



COMSATS University, Islamabad

# Assignment # 1

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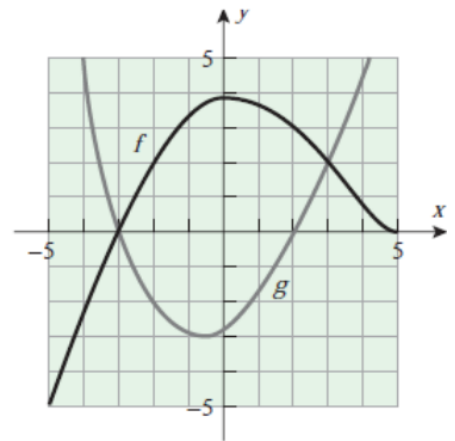
Course: Calculus (MTH-101)

Instructor: Dr. Tayyaba Ehsan

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## Question # 1

Use the graphs of the functions  $f$  and  $g$  in the accompanying figure to solve the following problems.



### Solution

- Value of  $f(-2)$  is **2**. And the value of  $g(3)$  is **2**. This is the value of y-axis for each given x-axis value.
- $f(x) = g(x)$  for these values:  **$\{-3, 3\}$** .
- $f(x) < 2$  for values of  $x$  **other than** the values of  $x$  in range:  **$[-2, 3]$** .
- Domain of  **$f$**  is  **$[-5, 5]$**  and range of  **$f$**  is  **$[-5, 4]$** .
- Domain of  **$g$**  is  **$[-4, 4]$**  and range of  **$g$**  is  **$[-3, 5]$** .

## Question # 2

Sketch the graph of the equation  $x^2 - 9y^2 = 0$

### Solution

We simplify the equation first:

3

$$x^2 = 9y^2$$

$$y = \pm \sqrt{\frac{x^2}{9}}$$

$$y = -\sqrt{\frac{x^2}{9}}$$

$$y = -\frac{x}{3}$$

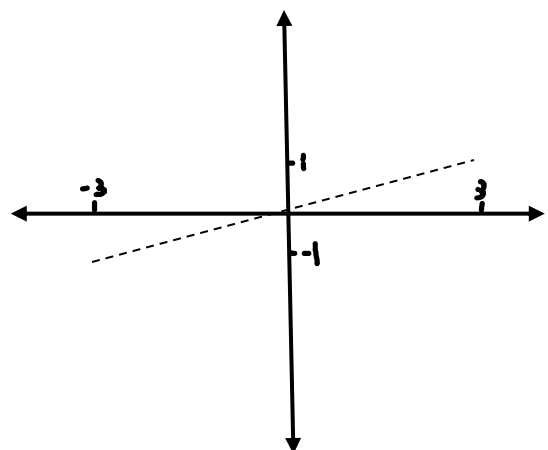
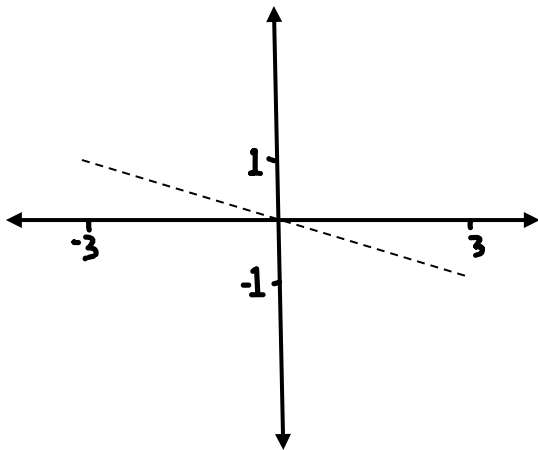
|

$$y = \sqrt{\frac{x^2}{9}}$$

$$y = \frac{x}{3}$$

X	-3	-2	-1	0	1	2	3
Y	1	0.7	0.3	0	-0.3	-0.7	-1

X	-3	-2	-1	0	1	2	3
Y	-1	-0.7	-0.3	0	0.3	0.7	1

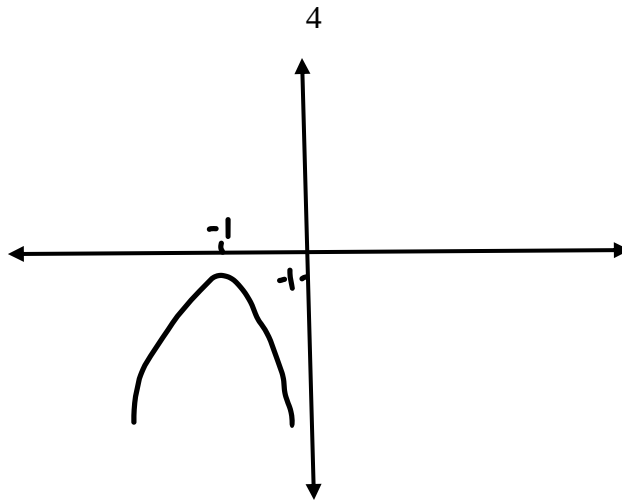


### Question # 3

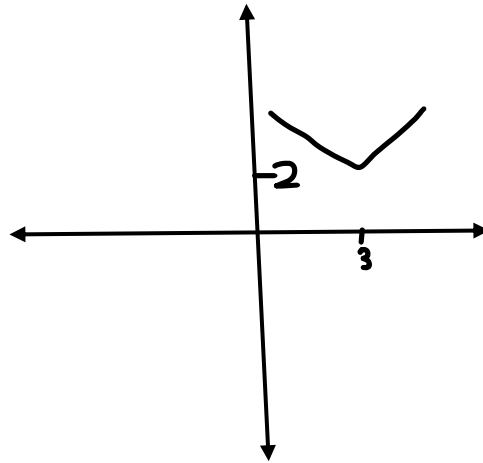
Sketch the graph of the equation by translating, reflecting, compressing, and stretching the graph of  $y$  appropriately, and then use the graphical utility to confirm your sketch is correct.

### Solution (part i)

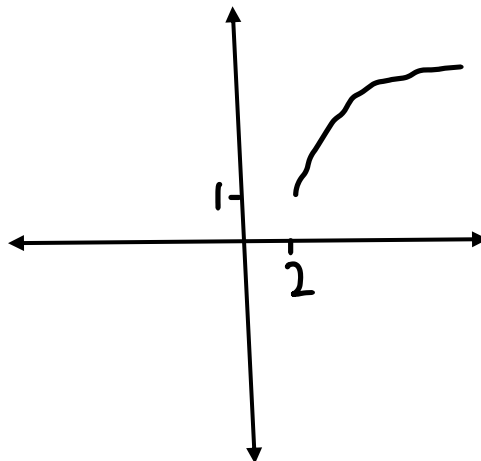
$$1. -2(x + 1)^2 - 1$$



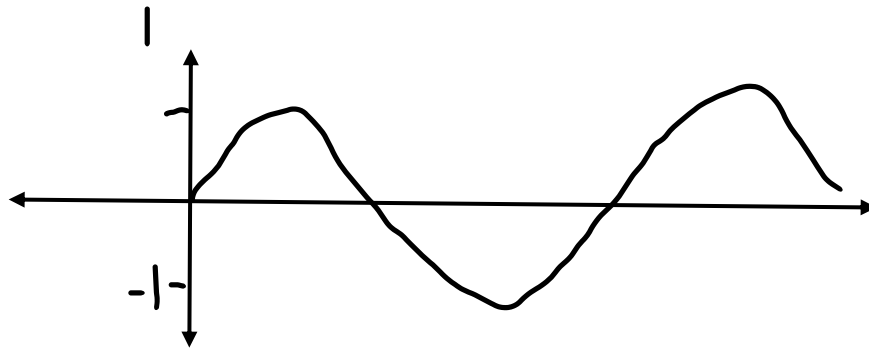
2.  $|2x - 3| + 2$



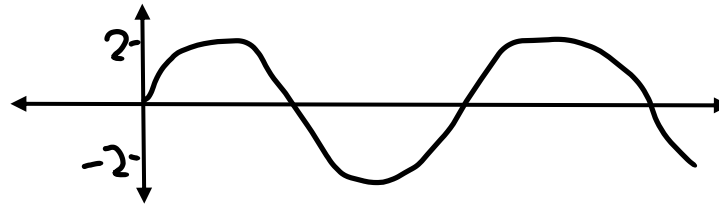
3.  $1 + 2\sqrt{x - 2}$



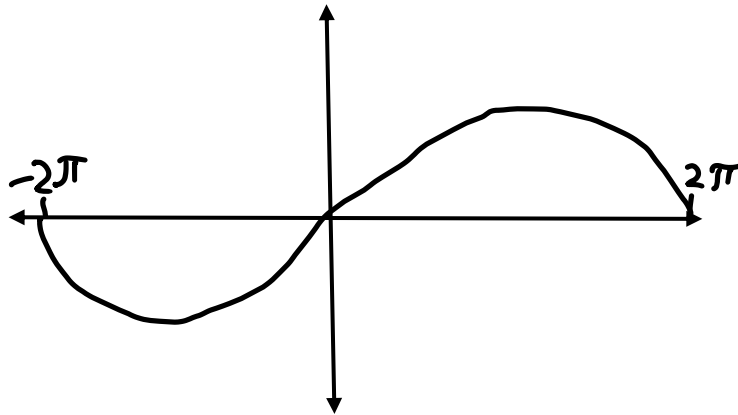
4.  $\sin(2x)$  |  $0 < x < 2\pi$



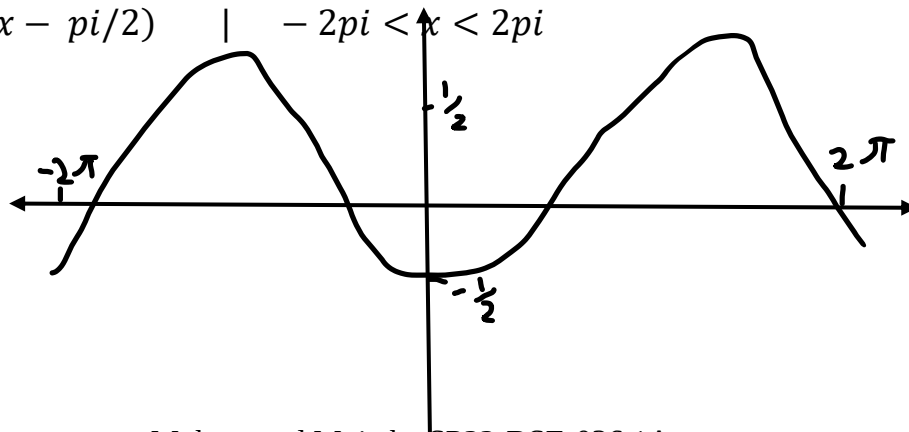
5.  $2\sin(x)$  |  $0 < x < 2\pi$



6.  $\sin\left(\frac{x}{2}\right)$  |  $-2\pi < x < 2\pi$



7.  $\frac{1}{2}\sin(x - \pi/2)$  |  $-2\pi < x < 2\pi$



8. The functions that are **not** one-to-one are (according to the above numbering): **1, 2, 4, 5, 6, 7.**

### Solution (part ii)

a.  $f(x) = \frac{x^3}{x^2+1}$  find  $x$  if  $f^{-1}(x) = 2$

$$\text{Since } f^{-1}(x) = 2 \rightarrow f(2) = x \rightarrow \frac{2^3}{2^2+1} = \frac{8}{5}$$

b.  $f(x) = -\sqrt{3-2x}$  find  $f^{-1}(x)$

$$y = -\sqrt{3-2x} \rightarrow y^2 = 3-2x \rightarrow -2x = y^2 - 3 \rightarrow x = (y^2 - 3)/(-2)$$

### Question # 4

Find the domain and range of the following.

### Solution

1.  $y = -2 + \sqrt{1-x}$

$$1-x \geq 0 \text{ and } 1 \geq x$$

$$\text{So, domain: } (-\infty, 1]$$

$$\text{And range: } [-2, \infty)$$

2.  $y = 3^{2-x} + 1$

$$\text{Domain: } (-\infty, \infty)$$

$$\text{Range: } (1, \infty)$$

3.  $y = \tan(2x - \pi)$

$$\text{We know that } 2x - \pi \neq 90$$

$$2x - \pi i = \frac{\pi i}{2} \rightarrow 2x = \frac{\pi i}{2} + \pi i \rightarrow 2x = \frac{3\pi i}{2}$$

$$x \neq \frac{3\pi i}{4}$$

$$\text{Domain: } \left[ \frac{\pi n}{2} + \frac{3\pi i}{4} \right] n_i \text{ (an integer)}$$

$$4. y = x^{2/5}$$

$$\text{Domain: } (-\infty, \infty)$$

$$\text{Range: } (0, \infty)$$

$$5. y = -1 + \sqrt[3]{2 - x}$$

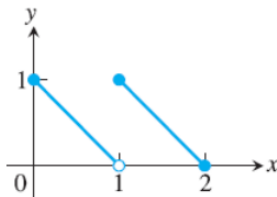
$$\text{Domain: } (-\infty, \infty)$$

$$\text{Range: } (-\infty, \infty)$$

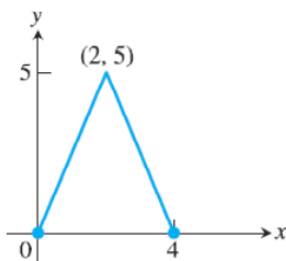
## Question # 5

Write the piecewise formula of the following.

### Solution



$$f(x) = \begin{cases} -x + 1 & 0 \leq x < 1 \\ -x + 2 & 1 \leq x \leq 2 \end{cases}$$



By applying formula:  $y = m(x - x')$

$$f(x) = \begin{cases} \frac{5}{2}x & 0 < x < 2 \\ -\frac{5}{2}x + 10 & 2 < x < 4 \end{cases}$$