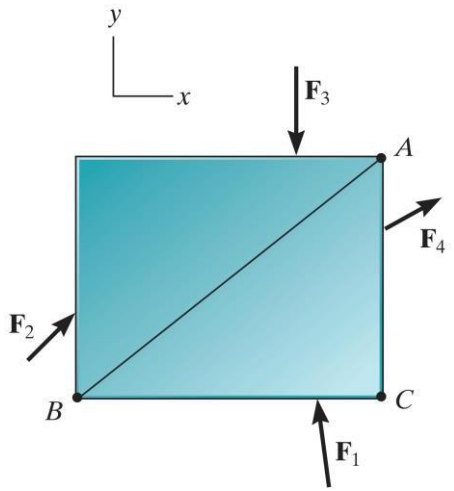
Engineering Mechanics: Statics

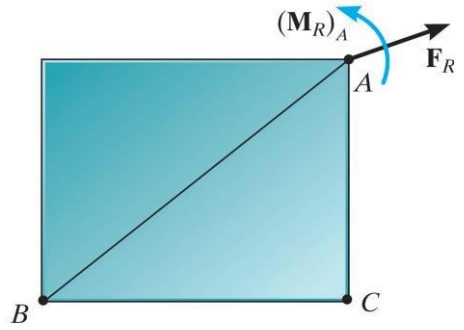
Chapter 5: Equilibrium of a Rigid Body

5.3 Equation of Equilibrium

$$\Sigma F_x = 0$$
$$\Sigma F_y = 0$$
$$\Sigma M_O = 0$$



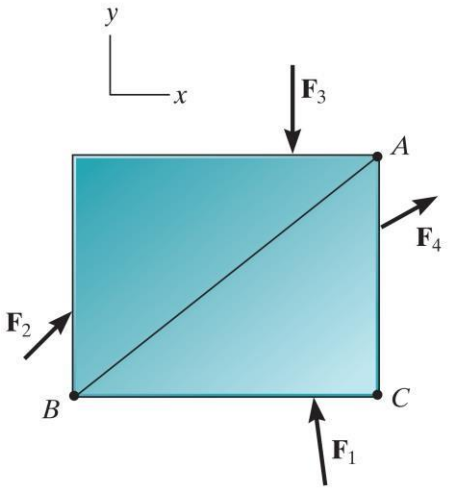
(a)



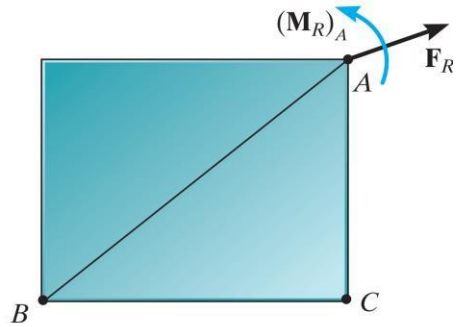
(b)

Equation of Equilibrium

$$\Sigma F_x = 0$$
$$\Sigma F_y = 0$$
$$\Sigma M_O = 0$$



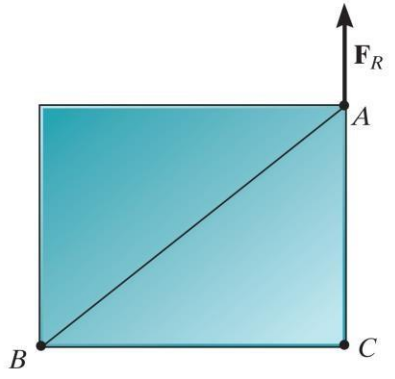
(a)



(b)

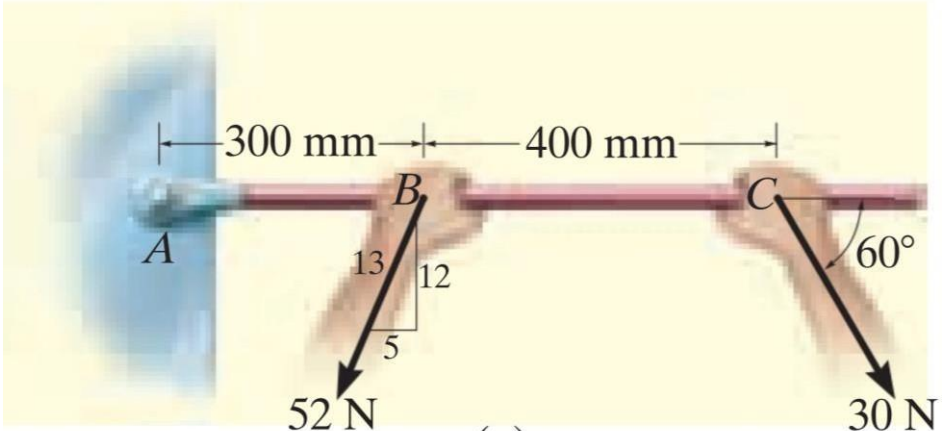
Alternative Sets of Equilibrium Equations.

$$\Sigma F_x = 0 \qquad \Sigma M_A = 0$$
$$\Sigma M_A = 0 \qquad \Sigma M_B = 0$$
$$\Sigma M_B = 0 \qquad \Sigma M_C = 0$$



Example

The box wrench is used to tighten the bolt at *A*. If the wrench does not turn when the load is applied to the handle, determine the torque or moment applied to the bolt and the force of the wrench on the bolt.



(a)

Equations of Equilibrium.

$$\overset{+}{\rightarrow}\Sigma F_x = 0; \qquad A_x - 52\left(\tfrac{5}{13}\right)\text{N} + 30\cos 60^\circ\,\text{N} = 0$$

$$A_x = 5.00\,\text{N} \qquad \textit{Ans.}$$

$$+\uparrow\Sigma F_y = 0; \qquad A_y - 52\left(\tfrac{12}{13}\right)\text{N} - 30\sin 60^\circ\,\text{N} = 0$$

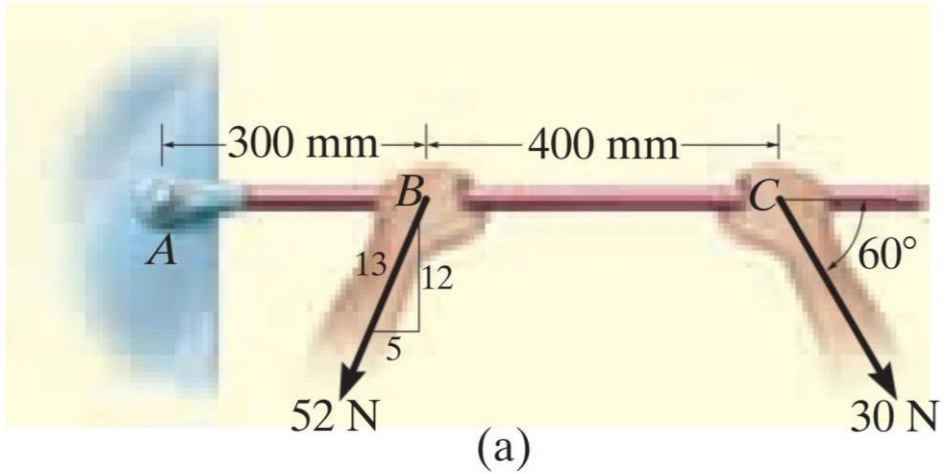
$$A_y = 74.0\,\text{N} \qquad \textit{Ans.}$$

$$\curvearrowleft+\Sigma M_A = 0; \quad M_A - \left[52\left(\tfrac{12}{13}\right)\text{N}\right](0.3\,\text{m}) - (30\sin 60^\circ\,\text{N})(0.7\,\text{m}) = 0$$

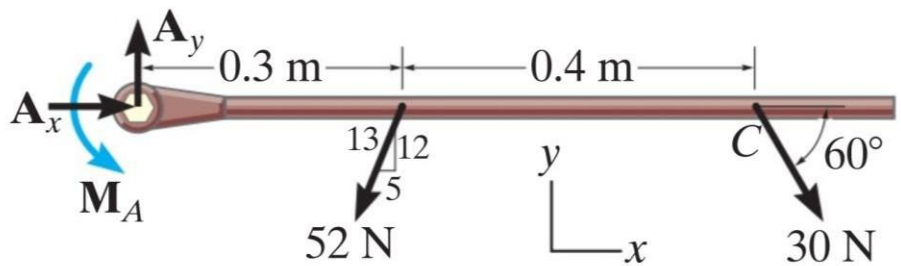
$$M_A = 32.6\,\text{N}\cdot\text{m} \qquad \textit{Ans.}$$

Note that $\mathbf{M}_A$ must be *included* in this moment summation. This couple moment is a free vector and represents the twisting resistance of the bolt on the wrench. By Newton's third law, the wrench exerts an equal but opposite moment or torque on the bolt. Furthermore, the resultant force on the wrench is

$$F_A = \sqrt{(5.00)^2 + (74.0)^2} = 74.1\,\text{N} \qquad \textit{Ans.}$$



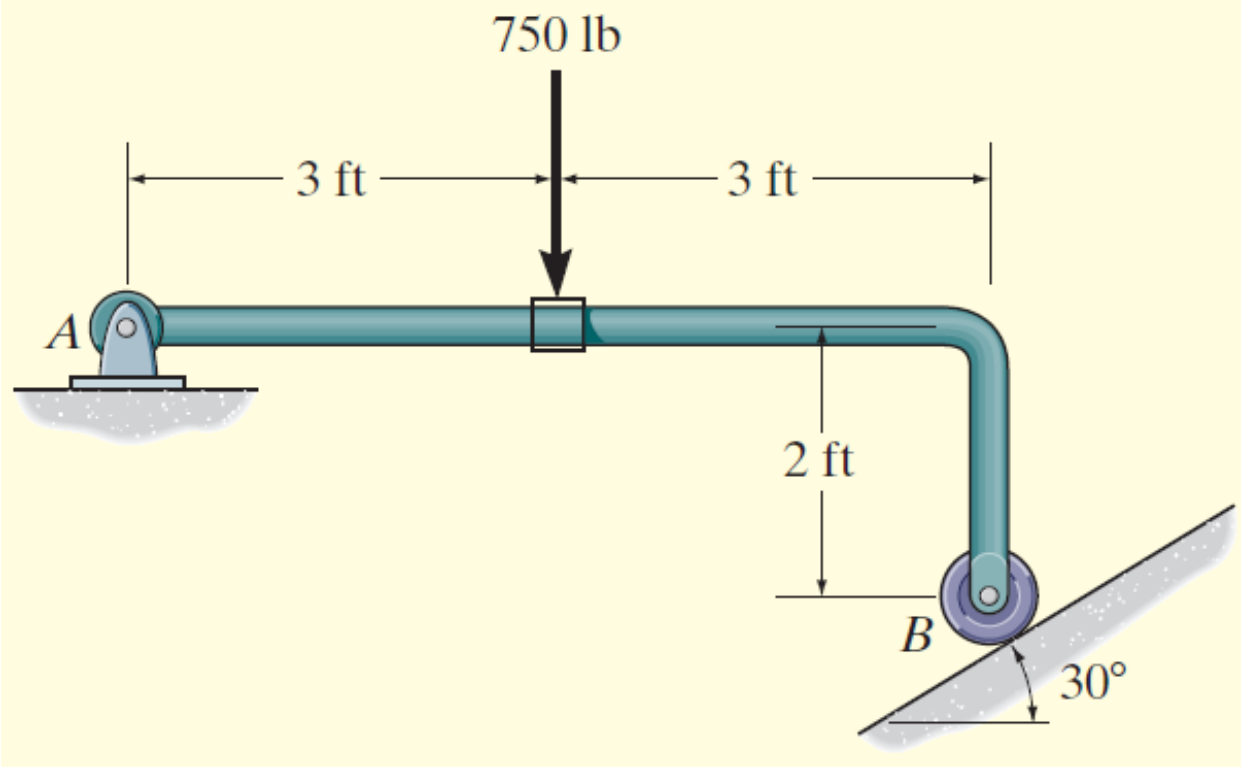
(a)



(b)

Example

Determine the horizontal and vertical components of reaction on the member at the pin A, and the normal reaction at the roller B in Fig. 5–16*a*.



$$\circlearrowleft + \Sigma M_A = 0;$$

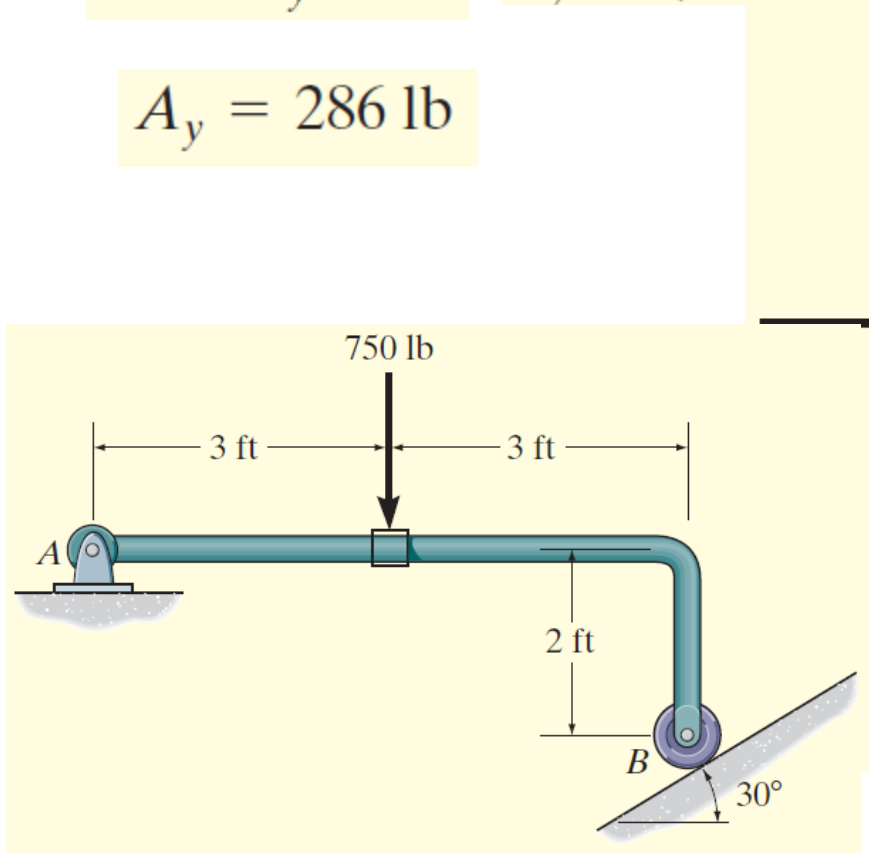
$$[N_B \cos 30°](6 \text{ ft}) - [N_B \sin 30°](2 \text{ ft}) - 750 \text{ lb}(3 \text{ ft}) = 0$$

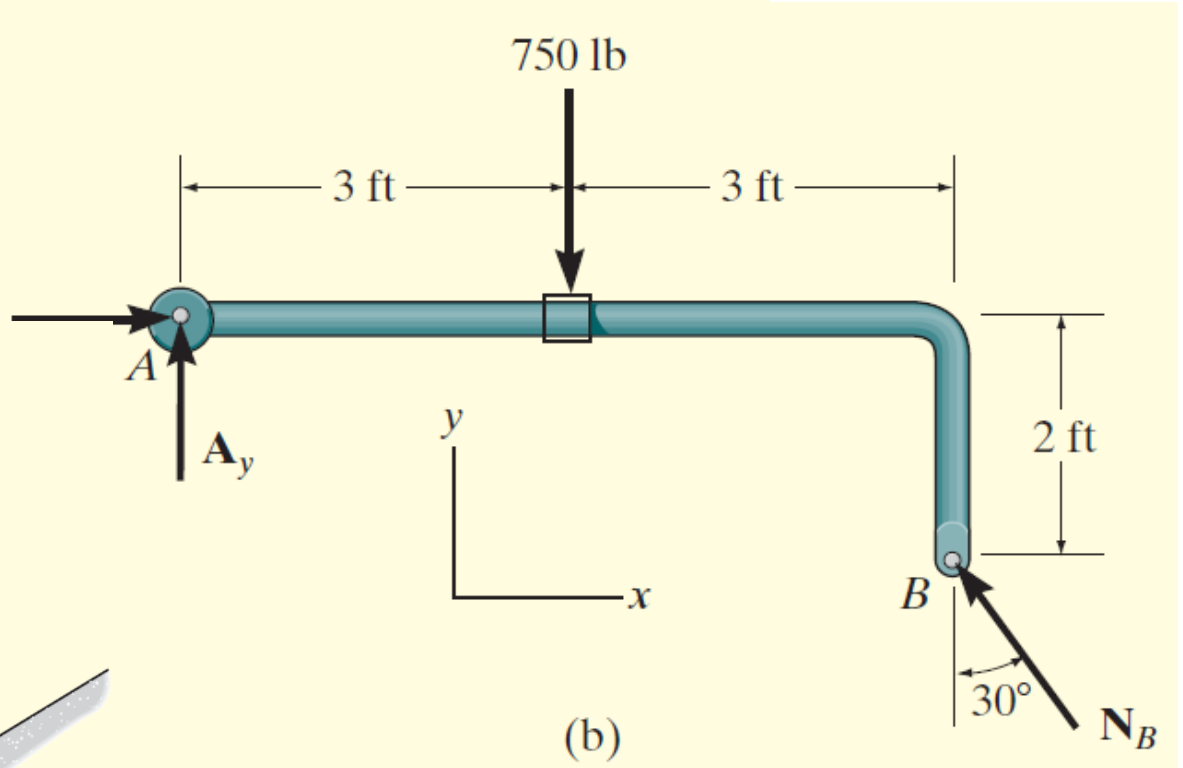
$$N_B = 536.2 \text{ lb} = 536 \text{ lb}$$

$$\pm\rightarrow \Sigma F_x = 0; \quad A_x - (536.2 \text{ lb}) \sin 30° = 0 \quad A_x = 268 \text{ lb}$$

$$+\uparrow \Sigma F_y = 0; \quad A_y + (536.2 \text{ lb}) \cos 30° - 750 \text{ lb} = 0$$

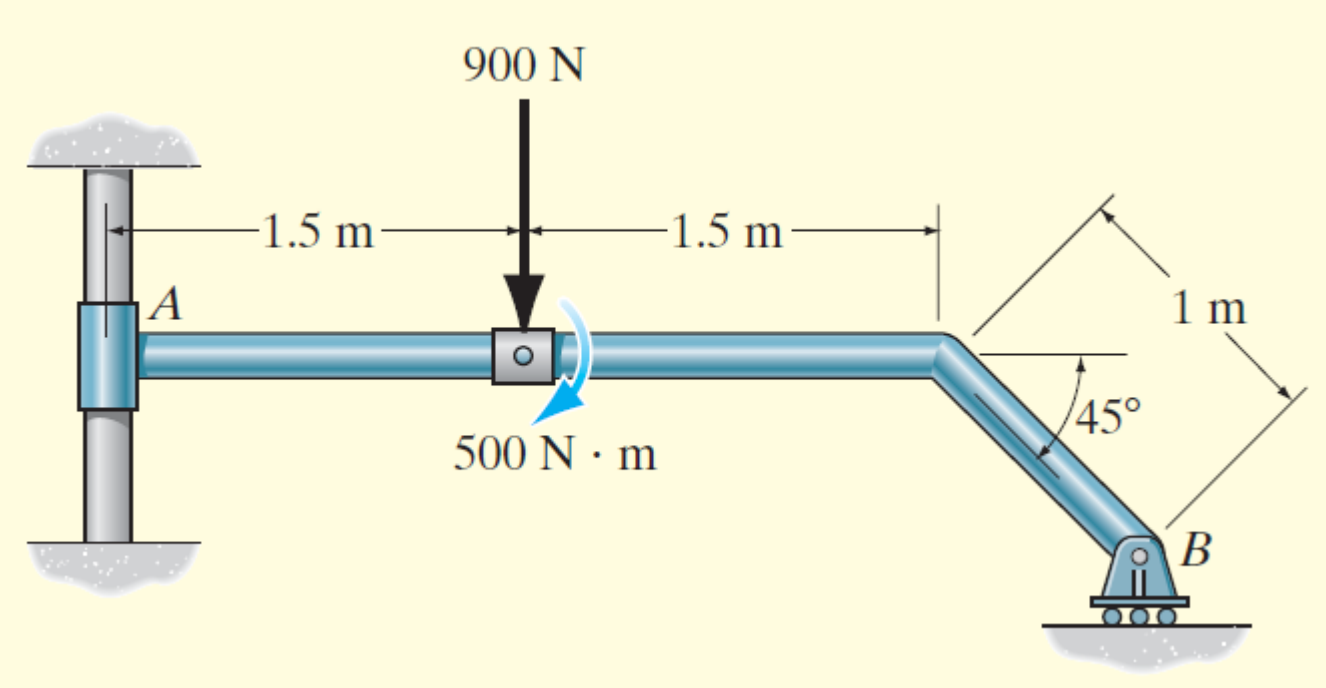
$$A_y = 286 \text{ lb}$$





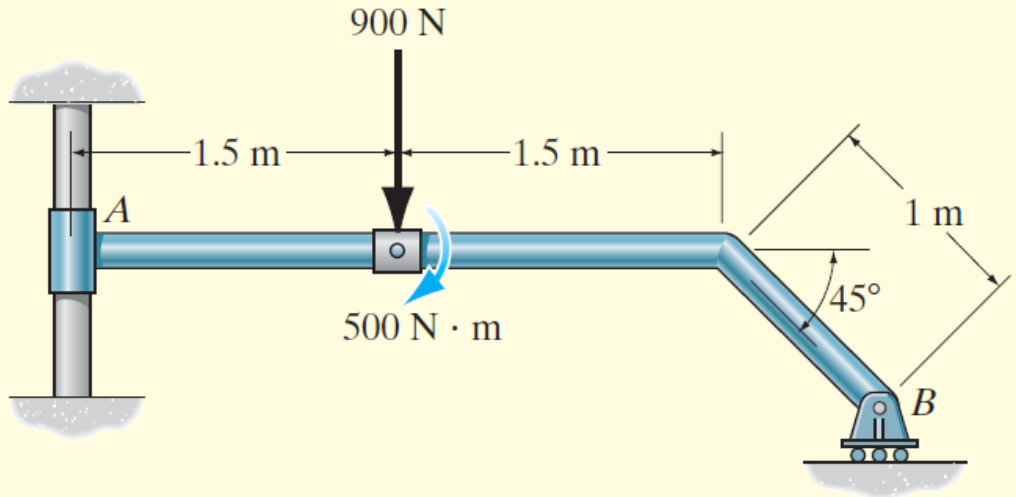
Example

Determine the support reactions on the member in Fig. 5–19*a*. The collar at *A* is fixed to the member and can slide vertically along the vertical shaft.



$$\pm\rightarrow \Sigma F_x = 0; \quad A_x = 0$$

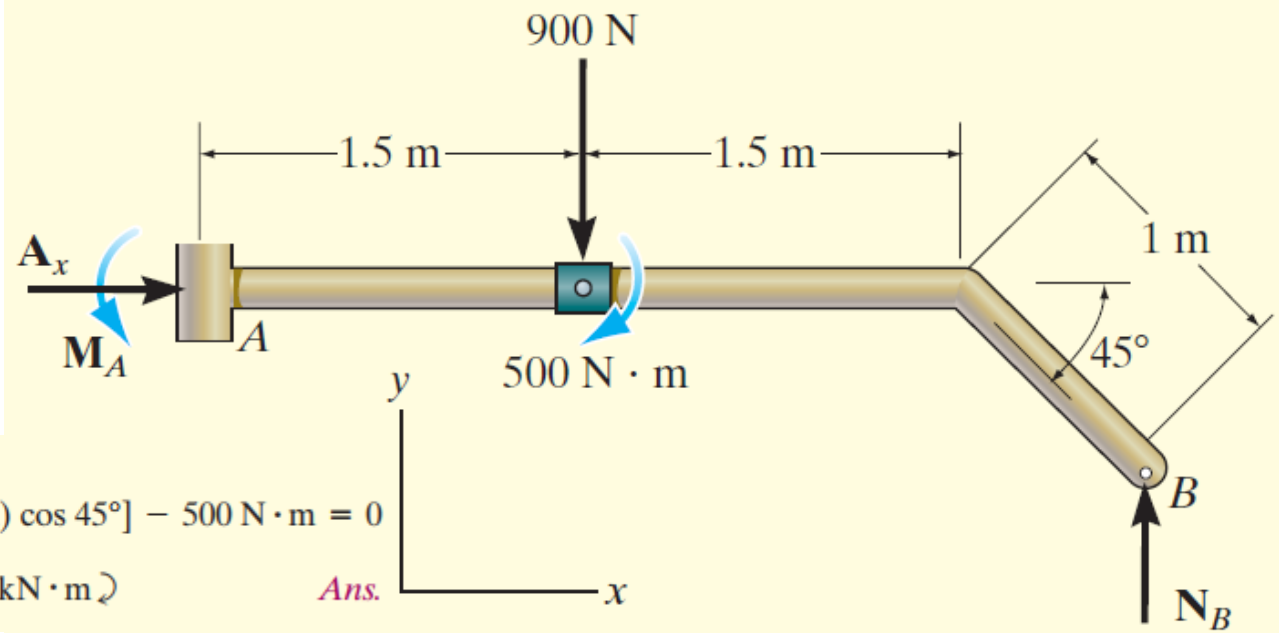
$$+\uparrow \Sigma F_y = 0; \quad N_B - 900\ \text{N} = 0$$



$$\circlearrowleft + \Sigma M_A = 0;$$

$$M_A - 900\ \text{N}(1.5\ \text{m}) - 500\ \text{N}\cdot\text{m} + 900\ \text{N}\,[3\ \text{m} + (1\ \text{m})\cos 45°] = 0$$

$$M_A = -1486\ \text{N}\cdot\text{m} = 1.49\ \text{kN}\cdot\text{m}\circlearrowright$$



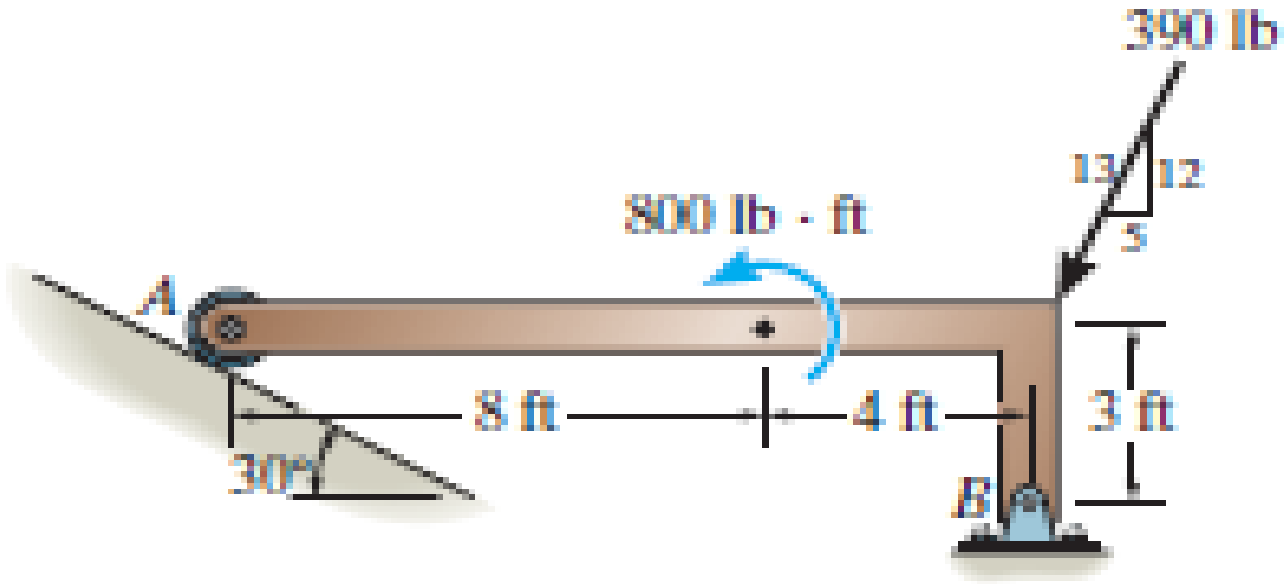
or

$\circlearrowleft + \Sigma M_B = 0;\ M_A + 900\ \text{N}\,[1.5\ \text{m} + (1\ \text{m})\cos 45°] - 500\ \text{N}\cdot\text{m} = 0$

$M_A = -1486\ \text{N}\cdot\text{m} = 1.49\ \text{kN}\cdot\text{m}\circlearrowright$ *Ans.*

Problem

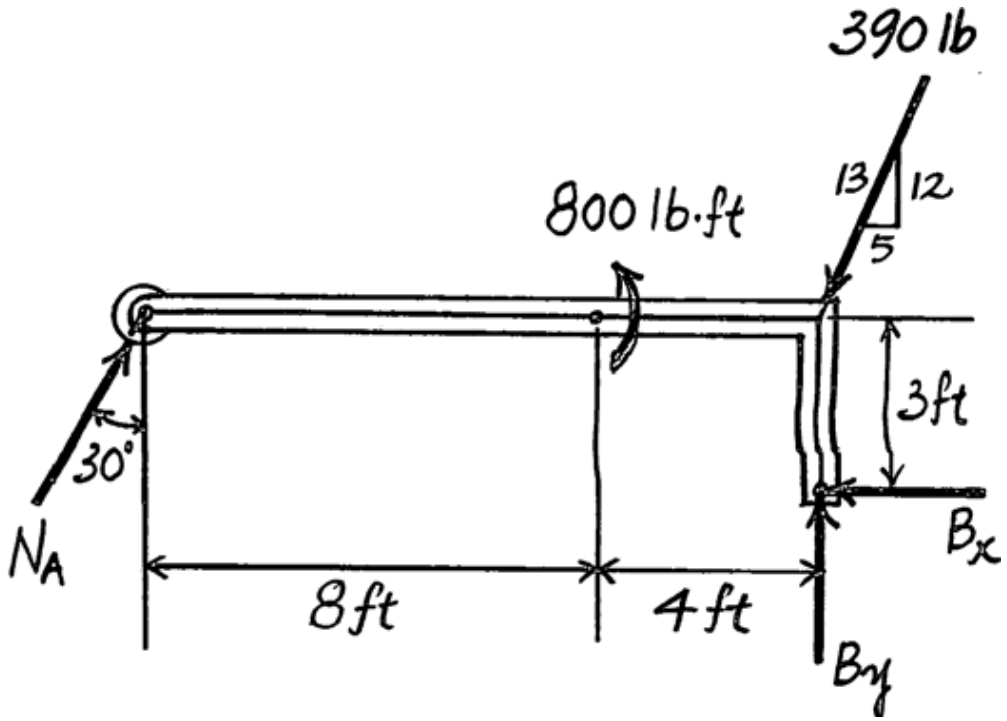
5–2. Draw the free-body diagram of member AB, which is supported by a roller at A and a pin at B. Explain the significance of each force on the diagram. (See Fig. 5–7*b*.)



Prob. 5–2

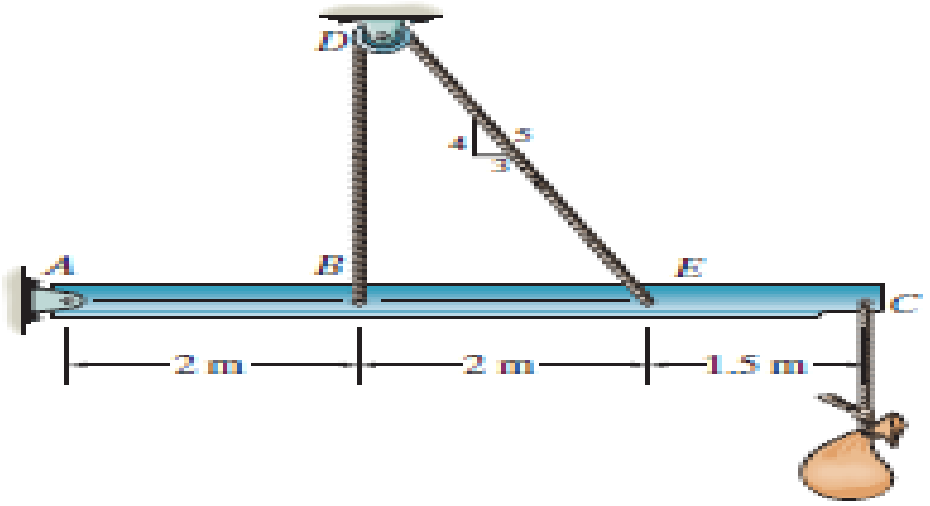
FBD

N_A force of plane on roller.
B_x, B_y force of pin on member.



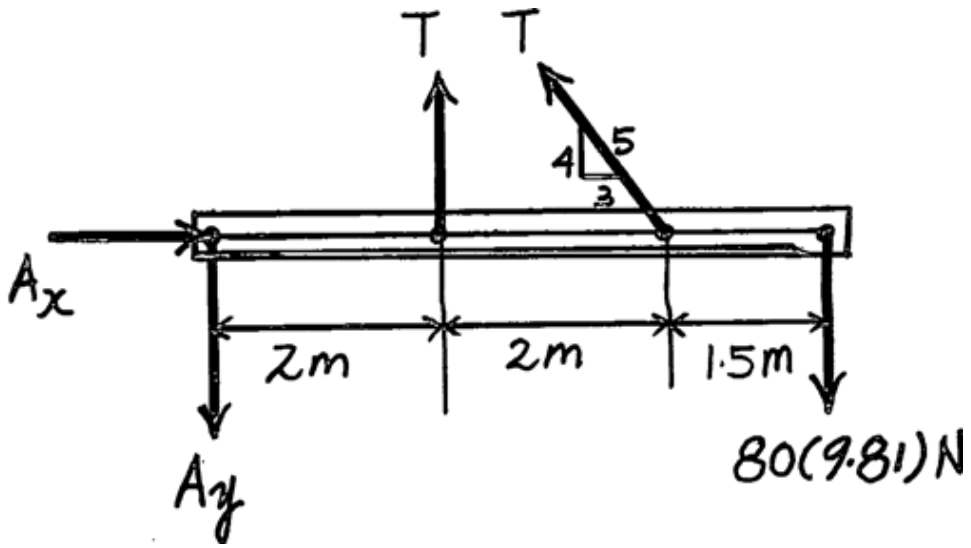
Problem

***5–4.** Draw the free-body diagram of the beam which supports the 80-kg load and is supported by the pin at A and a cable which wraps around the pulley at D. Explain the significance of each force on the diagram. (See Fig. 5–7*b*.)



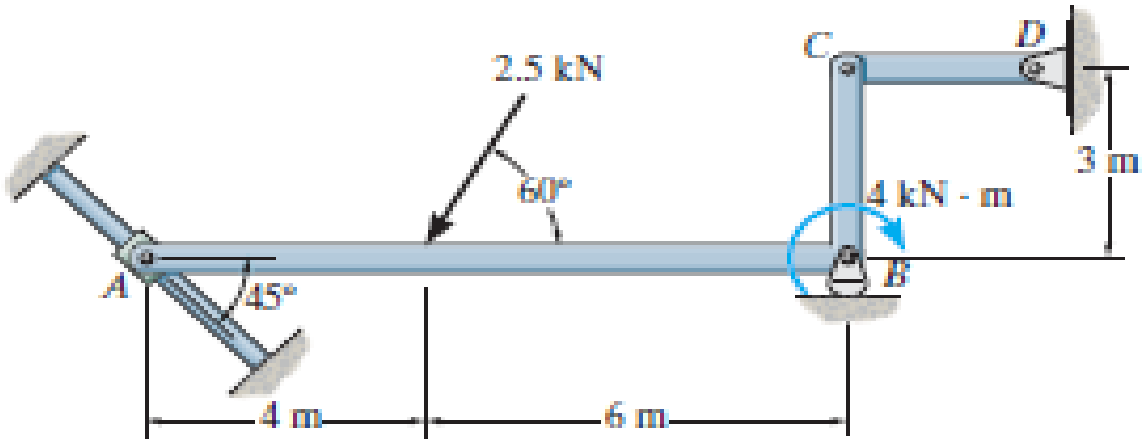
FBD

T force of cable on beam.
A_x, A_y force of pin on beam.
80(9.81)N force of cable on beam.



Problem

***5–8.** Draw the free-body diagram of member ABC which is supported by a smooth collar at A, roller at B, and short link CD. Explain the significance of each force acting on the diagram. (See Fig. 5–7*b*.)



Prob. 5–8

FBD

The Significance of Each Force :

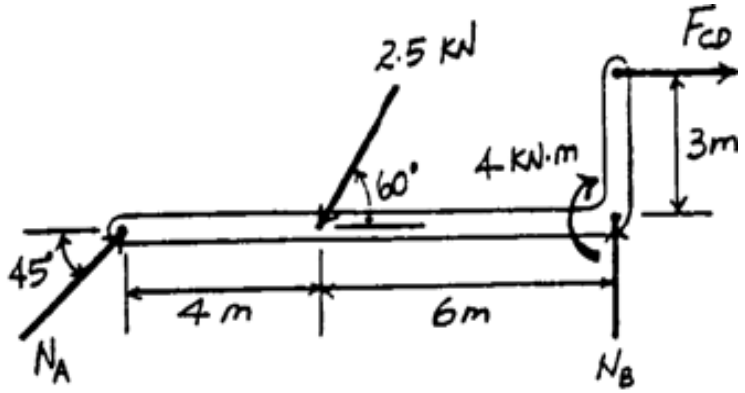
N_A is the smooth collar reaction on member ABC.

N_B is the roller support B reaction on member ABC.

F_{CD} is the short link reaction on member ABC.

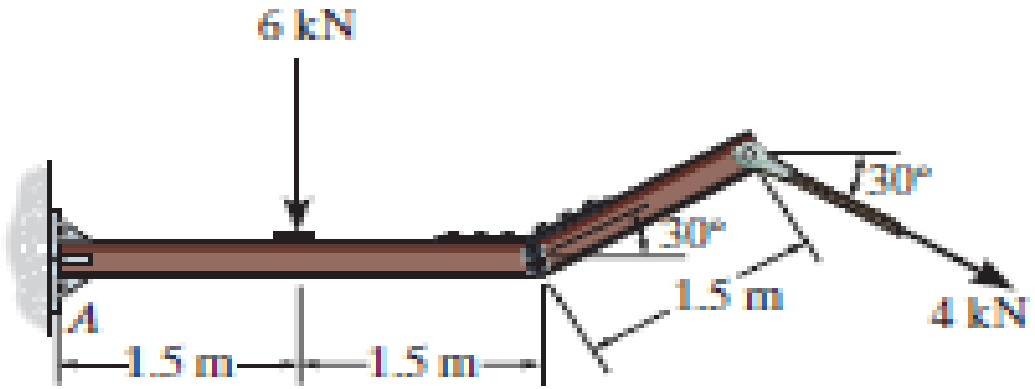
2.5 kN is the effect of external applied force on member ABC.

4 kN · m is the effect of external applied couple moment on member ABC.



Problem

5–10. Determine the components of the support reactions at the fixed support A on the cantilevered beam.



Prob. 5–10

SOLUTION

Equations of Equilibrium: From the free-body diagram of the cantilever beam, Fig. *a*, A_x, A_y, and M_A can be obtained by writing the moment equation of equilibrium about point A.

$$\stackrel{+}{\rightarrow} \Sigma F_x = 0; \qquad 4\cos 30^\circ - A_x = 0$$

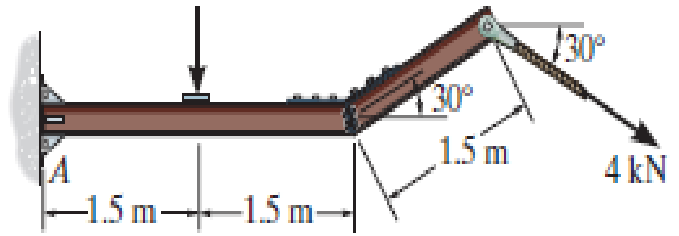
$$A_x = 3.46\ \text{kN} \qquad \textbf{Ans.}$$

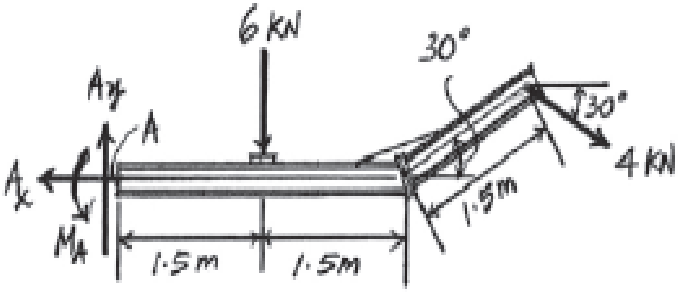
$$+\uparrow \Sigma F_y = 0; \qquad A_y - 6 - 4\sin 30^\circ = 0$$

$$A_y = 8\ \text{kN} \qquad \textbf{Ans.}$$

$$\zeta + \Sigma M_A = 0; M_A - 6(1.5) - 4\cos 30^\circ\,(1.5\sin 30^\circ) - 4\sin 30^\circ(3 + 1.5\cos 30^\circ) = 0$$

$$M_A = 20.2\ \text{kN}\cdot\text{m} \qquad \textbf{Ans.}$$

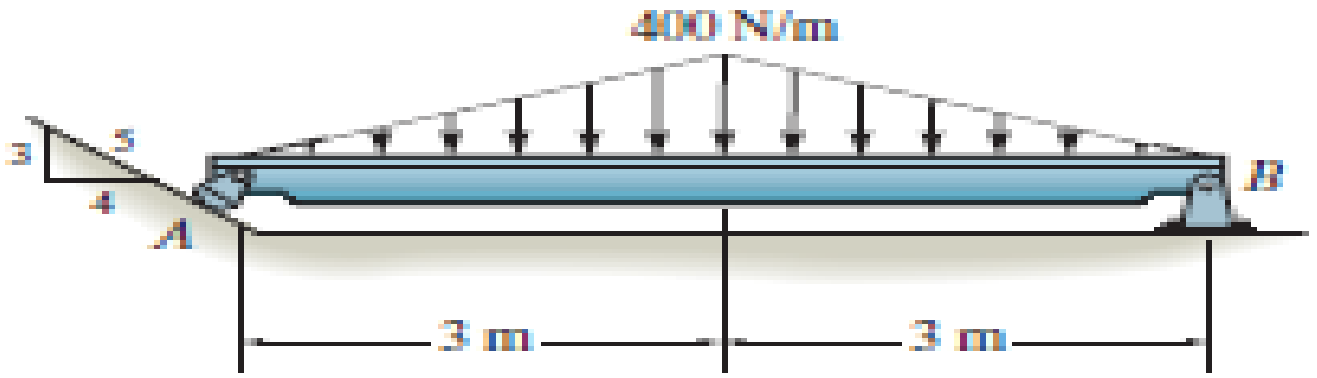




(a)

Problem

5–11. Determine the reactions at the supports.



Prob. 5–11

SOLUTION

Equations of Equilibrium. $\mathbf{N}_A$ and $\mathbf{B}_y$ can be determined directly by writing the moment equations of equilibrium about points B and A, respectively, by referring to the beam's FBD shown in Fig. a.

$$\zeta + \Sigma M_B = 0; \qquad \frac{1}{2}(400)(6)(3) - N_A\left(\frac{4}{5}\right)(6) = 0$$

$$N_A = 750 \text{ N} \qquad \textbf{Ans.}$$

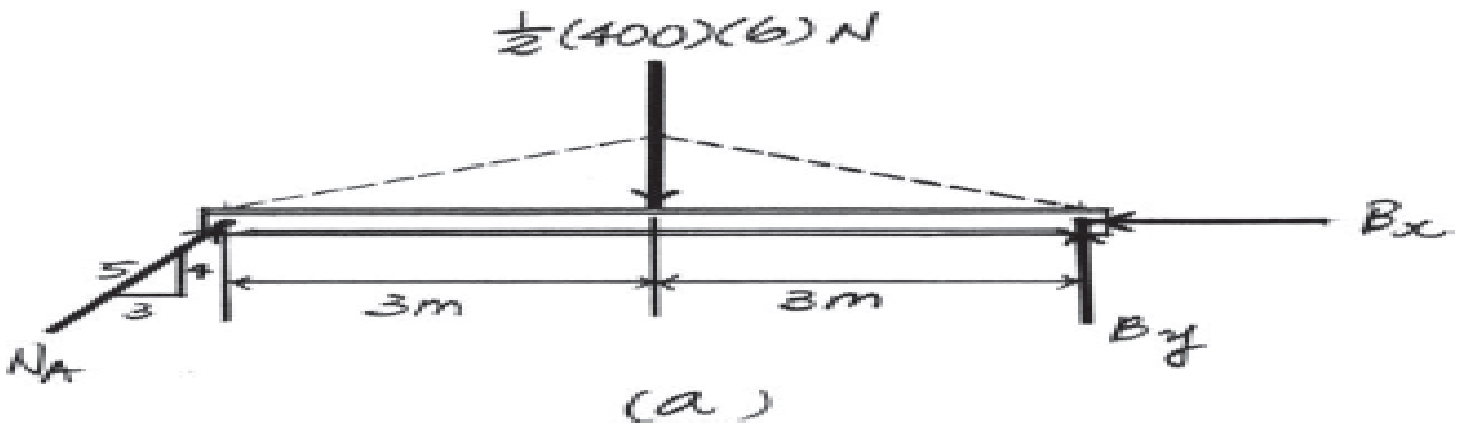
$$\zeta + \Sigma M_A = 0; \qquad B_y(6) - \frac{1}{2}(400)(6)(3) = 0$$

$$B_y = 600 \text{ N} \qquad \textbf{Ans.}$$

Using the result of $\mathbf{N}_A$ to write the force equation of equilibrium along the x axis,

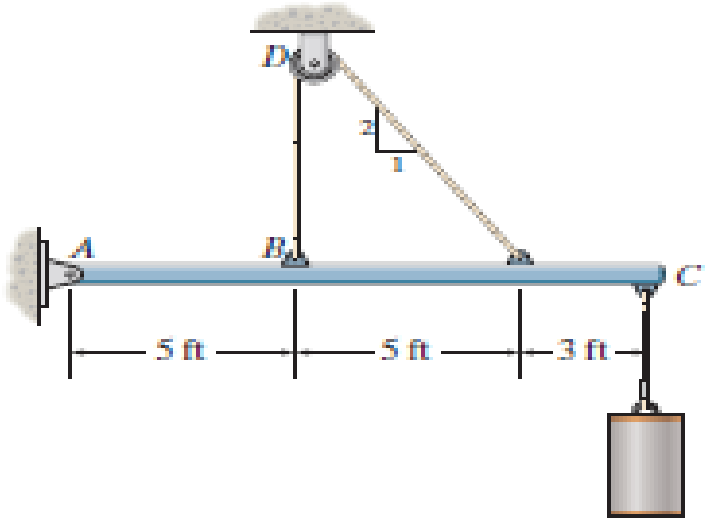
$$\pm \Sigma F_x = 0; \qquad 750\left(\frac{3}{5}\right) - B_x = 0$$

$$B_x = 450 \text{ N} \qquad \textbf{Ans.}$$



Problem

***5–16.** Determine the tension in the cable and the horizontal and vertical components of reaction of the pin A. The pulley at D is frictionless and the cylinder weighs 80 lb.



Prob. 5–16

SOLUTION

Equations of Equilibrium: The tension force developed in the cable is the same throughout the whole cable. The force in the cable can be obtained directly by summing moments about point A.

$$\zeta + \Sigma M_A = 0; \quad T(5) + T\left(\frac{2}{\sqrt{5}}\right)(10) - 80(13) = 0$$

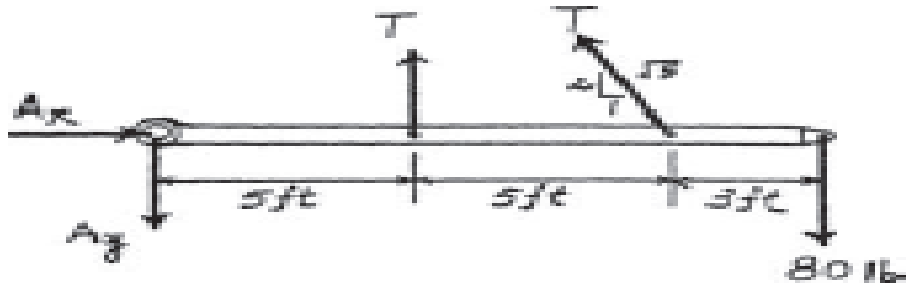
$$T = 74.583 \text{ lb} = 74.6 \text{ lb} \qquad \textbf{Ans.}$$

$$\pm \Sigma F_x = 0; \quad A_x - 74.583\left(\frac{1}{\sqrt{5}}\right) = 0$$

$$A_x = 33.4 \text{ lb} \qquad \textbf{Ans.}$$

$$+\uparrow \Sigma F_y = 0; \quad 74.583 + 74.583\left(\frac{2}{\sqrt{5}}\right) - 80 - B_y = 0$$

$$A_y = 61.3 \text{ lb} \qquad \textbf{Ans.}$$



Home Assignment

- F5-1, F5-2 & F5-4.