The Unit-Step Function

(pp 282 8th Ed HKD)

It is defined as "a function of time which is zero for all values of its argument less than zero and which is unity for all positive values of its argument."

Consider:

Represent unit step function by u.

Let the argument be $(t - t_0)$.

Then $u(t - t_0)$ must be zero for all values of t less than t_0.

— And it must be unity for all values of t greater than t_0.

— The operation of a switch in series with a battery is equivalent to a forcing function which is zero upto the instant that the switch is closed and is equal to the battery voltage thereafter.

— It has a break, or discontinuity, at the instant the switch is closed.

——— cont'd

— The value of the function at $t = t_0$ is not defined.

— We often indicate this by writing

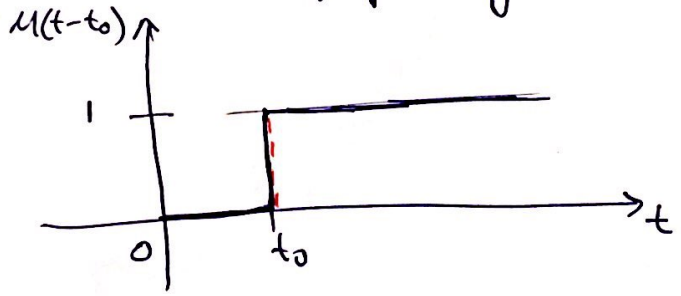
$$u(t_0^-) = 0 \quad \text{and}$$

$$u(t_0^+) = 1$$

— The concise mathematical definition of the unit-step forcing function is :-

$$u(t-t_0) = \begin{cases} 0 & t < t_0 \\ 1 & t > t_0 \end{cases}$$

— It can be shown graphically as



(Fig 8.26 The unit-step forcing function $u(t-t_0)$)

Note: The unit-step function is dimensionless.

— If we wish it to represent a voltage, it is necessary to multiply $u(t-t_0)$ by some constant voltage.

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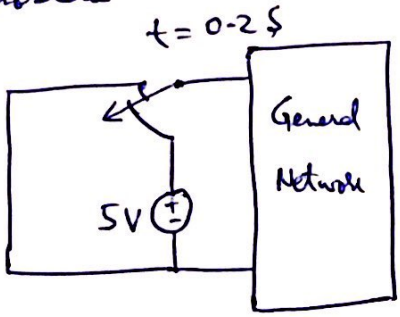
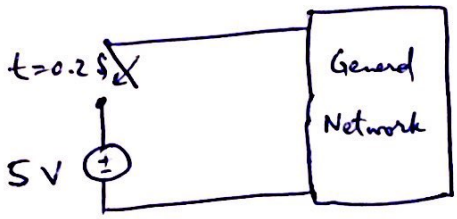
—contd (284)

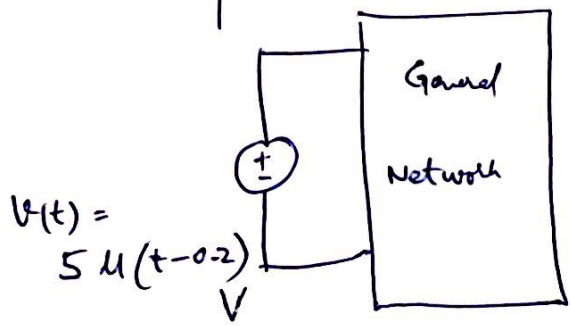
So the problem



or

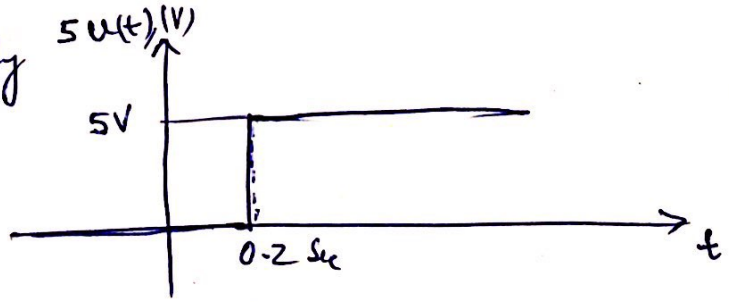


Can be represented as :-



Now $5u(t-0.2)V$ means it is a voltage source that is '0' before $t = 0.2$ Sec and a constant 5V after $t = 0.2$ Sec.

Graphically



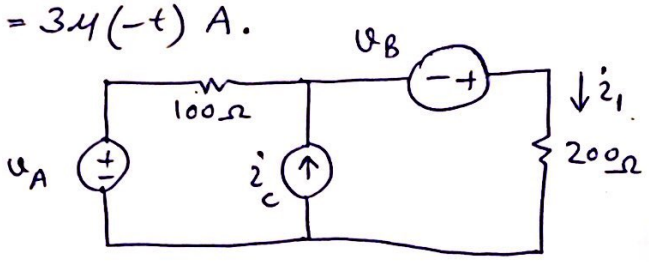
Note: The unit-step function is discontinuous at $t = 0$.

(Evaluations)

Prob 8.53 The Unit-Step Function

(pp 310 7th Ed HKD)

For the given circuit, determine i_1 at $t = -1.5$ s, -0.5 s, 0.5 s and at $t = 1.5$ s, when $v_A = 300\,u(t-1)$ V, $v_B = -120\,u(t+1)$ V and $i_C = 3\,u(-t)$ A.



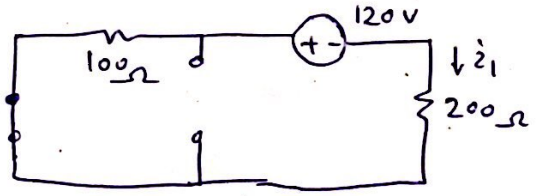
Solution: Let us find i_1 for $t = 0.5$ s.

At $t = 0.5$ s, $v_A = 300\,u(0.5-1) = 300\,u(-0.5) = 0$

$v_B = -120\,u(0.5+1) = -120$ V

$i_C = 3\,u(-0.5) = 0$ A

Now the circuit can be redrawn as:-



The polarity of 120 V has been reversed to cater for -ve sign.

Hence $i_1 = -\frac{120}{300} = -400$ mA

Similarly $i_1(-1.5\text{ s}) = 1$ A

$i_1(-0.5\text{ s}) = 600$ mA

$i_1(+1.5\text{ s}) = 600$ mA