Muhideen Ogunlowo Math426 Assignment 2

```
%% Exercises 1.1.3 Question a:
format long %This shows the format of the output
p=355/113; %This is a much better rational approximation to pi than 22/7
that is assumed
pi=22/7; %This assumes pi is 22/7
abs_accuracy=abs(p-pi) %This is the code showing the output of absolute
accuracy
```

```
abs_accuracy =
   0.001264222503160
```

```
rel_accuracy=abs(p-pi)/pi %This is the code showing the output of relative
accuracy
```

```
rel_accuracy =
     4.022526146419050e-04
```

```
accurate_digits=-log(rel_accuracy) %This is the code of the formula showing
the output of the number of accurate digits
```

```
accurate_digits =
   7.818430272070881
```

```
%% Exercises 1.1.3 Question b:
format long %This shows the format of the output
p=103638/32989; %This is a much better rational approximation to pi than
22/7 that is assumed
pi=22/7; %This assumes pi is 22/7
abs_accuracy=abs(p-pi) %This is the code of the formula showing the output
of absolute accuracy
```

```
abs_accuracy =
   0.001264490760990
```

```
rel_accuracy=abs(p-pi)/pi %This is the code of the formula showing the
output of relative accuracy
```

```
rel_accuracy =
     4.023379694057679e-04
```

```
accurate_digits=-log(rel_accuracy) %This is the code of the formula showing
the output of the number of accurate digits
```

```
accurate_digits =
   7.818218102637443
```

Exercise 1.1.4 a

According to IEEE 754 single precision, 23 binary bits are used for the mantissa 1+f in which f=2^(-23). Therefore, the first single precision number greater than 1 is $1+2^{-23}$. In converting $1+2^{-23}$ to base 10, we insert into MATLAB and get the following:

```
1+2^(-23)
```

```
ans =
   1.000000119209290
```

In base-10 terms, the first single precision number greater than 1 in this system is 1+2^(-23)= 1.000000119209290

Section 1.2 Exercise 1.2.1(b)

$f(x) = \log(x)$

$k_f(x) = \left|\frac{x * f'(x)}{f(x)}\right|$

$f'(x) = \frac{\mathrm{d}}{\mathrm{d}x}\log(x) = \frac{1}{x}$

$k_f(x) = \left|\frac{\frac{1}{x} * x}{\log(x)}\right|$

$k_f(x) = \frac{1}{|\log(x)|}$

As x approaches 1 from either side, $\log(x)$ --> 0. $k_f(x) = \frac{1}{|\log(x)|}$ is very large when $\log(x)$ is very close to 0. This happens precisely as x approaches 1. Therefore the condition number is large when x≈1, making conditioning poor as x approaches 1.

Section 1.2, Exercise 1.2.6

In deriving an expression for the relative condition number of a root of $\mathrm{ax}^2 + \mathrm{bx} + c = 0$, due to perturbations in b only, we will pick one root r and first consider what happens as we vary the co-efficient b. This suggests a scalar function $f(x) = b$

Therefore will we use the technique of implicit differentiation to find $\frac{\mathrm{dr}}{\mathrm{db}}$, while a and c are held fixed. Taking $\frac{d}{\mathrm{db}}$ of both sides and applying the chain rule, we get:

$$\frac{d}{\mathrm{db}}(\mathrm{ar}^2 + \mathrm{br} + c = 0)$$

$$\frac{d}{\mathrm{db}}(\mathrm{ar}^2) + \frac{d}{\mathrm{db}}(\mathrm{br}) + \frac{\mathrm{dc}}{\mathrm{db}} = 0$$

$$a * 2r * \frac{\mathrm{dr}}{\mathrm{db}} + r + b * \frac{\mathrm{dr}}{\mathrm{db}} = 0$$

Collect like terms and solve for $\frac{\mathrm{dr}}{\mathrm{db}}$

Solving for the derivative, we get

$$\frac{\mathrm{dr}}{\mathrm{db}}(2\mathrm{ar} + b) = -r$$

$$\frac{\mathrm{dr}}{\mathrm{db}} = \frac{-r}{(2\mathrm{ar} + b)}$$

$$\frac{\mathrm{dr}}{\mathrm{db}} = \frac{-r}{2\mathrm{ar} + b} = \frac{-r}{\pm\sqrt{b^2 - 4\mathrm{ac}}}$$

where in the last step I have applied the quadratic formula for root r. Finally, the condition number for the problem $f(b) = r$ is :

$$k(b) = \left|\frac{b}{r} * \frac{-r}{\pm\sqrt{b^2 - 4\mathrm{ac}}}\right|$$

$$k(b) = \left|\frac{-b}{\sqrt{b^2 - 4\mathrm{ac}}}\right|$$

$$k(b) = \left|\frac{b}{\sqrt{b^2 - 4\mathrm{ac}}}\right|$$

We can expect poor coniditioning for small discriminants, i.e, near double roots.