Qubit, M-Qubit, Interference, Gates

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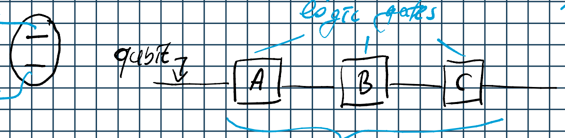
MULTI-QUBIT CIRCUITS

$|e\rangle, |1\rangle$

$|g\rangle, |0\rangle$

quantum logic gates

qubit

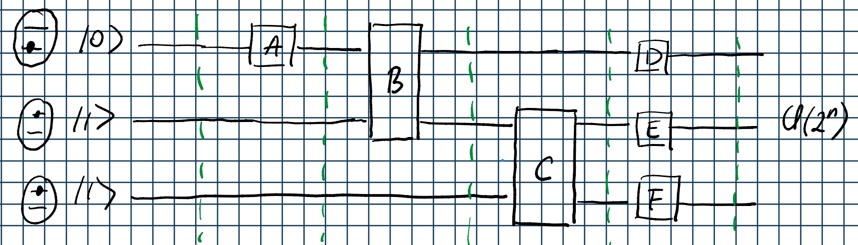


$U = CBA$

they are matrices $\Rightarrow$ Unitary matrices

$UU^\dagger = \mathbb{1}$: they form a group!

$U(2)$ complex group



$|0\rangle$, $|1\rangle$, $|1\rangle$ $\quad U(2^n)$

for all gates

$A \rightarrow U(2^1) \rightarrow U(2)$

$B \rightarrow U(2^2) \rightarrow U(4)$

total # of elementary gates is the size of the circuit

size of circuit $\rightarrow 6$

depth of circuit $\rightarrow 4$

# of qubit $\rightarrow 3$

WHY QUBITS?

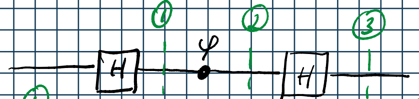
Define system by each qubit : ... $\quad 2^n$ configurations : $2^n \rightarrow n$ operations

each qubit is a state of the system

$|1\rangle \otimes |0\rangle \otimes \ldots \otimes |1\rangle = |1\rangle|0\rangle \ldots |1\rangle = |10\ldots1\rangle$

SINGLE-QUBIT INTERFERENCE

(Hadamard - Phase - Hadamard)



$H = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}, \quad P_\varphi = \begin{pmatrix} 1 & \\ & e^{i\varphi} \end{pmatrix}$

$H|0\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle) \qquad P_\varphi|0\rangle = |0\rangle$

$H|1\rangle = \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle) \qquad P_\varphi|1\rangle = e^{i\varphi}|1\rangle$

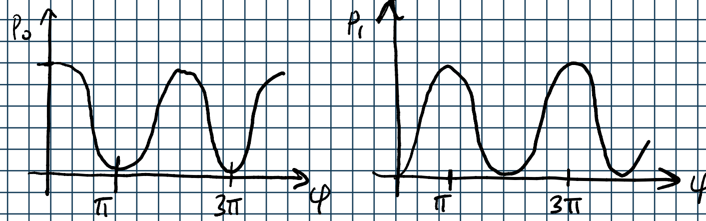
$$|0\rangle \xrightarrow{H} H|0\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle) \xrightarrow{P_\varphi} \frac{1}{\sqrt{2}}(|0\rangle + e^{i\varphi}|1\rangle) \xrightarrow{H} \frac{1}{2}\left[|0\rangle + |1\rangle + e^{i\varphi}(|0\rangle - |1\rangle)\right] = \frac{1}{2}\left[|0\rangle(1 + e^{i\varphi}) + |1\rangle(1 - e^{i\varphi})\right] = \cos\frac{\varphi}{2}|0\rangle - i\sin\frac{\varphi}{2}|1\rangle$$

Say output: $|\psi\rangle = \cos\frac{\varphi}{2}|0\rangle - i\sin\frac{\varphi}{2}|1\rangle$

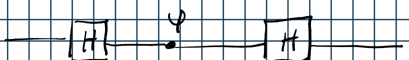
$$P = |\langle\psi\rangle|^2 = \langle\psi|\psi\rangle = \left(\cos\frac{\varphi}{2}\langle 0| + i\sin\frac{\varphi}{2}\langle 1|\right)\left(\cos\frac{\varphi}{2}|0\rangle - i\sin\frac{\varphi}{2}|1\rangle\right)$$

$$= \cos^2\frac{\varphi}{2}\langle 0|0\rangle - \cancel{\cos\frac{\varphi}{2} i\sin\frac{\varphi}{2}\langle 0|1\rangle} + \cancel{\cos\frac{\varphi}{2} i\sin\frac{\varphi}{2}\langle 1|0\rangle} + \sin^2\frac{\varphi}{2}\langle 1|1\rangle = \cos^2\frac{\varphi}{2} + \sin^2\frac{\varphi}{2}$$

$$\boxed{P_0 = \cos^2\frac{\varphi}{2} = \frac{1}{2}(1 + \cos\varphi) \qquad P_1 = \sin^2\frac{\varphi}{2} = \frac{1}{2}(1 - \sin\varphi)}$$



PAULI, CLIFFORD, T-GATE



$\frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$ H $\quad \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ X, σ_x $\quad \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$ Y, σ_y $\quad \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$ Z, σ_z $\quad \begin{pmatrix} 1 & 0 \\ 0 & -i \end{pmatrix}$ S $\quad \begin{pmatrix} 1 & 0 \\ 0 & e^{i\pi/4} \end{pmatrix}$ T

special phase gates (S, T)

PAULI GATES (X, Y, Z)

CLIFFORD GATES

$XZ = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$

$iXZ = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} = Y$

$X, \sigma_x \rightarrow$ bit-flip gate, NOT Gate

$Z, \sigma_z \rightarrow$ phase gate

$P_\varphi = \begin{pmatrix} 1 & 0 \\ 0 & e^{i\varphi} \end{pmatrix} \rightarrow$ special phase gate

$-\boxed{H}-\boxed{X}-\boxed{H}- \;=\; -\boxed{Z}-$

$-\boxed{H}-\boxed{Z}-\boxed{H}- \;=\; -\boxed{X}-$

$P_\varphi = \begin{pmatrix} 1 & 0 \\ 0 & e^{i\varphi} \end{pmatrix} \rightarrow$ special phase gate

$$\underbrace{\text{PAULI GATES}}_{\text{CLIFFORD GATES}}$$

$$-\boxed{H}-\boxed{Z}-\boxed{H}- \;=\; -\boxed{X}-$$