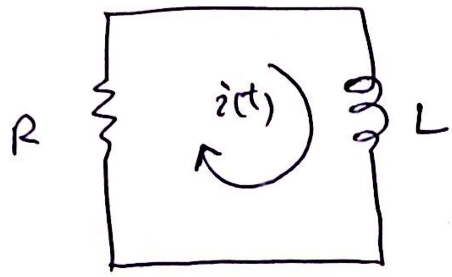
8.1 The Source-Free RL Circuit

(PP 261 8th Ed HKD)

Consider the circuit:-



Given
$i(t_0) = i(0) = I_0$

(Fig 8.1 A series RL Circuit)

— Applying KVL,

$$Ri + L\frac{di}{dt} = 0$$

or $$\frac{di}{dt} + \frac{R}{L}i = 0 \quad \text{——— (A)}$$

We're to solve this differential equation to determine $i(t)$.

— Assume that the solution is:

$$i(t) = Ae^{s_1 t}.$$

where A and s_1 are unknown constants.

——— contd

— Contd (261)

— Now $\frac{di}{dt} = A s_1 e^{s_1 t}$

— Hence (A) becomes

$$A s_1 e^{s_1 t} + \frac{R}{L} A e^{s_1 t} = 0$$

or $A e^{s_1 t}\left(s_1 + \frac{R}{L}\right) = 0$

— Therefore $s_1 + \frac{R}{L} = 0$

or $s_1 = -\frac{R}{L}$

Hence $i(t) = A e^{-R/L\, t}$

— At $t=0$

$$i(t_0) = A \times 1 = I_0 \quad \text{(Given)}$$

So $i(t) = I_0 e^{-R/L\, t}$, A

— This is the functional form of the source-free response.

— it is called "natural response"

— also called "transient response"

Note: The circuits with a single storage element are "first-order circuits".

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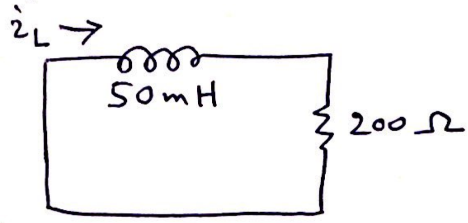
Example 8.1 The Source-Free RL Circuit

(PP 263 8th Ed HKD)

If $i_L = 2$ A at $t = 0$, find an expression for $i_L(t)$ valid for $t > 0$ and its value at $t = 200\ \mu s$.



Given $i_L(0) = 2$ A

Solution: We have learnt that the functional form of the source-free current shall be:

$$i_L(t) = I_0 e^{-\frac{R}{L}t} \text{ A}$$

Now := $R = 200\,\Omega$ and $L = 50\,mH$

and $I_0 = 2$ A ie $i_L(0) = I_0 = 2$ A (Given)

So $$i_L(t) = 2\, e^{-\frac{200}{50\times10^{-3}}t}$$

or $$i_L(t) = 2\, e^{-4000t} \text{ A} \quad t > 0$$

— $i_L(t)$ at $t = 200\,\mu s$ is:

$$i_L(200\times10^{-6}s) = 2\, e^{-4000\times200\times10^{-6}}$$

$$= 2\, e^{-0.8} = 2\times0.449$$

$$= 0.8987 \text{ A}$$

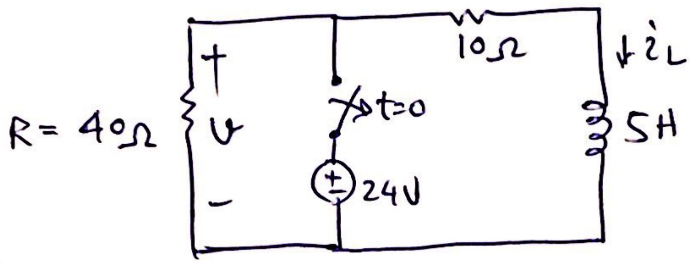
or $$= 898.7 \text{ mA}$$

(Example)

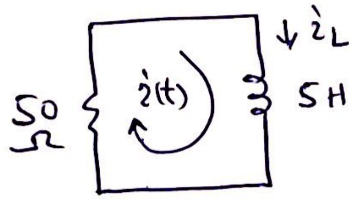
Example 8.2 The Source-Free RL Circuit

(PP 266 8th Ed HKD)

Find the voltage labelled 'v' at $t = 200\,ms$.



Solution: At $t \geq 0$ the circuit becomes



$\downarrow i_L(0) = \frac{24}{10} = 2.4\text{ A} = I_0$

— The functional form of the response is :-

$$i(t) = I_0\, e^{-R/L\, t}$$

or $i(t) = 2.4\, e^{-\frac{50}{5}t} = 2.4\, e^{-10t}$ A

So $v(200\,ms) = (\overbrace{\underline{40} \times -2.4\, e^{-10 \times 200 \times 10^{-3}}}^{R \times i(t)})$ V

or $v(200\,ms) = -96\, e^{-2} = -96 \times 0.1353$

Hence $v(200\,ms) = -12.99$ V

Note: Disregard the lengthier approach in text book.