Engineering Mechanics

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Research Interest: Photovoltaic systems, Power & Control, Sliding mode control (SMC)

Contents (Section 4.8)

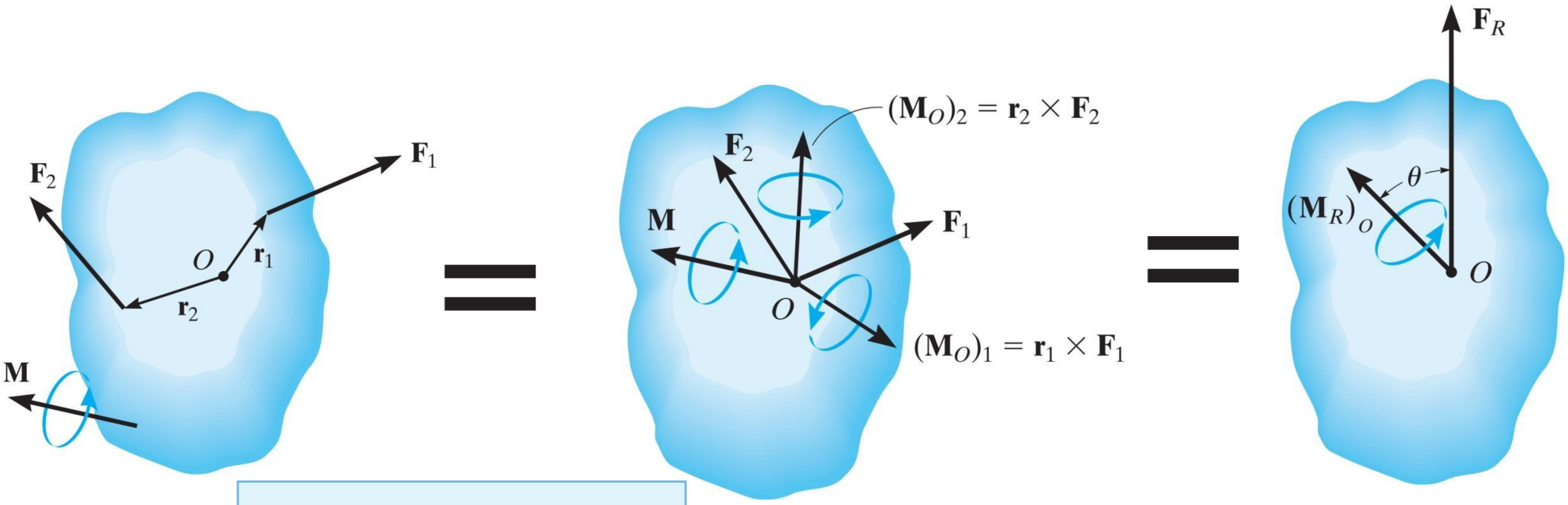
- Recap
- Further Simplification of a Force and Couple System

RECAP

Simplification of a Force and Couple System

Simplification of a Force and Couple System

System of Forces and Couple Moments.



$$\mathbf{F}_R = \Sigma \mathbf{F}$$
$$(\mathbf{M}_R)_O = \Sigma \mathbf{M}_O + \Sigma \mathbf{M}$$

$$(F_R)_x = \Sigma F_x$$
$$(F_R)_y = \Sigma F_y$$
$$(M_R)_O = \Sigma M_O + \Sigma M$$

4.8 Further Simplification of a Force and Couple System

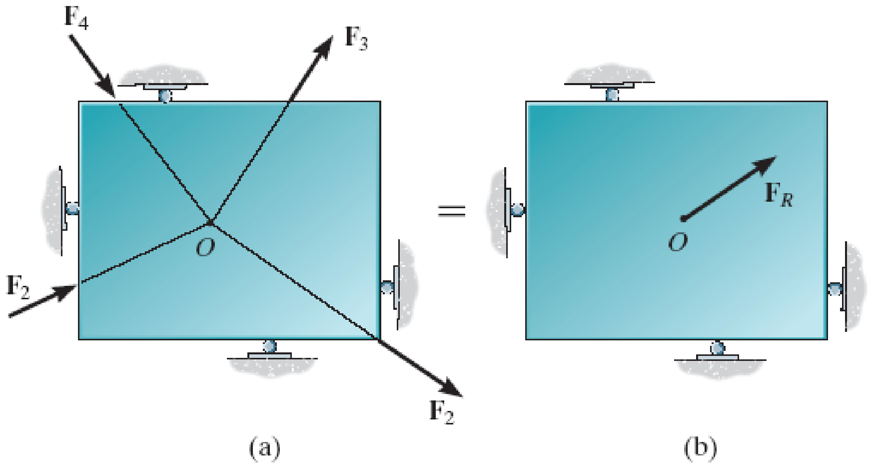
In the preceding section, we developed a way to reduce a force and couple moment system acting on a rigid body into an equivalent resultant force $\mathbf{F}_R$ acting at a specific point O and a resultant couple moment $(\mathbf{M}_R)_O$. The

force system can be further reduced to an equivalent single resultant force provided the lines of action of $\mathbf{F}_R$ and $(\mathbf{M}_R)_O$ are *perpendicular* to each other. Because of this condition, only concurrent, coplanar, and parallel force systems can be further simplified.

Further Simplification of a Force and Couple System

Concurrent Force System

- A *concurrent force system* is where lines of action of all the forces intersect at a common point *O*
- Then the force system produces **no moment** about this point. As a result, the equivalent system can be represented by a single resultan force



$$F_R = \sum F$$

The four cable forces are all concurrent at point O on this bridge tower. Consequently they produce no resultant moment there, only a resultant force $\mathbf{F}_R$. Note that the designers have positioned the cables so that $\mathbf{F}_R$ is directed *along* the bridge tower directly to the support, so that it does not cause any bending of the tower. (© Russell C. Hibbeler)

Further Simplification of a Force and Couple System

Coplanar Force System

- Lines of action of all the forces lie in the same plane
- Resultant force of this system also lies in this plane
- Besides, the moment of each of the forces about any point *O* is directed perpendicular to this plane.
- Thus, the resultant moment $(\mathbf{M}_R)_o$ and resultant force $\mathbf{F}_R$ will be *mutually perpendicular*.

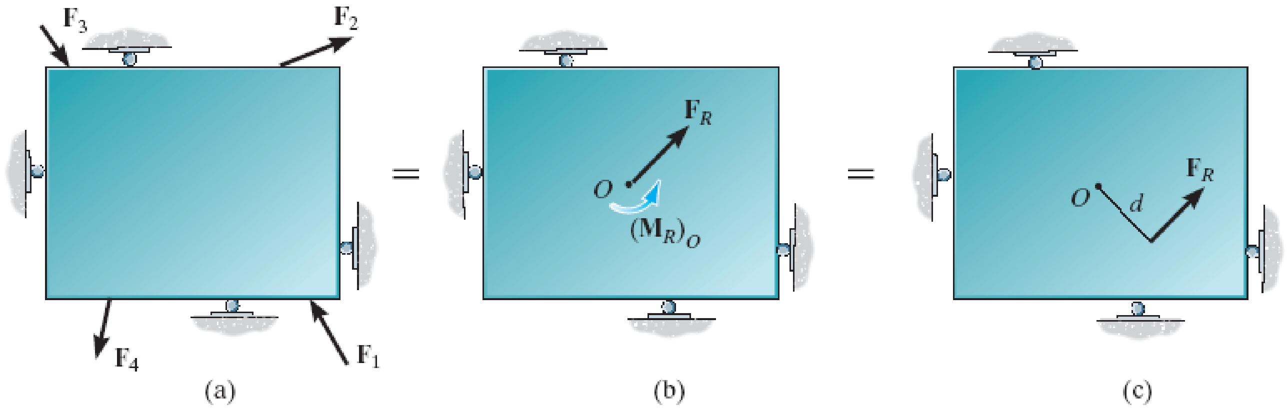
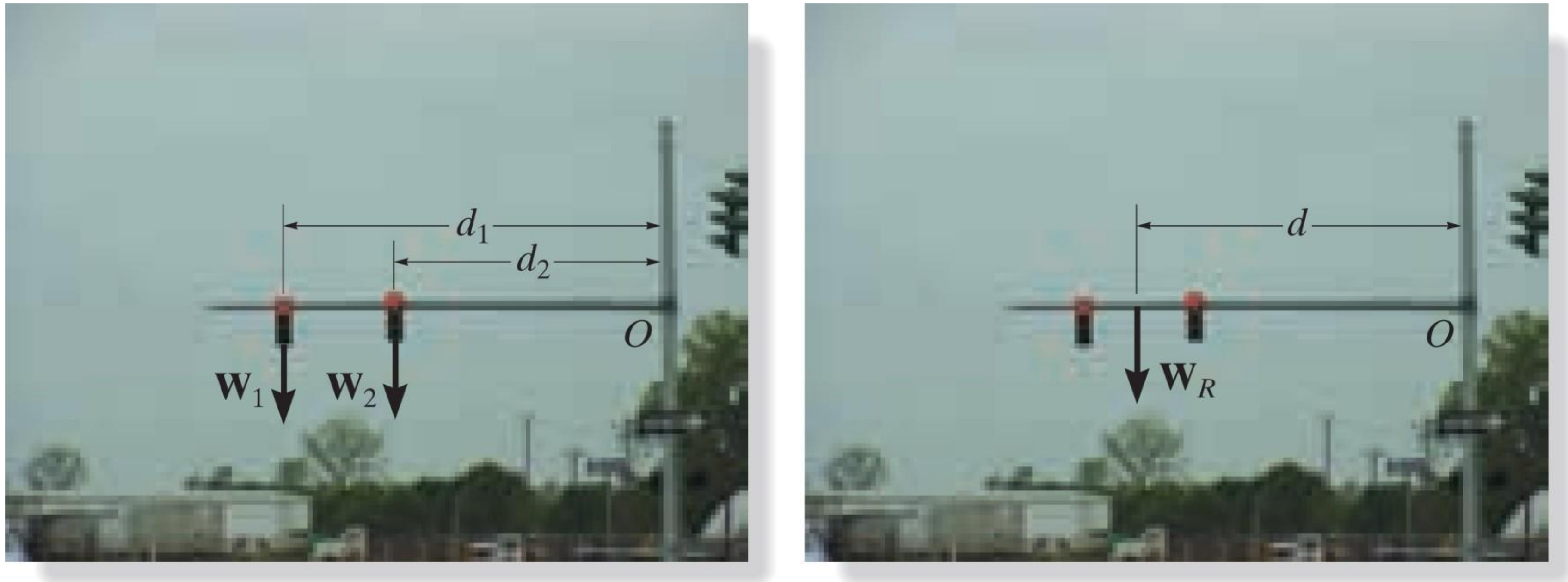


Fig. 4–41*b*. The resultant moment can be replaced by moving the resultant force $\mathbf{F}_R$ a perpendicular or moment arm distance d away from point O such that $\mathbf{F}_R$ produces the *same moment* $(\mathbf{M}_R)_O$ about point O, Fig. 4–41*c*. This distance d can be determined from the scalar equation $(M_R)_O = F_R d = \Sigma M_O$ or $d = (M_R)_O / F_R$.

Further Simplification of a Force and Couple System

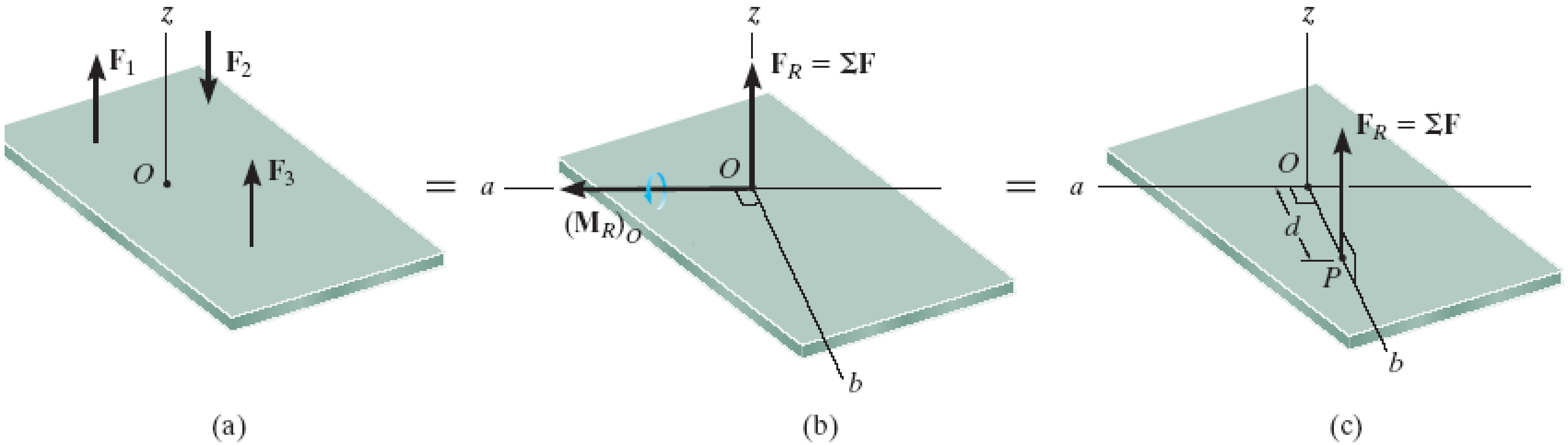
Coplanar Force System.



Here the weights of the traffic lights are replaced by their resultant force $W_R = W_1 + W_2$ which acts at a distance $d = (W_1 d_1 + W_2 d_2)/\ W_R$ from O. Both systems are equivalent. (© Russell C. Hibbeler)

Further Simplification of a Force and Couple System

<u>Parallel Force System</u>

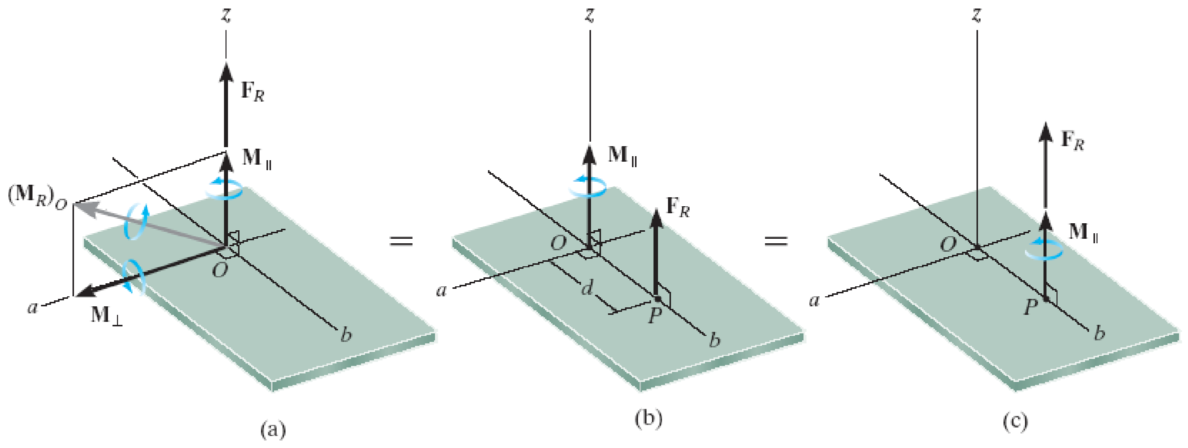
- Consists of forces that are all parallel to the *z* axis
- Resultant force at point *O* must also be parallel to this axis
- 

result, the force system can be further reduced to an equivalent single resultant force $\mathbf{F}_R$, acting through point P located on the perpendicular b axis, Fig. 4–42*c*. The distance d along this axis from point O requires $(M_R)_O = F_R d = \Sigma M_O$ or $d = \Sigma M_O / F_R$.

Further Simplification of a Force and Couple System

Reduction to a Wrench

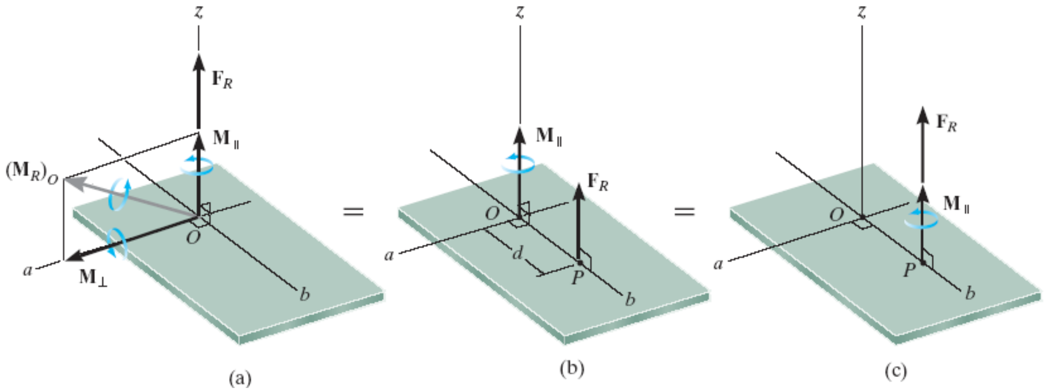
- In general, the force and couple moment system acting on a body will reduce to a single resultant force and a couple moment at o that are not perpendicular
- $\mathbf{F}_R$ will act at an angle θ from $\mathbf{M}_{Ro}$
- $\mathbf{M}_{Ro}$ can be resolved into one perpendicular $\mathbf{M}_\perp$ and the other $\mathbf{M}_\parallel$ parallel to line of action of $\mathbf{F}_R$
- $\mathbf{M}_\perp$ can be eliminated by moving $\mathbf{F}_R$ to point P, a distance d from **o** that lies on the axis bb', which is perpendicular to both **a-axis** and line of action of $\mathbf{F}_R$



Further Simplification of a Force and Couple System

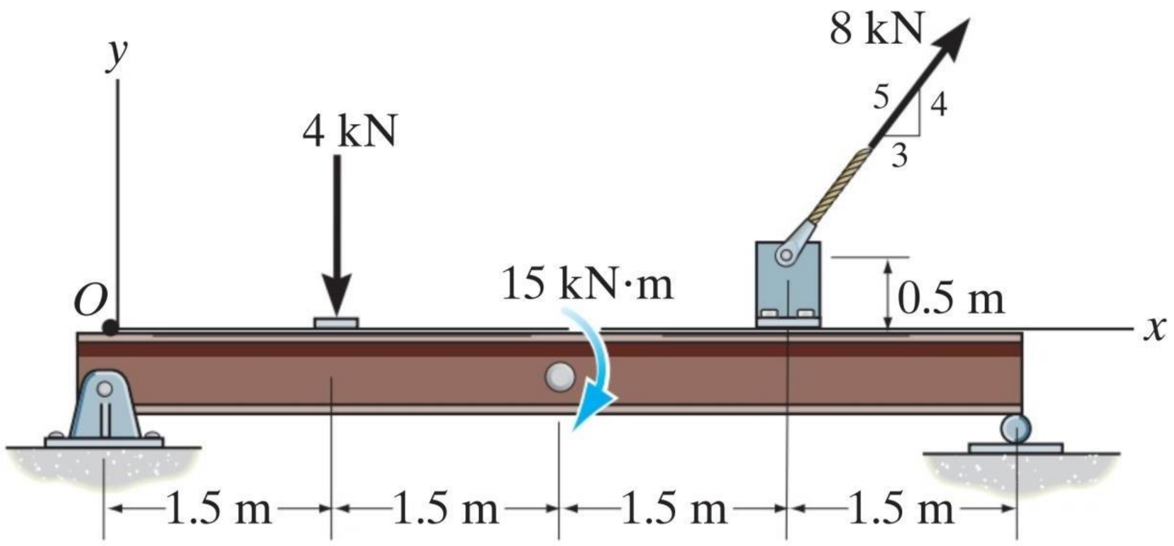
Reduction to a Wrench

- To maintain equivalency of loading, for distance from O to P, $d = M_{\perp}/F_R$
- When $\mathbf{F}_R$ is applied at P, moment of $\mathbf{F}_R$ tends to cause rotation in the same direction as $M_{\perp}$
- Since $\mathbf{M}_{\parallel}$ is a free vector, it may be moved to P so that it is collinear to $\mathbf{F}_R$
- Combination of collinear force and couple moment is called a wrench or screw
- Axis of wrench has the same line of action as the force
- Wrench tends to cause a translation and rotation about this axis

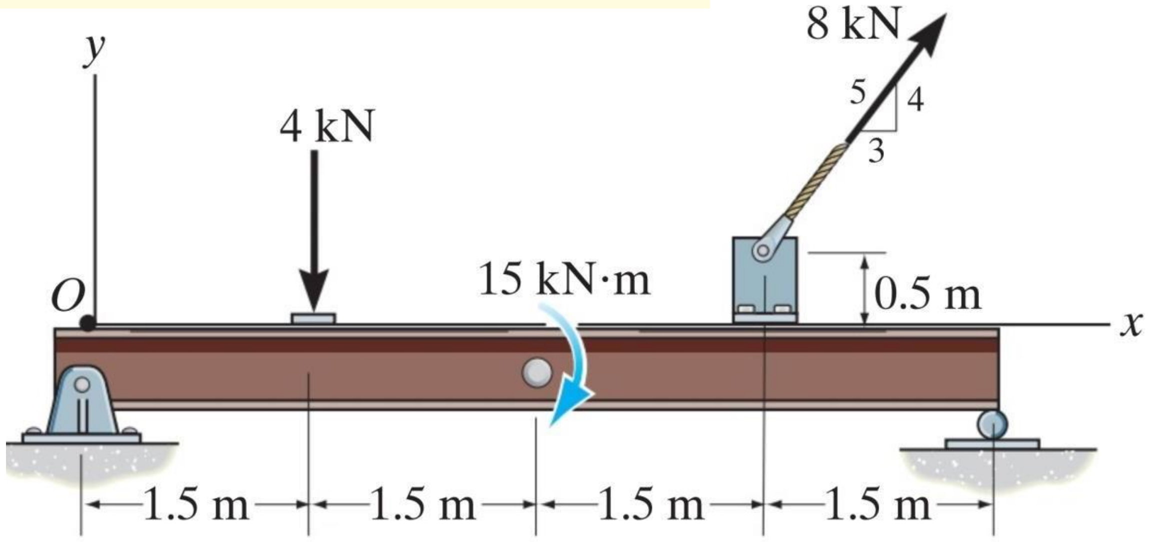


Example

Replace the force and couple moment system acting on the beam in Fig. 4–44*a* by an equivalent resultant force, and find where its line of action intersects the beam, measured from point *O*.



Replace the force and couple moment system acting on the beam in Fig. 4–44*a* by an equivalent resultant force, and find where its line of action intersects the beam, measured from point O.



Force Summation. Summing the force components,

$$\overset{+}{\rightarrow}(F_R)_x = \Sigma F_x; \qquad (F_R)_x = 8\text{ kN}\left(\tfrac{3}{5}\right) = 4.80\text{ kN} \rightarrow$$

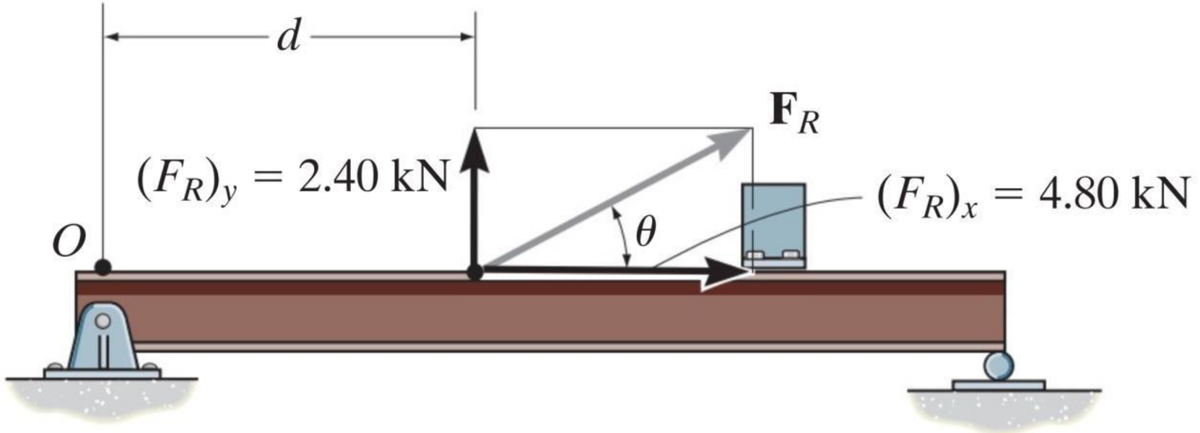
$$+\uparrow (F_R)_y = \Sigma F_y; \qquad (F_R)_y = -4\text{ kN} + 8\text{ kN}\left(\tfrac{4}{5}\right) = 2.40\text{ kN}\uparrow$$

From Fig. 4–44*b*, the magnitude of $\mathbf{F}_R$ is

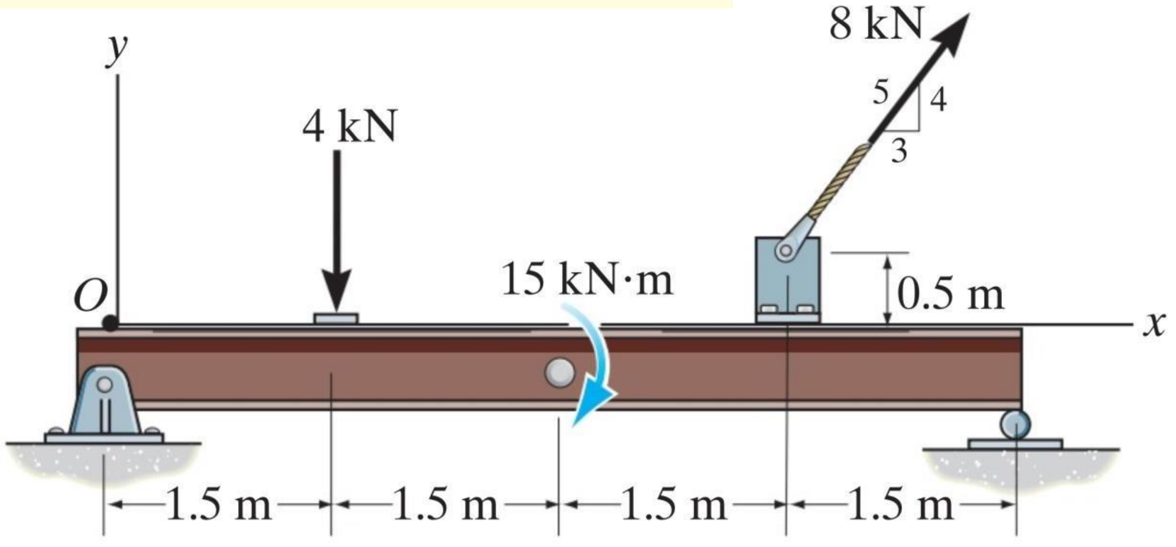
$$F_R = \sqrt{(4.80\text{ kN})^2 + (2.40\text{ kN})^2} = 5.37\text{ kN}$$

The angle θ is

$$\theta = \tan^{-1}\left(\frac{2.40\text{ kN}}{4.80\text{ kN}}\right) = 26.6^\circ$$



Replace the force and couple moment system acting on the beam in Fig. 4–44*a* by an equivalent resultant force, and find where its line of action intersects the beam, measured from point O.

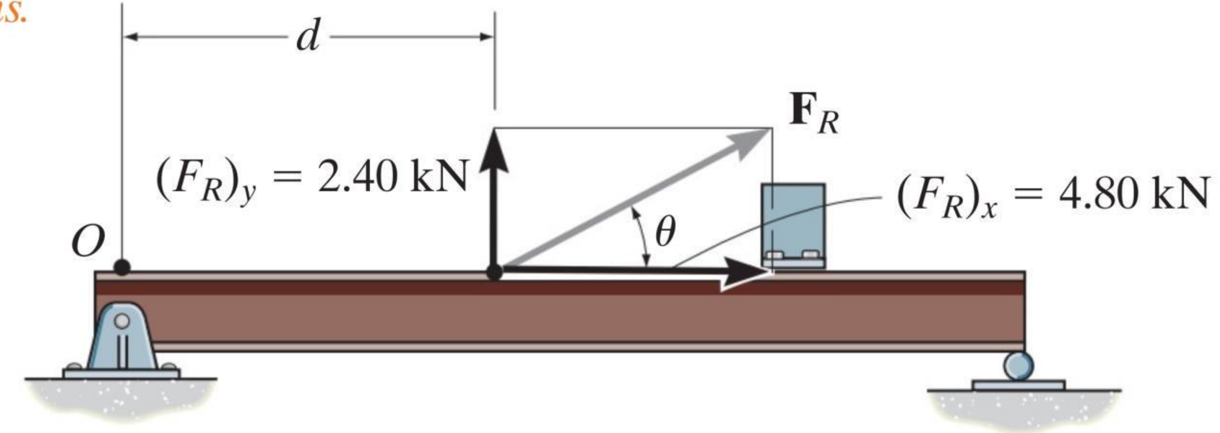


Moment Summation. We must equate the moment of $\mathbf{F}_R$ about point O in Fig. 4–44*b* to the sum of the moments of the force and couple moment system about point O in Fig. 4–44*a*. Since the line of action of $(\mathbf{F}_R)_x$ acts through point O, *only* $(\mathbf{F}_R)_y$ *produces a moment about this point.* Thus,

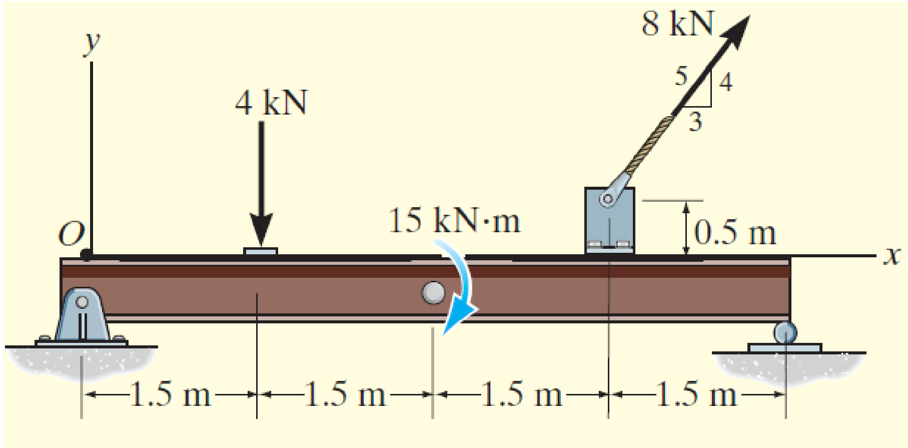
$$\circlearrowleft + (M_R)_O = \Sigma M_O; \qquad 2.40\text{ kN}(d) = -(4\text{ kN})(1.5\text{ m}) - 15\text{ kN}\cdot\text{m}$$

$$-\left[8\text{ kN}\left(\tfrac{3}{5}\right)\right](0.5\text{ m}) + \left[8\text{ kN}\left(\tfrac{4}{5}\right)\right](4.5\text{ m})$$

$$d = 2.25\text{ m}$$ *Ans.*



Force Summation.

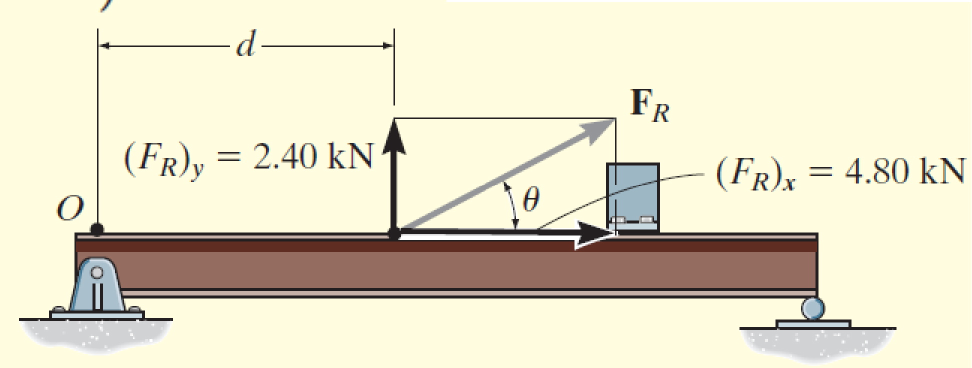


$$(F_R)_x = \Sigma F_x;$$
$$(F_R)_x = 8\text{ kN}\left(\tfrac{3}{5}\right) = 4.80\text{ kN}$$

$$(F_R)_y = \Sigma F_y;$$
$$(F_R)_y = -4\text{ kN} + 8\text{ kN}\left(\tfrac{4}{5}\right) = 2.40\text{ kN}$$

$$F_R = \sqrt{(4.80\text{ kN})^2 + (2.40\text{ kN})^2} = 5.37\text{ kN}$$

$$\theta = \tan^{-1}\left(\frac{2.40\text{ kN}}{4.80\text{ kN}}\right) = 26.6^\circ$$



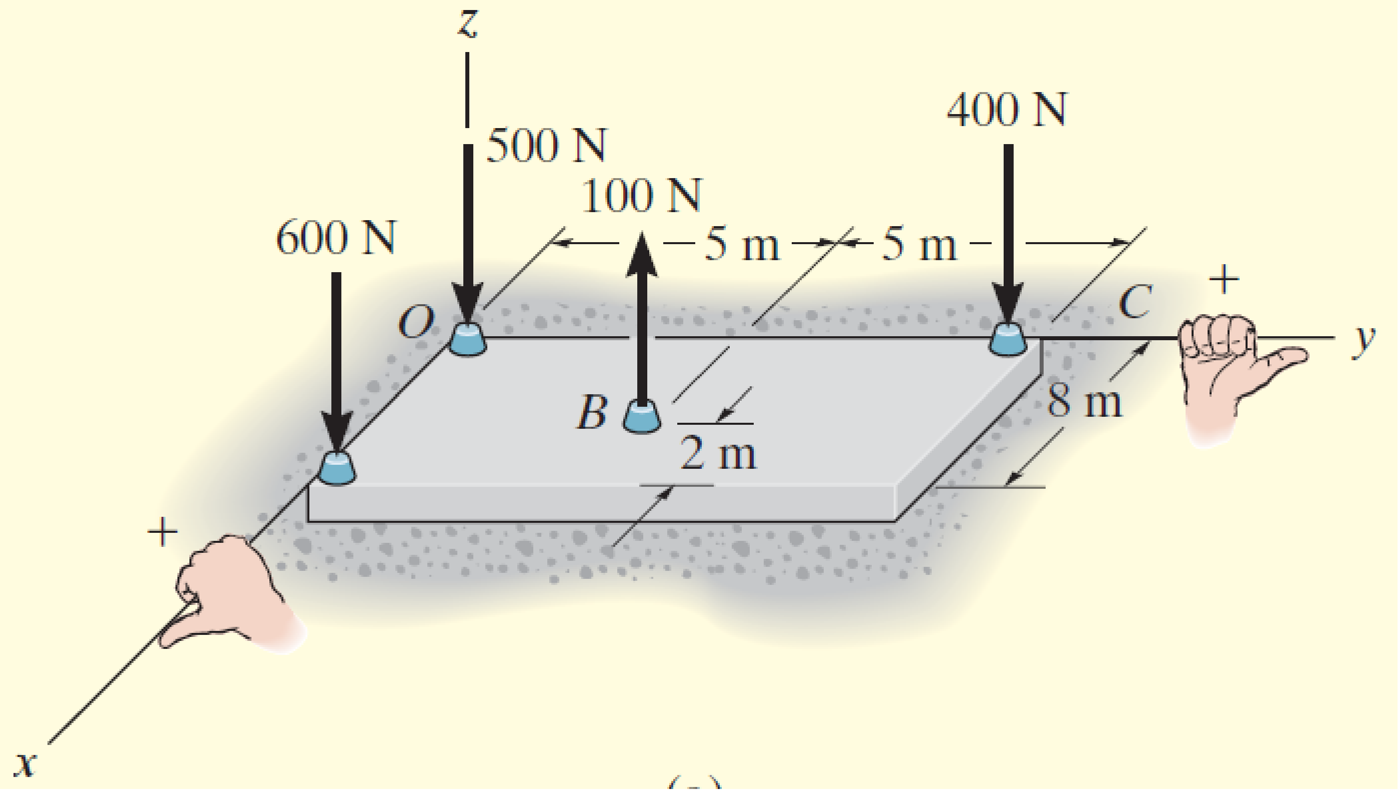
Moment Summation.

$$(M_R)_O = \Sigma M_O;$$ The moment of $\mathbf{F}_R$ about point O in Fig. 4–44*b* *equal* to the sum of the moments of the force and couple moment system about point O

$$2.40\text{ kN}(d) = -(4\text{ kN})(1.5\text{ m}) - 15\text{ kN}\cdot\text{m}$$
$$-\left[8\text{ kN}\left(\tfrac{3}{5}\right)\right](0.5\text{ m}) + \left[8\text{ kN}\left(\tfrac{4}{5}\right)\right](4.5\text{ m})$$
$$d = 2.25\text{ m}$$

Example

The slab in Fig. 4–46*a* is subjected to four parallel forces. Determine the magnitude and direction of a resultant force equivalent to the given force system and locate its point of application on the slab.



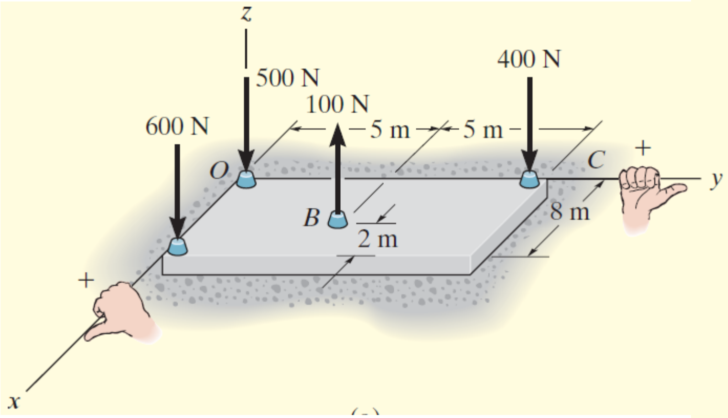
$$F_R = \Sigma F; \qquad F_R = -600\text{ N} + 100\text{ N} - 400\text{ N} - 500\text{ N}$$
$$= -1400\text{ N}$$

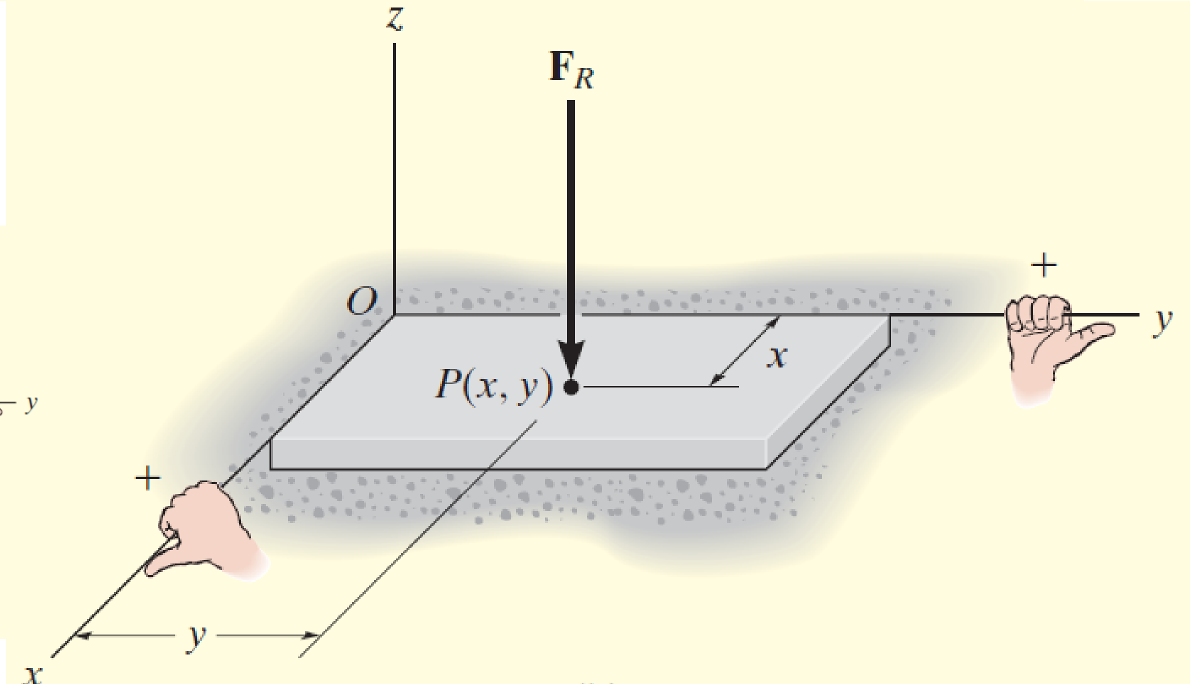
$(M_R)_x = \Sigma M_x;$ the moment about the x axis of the resultant force, Fig. 4–46b, to be equal to the sum of the moments about the x axis of all the forces in the system,

$$-(1400\text{ N})y = 600\text{ N}(0) + 100\text{ N}(5\text{ m}) - 400\text{ N}(10\text{ m}) + 500\text{ N}(0)$$
$$-1400y = -3500 \qquad y = 2.50\text{ m}$$

$(M_R)_y = \Sigma M_y;$ The moment arms are determined from the y coordinates since these coordinates represent the *perpendicular distances* from the x axis to the lines of action of the forces.

$$(1400\text{ N})x = 600\text{ N}(8\text{ m}) - 100\text{ N}(6\text{ m}) + 400\text{ N}(0) + 500\text{ N}(0)$$
$$1400x = 4200$$
$$x = 3\text{ m}$$

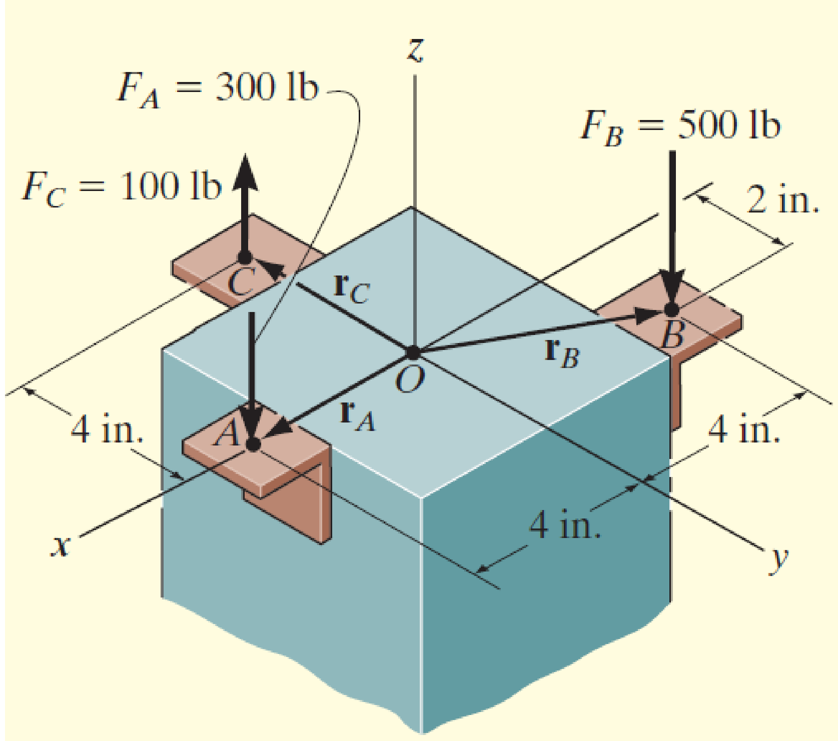




(b)

Example

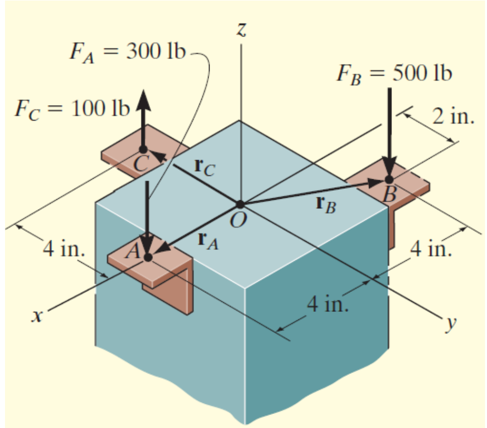
Replace the force system in Fig. 4–47*a* by an equivalent resultant force and specify its point of application on the pedestal.



Force Summation.

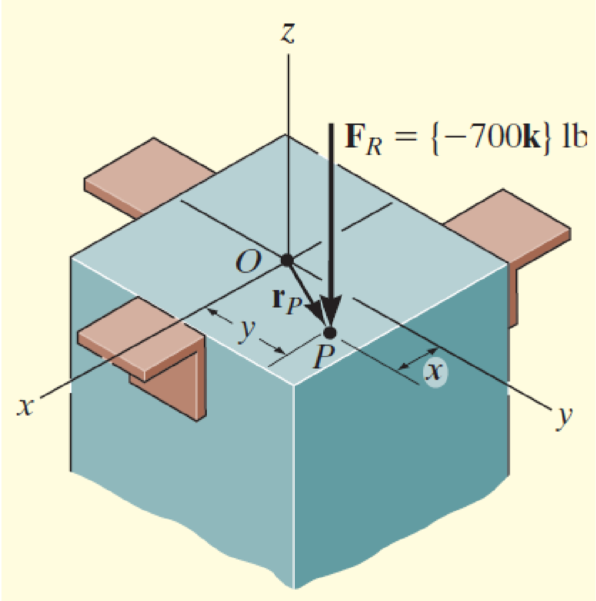
$$\mathbf{F}_R = \Sigma \mathbf{F};$$

$$\mathbf{F}_R = \mathbf{F}_A + \mathbf{F}_B + \mathbf{F}_C$$



$$= \{-300\mathbf{k}\}\text{ lb} + \{-500\mathbf{k}\}\text{ lb} + \{100\mathbf{k}\}\text{ lb}$$

$$= \{-700\mathbf{k}\}\text{ lb}$$



Location. Moments will be summed about point O. The resultant force is assumed to act through point P $(x, y, 0)$, Fig. 4–47*b*. Thus

$$(\mathbf{M}_R)_O = \Sigma \mathbf{M}_O;$$

$$\mathbf{r}_P \times \mathbf{F}_R = (\mathbf{r}_A \times \mathbf{F}_A) + (\mathbf{r}_B \times \mathbf{F}_B) + (\mathbf{r}_C \times \mathbf{F}_C)$$

$$(x\mathbf{i} + y\mathbf{j}) \times (-700\mathbf{k}) = [(4\mathbf{i}) \times (-300\mathbf{k})] + [(-4\mathbf{i} + 2\mathbf{j}) \times (-500\mathbf{k})] + [(-4\mathbf{j}) \times (100\mathbf{k})]$$

$$-700x(\mathbf{i} \times \mathbf{k}) - 700y(\mathbf{j} \times \mathbf{k}) = -1200(\mathbf{i} \times \mathbf{k}) + 2000(\mathbf{i} \times \mathbf{k}) - 1000(\mathbf{j} \times \mathbf{k}) - 400(\mathbf{j} \times \mathbf{k})$$

$$700x\mathbf{j} - 700y\mathbf{i} = 1200\mathbf{j} - 2000\mathbf{j} - 1000\mathbf{i} - 400\mathbf{i}$$

$$-700y = -1400$$
$$y = 2\text{ in.}$$
$$700x = -800$$
$$x = -1.14\text{ in.}$$

The - sign indicates that the x coordinate of point P is negative.

Home Assignment

- Example 4.18.