National University of Sciences & Technology
School of Electrical Engineering and Computer Science
Department of Basic Sciences

MATH-101: Calculus and Analytical Geometry (3+0): BEE2k20-ABC Fall 2020

Assignment 3	
CLO: 1 (Understand the concept of limit, continuity and derivative with its application to find extrema)	
Maximum Marks: 30	Instructor: Dr. Naila Amir
Announcement Date: 24^{th} December 2020	Due Date: 31^{st} December 2020

Instructions:

- Understanding the question is part of the assignment and copying is not allowed.
- Express your answer in the most simplified form. Direct calculations using calculator are not allowed, you need to show the detail of your work to get the maximum marks.
- This is an individual assignment.
- Assignment must be handwritten and properly scanned in a single pdf file. These two pages must be part of every assignment.
- Assignment must be properly tagged and is required to be submitted on MS teams.
- Assignment is not acceptable after deadline.

Tasks: Attempt all questions.

Students Name	CMS Id.	Section
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Total Marks	**Marks Obtained**	**Weight in 10**
30 Marks		

Q - 1: [CLO-1: 20 marks]

For the function:

$$f(x) = \frac{2+x^3}{25-x^2}\,.$$

a) Determine the domain of $f(x)$. (Note: write domain in interval notation)

b) x & y −Intercepts, vertical/horizontal/oblique asymptotes, and holes (if any).

c) Determine whether $f(x)$ is continuous on its domain and, if not, find and classify the discontinuities.

d) Determine the critical points and use the first derivative test to find local extrema. Employ the sign of $f'(x)$ to find intervals on which $f(x)$ is increasing or is decreasing.

e) Find the points of inflection for the given function and determine the concavity of $f(x)$.

f) Sketch the graph of $f(x)$.

Q - 2: [CLO-1: 10 marks]

A topless rectangular box with a square base has a volume of $1926\ cm^3$. The material for the base costs 3 dollars per cm^2, and the material for the sides cost 2 dollars per cm^2. What should be the dimensions of box to minimize its cost?

Q1

$$f(x) = \frac{2+x^3}{25-x^2}$$

a) Domain : $(-\infty,-5) \cup (-5,5) \cup (5,\infty)$

b) x, y intercepts:

→ For x intercept $(x,0)$: $\frac{2+x^3}{25-x^2} = 0$

$$x = \sqrt[3]{-2}$$

→ For y intercept $(0,y)$: $y = \frac{2+0}{25-0}$

$$y = \frac{2}{25}$$

→ Asymptotes :

- Vertical : $25 - x^2 = 0$

$$\boxed{x = \pm 5}$$

The numerator is not zero for either of these, hence, these are the vertical asymptotes.

- Horizontal : None
- Oblique :

$$\begin{array}{r} -x \\ 25-x^2 \overline{)\, 2+\cancel{x^3}} \\ \underline{-25x+\cancel{x^3}} \\ 25x+2 \end{array}$$

$$-x + \frac{25x+2}{25-x^2}$$

The rational term approaches 0 as limit approaches infinity.
Hence,

$\boxed{y = -x}$ is the oblique asymptote.

→ Continuity: Yes, the function is fully continuous on its whole domain. There is an essential discontinuity on both $x = -5$ and $x = 5$ but it is not part of the domain.

- Critical Points:

$$f'(x) = \frac{(3x^2)(25-x^2) - (2+x^3)(-2x)}{(25-x^2)^2}$$

$$= \frac{25(3x^2) - 3x^4 + 4x + 2x^4}{(25-x^2)^2}$$

$$= \frac{-x^4 + 75x^2 + 4x}{(25-x^2)^2}$$

Solving $-x^4 + 75x^2 + 4x = 0$

Since 0 is a root, we divide by x.

$$-x^3 + 75x + 4 = 0$$

Critical Points:

$x = 0$, $x = -8.63$, $x = 8.686$, $x = -0.0533$

- Increasing and Decreasing Intervals:

$(-\infty, -8.63) \cup (-8.63, -5) \cup (-5, -0.0533) \cup (-0.0533, 0)$ *

$\searrow -$ $\nearrow +$ $\nearrow +$ $\searrow -$

* $(0, 5) \cup (5, 8.686) \cup (8.686, \infty)$

$\nearrow +$ $\nearrow +$ $\searrow -$

- Inflection Point:

$$f'(x) = \frac{x(-x^3 + 75x + 4)}{(25 - x^2)^2}$$

$$= \frac{-x^4 + 75x^2 + 4x}{(25 - x^2)^2}$$

$$f''(x) = \frac{(-x^3 + 75x + 4)(25 - x^2)^2 - 2(25 - x^2)(-2x)(-x^4 + 75x^2 + 4x)}{(25 - x^2)^4}$$

$$= \frac{(25 - x^2)(-x^3 + 75x + 4) + 4x(-x^4 + 75x^2 + 4x)}{(25 - x^2)^3}$$

$$= \frac{(-x^3 + 75x + 4)(25 - x^2 + 4x^2)}{(25 - x^2)^3}$$

$$= \frac{(-x^3 + 75x + 4)(25 + 3x^2)}{(25 - x^2)^3}$$

$$= \frac{2(25x^3 + 6x^2 + 1875x + 50)}{(25 - x^2)^3}$$

$f''(x) = 0$

$x = -0.0266$

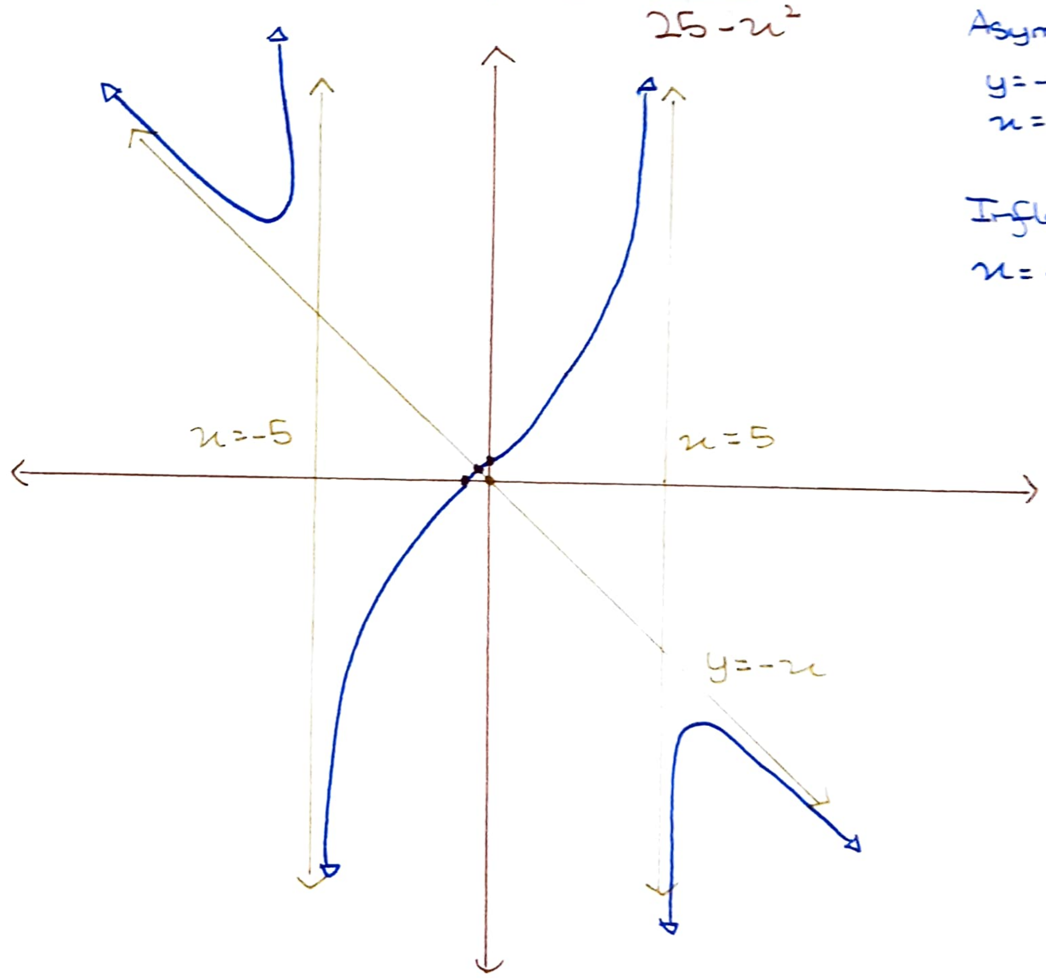
$f''(x) = \text{undefined}$

$x = \pm 5$

Intervals:

$(-\infty,-5)$	$\cup(-5,-0.0266)$	$\cup[-0.0266]$	$\cup(-0.0266,5)$	$\cup(5,\infty)$
+	−	0	+	−
		Inflection		
Upward	Downward		Upward	Downward

Graph: General Shape of $y=\dfrac{2+x^3}{25-x^2}$



Asymptotes:-

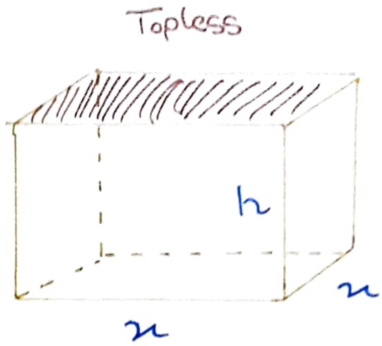
$y=-x$

$x=\pm 5$

Inflection:-

$x=-0.0266$

Q2



Volume $= x^2h$

Given, $V = 1926 \text{ cm}^3$

$$x^2h = 1926$$

$$\boxed{h = \frac{1926}{x^2}}$$

Cost function is;

$$C(x) = 3x^2 + 2(4xh)$$
$$= 3x^2 + 8xh$$
$$= 3x^2 + 8\left(\frac{1926}{x}\right)$$

$$\boxed{C(x) = 3x^2 + \frac{15408}{x}}$$

Taking the first derivative to find critical point.

$$C'(x) = 6x - \frac{15408}{x^2}$$

$$\therefore C'(x) = 0$$

$$6x - \frac{15408}{x^2} = 0$$

$$6x^3 = 15408$$

$$x^3 = 2568$$

$$\boxed{x = 13.694 \text{ cm}}$$

Using 2nd derivative to find the behaviour.

$$C''(x) = 6 + \frac{30816}{x^3}$$

$$C''(13.694) = 18 \quad > 0$$

As $C''(13.694) > 0$, it is a minima.

Using $x = 13.694$ to find h.

$\Rightarrow h = \frac{1926}{x^2}$

$h = 10.27$ cm

Hence, for minimum cost, square base's length should be 13.694 cm and the height, 10.27 cm.
