Drive Screw Linear Actuator

Submitted By:
Dharmender (193100061)
Mujahed Patil (193109009)

- Both contributed equally

Abstract

We worked on the problem "Modeling a drive screw linear actuator". This problem consists of mixed continuous-integer nonlinear programming which is different from the standard numerical nonlinear programming so, we decided to work on this problem. The purpose of this problem was to minimize the cost of a drive screw linear actuator. The cost function for the linear actuator consists of two parts, the fixed cost associated with manufacturing part, burden and the cost associated with modeling part. The content of this report deals mainly with the optimization of modeling part i.e. minimizing the volume of design, which results in minimizing the overall cost of linear actuator. An optimal solution was found for linear actuator model using MATLAB. A monotonicity analysis was also performed, which verified the results obtained using MATLAB.

I. Introduction

The proposed optimization project deals with the design optimization of linear actuator (drive screw). The drive screw modeled here is part of a power module assembly that converts rotary to linear motion, so that a given load is linearly oscillated at a specified rate. The device is used in a household appliance (a washing machine). The system assembly consists of an electric motor, drive gear, pinion (driven) gear, drive screw, load-carrying nut, and a chassis providing bearing surfaces and support. The model addresses only the design of the drive screw, schematically shown in Figure 1. The drive screw can be made out of metal or plastic. Only a metal screw model is presented below.

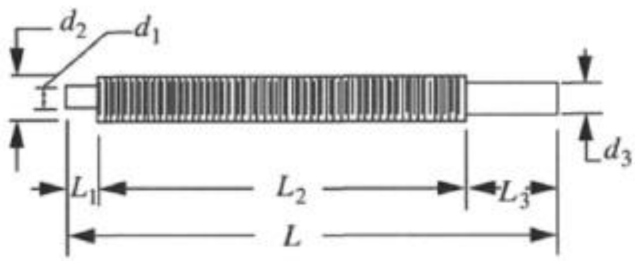

Figure 1- Schematic of drive screw design

Objective:

The objective of this optimization problem is to minimize the total product cost of the drive screw which consists of material cost and machining costs.

$$\text{Minimize } f = C_mV + MC$$

Here,
C_mV = material cost
MC = machining cost

The machining cost associated with the machining of metal drive screw are relatively insensitive to small changes in the design. Therefore, we will assume that the machining cost for the operations will remain constant. With this assumption, the objective function reduces to:

$$\text{Minimize } f = C_mV$$

Here,
C_m = \$/in^3
V = volume of drive screw

Outline:

II. Mathematical Model

Variables:

$x_1 = d_1$ = Diameter of shaft/gear screw interface
$x_2 = d_2$ = Mean diameter of threaded segment
$x_3 = d_3$ = Diameter of bearing surface segment
$x_4 = L_2$ = Length of threaded segment
$x_5 = L_3$ = Length of bearing surface segment
$x_6 = N_m$ = Number of teeth on motor gear (drive)
$x_7 = N_s$ = Number of teeth on screw gear (driven)
$x_8 = N_T$ = Number of threads per inch

Parameters:

Table 1. List of Parameters (Material Values for Stainless Steel)

C_m	material cost ($/in^3)
f	friction coefficient (0.35 for steel on plastic)
F_a	force required to snap fit screw into the chassis during assembly (6 lb.)
K	stress concentration factor (3)
L_1	length of gear/drive screw interface segment (0.405 in)
S	linear cycle rate (0.0583 in/s)
S_m	motor speed (300 rpm)
T_m	motor torque (2 in-ounces)
W	drive screw load (3 lb)
α_n	thread angle in normal plane (60°)
σ_{all}	maximum allowable bending stress (20,000 psi)
τ_{all}	maximum allowable shear stress (22,000 psi)

Mathematical Model:

Drive screw linear actuator
Objective function:

$$\text{Minimize} \quad f = C_m(\pi/4)(d_1^2L_1 + d_2^2L_2 + d_3^2L_3)$$

subjected to:

Inequality constraints

$g_1 = \frac{Mc_1}{I} \leq 20{,}000$

$g_9 = d_2 \leq 0.625$

$g_2 = \frac{KTc_3}{J} \leq 22{,}000$

$g_3 = L \geq 8.75$

$g_4 = L \leq 10.0$

$g_5 = L_2 \leq 7.523$

$g_6 = L_2 \geq 7.023$

$g_7 = L_3 \leq 1.6525$

$g_8 = L_3 \leq 1.1525$

$g_{10} = d_3 \leq d_2$

$g_{11} = d_1 \leq d_2$

$g_{12} = 5\left(\frac{N_m}{N_s}\right) - 0.0583N_T \leq 0$

$g_{13} = \left(3\,\frac{d_2}{2}\right)\left[\frac{(\pi(0.35)d_2 + N_T^{-1}\cos 60\,)}{(\pi d_2 \cos 60 - N_T^{-1} 0.35\,)}\right] \leq T$

$g_{14} = N_m \geq 8$

$g_{15} = N_s \leq 52$

$g_{16} = N_T \leq 24$

Geometric Constraints

$M = 3(0.405 + L_2 + L_3)$

$L = 0.405 + L_2 + L_3$

$T = T_m c_2 N_s / N_m$

$I = \pi(d_2^4/64)$

$J = \pi(d_2^4/32)$

$c_3 = d_1/2$

$C_1 = d_2/2$

Model Summary:

Objective

Minimize $f = C_m(\pi/4)(0.405x_1^2 + x_2^2 x_4 + x_3^2 x_5)$

Subjected to:

$g_1 = 38.88 + 96x_4 + 96x_5 - \pi\sigma_{all} x_2^3 \leq 0$

$g_2 = 6\left(\frac{x_7}{x_6}\right) - \pi\tau_{all} x_1^3 \leq 0$

$g_3 = 8.345 - x_4 - x_5 \leq 0$

$g_4 = -9.595 + x_4 + x_5 \leq 0$

$g_5 = L_2 - 7.523 \leq 0$

$g_6 = 7.023 - x_4 \leq 0$

$g_7 = x_5 - 1.6525 \leq 0$

$g_8 = 1.1525 - x_5 \leq 0$

$g_9 = x_2 - 0.625 \leq 0$

$g_{10} = x_3 - x_2 \leq 0$

$g_{11} = x_1 - x_2 \leq 0$

$g_{12} = 5\left(\frac{x_6}{x_7}\right) - 0.0583x_8 \leq 0$

$g_{13} = (1.5x_2)\left[\frac{(\pi(0.35)x_2 + 0.5x_8x_4)}{(\pi d_2 \cos 60 - x_8x_4 0.35)}\right] - 0.125(\frac{x_7}{x_6}) \leq 0$

$g_{14} = 8 - x_6 \geq 0$

$g_{15} = x_7 - 52 \leq 0$

$g_{16} = x_8 - 24 \leq 0$

Number of equality constraints: 0

Number of inequality constraints: 16

III. Optimization Problem

The objective of this optimization problem is to minimize the total cost of the drive screw linear actuator which consists of the material cost and manufacturing cost. Since machining costs are considered fixed for relatively small changes in the design. Therefor we will assume that the manufacturing costs remain constant. With this assumption, the objective function reduces to minimization volume of the drive screw.

Objective function:

Minimize f $= C_m$*(volume of drive screw)

C_m = material cost($/in^3)

Volume of drive screw V = $V_1 + V_2 + V_3$

V = $\frac{\pi}{4} d_1^2 L_1$

V = $\frac{\pi}{4} d_2^2 L_2$

V = $\frac{\pi}{4} d_3^2 L_3$

Therefore,

Min f = $C_m(\pi/4)(d_1^2 L_1 + d_2^2 L_2 + d_3^2 L_3)$

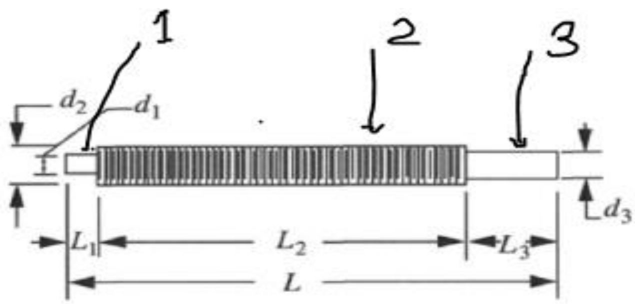

Fig. 2 Drive Screw

Constraints:

I. Bending moment constraint

Bending moment stress due to force required to snap drive screw into the chassis during assembly.

Bending strength against bending

$$\frac{Mc_1}{I} \leq \sigma_{allowable}$$

Here,

$\sigma_{allowable}$ =maximum allowable bending stress

M = $F_a L/2$ is the bending moment

F_a = force required to snap the drive screw

$L = L_1 + L_2 + L_3$

$C_1 = d_2/2$

$I = \pi(d_2^4/64)$

II. Shear Stress Due to Torsion:

During operation, a constraint against fatigue failure in shear plane due to applied torque.

$$KTc_3/J \leq \tau_{allowable}$$

Here,

$\tau_{allowable}$ = maximum allowable shear stress

K = stress concentration factor

T = applied torque

$c_3 = d_1/2$

$J = \pi(d_2^4/32)$

III. Torque constraint:

Torque requirement due to thread load

$$T = T_m c_2 N_s / N_m$$

Here,

T_m = motor torque

C_2 = 1/16(lb/ounce) conversion factor

N_s = number of teeth on screw gear (driven)

N_m = number of teeth on motor gear (drive)

IV. Speed Constraint:

To meet the specified linear cycle rate of oscillation, a speed constraint is imposed

$$(c_4 N_m S_m)/(N_s N_T) \leq S$$

Here,

$C_4 = 60^{-1}$ (no. of threads/rev)(min/s) conversion factor

S_m = motor speed(rpm)

N_T = number of threads per inch

S = linear cycle rate(in/s)

V. No-Slip Constraint:

For the screw to operate in a drive mode a non-slip constraint must be satisfied

$$\left(W \frac{d_2}{2}\right)\left[\frac{(\pi f d_2 + N_T^{-1} \cos \alpha_n)}{(\pi d_2 \cos \alpha_n - N_T^{-1} f)}\right] \leq T$$

Here,

W = drive screw load

f = friction coefficient

N_T^{-1} = number of screw thread

α_n = thread angle

VI. Other constraints are:

Minimum overall length of drive screw.

Maximum overall length of drive screw.

Maximum number of teeth on drive gear.

Minimum number of teeth on pinion gear.

Maximum number of threads per inch in order to mass produce.

Maximum diameter for threaded segment.

Minimum and maximum length for threaded segment.

Minimum and maximum length for lower bearing surface.

Types of variable:

d_1 , d_2 , d_3 , L_2 and L_3 are continuous and N_s , N_m , N_T are integer.

Algorithm:

We used genetic algorithm to solve this problem in MATLAB. Genetic algorithm can solve mixed integer optimization problems. Syntax of genetic algorithm is

```
x = ga(fun,nvars,A,b,[],[],lb,ub,nonlcon,IntCon,options)
```

Monotonicity Analysis:

Monotonicity Table 1:

	X_1	X_2	X_3	X_4	X_5	X_6	X_7	X_8
F	+	+	+	+	+	0	0	0
g_1	0	-	0	+	+	0	0	0
g_2	-	0	0	0	0	-	+	0
g_3	0	0	0	-	-	0	0	0
g_4	0	0	0	+	+	0	0	0
g_5	0	0	0	+	0	0	0	0
g_6	0	0	0	-	0	0	0	0
g_7	0	0	0	0	+	0	0	0
g_8	0	0	0	0	-	0	0	0
g_9	0	+	0	0	0	0	0	0
g_{10}	0	0	0	0	0	+	-	-
g_{11}	0	+	0	0	0	+	-	-
g_{12}	0	0	0	0	0	0	0	+
g_{13}	0	0	0	0	0	-	0	0
g_{14}	0	0	0	0	0	0	+	0

From above table we can see that variable x_3 is unbounded.

So,

$$x_3 = 0.1875$$

This the condition required for minimum bearing diameter which is required to provide additional thrust.

From above table g_2 is active from that we get,

$$x_2 = \left[\frac{38.88 + 96 \times x_4 \quad + 96 \times x_5}{\pi \times \sigma_{all}}\right]^{1/3}$$

Also g_2 is an active constraint,

$$x_1 = \left[\frac{6}{\pi \times \tau_{all}}\left(\frac{N_s}{N_m}\right)\right]^{1/3}$$

After this new objective function will be,

$$f = C_m(\pi/4)(0.405 \times \left[\frac{6}{\pi \times \tau_{all}}\left(\frac{N_s}{N_m}\right)\right]^{2/3} + \left[\frac{38.88 + 96 \times x_4 \quad + 96 \times x_5}{\pi \times \sigma_{all}}\right]^{2/3} x_4 + x_3^2 x_5)$$

Monotonicity Table 2:

	X_4	X_5	X_6	X_7	X_8
F	+	+	-	+	0
g_3	-	-	0	0	0
g_4	+	+	0	0	0
g_5	+	0	0	0	0
g_6	-	0	0	0	0
g_7	0	+	0	0	0
g_8	0	-	0	0	0
g_9	+	+	0	0	0
g_{10}	0	0	+	-	-
g_{11}	+	+	+	-	-
g_{12}	0	0	0	0	+
g_{13}	0	0	-	0	0
g_{14}	0	0	0	+	0

From above table if we,

Let g_3 and g_6 are active then we can see all the other constraints will be satisfied which are related to both x_5 and x_4.

So, we can say that both are active constraints and because of that constraints $g_3, g_4, g_5, g_6, g_7, g_8,$ g_9 will be eliminated from monotonicity analysis.

From this table we get the values of x_5 and x_4

$$x_4 = 7.023$$

$$x_5 = 1.322$$

$$x_2 = 0.2373$$

$$x_3 = 0.0677$$

New objective function will be

$$f = C_m(\pi/4)(0.405 \times \left[\frac{6}{\pi \times \tau_{all}}\left(\frac{N_s}{N_m}\right)\right]^{\frac{2}{3}} + 0.4421)$$

Monotonicity Table 3:

	X_6	X_7	X_8
F	-	+	0
g_{10}	+	-	-
g_{11}	+	-	-
g_{12}	0	0	+
g_{13}	-	0	0
g_{14}	0	+	0

Here for x_8 we can see that function is independent on its value, and there are both types of positive and negative signs (opposite), so for such a case constraint with opposite sign must be active.

Now there are two possibilities, either g_{10} and g_{12} are active or g_{12} and g_{11} but in both cases g_{12} has to be active.

$$g_{12} = 24$$

If we take g_{11} and g_{12} active we can see that constraint g_{10} will not be satisfied so these two will not be active but the same if we take g_{10} and g_{12} active we can see that constraint g_{11} will be satisfied.

From that we get,

$$\frac{N_s}{N_m} = 3.5735$$

Which means that above ratio is going to be constant and this is **multiple optimal solution**.

So, final value of objective function will be **0.34867**

But as per our constraints both N_s and N_m are the integers we will not get the most optimal solution, we have to choose the relative optimal pair of N_s andN_m.

So, for

$N_m = 8$ $N_s = 29$ And $N_m = 12$ $N_s = 43$

These are the possible solutions with integer values of N_s andN_m.

For $N_m = 8$ $N_s = 29$

Objective function value is **0.3487**

So, we can see that

$$error = 0.3487 - 0.34867 = 1.784 \times 10^{-5}$$

So, the error due to making integers to both variables is very minimum.

Also, now we can also check the model validity constraint.

$$\frac{L}{d_2} > 10$$

$$\frac{L}{d_2} = 36.92$$

Which is well above the limit.

IV. Results:

From the MATLAB using genetic algorithm, optimal solutions along with function value are:

```
Command Window
Optimization terminated: average change in the penalty fitness value less than options.FunctionTolerance
and constraint violation is less than options.ConstraintTolerance.
Final Objective: 0.34949
Solution
x1 = 0.069766
x2 = 0.23733
x3 = 0.1875
x4 = 7.0698
x5 = 1.2743
x6 = 8
x7 = 30
x8 = 23
>> output

output =

  struct with fields:

      problemtype: 'integerconstraints'
         rngstate: [1×1 struct]
      generations: 311
        funccount: 24961
          message: 'Optimization terminated: average change in the penalty fitness value less than options.FunctionTolerance↵a
    maxconstraint: 8.6956e-04
```

Fig. 3 Final optimal solutions using genetic algorithm

From monotonicity analysis, optimal solutions along with function value are:

X_1	0.0677
X_2	0.2373
X_3	0.1875
X_4	7.023
X_5	1.322
X_6/ X_7	3.5735
X_8	24

Function value = 0.3487

V. Discussions:

1) Optimum objective value is **0.34867,**

This is for

$$\frac{N_s}{N_m} = 3.5735$$

For,

$N_m = 8 \quad N_s = 29$

Objective function value is **0.3487**

So we can see that

$$error = 0.3487 - 0.34867 = 1.784 \times 10^{-5}$$

For,

$N_m = 12 \quad N_s = 43$

Objective function value is **0.34867**

$$error = 0.3486708 - 0.34867 = 8.1647 \times 10^{-7}$$

2) According to MATLAB;

Objective function value is **0.34949**

Which is very close to the results obtained by monotonicity analysis. But the small error occurred in MATLAB results is because the values of N_m andN_s.

So,

$$\frac{N_s}{N_m} = 3.75$$

Which is not the same as we obtained in the monotonicity analysis.

$$error = 0.34949 - 0.34867 = 8.2 \times 10^{-4}$$

3) Also, now we can also check the model validity constraint,

$$\frac{L}{d_2} > 10 \quad ; \qquad \frac{L}{d_2} = 36.92$$

Which is well above the limit.

VI. Conclusions:

1) Solution is multiple optimum, but as it is constrained by integer values of N_s and N_m we are limited to two to three solutions.
2) So, from above discussion we can conclude that error occurring due to approximating the integer values of N_s and N_m is very small and is in acceptable limits.
3) Also, the result of MATLAB programming is also small and due to the values of N_m andN_s, also value of $N_t = 23$ taken is little higher than what we found by monotonicity analysis ($N_t = 24$)
4) But as it is not coming in the objective function it is not affecting that much to the final value.

VII. Extra Work:

a) Done monotonicity analysis

VIII. References:

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