Project report on

Error Analysis in Trifilar Suspension for Mass Moment of Inertia Measurement

(ME721 – Design Engineering Lab)

Under guidance of

Prof. V. Kartik

(Mechanical Department)

Submitted by

Shubham Sabale - 193100011

Mujahed Patil - 193109009

Abhishek Kumar - 193104003

Error analysis in trifilar suspension for inertia measurement

Introduction:

A trifilar suspension system is used to measure the moment of inertia of the body by measuring period of oscillation of the body in the horizontal plane. Accurate calculation of the moment of inertia of an irregular body is made difficult by the large number of quantities involved in the analysis of trifilar suspension. In this project work various errors present in the trifilar suspension system are studied for their effect on final results and methods to minimize these errors are also effectively implemented.

Objective:

Calculation of moment of inertia of body using trifilar suspension and error analysis for misalignment of center of mass of body and plate on trifilar platform.

Methodology of the Trifilar Suspension:

A plate is suspended from three vertical wires, one at each corner. The body whose inertia is to be determined is placed on the plate, with its centre of mass at the centre of the plate. The plate is then twisted in the horizontal plane and released so that it performs free rotational oscillations about the centre of mass. For the purposes of this project, some simplifying assumptions are made all damping is neglected and the pendulum is taken to be governed by simple harmonic motion, the mass of the wires is assumed to be negligible, the wires are assumed to carry only axial loading and finally, the wires and plate are assumed to be inextensible and rigid, respectively, and only the rotational motion of the pendulum is taken into account to produce a one-DOF dynamic system.

Analysis of moment of inertia of body:

To determine the properties of the system as a whole (the plate and the body), it is necessary to solve the dynamic equation formed by Newton's second law for rotation.

$$T_z = I_{zz}\,\ddot{\theta}$$

Where T_z represents torque and I_{zz} represents moment of inertia of both plate and body together

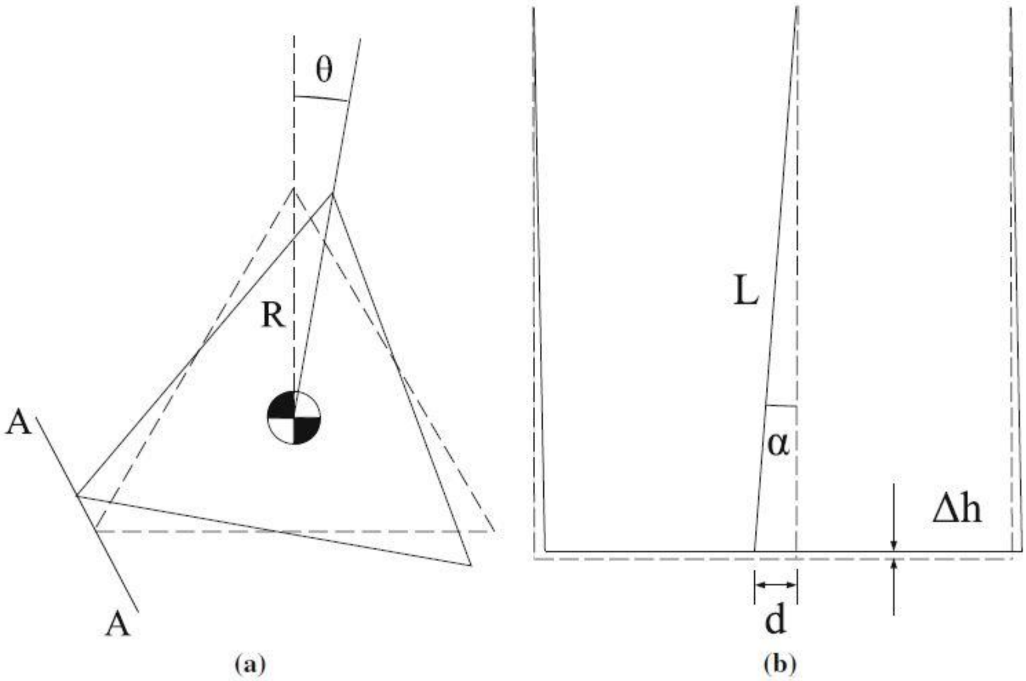


Fig 1 Base plate of trifilar suspension

For small angles

$$\sin\theta \approx \theta \qquad \sin\alpha \approx \alpha$$

$$\cos\theta \approx 1 \qquad \cos\alpha \approx 1$$

The two angles can be related by the horizontal displacement d as follows:

$$L\ \sin\alpha = d$$

$$\mathrm{R}\sin\theta = d$$

Thus we get $$L\,\alpha = R\,\theta$$

Weight supported by each of the three wires, W, to be computed easily using a static equilibrium equation in the vertical direction as

$$W = \frac{1}{3}\,Mg$$

Where M is the mass of the system and g is the gravitational acceleration.

The horizontal force acting on one corner of the plate opposes the displacement

$$F = -w \sin \alpha = -\frac{1}{3} Mg \ \sin \alpha$$

The torque on the plate is defined in terms of the three corner forces and the distance from the centre i.e. R.

$$T_z = 3RF = -RMg \sin \alpha$$

$$T_z = -\frac{R^2 Mg}{L} \theta$$

Substituting into the dynamic equation of motion gives us

$$\frac{R^2 Mg}{L} \theta + I_{zz} \ddot{\theta} = 0$$

Assuming simple harmonic motion, the solution can be found in terms of the angular frequency of oscillations as

$$I_{zz} = \frac{R^2 Mg \tau^2}{4\pi^2 L}$$

Where, τ is the period of oscillation

Mass and inertia can be split into two components belonging to plate and body as

$$M = m_p + m_b$$

$$I_{zz} = I_{pzz} + I_{bzz}$$

The moment of inertia of the body is found using

$$I_{bzz} = \frac{R^2 g \tau^2}{4\pi^2 L} \left(m_p + m_b\right) - I_{pzz}$$

From the above equation we observed that moment of inertia of the body is dependent on Mass of plate and body, distance between centre of plate and point of wire attachment (R), length of string, time period of oscillation.

Sources of Error:

1. Centre of mass misalignment:

All the above analysis is valid when centre of mass of body is perfectly aligned with centre of mass of plate. Precise determination of the centre of mass of irregular bodies can prove difficult, and aligning the centre of mass with the centre of a trifilar plate can be even more troublesome. Hence this misalignment error gets introduced in the trifilar suspension.

If the body is placed on the plate with its centre of mass misaligned, the value calculated for the moment of inertia is affected in two ways:

1. The inertia of the system is no longer simply a sum of the two component inertias, but must be calculated taking into account the distance of each component centre of mass from the system centre of mass.
2. The centre of mass is no longer located at the geometric centre of the plate. This will redefine the weight distribution between the wires and also the angle, α, through which each wire is displaced for a given plate rotation, θ. This in turn redefines the equation of motion.

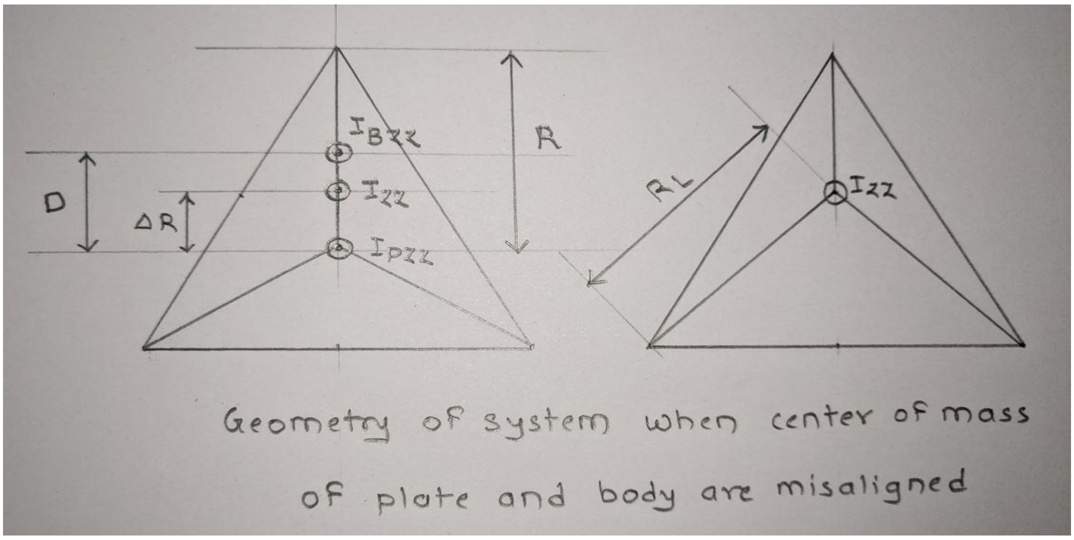


Fig. 2. Misaligned centre of mass of body and plate

Analysis of centre of mass misalignment:

Let centre of mass of body is displaced by distance D, as result of which the overall centre of mass of system is displaced by $\triangle$R.

We can write ratio of component of masses as

$$\Delta R = \frac{m_b}{m_p + m_b}\ D$$

We can apply parallel axis theorem as follows

$$I_{zz}\ =\ (I_{pzz} + m_p \Delta R^2) + (I_{bzz} + m_b (D - \Delta R)^2)$$

$$I_{zz}\ = I_{pzz} + I_{bzz} + \frac{m_p m_b D^2}{m_p + m_b}$$

Substituting this value of Izz in the original equation without any misalignment

$$I_{bzz}\ = \frac{R^2 g \tau^2}{4\pi^2 L}\left(m_p +\ m_b\right) - I_{pzz} - \frac{m_p m_b D^2}{m_p + m_b}$$

The above equation takes account of error caused in sum of moment of inertias due to misalignment of centre of mass of body and plate.

Now we will consider effect of weight distribution caused because of centre of mass misalignment

Let

$$R_s = R - \Delta R \ \text{ And } \frac{R_s}{R} = 1 - \gamma$$

$$\gamma = \frac{\Delta R}{R}$$

$$R_L = \sqrt{\left(\Delta R + \frac{R}{2}\right)^2 + \left(\frac{\sqrt{3}R}{2}\right)^2}$$

$$\frac{R_L}{R} = \sqrt{1 + \gamma + \gamma^2}$$

Because of this weight distribution changes as

$$W_s = \frac{Mg}{3}(1 + 2\gamma) \qquad\qquad W_L = \frac{Mg}{3}(1 - \gamma)$$

$$F_S = -W_s \alpha_S \qquad\qquad F_L = -W_L \alpha_L$$

$$T_Z = R_s F_S + 2 R_L F_L$$

After substituting above value in equation of motion we get

$$(1 - \gamma^2)\frac{R^2 Mg}{L}\,\theta + I_{zz}\ddot{\theta} = 0$$

By solving this differential equation and substituting in previously derived equation we get

$$I_{bzz} = \frac{R^2 g \tau^2}{4\pi^2 L}\left(m_p + m_b\right) - I_{pzz} - \frac{m_b D^2}{m_p + m_b} X \left(m_p + \frac{m_b g \tau^2}{4\pi^2 L}\right)$$

The final term in above equation represent the error because of misalignment

$$\epsilon_D = -\frac{m_b D^2}{m_p + m_b} X \left(m_p + \frac{m_b g \tau^2}{4\pi^2 L}\right)$$

There are two components of the error which are increased inertia of the system and the change in weight distribution.

2. Dimension measurement error:

If dimensions of system are not measured accurately then it will also introduce error in the calculation of moment of inertia of body which can be represented as follows

$$\frac{\Delta I_{ZZ}}{I_{ZZ}} = 2\frac{\Delta R}{R} + \frac{\Delta m}{m} + 2\frac{\Delta \tau}{\tau}$$

We need to consider this error also because it has value comparable with error introduced because of misalignment of centre of mass of body and plate.

Experimental setup:

The experimental setup contains following main components as shown in schematic representation.

1. base plate
2. support
3. strings

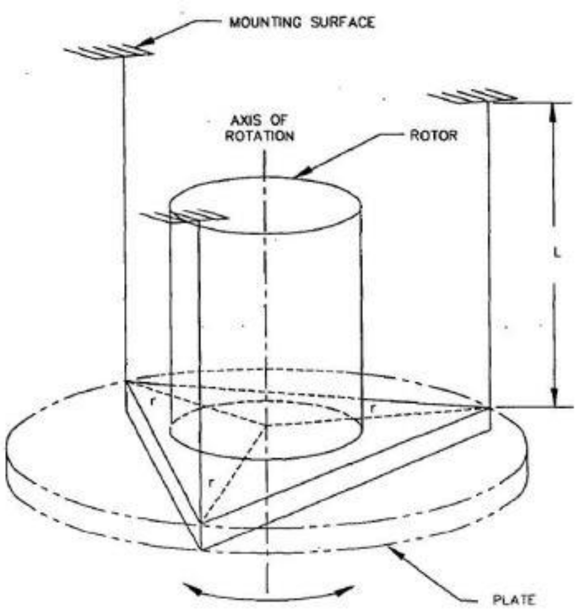


Fig 3. Schematic representation and experimental setup of trifilar suspension system

Specifications:

Mass of body = 1.032 kg

Radius of body = 38 mm

Mass of plate = 0.227 kg

Radius of plate = 100 mm

Distance between centre of plate and point of wire attachment (R) = 97 mm

Observation and Results:

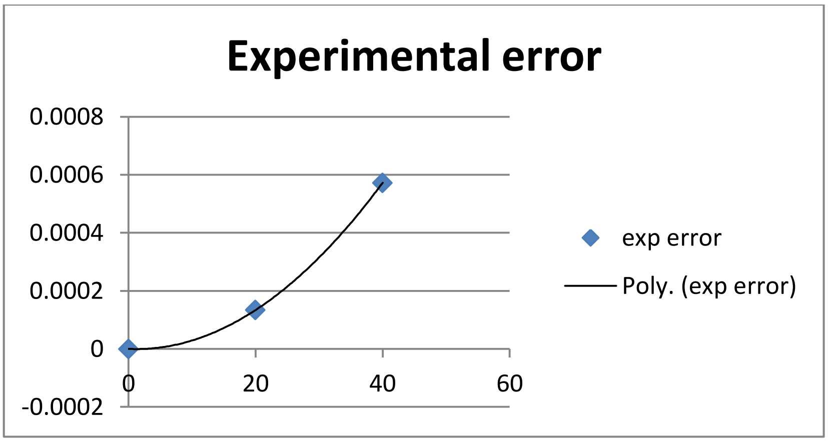
D : Misalignment in centre of mass in body and plate in mm

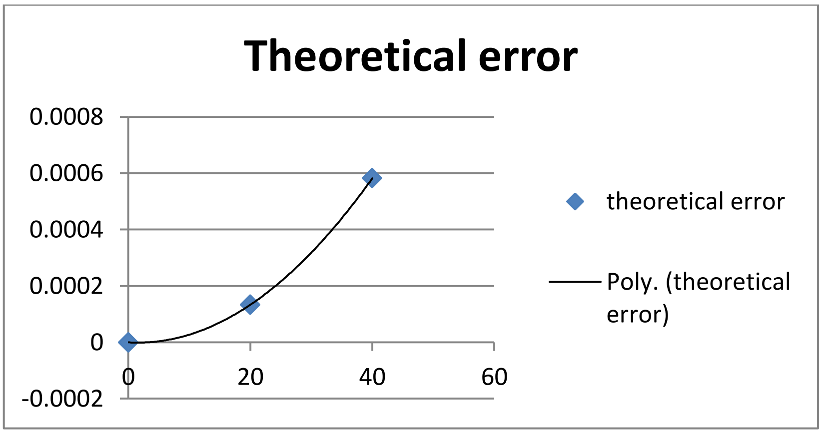
N : Number of oscillations

T : Time of observation in sec

τ : Time period of oscillations in sec

D	N	T	τ	Actual MOI @ D=0	Experimental MOI	Error Experimental	Error theoretical
0	17	10	0.58	7.877×10^{-4}	7.877×10^{-4}	0	0
20	33	20	0.60		9.23×10^{-4}	1.34×10^{-4}	1.332×10^{-4}
40	30	20	0.66		13.6×10^{-4}	5.7224×10^{-4}	5.82×10^{-4}





Conclusion:

From above results we can conclude that error in measurement of mass moment of inertial of body using trifilar suspension because of misalignment between centre of mass of body and plate is directly proportional to the square of misalignment distance (D). For body with unknown centre of mass we can use this principle to reduce misalignment by varying position of body over base plate such that time period of oscillation becomes minimum. In this way we can minimize error because of misalignment in trifilar suspension.