Holographic duality beyond AdS/CFT

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Contents

1 Introduction

Since the discovery of AdS/CFT duality [1], we have greatly furthered our understanding of quantum gravity in asymptotically AdS backgrounds. However, there are plenty of non-AdS geometries in our real worlds, including:

- The Kerr metric of a rotation black hole, which ... ;
- The asymptotically flat / Minkowski spacetime, which well approximates our current universe at a smaller length (and time) scale;
- The asymptotically de Sitter spacetime, which well approximates our current (and future) universe at a larger length scale, and also during the period of inflation;
- The FRW metric (or more precisely, the Friedmann-Lemaître-Robertson-Walker [2–5] metric), which describes the evolution of our homogeneous and isotropic universe from the big bang to its ... ;
- and more ...

Much less is known about quantum gravity in these backgrounds.

2 Brief review of $\mathrm{AdS}_3/\mathrm{CFT}_2$

3 Bottom-up approach: from asymptotic symmetry

4 Top-down approach: from string theory

String theory is a self-consistent theory of quantum gravity.

The first example of microscopic counting of black hole entropy, discovered by Strominger-Vafa [6], comes from the D1-D5-P system in string theory.

The first incarnation of holographic principle was realized by Maldacena [1], by a stack of D3 branes in type IIB string theory.

4.1 The D1-D5-P system

Let us look at the D1-D5-P brane configuration in type IIB string theory. This is well-reviewed in [7]. This configuration allows for an open string description and a closed string description.

Geometry	$\mathbb{R}^{4,1}$		S^1	$\mathcal{M}_4 = T^4, \mathrm{K3}$			
Direction	0	1, 2, 3, 4	5	6	7	8	9
$\#\,\mathrm{D5} = Q_5$	$\times$		$\times$	$\times$	$\times$	$\times$	$\times$
$\#\,\mathrm{D1} = Q_1$	$\times$		$\times$				
P	$\times$		$\times$				

Table 1. Brane configuration of the D1-D5-P system. Here we are considering type IIB string theory on flat 6D spacetime, with a compactified $x^5 \in S^1$ direction, along with an internal $\mathcal{M}_4$ manifold. We use "$\times$" to mark the directions x^μ that an object occupies. Here $\mu = 0, 1, \cdots, 9$.

The D5 branes wrap the compact $\mathcal{M}_4$, while the D1 branes are localized on $\mathcal{M}_4$. Both the D1 and D5 branes extend along the fifth direction x^5, which is compactified to a circle S^1 with a large radius.

Open string excitations on the branes carry momentum and winding. Due to the large radius of S^1, we can focus on the momentum modes P along $x^5 \in S^1$ and neglect the winding modes. On the other hand, we will neglect momentum modes along the $\mathcal{M}_4$ directions, since $\mathcal{M}_4$ is assumed to be compact and small.

In the IR limit, type IIB string theory is described by the low energy effective action of type IIB supergravity (SUGRA). The field content and the action of type IIB SUGRA are well reviewed in the literature; see e.g. Appendix H of [8]. In particular, there is a pair of 2-form gauge fields in type IIB SUGRA. One of them is the NS-NS field B_2, and the other is the R-R field C_2. The D1 branes are electrically charged under C_2, while the D5 branes are magnetically charged under C_2.

The bosonic part of the string frame action is then given by:

$$\frac{1}{16\pi G}\int \mathrm{d}^{10}x\,\sqrt{-g}\,\left(e^{-2\phi}\Big(R+4\,(\nabla\phi)^2-\frac{1}{12}H^2\Big)-\frac{1}{12}F^2\right), \tag{4.1}$$

$$H=\mathrm{d}B_2\,,\quad F=\mathrm{d}C_2 \tag{4.2}$$

After dimension reduction of the compact $\mathcal{M}_4$, the equations of motion lead to the following black string solution in 6D:

$$\begin{aligned}\mathrm{d}s^2&=(f_1f_5)^{-1/2}\left(-\,\mathrm{d}t^2+\mathrm{d}\phi^2+\frac{r_0^2}{r^2}\,(\cosh\sigma\,\mathrm{d}t+\sinh\sigma\,\mathrm{d}\phi)^2\right)\\&\quad+(f_1f_5)^{+1/2}\left(\frac{\mathrm{d}r^2}{1-r_0^2/r^2}+r^2\,\mathrm{d}\Omega_3^2\right)\end{aligned} \tag{4.3}$$

4.2 Type IIB string theory with NS-NS flux

5 Deformation of $\mathbf{AdS_3/CFT_2}$ in string theory

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