2/2/23

Control Systems

>>> Types of Control System

- Tracking / Servo control
  (Following a reference (changing))
- Regulating Control
  (Maintaining output)

>>> Open & Closed Loop Systems

Open → No feedback

inclusion of output in

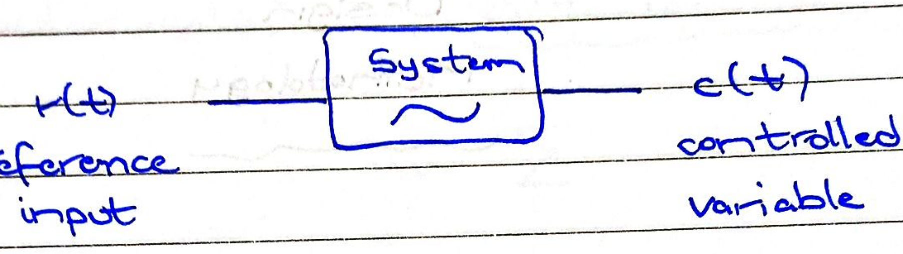
[SISO, MIMO, etc.]

Input → Processing → Out
Feedback

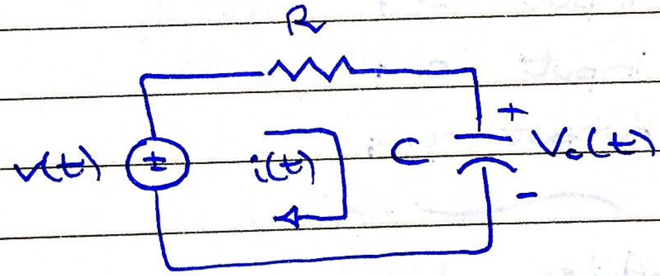
>>> Design → Steady state error
→ Speed of Response
→ Stability

Linear Control Systems

» We study control systems by developing mathematical models of designed system(s).



» RC Network



» $$v(t) = R\,i(t) + \frac{1}{C}\int_0^t i(\tau)\,d\tau$$

$$= R\frac{dq(t)}{dt} + \frac{1}{C}q(t)$$

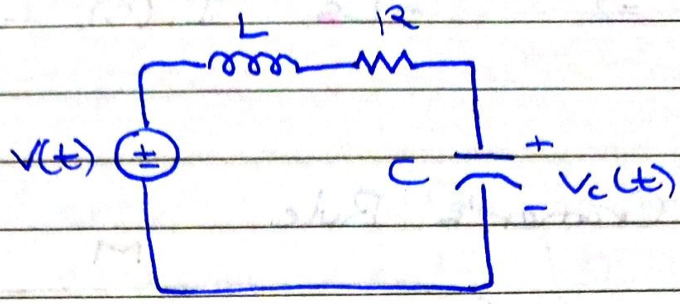
$$v(t) = RC\frac{dV_c(t)}{dt} + V_c(t)$$

→ LT :

$$V(s) = RC\,s\,V_c(s) + V_c(s)$$

$$\frac{V_c(s)}{V(s)} = \frac{1/RC}{s + 1/RC}$$

RLC Network



$$v(t) = L\frac{di(t)}{dt} + \frac{1}{C}\int_0^t i(\tau)\,d\tau + Ri(t)$$

$$= L\frac{d^2q(t)}{dt^2} + v_c(t) + R\frac{dq(t)}{dt}$$

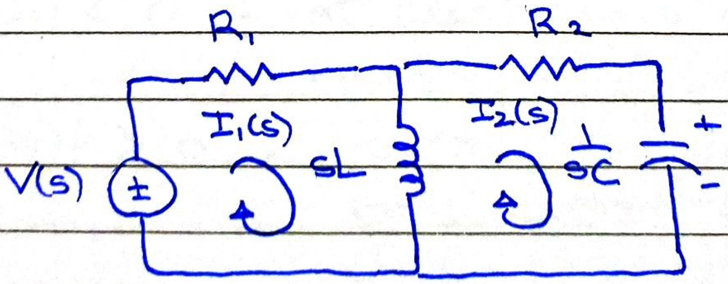
$$= LC\frac{d^2v_c(t)}{dt^2} + V_c(t) + RC\frac{dV_c(t)}{dt}$$

LT:

$$V(s) = LCs^2V_c(s) + V_c(s) + RCsV_c(s)$$

$$\frac{V_c(s)}{V(s)} = \frac{1}{(LCs^2 + RCs + 1)} = \frac{1/LC}{s^2 + \frac{R}{L}s + 1/LC}$$

Example 2.10



$$\text{I}: \quad -V(s) + R_1I_1(s) + sL(I_1(s) - I_2(s)) = 0$$

$$\text{II}: \quad 0 = R_2I_2(s) + \frac{1}{sC}I_2(s) + sL(I_2(s) - I_1(s))$$

$$\Rightarrow \quad (R_1 + Ls)\, I_1(s) - sL\, I_2(s) = V(s) \, ;$$

$$\Rightarrow \quad -Ls \quad I_1(s) + \left\{ R_2 + Ls + \frac{1}{sC} \right\} = 0$$

Cramer's Rule

$$\Rightarrow \quad \overbrace{\begin{bmatrix} R_1 + Ls & -Ls \\ -Ls & Ls + R_2 + 1/sC \end{bmatrix}}^{M} \begin{bmatrix} I_1(s) \\ I_2(s) \end{bmatrix} = \begin{bmatrix} V(s) \\ 0 \end{bmatrix}.$$

$$I_1(s) = \frac{\begin{bmatrix} V(s) & -Ls \\ 0 & Ls + R_2 + 1/sC \end{bmatrix}}{\Delta}, \text{ etc.}$$

, where Δ is determinant of M

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9/2/23

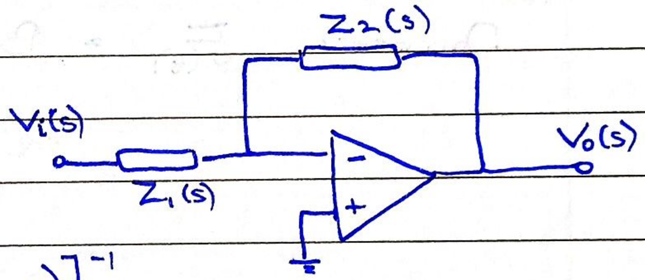
Linear Control Systems

Model of the System

- Electrical
- Mechanical
  - Translational
  - Rotational
- Electro mechanical

Inverting Op-Amp

$$\frac{V_o(s)}{V_i(s)} = -\frac{Z_2(s)}{Z_1(s)}$$



$$Z_2(s) = 220k + \frac{10M}{s}$$

$$Z_1(s) = \left[\left(\frac{1}{360k}\right) + (5.6\mu s)\right]^{-1}$$

$$= \left[\frac{1 + 2.016s}{360k}\right]^{-1}$$

$$= \frac{360k}{1 + 2.016s}$$

$$\frac{V_o(s)}{V_i(s)} = -\frac{Z_2(s)}{Z_1(s)} = \frac{\frac{220k\,s + 10M}{s}}{\frac{360k}{1 + 2.016s}}$$

$$= \frac{220k\,s + 443k\,s^2 + 10M + 20.16M\,s}{360k\,s}$$

Solve using variables first

$$= \frac{443k\,s^2 + 20.38M\,s + 10M}{360k\,s}$$

Mechanical System - Translational

I/P	O/P	System Params
F	x, v, a	M, K, B/f_v

Mass : $F_{(t)} = Ma = M\dfrac{dv}{dt} = M\dfrac{dx^2}{dt^2}$

Spring: $F_{(t)} = Kx_1(t)$ or $K(x_1(t) - x_2(t)]$

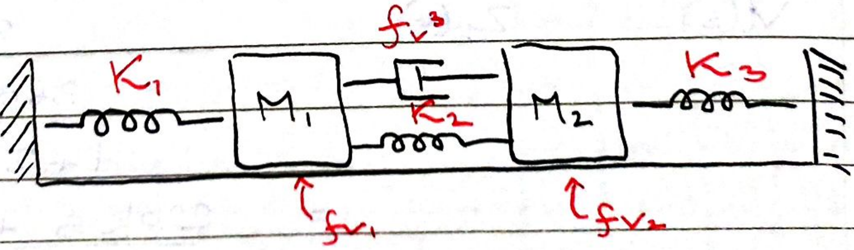
one displacement — two displacement (direction dependent)

Damper: $F_{(t)} = B\dfrac{dx_1(t)}{dt}$ or $B\left[\dfrac{dx_1(t)}{dt} - \dfrac{dx_2(t)}{dt}\right]$

one displacement — two displacement

Spring ⟷ Capacitor
Damper ⟷ Resistor
Mass ⟷ Inductor

- **Degree of Freedom** : Number of linearly independent motions



DoF = 2

① $K_1X_1(s) + s^2M_1X_1(s) + K_2(X_1(s) - X_2(s)) + sf_{v3}(X_1(s) - X_2(s)) + sf_{v1}X_1(s) = F(s)$

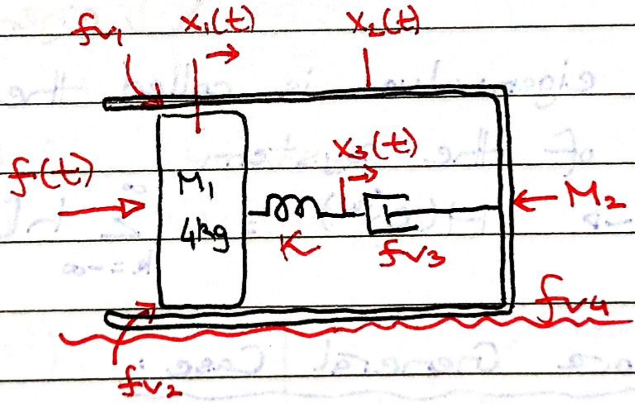
② $K_3 X_2(s) + s^2 M_2 X_2(s) + K_2(X_2(s) - X_1(s))$
$+ s f_{v_3} (X_2(s) - X_1(s)) + s f_{v_2} X_2(s) = 0$

→ Using free body diagrams

– forces due to x_2 motion

Do the same for M_2

Activity



Solve as an assignment

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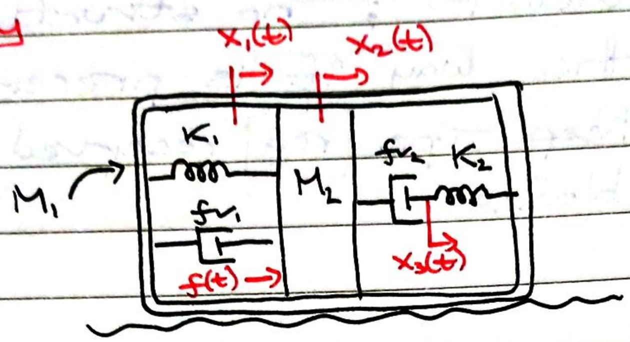
Linear Control Systems

Activity



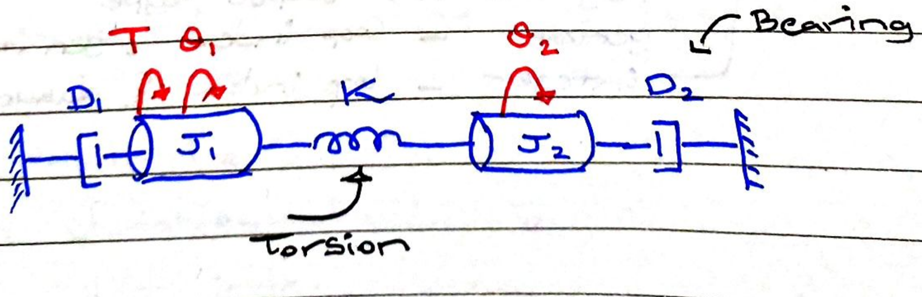
i: $M_1 s^2 X_1 + K_1(X_1 - X_2) + s fv_1 (X_1 - X_2) \ldots$
$+ K_2(X_1 - X_3) = 0$

ii: $M_2 s^2 X_2 + K_1(X_2 - X_1) + s fv_1 (X_2 - X) \ldots$
$+ s fv_2 (X_2 - X_3) =$

iii: $K_2(X_3 - X_1) + s fv_2 (X_3 - X_2) = 0$

Rotational System

$$\left\{ \begin{array}{ccc} F & \rightarrow & T \\ M & \rightarrow & J \\ X & \rightarrow & \theta \end{array} \right\}$$



i: $J_1 s^2 \theta_1(s) + K(\theta_1(s) - \theta_2(s)) + D_1 s \, \theta_1(s) = T(s)$

ii: $J_2 s^2 \theta_2(s) + K(\theta_2(s) - \theta(s)) + D_2 s \, \theta_2(s) = 0$

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Linear Control Systems

Electromechanical Systems

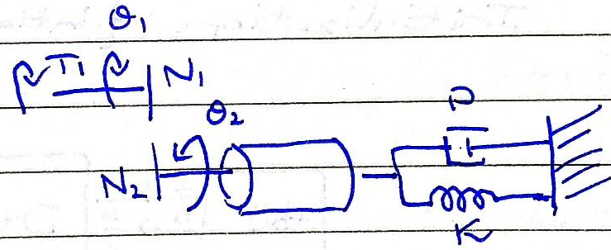
- Systems with gears $\begin{cases} \text{Teeth / Cogs} & N \\ \text{Radius} & r \\ \text{Angular Displacement} & \theta \end{cases}$

- $\dfrac{N_1}{N_2} = \dfrac{T_1}{T_2} = \dfrac{\theta_2}{\theta_1}$ } Analogous to N, V, I of transformers

- $N \propto r$: Bigger the gear, the more the teeth

- $r_1\theta_1 = r_2\theta_2 \Rightarrow \dfrac{r_1}{r_2} = \dfrac{\theta_2}{\theta_1}$

Example:



$$\frac{\theta_2(s)}{T_2(s)} = (Js^2 + K + Ds)^{-1} = \frac{1}{Js^2 + Ds + K}$$

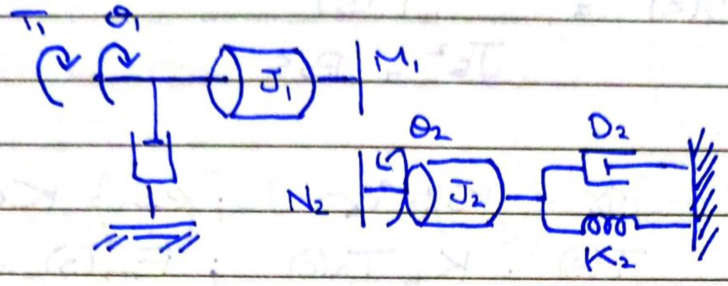
$$\frac{\theta_2(s)}{T_1(s)} = \frac{N_2/N_1}{Js^2 + Ds + K}$$

$$\frac{\theta_1(s)}{T_1(s)} = \frac{(N_2/N_1)^2}{Js^2 + Ds + K}$$

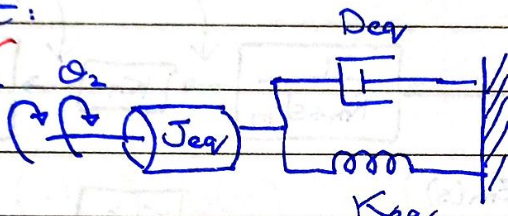
→ Reflection : Multiplication of :

$$\left(\frac{\text{No. of Teeth on Source}}{\text{No. of Teeth on Dest.}}\right)^2$$

Example 2.21:



Equivalent:



$$T_2 = T_1\left(\frac{N_2}{N_1}\right)$$

$$J_{eq} = J_1 (N_2/N_1)^2 + J_2$$

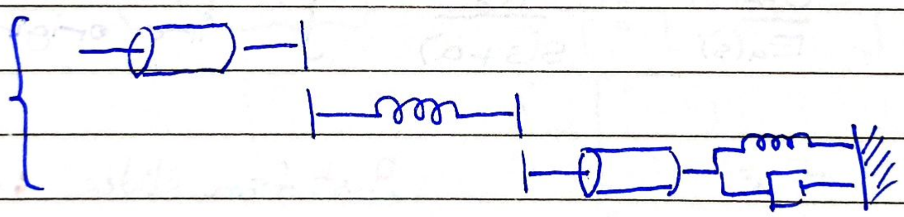
$$D_{eq} = D_1 (N_2/N_1)^2 + D_2$$

$$K_{eq} = K_2$$

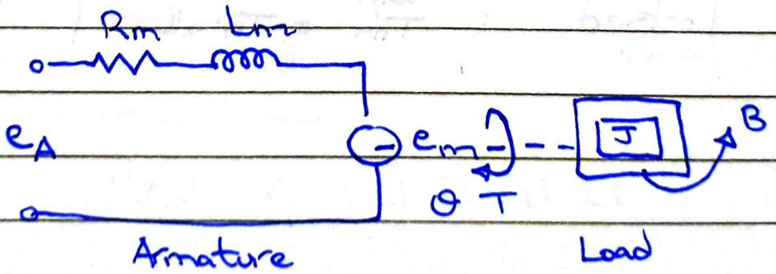
Example:

DOF = 2

Reflect



Motor System



$$E_a(s) = (R_m + sL_m) I_a(s) + E_m(s)$$

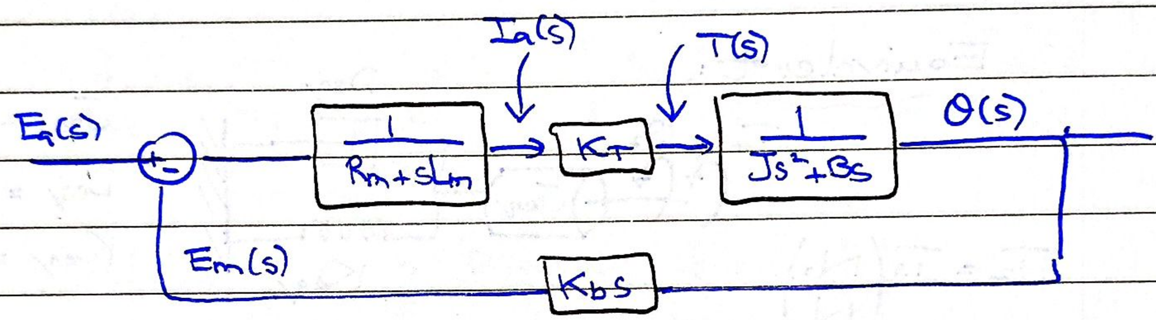
$$I_a(s) = \frac{E_a(s) - E_m(s)}{R_m + sL_m}$$

$$T(s) = (Js^2 + Bs)\,\theta(s)$$

$$\theta(s) = \frac{T(s)}{Js^2 + Bs}$$

$$\tau = K\phi\, i_A \quad ; \quad e_m = K\phi\,\omega$$

$$T(s) = K_t I_A(s) \quad ; \quad E_m(s) = K_b\, s\, \theta(s)$$



$$T(s) = (Js^2 + Bs)\,\theta(s) \quad ✓$$

$$\left.\frac{\theta_m(s)}{E_a(s)} = \frac{K}{s(s+a)}\right\} \text{ By substituting } T(s) \text{ back in original eqns}$$

Rest from slides ;-;

No Load :	$\omega_m = \omega_{nl}$, $T_m = 0$	Find constants K_b and $\frac{K_t}{R_A}$
Locked :	$\omega_m = 0$, $T_m = T_{stall}$	

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Linear Control Systems

$$\frac{K_t}{R_a} = \frac{I_{stall}}{e_a} \quad -i \quad ; \quad K_b = \frac{e_a}{\omega_{nl}} \quad -ii$$

$$\frac{\theta_m(s)}{E_a(s)} = \underbrace{\frac{K_t/R_a J_m}{s\left[s + \frac{1}{J_m}\left(D_m + \frac{K_t K_b}{R_a}\right)\right]}}$$

solve for constants using i and ii

Electro mechanical Systems

Linearization of Non-Linear Systems

Steps:
- Check where the non-linear component is, and write the non linear differential equation.
- Use Taylor series approximation to obtain linearity.

→ Taylor Series Approximation

slope m

$$f(x) = f(x_0) + f'(x_0)(\delta x)$$

Example: $f(x) = 5\cos(x)$; $x = \pi/2$

$$\Rightarrow f(x) = f(x_0) + f'(x)(\delta x)$$

$$f(x) = 5\cos(\pi/2) + 5(-\sin x_0)\Big|_{x_0 = \frac{\pi}{2}} (x - \pi/2)$$

$$5\cos(x) = -5\delta x$$

EXCELLENT

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Example $\dfrac{d^2x}{dt^2} + \dfrac{2dx}{dt} + \underbrace{\cos x}_{\text{Non linear component}} = 0$; $x_0 = \dfrac{\pi}{4}$

$f(x) = f(x_0) + f'(x_0)\,\delta x$

$\cos(x) = \cos(\pi/4) + (-\sin(\pi/4))\,\delta x$

$\Rightarrow \dfrac{d^2x}{dt^2} + \dfrac{2dx}{dt} + \left\{\dfrac{1}{\sqrt{2}} - \dfrac{1}{\sqrt{2}}\,\delta x\right\} = 0$

$\Rightarrow x - \pi/4 = \delta x \rightarrow x = \delta x + \pi/4$

- $\dfrac{d^2(\delta x + \pi/4)}{dt^2} + \dfrac{2d(\delta x + \pi/4)}{dt} + \dfrac{1}{\sqrt{2}} \ldots$

  $- \dfrac{1}{\sqrt{2}}\,\delta x = 0$

Example $J\dfrac{d^2\theta}{dt^2} + \dfrac{Mgl}{2}\sin\theta = \tau(t)$

$\therefore \theta = 0$

$f(x) = f(x_0) + f'(x_0)\,\delta x$

$\dfrac{Mgl}{2}\sin\theta = \dfrac{Mgl}{2}\sin(0) + \dfrac{Mgl}{2}\cos(0)\,\delta x$

$= \dfrac{Mgl}{2}\,\delta x$

$\Rightarrow J\dfrac{d^2(\theta_0 + \delta\theta)}{dt} + \dfrac{Mgl}{2}\,\delta\theta = \tau(t)$

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Linear Control Systems

Time Response of 1st / 2nd Order Systems

Analysis
- Stability
- Steady state error
- Transient response → 1st order, 2nd order

Inputs to the system for analysis: Known signals such as impulse, step, ramp, etc.

① $R(s) = 1/s \qquad G(s) = \dfrac{s+2}{s+5}$

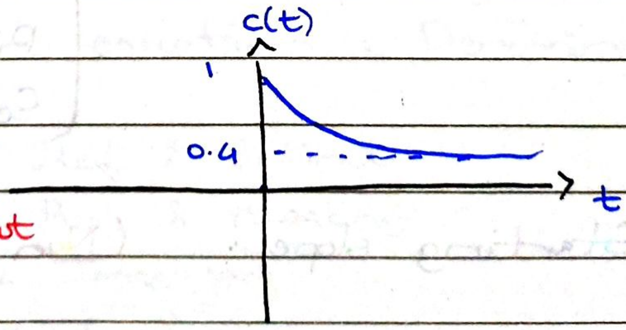
$$C(s) = \frac{1}{s}\left(\frac{s+2}{s+5}\right) = \frac{A}{s} + \frac{B}{s+5}$$

$$C(s) = \frac{2}{5s} + \frac{3}{5(s+5)}$$

natural response of the system

$$c(t) = \left[\frac{2}{5} + \frac{3}{5}e^{-5t}\right]u(t)$$

forced response coming from the applied input



② $R(s) = \dfrac{1}{s} \qquad G(s) = \dfrac{a}{s+a}$

$$C(s) = \frac{1}{s} \cdot \left(\frac{a}{s+a} \right)$$

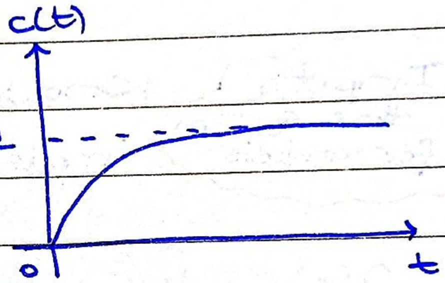
$$= \frac{A}{s} + \frac{B}{s+a}$$

$$= \frac{1}{s} + \frac{(-1)}{s+a}$$

$$c(t) = \underbrace{u(t)} - \underbrace{e^{-at}\, u(t)}$$

$$= [1 - e^{-at}]\, u(t)$$

Unity Gain
($t \to \infty$: $s \to 0$)
to find gain



when $t \to 1/a$: $c(t) = 0.63$
or 63 % of its ... final value
or natural response decays to 37%

For first order :
- Time constant τ
- Rise time τ_R from 0.1 to 0.9 of its final value
- Settling time τ_s $\pm 2\%$ of final value

Starting slope : (Non-zero)

$$c'(t) = \left.\frac{dc(t)}{dt}\right|_{t=0} = \left. a e^{-at} \right|_{t=0} = a$$

ate

→ Rise time :

$c(t) = 1 - e^{-at}$ $\left.\begin{array}{l} c(t) = 0.1 \\ c(t) = 0.9 \end{array}\right.$

$t_r = \frac{2.2}{a} = \boxed{2.2\tau}$ $\therefore \tau = 1/a$

→ Settling time:

$c(t) = 0.98$ and solve for t

$t_s = \frac{4}{a} = \boxed{4\tau}$ $\therefore \tau = 1/a$

Example: $k = 0.72$ $\tau = 0.15$

$C(s) = \frac{k}{\tau s+1} = \frac{0.72}{0.15s+1} \Rightarrow \boxed{\frac{4.8}{s+6.67}}$

$a_o = \frac{1}{\tau} = 6.67$ $b_o = \frac{k}{\tau} = 4.8$

Second Order Systems

Characteristic equation : Denominator = 0

- Roots are Real & distinct
- Roots are Real & repeating
- Roots are Imaginary
- Roots are Complex

Case 1: Roots (Real and Distinct)

$C(s) = \frac{9}{s^2+9s+9}$] overdamped case

day/date

Case 2: Roots (Real and Repeated)

$$C(s) = \frac{9}{s^2+6s+9}$$ critically damped

Case 3: Roots (Complex)

$$C(s) = \frac{9}{s^2+2s+9}$$ under-damped

Case 4: Roots (Imaginary)

$$C(s) = \frac{9}{s^2+9}$$ Undamped

ω_n : natural f. (without damping)

ζ : ratio of damping (how fast oscillations die down)

→ General Form

$$C(s) = \frac{K\omega_n^2}{s^2+2\zeta\omega_n s+\omega_n^2}$$

ζ:
- <1 Under damped
- $=1$ Critically damped
- >1 Overdamped
- $=0$ Undamped

Example:

a) $$G(s) = \frac{800}{s^2+12s+400}$$ (Compare)

$\omega_n = 20$ $K = 2$ $\zeta = 0.3$

→ Underdamped

ate

b) $G(s) = \dfrac{1200}{s^2 + 90s + 900}$

$\omega_n = 30 \qquad K = 1.33 \qquad \xi = 1.5$

$\rightarrow$ Overdamped

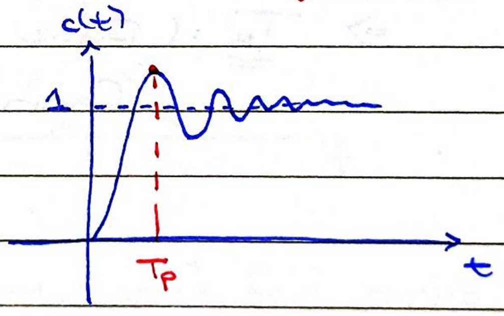
Roots:

$$s_{1,2} = -\xi\omega_n \pm \left[\sqrt{1-\xi^2}\right] j\omega_n$$

$$= -\underbrace{\sigma_d}_{\text{Exponential decay frequency}} \pm j\underbrace{\omega_d}_{\text{Damped frequency}}$$

2^{nd} Order

- Rise time T_r : time to go from $0.1 - 0.9$ of final value
- Peak time T_p : time to reach the first peak
- Percent overshoot $\%\,OS$: amount system overshoots the final value



$\Rightarrow$ T_p : Maxima through derivative

$\mathcal{L}\, c'(t) = s\,C(s)$

$\rightarrow \omega_n\sqrt{1-\xi^2}\; t = n\pi$ first peak $n = 1$

$$t = \frac{\pi}{\omega_n\sqrt{1-\xi^2}} = \boxed{\frac{\pi}{\omega_d}}$$

{On next LCS pages}

⇒ T_s : (Settling Time)

$$c(t) = 1 - \frac{1}{\sqrt{1-\xi^2}} e^{-\xi\omega_n t} \cos(\omega_n\sqrt{1-\xi^2}\, t + \phi)$$

$$\frac{1}{\sqrt{1-\xi^2}} \Rightarrow 0.02$$

$$T_s = \frac{4}{\xi\omega_n} = \boxed{\frac{4}{\sigma_d}}$$

⇒ % OS : $\frac{C_{max} - C_{final}}{C_{final}}$

$$C_{max} = c(T_p)$$

$$\% OS = \boxed{e^{-(\xi\pi/\sqrt{1-\xi^2})} \times 100}$$

⇒ T_R : (Rise Time)

Cannot be computed directly
Use normalized
approch

Example (Do it from HOME 😊

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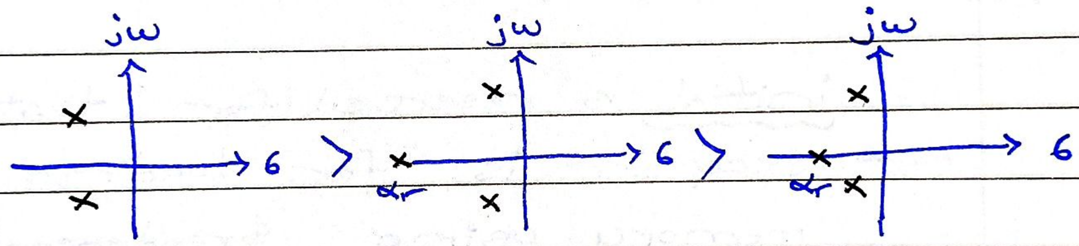
Linear Control Systems

First Order → τ, T_r, T_s

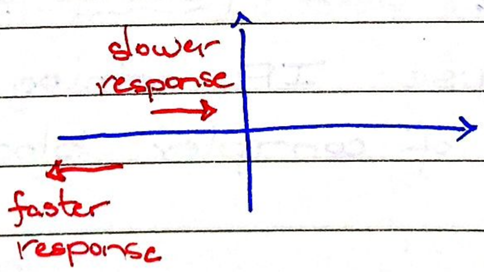
Second Order → $\underbrace{T_r, T_p, T_s, \%OS}$

- Underdamped system characteristics

- Approximation of Higher Order Systems
  - Addition of poles slows down the response compared to lower order system



Speed of Response



Five times rule.

If additional poles are at least "five times" away from dominant poles, approximations can be applied.

→ necessary but not sufficient

Addition of zero

$$\Rightarrow G(s) = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

- Adding zero at $-a$

$$G(s) = \frac{(s+a)\,\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2} = \overset{\text{derivative}}{\frac{s\omega_n^2}{\text{〃}}} + \overset{\text{scaled}}{\frac{a\omega_n^2}{\text{〃}}}$$

- Decreases / faster response in terms of T_p, but increases %OS
- Closer the zeros are to imaginary axis, higher the increase in %OS
  the effect in response

- Zeros at $+a$
- Response begins towards the negative direction (Non minimum - phase system)

- Zero cannot cancel pole in the right HP

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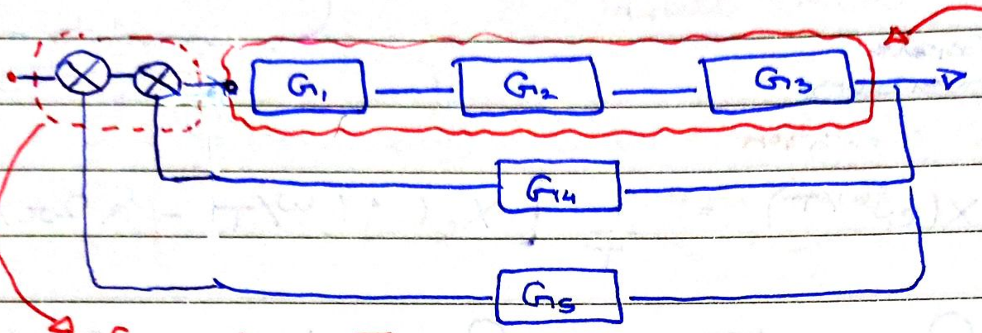
Linear Control Systems

Reduction of Multiple Sub Systems



Series - can be multiplied

$G_1G_2G_3$

Summing Junctions

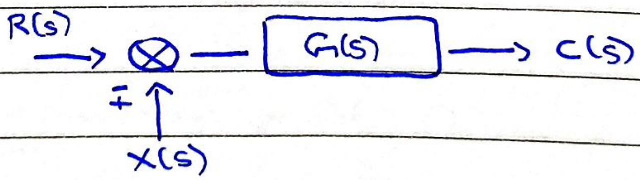
$$\Rightarrow T(s) = \frac{C(s)}{R(s)} = \frac{G(s)}{1 \pm G(s)H(s)}$$

$$\Rightarrow R(s) = E(s)\left[1 \boxed{\pm} H(s)G(s)\right]$$

+ : Negative fb

− : Positive fb

Moving a block

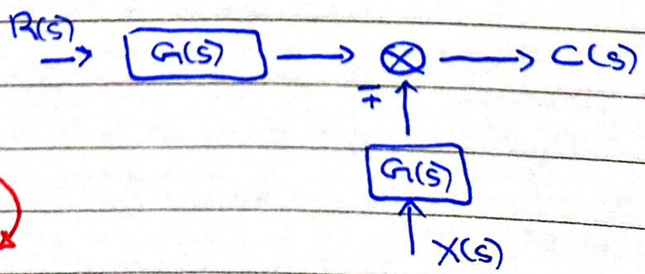


× G(s)

for summing junctions

left past

~ is equivalent to



For right past

$\times \frac{1}{G(s)}$

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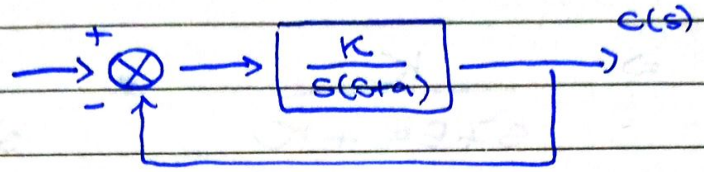
Linear Control Systems



$$\Rightarrow \quad \frac{Z(s)}{P(s)+Z(s)} = \frac{K}{s(s+a)+K} = \frac{K}{s^2+as+K}$$

Analysis

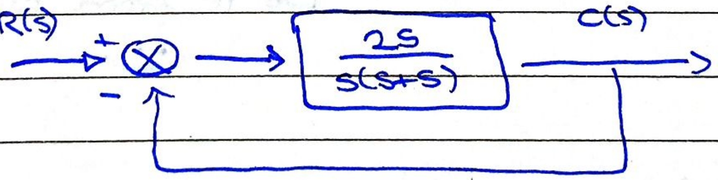
$$S_{1,2} = -\frac{a}{2} \pm \frac{\sqrt{a^2-4K}}{2}$$

$\rightarrow$ $0 < K < a^2/4$: Overdamped

$\rightarrow$ $K = a^2/4$: Critically Damped

$\rightarrow$ $K > a^2/4$: Under-damped

Example



$$\Rightarrow \quad \frac{25}{s^2+5s+25} \equiv \frac{K\omega_n^2}{s^2+2\zeta\omega_n s+\omega_n^2}$$

$\omega_n = 5$; $\zeta = 0.5$

$$T_p = \frac{\pi}{\omega_n\sqrt{1-\zeta^2}} = 0.725s$$

$$T_s = \frac{4}{\zeta\omega_n} = 1.6s$$

$$\%OS = e^{-\zeta\pi/\sqrt{1-\zeta^2}} \times 100$$

$$= 0.163 \times 100$$

$$= 16.3\ \%$$

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Example $\sim \frac{K}{s(s+5)}$ in unity fb

$$\Rightarrow \frac{K}{s^2+5s+K} = \frac{K\omega_n^2}{s^2+2\zeta\omega_n s+\omega_n^2}$$

$$\%OS = e^{-\zeta\pi/\sqrt{1-\zeta^2}} \times 100\%$$

$$\frac{10}{100} = e^{-\zeta\pi/\sqrt{1-\zeta^2}}$$

$$-2.302 = -\frac{\zeta\pi}{\sqrt{1-\zeta^2}}$$

$$\zeta = 0.591$$

$$\Rightarrow \begin{matrix} K = \omega_n^2 \\ K = 17.88 \end{matrix} \left\{ \begin{matrix} 2\zeta\omega_n = 5 \\ \omega_n = 4.229 \end{matrix} \right.$$

Signal Flow Graphs

Do it from slides

:)

Mason's Rule

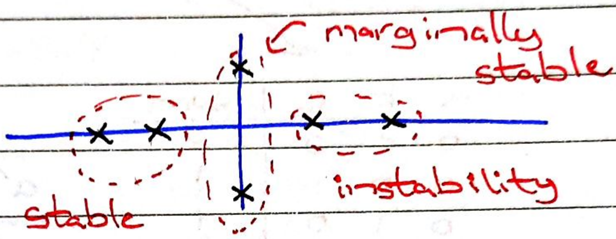
$$G(s) = \frac{C(s)}{R(s)} = \frac{\sum_k T_k \Delta_k}{\Delta}$$

Example from slide

Linear Control Systems

Stability

System output → natural response → should die down to 0 → for stability
System output → forced response



Also ; stable system follows BIBO

Feedback Example

$$T(s) \Rightarrow \frac{3}{3 + s(s+1)(s+2)}$$

$$\Rightarrow \frac{3}{(s^2+s)(s+2)+3} \Rightarrow \frac{3}{s^3 + 3s^2 + 2s + 3}$$

Pole locations : -2.67 , $-0.164 \pm j1.046$

left half plane : stable

$$\Rightarrow \frac{7}{s^3 + 3s^2 + 2s + 7} \Rightarrow -3.08 \, , \; 0.043 \pm j1.505$$

For higher order systems:

→ If poles lie in left half plane;
Denominator is of form:

$$(s + a_i)$$

• where a_i is either real or complex with +ive real part.

→ Signs of all $\overbrace{a_i}^{\text{coeffs of denom}}$'s must be same for stability / any sign change = unstable

→ If any power of s is missing, system is
- unstable
- marginally stable

Routh Table

$$\Rightarrow \frac{N(s)}{a_4 s^4 + a_3 s^3 + a_2 s^2 + a_1 s + a_0}$$

We look for no. of sign changes in the first column (for stability)

s^4	a_4	a_2	a_0	0
s^3	a_3	a_1	0	0
s^2	b_1: $-\frac{\begin{vmatrix} a_4 & a_2 \\ a_3 & a_1 \end{vmatrix}}{a_3}$	b_2: $-\frac{\begin{vmatrix} a_4 & a_0 \\ a_3 & 0 \end{vmatrix}}{a_3}$	b_3: $-\frac{\begin{vmatrix} a_4 & 0 \\ a_3 & 0 \end{vmatrix}}{a_3}$	
s^1	c_1: $-\frac{\begin{vmatrix} a_3 & a_1 \\ b_1 & b_2 \end{vmatrix}}{b_1}$	0	0	
s^0	$-\frac{\begin{vmatrix} b_1 & b_2 \\ c_1 & 0 \end{vmatrix}}{c_1}$			

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Linear Control Systems

Problem

$$P(s) = 3s^7 + 9s^6 + 6s^5 + 4s^4 + 7s^3 + 8s^2 + 2s + 6$$

s^7	3	6	7	2	0
s^6	9	4	8	6	0
s^5	4.667	4.333	0	0	0
s^4	-4.355	8	6	0	0
s^3	12.906	6.429	0	0	0
s^2	10.169	6	0	0	0
s	-1.18	0	0	0	0
s^0	6	0	0	0	0

4 sign changes ≡ 4 poles in RHP

Special Cases

- Entire Row is zero
- Zero in first column

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Example : $Q(s) = s^5 + s^4 + 2s^3 + 3s^2 + s + 4$

s^5	1	2	1	0
s^4	1	3	4	0
s^3	-1	-3	0	0
s^2	ε $\cancel{0}$	-4	0	0
s^1	$(-3\varepsilon + 4)/\varepsilon$	0	0	0
s^0	4	0	0	0

ε can be either a small -ive number or +ive ; no. of sign changes remains the same (2 sign changes)

Example . $Q(s) = s^5 + 2s^4 + 3s^3 + 6s^2 + 5s + 3$

s^5	1	3	5	0
s^4	2	6	3	0
s^3	$\cancel{0}$ ε	3.5	0	0
s^2	$(6\varepsilon - 7)/\varepsilon$	3	0	0
REEE s^1	* $\dfrac{3.5\left(\frac{6\varepsilon - 7}{\varepsilon}\right) - 3\varepsilon}{(6\varepsilon - 7)/\varepsilon}$	0	0	0
s^0	3	0	0	0

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$\star$ is: $\dfrac{(3.5)(6\varepsilon-7) - 3\varepsilon^2}{(6\varepsilon - 7)}$ ~

Use ε +ive, two sign changes

Example (maybe later)

→ Auxillary Polynomials only have roots that are symmetric about the origin.

→ First column values uptill Aux. Pol. follow normal rules; After it, we follow rules of symmetry.

Activity $s^6 + s^5 + 5s^4 + s^3 + 2s^2 - 2s - 8$

s^6		1	5	2	-8	0
s^5		1	1	-2	0	0
s^4		4	4	-8	0	0
s^3	16	0	8 0	0	0	0
s^2		2	-8	0	0	0
s		72	0	0	0	0
s^0		-8	0	0	0	0

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$$\text{Aux. P.L} = 4s^4 + 4s^2 - 8 = 0$$
$$= 16s^3 + 8s$$

LHP = 3
RHP = 1
IA = 2

Example $\quad P(s) = s^4 + 2s^3 + 4s^2 + 2s + K$

s^4	1	4	K
s^3	2	2	0
s^2	3	K	0
s	$(6-2K)/3$	0	0
s^0	K	0	0

$6 - 2K > 0 \Rightarrow K < 3$

$K > 0$

$\Rightarrow \boxed{0 < K < 3}$

Marginally stable at $K = 3$

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Linear Control Systems

Example $P(s) = s^4 + 2s^3 + 4s^2 + Ks + 6$

$$
\begin{array}{c|ccc}
s^4 & 1 & 4 & 6 \\
s^3 & 2 & K & 0 \\
s^2 & \frac{8-K}{2} & 6 & 0 \\
s^{\star} & \frac{\left(\frac{8K-K^2}{2} - 12\right)}{(8-K)/2} & 0 & 0 \\
s^0 & 6 & 0 & 0
\end{array}
$$

$$\star : \left.\frac{8K - K^2 - 24}{8 - K}\right\}$$

Gain can not be complex
System always has RHP poles

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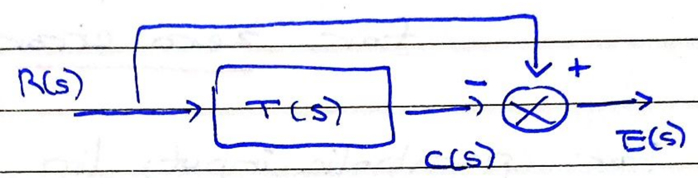
Linear Control Systems

- Step Input : Stationary
- Ramp Input : Tracking velocity
- Parabola : Tracking acceleration



$E(s) = R(s) - C(s)$

↳ $E(s)_{ss} = (1/K)\,C(s)_{ss}$ → With pure gain K, error can never be zero.

$\lim_{s \to 0} sF(s) \leadsto f(\infty)$

Example $\quad E(s) = R(s)\,[1 - T(s)]$ General Rep.

$\lim_{s \to 0} sR_s\,[1 - T(s)]$

↳ Sub values in slides

Example $\quad E(s) = \dfrac{R(s)}{1 + G(s)}$ Unity fb representation

$\lim_{s \to 0} \dfrac{sR(s)}{1 + G(s)}$

For step input ; $\dfrac{1}{1 + \lim_{s \to 0} G(s)}$ } for err to be zero, $G(s)$ should be ∞.

↳ $G(s)$ should have at least one pure integrator $(1/s)$.

For ramp input ; $\lim_{s \to 0} \left[\dfrac{1}{\cancel{s} + sG(s)} \right]$

$$\lim_{s \to 0} \frac{1}{sG(s)}$$

→ $G(s)$ should contain at least two pure integrators for zero error.

For parabolic input; $\lim_{s \to 0} \dfrac{1}{\cancel{s^2} + s^2 G(s)}$

$$\lim_{s \to 0} \frac{1}{s^2 G(s)}$$

→ $G(s)$ should contain at least three pure integrators for zero error.

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Linear Control Systems

$$e(\infty) = \lim_{s \to 0} s R(s) [1 - T(s)] \quad \text{(General Form)}$$

$$e(\infty) = \lim_{s \to 0} s \frac{R(s)}{1 + G(s)} \quad \text{(Unity feedback)}$$

Static Error Constant

Step / Position $\rightarrow$ $K_p = \lim_{s \to 0} G(s)$

Ramp / Velocity $\rightarrow$ $K_v = \lim_{s \to 0} s G(s)$

Parabola / Acceleration $\rightarrow$ $K_a = \lim_{s \to 0} s^2 G(s)$

System Type

If system is represented in unity feedback form:
number of integrators in forward path define system type

i.e. $n = 2$; Type 2 system

Example 7.6

Unity feedback $G(s) = \dfrac{K(s+5)}{s(s+6)(s+7)(s+8)}$

$K_v = 1/e(\infty)$ $\because$ $e(\infty)$ is in % ?

$K_v = \dfrac{1}{0.10} \Rightarrow \dfrac{K(5)}{(6)(7)(8)} = 10$

$K = \boxed{672}$

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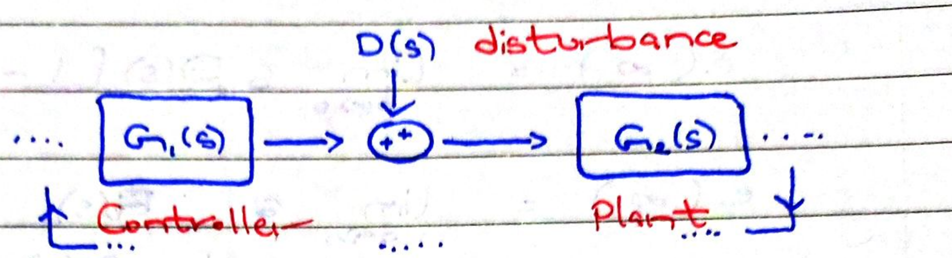
SS Error for Disturbances



$$E(s) = \underbrace{\frac{R(s)}{1+G_1(s)G_2(s)}}_{\text{due to input}} - \underbrace{\frac{D(s)G_2(s)}{1+G_1(s)G_2(s)}}_{\text{due to disturbance}}$$

$$e_\infty = \lim_{s\to 0} sE(s)$$

Example 7.7

$$D(s) = 1/s$$

$$e_d(\infty) = -1\Big/\left[\lim_{s\to 0} G_1(s) + 1/\lim_{s\to 0} G_2(s)\right]$$

$$= -\frac{1}{0+1000} = \frac{1}{1000}$$

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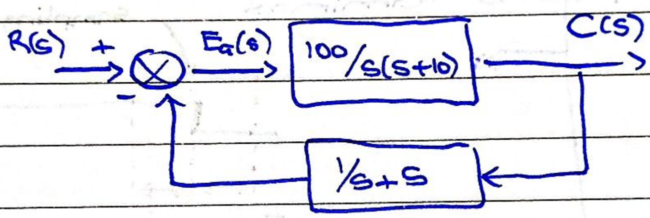
Linear Control System

Non-unity feedback

Convert given system to unity feedback
- Identify System Type
- Apply the rest of stuff :

Example 7.8



$$\text{Unity FB } G_{fb}(s) = \frac{G(s)}{1 + G(s)H(s) - G(s)}$$

$$= \frac{100/s(s+10)}{1 + \frac{100}{s(s+10)(s+5)} - \frac{100}{s(s+10)}}$$

$$= \frac{100}{s(s+10) + \frac{100}{s+5} - 100}$$

$$= \frac{100\,(s+5)}{s^3 + 15s^2 - 50s - 400}$$

→ Type 0

Apply unit step ;

$$K_p = \lim_{s \to 0} G(s) = -5/4$$

$$e_{ss} = \frac{1}{1+K_p} = -4$$

Example 7.9

$$c_{a_1}(\infty) = \lim_{s\to 0} \frac{s\,G_1(s)\,R(s)}{1+G_2(s)H_1(s)}$$

$$\longrightarrow \quad = \lim_{s\to 0} \frac{\cancel{s}\cdot 1/\cancel{s}}{1+\frac{100}{s(s+10)}\frac{1}{s+5}} \quad \Bigg\} \quad R(s) = 1/s \text{ unit step}$$

$$= \frac{s(s+10)(s+5)}{s(s+10)(s+5)+100}$$

$$= \boxed{0}$$

$$\longrightarrow \quad = \lim_{s\to 0} \frac{\cancel{s}\cdot 1/s^{\cancel{2}}}{1+\frac{100}{s(s+10)(s+5)}} = \lim_{s\to 0} \frac{1}{s+\frac{100}{(s+10)(s+5)}}$$

$$= \boxed{1/2}$$

Sensitivity

$$S_{F:P} = \lim_{\Delta P\to 0} \frac{\text{Fractional change in } F}{\text{Fractional change in } P}$$

$$= \lim_{\Delta P\to 0} \frac{\Delta F/F}{\Delta P/P}$$

$$= \lim_{\Delta P\to 0} \frac{P}{F}\frac{\Delta F}{\Delta P}$$

$$= \frac{P\,\delta F}{F\,\delta P}$$

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Linear Control Systems

Sensitivity

└ in slides

Root Locus

OLTF : $K / s(s+3)$

CLTF : $K / s^2 + 3s + K$ ← characteristic polynomial

$s^2 + 3s + K$

| $K = 1$ → $\underbrace{-0.381, -2.618}_{\text{over damped}}$

| $K = 10$ → $\underbrace{-1.5 \pm 2.78j}_{\text{underdamped}}$

| $K = 100$ → $\underbrace{-1.5 \pm 31.58j}_{\text{pole moves further on } j\omega \text{ axis ; } \omega_d \text{ changes}}$

└ with variations in K ; closed loop poles trace a path

→ $G(s) = N_G(s) / D_G(s)$

$H(s) = N_H(s) / D_H(s)$

Open Loop TF : $K G(s) H(s)$

Closed Loop TF : $\dfrac{K G(s)}{1 + K G(s) H(s)}$

$M = \dfrac{\pi \text{ zero lengths}}{\pi \text{ pole lengths}} = \prod_{i=1}^{m} |(s+z_i)| \Big/ \prod_{i=1}^{n} |(s+p_i)|$

└ system magnitude

$\theta = \sum_m \angle(s+z_i) - \sum_n \angle(s+p_i)$

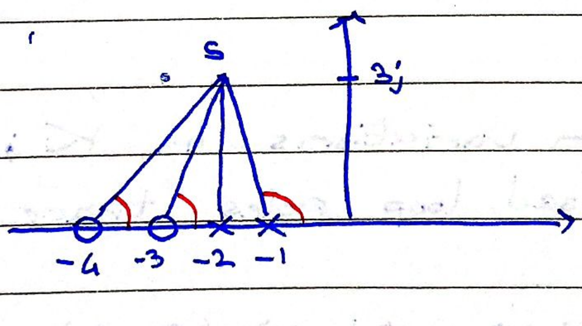
└ system angle

- $|KG(s)H(s)| = 1$ Magnitude criterion
- $\underbrace{\angle KG(s)H(s)}_{\text{OLTF}} = (2k+1)180^\circ$ Angle criterion

Example

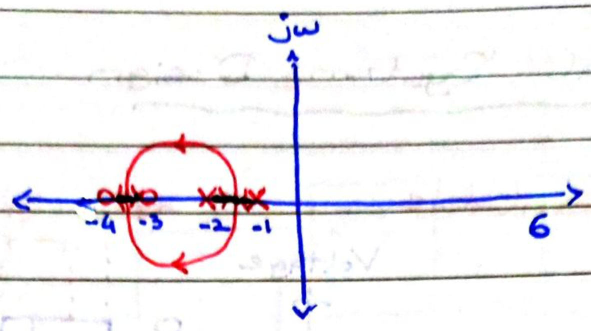
$$KG(s)\,H(s) = \frac{K(s+3)(s+4)}{(s+1)(s+2)}$$

Check $s = -2 + 3j$



$$\begin{aligned}\angle KG(s)H(s) &= \angle Z_1 + \angle Z_2 - \angle P_1 - \angle P_2 \\ &= 56.3^\circ + 71.56^\circ - 90^\circ - 108.43^\circ \\ &= -70.60^\circ\end{aligned}$$

Not an odd multiple of 180°



Sketching the root locus

1. Number of branches = Number of closed loop poles
2. Root locus is symmetric about real axis.
3. For $K > 0$, root locus exists to the left of an odd number of finite open loop poles and /or zeros.
4. Root locus begins at finite and infinite poles of $G(s)H(s)$ and ends at the finite and infinite zeros of $G(s)H(s)$.
5. Behaviour at infinity is described by asymptotes.

Real axis intercept $\quad \sigma_a = \dfrac{\sum \text{finite poles} - \sum \text{finite zeros}}{\#\text{finite poles} - \#\text{finite zeros}}$

$\theta_a = \dfrac{(2k+1)\pi}{\#\text{finite poles} - \#\text{finite zeros}}$

k is from 0 .... number of poles moving towards inf

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Linear Control Systems

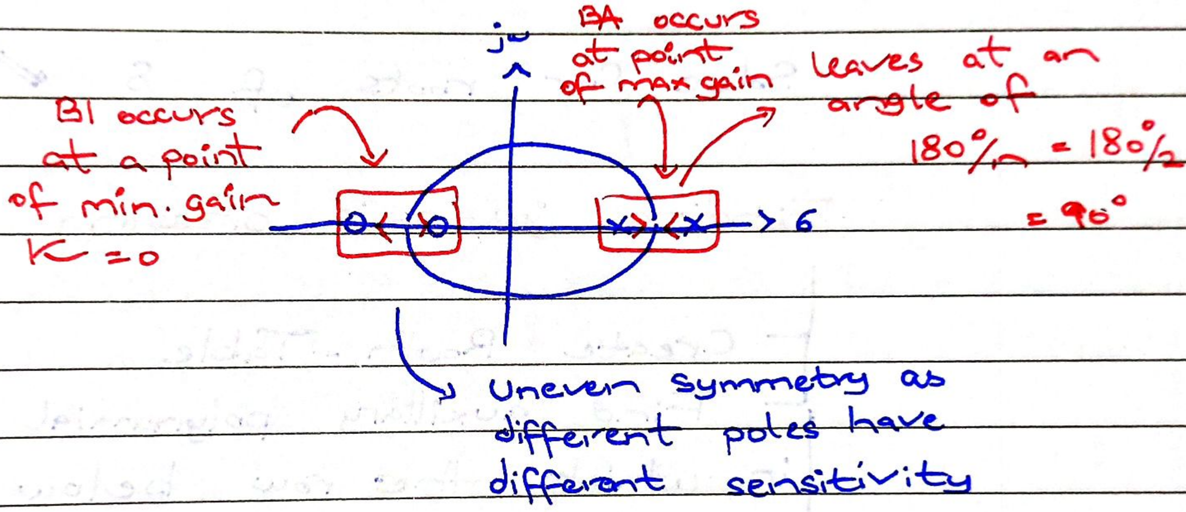
Breakaway / Breakin Points

jw - axis intercept

Angel of Arrival / Departure
ha!

↳ complex for zeros ↳ complex for poles

where poles enter / leave real axis



$$1 + KG(s)H(s) = 0 \Rightarrow K = -\frac{1}{G(s)H(s)}$$

Example:

$$KG(s)H(s) = \frac{K(s-3)(s-5)}{(s+1)(s+2)} = \frac{K(s^2-8s+15)}{s^2+3s+2}$$

Real axis: $6 = s \Rightarrow \frac{K(6^2-86+15)}{6^2+36+2} = -1$

$$K = \frac{-(6^2+36+2)}{6^2-86+15}$$

$\frac{dK}{ds} = \sim = 0$

Choose BA & BI points conforming to the sketch.

Transition Method

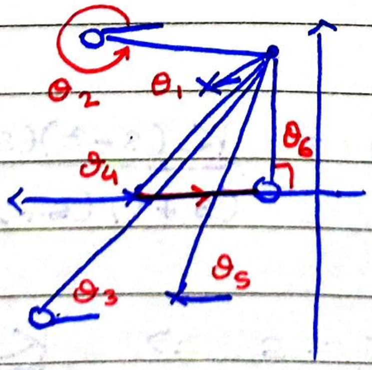
$$\sum_{1}^{m} 1/\delta + z_i = \sum_{1}^{n} 1/\delta + p_i$$

Solve for roots of δ ✓

Finding jw-axis crossing

- Create Routh-Table ↗ $K > 0$
- Find auxillary polynomial for K sub in which the row below it is ~ completely zero.
- Find roots of aux. polynomial

Angle of Departure / Arrival



$$\sum_{i} \angle z_i - \sum_{i} \angle p_i = (2k+1)\,180°$$

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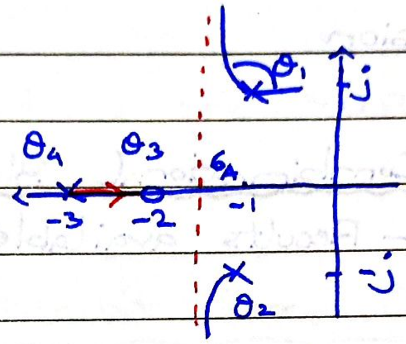
$$-\theta_1 - \theta_5 - \theta_4 + \theta_2 + \theta_3 + \theta_6 = (2k+1)180^\circ$$

Example

$$KG(s)H(s) = \frac{K(s+2)}{(s+3)(s^2+2s+2)}$$



$$\sigma_A = \frac{-3-1-j-1+j+2}{3-1} = -3/2$$

$$\theta = \frac{(2k+1)\pi}{3-1} \quad \Bigg] \quad \begin{aligned} \theta_1 &= \pi/2 \\ \theta_2 &= 3\pi/2 \end{aligned}$$

Keep θ_1 unknown and join all poles / zeros to it

$$\begin{aligned} -\theta_1 &= \theta_2 + \theta_4 - \theta_3 + 180^\circ \\ &= 90^\circ + 26.56^\circ - 45^\circ + 180^\circ \\ &= +251.56^\circ \end{aligned}$$

$$\theta_1 = -251.56^\circ \Rightarrow 108.4^\circ$$

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Linear Control Systems

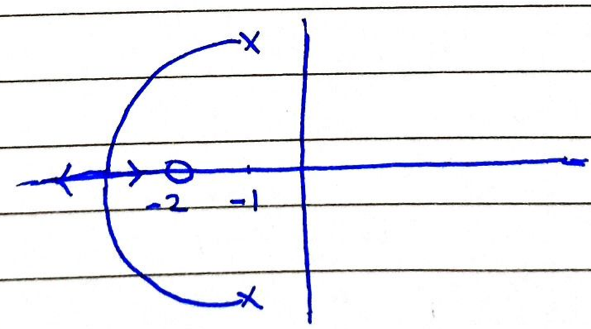
Variation of Parameters other than K

$\Rightarrow \dfrac{10}{(s+2)(s+p_1)}$ in CL : $\dfrac{10}{s^2+2s+p_1 s+2p_1+10}$

Convert to $\dfrac{K\,G(s)}{1+KG(s)H(s)}$ form

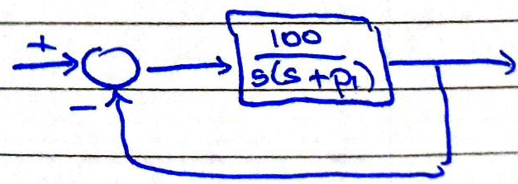
$\Rightarrow \dfrac{10}{(s^2+2s+10)+p_1(2+s)}$ $\Rightarrow \dfrac{10/s^2+2s+10}{1+\dfrac{p_1(s+2)}{s^2+2s+10}}$

Compare: $KG(s)H(s) = \dfrac{p_1(s+2)}{s^2+2s+10}$



Example 8.7

$\Rightarrow G(s) = \dfrac{100}{s(s+p_1)}$



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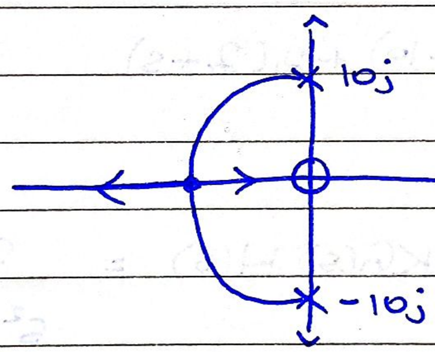
$$\Rightarrow \frac{100}{s^2 + P_1 s + 100} \Rightarrow \frac{100}{(s^2+100) + P_1 s}$$

$$\Rightarrow \frac{100}{(s^2+100)\left[1 + P_1 s / s^2+100\right]} \Rightarrow \frac{100/(s^2+100)}{1 + P_1 s / s^2+100}$$

OLTF : Comparing $\rightarrow$ $KG(s)H(s)$

$$\Rightarrow \frac{P_1 s}{s^2+100}$$



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Linear Control Systems

Design via Root Locus

→ Root Sensitivity

Fractional changes in closed loop pole to fractional change in parameter.

$$S_{s:K} = \frac{K}{s}\frac{\delta s}{\delta K}$$

$$\Delta s = s(S_{s:K})\frac{\Delta K}{K}$$

Example

$$KG(s)H(s) = \frac{K}{s(s+10)}$$

$$\frac{KG(s)}{1+KG(s)H(s)} = \frac{K}{s^2+10s+K}$$

→ $s^2 + 10s + K = 0$

$$2s\frac{\delta s}{\delta K} + 10\frac{\delta s}{\delta K} + 1 = 0$$

$$\frac{\delta s}{\delta K} = \frac{-1}{2s+10}$$

$$\boxed{S_{s:K} = \frac{K}{s}\left(\frac{-1}{2s+10}\right)}$$

Magnitude criterion : $K = \frac{1}{|G(s)H(s)|}$

$\Rightarrow \quad K = |s(s+10)|$

$K|_{s=-9.47} = 5 \quad ; \quad K|_{s=-5+5j} = 50$

$S_{s:K}|_{s=-9.47} = -0.059 \quad ; \quad S_{s:K}|_{s=-5+5j} = (1/\sqrt{2}) \angle -45°$

$\Delta s|_{s=-9.47} = 0.056 \quad ; \quad \Delta s|_{s=} \; j0.5$

Change in s for ΔK (10 % change in gain)

→ Design

└ to improve
- Transient response PD
- Steady state error PI
- Both of above PID

$$G(s) = \frac{1}{(s+1)(s+2)(s+3)}$$

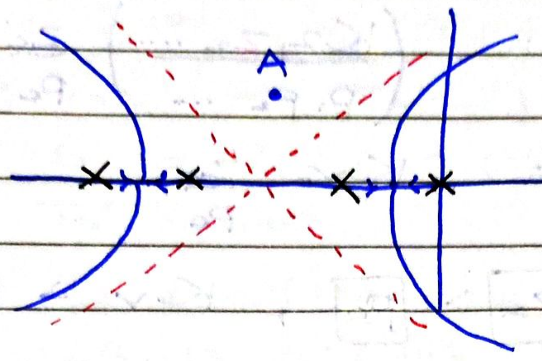
Type - 0 system (Finite error for step input)

└ for zero steady state error, add a pure integrator

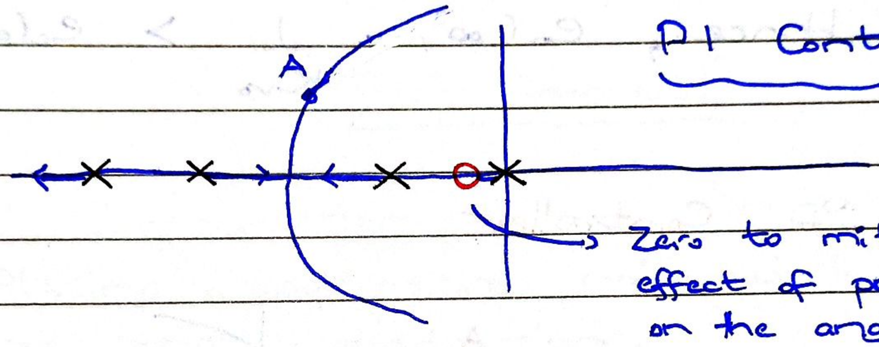
But design point A previously on the root locus can no longer be attained through just gain K

Addition of zeros ✓

Rough Sketch



PI Controller

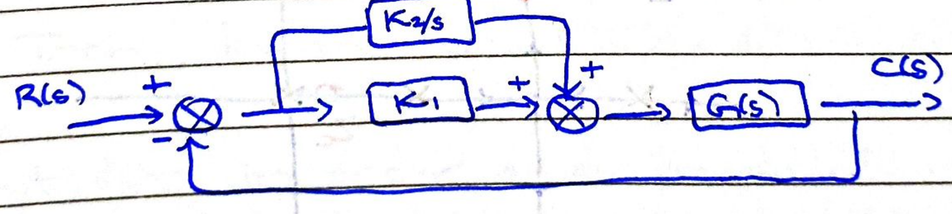


Zero to mitigate the effect of pole at origin on the angle criterion

PI Controller (Needs active networks)

$$G_c(s) = K_1 + \frac{K_2}{s} = \frac{K_1(s + K_2/K_1)}{s}$$

Pole at origin with zero its in near vicinity



↓ Practical Version

Lag Compensation (For passive components you can't insert pz at origin)

$$K_v = \lim_{s \to 0} sG(s)$$

$$K_{v_0} \underset{\text{Uncompensated}}{} = \frac{K z_1 z_2 \ldots}{p_1 p_2 \ldots}$$

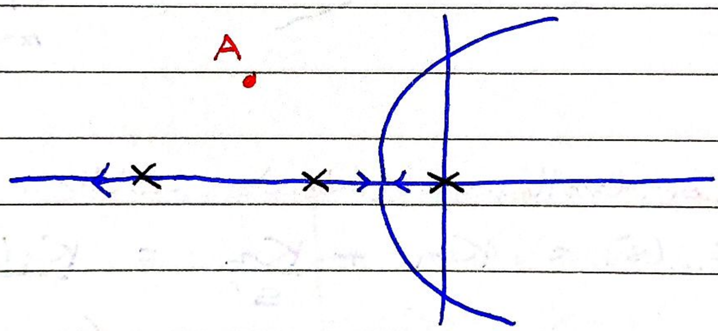
$$K_v = \left(\frac{K z_1 z_2 \ldots}{p_1 p_2 \ldots} \right) \frac{z_c}{p_c}$$

$$K_v = K_{v_0} \frac{z_c}{p_c}$$

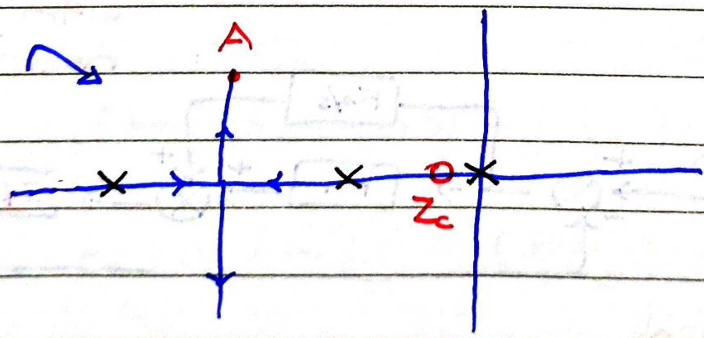
When $\boxed{z_c} > \boxed{p_c}$, $K_v > K_{v_0}$

Hence, $e_0(\infty) = \frac{1}{K_{v_0}} > e_c(\infty) = \frac{1}{K_v}$

PD Controller



$(s+z_c)$



Example

$KG(s)H(s) = \text{OLTF}$

ate

$16\%\ OS \rightarrow \zeta = 0.504$;

$T_s' = \frac{T_s}{3}$

$T_s = 3.302 \rightarrow T_s' = 1.107$

$\sigma_d = \frac{4}{T_s'} = \frac{4}{1.107} = 3.613$

$\omega_d = \sigma_d \tan(180 - \cos^{-1}(0.504)) = 6.193$

$s = -3.613 \pm j6.193$

Rest from slide

Lead Compensation

Placing a pole-zero pair whilst matching pz angle contribution.

$$\theta_c = \underbrace{\theta_2}_{\text{of zero}} - \underbrace{\theta_1}_{\text{of pole}}$$

PD Controller

- First design for transient response
- Then design for steady state error

If not done in order ; design point may vary → steady state error might increase when adding zero of PD

Lead → Lag
Design

date

PID : $\dfrac{(s+z_c')(s+z_c)}{s}$ Active

Lag-Lead : $\dfrac{(s+z_c')(s+z_c)}{(s+p_c')(s+p_c)}$ Passive

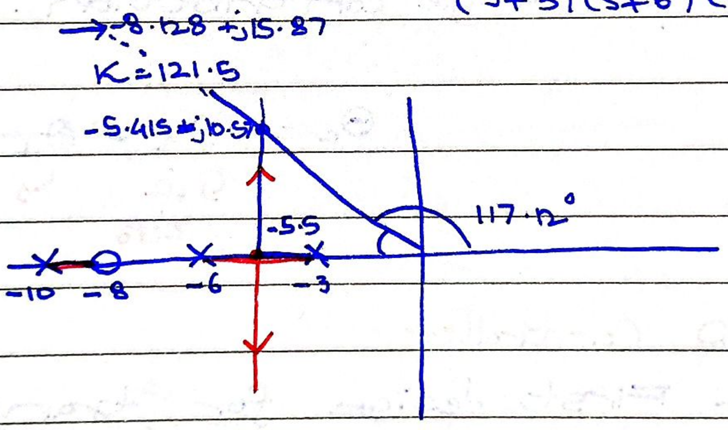
$$G_c(s) = K_1 + \frac{K_2}{s} + K_3 s$$

$$= \frac{K_1 s + K_2 + K_3 s^2}{s}$$

$$= \frac{K_3\left(s^2 + K_1/K_3\, s + K_2/K_3\right)}{s}$$

Example $KG(s)H(s) = \dfrac{K(s+8)}{(s+3)(s+6)(s+10)}$

$\zeta = 0.456$



$\rightarrow T_p = \dfrac{\pi}{\omega_d} = 0.297$ sec

$T_p' = 0.198$ s

$\omega_d' = 15.87 \rightarrow \omega_d = \sigma_d \tan(62.88°)$

$\sigma_d' = 8.128$

$s' = \sigma_d' \pm j\omega_d' = 8.128 \pm j15.87$

date

Angle Criterion => -197.94°

$$-197.94^\circ + \theta_c = -180^\circ$$

$$\theta_c = 17.94^\circ$$

$$\tan^{-1}(18.37^\circ) = \frac{15.87}{Z_c - 8.13} \Rightarrow Z_c = 55.92$$

Rest from slide

date

12/06/23

Linear Control Systems

From SS → TF

$$\dot{x} = Ax + Bu$$
$$y = Cx + Du$$

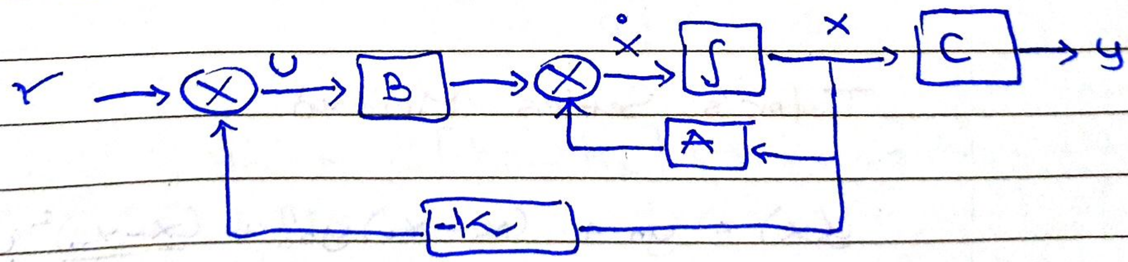
Laplace → $sX(s) = AX(s) + BU(s)$
$Y(s) = CX(s) + DU(s)$

$$T(s) = \frac{Y(s)}{U(s)} = C(sI - A)^{-1}B + D$$

Rest from book

Design via State Space {Do from book & slides}

$$\dot{x} = Ax + Bu$$
$$y = Cx$$



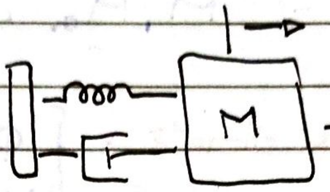
$$u = r - Kx$$

Open Loop } characteristic Polynomials : $\det(sI - A)$
Closed Loop } : $\det(sI - (A - BK))$

18/05/23

Linear Control Systems

Frequency Domain Analysis : Bode Plots

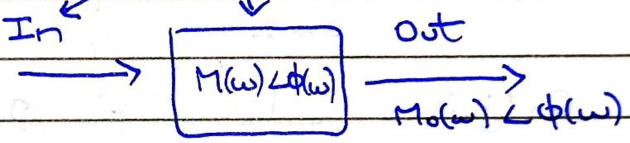


$x(t) = M_o \cos(\omega + \phi_o)$

$f(t) = M_i \cos(\omega t + \phi_i)$

$$M_o(\omega) \angle \phi_o(\omega) = M_i(\omega)\, M(\omega) \angle [\phi_i(\omega) + \phi(\omega)]$$

Magnitudes multiply
Phases add up



System with Sinusoidal Input

$$R(s) = \frac{As + B\omega}{s^2 + \omega^2}$$

$$r(t) = A\cos(\omega t) + B\sin(\omega t)$$
$$= \sqrt{A^2 + B^2}\cos[\omega t - \tan^{-1}(B/A)]$$

$$C(s) = R(s)\,G(s)$$
$$= \frac{As + B\omega}{s^2 + \omega^2} G(s)$$
$$= \text{PF Expansion} \Rightarrow \frac{As + B\omega}{(s+j\omega)(s-j\omega)} G(s)$$
$$= \frac{K_1}{s+j\omega} + \frac{K_2}{s-j\omega}$$

date

$$K_1 = \frac{M_1 M_2}{2} e^{-j(\phi_i + \phi_G)}$$

$$K_2 = \frac{M_1 M_2}{2} e^{j(\phi_i + \phi_G)}$$

$$c(t) = M_1 M_G \cos(\omega t + \phi_i + \phi_G)$$

Proves that : $\begin{Bmatrix} \phi_o = \phi_i + \phi \\ M_o(\omega) = M_i(\omega) M(\omega) \end{Bmatrix}$

Example :

$$G(s) = \frac{1}{s+2}$$

$$G(j\omega) = \frac{1}{j\omega + 2} = \frac{2 - j\omega}{\omega^2 + 4}$$

$$|G(j\omega)| = \frac{1}{\sqrt{\omega^2 + 4}}$$

$$\phi_G = -\tan^{-1}(\omega/2)$$

↳ - due to denominator

Critical frequency : Location of poles

Asymptotic Approximation :

$$G(s) = \frac{1}{s+a} = \frac{1}{a(\frac{s}{a} + 1)}$$

$$G(j\omega) = \frac{1}{j\omega + a}$$

/date

$\boxed{\omega \ll a}$

$$G(j\omega) = \frac{1}{a} \angle 0^\circ$$

$$20 \log (1/a) = -20 \log a = M$$
$$0^\circ = \theta$$

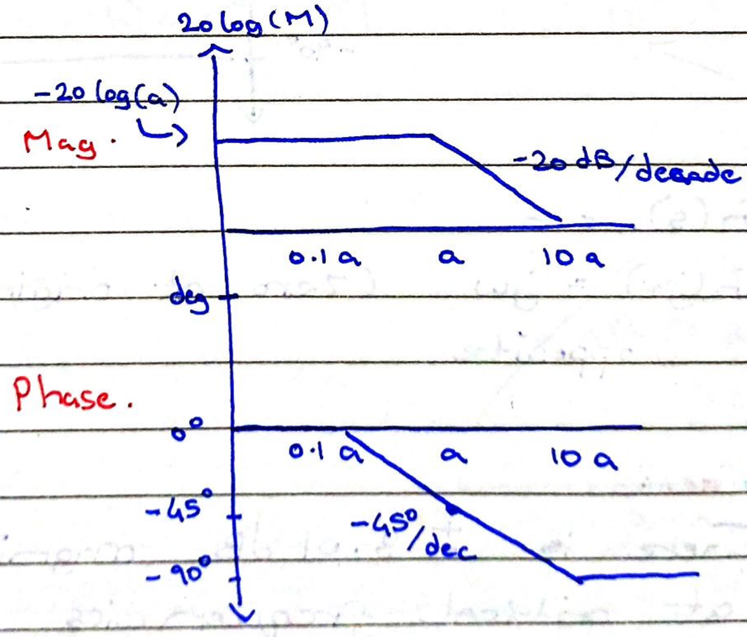
$\boxed{\omega \gg a}$

$$G(j\omega) = \frac{1}{\omega} \angle -90^\circ$$

$$20 \log (1/\omega) = -20 \log(\omega) = M$$
$$-90^\circ = \theta$$

$\boxed{\text{At } \omega = a}$

$$M = 20 \log (a)$$
$$\theta = -45^\circ$$



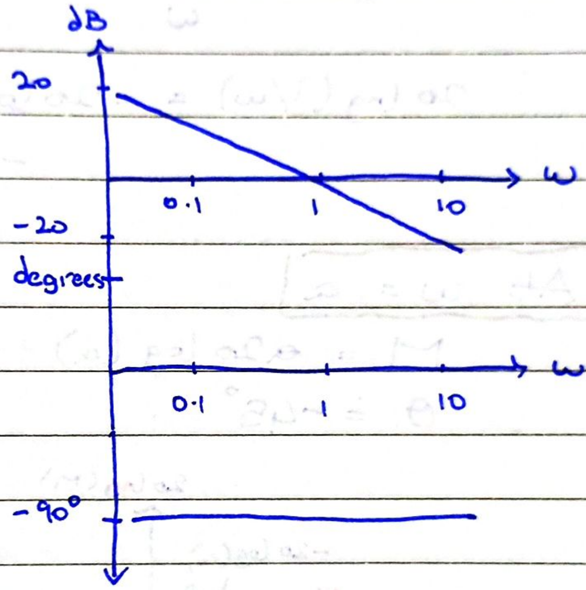
$G(j\omega) = j\omega + a$ (A zero) will be opposite ay/date

$G(s) = \frac{1}{s}$ (Pole at origin)

$G(j\omega) = \frac{1}{j\omega}$

$= \frac{1}{\omega} \angle -90°$

$M = -20\log(\omega) \angle -90°$



$G(s) = s$

$G(j\omega) = j\omega$ (zero at origin) will be opposite

Note:

There is ± 3.01 dB magnitude disparity (b/w approx and true) at critical frequencies for zeros and pole plots respectively

There is disparity of 5.72 degrees max at 1 decade above and below critical frequency.

EXCELLENT

$$G(s) = s^2 + 2\zeta\omega_n s + \omega_n^2$$

Plot for 2nd order system with no zeros

Critical frequency is ω_n

$$G(s) = \omega_n^2\left(\frac{s^2}{\omega_n^2} + 2\zeta\frac{s}{\omega_n} + 1\right)$$

$\boxed{\omega \ll \omega_n}$ $\boxed{\omega \gg \omega_n}$

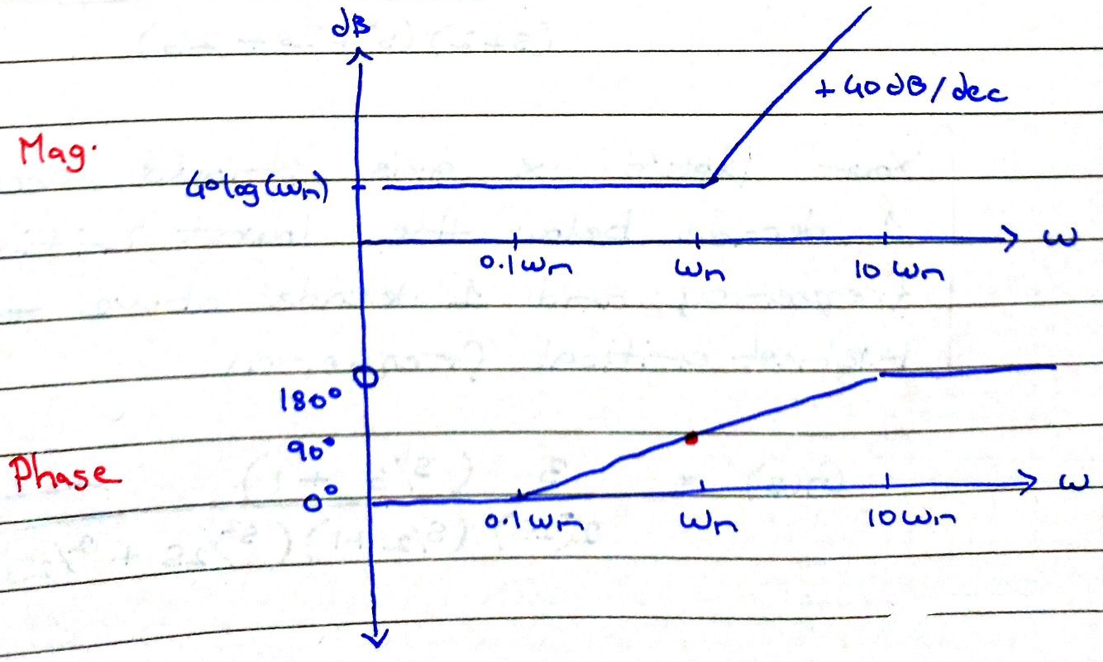
$$\Rightarrow G(j\omega) = -\omega^2 + 2j\zeta\omega_n\omega + \omega_n^2$$

$$G(j\omega) = 20\log(\omega_n^2)\angle 0^\circ$$
$$= 40\log(\omega_n)\angle 0^\circ$$

$$G(j\omega) = -\omega^2 = \omega^2\angle 180^\circ$$
$$= 40\log(\omega)\angle 180^\circ$$

$\boxed{\omega = \omega_n}$

$\theta = 90^\circ$



For a 2^{nd} order system with poles only ;

$$G(s) = \frac{1}{s^2 + 2\xi\omega_n s + \omega_n^2}$$

Reverse / opposite of $G(s)$ with 2^{nd} order zeros

Normalized Plots

make start at 0 dBs

as $20 \log(\omega_n^2 / \omega_n^2) = 0$ dB

In 2^{nd} order systems ; disparity is dependent on the value of ξ as at $[\omega = \omega_n]$

$$G(j\omega) = 2j\xi\omega_n^2$$

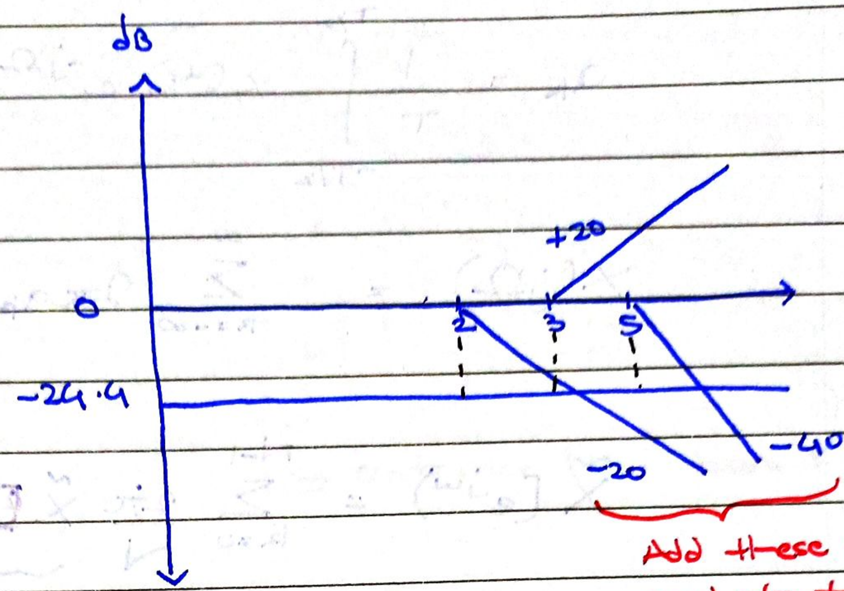
Example :

$$G(s) = \frac{(s+3)}{(s+2)(s^2+2s+25)}$$

Your plot's x axis should accomodate 1 decade below the lowest critical frequency and 1 decade above the highest critical frequency

$$G(s) = \frac{3 \, (s/3 + 1)}{2(25)\,(s/2 + 1)(s^2/25 + 2/25\, s + 1)}$$

$20 \log (3/(2(25))) = -24.4 \text{ dB}$



Add these up and start / place them on -24.4 dB line

Stability : Gain and Phase Margins

GM = ∞ means that system is always stable
(180°) wont ever be reached on the phase plot
[you do 0 dB − (Value at 180° Phase ω)]

PM = angle where magnitude plot crosses 0 dB

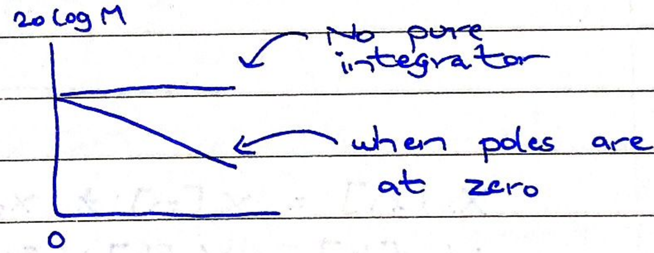
GM of 0 dB implies marginal stability
PM is also 0 .

Static error constants

LCS

Type {0} K_p

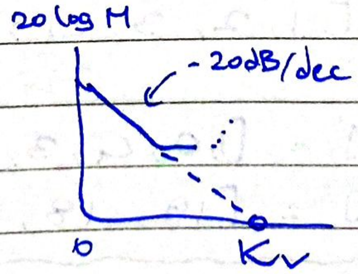
$$G(s) = K \frac{\prod_{i=1}^{n} (s+z_i)}{\prod_{i=1}^{m} (s+p_i)}$$



Initial Value : $20 \log_{10} \left(\frac{\prod_{i=1}^{n} z_i}{\prod_{i=1}^{m} p_i} \right)$

Type {1} K_v

$$G(s) = K \frac{\prod_{i=1}^{n} (s+z_i)}{s \prod_{i=1}^{m} (s+p_i)}$$



late

19 / 05 / 23

Linear Control Systems

⇒ Speed of Time Response

$20\log(M) = 3\,dB \Rightarrow M = \frac{1}{\sqrt{2}} \Big|_{\omega = \omega_{BW}}$

ω_{BW} frequency at which magnitude curve is 3dB down from its value at zero freq.

$$\omega_{BW} = \frac{\pi}{T_p\sqrt{1-\xi^2}}\left[\sqrt{(1-2\xi^2)+\sqrt{4\xi^4-4\xi^2+2}}\right]$$

$$\omega_{BW} = \frac{4}{T_s\,\xi}\left[\quad // \quad // \quad // \quad\right]$$

For phase margin:

$|G(j\omega)| = 1$

$$G(s) = \frac{\omega_n^2}{s(s+2\xi\omega_n)}$$

$$\phi_m = 90 - \tan^{-1}\left(\frac{\sqrt{-2\xi^2+\sqrt{1+4\xi^4}}}{2\xi}\right)$$

Continued ☆

Initial Value : $20\log\left(\frac{\prod_{i=1}^{n} z_i}{\omega_0 \prod_{i=1}^{m} p_i}\right)$

$$w = K \frac{\prod_{i=1}^{n} z_i}{\prod_{i=1}^{n} p_i}$$

Type {2} K_a

Initial Value : $20 \log \left(K \frac{\prod_{i=1}^{n} z_i}{\omega_b^2 \prod_{i=1}^{n} p_i} \right)$

$$w = \sqrt{K \frac{\prod_{i=1}^{n} z_i}{\prod_{i=1}^{n} p_i}}$$

$$= \sqrt{K_a}$$