TIME VARYING FIELDS AND MOTIONAL EMF

Time Varying Fields

- Until now, we have restricted our discussions to static, or time invariant Electric and Magnetic fields

- Next, we shall examine situations where electric and magnetic fields are dynamic, or time varying

- It should be mentioned first that in static EM fields, electric and magnetic fields are independent of each other

- Whereas in dynamic EM fields, the two fields are interdependent

- In other words, a time-varying electric field necessarily involves a corresponding time-varying magnetic field

Time Varying Fields

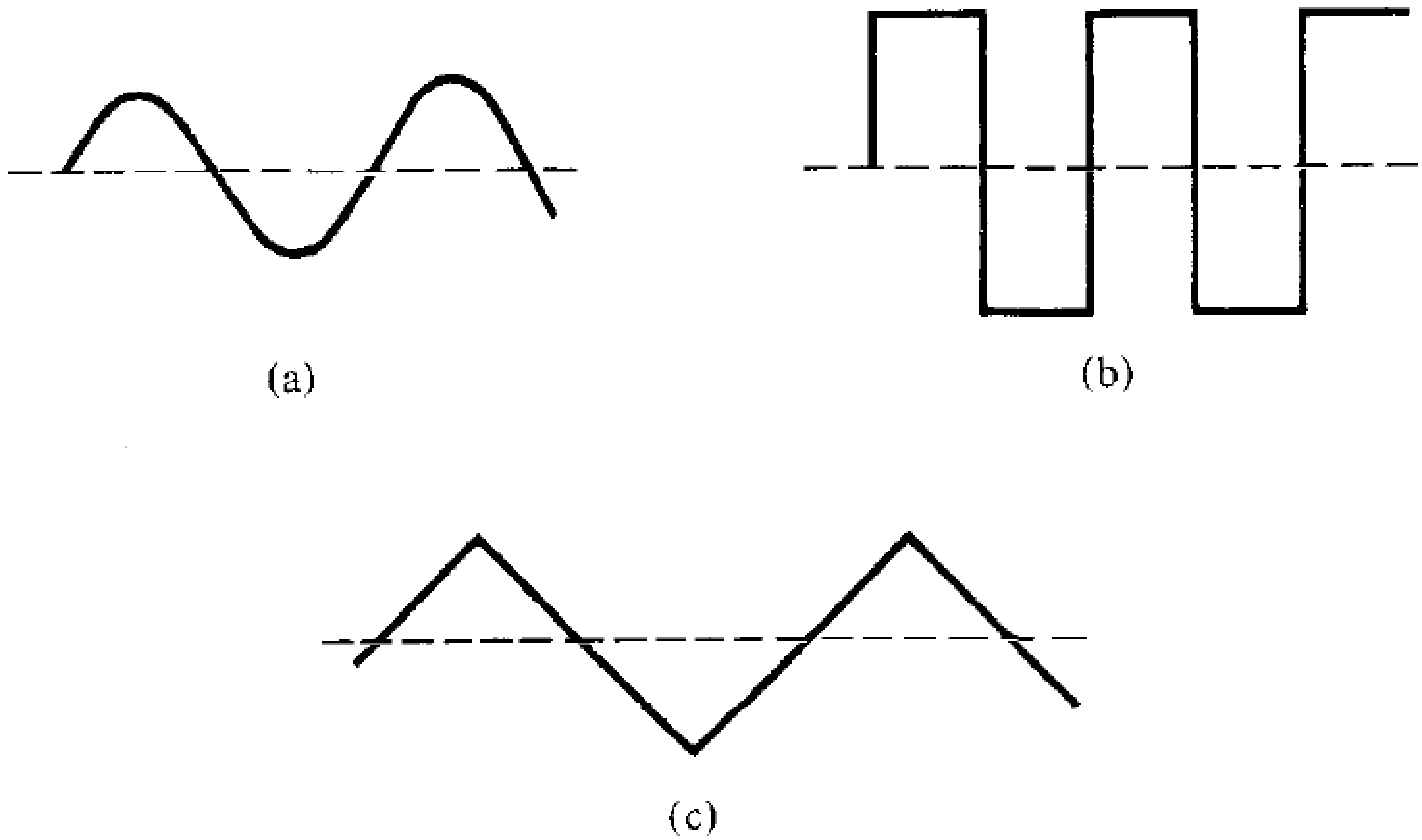
- Time-varying EM fields, represented by **E**(x, y, z, t) and **H**(x, y, z, t), are of more practical value than static EM fields

- Time-varying fields or waves are usually due to accelerated charges or time-varying currents such as sine or square waves

- Any pulsating current will produce radiation (time-varying fields)

Time Varying Fields

- Figure below shows examples of accelerated charges or time-varying currents



Time Varying Fields

- It is worth noting that pulsating current of the type shown in figure (b) is the cause of radiated emission in digital logic boards

- In summary:
  - Stationary charges $\longrightarrow$ Electrostatic fields
  - Steady currents $\longrightarrow$ Magnetostatic fields
  - Time-varying currents $\longrightarrow$ electromagnetic fields (or waves)

Faraday’s Law

- After Oersted's experimental discovery (upon which Biot-Savart and Ampere based their laws) that a steady current produces a magnetic field, it seemed logical to find out if magnetism would produce electricity

- In 1831, about 11 years after Oersted's discovery, Michael Faraday in London and Joseph Henry in New York discovered that a time-varying magnetic field would produce an electric current

- According to Faraday's experiments, a static magnetic field produces no current flow, but a time-varying field produces an induced voltage (called electromotive force or simply emf) in a closed circuit, which causes a flow of current

Faraday's Law

- The Faraday's law states that the induced emf, V_{emf} (in volts), in any closed **circuit** is equal to the time rate of change of the magnetic flux linkage by the circuit

- Mathematically, Faraday's law can be expressed as:

$$V_{\text{emf}} = -\frac{d\lambda}{dt} = -N\frac{d\Psi}{dt}$$

- where *N* is the number of turns in the circuit and *Ψ* is the flux through each turn

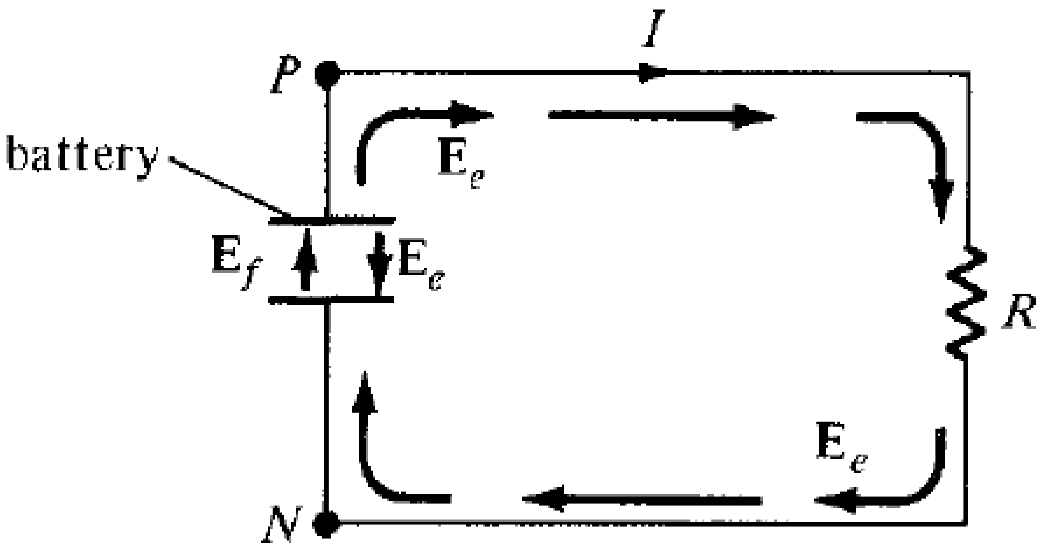
- *Lenz's law* states that the direction of current flow in the circuit is such that the induced magnetic field produced by the induced current will oppose the original magnetic field

Faraday's Law

- From Lenz's law, the negative sign shows that the induced voltage acts in such a way as to oppose the flux producing it
- Recall that we described an electric field as one in which electric charges experience force
- The electric fields considered so far are caused by electric charges; in such fields, the flux lines begin and end on the charges
- There are other kinds of electric fields not directly caused by electric charges
- These are emf-produced fields

Electromotive Force (emf)

- Consider the electric circuit in figure below, where the battery is a source of emf



- The electrochemical action of the battery results in an emf-produced field $\mathbf{E}_f$

- Due to the accumulation of charge at the battery terminals, an electrostatic field $\mathbf{E}_e$ $(-\nabla V)$ also exists

Electromotive Force (emf)

- The total electric field at any point is:

$$\mathbf{E} = \mathbf{E}_f + \mathbf{E}_e$$

- Note that $\mathbf{E}_f$ is zero outside the battery

- $\mathbf{E}_f$ and $\mathbf{E}_e$ have opposite directions in the battery

- The direction of $\mathbf{E}_e$ inside the battery is opposite to that outside it

- By integrating the above equation over the closed circuit, we get:

$$\oint_L \mathbf{E} \cdot d\mathbf{l} = \oint_L \mathbf{E}_f \cdot d\mathbf{l} + 0 = \int_N^P \mathbf{E}_f \cdot d\mathbf{l} \qquad \text{(through battery)}$$

- Where $\oint \mathbf{E}_e \cdot d\mathbf{l} = 0$ because $\mathbf{E}_e$ is conservative

Electromotive Force (emf)

- The emf of the battery is the line integral of the emf-produced field, that is:

$$V_{\text{emf}} = \int_N^P \mathbf{E}_f \cdot d\mathbf{l} = -\int_N^P \mathbf{E}_e \cdot d\mathbf{l} = IR$$

- The negative sign is because $\mathbf{E}_f$ and $\mathbf{E}_e$ are equal but opposite within the battery

- It is important to note that:

- An electrostatic field $\mathbf{E}_e$ cannot maintain a steady current in a closed circuit since:

$$\oint_L \mathbf{E}_e \cdot d\mathbf{l} = 0 = IR$$

- This means that an emf-produced field $\mathbf{E}_f$ is non-conservative

Transformer and Motional EMFs

- We now examine how Faraday's law links electric and magnetic fields

- For a circuit with a single turn (N = 1), we have:

$$V_{\text{emf}} = -\frac{d\Psi}{dt}$$

- In terms of **E** and **B**, the above equation may be written as:

$$V_{\text{emf}} = \oint_L \mathbf{E} \cdot d\mathbf{l} = -\frac{d}{dt}\int_S \mathbf{B} \cdot d\mathbf{S}$$

- where Ψ has been replaced by $\int_S \mathbf{B} \cdot d\mathbf{S}$ and S is the surface area of the circuit bounded by the closed path L

Transformer and Motional EMFs

- It is clear from above equation that in a time-varying situation, both electric and magnetic fields are present and are interrelated

- The variation of flux with time (as in previous equation) may be caused in three ways:

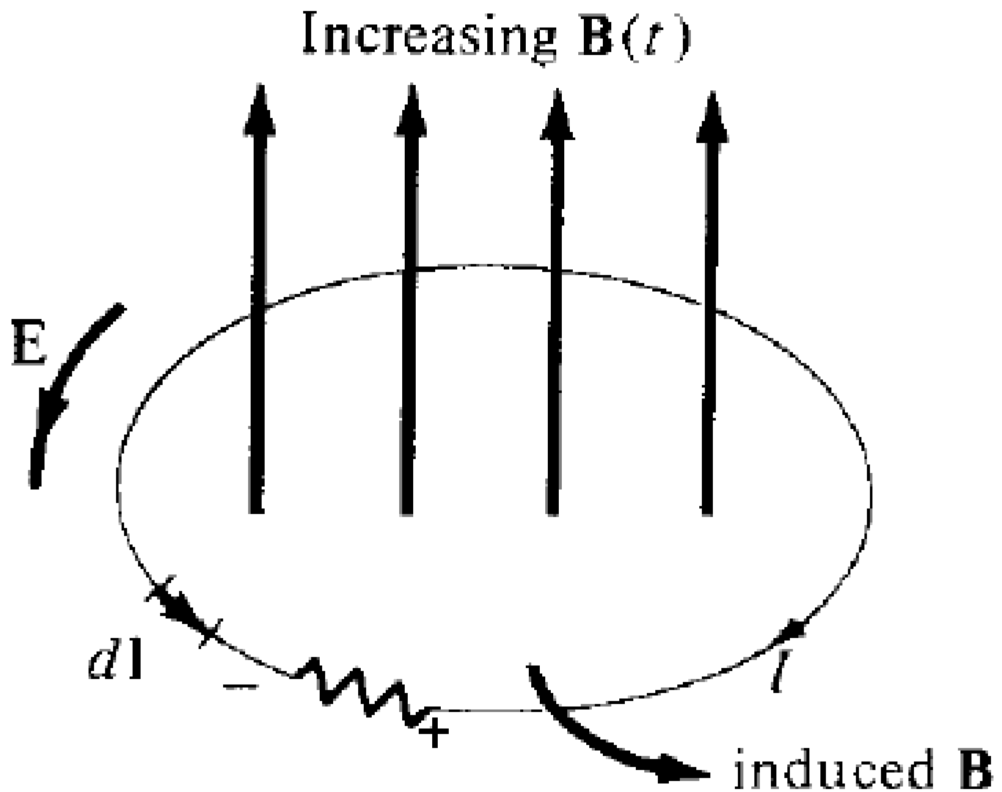
1. By having a stationary loop in a time-varying **B** field
2. By having a time-varying loop area in a static **B** field
3. By having a time-varying loop area in a time-varying **B** field

- Each of these will be considered separately.

Stationary Loop; Time-Varying B Field

- Figure below shows a stationary conducting loop in a time varying magnetic **B** field

$$V_{\text{emf}} = \oint_L \mathbf{E} \cdot d\mathbf{l} = -\int_S \frac{\partial \mathbf{B}}{\partial t} \cdot d\mathbf{S}$$



- This induced emf is often referred to as transformer emf in power analysis since it is due to transformer action

- Observe in the figure that the Lenz's law is obeyed

Stationary Loop; Time-Varying B Field

- By applying Stokes theorem to the emf equation, we obtain:

$$\int_S (\nabla \times \mathbf{E}) \cdot d\mathbf{S} = -\int_S \frac{\partial \mathbf{B}}{\partial t} \cdot d\mathbf{S}$$

- Therefore, we get: $$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

- This is second Maxwell's equation for time-varying fields

- It shows that the time varying **E** field is not conservative

$$(\nabla \times \mathbf{E} \neq 0)$$

- This implies that the work done in taking a charge about a closed path in a time-varying electric field, is due to the energy from the time-varying magnetic field

Moving Loop; Static B Field

- When a conducting loop is moving in a static **B** field, an emf is induced in the loop

- Recall that the force on a charge moving with uniform velocity **u** in a magnetic field **B** is:

$$\mathbf{F}_m = Q\mathbf{u} \times \mathbf{B}$$

- We define the motional electric field **E**$_m$ as:

$$\mathbf{E}_m = \frac{\mathbf{F}_m}{Q} = \mathbf{u} \times \mathbf{B}$$

- Consider a conducting loop, moving with uniform velocity **u** as consisting of a large number of free electrons, the emf induced is:

$$V_{\text{emf}} = \oint_L \mathbf{E}_m \cdot d\mathbf{l} = \oint_L (\mathbf{u} \times \mathbf{B}) \cdot d\mathbf{l}$$

Moving Loop; Time varying B Field

- In this case, both transformer emf and motional emf are present

- Hence we combine both the emfs as:

$$V_{\text{emf}} = \oint_L \mathbf{E} \cdot d\mathbf{l} = -\int_S \frac{\partial \mathbf{B}}{\partial t} \cdot d\mathbf{S} + \oint_L (\mathbf{u} \times \mathbf{B}) \cdot d\mathbf{l}$$