Engineering Mechanics

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CHAPTER 4
Force System Resultants

Contents (Section 4.4 and 4.5)

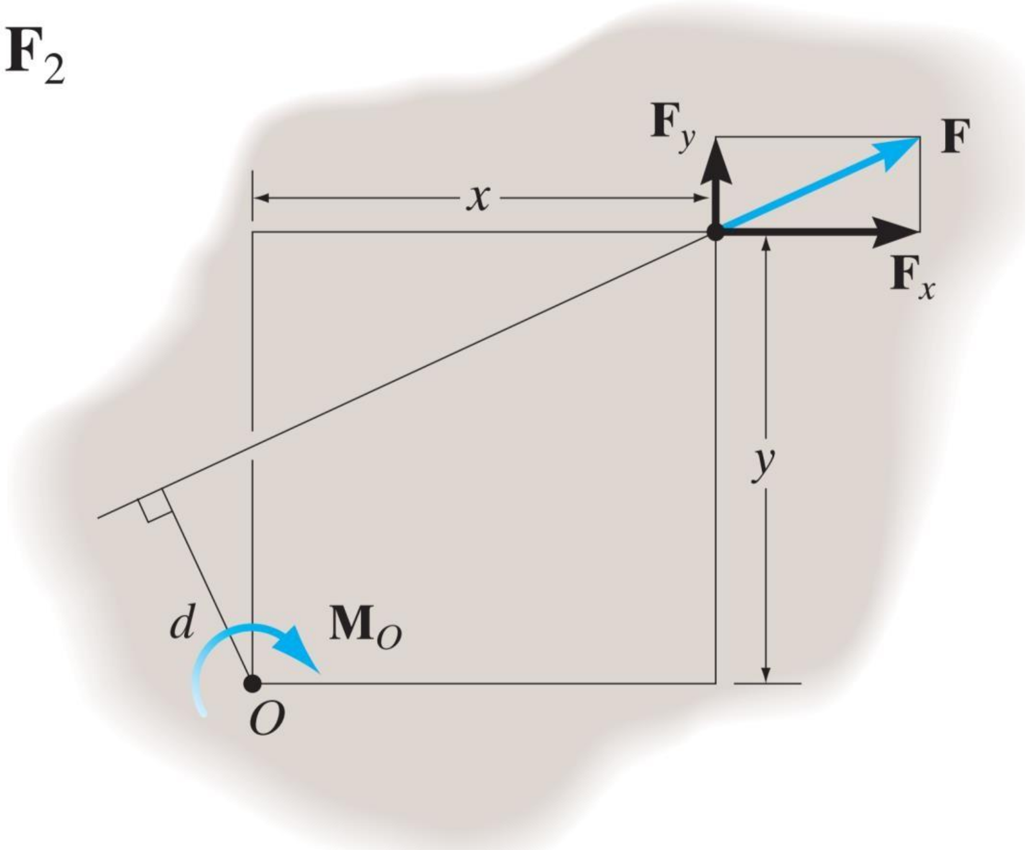
- Principle of Moment
- Moment of a Force about a Specified Axis

4.4 Principle of Moments (Varignon's Theorem)

The moment of a force about a point is equal to the sum of the moments of the components of the force about the point

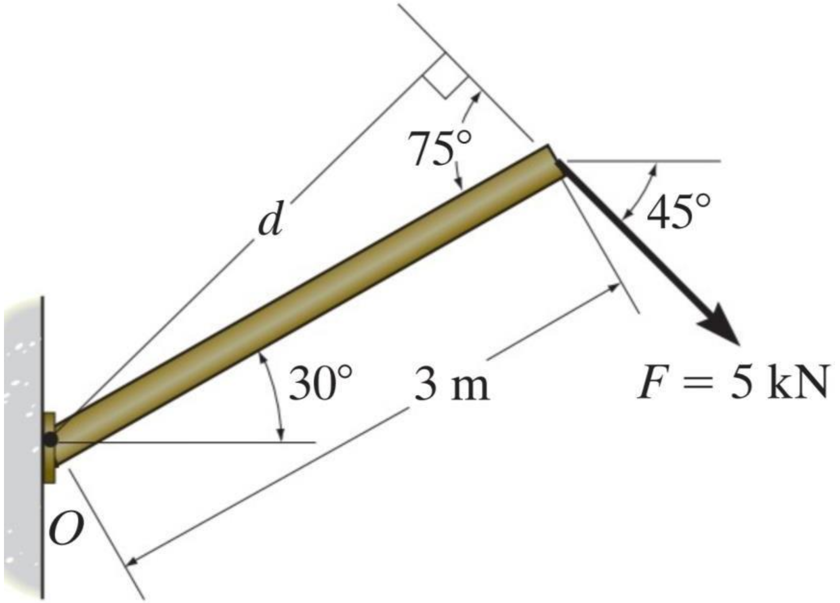
$$\mathbf{M}_O = \mathbf{r} \times \mathbf{F} = \mathbf{r} \times (\mathbf{F}_1 + \mathbf{F}_2) = \mathbf{r} \times \mathbf{F}_1 + \mathbf{r} \times \mathbf{F}_2$$

$$M_O = F_x y - F_y x$$

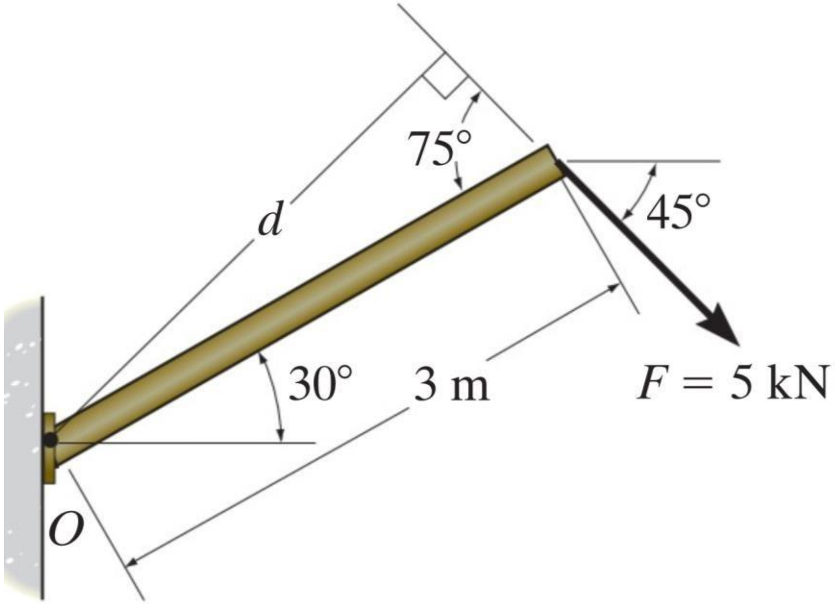


Example

Determine the moment of the force in Fig. 4–18*a* about point O.



Determine the moment of the force in Fig. 4–18*a* about point *O*.



SOLUTION I

The moment arm *d* in Fig. 4–18*a* can be found from trigonometry.

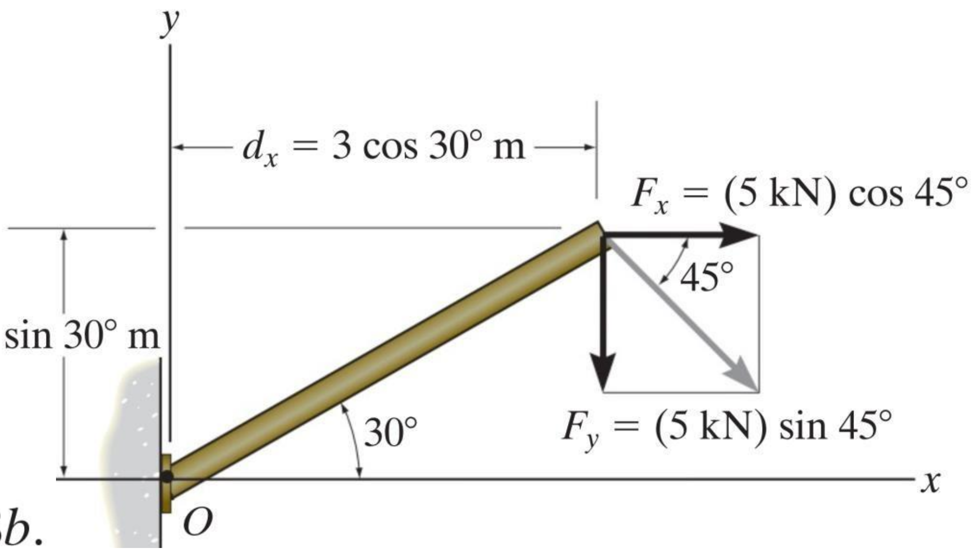
$$d = (3\text{ m})\sin 75^\circ = 2.898\text{ m}$$

Thus,

$$M_O = Fd = (5\text{ kN})(2.898\text{ m}) = 14.5\text{ kN}\cdot\text{m} \curvearrowright \qquad \textit{Ans.}$$

Since the force tends to rotate or orbit clockwise about point *O*, the moment is directed into the page.

Determine the moment of the force in Fig. 4–18*a* about point *O*.



SOLUTION II

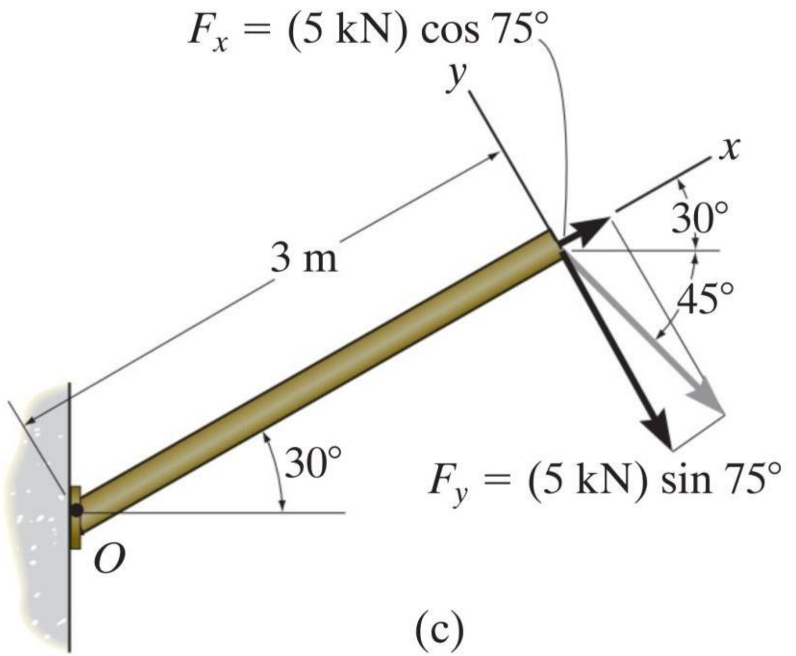
The x and y components of the force are indicated in Fig. 4–18*b*. Considering counterclockwise moments as positive, and applying the principle of moments, we have

$$\circlearrowleft + M_O = -F_x d_y - F_y d_x$$

$$= -(5 \cos 45° \text{ kN})(3 \sin 30° \text{ m}) - (5 \sin 45° \text{ kN})(3 \cos 30° \text{ m})$$

$$= -14.5 \text{ kN} \cdot \text{m} = 14.5 \text{ kN} \cdot \text{m} \circlearrowright \qquad \textit{Ans.}$$

Determine the moment of the force in Fig. 4–18*a* about point O.



(c)

SOLUTION III

The x and y axes can be set parallel and perpendicular to the rod's axis as shown in Fig. 4–18*c*. Here $\mathbf{F}_x$ produces no moment about point O since its line of action passes through this point. Therefore,

$$\circlearrowleft + M_O = -F_y d_x$$

$$= -(5 \sin 75° \text{ kN})(3 \text{ m})$$

$$= -14.5 \text{ kN} \cdot \text{m} = 14.5 \text{ kN} \cdot \text{m} \circlearrowright \qquad \textit{Ans.}$$

SOLUTION I

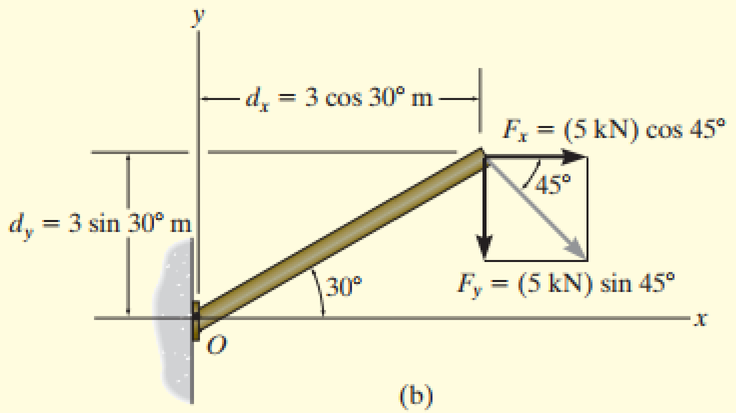
The moment arm d in Fig. 4–18*a* can be found from trigonometry.

$$d = (3 \text{ m}) \sin 75° = 2.898 \text{ m}$$

Thus,

$$M_O = Fd = (5\text{kN})(2.898 \text{ m}) = 14.5 \text{ kN} \cdot \text{m} \circlearrowright \qquad \textit{Ans.}$$

Since the force tends to rotate or orbit clockwise about point O, the moment is directed into the page.



(b)

SOLUTION II

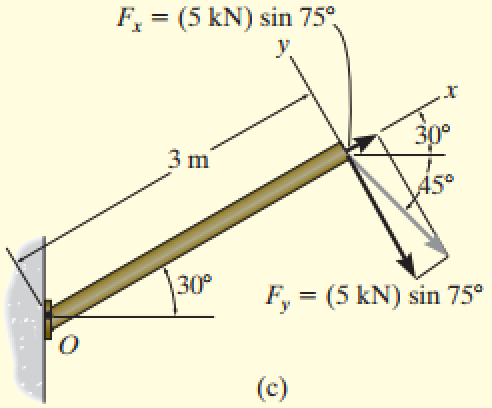
The x and y components of the force are indicated in Fig. 4–18*b*. Considering counterclockwise moments as positive, and applying the principle of moments, we have

$$\begin{aligned}
\circlearrowleft + M_O &= -F_x d_y - F_y d_x \\
&= -(5 \cos 45° \text{ kN})(3 \sin 30° \text{ m}) - (5 \sin 45° \text{ kN})(3 \cos 30° \text{ m}) \\
&= -14.5 \text{ kN} \cdot \text{m} = 14.5 \text{ kN} \cdot \text{m} \circlearrowright \qquad \textit{Ans.}
\end{aligned}$$

SOLUTION III

The x and y axes can be set parallel and perpendicular to the rod's axis as shown in Fig. 4-18*c*. Here $\mathbf{F}_x$ produces no moment about point O since its line of action passes through this point. Therefore,

$$\begin{aligned}
\circlearrowleft + M_O &= -F_y d_x \\
&= -(5 \sin 75° \text{ kN})(3 \text{ m}) \\
&= -14.5 \text{ kN} \cdot \text{m} = 14.5 \text{ kN} \cdot \text{m} \circlearrowright \qquad \textit{Ans.}
\end{aligned}$$

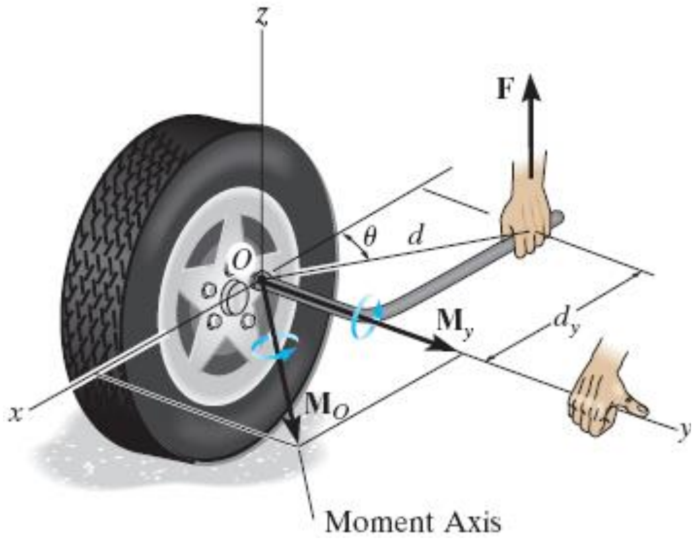


(c)

Fig. 4–18

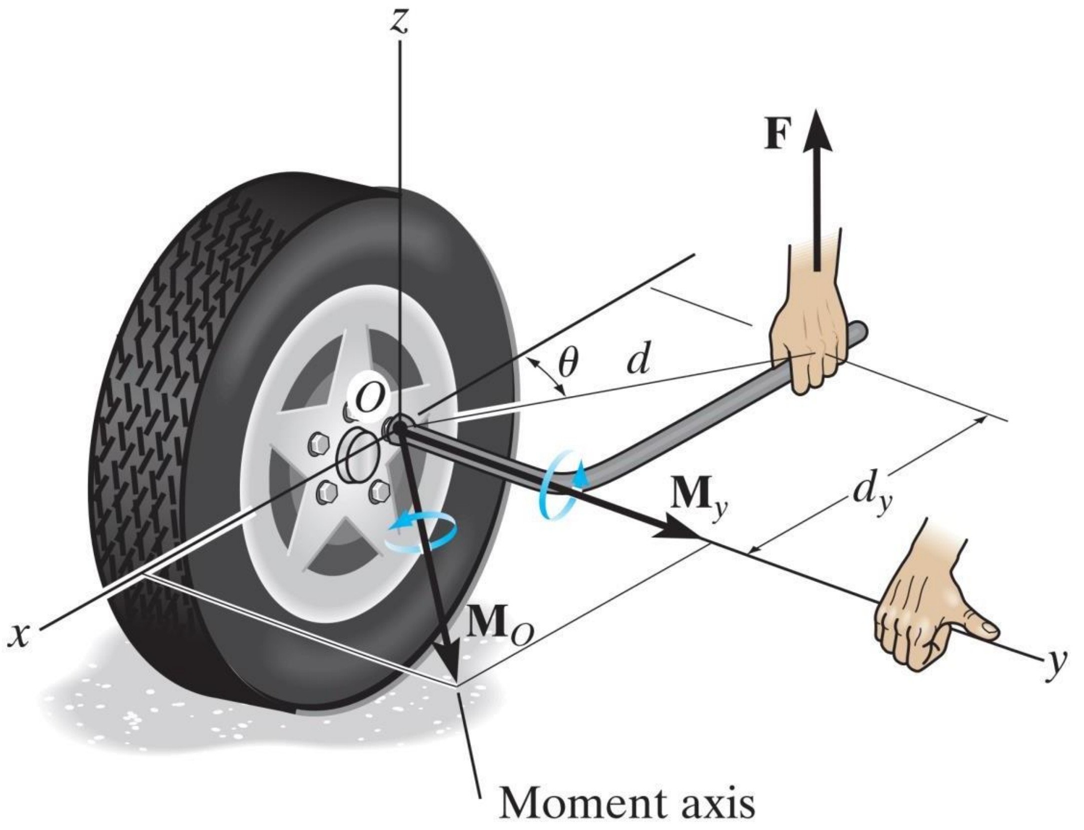
4.5 Moment of a Force about a Specified Axis

- The force applied to the wrench will create a tendency for the wrench and the nut to rotate about the *moment axis* passing through *O*; however,
- The nut can only rotate about the *y* axis. Therefore, to determine the turning effect, only the *y* component of the moment is needed, and the total moment produced is not important.
- A **scalar or vector analysis** is used to find the component of the moment along a specified axis that passes through the point



Moment of a Force about a Specified Axis

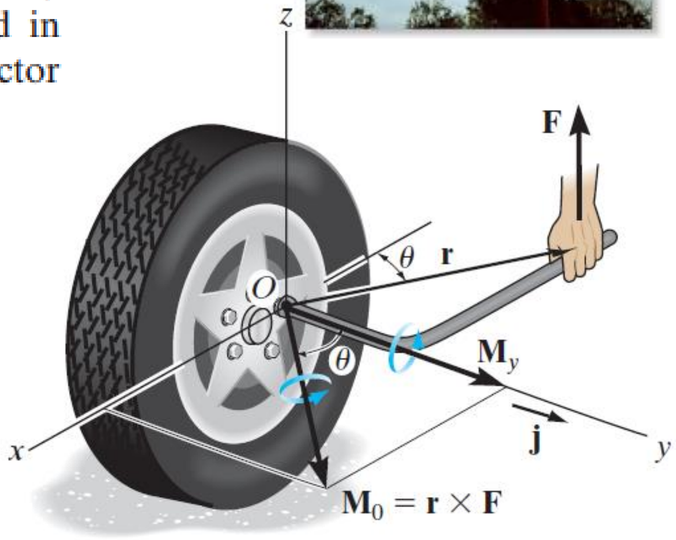
Scalar Analysis.



Vector Analysis. To find the moment of force **F** in Fig. 4–20*b* about the *y* axis using a vector analysis, we must first determine the moment of the force about *any point O* on the *y* axis by applying Eq. 4–7,

$$\mathbf{M}_O = \mathbf{r} \times \mathbf{F}$$

$\mathbf{M}_O = \mathbf{r} \times \mathbf{F}$. The component $\mathbf{M}_y$ along the *y* axis is the *projection* of $\mathbf{M}_O$ onto the *y* axis. It can be found using the *dot product* discussed in Chapter 2, so that $M_y = \mathbf{j} \cdot \mathbf{M}_O = \mathbf{j} \cdot (\mathbf{r} \times \mathbf{F})$, where **j** is the unit vector for the *y* axis.



$$M_y = \mathbf{j} \cdot \mathbf{M}_O$$
$$= \mathbf{j} \cdot (\mathbf{r} \times \mathbf{F})$$

We can generalize this approach by letting $\mathbf{u}_a$ be the unit vector that specifies the direction of the *a* axis shown in Fig. 4–21. Then the moment of **F** about the axis is $M_a = \mathbf{u}_a \cdot (\mathbf{r} \times \mathbf{F})$. This combination is referred to as the *scalar triple product*. If the vectors are written in Cartesian form, we have

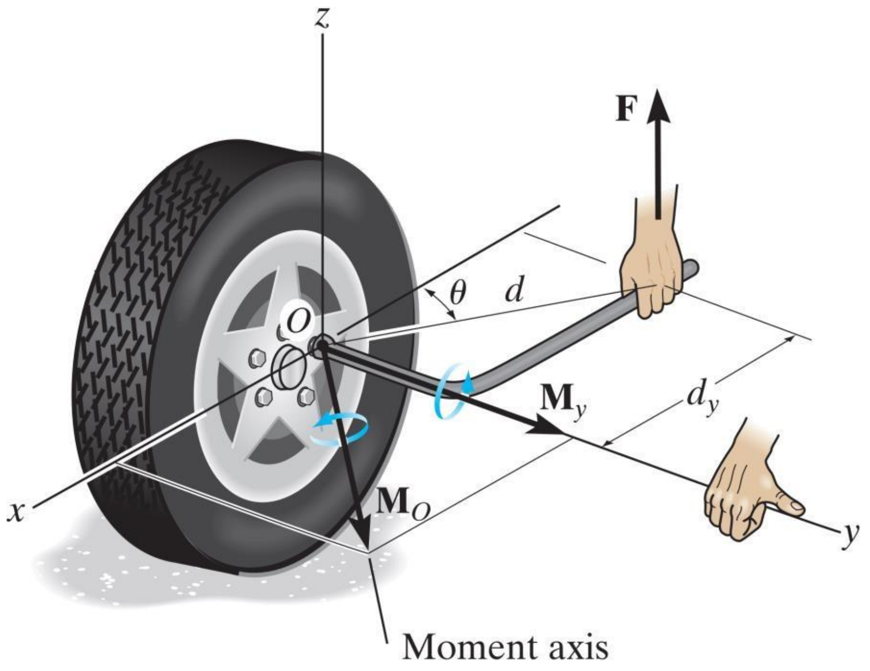
$$M_a = \mathbf{u}_a \cdot (\mathbf{r} \times \mathbf{F})$$

$$M_a = [u_{a_x}\mathbf{i} + u_{a_y}\mathbf{j} + u_{a_z}\mathbf{k}] \cdot \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ r_x & r_y & r_z \\ F_x & F_y & F_z \end{vmatrix}$$

$$= u_{a_x}(r_yF_z - r_zF_y) - u_{a_y}(r_xF_z - r_zF_x) + u_{a_z}(r_xF_y - r_yF_x)$$

Moment of a Force about a Specified Axis

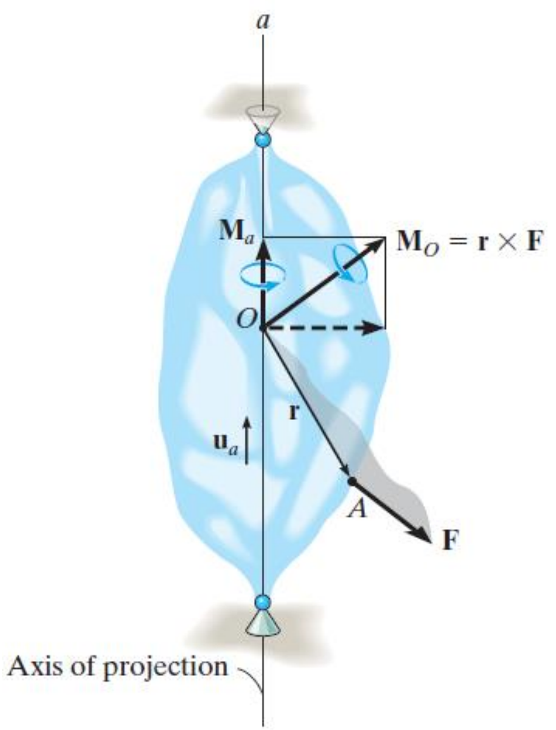
Vector Analysis.



$$M_a = [u_{a_x}\mathbf{i} + u_{a_y}\mathbf{j} + u_{a_z}\mathbf{k}] \cdot \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ r_x & r_y & r_z \\ F_x & F_y & F_z \end{vmatrix}$$

$$= u_{a_x}(r_yF_z - r_zF_y) - u_{a_y}(r_xF_z - r_zF_x) + u_{a_z}(r_xF_y - r_yF_x)$$

$$M_a = \mathbf{u}_a \cdot (\mathbf{r} \times \mathbf{F}) = \begin{vmatrix} u_{a_x} & u_{a_y} & u_{a_z} \\ r_x & r_y & r_z \\ F_x & F_y & F_z \end{vmatrix}$$



$$\mathbf{M}_a = M_a \mathbf{u}_a$$

$u_{a_x}, u_{a_y}, u_{a_z}$ represent the x, y, z components of the unit vector defining the direction of the a axis

r_x, r_y, r_z represent the x, y, z components of the position vector extended from *any point O* on the *a* axis to *any point A* on the line of action of the force

F_x, F_y, F_z represent the x, y, z components of the force vector.

When M_a is evaluated from Eq. 4–11, it will yield a positive or negative scalar. The sign of this scalar indicates the sense of direction of $\mathbf{M}_a$ along the a axis. If it is positive, then $\mathbf{M}_a$ will have the same sense as $\mathbf{u}_a$, whereas if it is negative, then $\mathbf{M}_a$ will act opposite to $\mathbf{u}_a$.

Once M_a is determined, we can then express $\mathbf{M}_a$ as a Cartesian vector, namely,

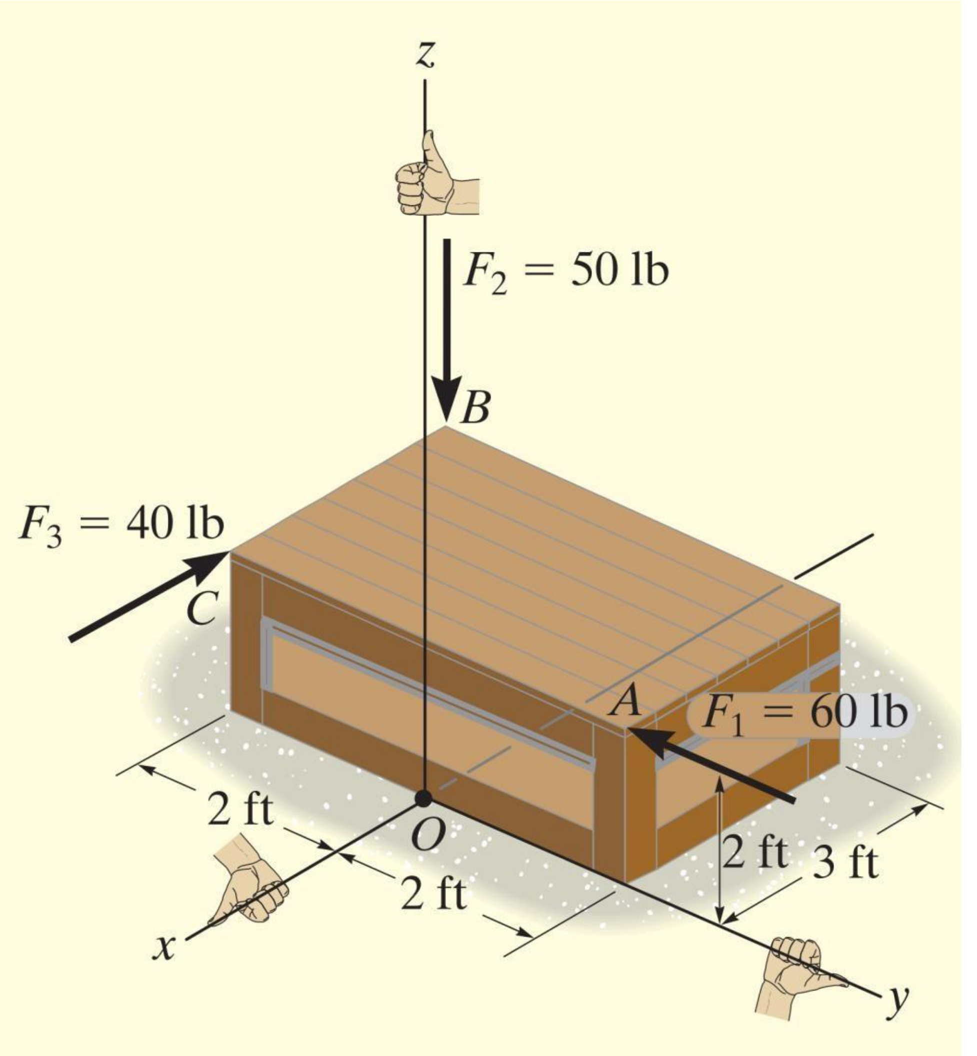
$$\mathbf{M}_a = M_a \mathbf{u}_a \tag{4–12}$$

Important Points

- The moment of a force about a specified axis can be determined provided the perpendicular distance d_a from the force line of action to the axis can be determined. $M_a = F d_a$.
- If vector analysis is used, $M_a = \mathbf{u}_a \cdot (\mathbf{r} \times \mathbf{F})$, where $\mathbf{u}_a$ defines the direction of the axis and $\mathbf{r}$ is extended from *any point* on the axis to *any point* on the line of action of the force.
- If M_a is calculated as a negative scalar, then the sense of direction of $\mathbf{M}_a$ is opposite to $\mathbf{u}_a$.
- The moment $\mathbf{M}_a$ expressed as a Cartesian vector is determined from $\mathbf{M}_a = M_a \mathbf{u}_a$.

Example

Determine the resultant moment of the three forces about the *x* axis, the *y* axis, and the *z* axis.

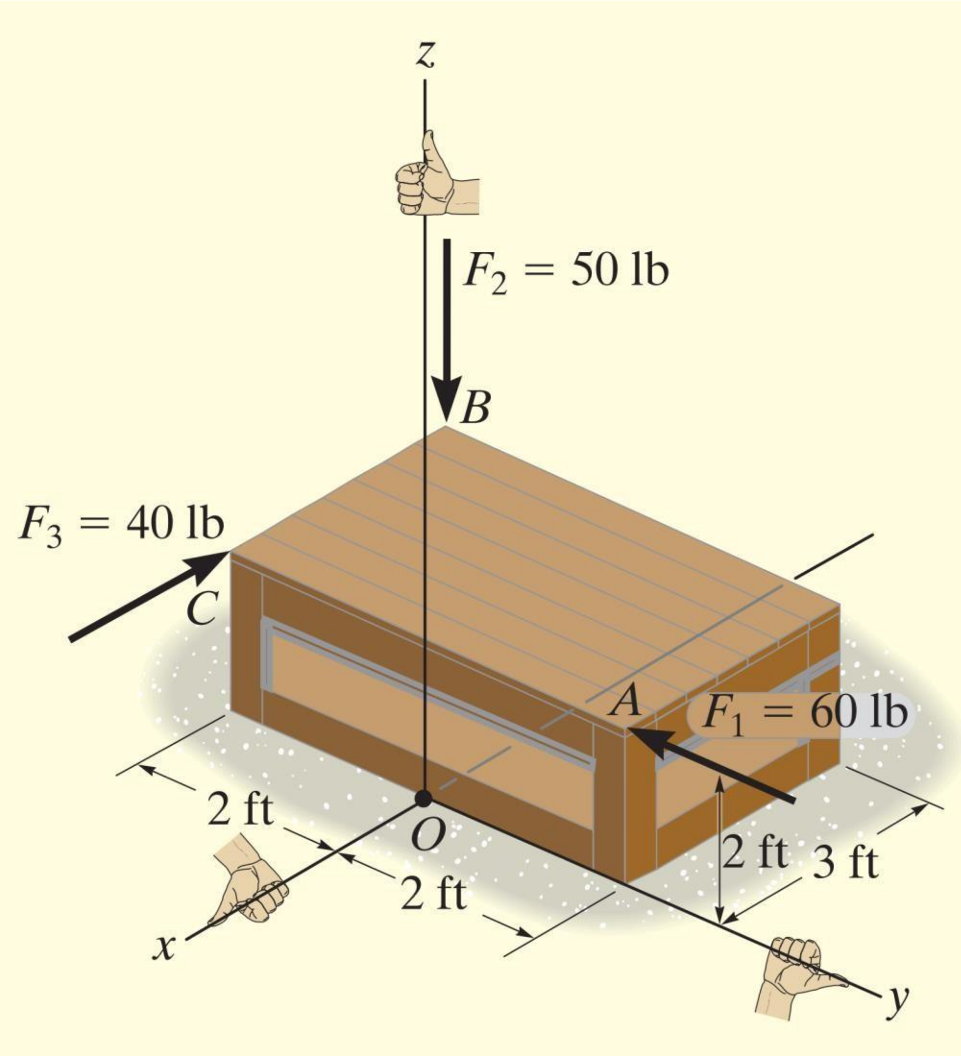


Determine the resultant moment of the three forces about the *x* axis, the *y* axis, and the *z* axis.

$$M_x = (60\text{ lb})(2\text{ ft}) + (50\text{ lb})(2\text{ ft}) + 0 = 220\text{ lb}\cdot\text{ft}$$

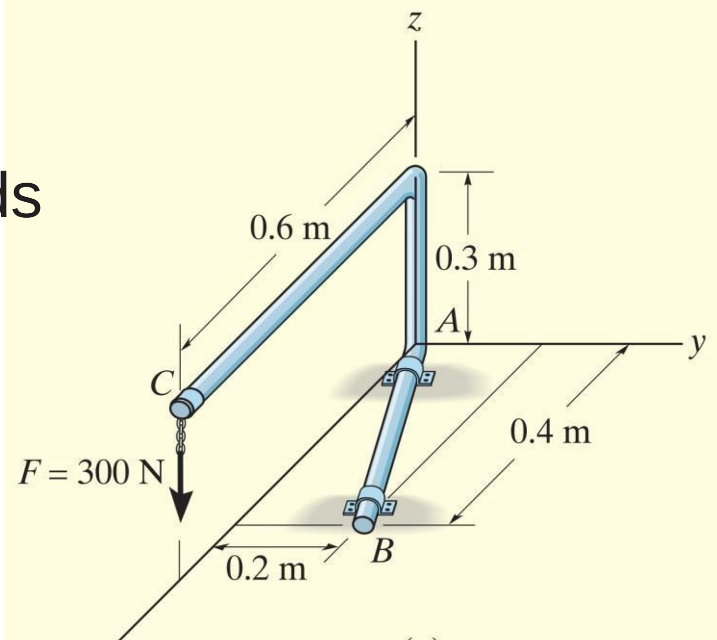
$$M_y = 0 - (50\text{ lb})(3\text{ ft}) - (40\text{ lb})(2\text{ ft}) = -230\text{ lb}\cdot\text{ft}$$

$$M_z = 0 + 0 - (40\text{ lb})(2\text{ ft}) = -80\text{ lb}\cdot\text{ft}$$



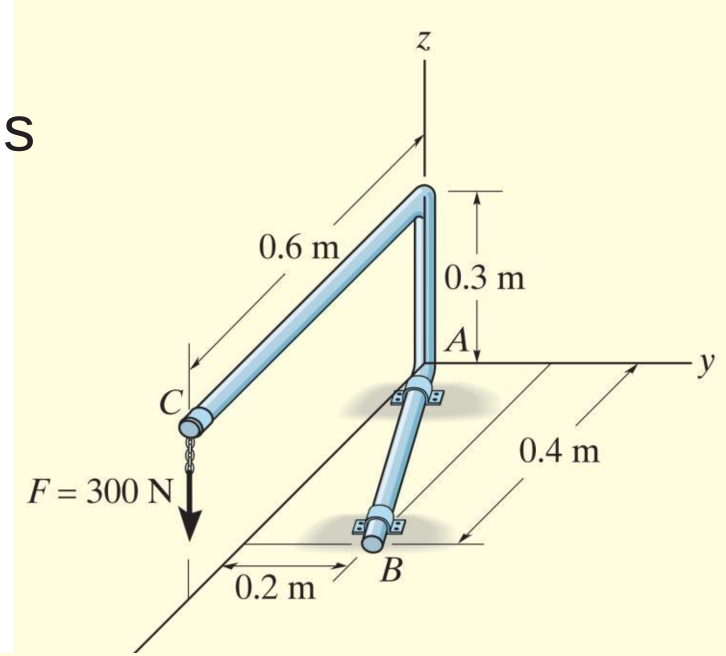
Example

Determine the moment $\mathbf{M}_{AB}$ produced by the force $\mathbf{F}$ which tends to rotate the rod about the *AB* axis.



(a)

Determine the moment $\mathbf{M}_{AB}$ produced by the force $\mathbf{F}$ which tends to rotate the rod about the *AB* axis.



$$\mathbf{u}_B = \frac{\mathbf{r}_B}{r_B} = \frac{\{0.4\mathbf{i} + 0.2\mathbf{j}\}\ \text{m}}{\sqrt{(0.4\ \text{m})^2 + (0.2\ \text{m})^2}} = 0.8944\mathbf{i} + 0.4472\mathbf{j}$$

$$\mathbf{r}_D = \{0.6\mathbf{i}\}\ \text{m}$$

$$\mathbf{F} = \{-300\mathbf{k}\}\ \text{N}$$

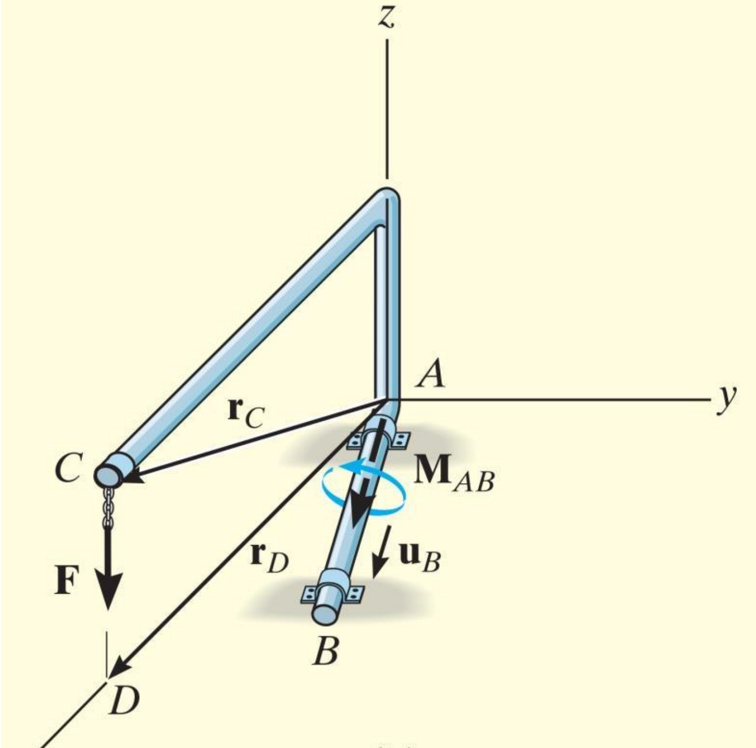
$$M_{AB} = \mathbf{u}_B \cdot (\mathbf{r}_D \times \mathbf{F}) = \begin{vmatrix} 0.8944 & 0.4472 & 0 \\ 0.6 & 0 & 0 \\ 0 & 0 & -300 \end{vmatrix}$$

$$= 0.8944[0(-300) - 0(0)] - 0.4472[0.6(-300) - 0(0)] + 0[0.6(0) - 0(0)]$$

$$= 80.50\ \text{N}\cdot\text{m}$$

$$\mathbf{M}_{AB} = M_{AB}\mathbf{u}_B = (80.50\ \text{N}\cdot\text{m})(0.8944\mathbf{i} + 0.4472\mathbf{j})$$

$$= \{72.0\mathbf{i} + 36.0\mathbf{j}\}\ \text{N}\cdot\text{m}$$



(b)

Home Assignment

- Example 4.9, F 4-13,4-16.