öz:1. $y''' + 3y'' - 4y = 0$ dif. denk. genel çözümünü bulunuz.

$\Rightarrow \quad r^3 + 3r^2 - 4 = 0$

$(r-1) \cdot (r^2 + 4r + 4) = 0$

$(r-1)(r+2)^2 = 0$

$r_1 = 1 \qquad r_2 = r_3 = -2$

$$
\begin{array}{r|l}
r^3 + 3r^2 - 4 & r - 1 \\
\hline
-r^3 \pm r^2 & r^2 + 4r + 4 \\
\hline
4r^2 - 4 \\
-4r^2 \pm 4r \\
\hline
4r - 4 \\
4r - 4 \\
\hline
0
\end{array}
$$

$y_1 = e^x \qquad y_2 = e^{-2x} \quad y_3 = xe^{-2x}$

$\Rightarrow \qquad y = c_1 y_1 + c_2 y_2 + c_3 y_3$

$$\boxed{y = c_1 e^x + c_2 e^{-2x} + c_3 x e^{-2x}}$$ genel çözümü elde edilir.

öz:2. $y^{IV} + 2y'' + y = 0$ dif. denk. genel çözümünü bulunuz.

$\Rightarrow \qquad r^4 + 2r^2 + 1 = 0$

$(r^2 + 1)^2 = 0$

$r_1 = r_2 = i, \quad r_3 = r_4 = -i$

$y_1 = \cos x \qquad y_3 = \sin x$

$y_2 = x \cdot \cos x \qquad y_4 = x \cdot \sin x$

$\Rightarrow \quad y = c_1 y_1 + c_2 y_2 + c_3 y_3 + c_4 y_4$

$y = c_1 \cos x + c_2 x \cos x + c_3 \sin x + c_4 x \sin x$

$$\boxed{y = (c_1 + c_2 x) \cos x + (c_3 + c_4 x) \sin x}$$

elde edilir.

Ör:3. $y^{IV} - 3y''' + 3y'' - y' = 0$, $y(0) = y'(0) = 0$

$$y''(0) = y'''(0) = 1$$

başlangıç değer problemini çözünüz.

$\Rightarrow$ $r^4 - 3r^3 + 3r^2 - r = 0$ $\Rightarrow$ karakteristik denk.

$r(r^3 - 3r^2 + 3r - 1) = 0$

$r \cdot (r-1)^3 = 0$

$r_1 = 0 \qquad r_2 = r_3 = r_4 = 1$

$y_1 = e^{0x} = 1 \qquad y_2 = e^x \quad y_3 = xe^x \quad y_4 = x^2 e^x$

$y = c_1 y_1 + c_2 y_2 + c_3 y_3 + c_4 y_4$

$$\boxed{y = c_1 + c_2 e^x + c_3 x e^x + c_4 x^2 e^x} \quad \text{genel çözüm}$$

$y' = c_2 e^x + c_3 e^x + c_3 x e^x + 2c_4 x e^x + c_4 x^2 e^x$

$y'' = c_2 e^x + c_3 e^x + c_3 e^x + c_3 x e^x + 2c_4 e^x + 2c_4 x e^x + 2c_4 x e^x + c_4 x^2 e^x$

$y'' = c_2 e^x + c_3(2e^x + xe^x) + c_4(2e^x + 4xe^x + x^2 e^x)$

$y''' = c_2 e^x + c_3(3e^x + xe^x) + c_4(6e^x + 6xe^x + x^2 e^x)$

$\boxed{c_3 = 2}$

$\boxed{c_1 = 2}$

$\boxed{c_2 = -2}$

$0 = c_1 + c_2 + 0 + 0 \Rightarrow \boxed{c_1 + c_2 = 0}$

$0 = c_2 + c_3 + 0 + 0 + 0 \Rightarrow \boxed{c_2 + c_3 = 0}$

$\begin{array}{r} -2/\ 2c_4 - c_2 = 1 \\ 6c_4 - 2c_2 = 1 \\ \hline -4c_4 + 2c_2 = -2 \\ 6c_4 - 2c_2 = 1 \end{array}$

$1 = c_2 + c_3(2) + c_4(2) \Rightarrow \boxed{c_2 + 2c_3 + 2c_4 = 1}$

$2c_4 = -1$

$1 = c_2 + c_3(3) + c_4(6) \Rightarrow c_2 + 3c_3 + 6c_4 = 1$

$\boxed{c_4 = -1/2}$

$$y = c_1 + c_2 e^x + c_3 x e^x + c_4 x^2 e^x$$

$$\boxed{y = 2 - 2e^x + 2xe^x - \frac{1}{2}x^2 e^x}$$

özel çözümü bulunur.

ÖR:4 : $y''' - y = 0$ denk. genel çözümünü bulunuz.

$$\Rightarrow \quad r^3 - 1 = 0$$

$$(r-1)(r^2 + r + 1) = 0$$

$\boxed{r_1 = 1}$

$$\Delta = b^2 - 4ac$$

$$\Delta = 1 - 4 = -3 \quad \Rightarrow \quad r_2 = \frac{-b + \sqrt{\Delta}}{2a} = \boxed{\frac{-1 + i\sqrt{3}}{2}}$$

$$\boxed{r_3 = \frac{-1 - i\sqrt{3}}{2}}$$

$$y_1 = e^x \qquad y_2 = e^{-x/2}\cos\left(\frac{\sqrt{3}}{2}x\right)$$

$$y_3 = e^{-x/2}\sin\left(\frac{\sqrt{3}}{2}x\right)$$

$$\Rightarrow \boxed{y = c_1 e^x + c_2 e^{-x/2}\cos\left(\frac{\sqrt{3}}{2}x\right) + c_3 e^{-x/2}\sin\left(\frac{\sqrt{3}}{2}x\right)} \quad \text{elde edilir.}$$

ÖR:5 : $y''' - 6y' + 9y = 0$ denk. genel çöz. bulunuz.

$$\Rightarrow \quad r^3 - 6r + 9 = 0$$

$$(r+3)(r^2 - 3r + 3) = 0$$

$\boxed{r_1 = -3}$ $\Delta = 9 - 12 = -3$

$$\boxed{r_2 = \frac{3 + i\sqrt{3}}{2}}$$

$$\boxed{r_3 = \frac{3 - i\sqrt{3}}{2}}$$

$$\begin{array}{r|l} r^3 - 6r + 9 & \underline{r + 3} \\ -r^3 \mp 3r^2 & r^2 - 3r + 3 \\ \hline -3r^2 - 6r + 9 & \\ -\mp 3r^2 \mp 9r & \\ \hline 3r + 9 & \\ 3r + 9 & \\ \hline 0 & \end{array}$$

$$y_1 = e^{-3x}, \quad y_2 = e^{3x/2}\cos\left(\tfrac{\sqrt{3}}{2}x\right) \quad y_3 = e^{3x/2}\sin\left(\tfrac{\sqrt{3}}{2}x\right) \quad \text{(20)}$$

$$y = c_1 e^{-3x} + c_2 e^{3x/2}\cos\left(\tfrac{\sqrt{3}}{2}x\right) + c_3 e^{3x/2}\sin\left(\tfrac{\sqrt{3}}{2}x\right)$$

elde edilir

__ör:6:__ $\quad y''' - 8y = 0 \quad , \quad y(0) = 0, \; y'(0) = 1, \; y''(0) = 0$

başlangıç değer problemini çözünüz.

$\Rightarrow \quad r^3 - 8 = 0 \quad \Rightarrow$ karak. denk.

$$r^3 - 2^3 = 0$$

$$y_1 = e^{2x} \qquad y_2 = e^{-y}\cos\sqrt{3}x$$
$$y_3 = e^{-x}\sin\sqrt{3}x$$

$$(r-2)(r^2 + 2r + 4) = 0$$

$\boxed{r_1 = 2} \qquad \Delta = 4 - 16 = -12$

$$r_2 = \frac{-2 + i2\sqrt{3}}{2} \quad \boxed{= -1 + i\sqrt{3}}$$

$$\boxed{r_3 = -1 - i\sqrt{3}}$$

$$y = c_1 y_1 + c_2 y_2 + c_3 y_3 = c_1 e^{2x} + c_2 e^{-x}\cos(\sqrt{3}x) + c_3 e^{-x}\sin(\sqrt{3}x)$$

elde edilir.

$\boxed{y(0) = c_1 + c_2 = 0}$

$$y'(0) = 2c_1 e^{2x} - c_2 e^{-x}\cos(\sqrt{3}x) - \sqrt{3}c_2 e^{-x}\sin(\sqrt{3}x) - c_3 e^{-x}\sin(\sqrt{3}x) + \sqrt{3}c_3 e^{-x}\cos\sqrt{3}x$$

$\boxed{1 = 2c_1 - c_2 + \sqrt{3}c_3} \qquad \boxed{0 = 4c_1 + 2c_2 - 2\sqrt{3}c_3}$

$$y''(0) = 4c_1 e^{2x} + c_2 e^{-x}\cos(\sqrt{3}x) + \sqrt{3}c_2 e^{-x}\sin(\sqrt{3}x) + \sqrt{3}c_2\sin(\sqrt{3}x) - 3c_2 e^{-x}\cos\sqrt{3}x$$
$$+ c_3 e^{-x}\sin(\sqrt{3}x) - \sqrt{3}c_3 e^{-x}\cos(\sqrt{3}x) + \sqrt{3}c_3 e^{-x}\cos\sqrt{3}x - 3c_3 e^{-x}\sin\sqrt{3}x$$

$$-2 = -4c_1 + 2c_2 - 2\sqrt{3}\,c_3$$

$$0 = 4c_1 - 2c_2 - 2\sqrt{3}\,c_3$$

$$-2 = -4\sqrt{3}\,c_3$$

$$c_3 = \frac{1}{2\sqrt{3}} \qquad \Rightarrow \qquad \boxed{c_3 = \frac{\sqrt{3}}{6}}$$

$$4c_1 - 2c_2 = 1.$$

$$2c_1 + 2c_2 = 0$$

$$\overline{}$$

$$6c_1 = 1$$

$$\boxed{c_1 = \tfrac{1}{6}} \qquad\qquad \boxed{c_2 = -\tfrac{1}{6}}$$

$$\boxed{y = \frac{1}{6}e^{2x} - \frac{1}{6}e^{-x}\cos(\sqrt{3}\,x) + \frac{\sqrt{3}}{6}e^{-x}\sin(\sqrt{3}\,x)}$$

özel çöz elde edilir.

ÖDEV SORULAR

1) $y^{IV} - 5y'' + 4y = 0$

2) $y^{VI} - 2y^{V} + 2y^{IV} - 2y''' + y'' = 0$

3) $y'' - 3y' + 2 = 0$

4) $y''' - 3y'' + 3y' - y = 0$

5) $y^{V} + 3y''' - 4y' = 0$