

Shor's 9-bit code

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$$|0\rangle \xrightarrow{\text{encode}} \frac{1}{\sqrt{2}} [(|1000\rangle + |1111\rangle)(|1000\rangle + |1111\rangle)(|1000\rangle + |1111\rangle)] = \frac{1}{\sqrt{2}} (|1000\rangle + |1111\rangle)^{\otimes 3} = |0_3\rangle$$

$$|1\rangle \xrightarrow{\text{encode}} \frac{1}{\sqrt{2}} [(|1000\rangle - |1111\rangle)(|1000\rangle - |1111\rangle)(|1000\rangle - |1111\rangle)] = \frac{1}{\sqrt{2}} (|1000\rangle - |1111\rangle)^{\otimes 3} = |1_3\rangle$$

$$|1_3\rangle = \alpha|10\rangle + \beta|11\rangle \rightarrow \alpha|10_3\rangle + \beta|11_3\rangle$$

with environment entangling to system $|e_3\rangle$

$$|e_3\rangle |1_3\rangle = \alpha|e_3\rangle |10_3\rangle + \beta|e_3\rangle |11_3\rangle \xrightarrow{\text{error}} (\text{entangle with the environment})$$

$$\Rightarrow |e_3\rangle |10_3\rangle \rightarrow |a_0\rangle |10\rangle + |a_1\rangle |11\rangle$$

$$|e_3\rangle |11_3\rangle \rightarrow |a_2\rangle |10\rangle + |a_3\rangle |11\rangle$$

entanglement with environment

$$|e_3\rangle (\alpha|10_3\rangle + \beta|11_3\rangle) = \frac{|e_3\rangle}{\sqrt{2}} [\alpha(|1000\rangle + |1111\rangle)^{\otimes 3} + \beta(|1000\rangle - |1111\rangle)^{\otimes 3}]$$

$$\Rightarrow \frac{|e_3\rangle}{\sqrt{2}} [\alpha(|1000\rangle + |1111\rangle)] + \frac{|e_3\rangle}{\sqrt{2}} [\beta(|1000\rangle - |1111\rangle)]$$

error on 1st qubit

$$\Rightarrow \frac{\alpha}{\sqrt{2}} [(|a_0\rangle |10\rangle + |a_1\rangle |11\rangle) \otimes |100\rangle + (|a_2\rangle |10\rangle + |a_3\rangle |11\rangle) \otimes |111\rangle] + \frac{\beta}{\sqrt{2}} [(|a_0\rangle |10\rangle + |a_1\rangle |11\rangle) \otimes |100\rangle - (|a_2\rangle |10\rangle + |a_3\rangle |11\rangle) \otimes |111\rangle]$$

$$\frac{\alpha}{\sqrt{2}} [(|a_0\rangle + |a_3\rangle)(|1000\rangle + |1111\rangle) + (|a_1\rangle - |a_2\rangle)(|1000\rangle - |1111\rangle) + (|a_1\rangle + |a_2\rangle)(|1100\rangle + |1011\rangle) + (|a_1\rangle - |a_2\rangle)(|1100\rangle - |1011\rangle)]$$

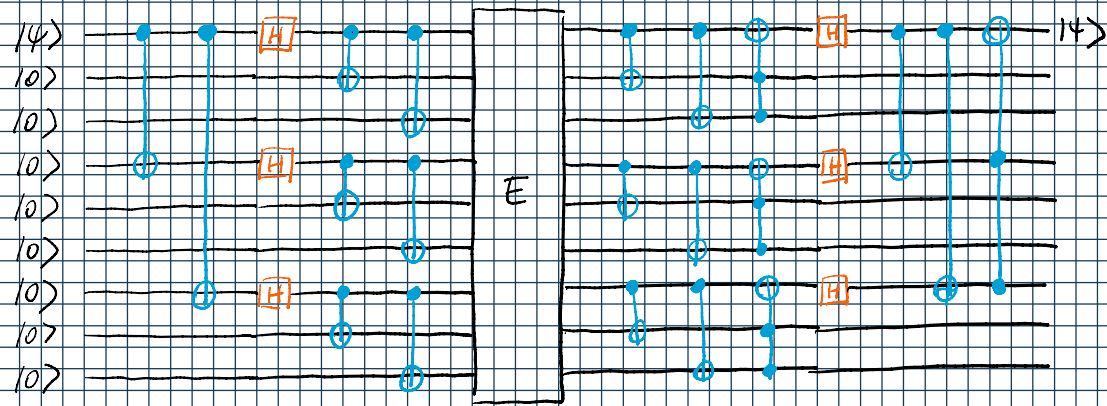
environment qubit

correct qubit: $|1000\rangle + |1111\rangle$

$|10_3\rangle$ without the last 6 qubits

$$\frac{\beta}{\sqrt{2}} [(|a_0\rangle + |a_3\rangle)(|1000\rangle - |1111\rangle) + (|a_1\rangle - |a_2\rangle)(|1000\rangle + |1111\rangle) + (|a_1\rangle + |a_2\rangle)(|1100\rangle - |1011\rangle) + (|a_1\rangle - |a_2\rangle)(|1100\rangle + |1011\rangle)]$$

$|11_3\rangle$ without the last 6 qubits



last 6 qubits are not entangled (assume 1st qubit has error)
 \Rightarrow ignore last 6 for simplicity

1) did entanglement with ancillary bit

2) measure ancillary bit

3) if no error; $\alpha(|1000\rangle + |1111\rangle) + \beta(|1000\rangle - |1111\rangle)$
 $|10_3\rangle \quad |11_3\rangle$

if ancillary bit tells otherwise:

wrong sign 2nd row; apply Z