Continuous Random Variable

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Continuous Random Variable

A continuous random variable can assume any value along a given interval of a number line.

Examples:

- Depth
- Volume
- Time
- Weight
- Length

Probability Density function

A function f(x) is called probability density function, abbreviated to p.d.f, or simply density function of the r.v X.

A p.d.f has the following properties

- f(x)$\geq o, \ for\ all\ x$
- $\int_{-\infty}^{\infty} f(x).d(x) = 1$

Distribution Function

F(x) is continuous and is differentiable everywhere except at isolated points in the given range.

$$F(x) = \int_{-\infty}^{x} f(x).\,d(x)$$

F(x) is used for calculating probabilities.

The probability that X takes on a value in the interval [c, d], c<d is given by

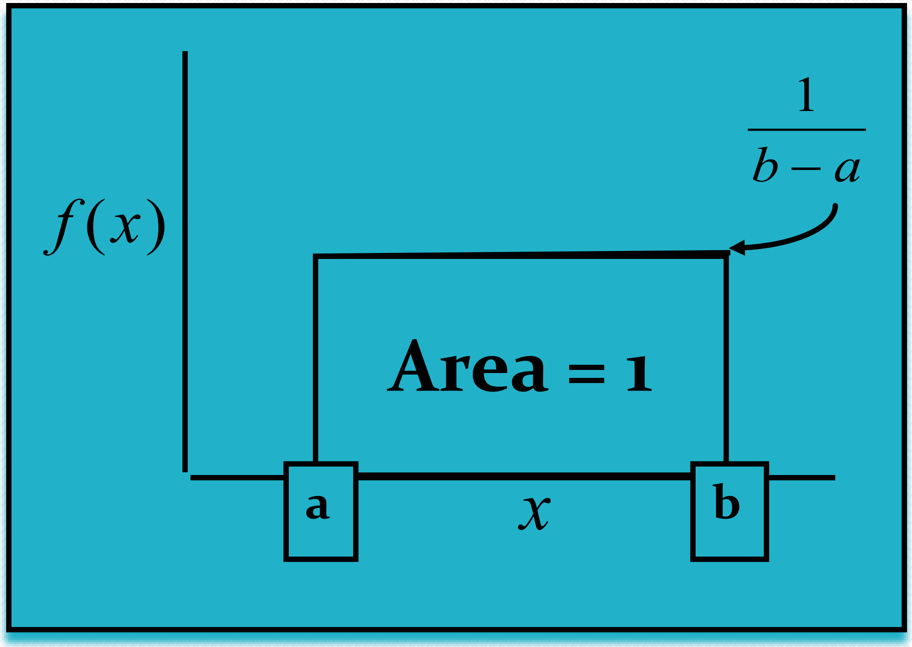
P(c<x ≤ d)=F(d)-F(c)

$= \int_{-\infty}^{d} f(x).\,d(x)$- $\int_{-\infty}^{c} f(x).\,d(x)$= $\int_{c}^{d} f(x).\,d(x)$

Example: Uniform Distribution

The uniform distribution is a continuous distribution in which the same height, of f(X), is obtained over a range of values.

$$f(x) = \begin{cases} \dfrac{1}{b-a} & \text{for} \quad a \le x \le b \\ 0 & \text{for} \quad \text{all other values} \end{cases}$$



Uniform Distribution

The cumulative distribution function of the uniform random variable X is given by

For any x such that $-\infty < x \leq 0$

$$F(x) = \int_{-\infty}^{x} 0 d(x) = 0$$

For any x such that $a \leq x \leq b$

$$F(x) = \begin{cases} 0 & for & x < 0 \\ \frac{x-a}{b-a} & for & a \leq x \leq b \\ 1 & for & x > b \end{cases}$$

$$F(x) = \int_{-\infty}^{a} 0.d(x) + \int_{a}^{x} \frac{1}{b-a} d(x) = \frac{x-a}{b-a}$$

For $x > b$

$$F(x) = \int_{-\infty}^{a} 0 d(x) + \int_{a}^{b} \frac{1}{b-a} d(x) + \int_{b}^{x} 0 d(x) = 1$$

Question: Uniform Distribution

The arrival time of a student in a class is uniformly distributed with first 10 minutes. Find the probabilities that

- the student arrive within first 5 minutes.
- the student arrive after 6 minutes.
- the student arrive between 2 to 7 minutes.
- the student arrive after 15 minutes.

Solution

The p.d.f of X is

$$f(x) = \begin{cases} \dfrac{1}{10} & for \quad 0 \le x \le 10 \\ 0 & for \quad \text{all other values} \end{cases}$$

The Cumulative distribution function of X is

$$F(x) = \begin{cases} 0 & for \quad x < 0 \\ \dfrac{x}{10} & for \quad 0 \le x \le 10 \\ 1 & for \quad \mathrm{x} > 10 \end{cases}$$

Solution

$$F(x) = \begin{cases} 0 & for \quad x < 0 \\ \dfrac{x}{10} & for \quad 0 \le x \le 10 \\ 1 & for \quad \mathrm{x} > 10 \end{cases}$$

- Find the probability that the student arrive within first 5 minutes.

- Find the probability that the student arrive after 6 minutes.

$$F(x)=\begin{cases} 0 & for \quad x<0 \\ \dfrac{x}{10} & for \quad 0\le x\le 10 \\ 1 & for \quad \mathrm{x}>10 \end{cases}$$

Solution

- Find the probability that the student arrive between 2 to 7 minutes.

- Find the probability that the student arrive after 15 minutes.

Question

A continuous random variable X has the following function

$$f(x)=\begin{cases} x & 0 < x \leq 1 \\ 2-x & 1 < x \leq 2 \\ 0 & elsewhere \end{cases}$$

- Check that f(x) is probability density function.
- Obtain the expression of distribution function.
- Find the following probabilities

P(x>$\frac{1}{2}$), P($\frac{1}{3} \leq x \leq \frac{1}{2}$), $P(\frac{2}{3} \leq x \leq \frac{5}{2}$), P(x=$\frac{2}{5}$), P(x<$\frac{7}{2}$)

Solution

- Check that f(x) is probability density function.

For p.d.f we verify following condition

$\int_{-\infty}^{\infty} f(x).d(x) = 1$

$$\int_{-\infty}^{0} 0.d(x) + \int_{0}^{1} x.d(x) + \int_{1}^{2} (2-x).d(x) + \int_{2}^{\infty} 0.d(x) = 1$$

- Obtain the expression of distribution function.

Solution

- Obtain the expression of distribution function.

$$F(x)=\begin{cases} \int_{-\infty}^{x} 0.\,d(x) & x \le 0 \\ \int_{-\infty}^{0} 0.\,d(x) + \int_{0}^{x} x.\,d(x) & 0 \le x < 1 \\ \int_{-\infty}^{0} 0.\,d(x) + \int_{0}^{1} x.\,d(x) + \int_{1}^{x} (2-x)d(x) & 1 \le x < 2 \\ \int_{-\infty}^{0} 0.\,d(x) + \int_{0}^{1} x.\,d(x) + \int_{1}^{2}(2-x).\,d(x) + \int_{2}^{x} 0.\,d(x) & x \ge 2 \end{cases}$$

Solution

- Find the following probabilities

$P(x>\frac{1}{2})=1-F(\frac{1}{2})$

$P(\frac{1}{3} \leq x \leq \frac{1}{2}) = F(\frac{1}{2}) - F(\frac{1}{3})$

$P\left(\frac{2}{3} \leq x \leq \frac{5}{2}\right) = F(\frac{5}{2})-F(\frac{2}{3})$

$P(x=\frac{2}{5})=F(\frac{2}{5})-F(\frac{2}{5})=0$

$P(x<\frac{7}{2})=F(\frac{7}{2})$

Question

Suppose that the battery failure time, measured in hours, has a probability density function, given by

$$\boldsymbol{f(x)} = \begin{cases} \dfrac{\mathbf{2}}{(\boldsymbol{x}+\mathbf{1})^{3}} & \boldsymbol{for\ x \geq 0} \\ \mathbf{0} & \boldsymbol{elsewhere} \end{cases}$$

- Find the probabilities of $\boldsymbol{P(0 \leq x \leq 10)}$, p(x<30), and p(x=25)?
- Develop distributions function of x?