***COMSATS UNIVERSITY, ISLAMABAD***

**Assignment . 4 Discrete Structures**

**By Mujtaba SP22-BSE-036**

**To Mam Mamoona Malik**

**Date: Jan 9, 2022**

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**Q1: Solution:** 1 + 2 + 2^2 + …. 2^n = 2^(n+1)-1

Step 1 (Basis): P(0) = 1

Step 2 (Inductuve): For the inductive hypothesis, we assume that P (k) is true for an arbitrary nonnegative integer k. That is, we assume that

1 + 2 + 22 +⋯+ 2k = 2k+1 − 1.

To carry out the inductive step using this assumption, we must show that when we assume that P (k) is true, then P (k + 1) is also true. That is, we must show that

1 + 2 + 22 +⋯+ 2k + 2k+1 = 2(k+1)+1 − 1 = 2k+2 – 1

assuming the inductive hypothesis P (k). Under the assumption of P (k), we see that

1 + 2 + 22 +⋯+ 2k + 2k+1 = (1 + 2 + 22 +⋯+ 2k) + 2k+1 IH = (2k+1 − 1) + 2k+1 = 2 ⋅ 2k+1 − 1 = 2k+2 – 1

**Q2: Solution:**

Because √2 = a/b, when both sides of this equation are squared, it follows that 2 = a2 b2 . Hence, 2b2 = a2. By the definition of an even integer it follows that a2 is even. We next use the fact that if a2 is even.

Furthermore, because a is even, by the definition of an even integer, a = 2c for some integer c. Thus, 2b2 = 4c2. Dividing both sides of this equation by 2 gives b2 = 2c2. By the definition of even, this means that b2 is even. Again using the fact that if the square of an integer is even, then the integer itself must be even, we conclude that b must be even as well.

We have now shown that the assumption of ¬p leads to the equation √2 = a/b, where a and b have no common factors, but both a and b are even, that is, 2 divides both a and b. Note that the statement that √2 = a/b, where a and b have no common factors, means, in particular, that 2 does not divide both a and b. Because our assumption of ¬p leads to the contradiction that 2 divides both a and b and 2 does not divide both a and b, ¬p must be false. That is, the statement p, "√2 is irrational," is true. We have proved that √2 is irrational.

**Q3: Solution:**

p(E | F ) = p(E ∩ F ) p(F ) .

Because E ∩ F = {0000, 0001, 0010, 0011, 0100}, we see that p(E ∩ F ) = 5/16.

Because there are eight bit strings of length four that start with a 0, we have p(F ) = 8/16 = 1/2. Consequently, p(E | F ) = 5/16 1/2 = 5 8

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