Holographic duality beyond AdS/CFT

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Contents

0 Introduction

Since the discovery of AdS/CFT duality [1], we have greatly furthered our understanding of quantum gravity in asymptotically AdS backgrounds. However, there are plenty of non-AdS geometries in our real worlds, including:

- The Kerr metric of a rotation black hole, which ... ;
- The asymptotically flat / Minkowski spacetime, which well approximates our current universe at a smaller length (and time) scale;
- The asymptotically de Sitter spacetime, which well approximates our current (and future) universe at a larger length scale, and also during the period of inflation;
- The FRW metric (or more precisely, the Friedmann-Lemaître-Robertson-Walker [2–5] metric), which describes the evolution of our homogeneous and isotropic universe from the big bang to its ... ;
- and more ...

Much less is known about quantum gravity in these backgrounds.

1 Brief review of $\mathrm{AdS}_3/\mathrm{CFT}_2$

2 Bottom-up approach: from asymptotic symmetry

3 Top-down approach: from string theory

String theory is a self-consistent theory of quantum gravity.

The first example of a microscopic counting of black hole entropy, discovered by Strominger-Vafa [6], comes from the D1-D5-P system in string theory.

The first incarnation of holographic principle was realized by Maldacena [1], by a stack of D3 branes in type IIB string theory.

3.1 The D1-D5-P system and its IR limit

Let us look at the D1-D5-P brane configuration in type IIB string theory. This is well-reviewed in [7]. This configuration allows for an open string description and a closed string description.

Geometry	$\mathbb{R}^{4,1}$		S^1	$\mathcal{M}_4 = T^4, \mathrm{K3}$			
Direction	0	1, 2, 3, 4	5	6	7	8	9
$\#\,\mathrm{D5} = Q_5$	$\times$		$\times$	$\times$	$\times$	$\times$	$\times$
$\#\,\mathrm{D1} = Q_1$	$\times$		$\times$				
P	$\times$		$\times$				

Table 1. Brane configuration of the D1-D5-P system. Here we are considering type IIB string theory on flat 6D spacetime, with a compactified $x^5 \in S^1$ direction, along with an internal $\mathcal{M}_4$ manifold. We use "$\times$" to mark the directions x^μ that an object occupies. Here $\mu = 0, 1, \cdots, 9$.

The D5 branes wrap the compact $\mathcal{M}_4$, while the D1 branes are localized on $\mathcal{M}_4$. Both the D1 and D5 branes extend along the fifth direction x^5, which is compactified to a circle S^1 with a large radius.

Open string excitations on the branes carry momentum and winding. Due to the large radius of S^1, we can focus on the momentum modes P along $x^5 \in S^1$ and neglect the winding modes. On the other hand, we will neglect momentum modes along the $\mathcal{M}_4$ directions, since $\mathcal{M}_4$ is assumed to be compact and small.

3.1.1 IR limit and the AdS_3 gravity

In the IR limit, type IIB string theory is described by the low energy effective action of type IIB supergravity. The field content and the action of type IIB supergravity are well reviewed in the literature; see e.g. Appendix H of [8]. In particular, there is a pair of

2-form gauge potentials in type IIB supergravity. One of them is the NS-NS field B_2, and the other is the R-R field C_2. The D1 branes are electrically charged under C_2, while the D5 branes are magnetically charged under C_2.

The bosonic part of the string frame action is then given by [citations needed]:

$$\frac{1}{16\pi G}\int \mathrm{d}^{10}x\,\sqrt{-g}\left(e^{-2\phi}\left(R+4\,(\nabla\phi)^2-\frac{1}{12}H^2\right)-\frac{1}{12}F^2\right), \tag{3.1}$$

$$H=\mathrm{d}B_2\,,\quad F=\mathrm{d}C_2 \tag{3.2}$$

where H and F are the 3-form field strengths, $H^2=H_{\mu\nu\rho}H^{\mu\nu\rho}\propto H\wedge\star H$, and similar for F^2. After dimension reduction of the compact $\mathcal{M}_4$, the equations of motion admit a *black string* solution in 6D, where the metric is given by [citations needed]:

WL: convention for the coefficients?

$$\begin{aligned}\mathrm{d}s^2 &= (f_1f_5)^{-1/2}\left(-\,\mathrm{d}t^2+\mathrm{d}\phi^2+\frac{r_0^2}{r^2}\left(\cosh\sigma\,\mathrm{d}t+\sinh\sigma\,\mathrm{d}\phi\right)^2\right)\\ &\quad+(f_1f_5)^{+1/2}\left(\frac{\mathrm{d}r^2}{1-r_0^2/r^2}+r^2\,\mathrm{d}\Omega_3^2\right),\quad \phi\cong\phi+2\pi R,\end{aligned} \tag{3.3}$$

$$\text{where}\quad f_1=1+\frac{r_1^2}{r^2},\quad f_5=1+\frac{r_5^2}{r^2} \tag{3.4}$$

The parameters in this supergravity solution can be related to the brane construction as follows:

- $\phi\equiv x^5$ is the compactified S^1 direction along the D1 brane, normalized such that $\phi\cong\phi+2\pi R$, where R is the large radius of the S^1 circle.

  Upon dimension reduction of the ϕ direction, this 6D black string solution will become a 5D black hole solution. In fact the resulting 5D black hole solution is precisely the Strominger-Vafa black hole [6], which serves as the first example of a microscopic counting of the black hole entropy.

- r_0 marks the horizon of the black string, and it is related to the open string momentum P attached to the branes: $P\propto r_0^2\sinh 2\sigma$.

- r_1^2 and r_5^2 are related to the charges Q_1 and Q_5.

We further note that the above black string solution is asymptotically flat, consistent with our brane construction in string theory. On the other hand, if we zoom in to the near horizon region of this black string solution, we discover an $\mathrm{AdS}_3\times S^3$ geometry. This can be achieved by setting:

$$\ell^2=r_1r_5,\quad r\mapsto\lambda\ell r,\quad r_0\mapsto\lambda\ell r_0,\quad t\mapsto t\ell/\lambda,\quad \phi\mapsto\phi\ell/\lambda, \tag{3.5}$$

where ℓ is the AdS radius, and now the ϕ coordinate is normalized such that $\phi\cong\phi+2\pi$. More specially,

- For extremal black string with $r_0 = 0$ and thus $P = 0$, the near horizon limit leads to the zero mass BTZ geometry, with an additional S^3 factor:

$$\mathrm{d}s^2 = \ell^2 \left(r^2(-\,\mathrm{d}t^2 + \mathrm{d}\phi^2) + \frac{\mathrm{d}r^2}{r^2} + \mathrm{d}\Omega_3^2 \right) \tag{3.6}$$

- For the near-extremal case with generic r_0, σ, the near horizon limit leads to the rotating BTZ geometry, again with an additional S^3 factor:

$$\mathrm{d}s^2 = \ell^2 \left(r^2(-\,\mathrm{d}t^2 + \mathrm{d}\phi^2) + \frac{\mathrm{d}r^2}{r^2 - r_0^2} + r_0^2 \left(\cosh\sigma\,\mathrm{d}t + \sinh\sigma\,\mathrm{d}\phi\right)^2 + \mathrm{d}\Omega_3^2 \right) \tag{3.7}$$

It is convenient to define the left and right-moving temperature:

$$T_L = \frac{1}{2\pi}\frac{r_0\, e^{\sigma}}{\ell^2}, \quad T_R = \frac{1}{2\pi}\frac{r_0\, e^{-\sigma}}{\ell^2} \tag{3.8}$$

On the other hand, the Hawking temperature T_H of this solution can be computed, and is given in terms of T_L, T_R as follows:

$$\frac{2}{T_H} = \frac{1}{T_L} + \frac{1}{T_R} \tag{3.9}$$

One can understand the IR supergravity solution as the result of "integrating out" the dynamics of the branes, including the open string excitations. This process deforms the background geometry, and we end up with a closed string theory on the black string background. ......

3.1.2 Worldvolume CFT$_2$ and the duality

On the other hand, we can consider the worldvolume theory of the brane construction. After dimension reduction of the compact $\mathcal{M}_4$, we have a $(1+1)$ dimensional QFT living on the D1-D5 branes. This is a supersymmetric gauge theory with $\mathcal{N} = (4,4)$ supersymmetry. Similar to our previous discussions, we can consider the IR limit of this system. It should flow to an IR fixed point, which is a $(1+1)$ dimensional superconformal field theory (SCFT$_2$).

The central charge of this SCFT$_2$ can be read off from the field contents of the worldvolume theory: the number of bosonic fields is given by:

$$4Q_1Q_5 \tag{3.10}$$

and the same amount for fermions. The factor Q_1Q_5 comes from the open string excitations between the D1 and D5 branes, while the factor of 4 comes from the fact that the D1 branes can move inside the D5 branes along the compact $\mathcal{M}_4$ directions. In the end we have the central charge:

WL: is this language accurate?

$$c = 1 \times 4Q_1Q_5 + \frac{1}{2} \times 4Q_1Q_5 = 6Q_1Q_5 \tag{3.11}$$

3.2 Type IIB string theory with NS-NS flux

4 Deformation of AdS_3/CFT_2 in string theory

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