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|  | | Assignment # 1 | | | | |  | |
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|  | | | | MujtabaSP22-BSE-036MalaikaSP22-BSE-025 |  | | | |
|  | | | | Oct 9, 2022—Discrete Structures—Mam Memoona Malik |  | | | |
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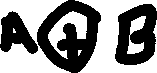
QUESTION . 1

(A)

1. Membership Table

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| A | B | A - B | B - A | A B |
| 0 | 0 | 0 | 0 | 0 |
| 0 | 1 | 0 | 1 | 1 |
| 1 | 0 | 1 | 0 | 1 |
| 1 | 1 | 0 | 0 | 0 |

**b.** Venn Diagram



**c.** Let A = {1,2,3,4} and B = {3,4,5,6} and C = {5,6,7,8}

B C = {3,4,7,8}

(A B) C = {1,2,7,8}

**d.** Let A B = (A B) – (A B)

By definition,

A B = {x|(x A v x B) A (x A n x B})}

= {(A u B) A (A n B)} 🡪 Answer

**e.** Let A B = (A - B) (B - A)

We know that,

A B = {x|(x A v x B) A (x A n x B})}

= {(x A - x B) U (x B n x A})}

= (A - B) (B - A))} 🡪 Answer

(B)



(C)

**a.**

|  |  |  |
| --- | --- | --- |
| A | B | A B |
| 0 | 0 | 0 |
| 0 | 1 | 1 |
| 1 | 0 | 1 |
| 1 | 1 | 0 |

**b.**



**c.**

A = {1,2,3,4} B = {3,4,5,6} C = {5,6,7,8}

B C = (B - C) (C - B)

= {3,4}

A B C = {1,2}

**d.**

A B = (A B) – (A B)

A B = (A - B) (B - A)

Let A B = (A B) – (A B)

From de-morgan law

A - (B C) = (A - B) n (A - C)

We can write it as ((A U B) - B U (B n A))

= A

QUESTION . 2

(A)

Draw the digraph of the relation.

1. The relation *R* = {(1, 2), (2, 3), (3, 4), (4, 1)} on {1, 2, 3, 4}
2. **2**

**4 3**

1. The relation *R*={(1, 2), (2, 1), (3, 3), (1, 1), (2, 2)} on *X* = {1, 2, 3}
2. **2**

**3**

(B)

1. **R** = { (a ,b) , (b ,a) , (b ,d) , (c ,d) , (a ,c) , (c ,c) } on { a , b , c , d}

**Domain** = {a,b,c}?

**Range** = {a,b,c,d}?

1. **R** = {(1,1),(2,2),(3,3),(3,5),(5,5),(5,4),(4,4),(4,3) } on {1,2,3,4,5}

**Domain** = {1,2,3,4,5}?

**Range** = {1,2,3,4,5}?

1. **R** = {(b ,c),(c ,b),(d ,d)} on {a ,b ,c ,d}

**Domain** = {b,c,d}?

**Range** = {b,c,d}?

(C)

Let R1 and R2 be the relations on {1, 2, 3, 4} given by

R1 = {(1, 1), (1, 2), (3, 4), (4, 2)}

R2 = {(1, 1), (2, 1), (3, 1), (4, 4), (2, 2)}.

List the elements of:

1. R1 R2

**R** = {(1,1),(1,2),(2,1),(2,2),(3,1),(3,4),(4,2),(4,4)}

1. R1  R2

**R** = {(1, 1)}

(D)

1. **Reflexive:** YES. Given string α ∈ X, α indeed has some common

substring of size 2 with itself, say its substring consisting of the first

two bits of α.

2. **Symmetric**: YES. If α, β ∈ X, and α has some substring of size 2 in

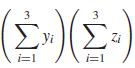
common with β, also β has that same substring in common with α.

4. **Transitive**: NO. Counterexample: 1110 R 1100 (both contain 11) and

1100 R 0001 (both contain 00), however 1110 R6 0001.

QUESTION . 3

(A)

1.  = (y+2y+3y)(z+2z+3z)

= 6y(6z)

2.

= 3 + 9 + 15 = 27

(B)

i. ii.

iii. iv.

Evaluate if n = 4:

a=6, b=14, c=4, d=14

QUESTION . 4

1. F(x) = 75 + ((x-400)/100) \* 0, 60), x > 9000

F(x) =

1. **Write the definition of “one-to-one” using logical notation (i.e., use, ∃, etc.)**

A function f: A → B is injective (or one-to-one) if it does not map different elements of A to the same element of B.

In logical notation: if ∀x∈A. ∀y∈A. (f(x) = f(y) =⇒ x = y).

1. **Write the definition of “onto” using logical notation (i.e., use, ∃, etc.).**

A function f: A → B is surjective (or onto) if every element in B is mapped to by an element in A.

In logical notation: if ∀y∈B.∃x∈A.(f(z) = y).

1. **Determine whether each of these functions is a bijection from R to R.**

**f (x) = 2x + 1**

Ans. Yes

**f (x) = x2 + 1**

Ans. No

**f (x) = x3**

Ans. Yes

**f (x) = (x2 + 1) / (x2 + 2)**

Ans. No

1. **Find at least three different sequences beginning with the terms 3, 5, 7 whose terms are generated by a simple formula or rule.**

a1 = 2(1) +1

**a1 = 3**

a2 = 2(2) +1

**a2 = 5**

a3 = 2(3) +1

a3 =

a4 = 2(4) +1

**a4 = 9**

a 5 = 2(5) +1

a5= 11

a6= 2(6) + 1

a6 = 13

:

:

an= 2(n)+1

So we generalize the formula that:

**an= 2(n)+1**

1. **Find the first five terms of the sequence defined by each of these recurrence relations and initial conditions.**
2. **an = 6an−1, a0 = 2**

an = 6an-1

a0 = 2

a1 = 6a1-1

=6a0, =6(2)

a1=12

a2= 6a2-1

=6a1, =6(12)

a2=72

a3 = 6a3-1

=6a2, =6(72)

a3=252

a4 = 6a4-1

=6a3, =6(252)

a4 =1512

1. **an = a2n−1, a1 = 2**

an = a2n−1, a1 = 2

a1=2

a2=a22-1

=a2(1), =(a1)2, =22

a2=4

a3=a23-1

=a2(2), =(a2)2, =42

a3=16

a4=a24-1

=a2(3), =(a3)2, =162

a4=256

a5=a25-1

=a2(4), =(a4)2, =2562

a5=65536

1. **Suppose that the number of bacteria in a colony triples every hour.**
   * 1. **Set up a recurrence relation for the number of bacteria after n hours have elapsed.**

According to the given condition:

Initial condition is a0=1

We can use the iterative approach to find formula for an, in which bacteria triples every hour so,

a1=3(a0), 3(1)

a1=3

a2=3(a1), 3(3)

a2=9

a3=3(a2), 3(9)

a3=27

:

:

an=3(an-1) (generalized term)

* + 1. **If 100 bacteria are used to begin a new colony, how many bacteria will be in the colony in 10 hours?**

According to the given condition, we have to find a10

Here, we have given that a0=100

a0=100

a1=3(100)

a1=300

a2=3(300)

a2=900

a3=3(900)

a3=2700

a4=3(2700)

a4=8100

a5=3(8100)

a5=24,300

a6=3(24,300)

a6=72,900

a7=3(72,900)

a7=218,700

a8=3(218,700)

a8=656,100

a9=3(656,100)

a9=1,968,300

a10=3(1,968,300)

a10=5,904,900

so, we generalize the formula that:

an=3(an-1)