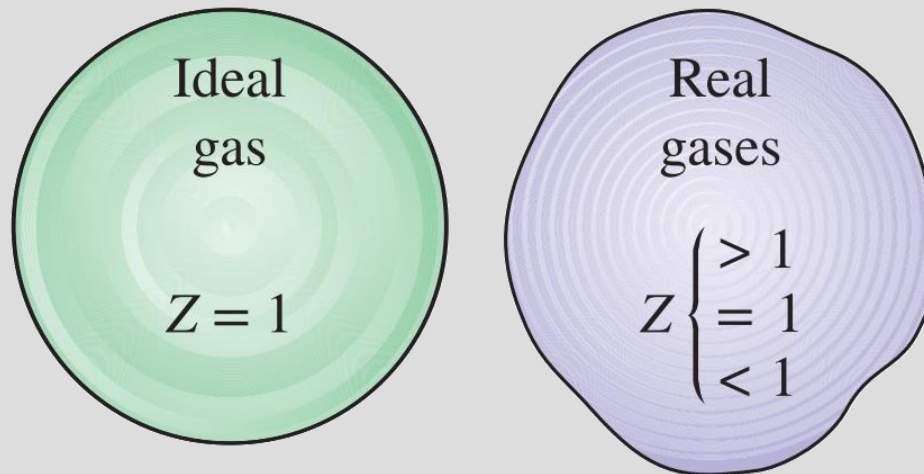Thermodynamics I

Lecture 15

Compressibility Chart (Ch-3)

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COMPRESSIBILITY FACTOR—A MEASURE OF DEVIATION FROM IDEAL-GAS BEHAVIOR

- The ideal-gas equation is very simple and thus very convenient to use.
- Gases deviate from ideal-gas behavior significantly at states near the **saturation region** and the **critical point**.

The compressibility factor is unity for ideal gases.

DEVIATION FROM IDEAL-GAS BEHAVIOR

- Deviation from ideal-gas behavior at a given temperature and pressure can accurately be accounted for by the introduction of a correction factor called the *compressibility factor, Z*:

$$Z = \frac{Pv}{RT} \implies Pv = ZRT \implies Z = \frac{v_{actual}}{v_{ideal}}$$

- where v_{ideal} = RT/P. Obviously, **Z = 1** for ideal gases.
- For **real gases** Z can be greater than or less than unity
- The **farther away Z is from unity**, the more the gas deviates from ideal-gas behavior.

Gases behave as an ideal gas at low densities (i.e., low pressure, high temperature).
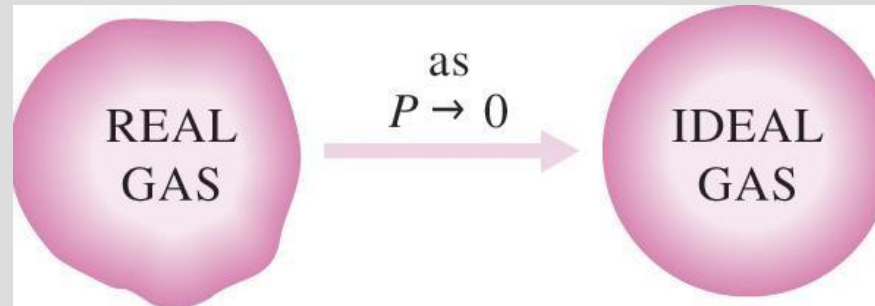
At very low pressures, all gases approach ideal-gas behavior (regardless of their temperature).

Is −100°C a low temperature?

**Question**: What is the criteria for low pressure   and high temperature?

Answer: The pressure or temperature of a gas   is high or low relative to its critical temperature or pressure.

Gases behave differently at a given temperature and pressure, but they behave very much the same at temperatures and pressures normalized with respect to their critical temperatures and pressures.

The normalization is done as

$$P_R = \frac{P}{P_{cr}} \text{ and } T_R = \frac{T}{T_{cr}}$$

Here P_R is called the **reduced pressure** and T_R the **reduced temperature**. The Z factor for all gases is approximately the same at the same reduced pressure and temperature. This is called the **principle of corresponding states**.
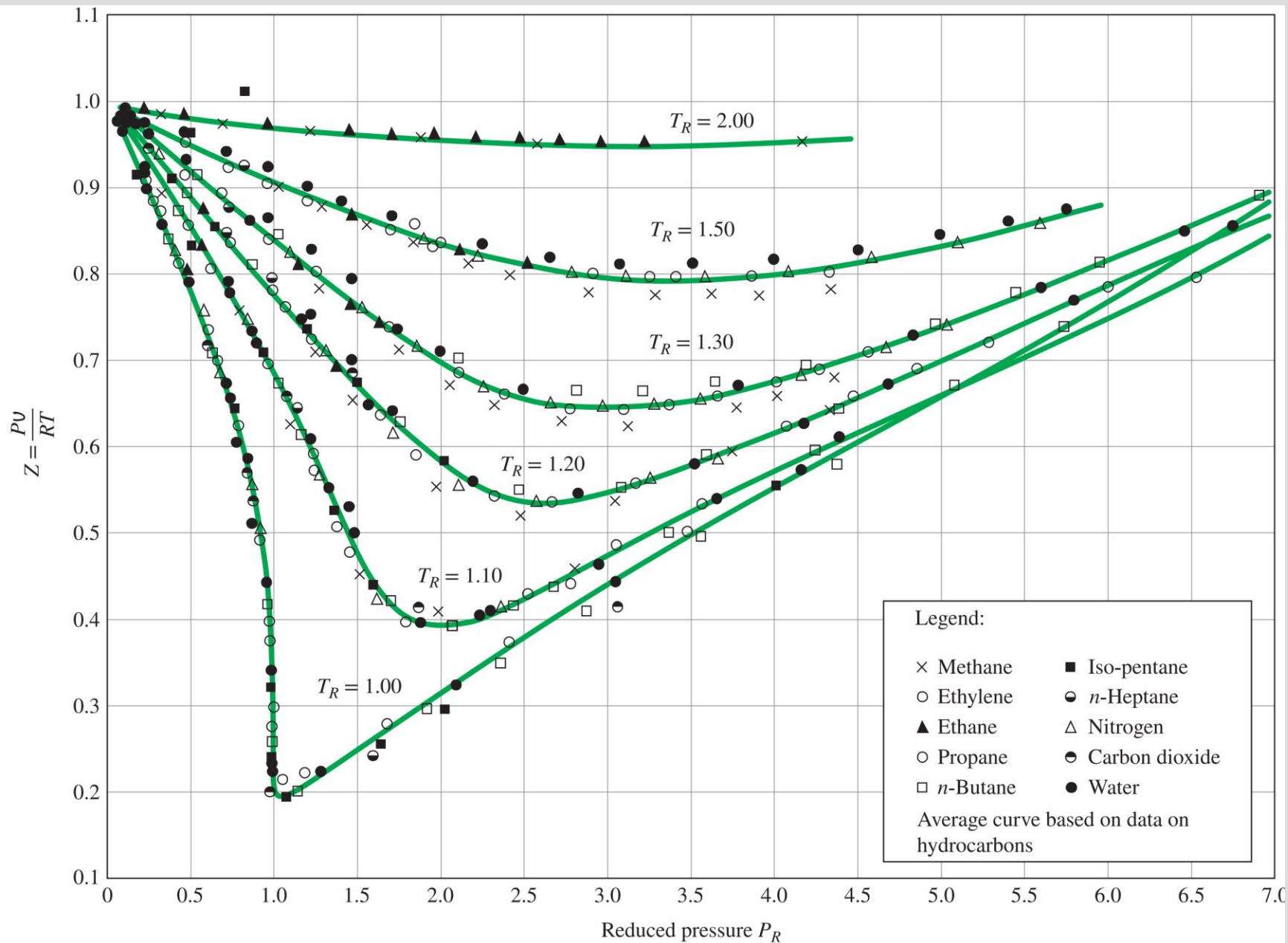
Comparison of Z factors for various gases.

The gases seem to obey the principle of corresponding states reasonably well. By curve-fitting all the data, we obtain the **generalized compressibility chart (A-15)** that can be used for all gases

The following observations can be made from the generalized compressibility chart:

1. At **very low pressures (PR << 1),** gases behave as ideal gases regardless of temperature.
2. At **high temperatures (TR > 2),** ideal-gas behavior can be assumed with good accuracy regardless of pressure (except when PR >> 1).
3. The **deviation of a gas** from ideal-gas behavior is greatest in the vicinity of the critical point.

T

Ideal-gas behavior

Nonideal-gas behavior

Ideal-gas behavior

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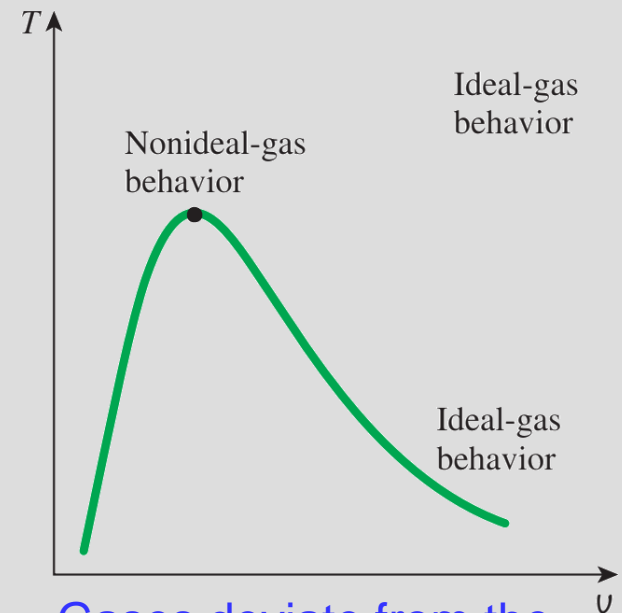
Gases deviate from the ideal-gas behavior the most in the neighborhood of the critical point.

- When **P** and **v**, or **T** and **v**, are given instead of **P** and **T**, the generalized compressibility chart can still be used to determine the third property, but it would involve tedious trial and error.
- It is necessary to define one more reduced properties called the **pseudo-reduced specific volume v$_R$** as

$$v_R = \frac{v_{actual}}{RT_{cr}/P_{cr}}$$

$$\left.\begin{array}{l} P_R = \dfrac{P}{P_{cr}} \\[2em] v_R = \dfrac{v}{RT_{cr}/P_{cr}} \end{array}\right\} \begin{array}{l} Z = \dots \\ (Fig.\ A\!-\!15) \end{array}$$

The compressibility factor can also be determined from a knowledge of P_R and v_R

Example 3-11

Determine the specific volume of refrigerant-134a at 1 MPa and 50°C, using (a) the ideal-gas equation of state and (b) the generalized compressibility chart. Compare the values obtained to the actual value of 0.021796 m³/kg and determine the error involved in each case.

$$R = 0.0815 \text{ kPa·m}^3/\text{kg·K} \qquad P_{cr} = 4.059 \text{ MPa} \qquad T_{cr} = 374.2 \text{ K}$$

(a) The specific volume of refrigerant-134a under the ideal-gas assumption is

$$v = \frac{RT}{P} = \frac{(0.0815 \text{ kPa·m}^3/\text{kg·K})(323 \text{ K})}{1000 \text{ kPa}} = \mathbf{0.026325 \text{ m}^3/\text{kg}}$$

(b) To determine the correction factor Z from the compressibility chart, we first need to calculate the reduced pressure and temperature:

$$\left.\begin{aligned} P_R &= \frac{P}{P_{cr}} = \frac{1 \text{ MPa}}{4.059 \text{ MPa}} = 0.246 \\ T_R &= \frac{T}{T_{cr}} = \frac{323 \text{ K}}{374.2 \text{ K}} = 0.863 \end{aligned}\right\} \quad Z = 0.84$$

Thus

$$v = Zv_{ideal} = (0.84)(0.026325 \text{ m}^3/\text{kg}) = \mathbf{0.022113 \text{ m}^3/\text{kg}}$$

Practice Problem

Determine the specific volume of superheated water vapor at 15 MPa and 350°C, using **(a)** the ideal-gas equation, **(b)** the generalized compressibility chart, and **(c)** the steam tables. Also determine the error involved in the first two cases.

$$R = 0.4615 \text{ kPa·m}^3/\text{kg·K},$$

$$v = \frac{RT}{P} = \frac{(0.4615 \text{ kPa·m}^3/\text{kg·K})(623.15 \text{ K})}{15{,}000 \text{ kPa}} = \mathbf{0.01917 \ m^3/kg}$$

$$T_{cr} = 647.1 \text{ K},$$

$$P_{cr} = 22.06 \text{ MPa}$$

$$\left.\begin{array}{l} P_R = \dfrac{P}{P_{cr}} = \dfrac{10 \text{ MPa}}{22.06 \text{ MPa}} = 0.453 \\[4mm] T_R = \dfrac{T}{T_{cr}} = \dfrac{673 \text{ K}}{647.1 \text{ K}} = 1.04 \end{array}\right\} \ Z = 0.65$$

$$v = Z v_{\text{ideal}} = (0.65)(0.01917 \text{ m}^3/\text{kg}) = \mathbf{0.01246\, m^3/kg}$$

$$\left.\begin{array}{l} P = 15 \text{ MPa} \\ T = 350°C \end{array}\right\} \ \boldsymbol{v = 0.01148 \ m^3/kg}$$

Practice Problem

Determine the pressure of water vapor at 600°F and 0.51431 ft³/lbm, using (*a*) the steam tables, (*b*) the ideal-gas equation, and (*c*) the generalized compressibility chart.

$$R = 0.5956 \text{ psia·ft}^3/\text{lbm·R} \qquad P_{cr} = 3200 \text{ psia} \qquad T_{cr} = 1164.8 \text{ R}$$

(*a*) The pressure at the specified state is determined from Table A–6E to be

$$\left. \begin{array}{l} v = 0.51431 \text{ ft}^3/\text{lbm} \\ T = 600°\text{F} \end{array} \right\} \quad P = \textbf{1000 psia}$$

(*b*) The pressure of steam under the ideal-gas assumption is determined from the ideal-gas relation to be

$$P = \frac{RT}{v} = \frac{(0.5956 \text{ psia·ft}^3/\text{lbm·R})(1060 \text{ R})}{0.51431 \text{ ft}^3/\text{lbm}} = \textbf{1228 psia}$$

$$\left. \begin{array}{l} v_R = \dfrac{v_{\text{actual}}}{RT_{cr}/P_{cr}} = \dfrac{(0.51431 \text{ ft}^3/\text{lbm})(3200 \text{ psia})}{(0.5956 \text{ psia·ft}^3/\text{lbm·R})(1164.8 \text{ R})} = 2.372 \\[3mm] T_R = \dfrac{T}{T_{cr}} = \dfrac{1060 \text{ R}}{1164.8 \text{ R}} = 0.91 \end{array} \right\} \quad P_R = 0.33$$

Thus,

$$P = P_R P_{cr} = (0.33)(3200 \text{ psia}) = \textbf{1056 psia}$$

Summary

- Pure substance
- Phases of a pure substance
- Phase-change processes of pure substances
  - ✓ Compressed liquid, Saturated liquid, Saturated vapor, Superheated vapor
  - ✓ Saturation temperature and Saturation pressure
- Property diagrams for phase change processes
  - ✓ The *T-v* diagram, The *P-v* diagram
- Property tables
  - ✓ Enthalpy
  - ✓ Saturated liquid, saturated vapor, Saturated liquid vapor mixture, Superheated vapor, compressed liquid
  - ✓ Reference state and reference values
- The ideal gas equation of state
  - ✓ Is water vapor an ideal gas?
- Compressibility factor