National University of Sciences & Technology
School of Electrical Engineering and Computer Science
Department of Humanities and Sciences

MATH-232: Complex Variables and Transforms (3+0): BEE2k20-12ABC Spring 2022

Assignment – 3	
CLO-3 (Evaluate Fourier and Z-transforms of a given function)	
Maximum Marks: 10 (5+5)	Instructor: Mr. Saeed Afzal
Announcement Date: 18th May 2022	Due Date: 25th May 2022

Instructions:

- Understanding the question is part of the assignment and copying is not allowed.
- Express your answer in the most simplified form. Direct calculations using calculator are not allowed, you need to show the detail of your work to get the maximum marks.
- This is an individual assignment.
- Assignment must be handwritten and properly arranged with page numbers These two pages must be part of every assignment.
- Assignment is not acceptable after deadline.

Tasks: Attempt all questions.

Students Name	NUST/Qalam ID	Section
Muhammad Umer	345834	BEE 12C

Total Marks	Marks Obtained
10 Marks	

Q – 1 (5 marks): Evaluate Fourier transform and sketch the magnitude and phase spectra of the function given by:

$$\boldsymbol{f(t) = \frac{3\sin\left(30\pi\left(t - \frac{1}{20}\right)\right)}{\pi\left(t - \frac{1}{20}\right)}\cos\left(300\pi t\right)}$$

Note: 5 marks are assigned for class participation.

Q1:

$$f(t) = \frac{3\sin(30\pi(t-1/20))}{\pi(t-1/20)}\cos 300\pi t$$

- We can break down $f(t)$ into a compound of different functions. From observation,

$$g(t) = \frac{3\sin(30\pi t)}{\pi t}$$

$$h(t) = g(t-1/20)$$

$$k(t) = h(t)\cos 300\pi t = f(t)$$

- Now, we can utilize known properties of fourier transform to get the transform of $f(t)$.

$$g(t) = \frac{3}{\pi}\left(\frac{30}{30}\right)\frac{\sin(30\pi t)}{t}$$

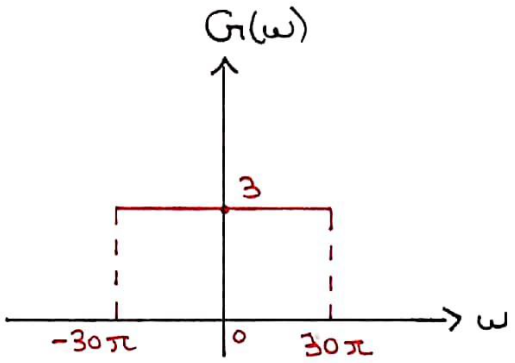
$$= 90\,\frac{\sin(30\pi t)}{30\pi t}$$

$$= 90\,\mathrm{sinc}(30\pi t)$$

-> We know that: $A\,\mathrm{sinc}(at) \longleftrightarrow \frac{A\pi}{a}\,\mathrm{rect}\left(\frac{\omega}{2a}\right)$

$$G(j\omega) = 3\,\mathrm{rect}\left(\frac{\omega}{60\pi}\right)$$

- The magnitude spectrum of which is:



- Using time shift property, we can find the transform of $h(t)$.

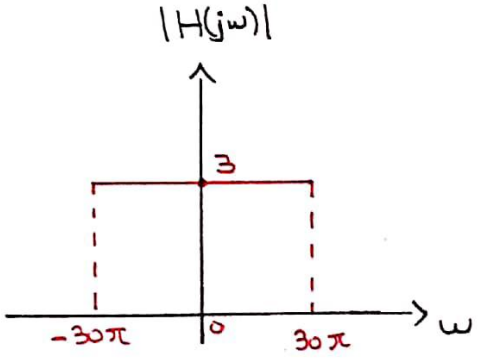
$$h(t) = g(t - 1/20)$$

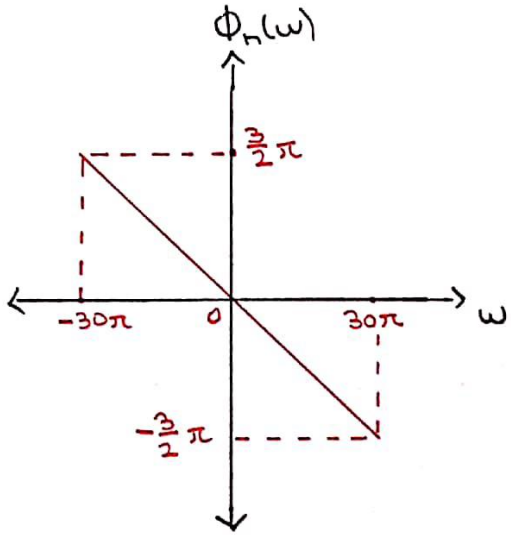
$$H(j\omega) = \left(e^{-j\omega/20}\right) 3 \text{ rect}\left(\frac{\omega}{60\pi}\right)$$

- $|H(j\omega)| = |G(j\omega)|$ and hence, magnitude spectrum is the same.

- $\phi_h(\omega) = -\frac{\omega}{20}$

- Spectrum of $H(j\omega)$ are as follows:





- Lastly for $f(t)$, we multiply $h(t)$ with $\cos(300\pi t)$.

$$f(t) = h(t)\cos(300\pi t)$$

$$= h(t)\left[\frac{e^{j300\pi t} + e^{-j300\pi t}}{2}\right]$$

$$F(j\omega) = \frac{1}{2}\left[H(j(\omega - 300\pi)) + H(j(\omega + 300\pi))\right]$$

$$F(j\omega) = \frac{1}{2}\left[e^{-j\omega/20}\ 3\ \text{rect}\left(\frac{\omega - 300\pi}{60\pi}\right) + e^{-j\omega/20}\ 3\ \text{rect}\left(\frac{\omega + 300\pi}{60\pi}\right)\right]$$

- Spectrum of $F(j\omega)$