ELECTROMAGNETIC WAVE PROPAGATION

Introduction

- Our goal is to derive EM wave motion in the following media:

1. Free space ($\sigma = 0, \varepsilon = \varepsilon_o, \mu = \mu_o$)
2. Lossless dielectrics ($\sigma = 0, \varepsilon = \varepsilon_r\varepsilon_o, \mu = \mu_r\mu_o$ or $\sigma \ll \omega\varepsilon$)
3. Lossy dielectrics ($\sigma \neq 0, \varepsilon = \varepsilon_r\varepsilon_o, \mu = \mu_r\mu_o$)
4. Good conductors ($\sigma \approx \infty, \varepsilon = \varepsilon_o, \mu = \mu_r\mu_o$ or $\sigma \gg \omega\varepsilon$)

- where ω is the angular frequency of the wave

- Case 3, for lossy dielectrics, is the most general case and will be considered first

- Remaining cases derived by changing the values of $\sigma, \varepsilon,$ and μ

Waves in General

- A clear understanding of EM wave propagation depends on a grasp of what waves are in general
- A **wave** is a function of both space and time
- Wave equation is derived from scalar potentials for time-varying fields and is a partial differential equation of the second order
- In one dimension, a scalar wave equation takes the form of:

$$\frac{\partial^2 E}{\partial t^2} - u^2 \frac{\partial^2 E}{\partial z^2} = 0$$

- where *u* is the wave velocity

Waves in General

- The solutions of the wave equation are of the form:

$$E^- = f(z - ut)$$

$$E^+ = g(z + ut)$$

- Or:

$$E = f(z - ut) + g(z + ut)$$

- where f and g denote any function of $z - ut$ and $z + ut$, respectively

- Examples of such functions include sin $k(z \pm ut)$, cos $k(z \pm ut)$ and $e^{jk(z \pm ut)}$, where k is a constant

- It can easily be shown that these functions all satisfy the wave equation

Waves in General

- If we particularly assume harmonic (or sinusoidal) time dependence $e^{j\omega t}$, the wave equation becomes:

$$\frac{d^2E_s}{dz^2} + \beta^2 E_s = 0$$

- where $\beta = \omega/u$ and E_s is the phasor form of E

- With the time factor inserted, the possible solutions to the above equation are:

$$E^+ = Ae^{j(\omega t - \beta z)}$$

$$E^- = Be^{j(\omega t + \beta z)}$$

- And:

$$E = Ae^{j(\omega t - \beta z)} + Be^{j(\omega t + \beta z)}$$

- Where A and B are real constants

Waves in General

- For the moment, lets consider the solution below:

$$E^+ = Ae^{j(\omega t - \beta z)}$$

- Taking the imaginary part of this equation, we have:

$$E = A \sin(\omega t - \beta z)$$

- This is a sine wave chosen for simplicity; a cosine wave would have resulted had we taken the real part

Waves in General

- Note the following characteristics of the solution of wave equation shown previously:

1. It is time harmonic because we assumed time dependence $e^{j\omega t}$ to arrive at the solution

2. *A* is called the amplitude of the wave and has the same units as *E*

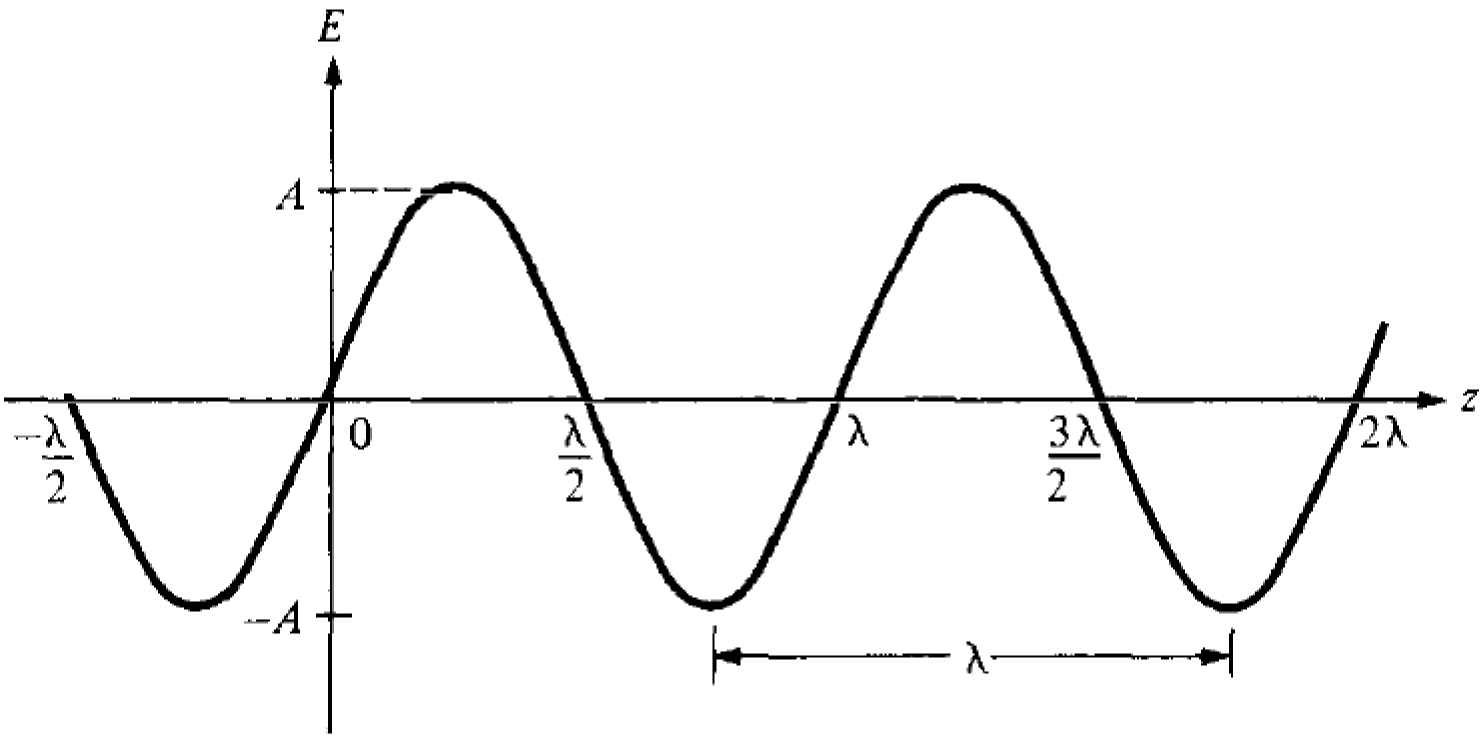
3. $(\omega t - \beta z)$ is the phase (in radians) of the wave; it depends on time t and space variable z

4. ω is the angular frequency (in radians/second); β is the phase constant or wave number (in radians/meter)

Waves in General

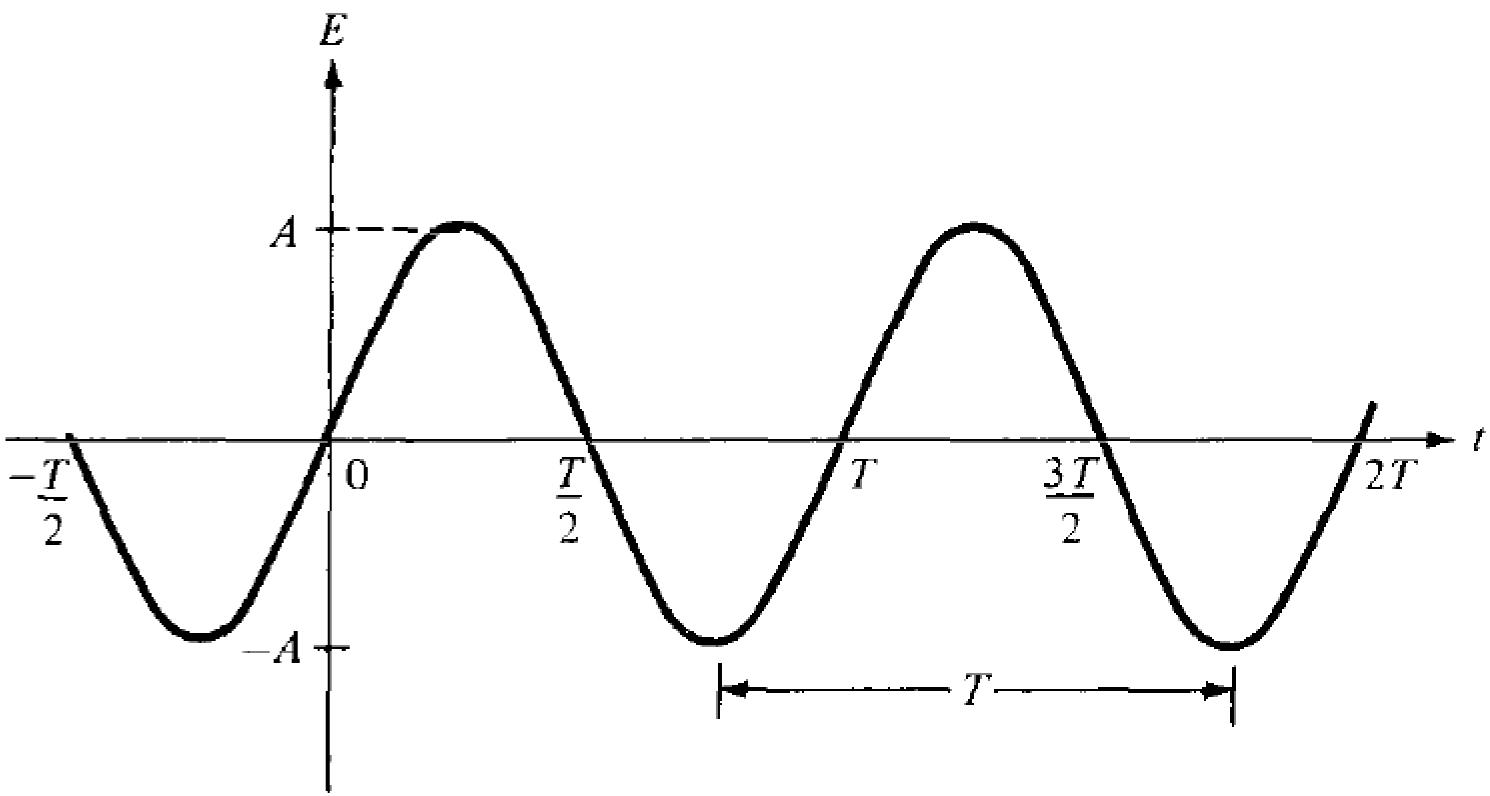
- Due to the variation of E with both time t and space variable z, we may plot E as a function of t by keeping z constant and vice versa

- Figure below is a plot for $E(z,\ t$ = constant)



Waves in General

- Figure below is a plot for $E(z = \text{constant}, t)$



Waves in General

- We observe that the wave takes distance λ to repeat itself and hence λ is called the wavelength (in meters)

- Also, the wave takes time T to repeat itself; consequently, T is known as the time period (in seconds)

- Since it takes time T for the wave to travel distance λ at the speed *u*, we expect:

$$\lambda = uT$$

- But T = *1/f*, where *f* is the frequency (the number of cycles per second) of the wave in Hertz (Hz), hence:

$$u = f\lambda$$

Waves in General

- We will now show that the wave equation below is traveling with a velocity *u* in the +z direction

$$E = A \sin(\omega t - \beta z)$$

- To do this, we consider a fixed point *P* on the wave and sketch the above equation at times $t = 0$, $T/4$, and $T/2$

- The point P is a point of constant phase with respect to a reference, therefore:

$$\omega t - \beta z = \text{constant}$$

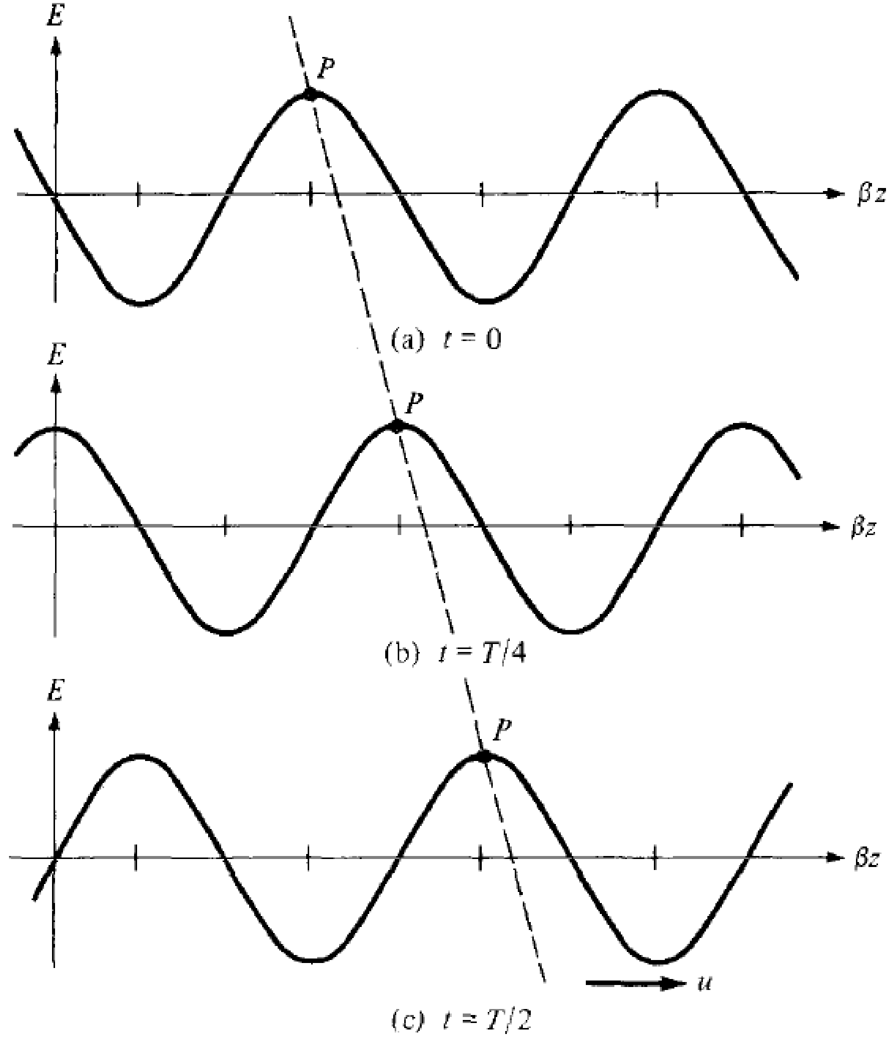
- By differentiating both sides w.r.t time, we get:

$$\frac{dz}{dt} = \frac{\omega}{\beta} = u$$

- Therefore, the wave travels with velocity *u* in the +z-direction

Waves in General

- From the figure, it is evident that as the wave advances with time, point *P* moves along +z-direction



Waves in General

➢ In summary, we note the following:

1. A wave is a function of both time and space
2. Though time t = 0 is arbitrarily selected as a reference for the wave, a wave is without beginning or end
3. A negative sign in $(\omega t \mp \beta z)$ is associated with a wave propagating in the +z-direction (forward traveling or positive-going wave)
4. Whereas a positive sign indicates that a wave is traveling in the $-z$-direction (backward traveling or negative going wave)

Problem-1

- The electric field in free space is given by:

$$\mathbf{E} = 50 \cos (10^8 t + \beta x)\, \mathbf{a}_y \text{ V/m}$$

a) Find the direction of wave propagation.
b) Calculate β and the time it takes to travel a distance of $\lambda/2$.
c) Sketch the wave at t = 0, T/4, and T/2