ELECTRIC FIELDS DUE TO CONTINUOUS CHARGE DISTRIBUTIONS - SURFACE CHARGE

Introduction

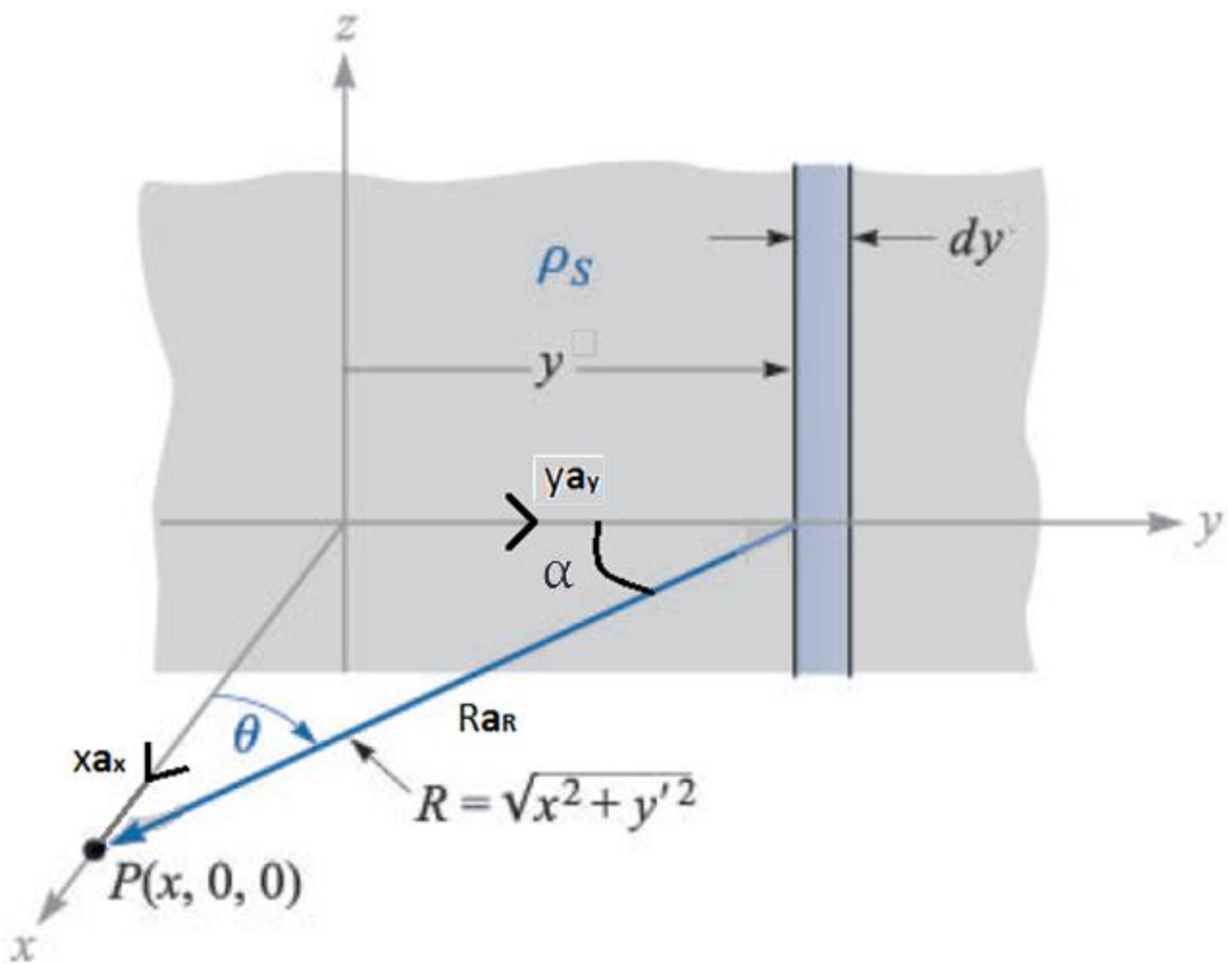
- Another basic charge configuration is the infinite sheet of charge having a uniform density of $\rho_s \mathrm{C/m^2}$
- Such a charge distribution may often be used to approximate the charge found on the plates of a parallel-plate capacitor
- For ease of derivation, the sheet of charge is considered to be infinite
- This is a good approximation since the distances involved in the measurement of fields are generally small compared to the dimensions of the sheet of charge

Sheet of Charge

- Consider an infinite sheet of charge with uniform charge density ρ_s in the *yz*-plane

- We consider a small portion of the sheet having height *dz* and width *dy*

- Let the point of observation be at P(x,0,0)



Sheet of Charge

- The charge associated with an elemental area *dS* is:

$$dQ = \rho_s dS \quad \text{where} \qquad dS = dydz$$

- The sheet of charge may be assumed to consist of infinite line charges that extend from $-\infty \; to + \infty$ along the z-axis

- We have for an infinite line charge:

$$\boldsymbol{E} = \frac{\rho_L \boldsymbol{a_R}}{2\pi\epsilon_o R}$$

- Therefore, the differential intensity due to the line charge is:

$$d\boldsymbol{E} = \frac{\rho_S dy \boldsymbol{a_R}}{2\pi\epsilon_o R}; \qquad \text{where } \rho_L = \rho_S dy$$

Sheet of Charge

- It may be observed from the figure that vector **R** is perpendicular to the line charge

- Therefore: $y\boldsymbol{a}_y + R\boldsymbol{a}_R = x\boldsymbol{a}_x$ or $R\boldsymbol{a}_R = x\boldsymbol{a}_x - y\boldsymbol{a}_y$

and

$$\boldsymbol{a}_R = \frac{x\boldsymbol{a}_x - y\boldsymbol{a}_y}{\sqrt{x^2 + y^2}}$$

- Substituting values in the equation for *d**E***, we get:

$$d\boldsymbol{E} = \frac{\rho_S dy(x\boldsymbol{a}_x - y\boldsymbol{a}_y)}{2\pi\epsilon_o(x^2 + y^2)}$$

Sheet of Charge

- The intensity due to all the line charges will be their vector sum

- When we take the vector sum, the y-component will cancel due to symmetry

- For every line charge along the positive y-axis, there is a line charge at the same distance along the negative y-axis

- So in the summation, the y-axis is ignored and limits are taken along z-axis from $-\infty \; to + \infty$

Sheet of Charge

- Therefore, we have the total electric field as:

$$\boldsymbol{E} = \int_{-\infty}^{\infty} \frac{\rho_S dyx\boldsymbol{a_x}}{2\pi\epsilon_o(x^2 + y^2)}$$

- We use change of variables to solve the above equation

$$tan\alpha = \frac{x}{y}$$

$$\Rightarrow y = xcot\alpha \qquad and \qquad dy = -xcosec^2\alpha d\alpha$$

$$when\ y = -\infty \qquad \Rightarrow cot\alpha = -\infty \quad so \quad \alpha = \pi$$

$$when\ y = \infty \qquad \Rightarrow cot\alpha = \infty, \qquad so \ \ \alpha = 0$$

Sheet of Charge

- So, we get:

$$\boldsymbol{E} = -\int_{\pi}^{0} \frac{\rho_S x^2 cosec^2 \alpha d\alpha \boldsymbol{a_x}}{2\pi\epsilon_o (1 + cot^2 \alpha) x^2}$$

Or

$$\boldsymbol{E} = \int_{0}^{\pi} \frac{\rho_S d\alpha \boldsymbol{a_x}}{2\pi\epsilon_o}$$

$$\boldsymbol{E} = \frac{\rho_S \boldsymbol{a_x}}{2\epsilon_o}$$

- We see that for an infinite sheet of charge, the intensity does not depend upon any coordinate

- So if we take P anywhere, the intensity will remain the same but the direction will change

Sheet of Charge

- If the point of observation is located at the back of the sheet, then we have:

$$\boldsymbol{E} = -\frac{\rho_S \boldsymbol{a_x}}{2\epsilon_o}$$

Or generally:

$$\boldsymbol{E} = \frac{\rho_S \boldsymbol{a_n}}{2\epsilon_o}$$

- Where $\mathbf{a_n}$ is a vector normal to the sheet

Problem-1

- A circular disk of radius a is uniformly charged with ρ_s C/m^2. If the disk lies on the z = 0 plane with its axis along the z-axis,
- (a) Show that at point (0, 0, h)

$$\mathbf{E} = \frac{\rho_s}{2\varepsilon_o}\left\{1 - \frac{h}{[h^2 + a^2]^{1/2}}\right\}\mathbf{a}_z$$

- (b) From this, derive the **E** field due to an infinite sheet of charge on the z = 0 plane