COMSATS University, Islamabad

Assignment # 1

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Course: Mathematics (MTH-100)

Instructor: Dr. Amna Nazeer

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Question # 1

Solve for x in the following problems

Solution

1. 7 + | 2x – 5 | = 4

$$7 + |2x - 5| = 4$$

$$|2x - 5| = 4 - 7$$

$$|2x - 5| = -3$$

The absolute number can never be negative, so there is no solution.

2. - | - 8 – 2x | = -12

$$-8 - 2x = 12 \quad | \quad -(-8 - 2x) = 12$$

$$-2x = 12 + 8 \quad | \quad 2x = 12 - 8$$

$$x = \frac{-20}{2} \quad | \quad x = \frac{4}{2}$$

$$x = -10 \quad | \quad x = 2$$

Therefore, the solution set is: $x = \{-10, 2\}$

3. $x^2 - 8x + 13 = 0$

For this question, we use quadratic formula:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Here a = 1, b = -8, c = 13

$$x = \frac{-(-8) + \sqrt{-8^2 - 4(1)(13)}}{2(1)} \quad | \quad x = \frac{-(-8) - \sqrt{-8^2 - 4(1)(13)}}{2(1)}$$

$$x = \frac{8 + \sqrt{64 - 52}}{2} \quad | \quad x = \frac{8 - \sqrt{64 - 52}}{2}$$

$$x = \frac{8 + \sqrt{12}}{2} \quad | \quad x = \frac{8 - \sqrt{12}}{2}$$

$$x = \frac{8 + 3.46}{2} \quad | \quad x = \frac{8 - 3.46}{2}$$

$$x = \frac{11.46}{2} \quad | \quad x = \frac{4.54}{2}$$

$$x = 5.73 \quad | \quad x = 2.27$$

Therefore, the solution set is: $x = \{5.73, 2.27\}$

4. $\frac{3}{x} + \frac{5}{x+2} = 2$

$$\frac{3(x+2) + 5x}{(x)(x+2)} = 2$$

$$\frac{3x + 6 + 5x}{x^2 + 2x} = 2$$

$$8x + 6 = 2x^2 + 4x$$

$$2x^2 - 4x - 6 = 0$$

Taking '2' common,

$$2(x^2 - 2x - 3) = 0$$

$$x^2 - 2x - 3 = 0$$

$$x^2 - 3x + x - 3 = 0$$

$$x(x-3) + (x-3) = 0$$

$$(x+1)(x-3) = 0$$

$$x + 1 = 0 \quad | \quad x - 3 = 0$$

$$x = -1 \quad | \quad x = 3$$

Therefore, the solution set is $\{-1,3\}$

Question # 2

Solve for x and write the solution in interval notation. Show the solution graphically on number line.

Solution

1. $|2 + 2x| > 0$

$$2x + 2 = 0$$

$$x = -\frac{2}{2}$$

$$x = -1$$

Check if x < -1:

$$|2(-2) + 2| > 0$$

$$|-4 + 2| > 0$$

$$|-2| > 0$$

$$2 > 0\ (true)$$

Check if x > -1:

$$|2(1) + 2| > 0$$

$$|2 + 2| > 0$$

$$|4| > 0$$

$$4 > 0\ (true)$$

Check if x = -1:

$$|2(-1) + 2| > 0$$

$$|-2 + 2| > 0$$

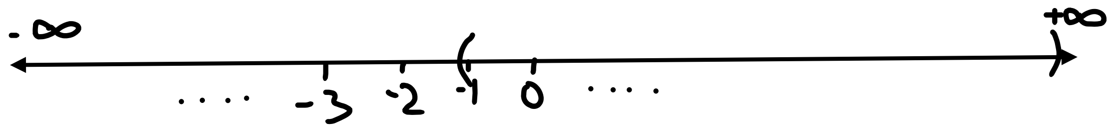
$$|0| > 0$$

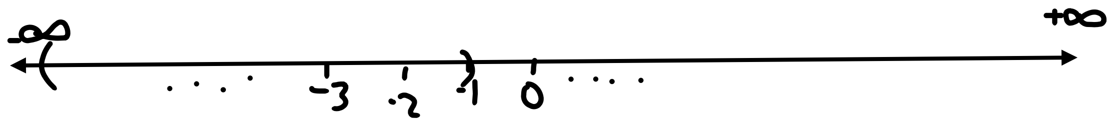
$$0 > 0\ (false)$$

In interval notation,

$$(-1, +\infty) \quad | \quad (-\infty, -1)$$

In line number,





2. $|4x - 2| \leq 17$

$$4x - 2 = \pm 17$$

$$4x - 2 = 17 \quad | \quad 4x - 2 = -17$$

$$4x = 19 \quad | \quad 4x = -15$$

$$x = 19/4 \quad | \quad x = -15/4$$

$$x = 4.75 \quad | \quad x = -3.75$$

Check if x < -3.75 & x > 4.75:

$$|4(-5) - 2| \leq 17 \quad | \quad |4(5) - 2| \leq 17$$

$$|-20 - 2| \leq 17 \quad | \quad |20 - 2| \leq 17$$

$$|-22| \leq 17 \quad | \quad |18| \leq 17$$

$$22 \leq 17 \quad | \quad 18 \leq 17$$

$$(false) \quad | \quad (false)$$

Check if $-3.75 \leq x \leq 4.75$:

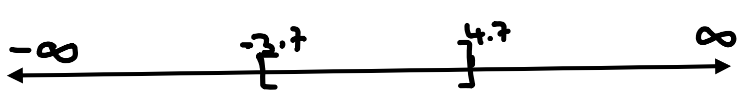
$$|4(2) - 2| \leq 17$$

$$|8 - 2| \leq 17$$

$$|6| \leq 17$$

$$6 \leq 17 \ (true)$$

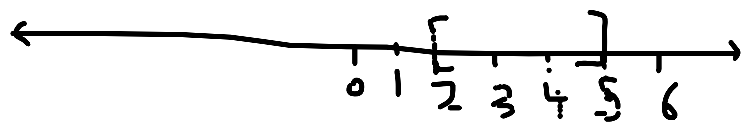
Solution set = [-3.75, 4.75]



3. $4 \leq 3x - 2 \leq 13$

$$4 \leq 3x - 2 \quad | \quad 3x - 2 \leq 13$$
$$6 \leq 3x \quad | \quad 3x \leq 15$$
$$6/3 \leq x \quad | \quad x \leq 15/3$$
$$x \geq 2 \quad | \quad x \leq 5$$

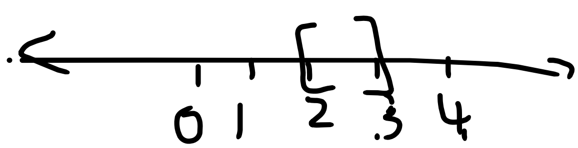
Therefore, $2 \leq x \leq 5$



4. $x^2 \leq 5x - 6$

Change into quadratic form,

$$x^2 - 3x - 2x + 6 = 0$$
$$x(x-3) - 2(x-3) = 0$$
$$(x-3)(x-2) = 0$$
$$(x-3) = 0 \quad | \quad (x-2) = 0$$
$$x = 3 \quad | \quad x = 2$$
$$2 \leq x \leq 3$$



5. $\frac{1+x}{1-x} \geq 1$

$$1 + x = 1(1 - x)$$
$$1 + x = 1 - x$$
$$2x = 0$$
$$x = 0$$

Check if x = 0:

$$\frac{1+0}{1-0} \geq 1$$
$$1 \geq 1 \ (true)$$

Check if x < 0:

$$\frac{1+(-1)}{1-(-1)} \geq 1$$
$$\frac{0}{2} \geq 1$$
$$0 \geq 1\ (false)$$

Check if $0 \leq x < 1$:

$$\frac{1+(0.5)}{1-(0.5)} \geq 1$$
$$\frac{1.5}{0.5} \geq 1$$
$$3 \geq 1\ (true)$$

Question # 3

Derive the quadratic formula to find the roots of quadratic equation.

$$ax^2 + bx + c = 0$$

Solution

Divide by a,

$$x^2 + \frac{b}{a}x + \frac{c}{a} = 0$$
$$x^2 + 2(x)\left(\frac{b}{2a}\right) + \left(\frac{b^2}{4a^2}\right) - \left(\frac{b^2}{4a^2}\right) + \frac{c}{a} = 0$$
$$\left(x + \frac{b}{2a}\right)^2 + \frac{c}{a} - \frac{b^2}{4a^2} = 0$$
$$\left(x + \frac{b}{2a}\right)^2 = -\frac{c}{a} + \frac{b^2}{4a^2}$$
$$\left(x + \frac{b}{2a}\right)^2 = \frac{-4ac + b^2}{4a^2}$$
$$x + \frac{b}{2a} = \pm\sqrt{\frac{-4ac + b^2}{4a^2}}$$
$$x = -\frac{b}{2a} \pm \sqrt{\frac{-4ac + b^2}{4a^2}}$$
$$x = \frac{-b \pm \sqrt[2]{b^2 - 4ac}}{2a}$$

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Assignment # 2

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Question # 1

Which of the points P(1,-2) or Q(8,9) is closer to the point A(5,3)?

Solution

We have three vectors $\vec{P}$, $\vec{Q}$ and $\vec{A}$. We want to know whether $\vec{P}$ or $\vec{Q}$ is closer to $\vec{A}$.

Distance from $\vec{P}$ to $\vec{A}$ is:

$$\vec{D}_{P\to A} = \big(5-1,3-(-2)\big) = (4,5)$$

$$D_{P\to A} = \sqrt{(4^2+5^2)} = \sqrt{(41)} = 6.4$$

Distance from $\vec{Q}$ to $\vec{A}$ is:

$$\vec{D}_{Q\to A} = (5-8,3-9) = (-3,-6)$$

$$D_{Q\to A} = \sqrt{(-3^2+-6^2)} = \sqrt{(45)} = 6.7$$

From above two calculations, it is clear that:

- ✓ Distance from $\vec{P}$ to $\vec{A}$ is *smaller*, hence it is closer to $\vec{A}$.

Question # 2

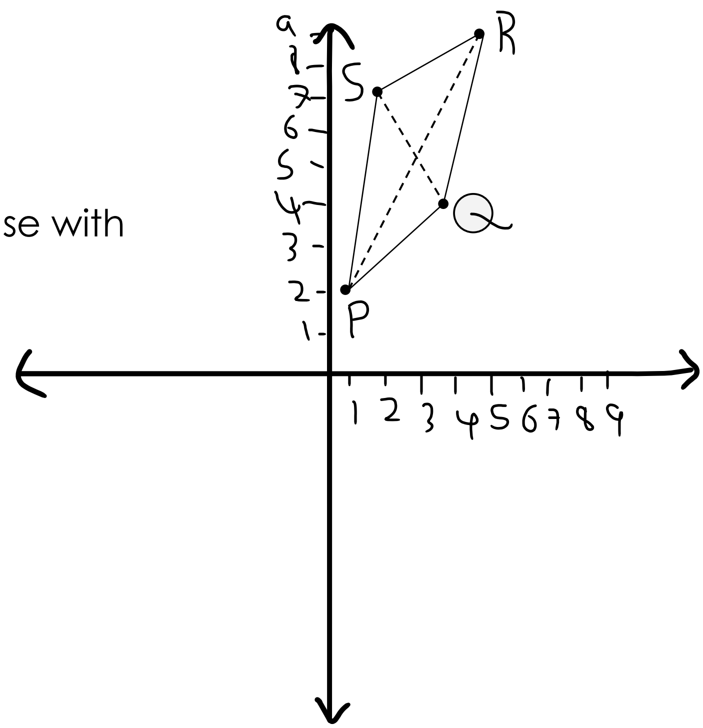
Show that the quadrilateral with vertices P(1,2), Q(4,4), R(5,9), and S(2,7) is a parallelogram by proving that its two diagonals bisect each other. (Show the quadrilateral on the coordinate plane).

Solution

To prove this, we need to find the mid-point of its diagonals. If mid-points are equal, it means that the two diagonals bisect each other.

Mid-point of any line is:

$$M_p = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$



Where x_1 and x_2 are initial and Final points respectively. Same case with Y_1 and y_2.

Mid-point of $\overline{PR}$ is:

$$M_P = \left(\frac{1+5}{2}, \frac{2+9}{2}\right)$$
$$M_P = (3, 5.5) \quad \ldots \quad \ldots \quad \ldots \quad (1)$$

Mid-point of $\overline{SQ}$ is:

$$M_P = \left(\frac{2+4}{2}, \frac{7+4}{2}\right)$$
$$M_P = (3, 5.5) \quad \ldots \quad \ldots \quad \ldots \quad (2)$$

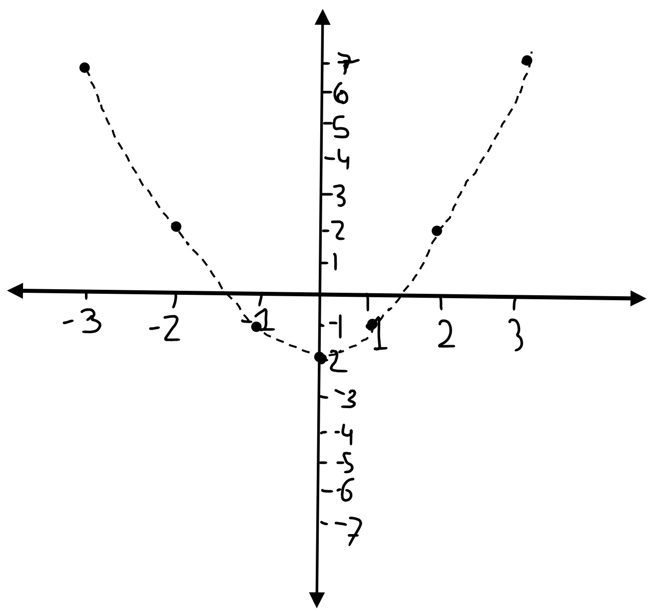
- ✓ Comparing (1) and (2), it is proved that it is a parallelogram.

Question # 3

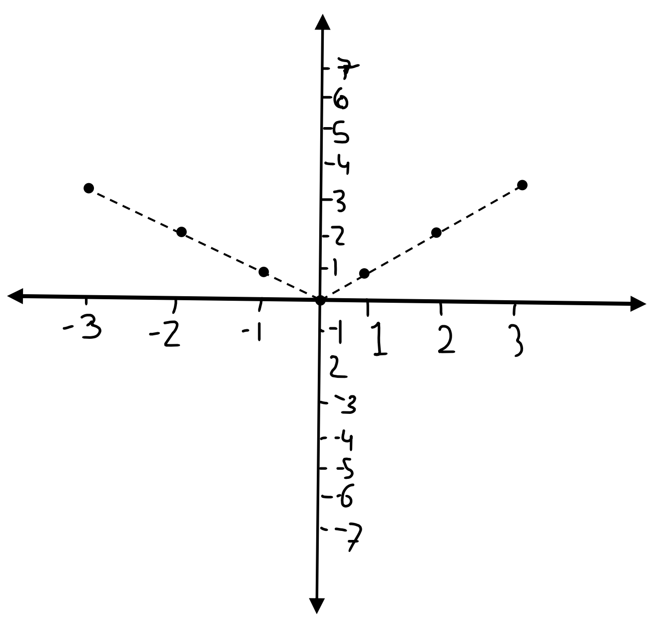
Sketch the graph of the equations $y = x^2 - 2$ and $y = |x|$ for $-3 \leq x \leq 3$

Solution

a) Graph for $y = x^2 - 2$ (in range $-3 \leq x \leq 3$) is:



b) Graph for $y = |x|$ (in range $-3 \leq x \leq 3$) is:



Question # 4

Write the equation of the line in slope-intercept form (y = mx + b). Identify slope and y-intercept of the line.

a) 8x − 9y = 0
b) 9x − 3y + 15 = 0

Solution

a) Slope-intercept form of 8x − 9y = 0:

$$8x - 9y = 0$$

$$9y = 8x + 0$$

$$y = \frac{(8x + 0)}{9}$$

$$y = \frac{8}{9}x + 0$$

Here, the slope is $\frac{8}{9}$ and y-intercept is 0, i.e. (0, 0).

b) Slope-intercept form of 9x − 3y + 15 = 0:

$$9\mathrm{x} - 3\mathrm{y} + 15 = 0$$

$$3y = 9x + 15$$

$$y = \frac{(9x + 15)}{3}$$

$$y = 3x + 5$$

Here, the slope is 3 and y-intercept is 5, i.e. (0, 5).

Question # 5

Find an equation of the line through the points (-1,2) and (3,-4) using point-slope form of the equation.

Solution

The point-slope form of the equation is: $\frac{y_2-y_1}{x_2-x_1} = m$

So,

$$m = \frac{(-4)-2}{3-(-1)}$$

$$m = \frac{-6}{4} = \frac{-3}{2}$$

We can also write this in the form $(y_2 - y_1) = m \cdot (x_2 - x_1)$:

$$y_2 = m \cdot (x_2 - x_1) + y_1$$

$$y_2 = \frac{-3}{2} \cdot (x_2 - (-1)) + 2$$

$$y_2 = \frac{-3}{2} x_2 \; - \frac{3}{2} + 2$$

$$y_2 = \frac{-3}{2} x_2 \; + \frac{1}{2}$$

Question # 6

Find an equation of the line that is perpendicular to the line 4x + 6y + 5 = 0 and passes through the origin.

Solution

- The slope m of parallel lines is equal.
- The slope m of perpendicular lines have *their* product = -1

For line 4x + 6y + 5 = 0:

$$6y = -4x - 5$$

$$y = \frac{-4}{6}x - \frac{5}{6}$$

So the slope of given line is: $m = \frac{-4}{6}$

The slope of perpendicular line is such that, when multiplied by this slope, yields -1:
So,

$$mw = -1$$

$$\frac{-4}{6}w = -1$$

$$w = \frac{3}{2} \dots \dots \dots (1)$$

The line must have y-intercept = (0, 0) in order for it to be passing through the origin. Therefore,

$$b = 0 \dots \dots \dots (2)$$

From the above discussion, we are synthesizing the equation for the perpendicular line:

$$y = wx + b$$

$$y = \frac{3}{2}x + 0$$

Or,

$$2y - 3x = 0$$