* Operation research is the study of scientific approach to decision-making. Through mathmetical modeling it seek to design improve and operate complex systems in the best possible way. OR is a set of activity which create value for services minimize negative factors transforming input into output to create a value.

* Relationship of OR & managers
  -> manager Recognize that the problem exists by analysing organiza-tional symptoms.
  -> manager & OR specialist decide what variables are involved
  -> OR specialist Investigate methods for solving the problem & find various solutions
  -> manager & OR specialist determine which solution is most effective
  -> manager choose the solution.

* Tools & Techniques
  -> Linear programming -> mathemetical model -> queue theory
  -> Inventory control -> Game theory -> Goal programming

* Applications
  -> OR is a problem solving & decision making technique. OR is considered a kit of scientific & programmable rules which provides management a quant-itive basis for decision concerning the operation under it's control.
    -> Allocation & Distribution in projects -> marketing -> finance
    -> Production & facilities planning -> programmed decision -> org. behaviour

* Limitation of OR.
(1) Magnitude of computation (2) Non-quantative factors
(3) Distance between user & analyst (4) Time & money cost
(5) Implementation

* Linear Programming problem is a decision making Problem in which decision maker has to decide level of different alternatives under some condition.
* Decision variable : A variable denoting the number of different alternative to be manufactured : $x_1, x_2 \dots x_n$
* objective function $z = c_1x_1 + c_2x_2 + \dots c_nx_n$
* Constraints : STCs
* Slack variable : Amount of unused resources $x_1+3x_2 \le 10 \rightarrow$ $x_1+3x_2+S_1=10$
* Surplus variable: Amount of shortage resources $2x_1-5x_2 \ge 20 \rightarrow$ $2x_1-5x_2-S_1=20$
* Dummy variable : variable added to make identity matrix : $D_1, D_2 \dots D_n$ if stc has $\le$ constant $\rightarrow +S_1$, $\ge$ Constant $\rightarrow -S_2+D_1$, $=$ Const $\rightarrow$ $+D_2$ where $D_2, D_1$ Take value of m where m is calculated as: if z is maximized m is negative else positive. m takes value of $10 * R$ where R is number of digit in highest coefficient in the ~~[illegible]~~ Obj function.
* The variables That makes identity matrix is known as basic variable. otherwise the variables are called non-basic variable.
* Simplex method
(1) Rewrite z & stc into standard form by converting stcs to equation.
(2) Put the values in simplex table

| Cb | xb | Cj | $C_1$ | $C_2$ | … | $C_n$ | R |
|---|---|---|---|---|---|---|---|
| | | C | $x_1$ | $x_2$ | … | $x_n$ | |

(3) Calculate $Z_j$ by taking sums of Cb × Col value.
(4) Calculate $Z_j - C_j$ which indicate the optimum solution. for max(z) soln is optimum if all $Z_j - C_j \ge 0$ & for min(z) if $Z_j - C_j \le 0$. if optimum stop.
(5) for max(z) Col having most negative $Z_j - C_j$ is key column else most pos.
(6) find the ratio of each row by ratio = const / key, do not calculate the ratio for if key col value = 0 or - Row having minimum ratio is key row / Leaving row

(7) for leaving row, new value = oldvalue / Key element
for others new value = old value − (key element × new value of leaving row)

(8) Repeat from (2) if optimum, write min/max value as value of $Z_j$ in C Column & Quantity of variables in Xb as Corresponding value in C Column. else Replace Cb value & xb As Key Col's basic var.

# if there is tie in $Z_j - C_j$, Choose left one & if there is tie in ratio, Choose right one.

# if ratio are infinite or negative, the problem is unbounded. There is no solution possible.

# if atleast one artificial variable remains with non zero value, it is infeasible solution.

* Primal & dual

(1) Normalize the given equations to make ≥ for min(z) or ≤ for max.

(2) you can leave = unchanged or ~~into~~ Convert it into both ≥ & ≤.

(3) Rewrite. min (z) → max(z), max(z) → min(z)

→ Coefficient of z becomes new constants. Constants of stc becomes new coefficient of vars in z.

→ Sign of stc is flipped/replaced to match as stated in (1)

→ Coefficient to stc are value of transpose of coeff. of old stc

→ if third inequality have '=' new third variable is unrestricted. and vice versa. others are ≥ 0

* Graphical method

(1) formulate problem, replace inequality with = & construct the graph & plot

(2) Point the lines to the direction based on their inequality signs.

(3) Identify feasible solution & find optimum points.

(4) Evaluate the obj function at optimum point to get required maximum value

* Transportation problem is a problem where we have to figure out source and the destination from various options and cost so that our cost is minimum.

# NWCM (North west corner method)

(1) Assign as much as possible to northwest cell & cancel the row/col if it can no longer demand or supply.

(2) repeat (1) untill all demand & supply are completed. This is initial feasible solution.

# Least cost method

(1) Allocate as much as possible to cell having lowest cost & cancel the row/col that can no longer demand or supply.

(2) repeat (1) untill all demand & supply are completed. This is initial feasible solution.

# Vogel's approximation method.

(1) Tabulate the given information & balance the problem if it's not already balanced.

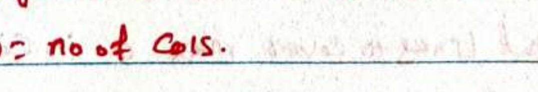
(2) Convert the table to regret table by subtracting all value with max value if we are asked to maximize profit

(3) Calculate penalty (difference of two least cost) for all cells & rows.

(4) Allocate the max possible quantity to the least cost cell having largest penalty (cost difference) & Repeat 3 & 4 untill all demand & supply are done.

* A problem is said to be degenerate if number of allocated cells ≠ (m+n-1) where m = no of rows, n = no of cols.

* A close path that starts from most negative $\Delta_{ij}$ & moves horizontally or vertically that may or may not turn in any allocated cell and ends to the starting point is called a loop. Shape can be:



# Testing optimality with modified distribution method (MODI)

(1) find initial feasible solution & test for degeneracy. if degenerate allocate 0 quantity to least cost unoccupied cell to make solution non-degenerate.

(2) Calculate $R_i$ and $K_j$ for all rows & cols where $C_{ij}$ = occupied cell cost

(3) Take $R_i$ or $K_j = 0$ first for a row/col having most allocation to make easy and calculate rest $R_i$ & $K_j$ based on $C_{ij} = R_i + K_j$

(4) Calculate improvement index $\Delta_{ij}$ for each non occupied cells as $\Delta_{ij} = C_{ij} - (R_i + K_j)$. if all $\Delta_{ij}$ are positive, The solution is optimal. else the solution is not optimum. if optimum, stop

(5) form a loop in the improvement indexes. assume starting is '+' every turning point changes the sign.

(6) subtract minimum allocation of '-' cell from all '-' cells & add to all '+' cells. This is modified solution. Repeat from (2) until solution is optimal.

# Transhipment Problem is similar to transportation except here we may have intermediate points. a plant can also supply to another storage or plant.

# The Assignment Problem is fundamental combination of optimization problem where we basically have to assign jobs to workers/machines based on cost so that the cost is min.

# Hungarian method

(1) Subtract each cells from highest value in cells to convert it into regret table if the problem requires max profit. make problem balanced first.

(2) Find Row min and subtract from all values in row. & same for Column

(3) Draw min straight lines to cover max 0 in cost matrix. if lines are equal to numbers of row, optimal solution is obtained. else subtract min cost from uncrossed cell from all uncrossed cell and add to double crossed cell & repeat (3) until optimal solution is obtained.

(4) Count the number of zero in first row/col. if there is only one zero, make a box and cancel all horizontal & vertical cells. it indicate allocation for corresponding worker/machine & job.

(5) Repeat (4) until all job are assigned. a box at last you can any if any row/col have two zero but initially try to skip those rows/cols.

* The analysis of waiting line of customers, their behaviour, Pattern of their arrival, departure and service etc. is called the queuing theory. The main objective of queuing theory is to minimize the waiting time in minimum operating cost.

* Service Process / Queuing models.
  (1) one queue, one service center facility (2) one queue, multiple service center facility.
  (3) Single queue, multistage facility (4) several queue single service center facility.
  (5) several queue, several service center facility.

* Arrival rate = $\lambda$ (Should be in number of customer per x time (usually day))
* Service rate = $\mu$ ( " " " " " " " " " " )
* Queue Length = $L_q$ (Average number of customer in queue) $\frac{\lambda}{\mu}\left(\frac{\lambda}{\mu-\lambda}\right)$
* System " = $L_s$ ( " " " " in System) $\frac{\lambda}{\mu-\lambda}$
* waiting time in queue = $T_q$ ( " Time before starting service) $\frac{\lambda}{\mu}\left(\frac{1}{\mu-\lambda}\right)$
* " " " System = $T_s$ ( " " " finishing " ) $\frac{1}{\mu-\lambda}$
* utilization factor = $P$ (The % of time system is busy) $\frac{\lambda}{\mu}$
* Idle time / Probability That system is free = $1-P$
* Probability That There are K customers in system : $P(n=K) = (1-P)(P)^K$
* " " " " atleast K " " " = $P(n \geq K)$ $(P)^K$
* " " " " more than K " " " = $P(n > K)$ $(P)^{K+1}$

# An inventory management system provides the organization structure and the operating policies for maintaining and controlling goods to be stocked. This system is responsible for ordering and receipt of goods, timing the order, placement and keeping track of what has been ordered, the quantity & from whom. Inventory management is one of the important functions under operation system. Basically inventory is the stock of any item or resource that is used by organization to create different types of outputs. Inventory refers to stock of raw materials, finished goods, component parts, supplies and work in progress etc.

# functions of Inventory control/management.

→ To meet anticipated customer demand → To smooth production requirements
→ To decouple operations → To protect against stockouts
→ To take advantage of order cycles → To take advantage of quantity discounts.

# Types of inventory

(1) Raw materials & purchased parts (2) Work in progress (3) finished goods
(4) Anticipated inventory (stock to satisfy the demand) (5) Buffer or safety stock
(6) decoupling inventory (decoupled due to unequal capacity of stages, repairs etc)
(7) MRO (inventories devoted to maintainance, repair and operating supplies to keep machine producting)

# Types of costs

(1) Holding/Carrying Cost : Cost of physically having item in the inventory, maintainance & security
(2) Ordering/Setup Cost : cost of ordering & receiving goods in inventory, telephone, mail, labor, etc
(3) Stockout Cost : opportunity cost lost due to stockout/shortage, goodwill, royalty switching cost etc
(4) Total Inventory Cost = Purchase Cost + ordering cost + Carrying Cost + Shortage Cost

# ABC Inventory System : ABC stands for always better control. So it is called always better control technique as this technique is used to control raw material, components, and WIPs.

Normally : A → 10-20% volume → 70-85% value has low safety stock
B → 20-30% " → 10-25% " has average " "
C → 60-70% " → 5-15% " has decent " "

# Steps of ABC inventory classification.

(1) Identify all items and calculate annual value as unitcost × annual consumption rate

(2) Arrange the items in descending order and express the annual value in percentage of the total value of all items and also calculate % based on cumulated annual consumption.

(3) determine class as per rule of ABC inventory system based on %.

# Calculation of different costs.

* A = Annual demand , C = Carrying cost per unit , O = Ordercost per order

(1) $EOQ = \sqrt{\frac{2AO}{C}}$

(2) Number of order = $A / EOQ$

(3) Total ordering cost = $AO / EOQ$ = TOC

(4) Total Carrying cost = $\frac{EOQ \times C}{2}$ = TCC

(5) Total Cost (Excluding Product cost) = TOC + TCC = TC

(6) Length of order cycle = $EOQ / A$ = cycle time

(7) Total cost (including Product cost) = TC + (EOQ × Purchase Price)

(8) Average inventory = $EOQ/2$

# The replacement theory is used in the decision making process of replacing a used equip-ment with a substitute. mostly a new equipment of better usage. This kind of replace-ment might be necessary due to the deteriorating property or failure or breakdown of particular equipment. Efficiency of any equipment gradually decreases & maintainance cost incre we need to find optimum replacement period to find the time when the equipment should be replaced.

# Replacement of item that deteriorate / maintainance cost increase assuming that the value of money does not change during the period.

# Replacement of item whose maintainance cost increase with time and also the value of money changes with time

# Group Replacement Policy.

* Replacement of item that deteriorate with time.

(1) Tabulate given data As follow: 1 | 2 | 3 | 4 | 5 | 6 | 7

1-> years of service

2-> Resale value

3-> Depreciation Cost = [Purchase price - Resale value]

4-> Annual maintainance cost

5-> Cummulative Annual " "

6-> Total Cost = Depreciation Cost + Cummulative Annual maintainance Cost

7-> Average Annual Cost = Total cost / year of service.

(2) Replace in the year where average annual cost is minimum. it is optimum Replacement period.

(3) As per policy: if the running & maintainance cost of the machine for the next year is more than the average annual cost of selected year, then replace at the end of the year.

(4) we have solved our replacement problem.

# Group and individual replacement

→ In individual replacement, an item is replaced, immediately after it's failure.

→ In group replacement, decision is taken as to when all the items must be replaced irrespective of the fact that the item have failed or not. With a provision that if item fails before group replacement time, it can be replaced individually.

# Steps

(1) find probability of failure for interval / week / months. $P_n = \frac{g_n - g_{n-1}}{100}$ where $g_n$ = given failure %. for the interval on problem (cumulated %) $\boxed{\sum_1^n P_n = 1}$

(2) find number of replaced items in interval $N_n = N \times P_n + N_{n-1} \times P_1 + \dots$ $\dots + n_1 \times P_{n-1}$ where $N$ = Total items

(3) find mean age of the items as $\sum_1^n n \times P_n$

(4) find Avg failure in each week $N/\text{mean age}$

(5) Avg cost of replacing item individually = avg failure of week × cost of indiv rep.

(6) find optimum cost of group replacement 1 | 2 | 3 | 4

1 → interval of service

2 → individual replacement cost of the week

3 → Cost of group replacement $C_i = C_{i-1}$ + individual replacement cost

$C_0 = N *$ group replacement cost.

4 → Average cost = Cost of group replacement / interval of service

(7) find minimum Avg group replacement cost that is optimum replacement period & cost.

(8) if optimum group replacement cost > avg individual replacement cost, individual replacement should be preferred.

# Game theory is a type of decision theory in which one choice of action is determined by / after taking into account all possible alternatives available to an opponent playing the same game rather than just by the possibility of several outcome results. The game theory problem is based upon minimax criteria. It implies that each player will act so as to maximize his minimum gain or minimize his maximum Loss.

# Characteristics of game theory.

(1) Change of strategy
 -> Game of strategy : Activities are defined by skills
 -> Game of chance : Activities are defined by chance
 -> Game of chance & strategy : mostly all general games are of this nature

(2) Number of person
 -> n person game / n player game : Number of players is N
 * Person may mean individual or a group.

(3) Numbers of Activities : finite & infinite

(4) " " Choices available to each Players : finite & infinite

(5) Information to the players about the past experience of other players:
 -> completely available -> Partially available -> Not available

(6) Payoff : quantative measure of satisfaction to a person get at the end of each play.

* Competitive game : A competitive situation in game.
 -> There are finite numbers of competitors $n \geq 2$
 -> Each player have a list of finite numbers of possible activity.

* Zero-sum & Non zero-sum Game
 -> if Gain of one player is Loss of another player. They make payment to each other

* Strategy : The strategy of a given player is a set of rule that he follow in any activity.
 -> Pure strategy : if a player know exactly what the other player is going to do. a deterministic situation occurs. The obj function is to maximize the gain.

-> Mixed Strategy : if a player is guessing as to which action is to be selected by other player on particular occasion. A probablistic situation occurs. The obj is to maximize the expected gain.

# Two person zero sum game

-> A game with only two player is called two person game. if a Loss of one player is equal to gain of another player is called two person zerosum game.

# Pay off matrix

-> Player A has m activity
-> " B " N "

$$\begin{array}{c|cccc} & 1 & \cdots & \cdots & m \\ \hline 1 & V_{11} & V_{12} & V_{13} & \\ \vdots & & & & \\ \vdots & & & & \\ n & & & & V_{nm} \end{array}$$

(A above the columns, B beside the rows)

* minmax- maxmin (Pure)

#(1) Find min value for each row and max value for each column.
(2) find max of min value and min of max value. if both are same, it is the value of game / Saddle point.

* odd / Arithmetic / Shortcut method ($2\times2$)

(1) find absolute differences of value in $r_1$ & put it against $r_2$ same for $C_1$ & $C_2$ & $R_2$, Let values $Cv_1$, $Cv_2$, $Rv_1$, $Rv_2$

(2) find $P_1, P_2, q_1, q_2$ as $P_1 = \frac{Rv_1}{Rv_1+Rv_2}$, $P_2 = \frac{Rv_2}{Rv_1+Rv_2}$, $q_1 = \frac{Cv_1}{Cv_1+Cv_2}$, $q_2 = \frac{Cv_2}{Cv_1+Cv_2}$

(3) find expected gain of A as value in cell 11 $\times P_1$ + value in cell 21 $\times P_2$ & so on

(4) find expected loss of B " " " cell 11 $\times q_1$ + cell 12 $\times q_2$ and so on

* Dominance method (Reduce size of matrix) : if $R_1 \leq R_3$ Remove $R_1$, if $C_1 \geq C_3$ Remove $C_1$

* Algebraic method $\begin{array}{cc} & B \\ A & \begin{bmatrix} a & b \\ c & d \end{bmatrix} \end{array}$

(1) A play's $(P_1, P_2)$, $P_1 = \frac{d-c}{(a+d)-(b+c)}$, $P_2 = 1 - P_1$

(2) B play's $(q_1, q_2)$, $q_1 = \frac{d-b}{(a+d)-(b+c)}$, $q_2 = 1 - q_1$

(3) value of game $V = \frac{a\cdot d - b\cdot c}{(a+d)-(b+c)}$

$\boxed{V = aP_1 + bP_2 \text{ or any eqn} \quad aP_1 + bP_2 = cP_1 + cP_2 \text{, substitute } P_2 = 1 - P_1}$

* Graphic ($n\times2$ or $2\times n$)

(1) Draw two vertical parallel line with scale on it. Plot all points as line on those two scale. From top, the lowest point is known as value of game. use formula to calculate intersection point of lines. Find value of transpose of those value as $aP_1 + bP_2$, $cP_1 + cP_2$) find $P_1, P_2$ $aq_1 + cq_2$, $bq_1 + dq_2$, find $q_1$ $q_2$