Bloch sphere & quantum error types

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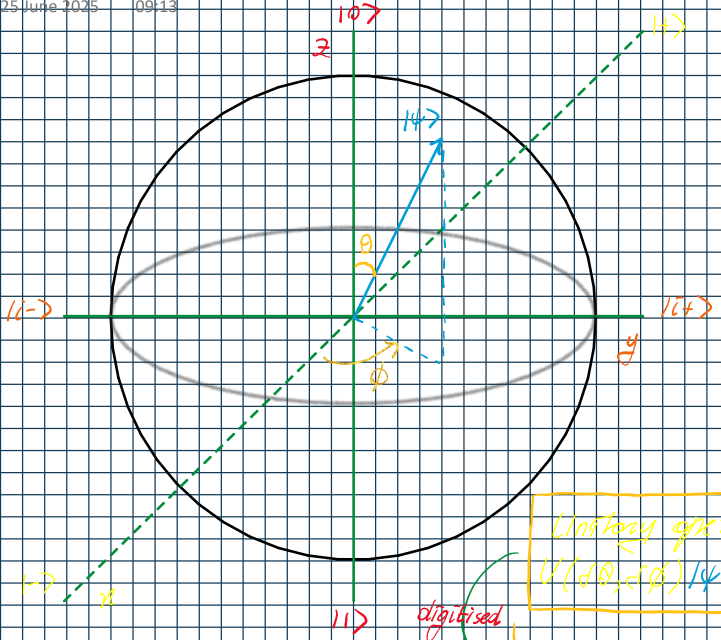

Basis vectors; $|0\rangle = \begin{bmatrix}1\\0\end{bmatrix}, |1\rangle = \begin{bmatrix}0\\1\end{bmatrix}$

$z \rightarrow |0\rangle, |1\rangle$

$x \rightarrow |+\rangle = 1/\sqrt{2}(|0\rangle + |1\rangle)$

$|-\rangle = 1/\sqrt{2}(|0\rangle - |1\rangle)$

$y \rightarrow |i+\rangle = 1/\sqrt{2}(|0\rangle + i|1\rangle)$

$|i-\rangle = 1/\sqrt{2}(|0\rangle - i|1\rangle)$

Geometric representation of general qubit state;

$$|\psi\rangle = \cos\frac{\theta}{2}|0\rangle + e^{i\phi}\sin\frac{\theta}{2}|1\rangle$$

Unitary operation $U(\delta\theta, \delta\phi)$ ①

$$U(\delta\theta,\delta\phi)|\psi\rangle = \cos\frac{\theta+\delta\theta}{2}|0\rangle + e^{i(\phi+\delta\phi)}\sin\frac{\theta+\delta\theta}{2}|1\rangle$$

digitised

②

$$U(\delta\theta,\delta\phi)|\psi\rangle = \alpha_I \mathbb{I}|\psi\rangle + \alpha_X X|\psi\rangle + \alpha_Z Z|\psi\rangle + \alpha_{XZ} XZ|\psi\rangle$$

Pauli Gates;

$$\mathbb{I} = \begin{bmatrix}1&0\\0&1\end{bmatrix}, X = \begin{bmatrix}0&1\\1&0\end{bmatrix}, Y = \begin{bmatrix}0&-i\\i&0\end{bmatrix}, Z = \begin{bmatrix}1&0\\0&-1\end{bmatrix}$$

$$U(\delta\theta,\delta\phi)|\psi\rangle = \alpha_I \mathbb{I}|\psi\rangle + \alpha_X X|\psi\rangle + \alpha_Y Y|\psi\rangle + \alpha_Z Z|\psi\rangle$$

$\alpha_{I,X,Y,Z}$: expansion coefficients

$$T = \begin{bmatrix}1&0\\0&e^{i\pi/4}\end{bmatrix} = \sqrt{S} = \sqrt[4]{Z}$$

Clifford Gates;

$H = 1/\sqrt{2}\begin{bmatrix}1&1\\1&-1\end{bmatrix}$ $H|0\rangle = |+\rangle$, $H|1\rangle = |-\rangle$ $H|+\rangle = |0\rangle$, $H|-\rangle = |1\rangle$ $HXH^+ = Z$, $HZH^+ = X$

$S = \begin{bmatrix}1&0\\0&e^{i\pi/2}\end{bmatrix} = \begin{bmatrix}1&0\\0&i\end{bmatrix} = \sqrt{Z}$ $SXS^+ = Y$ $SZS^+ = Z$

$$CNOT = \begin{bmatrix}1&0&0&0\\0&1&0&0\\0&0&0&1\\0&0&1&0\end{bmatrix}$$

— CNOT combinations —

P	$CNOT\ P\ CNOT^+$
$X \otimes I$	$X \otimes X$
$I \otimes X$	$I \otimes X$
$Z \otimes I$	$Z \otimes I$
$I \otimes Z$	$Z \otimes Z$

Unitary operation U can be used for addressing quantum errors but its original version ① defined for continuum of coherent errors

digitisation

With its digitised version ② now, a finite set of errors is gathered for any error. In other words, continuum dissected into parts

These dissected parts can be described by the Pauli Gates Clifford Gates...

Quantum error types

The result of digitisation of the errors, there are two fundamental quantum error types

Pauli X-type errors; $X|0\rangle = |1\rangle$, $X|1\rangle = |0\rangle$ X: bit-flips $X|\psi\rangle = \alpha X|0\rangle + \beta X|1\rangle = \alpha|1\rangle + \beta|0\rangle$

Pauli Z-type errors; $Z|0\rangle = |0\rangle$, $Z|1\rangle = -|1\rangle$ Z: phase-flips $Z|\psi\rangle = \alpha Z|0\rangle + \beta Z|1\rangle = \alpha|0\rangle - \beta|1\rangle$

Z; $|0\rangle, |1\rangle$ basis vectors are eigen vectors in Z

$$Z = \begin{bmatrix}1&1\\|0\rangle&-|1\rangle\\1&1\end{bmatrix} = \begin{bmatrix}1&0\\0&-1\end{bmatrix} \quad \lambda_1 = 1, \lambda_2 = -1$$