

```
%% Exercises 1.1.3 Question a:
format long %This shows the format of the output
p=355/113; %This is a much better rational approximation to pi than 22/7
that is assumed
pi=22/7; %This assumes pi is 22/7
abs_accuracy=abs(p-pi) %This is the code showing the output of absolute
accuracy
```

```
abs_accuracy =
    0.001264222503160
```

```
rel_accuracy=abs(p-pi)/pi %This is the code showing the output of relative
accuracy
```

```
rel_accuracy =
    4.022526146419050e-04
```

```
accurate_digits=-log(rel_accuracy) %This is the code of the formula showing
the output of the number of accurate digits
```

```
accurate_digits =
    7.818430272070881
```

```
%% Exercises 1.1.3 Question b:
format long %This shows the format of the output
p=103638/32989; %This is a much better rational approximation to pi than
22/7 that is assumed
pi=22/7; %This assumes pi is 22/7
abs_accuracy=abs(p-pi) %This is the code of the formula showing the output
of absolute accuracy
```

```
abs_accuracy =
    0.001264490760990
```

```
rel_accuracy=abs(p-pi)/pi %This is the code of the formula showing the
output of relative accuracy
```

```
rel_accuracy =
    4.023379694057679e-04
```

```
accurate_digits=-log(rel_accuracy) %This is the code of the formula showing
the output of the number of accurate digits
```

```
accurate_digits =
    7.818218102637443
```

Exercise 1.1.4 a

According to IEEE 754 single precision, 23 binary bits are used for the mantissa  $1+f$  in which  $f=2^{(-23)}$ .

Therefore, the first single precision number greater than 1 is  $1 + 2^{-23}$ . In converting  $1 + 2^{-23}$  to base 10, we insert into MATLAB and get the following:

```
1+2^(-23)
```

```
ans =  
1.000000119209290
```

In base-10 terms, the first single precision number greater than 1 in this system is  $1+2^{(-23)}=1.000000119209290$

## Section 1.2 Exercise 1.2.1(b)

Solving for the conditional number of  $\log(x)$

Assume  $f(x) = \log(x)$

Then the conditional number of  $f(x) = \log(x)$  is:

$$k_f(x) = \left| \frac{x * f'(x)}{f(x)} \right|$$

The derivative of  $f(x)$  is:

$$f'(x) = \frac{d}{dx} \log(x) = \frac{1}{x}$$

Replace  $f'(x)$  and  $f(x)$  by  $\frac{1}{x}$  and  $\log(x)$  respectively in the formula for conditional number

Therefore we have:

$$k_f(x) = \left| \frac{x * \frac{1}{x}}{\log(x)} \right|$$

Which means that the conditional number,  $k_f(x)$  is:

$$k_f(x) = \frac{1}{|\log(x)|}$$

As  $x$  approaches 1 from either side,  $\log(x) \rightarrow 0$ .  $k_f(x) = \frac{1}{|\log(x)|}$  is very large when  $\log(x)$  is very close to 0. This happens precisely as  $x$  approaches 1. Therefore the condition number is large when  $x \approx 1$ , making conditioning poor as  $x$  approaches 1.

## Section 1.2, Exercise 1.2.6

In deriving an expression for the relative condition number of a root of  $ax^2 + bx + c = 0$ , due to perturbations in  $b$  only, we will pick one root  $r$  and first consider what happens as we vary the co-efficient  $b$ . This suggests a scalar function  $f(x) = b$

Therefore will we use the technique of implicit differentiation to find  $\frac{dr}{db}$ , while  $a$  and  $c$  are held fixed. Taking  $\frac{d}{db}$  of both sides and applying the chain rule, we get:

$$\frac{d}{db}(ar^2 + br + c = 0)$$

$$\frac{d}{db}(ar^2) + \frac{d}{db}(br) + \frac{dc}{db} = 0$$

$$a * 2r * \frac{dr}{db} + r + b * \frac{dr}{db} = 0$$

Collect like terms and solve for  $\frac{dr}{db}$

Solving for the derivative, we get

$$\frac{dr}{db}(2ar + b) = -r$$

$$\frac{dr}{db} = \frac{-r}{(2ar + b)}$$

$$\frac{dr}{db} = \frac{-r}{2ar + b} = \frac{-r}{\pm \sqrt{b^2 - 4ac}}$$

where in the last step I have applied the quadratic formula for root  $r$ . Finally, the condition number for the problem  $f(b) = r$  is :

$$k(b) = \left| \frac{b}{r} * \frac{-r}{\pm \sqrt{b^2 - 4ac}} \right|$$

$$k(b) = \left| \frac{-b}{\sqrt{b^2 - 4ac}} \right|$$

$$k(b) = \left| \frac{b}{\sqrt{b^2 - 4ac}} \right|$$

We can expect poor conditioning for small discriminants, i.e, near double roots.