Engineering Mechanics : Statics

Chapter 5: Equilibrium of a Rigid Body

Chapter SUMMARY

- Conditions for Rigid Equilibrium
- Free-Body Diagrams
- Equations of Equilibrium
- Two and Three-Force Members
- Equilibrium in Three Dimensions
- Equations of Equilibrium

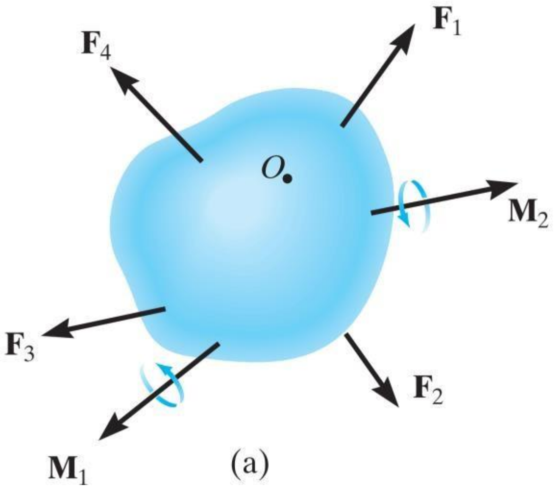
5.1 CONDITIONS FOR RIGID-BODY EQUILIBRIUM

- Equations of Equilibrium for Rigid Body

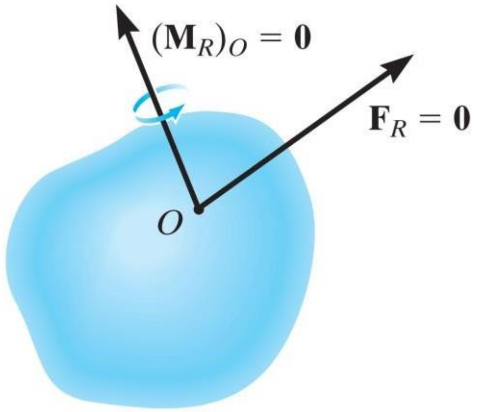
$$\sum \mathbf{F} = 0$$

$$\sum \mathbf{M}_O = 0$$

- A rigid body will remain in equilibrium provided the sum of all the external forces acting on the body = 0 and sum of moments of the external forces about a point = 0



(a)



$$F_R = \sum F = 0$$

$$(M_R)_O = \sum M_O = 0$$

5. 2 FREE-BODY DIAGRAMS

- FBD is the best method to represent all the known and unknown forces in a system
- FBD is a sketch of the outlined shape of the body, which represents it being isolated from its surroundings

5. 2 FREE-BODY DIAGRAMS

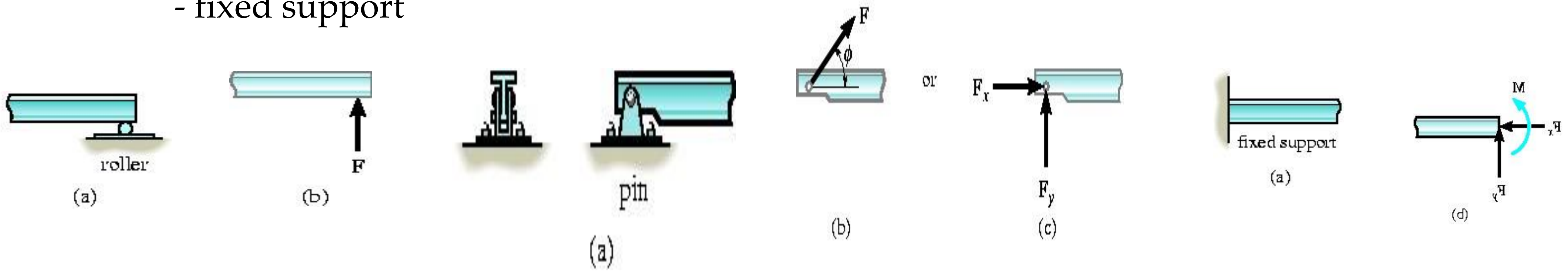
Support Reactions.

- If the support prevents the translation of a body in a given direction, then a force is developed on the body in that direction
- If rotation is prevented, a couple moment is exerted on the body
- Consider the three ways a horizontal member, beam is supported at the end

  - roller, cylinder

  - pin

  - fixed support



5. 3 EQUATIONS OF EQUILIBRIUM

- For equilibrium of a rigid body in 2D,

$$\sum F_x = 0; \sum F_y = 0; \sum M_O = 0$$

Alternative Sets of Equilibrium Equations

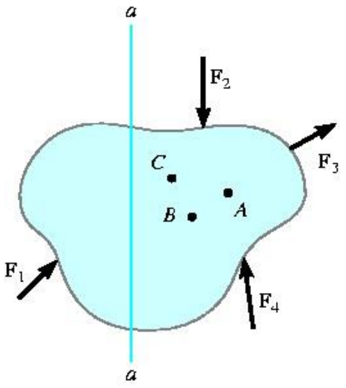
- For coplanar equilibrium problems, $\sum F_x = 0$; $\sum F_y = 0$; $\sum M_O = 0$ can be used
- Two alternative sets of three independent equilibrium equations may also be used

$$\sum F_a = 0; \sum M_A = 0; \sum M_B = 0$$

- A second set of alternative equations is

$$\sum M_A = 0; \sum M_B = 0; \sum M_C = 0$$

- Points A, B and C do not lie on the same line



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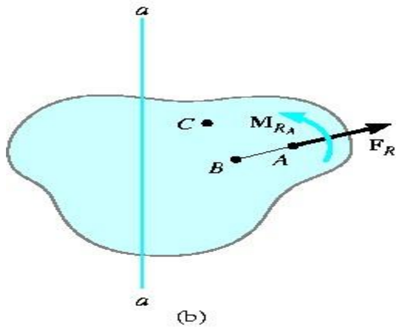

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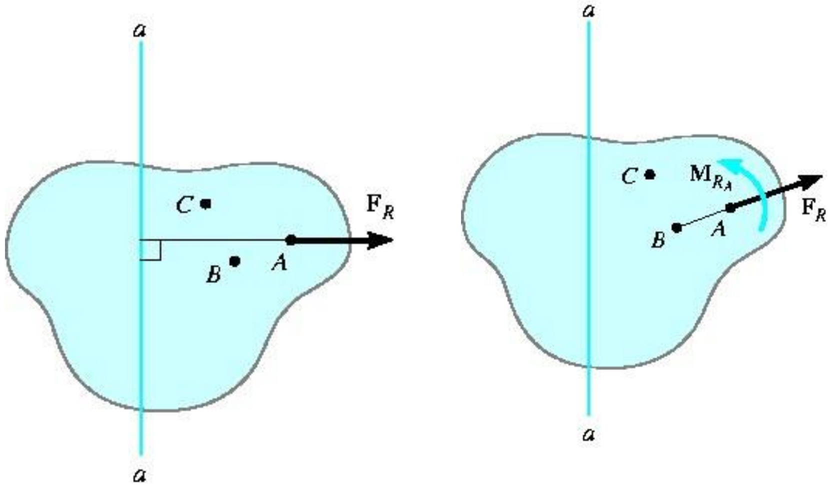


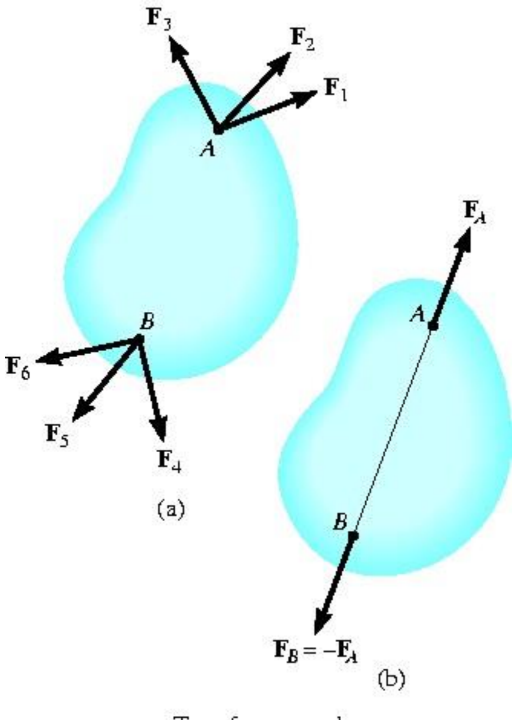
(c)

5. 4 Two- and three- force members

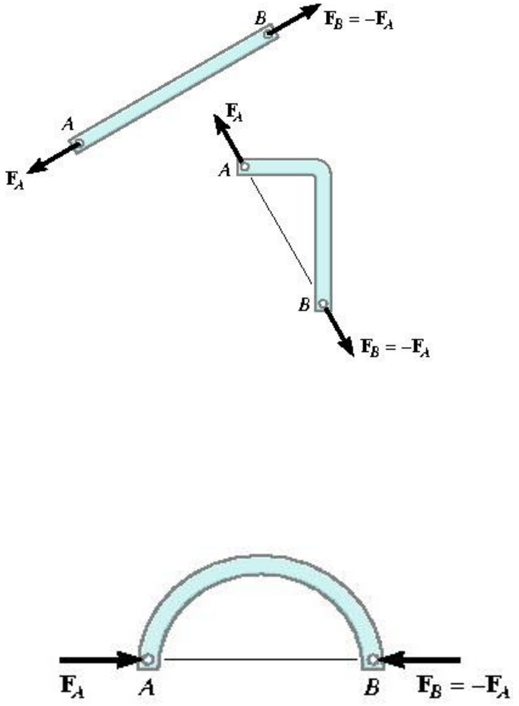
- Simplify some equilibrium problems by recognizing embers that are subjected top only 2 or 3 forces

Two-Force Members

- When a member is subject to no couple moments and forces are applied at only two points on a member, the member is called a two-force member



Two-force member

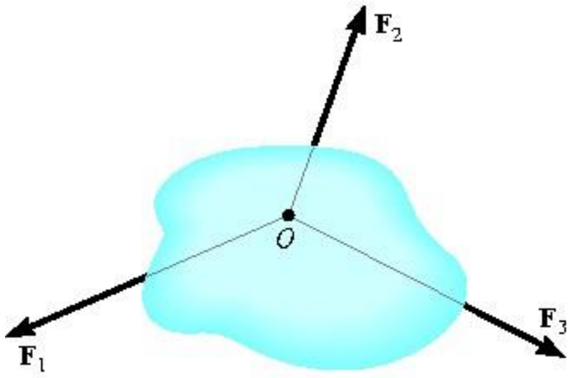


Two-force members

5. 4 Two- and three- force members

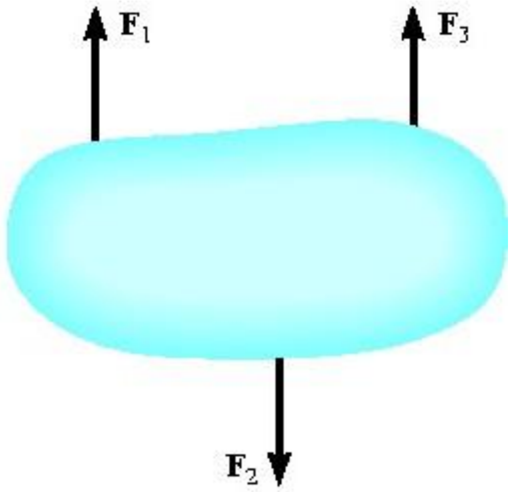
Three-Force Members

- If a member is subjected to only three forces, it is necessary that the forces be either concurrent or parallel for the member to be in equilibrium.



Concurrent forces

(a)



Parallel forces

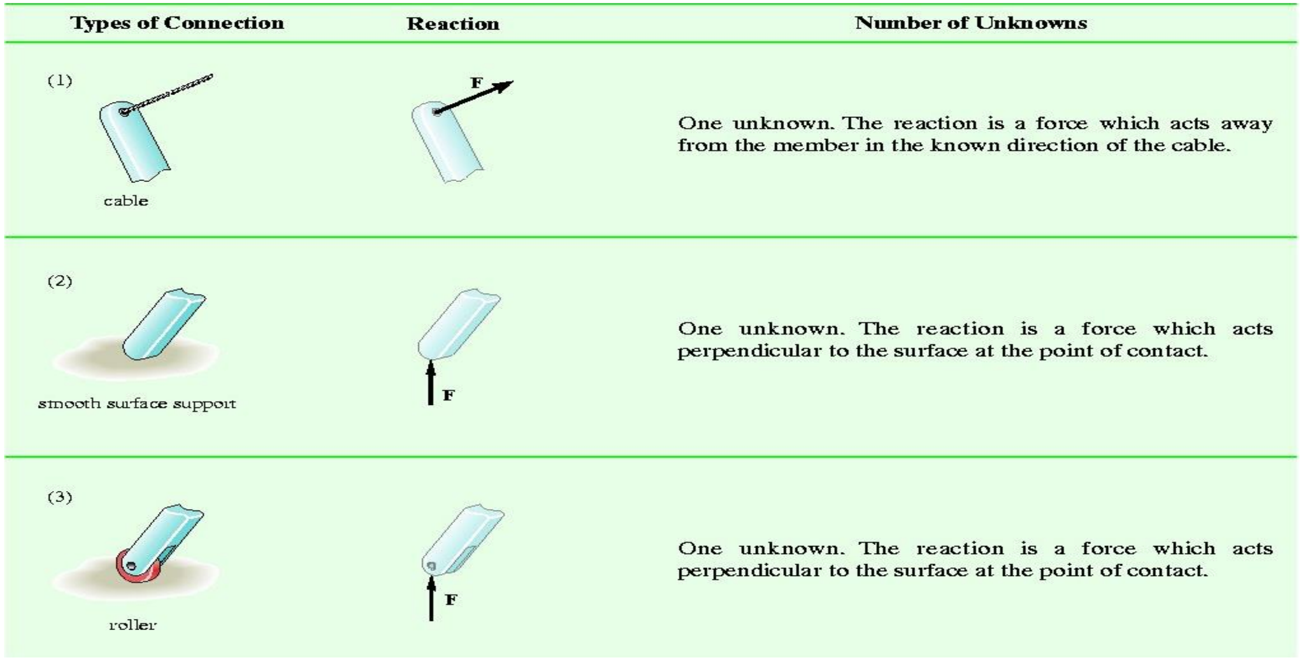
(b)

5. 5 EQUILIBRIUM IN THREE DIMENSIONS

Support Reactions.

As in the two-dimensional case:

- If a **support prevents translation** of a body in a given direction, then **a force is developed** on the body in the opposite direction.
- A couple moment is developed when rotation of the attached member is prevented
- The force's orientation is defined by the coordinate angles α, β and γ



Types of Connection	Reaction	Number of Unknowns
(1) cable	F	One unknown. The reaction is a force which acts away from the member in the known direction of the cable.
(2) smooth surface support	F	One unknown. The reaction is a force which acts perpendicular to the surface at the point of contact.
(3) roller	F	One unknown. The reaction is a force which acts perpendicular to the surface at the point of contact.

5. 6 EQUations of Equilibrium

Vector Equations of Equilibrium.

The two conditions for equilibrium of a rigid body may be expressed mathematically in vector form as

$$\begin{aligned} \Sigma \mathbf{F} &= \mathbf{0} \\ \Sigma \mathbf{M}_O &= \mathbf{0} \end{aligned}$$

Scalar Equations of Equilibrium. If all the external forces and couple moments are expressed in Cartesian vector form; we have

$$\Sigma \mathbf{F} = \Sigma F_x \mathbf{i} + \Sigma F_y \mathbf{j} + \Sigma F_z \mathbf{k} = \mathbf{0}$$

$$\Sigma \mathbf{M}_O = \Sigma M_x \mathbf{i} + \Sigma M_y \mathbf{j} + \Sigma M_z \mathbf{k} = \mathbf{0}$$

$$\begin{aligned} \Sigma F_x &= 0 \\ \Sigma F_y &= 0 \\ \Sigma F_z &= 0 \end{aligned} \qquad \begin{aligned} \Sigma M_x &= 0 \\ \Sigma M_y &= 0 \\ \Sigma M_z &= 0 \end{aligned}$$

Chapter review

Two Dimensions

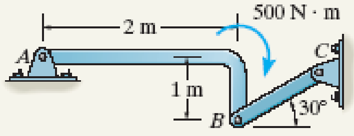
Before analyzing the equilibrium of a body, it is first necessary to draw its free-body diagram. This is an outlined shape of the body, which shows all the forces and couple moments that act on it.

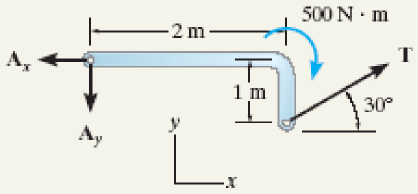
Couple moments can be placed anywhere on a free-body diagram since they are free vectors. Forces can act at any point along their line of action since they are sliding vectors.

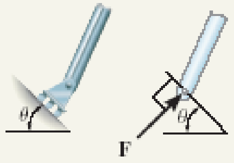
Angles used to resolve forces, and dimensions used to take moments of the forces, should also be shown on the free-body diagram.

Some common types of supports and their reactions are shown below in two dimensions.

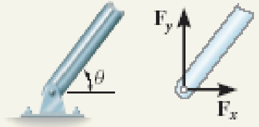
Remember that a support will exert a force on the body in a particular direction if it prevents translation of the body in that direction, and it will exert a couple moments on the body if it prevents rotation.
(*See pages 203–204.*)



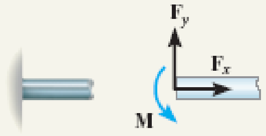




roller



smooth pin or hinge



fixed support

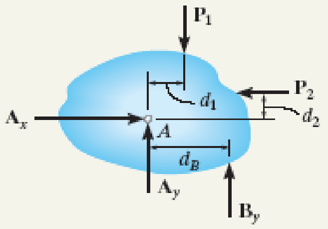
The three scalar equations of equilibrium can be applied when solving problems in two dimensions, since the geometry is easy to visualize.

$$\Sigma F_x = 0$$
$$\Sigma F_y = 0$$
$$\Sigma M_O = 0$$

Chapter review

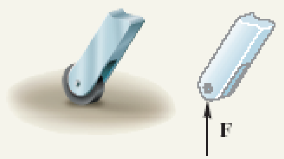
For the most direct solution, try to sum forces along an axis that will eliminate as many unknown forces as possible. Sum moments about a point A that passes through the line of action of as many unknown forces as possible. (*See pages 217 and 218.*)

$$\Sigma F_x = 0;$$
$$A_x - P_2 = 0 \qquad A_x = P_2$$
$$\Sigma M_A = 0;$$
$$P_2 d_2 + B_y d_B - P_1 d_1 = 0$$
$$B_y = \frac{P_1 d_1 - P_2 d_2}{d_B}$$

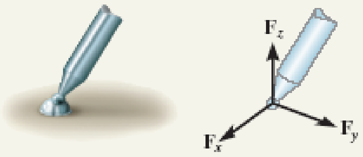


Three Dimensions

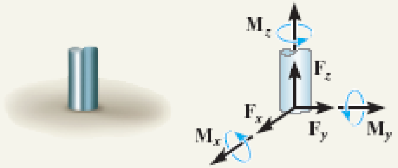
Some common types of supports and their reactions are shown in three dimensions.



roller



ball and socket



fixed support

In three dimensions, it is often advantageous to use a Cartesian vector analysis when applying the equations of equilibrium. To do this, first express each known and unknown force and couple moment shown on the free-body diagram as a Cartesian vector. Then set the force summation equal to zero. Take moments about a point O that lies on the line of action of as many unknown force components as possible. From point O direct position vectors to each force, and then use the cross product to determine the moment of each force. The six scalar equations of equilibrium are established by setting the respective **i**, **j**, and **k** components of these force and moment summations equal to zero.
(*See page 245.*)

$$\Sigma \mathbf{F} = \mathbf{0}$$
$$\Sigma \mathbf{M}_O = \mathbf{0}$$

$$\Sigma F_x = 0$$
$$\Sigma F_y = 0$$
$$\Sigma F_z = 0$$

$$\Sigma M_x = 0$$
$$\Sigma M_y = 0$$
$$\Sigma M_z = 0$$