Efficient modeling of timeouts in PRISM

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Abstract

Phase-type fitting remains the only way of modeling non-Markovian distributions within PRISM model checker. When analyzing models with phase-type distributions, it is problematic to obtain results of sufficient precision within reasonable time. We ran experimental computations and deduced a reasonable way of obtaining precise analysis results for phase-type fitted PRISM CTMCs. Lastly, we hint at an entirely different (and arguably better) approach to handling non-Markovian distributions.

Keywords: PRISM model checker, CTMC, deterministic timeout, phase-type distribution, modeling, analysis

1. Introduction

PRISM [1] and Storm [2] are popular tools for modeling and analysis of stochastic systems in continuous time. They use efficient algorithms for analysis of continuous time Markov chains (CTMC). This approach suffers from a severe restriction of modeling power — the time between transitions must be exponentially distributed. This restriction can be remedied by the use of phase-type distributions, which can approximate any general distribution with arbitrary accuracy by only using exponential distributions [3]. However, the use of phase-type distributions drastically increases the number of states within the CTMC.

In this paper, we experimentally evaluate the precision of the result and required computation time of various approaches to analysis of continuous time stochastic models with deterministic transitions (timeouts). The obtained results are then compared against results of our extension of PRISM capable of efficiently analyzing CTMC with non-Markovian alarms (ACTMC).

2. Experimental evaluation

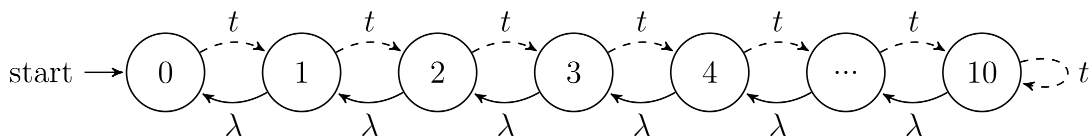

Figure 1: Model of a D/M/1/10 queue with production timeout $t = 0.1$ and service rate $\lambda = \frac{ln(2)}{timeout} \approx 6.931471805599453094$. The deterministic arrival event does not reset when the exponential service event occurs.

We have chosen to model a D/M/1/10 queue (deterministic arrivals, exponentially distributed service, single server, capacity of 10) as shown in Figure 1. The deterministic arrivals are approximated by a phase-type distribution, so the model remains a CTMC and can be analyzed by PRISM. The PRISM model we used is shown in Figure 2. We computed the steady-state distribution with various values of PRISM termination epsilon ϵ and the number of phases k.

```
ctmc

const int k;
const int qCapacity = 10;
const double timeout = 0.1;
const double lambda = 6.931471805599453094;

module main

qSize : [0..qCapacity] init 0;

[produce] (qSize <= qCapacity) -> (qSize' = min(qSize+1,qCapacity));
[consume] (qSize > 0) -> lambda: (qSize' = qSize - 1);

endmodule

module trigger

i : [1..k+1];

[] i < k -> k/timeout : (i'=i+1);
[produce] i = k -> k/timeout : (i'=1);

endmodule
```

Figure 2: PRISM CTMC model of the D/M/1/10 queue as shown in Figure 1 with a phase-type distribution approximating the arrival timeout. The phase-type module *trigger* is used as suggested on the PRISM website. Phase-type parameter k represents the number of phases. Increasing k should improve the approximation of the timeout.

2.1. Default absolute termination epsilon $\epsilon = 10^{-6}$

First, we used the PRISM default absolute termination epsilon $\epsilon = 10^{-6}$. The results are shown in Figure 3. For this ϵ, the obtained results are arguably insufficient. Increasing k does little to improve the precision, and only up to about $k \leq 50$, at which point increasing k further starts making the precision worse.

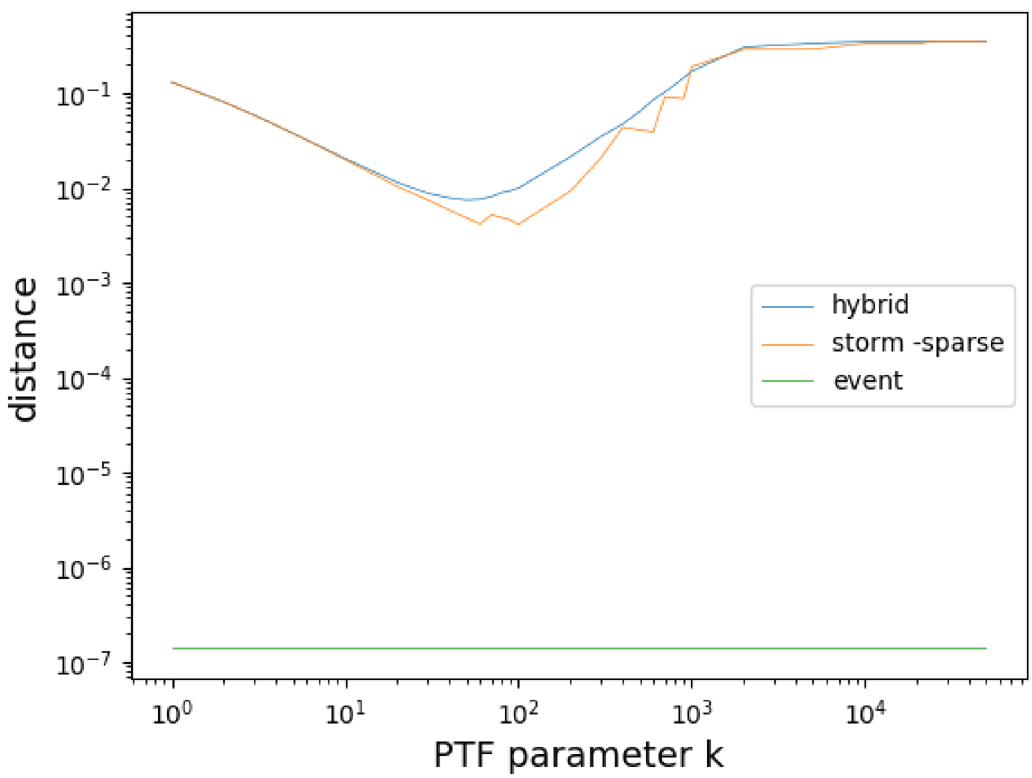

Figure 3: Maximum distance (infinity norm) of the steady-state probabilities from the reference high-precision ACTMC (10^{-20}) against various values of k for default absolute termination epsilon $\epsilon = 10^{-6}$. ACTMC results for $\epsilon = 10^{-6}$ are also compared to the reference and have a smaller error than the declared $\epsilon = 10^{-6}$.

2.2. Lowering absolute termination epsilon to $\epsilon = 10^{-10}$

For much lower $\epsilon = 10^{-10}$, the precision of the result improves only marginally in comparison to the default ϵ in Section 2.1. The results are shown in Figure 4.

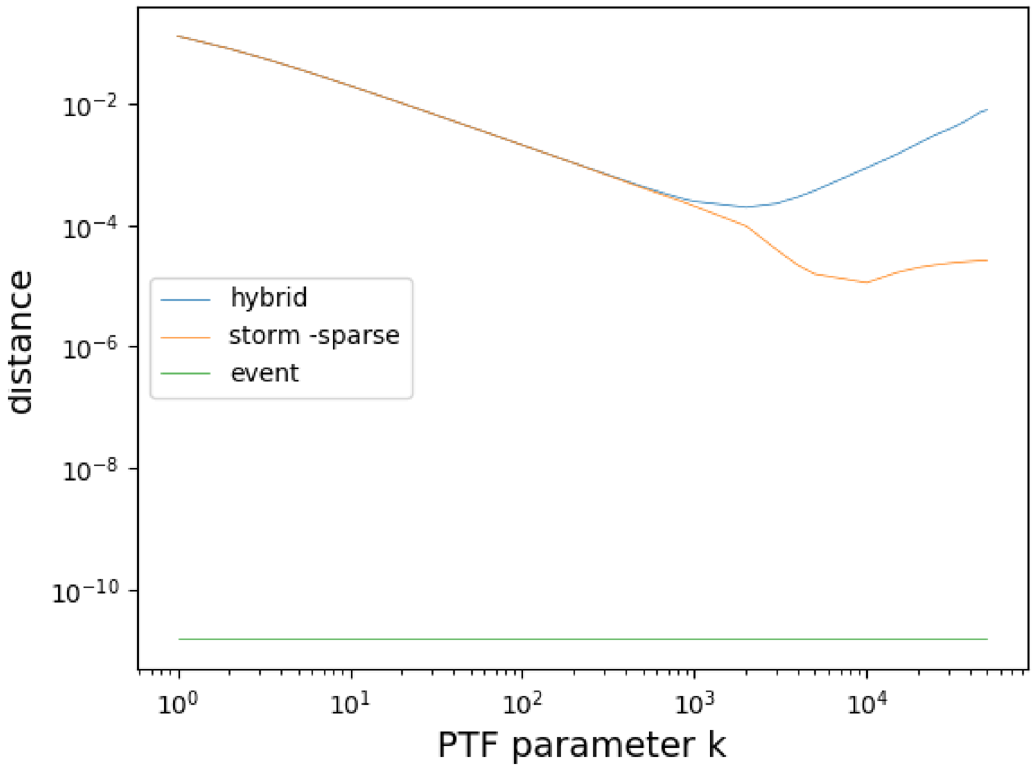

Figure 4: Maximum distance (infinity norm) of the steady-state probabilities from the reference high-precision ACTMC (10^{-20}) against various values of k for lowered absolute termination epsilon $\epsilon = 10^{-10}$. ACTMC results for $\epsilon = 10^{-10}$ are also compared to the reference and have a smaller error than the declared $\epsilon = 10^{-10}$.

2.3. Adjusting epsilon $\epsilon = 10^{-10}$ to k

Since increasing k increases the amount of phases (intermediate states) and each phase may have error up to ϵ, the actual potential error for each state of the queue is $\epsilon \cdot k$. To compensate for this, we devise a simple formula

$$\epsilon^{'} = \frac{\epsilon}{k}$$

where $\epsilon^{'}$ is the adjusted epsilon that should be given to PRISM if precision ϵ is desired for k phases. Using adjusted epsilon, the results get significantly better, as shown in Figure 5. What's more, the phase-type fitting results now seem to converge towards the reference steadily as k increases. This indicates that this approach is correct, and that the reference is trustworthy.

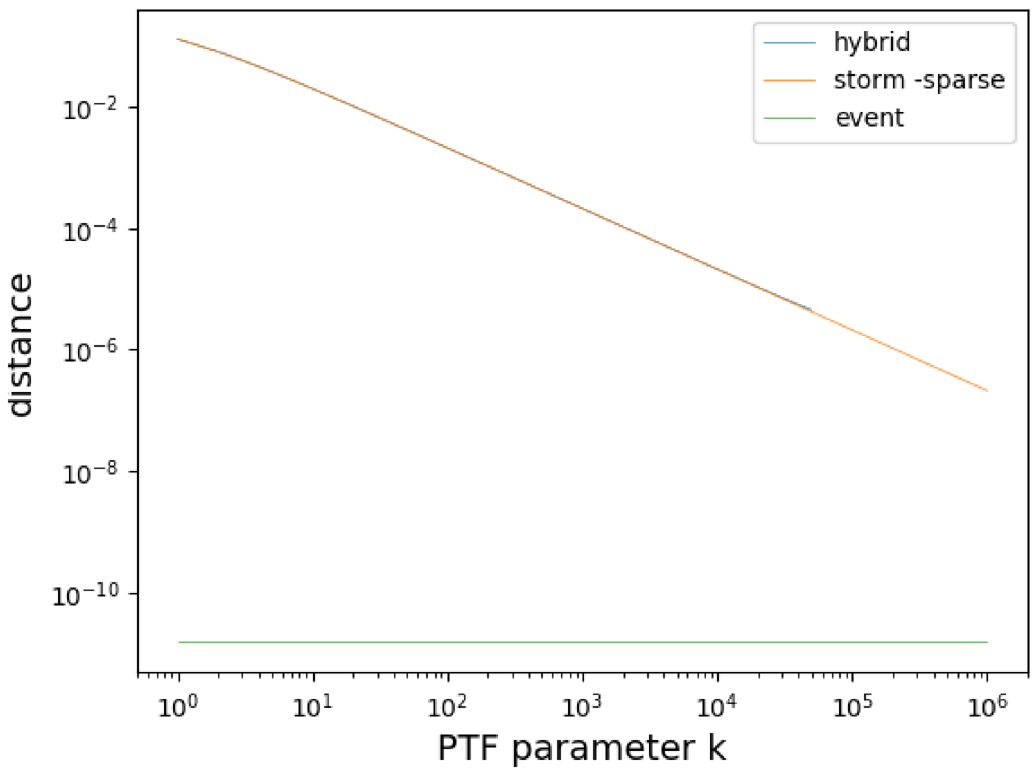

Figure 5: Maximum distance (infinity norm) of the steady-state probabilities from the reference high-precision ACTMC (10^{-20}) against various values of k for lowered and adjusted absolute termination epsilon $\epsilon = 10^{-10}$. ACTMC results for $\epsilon = 10^{-10}$ are also compared to the reference and have a smaller error than the declared $\epsilon = 10^{-10}$.

2.4. Required computation time to obtain precise results

Although we have shown it is possible to obtain relatively precise results using phase-type distributions, the precision comes at a rather steep cost in computation time. This is shown in Figure 6. Note that even though Storm is one of the fastest available tools, it still struggles to deliver good precision within reasonable time. The computations all ran on the same virtual machine under same conditions[1].

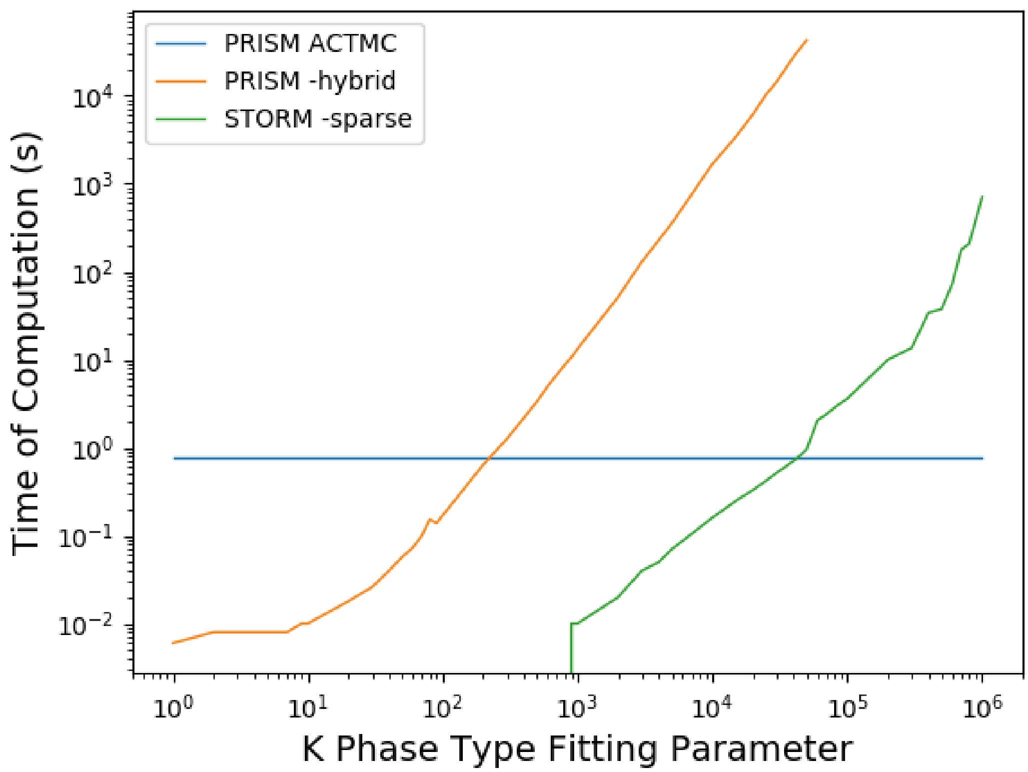

Figure 6: Time of computation in seconds of the steady-state probabilities against various values of k for lowered and adjusted absolute termination epsilon $\epsilon = 10^{-10}$, i.e. corresponding to the data shown in Figure 5. The computation of our PRISM ACTMC is constant in regards to k because it does not use phase-type fitting.

[1]Running in a virtual machine might have reduced the performance. The virtual machine we used is available at the Storm website.

3. Conclusion

We have presented a reliable method for high-precision analysis of PRISM CTMC models with phase-type distributions, that is, adjusting the termination epsilon by the number of phases. However, the large number of phases drastically increases the computation time.

For better performance, we suggest usage of more specialized tools that can deal with non-Markovian distributions directly, without phase-type fitting. Our PRISM ACTMC extension[2] has delivered results with sufficient precision a lot faster.

[1] M. Kwiatkowska, G. Norman, D. Parker, PRISM 4.0: Verification of probabilistic real-time systems, in: G. Gopalakrishnan, S. Qadeer (Eds.), Proc. 23rd International Conference on Computer Aided Verification (CAV'11), volume 6806 of *LNCS*, Springer, 2011, pp. 585–591.

[2] C. Dehnert, S. Junges, J. Katoen, M. Volk, A storm is coming: A modern probabilistic model checker, CoRR abs/1702.04311 (2017).

[3] P. Buchholz, J. Kriege, I. Felko, Input Modeling with Phase-Type Distributions and Markov Models: Theory and Applications, Springer Publishing Company, Incorporated, 2014.

[2] Our PRISM ACTMC extension is not fully completed yet and lacks optimization. It will be officially released later, but early development version is available at https://github.com/VojtechRehak/prism-gsmp.