Engineering Mechanics

Dr. Hina Gohar Ali

Hina.gohar@seecs.edu.pk

Office : IAEC building

Office Hours: Appointment through emails

Research Interest: Photovoltaic systems, Power & Control, Sliding mode control (SMC)

Contents (Section 5.1, 5.2)

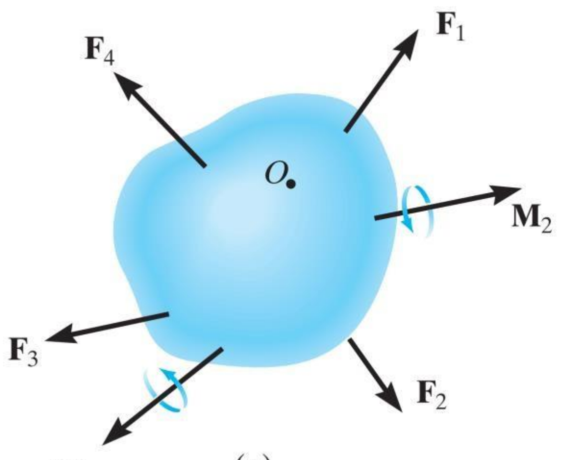
- Equilibrium of a Rigid Body
- Free Body Diagram
- Equations of Equilibrium

5.1 Conditions for Rigid-Body Equilibrium

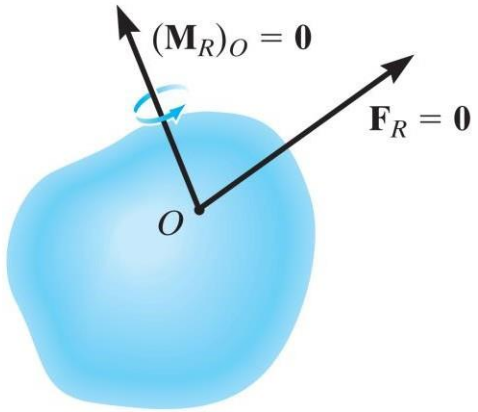
$$\mathbf{F}_R = \Sigma \mathbf{F} = \mathbf{0}$$

$$(\mathbf{M}_R)_O = \Sigma \mathbf{M}_O = \mathbf{0}$$

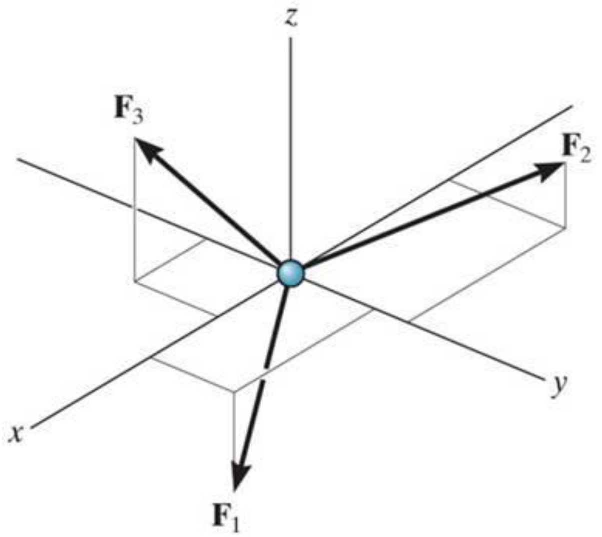
Necessary



(a)



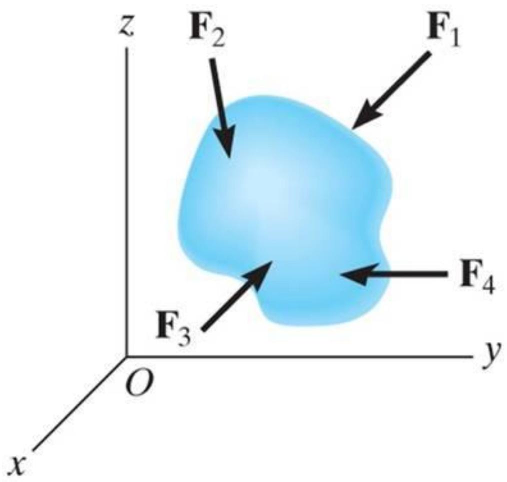
Conditions for Rigid-Body Equilibrium



Forces on a particle

In Chapter 3 we only considered forces acting on a particle (concurrent forces). In this case rotation is not a concern, so equilibrium could be satisfied by:

$$\boxed{\sum \boldsymbol{F} = 0 \text{ (no translation)}}$$



Forces on a rigid body

We will now consider cases where forces are not concurrent so we are also concerned that the rigid body does not rotate. In order for a rigid body to be in equilibrium, the net force as well as the net moment about any arbitrary point O must be equal to zero.

$$\sum \boldsymbol{F} = 0 \text{ (no translation)}$$

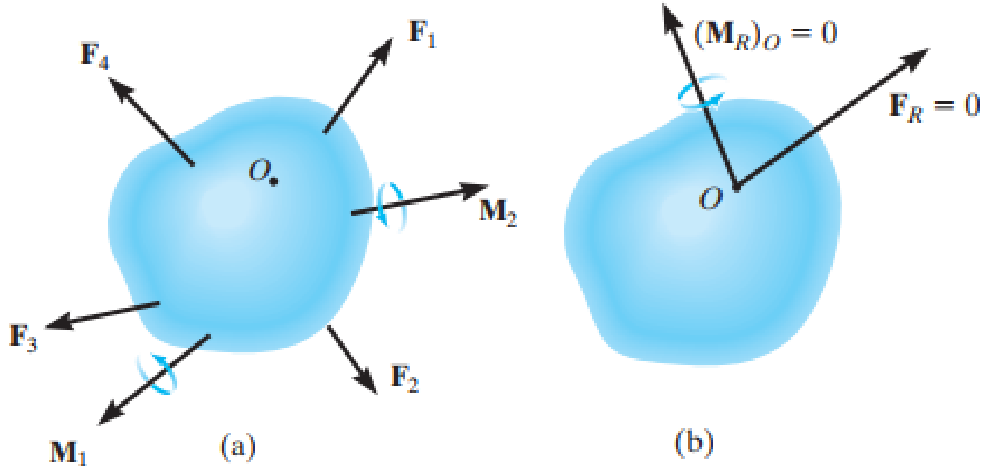
$$\text{and } \sum \boldsymbol{M_O} = 0 \text{ (no rotation)}$$

Conditions for Rigid-Body Equilibrium

- The equilibrium of a body is expressed as

$$F_R = \sum F = 0$$

$$(M_R)_O = \sum M_O = 0$$



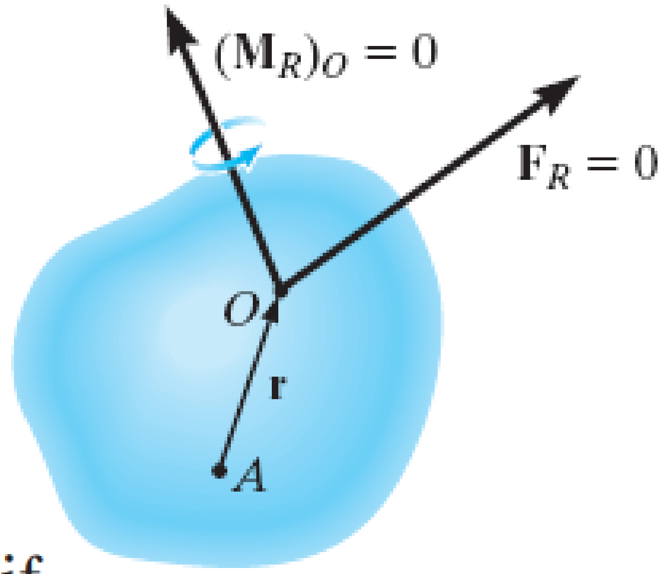
The first of these equations states that the sum of the forces acting on the body is equal to *zero*. The second equation states that the sum of the moments of all the forces in the system about point O, added to all the couple moments, is equal to *zero*.

Conditions for Rigid-Body Equilibrium

These two equations are not only necessary for equilibrium, they are also sufficient conditions. To show this

- Consider summing moments about some other point, such as point A, we require

$$\sum M_A = r \times F_R + (M_R)_O = 0$$

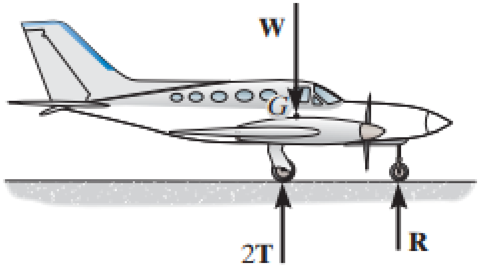


Since $\mathbf{r} \neq \mathbf{0}$, this equation is satisfied only if

$$F_R = \sum F = 0$$
$$(M_R)_O = \sum M_O = 0$$

Equilibrium in two Dimensions

In the first part of the chapter, we will consider the case where the force system acting on a rigid body lies in or may be projected onto a *single* plane and, furthermore, any couple moments acting on the body are directed perpendicular to this plane.



Free Body Diagrams

Support Reactions

- If a support prevents the translation of a body in a given direction, then a force is developed on the body in that direction.

- If rotation is prevented, a couple moment is exerted on the body.

5.2 Free-Body Diagrams

Equilibrium in 2D

The free body diagram is the most important single step in the solution of problems in mechanics

Equilibrium in 2D

Free-Body Diagrams

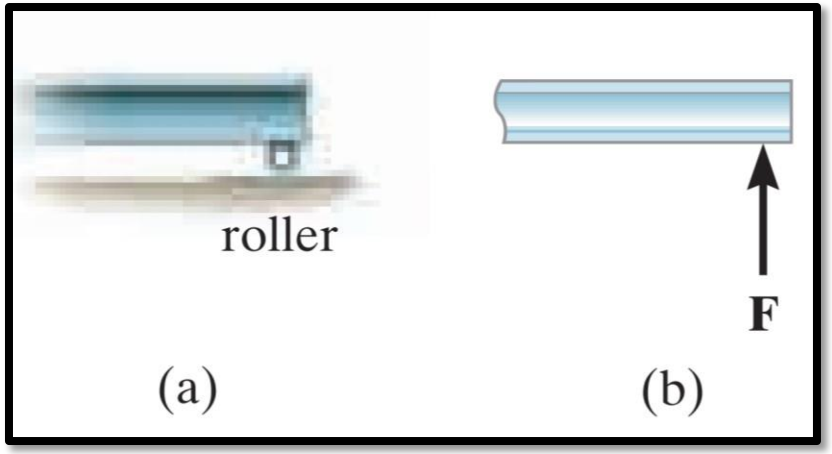
Reactive force:

Support Reactions.

- A support prevents the translation of a body in a given direction by exerting a force on the body in the opposite direction.

- A support prevents the rotation of a body in a given direction by exerting a couple moment on the body in the opposite direction.

For example, let us consider three ways in which a horizontal member, such as a beam, is supported at its end. One method consists of a *roller* or cylinder, Fig. 5–3*a*. Since this support only prevents the beam from *translating* in the vertical direction, the roller will only exert a *force* on the beam in this direction, Fig. 5–3*b*.



Equilibrium in 2D

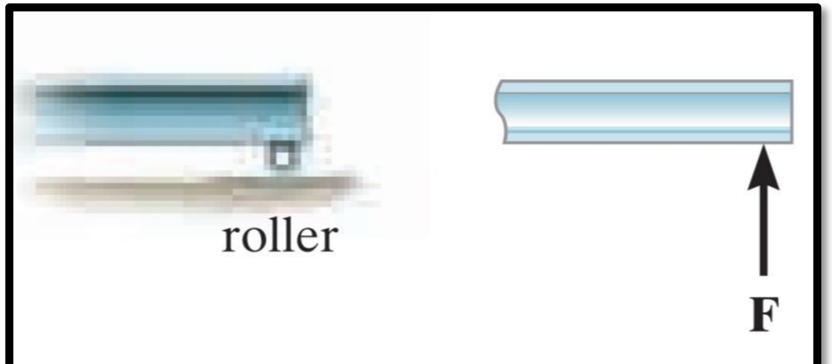
Free-Body Diagrams

Reactive force:

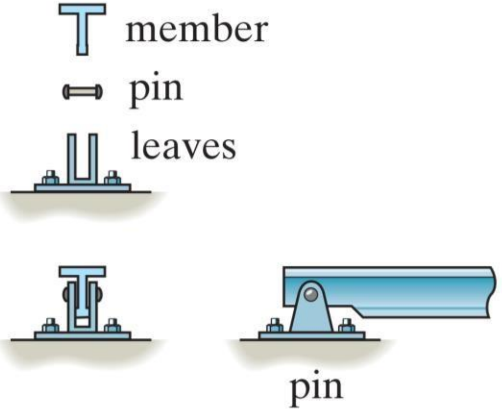
Support Reactions.

- A support prevents the translation of a body in a given direction by exerting a force on the body in the opposite direction.

- A support prevents the rotation of a body in a given direction by exerting a couple moment on the body in the opposite direction.

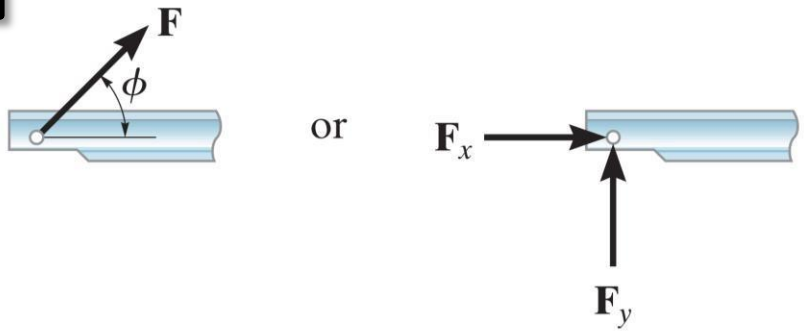


(a) (b)



(c)

The beam can be supported in a more restrictive manner by using a *pin*, Fig. 5-3c. The pin passes through a hole in the beam and two leaves which are fixed to the ground. Here the pin can prevent *translation* of the beam in *any direction* ϕ, Fig. 5-3*d*, and so the pin must exert a *force* **F** on the beam in this direction. For purposes of analysis, it is generally easier to represent this resultant force **F** by its two rectangular components $\mathbf{F}_x$ and $\mathbf{F}_y$, Fig. 5-3*e*. If F_x and F_y are known, then F and ϕ can be calculated.



(d) (e)

Equilibrium in 2D

Free-Body Diagrams

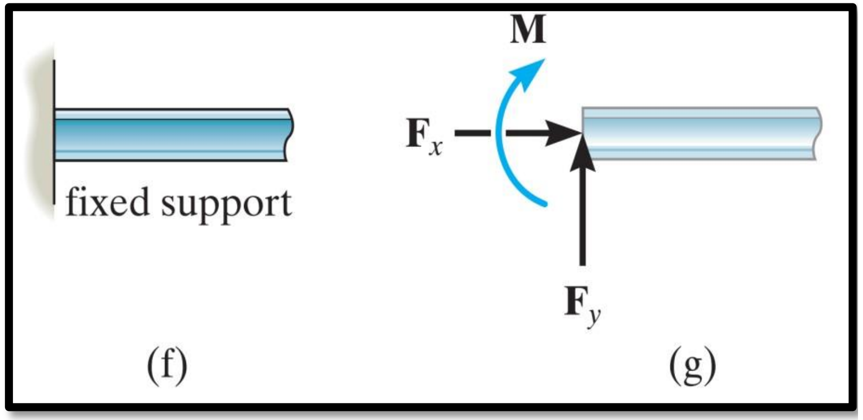
Reactive force:

Support Reactions.

- A support prevents the translation of a body in a given direction by exerting a force on the body in the opposite direction.

- A support prevents the rotation of a body in a given direction by exerting a couple moment on the body in the opposite direction.

The most restrictive way to support the beam would be to use a *fixed support* as shown in Fig. 5–3*f*. This support will prevent both *translation and rotation* of the beam. To do this a *force and couple moment* must be developed on the beam at its point of connection, Fig. 5–3*g*. As in the case of the pin, the force is usually represented by its rectangular components $\mathbf{F}_x$ and $\mathbf{F}_y$.



Equilibrium in 2D

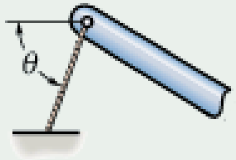
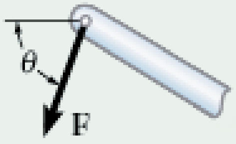
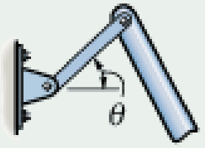
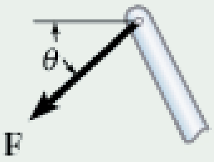
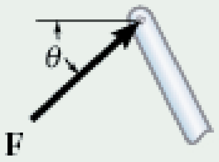
Support Reactions

The 2D reactions shown below are the ones shown in **Table 5-1** in the text. As a general rule:

1) if a **support prevents translation** of a body in a given direction, then **a force is developed** on the body in the opposite direction.

2) if **rotation is prevented, a couple moment** is exerted on the body in the opposite direction

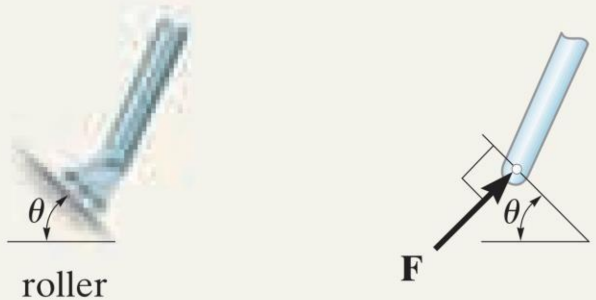
Table 5–1 Supports for Rigid Bodies Subjected to Two-Dimensional Force Systems

Types of Connection	Reaction			Number of Unknowns
(1)  cable				One unknown. The reaction is a tension force which acts away from the member in the direction of the cable.
(2)  weightless link		or		One unknown. The reaction is a force which acts along the axis of the link.

Free-Body Diagrams

Reactive force:

(3)



roller

One unknown. The reaction is a force which acts perpendicular to the surface at the point of contact.

(4)

rocker

One unknown. The reaction is a force which acts perpendicular to the surface at the point of contact.

Equilibrium in 2D

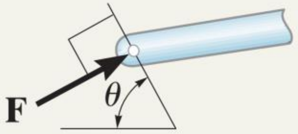
Support Reactions.

Free-Body Diagrams

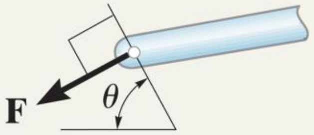
Reactive force:

(6)

roller or pin in
confined smooth slot



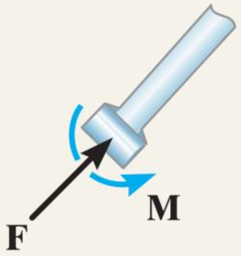
or



One unknown. The reaction is a force which acts perpendicular to the slot.

(9)

member fixed connected
to collar on smooth rod



Two unknowns. The reactions are the couple moment and the force which acts perpendicular to the rod.

Free Body Diagrams

Internal Forces

- External and internal forces can act on a rigid body

the internal forces that act between adjacent particles in a body always occur in collinear pairs such that they have the same magnitude and act in opposite directions (Newton's third law). Since these forces cancel each other, they will not create an *external effect* on the body. It is for this reason that the internal forces should not be included on the free-body diagram if the entire body is to be considered.

- For FBD, internal forces act between adjacent particles which are contained within the boundary of the FBD, are not represented

- Particles outside this boundary exert external forces on the system

Free Body Diagrams

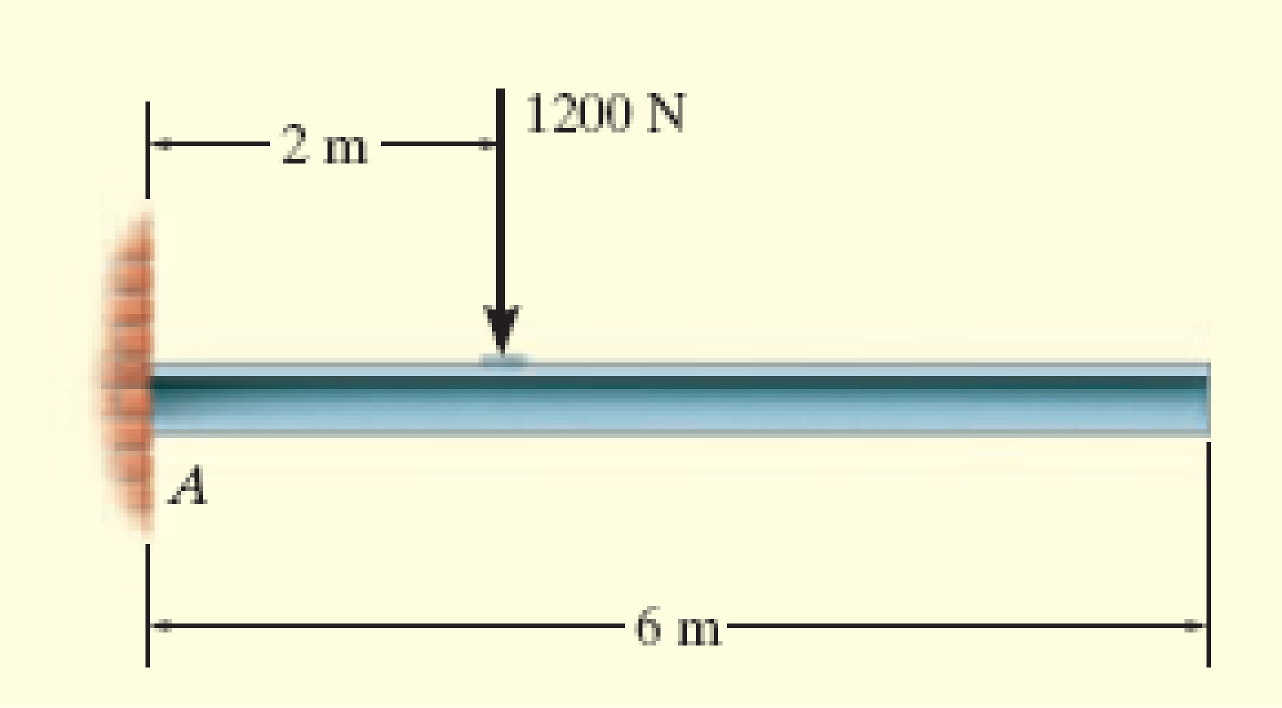
Internal Forces

For example, the engine shown in Fig. 5–4*a* has a free-body diagram shown in Fig. 5–4*b*. The internal forces between all its connected parts such as the screws and bolts, will cancel out because they form equal and opposite collinear pairs. Only the external forces $\mathbf{T}_1$ and $\mathbf{T}_2$, exerted by the chains and the engine weight **W**, are shown on the free-body diagram.

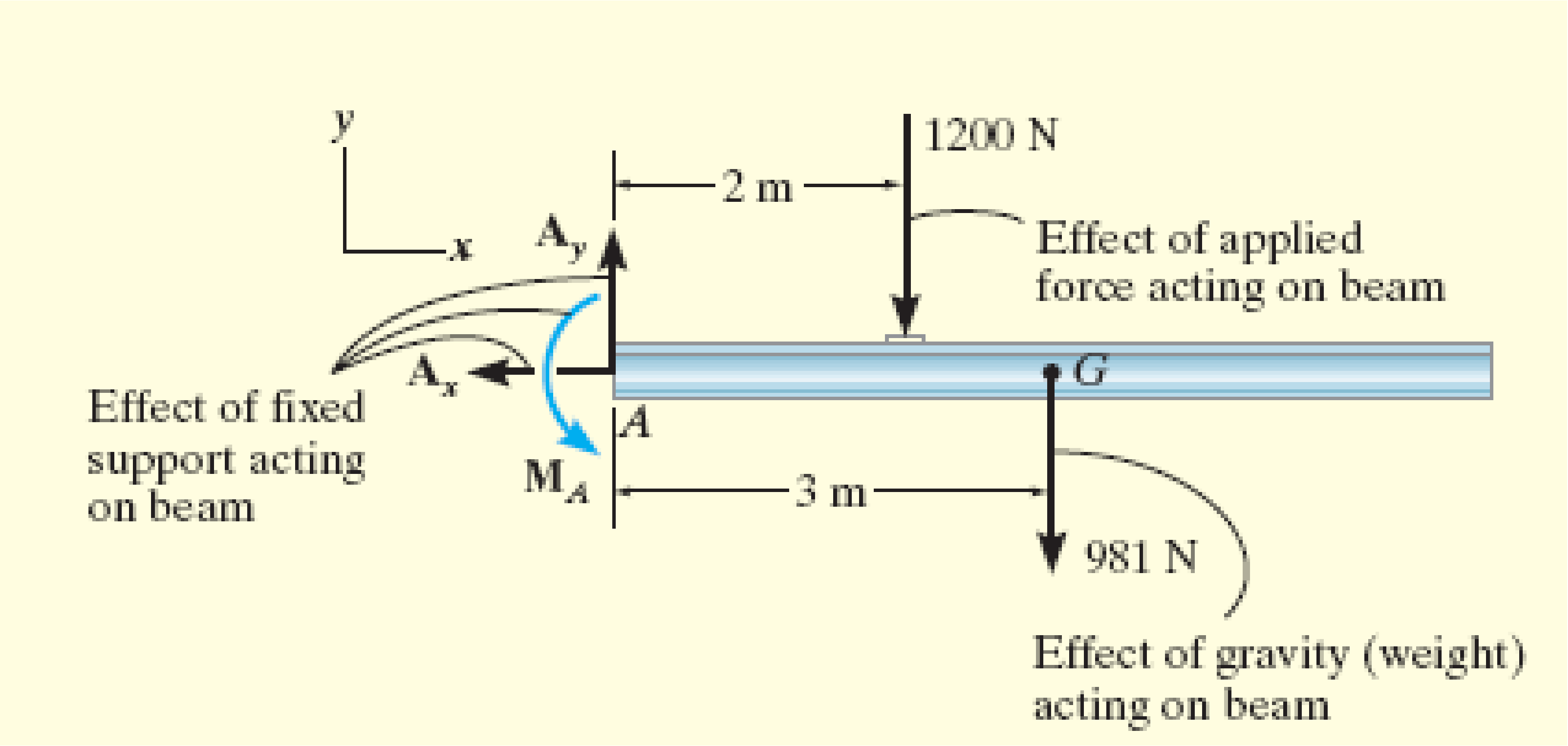

(a)

Example 1

Draw the free-body diagram of the uniform beam. The beam has a mass of 100kg.



Free-Body Diagram



Free-Body Diagram's Description

- Support at A is a fixed wall, the wall *exerts three reactions on the beam*. These three forces acting on the beam at A denoted as $\mathbf{A_x}$, $\mathbf{A_y}$, $\mathbf{M_A}$, drawn in an arbitrary direction
- Unknown magnitudes of these vectors
- Assume sense of these vectors
- For uniform beam,

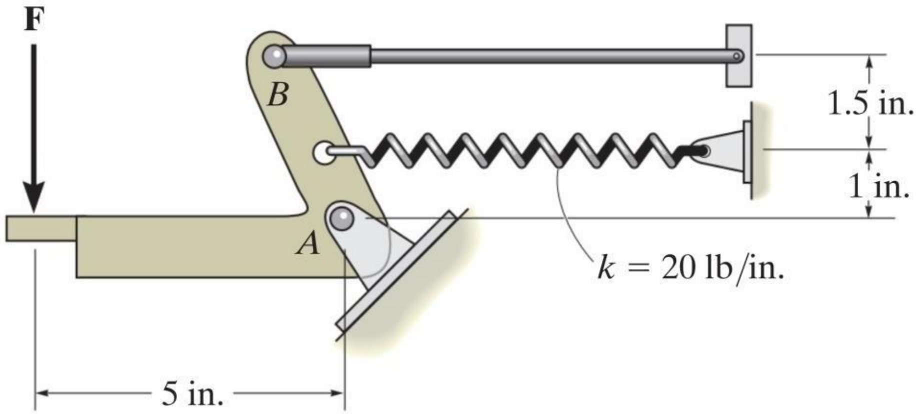
  Weight, W = 100(9.81) = 981N

  acting through beam's center of gravity, 3m from A

Example 2

Draw the free-body diagram of the foot lever shown in Fig. 5–8*a*. The operator applies a vertical force to the pedal so that the spring is stretched 1.5 in. and the force on the link at *B* is 20 lb.

(a)

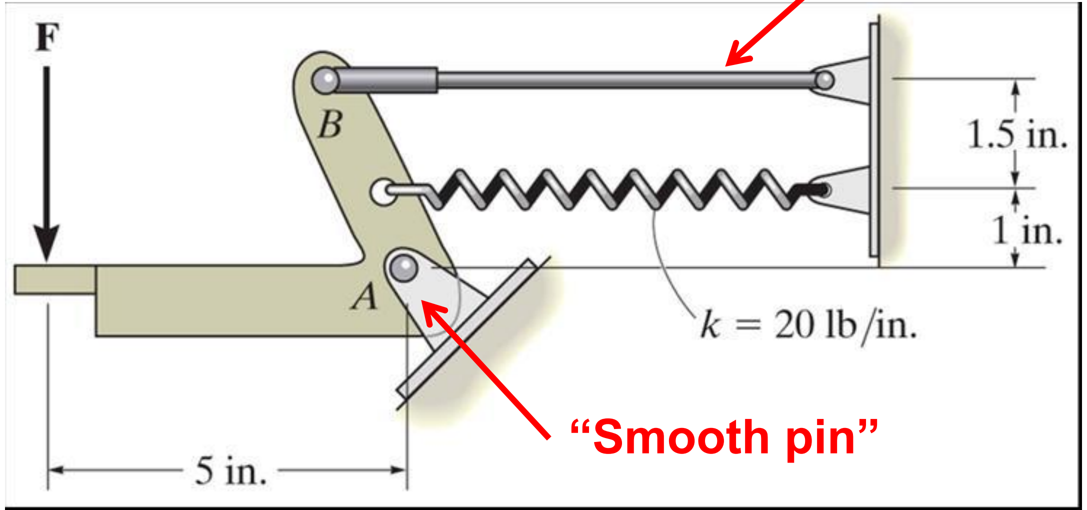


EXAMPLE: 2

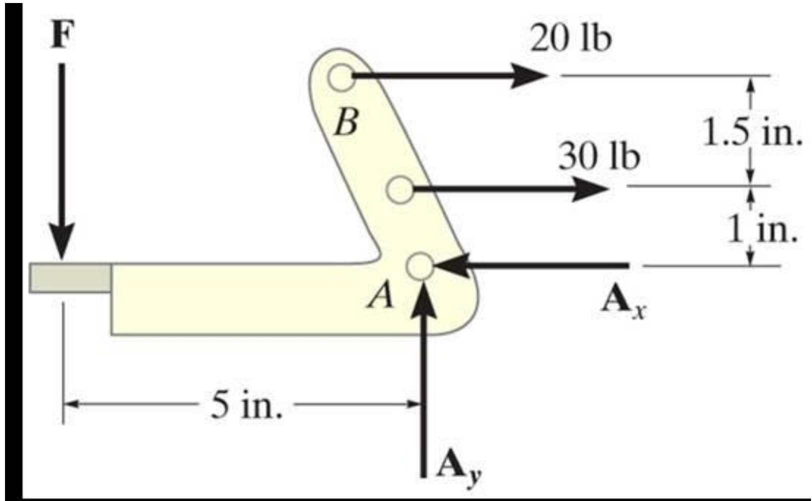
Given: The operator applies a vertical force to the pedal so that the spring is stretched 1.5 in. and the force in the short link at *B is 20 lb*.

Draw: A an idealized model and free-body diagram of the foot pedal.

“Weightless link” (see Table 5-1)



The idealized model



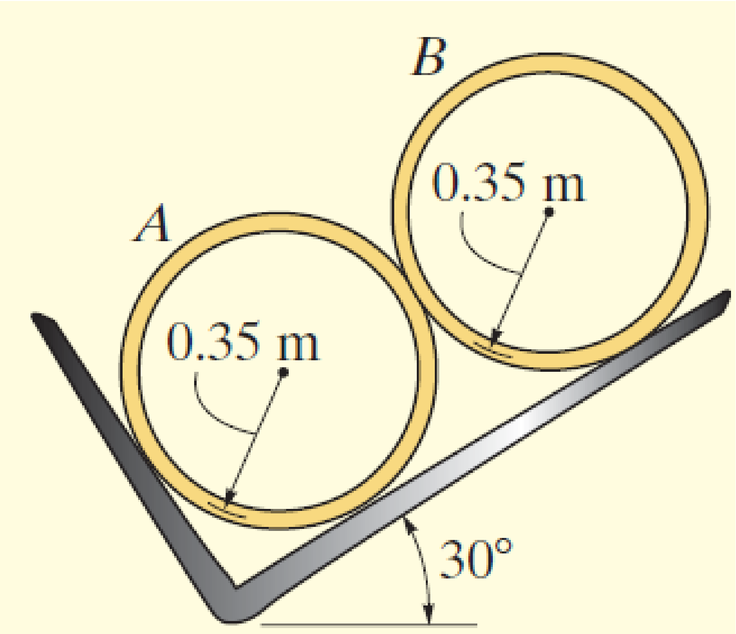
The free-body diagram

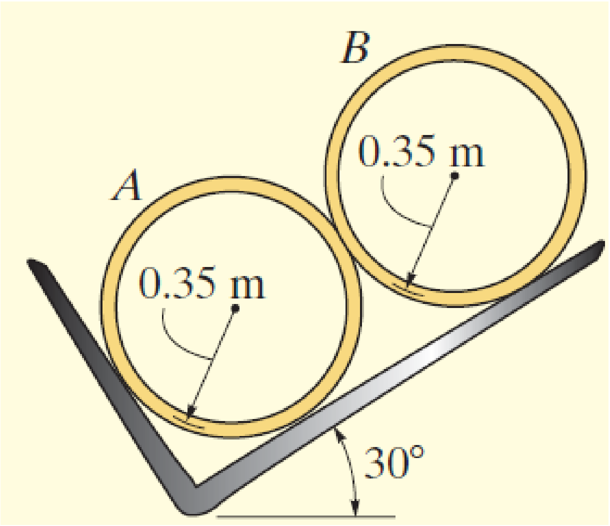
Free-Body Diagram's Description

By inspection of the photo the lever is loosely bolted to the frame at A. The rod at B is pinned at its ends and acts as a "short link." After making the proper measurements, the idealized model of the lever is shown in Fig. 5–8*b*. From this, the free-body diagram is shown in Fig. 5–8*c*. The pin support at A exerts force components $\mathbf{A}_x$ and $\mathbf{A}_y$ on the lever. The link at B exerts a force of 20 lb, acting in the direction of the link. In addition the spring also exerts a horizontal force on the lever. If the stiffness is measured and found to be $k = 20$ lb/in., then since the stretch $s = 1.5$ in., using Eq. 3–2, $F_s = ks = 20$ lb/in. (1.5 in.) $= 30$ lb. Finally, the operator's shoe applies a vertical force of $\mathbf{F}$ on the pedal. The dimensions of the lever are also shown on the free-body diagram, since this information will be useful when computing the moments of the forces. As usual, the senses of the unknown forces at A have been assumed. The correct senses will become apparent after solving the equilibrium equations.

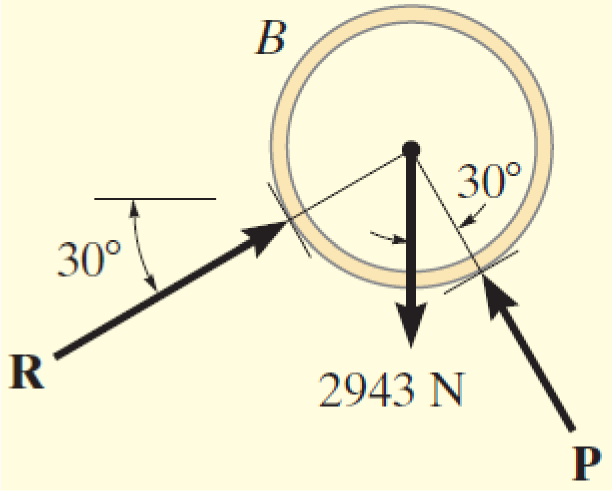
Example 3

Two smooth pipes, each having a mass of 300 kg, are supported by the forked tines of the tractor in Fig. 5–9*a*. Draw the free-body diagrams for each pipe and both pipes together.

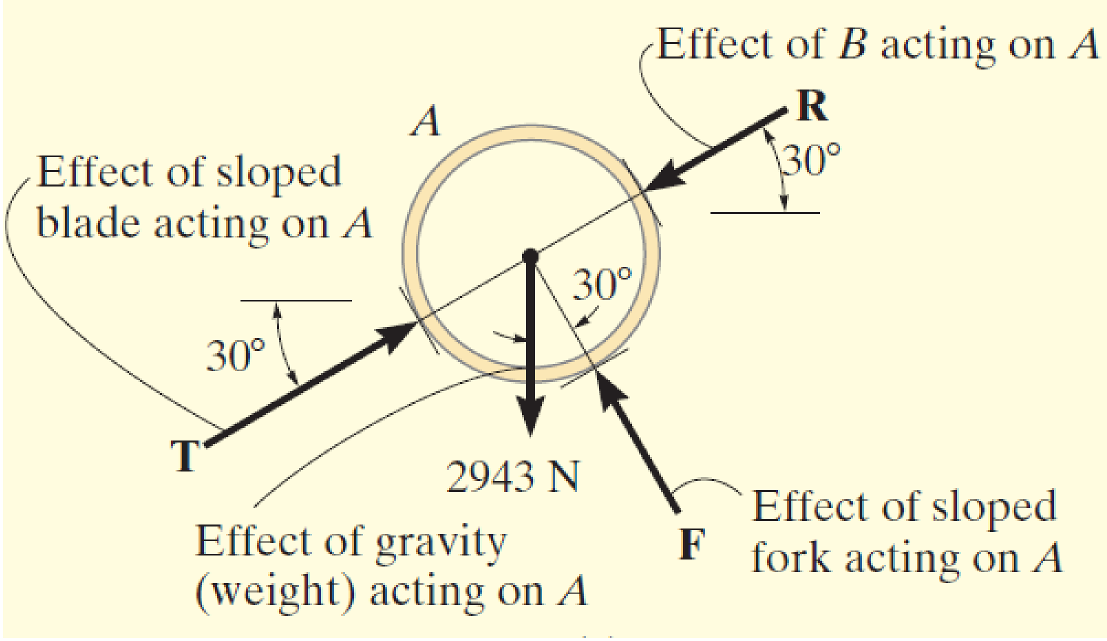




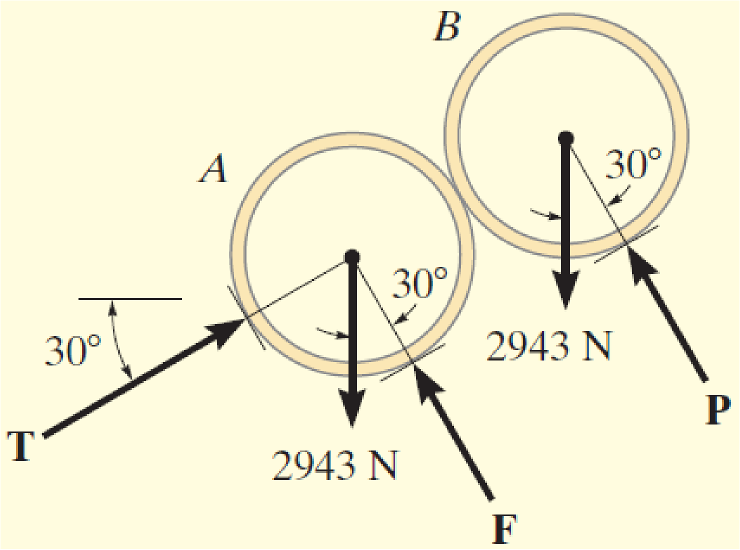
B
A
0.35 m
0.35 m
30°



B
30°
30°
R
2943 N
P



Effect of B acting on A
R
30°
A
Effect of sloped
blade acting on A
30°
30°
T
2943 N
Effect of gravity
(weight) acting on A
F
Effect of sloped
fork acting on A



B
A
30°
30°
30°
2943 N
P
T
2943 N
F

5.3 Equation of Equilibrium

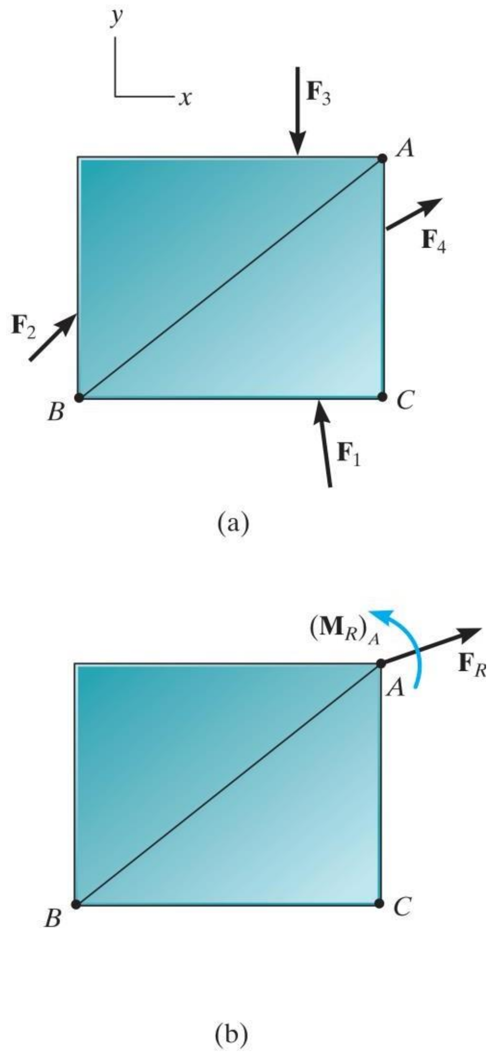
- For equilibrium of a rigid body in 2D,

$$\sum F_x = 0; \sum F_y = 0; \sum M_O = 0$$

- $\boldsymbol{\sum F_x}$ and $\boldsymbol{\sum F_y}$ represent sums of x and y components of all the forces
- $\boldsymbol{\sum M_O}$ represents the sum of the couple moments and moments of the force components

Equation of Equilibrium

$$\Sigma F_x = 0$$
$$\Sigma F_y = 0$$
$$\Sigma M_O = 0$$



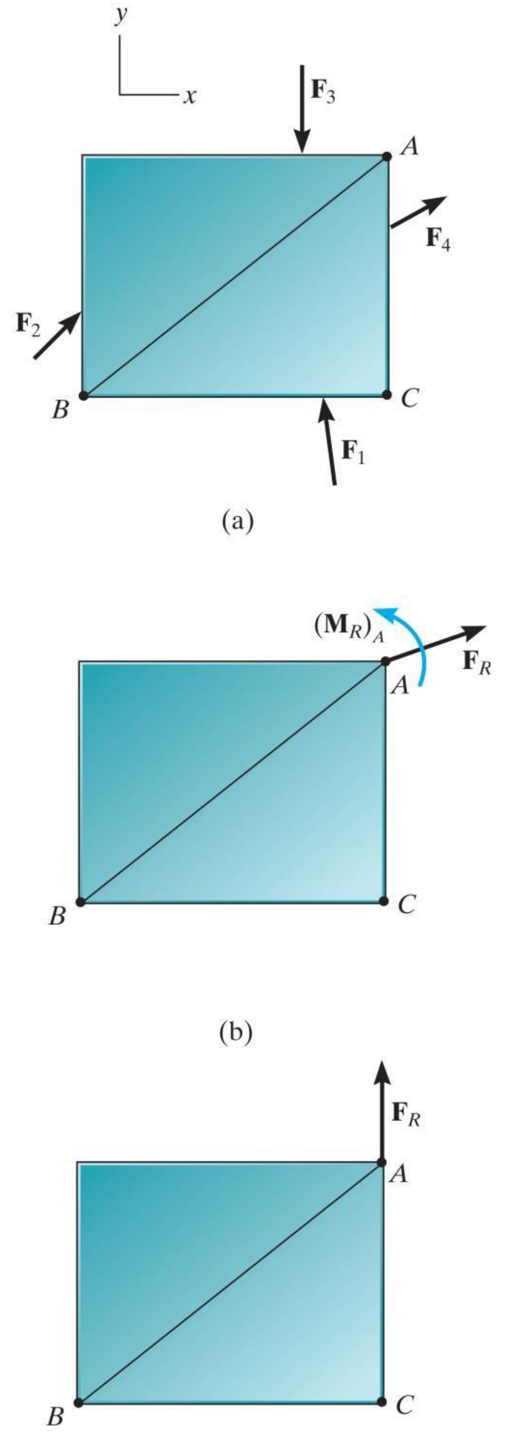
Equation of Equilibrium

$$\Sigma F_x = 0$$
$$\Sigma F_y = 0$$
$$\Sigma M_O = 0$$

Alternative Sets of Equilibrium Equations.

$$\Sigma F_x = 0$$
$$\Sigma M_A = 0$$
$$\Sigma M_B = 0$$

$$\Sigma M_A = 0$$
$$\Sigma M_B = 0$$
$$\Sigma M_C = 0$$



(a)
(b)

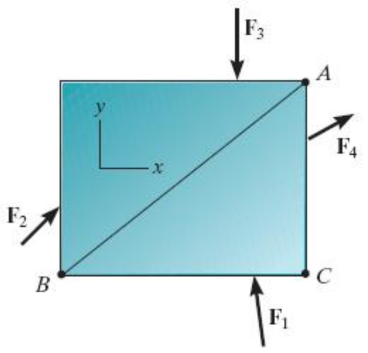
Equation of Equilibrium

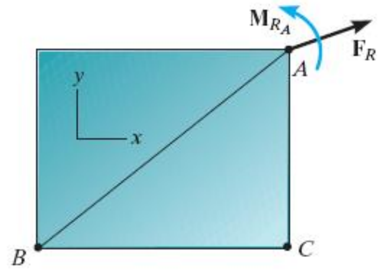
Alternative Sets of Equilibrium Equations

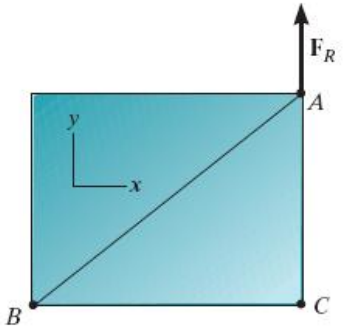
- For coplanar equilibrium problems,
  $\sum F_x = 0; \quad \sum F_y = 0; \sum M_O = 0$

- 2 alternative sets of 3 independent equilibrium equations,

  $\sum F_a = 0; \sum M_A = 0; \sum M_B = 0$

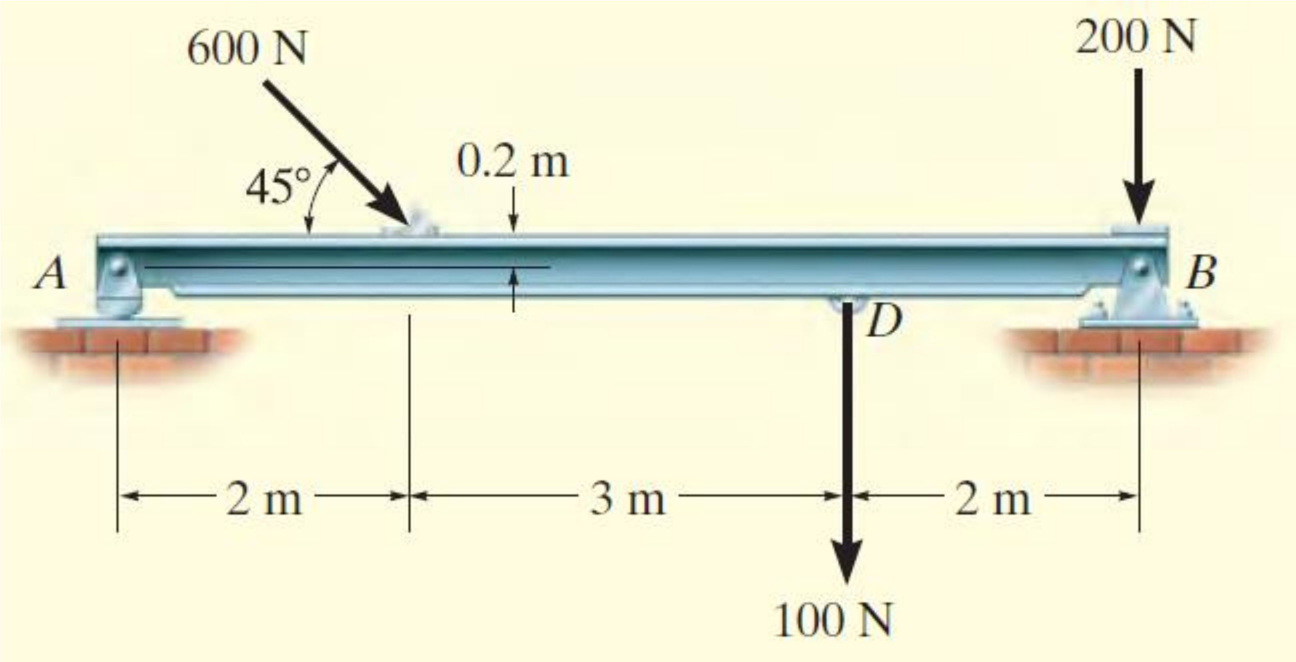


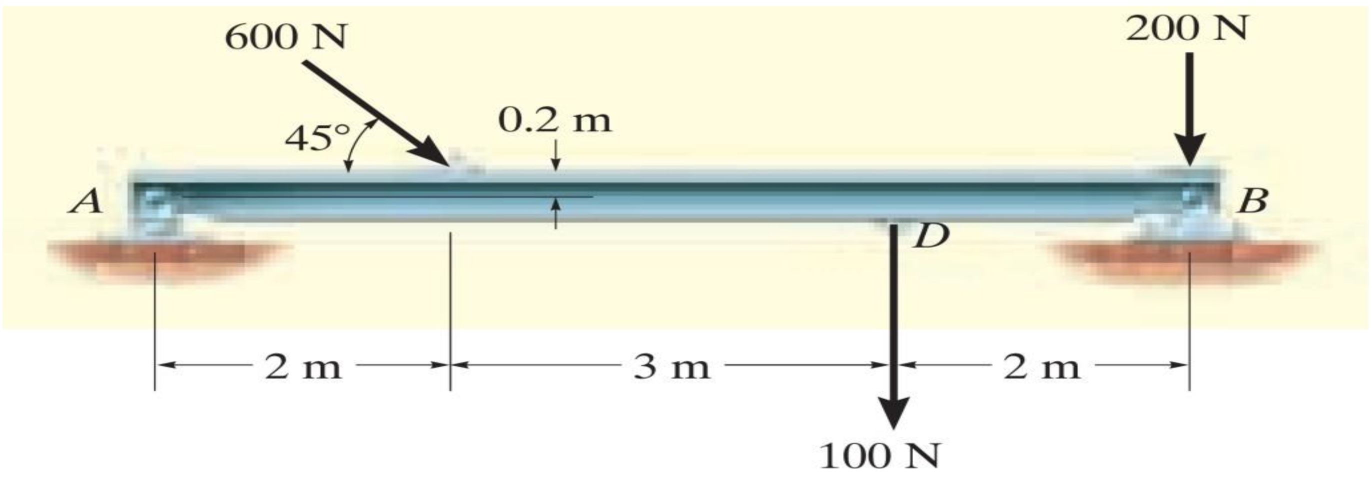


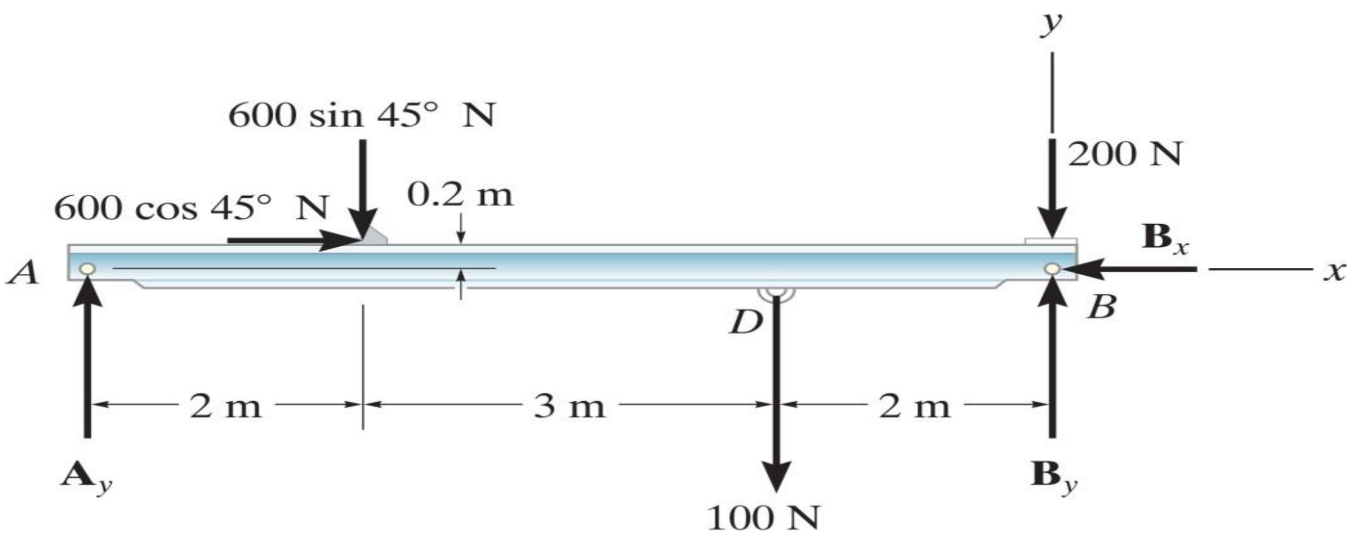


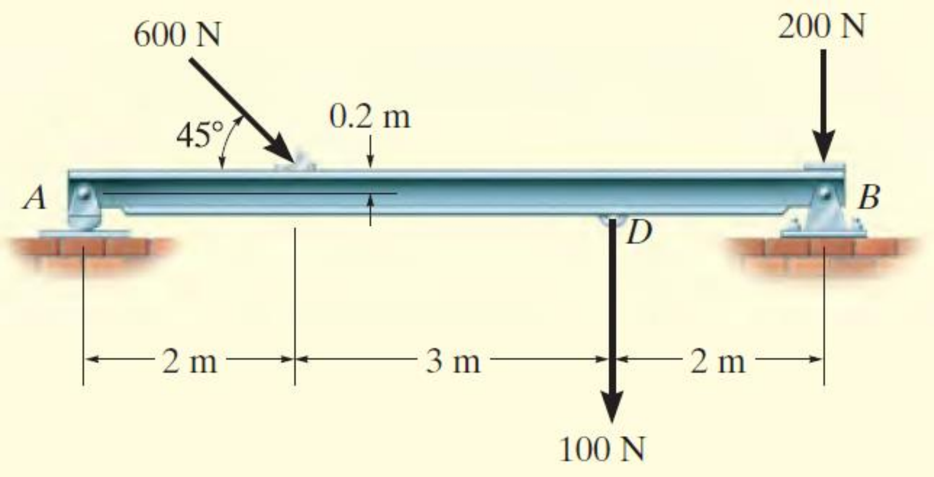
Example

Determine the horizontal and vertical components of reaction on the beam caused by the pin at *B* and the rocker at *A* as shown in Fig. 5–12*a*. Neglect the weight of the beam.




600 N
45°
0.2 m
200 N
A
B
D
2 m
3 m
2 m
100 N


y
600 sin 45° N
200 N
600 cos 45° N
0.2 m
B_x
A
x
B
D
2 m
3 m
2 m
A_y
B_y
100 N



$\xrightarrow{+} \Sigma F_x = 0;$

$$600 \cos 45° \text{ N} - B_x = 0$$

$$B_x = 424 \text{ N}$$

$\circlearrowleft + \Sigma M_B = 0;$

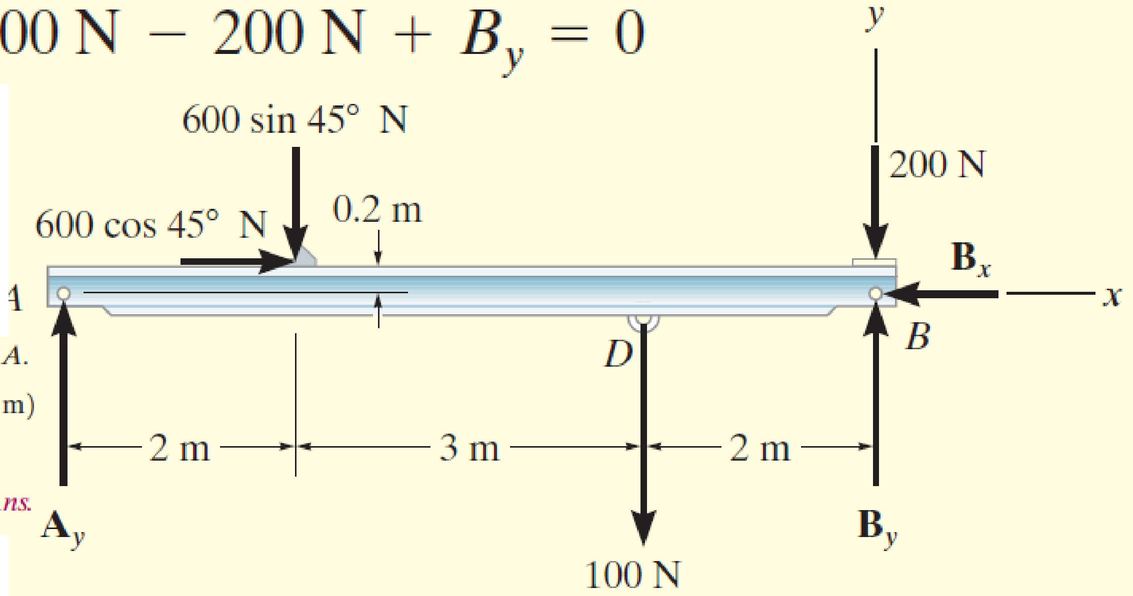
$$100 \text{ N}(2 \text{ m}) + (600 \sin 45° \text{ N})(5 \text{ m}) - (600 \cos 45° \text{ N})(0.2 \text{ m}) - A_y(7 \text{ m}) = 0$$

$$A_y = 319 \text{ N}$$

$+\uparrow \Sigma F_y = 0;$

$$319 \text{ N} - 600 \sin 45° \text{ N} - 100 \text{ N} - 200 \text{ N} + B_y = 0$$

$$B_y = 405 \text{ N}$$



NOTE: We can check this result by summing moments about point A.

$\circlearrowleft + \Sigma M_A = 0;$ $\quad -(600 \sin 45° \text{ N})(2 \text{ m}) - (600 \cos 45° \text{ N})(0.2 \text{ m})$

$-(100 \text{ N})(5 \text{ m}) - (200 \text{ N})(7 \text{ m}) + B_y(7 \text{ m}) = 0$

$B_y = 405 \text{ N}$ *Ans.*