Muhideen Ogunlowo Homework 6

Exercise 3.1.1

Given $y = f(x)$ is a differentiable non-negative real function, the differential, $\frac{d}{dx}(y) = f'(x)$ can be evaluated at a critical point x where $f'(x) = 0$ and it's minimum at a point where $f''(x) \geq 0$

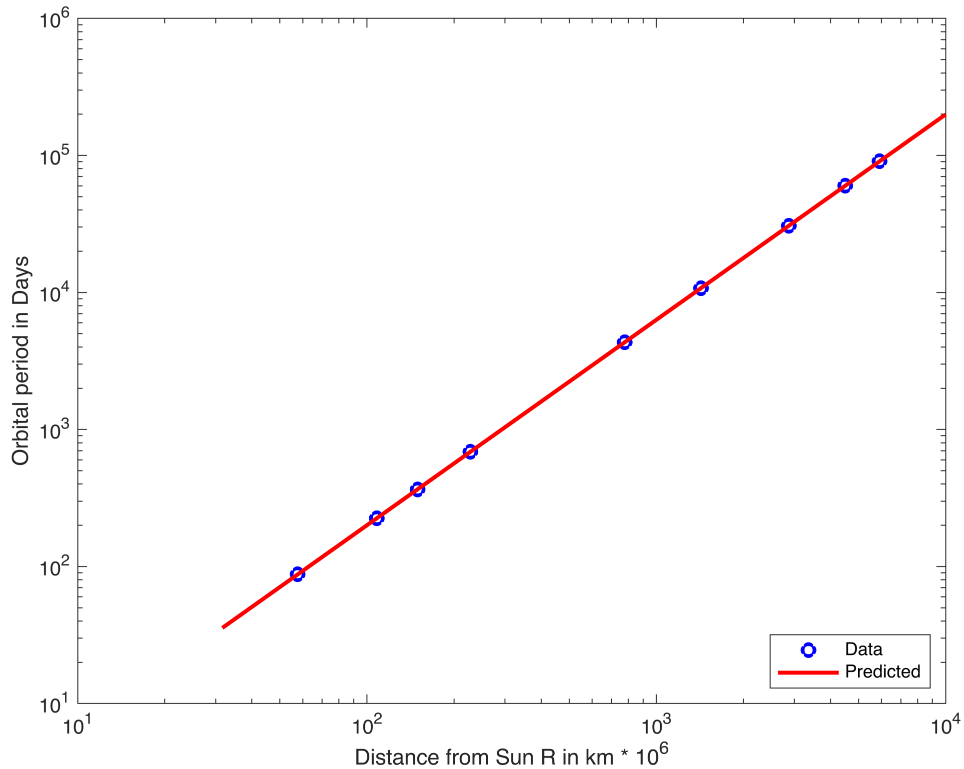
Given $f(x)$ is a differentiable non-negative real function, it's square,represented by the equation $p(x) = [f(x)]^2$ and differential of it's square represented by the equation $p'(x) = 2 * f'(x) * f(x)$ can be evaluated at a critical point where $[[f(x)]^2]' = 2 * f'(x) * f(x) = 0$. We can also determine that the second derivative of p(x): $p''(x) = 2 * (f'(x)^2 + 2f(x)f''(x)) \geq 0$. Hence by inspection, it is very clear that x is a local minima on both $y = f(x)$ and $p(x) = [f(x)]^2$

Exercise 3.1.6

```
close all, clear all, format compact, format short g  clc
%Data given from textbook
tau=[87.99;224.7;365.26;686.98;4332.4;10759;30684;60188;90710];
R=[57.59;108.11;149.57;227.84;778.14;1427;2870.3;4499.9;5909];
%logarithm of R and Tau
Q=log(R);
P=log(tau);
V=[Q.^0,Q]; %fitting matrix
C=V\P; %coefficients of linear fit
```

```
C = 2×1
     -1.6032
      1.4989
```

```
A=exp(C(1))
B= C(2)
RPlot=logspace(1.5,4,100);
TauP=A*RPlot.^B;
loglog(R,tau,'Ob',RPlot,TauP,'r-',"LineWidth",2); %loglogplot of R, tau,
RPlot,Tau as a function of A*R^B
%label of x and y-axis
xlabel('Distance from Sun R in km * 10^6'), ylabel('Orbital period in
Days')
legend("Data", "Predicted",'location', 'southeast')
```



Exercise 3.2.1

```
A=[2,-1;0,1;2,2]; %A given from textbook
```

```
A = 3×2
     2    -1
     0     1
     2     2
```

```
b=[1;-5;6]; %b given from textbook
```

```
b = 3×1
     1
    -5
     6
```

```
x=lsnormal(A,b) %application of least squares function
```

```
x = 2×1
       1.6364
      0.45455
```

Exercise 3.2.2

```
A=[1, -2,3]' %Matrix to be worked on
```

```
A = 3×1
     1
```

```
   -2
    3
```

```
Q=pinv(A) %pseudoinverse function
```

```
Q = 1×3
    0.071429     -0.14286      0.21429
```

```
R= (A'*A).^(-1)*(A')  %pseudoinverse formula
```

```
R = 1×3
    0.071429     -0.14286      0.21429
```

Exercise 3.2.4

If A is a non singular square matrix, then a is invertible

The Pseudoinverse of a matrix which is given to us as

$$A^{+} = (A^{T}A)^{-1}A^{T}$$

$$(A^{T}A)^{-1} = A^{-1}A^{T^{-1}}$$

$$A^{+} = A^{-1}{*}A^{T^{-1}} * A^{T}$$

$$A^{+} = A^{-1} * I$$

$$A^{+} = A^{-1}$$

Exercise 3.2.8 a

```
format long
m=10;
t= linspace(0,2*pi,m+1)';
beta=[1+10.^(0:-1:-8)]';%Betas starting from 2
for i= 1:length(beta); %for loop
    A=[t.^0, sin(t), cos(beta(i).*t)];
    CondNPartA(i,1)=cond(A)'; %condition number
end %end of loop
table(beta,CondNPartA) %table of beta coresponding to it's condition number
```

ans = 9×2 table

	beta	CondNPartA
1	2	1.4961672636...
2	1.1000000000...	1.7337953665...
3	1.0100000000...	1.5029635829...
4	1.0010000000...	1.4964624969...
5	1.0001000000...	1.4961927477...
6	1.0000100000...	1.4961697715...

	beta	CondNPartA
7	1.0000010000...	1.4961675140...
8	1.0000001000...	1.4961672886...
9	1.0000000100...	1.4961672661...

Exercise 3.2.8 b

```
format long
m=10;
t= linspace(0,2*pi,m+1)';
beta=[1+10.^(0:-1:-8)]';%Betas starting from 2
for i= 1:length(beta); %for loop
    A=[t.^0, sin(t).^2, cos(beta(i).*t).^2];
    CondNPartB(i,1)=cond(A)'; %condition numbers
end %end of loop
table(beta,CondNPartB) %table of beta coresponding to it's condition number
```

ans = 9×2 table

	beta	CondNPartB
1	2	4.577690725501474
2	1.1000000000...	8.922922247114583
3	1.0100000000...	88.894691866495...
4	1.0010000000...	8.859238289884349e+02
5	1.0001000000...	8.855895689529709e+03
6	1.0000100000...	8.855558325159177e+04
7	1.0000010000...	8.855524558813430e+05
8	1.0000001000...	8.855521175786896e+06
9	1.0000000100...	8.855520896240121e+07

Exercise 3.2.8 c

```
betaSubtract1=beta-1; %table of beta minus 1
table(betaSubtract1,CondNPartA,CondNPartB) %table of beta-1, condition
numbers of part a&b
```

ans = 9×3 table

	betaSubtract1	CondNPartA	CondNPartB
1	1	1.4961672636...	4.577690725501474
2	0.100000000000000	1.7337953665...	8.922922247114583
3	0.010000000000000	1.5029635829...	88.894691866495...
4	0.001000000000000	1.4964624969...	8.859238289884349e+02

	betaSubtract1	CondNPartA	CondNPartB
5	0.000100000000000	1.4961927477...	8.855895689529709e+03
6	0.000010000000000	1.4961697715...	8.855558325159177e+04
7	0.000001000000000	1.4961675140...	8.855524558813430e+05
8	0.000000100000000	1.4961672886...	8.855521175786896e+06
9	0.000000010000000	1.4961672661...	8.855520896240121e+07

The condition number grows in part(b) because the columns are getting closer and closer to being dependent. If $\beta = 1$, the cloumns will then be a linear combination of each other for part (b). This is because of the trigonometric identity $\sin^2(t) + \cos^2(t) = 1$. Part a is not close to being an identity, so the columns are not close to being linearly dependent.

Exercise 3.3.1

Using Part 2 of Theorem 3.3.1 we know that Q, a matrix with orthonormal columns, has the following property:

$$||\mathrm{Qx}||_2 = ||x||_2$$

$$||Q||_2 = \max_{||x||_2=1} ||\mathrm{Qx}||_2$$

$$\max_{||x||_2=1} ||\mathrm{Qx}||_2 = \max_{||x||_2=1} ||x||_2 = 1$$

Therefore,

$$||Q||_2 = \max_{||x||_2=1} ||x||_2 = 1$$

$$||Q||_2 = 1$$

Exercise 3.3.3

```
m=400;
t=linspace(-1,1,m+1)';
V=[t.^0 t t.^2, t.^3,t.^4];%Vandermode matrix
```

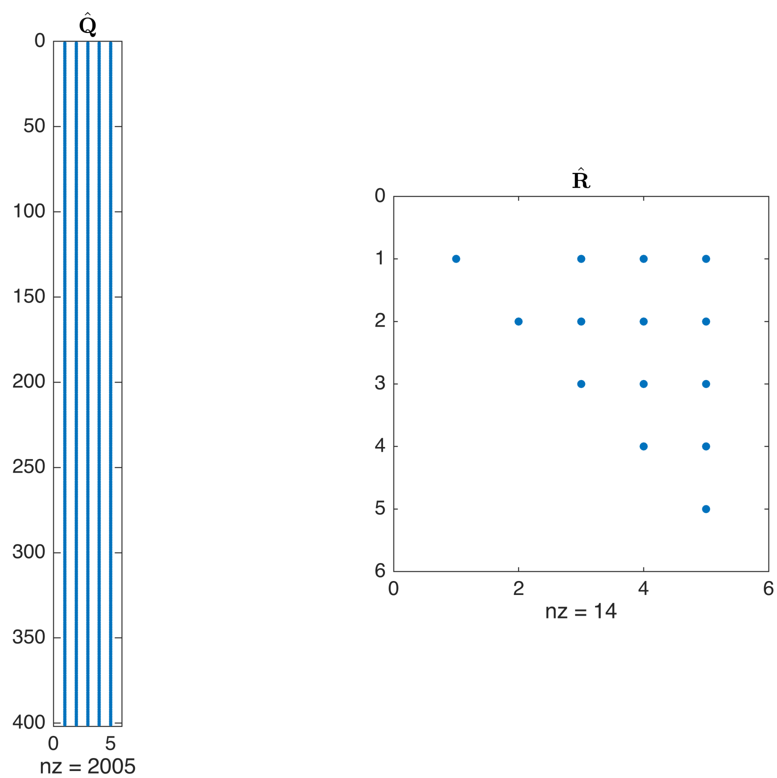
```
V = 401×5
   1.000000000000000  -1.000000000000000   1.000000000000000  -1.000000000000000 ···
   1.000000000000000  -0.995000000000000   0.990025000000000  -0.985074875000000
   1.000000000000000  -0.990000000000000   0.980100000000000  -0.970299000000000
   1.000000000000000  -0.985000000000000   0.970225000000000  -0.955671625000000
   1.000000000000000  -0.980000000000000   0.960400000000000  -0.941192000000000
   1.000000000000000  -0.975000000000000   0.950625000000000  -0.926859375000000
   1.000000000000000  -0.970000000000000   0.940900000000000  -0.912673000000000
   1.000000000000000  -0.965000000000000   0.931225000000000  -0.898632125000000
   1.000000000000000  -0.960000000000000   0.921600000000000  -0.884736000000000
   1.000000000000000  -0.955000000000000   0.912025000000000  -0.870983875000000
       ⋮
```

```
[Qhat, Rhat]= qr(V,0)
```

```
Qhat = 401×5
```

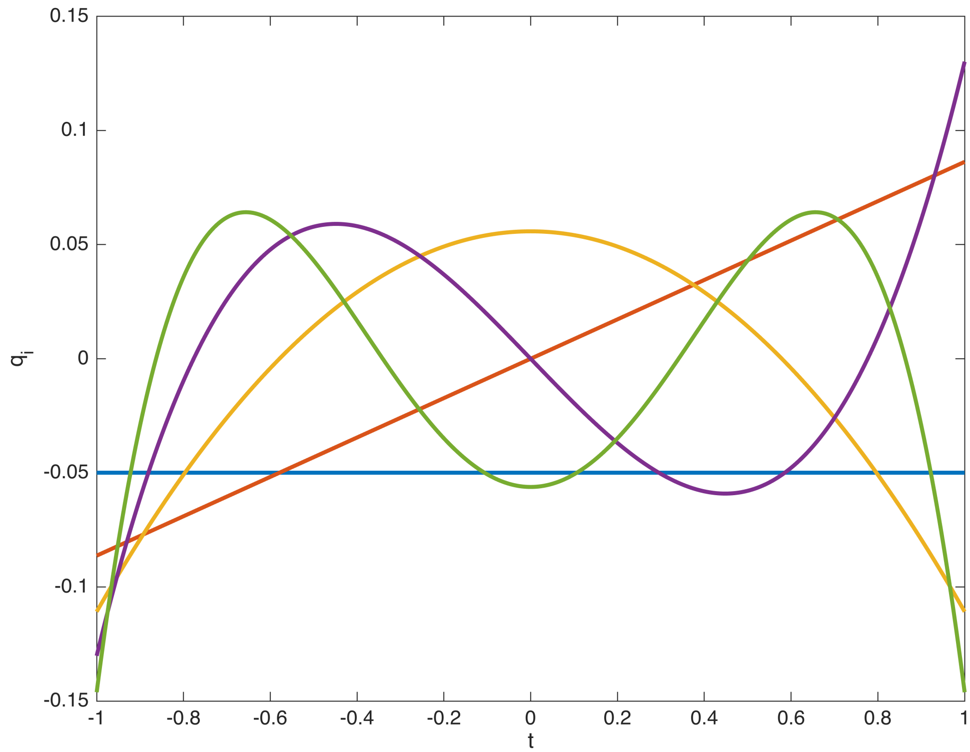
```
  -0.049937616943892  -0.086279061053211  -0.110831627270585  -0.130160312035993 ···
  -0.049937616943892  -0.085847665747944  -0.109169152861526  -0.126255502674913
  -0.049937616943892  -0.085416270442678  -0.107515011657526  -0.122399625761968
  -0.049937616943892  -0.084984875137412  -0.105869203658583  -0.118592435405456
  -0.049937616943892  -0.084553479832146  -0.104231728864698  -0.114833685713680
  -0.049937616943892  -0.084122084526880  -0.102602587275871  -0.111123130794941
  -0.049937616943892  -0.083690689221614  -0.100981778892102  -0.107460524757539
  -0.049937616943892  -0.083259293916348  -0.099369303713390  -0.103845621709774
  -0.049937616943892  -0.082827898611082  -0.097765161739737  -0.100278175759949
  -0.049937616943892  -0.082396503305816  -0.096169352971142  -0.096757941016363
     ⋮

Rhat = 5×5
 -20.024984394500787                   0  -6.708369772157765  -0.000000000000000 ···
                   0  11.590297766583028  -0.000000000000000   6.988891541461234
                   0                   0  -6.000092359705807   0.000000000000000
                   0                   0                   0   3.050123296341027
                   0                   0                   0                   0
```

```
figure%inserting figure
subplot(1,2,1)%subplot
spy(Qhat)%display of Qhat
title('$\hat{\mathbf{Q}}$',"Interpreter", 'latex') %title
subplot(1,2,2)
spy(Rhat)%display of Rhat
title('$\hat{\mathbf{R}}$',"Interpreter", 'latex') %title
```




```
figure %inserts figure
```

```
plot(t,Qhat,"LineWidth",2)%plot on graph
xlabel('t'),ylabel("q_i") %labeling
```



Exercise 3.3.6

Suppose A is an m x n matrix with m > n.

A= $\hat{Q}\hat{R}$ (thin QR factorization)

Therefore: $\hat{Q}$ is an m x n ONC matrix, $\hat{R}$ is an n x n, upper triangular matrix.

$$A^T = \left(\widehat{Q}\widehat{R}\right)^T$$
$$A^T = \widehat{R}^T\widehat{Q}^T$$
$$A^+ = (A^TA)^{-1}A^T$$
$$A^+ = \left(\widehat{R}^T\widehat{Q}^TQ\widehat{R}\right)^{-1}\widehat{R}^T\widehat{Q}^T$$
$$\widehat{Q}^T\widehat{Q} = I$$

Therefore :

$$A^+ = \left(\widehat{R}^TI\widehat{R}\right)^{-1}\widehat{R}^T\widehat{Q}^T$$
$$A^+ = \left(\widehat{R}^T\widehat{R}\right)^{-1}\widehat{R}^T\widehat{Q}^T$$
$$A^+ = \widehat{R}^{-1}\widehat{R}^{T^{-1}}\widehat{R}^T\widehat{Q}^T$$
$$A^+ = \widehat{R}^{-1}I\widehat{Q}^T$$
$$A^+ = \widehat{R}^{-1}\widehat{Q}^T$$

Exercise 3.2.1

```
function x = lsnormal (A,b)
% LSNORMAL Solve linear least squares by normal equations.
%Input:
%A coefficient matrix (m by n, m>n)
%b right-hand side (m by 1)
%Output:
% x minimizer of ||b-Ax||
N= A'*A;
Z=A'*b;
R=chol(N);
w=forwardsub(R',Z);
x=backsub (R,w);

end
```