Assignment . 2

Mujtaba
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Discrete Structures

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Mam Memoona Malik

Question . 1

(a)

p : It is below freezing.
q : It is snowing.

I. It is below freezing and snowing : **($p \wedge q$)**
II. It is below freezing but not snowing : **($p \wedge \neg q$)**
III. It is not below freezing and it is not snowing : **($\neg p \wedge \neg q$)**
IV. It is either snowing or below freezing (or both) : **($p \vee q$)**
V. If it is below freezing, it is also snowing : **($p \wedge q$)**
VI. Either it is below freezing or it is snowing, but it is not snowing if it is below freezing : **($p \oplus q$)**
VII. That it is below freezing is necessary and sufficient for it to be snowing : **($p \rightarrow q$)**

(b)

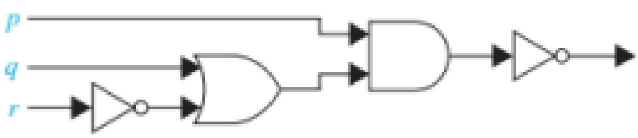
p: The user enters a valid password.
q: Access is granted.
r: The user has paid the subscription fee.

I. "The user has paid the subscription fee, but does not enter a valid password.": **($r \wedge \neg p$)**
II. "Access is granted whenever the user has paid the subscription fee and enters a valid password.": **$q \rightarrow (p \wedge r)$**
III. "Access is denied if the user has not paid the subscription fee.": **$\neg q \rightarrow \neg r$**
IV. "If the user has not entered a valid password but has paid the subscription fee, then access is granted.": **$(\neg p \rightarrow \wedge r) \rightarrow q$**

Question . 2

(a)

Output of this circuit:



This circuit translates to: **¬((q v ¬r) ^ p)**

(b)

Show that $\neg(p \rightarrow q) \rightarrow \neg q$ is a tautology using equivalence laws.

Solution:

p	q	$p \rightarrow q$	$\neg(p \rightarrow q)$	$\neg q$	$\neg(p \rightarrow q) \rightarrow \neg q$
1	1	1	0	0	1
1	0	0	1	1	1
0	1	1	0	0	1
0	0	1	0	1	1

Above table proves this relation.

Question . 3

Let C(x) be the statement "x has a cat," let D(x) be the statement "x has a dog," and let F (x) be the statement "x has a ferret." Express each of these statements in terms of C(x), D(x), F (x), quantifiers, and logical connectives. Let the domain consist of all students in your class.

I. A student in your class has a cat, a dog, and a ferret : $\exists \boldsymbol{x}\ (\boldsymbol{C}(\boldsymbol{x}) \wedge \boldsymbol{D}(\boldsymbol{x}) \wedge \boldsymbol{F}(\boldsymbol{x}))$
II. All students in your class have a cat, a dog, or a ferret : $\forall \boldsymbol{x}(\boldsymbol{C}\ (\boldsymbol{x}) \vee \boldsymbol{D}\ (\boldsymbol{x}) \vee \boldsymbol{F}\ (\boldsymbol{x}))$
III. Some student in your class has a cat and a ferret, but not a dog : **∃x(C(x)∧F(x)∧¬D(x))**

IV. No student in your class has a cat, a dog, and a ferret :
This is the negation of part(I): $\neg\exists x(C(x) \wedge D(x) \wedge F(x))$.

V. For each of the three animals, cats, dogs, and ferrets, there is a student in your class who has this animal as a pet :
Here the owners of these pets can be different: $(\exists x C(x)) \wedge (\exists x D(x)) \wedge (\exists x\ F(x))$. There is no harm in using the same dummy variable, but this could also be written, for example, as : $(\exists x C(x)) \wedge (\exists y D(y)) \wedge (\exists z\ F(z)$

Question . 4

(a)

What rule of inference is used in each of the following arguments?

I. Alice is a mathematics major. Therefore, Alice is either a mathematics major or a computer science major. **Answer: Addition.**

II. Jerry is a mathematics major and a computer science major. Therefore, Jerry is a mathematics major. **Answer: Simplification.**

III. If it is rainy, then the pool will be closed. It is rainy. Therefore, the pool is closed. **Answer: Modus ponens.**

IV. If it snows today, the university will close. The university is not closed today. Therefore, it did not snow today. **Answer: Modus tollens.**

V. If I go swimming, then I will stay in the sun too long. If I stay in the sun too long, then I will sunburn. Therefore, if I go swimming, then I will sunburn. **Answer: Hypothetical syllogism.**

(b)

This argument form shows that the premises lead to the desired conclusion.
Step Reason

1. $p \rightarrow q$ Premise
2. $\neg q \rightarrow \neg p$ Contrapositive of (1)
3. $\neg p \rightarrow r$ Premise
4. $\neg q \rightarrow r$ Hypothetical syllogism using (2) and (3)
5. $r \rightarrow s$ Premise
6. $\neg q \rightarrow s$ Hypothetical syllogism using (4) and (5)

(c)

a) If I play hockey, then I am sore the next day. $h \rightarrow s$
I use the whirlpool if I am sore. $s \rightarrow w$
I did not use the whirlpool. $\neg w$
h: I play hockey.
s: I am sore.
w: I use the whirlpool.

b) If I work, it is either sunny or partly sunny. $\forall x(W(x) \rightarrow (S(x) \lor P(x)))$
I worked last Monday or I worked last Friday. $W(Monday) \lor W(Friday)$
It was not sunny on Tuesday. $\neg S(Tuesday)$
It was not partly sunny on Friday. $\neg P(Friday)$
W(x): I work on x.
S(x): It is sunny on x.
P(x): It is partly sunny on x.

c) All insects have six legs. $\forall x[I(x) \rightarrow L(x)]$
Dragonflies are insects. $\forall x(D(x) \rightarrow I(x))$
Spiders do not have six legs. $\forall x(S(x) \rightarrow \neg L(x))$
Spiders eat dragonflies. $\forall x((S(x) \land D(y) \rightarrow E(x, y))$
I(x): x is an insect.
D(x): x is a dragonfly.
L(x): x has six legs.
S(x): x is a spider.
E(x, y): x eats y.

d) Every student has an internet account. $\forall x(S(x) \rightarrow I(x))$
Homer does not have an internet account. $\neg I(Homer)$
Maggie has an internet account. $I(Maggie)$
S(x): x is a student.
I(x): x has an internet account.

e) All foods that are healthy to eat do not taste good. $\forall x(H(x) \rightarrow \neg G(x))$
Tofu is healthy to eat. H(tofu)
You only eat what tastes good. $\forall x(E(x) \leftrightarrow G(x))$
You do not eat tofu. ¬E(tofu)
Cheeseburgers are not healthy to eat. ¬H(cheeseburger)
H(x): x is healthy to eat.
G(x): x tastes good.
E(x): You eat x

f) I am either dreaming or hallucinating. $d \lor h$
I am not dreaming. $\neg d$
If I am hallucinating, I see elephants running down the road. $h \rightarrow e$
d: I am dreaming.
h: I am hallucinating.
e: I see elephants running down the road.

(d)

a) Everyone enrolled in the university has lived in a dormitory. Mia has never lived in a dormitory. Therefore Mia has not enrolled in the university.
Let E(x) be the proposition that x is enrolled in the university, D(x) be the proposition that x has lived in a dorm, and D(M) be the proposition that Mia has lived in a dormitory.
Step Reason
1. $\forall x (E(x) \rightarrow D(x))$ Hypothesis
2. ¬D(M) Hypothesis
3. ¬E(M) Modus tollens using (3) and (2)
This is correct, using universal instantiation and modus tollens.

b) A convertible car is fun to drive. Isaac's car is not a convertible. Therefore, Isaac's car is not fun to drive.
This is not correct. After applying universal instantiation, it contains the fallacy of denying the hypothesis.
Let C(x) be the proposition that x is a convertible, D(x) be the proposition that x is fun to drive, and C(I) is the proposition that Isaac's car is not a convertible.
Step Reason

1. $\forall x (C(x) \rightarrow D(x))$ Hypothesis
2. $\neg C(I)$ Hypothesis
3. $\neg C(I) \rightarrow \neg D(I)$ Fallacy because the inverse of (1) is not equivalent to (1)
(universal instantiation would say C(I), not $\neg$ C(I))

c) Quincy likes all action movies. Quincy likes the movie Eight Men Out. Therefore, Eight Men Out is an action movie.
Let x be the set of movies. Let A(x) be the proposition that x is an action movie, Q(x) be the proposition that Quincy likes the movie, and Q(E) be the proposition that Quincy likes Eight Men Out.
Step Reason
1. $\forall x (A(x) \rightarrow Q(x))$ Hypothesis
2. Q(E) Hypothesis
3. $Q(E) \rightarrow A(E)$ Fallacy because the converse of (1) is not equivalent to (1)
(universal instantiation would say A(E), not Q(E); it affirms
the conclusion according to the author)

d) All lobstermen set at least a dozen traps. Hamilton is a lobsterman. Therefore, Hamilton sets at least a dozen traps.
Let L(x) be the proposition that x is a lobsterman, and T(x) be the proposition that x sets at least a dozen traps, and L(H) be the proposition that Hamilton is a lobsterman.
Step Reason
1. $\forall x (L(x) \rightarrow T(x))$ Hypothesis
2. L(H) Hypothesis and instantiation
3. $L(H) \rightarrow T(H)$ Conclusion by modus ponens (correct, using universal instantiation and modus ponens).