- Course: **EE383 Instrumentation and Measurements**
- Session: Fall 2022

  Lectures: Week 5
- Course Instructor: Dr. Shahzad Younis

INSTRUMENTATION
& MEASUREMENT

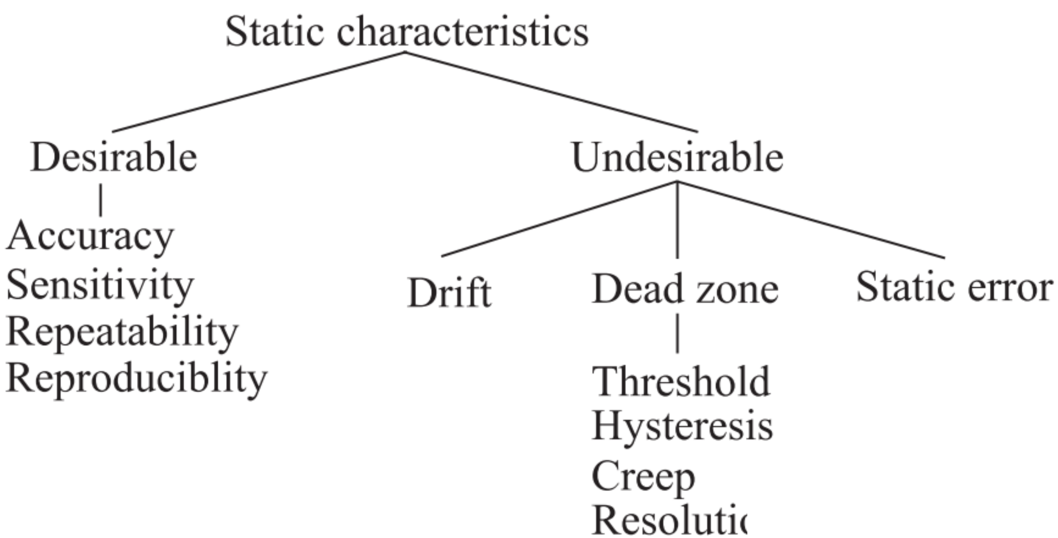
Week 5

- **Chapter 3**

  Estimation of Static Errors & Reliability

Review

- ❑ **Characteristics relating the steady-state (achieved) of an instrument**
- ❑ **Measurement of quantities which are constant or vary very slowly with time**



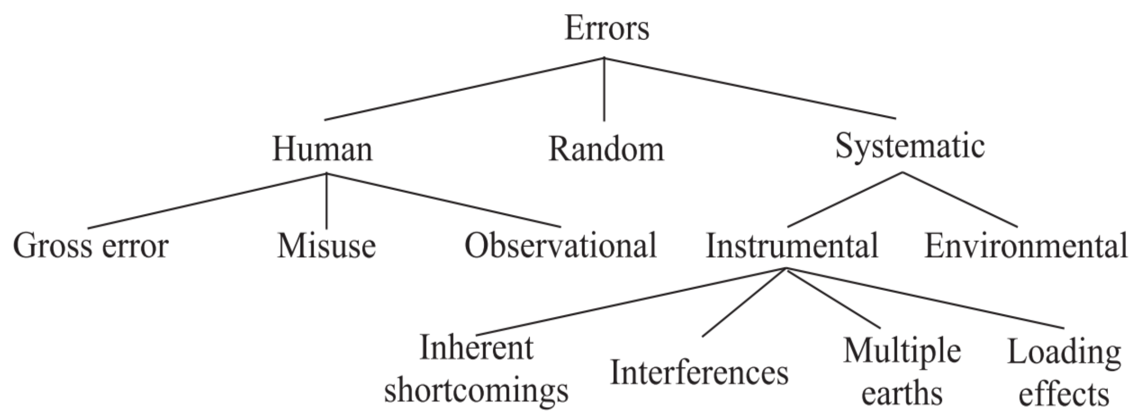


Fig. 2.7 The error tree.

Estimation of Static Errors

- **While reporting a measured value of a quantity, it is necessary to indicate the possible error in the measurement**

  - **How do estimate the measurement error?**

Why estimation of static errors?

Measurements involve error.

While reporting the measured values it is necessary to indicate a ***range of possible error***.

How to estimate the range of possible error?

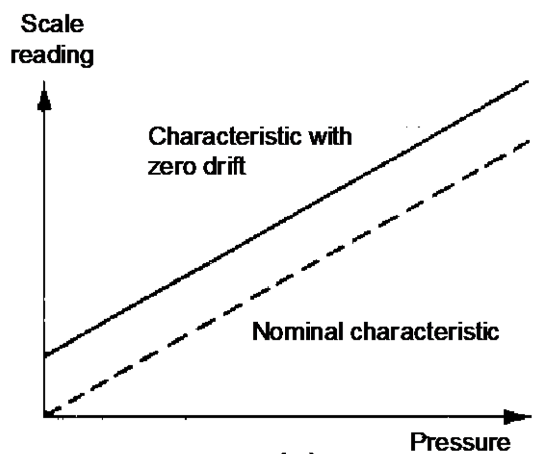
In this chapter we will find the answer

Contents

Statistical Treatment

Statistical Treatment

- ❑ **Systematic errors**
  - ❑ **Systematic errors can be removed by calibration (e.g. removing bias) and controlling environmental conditions**
  - ✓ **For example, zero drift or bias**



Statistical Treatment

- Random errors
  - Random variations of in the measured quantity due to unknown causes
  - Cannot be corrected by calibration or control

Statistical Treatment

- **Random errors**
  - **Random variations of in the measured quantity due to unknown causes**
  - **Cannot be corrected by calibration or control**

  - **Solution:** **take a number of readings and apply statistics to obtain the best approximation of the true value**

    - **Statistical treatment**

Statistical Treatment

- **multi-sample test**
  - **different instruments, different observers and different methods**
- **single-sample test**
  - **instrument, observer and method remaining the same, the data have been acquired at different times**

Statistical Treatment

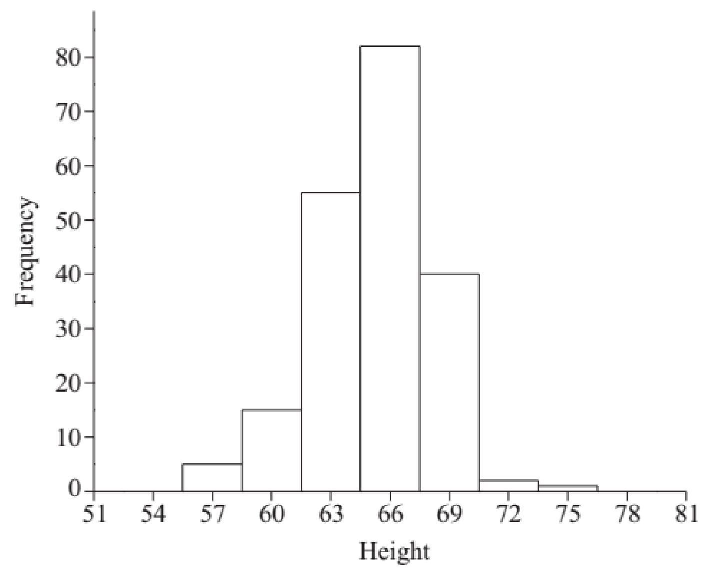
Statistical analysis of measurement data help us to get a better insight into it and estimate errors.

Suppose heights of 200 college students is measured and recorded in inches. How to conveniently summarize that data?

By making a frequency distribution graph

Statistical Treatment

- **Statistical methods are employed to estimate random errors**
  - **Frequency distribution of the height of 200 students.**
- **Height is divided in classes or cells**



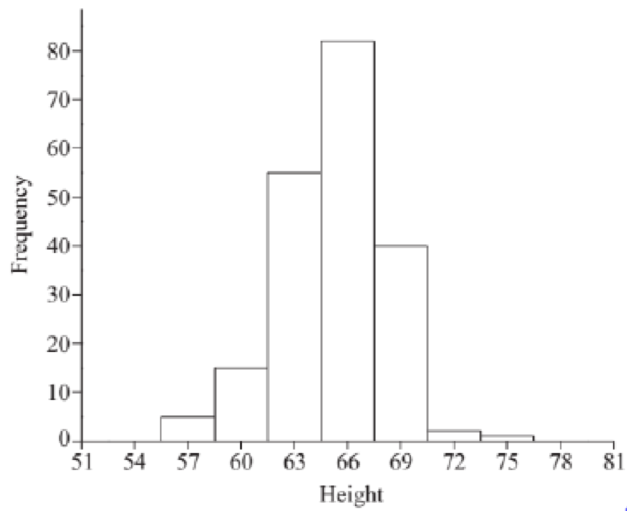
➢ **Histogram or frequency distribution**

Statistical Treatment

Height of one student maybe 62.35 inches. Another may be 58.75 inches.

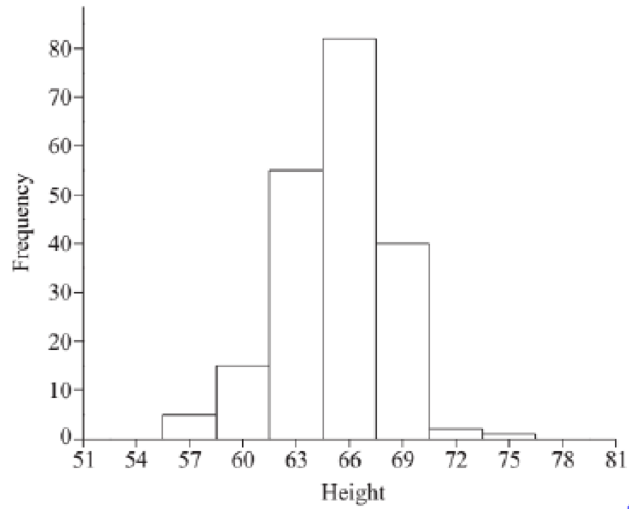
No two students may have the exact same height.

To make a histogram we divide the height of students within a few classes or cells such as 56-58 inches, 58-61 inches and so on.



How to characterize the frequency distribution data?

Statistical Treatment



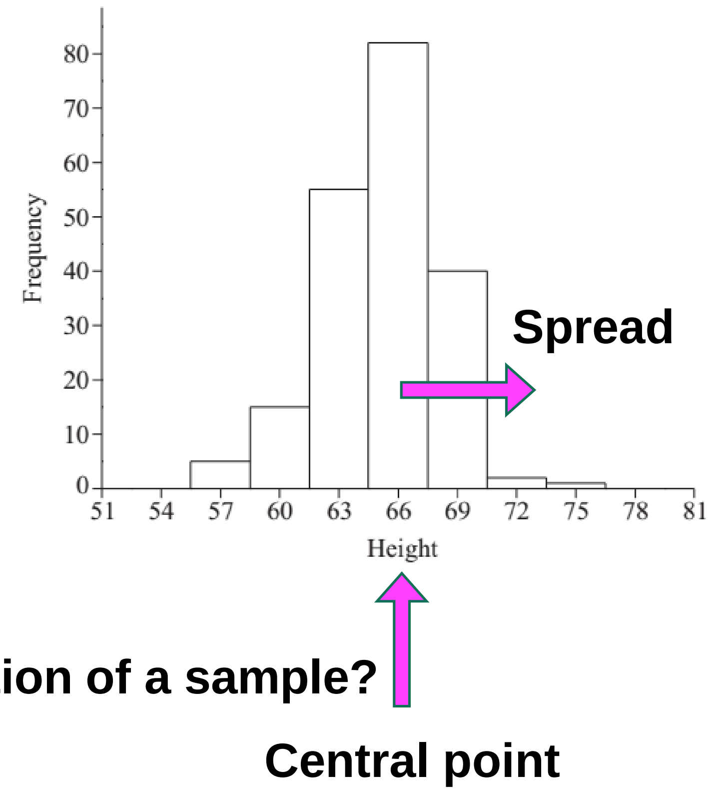
We can characterize the frequency distribution with two descriptive measures.

Central point/ tendency of distribution

Spread

Statistical Treatment

- Statistical methods are employed to estimate random errors
    - Frequency distribution of the height of 200 students.
- Height is divided in classes or cells



- Histogram or frequency distribution
    - how to characterize the frequency distribution of a sample?
        1. Central point of the distribution
        2. Spread

Measures of Central Tendency

- **A central point or average is a value which is a representative of a set of data**
- **Six types of averages**
  1. Mode
  2. Median
  3. Arithmetic mean or simply, mean
  4. Geometric mean
  5. Harmonic mean
  6. Root mean square

Measures of Central Tendency: Root Mean Square

- ❑ **The root mean square (rms) or quadratic mean of a set of *n* numbers x_1, x_2, . . . , x_n**

$$\mathrm{rms} = \sqrt{\frac{1}{n}\sum_{i=1}^{n} x_i^2}$$

- ❑ **Such averaging is quite common in engineering and physical applications**
  - ❑ **Any example?**

Electrical engineers often need to know the power, *P*, dissipated by an electrical resistance, *R*. It is easy to do the calculation when there is a constant current, *I*, through the resistance. For a load of *R* ohms, power is defined simply as:

$$P = I^2 R.$$

However, if the current is a time-varying function, *I*(*t*), this formula must be extended to reflect the fact that the current (and thus the instantaneous power) is varying over time.

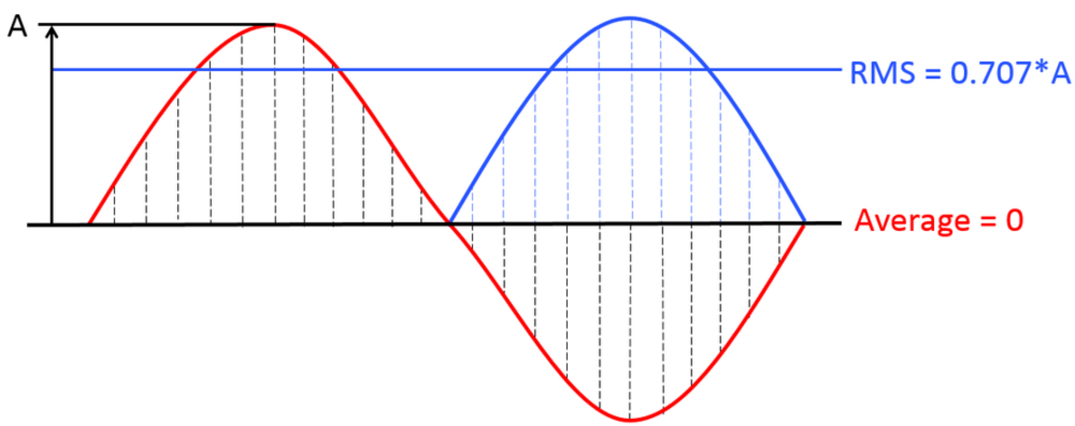
$$= I_{\mathrm{RMS}}^2 R \qquad \text{by definition of root-mean-square}$$

Measures of Central Tendency: Root Mean Square

- **The root mean square (rms) or quadratic mean of a set of *n* numbers x_1, $x_2, \ldots, x_n$**

$$\text{rms} = \sqrt{\frac{1}{n}\sum_{i=1}^{n} x_i^2}$$

- **Such averaging is quite common in engineering and physical applications**
  - **Any example?**

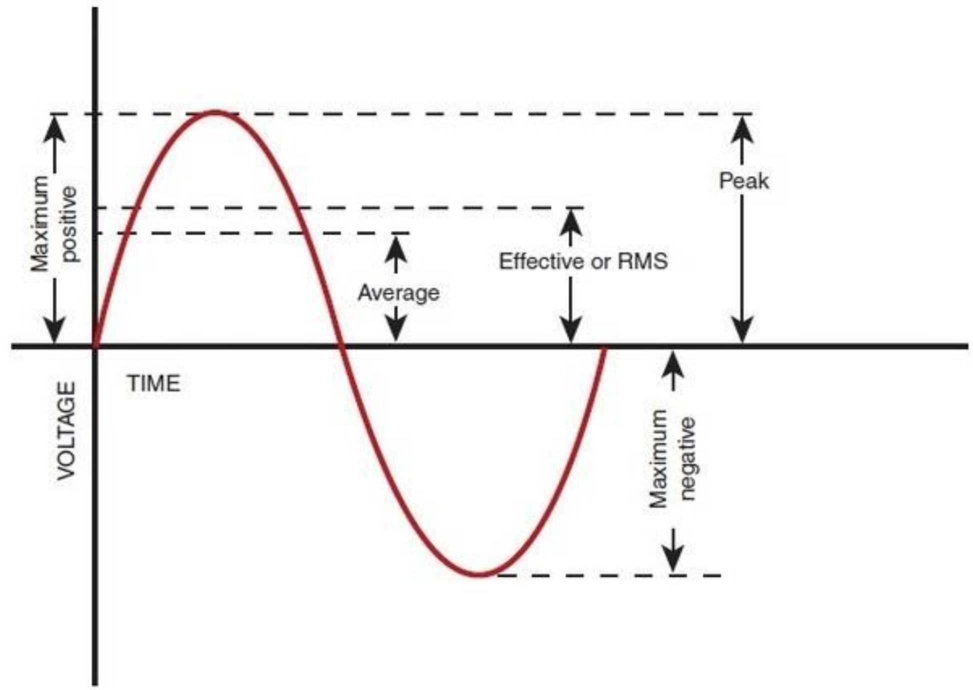


RMS Concepts

General equation for the RMS value of a periodic function.

$$V_{RMS} = \underbrace{\sqrt{\underbrace{\frac{1}{T}\int_0^T \underbrace{[V(t)]^2}_{\text{Square}} dt}_{\text{Mean}}}}_{\text{Root}}$$

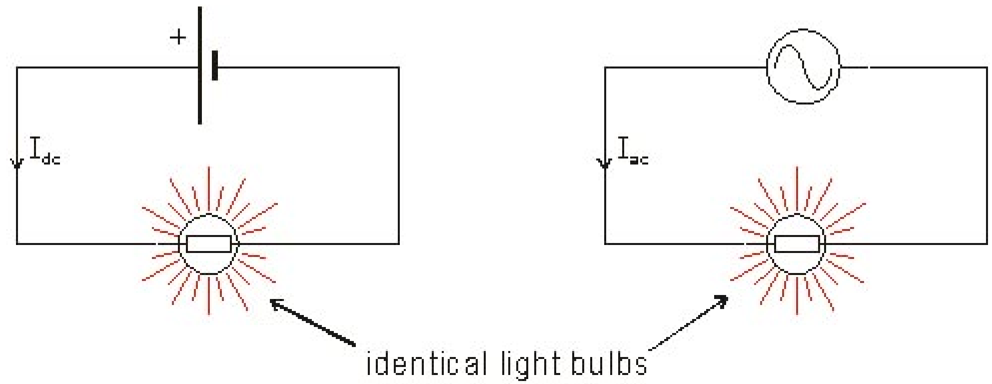
$$\boxed{\mathrm{rms} = \sqrt{\frac{1}{n}\sum_{i=1}^{n} x_i^2}}$$



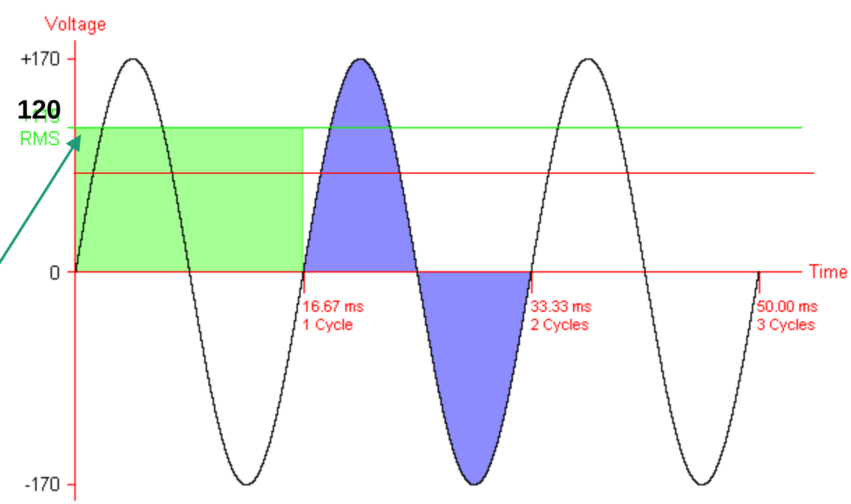
In everyday **use**, **AC** voltages (and currents) are always given as **RMS values** because this allows a sensible comparison to be made with steady DC voltages (and currents), such as from a battery. For example, a 6V **AC** supply means 6V **RMS** with the peak voltage about 8.6V.

Attempts to find an average value of AC would directly provide you the answer zero. Hence, **RMS** values are used. They help to find the effective value of AC (voltage or current). This **RMS** is a mathematical quantity (used in many math fields) used to compare both alternating and direct currents (or voltage).

RMS Concepts

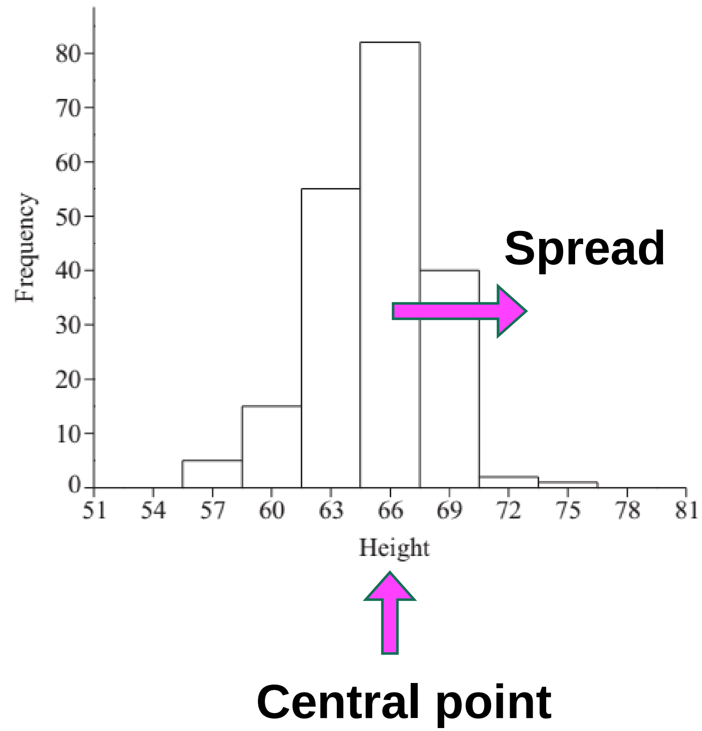


In order to relate both, we have nothing to use better than the RMS value. The direct voltage for the bulb is 115 V while the alternating voltage is 170 V. Hence, Vrms=Vdc=Vac/√2= **120 V**



Statistical Treatment

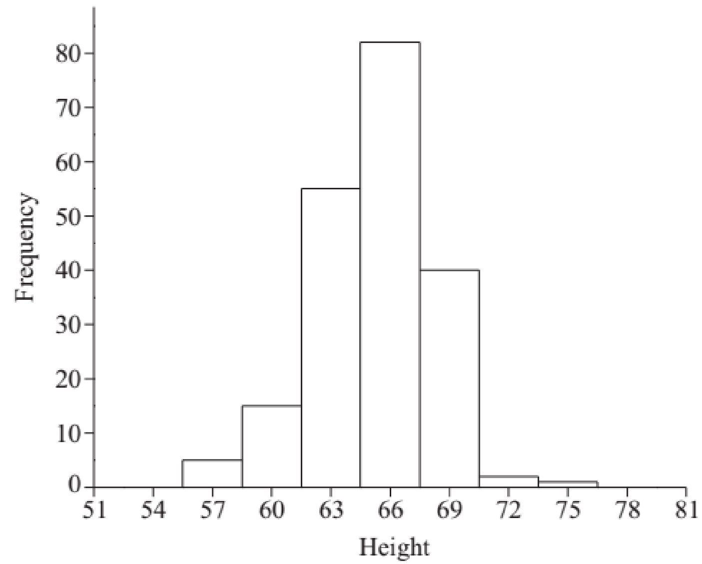
- Histogram or frequency distribution of the height of 200 students.



1. Measures of central tendency
2. Measures of spread

Measures of Spread

- **Histogram or frequency distribution of the height of 200 students.**

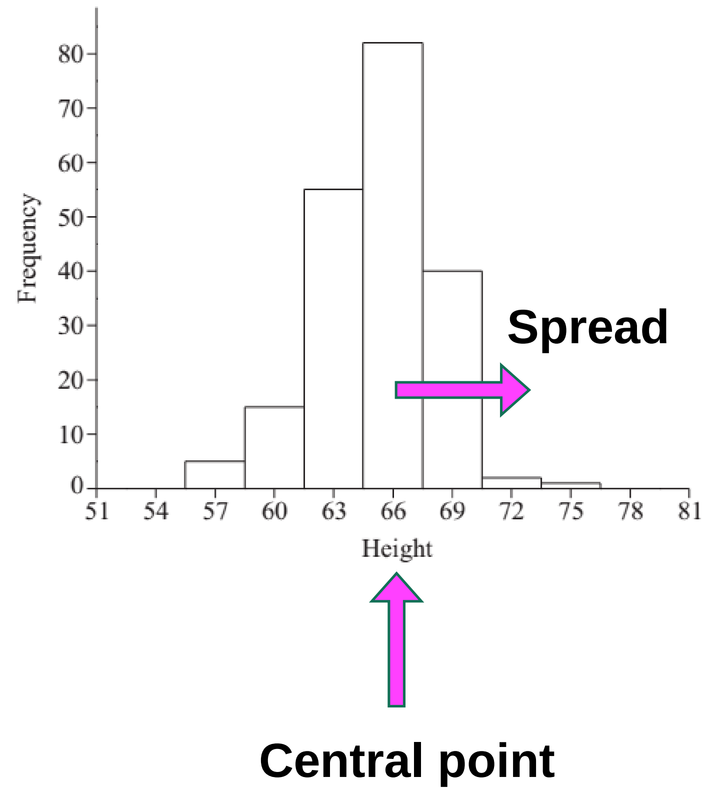


- **Apart from the average, another important characteristics is spread**
  - ✓ **How the observations *spread out or disperse* around the average value is another important statistic.**

Measures of Spread

- Several measures of spread or dispersion

    1. Deviation
    2. Mean absolute deviation
    3. Variance
    4. Standard deviation



Measures of Spread: Deviation

- **The scatter of an individual datum from the mean**

Usually written as d_i, deviation is the scatter of an individual datum from the mean. Symbolically,

$$d_i = x_i - \mu$$

$$\Sigma d_i = 0$$

Measures of Spread: Mean Absolute Deviation

- **Mean absolute deviation, *D*, of a set of data is defined as the average of absolute values of deviations**

$$D = \frac{1}{n}\sum_{i=1}^{n}|d_i| = \frac{1}{n}\sum_{i=1}^{n}|x_i - \mu|$$

- **If the data x_1 , x_2 , . . . , x_k occur f_1 , f_2, . . . , f_k times respectively**

$$D = \frac{1}{n}\sum_{i=1}^{n}f_i|x_i - \mu|$$

- **Mean Absolute Deviation** *also known as* ***Average Deviation***

Measures of Spread

Mean Absolute Deviation/Average Deviation

Example 3.12

Find the mean absolute deviation of heights of 100 male students of a class as given in table below.

Height (in)	60 – 62	63 – 65	66 – 68	69 – 71	72 – 74
No. of students	5	18	42	27	8

Measures of Spread: Mean Absolute Deviation

$$D = \frac{1}{n}\sum_{i=1}^{n} f_i|x_i - \mu|$$

Example 3.12
Find the mean absolute deviation of heights of 100 male students of a class as given in table below.

Height (in)	60 – 62	63 – 65	66 – 68	69 – 71	72 – 74
No. of students	5	18	42	27	8

Solution
Here, the arithmetic mean

$$\mu = \frac{(61\times 5)+(64\times 18)+(67\times 42)+(70\times 27)+(73\times 8)}{5+18+42+27+8}$$
$$= 67.45 \text{ in}$$

$$\boxed{\mu = \frac{f_1x_1 + f_2x_2 + \cdots + f_kx_k}{f_1 + f_2 + \cdots + f_k} = \frac{1}{n}\sum_{i=1}^{n} f_ix_i}$$

The rest of the calculation is presented in the following table:

Height (in)	$\lvert x_i - \mu\rvert = \lvert x_i - 67.45\rvert$	f_i	$f_i\lvert x_i - \mu\rvert$
60–62	6.45	5	32.25
63–65	3.45	18	62.10
66–68	0.45	42	18.90
69–71	2.55	27	68.85
72–74	5.55	8	44.40
	Σ =	**100**	**226.50**

Therefore, the mean absolute deviation $D = \frac{226.50}{100} = 2.26$ in

Measures of Spread

Variance

Intuitively, the mean absolute deviation is a good measure of spread; but it is mathematically intractable. One difficulty is the problem of differentiating an absolute value function. To obviate this difficulty, the *variance*, which is nothing but the mean squared deviation, was defined as

$$D = \frac{1}{n}\sum_{i=1}^{n} f_i |x_i - \mu|$$

Variance is the mean squared deviation.

$$\text{Variance} = \frac{1}{n}\sum_{i=1}^{n}(x_i - \mu)^2$$

As per statistical theories *sample* denotes the set of data less than or equal to 20 while *population* denotes set of data more than 20.

Divisor should be n-1 when calculating variance of a sample

Measures of Spread: Variance

- Mean squared deviation (MSD)

$$\text{Variance} = \frac{1}{n}\sum_{i=1}^{n}(x_i - \mu)^2$$

- Property: additive
- Variance or MSD is a good measure of dispersion

Measures of Spread: Standard Deviation

- **Standard deviation: ROOT-MEAN-SQUARED deviations**
- **Defined in two ways**
  - **For a sample, *n* ≤ 20**
  - **For a population, *n* > 20**

Sigma = $\sigma = \sqrt{\frac{\Sigma d_i^2}{n}} \qquad n > 20$

$$s = \sqrt{\frac{\Sigma d_i^2}{n-1}} \qquad n \leq 20$$

- **Same unit as the measured variable**

Measures of Spread: Standard Deviation

- **Standard deviation: ROOT-MEAN-SQUARED deviations**
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$$\sigma = \sqrt{\frac{\Sigma d_i^2}{n}} \qquad n > 20$$

$$s = \sqrt{\frac{\Sigma d_i^2}{n-1}} \qquad n \leq 20$$

- **Same unit as the measured variable**
- **Variance: squared standard deviation, σ^2 or s^2**

Measures of Spread

Example 3.13
A set of 10 independent measurements were made to determine the diameter of the bob of a simple pendulum. The measured values in cm were: 1.570, 1.597, 1.591, 1.562, 1.577, 1.580, 1.564, 1.586, 1.550 and 1.575. Determine (a) the arithmetic mean, (b) the average deviation, (c) the standard deviation, and (d) the variance.

Measures of Spread

Example 3.13

A set of 10 independent measurements were made to determine the diameter of the bob of a simple pendulum. The measured values in cm were: 1.570, 1.597, 1.591, 1.562, 1.577, 1.580, 1.564, 1.586, 1.550 and 1.575. Determine (a) the arithmetic mean, (b) the average deviation, (c) the standard deviation, and (d) the variance.

Solution

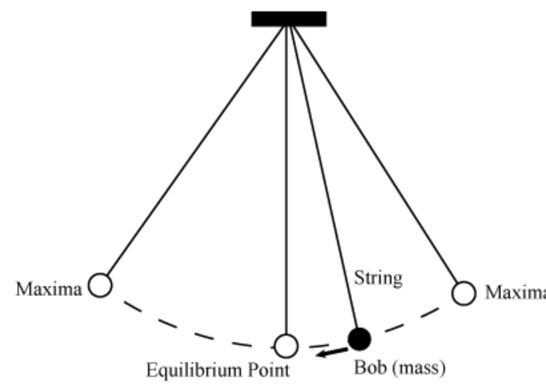
The calculation is presented in a tabular form below with the last row in bold face letters indicating sums of corresponding columns:

$$d_i = x_i - \mu$$

x_i	$\mid d \mid$	d^2
1.570	0.005	0.000025
1.597	0.022	0.000484
1.591	0.016	0.000256
1.562	0.013	0.000169
1.577	0.002	0.000004
1.580	0.005	0.000025
1.564	0.011	0.000121
1.586	0.011	0.000121
1.550	0.025	0.000625
1.575	0.000	0.000000
15.752	**0.110**	**0.001866**
$\mu = 1.575$	$D = 0.011$	$s = 0.0143$
		$s^2 = 0.000204$

$$\mu = \frac{1}{n}(x_1 + x_2 + \cdots + x_n) = \frac{1}{n}\sum_{i=1}^{n} x_i$$

$$D = \frac{1}{n}\sum_{i=1}^{n} |d_i| = \frac{1}{n}\sum_{i=1}^{n} |x_i - \mu|$$



The measurement can thus be reported as 1.575 ± 0.014 cm where the indicated error is the standard deviation.

Contents

- 3.1 → Definition of parameters
- 3.2 → Limiting Error
- 3.3 → Statistical Treatment
- 3.4 → Error Estimates from the normal(Gaussian) distribution
- 3.7 → Reliability Test

Error Estimation using Normal Distribution

- **Normal or Gaussian Distribution**
  - **For a large number of measurements, the frequency distribution or histogram assumes a bell-shaped curve called the normal or Gaussian curve**

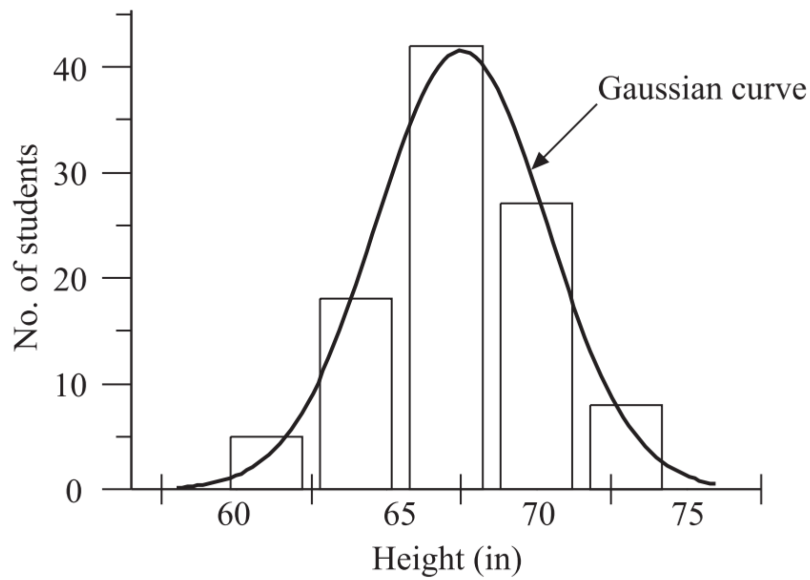
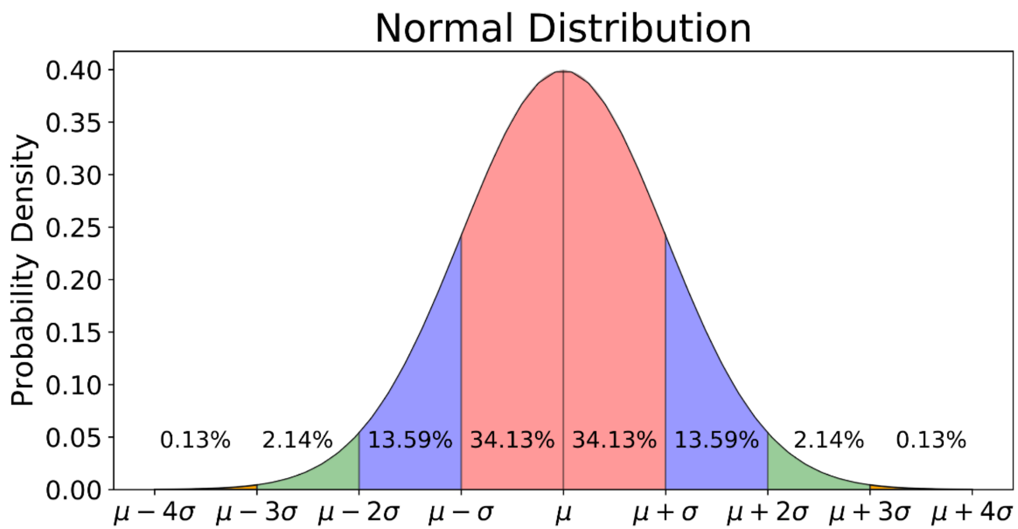


Fig. 3.2 Height vs. number of students of a class showing a Gaussian distribution.

- **Gaussian distribution curve for a typical height measurement of students of a class**

Error Estimation using Normal Distribution

❑ Normal or Gaussian Distribution

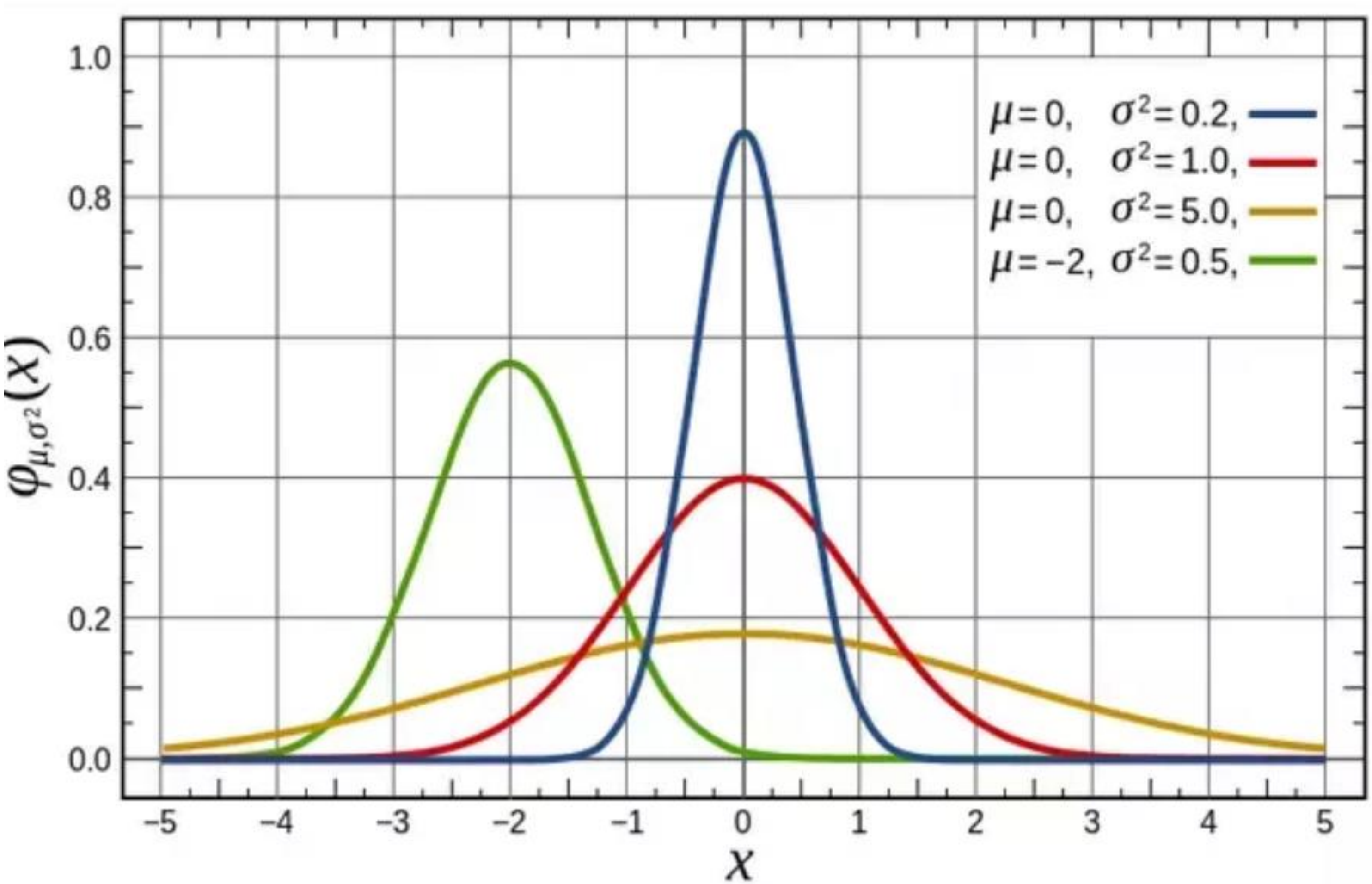


The errors made in physical measurements have a normal distribution.

$$y = \frac{1}{\sigma\sqrt{2\pi}} \exp\left\{-\frac{1}{2}\frac{(x-\mu)^2}{\sigma^2}\right\}$$

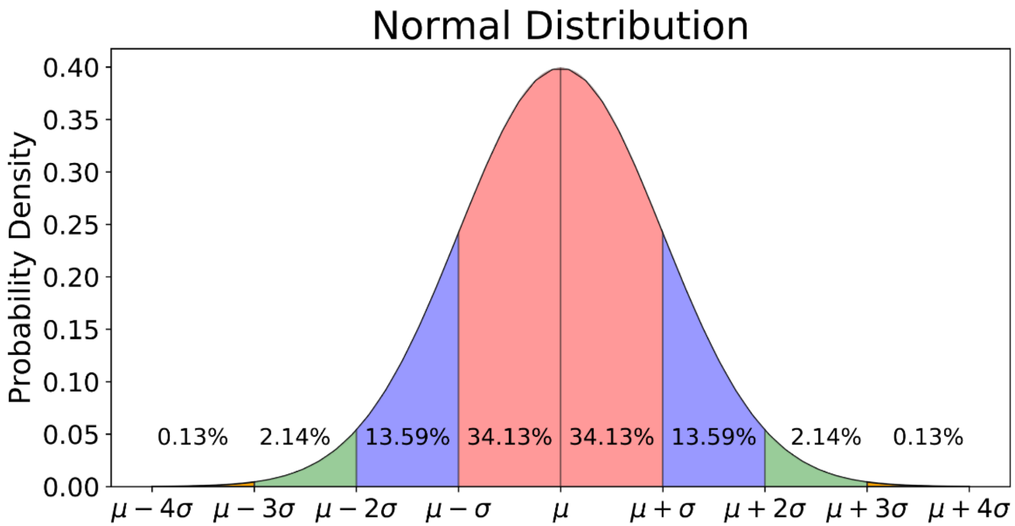
- **where μ is the mean,σ is the standard deviation, and π~= 3.14159**

Error Estimation using Normal Distribution



Error Estimation using Normal Distribution

- **Normal or Gaussian Distribution**



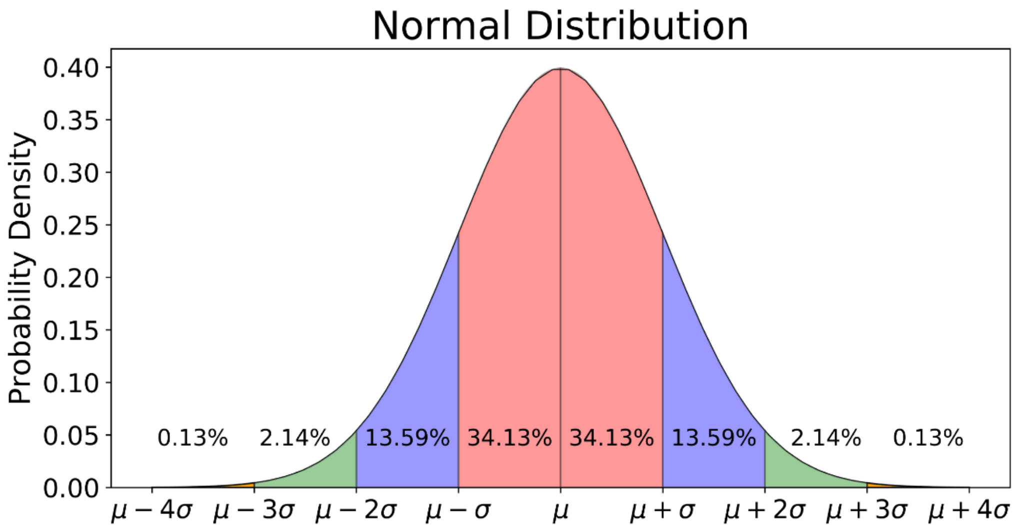
$$y = \frac{1}{\sigma\sqrt{2\pi}} \exp\left\{-\frac{1}{2}\frac{(x-\mu)^2}{\sigma^2}\right\}$$

- **for μ = 0 and σ= 1, the normal distribution**

$$y = \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{1}{2}x^2\right)$$

Error Estimation using Normal Distribution

- **Normal or Gaussian Distribution**



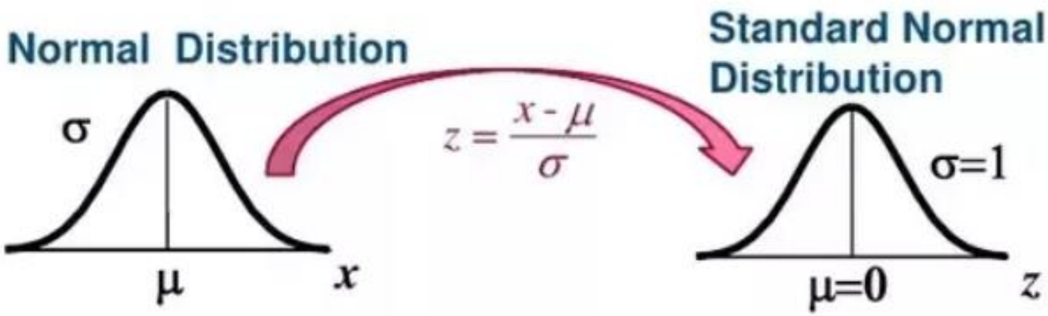
$$y = \frac{1}{\sigma\sqrt{2\pi}} \exp\left\{-\frac{1}{2}\frac{(x-\mu)^2}{\sigma^2}\right\}$$

- **By defining z = (x−μ) /σ**

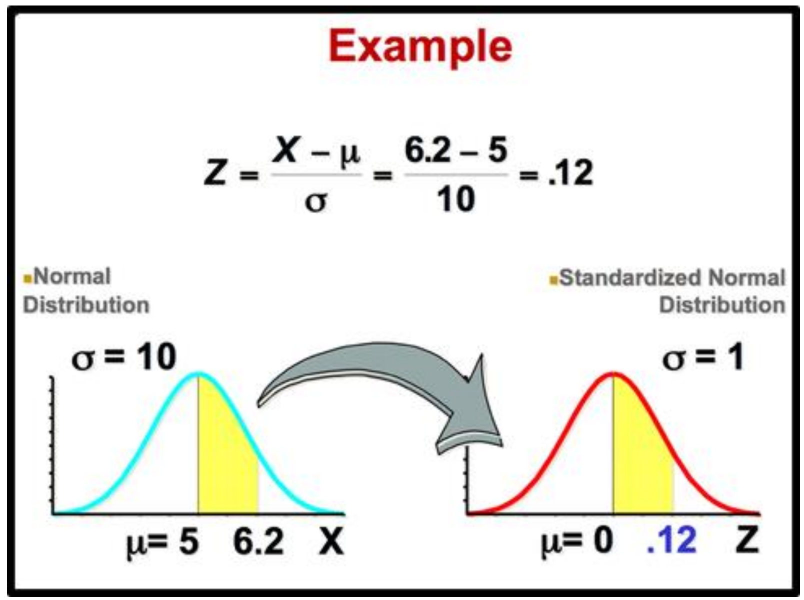
$$y = \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{1}{2}z^2\right)$$

The Standard Normal Distribution

- If each data value of a normally distributed random variable x is transformed into a z-score, the result will be the standard normal distribution.



- Use the Standard Normal Table to find the cumulative area under the standard normal curve.



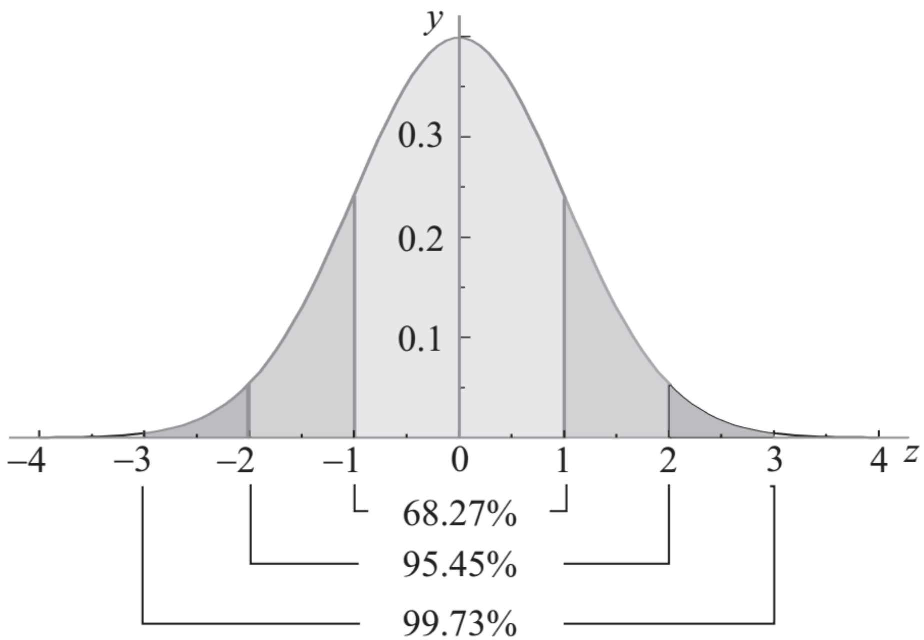
The Standard Normal Probability Distribution

- The standard normal distribution is a normal distribution with a **mean of 0** and a **standard deviation of 1**.
- It is also called the ***z*** **distribution**.
- A ***z*-value** is the signed distance between a selected value, designated *x*, and the population mean, μ, divided by the population standard deviation, σ.
- The formula is:

$$z = \frac{X - \mu}{\sigma}$$

Error Estimation using Normal Distribution

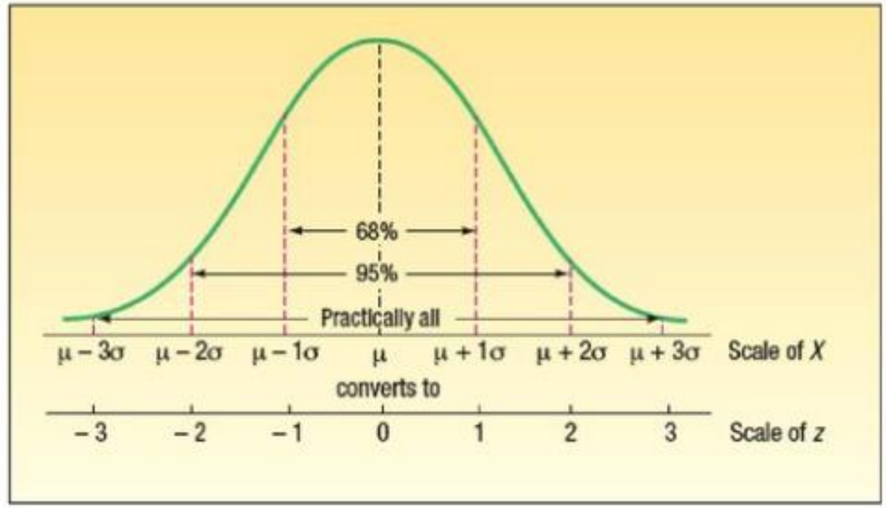
- **<u>Standard form</u> of the normal distribution**



$$y = \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{1}{2}z^2\right)$$

The Empirical Rule – Verification

- For $z=1.00$, the table's value is 0.3413; times 2 is 0.6826.
- For $z=2.00$, the table's value is 0.4772; times 2 is 0.9544.
- For $z=3.00$, the table's value is 0.4987; times 2 is 0.9974.



Areas Under the Normal Curve Using a Standard Normal Table

z	0.00	0.01	0.02	0.03	0.04	0.05	...
1.3	0.4032	0.4049	0.4066	0.4082	0.4099	0.4115	
1.4	0.4192	0.4207	0.4222	0.4236	0.4251	0.4265	
1.5	0.4332	0.4345	0.4357	0.4370	0.4382	0.4394	
1.6	0.4452	0.4463	0.4474	0.4484	0.4495	0.4505	
1.7	0.4554	0.4564	0.4573	0.4582	0.4591	0.4599	
1.8	0.4641	0.4649	0.4656	0.4664	0.4671	0.4678	
1.9	0.4713	0.4719	0.4726	0.4732	0.4738	0.4744	
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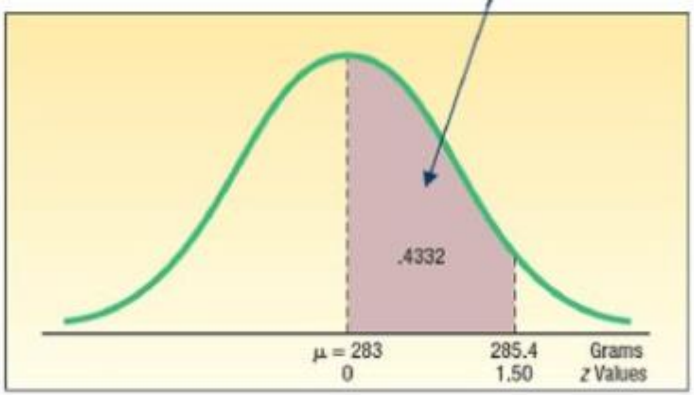
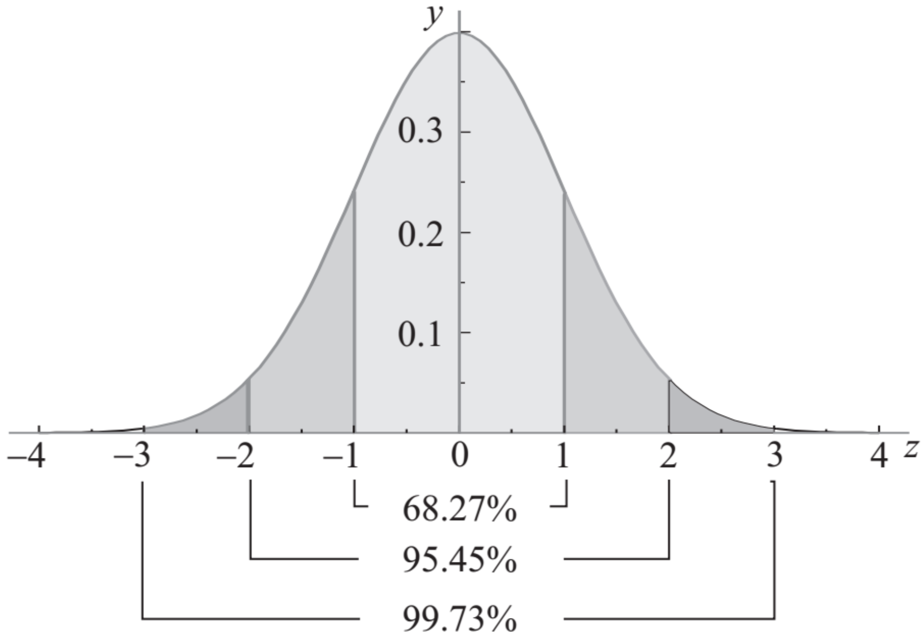


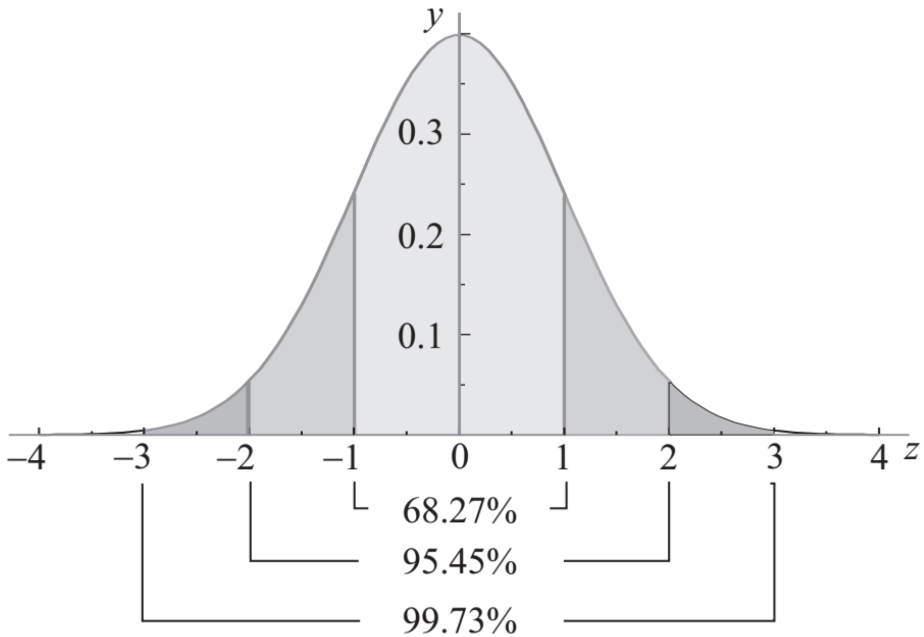
Error Estimation using Normal Distribution

- **Standard form of the normal distribution**
  - **Total area under the curve is equal to 1**



Interval in z-parameter	*Area under the interval*	*Standard deviation limits*	*Probability of lying a particular value within the limits*
$-1 \leq z \leq 1$	68.27%	$\pm\sigma$	68.27%
$-2 \leq z \leq 2$	95.45%	$\pm 2\sigma$	95.45%
$-3 \leq z \leq 3$	99.73%	$\pm 3\sigma$	99.73%

Error Estimation using Normal Distribution



- **Probability of occurrence**
  - **Probability: the area under the curve**
  - **What is probability of occurrence of a particular value between limits z_1 and z_2?**

$$\Pr\{z_1 \le z \le z_2\} = \int_{z_1}^{z_2} \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{z^2}{2}\right) dz$$

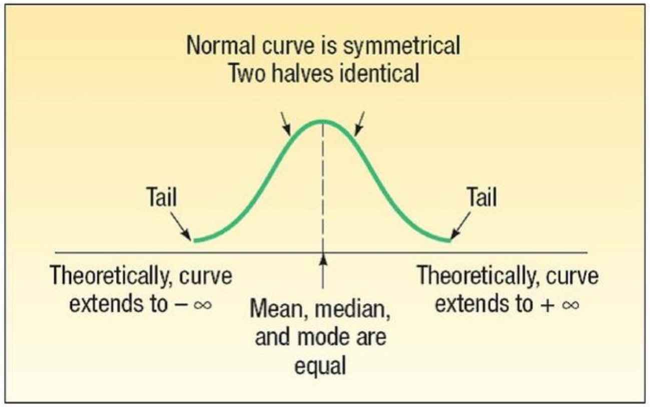
Error Estimation using Normal Distribution

$$\Pr\{z_1 \le z \le z_2\} = \int_{z_1}^{z_2} \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{z^2}{2}\right) dz$$

The probability of occurrence of a particular value between limits z_1 and z_2 given by the standard normal distribution is

$$\Pr\{z_1 \le z \le z_2\} = \int_{z_1}^{z_2} \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{z^2}{2}\right) dz \qquad (3.5)$$

Areas under this curve bounded by the ordinates at $z = 0$ and at any positive value of z are given in tabular form in Table 1, Appendix D at page 865. From such a table the area between any two z-values can be found by using the symmetry property of the curve, namely, $f(z) = f(-z)$. Thus, the probability of occurrence of a particular value within a certain limit can be found out from this table.

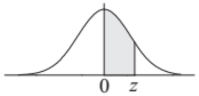


APPENDIX D

Statistical Tables

Table D.1 Areas for a standard normal distribution

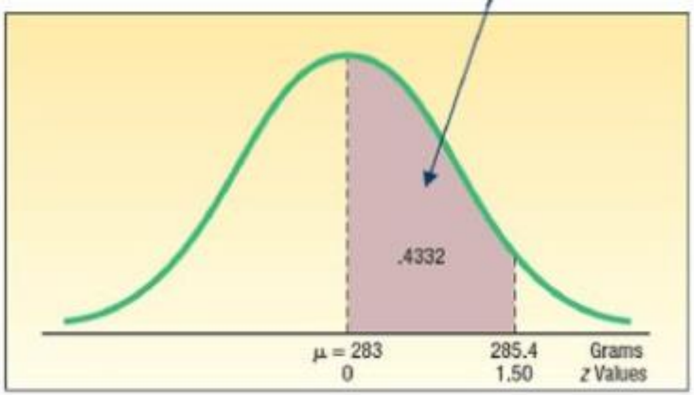
An entry in the table is the area under the curve, between $z = 0$ and $+z$ as shown in the adjacent figure. Areas for $-z$ are obtained from symmetry.



	Second decimal place of z									
z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.0000	.0040	.0080	.0120	.0160	.0199	.0239	.0279	.0319	.0359
0.1	.0398	.0438	.0478	.0517	.0557	.0596	.0636	.0675	.0714	.0754
0.2	.0793	.0832	.0871	.0910	.0948	.0987	.1026	.1064	.1103	.1141
0.3	.1179	.1217	.1255	.1293	.1331	.1368	.1406	.1443	.1480	.1517
0.4	.1554	.1591	.1628	.1664	.1700	.1736	.1772	.1808	.1844	.1879
0.5	.1915	.1950	.1985	.2019	.2054	.2088	.2123	.2157	.2190	.2224
0.6	.2258	.2291	.2324	.2357	.2389	.2422	.2454	.2486	.2518	.2549
0.7	.2580	.2612	.2642	.2673	.2704	.2734	.2764	.2794	.2823	.2852
0.8	.2881	.2910	.2939	.2967	.2996	.3023	.3051	.3078	.3106	.3133
0.9	.3159	.3186	.3212	.3238	.3264	.3289	.3315	.3340	.3365	.3389
1.0	.3413	.3438	.3461	.3485	.3508	.3531	.3554	.3577	.3599	.3621
1.1	.3463	.3665	.3686	.3708	.3729	.3749	.3770	.3790	.3810	.3830
1.2	.3849	.3869	.3888	.3907	.3925	.3944	.3962	.3980	.3997	.4015
1.3	.4032	.4049	.4066	.4082	.4099	.4115	.4131	.4147	.4162	.4177
1.4	.4192	.4207	.4222	.4236	.4251	.4265	.4279	.4292	.4306	.4319
1.5	.4332	.4345	.4357	.4370	.4382	.4394	.4406	.4418	.4429	.4441
1.6	.4452	.4463	.4474	.4484	.4495	.4505	.4515	.4525	.4535	.4545
1.7	.4554	.4564	.4573	.4582	.4591	.4599	.4608	.4616	.4625	.4633
1.8	.4641	.4649	.4656	.4664	.4671	.4678	.4686	.4693	.4699	.4706
1.9	.4713	.4719	.4726	.4732	.4738	.4744	.4750	.4756	.4761	.4767
2.0	.4722	.4778	.4783	.4788	.4793	.4798	.4803	.4808	.4812	.4817
2.1	.4821	.4826	.4830	.4834	.4838	.4842	.4846	.4850	.4854	.4857
2.2	.4861	.4864	.4868	.4871	.4875	.4878	.4881	.4884	.4887	.4890
2.3	.4893	.4896	.4898	.4901	.4904	.4906	.4909	.4911	.4913	.4916
2.4	.4918	.4920	.4922	.4925	.4927	.4929	.4931	.4932	.4934	.4936
2.5	.4938	.4940	.4941	.4943	.4945	.4946	.4948	.4949	.4951	.4952
2.6	.4953	.4955	.4956	.4957	.4959	.4960	.4961	.4962	.4963	.4964
2.7	.4965	.4966	.4967	.4968	.4969	.4970	.4971	.4972	.4973	.4974
2.8	.4974	.4975	.4976	.4977	.4977	.4978	.4979	.4979	.4980	.4981
2.9	.4981	.4982	.4982	.4983	.4984	.4984	.4985	.4985	.4986	.4986
3.0	.4987	.4987	.4987	.4988	.4988	.4989	.4989	.4989	.4990	.4990
3.1	.4990	.4991	.4991	.4991	.4992	.4992	.4992	.4992	.4993	.4993
3.2	.4993	.4993	.4994	.4994	.4994	.4994	.4994	.4995	.4995	.4995
3.3	.4995	.4995	.4995	.4996	.4996	.4996	.4996	.4996	.4996	.4997
3.4	.4997	.4997	.4997	.4997	.4997	.4997	.4997	.4997	.4997	.4998

Areas Under the Normal Curve Using a Standard Normal Table

z	0.00	0.01	0.02	0.03	0.04	0.05	. . .
1.3	0.4032	0.4049	0.4066	0.4082	0.4099	0.4115	
1.4	0.4192	0.4207	0.4222	0.4236	0.4251	0.4265	
1.5	0.4332	0.4345	0.4357	0.4370	0.4382	0.4394	
1.6	0.4452	0.4463	0.4474	0.4484	0.4495	0.4505	
1.7	0.4554	0.4564	0.4573	0.4582	0.4591	0.4599	
1.8	0.4641	0.4649	0.4656	0.4664	0.4671	0.4678	
1.9	0.4713	0.4719	0.4726	0.4732	0.4738	0.4744	
.							
.							
.							



Error Estimation using Normal Distribution

$$\Pr\{z_1 \le z \le z_2\} = \int_{z_1}^{z_2} \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{z^2}{2}\right) dz$$

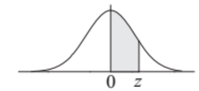
- ❑ **How to calculate?**
- ❑ **Table**
- ▪ **Probability of occurrence of a particular value can be found out from the table**
- ▪ **Area under the curve bounded by the ordinates at *z* = 0 and at any positive value of *z***
- ▪ **Area between any two z-values can be found by using the symmetry property of the curve:** $-f(z) = f(-z)$

APPENDIX D

Statistical Tables

Table D.1 Areas for a standard normal distribution

An entry in the table is the area under the curve, between $z = 0$ and $+z$ as shown in the adjacent figure. Areas for $-z$ are obtained from symmetry.



z	Second decimal place of z									
	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.0000	.0040	.0080	.0120	.0160	.0199	.0239	.0279	.0319	.0359
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2.4	.4918	.4920	.4922	.4925	.4927	.4929	.4931	.4932	.4934	.4936
2.5	.4938	.4940	.4941	.4943	.4945	.4946	.4948	.4949	.4951	.4952
2.6	.4953	.4955	.4956	.4957	.4959	.4960	.4961	.4962	.4963	.4964
2.7	.4965	.4966	.4967	.4968	.4969	.4970	.4971	.4972	.4973	.4974
2.8	.4974	.4975	.4976	.4977	.4977	.4978	.4979	.4979	.4980	.4981
2.9	.4981	.4982	.4982	.4983	.4984	.4984	.4985	.4985	.4986	.4986
3.0	.4987	.4987	.4987	.4988	.4988	.4989	.4989	.4989	.4990	.4990
3.1	.4990	.4991	.4991	.4991	.4992	.4992	.4992	.4992	.4993	.4993
3.2	.4993	.4993	.4994	.4994	.4994	.4994	.4994	.4995	.4995	.4995
3.3	.4995	.4995	.4995	.4996	.4996	.4996	.4996	.4996	.4996	.4997
3.4	.4997	.4997	.4997	.4997	.4997	.4997	.4997	.4997	.4997	.4998

865

Let's start with Simple Example..

The weekly incomes of shift foremen in the glass industry follow the normal probability distribution with a **mean of $1,000** and a **standard deviation of $100**.

What is the ***z* value** for the income, let's call it *x*, of a foreman who earns **$1,100** per week? For a foreman who earns **$900** per week?

For $x = \$1{,}100$:

$$z = \frac{x - \mu}{\sigma} = \frac{\$1{,}100 - \$1{,}000}{\$100} = 1.00$$

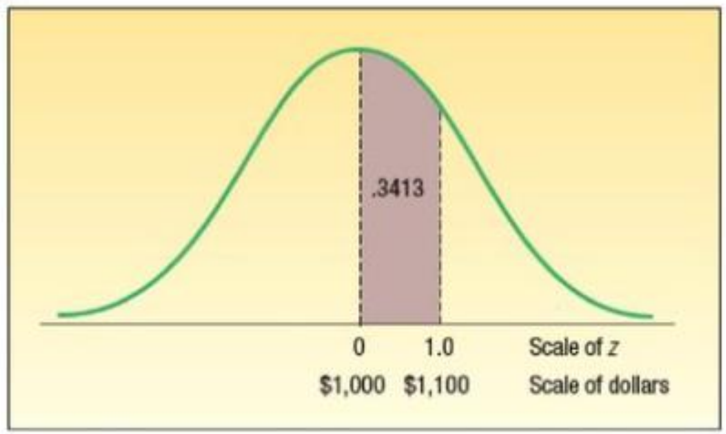
For $x = \$900$:

$$z = \frac{x - \mu}{\sigma} = \frac{\$900 - \$1{,}000}{\$100} = -1.00$$

Let's start with Simple Example..

$$z = \frac{x - \mu}{\sigma} = \frac{\$1{,}100 - \$1{,}000}{\$100} = 1.00$$

z	0.00	0.01	0.02
⋮	⋮	⋮	⋮
0.7	.2580	.2611	.2642
0.8	.2881	.2910	.2939
0.9	.3159	.3186	.3212
1.0	.3413	.3438	.3461
1.1	.3643	.3665	.3686
⋮	⋮	⋮	⋮



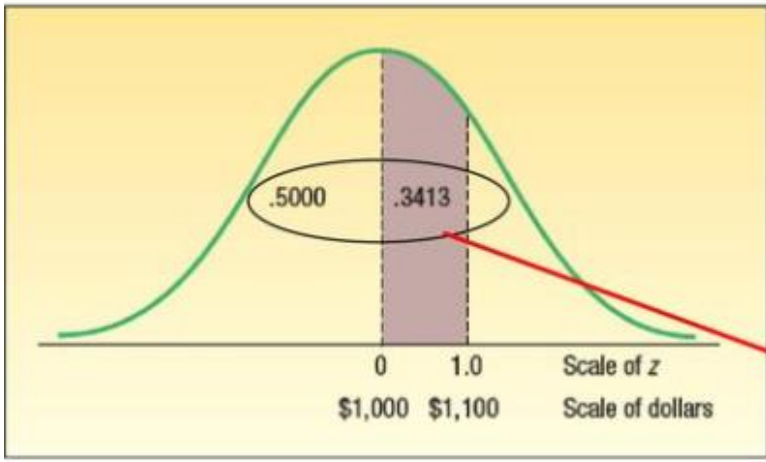
Finding Areas for z Using Excel

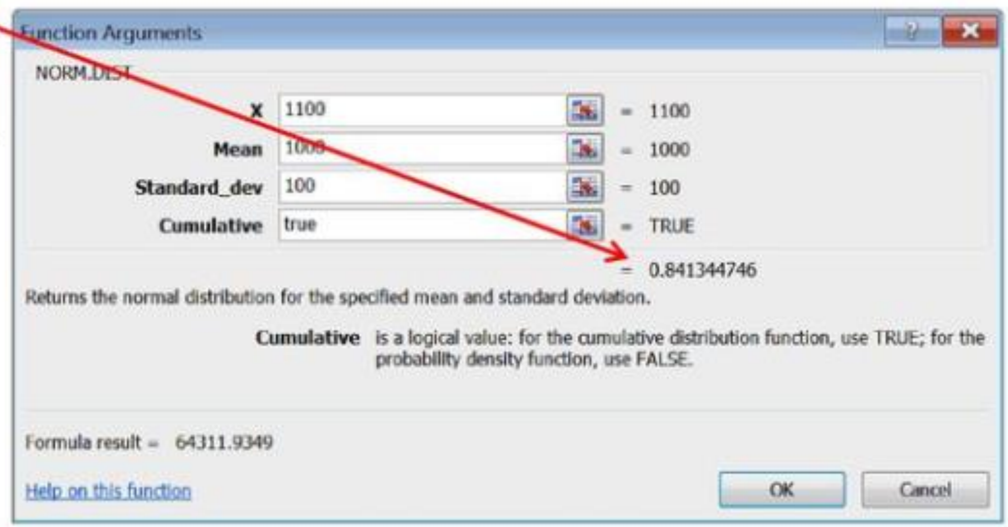
The Excel function:

=NORM.DIST(x,Mean,Standard_dev,Cumu)

=NORM.DIST(1100,1000,100,true)

calculates the probability (area) for z=1.



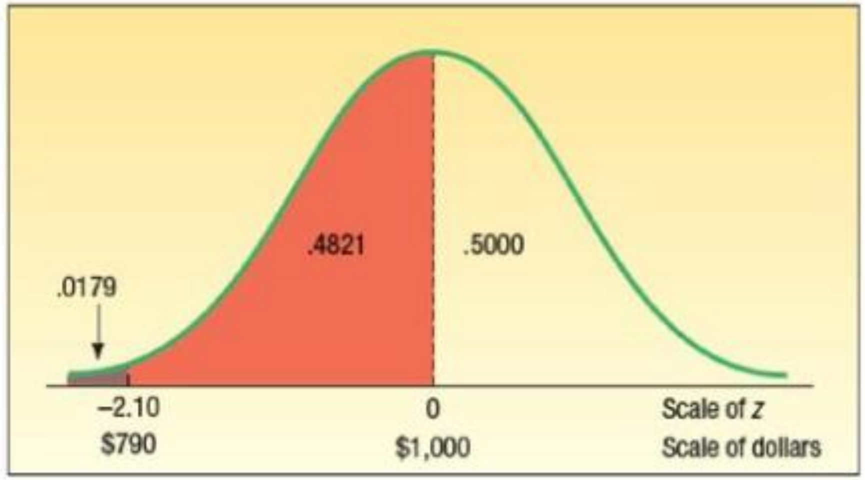


Example 2

Refer to the information regarding the weekly income of shift foremen in the glass industry. The distribution of weekly incomes follows the normal probability distribution with a mean of $1,000 and a standard deviation of $100.

What is the probability of selecting a shift foreman in the glass industry whose income is less than $790?

$$z = \frac{x - \mu}{s} = \frac{\$790 - \$1{,}000}{\$100} = -2.10$$



The probability of selecting a shift foreman with income less than $790 is 0.5 - .4821 = .0179.

Summarize:
Error Estimation from Normal or Gaussian Distribution

The errors made in physical measurements have a normal distribution.

$$y = \frac{1}{\sigma\sqrt{2\pi}} \exp\left\{-\frac{1}{2}\frac{(x-\mu)^2}{\sigma^2}\right\}$$

μ **: mean**
σ: standard deviation

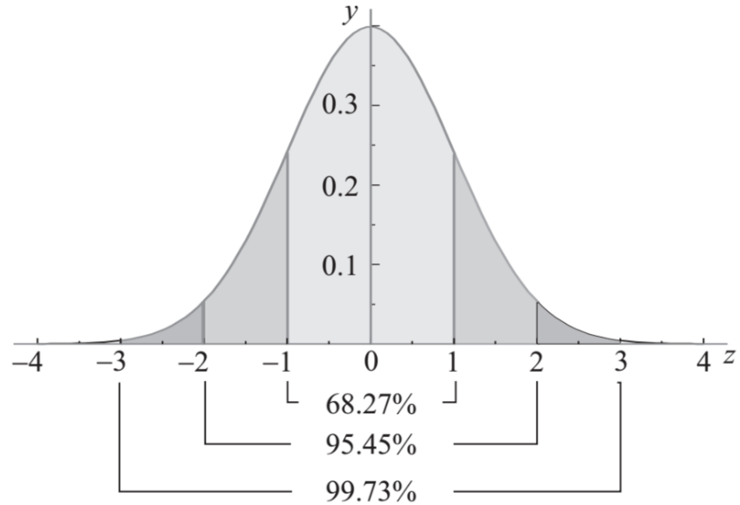
Substituting

μ **: 0**
σ: 1
z= x-*μ*/ σ

$$y = \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{1}{2}z^2\right)$$

$\longrightarrow$ ***Standard form of normal distribution***

Error Estimation using Normal Distribution

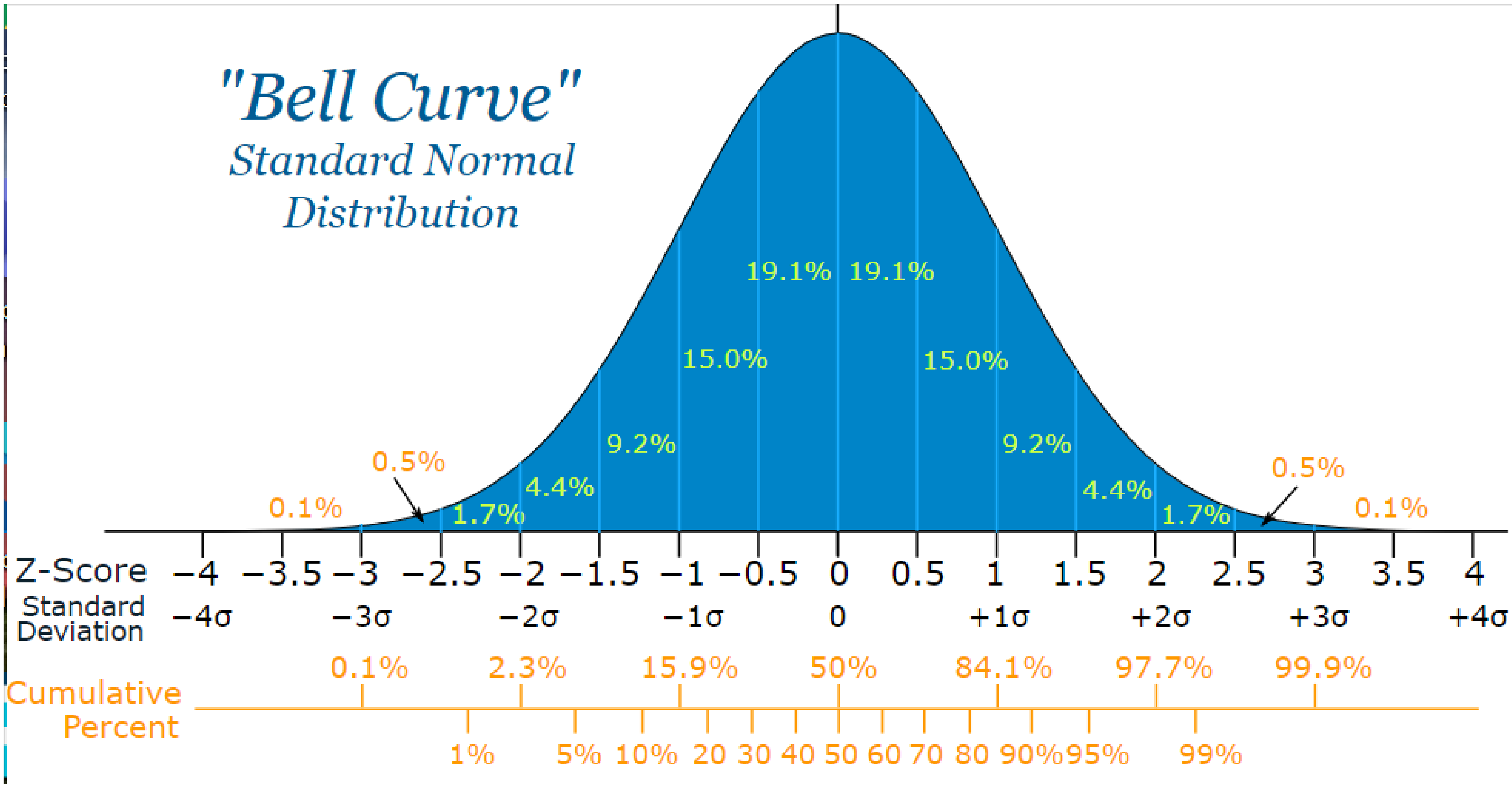


- **Properties of the normal distribution**

1. **The normal curve is symmetrical about the mean *μ***
2. **The mean is at the middle and it divides the area into halves**
3. **The total area under the curve is equal to 1**
4. **The curve is completely determined by its mean and standard deviation *σ* (or variance σ^2**
5. **The probability of occurrence of a particular value between limits *z*1 and *z*2**

$$\Pr\{z_1 \leq z \leq z_2\} = \int_{z_1}^{z_2} y \, dz$$

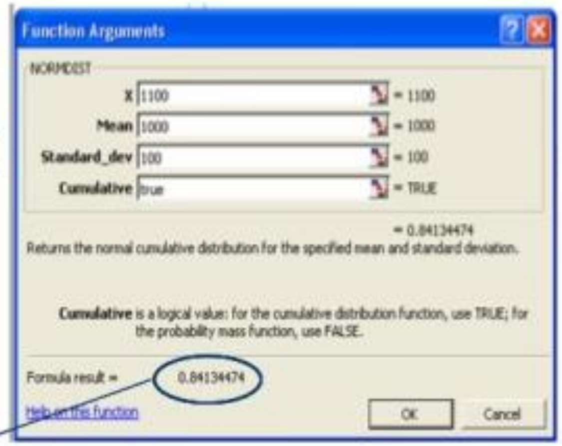
Summary

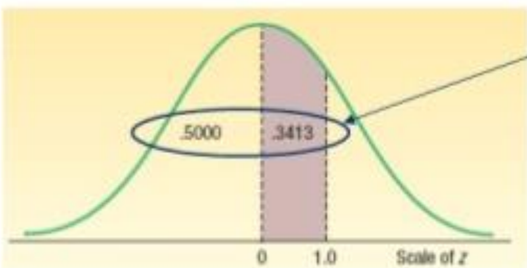


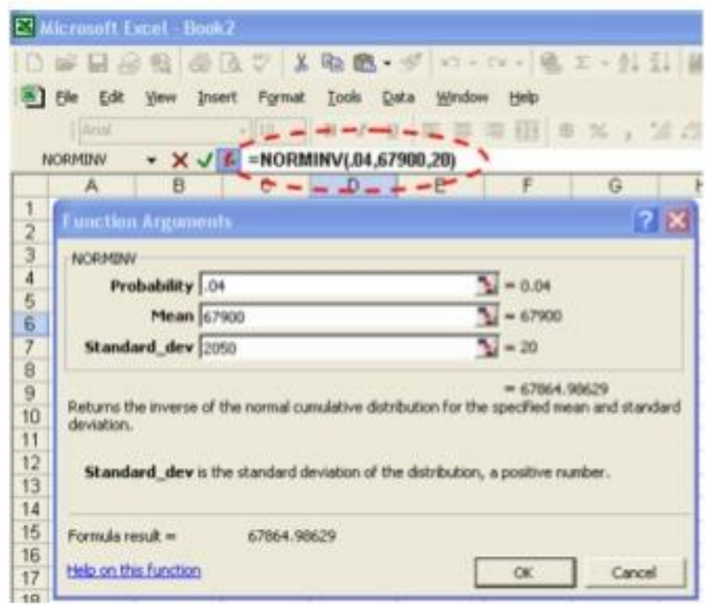
"Bell Curve"
Standard Normal Distribution
19.1% 19.1%
15.0% 15.0%
9.2% 9.2%
0.5% 4.4% 4.4% 0.5%
0.1% 1.7% 1.7% 0.1%
Z-Score −4 −3.5 −3 −2.5 −2 −1.5 −1 −0.5 0 0.5 1 1.5 2 2.5 3 3.5 4
Standard Deviation −4σ −3σ −2σ −1σ 0 +1σ +2σ +3σ +4σ
Cumulative Percent
0.1% 2.3% 15.9% 50% 84.1% 97.7% 99.9%
1% 5% 10% 20 30 40 50 60 70 80 90% 95% 99%

Finding Areas for Z Using Excel

The Excel function
=NORMDIST(x,Mean,Standard_dev,Cumu)
=NORMDIST(1100,1000,100,true)
generates area (probability) from
Z=1 and below



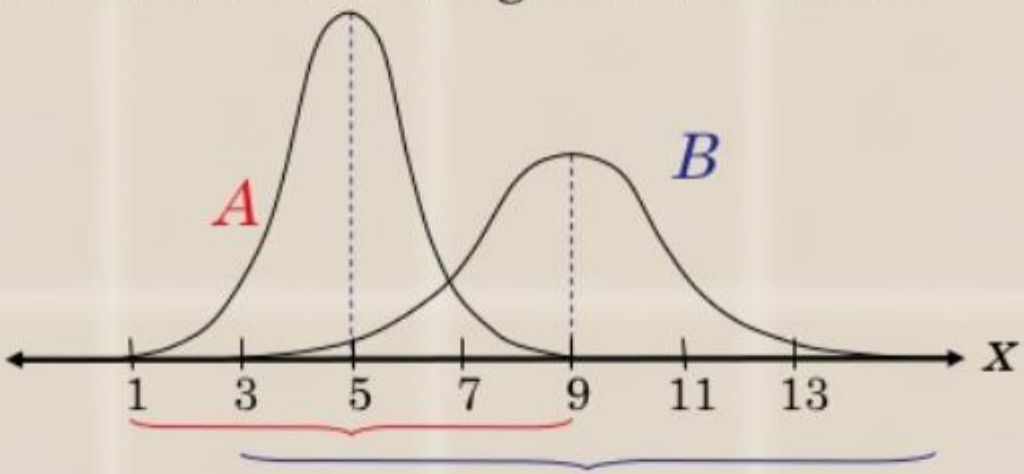




Concept Checks ?

Example:

2. Which curve has the greater mean?
3. Which curve has the greater standard deviation?



Concept Checks ?

Example:

2. Which curve has the greater mean?
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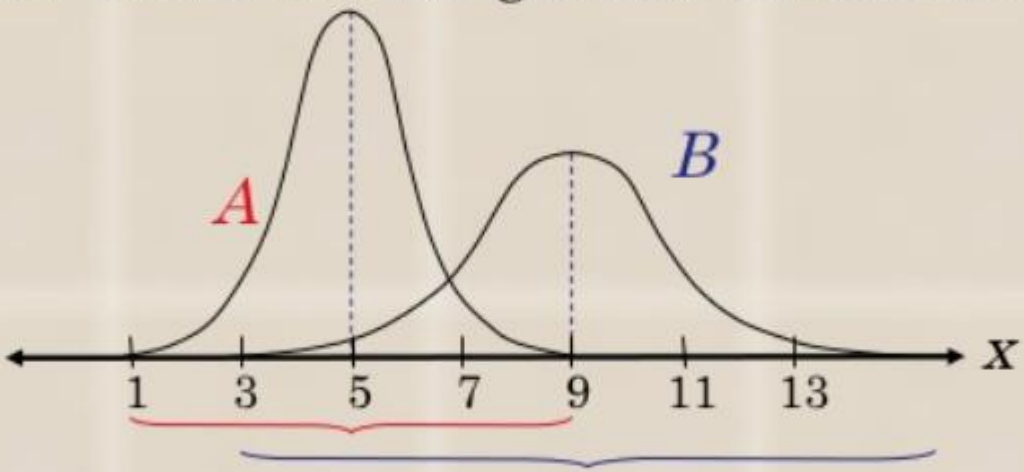


The line of symmetry of curve A occurs at $x = 5$. The line of symmetry of curve B occurs at $x = 9$. Curve B has the greater mean.

Curve B is more spread out than curve A, so curve B has the greater standard deviation.

Error Estimation from Normal or Gaussian Distribution

Example 3.14

Bolts produced in a factory have the specification 10 ± 0.2 cm where the specified error is the standard deviation. If one bolt is picked up at random from a heap, what is the probability that its length will lie between 9.9 and 10.1 cm?

Error Estimation using Normal Distribution

Example 3.14

Bolts produced in a factory have the specification 10$\pm$ 0.2 cm where the specified error is the standard deviation. If one bolt is picked up at random from a heap, what is the probability that its length will lie between 9.9 and 10.1 cm?

Solution

Here the probability may be expressed as $\Pr\{9.9 \leq x \leq 10.1\}$. This may be written in the standardised form as

$$\Pr\left\{\frac{9.9-10}{0.2} \leq \frac{x-10}{0.2} \leq \frac{10.1-10}{0.2}\right\} = \Pr\{-0.5 \leq z \leq 0.5\}$$

$$= 0.383 \quad \text{(the table lists 0.1915 for } z = 0.5\text{).}$$

Thus, the probability is 38.3%.

Error Estimation from Normal or Gaussian Distribution

Example 3.16
The average life of a certain type of instrument is 10 years, with a standard deviation of 2 years. If the manufacturer is willing to replace only 3% of the instruments that fail, how long a warranty should he offer, assuming that the lives of the instruments follow a normal distribution?

$$z = \frac{x - \mu}{\sigma}$$

Error Estimation using Normal Distribution

Example 3.16

The average life of a certain type of instrument is 10 years, with a standard deviation of 2 years. If the manufacturer is willing to replace only 3% of the instruments that fail, how long a warranty should he offer, assuming that the lives of the instruments follow a normal distribution?

Solution

Here, $\mu = 10$ and $\sigma = 2$. The corresponding normal distribution curve is given in Fig. 3.4.

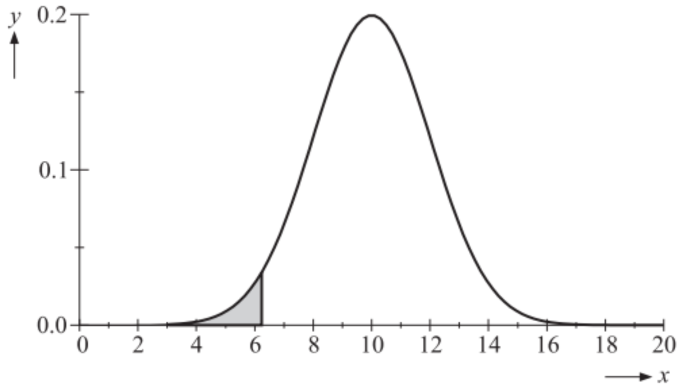


Fig. 3.4 Standard normal curve for $\sigma =$ and $\mu = 10$ (Example 3.16). The warranty is provided for the years that lie within the shaded area.

Let X be the life of an instrument and x be the warranty period. We need to find the value (in years) that will give us the bottom 3% of the distribution, i.e. the shaded area in Fig. 3.4. These are the instruments that the manufacturer is willing to replace under the warranty. Stated mathematically,

$$\Pr\{X < x\} = 0.03$$

The area of the lower half of the curve is 0.5, the total area being 1. So, to find the z value from the table, we need to know the area of the remaining portion which is $(0.5 - 0.03) = 0.47$. The corresponding nearest value, 0.4699, in the table is $z = -1.88$.

Since $z = \dfrac{x-\mu}{\sigma}$, we can write:

$$\frac{x-10}{2} = -1.88$$

$$\Rightarrow \qquad x = 10 - (2)(1.88) = 6.24$$

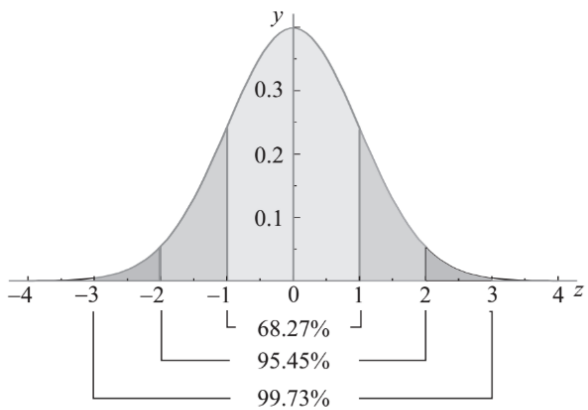
So the warranty period should be 6.24 years.

Probable Error

Gaussian distribution helps us estimate the probable error of any measurement. The probable error is defined as the 50% confidence limit of the population parameters corresponding to a measured value. Let us elaborate it.

- ❑ **Taken *n* measurements of a quantity and resultant standardised deviation is +/- ζ.**
  - ❑ **For what value of ζ, there will be a 50% chance that any value chosen at random will lie between −ζ and +ζ?**
  - ❑ **Or, for what value of the limits −ζ and ζ, the area under the standard normal curve be 0.5?**

$$\frac{1}{\sqrt{2\pi}}\int_{-\zeta}^{+\zeta}\exp\left(-\frac{1}{2}z^2\right)dz=\frac{1}{2}?$$



Error Estimation from Normal or Gaussian Distribution

Probable Error

Amongst a given set of measurements 50 % of the readings will not have an error greater than the probable error.

Such readings would lie within the normal distribution curve between ordinates + ζ and $-\zeta$.

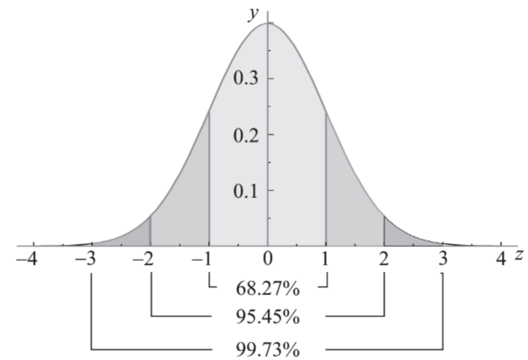
$$\frac{1}{\sqrt{2\pi}}\int_{-\zeta}^{+\zeta} \exp\left(-\frac{1}{2}z^2\right) dz = \frac{1}{2}$$

How to find ζ?

Probable Error

- *n* measurements of a quantity
  - For what value of ζ, there will be a 50% chance that any value chosen at random will lie between $-\zeta$ and $+\zeta$?
  - Or, for what value of the limits $-\zeta$ and ζ, the area under the standard normal curve be 0.5?

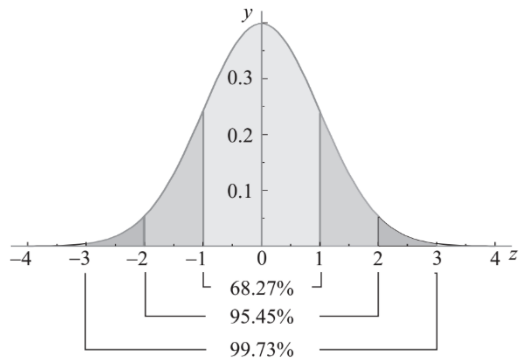
$$\frac{1}{\sqrt{2\pi}}\int_{-\zeta}^{+\zeta}\exp\left(-\frac{1}{2}z^2\right)dz=\frac{1}{2}?$$



Solution $\Rightarrow$ $\zeta = \pm 0.6745$

Probable Error

- n measurements of a quantity
  - For what value of ζ, there will be a 50% chance that any value chosen at random will lie between $-\zeta$ and $+\zeta$?
  - Or, for what value of the limits $-\zeta$ and ζ, the area under the standard normal curve be 0.5?



$$\frac{1}{\sqrt{2\pi}}\int_{-\zeta}^{+\zeta}\exp\left(-\frac{1}{2}z^2\right)dz = \frac{1}{2}?$$

Solution ⇨ $\zeta = \pm 0.6745$

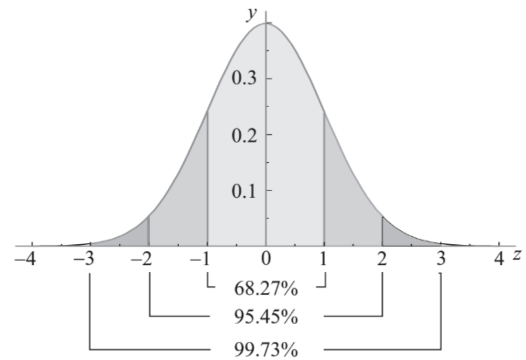
- Since $\zeta = (x-\mu)/\sigma$, the corresponding deviation r which is called the probable error is

$$r = x - \mu = \pm 0.6745\sigma$$

Probable Error

- ***n* measurements of a quantity**
  - **For what value of ζ, there will be a 50% chance that any value chosen at random will lie between −ζ and +ζ?**
  - **Or, for what value of the limits −ζ and ζ, the area under the standard normal curve be 0.5?**

$$\frac{1}{\sqrt{2\pi}}\int_{-\zeta}^{+\zeta}\exp\left(-\frac{1}{2}z^2\right)dz = \frac{1}{2}?$$



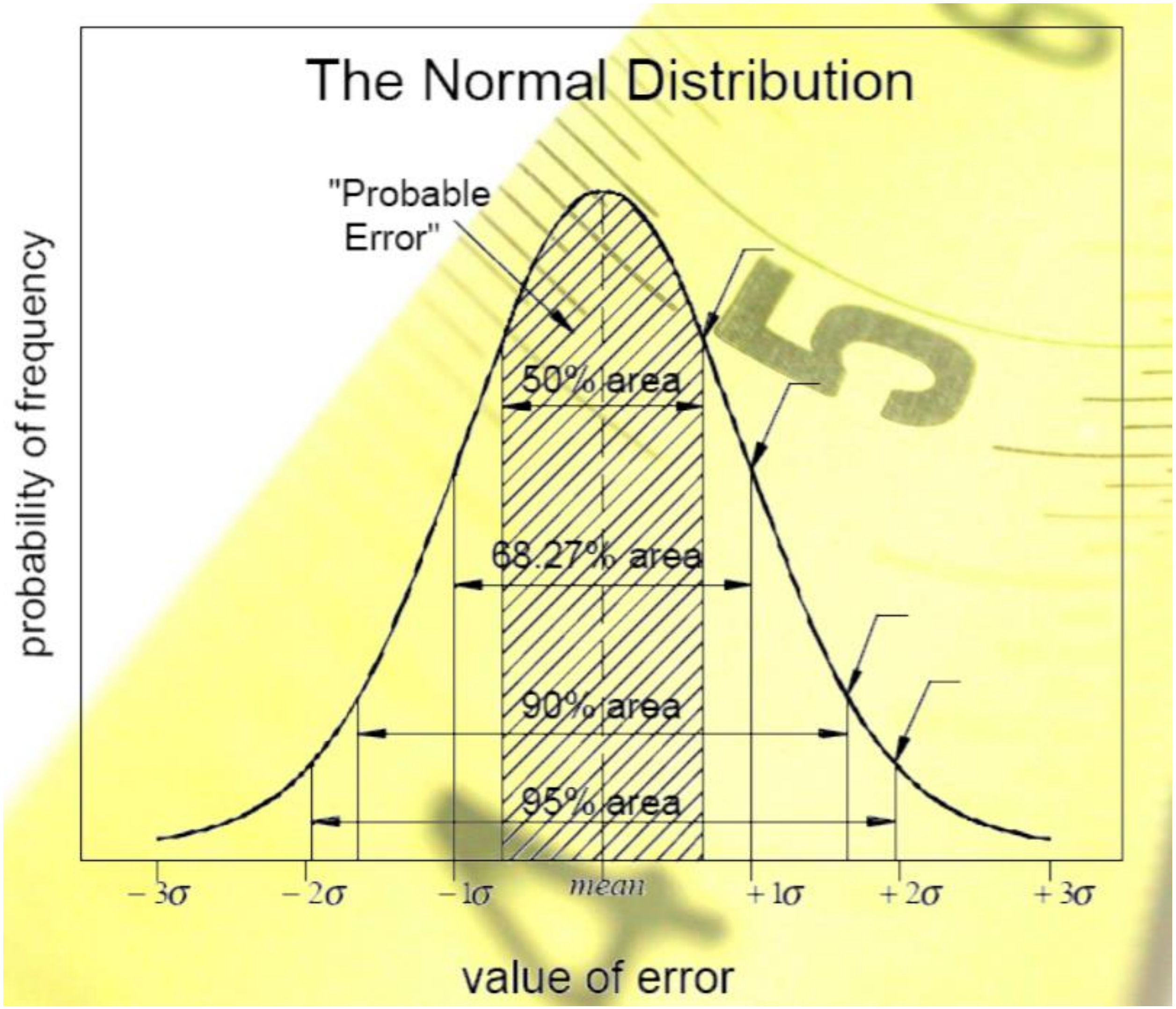
Solution $\zeta = \pm 0.6745$

- **Since ζ = (x−μ) /σ , the corresponding deviation *r* which is called the probable error is**

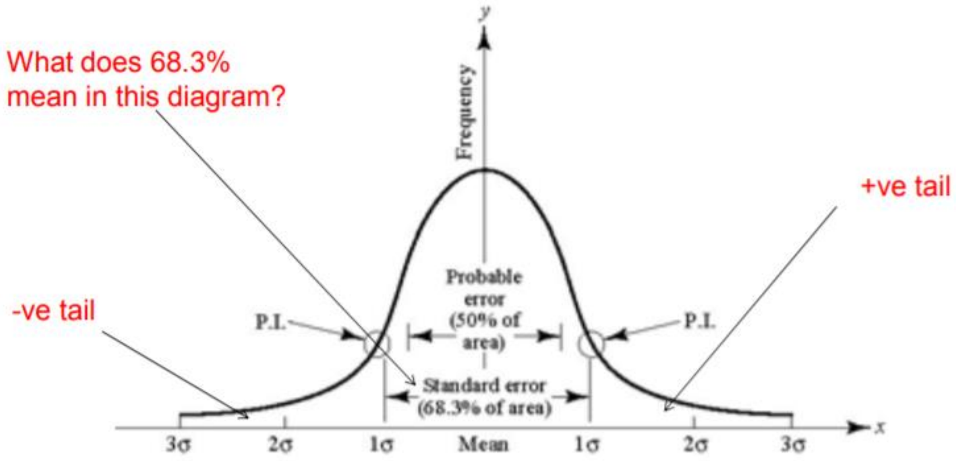
$$r = x - \mu = \pm 0.6745\sigma$$

- **Scaled standard deviation, scaling factor being 0.6745**

Probable Error (50%)?



Probability of Error?



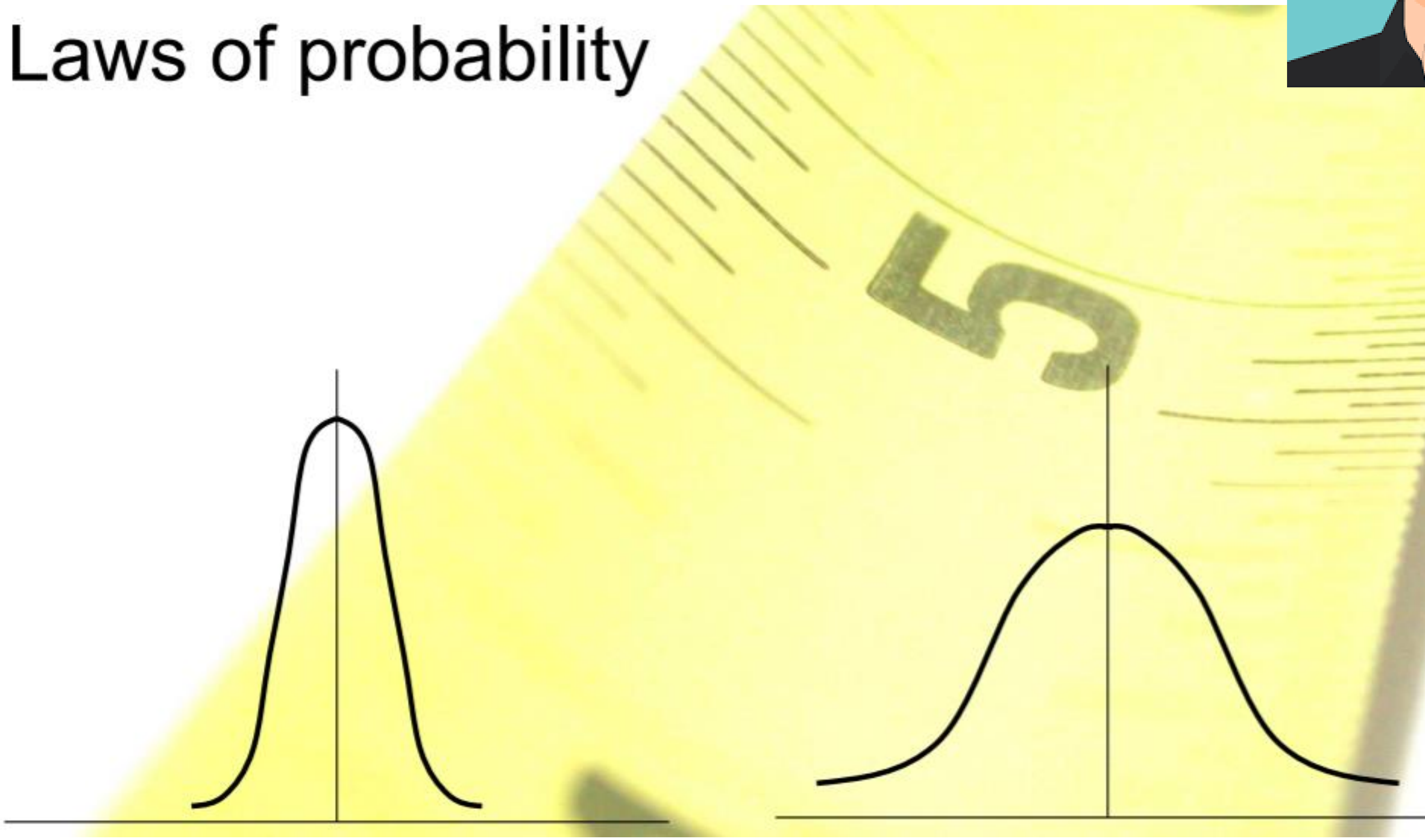
Error range	Associated Probability, %
$\pm 0.50\sigma$	38.3
$\pm 0.6745\sigma$	**50 (Probable Error)**
$\pm 1.00\sigma$	**68.3 (Standard Deviation)**
$\pm 1.6449\sigma$	90
$\pm 1.9599\sigma$	**95 (Maximum Anticipated Error)**
$\pm 2.00\sigma$	95.4
$\pm 3.00\sigma$	99.7
$\pm 3.29\sigma$	99.9

Note: In some surveying literature, the terms "standard deviation" and "standard error" are used interchangeably.

Normal Distribution Percentage Errors		
E_{50}	=	0.6745σ
E_{68}	=	1.0σ
E_{90}	=	1.6449σ
E_{95}	=	1.9599σ
$E_{max\ probable}$	=	3.0σ

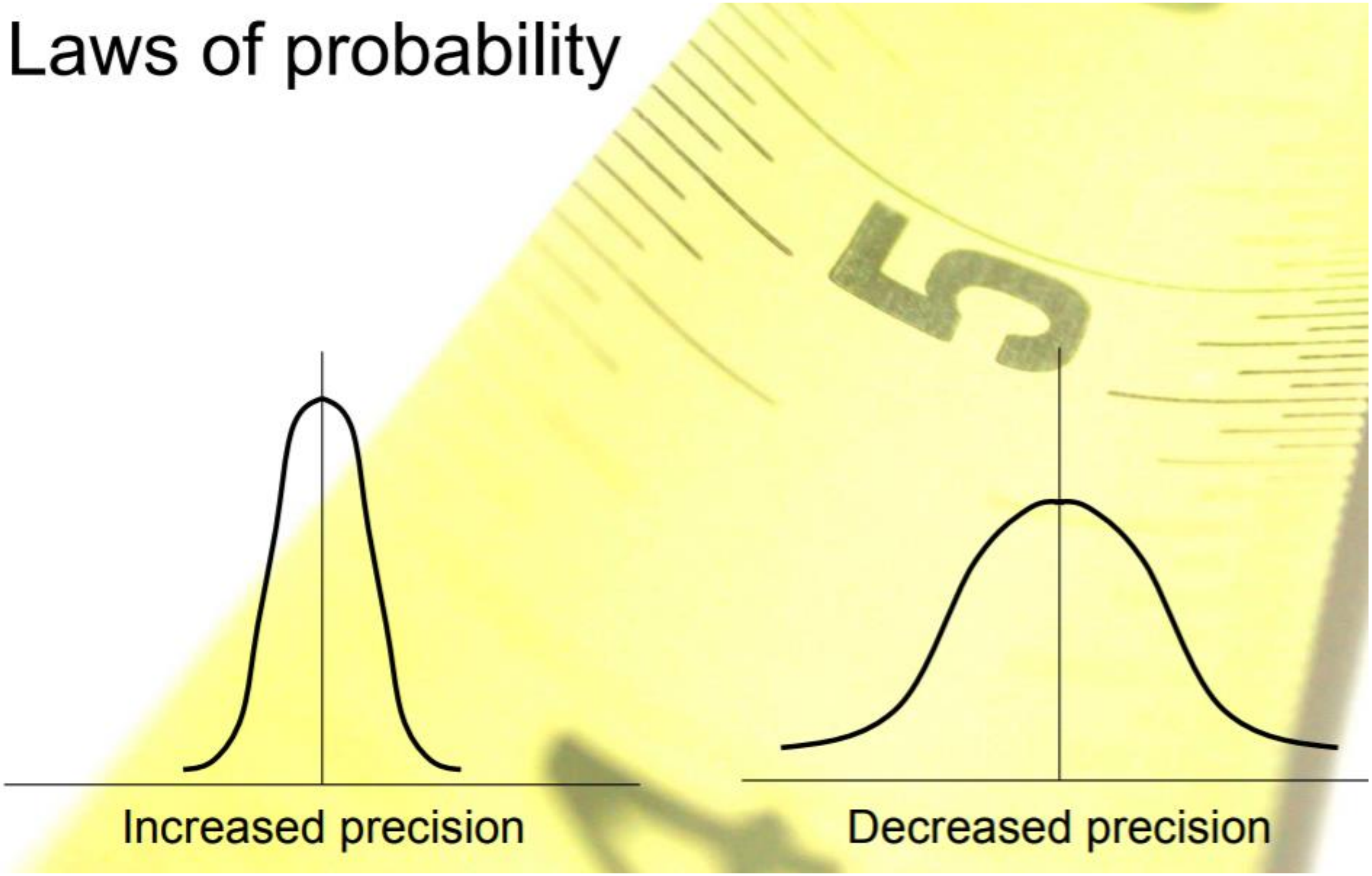
Why Doing that?

- Laws of probability



Why Doing that?

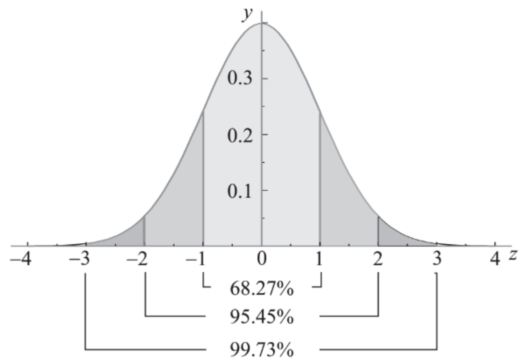
- Laws of probability



Probable Error

- ***n*** **measurements of a quantity**
  - **For what value of ζ, there will be a 50% chance that any value chosen at random will lie between−ζ and +ζ?**
  - **Or, for what value of the limits−ζ and ζ, the area under the standard normal curve be 0.5?**

$$\frac{1}{\sqrt{2\pi}}\int_{-\zeta}^{+\zeta}\exp\left(-\frac{1}{2}z^2\right)dz = \frac{1}{2}?$$



Solution $\zeta = \pm 0.6745$

- **Since ζ = (x−μ) /σ , the corresponding deviation *r* which is called the probable error is**

$$r = x - \mu = \pm 0.6745\sigma$$

❖ **There is 50% probability or even chance that an observation will have a random error no greater than ± r**

Probable Error

- **Probable error of the combinations**
  - **If the final result *X* is generated from individual measurands *p*, *q*, *r*, . . .**

$$X = f(p, q, r, \ldots)$$

$$r_X = \sqrt{\left(\frac{\partial X}{\partial p}\right)^2 r_p^2 + \left(\frac{\partial X}{\partial q}\right)^2 r_q^2 + \left(\frac{\partial X}{\partial r}\right)^2 r_r^2 + \cdots}$$

  - **The probable error of the combination: root-sum square (rss) formula**

Probable Error

Example 3.18

In a parallel circuit the current in one branch, I_1, is (100±2) A and in the other, I_2, is (200±5) A. Determine the total current considering errors as (a) limiting error, and (b) probable error.

Solution

(a) Total current, $I = I_1 + I_2$. Therefore,

$$\delta I = \pm(\delta I_1 + \delta I_2)$$
$$= \pm(2+5) = \pm 7 \text{ A}$$

So, $$I = 300 \pm 7 \text{ A}$$

(b) If the errors are probable errors, we have

$$r_I = \sqrt{\left(\frac{\partial I}{\partial I_1}\right)^2 r_{I_1}^2 + \left(\frac{\partial I}{\partial I_2}\right)^2 r_{I_2}^2}$$

Now, $$\frac{\partial I}{\partial I_1} = \frac{\partial I}{\partial I_2} = 1 \quad \text{and} \quad r_{I_1} = \pm 2, \; r_{I2} = \pm 5$$

$$r_I = \sqrt{4+25} \cong \pm 5.4 \text{ A}$$

Therefore, $$I = 300 \pm 5.4 \text{ A}$$

Note: The probable error gives a more optimistic estimate of the error than the limiting error.

Probable Error

Example 3.20

In a circuit the value of the current I is 10 A $\pm$ 1% and the voltage V across a resistor R of value 10 $\Omega \pm$ 0.1% is 100 V $\pm$ 1%, where the indicated errors are probable errors. The power consumed by the circuit can be calculated from the relations (a) $P = V^2/R$, and (b) $P = VI$. Calculate the probable errors in both the methods.

Solution

(a) Here,

$$\frac{\partial P}{\partial V} = \frac{2V}{R} = \frac{2 \times 100}{10} = 20, \qquad \frac{\partial P}{\partial R} = -\frac{V^2}{R^2} = -\frac{(100)^2}{(10)^2} = -100$$

$$r_V = \pm 1 \qquad\qquad r_R = \pm 0.01$$

From these we get,

$$\begin{aligned} r_P &= \sqrt{\left(\frac{\partial P}{\partial V}\right)^2 r_V^2 + \left(\frac{\partial P}{\partial R}\right)^2 r_R^2} \\ &= \sqrt{20^2 \times 1^2 + 100^2 \times 0.01^2} \\ &= \sqrt{401} \cong \pm 20.02 \text{ W} \end{aligned}$$

Hence $$P = 1000\,\text{W} \pm 2.002\%$$

(b) In this case,

$$\frac{\partial P}{\partial V} = I = 10, \quad \frac{\partial P}{\partial I} = V = 100, \quad r_V = \pm 1, \quad \text{and} \quad r_I = \pm 0.1$$

Therefore, $$r_P = \sqrt{\left(\frac{\partial P}{\partial V}\right)^2 r_V^2 + \left(\frac{\partial P}{\partial I}\right)^2 r_I^2}$$

$$= \sqrt{10^2 \times 1^2 + 100^2 \times 0.1^2} = \sqrt{200} \cong 14.14 \text{ W}$$

Thus $$P = 1000 \text{ W} \pm 1.414\%$$

Note: This example shows how to choose parameters of a quantity so that the error in its measurement is a minimum.

Contents

Reliability Principles

Three terms are defined in the context of reliability of measurement systems.

→Reliability

→Unreliability

→Mean Failure Rate

Reliability Principles

Reliability

Reliability *R (t)* of ***measurement system*** is defined as the probability that it will operate

→To an agreed level of performance

→For specified period of time

→Under specified conditions

→When used for the manner and purpose for which it is intended.

Example

Suppose, the agreed level of performance of a voltmeter is an accuracy of $\pm 2\%$ and the warranty period is 1 year. It should not be used above 40° and its maximum input should be 220 V. As long as the instrument is used in the specified conditions and it gives readings within this specified error limits, we consider it reliable. If it does not, although the system may be otherwise all right, it will be considered to have failed.

Because reliability is defined as a probability, its value always lies between 0 and 1. Quantitatively, reliability is defined as

$$R(t) = \Pr\ \{0 \text{ failures in } [0,t] | \text{ no failure at } t = 0\}$$

Reliability Principles

Reliability

n_0: total number of elements

n_s: number of elements working correctly at time t

n_f: number of elements that have failed after time t

$$R(t) = \frac{n_s}{n_0} = \frac{n_s}{n_s + n_f} \qquad (3.19)$$

We have denoted $R(t)$ as a function of time because it is a common experience that an instrument that has just been checked and calibrated has $R(t) = 1$, but after a lapse of, say, six months it might have a reliability of 0.9. It is also a common experience that the reliability of a system always decreases with time.

Reliability of system always decreases with time

Reliability Principles

Unreliability

Is the complement of reliability

<reliability>

$$\boxed{R(t) = \frac{n_s}{n_0} = \frac{n_s}{n_s + n_f}}$$

<unreliability>

$$\boxed{F(t) = \frac{n_f}{n_0} = \frac{n_f}{n_s + n_f}}$$

Since an element or system can only have failed or not failed, the total probability, that is, the sum of reliability and unreliability, must be unity. Thus,

$$R(t) + F(t) = 1$$

Unreliability of system always increases with time

Reliability Principles

Mean Failure Rate

The mean failure rate λ is average of faults per device per unit time.

λ = Average Faults/device/unit time

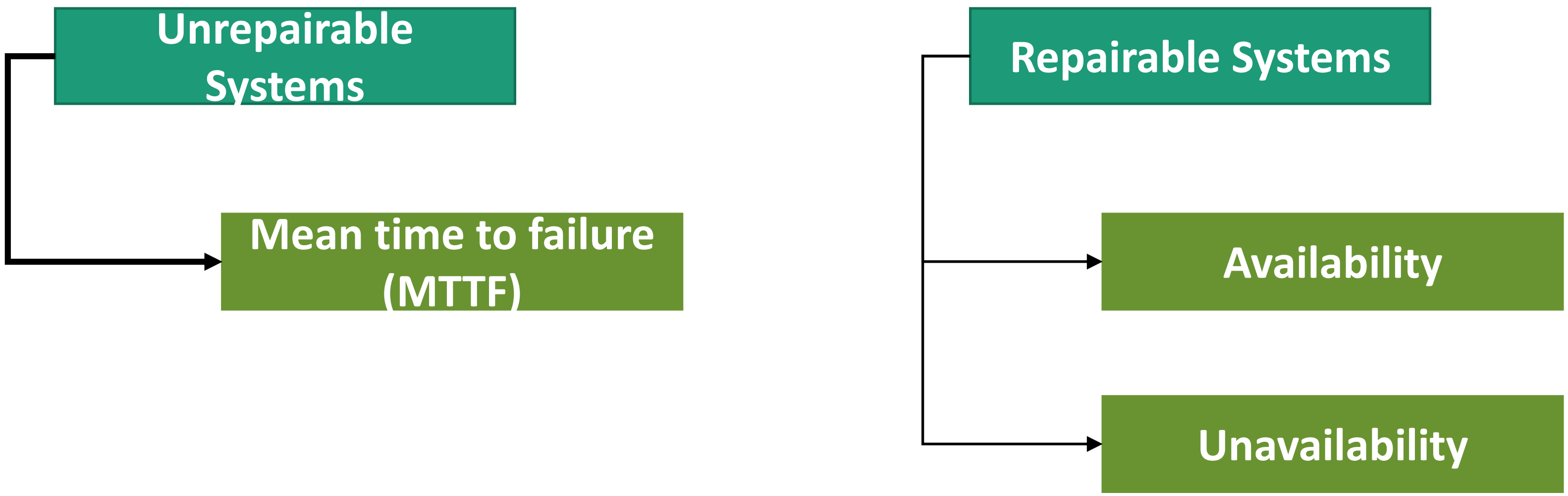
Commonly expressed in failures per year or failure per million hour (FPMH).

Example

television has a failure rate of five per million hours, it means that one can watch one million one hour-long television shows and may experience a failure during only five shows.

Reliability Principles

Measurement systems are of two types:



Reliability Principles

Unrepairable systems are discarded once a fault occurs

→Artificial Satellites

→Missiles

→Microprocessor

→Hard disc drives, etc.

Reliability Principles

Unrepairable systems

Mean time to failure (MTTF)

Failure rates are calculated from MTTF data. Suppose, n_0 number of new devices comprise an unrepairable system. The devices and their conditions are identical and they are allowed to operate until they fail. The time taken for each device to fail is noted and once it fails, it is taken out of service. The average of these times, when all n_0 devices have failed, is the MTTF. Thus, if we say the total survival time or 'up time' for the i-th failure is T_i, then

$$\text{MTTF} = \frac{\text{Total up time}}{\text{Total number of failures}} = \frac{\sum_{i=1}^{n_0} T_i}{n_0} \tag{3.20}$$

Obviously then,

$$\lambda = \frac{\text{Total number of failures}}{\text{Total up time}} = \frac{n_0}{\sum_{i=1}^{n_0} T_i} = \frac{1}{\text{MTTF}} \tag{3.21}$$

If the units of λ are number of failures per hour, then those of MTTF are hours.

Reliability Principles

Unrepairable systems

Mean time to failure (MTTF)

We observe that at time $t = 0$, n_0 devices survive and at $t = T$, 0 devices survive. So, from Eq. (3.19) the reliability $R(i)$ is

$$R(i) = \frac{\text{Number of devices survived at } t = T_i}{\text{Total number of devices at } t = 0} = \frac{n_0 - i}{n_0} \tag{3.22}$$

From Eq. (3.22) we can construct a survival table as shown in Table 3.2. The $R(i)$ vs. t plot

Table 3.2 Survival table of devices

Time (t)	*Number of survivors*	*Reliability* $[R(i)]$
0	n_0	1
T_1	$n_0 - 1$	$\frac{n_0 - 1}{n_0}$
$\vdots$	$\vdots$	$\vdots$
T_i	$n_0 - i$	$\frac{n_0 - i}{n_0}$
$\vdots$	$\vdots$	$\vdots$
T	0	0

looks like Fig. 3.12 where rectangles have heights of $1/n_0$.

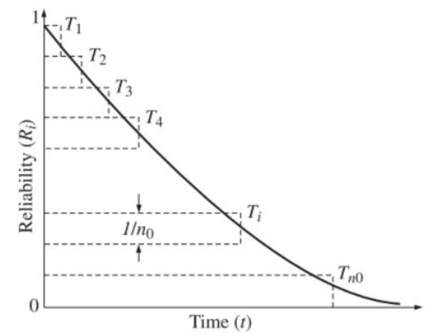


Fig. 3.12 Reliability vs. time plot for unrepairable systems.

The area under the curve is given by

$$\text{Area} = \left(T_1 \cdot \frac{1}{n_0}\right) + \left(T_2 \cdot \frac{1}{n_0}\right) + \left(T_3 \cdot \frac{1}{n_0}\right) + \cdots + \left(T_{n0} \cdot \frac{1}{n_0}\right) = \frac{\sum_{i=1}^{n_0} T_i}{n_0} \tag{3.23}$$

Comparing Eqs. (3.20) and (3.23), we find that the area is the MTTF itself. By making $n_0 \to \infty$, we can write Eq. (3.23) as

$$\text{MTTF} = \int_0^\infty R(t)\, dt \tag{3.24}$$

Reliability $R(t)$ and failure rate λ are related. Let us consider a simple analysis here.

Reliability Principles

Repairable systems In this case, the failed components are repaired or replaced and the system is put back into service.

Useful metrics are

→ Mean down time (MDT)

A particularly useful metric for repairable systems is the *mean time between failures* (MTBF). Let n_0 identical repairable systems or devices be tested over a period of time T. Once a fault occurs, it is recorded, the device repaired and put back into service. If T_i' be the down time of the i-th failure (Fig. 3.13), the total down time for n_f failures is $\sum_{i=1}^{n_f} T_i'$ and the *mean down time* (MDT) is

$$\mathrm{MDT} = \frac{\sum_{i=1}^{n_f} T_i'}{n_f}$$

Fig. 3.13 Failure pattern of repairable systems.

Reliability Principles

Repairable systems

Obviously, the total up time is then

$$\text{Total up time} = n_0 T - \sum_{i=1}^{n_f} T_i' = n_0 T - n_f(\text{MDT})$$

The MTBF is, therefore,

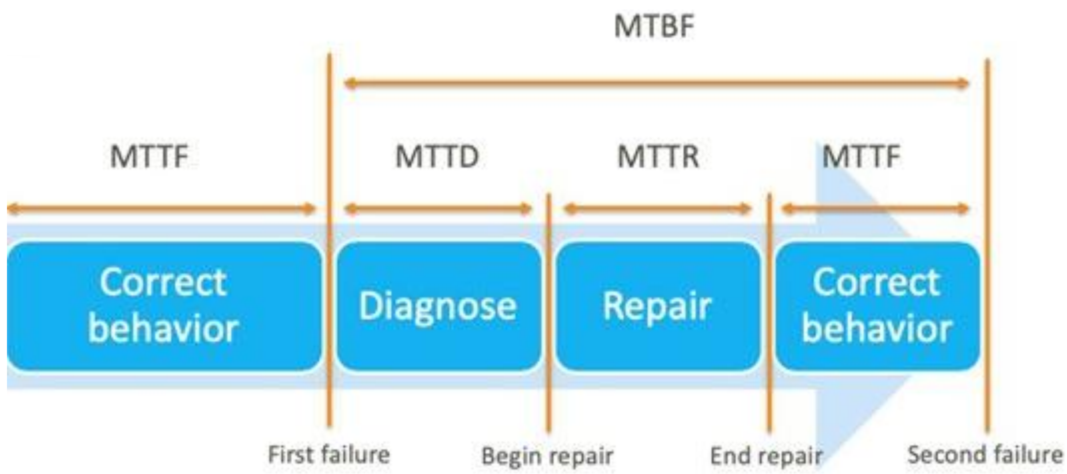
$$\text{MTBF} = \frac{\text{Total up time}}{\text{Total number of failures}} = \frac{n_0 T - n_f(\text{MDT})}{n_f} \tag{3.32}$$

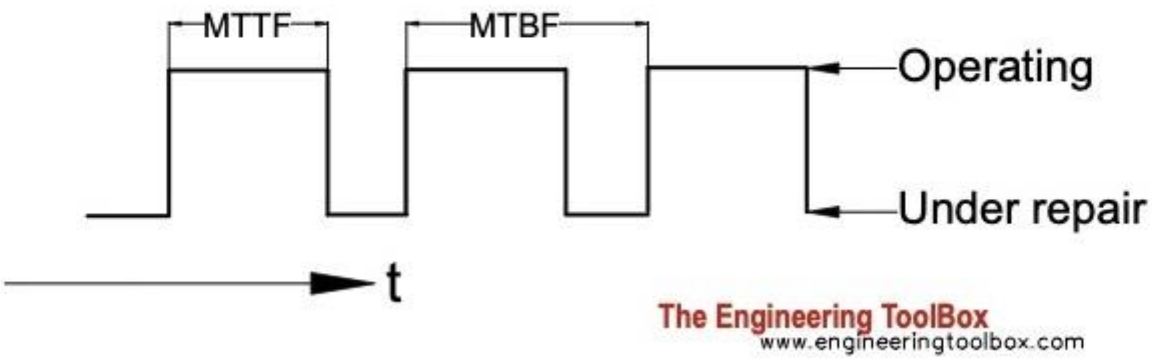
and the corresponding mean failure rate is

$$\lambda = \frac{1}{\text{MTBF}} \tag{3.33}$$

$$= \frac{n_f}{n_0 T - n_f(\text{MDT})} \tag{3.34}$$

MTTF and MTBF?





Example

$$\text{Total up time} = n_0T - \sum_{i=1}^{n_f} T_i' = n_0T - n_f(\text{MDT})$$

The MTBF is, therefore,

$$\text{MTBF} = \frac{\text{Total up time}}{\text{Total number of failures}} = \frac{n_0T - n_f(\text{MDT})}{n_f} \quad (3.32)$$

Example 3.30

Calculate the MTBF and the mean failure rate if 100 faults were recorded for 300 transducers of a system during 1.5 years, the mean down time being 1 day.

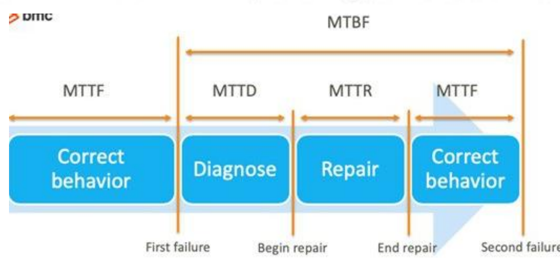
Solution

$\text{MDT} = 1 \text{ day} = \frac{1}{365}$ yr. Therefore, from Eq. (3.32)

$$\text{MTBF} = \frac{(300)(1.5) - (100)(1/365)}{100} = 4.497 \text{ yrs}$$

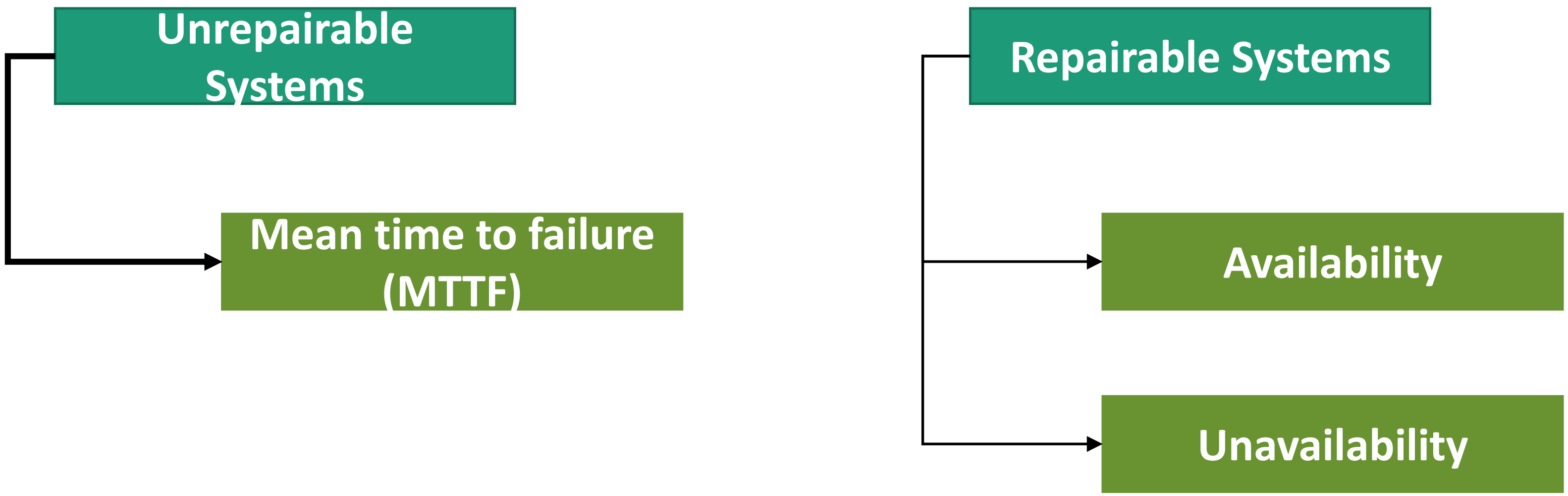
$$\text{Mean failure rate } \lambda = \frac{1}{4.497} = 0.222 \text{ per yr}$$

Note: Sometimes, the vendors mention a term *mean time to repair* (MTTR) which is lower than MDT because their MTTR is based on the assumption that a fully trained technician, complete with appropriate spares and test equipment, is ready for 24 h a day and that failed equipment is immediately available for repairs. So, the MTTR is rather an optimistic estimate while MDT, being the sum of MTTR and other delays, is more realistic.



Reliability Principles

Measurement systems are of two types:



Reliability Principles

Repairable systems

Availability

The availability A is the probability that a system will be functioning correctly when needed. In other words, it is the fraction of the total up time during a test interval. That is,

$$\begin{aligned} A &= \frac{\text{Total up time}}{\text{Test interval}} \\ &= \frac{\text{Total up time}}{\text{Total up time} + \text{Total down time}} \\ &= \frac{(n_f)(\text{MTBF})}{(n_f)(\text{MTBF}) + (n_f)(\text{MDT})} \\ &= \frac{\text{MTBF}}{\text{MTBF} + \text{MDT}} \end{aligned} \tag{3.35}$$

Example

Example 3.30
Calculate the MTBF and the mean failure rate if 100 faults were recorded for 300 transducers of a system during 1.5 years, the mean down time being 1 day.

Solution
MDT = 1 day = $\frac{1}{365}$ yr. Therefore, from Eq. (3.32)

$$\text{MTBF} = \frac{(300)(1.5) - (100)(1/365)}{100} = 4.497 \text{ yrs}$$

$$\text{Mean failure rate } \lambda = \frac{1}{4.497} = 0.222 \text{ per yr}$$

Example 3.31
Calculate the availability of the system of Example 3.30.

Solution
In Example 3.30, we had MTBF = 4.497 yrs and MDT = (1/365) yr. Therefore, from Eq. (3.35)

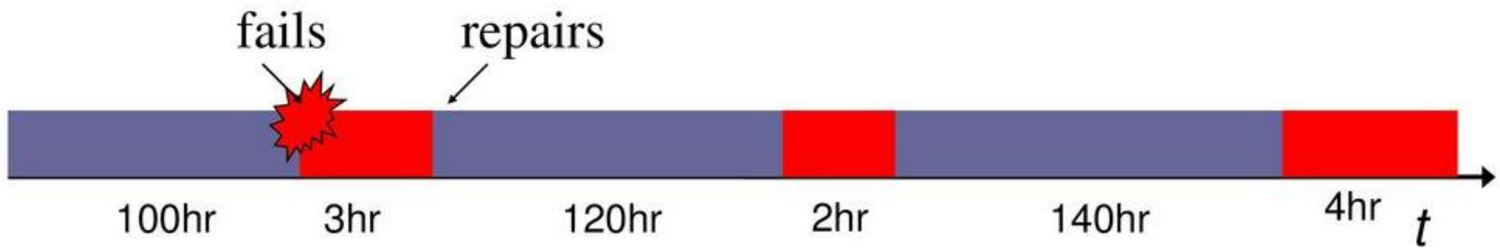
$$A = \frac{4.497}{4.497 + (1/365)} = 0.999$$

In other words, the availability is 99.9%.

Note: An availability of 99.9% is often called *three nines availability*. The one nine availability is not 9%, but 90%, two nines, 99% and five nines, 99.999%.

Concept Checks

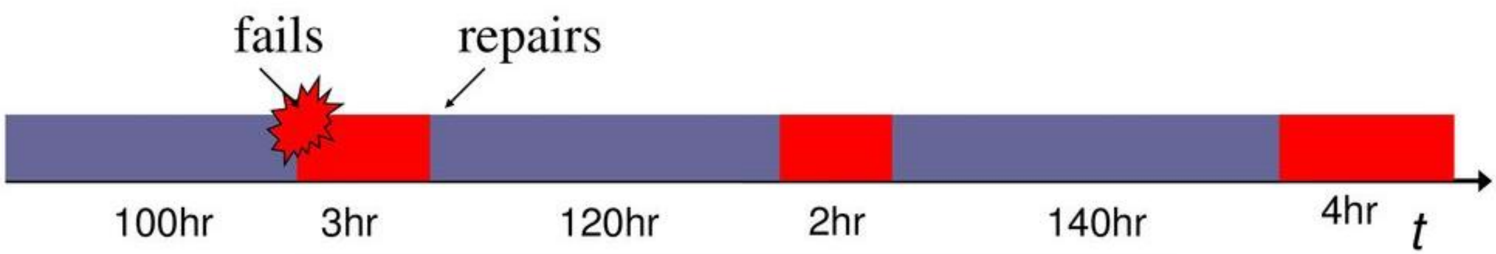
- What is Availability of this system ?



Concept Checks

Availability & MTTF & MTTR

- What is Availability of this system ?



$$A = MTTF / (MTTF+MTTR)$$

$$MTTF = (100 + 120 + 140) / 3 = 120 \text{ (hr/case)}$$
$$MTTR = (3 + 2 + 4) / 3 = 3 \text{ (hr/case)}$$

$$A = 120 / 120+3$$
$$= 120/123 = 0.975$$

Reliability Principles

Repairable systems

Unavailability

The unavailability U is the complement of availability. Since these are probabilities,

$$U = 1 - A$$
$$= \frac{\text{MDT}}{\text{MTBF} + \text{MDT}}$$

Substituting $1/\lambda$ for MTBF [see Eq. (3.33)], we get

the corresponding mean failure rate is

$$\lambda = \frac{1}{\text{MTBF}} \tag{3.33}$$

$$U = \frac{\lambda(\text{MDT})}{1 + \lambda(\text{MDT})} \approx \lambda(\text{MDT})$$

assuming $\lambda(\text{MDT}) \ll 1$.

Hazard rate

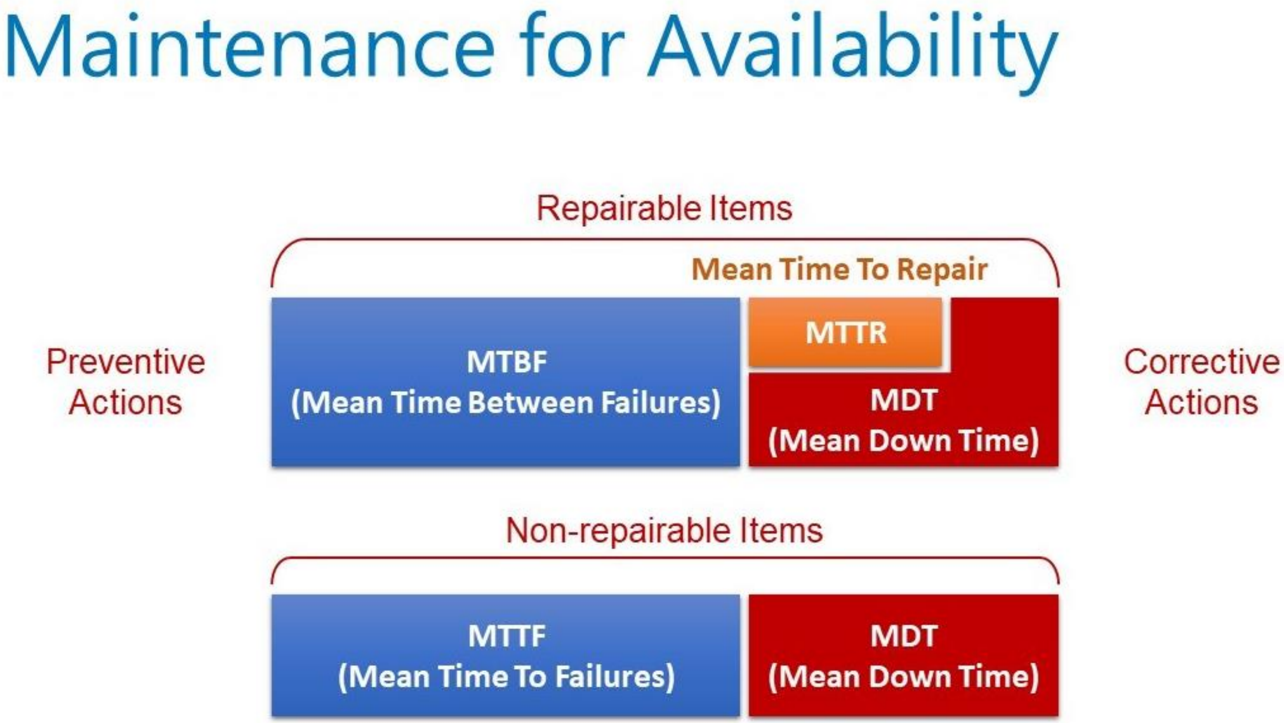
Often a *hazard rate* or *instantaneous failure rate* is defined if λ is not constant. Written as $\lambda(t)$, it is defined as

$$\lambda(t) = \frac{\text{Failure probability}}{\Delta t} = \frac{\Delta n_f}{n_0 \Delta t}$$

where Δt is a small span of time.

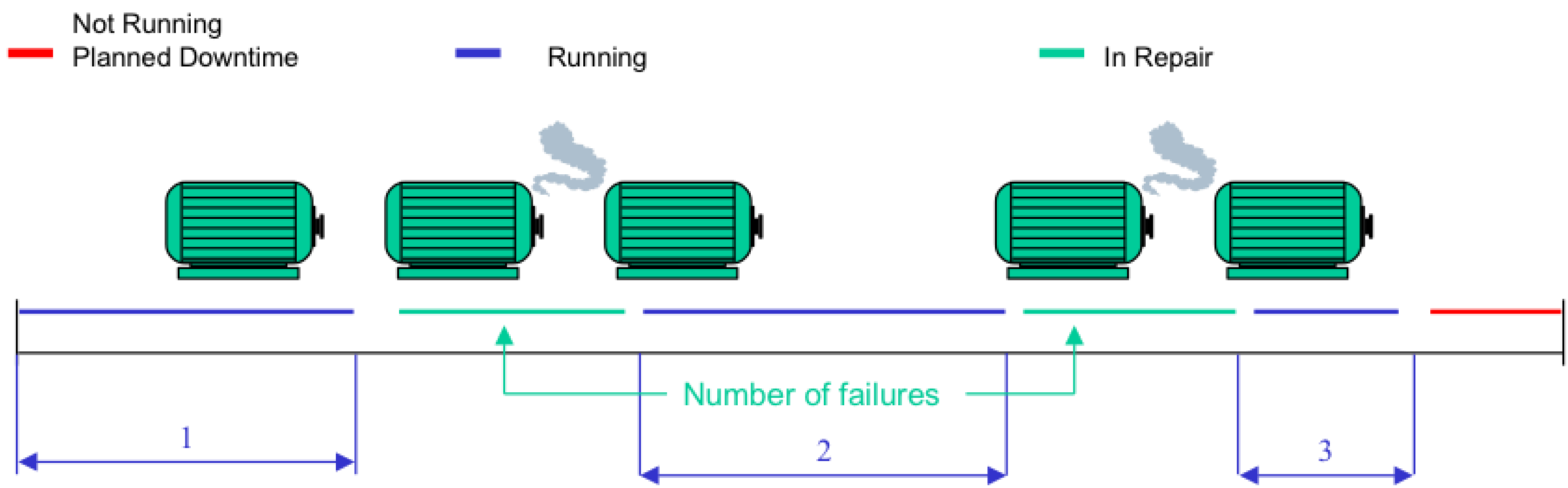
Concept Checks

MTTF and MTBF are for which Systems/Items…?



Concept Checks

MTBF & Reliability relationship?



Total Operating Time = 1 + 2 + 3

$$\text{M.T.B.F} = \frac{\text{Total operating Time}}{\text{Number of failures}}$$

Comparing Eqs. (3.20) and (3.23), we find that the area is the MTTF itself. By making $n_0 \to \infty$, we can write Eq. (3.23) as

$$\text{MTTF} = \int_0^\infty R(t)\,dt \tag{3.24}$$

Reliability $R(t)$ and failure rate λ are related. Let us consider a simple analysis here.

As reliability increases MTBF also increases

Queries

Thanks!