Digital Logic Design

Lecture No 06 : Signed Number Arithmetics

BEE-12CD

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Review of Last Lecture

- Subtraction using r's Complement?
  - If $M \geq N$, the sum will produce an end carry, r^n, which can be discarded; what is left is the result of $M - N$.
  - If $M < N$, the sum does not produce an end carry and is equal to $r^n - (N - M)$, which is the r's complement of $(N - M)$. To obtain the answer in a familiar form, take the r's complement of the sum and place a negative sign in front
  - If both are negative numbers, then end carry will occur, discard the carry, take complement of sum and place a negative sign

- Subtraction using r-1's Complement?
  - If an end carry occurs add 1
  - If there is no end carry take (r-1)'s complement of the result obtained and place a negative sign
  - If both are negative numbers, then end carry will occur, add the carry, take complement of sum and place a negative sign

Unsigned Binary Numbers Verses Signed Numbers

- Unsigned Number
  - Last digital is part of magnitude
- Signed Number
  - Last digital is a sign bit
  - 0 for Positive, 1 for Negative
  - Three ways of representation
    - Sign Magnitude
    - Signed r's Complement
    - Signed r-1's Complement
  - All positive numbers are same in three signed numbers systems

- Example. 10101
- Unsigned Number
  - 10101 is 21 in decimal
- Signed Number
  - Three ways of representation
    - Sign Magnitude 10101 is -5 in decimal
    - Signed r's Complement 10101 is -11 in decimal
    - Signed r-1's Complement is -10 in decimal

Signed Binary Numbers Range

How many numbers can be represented by four bits?

Decimal	Signed-2's complement	Signed-1's complement	Signed Magnitude
+7	0111	0111	0111
+6	0110	0110	0110
+5	0101	0101	0101
+4	0100	0100	0100
+3	0011	0011	0011
+2	0010	0010	0010
+1	0001	0001	0001
+0	0000	0000	0000
-0	------	1111	1000
-1	1111	1110	1001
-2	1110	1101	1010
-3	1101	1100	1011
-4	1100	1011	1100
-5	1011	1010	1101
-6	1010	1001	1110
-7	1001	1000	1111
-8	1000	------	------

Signed Numbers Summary

- Range of numbers
  - Signed r's complement?
    - $-(r^{n-1})$ to $+(r^{n-1}-1)$ and no -0
  - Signed r-1's complement?
    - $-(r^{n-1}-1)$ to $+(r^{n-1}-1)$

Arithmetic Addition (Signed-Magnitude System)

- The addition of two signed binary numbers in the signed-magnitude system follows the rules of ordinary arithmetic
- If the signs are the same we add the two magnitudes and give the sum the common sign
- If the signs are different we subtract the smaller magnitude from the larger and give the result the sign of the larger magnitude

Arithmetic Addition (Signed 2's Complement system)

- This system doesn't require the comparison of the signs and the magnitudes (as in signed-magnitude system), but only addition.
- The addition of two signed binary numbers with negative numbers represented in signed-2's complement form is obtained from addition of the two numbers, including their sign bits.
- A carry out of the sign-bit position is discarded.
- If the sum is negative, it will be in 2's complement form.

Example Arithmetic (Signed 2's Complement)

Arithmetic 9 and 11

+ 9		00001001		- 9		11110111
+11		00001011		+11		00001011
+20		00010100		+ 2		00000010
+ 9		00001001		- 9		11110111
-11		11110101		-11		11110101
- 2		11111110		-20		11101100

Add -100 and -56

Sign 64 32 16 8 4 2 1

+100 0 1 1 0 0 1 0 0

+56 0 0 1 1 1 0 0 0

0

1 0 0 1 1 1 0 0

+ 1 1 0 0 1 0 0 0

1) 0 1 1 0 0 1 0 0

overflow.

128 64 32 16 8 4 2 1

0 1 0 0 1 1 1 0 0

= +56.

Ans = -156.

Arithmetic Subtraction

- Subtraction can be performed by simply converting the equation into an addition formula.
  - Take the 2's complement of the subtrahend (including the sign bit) and add it to the minuend (including the sign bit)
  - A carry out of the sign bit position is discarded
  - Note: Subtraction operation can be changed to an addition operation if the sign of the subtrahend is changed. This is easily done by taking it's 2's complement

Example

- Consider the subtraction (–6) –(–13) = +7
- In binary with eight bits the same is written as (11111010 – 11110011)
- This subtraction is changed to addition by taking 2's complement of the subtrahend (– 13) to give (+ 13)
- In binary this is 11111010 + 00001101 = 100000111
- Removing the end carry, we obtain the correct answer: 00000111(+7)

The End