Modeling and Analysis of Dynamic Systems

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Outline

1. Lecture 12: Case Study - Geostationary Satellite
   - Introduction
   - Nonlinear Model
   - System Analysis

Outline

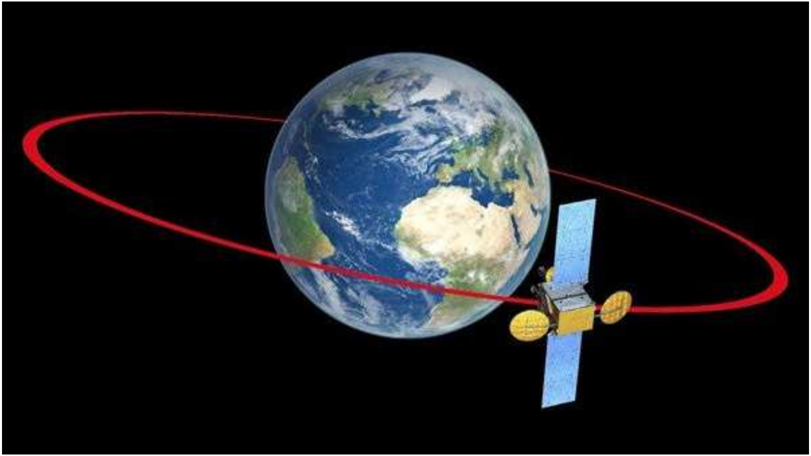
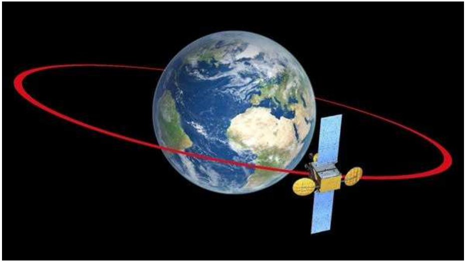


Figure: www.esa.int



Geostationary orbit

- around the Earth in the Equatorial plane,
- exactly the sideral Earth rotational speed: 23h 56min 4.1s,
- from Earth, satellite is seen at exact same location in sky.

$\Rightarrow$ Very useful for communication purposes.

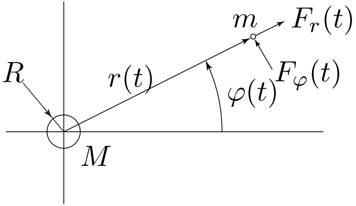
Importance of modeling for:

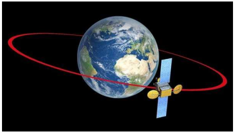
- understanding the basic properties and limitations of satellite dynamics,
- the design of an orbit-stabilizing controller.

Main steps followed:

1. derivation of the satellite motion,
2. linearization around the orbit trajectory,
3. study stability, observability, controllability.

Problem Definition

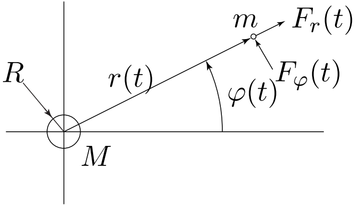


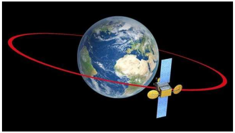


Notations:

R: radius of the Earth [m]
M: mass of the Earth [kg]
m: mass of the satellite [kg]
$r(t)$: dist. Earth center to satellite [m]
$\varphi(t)$: orbit angle of the satellite [rad]
$F_r(t)$: radial force [N]
$F_\varphi(t)$: tangential force [N]

Problem Definition





Assumptions:

- No other celestial bodies considered (only Earth gravitational force considered).
- $M \gg m \;\Rightarrow$ C.O.G located at center of Earth.
- satellite always remains in the equatorial plane $\Rightarrow$ 2 variables are sufficient to describe its position: $r(t)$ and $\varphi(t)$.
- the attitude (orientation) of the satellite is kept constant (by an inner control system) $\Rightarrow F_r(t)$ and $F_\varphi(t)$ independant.

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Building Up the Nonlinear Model

Step 1: Inputs & Outputs

- Inputs:
  - radial force $F_r(t)$
  - tangential force $F_\varphi(t)$
- Outputs:
  - Earth to satellite distance $r(t)$
  - orbit angle: $\varphi(t)$

Building Up the Nonlinear Model

Step 2: Energies involved in the satellite motion

- kinetic energy T

$$T(r, \dot{r}, \dot{\varphi}) = \frac{1}{2} m \, \dot{r}^2 + \frac{1}{2} \, m \, (r \, \dot{\varphi})^2$$

- potential energy V: is due to the Earth Gravitational field $\boldsymbol{\mathcal{G}}_{Earth}$

$$\boldsymbol{\mathcal{G}}_{Earth}(r) = \frac{G\,M}{r^2} \boldsymbol{u}$$

$$\boldsymbol{F}_{grav}(r) = m \boldsymbol{\mathcal{G}}_{Earth}$$

Universal gravitation constant: $G = 6.673 \cdot 10^{-11} \; m^3/(s^2 kg)$,
Earth mass: $M = 5.97410^{24}$ kg,
Earth radius: $R = 6.36710^6$ m. Satellite mass: $m \ll M$

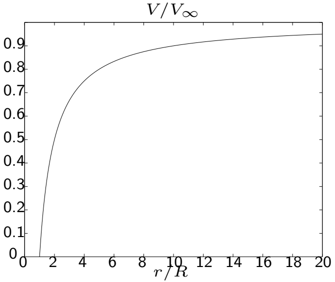
Building Up the Nonlinear Model

Step 2: Energy needed to bring the satellite from altitude R to r

$$
\begin{aligned}
V(r) = \int_R^r F(\rho)\, d\rho \quad &= \quad \int_R^r G \frac{M\, m}{\rho^2}\, d\rho \\
&= \quad G\, M\, m \int_R^r \frac{1}{\rho^2}\, d\rho \\
&= \quad G\, M\, m \left[-\frac{1}{\rho} \right]_R^r \\
&= \quad G\, M\, m \left(\frac{1}{R} - \frac{1}{r} \right), \quad r > R
\end{aligned}
$$

Remark: Potential energy of satellite:
- only a function of the distance r, $\Rightarrow V(r)$.
- is zero at Earth surface (arbitrary but convenient choice)

$$V(r) = G\,M\,m\left(\frac{1}{R} - \frac{1}{r}\right), \quad r > R$$



The potential energy:

is 0 on the surface of the earth (arbitrary but convenient choice)

and reaches 90% of its maximum value: $V_\infty = \frac{GMm}{R}$, at a distance of only 10 times the radius of the Earth.

Minimum Speed to Reach Orbit

Question: Which minimum speed should the rocket reach to overcome the Earth gravitation force to place the satellite in its orbit?

Assume a rapid acceleration at the start from standstill (take off) to a speed v_0, **the energy balance** is

$$\frac{1}{2}\, m\, v_0^2 = G\, M\, m \left(\frac{1}{R} - \frac{1}{r} \right) = (\Delta V_{grav})_{R \to r}$$

kinetic energy that a satellite must have to reach a certain orbit height $r - R$.
$(\Delta V_{grav})_{R \to r}$ is the change in potential energy from dist. R to r.

The minimum speed needed to reach altitude r is thus:

$$v_0(r) = \sqrt{2\, G\, M \left(\frac{1}{R} - \frac{1}{r} \right)}$$

Escape Velocity : v_{esc} or v_{∞}

Definition 1

It is the minimum velocity that an object should have, such that without additional acceleration (no propulsion anymore), this object will move away from massive body, without being pulled back.

Definition 2

It is the velocity of an object, such that its kinetic energy at a certain point in space A is equal its gravitational energy at that point A.

Definition 3

The velocity that is required to completely leave the influence of the gravitation field of the Earth.

Escape Velocity

How to compute the escape velocity?

$$v_\infty = \lim_{r\to\infty} v_0(r) = \sqrt{2\,G\,M\left(\frac{1}{R}-\frac{1}{r}\right)}$$

$$\begin{aligned} v_\infty = \sqrt{\frac{2\,G\cdot M}{R}} \quad &\approx \quad 1.12\cdot 10^4 \ \mathrm{m/s} \\ &\approx \quad 11.2\,\mathrm{km/s} \\ &\approx \quad 40300\,\mathrm{km/h} \end{aligned}$$

Building Up the Nonlinear Model

Step 3: Lagrange formalism

The dynamics of the satellite are formulated using Lagrange's method:

$$\frac{d}{dt}\left[\frac{\partial L}{\partial \dot{r}}\right] - \frac{\partial L}{\partial r} = F_r$$

and

$$\frac{d}{dt}\left[\frac{\partial L}{\partial \dot{\varphi}}\right] - \frac{\partial L}{\partial \varphi} = F_\varphi \cdot r$$

with the Lagrange function $L = T - V$ (difference of the kinetic and potential energies):

$$L = \frac{1}{2} m\, \dot{r}^2 + \frac{1}{2}\, m\, (r\, \dot{\varphi})^2 - G\, M\, m \left(\frac{1}{R} - \frac{1}{r}\right)$$

Building Up the Nonlinear Model

$$L = \frac{1}{2} m\, \dot{r}^2 + \frac{1}{2}\, m\, (r\, \dot{\varphi})^2 - G\, M\, m \left(\frac{1}{R} - \frac{1}{r} \right)$$

$$\frac{\partial L}{\partial \dot{r}} = m\, \dot{r} \qquad \frac{\partial L}{\partial \dot{\varphi}} = m\, r^2\, \dot{\varphi}$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{r}} \right) = m\, \ddot{r} \qquad \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\varphi}} \right) = m\, r^2\, \ddot{\varphi} + 2\, m\, r\, \dot{\varphi}\, \dot{r}$$

$$\frac{\partial L}{\partial r} = m\, r\, \dot{\varphi}^2 - G\, M\, m\, \frac{1}{r^2} \qquad \frac{\partial L}{\partial \varphi} = 0$$

$$\begin{aligned} m \cdot \ddot{r} &= m \cdot r \cdot \dot{\varphi}^2 - G \cdot M \cdot m \cdot \frac{1}{r^2} + F_r \\ m \cdot r^2 \cdot \ddot{\varphi} &= -2\, m \cdot r \cdot \dot{\varphi} \cdot \dot{r} + F_{\varphi} \cdot r \end{aligned}$$

Building Up the Nonlinear Model

In order to simplify the notations, let's introduce the control accelerations

$$u_r = \frac{F_r}{m}, \quad \text{respectively} \quad u_\varphi = \frac{F_\varphi}{m}$$

yields

$$\begin{aligned} \ddot{r} &= r\,\dot{\varphi}^2 - G \cdot M\,\frac{1}{r^2} + u_r \\ \ddot{\varphi} &= -2\,\dot{\varphi}\,\dot{r}\,\frac{1}{r} + \frac{1}{r}\,u_\varphi \end{aligned}$$

Building Up the Nonlinear Model

Circular Orbit

For geostationary conditions, the periodic reference orbit must be chosen to be:

- circular
- with constant angular velocity, i.e.,

$$u_r = 0, \quad \ddot{r} = 0, \quad \dot{r} = 0, \quad r = r_0$$

$$u_\varphi = 0, \quad \ddot{\varphi} = 0, \quad \dot{\varphi} = \omega_0, \quad \varphi = \omega_0 \cdot t$$

What are the satellite turn rate ω_0, and the turn radius r_0?

Building Up the Nonlinear Model: Circular Orbit

Orbit turn rate: ω_0

The satellite turn rate is $\omega_0 = 2\pi/\text{day}$
taking 1 sidereal day = 23h 56min 4.1 s, you find:

$$\omega_0 = 7.2910^{-5} \text{ rad/s}$$

Building Up the Nonlinear Model: Circular Orbit

Geostationary orbit radius: r_0, $(F_{centripetal} = F_{gravitational}$

$$r_0 = \left(\frac{G \cdot M}{\omega_0^2}\right)^{1/3} \approx 4.22 \ldots 10^7 \ m$$

Remarks:

r_0 is approximately 6.2 times the radius of the earth.

The energy required is more than 80% of the escape energy.

The resulting tangential speed

$v_\varphi = r_0 \omega_0 = 3.06 \, m/s \approx 10800 \, km/h$

How to Launch a Satellite?

What is an energy efficient trajectory to place a geostationary satellite to orbit?

http://www.planetary.org/blogs/jason-davis/20140116-how-to-get-a-satellite-to-gto.html

https://youtu.be/COCAIPtVA2M

Outline

Building Up the Nonlinear Model

State-Space Formulation

In order not to have second-order ODE to work with, the system is put into a state space form, such that only 1st order time-derivatives appear.

$$x_1(t) = r, \quad x_2(t) = \dot{r}, \quad u_1(t) = u_r$$

$$x_3(t) = \varphi, \quad x_4(t) = \dot{\varphi}, \quad u_2(t) = u_\varphi$$

$$x(t) = \begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \\ x_4(t) \end{bmatrix}, \quad u(t) = \begin{bmatrix} u_1(t) \\ u_2(t) \end{bmatrix}$$

Nonlinear ODE

$$\frac{d}{dt}x(t) = f(x(t), u(t))$$

where

$$f(x(t)) = \begin{bmatrix} x_2(t) \\ x_1 \cdot x_4^2(t) - G \cdot M / x_1^2(t) + u_1(t) \\ x_4(t) \\ -2\, x_2(t) \cdot x_4(t)/x_1(t) + u_2(t)/x_1(t) \end{bmatrix}$$

Measurements Equations

$$y(t) = h(x(t))$$

where

$$h(x(t)) = \begin{bmatrix} x_1(t)/r_0 \\ x_3(t) \end{bmatrix}$$

Notice: $y_1(t)$ is the *normalized* radius.

Model Linearization

The model is now linearized around the nominal orbit

$$x_0(t) = \begin{bmatrix} r_0 \\ 0 \\ \omega_0 \cdot t \\ \omega_0 \end{bmatrix}, \quad u_0(t) = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

Notice that in this case the nominal trajectory

- is *not* an equilibrium point
- but a periodic solution of the system equations.

However, introducing the notation

$$x(t) = x_0(t) + \delta x(t), \quad u(t) = u_0(t) + \delta u(t), \quad y(t) = y_0(t) + \delta y(t)$$

yields

$$\begin{aligned} \frac{d}{dt}x_0(t) + \frac{d}{dt}\delta x(t) &= f(x_0(t) + \delta x(t), u_0(t) + \delta u(t)) \\ &\approx f(x_0(t), u_0(t)) + \left.\frac{\partial f}{\partial x}\right|_{x_0(t),u_0(t)} \cdot \delta x(t) \\ &\quad + \left.\frac{\partial f}{\partial u}\right|_{x_0(t),u_0(t)} \cdot \delta u(t) \end{aligned}$$

Since, by construction

$$\frac{d}{dt}x_0(t) = f(x_0(t), u_0(t))$$

the linearized system is described by the equation

$$\frac{d}{dt}\delta x(t) = \left.\frac{\partial f}{\partial x}\right|_{x_0(t),u_0(t)} \cdot \delta x(t) + \left.\frac{\partial f}{\partial u}\right|_{x_0(t),u_0(t)} \cdot \delta u(t)$$

The computation of the Jacobian matrices yields the following results

$$\frac{\partial f}{\partial x} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ x_4^2 + 2\,G \cdot M/x_1^3 & 0 & 0 & 2\,x_1 \cdot x_4 \\ 0 & 0 & 0 & 1 \\ 2\,x_2 \cdot x_4/x_1^2 - u_2/x_1^2 & -2\,x_4/x_1 & 0 & -2\,x_2/x_1 \end{bmatrix}$$

$$\frac{\partial f}{\partial u} = \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 0 \\ 0 & 1/x_1 \end{bmatrix}$$

Along the periodic nominal solution, we get

$$\left.\frac{\partial f}{\partial x}\right|_{x_0(t),u_0(t)} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 3\,\omega_0^2 & 0 & 0 & 2\,r_0 \cdot \omega_0 \\ 0 & 0 & 0 & 1 \\ 0 & -2\,\omega_0/r_0 & 0 & 0 \end{bmatrix},$$

$$\left.\frac{\partial f}{\partial u}\right|_{x_0(t),u_0(t)} = \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 0 \\ 0 & 1/r_0 \end{bmatrix}$$

Remark: In this special case the resulting system matrices are time *invariant*. This is not true in general. When linearizing a nonlinear system around a non-constant equilibrium orbit $x_e(t), u_e(t)$ the resulting system matrices will be known functions of time $\{A(t), B(t), C(t), D(t)\}$.

System Analysis

System in standard notation

$$\dot{x}(t) = A \cdot x(t) + B \cdot u(t), \quad y(t) = C \cdot x(t)$$

with

$$A = \left.\frac{\partial f}{\partial x}\right|_{x_0(t),u_0(t)}, \quad B = \left.\frac{\partial f}{\partial u}\right|_{x_0(t),u_0(t)}$$

and

$$C = \begin{bmatrix} 1/r_0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

For the system $\{A, B, C, 0\}$ defined above the following questions will be discussed below:

1. What are the stability properties of the system?
2. Is the system controllable and observable?
3. Which actuators and sensors are most important?
4. What are the main dynamic properties of the system?

System Stability

Eigenvalues of the matrix A are the roots of

$$\det(s \cdot I - A) = \det \begin{bmatrix} s & -1 & 0 & 0 \\ -3\,\omega_0^2 & s & 0 & -2\,r_0 \cdot \omega_0 \\ 0 & 0 & s & -1 \\ 0 & 2\,\omega_0/r_0 & 0 & s \end{bmatrix}$$

System Stability

$$
\begin{aligned}
\det(s \cdot I - A) &= s \cdot \det \begin{bmatrix} s & -1 & 0 \\ -3\,\omega_0^2 & s & -2\,r_0 \cdot \omega_0 \\ 0 & 2\,\omega_0/r_0 & s \end{bmatrix} \\
&= s \cdot \left[s \cdot (s^2 + 4\,\omega_0^2) - (-1) \cdot (-3\,\omega_0^2 \cdot s) \right] \\
&= s^2 \cdot (s^2 + \omega_0^2)
\end{aligned}
$$

System Stability

$$\det(s \cdot I - A) = s^2 \cdot (s^2 + \omega_0^2)$$

The roots are $\{0, 0, +j\,\omega_0, -j\,\omega_0\}$.

- The eigenvalue pair $\pm j\,\omega_0$ shows that the system includes oscillatory modes with eigenfrequency equal to the angular speed of the satellite: ω_0.
- The double root in the origin indicates that the system might be unstable.

One way to see that is to analyze the eigenstructure of the system matrix A. The rank of the matrix

$$M = (s \cdot I - A)|_{s=0}$$

is rank(M)=3. $\Rightarrow$ there is only one eigenvector associated with the double eigenvalue $s = 0$.

Conclusion

Therefore, the matrix A is cyclic and, as it was shown in the main text, this means that the linearized system is unstable (behavior of two integrators in series).

Remark : The same result is obtained when the system's transfer functions will be computed.

Controllability and Observability

Recall of the linear system

$$\dot{\boldsymbol{x}} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 3\,\omega_0^2 & 0 & 0 & 2\,r_0 \cdot \omega_0 \\ 0 & 0 & 0 & 1 \\ 0 & -2\,\omega_0/r_0 & 0 & 0 \end{bmatrix} \cdot \boldsymbol{x} + \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 0 \\ 0 & 1/r_0 \end{bmatrix} \cdot \boldsymbol{u}$$

and

$$\boldsymbol{y} = \begin{bmatrix} 1/r_0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \cdot \boldsymbol{x}$$

Controllability matrix $\mathcal{R}$

$$\mathcal{R} = [B, A\cdot B, A^2\cdot B, A^3\cdot B] = \begin{bmatrix} 0 & 0 & 1 & 0 & \ldots \\ 1 & 0 & 0 & 2\,\omega_0 & \ldots \\ 0 & 0 & 0 & 1/r_0 & \ldots \\ 0 & 1/r_0 & -2\,\omega_0/r_0 & 0 & \ldots \end{bmatrix}$$

This system is completely controllable.

In fact the four first columns already are linearly independent (the determinant of that submatrix is $-1/r_0^2$).

Failure Scenario 1: Radial Thruster Failure

This yields input matrix $\boldsymbol{b}_2 = [0,\ 0,\ 0,\ 1/r_0]^T$. The corresponding controllability matrix has the form

$$\mathcal{R}_2 = \begin{bmatrix} 0 & 0 & 2\,\omega_0 & 0 \\ 0 & 2\,\omega_0 & 0 & -2\,\omega_0^3 \\ 0 & 1/r_0 & 0 & -4\,\omega_0/r_0 \\ 1/r_0 & 0 & -4\,\omega_0^2/r_0 & 0 \end{bmatrix}$$

The determinant of this matrix is $-12\,\omega_0^4/r_0^2$, i.e., not zero on the reference orbit.

Accordingly, the satellite remains completely controllable even if the radial thruster fails.

Failure Scenario 2: Tangential Thruster Failure

This yields $\boldsymbol{b}_1 = [0,\ 1,\ 0,\ 0]^T$ and the controllability matrix

$$\mathcal{R}_1 = \begin{bmatrix} 0 & 1 & 0 & -\omega_0^2 \\ 1 & 0 & -\omega_0^2 & 0 \\ 0 & 0 & -2\,\omega_0/r_0 & 0 \\ 0 & -2\,\omega_0/r_0 & 0 & 2\,\omega_0^3/r_0 \end{bmatrix}$$

in this case has only three independent columns (the fourth column is equal to $-\omega_0^2$ times the second column), i.e.,

In case of tangential-thruster failure

the system is no longer completely controllable.

Observability

$$\mathcal{O} = \begin{bmatrix} C \\ C \cdot A \\ C \cdot A^2 \\ C \cdot A^3 \end{bmatrix} = \begin{bmatrix} 1/r_0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1/r_0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ \cdots & \cdots & \cdots & \cdots \end{bmatrix}$$

The system with no faults is completely observable.

Fault Scenario 1: Radial Sensor Failure r_{meas}

If the radial sensor fails, the measurement matrix reduces to $\boldsymbol{c}_2 = [0, \ 0, \ 1, \ 0]$ and the resulting observability matrix $\mathcal{O}_2$ has the form

$$\mathcal{O}_2 = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & -2\,\omega_0/r_0 & 0 & 0 \\ -6\,\omega_0^3/r_0 & 0 & 0 & -4\,\omega_0^2 \end{bmatrix}$$

This matrix is regular (its determinant is $-12\,\omega_0^4/r_0^2$). Accordingly, the system remains completely observable even if the radial sensor fails.

Fault Scenario 2: Tangential Sensor Failure φ_{meas}

If the tangential sensor fails, the measurement matrix reduces to $c_1 = [1,\ 0,\ 0,\ 0]$ and the resulting observability matrix $\mathcal{O}_1$ has the form

$$\mathcal{O}_1 = \begin{bmatrix} 1/r_0 & 0 & 0 & 0 \\ 0 & 1/r_0 & 0 & 0 \\ 3\,\omega_0^2/r_0 & 0 & 0 & 2\,r_0 \cdot \omega_0 \\ 0 & -\omega_0^2/r_0 & 0 & 0 \end{bmatrix}$$

Obviously, this matrix is singular (the third row is zero).
Accordingly, the system is not completely observable in this case.

Conclusions on the satellite design

This analysis is helpful for the design of the satellite.

The main result is:

- tangential actuator and sensor are more important than radial ones.
- If radial actuator and sensor fail, the satellite stability is still guaranteed (control reconfiguration).

Transfer Function

$$P(s) = \begin{bmatrix} P_{11}(s) & P_{12}(s) \\ P_{21}(s) & P_{22}(s) \end{bmatrix} = C(sI - A)^{-1}B$$

$$\Rightarrow P(s) = \begin{bmatrix} \dfrac{1}{s^2 + \omega_0^2} & \dfrac{2\,\omega_0}{s \cdot (s^2 + \omega_0^2)} \\ \dfrac{-2\,\omega_0}{r_0 \cdot s \cdot (s^2 + \omega_0^2)} & \dfrac{s^2 - 3\,\omega_0^2}{r_0 \cdot s^2 \cdot (s^2 + \omega_0^2)} \end{bmatrix}$$

Note that even if the system *is* stabilizable with only the tangential thruster and sensor working,

- the control problem is difficult,
- because the corresponding SISO system transfer function $P_{22}(s)$ has a non-minimumphase zero at $\sqrt{3} \cdot \omega_0$ (limits the attainable crossover frequency to about 0.85 ω_0.

Remark:

The MIMO system has no finite transmission zeros. This can be seen from

$$Z(s) = \begin{bmatrix} (s \cdot I - A) & -B \\ C & D \end{bmatrix}$$

which turns out to be

$$\det Z(s) = \frac{1}{r_0}$$

Accordingly, no finite s yields a determinant equal to zero and, therefore, no finite zero exists. In other words, the MIMO system $P(s)$ is minimumphase. Compared to the SISO design using only $P_{22}(s)$, the design of a suitable controller using both inputs and outputs will yield a much better closed-loop system performance in terms of bandwidth, response times, etc.

Two simulations

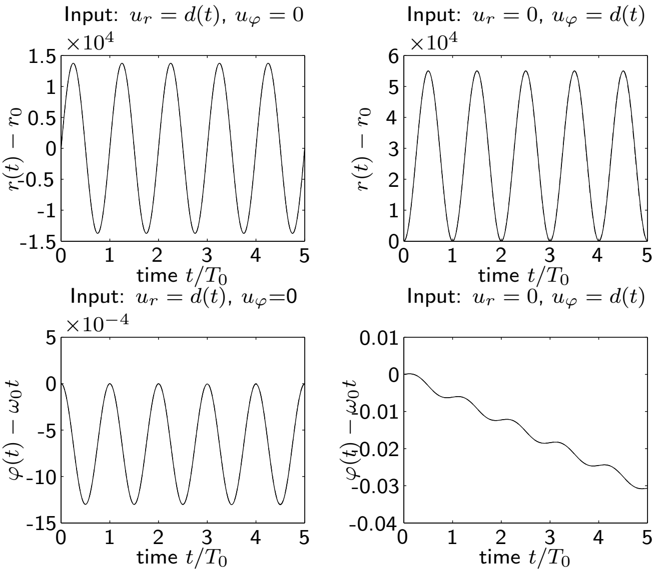
The main objective is to convey a first impression of the differences between the linear system behavior, as analyzed above, and the nonlinear system behavior.

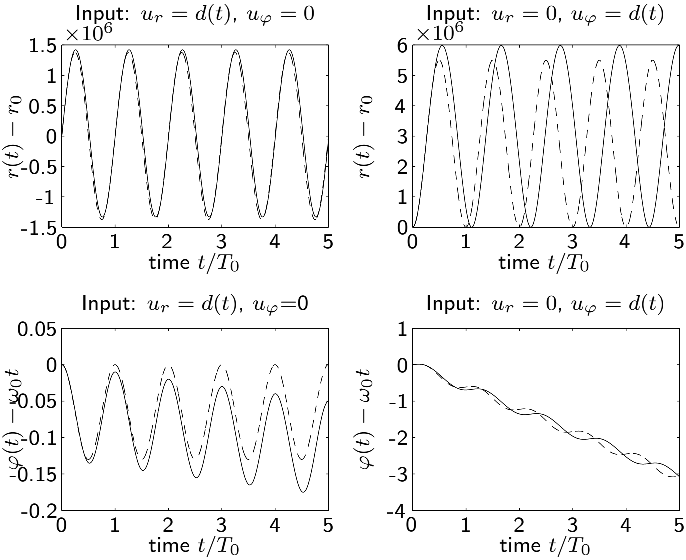
The system is assumed to be in its reference state for all $t < 0$. At that moment a test signal

$$d(t) = \begin{cases} d_0 & 0 \leq t < 1 \\ 0 & 1 \leq t \end{cases}$$

is applied.

In a first case d_0 is chosen rather small and in a second case very large, in order to clearly show the differences between the linear an the nonlinear system behavior.


Input: $u_r = d(t)$, $u_\varphi = 0$
$\times 10^4$
1.5
1
0.5
0
-0.5
-1
-1.5
$r(t) - r_0$
0
1
2
3
4
5
time t/T_0
Input: $u_r = 0$, $u_\varphi = d(t)$
$\times 10^4$
6
5
4
3
2
1
0
$r(t) - r_0$
time t/T_0
Input: $u_r = d(t)$, u_φ=0
$\times 10^{-4}$
5
0
-5
-10
-15
$\varphi(t) - \omega_0 t$
time t/T_0
Input: $u_r = 0$, $u_\varphi = d(t)$
0.01
0
-0.01
-0.02
-0.03
-0.04
$\varphi(t) - \omega_0 t$
time t/T_0


Input: $u_r = d(t)$, $u_\varphi = 0$
$\times 10^6$
1.5
1
0.5
0
-0.5
-1
-1.5
$r(t) - r_0$
0 1 2 3 4 5
time t/T_0
Input: $u_r = 0$, $u_\varphi = d(t)$
$\times 10^6$
6
5
4
3
2
1
0
$r(t) - r_0$
0 1 2 3 4 5
time t/T_0
Input: $u_r = d(t)$, u_φ=0
0.05
0
-0.05
-0.1
-0.15
-0.2
$\varphi(t) - \omega_0 t$
0 1 2 3 4 5
time t/T_0
Input: $u_r = 0$, $u_\varphi = d(t)$
1
0
-1
-2
-3
-4
$\varphi(t) - \omega_0 t$
0 1 2 3 4 5
time t/T_0

Next lecture + Upcoming Exercise

Next lecture

- Zero dynamics
- Nonlinear systems

Thank you for your attention.