Proving a triangle is a right triangle

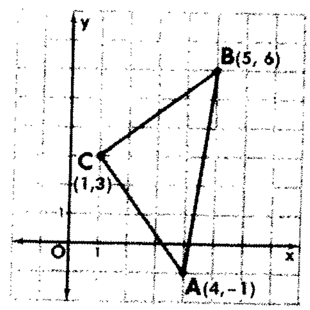
Method 1: Show two sides of the triangle are perpendicular by demonstrating their slopes are opposite reciprocals.

Method 2: Calculate the distances of all three sides and then test the Pythagorean's theorem to show the three lengths make the Pythagorean's theorem true.

Example 1:

Given: The triangle with vertices $A(4, -1)$, $B(5, 6)$, and $C(1, 3)$.

Show: $\triangle ABC$ is an isosceles right triangle.

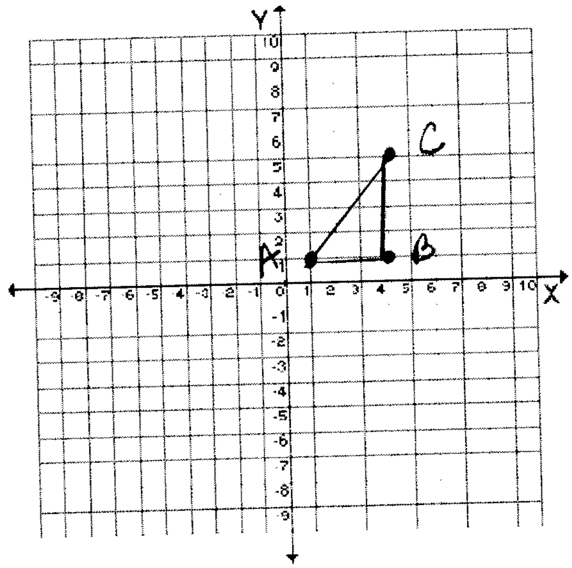


See notes on 1/11/2011

Example 2:
Prove that the polygon with coordinates A(1, 1), B(4, 5), and C(4, 1) is a right triangle.

<ins>method 1: Slopes</ins>

$m = \dfrac{\Delta y}{\Delta x}$



$m_{\overline{AB}} = \dfrac{0}{3} = 0$

$m_{\overline{BC}} = \dfrac{4}{0} = \text{undefined}$

$\overline{AB} \perp \overline{BC}$

$\therefore \Delta ABC$ is a right Δ with a right $\angle$ at B.

Example 3:
Prove that the polygon with coordinates A(5, 6), B(8, 5), and C(2, -3) is a right triangle.

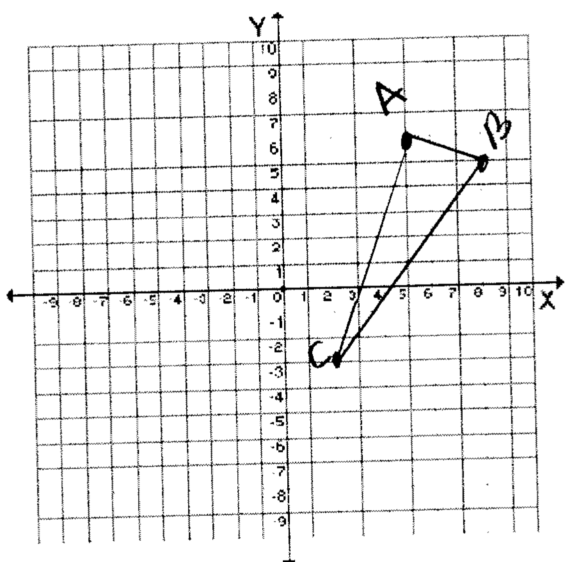
<ins>method 2: Distance</ins>

$d = \sqrt{\Delta x^2 + \Delta y^2}$

$AB = \sqrt{(-3)^2 + (1)^2} = \sqrt{10}$

$BC = \sqrt{(6)^2 + (8)^2} = \sqrt{100} = 10$

$AC = \sqrt{(3)^2 + (9)^2} = \sqrt{90} = 3\sqrt{10}$



$c^2 = a^2 + b^2$

$(\sqrt{100})^2 = (\sqrt{10})^2 + (\sqrt{90})^2$

$100 = 10 + 90$

$100 = 100$

$\therefore \Delta ABC$ is a right Δ.

Proving a Quadrilateral is a Parallelogram

Method 1: Show that the diagonals bisect each other by showing the midpoints of the diagonals are the same

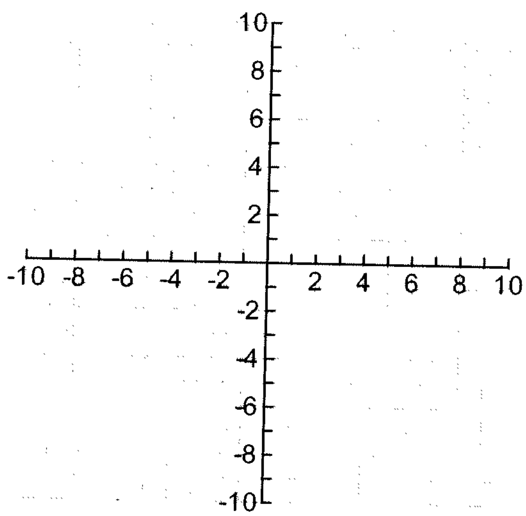
Method 2: Show both pairs of opposite sides are parallel by showing they have equal slopes.

Method 3: Show both pairs of opposite sides are equal by using distance.

Method 4: Show one pair of sides is both parallel and equal.

Examples

1. Prove that the quadrilateral with the coordinates L(-2,3), M(4,3), N(2,-2) and O(-4,-2) is a parallelogram.



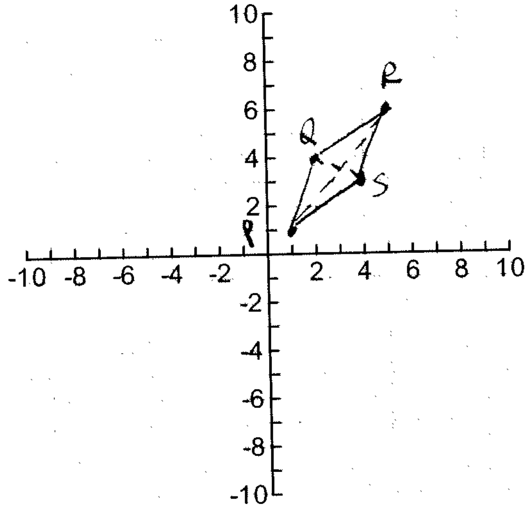
See notes on 1/11/2011

2. Prove that the quadrilateral with the coordinates P(1,1), Q(2,4), R(5,6) and S(4,3) is a parallelogram.

$$M_p = \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$$

OR

$$M_p = (\bar{x}, \bar{y})$$



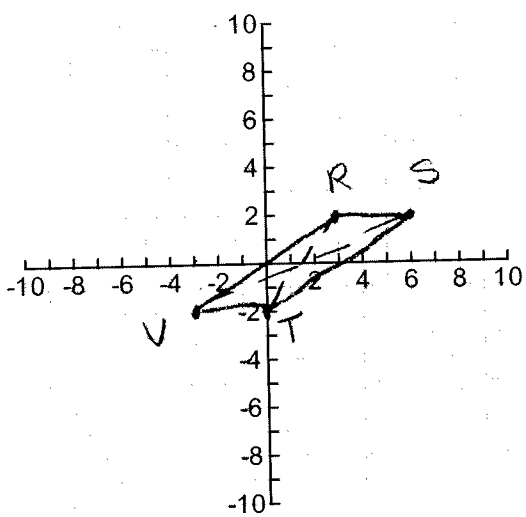
M_p of $\overline{QS} = \left(\frac{2+4}{2}, \frac{4+3}{2}\right) = (3, 7/2)$

M_p of $\overline{PR} = \left(\frac{1+5}{2}, \frac{1+6}{2}\right) = (3, 7/2)$

∴ Quad PQRS is a ▱ b/c diags bisect each other.

3. Prove that the quadrilateral with the coordinates R(3,2), S(6,2), T(0,-2) and U(-3,-2) is a parallelogram.

$$M_p = (\bar{x}, \bar{y})$$



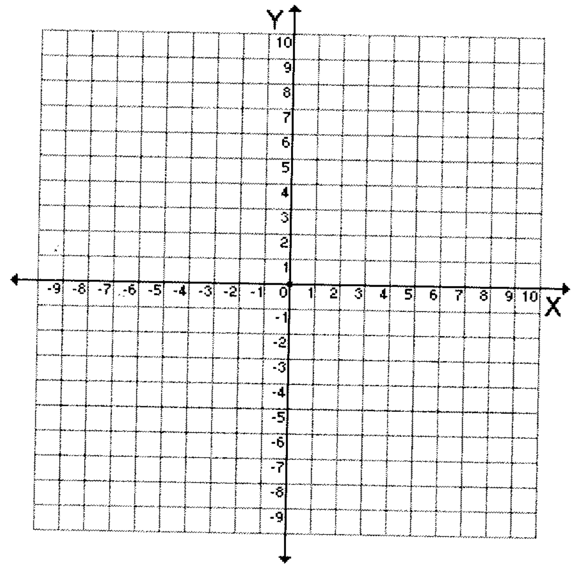
M_p of $\overline{RT} = \left(\frac{3+0}{2}, \frac{2+-2}{2}\right) = \left(\frac{3}{2}, 0\right)$

M_p of $\overline{US} = \left(\frac{-3+6}{2}, \frac{-2+2}{2}\right) = \left(\frac{3}{2}, 0\right)$

∴ Quad RSTU is a ▱ b/c diags bisect each other.

Homework

1. Triangle *TRI* has vertices *T*(15, 6), *R*(5, 1), and *I*(5, 11). Use coordinate geometry to prove that triangle *TRI* is isosceles.



$$D = \sqrt{\Delta x^2 + \Delta y^2}$$

$$TR = \sqrt{(10)^2 + (5)^2} = \sqrt{125} = 5\sqrt{5}$$

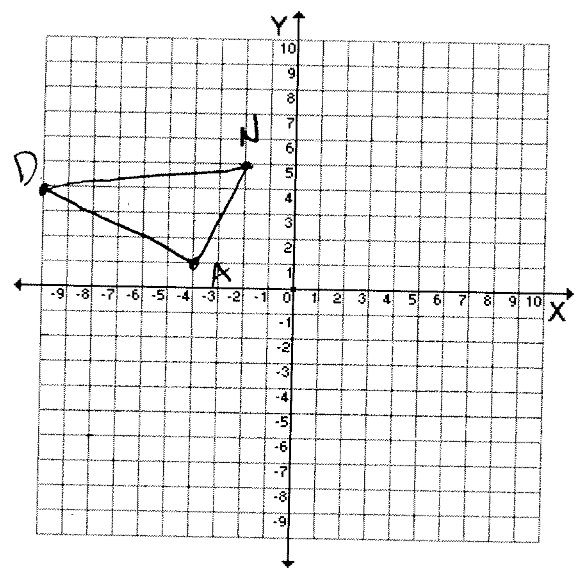
$$RI = \sqrt{(0)^2 + (-10)^2} = \sqrt{100} = 10$$

$$TI = \sqrt{(10)^2 + (-5)^2} = \sqrt{125} = 5\sqrt{5}$$

$$TR = TI$$

$\therefore$ ΔTRI is isosceles.

2. Triangle *DAN* has coordinates *D*(−10, 4), *A*(−4, 1), and *N*(−2, 5). Using coordinate geometry, prove that triangle *DAN* is a right triangle.



$$m = \frac{\Delta y}{\Delta x}$$

$$m_{\overline{DA}} = \frac{3}{-6} = \frac{-1}{2}$$

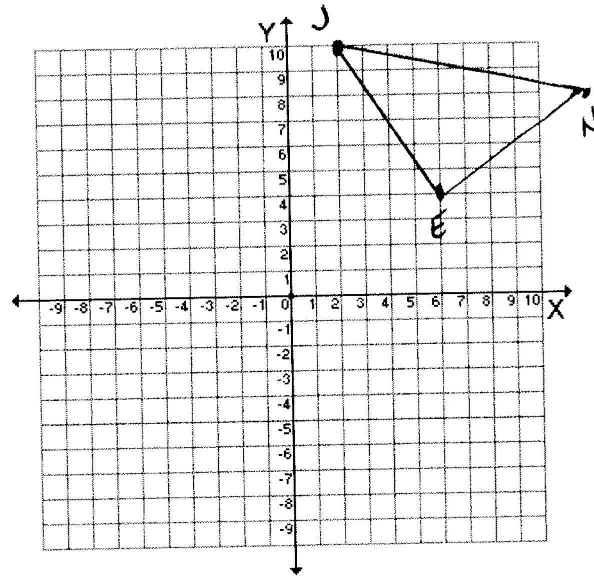
$$m_{\overline{AN}} = \frac{4}{2} = \frac{2}{1}$$

$$\frac{-1}{2} \cdot \frac{2}{1} = \frac{-2}{2} = -1$$

$$\overline{DA} \perp \overline{AN}$$

$\therefore$ ΔDAN is a right triangle with a right $\angle$ at A.

3. The vertices of triangle *JEN* are *J*(2, 10), *E*(6, 4), and *N*(12, 8). Use coordinate geometry to prove that triangle *JEN* is an isosceles right triangle.



$D=\sqrt{\Delta x^2+\Delta y^2}$

$JE=\sqrt{(-4)^2+(6)^2}=\sqrt{52}=2\sqrt{13}$

$EN=\sqrt{(6)^2+(4)^2}=\sqrt{52}=2\sqrt{13}$

$JN=\sqrt{(10)^2+(2)^2}=\sqrt{104}=2\sqrt{26}$

$JE=EN$

$c^2=a^2+b^2$

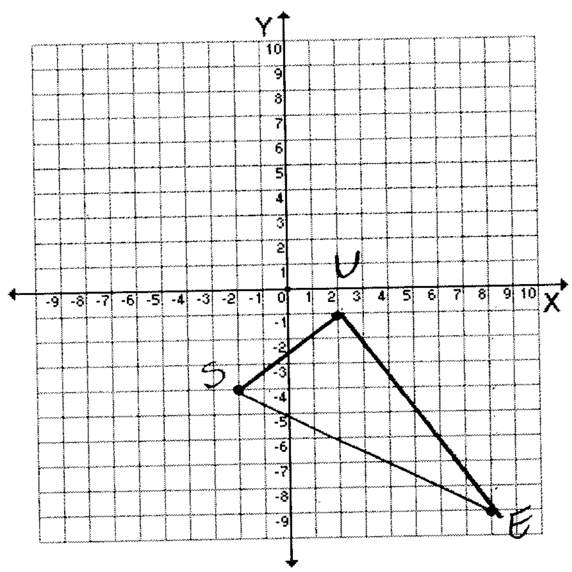
$(\sqrt{104})^2=(\sqrt{52})^2+(\sqrt{52})^2$

$104=52+52$

$104=104$

$\therefore$ ΔJEN is an isos right triangle with a right $\measuredangle$ at E.

4. The coordinates of the vertices of triangle *SUE* are *S*(−2, −4), *U*(2, −1) and *E*(8, −9). Using coordinate geometry, prove that
a. triangle *SUE* is a right triangle.
b. triangle *SUE* is *not* an isosceles right triangle.



$D=\sqrt{\Delta x^2+\Delta y^2}$

$SU=\sqrt{(4)^2+(3)^2}=\sqrt{25}=5$

$UE=\sqrt{(-6)^2+(8)^2}=\sqrt{100}=10$

$SE=\sqrt{(10)^2+(5)^2}=\sqrt{125}=5\sqrt{5}$

a. $c^2=a^2+b^2$

$(\sqrt{125})^2=(\sqrt{100})^2+(\sqrt{25})^2$

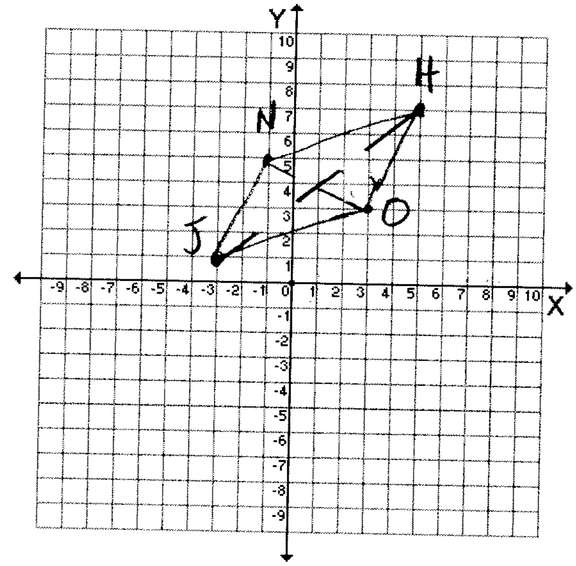
$125=125$

$\therefore$ ΔSUE is a right Δ.

b. $SU \neq UE$

$\therefore$ ΔSUE is not an isosceles right Δ.

5. The vertices of quadrilateral *JOHN* are $J(-3,1)$, $O(3,3)$, $H(5,7)$, and $N(-1,5)$. Use coordinate geometry to prove that quadrilateral *JOHN* is a parallelogram.



$Mp = (\overline{x}, \overline{y})$

$Mp \text{ of } \overline{JH} = \left(\frac{-3+5}{2}, \frac{1+7}{2}\right) = (1,4)$

$Mp \text{ of } \overline{NO} = \left(\frac{3+-1}{2}, \frac{3+5}{2}\right) = (1,4)$

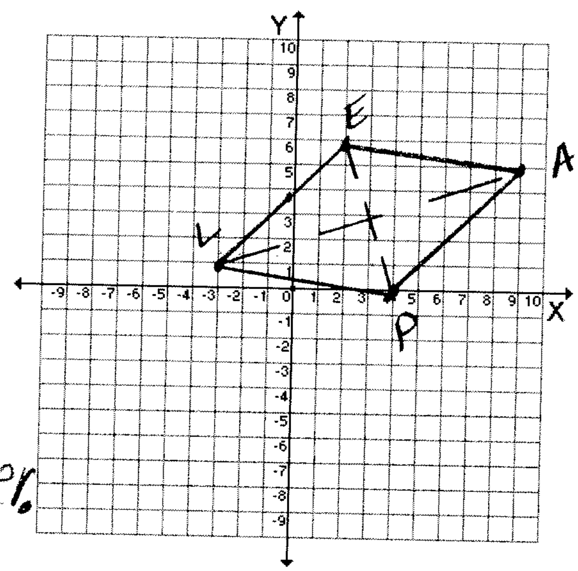
$\therefore$ Quad JOHN is a ▱ b/c diags bisect each other.

6. Prove that quadrilateral LEAP with the vertices L(-3,1), E(2,6), A(9,5) and P(4,0) is a parallelogram.

$Mp = (\overline{x}, \overline{y})$

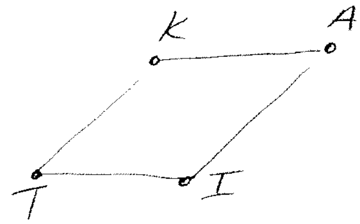
$Mp \text{ of } \overline{LA} = \left(\frac{-3+9}{2}, \frac{1+5}{2}\right) = (3,3)$

$Mp \text{ of } \overline{EP} = \left(\frac{2+4}{2}, \frac{6+0}{2}\right) = (3,3)$



$\therefore$ Quad LEAP is a ▱ b/c diags bisect each other.

7. The vertices of quadrilateral $KAIT$ are $K(0, 0)$, $A(a, 0)$, $I(a + b, c)$, and $T(b, c)$. Use coordinate geometry to prove that quadrilateral $KAIT$ is a parallelogram.



$Mp = (\overline{x}, \overline{y})$

Mp of $\overline{TA}$

$\left(\frac{a+b}{2}, \frac{0+c}{2}\right) = \left(\frac{a+b}{2}, \frac{c}{2}\right)$

Mp of $\overline{KI}$

$\left(\frac{0+(a+b)}{2}, \frac{0+c}{2}\right) = \left(\frac{a+b}{2}, \frac{c}{2}\right)$

$\therefore$ Quad KAIT is a $\square$

b/c diags bisect each other.

Proving a Quadrilateral is a Rectangle

Prove that it is a parallelogram first, then:

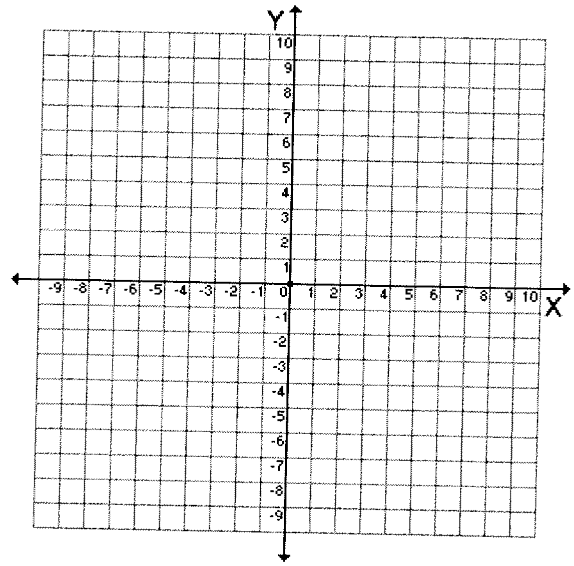
Method 1: Show that the diagonals are congruent.

Method 2: Show that it has a right angle by using slope.

Examples:

1. Prove a quadrilateral with vertices G(1,1), H(5,3), I(4,5) and J(0,3) is a rectangle.

See class notes on 1/11/2011



2. The vertices of quadrilateral COAT are C(0,0), O(5,0), A(5,2) and T(0,2). Prove that COAT is a rectangle.

$Mp = (\bar{x}, \bar{y})$

$D = \sqrt{\Delta x^2 + \Delta y^2}$

Mp of $\overline{CA} = \left(\frac{0+5}{2}, \frac{0+2}{2}\right) = \left(\frac{5}{2}, 1\right)$

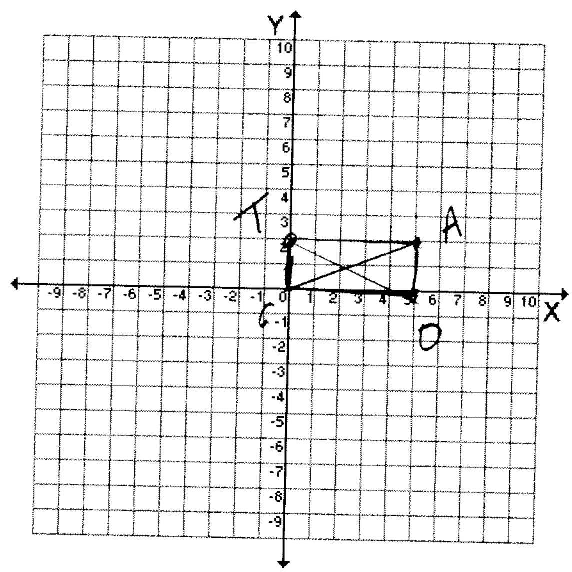
Mp of $TO = \left(\frac{5+0}{2}, \frac{0+2}{2}\right) = (5/2, 1)$

$\therefore$ Quad COAT is a ▱ b/c diags bisect eachother.

$TO = \sqrt{(-5)^2 + (2)^2} = \sqrt{29}$

$CA = \sqrt{(5)^2 + (2)^2} = \sqrt{29}$

$TO = CA$



$\therefore$ ▱COAT is a $\boxed{R}$ b/c diags are equal.

Proving a Quadrilateral is a Rhombus

Prove that it is a parallelogram first, then:

Method 1: Prove that the diagonals are perpendicular.

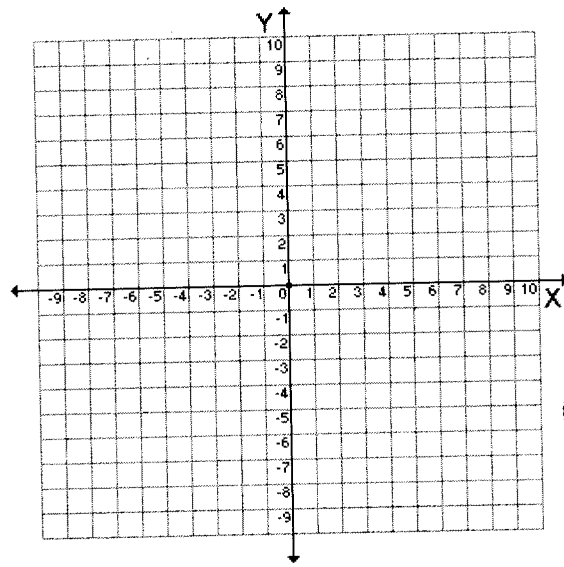
Method 2: Prove that a pair of adjacent sides are equal.

Method 3: Prove that all four sides are equal.

Examples:

1. Prove that a quadrilateral with the vertices A(-2,3), B(2,6), C(7,6) and D(3,3) is a rhombus.

See class notes on 1/11/2011



2. Prove that the quadrilateral with the vertices A(-1,4), B(2,6), C(5,4) and D(2,2) is a rhombus.

$Mp = (\bar{x}, \bar{y})$ $\quad m = \frac{\Delta y}{\Delta x}$

$Mp \text{ of } \overline{AC} = \left(\frac{-1+5}{2}, \frac{4+4}{2}\right) = (2,4)$

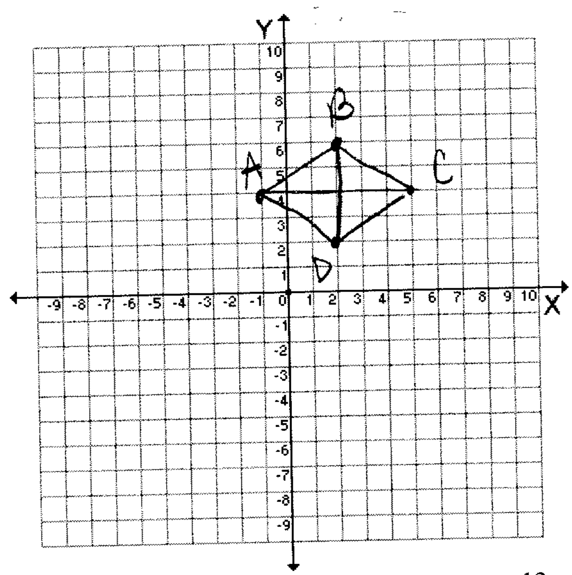
$Mp \text{ of } \overline{BD} = \left(\frac{2+2}{2}, \frac{6+2}{2}\right) = (2,4)$

∴ Quad ABCD is a ▱ b/c diags bisect each other.

$m_{\overline{AC}} = \frac{0}{6} = 0$

$m_{\overline{BD}} = \frac{4}{0} = \text{undefined}$

$\overline{AC} \perp \overline{BD}$



∴ ▱ABCD is a rhombus b/c diags are ⊥.

Proving that a Quadrilateral is a Square

There are many ways to do this. I recommend proving the diagonals bisect each other (parallelogram), are equal (rectangle) and perpendicular (rhombus).

Examples:

1. Prove that the quadrilateral with vertices A(0,0), B(4,3), C(7,-1) and D(3,-4) is a square.

$m = \frac{\Delta y}{\Delta x}$ $d = \sqrt{\Delta x^2 + \Delta y^2}$

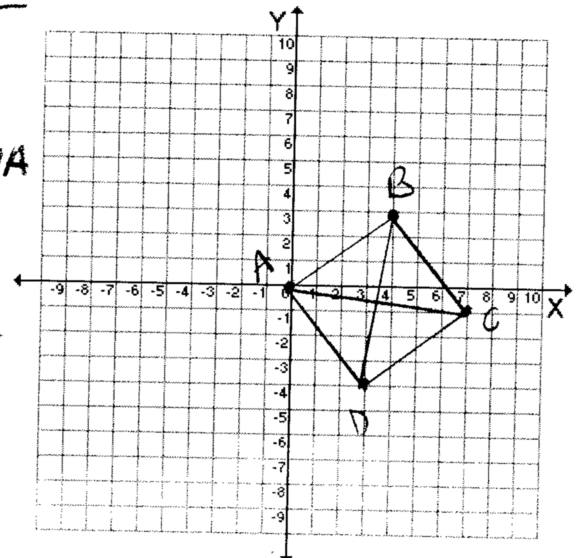
$AB = \sqrt{(4)^2 + (3)^2} = 5$

$BC = \sqrt{(-3)^2 + (4)^2} = 5$

$CD = \sqrt{(4)^2 + (3)^2} = 5$

$DA = \sqrt{(-3)^2 + (4)^2} = 5$

$AB = BC = CD = DA$



∴ Quad ABCD is a ▱ b/c 2 pairs of opposite sides are equal

∴ Quad ABCD is a rhombus b/c it is equilateral

$M_{\overline{AB}} = \frac{3}{4}$, $M_{\overline{DA}} = \frac{4}{-3}$; $\frac{3}{4} \cdot \frac{4}{-3} = \frac{12}{-12} = -1$; $\overline{AB} \perp \overline{DA}$

∴ ABCD is a square b/c it is a rhombus w/ a right ∠.

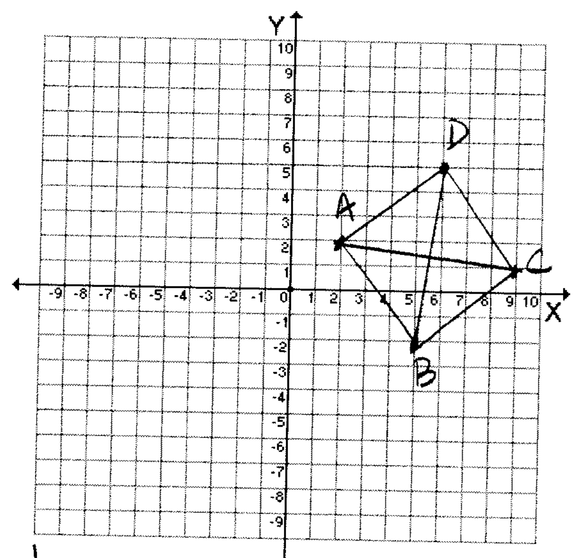
2. Prove that the quadrilateral with vertices A(2,2), B(5,-2), C(9,1) and D(6,5) is a square.

$M_P = (\bar{x}, \bar{y})$ $m = \frac{\Delta y}{\Delta x}$ $d = \sqrt{\Delta x^2 + \Delta y^2}$

Mp of $\overline{BD} = \left(\frac{5+6}{2}, \frac{-2+5}{2}\right) = (11/2, 3/2)$

Mp of $\overline{AC} = \left(\frac{2+9}{2}, \frac{2+1}{2}\right) = (11/2, 3/2)$

∴ Quad ABCD is a ▱ b/c diags bisect each other.



$M_{\overline{BD}} = \frac{7}{1}$, $M_{\overline{AC}} = \frac{1}{-7}$; $\frac{7}{1} \cdot \frac{1}{-7} = \frac{7}{-7} = -1$; $\overline{BD} \perp \overline{AC}$

∴ ▱ ABCD is a rhombus b/c diags ⊥

$BD = \sqrt{(1)^2 + (7)^2} = \sqrt{50} = 5\sqrt{2}$

$AC = (-7)^2 + (1)^2 = \sqrt{50} = 5\sqrt{2}$

$BD = AC$

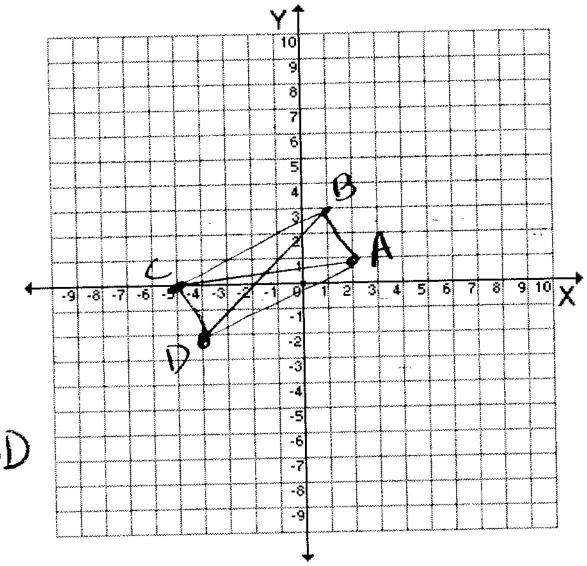
∴ ▱ ABCD is a rectangle b/c diags =

∴ ABCD is a square b/c it is a rhombus and a rectangle.

Homework

1. Prove that quadrilateral ABCD with the vertices A(2,1), B(1,3), C(-5,0), and D(-4,-2) is a rectangle.

$M_p = (\overline{x}, \overline{y}) \qquad d = \sqrt{\Delta x^2 + \Delta y^2}$



M_p of $\overline{AC} = \left(\frac{2+-5}{2}, \frac{1+0}{2}\right) = \left(\frac{-3}{2}, \frac{1}{2}\right)$

M_p of $\overline{BD} = \left(\frac{1+-4}{2}, \frac{3+-2}{2}\right) = \left(\frac{-3}{2}, \frac{1}{2}\right)$

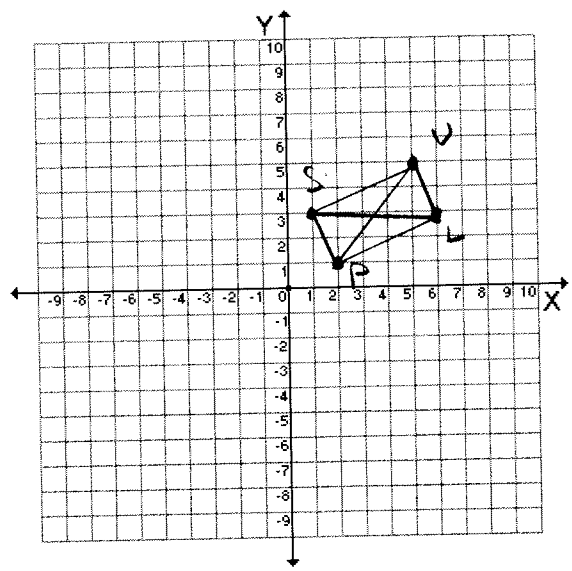
$\therefore$ Quad ABCD is a ▱ b/c diags bisect each other.

$AC = \sqrt{(7)^2 + (1)^2} = \sqrt{50} = 5\sqrt{2}$
$BD = \sqrt{(-5)^2 + (5)^2} = \sqrt{50} = 5\sqrt{2}$
$\Big\}$ $AC = BD$

$\therefore$ ▱ABCD is a rectangle b/c diags =.

2. Prove that quadrilateral PLUS with the vertices P(2,1), L(6,3), U(5,5), and S(1,3) is a rectangle.

$M_p = (\overline{x}, \overline{y}) \qquad d = \sqrt{\Delta x^2 + \Delta y^2}$



M_p of $\overline{UP} = \left(\frac{2+5}{2}, \frac{1+5}{2}\right) = (7/2, 3)$

M_p of $\overline{SL} = \left(\frac{6+1}{2}, \frac{3+3}{2}\right) = (7/2, 3)$

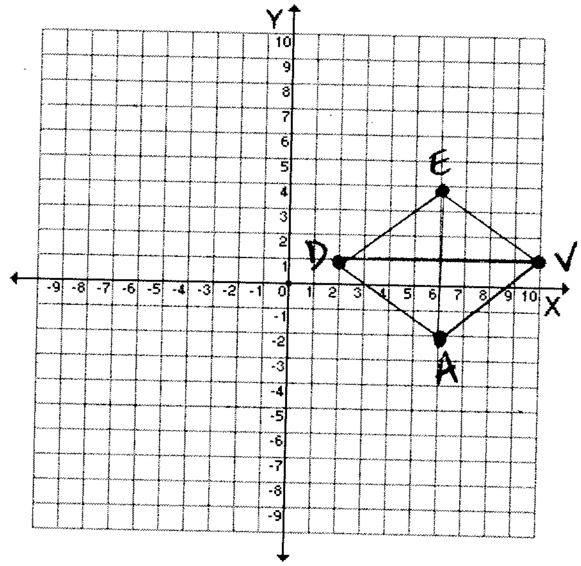
$\therefore$ Quad PLUS is a ▱ b/c diags bisect each other.

$UP = \sqrt{(3)^2 + (4)^2} = \sqrt{25} = 5$
$SL = \sqrt{(5)^2 + (0)^2} = \sqrt{25} = 5$
$\Big\}$ $UP = SL$

$\therefore$ ▱ABCD is a rectangle b/c diags =.

3. Prove that quadrilateral DAVE with the vertices D(2,1), A(6,-2), V(10,1), and E(6,4) is a rhombus.

$Mp = (\bar{x}, \bar{y}) \quad m = \frac{\Delta y}{\Delta x}$



Mp of $\overline{EA} = \left(\frac{6+6}{2}, \frac{-2+4}{2}\right) = (6,1)$

Mp of $\overline{DV} = \left(\frac{2+10}{2}, \frac{1+1}{2}\right) = (6,1)$

$\therefore$ Quad DAVE is a ▱ b/c diags bisect each other

$M_{\overline{EA}} = \frac{6}{0} =$ undefined

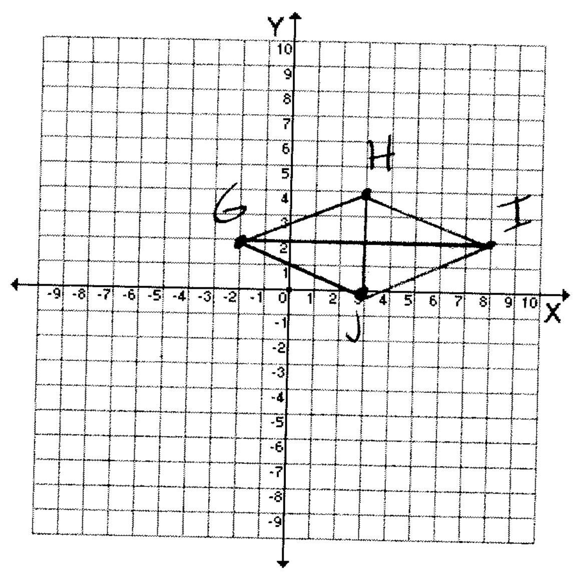
$M_{\overline{DV}} = \frac{0}{8} = 0$

$\overline{EA} \perp \overline{DV}$

$\therefore$ ▱DAVE is a rhombus b/c diags $\perp$.

4. Prove that quadrilateral GHIJ with the vertices G(-2,2), H(3,4), I(8,2), and J(3,0) is a rhombus.

$Mp = (\bar{x}, \bar{y}) \quad m = \frac{\Delta y}{\Delta x}$



Mp of $\overline{GI} = \left(\frac{-2+8}{2}, \frac{2+2}{2}\right) = (3,2)$

Mp of $\overline{HJ} = \left(\frac{3+3}{2}, \frac{4+0}{2}\right) = (3,2)$

$\therefore$ Quad GHIJ is a ▱ b/c diags bisect each other.

$M_{\overline{GI}} = \frac{0}{10} = 0$

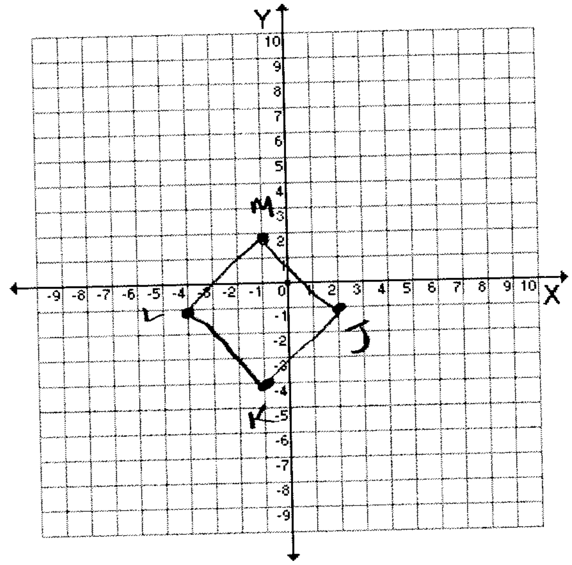
$M_{\overline{HJ}} = \frac{4}{0} =$ undefined

$\overline{GI} \perp \overline{HJ}$

$\therefore$ ▱GHIJ is a rhombus b/c diags $\perp$

5. Prove that a quadrilateral with vertices J(2,-1), K(-1,-4), L(-4,-1) and M(-1, 2) is a square.

$Mp = (\bar{x}, \bar{y}) \quad m = \frac{\Delta y}{\Delta x} \quad d = \sqrt{\Delta x^2 + \Delta y^2}$



Mp of $\overline{LJ} = \left(\frac{2+-4}{2}, \frac{-1+-1}{2}\right) = (-1,-1)$

Mp of $MK = \left(\frac{-1+-1}{2}, \frac{-4+2}{2}\right) = (-1,-1)$

∴ Quad JKLM is a ▱ b/c diags bisect each other.

$m_{\overline{LJ}} = \frac{0}{6} = 0 \quad m_{\overline{MK}} = \frac{6}{0} = \text{undefined}$

$\overline{LJ} \perp \overline{MK}$

∴ ▱JKLM is a rhombus b/c diags ⊥

$LJ = \sqrt{(6)^2 + (0)^2} = 6 \quad MK = \sqrt{0^2 + 6^2} = 6$

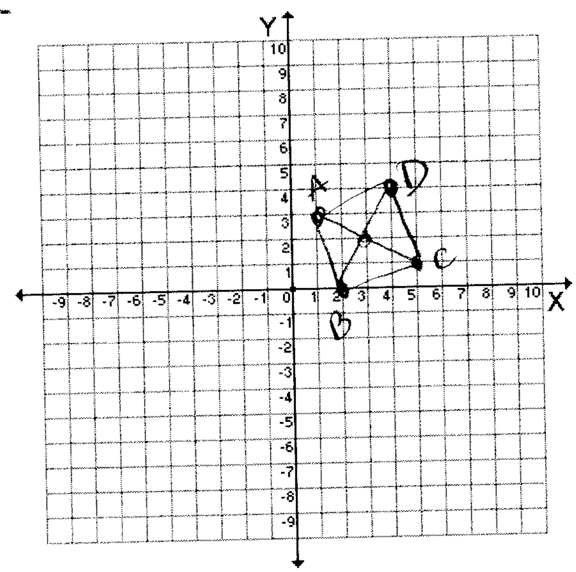
$LJ = MK$

∴ ▱JKLM is a rect. b/c diags =

∴ JKLM is a square b/c it is a rhombus and a rectangle.

6. Prove that ABCD is a square if A(1,3), B(2,0), C(5,1) and D(4,4).

$Mp = (\bar{x}, \bar{y}) \quad m = \frac{\Delta y}{\Delta x} \quad d = \sqrt{\Delta x^2 + \Delta y^2}$



$AD = \sqrt{(3)^2 + (1)^2} = \sqrt{10}$

$DC = \sqrt{(-1)^2 + (3)^2} = \sqrt{10}$

$BC = \sqrt{(3)^2 + (1)^2} = \sqrt{10}$

$BA = \sqrt{(-1)^2 + (3)^2} = \sqrt{10}$

$AD = DC = BC = BA$

∴ Quad ABCD is a ▱ b/c it has 2 pairs of opposite equal sides.

∴ Quad ABCD is a rhombus b/c it is equilateral.

$M_{\overline{AD}} = \frac{1}{3}$

$M_{\overline{BA}} = \frac{3}{-1} = \frac{-3}{1}$

$\frac{1}{3} \cdot \frac{-3}{1} = \frac{-3}{3} = -1$

$\overline{AD} \perp \overline{BA}$

∴ ABCD is a square b/c it is a rhombus with a right ∡.

Proving a Quadrilateral is a Trapezoid

Show one pair of sides are parallel (same slope) and one pair of sides are not parallel (different slopes).

Proving a Quadrilateral is an Isosceles Trapezoid

Prove that it is a trapezoid first, then:

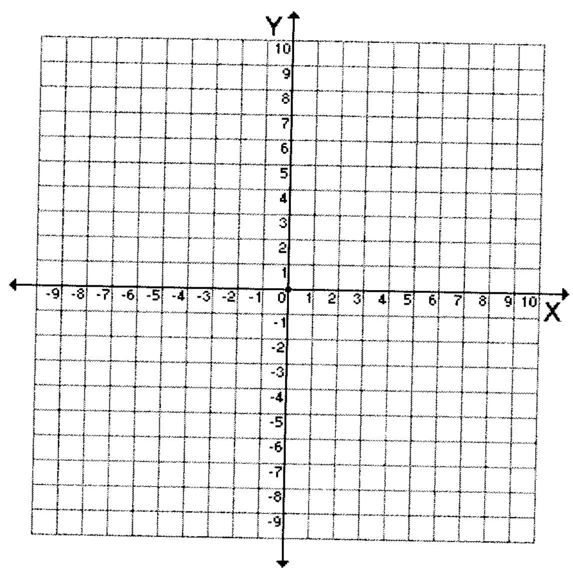
Method 1: Prove the diagonals are congruent using distance.

Method 2: Prove that the pair of non parallel sides are equal.

Examples:

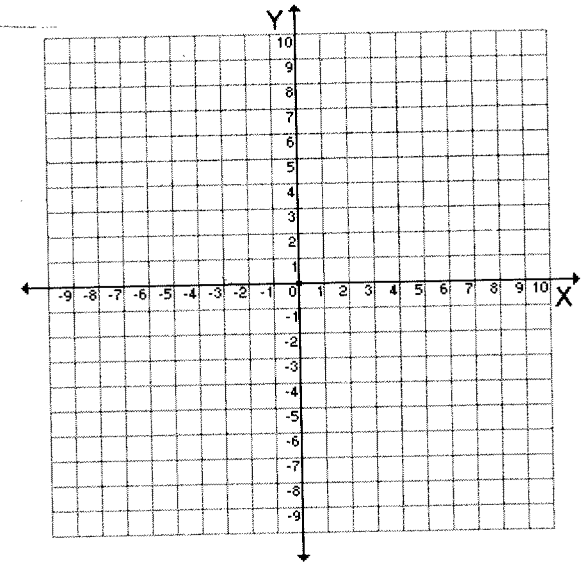
1. Prove that KATE a trapezoid with coordinates K(1,5), A(4,7), T(7,3) and E(1,-1).

See notes on 1/13/2011

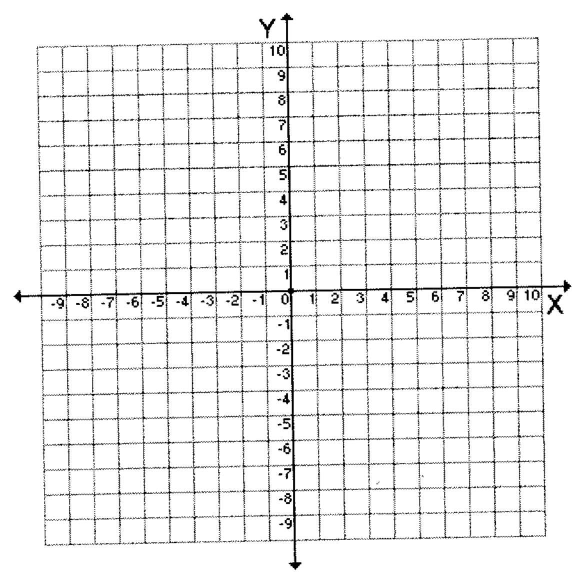


2. Prove that quadrilateral MILK with the vertices M(1,3), I(-1,1), L(-1, -2), and K(4,3) is an isosceles trapezoid.

see class notes on 1/13/2011

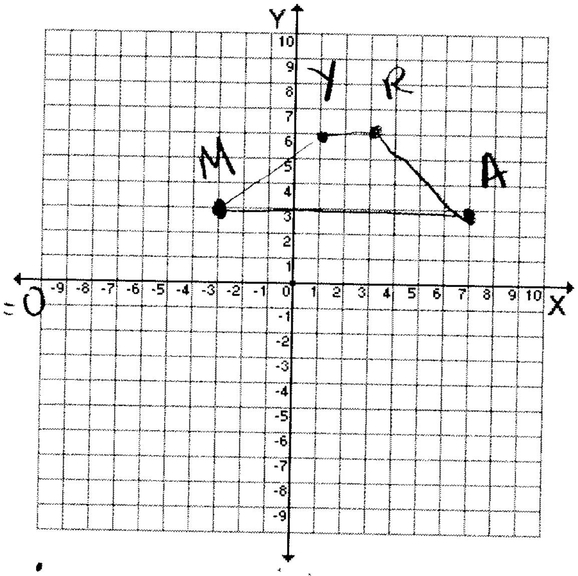


3. Prove that the quadrilateral with the vertices C(-3,-5), R(5,1), U(2,3) and D(-2,0) is a trapezoid but not an isosceles trapezoid.



Homework

1. The vertices of quadrilateral $MARY$ are $M(-3, 3)$, $A(7, 3)$, $R(3, 6)$, and $Y(1, 6)$. Use coordinate geometry to prove that quadrilateral $MARY$ is an isosceles trapezoid.



$M = \frac{\Delta y}{\Delta x}$ $\quad d = \sqrt{\Delta x^2 + \Delta y^2}$

$M_{\overline{MY}} = \frac{3}{4}$ $\quad M_{YR} = \frac{0}{2} = 0$ $\quad M_{RA} = \frac{3}{-4}$ $\quad M_{MA} = \frac{0}{10} = 0$

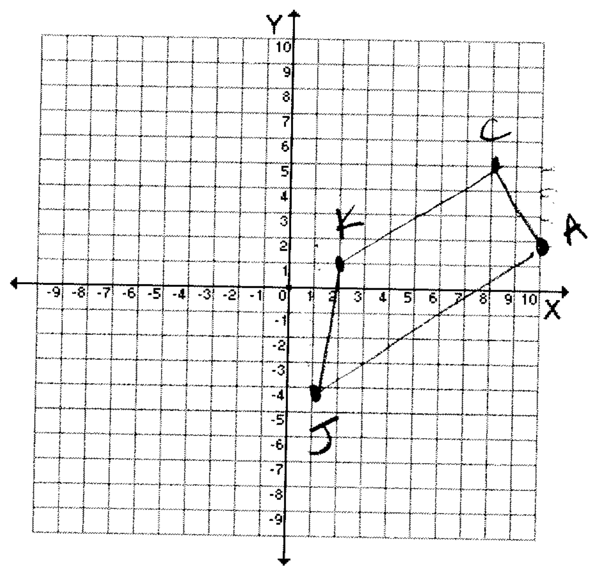
$\overline{YR} \parallel \overline{MA}$, $\overline{MY} \nparallel \overline{RA}$

∴ Quad MARY is a trapezoid b/c it has only <u>one</u> set of parallel sides

$$\left.\begin{aligned} MY &= \sqrt{(4)^2 + (3)^2} = \sqrt{25} = 5 \\ RA &= \sqrt{(-4)^2 + (3)^2} = \sqrt{25} = 5 \end{aligned}\right\} MY = RA$$

∴ trapezoid MARY is isosceles b/c the legs are =.

2. Quadrilateral $JACK$ has vertices $J(1, -4)$, $A(10, 2)$, $C(8, 5)$, and $K(2, 1)$. Use coordinate geometry to prove that
   a. quadrilateral $JACK$ is a trapezoid.
   b. quadrilateral $JACK$ is *not* isosceles.



$M = \frac{\Delta y}{\Delta x}$ $\quad d = \sqrt{\Delta x^2 + \Delta y^2}$

$M_{JK} = \frac{5}{1}$ $\quad M_{\overline{CA}} = \frac{3}{-2}$

$M_{\overline{KC}} = \frac{4}{6} = \frac{2}{3}$ $\quad M_{\overline{JA}} = \frac{6}{9} = \frac{2}{3}$

$\overline{KC} \parallel \overline{JA}$, $\overline{JK} \nparallel \overline{CA}$

∴ Quad JACK is a trapezoid b/c it has only <u>one</u> set of parallel sides.

$$\left.\begin{aligned} JK &= \sqrt{(1)^2 + (5)^2} = \sqrt{26} \\ CA &= \sqrt{(-2)^2 + (3)^2} = \sqrt{13} \end{aligned}\right\} JK \neq CA$$

∴ Quad JACK is not isosceles

3. Triangle *ART* has vertices $A(a, b)$, $R(a + c, b)$, and $T(a + \frac{c}{2}, b + d)$. Use coordinate geometry to prove that triangle *ART* is isosceles.

$$d = \sqrt{\Delta x^2 + \Delta y^2}$$

$$TA = \sqrt{[a-(a+c/2)]^2 + [b-(b+d)]^2} = \sqrt{(c/2)^2 + (-d)^2} = \sqrt{\frac{c^2}{4} + d^2}$$

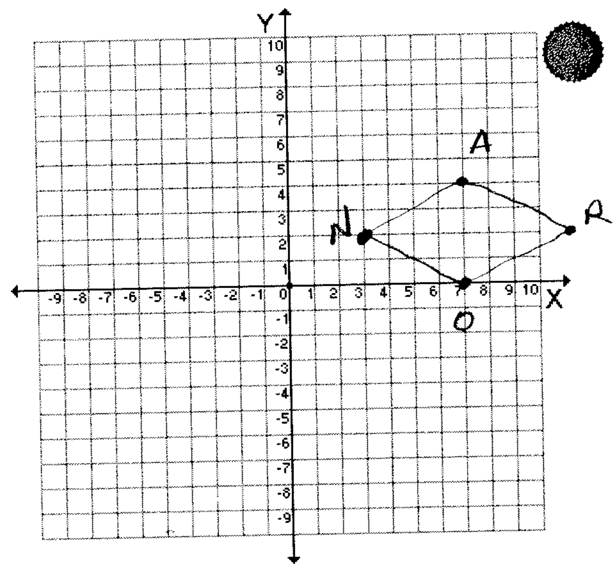
$$TR = \sqrt{[(a+c)-(a+c/2)]^2 + (b-(b+d))^2} = \sqrt{(+c/2)^2 + (-d)^2} = \sqrt{\frac{c^2}{4} + d^2}$$

$$AR = \sqrt{(a-(a+c))^2 + (b-b)^2} = \sqrt{(-c)^2 + (0)^2} = c$$

$TA = TR$

$\therefore \triangle ART$ is isosceles

4. Quadrilateral *NORA* has vertices $N(3, 2)$, $O(7, 0)$, $R(11, 2)$, and $A(7, 4)$.
Use coordinate geometry to prove that
a. quadrilateral *NORA* is a rhombus.
b. quadrilateral *NORA* is *not* a square.



$Mp = (\bar{x}, \bar{y})$ $\quad M = \frac{\Delta y}{\Delta x}$ $\quad d = \sqrt{\Delta x^2 + \Delta y^2}$

Mp of $\overline{NR} = \left(\frac{3+11}{2}, \frac{2+2}{2}\right) = (7, 2)$

Mp of $AO = \left(\frac{7+7}{2}, \frac{0+4}{2}\right) = (7, 2)$

$\therefore$ NORA is a $\square$ b/c diags bisect each other.

$$\left.\begin{array}{l} M_{\overline{NR}} = \frac{0}{8} = 0 \\ M\,\overline{AO} = \frac{4}{0} = \text{undefined} \end{array}\right\} \overline{NR} \perp \overline{AO}$$

$\therefore \square$NORA is a rhombus b/c diags $\perp$

$NR = \sqrt{(8)^2 + (0)^2} = 8$

$AO = \sqrt{(0)^2 + (4)^2} = 4$

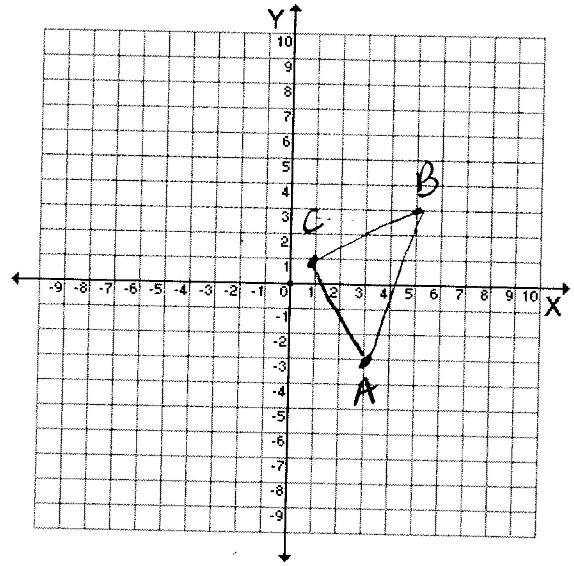
$NR \neq AO$

$\therefore$ Quad NORA is not a square b/c it is not a rectangle and a rhombus

Practice with Coordinate Proofs

1. The vertices of ΔABC are $A(3,-3)$, $B(5,3)$ and $C(1,1)$. Prove by coordinate geometry that ΔABC is an isosceles right triangle.

$D = \sqrt{\Delta x^2 + \Delta y^2}$



$AC = \sqrt{(-2)^2 + (4)^2} = \sqrt{20} = 2\sqrt{5}$

$BC = \sqrt{(4)^2 + (2)^2} = \sqrt{20} = 2\sqrt{5}$

$AB = \sqrt{(2)^2 + (6)^2} = \sqrt{40} = 2\sqrt{10}$

$AC = BC$

$\therefore$ ΔABC is isosceles

$c^2 = a^2 + b^2$

$(\sqrt{40})^2 = (\sqrt{20})^2 + (\sqrt{20})^2$

$40 = 20 + 20$

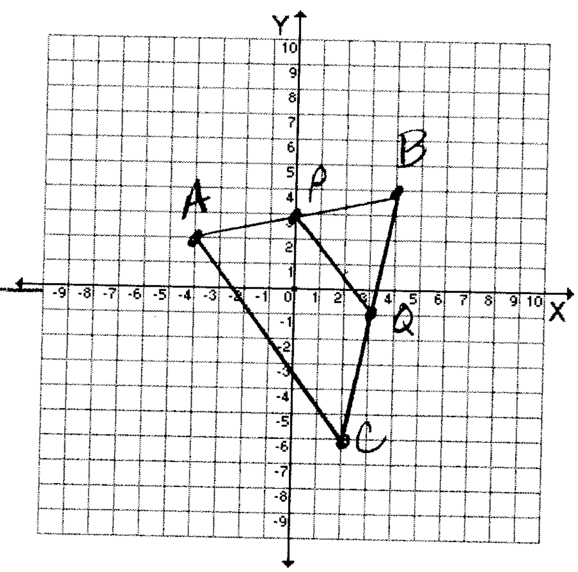
$40 = 40$

$\therefore$ ΔABC is an isosceles right triangle.

2. Given ΔABC with vertices $A(-4,2)$, $B(4,4)$ and $C(2,-6)$, the midpoints of AB and BC are P and Q, respectively, and PQ is drawn. Prove by coordinate geometry:

   a. $PQ \parallel AC$
   b. $PQ = \frac{1}{2} AC$

$M_p = (\bar{x}, \bar{y})$ $\quad D = \sqrt{\Delta x^2 + \Delta y^2}$ $\quad M = \frac{\Delta y}{\Delta x}$



M_p of $AB = \left(\frac{-4+4}{2}, \frac{2+4}{2}\right) = (0,3) = P$

M_p of $BC = \left(\frac{4+2}{2}, \frac{4-6}{2}\right) = (3,-1) = Q$

a. $m_{\overline{PQ}} = \frac{4}{-3}$

$m_{\overline{AC}} = \frac{8}{-6} = -\frac{4}{3}$ $\Big\}$ $PQ \parallel AC$

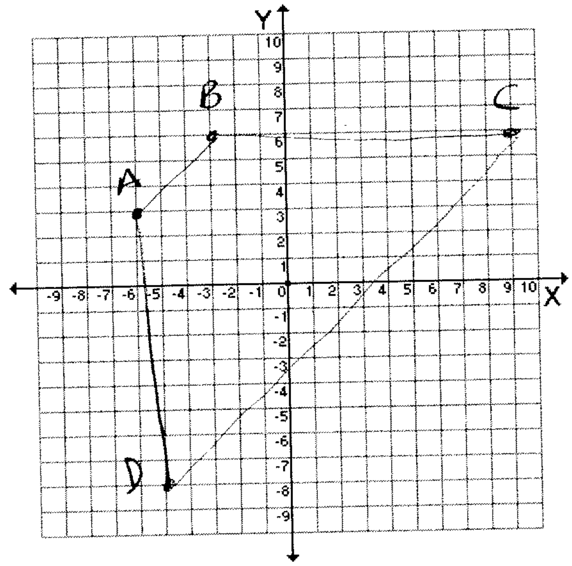
$\therefore$ $\overline{PQ} \parallel \overline{AC}$ b/c their slopes are equal.

$PQ = \sqrt{(-3)^2 + (4)^2} = \sqrt{25} = 5$

$AC = \sqrt{(6)^2 + (8)^2} = \sqrt{100} = 10$ $\Big\}$ $5 = \frac{1}{2}(10)$

$\therefore$ $PQ = \frac{1}{2} AC$

3. Quadrilateral ABCD has vertices A(-6,3), B(-3,6), C(9,6) and D(-5,-8). Prove that quadrilateral ABCD is:
   a. a trapezoid
   b. not an isosceles trapezoid



$m = \frac{\Delta y}{\Delta x}$ $\quad d = \sqrt{\Delta x^2 + \Delta y^2}$

$m_{\overline{AB}} = \frac{3}{3} = 1$ $\quad m_{DC} = \frac{14}{14} = 1$

$\overline{BC} = \frac{0}{12} = 0$ $\quad m_{DA} = \frac{11}{-1} = \frac{-11}{1}$

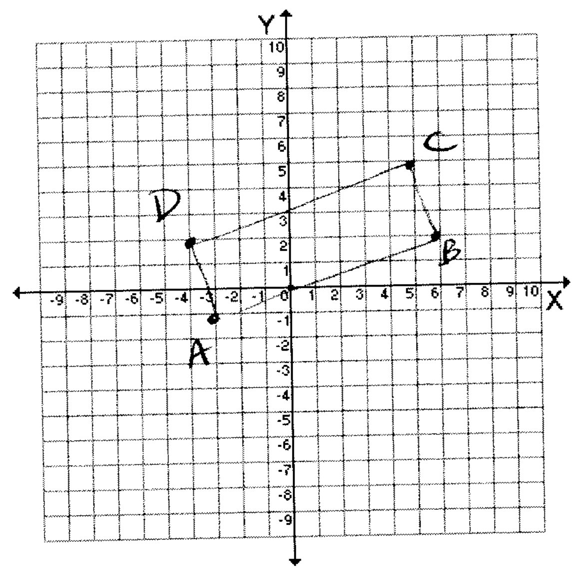
$\overline{AB} \parallel \overline{DC}$, $\overline{BC} \nparallel DA$

a) ∴ Quad ABCD is a trapezoid b/c it has only one set of parallel sides

b) $BC = \sqrt{12^2 + 0^2} = 12$
$DA = \sqrt{(-1)^2 + 11^2} = \sqrt{122}$ $\quad BC \neq DA$

∴ Quad ABCD is not an isosceles trapezoid

4. The vertices of quadrilateral ABCD are A(-3,-1), B(6,2), C(5,5) and D(-4,2). Prove that quadrilateral ABCD is a rectangle.



$Mp = (\bar{x}, \bar{y})$ $\quad d = \sqrt{\Delta x^2 + \Delta y^2}$

Mp of $\overline{DB} = \left(\frac{6+-4}{2}, \frac{2+2}{2}\right) = (1, 2)$

Mp of $\overline{AC} = \left(\frac{-3+5}{2}, \frac{-1+5}{2}\right) = (1, 2)$

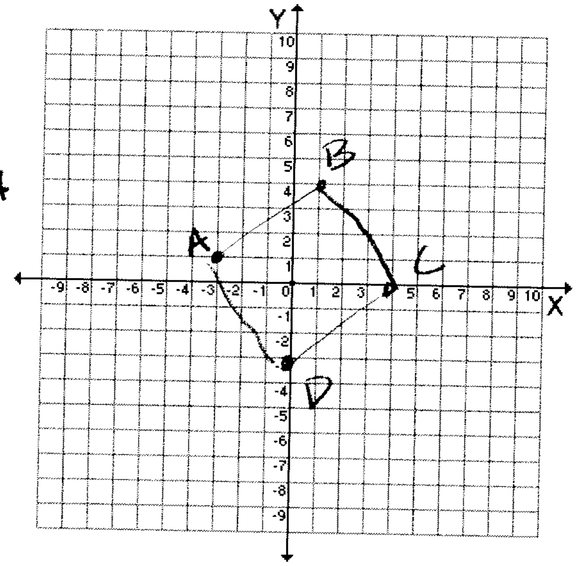
∴ Quad ABCD is a ▱ b/c diags bisect each other.

$DB = \sqrt{(10)^2 + (0)^2} = 10$
$AC = \sqrt{(8)^2 + (6)^2} = \sqrt{100} = 10$ $\quad DB = AC$

∴ ▱ABCD is a rectangle b/c diags are equal.

5. The vertices of quadrilateral ABCD are A(-3,1), B(1,4), C(4,0) and D(0,-3). Prove that quadrilateral ABCD is a square.

$D = \sqrt{\Delta x^2 + \Delta y^2}$ $\quad m = \frac{\Delta y}{\Delta x}$



$AB = \sqrt{(4)^2 + (3)^2} = 5$

$BC = \sqrt{(-3)^2 + (4)^2} = 5$

$CD = \sqrt{(4)^2 + (3)^2} = 5$

$DA = \sqrt{(3)^2 + (4)^2} = 5$

$AB = BC = CD = DA$

∴ Quad ABCD is a ▱ b/c 2 pairs of opp sides are =.

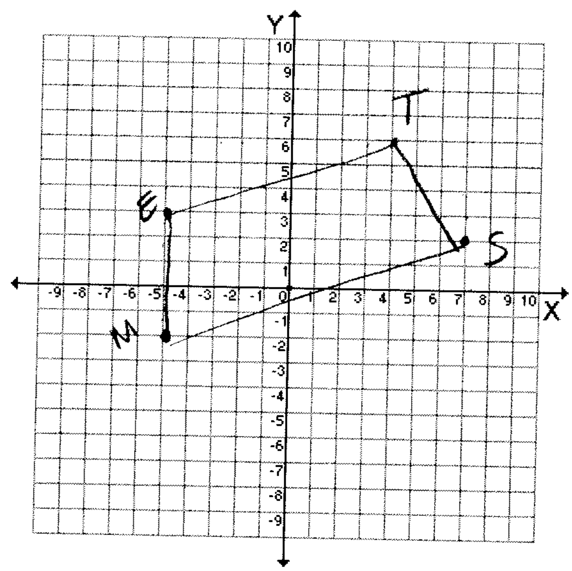
∴ Quad ABCD is a rhombus b/c it is equilateral.

$m_{\overline{AB}} = \frac{3}{4}$ $\quad m_{\overline{DA}} = \frac{4}{-3}$; $\frac{3}{4} \cdot \frac{4}{-3} = \frac{12}{-12} = -1$; $\overline{AB} \perp \overline{DA}$

∴ Quad ABCD is a square b/c it is a rhombus w/ a right ∡.

6. Quadrilateral METS has vertices M(-5, -2), E(-5,3), T(4,6) and S(7,2). Prove by coordinate geometry that quadrilateral METS is an isosceles trapezoid.

$m = \frac{\Delta y}{\Delta x}$ $\quad d = \sqrt{\Delta x^2 + \Delta y^2}$



$m_{\overline{ME}} = \frac{5}{0}$ = undefined $\quad m_{ET} = \frac{3}{9} = \frac{1}{3}$

$m_{TS} = \frac{4}{-3}$ $\quad m_{\overline{MS}} = \frac{4}{12} = 1/3$

$\overline{ET} \parallel \overline{MS}$, $\overline{ME} \nparallel \overline{TS}$

∴ METS is a trapezoid b/c it has only <u>one</u> set of parallel sides

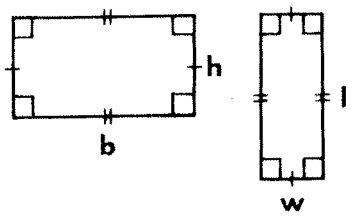
$ME = \sqrt{(0)^2 + (5)^2} = 5$

$TS = \sqrt{(-3)^2 + (4)^2} = \sqrt{25} = 5$

$ME = TS$

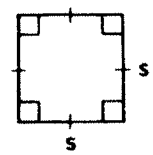
∴ METS is an isosceles trapezoid.

1. Rectangle:



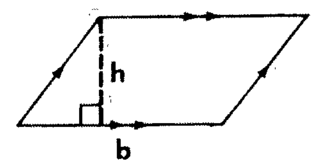
$A = bh$ or $A = lw$

2. Square:



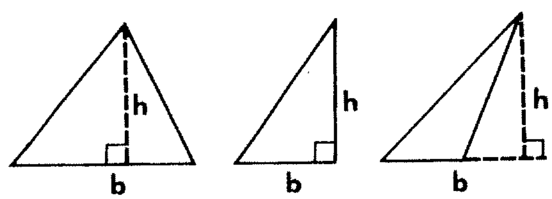
$A = s^2$

3. Parallelogram:



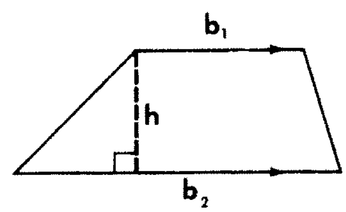
$A = bh$

4. Triangle:



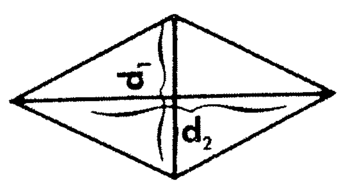
$A = \frac{1}{2}bh$

5. Trapezoid:



$A = \frac{1}{2}(b_1 + b_2)h$

6. Rhombus:



$A = \frac{1}{2}d_1 \cdot d_2$

Areas and Coordinates

To find areas of polygons in coordinate geometry, we use the area formulas previously developed. When a figure has one or more sides parallel to either of the axes, the process of finding its area usually is simpler.

EXAMPLE 1. The vertices of rhombus $ABCD$ are $A(1, 5)$, $B(-1, 2)$, $C(1, -1)$, and $D(3, 2)$. Graph rhombus $ABCD$ and find its area.

Solution:

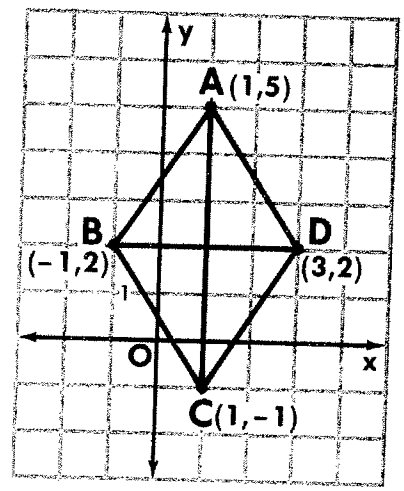
(1) Find the lengths of the diagonals of $ABCD$. $d_1 = AC = 6$, and $d_2 = BD = 4$.

(2) Area of rhombus $= \frac{1}{2}d_1d_2$

$$= \tfrac{1}{2} \cdot 6 \cdot 4$$

$$= 12$$

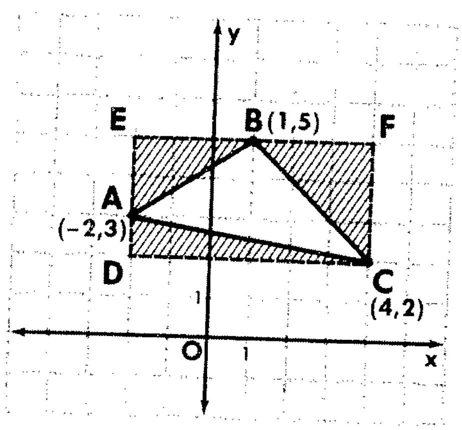
Answer: The area of rhombus $ABCD$ is 12 square units.



EXAMPLE 2. Find the area of a triangle whose vertices are $A(-2, 3)$, $B(1, 5)$, and $C(4, 2)$.

Method 1: Enclose the figure in a rectangle.

Through the uppermost and lowermost points, here B and C, draw lines parallel to the x-axis. Through the points farthest to the left and farthest to the right, here A and C, draw lines parallel to the y-axis. Thus, rectangle $CDEF$ is formed.



The area of $\triangle ABC$ is equal to the area of rectangle $CDEF$ minus the sum of the areas of right triangles $\triangle CDA$, $\triangle BEA$, and $\triangle BFC$.

Note that each of the sides of rectangle $CDEF$ and each of the legs of the right triangles is parallel to one of the axes. Thus, the lengths of these sides can be found by counting units.

(1) Area of rectangle $CDEF = DC \times DE = 6 \times 3 = 18$

(2) Area of rt. $\triangle CDA = \frac{1}{2}DC \times DA = \frac{1}{2}(6)(1) = 3$

(3) Area of rt. $\triangle BEA = \frac{1}{2}BE \times EA = \frac{1}{2}(3)(2) = 3$

(4) Area of rt. $\triangle BFC = \frac{1}{2}BF \times FC = \frac{1}{2}(3)(3) = 4.5$

(5) Area of $\triangle ABC = \begin{pmatrix}\text{area of} \\ \text{rect. } CDEF\end{pmatrix} - \begin{pmatrix}\text{area of} \\ \triangle CDA\end{pmatrix} + \begin{pmatrix}\text{area of} \\ \triangle BEA\end{pmatrix} + \begin{pmatrix}\text{area of} \\ \triangle BFC\end{pmatrix}$

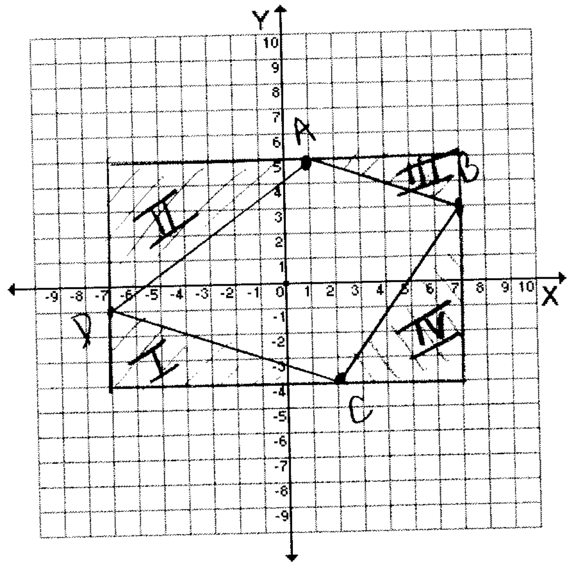
$$= 18 - (3 + 3 + 4.5)$$

$$= 18 - 10.5$$

Area of $\triangle ABC = 7.5$

Practice

1. Find the area of trapezoid ABCD if the vertices are A(1,5), B(7,3), C(2,-4) and D(-7,-1).



Area of rect = L x W = 14 x 9 = 126

Area of rt ΔI = ½bh = 0.5(9)(3) = 13.5

Area of rt ΔII = ½bh = 0.5(8)(6) = 24

Area of rt ΔIII = ½bh = 0.5(6)(2) = 6

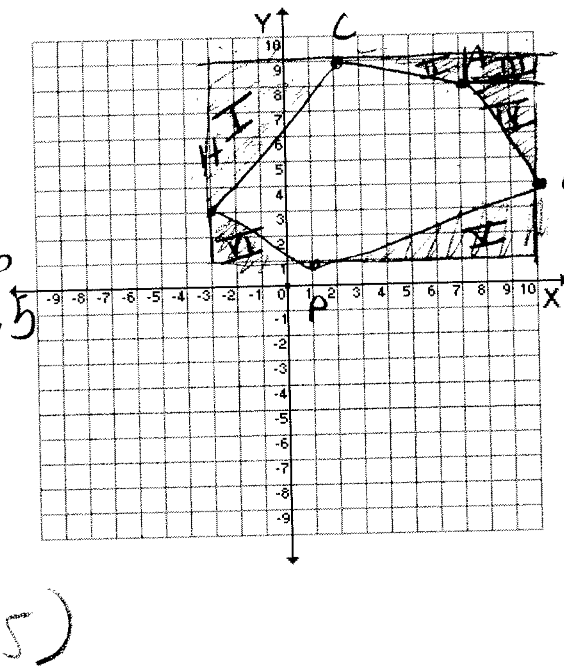
Area of rt ΔIV = ½bh = 0.5(5)(7) = 17.5

Area of trap ABCD = 126 − (13.5 + 17.5 + 24 + 6)

= 126 − 61

$\boxed{\text{Area of } \square ABCD = 65 \text{ squ}}$

2. If the coordinates of the vertices of polygon PEACH are P(1,1), E(10,4), A(7,8), C(2,9) and H(-3,3), what is the area of pentagon PEACH?



Area of rect = L x W = 13 x 8 = 104

Area of rt Δ = 0.5bh = 0.5(5)(6) = 15

Area of rt Δ_{II} = 0.5bh = 0.5(5)(1) = 2.5

Area of rect_{III} = L x W = 3 x 1 = 3

Area of rt Δ_{IV} = 0.5bh = 0.5(3)(4) = 6

Area of rt Δ_{V} = 0.5bh = 0.5(9)(3) = 13.5

Area of rt Δ_{VI} = 0.5bh = 0.5(4)(2) = 4

Area of Pentagon ABCDE = 104 − (15 + 3 + $\cancel{6+4}$ + 2.5 + 13.5)

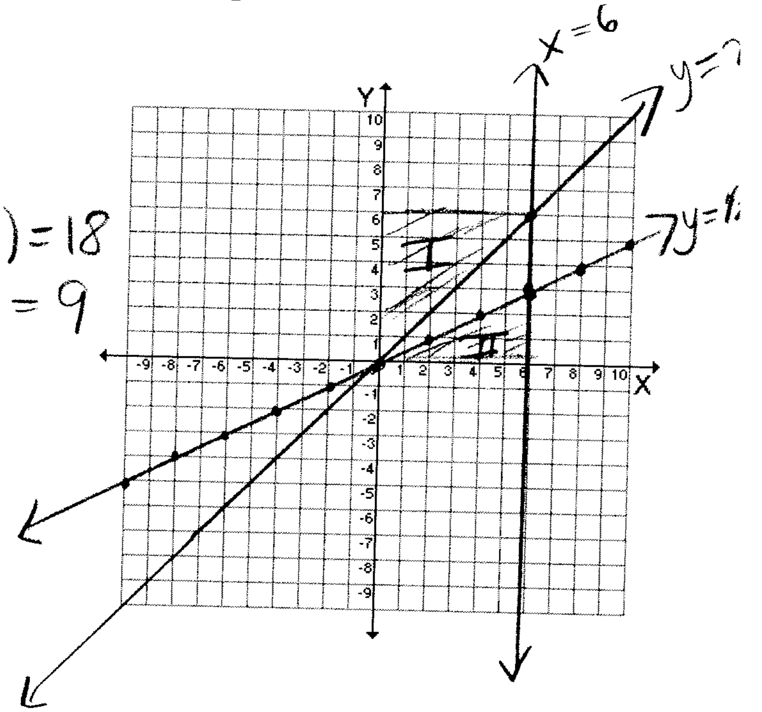
= 104 − 54

= 50 squ

3. **a.** On the same set of axes, draw the graph of each of the following equations: (1) $x = 6$, (2) $y = x$, (3) $y = \frac{1}{2}x$.
b. The points of intersection of the graphs drawn in part **a** are the vertices of a triangle. Find the coordinates of these vertices.
c. Find the area of the triangle described in part **b**.

b) (0,0)(6,6)(6,3)

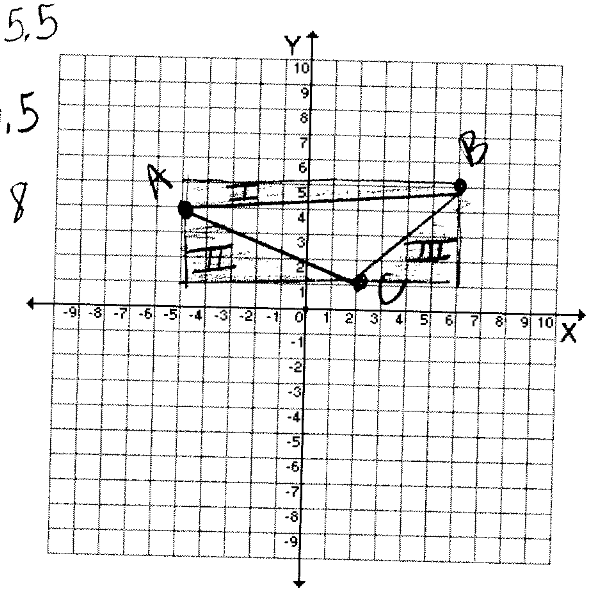
c. Area of square $= s^2 = 6^2 = 36$
Area of rt $\Delta_{I} = 0.5bh = 0.5(6)(6) = 18$
Area of rt $\Delta_{II} = 0.5bh = 0.5(6)(3) = 9$



Area of Δ $= 36 - (18 + 9)$
$= 36 - 27$
$\boxed{= 9 \text{ squ}}$

4. Find the area of a triangle whose vertices are (-5,4), (2,1) and (6,5).

Area of rect $= L \times W = 11 \times 4 = 44$
Area of rt $\Delta_{I} = 0.5bh = 0.5(11)(1) = 5.5$
Area of rt $\Delta_{II} = 0.5bh = 0.5(7)(3) = 10.5$
Area of rt $\Delta_{III} = 0.5bh = 0.5(4)(4) = 8$



Area of $\Delta = 44 - (5.5 + 10.5 + 8)$
$= 44 - 24$
$\boxed{= 20 \text{ squ}}$

Answers to Areas in Coordinate Geometry HW

1. a. 40 **b.** 30 **2. a.** 12 **b.** 28

3. a. 48 **b.** 22 **c.** 12 **d.** 15

4. a. 51 **b.** $\frac{51}{2}$

5. b. parallelogram **c.** 40

6. a. $CD = 8$, $DE = 6$, $CE = 10$

c. trapezoid **d.** 52

7. a. 24 **b.** 10 **c.** 60 **d.** $\frac{221}{2} = 110\frac{1}{2}$

8. a. 16 **b.** 40 **c.** 6 **d.** 8 **e.** 15

f. 100

9. a. 22 **b.** 20 **c.** 10 **d.** 20 **e.** 9 **f.** 14

10. $31\frac{1}{2}$ **11. a.** 22 **b.** 40 **c.** 5 **d.** 30

12. a. 6 **b.** 24 **13. a.** 9 **b.** 27

14. a. 30 **b.** 10 **c.** 6

15. a. 25

b. slope of $\overleftrightarrow{DE} = \frac{3}{4}$, slope of $\overleftrightarrow{EF} = -\frac{4}{3}$; $\overleftrightarrow{DE} \perp \overleftrightarrow{EF}$

c. $DE = 5$, $EF = 10$

d. $A = \frac{1}{2}(5)(10) = 25$

16. b. slope of $\overleftrightarrow{AC} = \frac{1}{2}$,

slope of $\overleftrightarrow{BC} = -2$; $\overleftrightarrow{AC} \perp \overleftrightarrow{BC}$ **c.** 30

17. a. $(-3, 6)$ **b.** 32 **c.** $(1, 5)$

d. 8 **e.** 1:2 **f.** 1:4

18. a. $AB = BC = CD = DA = 5$

b. 24 **c.** $\frac{24}{5}$

19. a. $AB = BC = CD = DA = 10$

b. 96 **c.** 9.6

20. a. $PQ = 5$, $QR = 5$, $PR = 6$ **b.** 12

c. $(-1, \frac{1}{2})$ **d.** $(-1, -\frac{5}{2})$ **e.** 3 **f.** 1:4