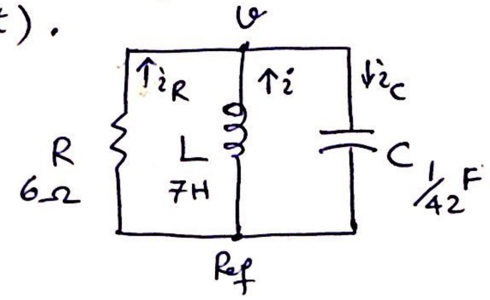
Example: The Overdamped Parallel RLC Circuit : $\alpha > \omega_0$

(PP 326 8th Ed HKD)

Determine $v(t)$.



Given:
$v(0) = 0\ V$
and $i(0) = 10\ A$
(as defined in figure)

Solution: We identify it as a parallel RLC circuit so;

$$\alpha = \frac{1}{2RC} = \frac{1}{2 \times 6 \times \frac{1}{42}} = 3.5 \qquad \left(\alpha = \frac{R}{2L} \text{ — series}\right)$$

$$\text{and } \omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{7 \times \frac{1}{42}}} = 2.45 \qquad \text{(resonant frequency)}$$

$$\text{Now } s_1 = -\alpha + \sqrt{\alpha^2 - \omega_0^2} = -1$$

$$\text{and } s_2 = -\alpha - \sqrt{\alpha^2 - \omega_0^2} = -6$$

— As $\alpha > \omega_0$ so;

the general form of the response is:

$$v(t) = A_1 e^{s_1 t} + A_2 e^{s_2 t}$$

$$\text{or } v(t) = A_1 e^{-t} + A_2 e^{-6t}$$

— To determine A_1 and A_2;

we know $v(0) = 0$

$$\text{So } v(t) = v(0) = 0 = A_1 + A_2$$

contd

— contd (326)

Hence $A_1 + A_2 = 0$ ——— (1)

Also the derivative of $v(t)$

$$v(t) = A_1 e^{-t} + A_2 e^{-6t}$$

$$\frac{dv}{dt} = -A_1 e^{-t} - 6A_2 e^{-6t}$$

— At $t=0$

$$\left.\frac{dv}{dt}\right|_{t=0} = -A_1 - 6A_2$$

— We know $i_c = C\frac{dv}{dt}$

or $\frac{dv}{dt} = \frac{1}{C} i_c$

— Now KCL must hold at any instant in time,

thus $i(0) + i_R(0) = i_c(0)$

Given: $\left\{ \begin{array}{l} i(0) = 10 \text{ and} \\ v(0) = 0 \end{array} \right\}$

$10 + \frac{v(0)}{R} = i_c(0)$

So $10 + 0 = i_c(0)$

$i_c(0) = 10$

— Hence $\left.\frac{dv}{dt}\right|_{t=0} = \frac{i_c(0)}{C} = \frac{10}{1/42} = 420 \text{ V/s}$

— So we get $-A_1 - 6A_2 = 420$ ——— (2)

——— contd

— contd (327)

Now solving (1) and (2)

$A_1 = 84$ and

$A_2 = -84$

— Therefore $v(t) = 84e^{-t} - 84e^{-6t}$

or $v(t) = 84(e^{-t} - e^{-6t})$ V