MCAC 201 Data Structures (Practice Qs)

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July 5, 2022

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1 Lab Exercise Insertion Sort

Consider the following algorithm for insertion sort

Algorithm 1: Insertion sort(A, n)

input : Array: $A[1], A[2]..., A[n]$
output: Sorted array; $A[1] \leq A[2] \leq ..... \leq A[n]$
for *i: 2 to n* **do**
  Insert$(A[1 \dots i-1], i)$ /* function searches for an appropriate location j to insert $A[i]$ in $A[1 \dots i-1]$ so that $A[1 \dots i]$ is sorted. It also inserts $A[i]$ in the j^{th} location.
end

1. What are the possible locations that $A[i]$ may take in the i^{th} iteration?

2. What is the number of comparisons performed by IS to insert $A[i]$ in location j. Give your answer in terms of i and j.

3. Run Insert$(A[1...i-1], i)$ for $i = 4$ for all possible permutations of 1, 2, 3, 4. For every $j = 1, 4$, list down the instances (the permutations) in which $A[i]$ is inserted at the j^{th} location. What do you observe? Now argue: the probability that $A[i]$ will be inserted at the j^{th} location is $1/i$.

   (a) For every $i = 1, 4$, Compute the average/expected number of comparisons performed by IS to insert $A[i]$. Average is taken over all possible permutations.

   (b) For every $i = 1, 4$, compute the probability that in the i^{th} iteration, $A[i]$ will be inserted in the j^{th} location $j = 1 \dots ?$. Make a 2d table P with i at the rows and j at the columns where $P[i, j]$ stores the above probability.

(c) Repeat part (a) using the probabilities computed in part (b).

4. Consider a random input sequence. What is the probability that in the i^{th} iteration, $A[i]$ will be inserted in the j^{th} location for $j = 1 \ldots i$? Give your answer in terms of i (and j if required).