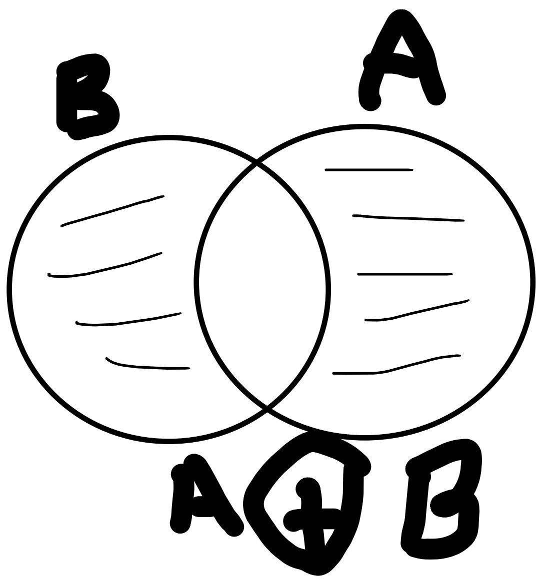
QUESTION . 1

<u>(A)</u>

<u>a.</u> Membership Table

A	B	A - B	B - A	A ⊕ B
0	0	0	0	0
0	1	0	1	1
1	0	1	0	1
1	1	0	0	0

<u>b.</u> Venn Diagram



<u>c.</u> Let A = {1,2,3,4} and B = {3,4,5,6} and C = {5,6,7,8}

B ⊕ C = {3,4,7,8}

(A ⊕ B) ⊕ C = {1,2,7,8}

<u>d.</u> Let A ⊕ B = (A ∪ B) – (A ∩ B)

By definition,

A ⊕ B = {x|(x ∈ A v x ∈ B) A¬ (x ∈ A n x ∈ B})}

= {(A u B) A¬ (A n B)} → Answer

e. Let $A \oplus B = (A - B) \cup (B - A)$

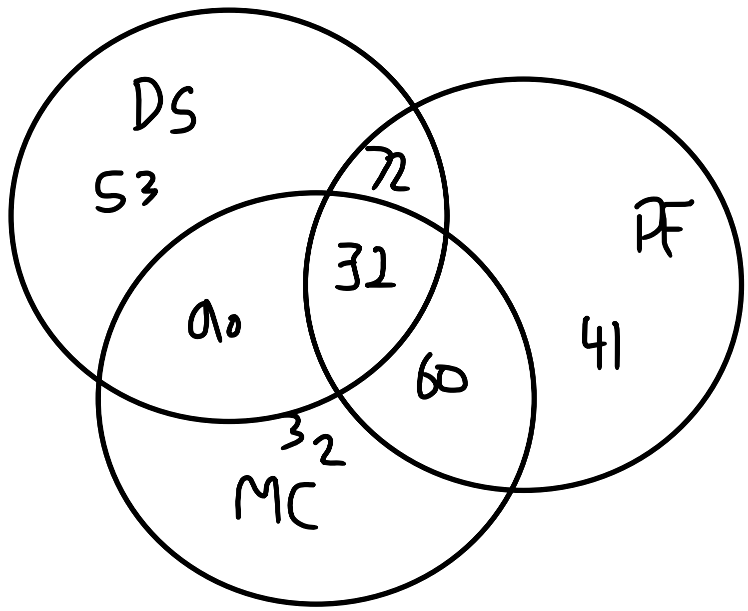
We know that,

$A \oplus B = \{x|(x \in A \text{ v } x \in B) \text{ A}\neg (x \in A \text{ n } x \in B\})\}$

$= \{(x \in A \wedge - x \in B) \text{ U } (x \in B \text{ n } x \in A\})\}$

$= (A - B) \cup (B - A))\}$ → Answer

(B)

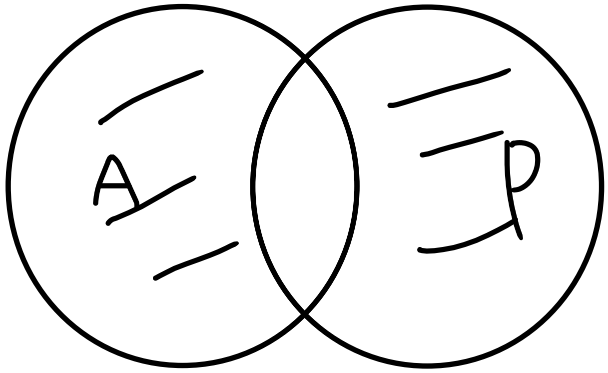


(C)

a.

A	B	$A \oplus B$
0	0	0
0	1	1
1	0	1
1	1	0

b.



c.

$A = \{1,2,3,4\}$ $B = \{3,4,5,6\}$ $C = \{5,6,7,8\}$

$B \oplus C = (B - C) \cup (C - B)$

$= \{3,4\}$

$A \oplus B \oplus C = \{1,2\}$

d.

$A \oplus B = (A \cup B) – (A \cap B)$

$A \oplus B = (A - B) \cup (B - A)$

Let $A \oplus B = (A \cup B) – (A \cap B)$

From de-morgan law

$A - (B \cup C) = (A - B)\ n\ (A - C)$

We can write it as $((A\ U\ B) - B\ U\ (B\ n\ A))$

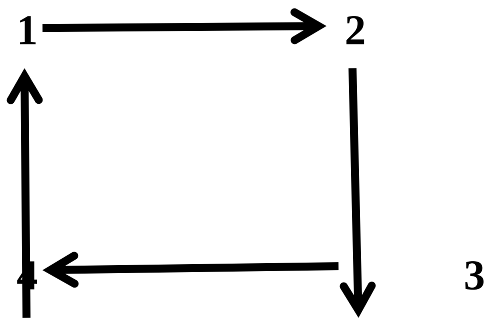
$= A$

QUESTION . 2

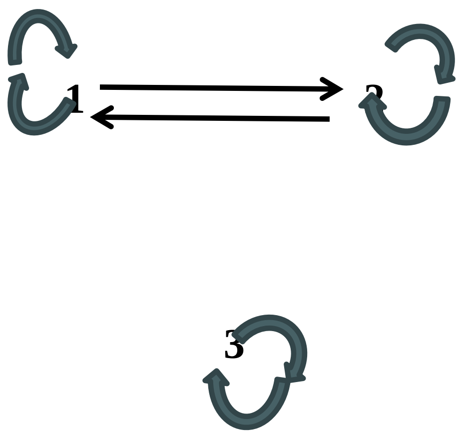
(A)

Draw the digraph of the relation.

i. The relation R = {(1, 2), (2, 3), (3, 4), (4, 1)} on {1, 2, 3, 4}



ii. The relation R={(1, 2), (2, 1), (3, 3), (1, 1), (2, 2)} on X = {1, 2, 3}



(B)

i. **R** = { (a ,b) , (b ,a) , (b ,d) , (c ,d) , (a ,c) , (c ,c) } on { a , b , c , d}
Domain =
Range =

ii. **R** = {(1,1),(2,2),(3,3),(3,5),(5,5),(5,4),(4,4),(4,3) } on {1,2,3,4,5}
Domain =
Range =

iii. **R** = {(b ,c),(c ,b),(d ,d)} on {a ,b ,c ,d}
Domain =
Range =

(C)

Let R_1 and R_2 be the relations on {1, 2, 3, 4} given by

R_1 = {(1, 1), (1, 2), (3, 4), (4, 2)}
R_2 = {(1, 1), (2, 1), (3, 1), (4, 4), (2, 2)}.
List the elements of:

i. $R_1 \cup R_2$

R = {(1,1),(1,2),(2,1),(2,2),(3,1),(3,4),(4,2),(4,4)}

ii. $R_1 \cap R_2$

R = {(1, 1)}

(D)

1. **Reflexive:** YES. Given string $\alpha \in X$, α indeed has some common substring of size 2 with itself, say its substring consisting of the first two bits of α.
2. **Symmetric**: YES. If $\alpha, \beta \in X$, and α has some substring of size 2 in common with β, also β has that same substring in common with α.
4. **Transitive**: NO. Counterexample: 1110 R 1100 (both contain 11) and 1100 R 0001 (both contain 00), however 1110 R6 0001.

QUESTION . 3

(A)

1. $\left(\sum_{i=1}^{3} y_i\right)\left(\sum_{i=1}^{3} z_i\right)$ = (y+2y+3y)(z+2z+3z)

= 6y(6z)

2. $\sum_{i=-1}^{1} \sum_{j=0}^{2} (2i + 3j) = \sum_{i=-1}^{1} (2i + 2i + 3 + 2i + 6)$

= 3 + 9 + 15 = 27

(B)

i. $\sum_{i=1}^{n} \sum_{j=1}^{i} (x + 1)$ ii. $\sum_{i=1}^{n} \sum_{j=1}^{i} \sum_{k=1}^{i} (x + 1)$

iii. $\sum_{i=1}^{n} \sum_{j=1}^{i} \sum_{k=1}^{j} (x + 1)$ iv.
$\sum_{i=1}^{n} \sum_{j=1}^{i} \sum_{k=1}^{j} \sum_{l=1}^{i} (x + 1)$

Evaluate if n = 4:
a=6, b=14, c=4, d=14

QUESTION . 4

a) F(x) = 75 + ((x-400)/100) * 0, 60), x > 9000

$$F(x) = \begin{cases} 75, x \leq 4000 \\ 75 + \left[\left(\frac{x-4000}{100}\right) * 0{,}60\right], x > 4000 \end{cases}$$

b) Write the definition of "one-to-one" using logical notation (i.e., use, ∃, etc.)

A function f: A → B is injective (or one-to-one) if it does not map different elements of A to the same element of B.

In logical notation: if ∀x∈A. ∀y∈A. (f(x) = f(y) =⇒ x = y).

c) Write the definition of "onto" using logical notation (i.e., use, ∃, etc.).

A function f: A → B is surjective (or onto) if every element in B is mapped to by an element in A.

In logical notation: if ∀y∈B.∃x∈A.(f(z) = y).

d) Determine whether each of these functions is a bijection from R to R.

f (x) = 2x + 1

Ans. Yes

f (x) = x2 + 1

Ans. No

f (x) = x3

Ans. Yes

f (x) = (x2 + 1) / (x2 + 2)

Ans. No

e) Find at least three different sequences beginning with the terms 3, 5, 7 whose terms are generated by a simple formula or rule.

a_1 = 2(1) +1

a_1 = 3

$a_2 = 2(2) + 1$
$a_2 = 5$
$a_3 = 2(3) + 1$
$a_3 =$
$a_4 = 2(4) + 1$
$a_4 = 9$
$a_5 = 2(5) + 1$
$a_5 = 11$
$a_6 = 2(6) + 1$
$a_6 = 13$
:
:
$a_n = 2(n)+1$

So we generalize the formula that:
$a_n = 2(n)+1$

f) Find the first five terms of the sequence defined by each of these recurrence relations and initial conditions.

a) an = 6an−1, a0 = 2

$a_n = 6a_{n-1}$
$\underline{a_0 = 2}$
$a_1 = 6a_{1-1}$
$=6a_0, =6(2)$
$\underline{a_1=12}$
$a_2 = 6a_{2-1}$
$=6a_1, =6(12)$
$\underline{a_2=72}$
$a_3 = 6a_{3-1}$
$=6a_2, =6(72)$
$\underline{a_3=252}$
$a_4 = 6a_{4-1}$
$=6a_3, =6(252)$
$\underline{a_4 = 1512}$

b) an = a²n−1, a1 = 2

$an = a^2n-1, a1 = 2$

$a1=2$

$a_2=a^2{}_{2-1}$

$=a^2(_1), =(a_1)^2, =2^2$

$a_2=4$

$a_3=a^2{}_{3-1}$

$=a^2(_2), =(a_2)^2, =4^2$

$a_3=16$

$a_4=a^2{}_{4-1}$

$=a^2(_3), =(a_3)^2, =16^2$

$a_4=256$

$a_5=a^2{}_{5-1}$

$=a^2(_4), =(a_4)^2, =256^2$

$a_5=65536$

g) Suppose that the number of bacteria in a colony triples every hour.

i. Set up a recurrence relation for the number of bacteria after n hours have elapsed.

According to the given condition:

Initial condition is a0=1

We can use the iterative approach to find formula for an, in which bacteria triples every hour so,

$a_1=3(a0), 3(1)$

$a_1=3$

$a_2=3(a1), 3(3)$

$a_2=9$

$a_3=3(a2), 3(9)$

$a_3=27$

:

:

$a_n=3(an-1)$ (generalized term)

ii. **If 100 bacteria are used to begin a new colony, how many bacteria will be in the colony in 10 hours?**

According to the given condition, we have to find a10
Here, we have given that a0=100
$\underline{a_0=100}$
$a_1=3(100)$
$\underline{a_1=300}$
$a_2=3(300)$
$\underline{a_2=900}$
$a_3=3(900)$
$\underline{a_3=2700}$
$a_4=3(2700)$
$\underline{a_4=8100}$
$a_5=3(8100)$
$\underline{a_5=24,300}$
$a_6=3(24,300)$
$\underline{a_6=72,900}$
$a_7=3(72,900)$
$\underline{a_7=218,700}$
$a_8=3(218,700)$
$\underline{a_8=656,100}$
$a_9=3(656,100)$
$\underline{a_9=1,968,300}$
$a_{10}=3(1,968,300)$
$\underline{a_{10}=5,904,900}$
so, we generalize the formula that:
$a_n=3(a_{n-1})$