Holographic duality beyond AdS/CFT

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Contents

0 Introduction

Since the discovery of AdS/CFT duality [1], we have greatly furthered our understanding of quantum gravity in asymptotically AdS backgrounds. However, there are plenty of non-AdS geometries in our real worlds, including:

- The Kerr metric of a rotation black hole, which ... ;
- The asymptotically flat / Minkowski spacetime, which well approximates our current universe at a smaller length (and time) scale;
- The asymptotically de Sitter spacetime, which well approximates our current (and future) universe at a larger length scale, and also during the period of inflation;
- The FRW metric (or more precisely, the Friedmann-Lemaître-Robertson-Walker [2–5] metric), which describes the evolution of our homogeneous and isotropic universe from the big bang to its ... ;
- and more ...

Much less is known about quantum gravity in these backgrounds.

1 Brief review of AdS_3/CFT_2

2 Bottom-up approach: from asymptotic symmetry

3 Top-down approach: from string theory

String theory is a self-consistent theory of quantum gravity.

The first example of microscopic counting of black hole entropy, discovered by Strominger-Vafa [6], comes from the D1-D5-P system in string theory.

The first incarnation of holographic principle was realized by Maldacena [1], by a stack of D3 branes in type IIB string theory.

3.1 The D1-D5-P system

Let us look at the D1-D5-P brane configuration in type IIB string theory. This is well-reviewed in [7]. This configuration allows for an open string description and a closed string description.

Geometry	$\mathbb{R}^{4,1}$		S^1	$\mathcal{M}_4 = T^4, \text{K3}$			
Direction	0	1, 2, 3, 4	5	6	7	8	9
$\#\,\text{D5} = Q_5$	×		×	×	×	×	×
$\#\,\text{D1} = Q_1$	×		×				
P	×		×				

Table 1. Brane configuration of the D1-D5-P system. Here we are considering type IIB string theory on flat 6D spacetime, with a compactified $x^5 \in S^1$ direction, along with an internal $\mathcal{M}_4$ manifold. We use "×" to mark the directions x^μ that an object occupies. Here $\mu = 0, 1, \cdots, 9$.

The D5 branes wrap the compact $\mathcal{M}_4$, while the D1 branes are localized on $\mathcal{M}_4$. Both the D1 and D5 branes extend along the fifth direction x^5, which is compactified to a circle S^1 with a large radius.

Open string excitations on the branes carry momentum and winding. Due to the large radius of S^1, we can focus on the momentum modes P along $x^5 \in S^1$ and neglect the winding modes. On the other hand, we will neglect momentum modes along the $\mathcal{M}_4$ directions, since $\mathcal{M}_4$ is assumed to be compact and small.

In the IR limit, type IIB string theory is described by the low energy effective action of type IIB supergravity (SUGRA). The field content and the action of type IIB SUGRA are well reviewed in the literature; see e.g. Appendix H of [8]. In particular, there is a pair of 2-form gauge potentials in type IIB SUGRA. One of them is the NS-NS field B_2, and the other is the R-R field C_2. The D1 branes are electrically charged under C_2, while the D5 branes are magnetically charged under C_2.

The bosonic part of the string frame action is then given by:

$$\frac{1}{16\pi G}\int \mathrm{d}^{10}x\,\sqrt{-g}\left(e^{-2\phi}\Big(R+4\,(\nabla\phi)^2-\frac{1}{12}H^2\Big)-\frac{1}{12}F^2\right), \tag{3.1}$$

$$H=\mathrm{d}B_2\,,\quad F=\mathrm{d}C_2 \tag{3.2}$$

where H and F are the 3-form field strengths, $H^2=H_{\mu\nu\rho}H^{\mu\nu\rho}\propto H\wedge\star H$, and similar for F^2. After dimension reduction of the compact $\mathcal{M}_4$, the equations of motion admit the following black string solution in 6D [citations needed]:

$$\begin{aligned}\mathrm{d}s^2=&\,(f_1f_5)^{-1/2}\left(-\mathrm{d}t^2+\mathrm{d}\phi^2+\frac{r_0^2}{r^2}\left(\cosh\sigma\,\mathrm{d}t+\sinh\sigma\,\mathrm{d}\phi\right)^2\right)\\&+(f_1f_5)^{+1/2}\left(\frac{\mathrm{d}r^2}{1-r_0^2/r^2}+r^2\,\mathrm{d}\Omega_3^2\right),\qquad f_1=1+\frac{r_1^2}{r^2},\quad f_5=1+\frac{r_5^2}{r^2},\end{aligned} \tag{3.3}$$

We note that the above black string solution is asymptotically flat, consistent with our brane construction in type IIB string theory. More specifically, parameters in this supergravity solution can be related to the brane construction in the following way:

- $\phi\equiv x^5$ is the compactified S^1 direction along the D1 brane, normalized such that $\phi\cong\phi+2\pi R$.

  Upon dimension reduction of the ϕ direction, this 6D black string solution will become a 5D black hole solution. In fact the resulting 5D black hole solution is precisely the Strominger-Vafa black hole [6], which serves as the first example of microscopic counting of the black hole entropy.

- r_0 marks the horizon of the black string, and it is related to the open string momentum P attached to the branes: $P\propto r_0^2\sinh 2\sigma$.

- r_1^2 and r_5^2 are related to the charges Q_1 and Q_5.

3.2 Type IIB string theory with NS-NS flux

4 Deformation of $\mathbf{AdS_3/CFT_2}$ in string theory

References

[1] J.M. Maldacena, *The Large N limit of superconformal field theories and supergravity*, *Adv. Theor. Math. Phys.* **2** (1998) 231 [hep-th/9711200].

[2] A. Friedmann, *On the Possibility of a world with constant negative curvature of space*, *Z. Phys.* **21** (1924) 326.

[3] G. Lemaitre, *The expanding universe*, *Annales Soc. Sci. Bruxelles A* **53** (1933) 51.

[4] H.P. Robertson, *Kinematics and World-Structure*, *Astrophys. J.* **82** (1935) 284.

[5] A.G. Walker, *On Milne's Theory of World-Structure*, *Proceedings of the London Mathematical Society* **42** (1937) 90.

[6] A. Strominger and C. Vafa, *Microscopic origin of the Bekenstein-Hawking entropy*, *Phys. Lett. B* **379** (1996) 99 [hep-th/9601029].

[7] J.R. David, G. Mandal and S.R. Wadia, *Microscopic formulation of black holes in string theory*, *Phys. Rept.* **369** (2002) 549 [hep-th/0203048].

[8] E. Kiritsis, *Introduction to superstring theory*, vol. B9 of *Leuven notes in mathematical and theoretical physics*, Leuven U. Press, Leuven (1998), [hep-th/9709062].