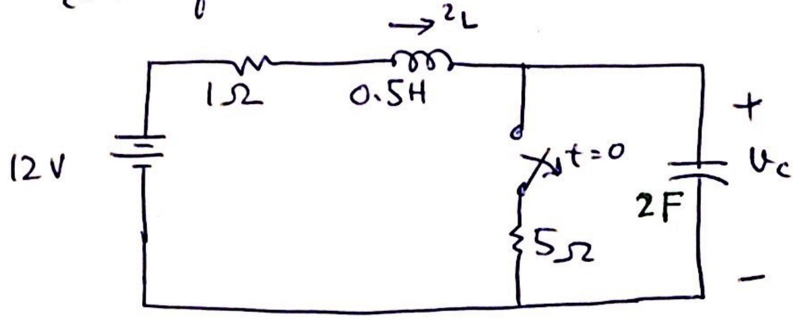
✓ (Conceptual)
(Yes)

Voltage Driven Series

Prob 9.61 The Complete Response of the RLC Circuit

(Like 9.56 8th) (PP 367 7th Ed HKD)

Find $v_c(t)$ for $t > 0$.



Solution: For a series RLC circuit :- (Turning off the independent source)

$$\alpha = \frac{R}{2L} = \frac{1}{1} = 1$$

and $$w_o^2 = \frac{1}{LC} = 1 \quad \text{so } w_o = 1$$

$$\alpha = w_o \quad \text{so critically damped.}$$

Now $$V_{c,f} = 12 \text{ V}$$

and $$v_c(0^-) = \left(\frac{5}{5+1}\right) \times 12 = 10 \text{ V} = v_c(0^+)$$

also $$i_L(0^-) = \frac{12}{6} = 2 \text{ A} = i_L(0^+)$$

We know $$f_n(t) = e^{-\alpha t}(A_1 t + A_2)$$

$$f_f(t) = 12$$

So $$v_c(t) = 12 + e^{-\alpha t}(A_1 t + A_2)$$

$$v_c(0) = 12 + e^{-1 \times 0}(A_1 \times 0 + A_2) = 10$$

Therefore $A_2 = -2$

Hence $$v_c(t) = 12 + e^{-t}(A_1 t - 2)$$

———— contd

(Conceptual)
(Yes)

— contd (367)

Now $i_C = C\dfrac{dv_c}{dt}$

$$\left.\frac{dv_c(0^+)}{dt}\right|_{t=0^+} = \frac{1}{C} i_C(0^+) = \frac{1}{2} i_C(0^+)$$

As $i_L(0^-) = i_L(0^+) = 2 = i_C(0^+)$

So $\frac{1}{2} i_C(0^+) = \frac{1}{2} \times 2 = 1$

— And $\dfrac{dv_c}{dt} = e^{-t} \times A_1 + A_1 t \times -e^{-t} - 2(-)e^{-t}$

$$= A_1 e^{-t} - A_1 t e^{-t} + 2e^{-t}$$

$$\left.\frac{dv_c(0)}{dt}\right|_{t=0} = A_1 + 2$$

Finally $A_1 + 2 = 1$

$$A_1 = -1$$

Therefore $v_c(t) = 12 + e^{-t}(-1t - 2)$

or $v_c(t) = 12 - e^{-t}(t+2)$, V $\quad t > 0$

(Note: i_C can be calculated if required)
or i_L
which is $= C\dfrac{dv_c}{dt}$

Notice no steady-state (forced) current due to presence of capacitor.