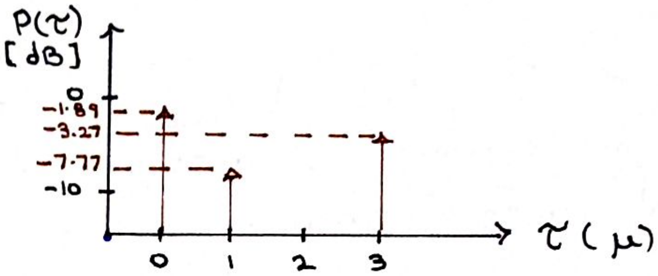
$$= \frac{0.8^2 + 0.9^2 + 0.7^2}{3} = 0.646\ W$$

Similarly,

$$P(1\mu) = \frac{0.4^2 + 0.5^2 + 0.3^2}{3} = \underline{0.167\ W}$$

$$P(2\mu) = 0\ W$$

$$P(3\mu) = \frac{0.7^2 + 0.75^2 + 0.6^2}{3} = \underline{0.470\ W}$$



b) RMS Delay Spread | $\bar{\tau} = \frac{\Sigma \tau P(\tau)}{\Sigma(P(\tau))}$, $\bar{\tau}^{(2)} = \frac{\Sigma \tau^2 P(\tau)}{\Sigma(P(\tau))}$

$$\bar{\tau} = \frac{(0)(0.646) + (1)(0.167) + (3)(0.470)}{0.646 + 0.167 + 0.470}\ [1e-6]\ s$$

$$= 1.229\ \mu s$$

$$\bar{\tau}^{(2)} = \frac{(0)^2(0.646) + (1)^2(0.167) + (3)^2(0.470)}{0.646 + 0.167 + 0.470}\ [1e-6]^2\ s$$

$$= 3.427\ ps$$

» $\sigma_\tau = \sqrt{\bar{\tau}^{(2)} - (\bar{\tau})^2} = \boxed{1.384\ \mu s}$

- Problem 4 | $P(\tau) = \delta(\tau) + 0.5(\tau - 300ns)$, $T_s = 10^{-5} s$

a) Mean Excess Delay

$$\bar{\tau} = \frac{\int_0^\infty \tau P(\tau) d\tau}{\int_0^\infty P(\tau) d\tau}$$

Using impulse property; $\int_{-\infty}^{\infty} f(x)\,\delta(x-k)\,dx = f(k)$

$$\gg \bar{\tau} = \frac{\cancel{\int_0^\infty \tau\,\delta(\tau)\,d\tau}^{\,0} + 0.5\int_0^\infty \tau\,\delta(\tau - 300\text{ns})\,d\tau}{\int_0^\infty \delta(\tau)\,d\tau + 0.5\int_0^\infty \delta(\tau-300\text{ns})\,d\tau}$$

$$\bar{\tau} = \frac{[0.5\,(300\,\text{ns})]}{1.5} = \boxed{100\text{ ns}}$$

b) RMS Delay Spread

$$\bar{\tau}^{(2)} = \frac{\int_0^\infty \tau^2 P(\tau)\,d\tau}{\int_0^\infty P(\tau)\,d\tau}$$

In a similar fashion; using impulse property, we get:

$$\gg \bar{\tau}^{(2)} = \frac{\cancel{\int_0^\infty \tau^2 \delta(\tau)\,d\tau}^{\,0} + 0.5\int_0^\infty \tau^2\delta(\tau - 300\text{ns})\,d\tau}{\int_0^\infty \delta(\tau)\,d\tau + 0.5\int_0^\infty \delta(\tau-300\text{ns})\,d\tau}$$

$$= \frac{[0.5\,(300\,\text{ns})^2]}{1.5} = 30\text{e}-15\text{ s}$$

$$\gg \sigma_\tau = \sqrt{\bar{\tau}^{(2)} - (\bar{\tau})^2} = \boxed{141\text{ ns}}$$

c) Classification

$T_s = 10^{-6}$ s ; As $T_s \gg \sigma_\tau$, the given channel is undergoing $\boxed{\text{flat}}$ fading. Moreover, as the channel is described as static, it is also that of $\boxed{\text{slow}}$ fading.

- Problem 5 | $\bar{\tau} = \frac{\Sigma \tau P(\tau)}{\Sigma P(\tau)}$, $\bar{\tau}^{(2)} = \frac{\Sigma \tau^2 P(\tau)}{\Sigma P(\tau)}$

a) $B_{c,50}$ and $B_{c,90}$

- Indoor Channel

$\therefore$ 0 dB → 1 W ; −10 dB → 0.1 W ; −20 dB → 0.01 W

» $\bar{\tau} = \frac{(0)(1) + (50)(1) + (75)(0.1) + (100)(0.01)}{1 + 1 + 0.1 + 0.01} [e-9]s$

$= 27.72 \text{ ns}$

» $\bar{\tau}^{(2)} = \frac{(0)^2(1) + (50)^2(1) + (75)^2(0.1) + (100)^2(0.01)}{1+1+0.1+0.01} [e-9]^2 s$

$= 1.498 \text{ e-15 s}$

» $\sigma_\tau = \sqrt{\bar{\tau}^{(2)} - (\bar{\tau})^2} = 27.02 \text{ ns}$

» $B_{c,90 \text{ (indoors)}} = \frac{1}{50\sigma_\tau} = \boxed{740 \text{ kHz}}$

» $B_{c,50 \text{ (indoors)}} = \frac{1}{5\sigma_\tau} = \boxed{7.4 \text{ MHz}}$

- Outdoor Channel

» $\bar{\tau} = \frac{(0)(0.01) + (5)(0.1) + (10)(1)}{0.01 + 0.1 + 1} [e-6] s$

$= 9.459 \text{ µs}$

» $\bar{\tau}^{(2)} = \frac{(0)^2(0.01) + (5)^2(0.1) + (10)^2(1)}{0.01 + 0.1 + 1} [e-6]^2 s$

$= 92.34 \text{ ps}$

» $\sigma_\tau = \sqrt{\bar{\tau}^{(2)} - (\bar{\tau})^2} = 1.671 \text{ µs}$

» $B_{c,90,(outdoors)} = \frac{1}{50\,\sigma_\tau} = \boxed{11.827\ kHz}$

» $B_{c,50,(indoors)} = \frac{1}{5\,\sigma_\tau} = \boxed{118.273\ kHz}$

b) Use / Need for an Equalizer

- Indoor Channel

$B_{c,50\ (indoor)} = 7.4\ MHz$

Comparing against the given standards ;

As :

$$200\ kHz < 1.25\ MHz < 5\ MHz < \underbrace{7.4\ MHz}_{B_{c,50}} < 20\ MHz$$

GSM CDMA WCDMA LTE

Excluding LTE, indoor channel is suitable for all other standards without the use of an equalizer.

- Outdoor Channel

$B_{c,50,(outdoor)} = 118.273\ kHz$

Comparing against the given standards ;

As :

$$\underbrace{118.273\ kHz}_{B_{c,50}} < 200\ kHz < 1.25\ MHz < 5\ MHz < 20\ MHz$$

GSM CDMA WCDMA LTE

Hence, no standard is suitable for use without an equalizer for outdoor channel.