It is difficult to measure the "quality" of posterior samples in MCMC. The paper studies these problems in the context of bidirectional Monte Carlo. The authors run annealed importance sampling chains from prior to posterior and vice versa to upper bound the symmetrize KL divergence between true posterior and approximate samples distribution. This is incorporated into BREAD, a protocol for validating the relevance of simulated data experiments to real datasets.

The method of evaluating the "quality" of a posterior extends the bidirectional Monte Carlo method recently developed to evaluate MCMC-based inference algorithms by bounding symmetrize KL divergence of approximate samples to the true posterior.

One issue with using BDMC is that rigorous bounds can only be achieved with an exact posterior sample. However, that is not possible with real world datasets, and so they develop a protocol BREAD for using BDMC to diagnose inference quality on real-world data.

There are two criteria for a sampling based approximate inference algorithm that draws samples representative of the true posterior. (1) Approximate distribution should cover high probability regions of the posterior (2) Avoid placing mass where there is little. The Jeffrey's divergence is a measure of both of these stipulations.

One problem is that hyper parameters are often assigned weakly informative prior in order to avoid biasing inference. However, datasets generated from hyper parameters sampled from very broad priors don't resemble real datasets which prevents us from generalizing conclusions derived from simulated data.

This can be circumvented by performing approximate posterior inference on the real dataset. Then simulate parameters from the forward model. Then to obtain the posterior sample for reverse chain we start from the estimated hyper parameter and then run a few MCMC transitions. This should heuristically representative of the posterior distribution unless the prior on the hyper parameter is not represented by the estimated hyper parameters.