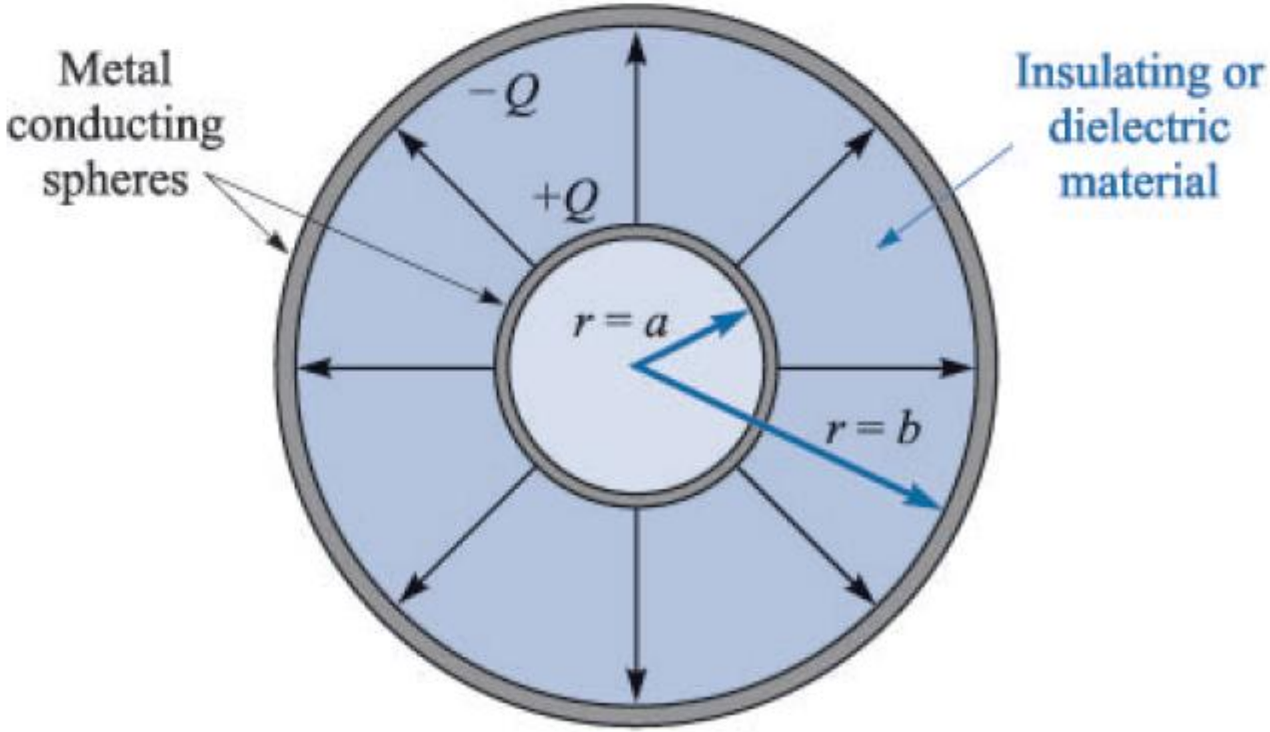
ELECTRIC FLUX DENSITY

AND
GAUSS LAW

Electric Flux Density

- Faraday used the equipment shown in figure below to study static electric fields

- The inner sphere was given a positive charge and the outer sphere was discharged by connecting it momentarily to ground



Electric Flux Density

- It was found that the total charge on the outer sphere was equal in magnitude to the original charge placed on the inner sphere

- This was true regardless of the dielectric material separating the two spheres

- It was concluded that there was some sort of displacement from the inner sphere to the outer sphere which was independent of the medium

- We now refer this flux as displacement flux or simply electric flux

Electric Flux Density

- Faraday's experiments also showed that a larger positive charge on the inner sphere induced a correspondingly larger negative charge on the outer sphere

- Hence there exists a direct proportionality between the electric flux and the charge on the inner sphere

- The constant of proportionality is dependent on the system of units involved and for SI units, it is 1

- If the electric flux is denoted by ψ and the total charge on the inner sphere by Q, then from Faraday experiment:

$$\Psi = Q$$

Electric Flux Density

- We can obtain more quantitative information by considering an inner sphere of radius *a* and outer sphere of radius *b*, with charges of *Q* and -*Q*

- So at the surface of the inner sphere, ψ coulombs of electric flux are produced by the charge *Q* (= ψ) distributed uniformly over a surface having an area of:

$$4\pi a^2 \text{ m}^2$$

- The density of the flux at this surface is called *electric flux density* and is denoted by **D**, mathematically:

$$Q/4\pi a^2 \text{ C/m}^2$$

Electric Flux Density

- The direction of **D** at a point is the direction of the flux lines at that point and the magnitude is given by the number of flux lines crossing a surface normal to the lines divided by the surface area

- At a radial distance *r*, where $a \leq r \leq b$

$$\mathbf{D} = \frac{Q}{4\pi r^2}\mathbf{a}_r$$

- If we now let the inner sphere become smaller and smaller, while still retaining a charge of *Q*, it becomes a point charge, but the electric flux density is still given by the above equation

- The electric field intensity is given as: $\mathbf{E} = \frac{Q}{4\pi\epsilon_0 r^2}\mathbf{a}_r$

- Therefore, we have in free space: $\mathbf{D} = \epsilon_0\mathbf{E}$

Electric Flux Density

- The electric flux ψ in terms of **D** may be obtained using the surface integral:

$$\Psi = \int \mathbf{D} \cdot d\mathbf{S}$$

- All the formulas derived for **E** from Coulomb's law can be used in calculating **D**, except that we must multiply those formulas by ϵ_o

- So for an infinite sheet of charge, we have:

$$\mathbf{D} = \frac{\rho_S}{2} \mathbf{a}_n$$

- And for a volume charge distribution, we have:

$$\mathbf{D} = \int \frac{\rho_v \, dv}{4\pi R^2} \mathbf{a}_R$$

Gauss Law

- Gauss's law states that the total electric flux ψ through any closed surface is equal to the total charge enclosed by that surface:

$$\Psi = Q_{\text{enc}}$$

- That is:

$$\Psi = \oint d\Psi = \oint_S \mathbf{D} \cdot d\mathbf{S}$$

$$= \text{Total charge enclosed } Q = \int \rho_v \, dv$$

- Or:

$$Q = \oint_S \mathbf{D} \cdot d\mathbf{S} = \int_v \rho_v \, dv$$

Gauss Law

- By applying divergence theorem:

$$\oint_S \mathbf{D} \cdot d\mathbf{S} = \int_v \nabla \cdot \mathbf{D}\, dv$$

- Comparing the two volume integrals above:

$$\rho_v = \nabla \cdot \mathbf{D}$$

- This is the **first** of the four Maxwell's equations

- The equation states that the volume charge density is the same as the divergence of the electric flux density

Gauss Law – Important Points

- Integral form of Gauss law:

$$Q = \oint_S \mathbf{D} \cdot d\mathbf{S} = \int_v \rho_v \, dv$$

- Differential or point form of Gauss law:

$$\rho_v = \nabla \cdot \mathbf{D}$$

- Gauss's law provides an easy means of finding **E** or **D** for symmetrical charge distributions

- Examples of symmetrical charge distributions are a point charge, an infinite line charge, an infinite cylindrical surface charge, and a spherical distribution of charge

Problem-1

- A uniform volume charge density of 80 $\mu C/m^3$ is present throughout the region 8mm< r <10mm. Let ρ_v = 0 for 0< r <8mm.

- a) Find the total charge inside the spherical surface r = 10 mm:
- b) Find D_r at r = 10 mm:
- c) If there is no charge for r > 10 mm, find Dr at r = 20 mm:

Problem-2

- Volume charge density is located as follows: $\rho_v = 0$ for $\rho < 1$ mm and $\rho > 2$ mm, $\rho_v = 4\rho\ \mu C/m^3$ for $1 < \rho < 2$ mm.

- a) Calculate the total charge in the region $0 < \rho < \rho 1$, $0 < z < L$, where $1 < \rho 1 < 2$ mm:
- b) Determine D_ρ at $\rho = \rho 1$: