ate

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Communication Systems

Consider $x(t)$

$$\Rightarrow E_x = \int_{-\infty}^{\infty} |x(t)|^2 \, dt \quad \Rightarrow P_x = \lim_{T \to \infty} \frac{1}{T} \int_{-T/2}^{T/2} |x(t)|^2 dt$$

i. $x(t) = \begin{cases} e^{-t}, & t \geq 0 \\ 0, & t < 0 \end{cases}$

$$E_x = \int_0^{\infty} e^{-2t} \, dt = \frac{1}{2}$$

>>> Periodic Signals

Represented as:

$$x(t) = x(t + kT) \quad \forall t, \forall k \in \mathbb{Z}$$

ii. Prove $\sin(t)$ is periodic

$$x(t) = \sin(2\pi f t)$$

$$x(t + kT) = \sin(2\pi f (t + kT))$$

$$\Rightarrow x(t + kT) = \sin(2\pi f t + 2\pi f kT)$$

ignore → $= \sin(2\pi f t)\cos(2\pi f kT) + \cos(2\pi f t)\sin(2\pi f kT)$

$$= \sin(2\pi f t + k 2\pi)$$

$$= \sin(2\pi f t)$$

Power of Periodic Signal: $P_x = \frac{1}{T} \int_{-T/2}^{T/2} |x(t)|^2 dt$

date

iii $x(t) = A\cos(2\pi F t)$, where $T = 1/F$

$$P_x = \frac{1}{T}\int_{-T/2}^{T/2} A^2\cos^2(2\pi F t)\,dt$$

$$= \frac{A^2}{T}\int_{-T/2}^{T/2} \frac{1+\cos(4\pi F t)}{2}\,dt$$

$$= \frac{A^2}{T}\left[\int_{-T/2}^{T/2}\frac{1}{2}\,dt + \frac{1}{2}\int_{-T/2}^{T/2}\cos(4\pi F t)\,dt\right]$$

$$= \frac{A^2}{T}\left[\frac{T}{2} + \frac{1}{2}\left[\frac{\sin(4\pi F t)}{4\pi F}\right]_{-T/2}^{T/2}\right]$$

$$= \frac{A^2}{T}\left[\frac{T}{2} + \cancel{\frac{1}{8\pi F}\left(\sin(2\pi) - \sin(-2\pi)\right)}^{\,0}\right]$$

$$= \frac{A^2}{2}$$

: Does not depend on phase (shift)

Frequency Domain Representation of a Signal

$$x(t) = \sum_{k=-\infty}^{\infty} C_k e^{j2\pi k F_0 t} \quad ; \quad F_0 = 1/T$$

$$C_l = \frac{1}{T}\int_{-T/2}^{T/2} x(t)\, e^{-j2\pi l F_0 t}\,dt$$

$$= \sum_{k=-\infty}^{\infty} C_k \frac{1}{T}\int_{-T/2}^{T/2} e^{j2\pi(k-l)F_0 t}\,dt$$

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- $k = l \Rightarrow \int = 1$
- $k \neq l \Rightarrow \frac{1}{T}\int_{-T/2}^{T/2} e^{j2\pi k F_0 t - j2\pi l F_0 t}\, dt$

$$\Rightarrow \frac{1}{T}\left\{\int_{-T/2}^{T/2} \cos(2\pi(k-l)F_0 t) \ldots + j\sin(2\pi(k-l)F_0 t)\, dt\right\}$$

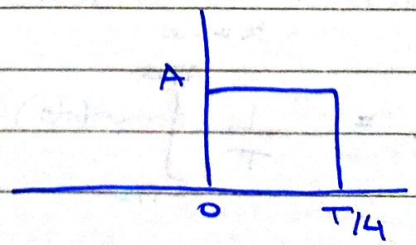
$$\Rightarrow \frac{1}{T}\left[\cancel{\frac{\sin(2\pi(k-l)F_0 t)}{2\pi(k-l)F_0}}\Bigg|_{-T/2}^{T/2} + j\left(-\frac{\cos(2\pi(k-l)F_0 t)}{2\pi(k-l)F_0}\right)\Bigg|_{-T/2}^{T/2}\right]$$

$$\Rightarrow \frac{-j}{T}\left[\cancel{\frac{\cos(\pi(k-l))}{2\pi(k-l)F_0}} - \cancel{\frac{\cos(-\pi(k-l))}{2\pi(k-l)F_0}}\right]$$

$$\Rightarrow 0$$

Hence, $\frac{1}{T}\int_{-T/2}^{T/2} e^{j2\pi(k-l)F_0 t}\, dt = \begin{cases} 1, & k = l \\ 0, & k \neq l \end{cases}$

<u>V</u>: Consider signal:



$$C_k = \frac{1}{T}\int_{-T/2}^{T/2} x(t)\, e^{-j2\pi k F_0 t}\, dt$$

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$$= \frac{1}{T}\int_{0}^{T/4} A e^{-j2\pi k f_0 t}\, dt$$

$$= \frac{A}{\pi k} \sin\left(\frac{\pi k}{4}\right) e^{-j\pi k/4}$$

$$C_k = \begin{cases} A/4 & , \ l = 0 \\ \frac{A}{\pi k} \sin\left(\frac{\pi k}{4}\right) e^{-j\pi k/4} & , \ l \neq 0 \end{cases}$$

$$|C_k| = \left| \frac{A}{\pi k} \sin\left(\frac{\pi k}{4}\right) \right|$$ magnitude spectrum

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Communication Systems

Power of a Periodic Signal

$$P = \sum_{k=-\infty}^{\infty} |c_k|^2 \quad \{\text{Parseval's Theorem}\}$$

Fourier Transform

$$x(t) = \int_{-\infty}^{\infty} X(F)\, e^{j2\pi Ft}\, dF$$

$$X(F) = \int_{-\infty}^{\infty} x(t)\, e^{-j2\pi Ft}\, dt$$

- Modulation Property

Carrier frequency

$$x_m(t) = x(t)\, e^{j2\pi F_c t}$$

modulated signal

- By using modulation, we can increase the energy of the signal and not rely on amplifiers.
- Provides a large guardband.
- Don't need tall antennas.

$$X_m(F) = \int_{-\infty}^{\infty} x(t)\, e^{-j2\pi(F-F_c)t}\, dt$$

$$\Rightarrow X_m(F) = X_m(F - F_c)$$

Modulation in time = Shift in frequency

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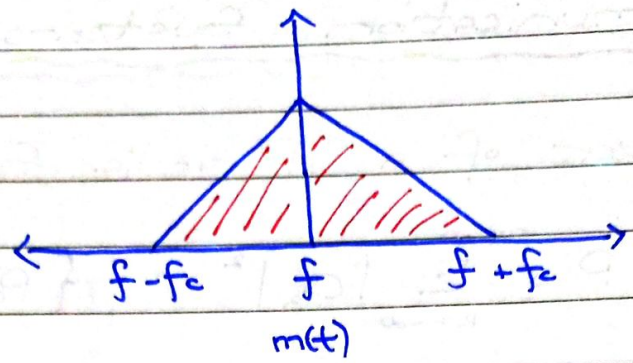


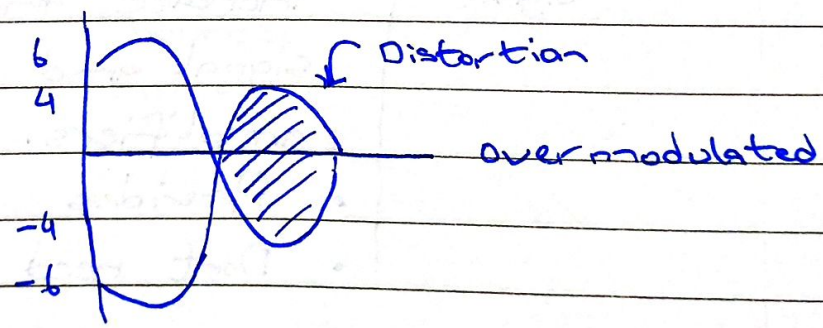
Amplitude Modulation

Carrier: $c(t) = A_c \cos(2\pi F_c t)$
Message: $m(t)$
Modulated: $x(t) = (1 + k_a m(t)) \times A_c \cos(2\pi F_c t)$

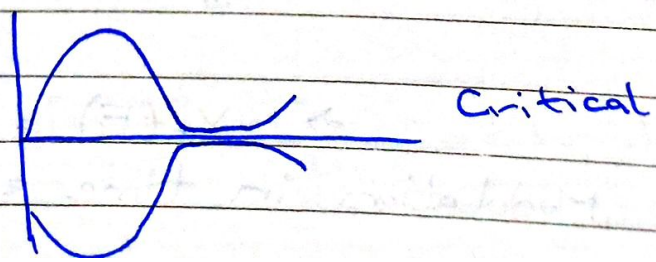
$\mu = k_a m(t)$ } modulation index
k_a = sensitivity
$1 + (k_a) m(t)$ } envelope

Cases:

$k_a = 1 \rightarrow m = 5 \quad | \begin{smallmatrix} +6 \\ -4 \end{smallmatrix}$



$k_a = 1/5 \rightarrow m = 5 \quad | \begin{smallmatrix} +2 \\ 0 \end{smallmatrix}$



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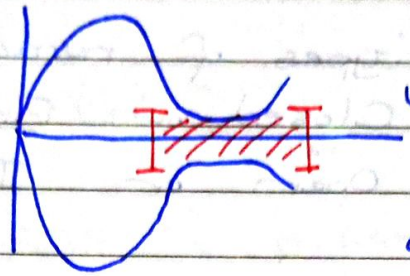
$k = 1/6 \rightarrow m = 5 \begin{vmatrix} +1.8 \\ -0.2 \end{vmatrix}$



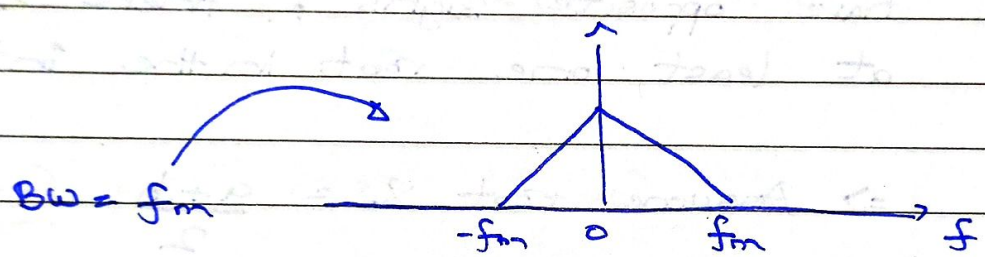
under modulated

best ;

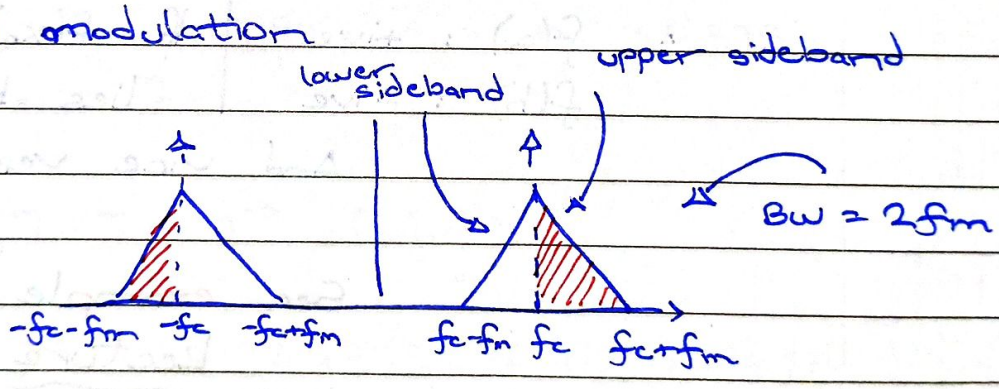
allows for

channel impairments

Undermodulated for $\mu = k_a m(t) < 1$.



After AM modulation



Envelope Detector

Diode : for rectification of mirror image

RC filter : for removing ripples

$\tau = R_L C$: $\frac{1}{F_c} \ll \tau \ll \frac{1}{F_m}$

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Communication System

$$x(t) = (1 + k_a m(t)) A_c \cos(2\pi F_c t)$$
$$= \underbrace{A_c \cos(2\pi F_c t)}_{\text{Pure Carrier}} + \underbrace{k_a m(t) A_c \cos(2\pi F_c t)}_{\text{Carrier modulated Part}}$$

Power & Efficiency of AM signal

Pure Carrier : $\frac{1}{2} A_c^2$

Rest from lecture : Theory

Double Sideband Modulation (DSB - SC)

$$x(t) = A_c m(t) \cos(2\pi F_c t)$$

Suppressed Carrier (to save power)

$$\Rightarrow \quad \cos(2\pi F_c t) \longleftrightarrow \frac{1}{2}\left[\delta(F - F_c) + \delta(F + F_c)\right]$$

→ Synchronous Demodulation

$$r(t) = x(t) \cdot \cos(2\pi F_c t)$$
$$= A_c m(t) \cos^2(2\pi F_c t)$$
$$= \frac{A_c m(t)}{2} + \frac{1}{2} A_c m(t) \cos(4\pi F_c t)$$

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Communication Systems

Non - coherent Demodulation

$$r(t) = x(t)\cos(2\pi F_c t + \phi)$$
$$= A_c\, m(t) \underbrace{\cos(2\pi F_c t)\cos(2\pi F_c t + \phi)}$$

$$\therefore \cos(\alpha)\cos(\beta) = \frac{1}{2}\left[\cos(\alpha-\beta) + \cos(\alpha+\beta)\right]$$

$$= \frac{A_c m(t)}{2}\cos\phi + \frac{A_c m(t)}{2}\cos(4\pi F_c t + \phi)$$

ϕ control {VCO}: Acts as a stabilizer

Using LPF : $r(t) = \frac{A_c m(t)}{2}\cos\phi$ phase error

- Higher Product Modulator
- Lower Product Modulator : $r(t) = \frac{A_c m(t)}{2}\sin\phi$
- Phase discriminator : $\frac{A_c^2 m(t)^2}{4}\sin\phi\cos\phi$

VCO : phase synchronization is achieved and phase offset is driven to zero.

Problem 3.18

$m(t) = 20\cos(2\pi t)$ V ; $f_m = 1$ Hz

$c(t) = 50\cos(100\pi t)$ V ; $f_c = 50$ Hz

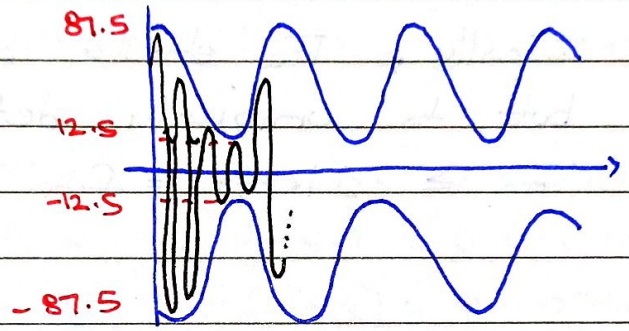
te

AM: 75 % $\Rightarrow \mu = 0.75$

$$k_a = \frac{\mu}{A_m} = \frac{0.75}{20} = 0.0375$$

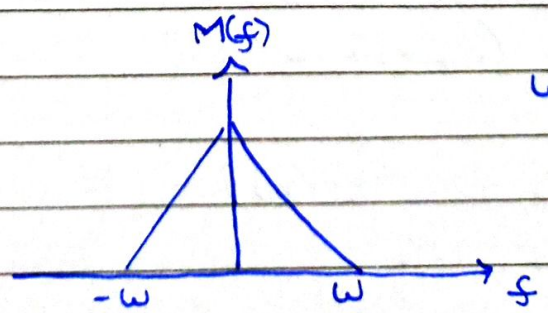
$$s(t) = [1 + (0.0375)\,20\cos(2\pi t)]\cos(100\pi t)\,(50)$$
$$= 50\,[1 + 0.75\cos(2\pi t)]\cos(100\pi t)$$

$\pm$ 87.5
$\pm$ 12.5



$$\text{Power} = 50\cos(100\pi t) + 37.5\cos(2\pi t)\cos(100\pi t)$$
$$= 50\cos(100\pi t) + \frac{37.5}{2}\left[\cos(102\pi t) + \cos(98\pi t)\right]$$
$$= \frac{1}{2}\frac{(50)^2}{100} + \frac{1}{2}\frac{(18.75)^2}{100} + \frac{1}{2}\frac{(18.75)^2}{2}$$
$$= 16.02\ W$$

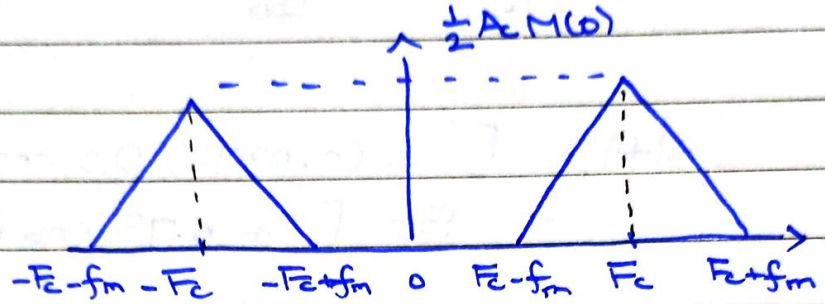
Problem 3.23



W: 1 kHz

$$c(t) = A_c\cos(2\pi f_c t)$$

$$s(t) = A_c \cos(2\pi F_c t)\, m(t)$$
$$= A_c \frac{1}{2}\left[M(f - F_c) + M(f + F_c) \right]$$



→ Ideally, F_c should be $\geq 2f_m$,
but to uniquely determine $m(t)$,
$F_c \geq 1\,kHz = f_m$

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Communication System

- Due to zero crossing in DSB-SC, envelope detector is infeasible. We employ coherent detector instead.
- DSB-SC offers no advantage over AM in terms of bandwidth.

→ Non-coherent detector

output ⇒ After LPF : $r(t) = \frac{A_c}{2} m(t) \cos\phi$

cases -
$$\begin{cases} \text{when } \phi = 0 \text{ ; } r(t) = \frac{A_c}{2} m(t) \text{ but} \\ r(t) = 0 \text{ when } \phi = \pm\pi/2 \end{cases}$$

Quadrature Carrier Multiplexing (QAM or QCM)

Two sources : $m_1(t)$ and $m_2(t)$

→ $s(t) = \underbrace{A_c m_1(t) \cos(2\pi f_c t)}_{①} + \underbrace{A_c m_2(t) \sin(2\pi f_c t)}_{②}$

⇒ Demodulated : $s(t) A_c' \cos(2\pi F_c t)$
and $s(t) A_c' \sin(2\pi F_c t)$

- which, after LPF, result in
  → $\frac{1}{2} A_c A_c' m_1(t)$
  → and $\frac{1}{2} A_c A_c' m_2(t)$

, respectively.

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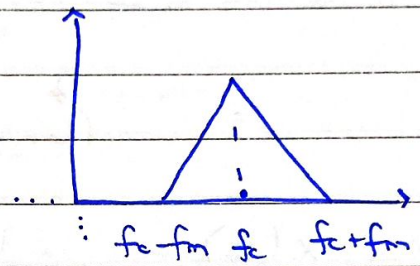
Communication Systems

⇒ $x(t) = 5\sin(2\pi 10^3 t)\sin(2\pi 10^6 t)$

① $x(t) = m(t) \cdot c(t)$

where, $c(t) = 5\sin(2\pi 10^6 t)$

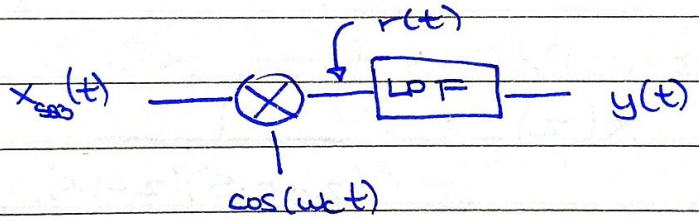
$m(t) = \sin(2\pi 10^3 t)$



USB $= 10^6 + 10^3$ Hz

LSB $= 10^6 - 10^3$ Hz

⇒ ②



$$r(t) = x_{SSB}(t) \cdot \cos(\omega_c t)$$

$$= [x(t)\cos(\omega_c t) \mp \hat{x}(t)\sin(\omega_c t)](\cos \omega_c t)$$

$$= x(t)\cos^2(\omega_c t) \mp \hat{x}(t)\sin(\omega_c t)\cos(\omega_c t)$$

$$= x(t)\left(\frac{1+\cos(2\omega_c t)}{2}\right) \ldots$$

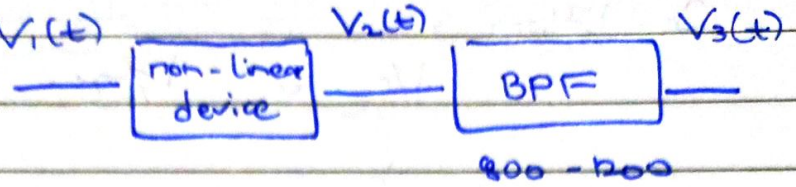
$$\mp \hat{x}(t)\left[\frac{1}{2}\left(\sin(2\omega_c t) + \cancel{\sin(0)}^{\,0}\right)\right]$$

$$= \frac{1}{2}\left[x(t) + \cancel{x(t)\cos(2\omega_c t)}^{\,\text{LPF}} + \cancel{\sin(2\omega_c t)\,\hat{x}(t)}^{\,\text{LPF}}\right]$$

$$y(t) = \frac{x(t)}{2}$$

=>

③



$$V_1(t) = 10\cos(2000\pi t) + 4\sin(200\pi t)$$

$$V_2(t) = V_1(t) + 0.1\, V_1^2(t)$$

$$= 10\cos(2000\pi t) + 4\sin(200\pi t) \ldots$$
$$+ 0.1\left[100\cos^2(2000\pi t) + 16\sin^2(200\pi t) + 2(40)\cos(2000\pi t)\sin(200\pi t)\right]$$

$$= 10\cos(2000\pi t) + 4\sin(200\pi t) \ldots$$
$$+ 0.1\left[\frac{100}{2}(1+\cos(4000\pi t)) \ldots\right.$$
$$+ \frac{16}{2}(1-\cos(400\pi t)) \ldots$$
$$\left.+ \frac{80}{2}\left(\sin(2200\pi t) - \sin(1800\pi t)\right)\right]$$

$$= 10\cos(2000\pi t) + 4\sin(200\pi t) \ldots$$
$$+ 0.1\left[50 + 50\cos(4000\pi t) + 8 - 8\cos(400\pi t) \ldots + 40\sin(2200\pi t) - 40\sin(1800\pi t)\right]$$

$$= 10\cos(2000\pi t) + 4\sin(200\pi t) + 5.8 \ldots$$
$$+ 5\cos(4000\pi t) - 0.8\cos(400\pi t) \ldots$$
$$+ 4\sin(2200\pi t) - 4\sin(1800\pi t)$$

BPF

$$V_3(t) = \left[10\cos(2000\pi t) + 4\sin(2200\pi t) \ldots - 4\sin(1800\pi t)\right]$$

Problem

$$x(t) = \cos(200\pi t) + 4\cos(4000\pi t)$$
$$c(t) = \cos(2\pi f_c t)$$
$$f_c = 1\ \text{MHz}$$

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Condition :

$$1/f_c \ll \tau \ll 1/f_m$$

$$\cos\alpha \cos\beta = \frac{1}{2}\left(\cos(\alpha+\beta) + \cos(\alpha-\beta)\right)$$

$$x(t) = m(t)\, c(t)$$

$$= \frac{4}{4}\left[\cos(200\pi t) + 4\cos(4000\pi t)\right]$$

$$= \cos(\alpha)\cos(\beta) \quad \times$$

both ignore

$f_m = 2000$ Hz (Higher f component)

$$1/1M \ll \tau \ll 1/2000$$

$$1\mu s \ll \tau \ll 0.5\,ms$$

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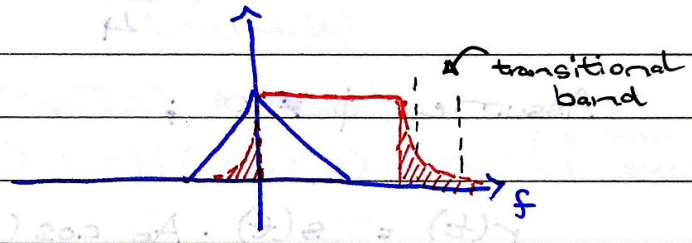
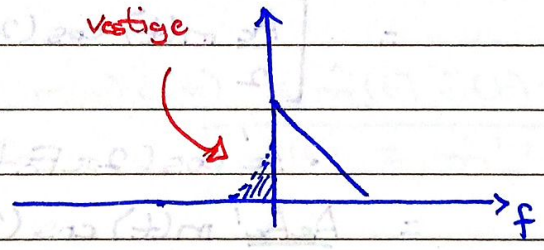
Communication Systems

s(t)	m(t)
modulated	message
passband	baseband
signal	signal

High pass filter ⟼ USB
Low pass filter ⟼ LSB

} more on that later

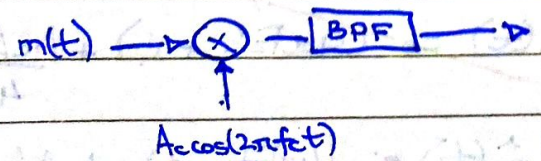




SSB signals are mostly used for audio signals due to inherent energy gap.

Modulators for SSB

Frequency Discrimination



Phase Discrimination

Similar to costas, however, with Hilbert transform incorp.

Problem 3.13

→ Show that $V_o(t) = \frac{A_c A_c'}{4} m(t)$

→ $s(t) = \frac{A_c}{2} \cos(2\pi f_c t)\, m(t) \mp \frac{A_c}{2} \sin(2\pi f_c t)\, m(t)$

→ $\underline{r(t) = s(t) \cdot A_c \cos(2\pi F_c)}$

Trigonometry

$$\left.\frac{A_c^2}{4} m(t)\right\} \text{After LPF}$$

Assume $\phi \neq 0$;

$$r(t) = s(t) \cdot A_c \cos(2\pi F_c t + \phi)$$

$$= \left[\frac{A_c}{2} m(t) \cos(2\pi F_c t) \mp \frac{A_c}{2} \hat{m}(t) \sin(2\pi F_c t)\right] \cdot A_c' \cos(2\pi F_c t + \phi)$$

$$= \frac{A_c A_c'}{2} m(t) \cos(2\pi F_c t) \cos(2\pi F_c t + \phi)$$

$$\mp \frac{A_c A_c'}{2} \hat{m}(t) \sin(2\pi F_c t) \cos(2\pi F_c t + \phi)$$

$$= \frac{A_c A_c'}{2} m(t) \left[\frac{1}{2}\left(\cos(4\pi F_c t + \phi) + \cos(\phi)\right)\right]$$

$$\mp \frac{A_c A_c'}{2} \hat{m}(t) \left[\frac{1}{2}\left(\sin(4\pi F_c t + \phi) - \sin(\phi)\right)\right]$$

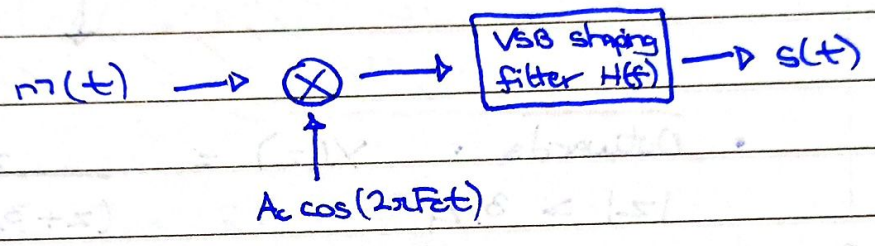
$$= \cancel{\frac{A_c A_c'}{4} m(t) \cos(4\pi F_c t + \phi)} + \frac{A_c A_c'}{4} m(t) \cos(\phi)$$

$$\mp \cancel{\frac{A_c A_c'}{4} \hat{m}(t) \sin(4\pi F_c t + \phi)} \pm \frac{A_c A_c'}{4} \sin\phi\, \hat{m}(t)$$

$$= \underbrace{\left[\frac{A_c A_c'}{4} m(t) \cos\phi \pm \frac{A_c A_c'}{4} \hat{m}(t) \sin\phi \right]}_{\text{After LPF}}$$

VSB Modulation (Sideband Shaping Filter)

└ VSB for both $\overbrace{\text{video and}}^{\text{low energy gap}}$ audio signals



$$s(t) = m(t)\cos(2\pi F_c t) * h(t)$$

$$\rightarrow H(F - F_c) + H(F + F_c) = 1 \quad \left(\text{if VSB filter is symmetric}\right)$$

Problem

SSB:

$$s(t) = m(t)\,c(t) \mp \hat{m}(t)\,\hat{c}(t)$$

$$\text{Envelope} = \sqrt{m^2(t) \mp \hat{m}^2(t)}$$

DSB-SC:

$$s(t) = m(t)\cos(\omega_c t + \theta)$$

$$= m(t)\left[\cos(\omega_c t)\cos(\theta) - \sin(\omega_c t)\sin\theta\right]$$

$$\text{Envelope} = |x(t)|$$

/date

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Communication Systems

Complex Envelope

baseband → original signal

$$s(t) = S_I(t)\,c(t) - S_Q(t)\,\hat{c}(t)$$

passband → modulated signal

generic form

$$s(t) = S_I(t)\cos(2\pi F_c t) \ldots - S_Q(t)\sin(2\pi F_c t)$$

→ $\tilde{c}(t) = e^{j2\pi F_c t} = \cos(2\pi F_c t) + j\sin(2\pi F_c t)$

→ $x(t) + j\hat{x}(t)$: x : modulated

$$\tilde{x}(t) = (x(t) + j\hat{x}(t))\,e^{-j2\pi F_c t}$$

- Complex envelope is just the frequency shifted version of pre-envelope.

→ (Pre-envelope) $2 \times X_p(f)$ → +ive frequency

→ (Complex envelope) Pre-envelope $\times\, e^{-j\omega_c t}$

Examples:

- $A_c \cos(\omega_c t) = A_c\left(\frac{1}{2}\right)\left(e^{j\omega_c t} + e^{-j\omega_c t}\right)$

$= A_c\, e^{j\omega_c t}$ Pre-envelope

$= A_c\, e^{j\omega_c t}\, e^{-j\omega_c t}$

$= A_c$ Complex envelope

- $A_c(1+m(t))\cos(\omega_c t) = A_c(1+m(t))\left(\frac{1}{2}\right)\left[e^{j\omega_c t} \ldots + e^{-j\omega_c t}\right]$

$\Rightarrow \frac{A_c}{2}(1+m(t))\left[e^{j\omega_c t} + e^{-j\omega_c t}\right]$

$\Rightarrow A_c(1+m(t))\, e^{j\omega_c t}$ Pre-envelope

- $\cos(2\pi 103 t)\cdot u(t) = \frac{u(t)}{2}\left[e^{j2\pi 103 t} + e^{-j2\pi 103 t}\right]$

↳ centred at 100

$\Rightarrow u(t)\, e^{j2\pi 103 t}$ Pre-envelope

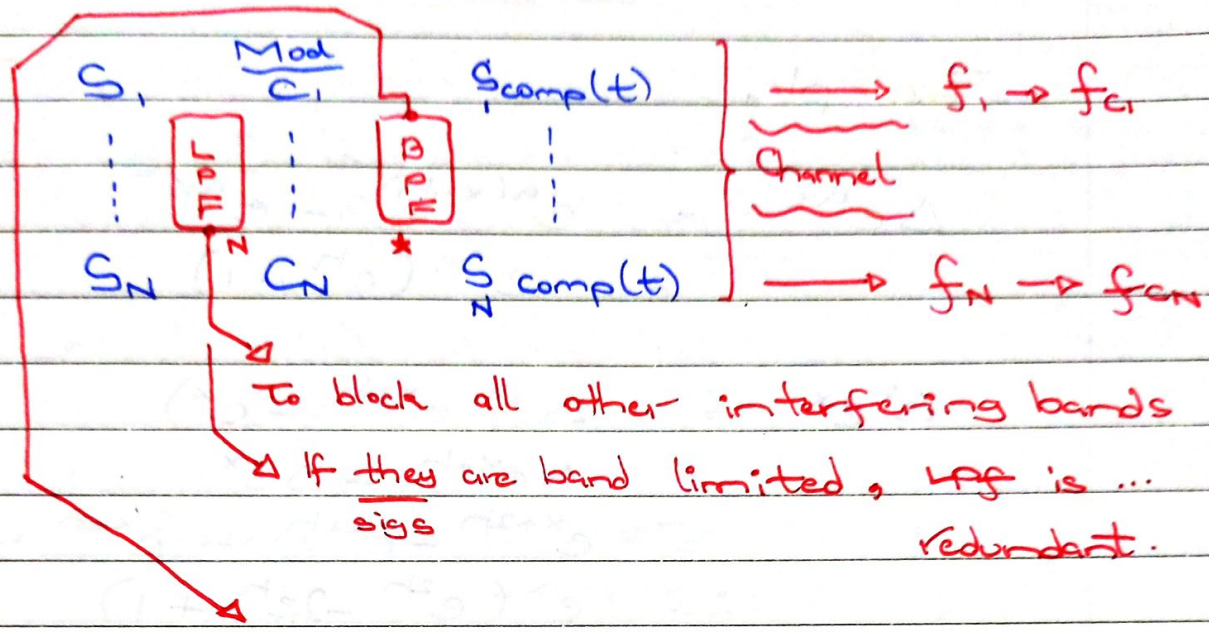
$\Rightarrow u(t)\, e^{j2\pi 103 t} \cdot e^{-j2\pi 100 t}$

$\Rightarrow u(t)\, e^{j2\pi 3 t} = u(t)\, e^{j6\pi t}$ Complex envelope

ay/date

6/03/23

Communication Systems



To block all other interfering bands

If they are band limited, LPF is ... redundant.
sigs

To separate / divide frequencies

FDMA

~ To transmit n number of modulating signals

Angle Modulation (FM + PM)

$$s(t) = A_c \cos \theta_i(t)$$

$$\theta_i(t) = 2\pi f_c t + \phi_i$$

$$= 2\pi f_c t + k_p m(t)$$

$$f_i(t) = f_c + k_f\, m(t)$$

$$\begin{cases} s(t)_{PM} = A_c \cos(2\pi f_c t + k_p m(t)) \\ s(t)_{FM} = A_c \cos\left[2\pi f_c t + 2\pi k_f \int_0^t m(t)\, dt\right] \end{cases}$$

late

→ Definitions:

- Modulation Index $= \Delta f / f_m = \beta$
- Phase Deviation $\Delta\theta$
- Frequency Deviation Δf

AM violates the principle of superposition

Do definitions from slides / book

Example 1

$$s(t) = 6\cos[2\pi 10^6 t + 4\sin(2\pi 10^3 t)]$$

$$\Delta f_{max} = \max|f_i - f_c|$$

$$f_i = f_c + k_p m(t)$$

$$f_i = \frac{1}{2\pi}\frac{d}{dt}\theta_i(t) = 2\pi 10^6 + 4\cos(2\pi 10^3 t)\, 2\pi 10^3$$

$$= 10^6 + 4\times 10^3 \cos(2\pi 10^3 t)$$

$$\Rightarrow k_p m(t) = 4\times 10^3 \cos(2\pi 10^3 t)$$

$$\Delta f_{max} = \max|4\times 10^3 \cos(2\pi 10^3 t)|$$

$$= 4\times 10^3$$

$$\Delta\phi_{max} = 4$$

Ratio $\dfrac{\Delta f_{max}}{\Delta\phi_{max}} = 1\times 10^3$ Hz / rad

EXCELLENT

Example 2:

PM signal $\Rightarrow$ $s(t) = 10\cos(\omega_c t + 5\sin 2000\pi t)$

$\hookrightarrow$ $s(t) = A_c \cos(2\pi f_c t + k_p m(t))$

$k_p = 10$

$\rightarrow$ $k_p m(t) = 5\sin(2000\pi t)$

$m(t) = 0.5 \sin(2000\pi t)$

Example 3:

$\hookrightarrow$ $m(t) = 0.5 \sin(3000t) + 1 \sin(2000\pi t)$

Example 4:

$$\phi_{em}(t) = 10\cos(\omega_c t + 5\sin 3000t + 10\sin 2000\pi t)$$

a) Power: $A_c = 10$; $A_c^2/2 = 50$ W

b) $f_i = \frac{1}{2\pi}\frac{d}{dt}\theta_i(t) = \frac{1}{2\pi}\Big[\omega_c + 15000\cos(3000t) + \dots$

$20000\pi\cos(2000\pi t)\Big]$

$= f_c + \frac{7500}{\pi}\cos(3000t) + \dots$

$10000\cos(2000\pi t)$

$\Delta f_{max} = \max|f_i - f_c|$

$= \frac{7500}{\pi} + 10000$

$= 12387.32$ Hz

c) Deviation ratio $\beta = \frac{\Delta f}{f_m}$ → Higher f_m

$$= \frac{12387.32}{1000}$$

$$= 12.387$$

d) Phase Deviation $\Delta\phi$

$$\Delta\phi = \max |\phi_i - 2\pi f_c t|$$

$$= \max |\cancel{2\pi f_c t} - \cancel{2\pi f_c t} + 5\sin 3000t + 10\sin 2000\pi t|$$

$$= 15 \text{ rad}$$

e) Bandwidth of $\phi_{em}(t)$

└ Left for later

→ Δf can also be found as:

$$\Delta f = [\beta_1 f_{m_1} + \beta_2 f_{m_2} \ldots\ldots] / 2\pi$$

provided signal components are all sine or cosine.

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Communication Systems

Example 4

$$m(t) = \text{sinc}(4\pi 10^4 t)$$

$k_f = 10^3$ Hz/volts ; $f_c = 1$ MHz

$\Rightarrow f_i = f_c + k_f\, m(t)$ — max sinc is 1

$= 10^6 + 10^3 \boxed{m(t)}$ — $k_f A_m = \Delta f_{max}$

$= \underline{10^6 + 10^3 \text{ Hz}}$

Example 5

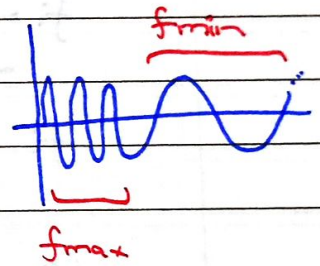
Carrier swing = 10 kHz

$f_m = 8$ kHz

$f_{imax} = f_c + \Delta f_{max}$

$f_{imin} = f_c - \Delta f_{max}$

or $f_c + \Delta f_{min}$

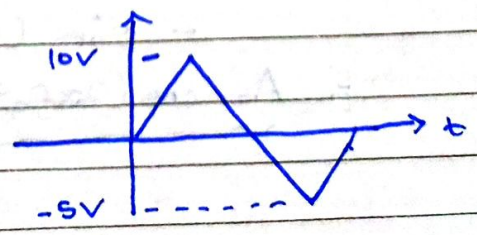


$\Rightarrow f_{imax} - f_{imin} = 2\Delta f_{max}$ ← Carrier Swing

$\Rightarrow 2\Delta f_{max} = 10$ kHz $\Rightarrow \Delta f_{max} = 5$ kHz

$\beta = \dfrac{\Delta f_{max}}{f_m} = \dfrac{5k}{8k} = \boxed{0.625}$

Example ~



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$\Rightarrow f_{imax} - f_{imin} = ?$

$k = 5 \text{ Hz / volt}$

$\Delta f_{max} = k_f A_m = 5 \times 10 = 50 \text{ Hz}$

$\Delta f_{min} = k_f A_m = 5 \times -5 = -25 \text{ Hz}$

$$\Rightarrow f_{imax} - f_{imin} = \Delta f_{max} - \Delta f_{min}$$
$$= 50 - (-25)$$
$$= 75 \text{ Hz}$$

- Narrowband FM

$$m(t) = A_m \cos(2\pi f_m t)$$

$$f_i = f_c + k_f m(t)$$
$$= f_c + k_f A_m \cos(2\pi f_m t)$$
$$f_i = f_c + \Delta f \cos(2\pi f_m t)$$

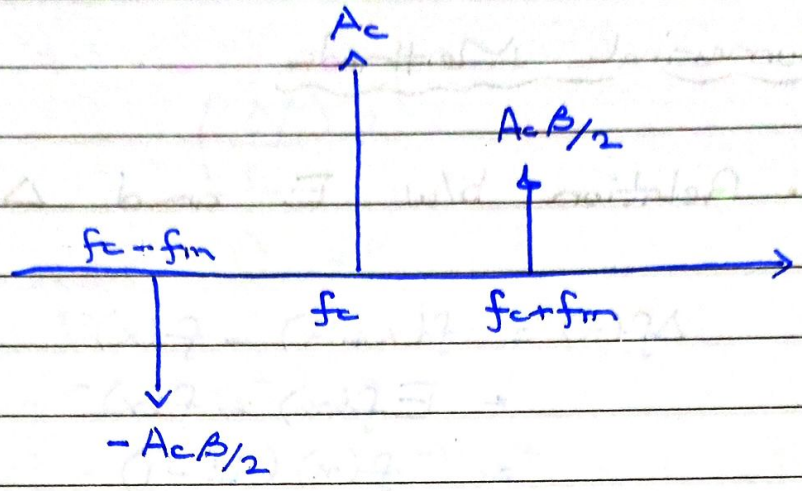
$$\theta_i = 2\pi f_c t + \frac{\Delta f}{f_m} \sin(2\pi f_m t)$$

$$s(t)_{FM} = A_c \cos \theta_i$$
$$= A_c \cos\left[\underbrace{2\pi f_c t}_{A} + \underbrace{\beta \sin(2\pi f_m t)}_{B}\right]$$

$\cos(\alpha + \beta)$

≈ 1 for $\beta \ll 1$

$$= A_c \left[\cos(2\pi f_c t) \cos(\beta \sin(2\pi f_m t)) \ldots - \sin(2\pi f_c t) \sin(\beta \sin(2\pi f_m t))\right]$$
$$= A_c \cos(2\pi f_c t) - A_c \sin(2\pi f_c t)\, \beta \sin(2\pi F_m t)$$



$$\text{Bandwidth} = \cancel{f_c} + f_m - \cancel{f_c} + f_m$$
$$= 2f_m$$

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Communication Systems

Wideband FM

Modulation index $\beta > 1$

L More later

Frequency Modulated Signal

For linear: $s(t) = s_i(t)\cos(2\pi f_c t) - s_Q(t)\sin(2\pi f_c t)$

For non linear: $s(t) = a(t)\cos(2\pi f_c t + \phi(t))$

Derivation
$$\begin{cases} [s_i^2(t) + s_Q^2(t)]^{1/2} \\ \phi(t) = \tan^{-1}\left(\dfrac{s_Q(t)}{s_i(t)}\right) \end{cases}$$

$$s(t) = a(t)\cos(2\pi f_c t + \phi(t))$$
$$= a(t)\left(\cos(2\pi f_c t)\cos(\phi(t)) - \sin(2\pi f_c t)\sin(\phi(t))\right)$$

$$\cos\phi(t) = \frac{s_Q(t)}{a(t)}$$

$$\sin\phi(t) = \frac{s_I(t)}{a(t)}$$

$$\Rightarrow s(t) = s_I(t)\cos(2\pi f_c t) - s_Q(t)\sin(2\pi f_c t)$$

Drill Problem

$$\Rightarrow s(t) = A_c\cos(2\pi F_c t) - \beta A_c \sin(2\pi F_c t)\sin(2\pi F_m t)$$

date

$$a(t) = \sqrt{A_c^2 + \beta^2 A_c^2 \sin^2(2\pi f_m t)}$$
$$= A_c \sqrt{1 + \beta^2 \sin^2(2\pi f_m t)}$$

$$\max(a(t)) = A_c\sqrt{1+\beta^2}$$
$$\min(a(t)) = A_c$$
$$\text{or} \quad \frac{A_{max}}{A_{min}} = \sqrt{1+\beta^2} = 1 + \frac{1}{2}\beta^2$$

- Wideband. Returned

$$s_{FM}(t) = A_c \cos(2\pi F_c t + \beta \sin 2\pi F_m t)$$

$$\Rightarrow \quad s(t) = A_c \cos\theta \quad ; \quad e^{j\theta} = \cos\theta + j\sin\theta$$
$$\Rightarrow \quad A_c \, Re\{e^{j\theta}\} = s(t)$$
$$\Rightarrow \quad A_c \, Re\{e^{j(2\pi F_c t + \beta \sin 2\pi F_m t)}\}$$
$$\Rightarrow \quad \tilde{s}(t) = A_c e^{j\beta \sin(2\pi F_m t)}$$

Periodic sin

$$\hookrightarrow \quad s(t + 1/F_m) = e^{j\beta \sin(2\pi F_m t + 2\pi)}$$
$$s(t + 1/F_m) = e^{j\beta \sin(2\pi F_m t)}$$

} Periodic Expression with $1/f_m$

$$\Rightarrow \quad \tilde{s}(t) = \sum_{n=-\infty}^{\infty} c_n e^{j2\pi n f_m t}$$

$$c_n = f_m \int_{-1/2f_m}^{1/2f_m} \tilde{s}(t) e^{(-j2\pi n f_m t)} dt$$
$$= f_m \int_{-1/2f_m}^{1/2f_m} e^{j\beta \sin(2\pi F_m t)} e^{-j2\pi n f_m t} dt$$

~ Solving

$$c_n = \frac{A_c}{2\pi} \int_{-\pi}^{\pi} e^{j[\beta \sin\theta - n\theta]} d\theta$$

Bessel function (except A_c)

EXCELLENT

Bessel function is denoted by :

$$J_n(\beta)$$

$$\Rightarrow \quad c_n = A_c J_n(\beta)$$

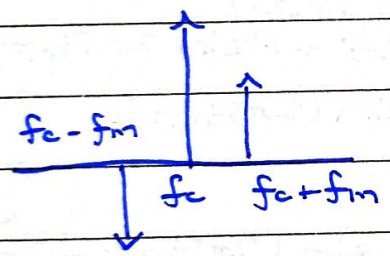
$$\Rightarrow \quad \tilde{s}(t) = A_c \sum_{n=-\infty}^{\infty} J_n(\beta) e^{j2\pi n f_m t}$$

$\rightarrow$ $J_0(x) > J_1(x) > J_2(x) \ldots$

$\rightarrow$ $J_{-n}(x) = (-1)^n J_n(x)$

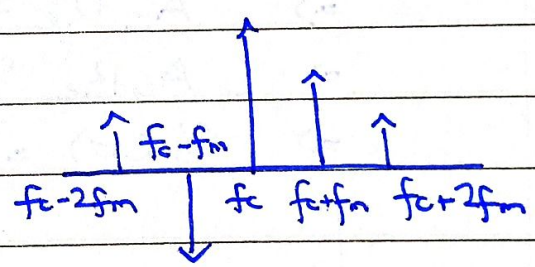
$\rightarrow$ $\sum_n J_n^2(x) = 1$

NBFM



$BW = 2f_m$

WBFM



$BW = 4f_m$

$\ldots A_c J_2(\beta) \cos(2\pi(f_c - 2f_m)t) -$

$A_c J_1(\beta) \cos(2\pi(f_c - f_m)t) +$

$A_c J_0(\beta) \cos(2\pi f_c t) + \ldots$

$A_c J_1(\beta) \cos(2\pi(f_c + f_m)t) \ldots$

$$BW = 2(\Delta f + f_m)$$

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Communication Systems

→ Example

$f_c = 10\,MHz$, $\Delta f = 50\,kHz$

↳ Calculate BW :

i) $f_m = 5\,kHz$

ii) $f_m = 50\,kHz$

iii) $f_m = 250\,kHz$

→ i) $\beta = 10$;

$BW = 2(\Delta f + f_m)$

$= 110\,kHz$

→ ii) $\beta = 1$

$BW = 2(\Delta f + f_m)$

$= 200\,kHz$

→ iii) $\beta = 0.2 < 1$ (NBFM)

$BW = 2f_m$

$= 500\,kHz$

Power

→ $\sum_{n=-\infty}^{\infty} \frac{A_c^2}{2} J_n^2(\beta)$

[↳ of FM $= \frac{A_c^2}{2}$

↳ of carrier

↳ consider $J_0(\beta)$

→ $A_c^2 J_0^2(\beta)/2$]

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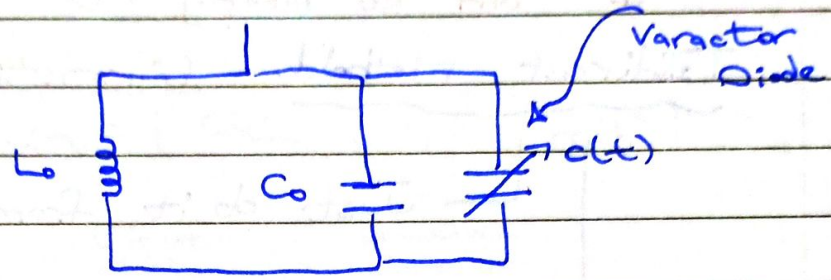
Communication Systems

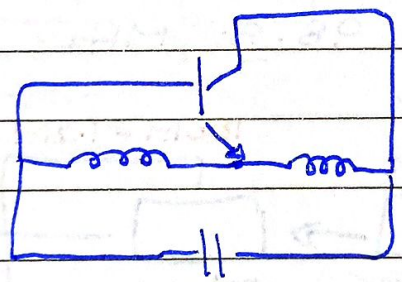
Generation of FM Signal

└ Direct Method



oscillator

$$\Rightarrow f = \frac{1}{2\pi\sqrt{L_o(C_o + c(t))}}$$



Varactor Diode (Transistor)

$$\Rightarrow f_i = f_c + k_f\, m(t)$$

$$\Rightarrow f_c = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{Lc(t)}} = \frac{1}{2\pi\sqrt{L(C - km(t))}}$$

$$\therefore c(t) = C - k\, m(t)$$

$$\Rightarrow f_c = f_o\left[1 - \frac{k}{C} m(t)\right]^{-1/2}$$

$$= f_o\left[1 + \frac{k}{2C} m(t)\right] \qquad \boxed{k_f = \frac{f_o k}{2C}}$$

$$= f_o + k_f\, m(t)$$

Limitation:

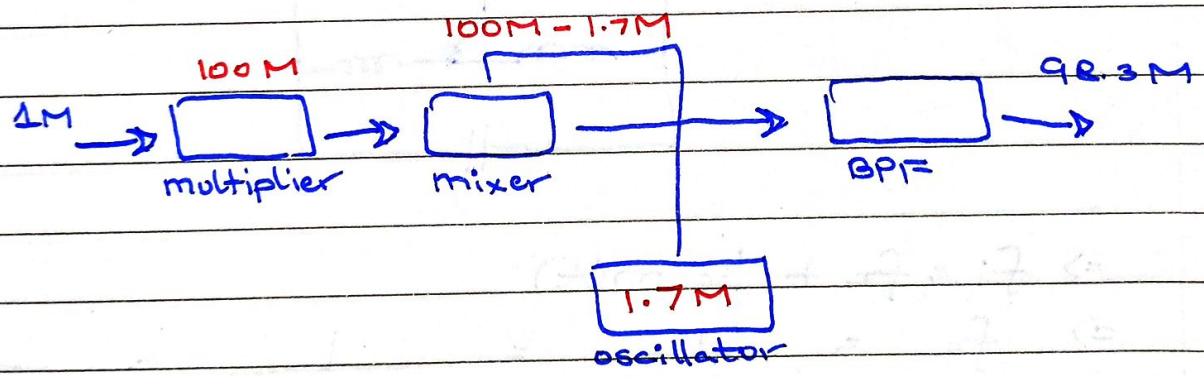
- Stability
- Tendency to drift
- Distortion

I am so sleepy ~

Indirect Method (Armstrong)

- Just do it from book / slide.
- Drawbacks: Noise gets amplified

$$\begin{cases} f_c = 1 \text{ MHz} \\ \text{Station} = 98.3 \text{ MHz} \end{cases}$$



Reciever:

$$s(t) = A_c \cos(2\pi f_c t + 2\pi k_f \int m(t)\,dt)$$

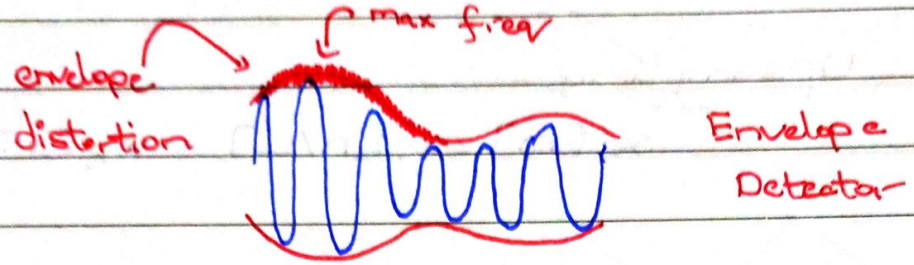
$$\frac{d\,s(t)}{dt} = -A_c\left[\sin(2\pi f_c t + 2\pi k_f \int m(t)\,dt)\right] \cdot \ldots$$

$$\left[2\pi f_c + 2\pi k_f\, m(t)\right]$$

$$= -A_c\, 2\pi \left[f_c + k_f\, m(t)\right]\left[\sin(2\pi f_c t + 2\pi k_f \int m(t)\,dt)\right]$$

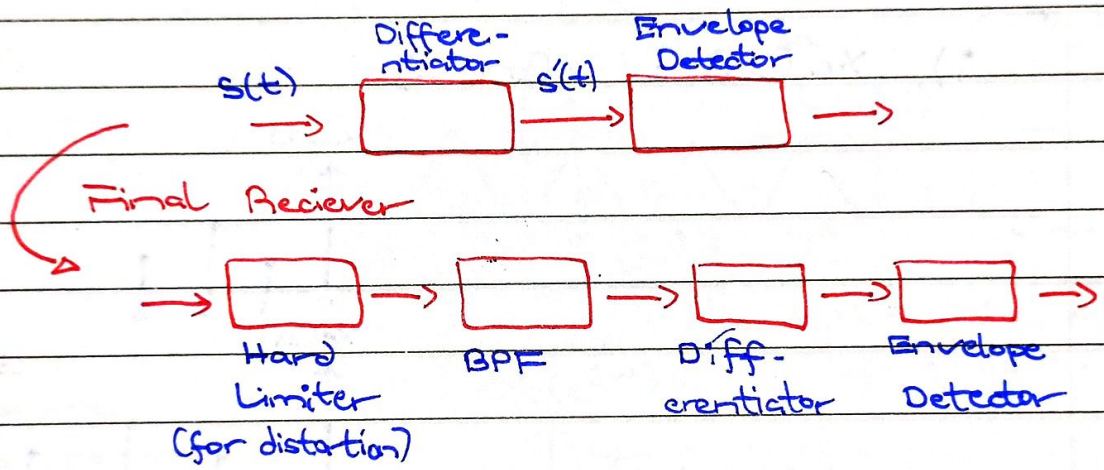
$$= -A_c 2\pi f_c \left[1 + \frac{k_f}{f_c} m(t)\right]\left[\sin(2\pi f_c t + 2\pi k_f \int m(t)\,dt)\right]$$

$\Rightarrow \quad A_c [1 + k_a m(t)]$ } envelope form



$s(t) = A_c \cos\theta$

$s'(t) = -A_c \sin\theta \, [2\pi f_c + 2\pi k_f m(t)]$



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Communication Systems

$\beta = \frac{\Delta f}{f_m}$ → ranges from [60 - 15 kHz]

$D = \frac{\Delta f}{f_m} = \frac{75k}{15k} = 5$ } when f_m is at maximum; $f_m = W$; $\beta = D$

4.12

$J_0(\beta) = 0$; $\beta = 2.4, 5.5, 8.6, 11.8$

- $f_m = 1$ kHz ; $A_m = 2$ V
- $\beta = \frac{k_f A_m}{f_m}$ => $\boxed{1.2 \text{ kHz/V}} = k_f$
- $\beta = 5.5$ now

$A_m = \frac{f_m \beta}{k_f} = \boxed{4.583 \text{ V}}$

4.13

$f_c = 100$ MHz $k_f = 25$ kHz/V

$A_m = 20$ V $f_m = 100$ kHz

$\Rightarrow \beta = \frac{k_f A_m}{f_m} = 5$

$\Rightarrow BW = 2(\Delta f + f_m)$

$= 2(k_f A_m + f_m)$

$= 1.2$ MHz

ate

- Assume $A_m = 40$;

$$\beta = 10 \quad ; \quad BW = 2(1+\beta)f_m = 2.2\,MHz$$

- Assume $f_m = 200\,kHz$;

$$\beta = 2.5 \quad ; \quad BW = 2(1+\beta)f_m = 1.4\,MHz$$

Example

$$s(t) = 100\cos(2\pi 10^6 t + 6\sin 4\pi 10^3 t)$$

Δf , BW , P_t

Repeat with doubled f_m

$\Rightarrow A_c = 100$; $f_m = 2\times 10^3\,Hz$

$f_c = 10^6\,Hz$

$\Rightarrow \Delta f = 6\times 2\times 10^3 = 12000\,Hz$

$\Rightarrow \beta = \frac{\Delta f}{f_m} = 6$

$\Rightarrow BW = 2(\Delta f + f_m) = 28000\,Hz$

$\Rightarrow P_t = \frac{A_c^2}{2} = 5000\,W$

Doubled f_m : $\beta = 3$ } ~~unchanged~~ Δf only changes due to A_m

$\Rightarrow \Delta f$ = unchanged

$\Rightarrow BW = 2(\Delta f + 2f_m) = 32000\,Hz$

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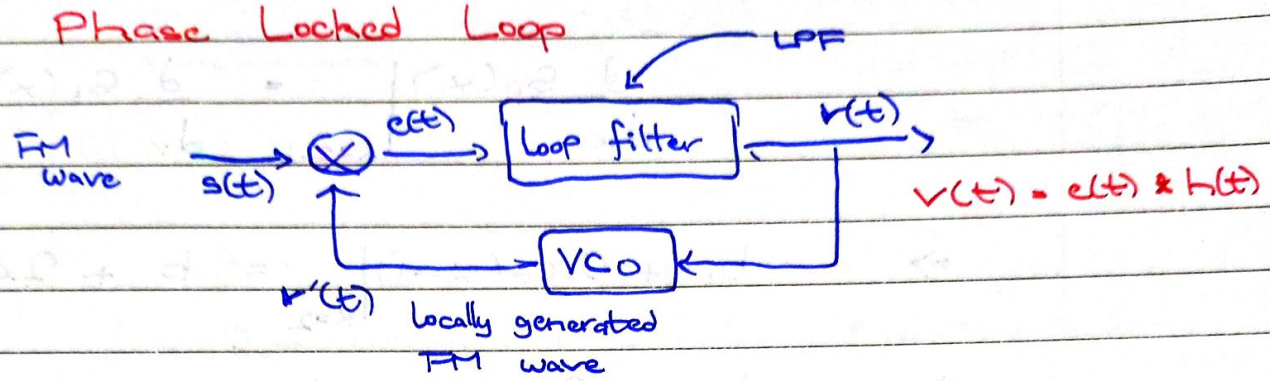
Communication Systems

Phase Locked Loop



if $e(t) = 0$; $\phi_1(t) = \phi_2(t)$

$$s(t) = A_c \cos\left(2\pi f_c t + 2\pi k_f \int m(t)\,dt\right)$$
$$v(t) = A_v \cos\left(2\pi f_c t + 2\pi k_v \int v(t)\,dt\right)$$

if $e(t) > 0$;

< 0 ;

Expanding → $e(t) = 0$: → phase lock

$$\phi_1(t) = 2\pi k_f \int m(t)\,dt$$
$$\phi_2(t) = 2\pi k_v \int v(t)\,dt$$

$$e(t) = s(t) * (-v(t))$$
$$= A_c \cos(2\pi f_c t + \phi_1(t)) * (-A_v \sin(2\pi f_c t + \phi_2(t)) \ldots$$
$$= +\frac{A_c A_v}{2}\left[\underbrace{\sin(4\pi f_c t + \phi_1(t) + \phi_2(t))}_{\text{LPF} \to 0} + \underbrace{\sin(\phi_1(t) - \phi_2(t))}_{\sin(x) \approx x \text{ when } x \text{ is small}}\right] \ldots$$

$$\therefore \phi_e(t) = \phi_1(t) - \phi_2(t)$$

$$v(t) = \overbrace{\frac{A_c A_v}{2}\phi_e(t)}^{e(t)} * H(t) \quad \text{— i}$$

ate

$$\phi_e(t) = \phi_1(t) - 2\pi k_v \int \frac{A_c A_v}{2} \phi_e(t) * h(t)\, dt$$

$$\frac{d}{dt}\phi_e(t) = \frac{d}{dt}\phi_1(t) - 2\pi k_v \frac{A_c A_v}{2} \phi_e(t) * h(t)$$

$$j\omega \phi_e(\omega) = j\omega \phi_1(\omega) - \frac{A_c A_v k_v}{2} \phi_e(\omega) H(\omega)$$

$$j\omega \phi_e(\omega) + k_v \frac{A_c A_v}{2} \phi_e(\omega) H(\omega) = j\omega \phi_1(\omega)$$

$$\phi_e(\omega)\left[j\omega + \frac{K_v A_c A_v}{2} H(\omega)\right] = j\omega \phi_1(\omega)$$

$$\phi_e(\omega)\left[1 + k_v \frac{A_c A_v H(\omega)}{2j\omega}\right] = \phi_1(\omega)$$

$$\phi_e(\omega) = \frac{\phi_1(\omega)}{1 + k_v A_c A_v H(\omega)/2j\omega}$$

$$\approx \frac{2j\omega\, \phi_1(\omega)}{k_v A_c A_v H(\omega)}$$

$$V(\omega) = \frac{1}{k_v}\left[j\omega \phi_1(\omega)\right]$$ — from i

Inverse FT : $v(t) = \frac{1}{k_v}\frac{d}{dt}\phi_1(t)$

$\therefore \phi_1(t) = 2\pi k_f \int m(t)\, dt$

$\Rightarrow v(t) = \frac{1}{k_v} 2\pi k_f\, m(t)$

$= 2\pi \frac{k_f}{k_v} m(t)$

$$\boxed{v(t) \propto m(t)}$$

FM Stereo Mux

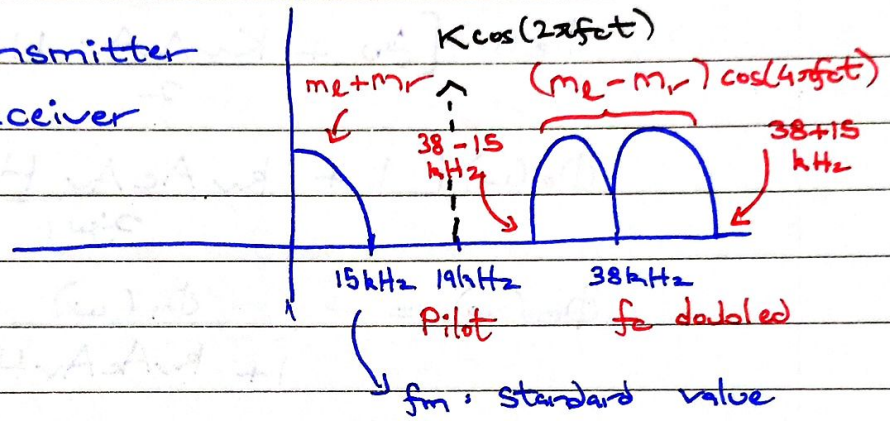
$$m(t) = [m_l(t) + m_r(t)] + \underbrace{[(m_l(t) - m_r(t)) \cdots \cos(4\pi f_c t)]}_{\text{modulated signal}}$$

$$+ K \cos(2\pi f_c t)$$

Pilot : 19 kHz

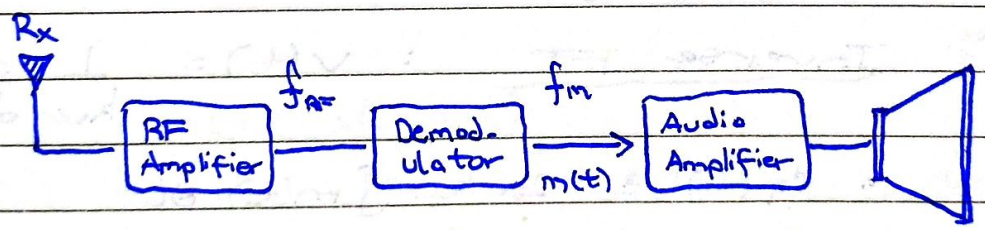
Emphasis: Transmitter

De-emphasis: Receiver



AM Receiver

- TRF
- Superheterodyne



i. selectivity : select desired freq.

ii : sensitivity : amplify weak signals

iii : fidelity . reproduce $m(t)$

Quality factor $= f_{rf} / BW$ = TRF / Bandwidth

- Purpose is to reduce adjacent channel interference.

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Communication Systems

TRF

$Q = \frac{f_{RF}}{BW}$

Results in adjacent channel interference whilst maintaining Q factor

Super heterodyne Rx

high components

mixing

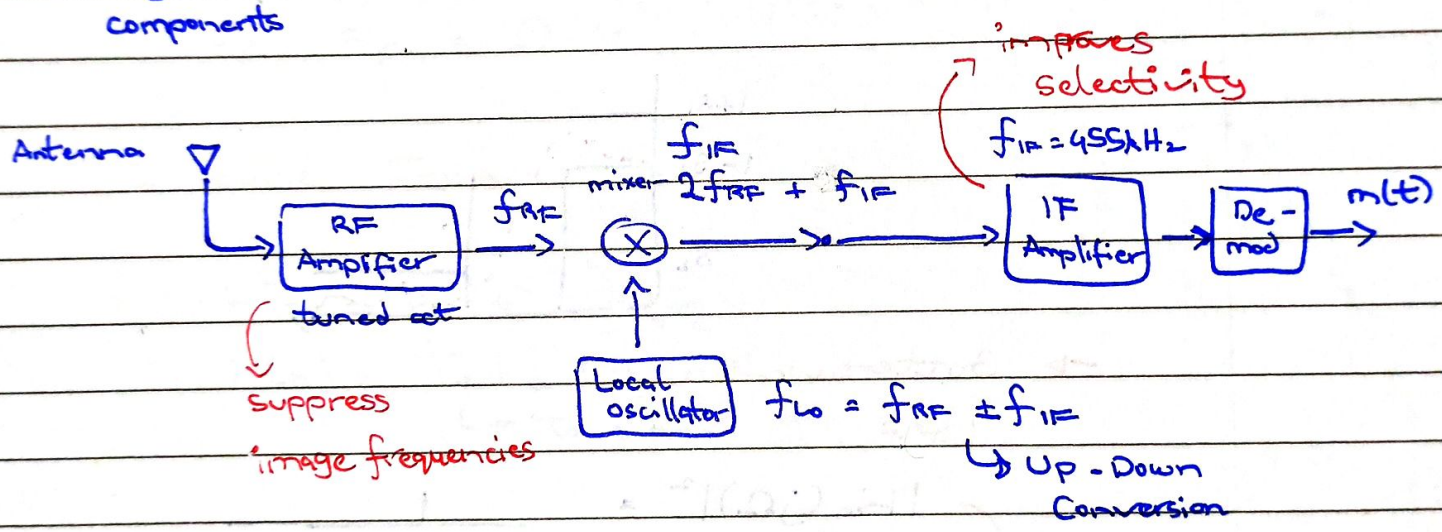
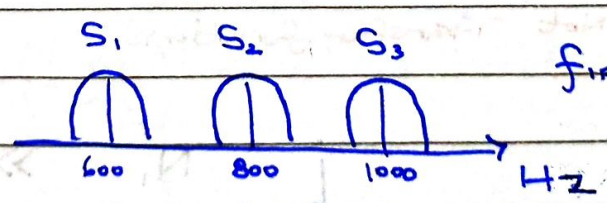


Image Frequencies

$f_{image} = f_{RF} + 2f_{IF}$

$= 1000 + 2(455)$

$= 1910$ kHz



$f_{IF} = 500$ Hz

$f_{LO} = f_{RF} + f_{IF}$

$= 800 + 500 = 1300$ Hz

y/date

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Communication Systems

4.8-1

$f_{RF} = 1500$ kHz

$f_{IF} = 455$ kHz

$f_{image} = f_{RF} + 2f_{IF} = 1500 + 910 = 2410$ kHz

lies outside RF range 0.5 — 1.6 M

$f_{image} = f_{RF} - 2f_{IF} = 1500 - 910 = 590$ kHz

1500 is the $\boxed{\text{image frequency}}$ of 590 kHz.

→ $f_{LO} = f_{RF} + f_{IF} = 590 + 455 = 1045$ kHz

→ Difference ⇒ $1500 - 1045 = 455$ kHz

4.8-2 : Range (1 – 30 MHz)

$f_{IF} = 8$ MHz $\quad f_{RF} = 10$ MHz

$f_{image} = f_{RF} \pm 2f_{IF} \overset{+}{=} 10 + 2(8) = 26$ MHz

$f_{LO} = f_{RF} + f_{IF} = 18$ MHz

4.18 : Range (88 – 108 MHz)

$f_{IF} = 10.7$ MHz

$f_{LO\,min} = f_{RF\,min} + f_{IF} = 98.7$ MHz

$f_{LO\,max} = f_{RF\,max} + f_{IF} = 118.7$ MHz

$f_{image\,min} = f_{RF\,min} + 2f_{IF} = 109.4$ MHz

$f_{image\,max} = f_{RF\,max} + 2f_{IF} = 129.4$ MHz

out side the spectrum
∴ no cross talk

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Example

$f_{IF} = 20\ MHz \qquad f_{LO} = 4.5\ GHz$

$\hookrightarrow f_{RF} > f_{LO}$; find f_{image}

$f_{LO} = f_{RF} - f_{IF} \Rightarrow f_{RF} = 4520\ MHz$

$f_{image} = f_{RF} - 2f_{IF} = 4480\ MHz$

Example : Range (88 - 98 MHz)

$\hookrightarrow f_{IF} \leq f_{LO}$

$f_{image} = f_{RF} + 2f_{IF}$

$99 = 88 + 2f_{IF}$

$f_{IF} = \dfrac{99 - 88}{2} = \dfrac{11}{2} = \boxed{5.5\ MHz}$

date

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Communication Systems

Analog

- AM
- Angle Modulation [FM, PM]
- Pulse Analog Modulation
  - PAM, PPM, PWM

c(t) carrier is discrete
m(t) remains continuous

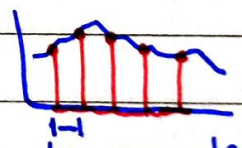
Digital [Low freq. / High freq.]

Why Digital? To allow for digital signal processing / compute capabilities
* (Transducers)

A → D:

Sampling using impulses

- $1/t_s = f_s$ should be at least twice the frequency of signal to be sampled

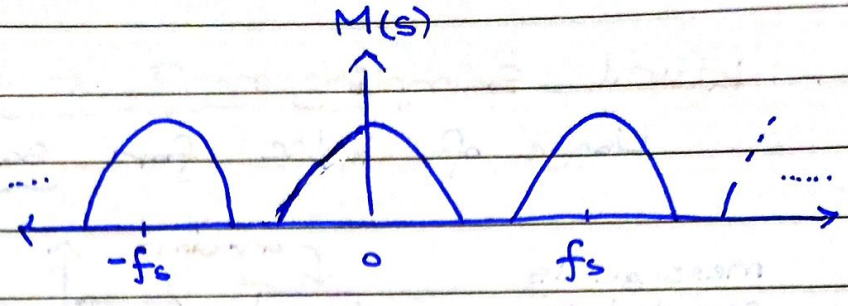


t_s sampling time ~

$$\Rightarrow m(t)\,c(t) = m'(t)$$

↳ sampled version

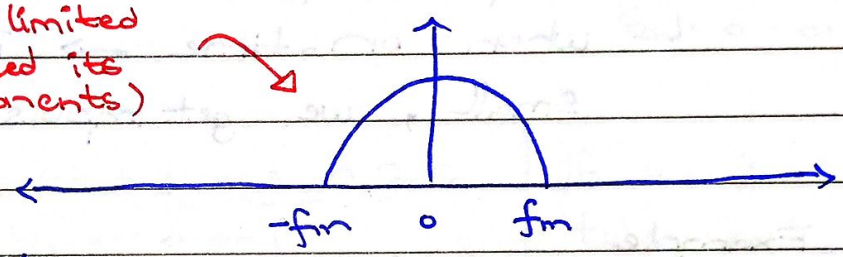
$$m'(t) = \sum_{n=-\infty}^{\infty} m(t - nT_s)$$



$$m'(f) = m(f) * f_s \sum_{n=-\infty}^{\infty} m(f - nf_s)$$

$$= f_s \sum_{n=-\infty}^{\infty} m(f - nf_s)$$

Band limited (we need its components)

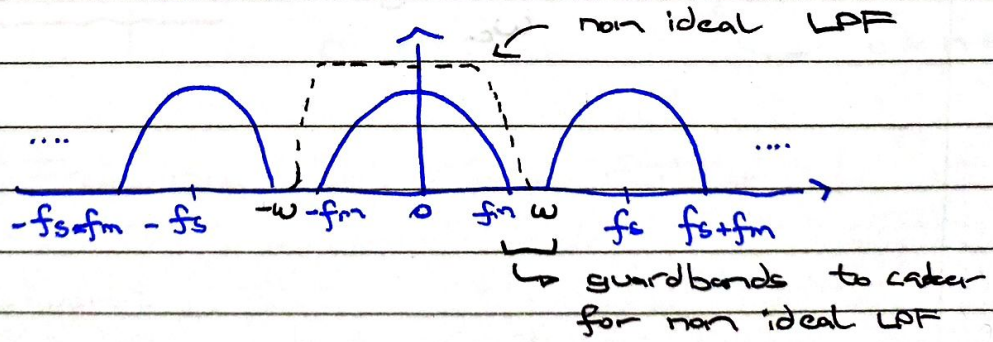


can be reconstructed using low pass filter

$\rightarrow f_s \geq 2f_m$

$\rightarrow T_s \leq \dfrac{1}{2f_m}$

Nyquist Criterion



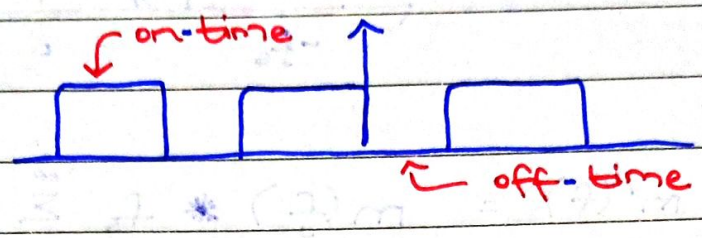
- With the increase in number of UEs, guardband congestion occurs (for which we need to move to 5G, 6G, etc.)

date

Natural Sampling :

Usage of Pulse for sampling

message is sampled for duration of on time



Instantaneous Sampling

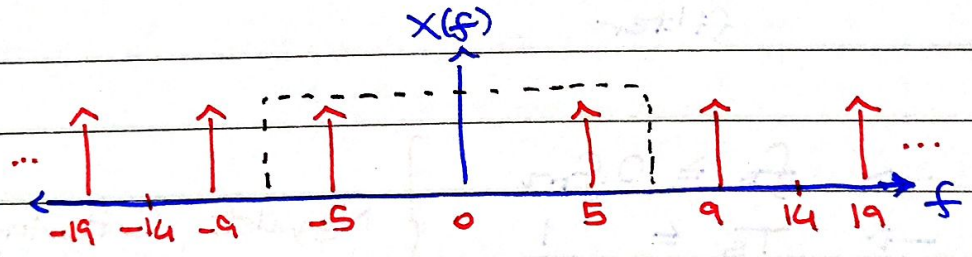
Usage of Impulses

↳ when on-time → infinitesimally small, we get impulses

Example

$x(t) = 10 \cos 10\pi t$ $\quad f_s = 14\,Hz$

$f_m = 5\ Hz$



Ideal LPF ↓ Range : 6 — 8 Hz

ω_c

ay/date

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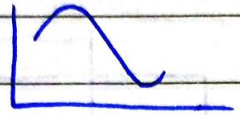
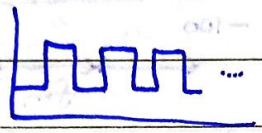
Communication Systems

Natural & Flat Top Sampling (PAM)

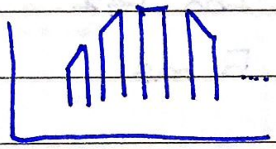
m(t)



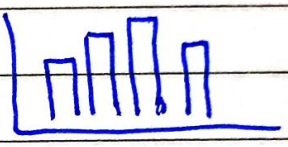
c(t)



Natural



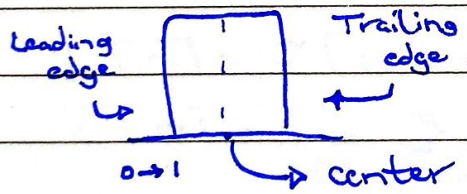
Flat Top



$m(t) \rightarrow$ Sampler $\rightarrow$ Pulse stretcher $\rightarrow r(t)$

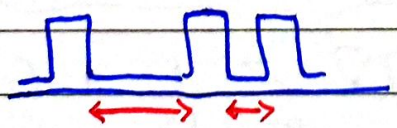
$r(t) \rightarrow$ LPF $\rightarrow m(t)$

PWM



Width changes in accordance with the message signal (leading / trailing edge)

PPM

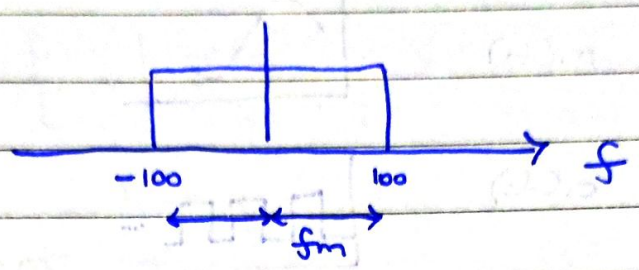


Position changes in accordance with the message signal

date

Problem

a) $g(t) = \text{sinc}(200t)$



$f_s = 2f_m = 200 \text{ Hz}$

$T_s = 1/f_s = 5 \text{ ms}$

c) $g(t) = \text{sinc}(200t) + \text{sinc}^2(200t)$

$f_m = 200 \text{ Hz}$ $\quad f_s = 2f_m = 400 \text{Hz}$

$T_s = 1/f_s = 2.5 \text{ ms}$

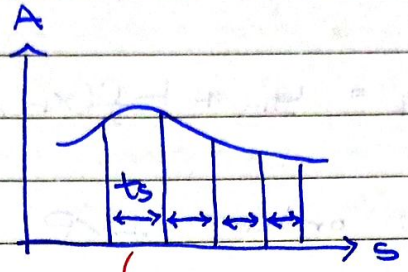
day/date

15/05/23

Communication Systems

Quantization (discretizing the amplitude)

Recap



sampling time can only be reduced to a limit ~

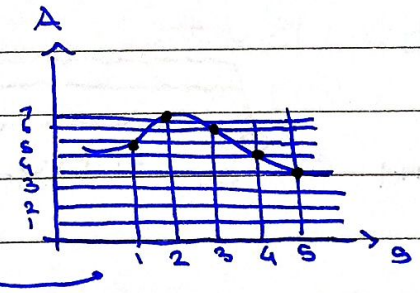
as

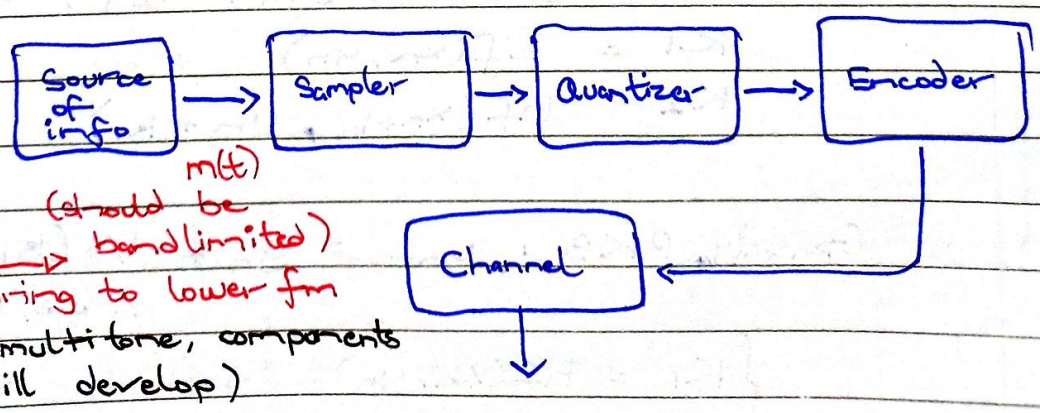
$\downarrow t_s \propto \frac{1}{f_s \uparrow}$, but BW is limited and a resource

$m(t) \rightarrow \underbrace{m(kT_s)}_{\text{sampled}}$

Continuation

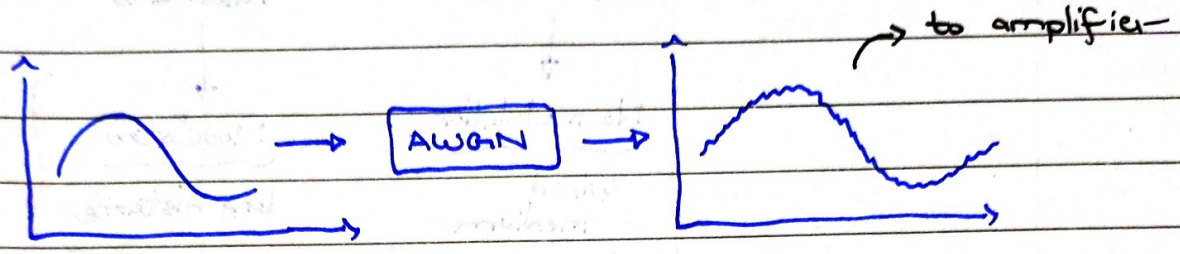
both axis discretized

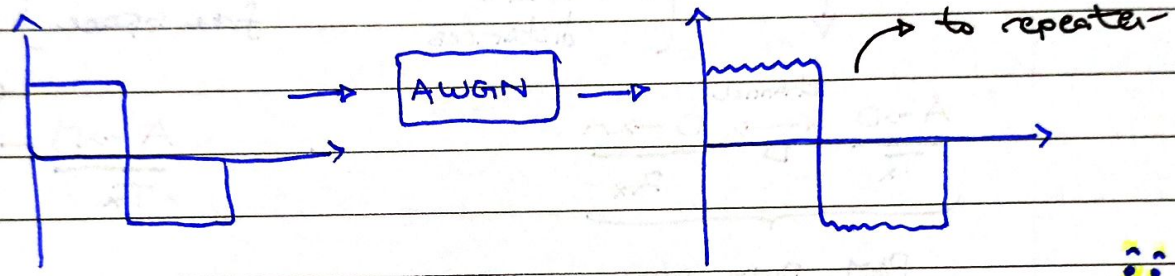




y/date

Encoder & Decoder implement checksum (data integrity) at transport layer to detect and correct bit errors

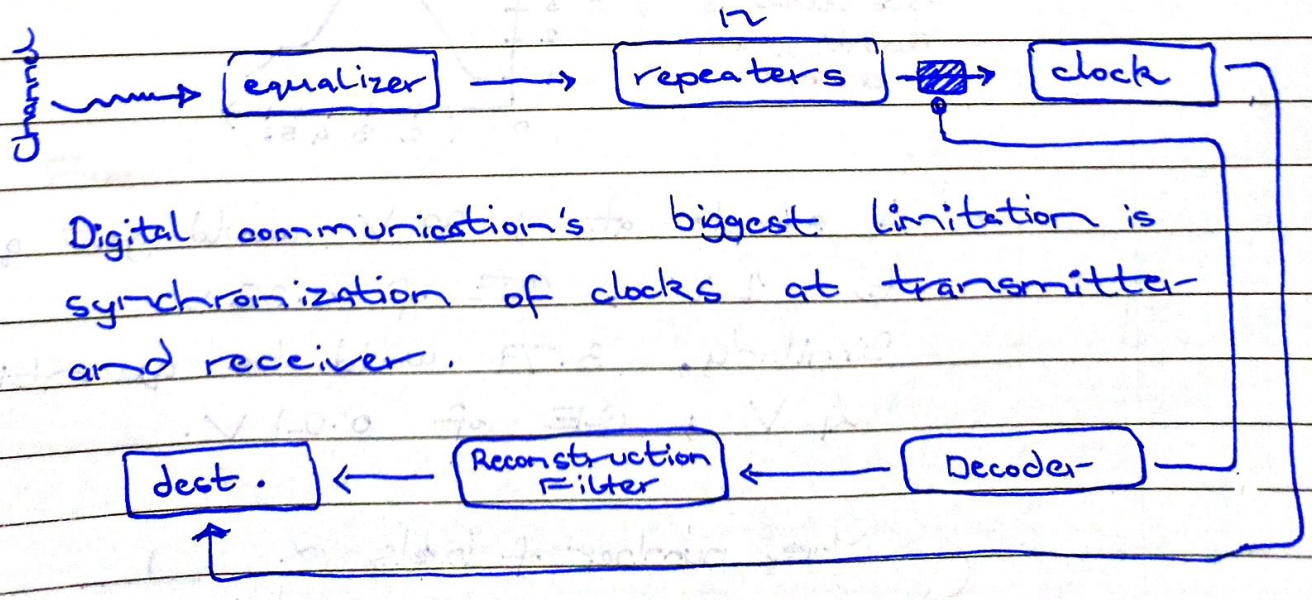




Repeaters (regenerative circuits) have the capacity to reduce noise

Amplifiers, however, boost both the signal and noise

at reciever we have



Digital communication's biggest limitation is synchronization of clocks at transmitter and receiver.

DC

- Baseband → No modulation → wired medium & short distances
- Passband → Modulated → long distance → free space

Baseband: A→D (Tx) ⟷ (channel, D) D→A (Rx)

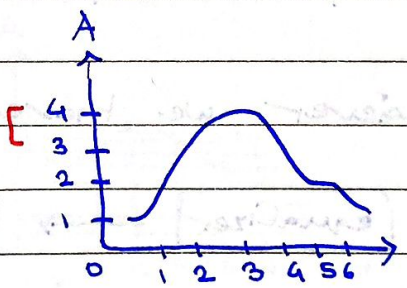
PAM, PWM, PPM, PDM

Passband: A→D (Tx) → A (channel) → D/A (As per requirement at Rx)

Things we did on (prior 2) pages

Quantization:

Difference b/w two levels is quantization error



- A sample at 1.33 V would get quantized to 1 V ; QE of 0.33 V
  Similarly, 3.79 would be quantized to 4 V ; QE of 0.21 V.

More number of levels $\propto \frac{1}{\text{less the QE}}$

↳ mean more bits
↳ more freq (band limitation)

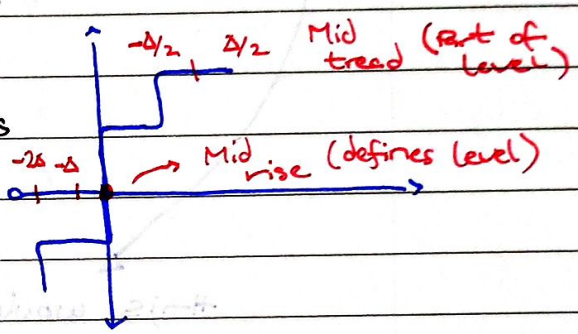
y/date

Types of Quantizers

- Uniform — fixed step size
- Non-uniform — variable step size

$$\Delta \text{ Step size} := \frac{\text{highest} - \text{lowest}}{n \text{ steps or levels}}$$

Uniform
- Mid-rise : Even no. of levels
- Mid-tread : Odd no. of levels

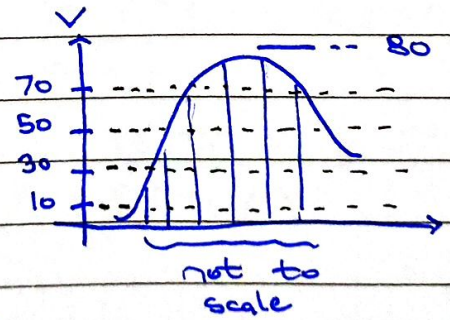


$$\text{levels} := 2^{\text{no. of bits}}$$

Non-Uniform : Step size varies according to variation in signal. {optimal quantization}

$$\Delta = \frac{V_{max} - V_{min}}{L}$$

Example $(L = 4)$



$$\Delta = \frac{80 - 0}{4} = 20$$

$x(kT_s)$	$t=0$	$t=1$	$t=2$	$t=3$	$t=4$	$t=5$
Sampler Output :	15	24.5	42.5	55.5	62.5	67.5
Quantizer :	10	30	50	50	70	70

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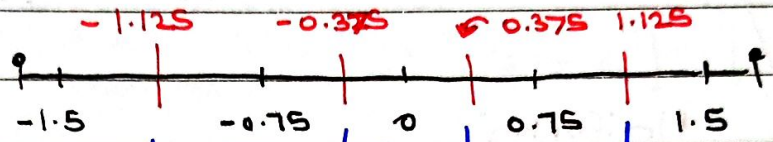
Example $\Delta = 0.75$ $L = \{-1.5, -0.75, 0.75, 1.5\}$

$x(kT_s)$	1.2	-0.2	-0.5	0.4	0.89	1.3	...
x_Q : Q_{out} (quantizer)	1.5	-0.75	-0.75	0.75	0.75	1.5	...
Q_E	0.3						
bits	11	01	01	10	10	11	...



These are the correct levels

this would be as follows

$\{1.125 \quad -0.375 \quad -0.375 \quad 0.375 \quad 1.125 \quad 1.125\}$

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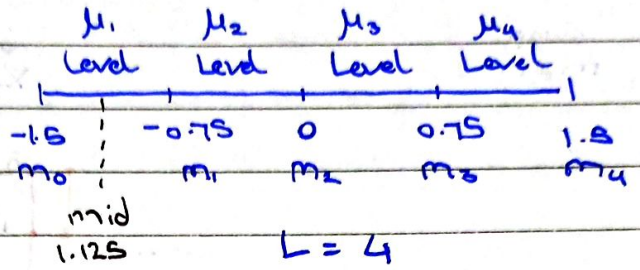
Communication Systems

Optimal Quantizer
or Lloyd Max Quantizer

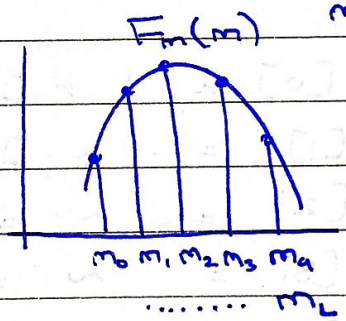


$L = 4$

QL → Quantization levels

QI → Quantization intervals

Goal is to minimize Quantization error



m is number of samples, 0, 1 ... L

$$= \frac{\partial}{\partial \mu} \int_{m_{k-1}}^{m_k} (\mu_k - m)^2 F_m(m)\, dm$$

μ_k: quantized value (to map on)

m: sampled value

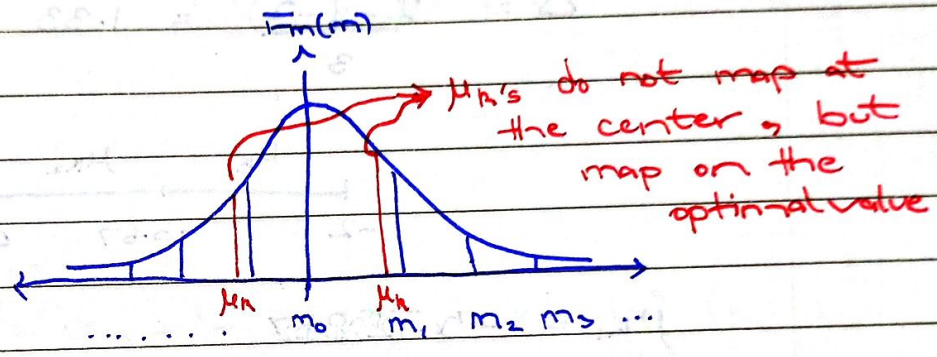
$$\Rightarrow \int_{m_{k-1}}^{m_k} \frac{\partial}{\partial \mu} (\mu_k - m)^2 F_m(m)\, dm = 0$$

$$\Rightarrow \int_{m_{k-1}}^{m_k} (\mu_k - m) F_m(dm) = 0$$

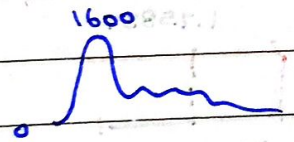
y/date

$$\Rightarrow \int_{m_{k-1}}^{m_k} \mu_k F_m(m)\, dm = \int_{m_{k-1}}^{m_k} m F_m(m)\, dm$$

$$\Rightarrow \mu_k = \frac{\int_{m_{k-1}}^{m_k} m F_m(m)\, dm}{\int_{m_{k-1}}^{m_k} F_m(m)\, dm}$$



Analysis



$L = 16$

$\Delta = 100$

which shows that 50 % of values will be wasted

Companding (before quantization)

→ compression ≫ expansion

→ why > decreases amplitude so step size decreases and QE decreases

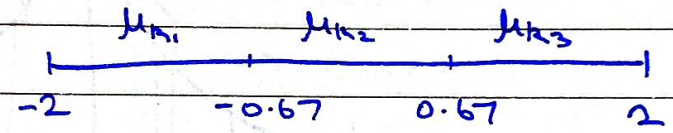
ate

Example (3 level mid-tread quantizer)

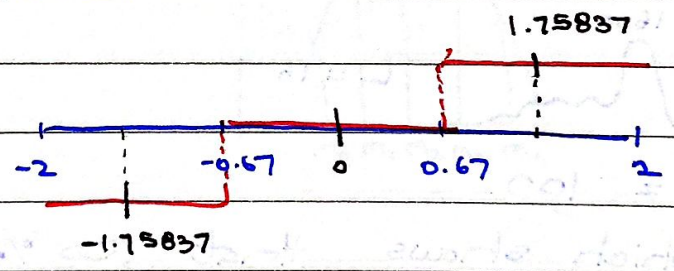
$$F_m(m) = (1 - m^2) \qquad [-2, 2]$$

$$\mu_k = \frac{\int m F_m(m)\, dm}{\int F_m(m)\, dm}$$

$$\Delta = \frac{2-(-2)}{3} = 1.33$$



$\mu_{k_1} = -1.75837$

$\mu_{k_2} = 0$

$\mu_{k_3} = 1.75837$