Engineering Mechanics

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Contents (Section 2.9)

- Recap
- Dot Product
- Examples
- Review

RECAP

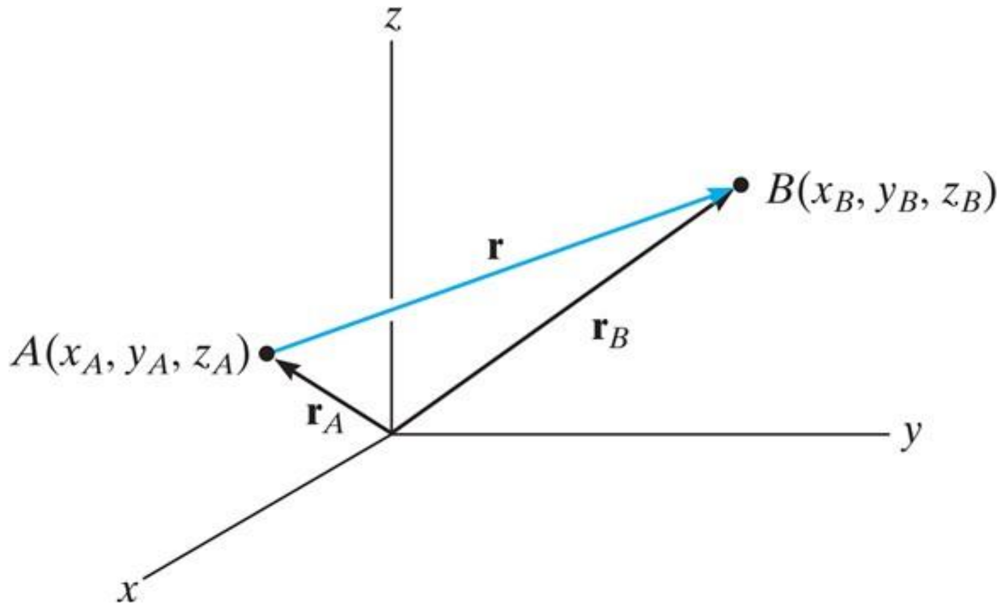
Cartesian Vectors

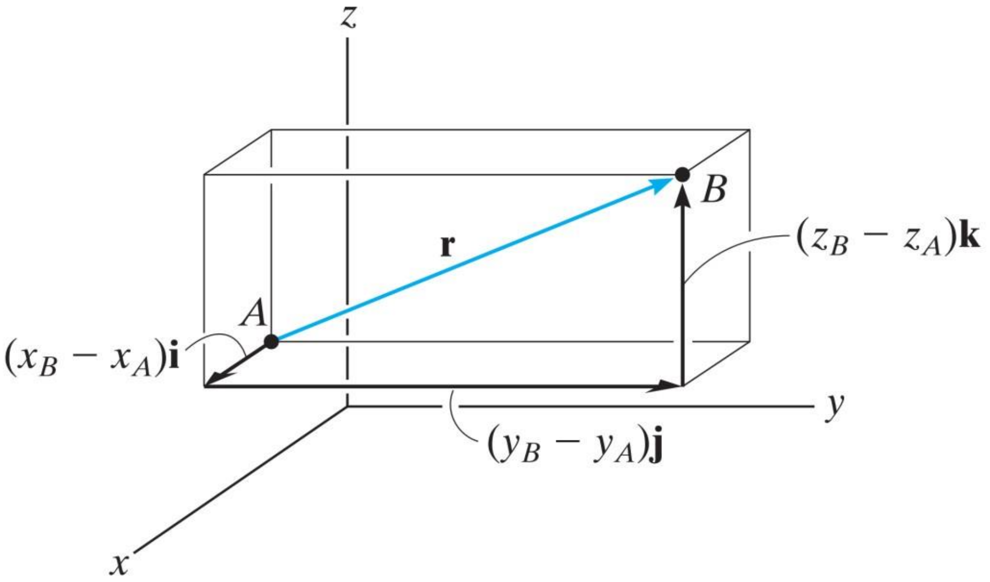
Position Vector (General Case)

$$\mathbf{r}_A + \mathbf{r} = \mathbf{r}_B$$

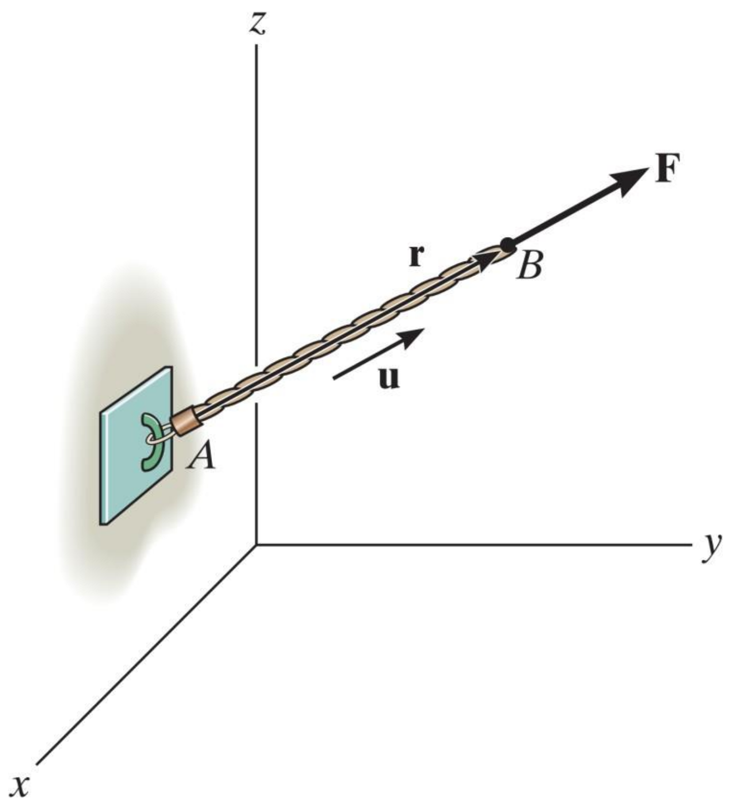
$$\mathbf{r} = \mathbf{r}_B - \mathbf{r}_A = (x_B\mathbf{i} + y_B\mathbf{j} + z_B\mathbf{k}) - (x_A\mathbf{i} + y_A\mathbf{j} + z_A\mathbf{k})$$

$$\boxed{\mathbf{r} = (x_B - x_A)\mathbf{i} + (y_B - y_A)\mathbf{j} + (z_B - z_A)\mathbf{k}}$$



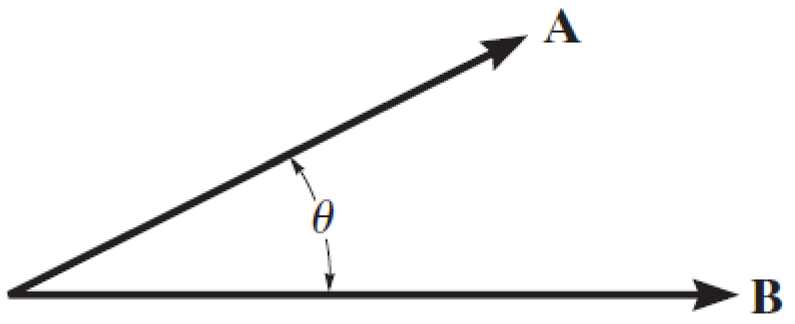


Force Vector Directed Along A Line



$$\mathbf{F} = F\,\mathbf{u} = F\left(\frac{\mathbf{r}}{r}\right) = F\left(\frac{(x_B - x_A)\mathbf{i} + (y_B - y_A)\mathbf{j} + (z_B - z_A)\mathbf{k}}{\sqrt{(x_B - x_A)^2 + (y_B - y_A)^2 + (z_B - z_A)^2}}\right)$$

Dot Product



The *dot product* of vectors **A** and **B**, written **A · B**, and read "**A** dot **B**" is defined as the product of the magnitudes of **A** and **B** and the cosine of the angle θ between their tails, Fig. 2–41. Expressed in equation form,

$$\boxed{\mathbf{A}\cdot\mathbf{B} = AB\cos\theta}$$

The dot product is often referred to as the *scalar product* of vectors since the result is a *scalar* and not a vector.

$$\mathbf{A}\cdot\mathbf{B} = (A_x\mathbf{i} + A_y\mathbf{j} + A_z\mathbf{k})\cdot(B_x\mathbf{i} + B_y\mathbf{j} + B_z\mathbf{k})$$

Carrying out the dot-product operations, the final result becomes

$$\boxed{\mathbf{A}\cdot\mathbf{B} = A_xB_x + A_yB_y + A_zB_z}$$

$$\mathbf{i}\cdot\mathbf{i} = (1)(1)\cos 0^\circ = 1$$

$$\mathbf{i}\cdot\mathbf{j} = (1)(1)\cos 90^\circ = 0$$

Laws of Operation.

1. Commutative law: $\mathbf{A}\cdot\mathbf{B} = \mathbf{B}\cdot\mathbf{A}$
2. Multiplication by a scalar: $a(\mathbf{A}\cdot\mathbf{B}) = (a\mathbf{A})\cdot\mathbf{B} = \mathbf{A}\cdot(a\mathbf{B})$
3. Distributive law: $\mathbf{A}\cdot(\mathbf{B}+\mathbf{D}) = (\mathbf{A}\cdot\mathbf{B}) + (\mathbf{A}\cdot\mathbf{D})$

Dot Product

$$\boxed{\mathbf{A} \cdot \mathbf{B} = AB \cos \theta}$$

1. Commutative law: $\mathbf{A} \cdot \mathbf{B} = \mathbf{B} \cdot \mathbf{A}$
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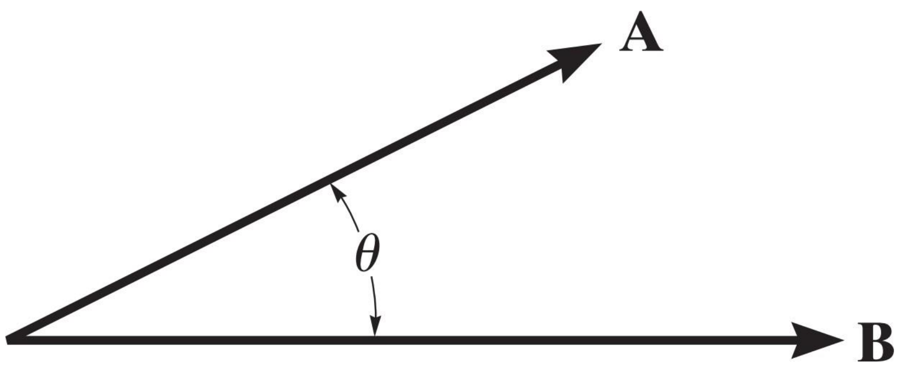
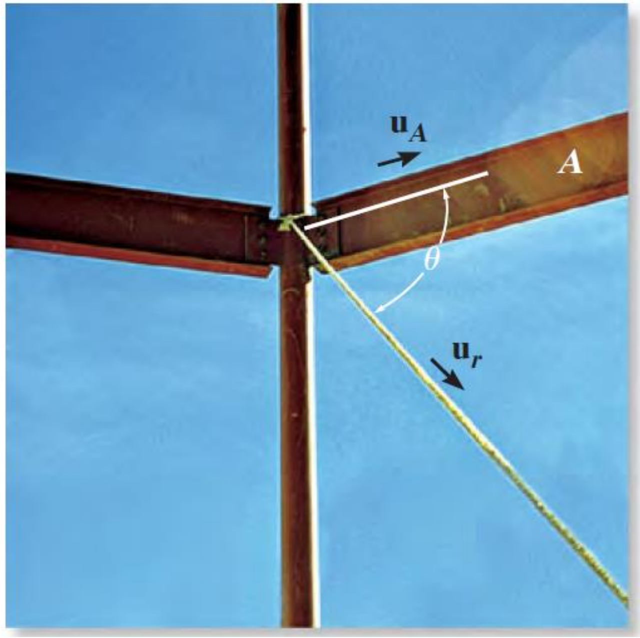


Dot Product

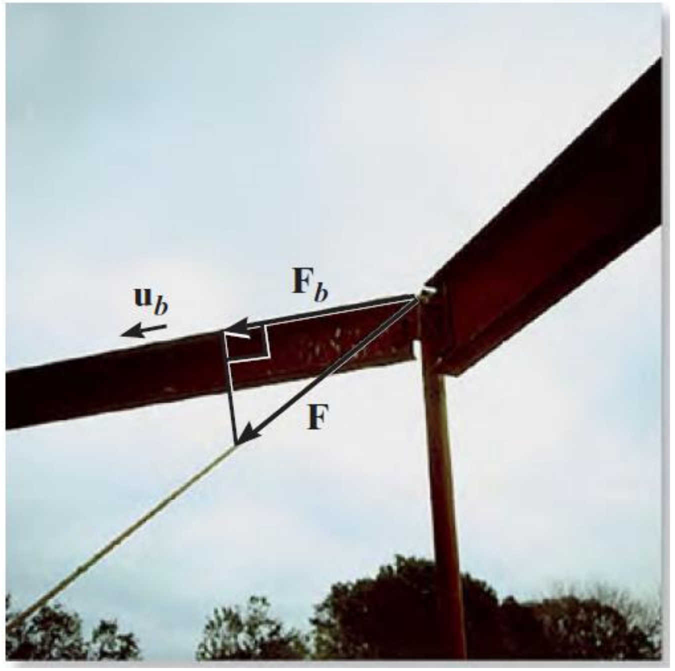
$$\begin{aligned}\mathbf{A}\cdot\mathbf{B} &= (A_x\mathbf{i} + A_y\mathbf{j} + A_z\mathbf{k})\cdot(B_x\mathbf{i} + B_y\mathbf{j} + B_z\mathbf{k}) \\ &= A_xB_x(\mathbf{i}\cdot\mathbf{i}) + A_xB_y(\mathbf{i}\cdot\mathbf{j}) + A_xB_z(\mathbf{i}\cdot\mathbf{k}) \\ &\quad + A_yB_x(\mathbf{j}\cdot\mathbf{i}) + A_yB_y(\mathbf{j}\cdot\mathbf{j}) + A_yB_z(\mathbf{j}\cdot\mathbf{k}) \\ &\quad + A_zB_x(\mathbf{k}\cdot\mathbf{i}) + A_zB_y(\mathbf{k}\cdot\mathbf{j}) + A_zB_z(\mathbf{k}\cdot\mathbf{k})\end{aligned}$$

$$\boxed{\mathbf{A}\cdot\mathbf{B} = A_xB_x + A_yB_y + A_zB_z}$$

Applications. The dot product has two important applications in mechanics.



The angle θ between the rope and the connecting beam can be determined by formulating unit vectors along the beam and rope and then using the dot product



The projection of the cable force $\mathbf{F}$ along the beam can be determined by first finding the unit vector $\mathbf{u}_b$ that defines this direction. Then apply the dot product, $F_b = \mathbf{F} \cdot \mathbf{u}_b$.

Applications. The dot product has two important applications in mechanics.

The angle formed between two vectors or intersecting lines.

$$\theta = \cos^{-1}\left(\frac{\mathbf{A}\cdot\mathbf{B}}{AB}\right) \qquad 0^\circ \le \theta \le 180^\circ$$

The components of a vector parallel and perpendicular to a line.

$$A_a = A\cos\theta$$

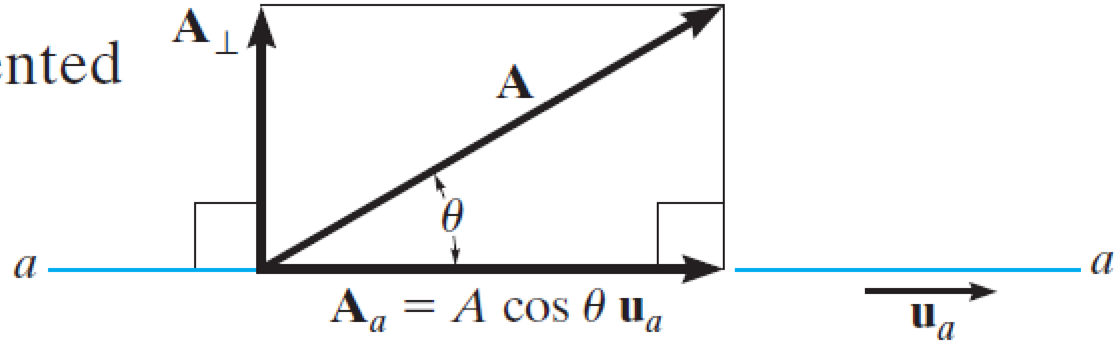
This component

is sometimes referred to as the *projection* of $\mathbf{A}$ onto the line

$$A_a = A\cos\theta = \mathbf{A}\cdot\mathbf{u}_a$$

The component $\mathbf{A}_a$ represented as a *vector* is therefore

$$\mathbf{A}_a = A_a\,\mathbf{u}_a$$



The component of **A** that is perpendicular to line *aa* can also be obtained

Since

$$\mathbf{A} = \mathbf{A}_a + \mathbf{A}_\perp$$

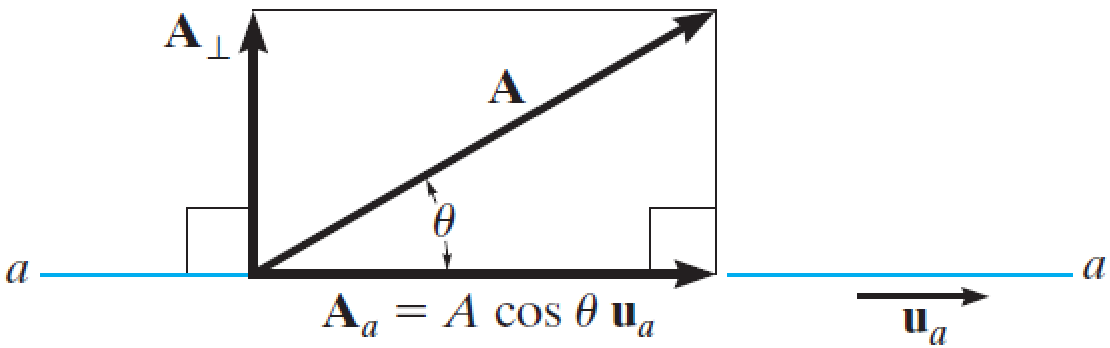
$$\mathbf{A}_\perp = \mathbf{A} - \mathbf{A}_a$$

$$A_\perp = \sqrt{A^2 - A_a{}^2}$$

Alternatively,

$$A_\perp = A \sin\theta$$

$$\theta = \cos^{-1}(\mathbf{A} \cdot \mathbf{u}_A / A),$$



Dot Product

Important Points

- The dot product is used to determine the angle between two vectors or the projection of a vector in a specified direction.
- If vectors $\mathbf{A}$ and $\mathbf{B}$ are expressed in Cartesian vector form, the dot product is determined by multiplying the respective x, y, z scalar components and algebraically adding the results, i.e., $\mathbf{A} \cdot \mathbf{B} = A_x B_x + A_y B_y + A_z B_z$.
- From the definition of the dot product, the angle formed between the tails of vectors $\mathbf{A}$ and $\mathbf{B}$ is $\theta = \cos^{-1}(\mathbf{A} \cdot \mathbf{B}/AB)$.
- The magnitude of the projection of vector $\mathbf{A}$ along a line aa whose direction is specified by $\mathbf{u}_a$ is determined from the dot product $A_a = \mathbf{A} \cdot \mathbf{u}_a$.

Example

Determine the magnitudes of the projection of the force **F** in Fig. 2–44 onto the u and v axes.

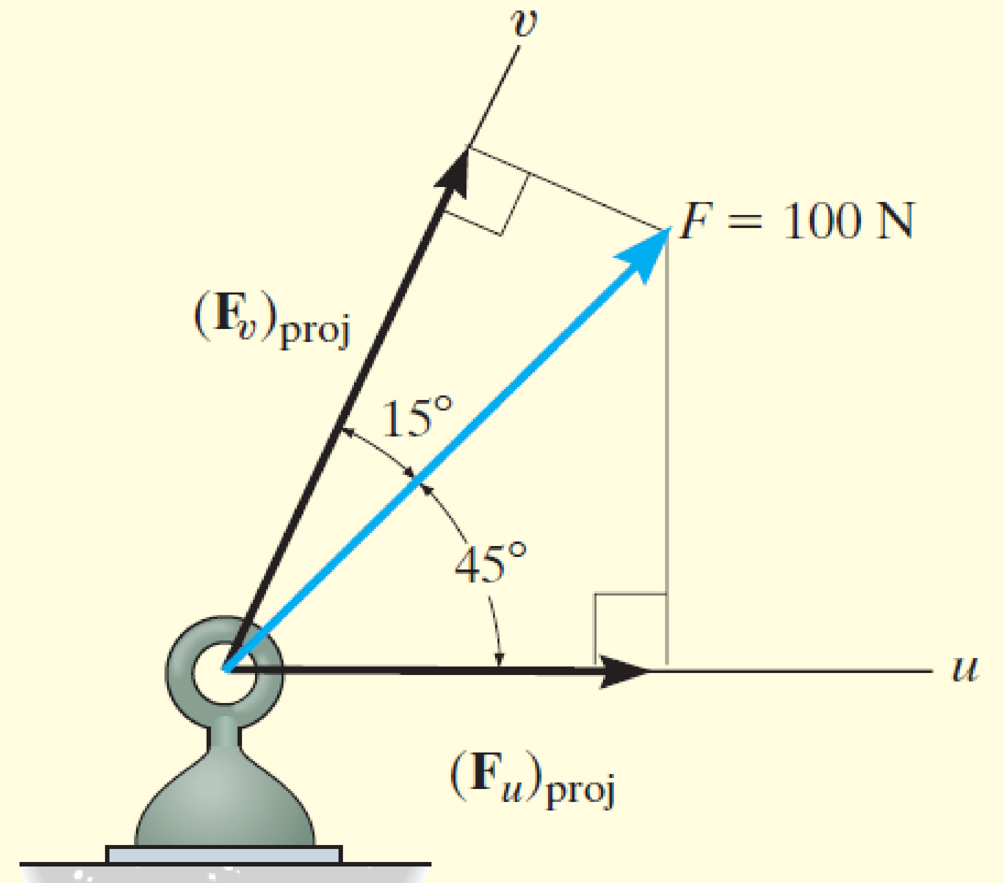


Fig. 2–44

Example

The magnitudes of the projections of **F** onto the *u* and *v* axes can be obtained by trigonometry:

$$(F_u)_{\text{proj}} = (100\ \text{N})\cos 45^\circ = 70.7\ \text{N}$$

$$(F_v)_{\text{proj}} = (100\ \text{N})\cos 15^\circ = 96.6\ \text{N}$$

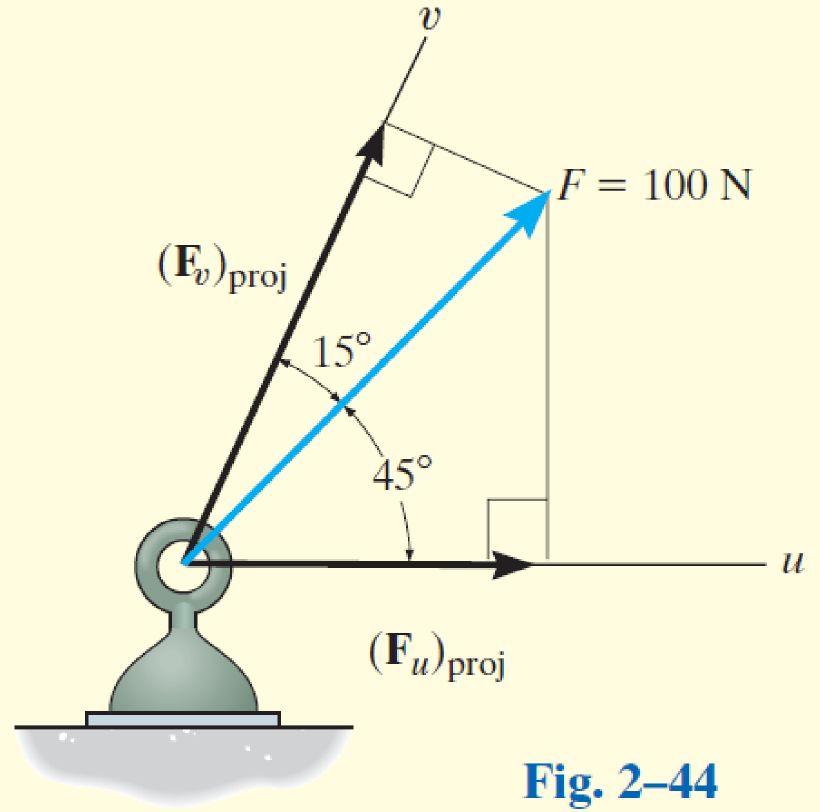
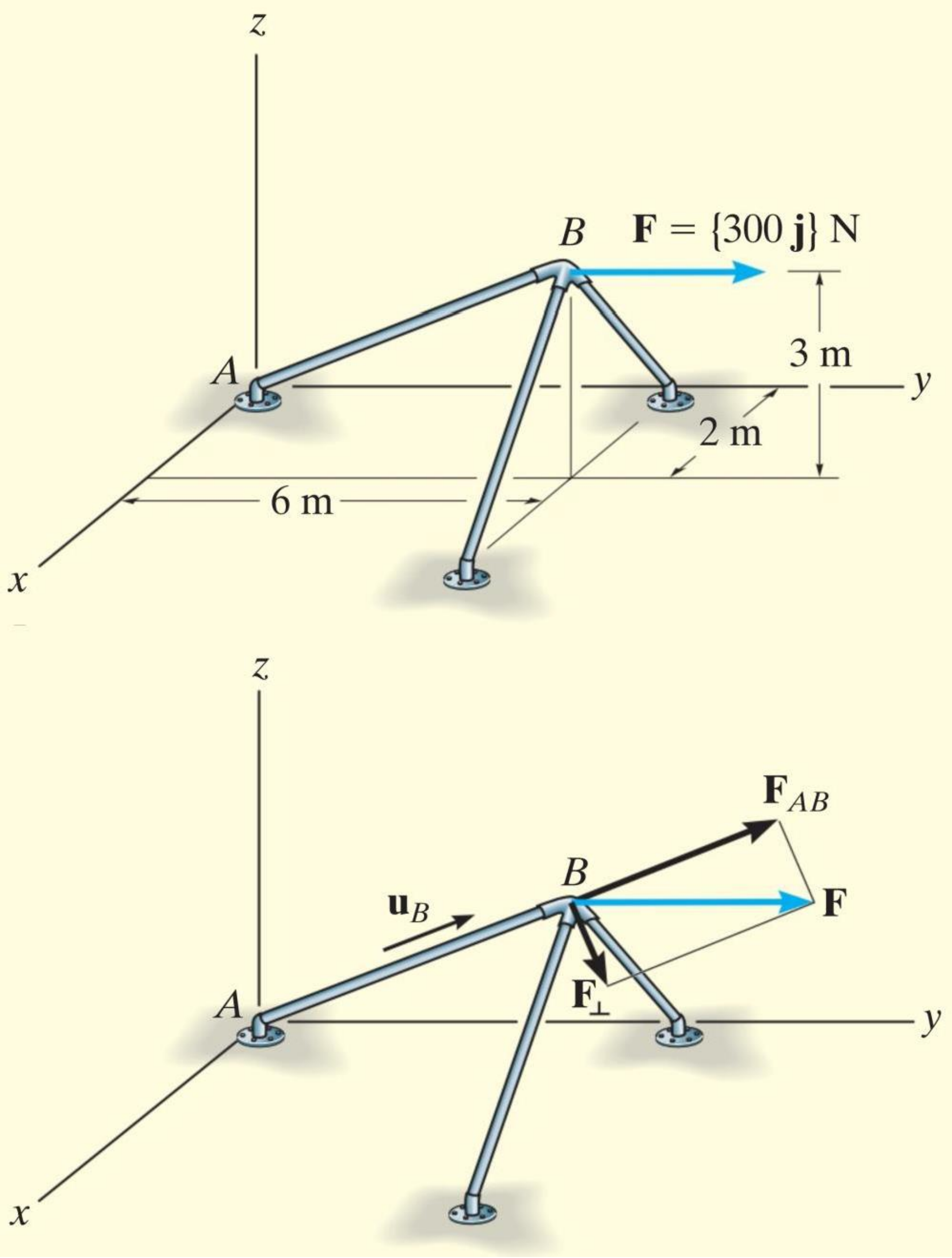


Fig. 2–44

NOTE: These projections are not equal to the magnitudes of the components of force **F** along the *u* and axes found from the parallelogram law. They will only be equal if the *u* and axes are *perpendicular* to one another.

Example

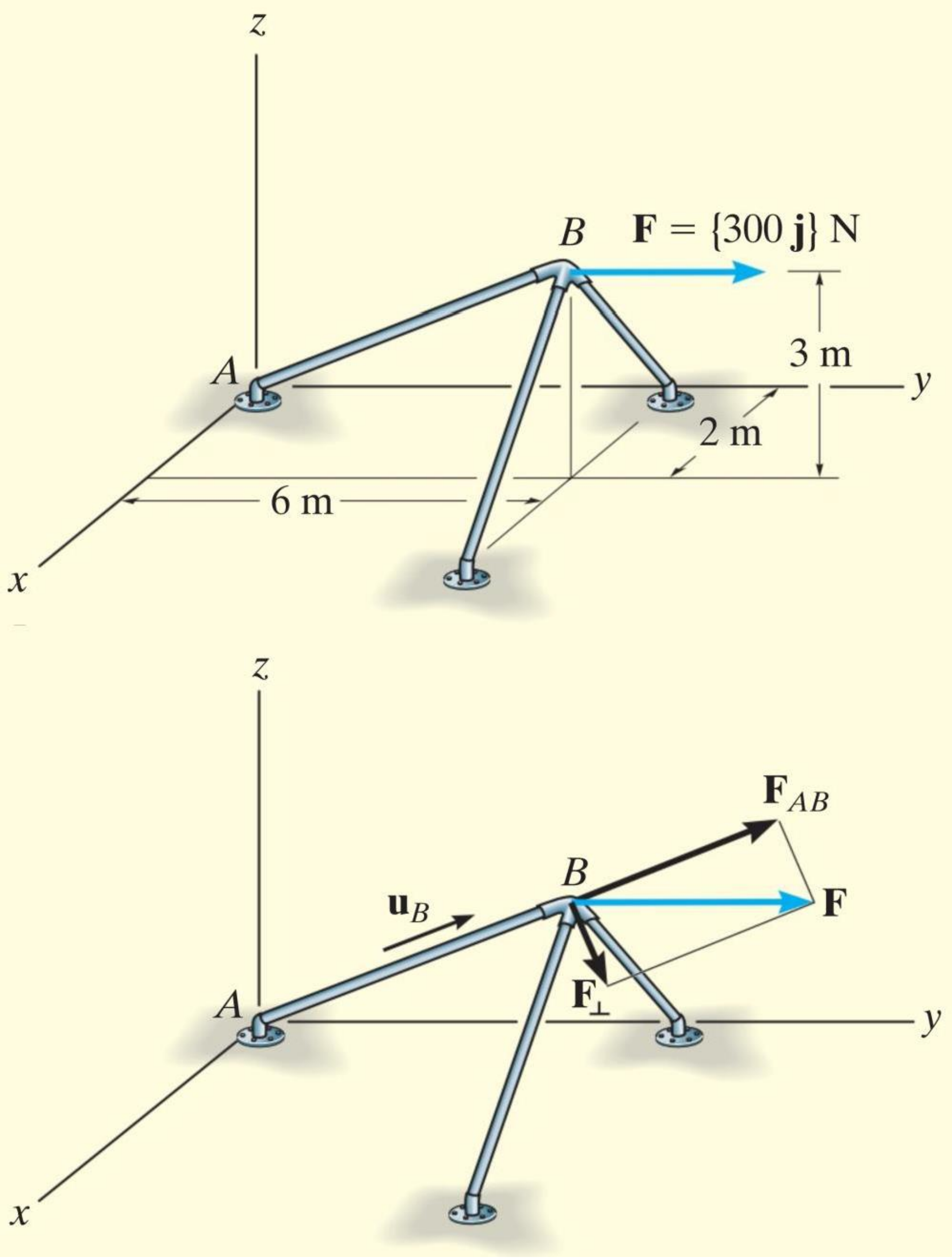
The frame shown is subjected to a horizontal force **F** = {300**j**} N. Determine the magnitudes of the components of this force parallel and perpendicular to member *AB*.



Example

The frame shown is subjected to a horizontal force **F** = {300**j**} N. Determine the magnitudes of the components of this force parallel and perpendicular to member *AB*.

$$\mathbf{u}_B = \frac{\mathbf{r}_B}{r_B} = \frac{2\mathbf{i} + 6\mathbf{j} + 3\mathbf{k}}{\sqrt{(2)^2 + (6)^2 + (3)^2}} = 0.286\mathbf{i} + 0.857\mathbf{j} + 0.429\mathbf{k}$$

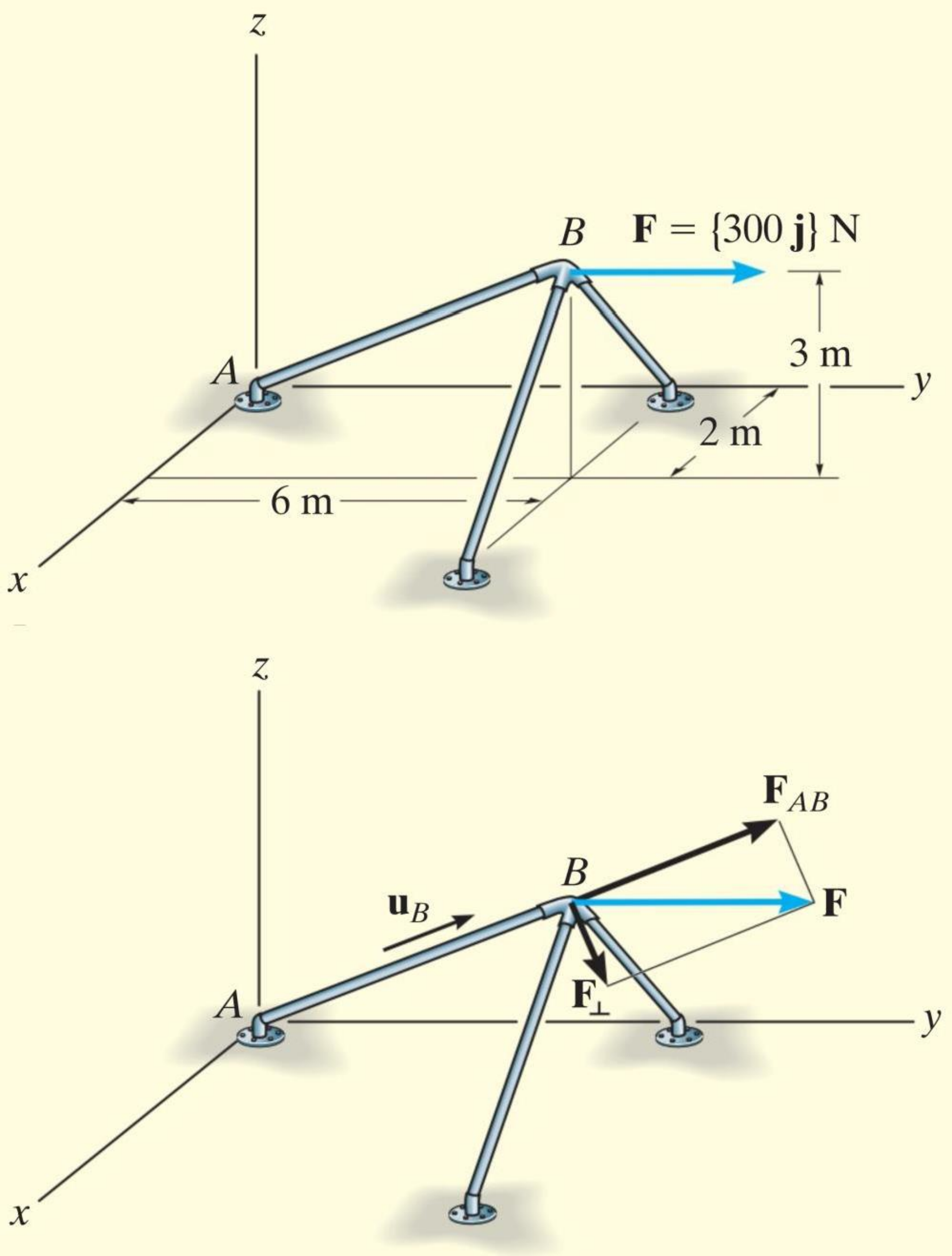


Example

$$\mathbf{F}_{AB} =$$
$$=$$

$$F_{\perp} =$$
$$=$$

$$\mathbf{F}_{\perp} =$$
$$=$$

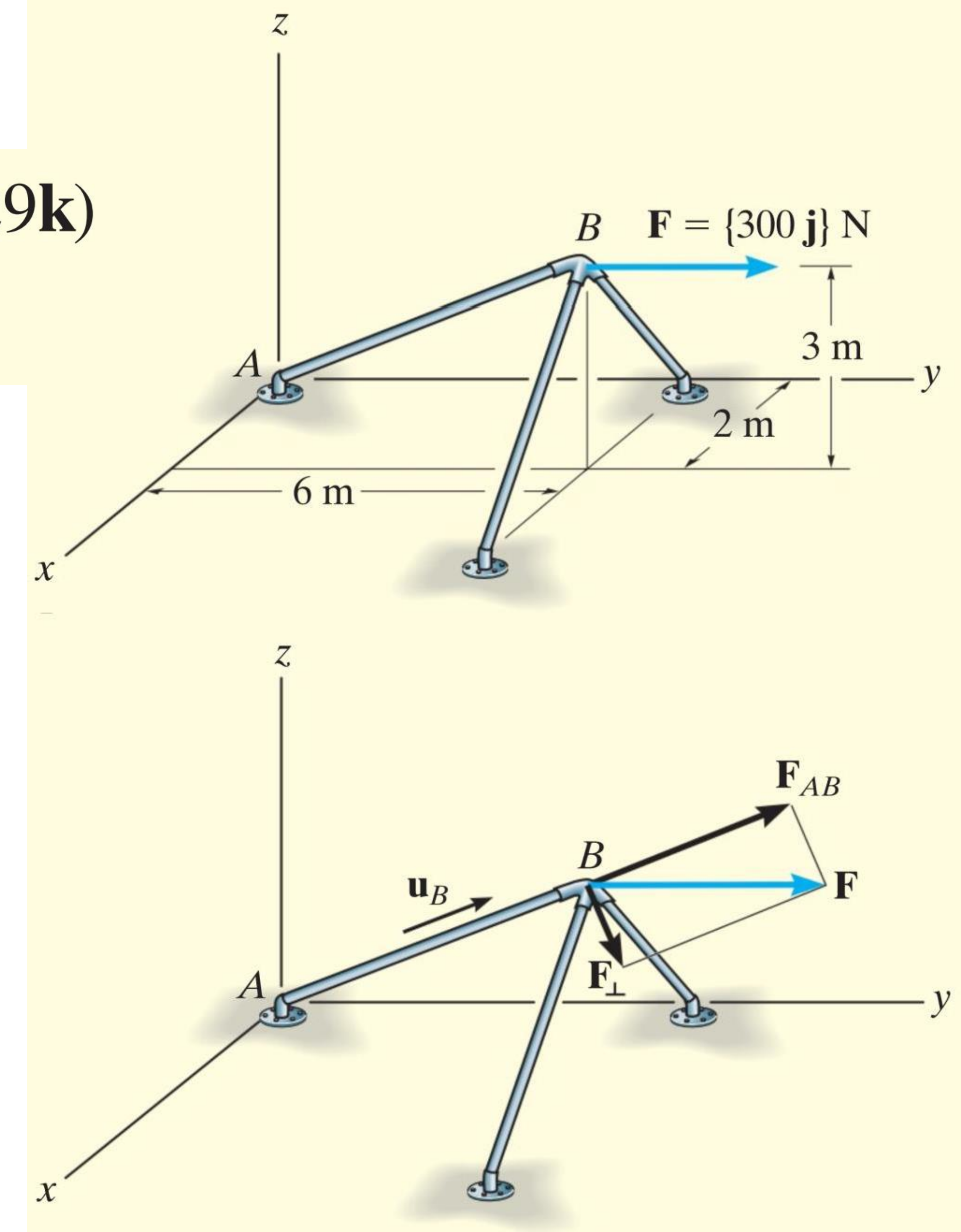


Example

$$\mathbf{F}_{AB} = F_{AB}\mathbf{u}_B = (257.1\text{ N})(0.286\mathbf{i} + 0.857\mathbf{j} + 0.429\mathbf{k})$$
$$= \{73.5\mathbf{i} + 220\mathbf{j} + 110\mathbf{k}\}\text{N}$$

$$F_{\perp} =$$
$$=$$

$$\mathbf{F}_{\perp} =$$
$$=$$

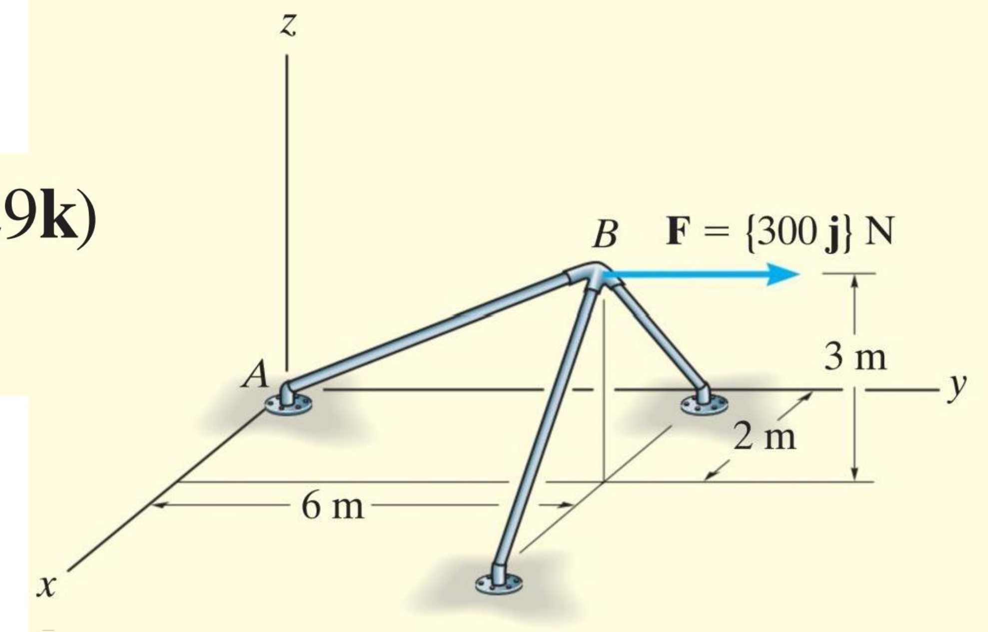


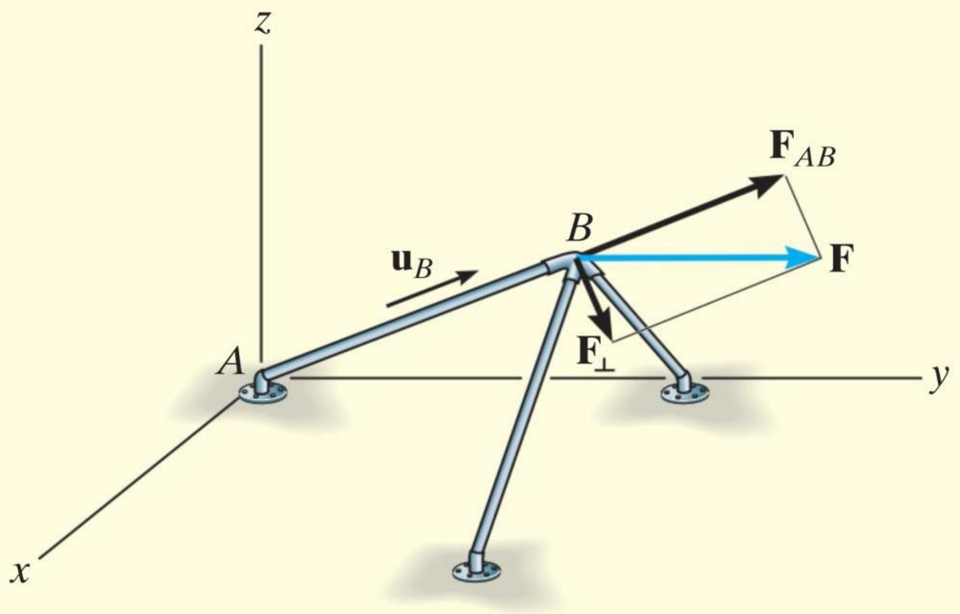
Example

$$\mathbf{F}_{AB} = F_{AB}\mathbf{u}_B = (257.1\text{ N})(0.286\mathbf{i} + 0.857\mathbf{j} + 0.429\mathbf{k})$$
$$= \{73.5\mathbf{i} + 220\mathbf{j} + 110\mathbf{k}\}\text{ N}$$

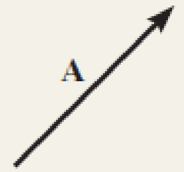
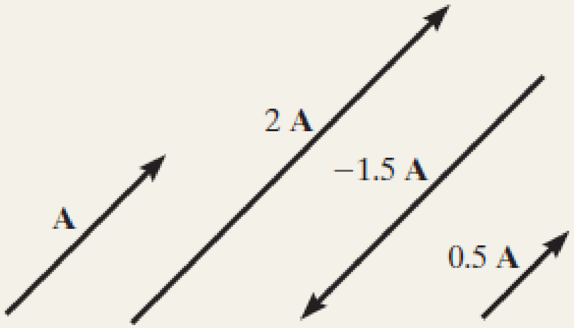
$$F_{\perp} = \sqrt{F^2 - F_{AB}^2} = \sqrt{(300\text{ N})^2 - (257.1\text{ N})^2}$$
$$= 155\text{ N}$$

$$\mathbf{F}_{\perp} = \mathbf{F} - \mathbf{F}_{AB} = 300\mathbf{j} - (73.5\mathbf{i} + 220\mathbf{j} + 110\mathbf{k})$$
$$= \{-73.5\mathbf{i} + 79.6\mathbf{j} - 110\mathbf{k}\}\text{ N}$$



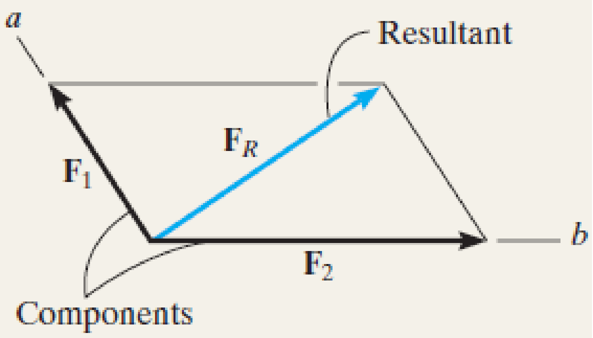


Chapter Review

A scalar is a positive or negative number; e.g., mass and temperature. A vector has a magnitude and direction, where the arrowhead represents the sense of the vector.		
Multiplication or division of a vector by a scalar will change only the magnitude of the vector. If the scalar is negative, the sense of the vector will change so that it acts in the opposite sense.		
If vectors are collinear, the resultant is simply the algebraic or scalar addition.	$R = A + B$	

Parallelogram Law

Two forces add according to the parallelogram law. The *components* form the sides of the parallelogram and the *resultant* is the diagonal.

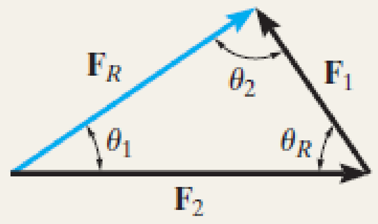


To find the components of a force along any two axes, extend lines from the head of the force, parallel to the axes, to form the components.

To obtain the components or the resultant, show how the forces add by tip-to-tail using the triangle rule, and then use the law of cosines and the law of sines to calculate their values.

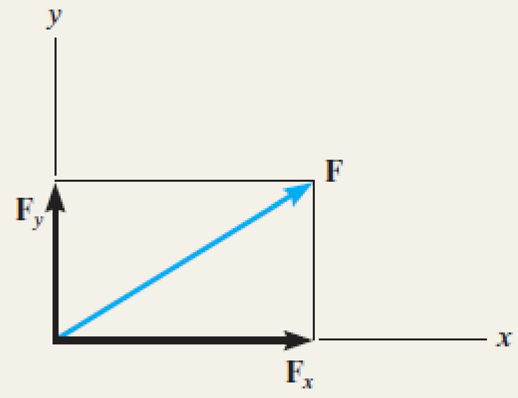
$$F_R = \sqrt{F_1^{\,2} + F_2^{\,2} - 2\,F_1F_2\cos\theta_R}$$

$$\frac{F_1}{\sin\theta_1} = \frac{F_2}{\sin\theta_2} = \frac{F_R}{\sin\theta_R}$$



Rectangular Components: Two Dimensions

Vectors $\mathbf{F}_x$ and $\mathbf{F}_y$ are rectangular components of $\mathbf{F}$.



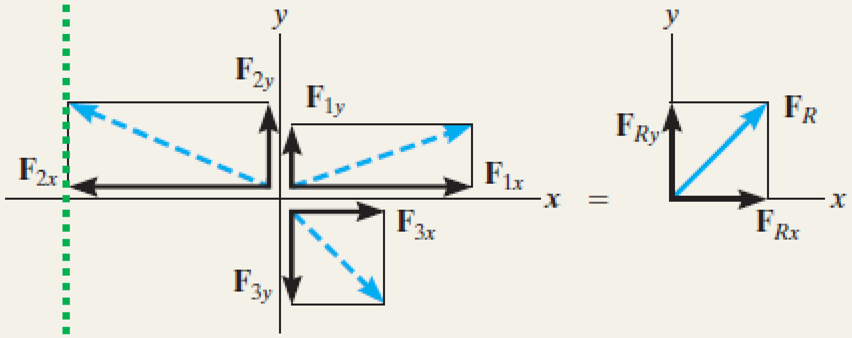
The resultant force is determined from the algebraic sum of its components.

$$F_{Rx} = \Sigma F_x$$

$$F_{Ry} = \Sigma F_y$$

$$F_R = \sqrt{(F_{Rx})^2 + (F_{Ry})^2}$$

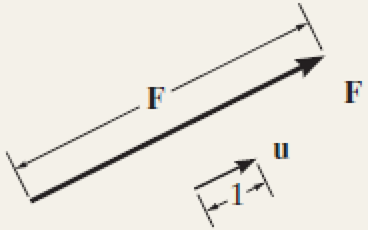
$$\theta = \tan^{-1}\left|\frac{F_{Ry}}{F_{Rx}}\right|$$



Cartesian Vectors

The unit vector **u** has a length of one, no units, and it points in the direction of the vector **F**.

$$\mathbf{u} = \frac{\mathbf{F}}{F}$$



A force can be resolved into its Cartesian components along the x, y, z axes so that $\mathbf{F} = F_x\mathbf{i} + F_y\mathbf{j} + F_z\mathbf{k}$.

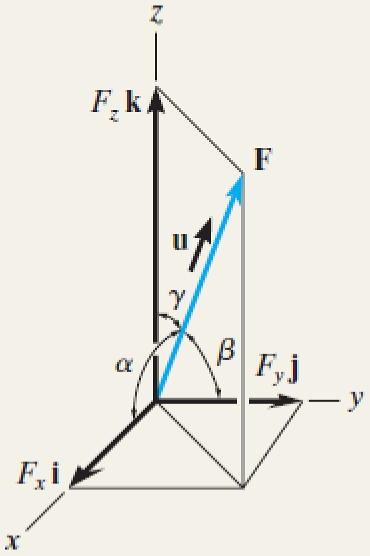
The magnitude of **F** is determined from the positive square root of the sum of the squares of its components.

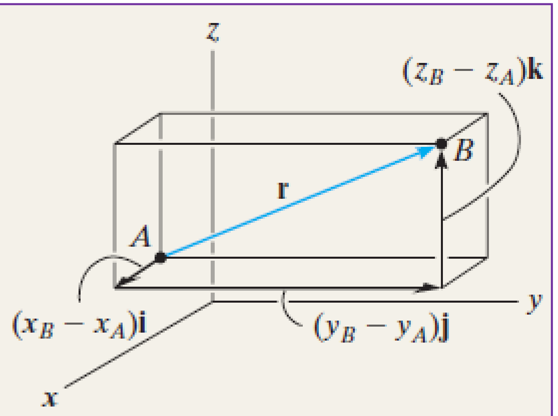
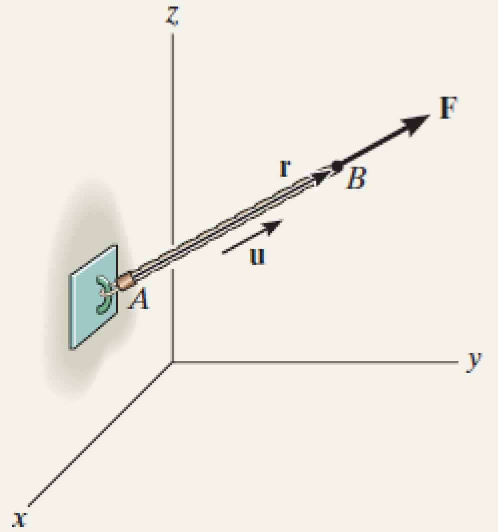
$$F = \sqrt{F_x^2 + F_y^2 + F_z^2}$$

The coordinate direction angles α, β, γ are determined by formulating a unit vector in the direction of **F**. The x, y, z components of **u** represent $\cos\alpha, \cos\beta, \cos\gamma$.

$$\mathbf{u} = \frac{\mathbf{F}}{F} = \frac{\mathbf{F_x}}{\mathbf{F}}\mathbf{i} + \frac{F_y}{F}\mathbf{j} + \frac{F_z}{F}\mathbf{k}$$

$$\mathbf{u} = \cos\alpha\,\mathbf{i} + \cos\beta\,\mathbf{j} + \cos\gamma\,\mathbf{k}$$

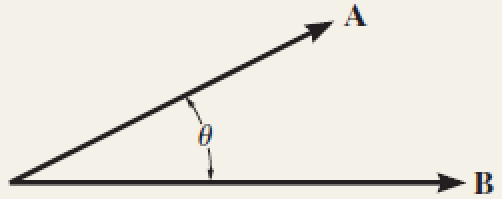


The coordinate direction angles are related so that only two of the three angles are independent of one another.	$\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$	
To find the resultant of a concurrent force system, express each force as a Cartesian vector and add the **i**, **j**, **k** components of all the forces in the system.	$\mathbf{F}_R = \Sigma\mathbf{F} = \Sigma F_x\mathbf{i} + \Sigma F_y\mathbf{j} + \Sigma F_z\mathbf{k}$	
Position and Force Vectors A position vector locates one point in space relative to another. The easiest way to formulate the components of a position vector is to determine the distance and direction that one must travel along the *x*, *y*, and *z* directions—going from the tail to the head of the vector.	$\mathbf{r} = (x_B - x_A)\mathbf{i} + (y_B - y_A)\mathbf{j} + (z_B - z_A)\mathbf{k}$	
If the line of action of a force passes through points A and B, then the force acts in the same direction as the position vector **r**, which is defined by the unit vector *u*. The force can then be expressed as a Cartesian vector.	$\mathbf{F} = F\mathbf{u} = F\left(\frac{\mathbf{r}}{r}\right)$	

Dot Product

The dot product between two vectors **A** and **B** yields a scalar. If **A** and **B** are expressed in Cartesian vector form, then the dot product is the sum of the products of their x, y, and z components

$$\mathbf{A} \cdot \mathbf{B} = AB\cos\theta = A_xB_x + A_yB_y + A_zB_z$$

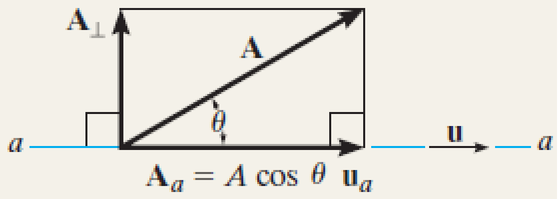


The dot product can be used to determine the angle between **A** and **B**.

$$\theta = \cos^{-1}\left(\frac{\mathbf{A} \cdot \mathbf{B}}{AB}\right)$$

The dot product is also used to determine the projected component of a vector **A** onto an axis aa defined by its unit vector $\mathbf{u}_a$.

$$\mathbf{A}_a = A\cos\theta\,\mathbf{u}_a = (\mathbf{A} \cdot \mathbf{u}_a)\mathbf{u}_a$$



Home Assignment

- Example 2.18 & Problem 2-106