Probability, P-Amp, Ramsey

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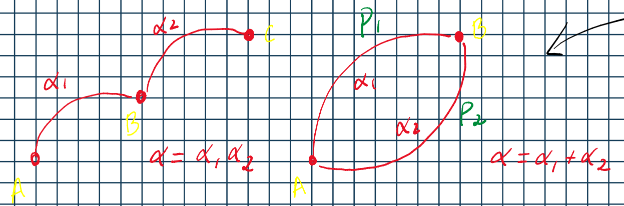


system evolving from state A to B

For a classical system Kolmogorov prob. formalism

$$P = P_1 + P_2$$

Quantum theory:

we first need to find prob. amplitude; α

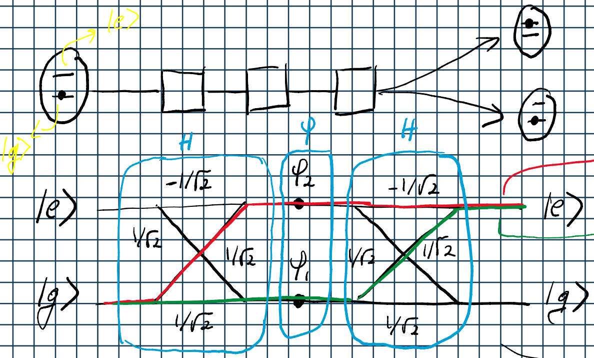
$$P = |\alpha|^2 \quad \text{add prob. amp.s not prob.s}$$

$$\alpha_1 = |\alpha_1| e^{i\varphi_1}$$
$$\alpha_2 = |\alpha_2| e^{i\varphi_2}$$
$$e^{i\varphi} = \cos\varphi + i\sin\varphi$$

$$P = |\alpha|^2 = |\alpha_1 + \alpha_2|^2 = |\alpha_1|^2 + |\alpha_2|^2 + \alpha_1\alpha_2^* + \alpha_2\alpha_1^*$$
$$= |\alpha_1|^2 + |\alpha_2|^2 + |\alpha_1\alpha_2| e^{i(\varphi_1-\varphi_2)} + |\alpha_1\alpha_2| e^{i(\varphi_2-\varphi_1)}$$
$$= |\alpha_1|^2 + |\alpha_2|^2 + |\alpha_1\alpha_2| \left(e^{i(\varphi_1-\varphi_2)} + e^{-i(\varphi_1-\varphi_2)}\right)$$
$$= |\alpha_1|^2 + |\alpha_2|^2 + 2|\alpha_1\alpha_2| \cos(\varphi_1-\varphi_2) = \underbrace{P_1 + P_2}_{\text{classical part}} + \underbrace{2\sqrt{P_1P_2}\cos(\varphi_1 - \varphi_2)}_{\text{quantum interference term}}$$

quantum interference term; it can be positive, negative depending on phase

— RAMSEY INTERFEROMETRY —



$$\alpha = \frac{1}{\sqrt{2}} e^{i\varphi_2}\left(-\frac{1}{\sqrt{2}}\right) + \frac{1}{\sqrt{2}} e^{i\varphi_1}\left(\frac{1}{\sqrt{2}}\right)$$

$|g\rangle \to |e\rangle$

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$$P = |\alpha|^2 = \frac{1}{4} + \frac{1}{4} - \frac{1}{4} 2\cos(\varphi_2 - \varphi_1) = \frac{1}{2} - \frac{1}{2}\cos\varphi; \quad = \frac{1}{2}(1-\cos\varphi) = \sin^2\varphi/2$$

Ramsey interferometry via matrix & circuit diagrams;

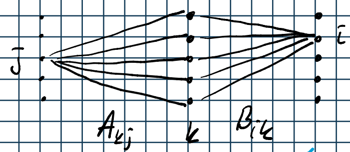
$$\overset{H}{\frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}} \overset{\varphi}{\begin{pmatrix} e^{i\varphi_2} & \\ & e^{i\varphi_1} \end{pmatrix}} \overset{H}{\frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}} = \begin{pmatrix} \cos\varphi/2 & -i\sin\varphi \\ i\sin\varphi/2 & \cos\varphi/2 \end{pmatrix}$$

$$\text{—}\boxed{H}\text{—}\overset{\varphi}{\bullet}\text{—}\boxed{H}\text{—}$$

— CIRCUIT DIAGRAMS & MATIX MULTIPLICATION

$$\underbrace{\text{—}\boxed{A}\text{—}\boxed{B}\text{—}}_{U = BA} \equiv \text{—}\boxed{U}\text{—}$$

$$U_{ij} = \sum_k A_{ik} B_{kj}$$

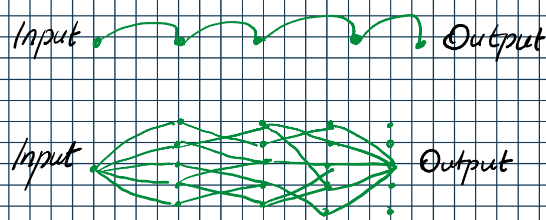


$$\underbrace{\text{—}\boxed{A}\text{—}\boxed{B}\text{—}\boxed{C}\text{—}}_{CBA}$$

$$\underbrace{\text{—}\boxed{H}\text{—}\overset{\varphi}{\bullet}\text{—}\boxed{H}\text{—}}_{H\varphi H}$$

DETERMINISTIC, PROBABILISTIC & QUANTUM COMPUTING

DETERMINISTIC



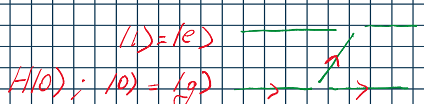
classic prob.: $P = P_1 + P_2 + \ldots$

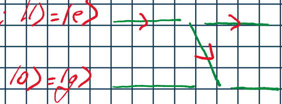
quantum prob.: $P = P_1 + P_2 + \ldots + 2\sqrt{P_1P_2}\cos\varphi + \ldots$

GATE-STATE EXAMPLES

$|g\rangle \to |0\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$ $\quad H = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$;

$|e\rangle \to |1\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$





$$H|0\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}\begin{pmatrix} 1 \\ 0 \end{pmatrix} = \frac{1}{\sqrt{2}}\left\{\begin{pmatrix} 1 \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 1 \end{pmatrix}\right\} = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$$

$$H|1\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}\begin{pmatrix} 0 \\ 1 \end{pmatrix} = \frac{1}{\sqrt{2}}\left\{\begin{pmatrix} 1 \\ 0 \end{pmatrix} - \begin{pmatrix} 0 \\ 1 \end{pmatrix}\right\} = \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)$$

$$H(H|0\rangle) = |0\rangle \qquad H(H|1\rangle) = |1\rangle$$