DTFS PROPERTIES AND PROBLEMS

DT Fourier Series Pair

$$x[n] = \sum_{k=<N>} a_k e^{jk\omega_0 n} \qquad \text{(Synthesis equation)}$$

$$a_k = \frac{1}{N} \sum_{n=<N>} x[n] e^{-jk\omega_0 n} \qquad \text{(Analysis equaiton)}$$

DTFS Properties - Multiplication

- The product of two CT signals of period T results in a periodic signal with period T, whose Fourier series coefficients is the *convolution* of the sequences of Fourier series coefficients of the two signals being multiplied.

- The product of two DT periodic sequences, both of period N, results in a periodic product with period N with Fourier coefficients, d_k, of the form:

$$x[n]y[n] \xleftrightarrow{FS} d_k = \sum_{l=<N>} a_l b_{k-l}$$

- i.e., as a circular or periodic convolution of the Fourier series coefficients of the two sequences.

DTFS Properties – First Difference

- First difference of a periodic sequence of period N is analogous to the differentiation property of the CT Fourier series
- First difference operation is defined as:

$$y[n] = x[n] - x[n-1]$$

with Fourier series coefficients:

$$x[n] \xleftrightarrow{FS} a_k; \quad y[n] \xleftrightarrow{FS} b_k$$

- By applying the time-shifting and linearity properties, we get the following:

$$y[n] = x[n] - x[n-1] \xleftrightarrow{FS} b_k = (1 - e^{-jk(2\pi/N)})a_k$$

DTFS Properties – Parseval's Relation

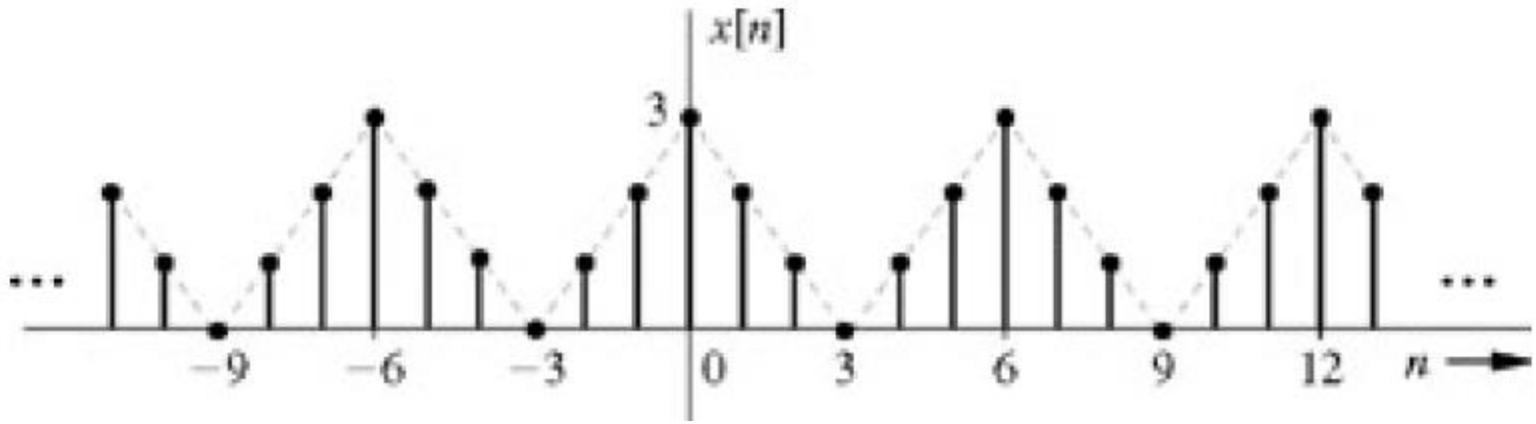
- For DT periodic signals, Parseval's relation is:

$$\frac{1}{N}\sum_{n=<N>}|x[n]|^2=\sum_{k=<N>}|a_k|^2$$

- where a_k are the Fourier series coefficients of $x[n]$ and N is the period.
- Parseval's relation shows that the average power in one period of a periodic signal equals the sum of the average power in all of its harmonic components.

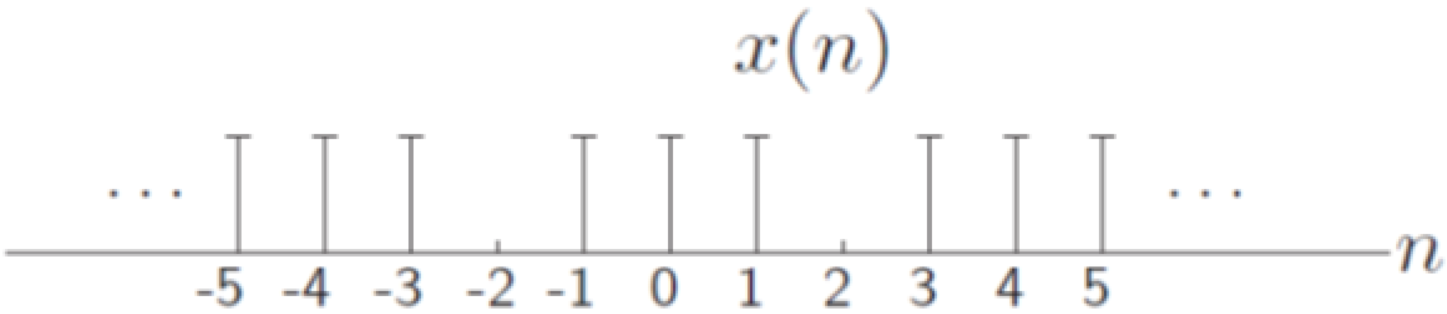
Problem-1

- Find the discrete-time Fourier series of the signal below.



Problem-2

- Find the discrete-time Fourier series of the signal below.



Problem-3

- Odd signal

Consider the periodic signal

$$\tilde{x}(0) = 1$$

$$\tilde{x}(1) = 1$$

$$\tilde{x}(2) = 1$$

$$\tilde{x}(3) = 0$$

and $\tilde{x}(n) = \tilde{x}(n-4)$, all n

END