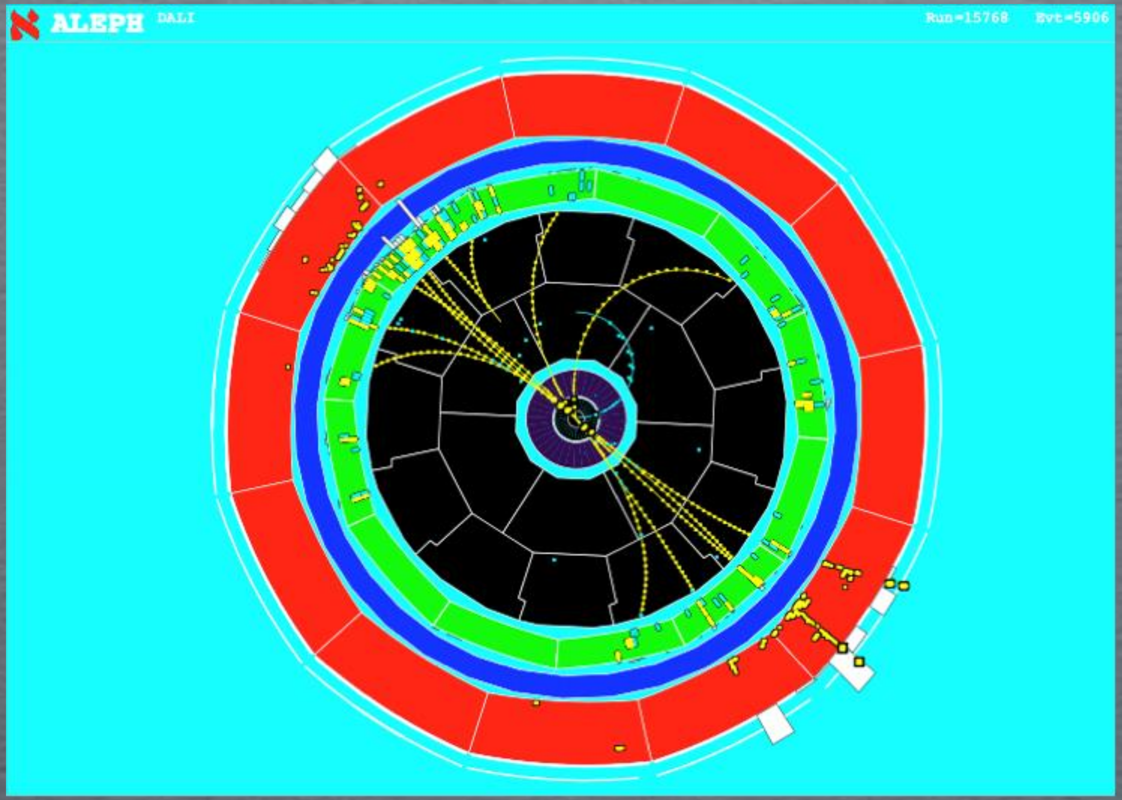
Resummation of jet observables in QCD



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NIKHEF – 19-20 November 2019 – Amsterdam

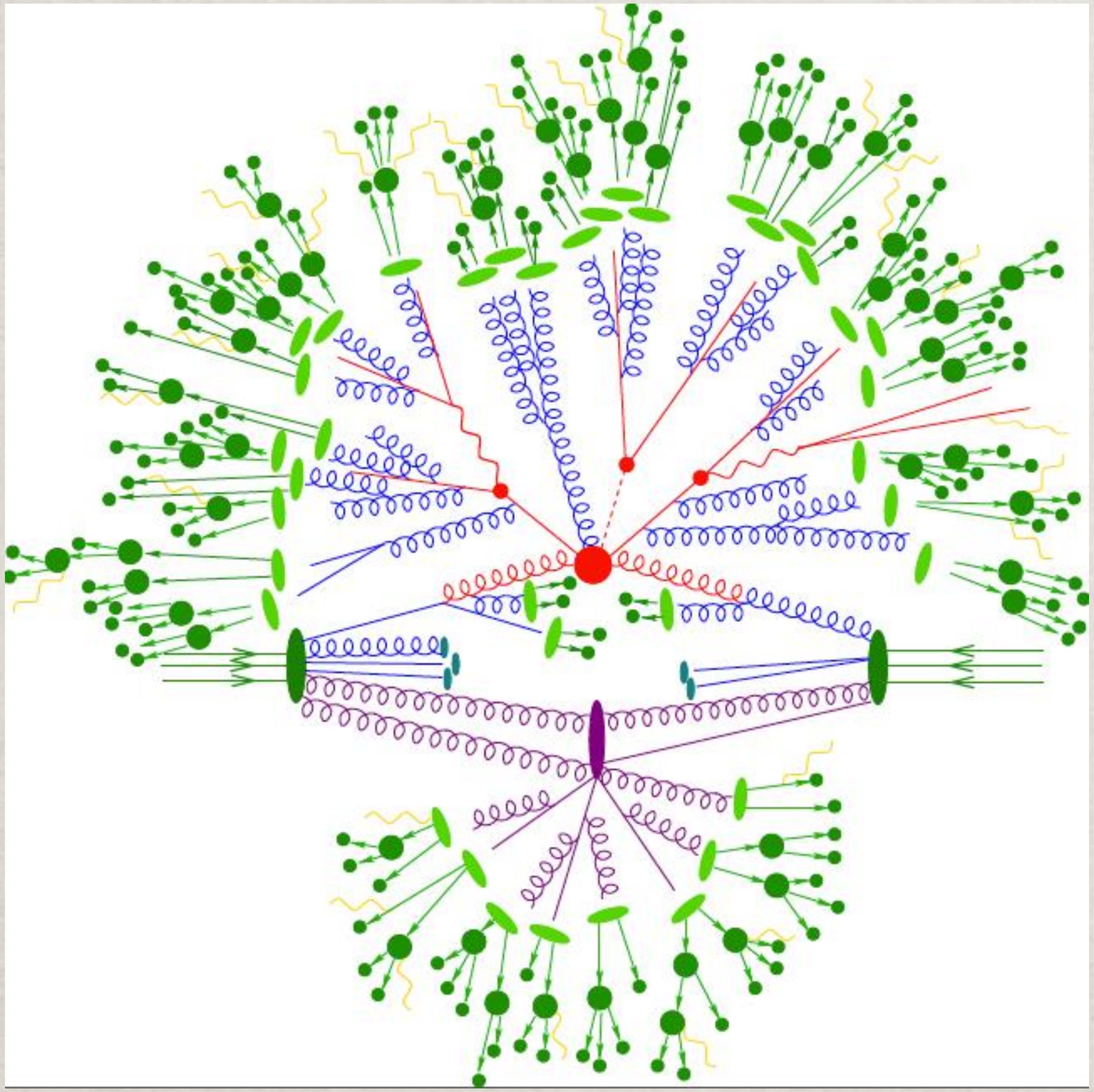
Outline

- Brief introduction to jets and jet algorithms
- **General principles of resummation of jet observables in QCD**
- Non-perturbative effects: hadronisation and underlying event
- Discussion on theory uncertainties

Introduction to jets

Events at hadron colliders

- The description of LHC events involves different levels

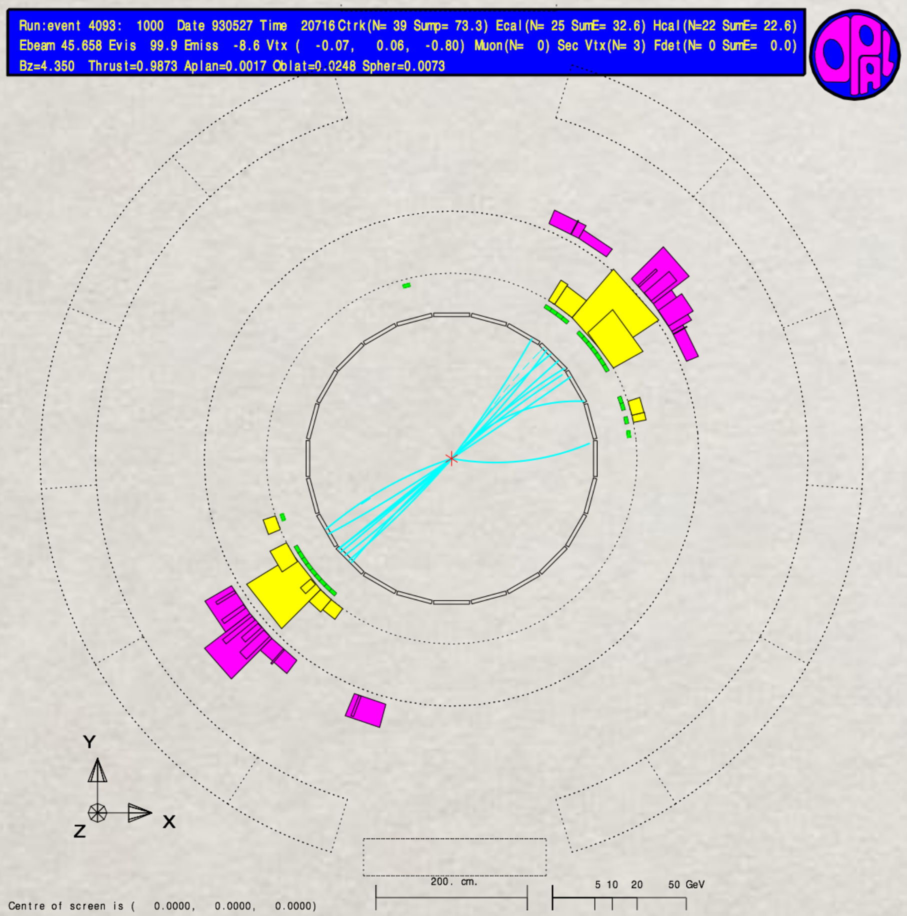


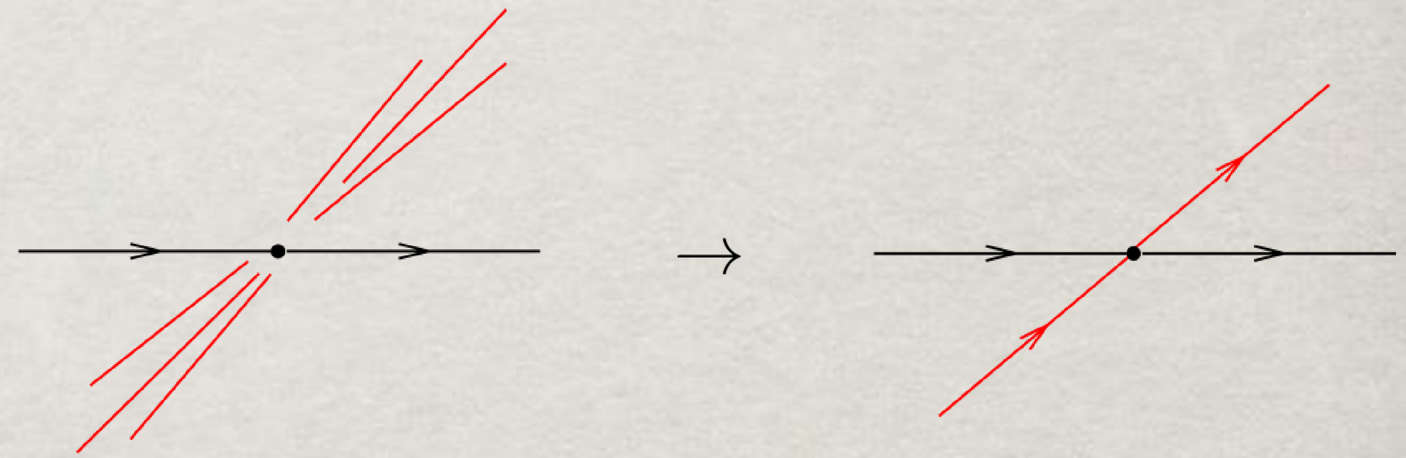
- A hard event, with well separated partons $\Rightarrow$ fixed-order QCD
- Radiation of secondary soft and collinear gluons from the hard partons $\Rightarrow$ Monte Carlo, all-order resummation
- Hadronisation $\Rightarrow$ Monte Carlo or analytical models
- Scattering of proton remnants, underlying event, etc.

- The main challenge is to find “good” observables that relate high-multiplicity events to the dynamics of the underlying quarks and gluons

JETTY EVENTS

- High-energy hadronic events contain highly collimated sprays of hadrons, the jets



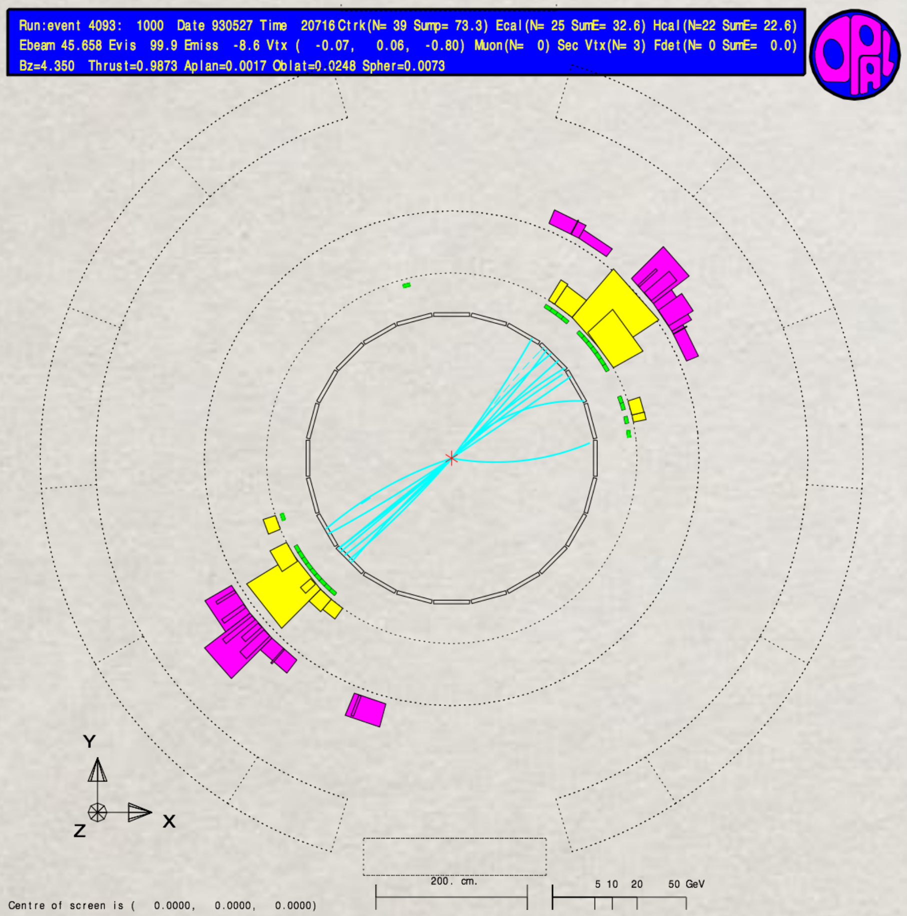


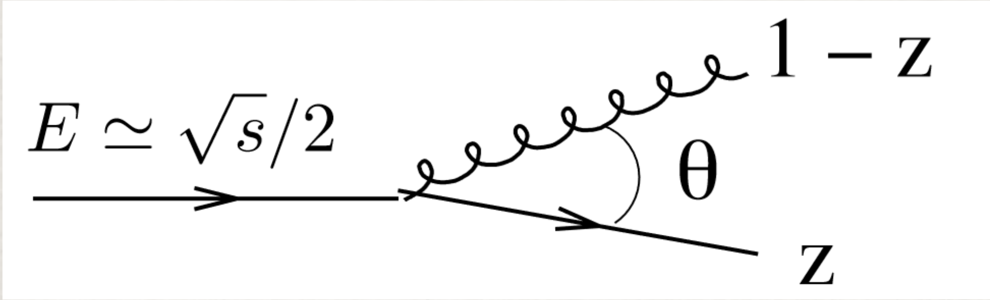
- Jets are the natural objects that we would like to associate with quarks and gluons, using the equality 1 jet = 1 parton

- We need to find a mathematical definition of jets that relate what we observe to what we can compute in QCD

Infrared and collinear safety

- Inside a QCD jet we find a number of soft and collinear splittings





$$E_q \simeq zE \qquad\qquad E_g \simeq (1-z)E$$

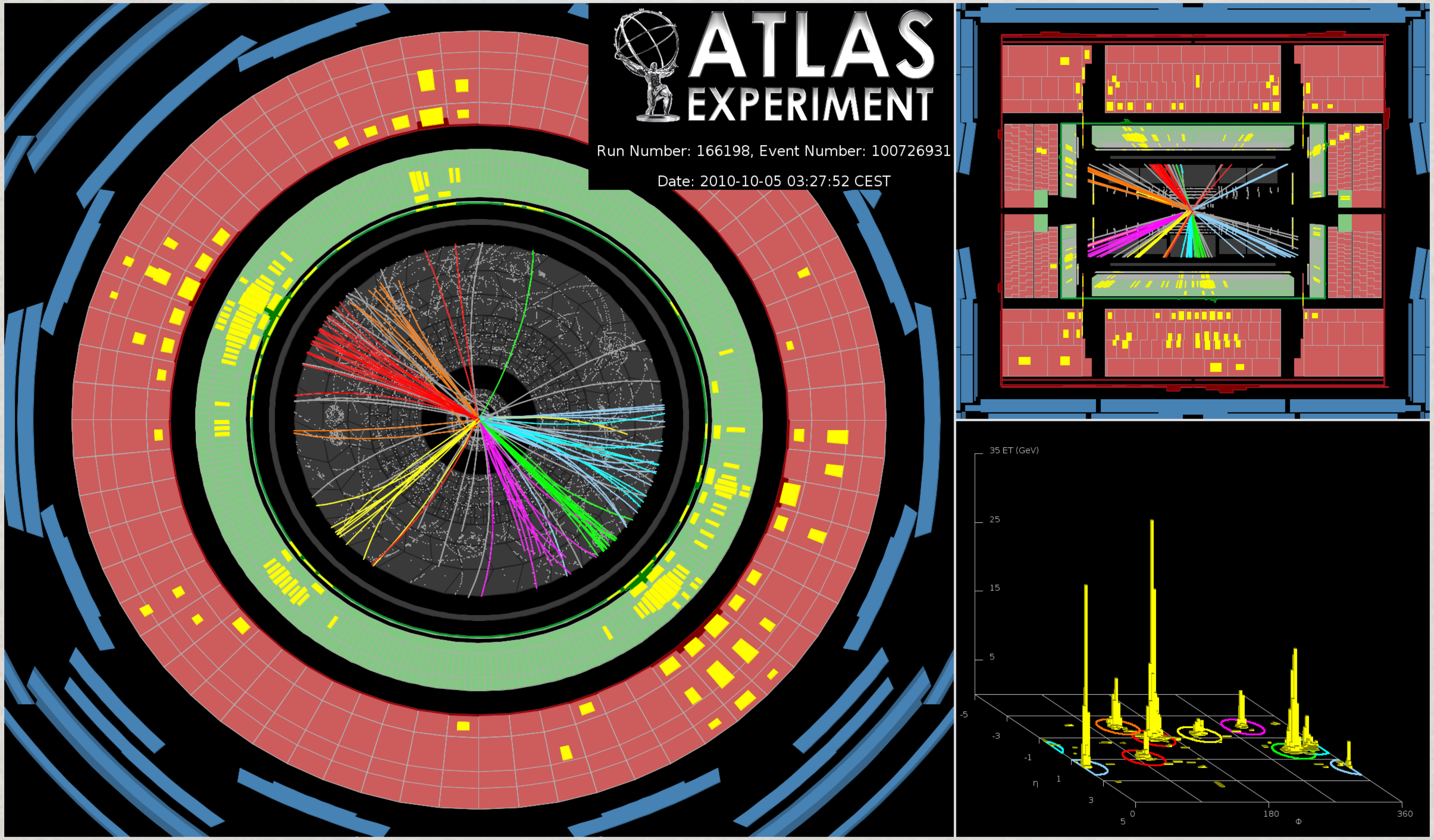
$$dP_{q\to qg} = C_F \frac{\alpha_s}{2\pi}\frac{d\theta^2}{\theta^2}dz\frac{1+z^2}{1-z}$$

Splitting probabilities are singular when emissions are soft ($z \to 1$) or collinear ($\theta \to 0$). These singularites cancel with virtual corrections if jet observables are insensitive

- the addition of any number of soft partons (IR safety)
- an arbitrary number of collinear splittings (collinear safety)

Cone algorithms

- At hadron colliders, jets appear as localised deposits of energy



- Cone algorithms: draw circles in the η-ϕ plane and build a jet out of the hadrons (or calorimetric cells) in each circle
- Problem: it is very difficult to translate this idea into an IRC safe algorithm

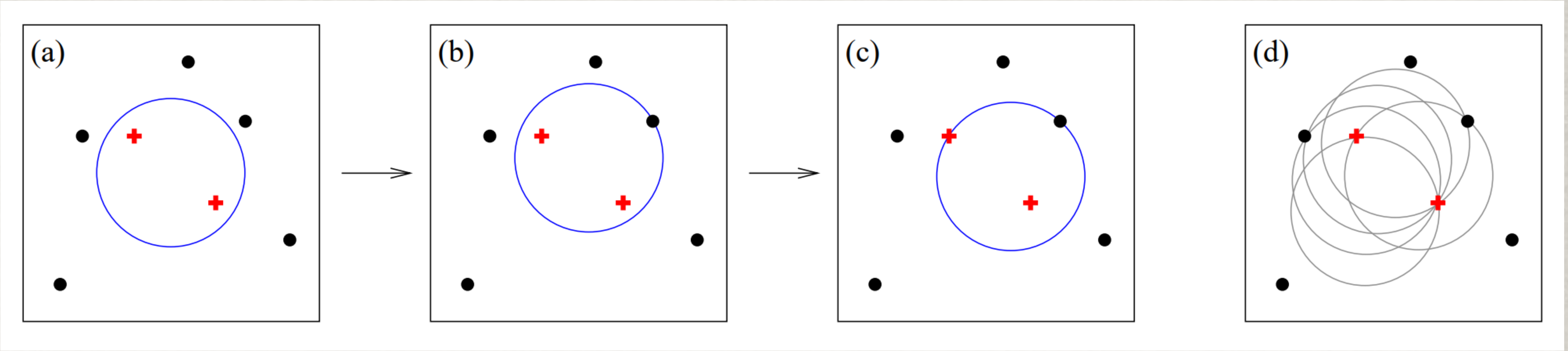
IRC SAFE CONE ALGORITHM

- A "stable" cone J is a set of particles such that

$$(y_i - y_J)^2 + (\phi_i - \phi_J)^2 < R^2 \qquad\qquad p_J = \sum_{i \in J} p_i$$

- All stable cones with transverse momentum above a given threshold are IRC safe objects $\Rightarrow$ a cone algorithm is a procedure to find all stable cones, and deal with overlapping cones

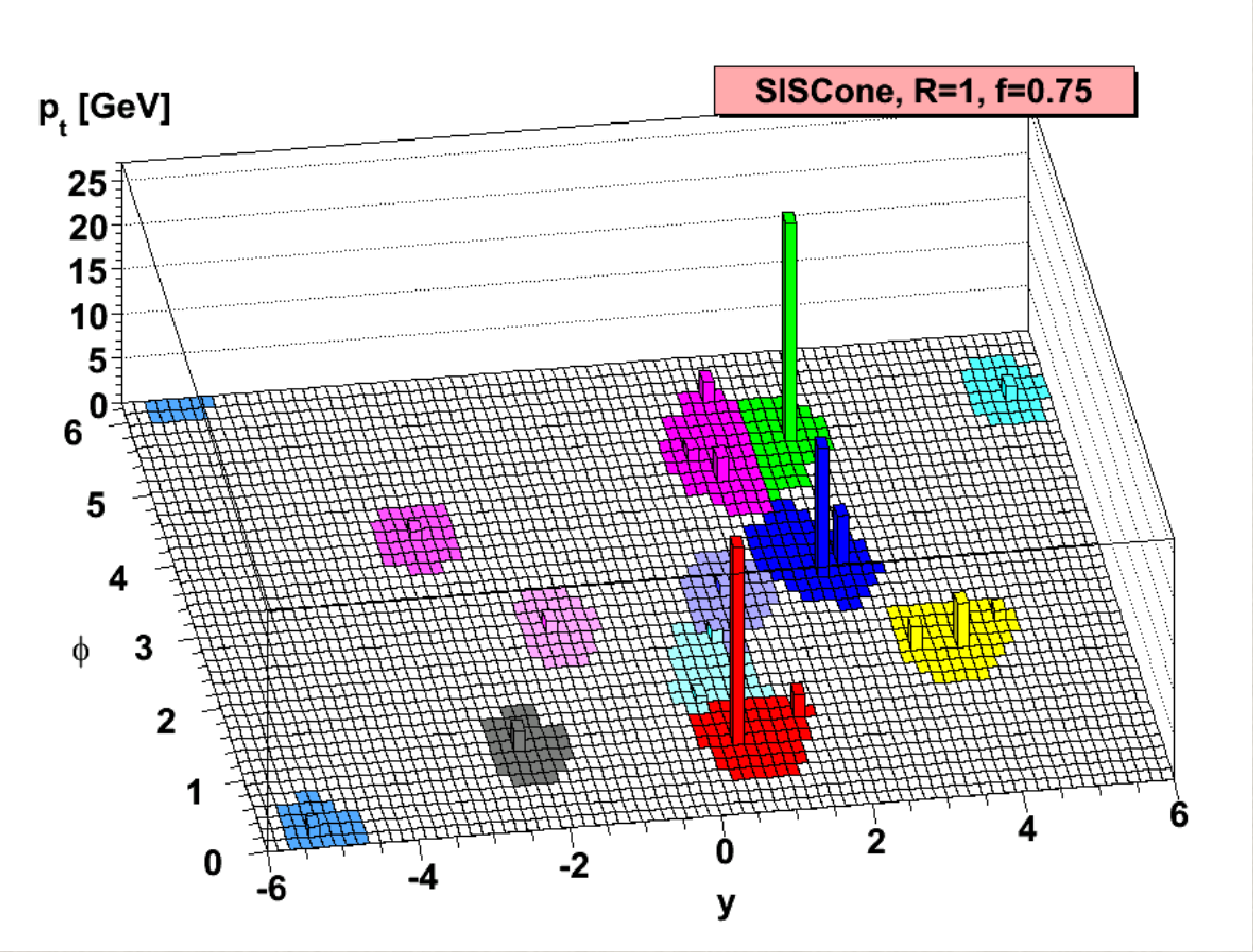
- Using points in the η-ϕ plane as "seeds" to construct stable cones lead to IRC unsafe procedures $\Rightarrow$ modern cone algorithms are seedless (SISCone)



[Salam Soyez 0704.0292]

IRC safe cone algorithm

- Problem: a perfect cone algorithm does not produce jets that are perfect circles of radius R in the η-ϕ plane!



Sequential algorithms

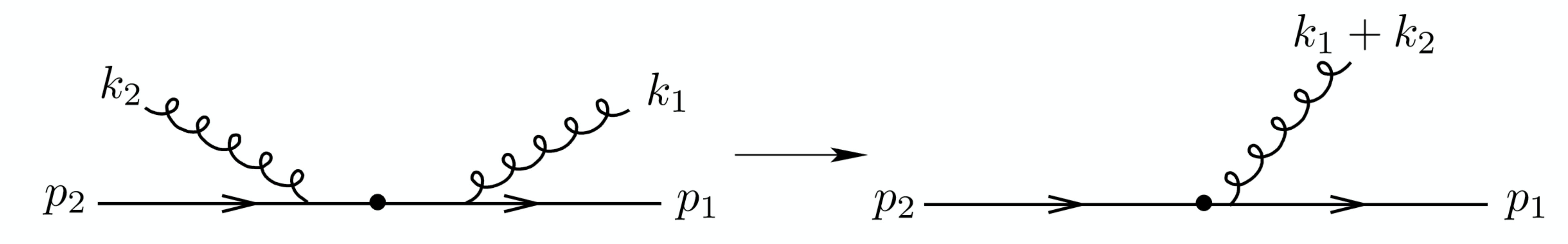
- Sequential algorithms are more popular in e^+e^- annihilation, where one has full information on hadron momenta
- First-ever sequential algorithm is the JADE algorithm: For any pair of particles, p_i, p_j, find the minimum "distance"

$$y_{ij} = \frac{(p_i + p_j)^2}{Q^2}$$

- If $y_{ij} < y_{\mathrm{cut}}$, merge particles p_i, p_j into a jet
- Repeat until all pairs have $y_{ij} > y_{\mathrm{cut}}$. The number of jets is the number of particle left

Sequential algorithms

- Problem: the distance measure of the JADE algorithm is the invariant mass of two partons. The JADE can cluster together two soft particles collinear to different legs, leading to spurious large-angle soft jets



- Solution: Durham (a.k.a. k_t) algorithm. The distance measure is the relative transverse momentum of the softer particle with respect to the harder one

[Catani Dokshitzer Olsson Turnock Webber PLB 269 (1991) 432]

$$y_{ij} = \frac{2\min\{E_i^2, E_j^2\}}{Q^2}(1-\cos\theta_{ij})$$

- The Cambridge algorithm is a more sophisticated version that uses angles only to determine the clustering sequence

[Dokshitzer Leder Moretti Webber hep-ph/9707323]

Generalised k_T algorithms

In hadron collisions, the most-used algorithms are the generalised-kt algorithms [Cacciari Salam Soyez 0802.1189]

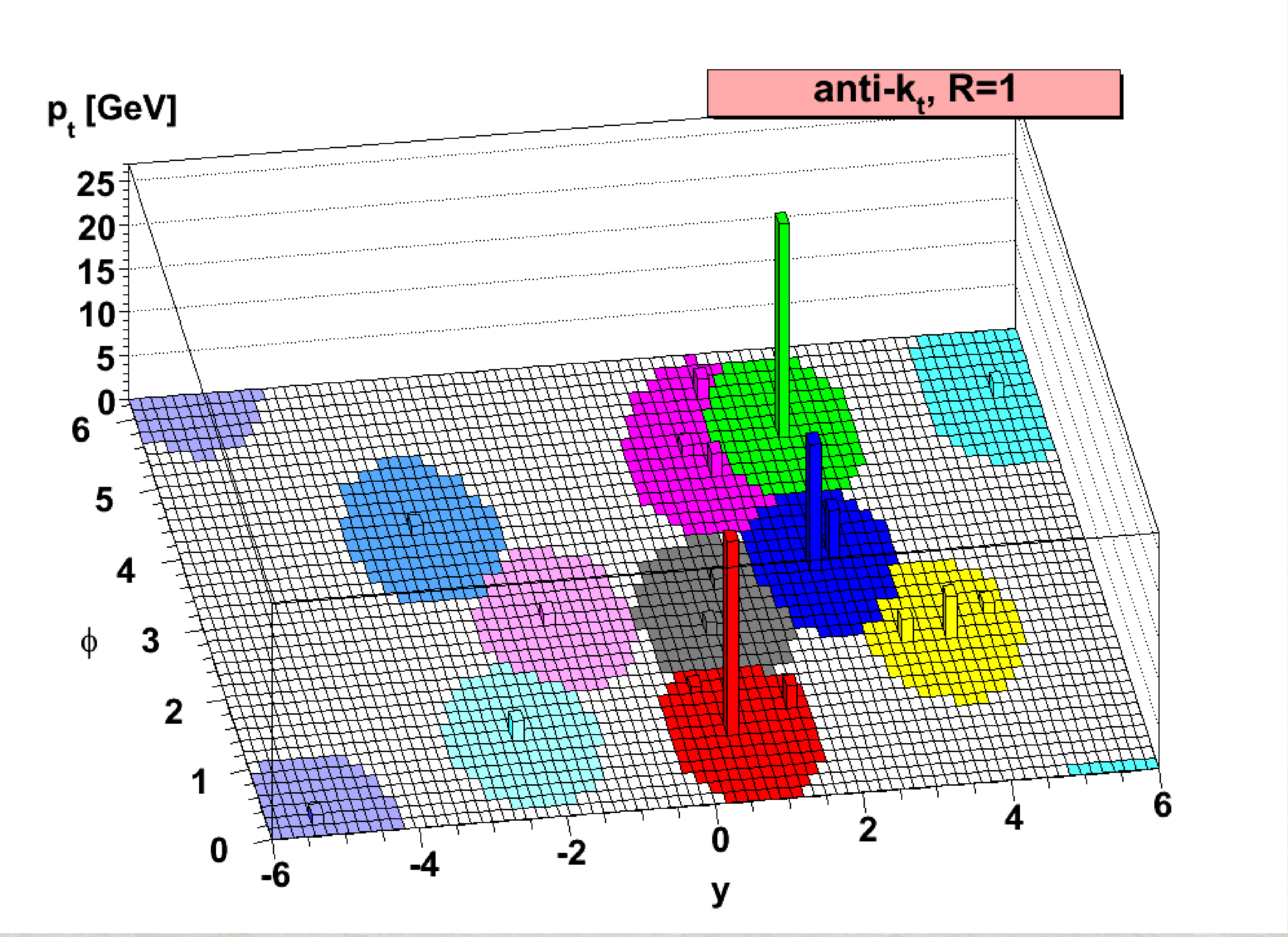
- One finds the minimum over all particles of the distances

$$d_{ij} = \frac{\min\{k_{ti}^{2p}, k_{tj}^{2p}\}}{R^2}\Delta R_{ij}^2 \qquad d_{iB} = k_{ti}^{2p} \qquad \Delta R_{ij}^2 = (y_i - y_j)^2 + (\phi_i - \phi_j)^2$$

- If the minimum distance is d_{iB} or d_{jB}, then p_i or p_j is a jet and is removed from the list of particles, otherwise p_i and p_j are merged into a jet
- The procedure is repeated until no particles are left
- The parameter R is the jet "radius". If there are only two particles, they are in the same jet only if $R_{ij} < R$
- The parameter p identifies the algorithm, and usually assumes values 1 (k_t algorithm), 0 (Cambdridge-Aachen algorithm) and -1 (anti- k_t algorithm)

The anti-k_T algorithm

- The hard jets reconstructed with the anti- k_t algorithm are perfect circles with radius R in the η-ϕ plane!



IRC safe jets

- After a sequential clustering, all jets with $p_t > p_{t,\min}$ are IRC safe
- IRC safe jet cross sections can be safely computed in massless QCD $\Rightarrow$ non-perturbative effects associated to quark masses are power suppressed

$$d\sigma_{pp\to\text{jets}}\left(\alpha_s(p_{t,\min}), \frac{m}{p_{t,\min}}\right) = d\sigma_{pp\to\text{jets}}\left(\alpha_s(p_{t,\min}), 0\right) + \mathcal{O}\left(\left(\frac{m}{p_{t,\min}}\right)^p\right)$$

- Collimation of the jets is naturally explained in terms of the collinear enhancement of QCD matrix elements

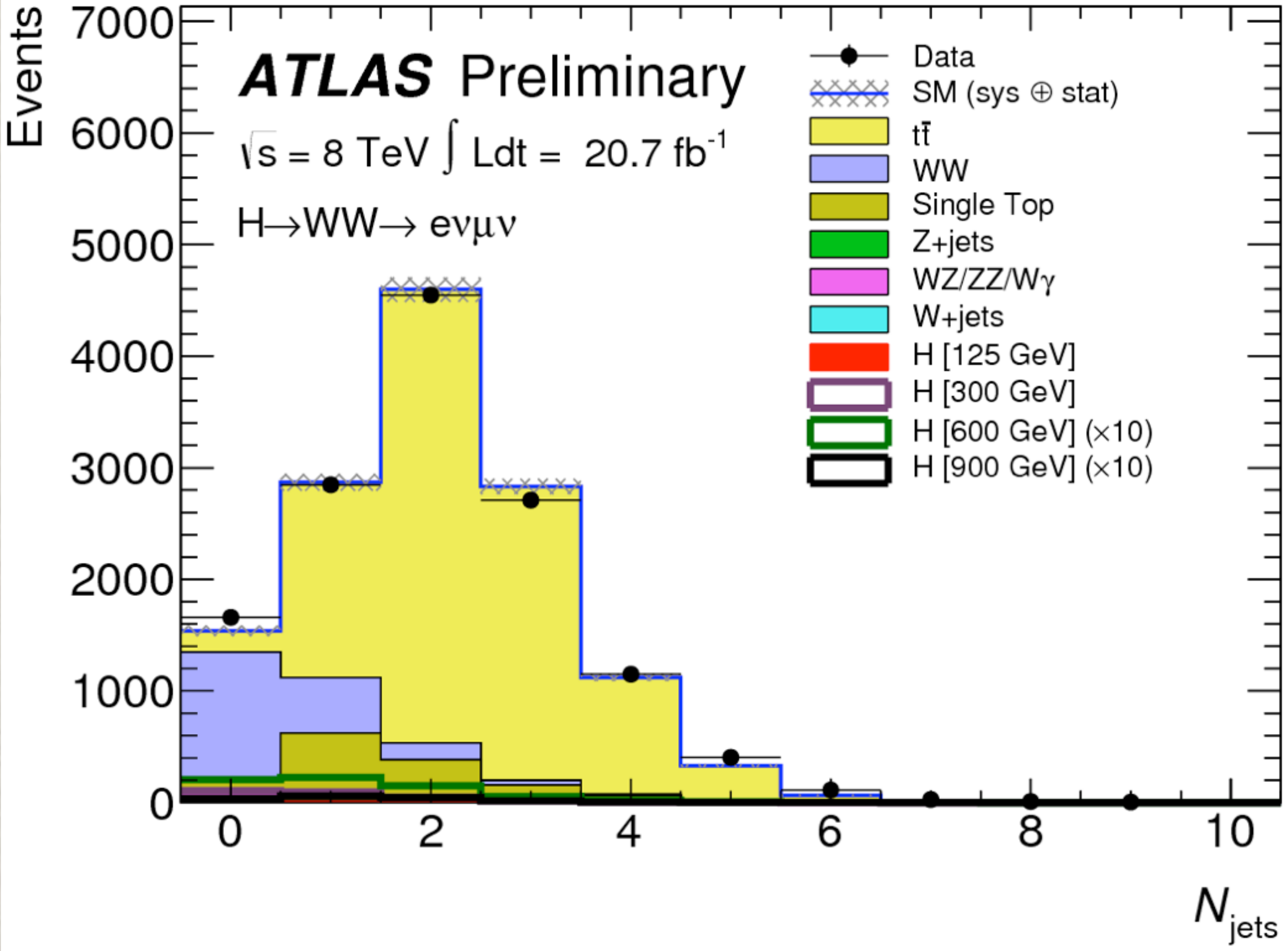
$$dP_{q\to qg} = C_F \frac{\alpha_s[z(1-z)\theta p_{t,\min}]}{2\pi}\frac{d\theta^2}{\theta^2} dz \frac{1+z^2}{1-z}$$

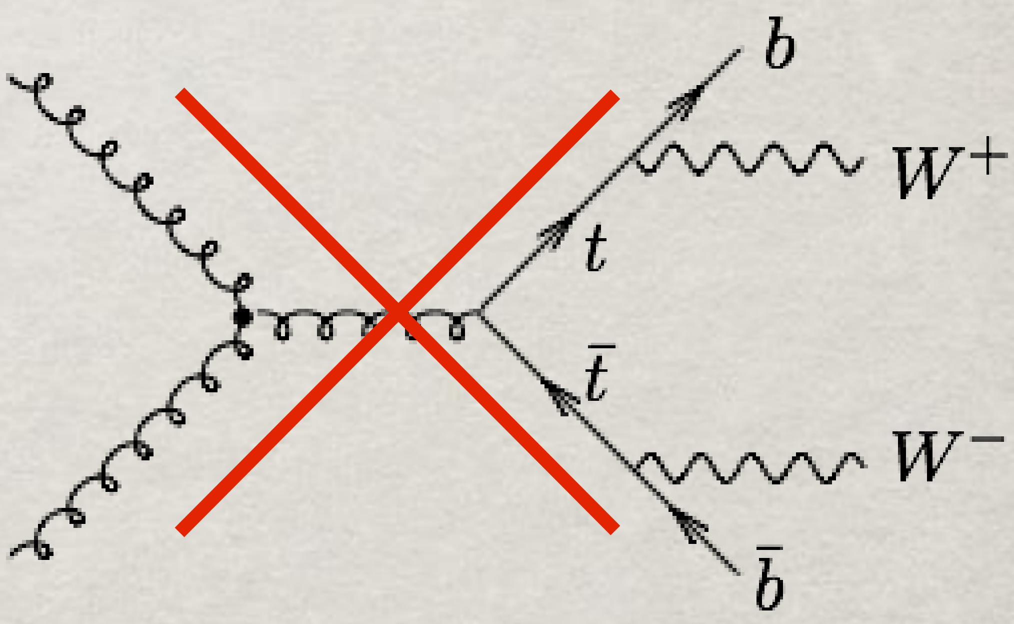
- Due to asymptotic freedom, $\alpha_s(p_{t,\min}) \to 0$ for $p_{t,\min} \to \infty$: the higher the energy, the more collimated the jets $\Rightarrow$recovery of the equality 1jet = 1 parton in the high-energy limit

Resummation

Jet vetoes

- In many situations (e.g. WW production), one puts a jet-veto to eliminate overwhelming top-antitop background

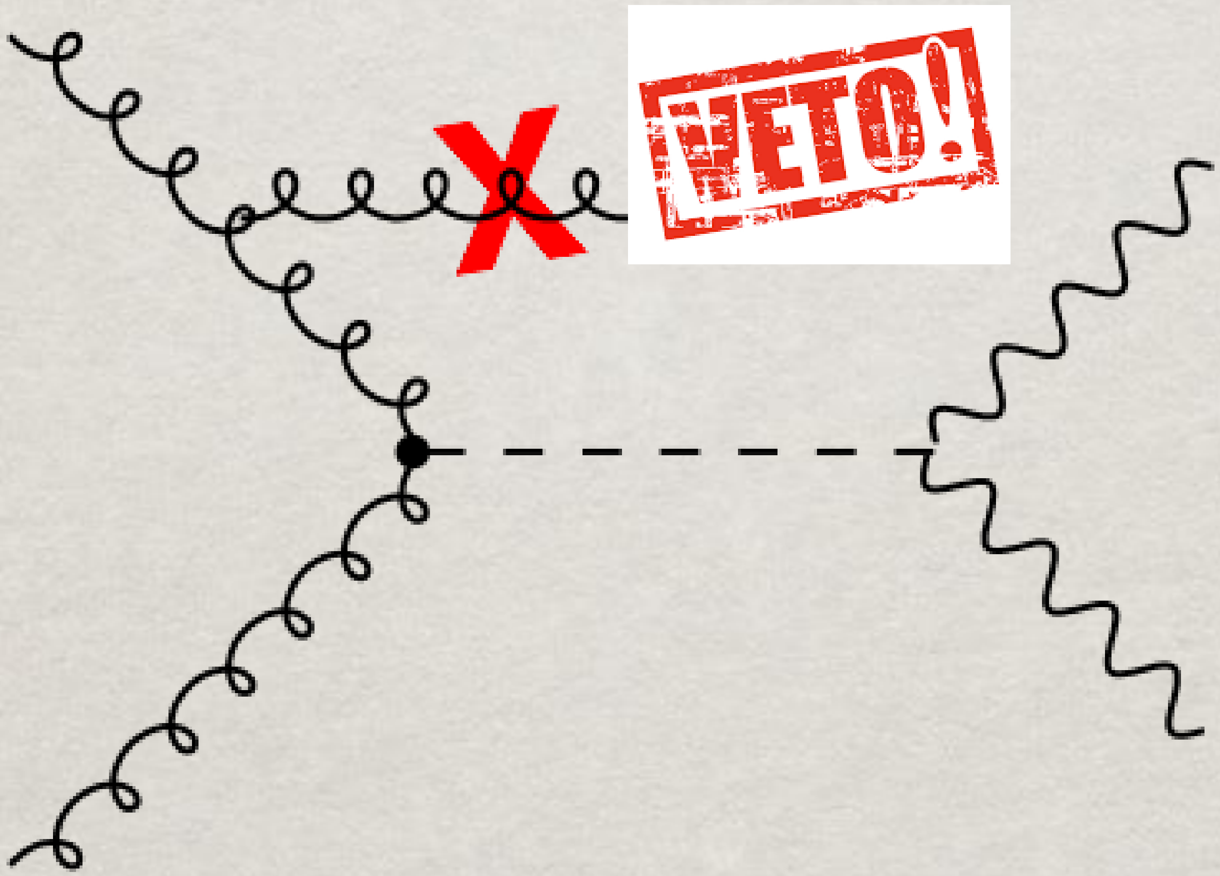




- The main object of study is the zero-jet cross section $\sigma_{0\text{-jet}}(p_{\mathrm{t,veto}})$, obtained by imposing that all jets have $p_t < p_{\mathrm{t,veto}}$

Two-scale problems

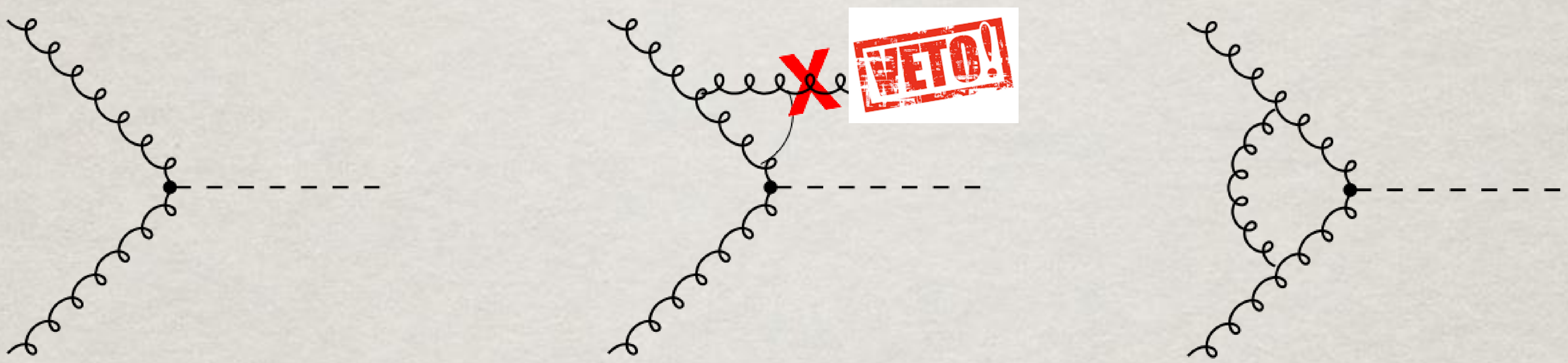
- The zero-jet cross section is characterised by two scales, the mass of the produced object M and the jet resolution $p_{\rm t,veto}$



- In QCD, large logarithms such as $\ln(M/p_{\rm t,veto})$ appear whenever the phase space for the emission of soft and/or collinear gluons is restricted

One gluon emission

- Example: veto one soft ($E \ll M$) and collinear ($\theta \ll 1$) gluon



BORN **REAL** **VIRTUAL**

soft collinear

$$\sigma_0 \left[1 \quad + \quad C_A \frac{\alpha_s}{\pi} \int \frac{dE}{E} \frac{d\theta^2}{\theta^2} \Theta \left(p_{\mathrm{t,veto}} - E\theta \right) \quad - \quad C_A \frac{\alpha_s}{\pi} \int \frac{dE}{E} \frac{d\theta^2}{\theta^2} \right]$$

factorisation of soft radiation

$$\boxed{\sigma_{\text{0-jet}} = \sigma_0 \left[1 - 2 C_A \frac{\alpha_s}{\pi} \ln^2 \left(\frac{M}{p_{\mathrm{t,veto}}} \right) \right]}$$

All-order resummation

- All-order resummation of large logarithms $\Rightarrow$ reorganisation of the perturbative series in the region $\alpha_s L \sim 1$, with e.g. $L \equiv \ln(M/p_{\mathrm{t,veto}})$

$$\sigma_{0-\mathrm{jet}} \sim \sigma_0\, e^{\underbrace{Lg_1(\alpha_s L)}_{\mathrm{LL}}} \times \left(\underbrace{G_2(\alpha_s L)}_{\mathrm{NLL}} + \underbrace{\alpha_s G_3(\alpha_s L)}_{\mathrm{NNLL}} + \ldots \right)$$

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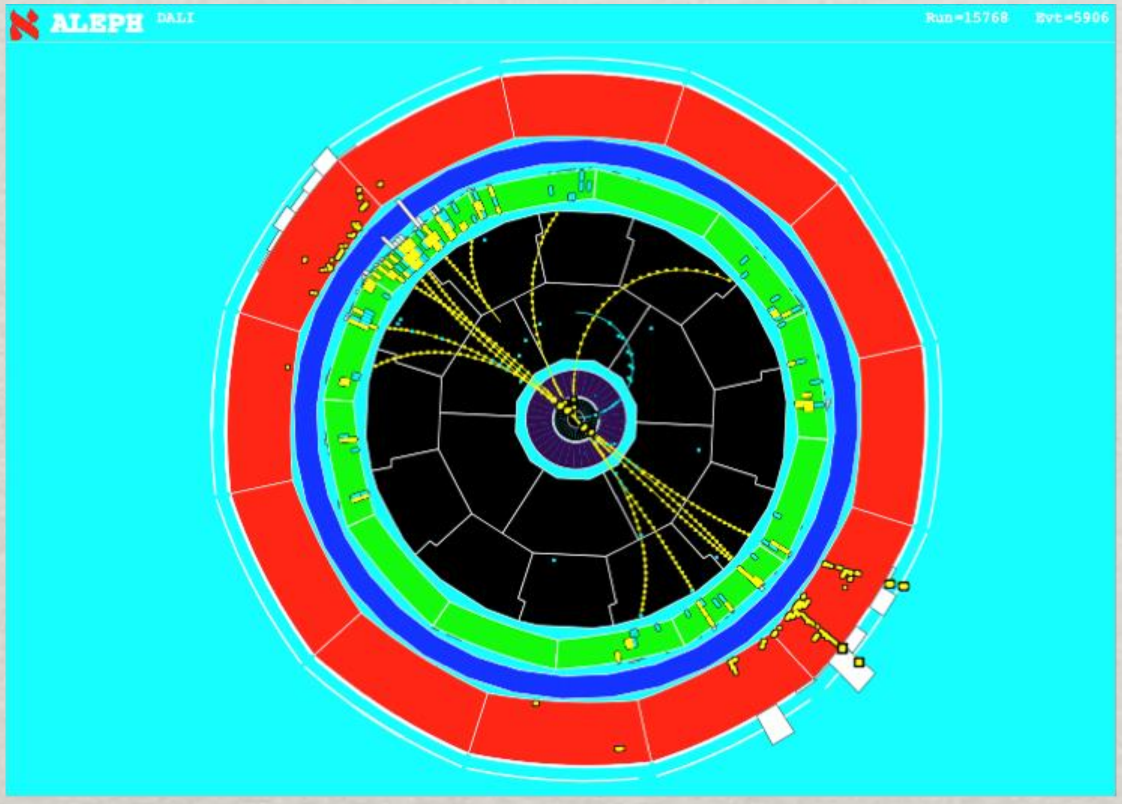
Final-state observables

- We consider a generic IRC safe final-state observable, a function $V(p_1, \ldots, p_n)$ of all final-state momenta $p_1, \ldots, p_n$

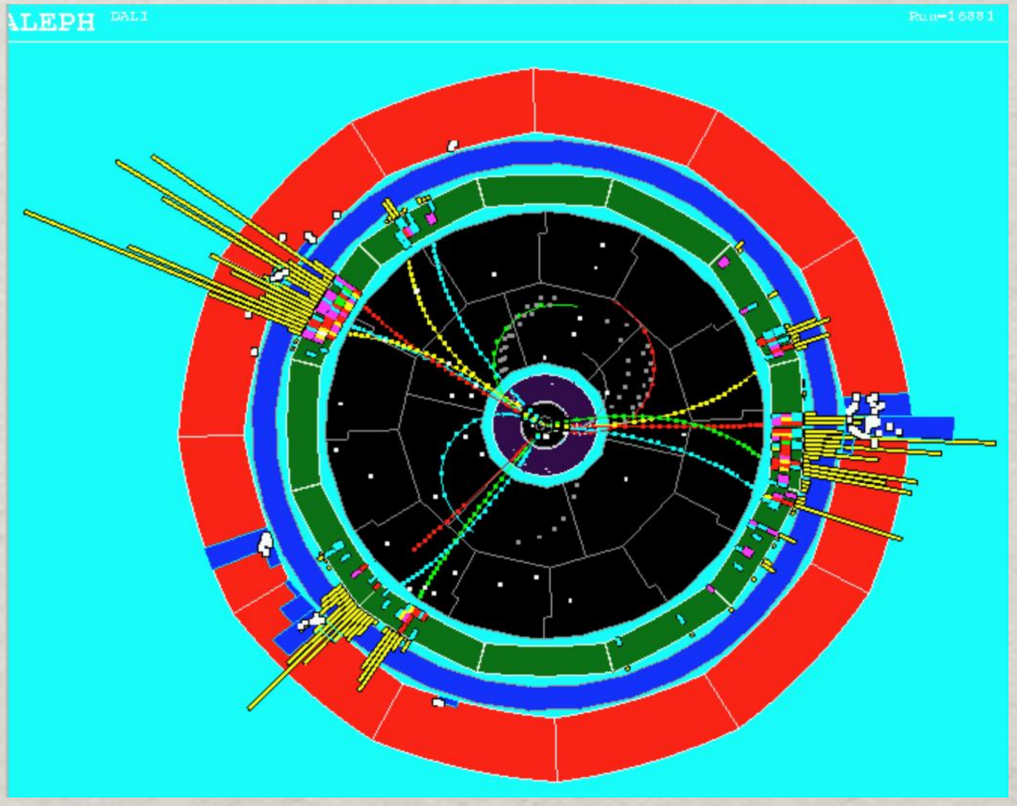
- Example: leading jet transverse momentum in Higgs production or thrust in $e^+e^- \to \mathrm{hadrons}$

$$\frac{p_{\mathrm{t,max}}}{m_H} = \max_{j \in \mathrm{jets}} \frac{p_{t,j}}{m_H} \qquad\qquad T \equiv \max_{\vec{n}} \frac{\sum_i |\vec{p}_i \cdot \vec{n}|}{\sum_i |\vec{p}_i|}$$

Pencil-like events $T \lesssim 1$



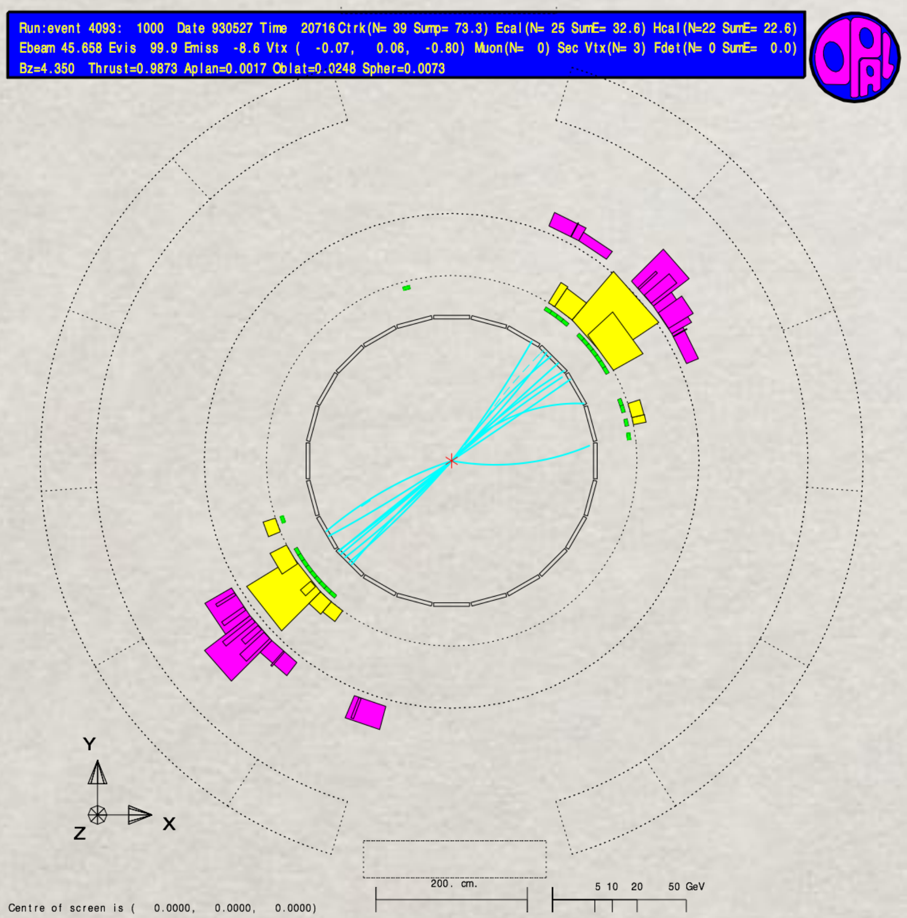
Planar events $T \gtrsim 2/3$



DEPARTURE FROM THE BORN LIMIT

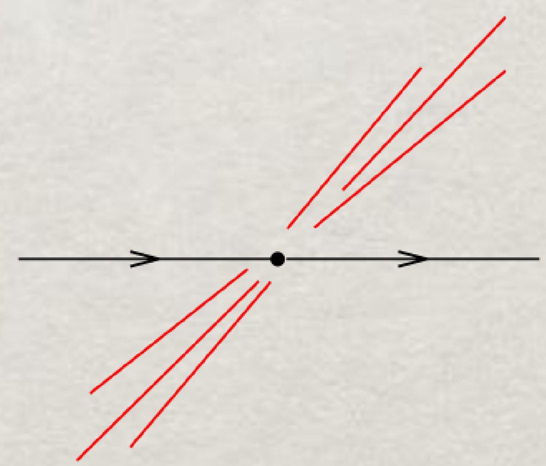
- Final-state observables have the property that, for configurations close to the Born limit (e.g. a back-to-back $q\bar{q}$ pair), their value is close to zero

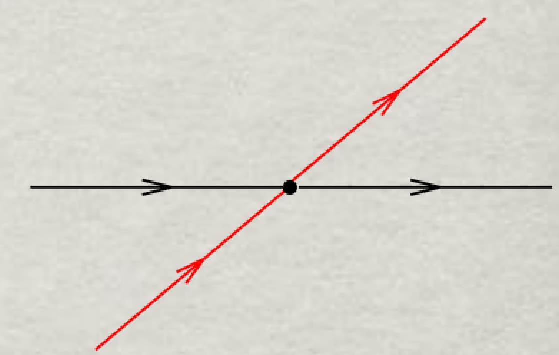
- Example: in two-jet events, one minus the thrust is the sum of the invariant masses of the two jets, which vanishes in the Born limit



$$\Sigma(v) = \mathrm{Prob}[V(p_1, \ldots, p_n) < v]$$

$$v \ll 1$$

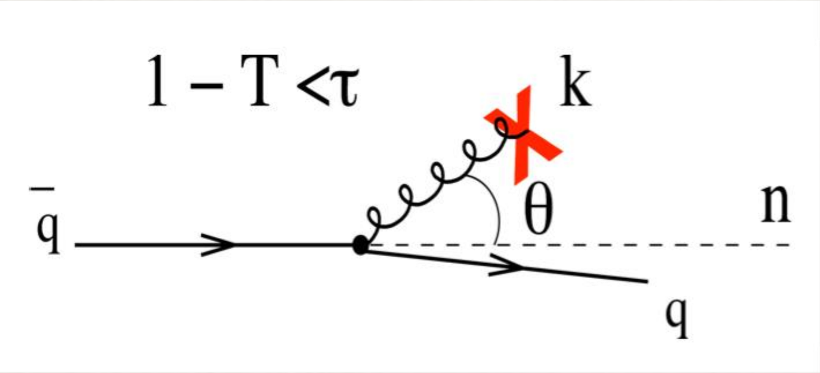




- To quantify the departure from the Born limit, we consider $\Sigma(v)$, the fraction of events such that $V(p_1, \ldots, p_n) < v$

The Lund plane

- Soft-collinear emissions can be visualised as points in the Lund plane

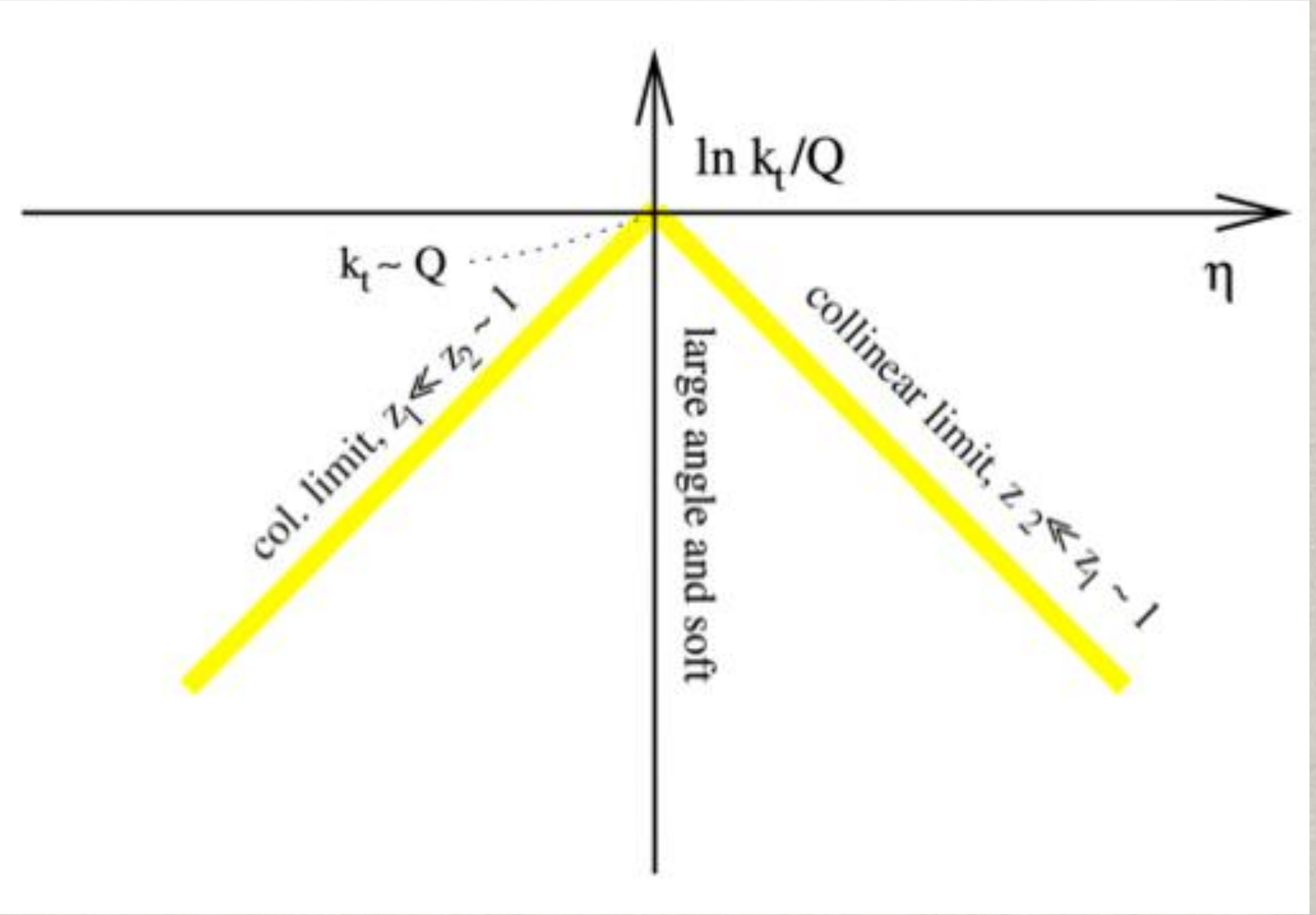


Sudakov decomposition

$$P_1 = \frac{Q}{2}(1, \vec{n}) \qquad P_2 = \frac{Q}{2}(1, -\vec{n})$$

$$k = z^{(1)} P_1 + z^{(2)} P_2 + k_t$$

$$\eta = \frac{1}{2} \ln \left(\frac{z^{(1)}}{z^{(2)}} \right) \simeq \ln \frac{1}{\theta} \qquad z_1 \gg z_2$$



- Collinear limit

$$z_1, z_2 < 1 \Rightarrow |\eta| < \ln\left(\frac{Q}{k_t}\right)$$

- Soft-collinear matrix element

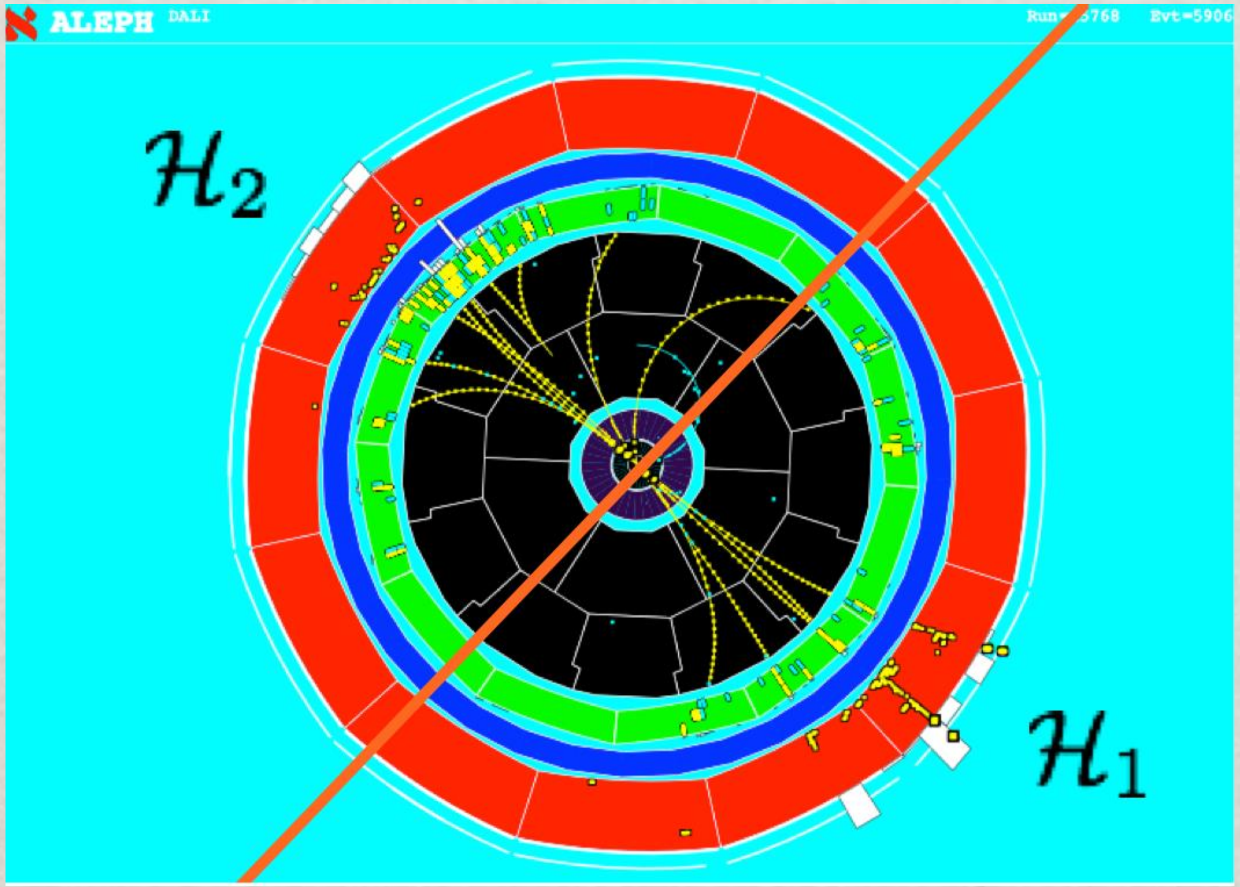
$$[dk] M^2(k) \simeq 2 C_F \frac{\alpha_s}{\pi} \frac{dk_t}{k_t} d\eta \frac{d\phi}{2\pi}$$

The thrust in the Lund plane

- Behaviour of the thrust in the soft-collinear limit

recoiling $q\bar{q}$pair

$$1 - T(\{\tilde{p}\}, k_1, \ldots, k_n) \simeq \sum_i \frac{k_{ti}}{Q} e^{-|\eta_i|} + \sum_{\ell=1,2} \frac{1}{Q^2} \frac{\left|\sum_{i\in\mathcal{H}_\ell} \vec{k}_{ti}\right|^2}{1 - \sum_{i\in\mathcal{H}_\ell} z_i^{(\ell_i)}}$$



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- Soft and collinear

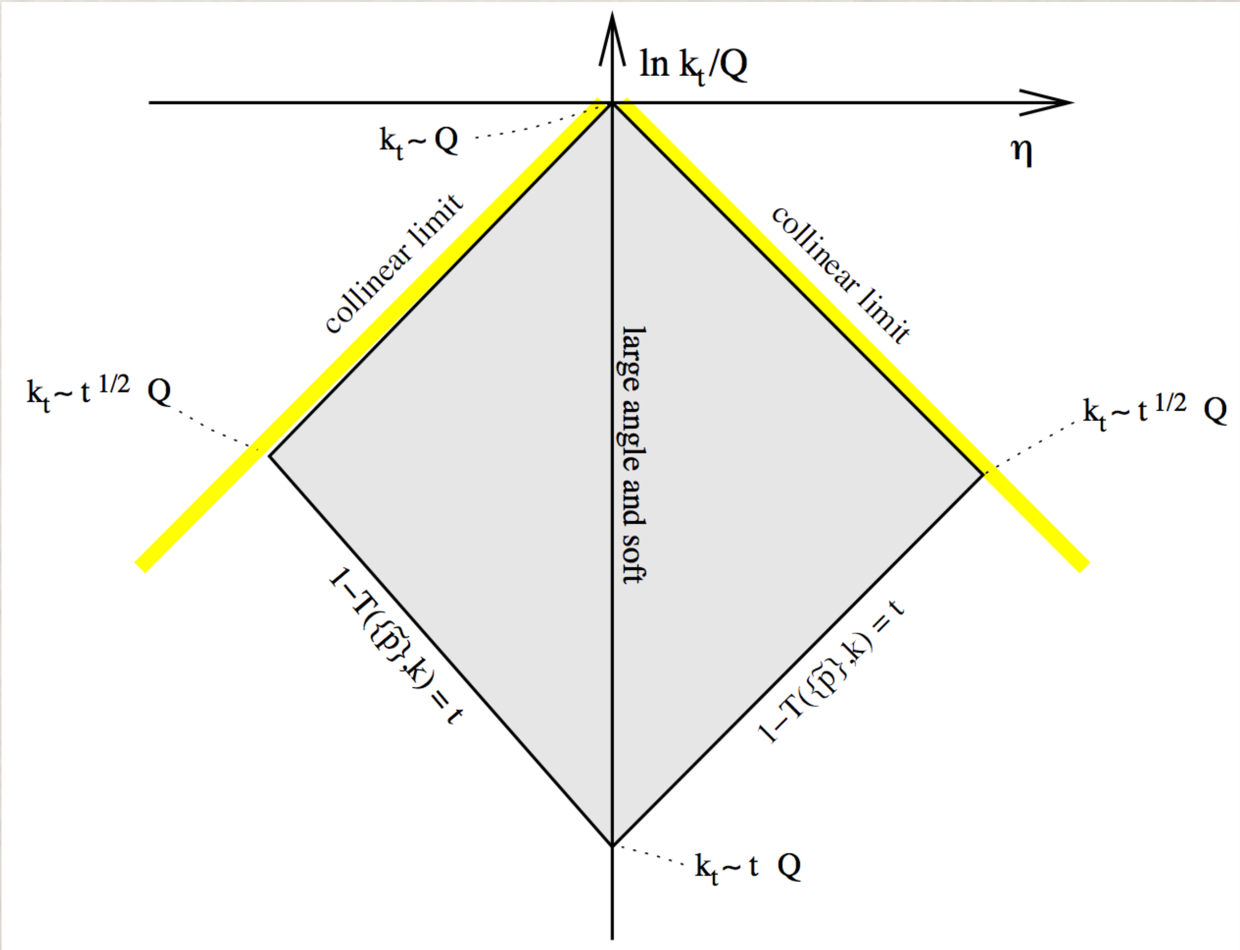
$$1 - T(\{\tilde{p}\}, k) \simeq \frac{k_t}{Q} e^{-|\eta|}$$

- Soft and large angle

$$1 - T(\{\tilde{p}\}, k) \sim k_t$$

- Hard and collinear

$$1 - T(\{\tilde{p}\}, k) \sim k_t^2$$



An observable in the Lund plane

- Behaviour of an IRC safe observable in the soft-collinear limits

- Soft and collinear to leg $\ell = 1, 2$

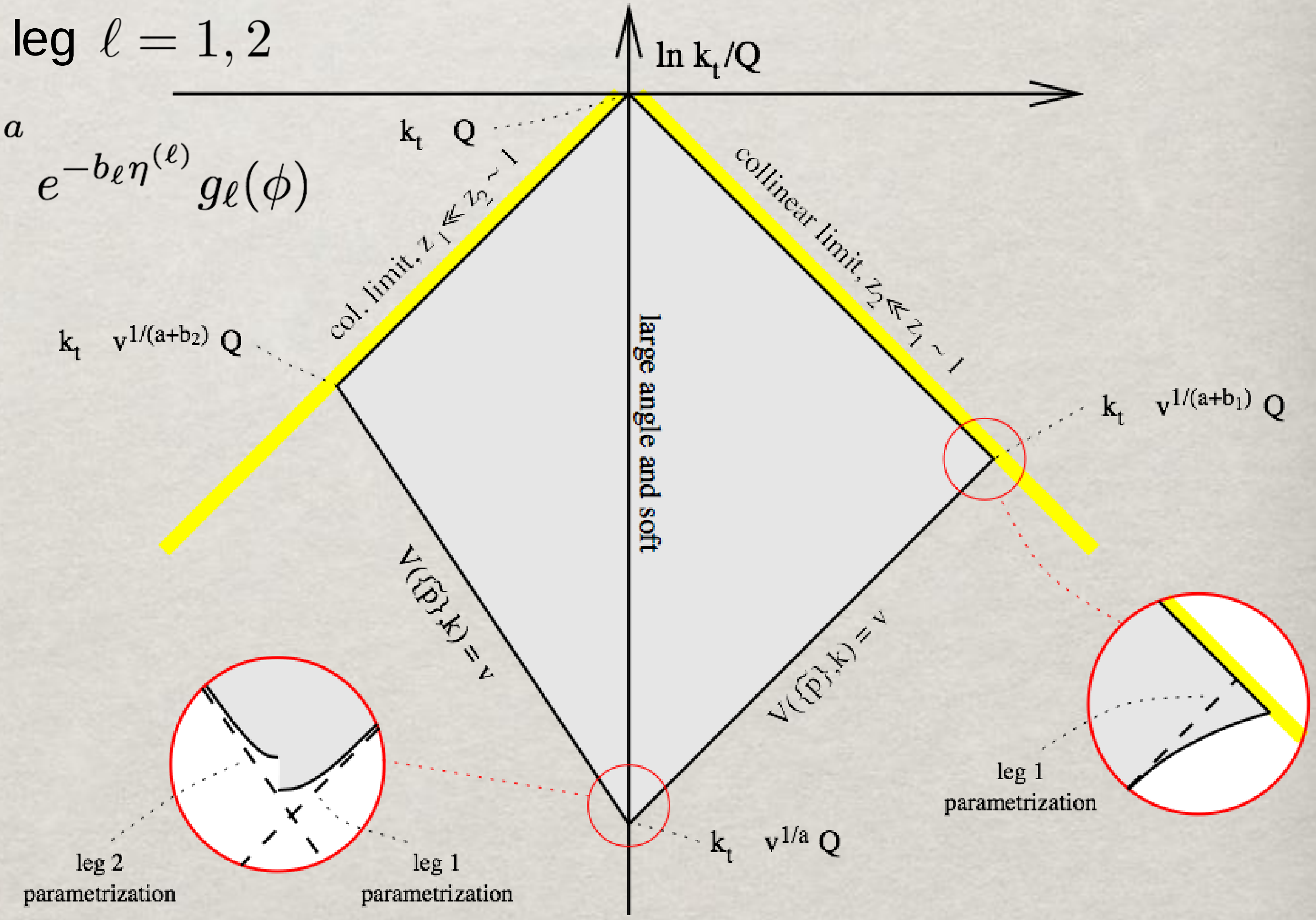
$$V(\{\tilde{p}\}, k) \simeq d_\ell \left(\frac{k_t}{Q}\right)^a e^{-b_\ell \eta^{(\ell)}} g_\ell(\phi)$$

- Soft and large angle

$$V(\{\tilde{p}\}, k) \sim k_t^a$$

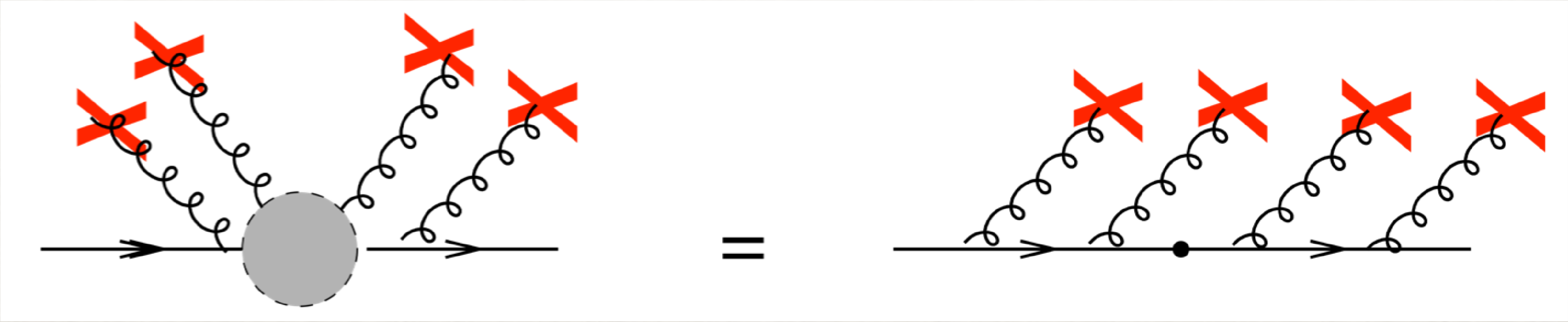
- Hard and collinear

$$V(\{\tilde{p}\}, k) \sim k_t^{a+b_\ell}$$



Multiple soft-collinear emissions

- We first consider an ensemble of soft-collinear emissions widely separated in angle (rapidity)
- Due to QCD coherence, the multi-gluon matrix element factorises into the product of single-emission matrix elements



- Contribution of multiple soft-collinear emissions to $\Sigma(v)$

$$\Sigma(v) = e^{-\int [dk] M^2(k)} \sum_{n=0}^{\infty} \frac{1}{n!} \int \prod_i [dk_i] M^2(k_i)\, \Theta(v - V(\{\tilde{p}\}, k_1, \ldots, k_n))$$

virtual corrections, ensure that the inclusive sum over all emissions gives one

Sudakov form factor

- Strategy: split the exponent in two parts

$$\int [dk] M^2(k) = \int_v [dk] M^2(k) + \int^v [dk] M^2(k) \qquad \int_v [dk] M^2(k) \equiv R(v)$$

$$\Sigma(v) = e^{-R(v)} \left\{ e^{-\int^v [dk] M^2(k)} \sum_{n=0}^{\infty} \frac{1}{n!} \int \prod_i [dk_i] M^2(k_i)\, \Theta(v - V(\{\tilde{p}\}, k_1, \ldots, k_n)) \right\}$$

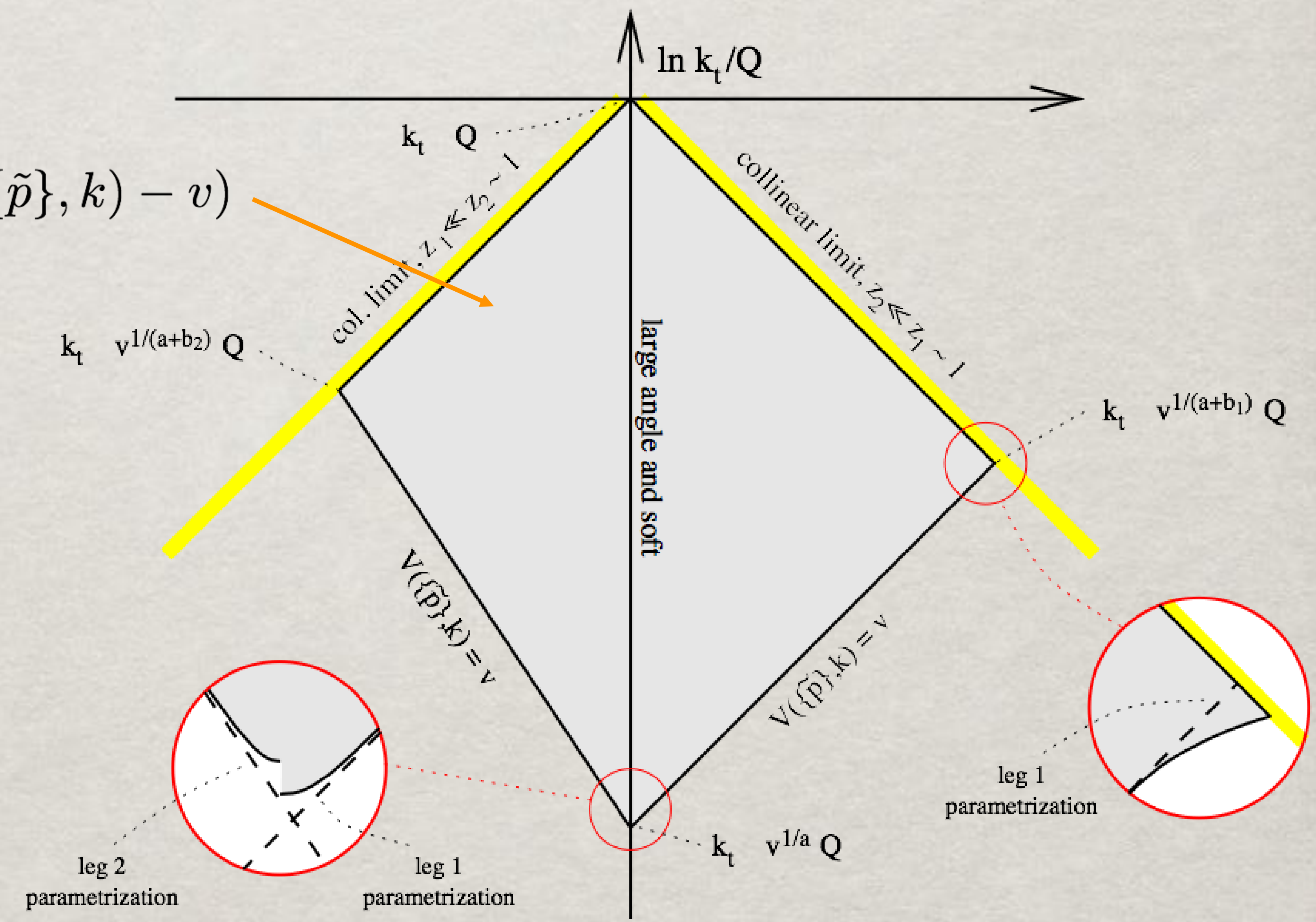
Sudakov form factor

multiple-emission correction

Double logarithmic radiator

- The Sudakov exponent, a.k.a. as "radiator", is just the area of the shaded region in the Lund plane

$$R(v) = \int [dk] M^2(k)\, \Theta(V(\{\tilde{p}\}, k) - v)$$



- Since it is an area in the Lund plane, its contribution is double logarithmic

Perfect exponentiation

- Consider an observable that takes contribution only from the emission for which $V(\{\tilde{p}\},k)$ is the largest

$$V(\{\tilde{p}\},k_1,\ldots,k_n) = \max_i V(\{\tilde{p}\},k_i)$$, e.g. $$\frac{p_{\mathrm{t,max}}}{m_H} = \max_{j\in\mathrm{jets}} \frac{p_{t,j}}{m_H}$$

$$\Theta\left(v - V(\{\tilde{p}\},k_1,\ldots,k_n)\right) = \prod_{i=1}^{n} \Theta\left(v - V(\{\tilde{p}\},k_i\right)$$

$$\Sigma(v) = e^{-R(v)} \left\{ e^{-\int^v [dk] M^2(k)} \underbrace{\sum_{n=0}^{\infty} \frac{1}{n!} \int \prod_{i=1}^{n} [dk_i] M^2(k_i) \Theta\left(v - V(\{\tilde{p}\},k_i)\right)}_{=e^{\int^v [dk] M^2(k)}} \right\} = e^{-R(v)}$$

- The cumulative distribution for such observables is a Sudakov form factor

- Interpretation of the Sudakov form factor: probability that all emissions have $V(\{\tilde{p}\},k_i) < v$

ADDITIVE OBSERVABLES

- Consider an observable that, in the soft and collinear limit, is the sum of the contributions of individual emissions

$$V(\{\tilde{p}\}, k_1, \dots, k_n) = \sum_i V(\{\tilde{p}\}, k_i) \quad \text{, e.g.} \quad 1 - T(\{\tilde{p}\}, k_1, \dots, k_n) \simeq \sum_i \frac{k_{ti}}{Q} e^{-|\eta_i|}$$

$$\Theta\left(v - V(\{\tilde{p}\}, k_1, \dots, k_n)\right) = \Theta\left(\cancel{v} - \sum_{i=1}^{n} \underbrace{V(\{\tilde{p}\}, k_i)}_{\equiv \cancel{v}\zeta_i}\right) = \Theta\left(1 - \sum_{i=1}^{n} \zeta_i\right)$$

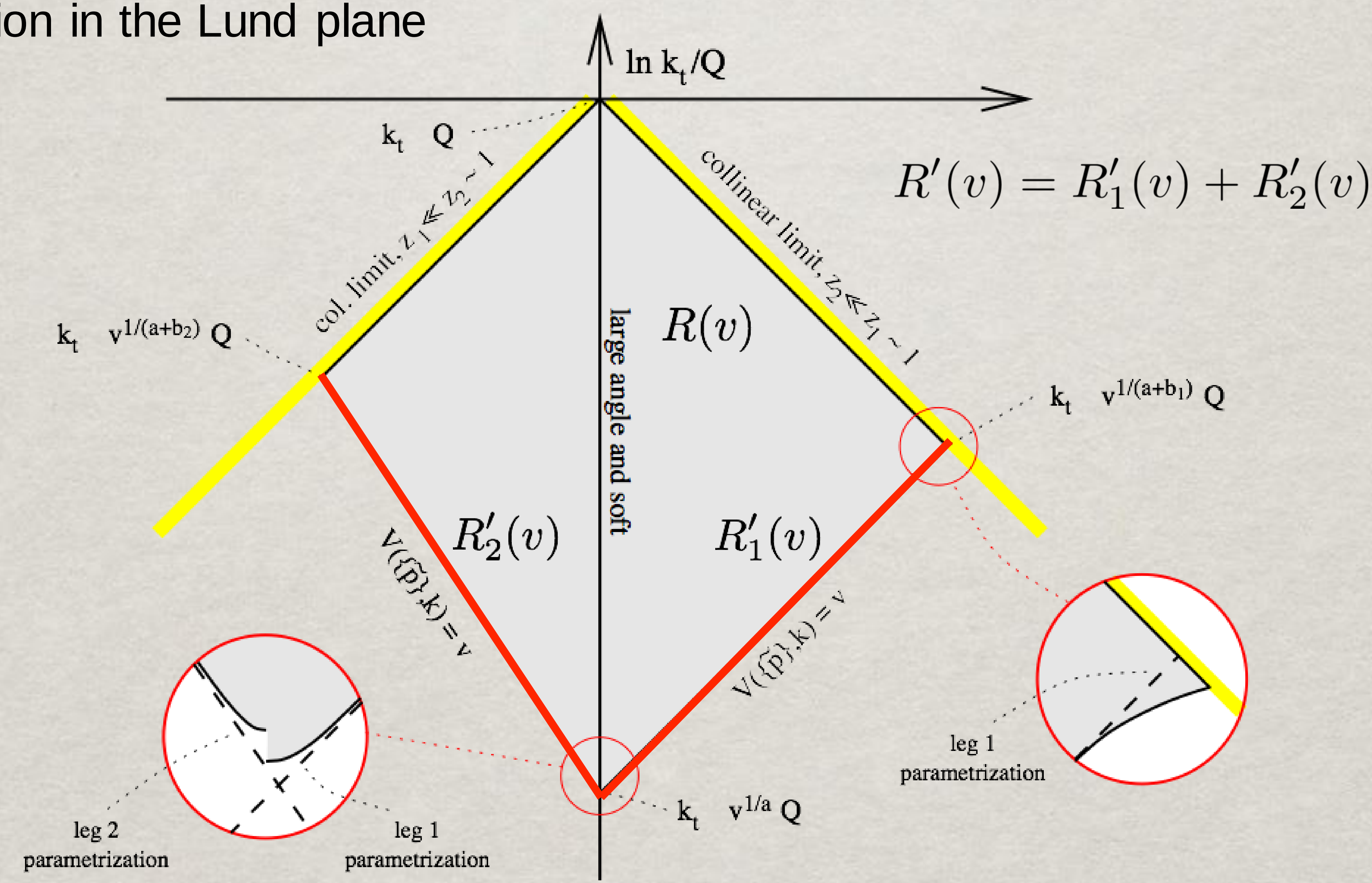
- Change of variable

$$R(v) \simeq \int_v^\infty [dk] M^2(k) = \int_0^\infty \frac{d\zeta}{\zeta} R'(\zeta v) \Theta(\zeta - 1) \quad \Rightarrow \quad [dk] M^2(k) \to \frac{d\zeta}{\zeta} R'(\zeta v)$$

- The function $R'(v) = -v \dfrac{dR}{dv} \sim \alpha_s L$ is single-logarithmic

Single logarithmic functions

- The logarithmic derivative of the radiator is the boundary of the shaded region in the Lund plane



- Since it is a line in the Lund plane, its contribution is single logarithmic

ADDITIVE OBSERVABLES

- Consider an observable that, in the soft and collinear limit, is the sum of the contributions of individual emissions

$$V(\{\tilde{p}\},k_1,\ldots,k_n)=\sum_i V(\{\tilde{p}\},k_i)$$, e.g. $$1-T(\{\tilde{p}\},k_1,\ldots,k_n)\simeq\sum_i \frac{k_{ti}}{Q}e^{-|\eta_i|}$$

- Multiple-emission correction

$$\mathcal{F}_{\rm sc}(v)\equiv e^{-\int_\epsilon^1 \frac{d\zeta}{\zeta}R'(\zeta v)}\sum_{n=0}^{\infty}\frac{1}{n!}\int\prod_{i=1}^{n}\int_\epsilon^\infty\frac{d\zeta_i}{\zeta_i}R'(\zeta_i v)\Theta\left(1-\sum_{i=1}^{n}\zeta_i\right)$$

cutoff

- Integral over ζ_i is finite $\Rightarrow \zeta_i\sim 1\Rightarrow R'(\zeta v)\simeq R'(v)+\mathcal{O}(\alpha_s)$

$$\mathcal{F}_{\rm sc}(v)\simeq\epsilon^{R'}\sum_{n=0}^{\infty}\frac{(R')^n}{n!}\int\prod_{i=1}^{n}\int_\epsilon^\infty\frac{d\zeta_i}{\zeta_i}\Theta\left(1-\sum_{i=1}^{n}\zeta_i\right)=\frac{e^{-\gamma_E R'}}{\Gamma(1+R')}$$

NLL function

Recursive IRC safety condition 1

- The first crucial property that ensures that $\mathcal{F}_{\rm sc}(v)$ does not give rise to double logarithms is that $V(\{\tilde{p}\},k_1,\dots,k_n)\sim v$ whenever $V(\{\tilde{p}\},k_i)\sim v$

$$\lim_{v\to 0}\frac{V(\{\tilde{p}\},k_1,\dots,k_n)}{v}=\sum_{i=1}^{n}\zeta_i \quad \text{(finite and non-zero)}$$

- In general, for each soft emission collinear to leg ℓ, we can perform the change of variables

$$V(\{\tilde{p}\},k)=\zeta v \qquad \eta^{(\ell)}=\xi^{(\ell)}\,\eta^{(\ell)}_{\rm max} \qquad \eta^{(\ell)}_{\rm max}\sim\ln\frac{1}{v}$$

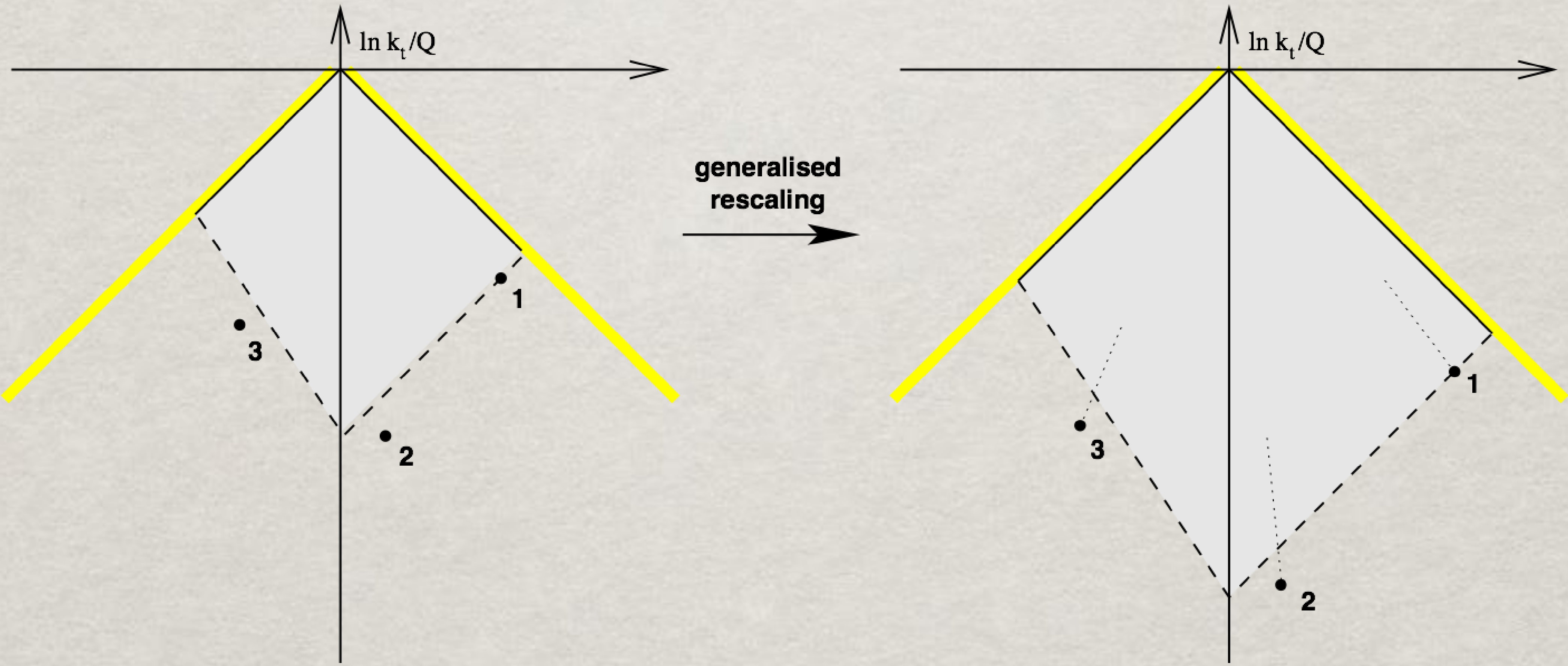
- For a general observable, for fixed $\zeta_i,\ell_i,\xi^{(\ell_i)},\phi^{(\ell_i)}$, we must have

$$\lim_{v\to 0}\frac{V(\{\tilde{p}\},k_1,\dots,k_n)}{v}=f_V(\{\zeta_1,\ell_1,\xi^{(\ell_1)},\phi^{(\ell_1)}\},\dots,\{\zeta_n,\ell_n,\xi^{(\ell_n)},\phi^{(\ell_n)}\})$$

Recursive IRC safety condition 1

- The requirement that the observable scales in the same way irrespectively of the number of emission is formalised as follows

$$\lim_{v\to 0} \frac{V(\{\tilde{p}\}, k_1, \ldots, k_n)}{v} = \text{finite and non-zero}$$



- This is the first of the requirements known as “recursive” IRC safety
- rIRC safe observables are the only ones that can be resummed so far

Recursive IRC safety condition 2a

- The second crucial property that ensures that $\mathcal{F}_{\rm sc}(v)$ does not give double logarithms is that the integral over ζ_i is finite
- This means that we can neglect all emissions with $V(\{\tilde{p}\}, k_i) < \epsilon v$, with the cutoff $\epsilon \gg v$, independent of v

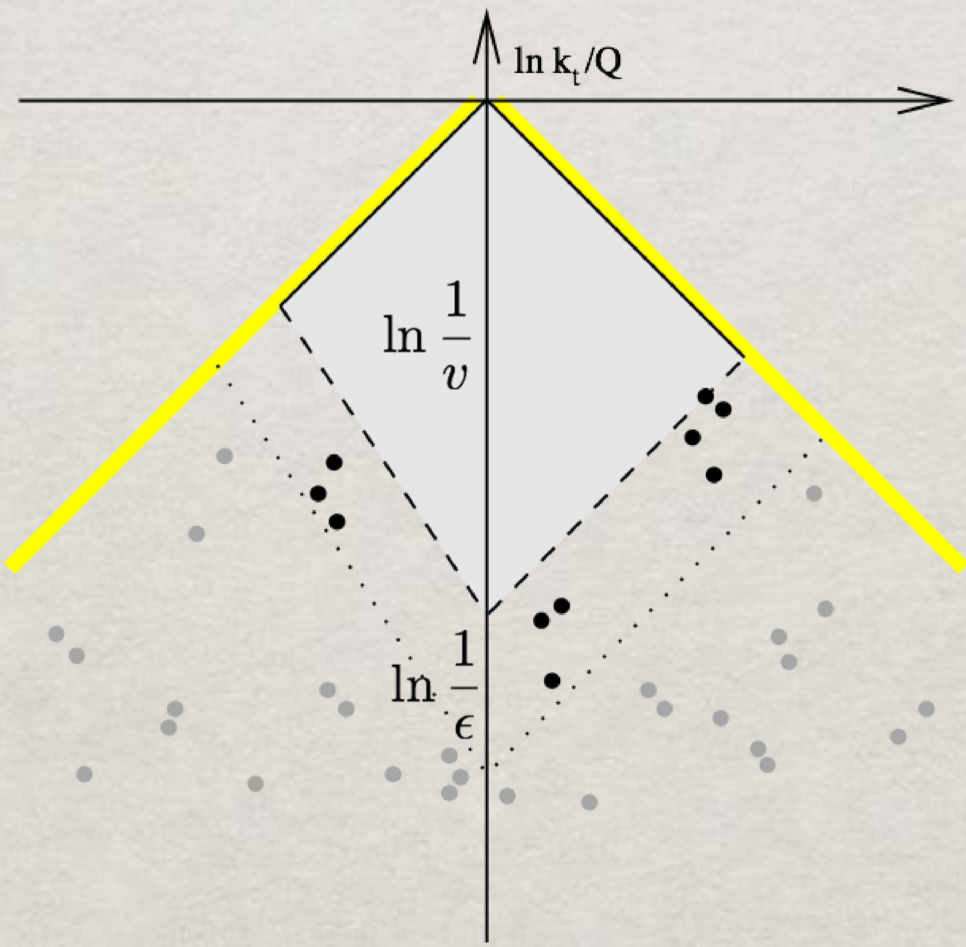
$$[dk]M^2(k) \simeq \sum_\ell \underbrace{R'_\ell(v)}_{\sim \alpha_s L} \frac{d\zeta}{\zeta} d\xi^{(\ell)} \frac{d\phi}{2\pi} \qquad R' = \sum_\ell R'_\ell = -v\frac{dR}{dv}$$

$$\mathcal{F}_{\rm sc}(v) \simeq \epsilon^{R'} \sum_{n=0}^{\infty} \frac{1}{n!} \int \prod_{i=1}^{n} \left(\int_\epsilon^\infty \frac{d\zeta_i}{\zeta_i} \sum_{\ell_i} R'_{\ell_i} \int_0^1 d\xi^{(\ell_i)} \int_0^{2\pi} \frac{d\phi_i^{(\ell_i)}}{2\pi} \right) \times$$
$$\times\, \Theta\left(1 - \lim_{v\to 0} \frac{V(\{\tilde{p}\}, k_1, \ldots, k_n)}{v}\right) = \mathcal{F}_{\rm NLL}(R')$$

Recursive IRC safety condition 2a

- The fact that we can neglect emissions with $\zeta_i < \epsilon$ is expressed formally by rIRC safety condition 2a

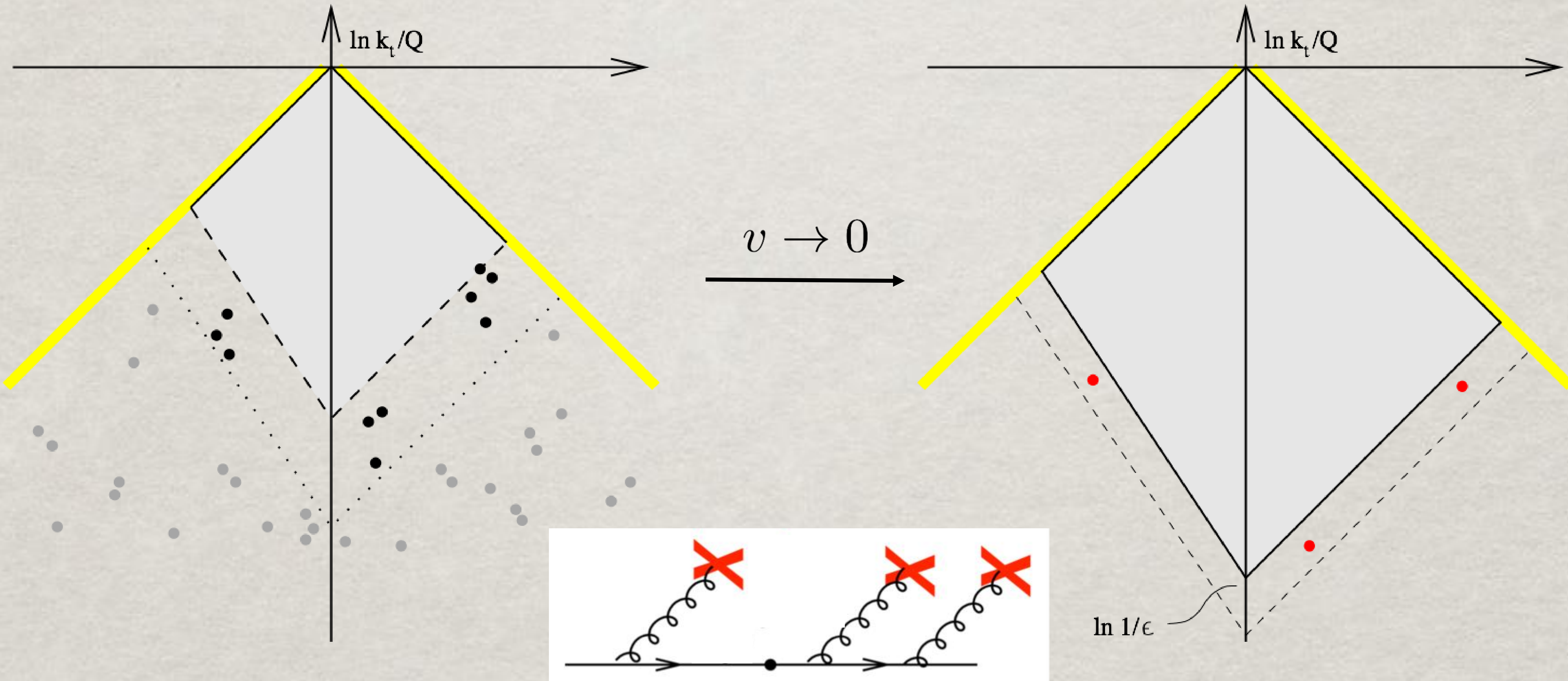
$$\lim_{\zeta_{n+1}\to 0} \lim_{v\to 0} \frac{V(\{\tilde{p}\}, k_1, \ldots, k_n, k_{n+1})}{v} = \lim_{v\to 0} \frac{V(\{\tilde{p}\}, k_1, \ldots, k_n)}{v}$$



- Note the order of the limits: the reversed limit trivially holds because the observable is IRC safe!

Recursive IRC safety condition 2b

- This condition ensures that the contribution of correlated gluon emissions, hard collinear and soft large-angle emissions is beyond NLL



- At NLL accuracy, relevant emissions are soft and collinear, widely separated in angle, and in a strip of size $\ln v \times \ln \epsilon$

- The strip is a line in the Lund plane, hence a single logarithmic contribution

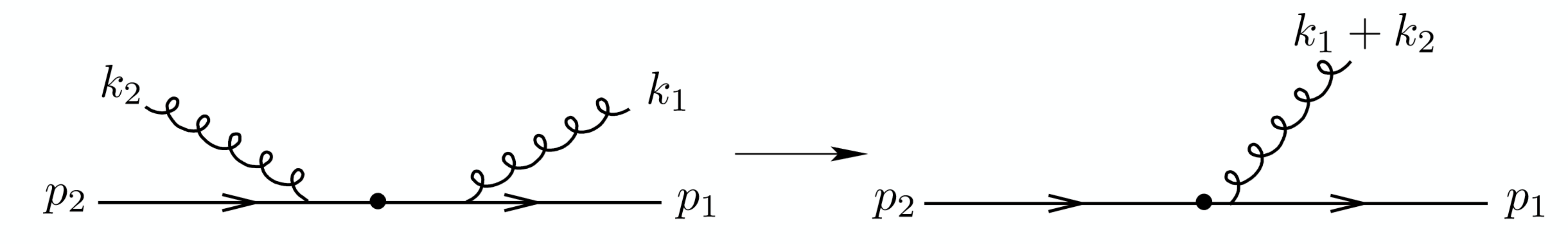
Non-exponentiating observables

- In the case of the two-jet rate in the JADE algorithm, double logarithms do not exponentiate [Brown Stirling PLB 252 (1990) 657]

$$\Sigma(y_{\rm cut}) = 1 - \frac{C_F\alpha_s}{\pi}\ln^2\left(\frac{1}{y_{\rm cut}}\right) + \frac{1}{2!}\times\frac{5}{6}\times\left(\frac{C_F\alpha_s}{\pi}\ln^2\left(\frac{1}{y_{\rm cut}}\right)\right)^2 + \dots$$

- This is due to the peculiar way JADE performs sequential recombinations

$$y_{ij} = \frac{(p_i+p_j)^2}{Q^2}$$

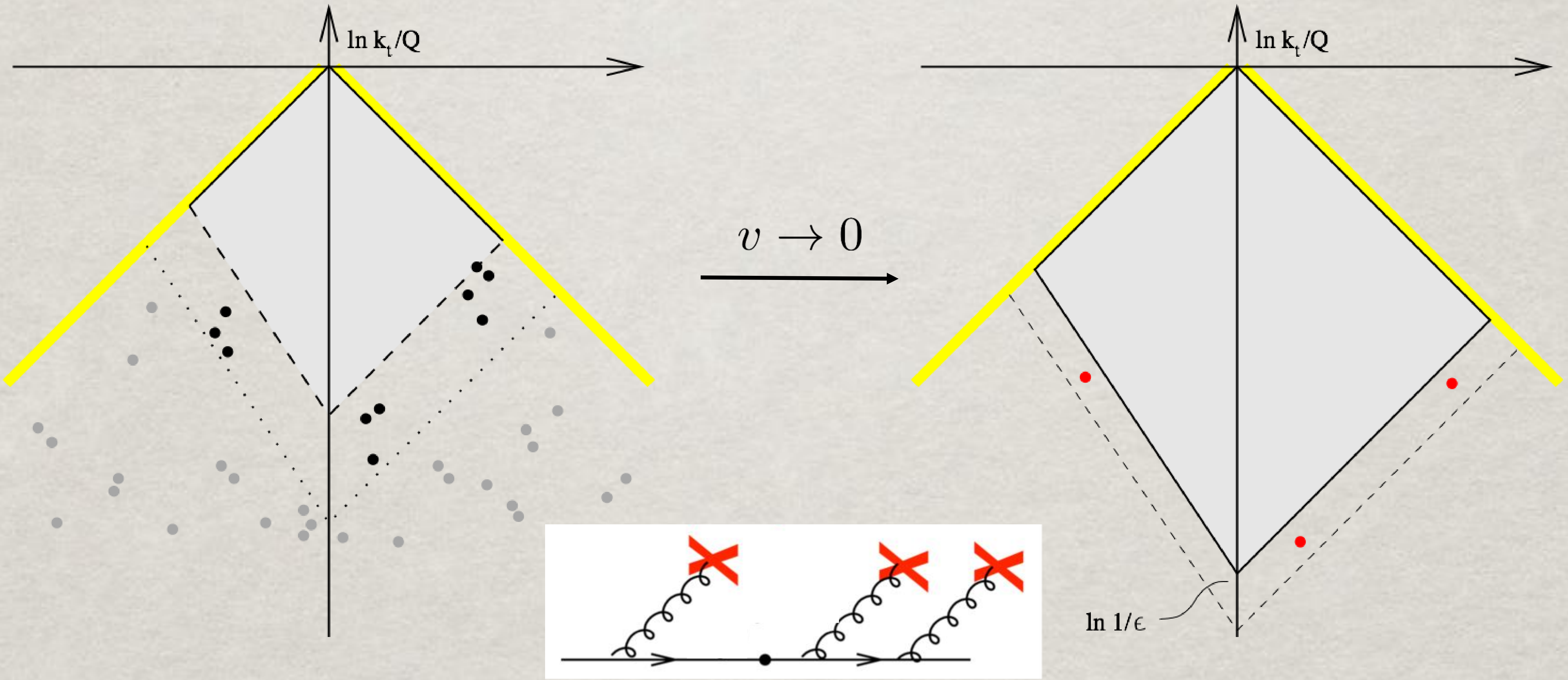


- The JADE algorithm is able to recombine together two soft emissions collinear to two different legs ⇒ violation of rIRC safety

General NLL resummation

- NLL resummation of rIRC safe observables can be performed with a universal master formula

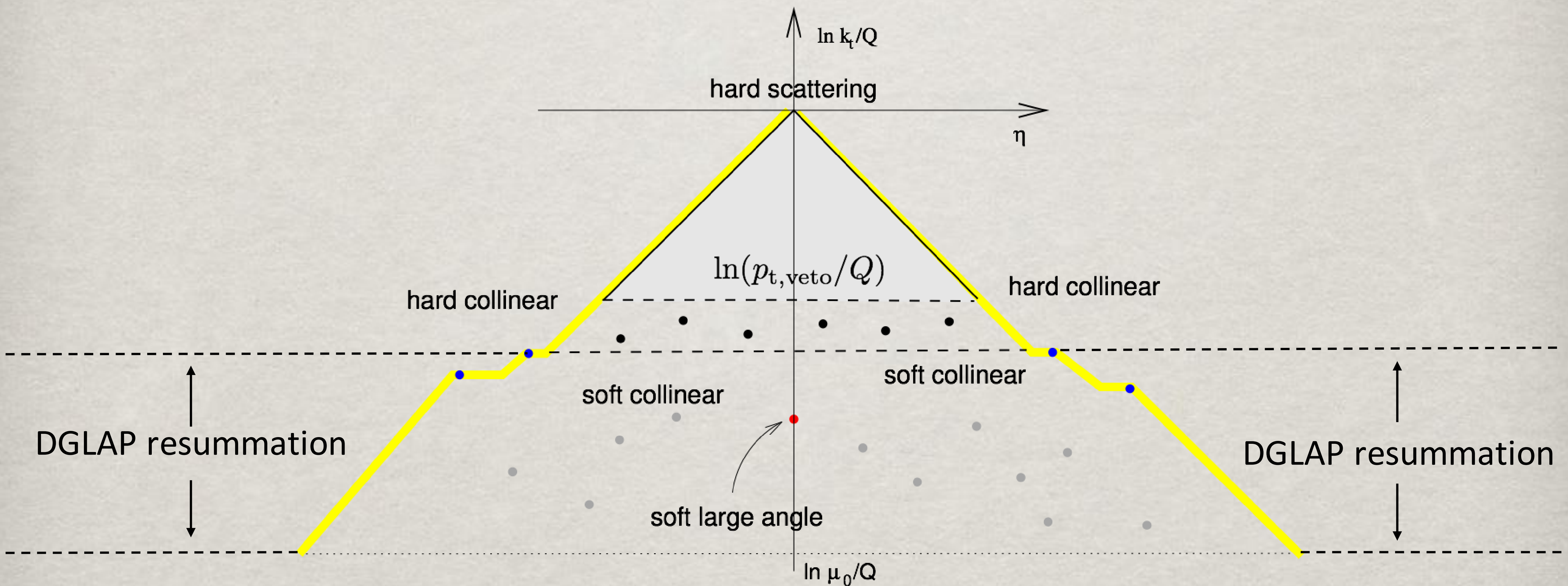
[Banfi Salam Zanderighi hep-ph/0407286]



$$\Sigma(v) = e^{-R(v)} \underbrace{\left\{ \epsilon^{R'} \sum_{n=0}^{\infty} \frac{1}{n!} \int \prod_{i=1}^{n} \left(\sum_{\ell_i} R'_{\ell_i} \int_{\epsilon}^{\infty} \frac{d\zeta_i}{\zeta_i} \int_0^1 d\xi_i^{(\ell_i)} \int_0^{2\pi} \frac{d\phi}{2\pi} \right) \Theta\left(1 - \lim_{v\to 0} \frac{V(\{\tilde{p}\}, k_1, \ldots, k_n)}{v}\right) \right\}}_{\text{single-logarithmic correction } \mathcal{F}_{\mathrm{NLL}}(R')}$$

Higgs plus zero jets at NLL

- In the presence of initial state radiation, the zero-jet cross section inclusive with respect to hard-collinear emission up to the scale $p_{\mathrm{t,veto}}$



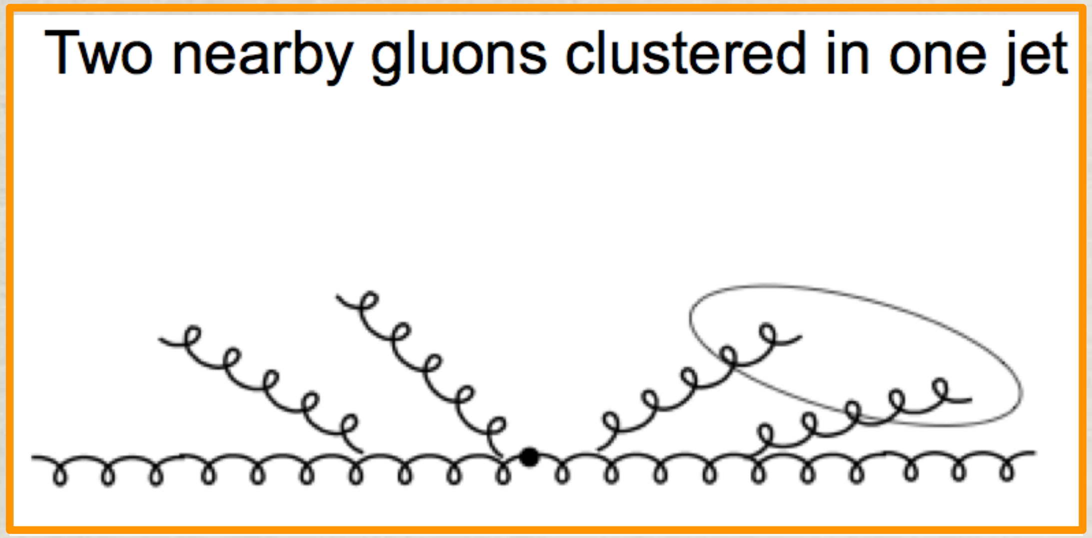
- No k_t-type algorithm can recombine gluons that are widely separated in angle $\Rightarrow$ perfectly exponentiating observable

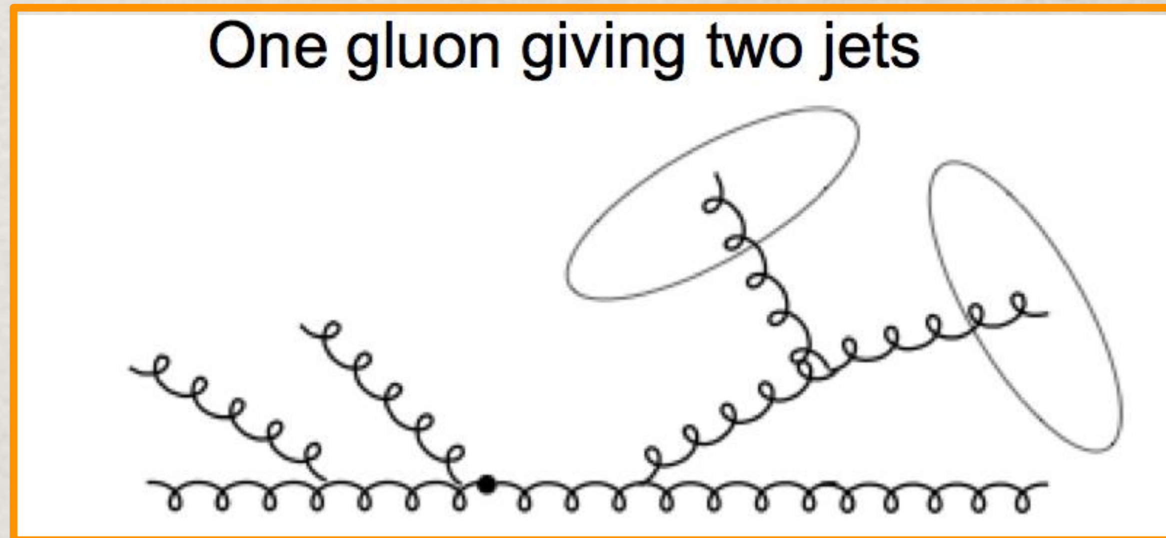
$$\sigma_{0-\mathrm{jet}} \simeq \mathcal{L}_{gg}(p_{\mathrm{t,veto}})\, e^{-R(p_{\mathrm{t,veto}})}$$

Higgs plus zero jets at NNLL

- Jet recombination effects start to matter at NNLL accuracy

[Banfi Monni Salam Zanderighi 1206.4998]





$$\sigma_{0-\mathrm{jet}} \simeq \mathcal{L}_{gg}(p_{\mathrm{t,veto}}) \left(1 + \underbrace{\alpha_s(p_{\mathrm{t,veto}})\, R'(p_{\mathrm{t,veto}})\, f(R)}_{\mathrm{NNLL}} \right) e^{-R(p_{\mathrm{t,veto}})}$$

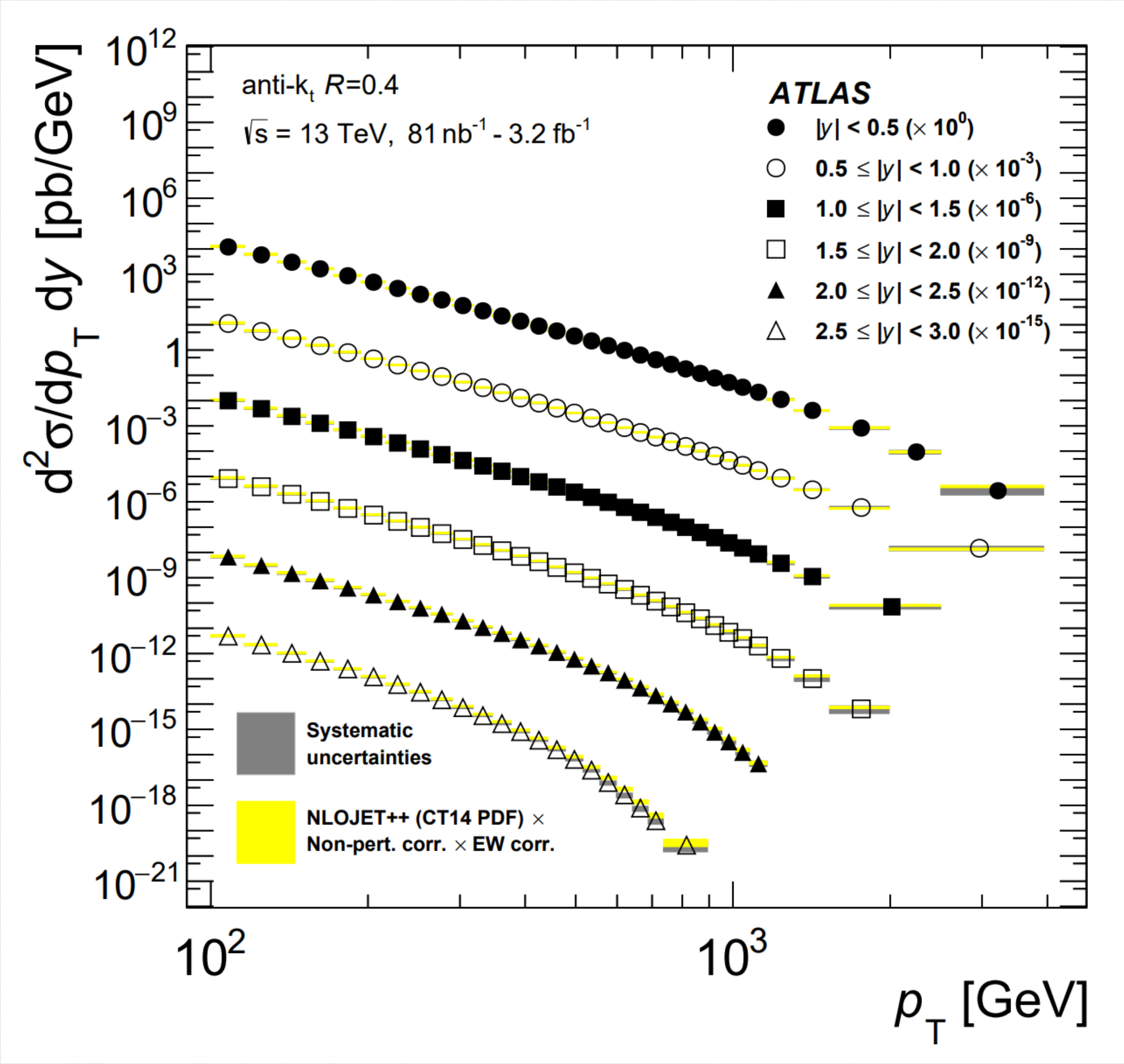
- The function $f(R) \sim \ln R$ since the jet radius provides an effective cutoff to the collinear singularity in gluon splitting. Leading logarithm of the jet radius can be also resummed at all orders

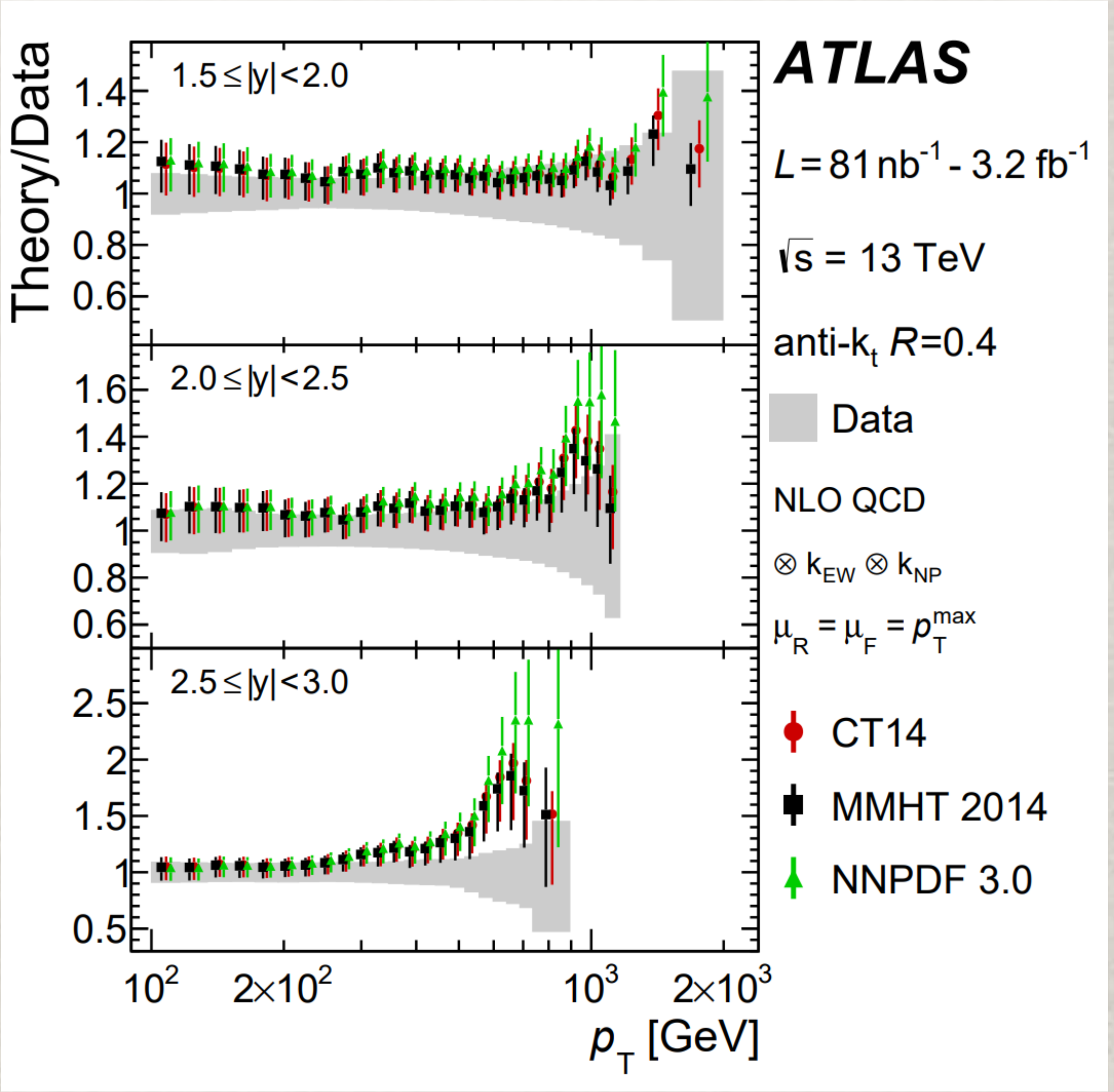
[Dasgupta Dreyer Salam Soyez 1411.5182]

Hadronisation and Underlying event

Inclusive jet p_T spectra

- The inclusive jet p_t spectra are important observables for measurements of the strong coupling as well as for fits of parton distribution functions





- Any meaningful comparison to data includes estimates of non-perturbative effects from hadronisation and underlying event

Non-perturbative shift

- Non-perturbative effects amount typically in shifts of perturbative distributions

$$p_{\mathrm{t,jet}} \simeq p_t + \epsilon \qquad \epsilon \ll p_t$$

- Suppose ϵ is distributed according to some non-perturbative distribution $f_{\mathrm{NP}}(\epsilon)$

$$\frac{d\sigma}{dp_{\mathrm{t,jet}}} = \int d\epsilon\, f_{\mathrm{NP}}(\epsilon)\, \frac{d\sigma_{\mathrm{PT}}}{dp_t} \delta(p_{\mathrm{t,jet}} - p_t - \epsilon)$$

- Since $f_{\mathrm{NP}}(\epsilon)$ peaks at values $\epsilon \ll p_t$ we can approximate

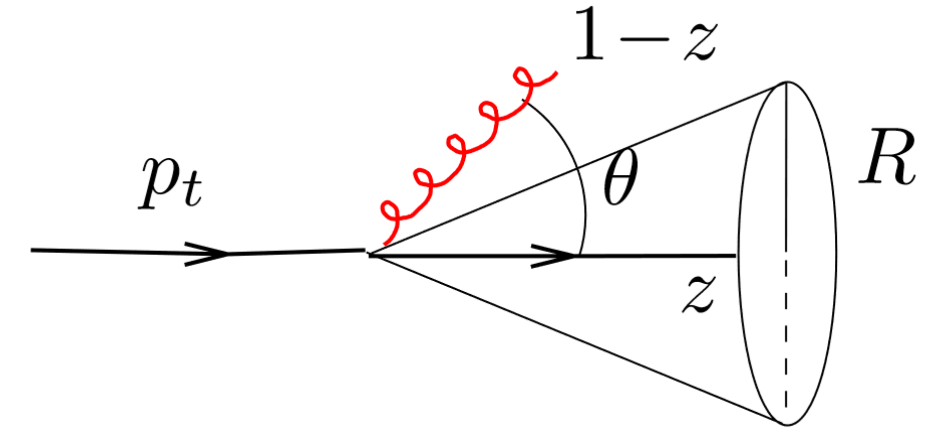
$$\frac{d\sigma}{dp_{\mathrm{t,jet}}} \simeq \int d\epsilon\, f_{\mathrm{NP}}(\epsilon)\, \frac{d\sigma_{\mathrm{PT}}(p_t)}{dp_t} \delta(p_{\mathrm{t,jet}} - p_t - \langle\epsilon\rangle) = \frac{d\sigma_{\mathrm{PT}}(p_{\mathrm{t,jet}} - \langle\epsilon\rangle)}{dp_t}$$

- As a first approximation, we need to compute the average of the difference in transverse momentum with and without NP effects

Hadronisation

- Emission of a soft gluon ($z \simeq 1$) from a hard parton in the triggered jet

[Dasgupta Magnea Salam 0712.3014]



$$dP_{q\to qg} \simeq \frac{d\theta^2}{\theta^2} dz \frac{\alpha_s[(1-z)\theta p_t]}{2\pi} \frac{2C_F}{1-z}$$

- A gluon emitted outside the jet causes a p_t loss $\delta p_t \simeq p_t - z p_t = (1-z) p_t$

$$\langle \delta p_t \rangle_q \simeq -\int_R \frac{d\theta}{\theta} dz \underbrace{p_t(1-z)}_{\delta p_t} \frac{\alpha_s[(1-z)\theta p_t]}{\pi} \frac{2C_F}{1-z}$$

- The argument of the coupling $k_t \simeq (1-z)\theta p_t$ is just the relative transverse momentum of the soft gluon with respect to the parent quark

- The coupling becomes large when the transverse momentum of the gluon is small, so we need to give a meaning to this low-energy contribution

Hadronisation

- Expanding $\alpha_s(k_t)$ around $\alpha_s(p_t) \ll 1$ leads to a factorially divergent series

$$\langle \delta p_t \rangle_q \simeq -\int_R \frac{d\theta}{\theta} dz \underbrace{p_t(1-z)}_{\delta p_t} \frac{\alpha_s[(1-z)\theta p_t]}{\pi} \frac{2C_F}{1-z} \simeq -\frac{2C_F}{R} \frac{\alpha_s(p_t)}{\pi} \sum_{n=0}^{\infty} n!(2\beta_0 \alpha_s(p_t))^n$$

- For $k_t < \mu_I \sim 1\,\mathrm{GeV}$ (pre-)hadrons are formed: their distribution in rapidity and azimuth is the same as soft gluons. We model them as gluons emitted with an effective coupling $\alpha_s^{\mathrm{NP}}(k_t)$

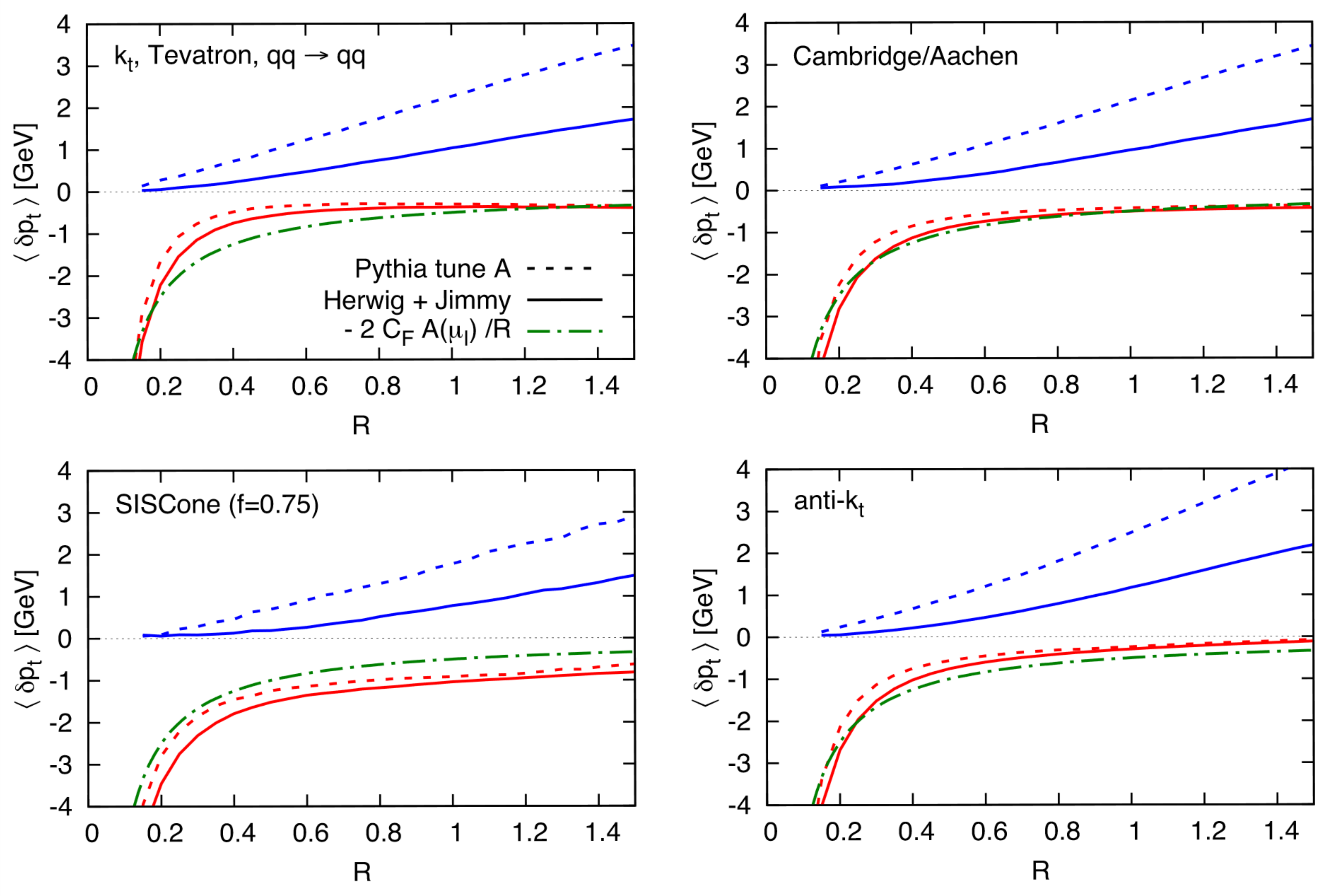
- Hadronisation corrections to IRC collinear safe observables are proportional to moments of this effective coupling, in our case

[Dasgupta Magnea Salam 0712.3014]

$$\langle \delta p_t \rangle_{q,h} \simeq -\frac{2C_F}{R} \int_0^{\mu_I} dk_t \frac{\alpha_s(k_t)}{\pi} = -\frac{2C_F}{R} \mathcal{A}(\mu_I)$$

Hadronisation on jet p_T

- If hadronisation corrections are determined only by soft radiation ($z \to 1$), the 1/R behaviour should be universal: this is confirmed by comparison with Monte Carlo event generators [Dasgupta Magnea Salam 0712.3014]



Quantifying hadronisation

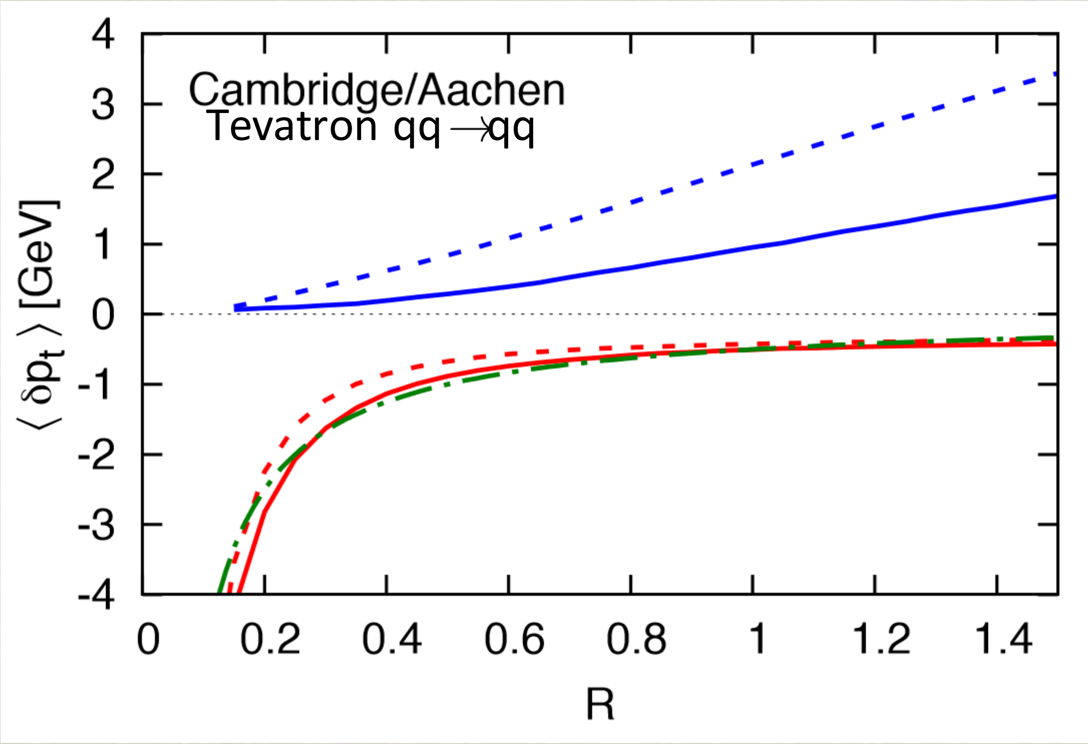
- The NP parameter $\mathcal{A}(\mu_I)$ is the same appearing in event shape distributions in e^+e^- annihilation $\Rightarrow 2C_F\mathcal{A}(2\,\mathrm{GeV}) \simeq 0.5\,\mathrm{GeV}$

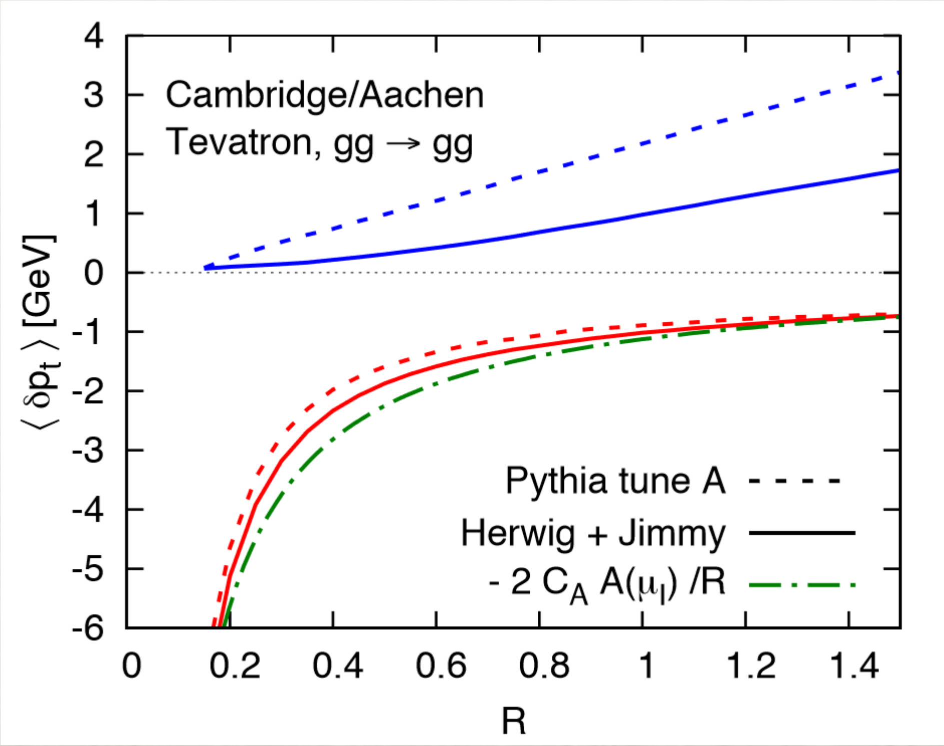
[Dasgupta Magnea Salam 0712.3014]

$$\langle \delta p_t \rangle_{q,h} \simeq -\frac{2C_F}{R}\mathcal{A}(\mu_I) \simeq -\frac{0.5\,\mathrm{GeV}}{R}$$

$$\langle \delta p_t \rangle_{g,h} \simeq -\frac{2C_A}{R}\mathcal{A}(\mu_I) \simeq 2\langle \delta p_t \rangle_{q,h}$$

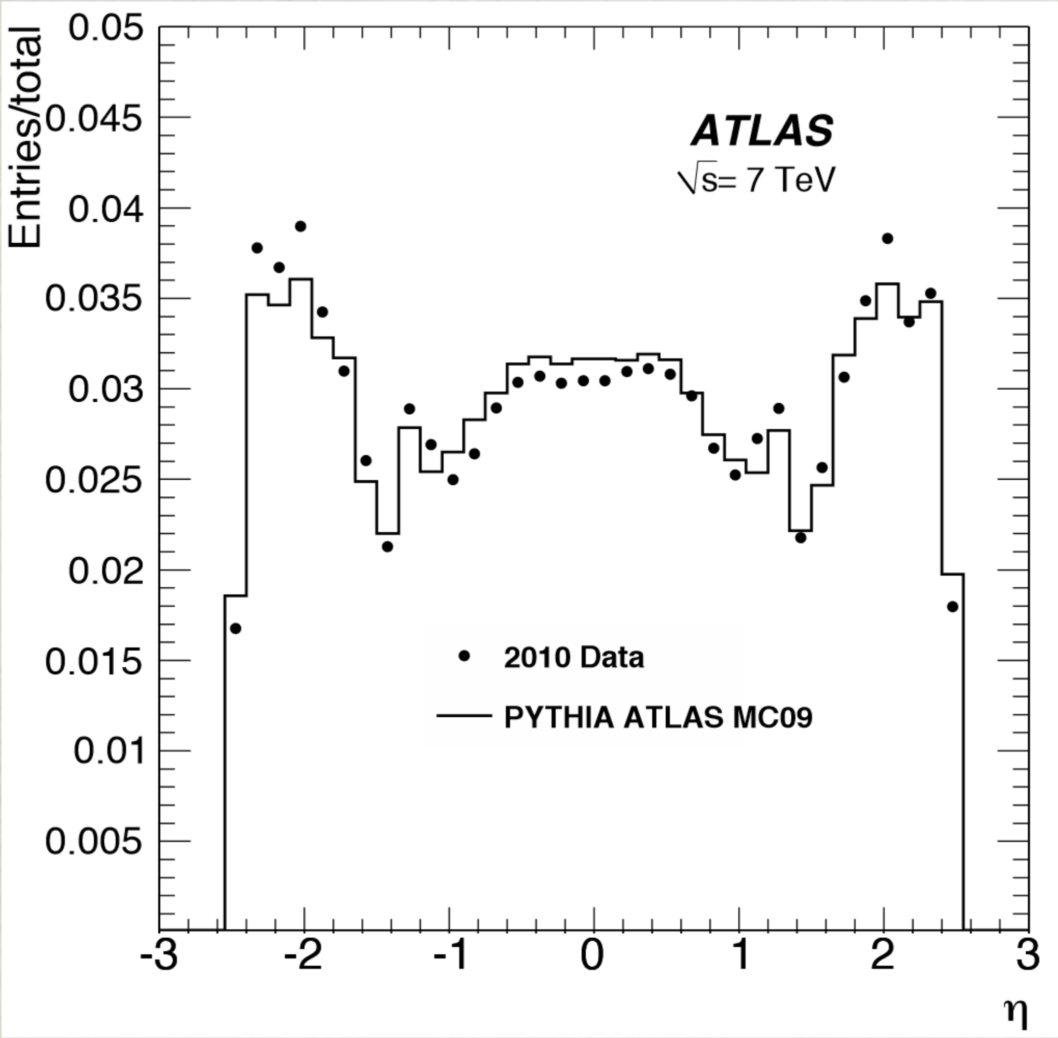
- We can then quantitatively estimate hadronisation effects on quark and gluon jets $\Rightarrow$ agreement with Monte Carlo event generators





The underlying event

- An additional source of hadrons is secondary collisions of primary hadrons (underlying event) or primary collisions of other protons in the same bunch (pile-up)



- Such hadrons can get clustered into hard jet causing distortions in the expected behaviour of jet observables

- Data itself suggest that secondary hadrons are uniformly distributed in rapidity (with respect to the beam) and azimuth

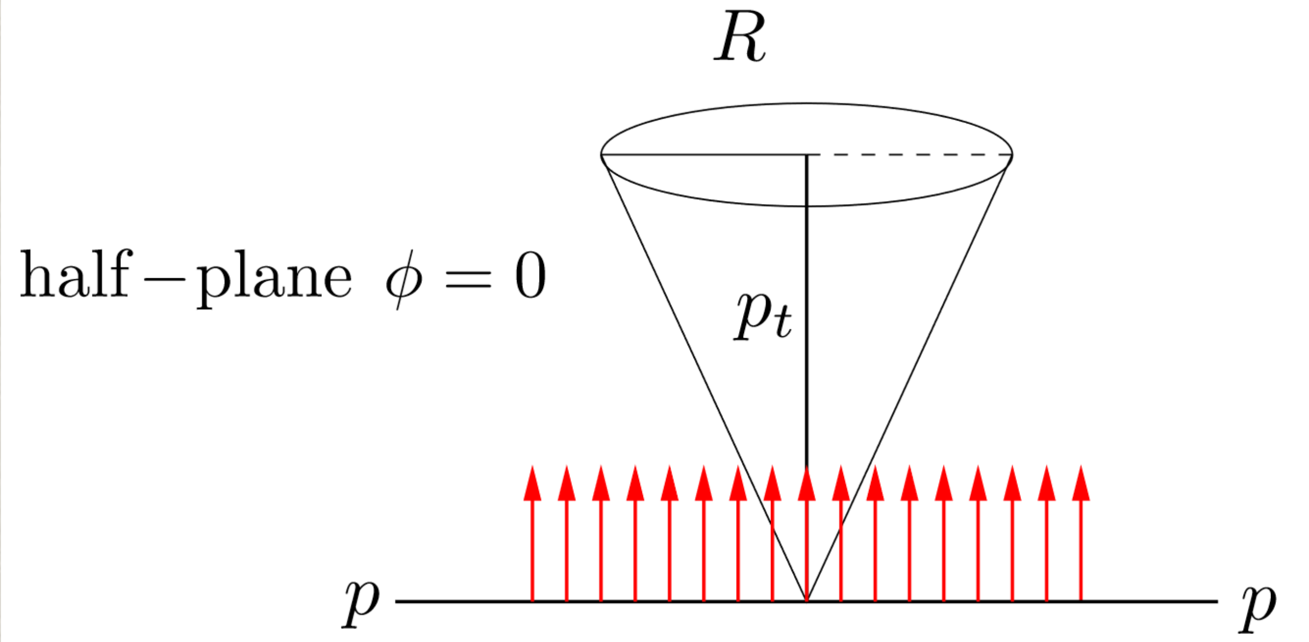
- There are various ideas on how to extract the properties of the underlying event from data

[Field hep-ph/0201192]

[Cacciari Salam Sapeta 0912.4926]

UNDERLYING EVENT AND JET P_T

- Hadrons from the UE/pile-up clustered with a jet produce an average change in the jet transverse momentum [Dasgupta Magnea Salam 0712.3014]



$$\delta p_{\mathrm{t,UE}} = |\vec{p}_t + \vec{p}_{\mathrm{t,UE}}| - p_t$$
$$\Downarrow$$
$$\delta p_{\mathrm{t,UE}} \simeq p_{\mathrm{t,UE}} \cos\phi$$

- Assuming the distribution of secondary hadrons gives ρ extra transverse momentum per unit rapidity and azimuth, we get

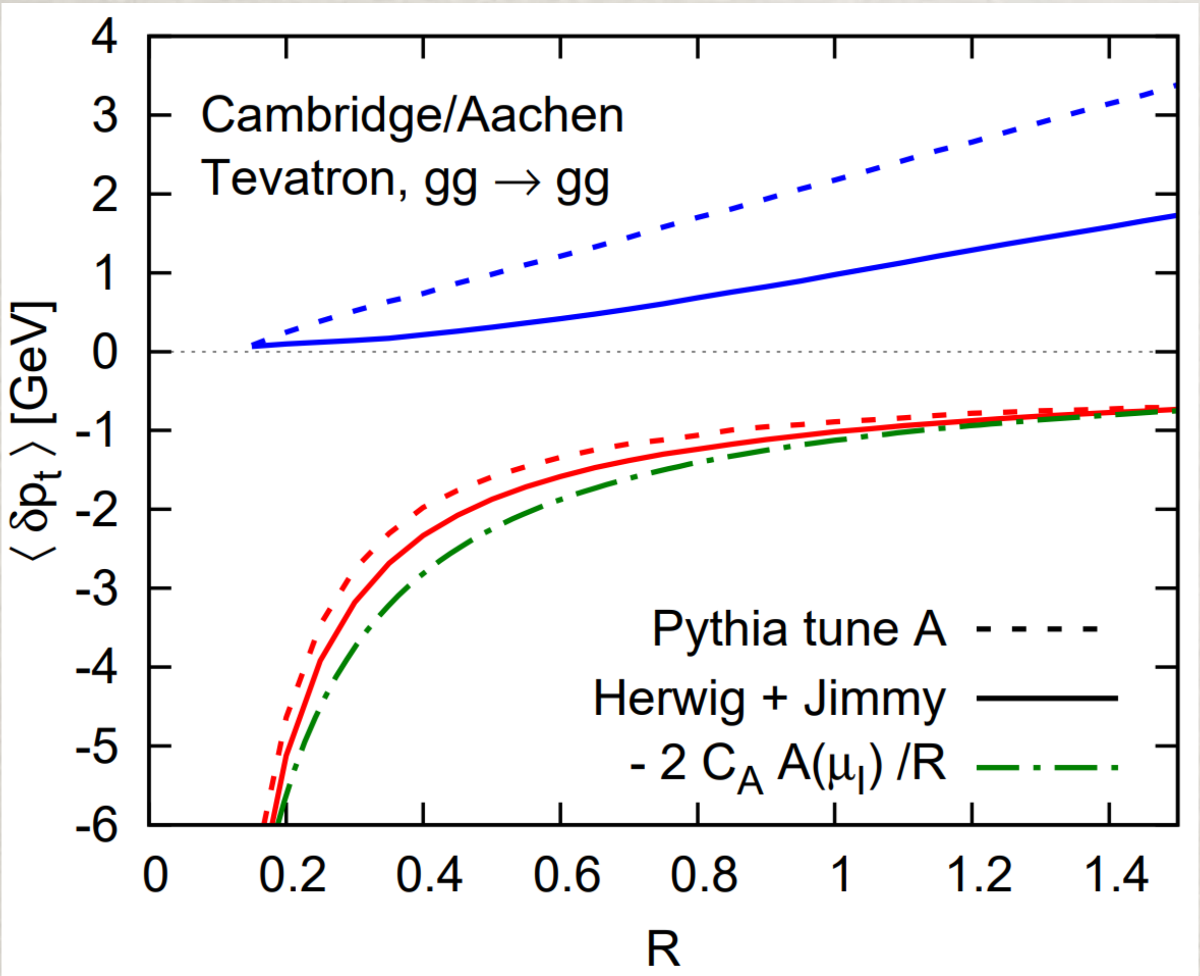
$$\langle \delta p_t \rangle_{\mathrm{UE}} \simeq \int_0^{2\pi} \frac{d\phi}{2\pi} \int_{-\infty}^{\infty} dy \, \underbrace{\rho \cos\phi}\, \Theta\left[R^2 - (y^2 + \phi^2)\right] = 2\pi\rho R J_1(R)$$

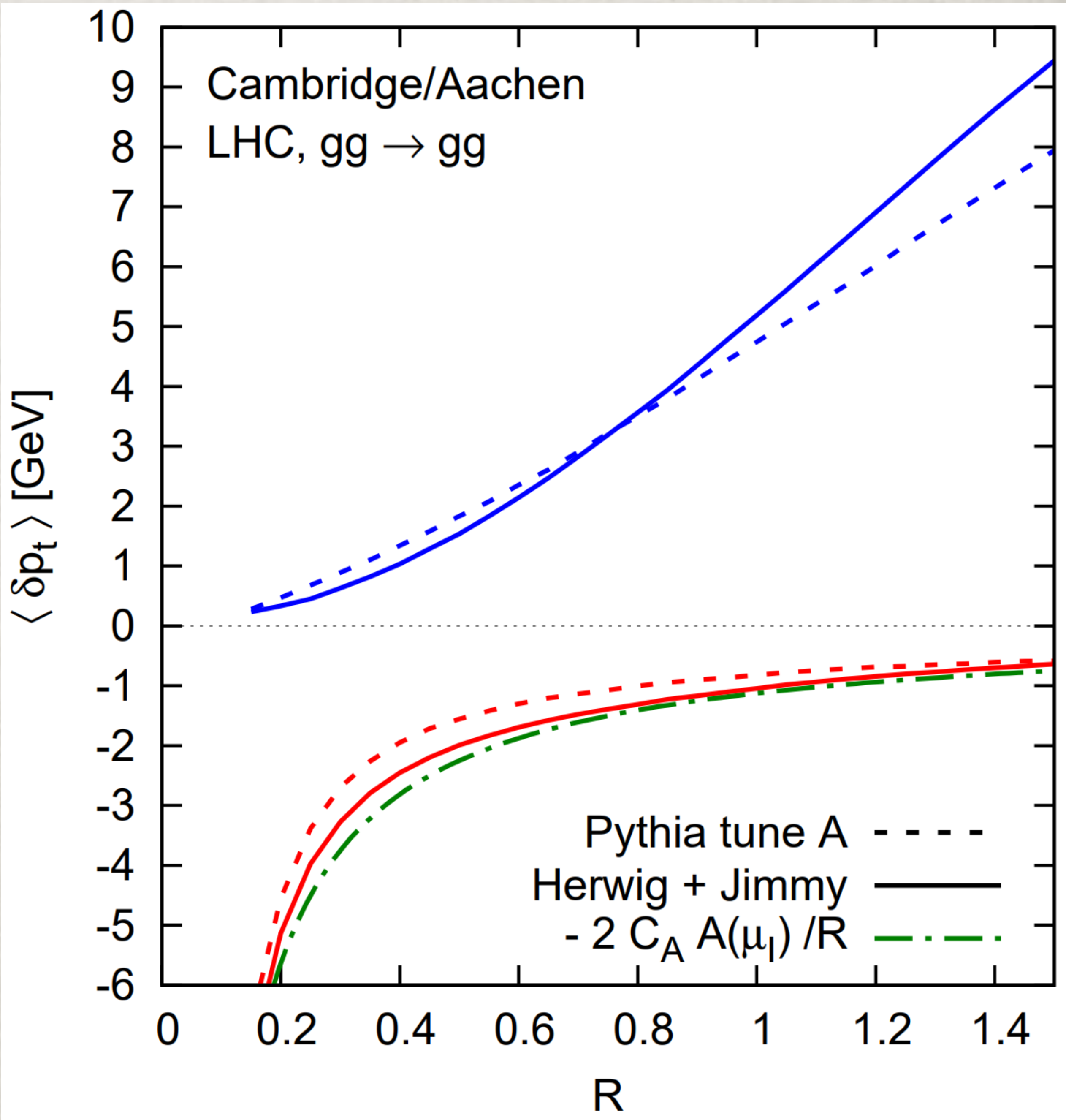
Small jet radius: $\boxed{\langle \delta p_t \rangle_{\mathrm{UE}} \simeq \rho\pi R^2 \left(1 - \frac{R^2}{8} + \dots\right)}$

Comparison with Monte Carlo

- Different event generators give $\langle \delta p_t \rangle_{\mathrm{UE}} \simeq \rho \pi R^2$, with different values of ρ

[Dasgupta Magnea Salam 0712.3014]





$$2\pi\rho \to 2-4\,\mathrm{GeV} \quad (\mathrm{Tevatron})$$
$$2\pi\rho \to 8-10\,\mathrm{GeV} \quad (\mathrm{LHC}\ 14\,\mathrm{TeV})$$

- Underlying event contribution is up to 10 times bigger than hadronisation

OPTIMAL JET RADIUS

- In precision studies, the jet PT distribution is computed, and one needs to find the radius that minimise the sum of hadronisation and UE corrections

[Dasgupta Magnea Salam 0712.3014]

$$0 = \frac{d}{dR}\left(\langle \delta p_t \rangle_h^2 + \langle \delta p_t \rangle_{\mathrm{UE}}^2\right) \Rightarrow \boxed{R = \sqrt{2}\left(\frac{C_{F/A}\mathcal{A}(2\,\mathrm{GeV})}{2\pi\rho}\right)}$$

	quark jets	gluon jets
Tevatron	0.5	0.7
LHC	0.4	0.5

- The optimal radius for gluon jets is larger because gluons tend to radiate more, and then lose more soft large angle hadrons
- Recent studies try to quantitative assess these effects using LHC data

[Dasgupta Dreyer Salam Soyez 1602.01110]

Theoretical uncertainties

Renormalisation and factorisation scales

- Any fixed-order cross section in hadron collisions depends on two unphysical parameters, the renormalisation and factorisation scales

$$d\sigma_{pp\to X} \sim \sum_{i,j} f_{i/p}(\underbrace{\mu_F}_{\text{factorisation scale}}) \otimes f_{j/p}(\mu_F) \otimes d\hat{\sigma}_{ij\to X}\left(\alpha_s(\underbrace{\mu_R}_{\text{renormalisation scale}}), \frac{\mu_F}{\mu_R}, \ldots\right)$$

- Varying these scales produces higher order contributions, e.g

$$\alpha_s^n(x\mu_R) = \alpha_s^n + (n\,\beta_0 \ln x)\,\alpha_s^{n+1}(\mu_R)$$

Scale uncertainties

- Varying renormalisation and factorisation scales is a natural way to estimate theoretical uncertainties

$$d\sigma_{pp\to X}(x\mu_R, y\mu_F) = \underbrace{d\sigma_{pp\to X}(\mu_R,\mu_F)}_{\sim\alpha_s^n} + \mathcal{O}(\alpha_s^{n+1})$$

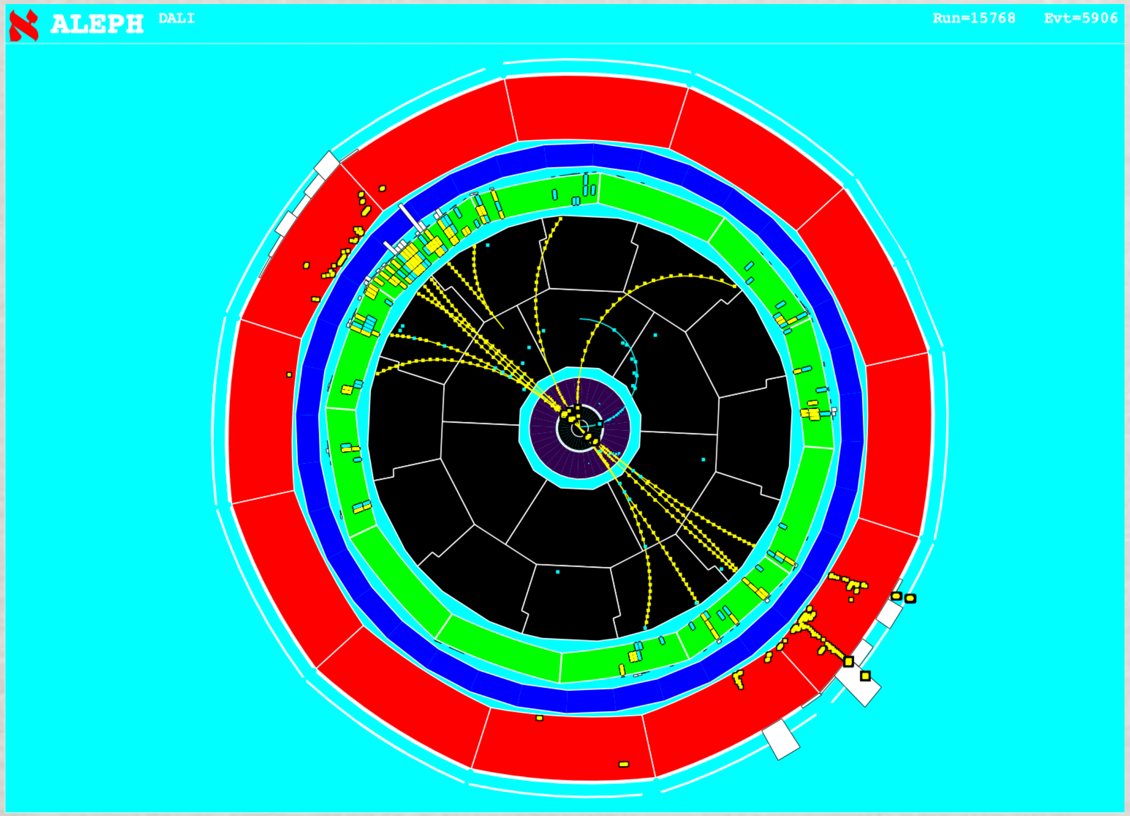
- Relevant questions are
  - how to choose the default (central) value of μ_R and μ_F ?
  - over what range should we vary these scales?
  - how should we add uncertainties?

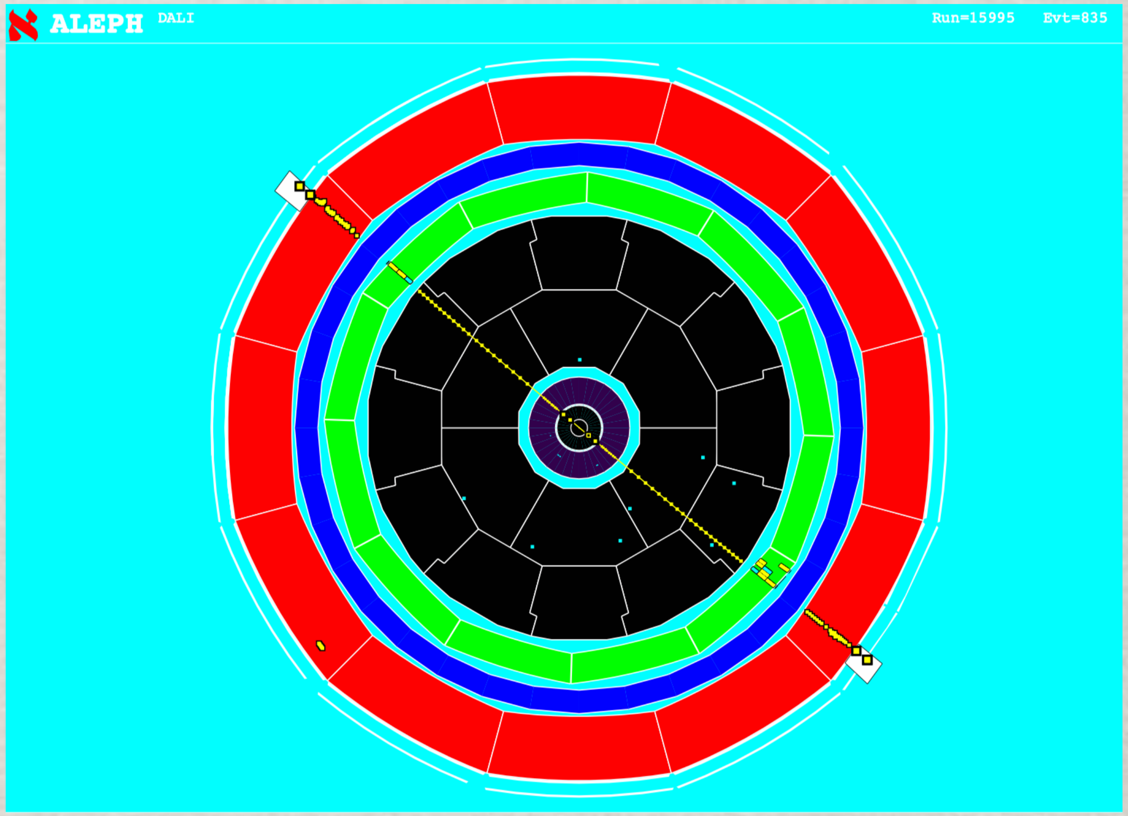
There is no theoretically sound answer to any of these questions. Nevertheless, we will reflect on directions to make sensible choices

A short-distance observable

- Consider a simple counting observable in e^+e^- annihilation

$$R = \frac{\sigma(e^+e^- \to \text{hadrons})}{\sigma(e^+e^- \to \mu^+\mu^-)}$$



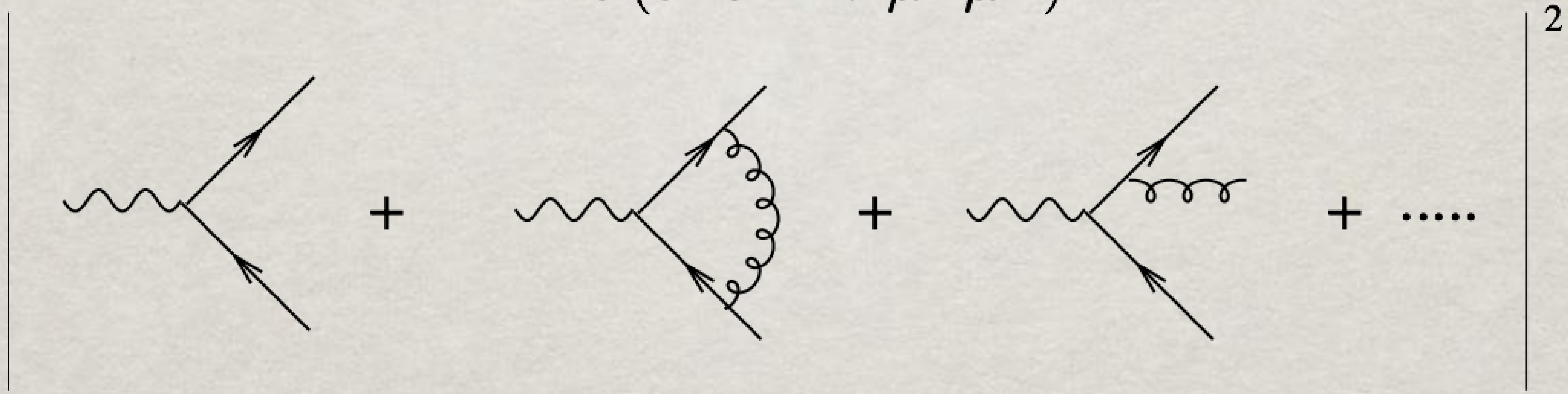


THE RATIO R IN QCD

- Since the ratio *R* is infrared and collinear safe, it admits a massless limit

$$R\left(\alpha_s(\mu_R^2), \frac{Q^2}{\mu_R^2}, \frac{m_q^2}{\mu_R^2}\right) = \hat{R}\left(\alpha_s(\mu_R^2), \frac{Q^2}{\mu_R^2}\right) + \mathcal{O}\left(\left(\frac{m_q^2}{Q^2}\right)^p\right)$$

$$\hat{R} = \frac{\sigma(e^+e^- \to \text{partons})}{\sigma(e^+e^- \to \mu^+\mu^-)}$$



- Since $\hat{R}$ depends on the single scale Q, independence of $\hat{R}$ on the renormalisation scale leads to

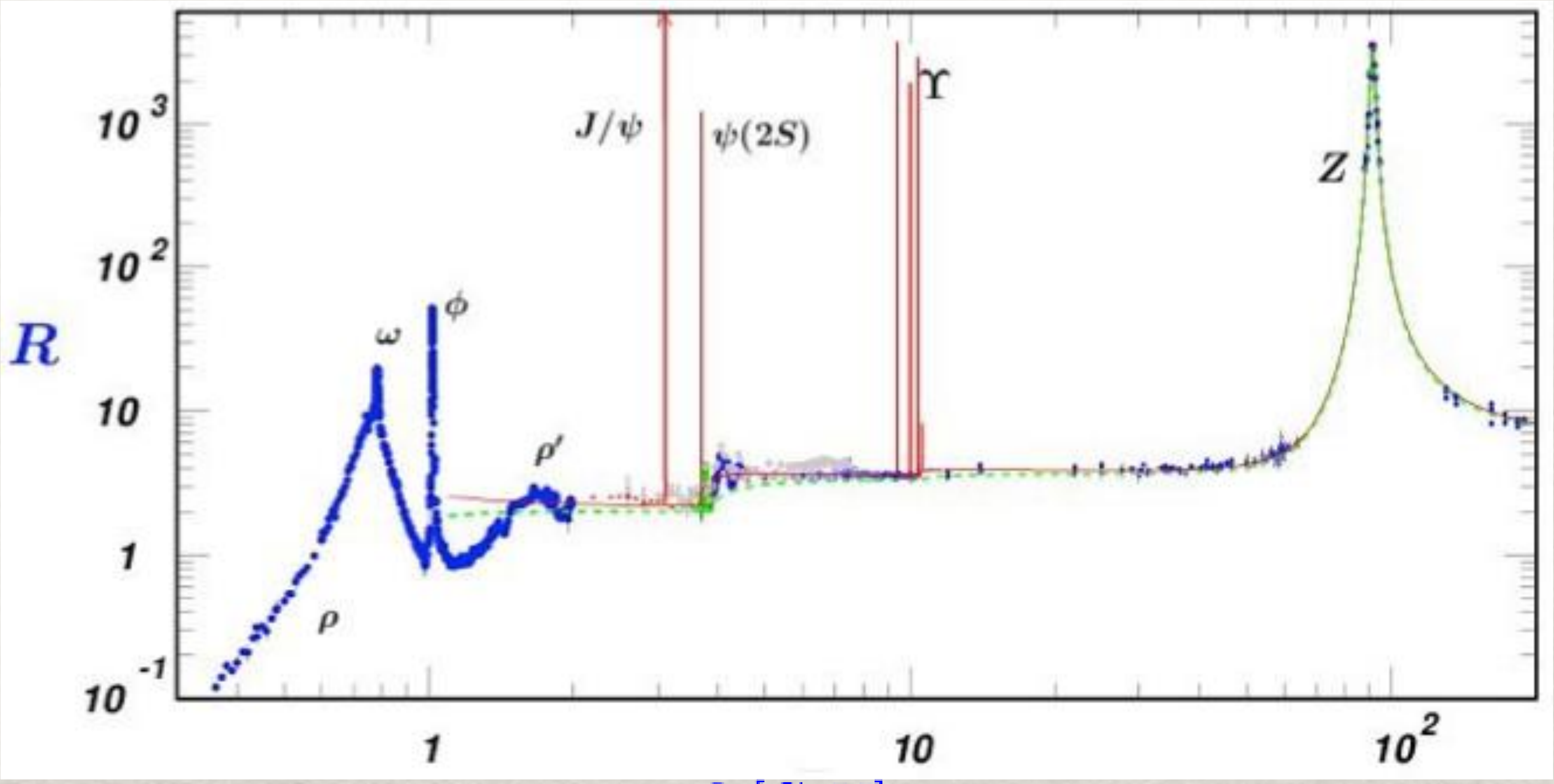
$$\hat{R}\left(\alpha_s(\mu_R^2), \frac{Q^2}{\mu_R^2}\right) = \hat{R}\left(\alpha_s(Q^2), 1\right)$$

Setting the central scale

- For a one-scale observable like $\hat{R}$, the dependence on μ_R appears in the following form

$$\hat{R}\left(\alpha_s(\mu_R), \frac{Q^2}{\mu_R^2}\right) = R_0 + R_1\alpha_s(\mu_R^2) + \left(R_1\beta_0 \ln\frac{Q^2}{\mu_R^2} + R_2\right)\alpha_s^2(\mu_R^2) + \mathcal{O}(\alpha_s^3)$$

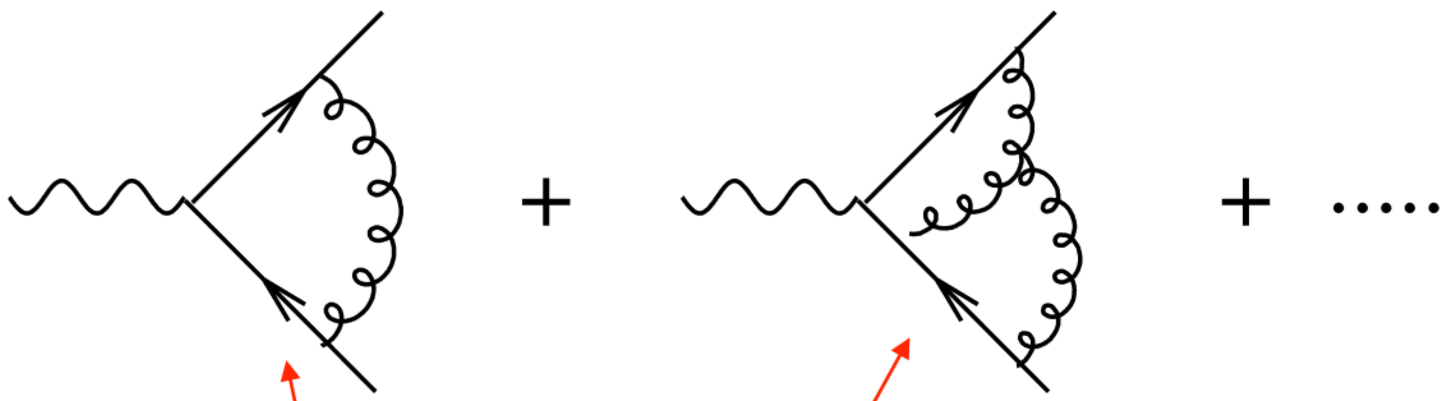
- Choosing $\mu_R^2 = Q^2$ resums terms $\ln(\mu_R^2/Q^2)$ at all orders in $\alpha_s(Q^2)$



Q [GeV]

RENORMALISATION SCALE

- The renormalisation scale is the price we pay when we get rid of UV divergences through renormalisation



$$\underbrace{\alpha_s(\Lambda) \times \left(1 + c_1 \alpha_s(\Lambda) \ln \frac{\Lambda^2}{\mu_R^2}\right)}_{\equiv \alpha_s(\mu_R)} + \ldots$$

- The renormalised coupling $\alpha_s(\mu_R)$ contains all quantum fluctuations with virtuality larger than μ_R

- Renormalisation scale should be of the order of the highest scale (e.g. the centre of mass energy in electron-positron collisions)

Central value in hadron collisions

- In hadronic collisions, especially in multi-jet events, observables depend on multiple scales

$$\sigma\left(\alpha_s(\mu_R^2), \frac{s_1}{\mu_R^2}, \ldots, \frac{s_1}{\mu_R^2}\right) = \underbrace{\sigma_0}_{\sim\alpha_s^n} \underbrace{+\,\alpha_s\beta_0 \ln \frac{s_1 \ldots s_n}{\mu_R^{2n}}}_{\sim\alpha_s^{n+1}+\sigma_1} + \mathcal{O}(\alpha_s^{n+2})$$

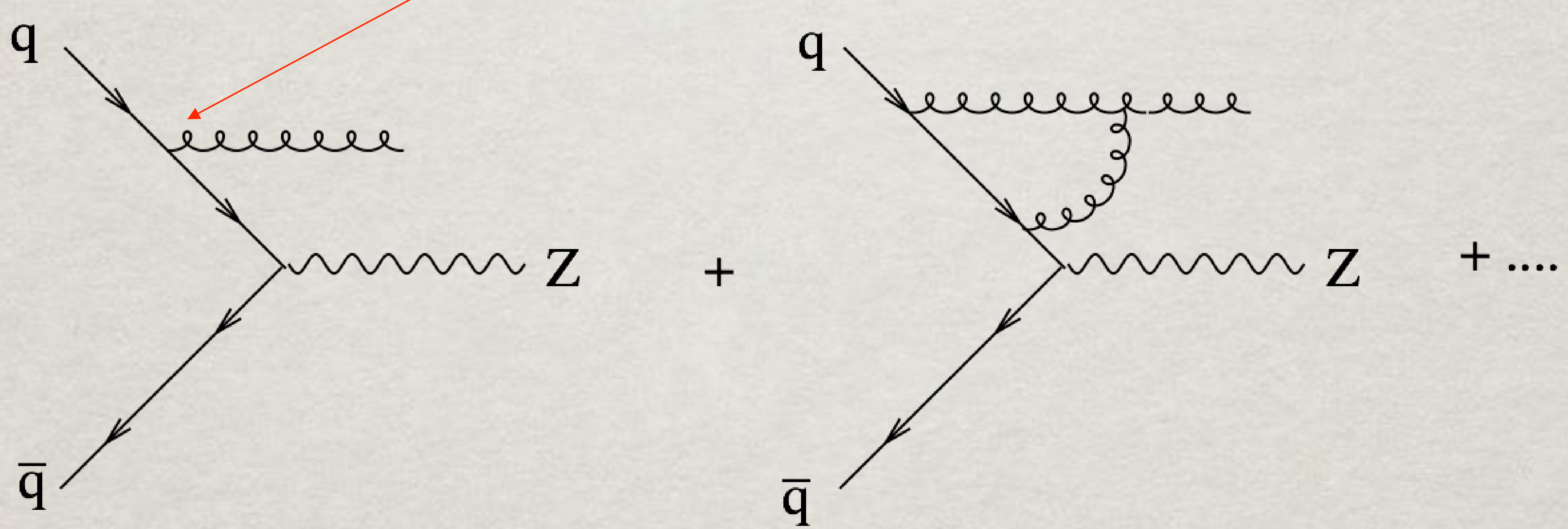
- The analogous strategy to the ratio R would be to cancel the logarithm of μ_R that appears at one loop $\Rightarrow$ $\mu_R^2 = (s_1\, s_2 \ldots s_n)^{1/n}$

Possible problems with such a choice

- In multi-scale observables, these are not the only logarithms around, so there can be extra logs of ratios of scales $\Rightarrow$ soft-collinear resummation
- This log-enhanced term has the same kinematics as tree-level, and does not account for the physics of extra-gluon radiation $\Rightarrow$ physical meaning of renormalisation scale?

Scale of soft radiation

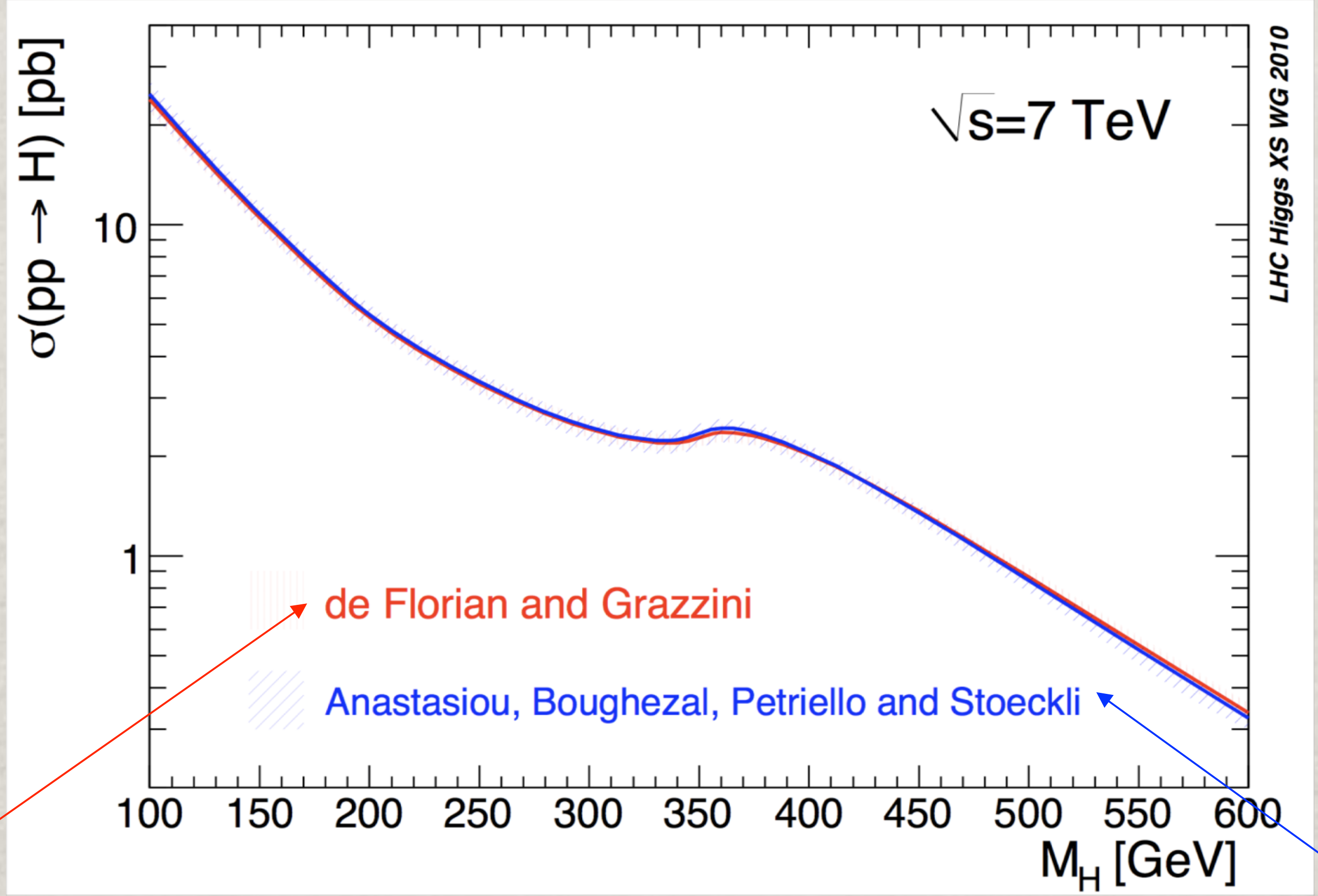
- For one emission at fixed transverse momentum k_t (one-scale problem), one can show that $\alpha_s = \alpha_s(k_t)$ resums $\ln(\mu_R^2/k_t^2)$ at all orders



- Physical meaning: integrate out of all quantum fluctuations from the cutoff to the scale k_t into the running coupling $\alpha_s(k_t)$

STRATEGIES FOR CENTRAL VALUE

- With QCD emissions, set the renormalisation scale close to the upper bound of transverse momentum of emitted gluons
- This scale can be empirically set by imposing the best convergence of the perturbative series



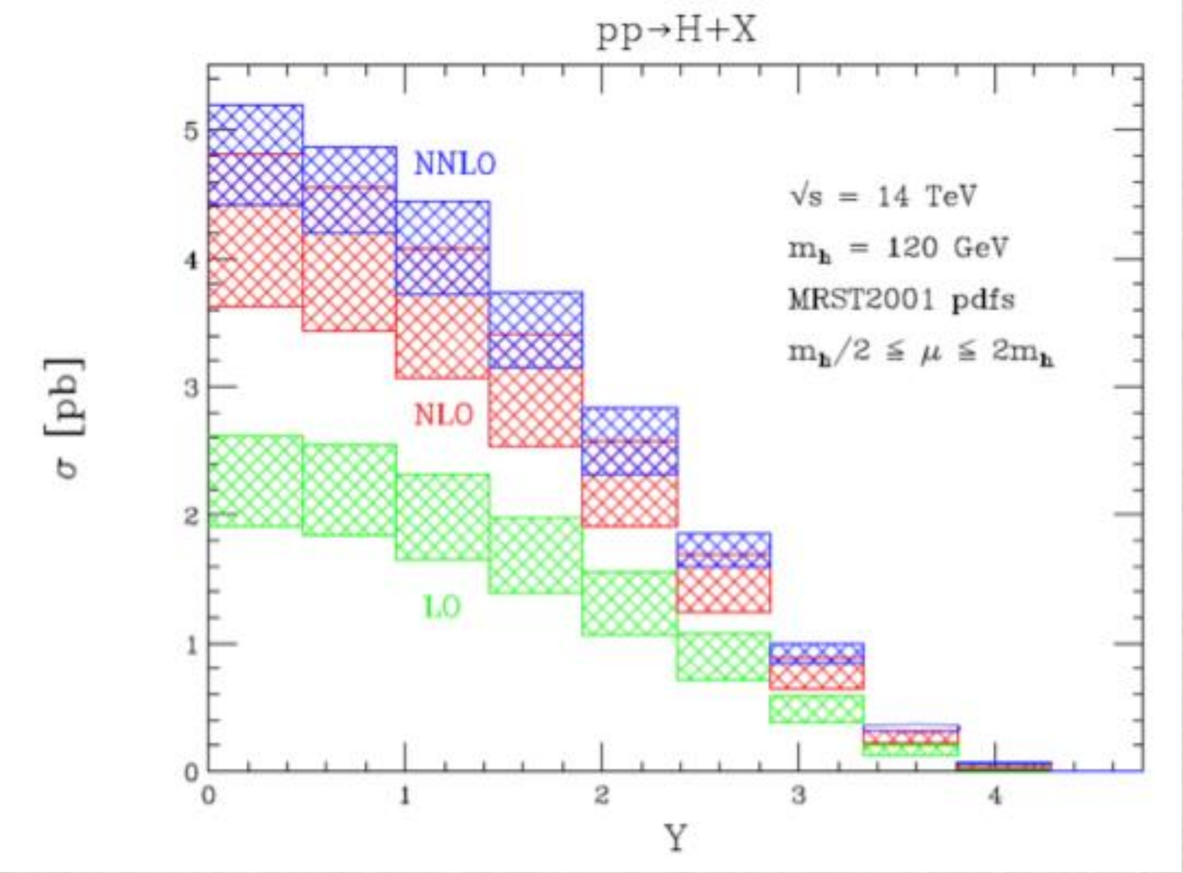
NNLL threshold resummation central scale M_H NNLO fixed-order central scale $M_H/2$

- Threshold resummations of $\ln(M/\hat{s})$ help stabilise the perturbative series

Renormalisation scale variations

- Varying the renormalisation scale by a factor is sensible only after one has identified a suitable central scale, otherwise $\sigma_1$will contain large logarithms of the typical scale and μ_R

$$\sigma_0(x\mu_R,\dots) = \underbrace{\sigma_0(\mu_R,\dots)}_{\sim\alpha_s^n} + \underbrace{(n\beta_0 \ln x)\,\alpha_s(\mu_R)\,\sigma_0(\mu_R,\dots) + \sigma_1(\mu_R,\dots)}_{\sim\alpha_s^{n+1}} + \dots$$

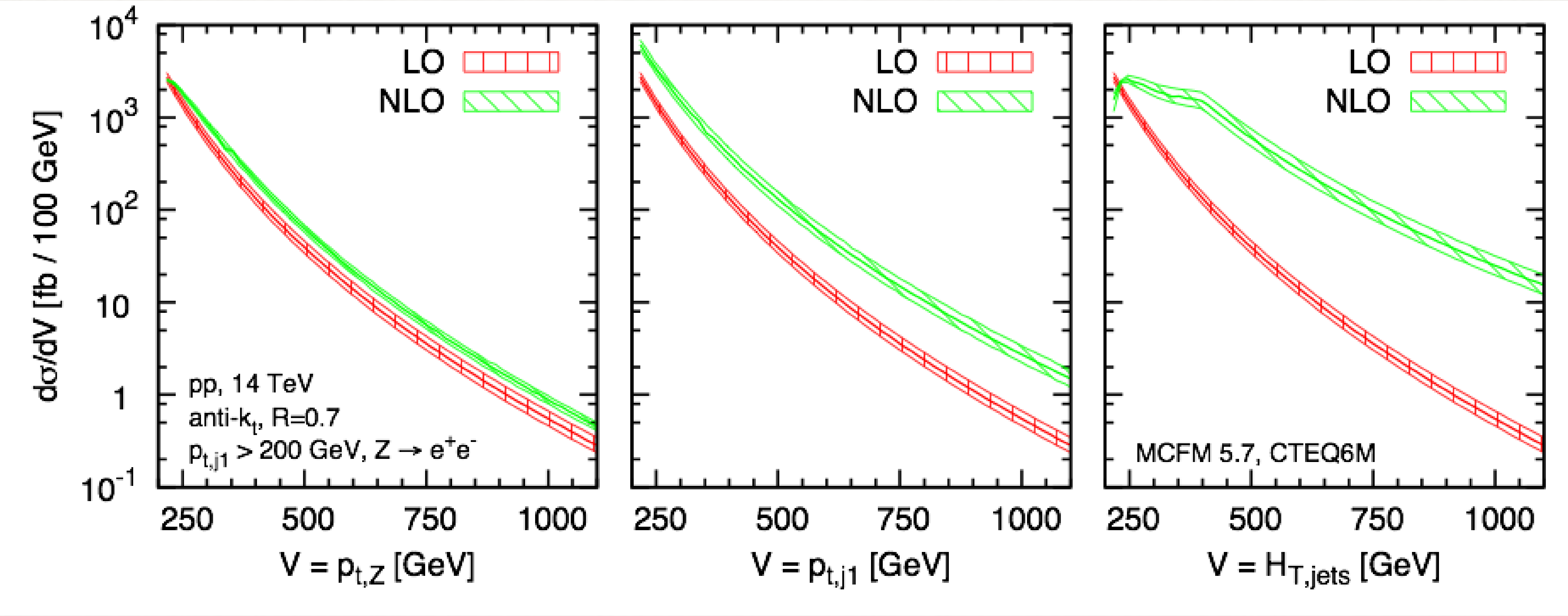


[Anastasiou Melnikov Petriello hep-ph/0409088]

- This procedures gives an estimate of missing higher orders as long as K-factors (e.g. σ_1/σ_0) are not too large

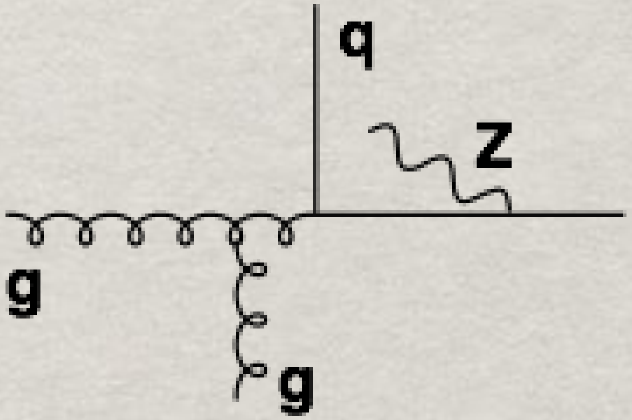
Large K-factors

- Large K-factors can have different origins than a bad choice of default renormalisation scale



- Example: some large K-factors arise from the the opening of new partonic channels and require an ad-hoc treatment

[Rubin Salam Sapeta 1006.2144]

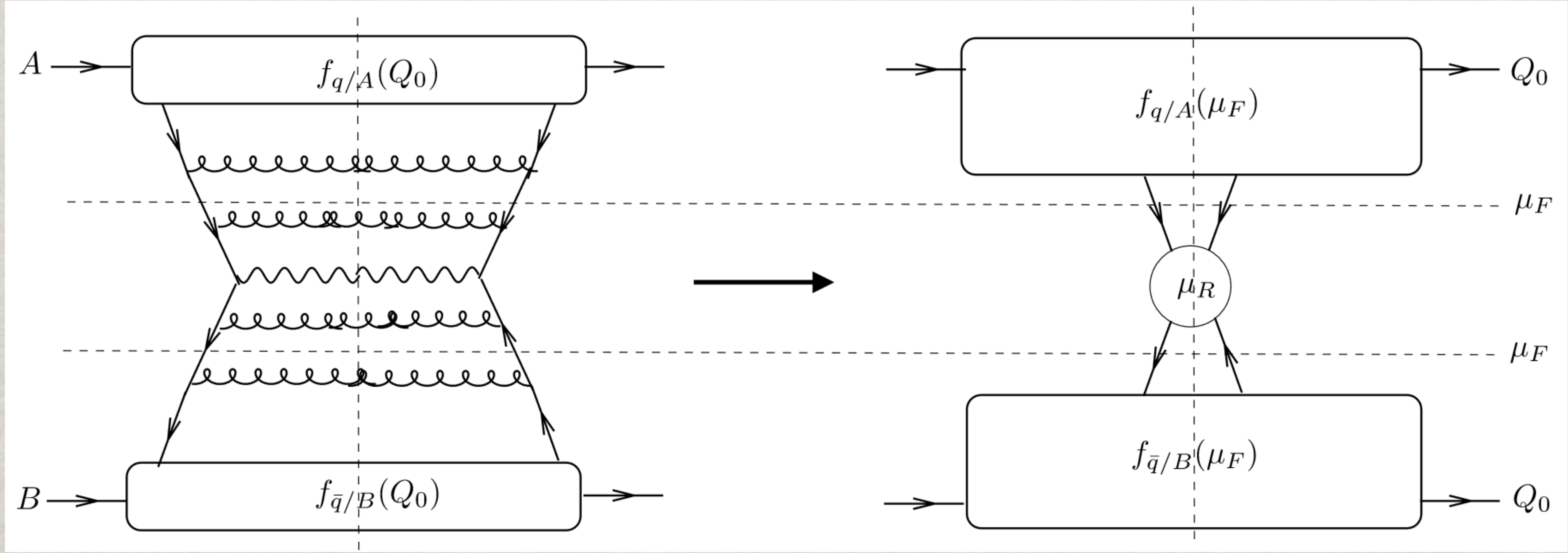


Factorisation scale

- General form of a cross section in hadronic collisions

$$d\sigma_{AB\to X} \sim \sum_{i,j} f_{i/A}(\mu_F) \otimes f_{j/B}(\mu_F) \otimes d\hat{\sigma}_{ij\to X}\left(\alpha_s(\mu_R), \frac{\mu_F}{\mu_R}, \ldots\right)$$

- Emissions collinear to hadron A with transverse momenta up to μ_F are not observed (unresolved) and included in $f_{i/A}(\mu_F)$



- Factorisation scale should be of the order of the lowest scale

ASYMMETRIC SCALE VARIATION

- Theoretically, there is no reason why renormalisation and factorisation scale should be the same, because they incorporate different physics effects
- Current strategy: perform a 7-point renormalisation and factorisation scale variation around a central scale Q_0

$$\frac{1}{2} \leq \frac{\mu_{R,F}}{Q_0} \leq 2 \qquad\qquad \frac{1}{2} \leq \frac{\mu_R}{\mu_F} \leq 2$$

- Example: Higgs cross section

symmetric scales	7-point scales
$\sigma_{pp\to H} = 44.8\,^{+0.2}_{-1.5}\,\mathrm{pb}$	$\sigma_{pp\to H} = 44.8\,^{+0.2}_{-1.8}\,\mathrm{pb}$

[AB Caola Dreyer Monni Salam Zanderighi Dulat 1511.02886]

- Asymmetric scale variations give more conservative uncertainties
- Need to worry about consistency with pdf determinations?

Resummation uncertainties

- In hadron collisions, the zero-jet cross section has the form

$$d\sigma_{0-\text{jet}} \sim \mathcal{L}(p_{\text{t,veto}})\, e^{Lg_1(\alpha_s L)+g_2(\alpha_s L)+\alpha_s g_3(\alpha_s L)} \left(1+\alpha_s \delta\mathcal{F}(\alpha_s L)\right)$$

  - Dependence of μ_R in the strong coupling $\alpha_s = \alpha_s(\mu_R)$
  - Dependence on μ_F in the “luminosity” $\mathcal{L}(p_{\text{t,veto}})$

- Theory uncertainties can be evaluated by performing 7-point scale variations around a central scale (e.g. $Q_0 = m_H/2$)

$$\frac{1}{2} \leq \frac{\mu_{R,F}}{Q_0} \leq 2 \qquad\qquad \frac{1}{2} \leq \frac{\mu_R}{\mu_F} \leq 2$$

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- The logarithm L depends on the ratio between the veto scale $p_{\mathrm{t,veto}}$, and a "resummation" scale Q, the higher scale up to which resummation is valid
- The scale Q can be varied around the central scale Q_0

- Resummations contain double logarithms of Q

$$Lg_1(\alpha_s L) \sim \alpha_s \ln^2(Q/p_{\mathrm{t,veto}})$$

- Variations of renormalisation and factorisation scale give rise to single logarithms $\ln(\mu_R/\mu_0)$ and $\ln(\mu_F/\mu_0) \Rightarrow$ reduce range of Q-variation?

$$\frac{2}{3} \leq \frac{Q}{Q_0} \leq \frac{3}{2}$$

Resummation uncertainties

- Most accurate predictions for Higgs plus zero jets is NNLL resummation matched to NNNLO

[AB et al 1511.02886]

$$\epsilon(p_{t,\mathrm{veto}}) \equiv \frac{\sigma_{0-\mathrm{jet}}(p_{t,\mathrm{veto}})}{\sigma_{\mathrm{tot}}}$$

$$\frac{1}{2} \leq \frac{\mu_{R,F}}{Q_0} \leq 2$$

$$\frac{1}{2} \leq \frac{\mu_R}{\mu_F} \leq 2$$

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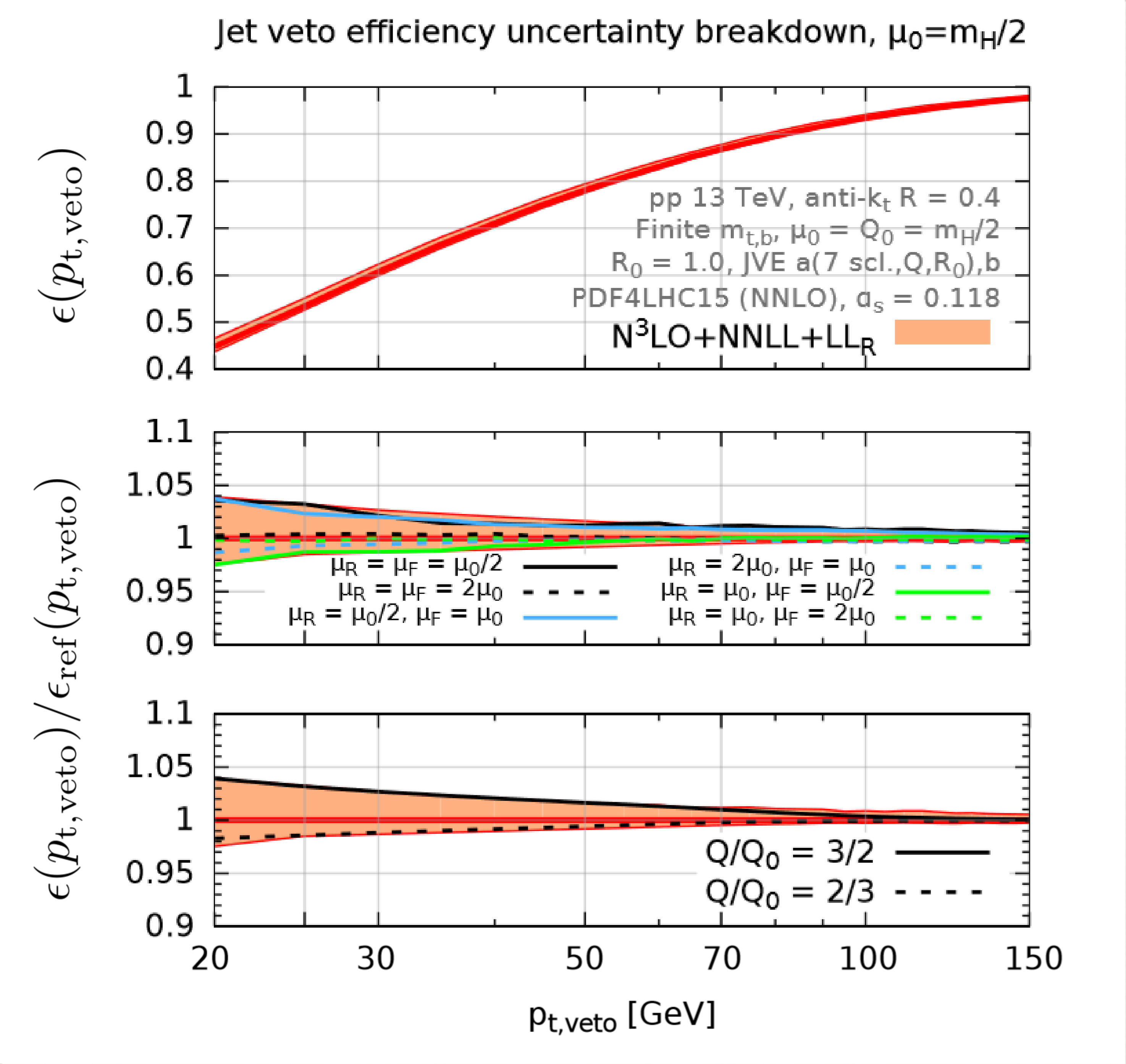
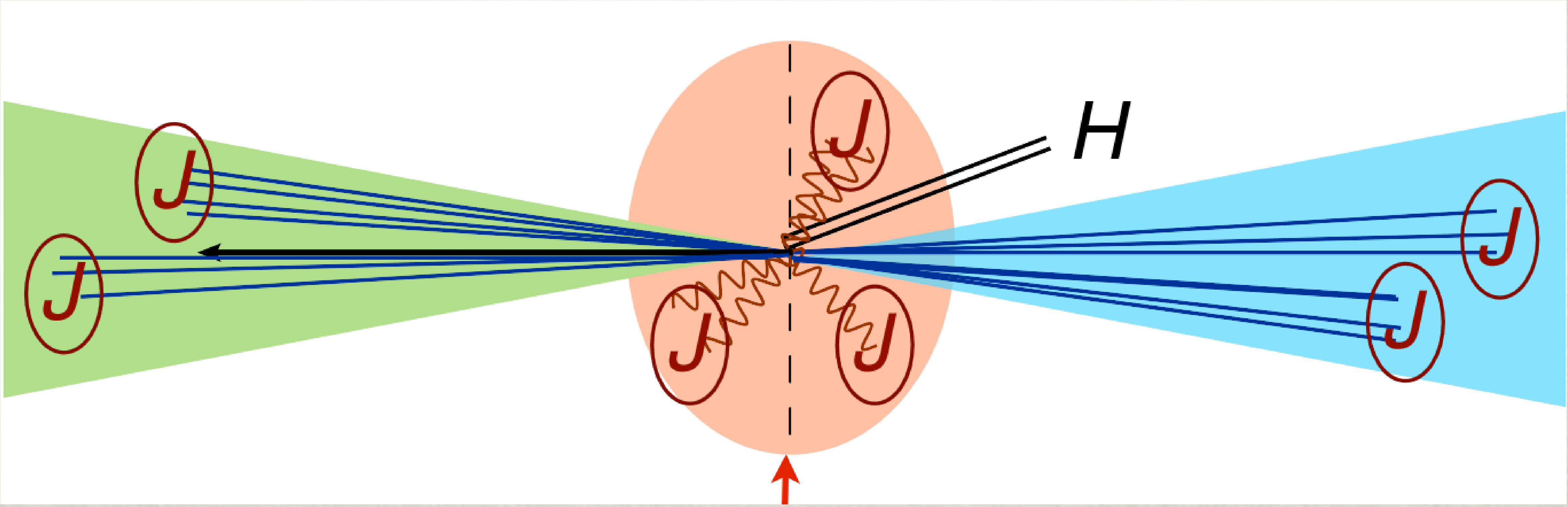


UNCERTAINTIES IN SCET

- In Soft-Collinear Effective Theory (SCET), resummation proceeds through refactorisation in hard and beam functions

[Becher Neubert 1205.3806]

$$\sigma_{0-\mathrm{jet}} \sim \mathcal{B}_g(\mu_J) \otimes \mathcal{B}_g(\mu_J) \otimes \mathcal{H}(m_H, \mu_H)$$

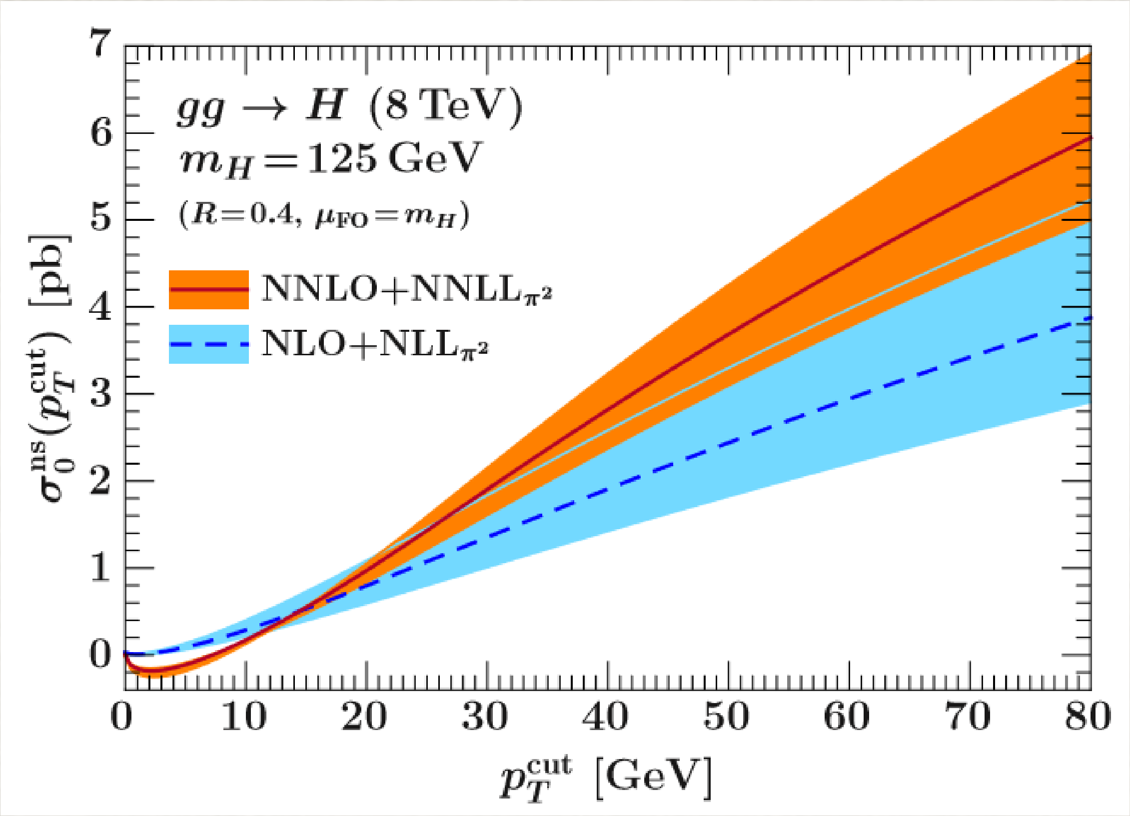


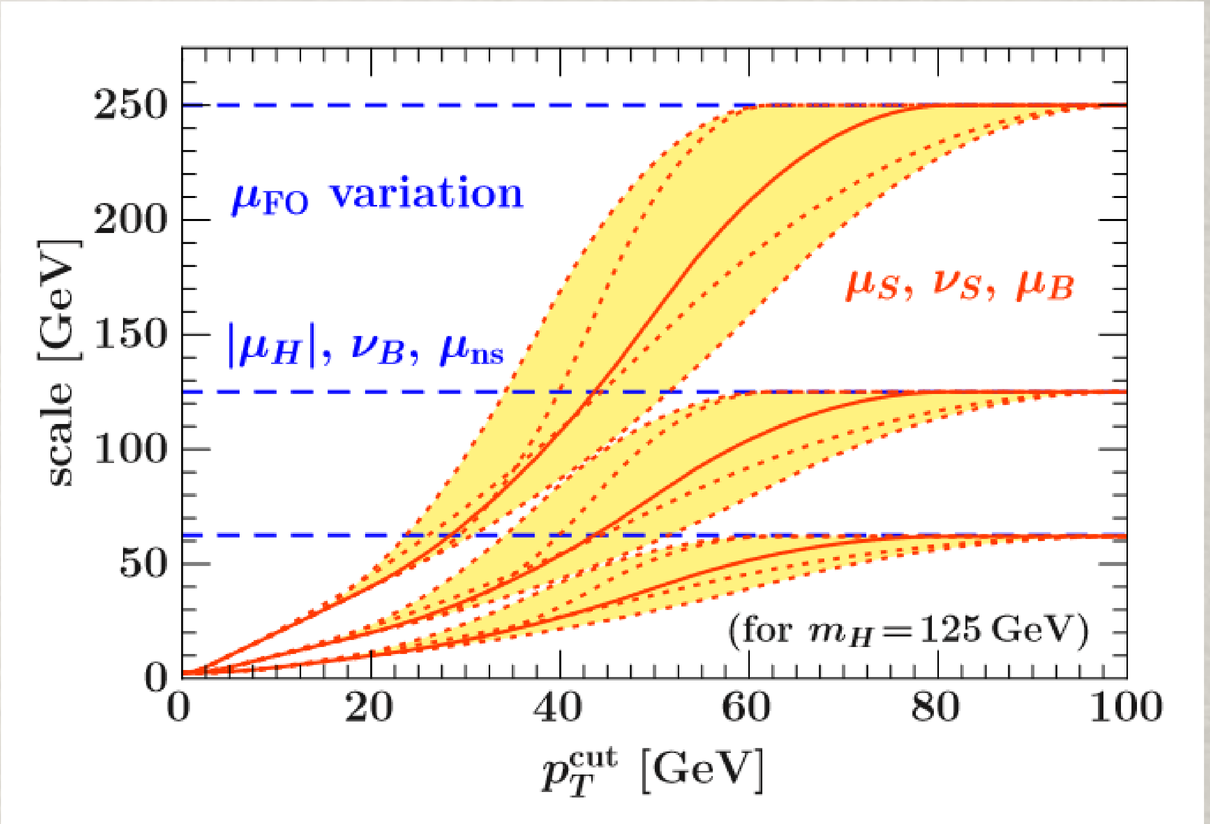
- Each function is characterised by a scale, and there is a computable relation between different scales

Uncertainties in SCET

- Resummation is achieved by setting the scales in the beam and hard functions to appropriate values that depend on the observable one wants to resum, e.g. $p_T^{\rm cut} \equiv p_{\rm t,veto}$

[Stewart Tackmann Walsh Zuberi 1307.1808, Becher Neubert Rothen 1205.3806]





- Uncertainties are estimated by varying the way scales move from low to high values $\Leftrightarrow$ profile scales

Learning outcomes

At the end of these lectures, you should be able to

- Understand the difference between cone and sequential jet algorithms
- Carry out simple resummations at NLL accuracy
- Give approximate estimates of the effect of hadronisation and underlying event on high-energy observables
- Give arguments for the choice of scales in fixed-order and resummed calculations

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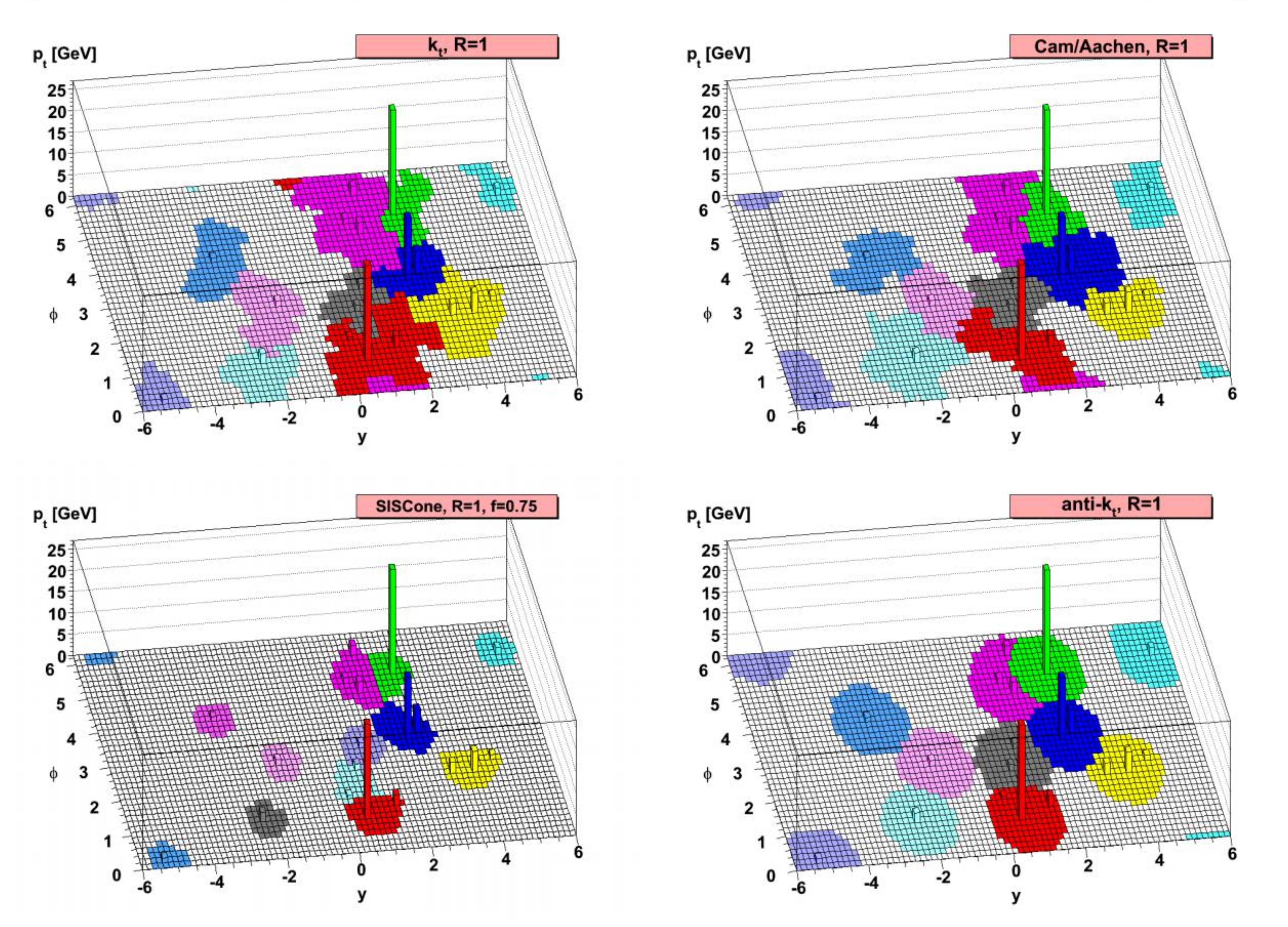
Thank you for your attention!

Extra

Catchment area of jets

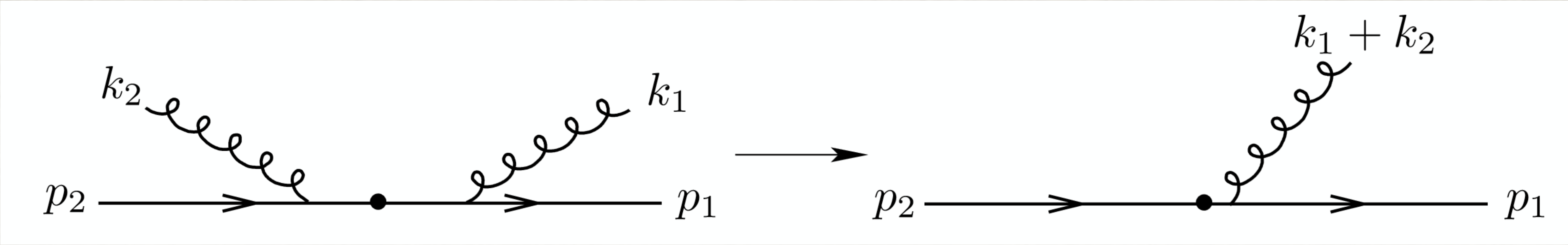


Failure of rIRC safety condition 1

- The way in which the JADE algorithm performs sequential recombinations changes the scaling properties of the three-jet resolution

$$y_{k_1 p_1} = y_3(\{\tilde{p}\}, k_1) = \frac{(k_1+p_1)^2}{Q^2} \simeq \frac{k_{t1}}{Q} e^{-\eta_1} = y_{\rm cut}$$

$$y_{k_2 p_2} = y_3(\{\tilde{p}\}, k_2) = \frac{(k_2+p_2)^2}{Q^2} \simeq \frac{k_{t2}}{Q} e^{+\eta_2} = y_{\rm cut}$$



$$y_{k_1 k_2} = \frac{(k_1+k_2)^2}{Q^2} \simeq y_{\rm cut}^{2-\xi_1-\xi_2} < y_{\rm cut} \quad \Leftrightarrow \quad \xi_1+\xi_2<1$$

$$\frac{y_3(\{\tilde{p}\}, k_1, k_2)}{y_{\rm cut}} = y_{\rm cut}^{1-\xi_1-\xi_2} \Rightarrow \text{depends on } y_{\rm cut} \Rightarrow \mathcal{F}_{\rm sc}(y_{\rm cut}) \text{ gives double logs}$$

Monte Carlo resummation

- The normalisation of independent soft-collinear emission suggests a Markov-chain procedure to compute $\mathcal{F}_{\rm NLL}(R')$

$$1 = \epsilon^{R'} \sum_{n=0}^{\infty} \frac{(R')^n}{n!} \int_{\epsilon}^{1} \prod_{i=1}^{n} \frac{d\zeta_i}{\zeta_i}$$

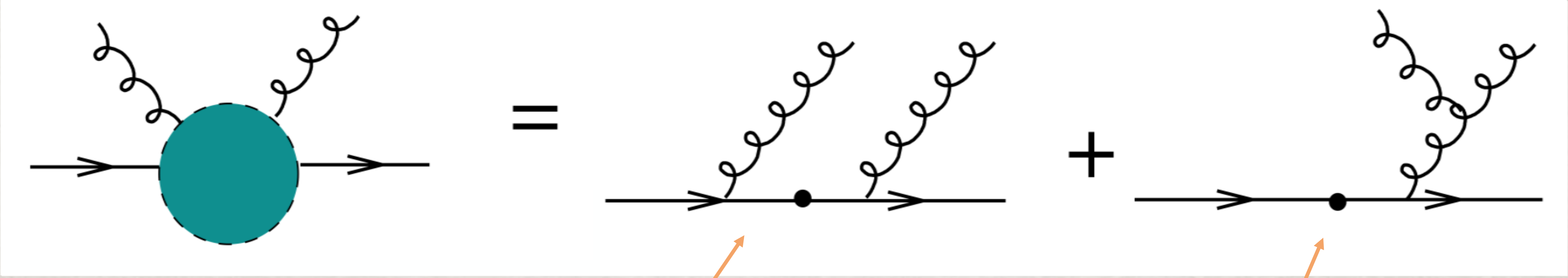
$$= \epsilon^{R'} + \int_{\epsilon}^{1} \underbrace{R' \frac{d\zeta_1}{\zeta_1} \zeta_1^{R'}}_{dP(\zeta_1)} \left[\left(\frac{\epsilon}{\zeta_1}\right)^{R'} + \int_{\epsilon}^{\zeta_1} \underbrace{R' \frac{d\zeta_2}{\zeta_2} \left(\frac{\zeta_2}{\zeta_1}\right)^{R'}}_{dP(\zeta_2)} \left[\left(\frac{\epsilon}{\zeta_2}\right)^{R'} + \dots \right.\right.$$

- Generate $\zeta_i < \zeta_{i-1}$ with probability $P(\zeta_i) = \left(\frac{\zeta_i}{\zeta_{i-1}}\right)^{R'}$
- If $\zeta_i < \epsilon$ stop
- Otherwise, generate ζ_{i+1}

Additive observable: $\mathcal{F}_{\rm NLL}(R') = \epsilon^{R'} \sum_{n=0}^{\infty} \frac{(R')^n}{n!} \prod_{i=1}^{n} \int_{\epsilon}^{1} \frac{d\zeta_i}{\zeta_i} \Theta\left(1 - \sum_{i=1}^{n} \zeta_i\right)$

Two-gluon correlated emission

- Consider the most singular case of two soft gluons strongly ordered in energy



Independent emission

Correlated emission, singular only when the gluons are close in angle

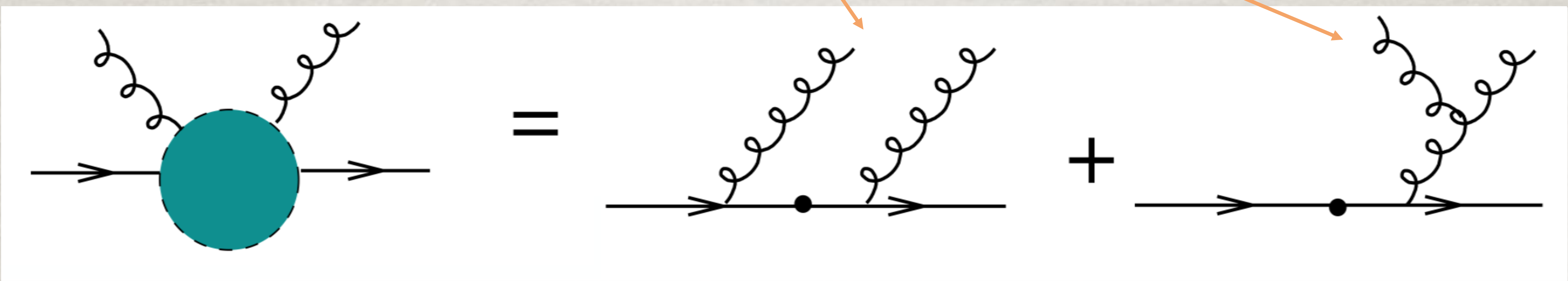
$$
\begin{aligned}
M^2(k_1,k_2) &= M^2(k_1)M^2(k_2) \\
&+ C_A C_F \boxed{\left(\frac{(p_1p_2)}{(p_1k_1)(k_1k_2)(k_2p_2)} + \frac{(p_1p_2)}{(p_2k_1)(k_1k_2)(k_2p_1)} - \frac{(p_1p_2)}{(pk_1)(k_1p_2)}\frac{(p_1p_2)}{(pk_2)(k_2p_2)}\right)}
\end{aligned}
$$

- For gluons widely separated in angle, only independent emission survives

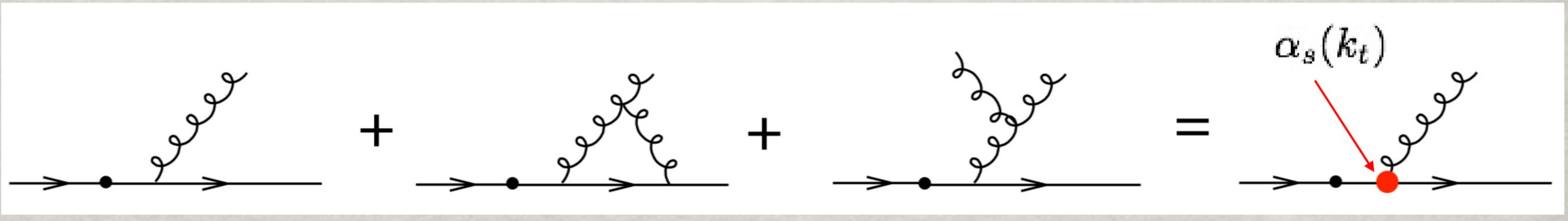
Two-gluon correlated emission

- The matrix element for two soft-collinear gluons can always be written as the sum of an independent and correlated emission part

$$M^2(k_1,k_2) = M^2(k_1)M^2(k_2) + \tilde{M}^2(k_1,k_2)$$



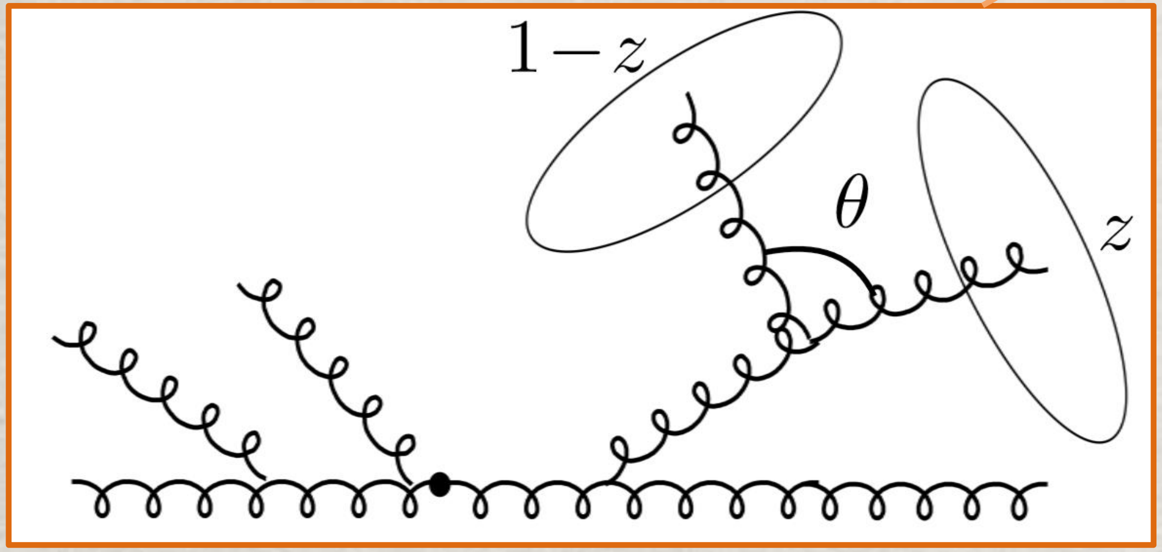
- The correlated emission part, if integrated inclusively, is combined with the one-loop one-gluon matrix element to give the running coupling in a physical renormalisation scheme

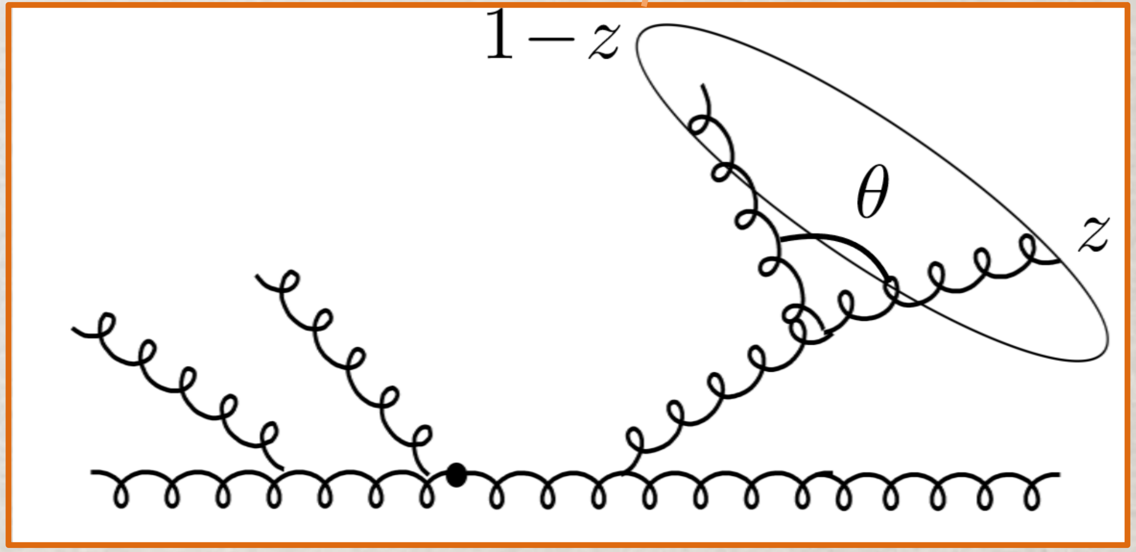


Two-gluon correlated emission

- The remainder after the extraction of the coupling

$$\int_{\epsilon v}[dk_1]\int_{\epsilon v}[dk_2]\tilde{M}^2(k_1,k_2)\left[\Theta\left(v-V(\{\tilde{p}\},k_1,k_2)\right)-\Theta\left(v-V(\{\tilde{p}\},k_1+k_2)\right)\right]$$





- Example: in a jet-rate, the two gluons can be clustered into different jets

$$\int_{\epsilon v}[dk]M^2(k)\times\left[C_A\int\frac{d\theta^2}{\theta^2}\int\frac{dz}{z(1-z)}\frac{\alpha_s[z(1-z)\theta k_t]}{2\pi}\right]$$

- Potential source of double logarithms, which are however absent for a rIRC safe observable