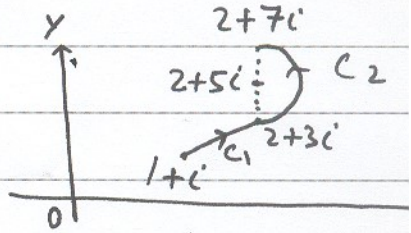
NUST School of Electrical Engineering & Computer Science

Complex Variables & Transforms - MATH 232 - Problem Sheet. NO. 3

Q-1. Evaluate $\int_C (2z+1)\,dz$, C is C_1 followed by C_2 as:



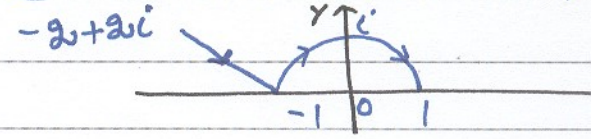
C_1: Line segment between $1+i$ & $2+3i$

C_2: Semicircle centered at $2+5i$

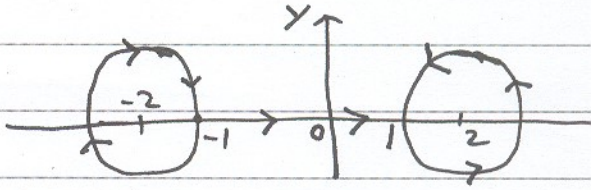
i) by Parametrization ii) using the fact $\frac{d}{dz}(z^2+z) = 2z+1$ $\quad (-44+32i)$

Q-2. a) Parametrize the circle $|z-2i| = 4$ traversed once in the clockwise direction strating from the point $z = 4+2i$ $\quad (C: z(t) = 4e^{-it} + 2i,\ 0 \le t < 2\pi)$

b) Parametrize the Contour indicated in the figure.



c) Parametrize the bar bell-shaped Contour shown in figure. It has inital point -1 & terminal point 1.

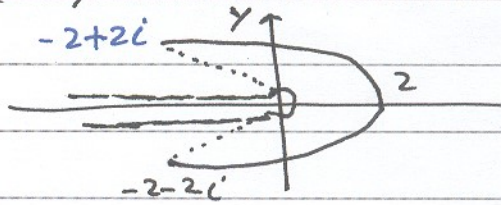


Q-3 Discuss the motion of a fluid having complex potential $\Omega(z) = ik\ln z$, $k > 0$.

Q-4. Let C be a parametrized curve as $C: z(t) = (2-t^2) + ti$, $-2 \le t \le 2$.

Evaluate the following integrals:

$$\int_C z^{1/2}\,dz, \quad \int_C (\log z)\,dz, \quad \int_C \frac{dz}{1+z^2}$$



$$\int_C z^{1/2}\,dz = \frac{z^{3/2}}{3/2}\Big|_{-2-2i}^{-2+2i} = -\frac{16}{3}i\,(2)^{1/4}\sin(\pi/8)$$

$$\int_C \log z\,dz = (z\log z - z)\Big|_{-2-2i}^{-2+2i} = 2i\log 8 - 6\pi - 4i$$

$$\int_C \frac{dz}{1+z^2} = \frac{1}{2i}\int_C \left[\frac{1}{z-i} - \frac{1}{z+i}\right] dz = \frac{1}{2i}\left(\log(z-i) - \log(z+i)\right)\Big|_{-2-2i}^{-2+2i}$$

$$\int_C \log z\,dz = \frac{i}{2}\log\left(\frac{13}{5}\right).$$