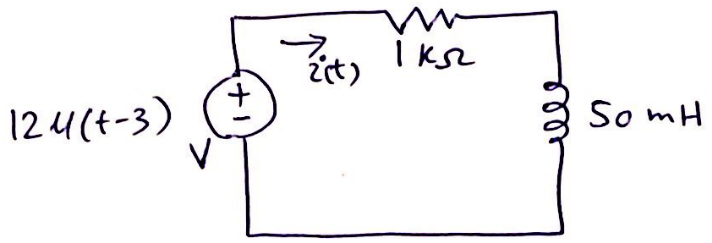
Example 8·7 Driven RL Circuits

(PP 288 8th Ed HKD) (PP288)

Find $i(t)$ for $t = \infty, 3^-, 3^+$ and 100 μs after the source changes value. (Note: Source changes value at $t = 3^+$)



Note: The source changes value at $t = 3$; $t = 3^-$ indicates the instant just before the source changes value. Also $t = 3^+$ indicates the instant just after the source changes value.

Solution: At $t = \infty$:

$$i(\infty) = \frac{12}{1 \times 10^3} = 12\,mA$$

At $t = 3^-$

$$i(3^-) = 0$$

At $t = 3^+$

$$i(3^+) = 0$$

as inductor current cannot change instantaneously; although resistor current can.

We now determine

$$i(t);$$

contd

—contd (288)

The time constant $\tau = \dfrac{L_{eq}}{R_{eq}}$ (After all independent sources are eliminated)

$$= \frac{50\times10^{-3}}{1\times10^{3}} = 5\times10^{-6}\ s$$

or $\dfrac{1}{\tau} = 20{,}000$

We know for a dc driven RL circuit:

$$i(t) = i(\infty) + \Big[\underset{3^{+}}{i(0^{+})} - i(\infty)\Big]e^{-t/\tau}$$

$$i(t) = 12 + (0-12)\,e^{-20{,}000\,t}$$

So at $t = 0.0001\ s$ (i.e $100\times10^{-6}\ s$)

Recalling
$0.1 \rightarrow 10^{-1}$
$0.01 \quad 10^{-2}$
$0.001 \quad 10^{-3}$
$0.0001 \quad 10^{-4}$

So $i(0.0001\ s) = 12 - 12\,e^{-20{,}000\times\frac{1}{10000}}$

$$= 12 - 12e^{-2}$$

$$= 12 - 12\times0.135 \quad (e^{-2} = 0.135)$$

$$= 12 - 1.62 = 10.38\ mA$$

Note: If we replace t with $t-3$, then it will be $i(3.0001) = 10.38\ mA$