INTERFERENCE

Two wave sources are coherent if their frequency & waveform are identical & their phase difference is constant

Interference is a phenomenon in which 2 or more coherent waves travelling in the same direction superpose to form a resultant wave of greater, lower, or the same amplitude

METHODS OF PRODUCING INTERFERENCE

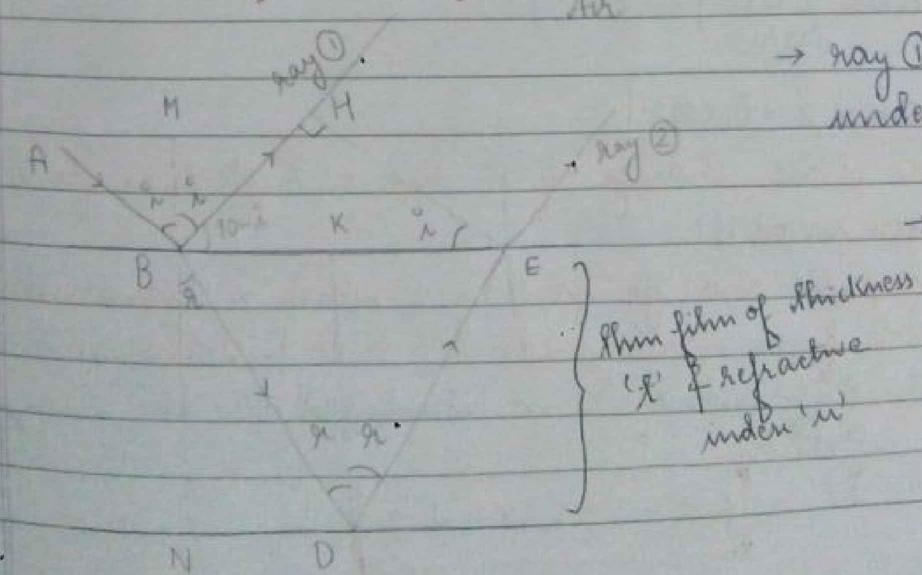
By Division of Wavefront
- → Young's Double Slit
- → Fresnel's biprism
- → Lloyd's Mirror

By division of Amplitude
- → Thin film Interference
- → Newton's ring
- → Wedge shape film interference

Types of Interference:
- → CONSTRUCTIVE: Intensity is maximum. Crests or Troughs of the 2 waves superpose
- → DESTRUCTIVE: Min intensity. Crest of one wave & trough of the other superpose

★ INTERFERENCE IN THIN FILM OF UNIFORM THICKNESS

→ ① Due to Reflected Light:



→ ray ① & ray ② will undergo interference at ∞

→ The ray AB is partially reflected & partially transmitted at B

→ Beyond the normal HE, path of ray ① & ray ② will be same

→ Path Difference b/w the 2 waves: $\Delta x = (BD + DE)_{\text{in film}} - (BH)_{\text{in air}}$

$\rightarrow \Delta x = (BD+DE)\mu - (BH)$

ΔBDK & ΔDKE are congruent $\rightarrow BK = KE$ & $BD = DE$

$\rightarrow \boxed{\Delta x = 2\mu BD - BH}$

in ΔBDK, $\cos r = \frac{DK}{BD} = \frac{t}{BD}$ $\rightarrow \boxed{BD = \frac{t}{\cos r}}$

in ΔBHE, $\sin i = \frac{BH}{BE}$ $\rightarrow BH = BE \sin i$
$= 2\, BK \sin i$

Now, in ΔBDK, $\tan r = \frac{BK}{KD} = \frac{BK}{t}$

$\rightarrow BK = t \tan r$

$\Rightarrow BH = 2t \tan r \sin i$

$$\Rightarrow \Delta x = \frac{2\mu t}{\cos r} - 2t \tan r \sin i$$

$$= \frac{2\mu t}{\cos r} - 2t \sin i \frac{\sin r}{\cos r} \frac{\sin r}{\sin r}$$

using Snell's law, $\frac{\sin i}{\sin r} = \mu$

$$\rightarrow \Delta x = \frac{2\mu t}{\cos r} \{1 - \sin^2 r\}$$

$$\rightarrow \boxed{\Delta x = 2\mu t \cos r}$$

★ Ray 1 suffers phase change of π due to reflection from denser medium at B.

There is no phase change for ray 2 which is transmitted at B & reflected from rarer medium at D & transmitted at E.

∴ Net path difference of $\lambda/2$ is introduced b/w the 2 rays due to reflection

$$\Rightarrow \boxed{\Delta x = 2\mu t \cos r \pm \frac{\lambda}{2}}$$

★ Interference pattern in the reflected & transmitted system is complementary

★ <u>CONSTRUCTIVE INTERFERENCE OR MAXIMA :</u>

path difference, $\Delta x = n\lambda$

$\rightarrow 2\mu t \cos r \pm \frac{\lambda}{2} = n\lambda$

★ $\rightarrow \boxed{2\mu t \cos r = (2n \pm 1)\frac{\lambda}{2}}$

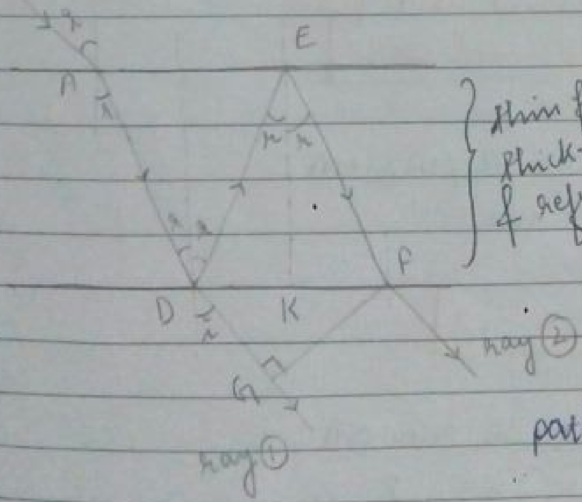
★ <u>DESTRUCTIVE INTERFERENCE OR MINIMA :</u>

path difference, $\Delta x = (2n \pm 1)\frac{\lambda}{2}$

$\rightarrow 2\mu t \cos r \pm \frac{\lambda}{2} = (2n \pm 1)\frac{\lambda}{2}$

★ $\boxed{2\mu t \cos r = n\lambda}$

→ ② Due to <u>Transmitted Light</u> :



thin film of thickness 't' & refractive index 'μ'

ray ① & ray ② will undergo interference at ∞

Ray AD is partially reflected & partially transmitted at D

path difference b/w the 2 waves, $\Delta x = (DE + EF)_{in\ film} - (DG)_{air}$

$\rightarrow \Delta x = (DE + EF)\mu - DG$

ΔDEK & ΔFEK are congruent $\rightarrow DE = EF$ & $DK = KF$

$\rightarrow \boxed{\Delta x = 2\mu(DE) - DG}$

In ΔDEK, $\cos r = \frac{EK}{DE} = \frac{t}{DE}$ $\rightarrow \boxed{DE = \frac{t}{\cos r}}$

In ΔDFG, $\sin i = \frac{DG}{DF} = \frac{DG}{DK+KF} = \frac{DG}{2DK}$

$\rightarrow DG = 2DK \sin i$

In ΔDEK, $\tan r = \frac{DK}{EK} \rightarrow DK = t \tan r$

$\rightarrow \boxed{DG = 2t \tan r \sin i}$

$\rightarrow \Delta x = \frac{2\mu t}{\cos r} - 2t \sin i \frac{\sin r}{\cos r} \frac{\sin r}{\sin r}$

$= \frac{2\mu t}{\cos r}\{1 - \sin^2 r\}$

$\rightarrow \boxed{\Delta x = 2\mu t \cos r}$

★ There is no phase change for ray 1 as it gets transmitted at D
Ray 2 also does not undergo phase change as it is reflected from rarer medium at D & E

★ $\Rightarrow \boxed{\Delta x = 2\mu t \cos r}$

★ CONSTRUCTIVE INTERFERENCE OR MAXIMA:

path difference, $\Delta x = n\lambda$

★ $\rightarrow \boxed{2\mu t \cos r = n\lambda}$

★ DESTRUCTIVE INTERFERENCE OR MINIMA:

path difference $\Delta x = (2n \pm 1)\frac{\lambda}{2}$

★ $\rightarrow \boxed{2\mu t \cos r = (2n \pm 1)\frac{\lambda}{2}}$

REFLECTED LIGHT

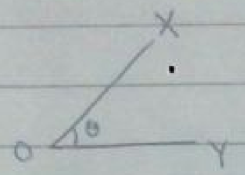
★ if $t \gg \lambda \rightarrow$ no interference takes place; only general illumination

★ if $t \ll \lambda \rightarrow$ path diff $\approx \frac{\lambda}{2} \rightarrow$ condition for destructive interference

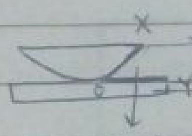
↳ film appears dark

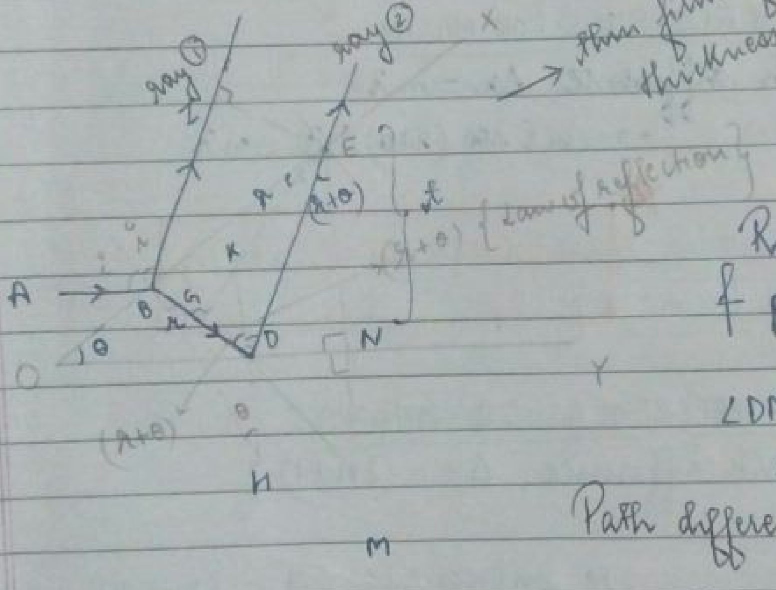
INTERFERENCE IN THIN FILM OF NON-UNIFORM THICKNESS

→ WEDGE SHAPED THIN FILM



When 2 plane surfaces OX & OY are slightly inclined to each other at a small angle, wedge shape is formed. It always has non-uniform thickness.





Ray AB is partially reflected & partially transmitted at B

$EN = NM = t$

$\angle DMN = \angle DEN = \angle HDM = (r+\theta)$

$\angle GEB = r$

Path difference, $\Delta x = (BD + DE)_{\text{in film}} - (BZ)_{\text{in air}}$

$$\boxed{\Delta x = (BG + GD + DE)\mu - BZ} \quad -(1)$$

from ΔBZE, $\sin i = \frac{BZ}{BE}$

from ΔBGE, $\sin r = \frac{BG}{BE}$

using Snell's law, $\mu = \frac{\sin i}{\sin r} = \frac{BZ}{BG}$

$\rightarrow BZ = \mu(BG)$

putting this value of BZ in (1), $\Delta x = \cancel{(BG)\mu} + (GD + DE)\mu - \cancel{(\mu)BG}$

$\Delta DEN \cong \Delta DMN$

$\rightarrow \Delta x = (GD + DE)\mu = (GD + DM)\mu$

$\rightarrow \Delta x = (GM)\mu$

in ΔGEM, $\cos(r+\theta) = \frac{GM}{EM}$

$\rightarrow GM = 2t\cos(r+\theta)$

$\rightarrow \boxed{\Delta x = 2\mu t\cos(r+\theta)}$

Ray (1) suffers a phase change by π due to reflection from denser medium at B

$\rightarrow \boxed{\Delta x = 2\mu t\cos(r+\theta) \pm \frac{\lambda}{2}}$

★ CONSTRUCTIVE INTERFERENCE OR MAXIMA:

path difference, $\Delta x = n\lambda$

$\rightarrow 2\mu t\cos(r+\theta) \pm \frac{\lambda}{2} = n\lambda$

★ $\rightarrow \boxed{2\mu t\cos(r+\theta) = (2n\pm1)\frac{\lambda}{2}}$

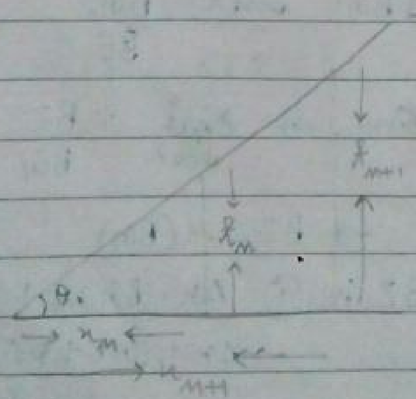
★ DESTRUCTIVE INTERFERENCE OR MINIMA:

path difference, $\Delta x = (2n\pm1)\frac{\lambda}{2}$

$\rightarrow 2\mu t\cos(r+\theta) \pm \frac{\lambda}{2} = (2n\pm1)\frac{\lambda}{2}$

★ $\boxed{2\mu t\cos(r+\theta) = n\lambda}$

FRINGE WIDTH:



Fringe width is the distance b/w two consecutive dark fringes.

If the n^{th} dark fringe is formed at a distance x_n from the edge of the wedge shaped film where the thickness is t_n then:

$t_n = x_n \tan\theta$ — (1)

Similarly, for $(n+1)^{th}$ dark fringe : $t_{n+1} = x_{n+1}\tan\theta$ — (2)

for normal incidence, $i=0 \rightarrow r=0$

$\therefore$ Condition for n^{th} dark fringe is :

$$2\mu t_n \cos\theta = n\lambda$$

Substituting for t_n from (1),

$$2\mu x_n \tan\theta . \cos\theta = n\lambda$$

$$\rightarrow \boxed{2\mu x_n \sin\theta = n\lambda} \quad \text{— (3)}$$

for $(n+1)^{th}$ dark fringe : $2\mu t_{n+1}\cos\theta = (n+1)\lambda$

substituting ~~form~~ t_{n+1} from (2),

$$2\mu x_{n+1} \tan\theta . \cos\theta = (n+1)\lambda$$

$$\rightarrow \boxed{2\mu x_{n+1}\sin\theta = (n+1)\lambda} \quad \text{— (4)}$$

Subtracting (3) from (4)

$$2\mu\{x_{n+1} - x_n\}\sin\theta = \lambda$$

fringe width, $\beta = x_{n+1} - x_n$

$$\rightarrow 2\mu\beta\sin\theta = \lambda$$

$$\rightarrow \boxed{\beta = \frac{\lambda}{2\mu\sin\theta}}$$

for small angle of the wedge, $\sin\theta \approx \theta$

$$\rightarrow \boxed{\beta = \frac{\lambda}{2\mu\theta}}$$

for air film, $\mu = 1 \rightarrow \beta_{air} = \frac{\lambda}{2\theta}$

Q. When seen by reflected light, why does an excessively thin film appear to be perfectly black when illuminated by white light

The path difference in reflected light is:

$$\Delta x = 2\mu t \cos r \pm \frac{\lambda}{2}$$

If thickness 't' is very small, $2\mu t \cos r$ can be neglected

$\rightarrow$ path difference, $\Delta x = \pm \frac{\lambda}{2}$

There is destructive interference

$\therefore$ An excessively thin film appears dark in reflected light

Q. A thin film illuminated by white light appears coloured when observed in reflected light

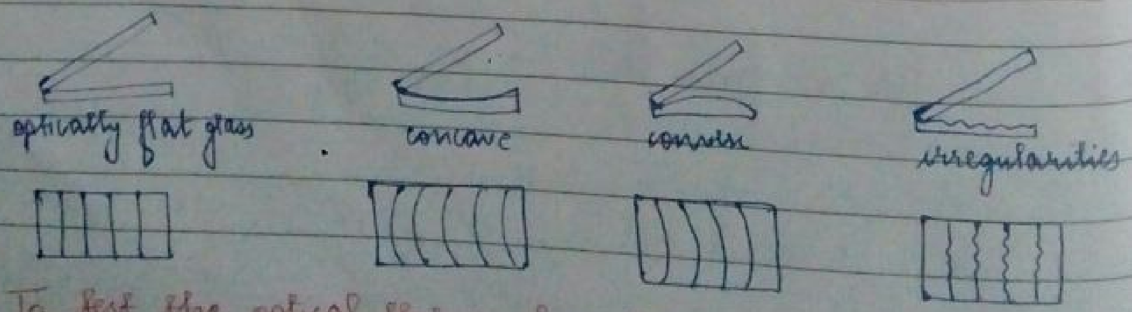
Condition for constructive interference in reflected light:

$$2\mu t \cos r = (2n \pm 1)\frac{\lambda}{2}$$

If white light is made incident on a thin film, constructive interference condition will be satisfied for different wavelengths at different angles of refraction 'r' & hence at different angles of incidence

Hence different colours will be seen at different angles in reflected light.

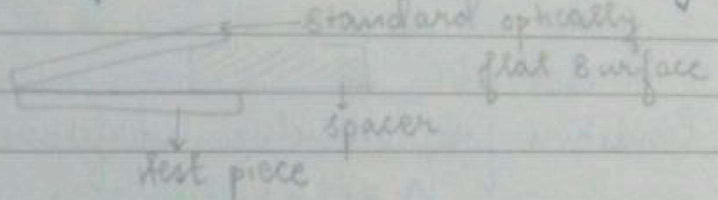
APPLICATIONS OF INTERFERENCE



→ To test the optical flatness of surfaces

Explain how interference pattern can be used for testing the optical flatness of surfaces

A wedge shaped air film is formed b/w the test piece & a standard optically flat surface by keeping them in contact at one edge & separating them by a spacer at the other edge



→ It is then illuminated by a monochromatic source of light

→ If the test piece is optically flat, the interference pattern will consist of straight fringes of equal width

If the test piece is not optically flat, the fringes will have irregular shape

→ When irregular fringes are observed, the test piece is polished & again tested till fringes of equal width are observed

→ Used in anti-reflection or non-reflection coating

Explain with a suitable diagram how the principle of interference is used in anti-reflection coating. Derive an expression for its thickness.

The anti reflection coatings reduce intensity of reflected light by destructive interference & hence enhance transmission.

Purpose of Anti-Reflection Coating

→ To enhance or ↑ transmittivity of material like glass

→ To ↓ loss of light due to reflection

$\boxed{\text{Coating: } \mu_{air} < \mu_{film} < \mu_{glass}}$

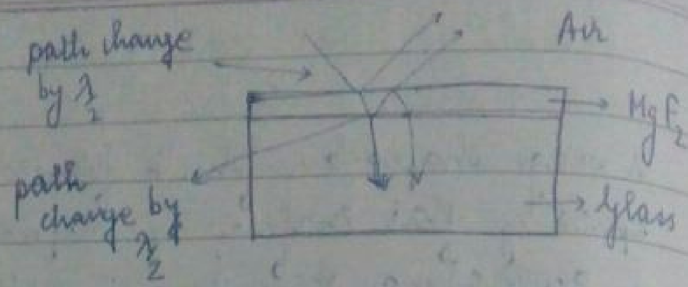
Commonly used coating: $MgCl_2$ or MgF_2

★ To enhance reflectivity, $\boxed{t = \frac{\lambda}{2\mu}}$

& $\boxed{\mu_{air} < \mu_{film} < \mu_{glass}}$

According to thin parallel film,

path diff: $\Delta = 2\mu t \cos r$



for normal incidence,

$r = 0$

$\rightarrow \Delta = 2\mu t$

Condition for Destructive Interference:

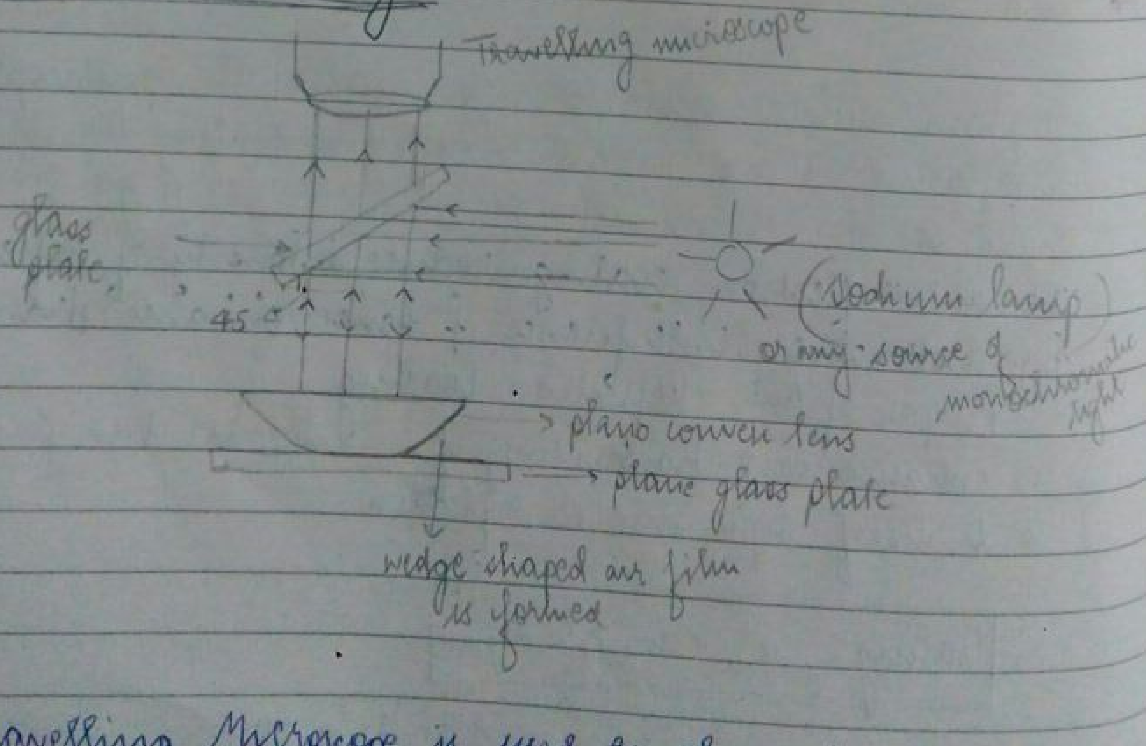
$$\Delta = (2n \pm 1)\frac{\lambda}{2}$$

$$\text{or } \Delta = \frac{\lambda}{2}$$

$$\rightarrow 2\mu t = \frac{\lambda}{2}$$

★ $$\boxed{t = \frac{\lambda}{4\mu}}$$

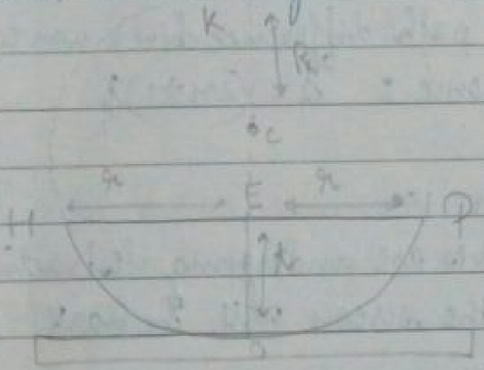
Newton's Rings



Travelling Microscope is used to observe Newton's rings

→ Alternate ~~bright~~ dark & ~~dark~~ bright circular fringes/rings.

An interference pattern is also observed on the glass plate below the lens due to transmission of light. This pattern will be exact opposite of the pattern due to reflection. Instead of central dark, central bright is observed.



Let 'R' be the radius of curvature of plano convex lens 'L'.
Condition for bright fringe:

$$2\mu t\cos r \pm \frac{\lambda}{2} = n\lambda$$

$$\rightarrow \boxed{2\mu t\cos r = (2n+1)\frac{\lambda}{2}}$$

For normal incidence, $\angle i = \angle r = 0$; for air film, $\mu = 1 \rightarrow \boxed{2t = (2n+1)\frac{\lambda}{2}}$

From Geometry, $HE \times EP = KE \times EO \Rightarrow r \times r = (KO - EO) \times EO$

$$\rightarrow r^2 = (2R - t) \times t = 2Rt$$

$$\rightarrow t = \frac{r^2}{2R} \rightarrow \frac{2r_n^2}{2R} = (2n+1)\frac{\lambda}{2}$$

$$\rightarrow r_n = \sqrt{\frac{(2n+1)\lambda R}{2}}$$

→ Diameter of n^{th} bright ring $D_n = 2r_n$

$$= \boxed{2\sqrt{\frac{(2n+1)\lambda R}{2}}}$$

Similarly, diameter of n^{th} dark ring, $\boxed{D_n = 2\sqrt{n\lambda R}}$

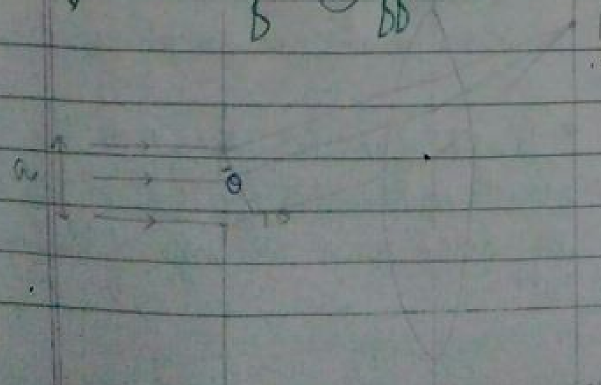
DIFFRACTION

Diffraction is defined as the bending of light around sharp edges of objects into the geometrical shadow.
For visible diffraction, the size of obstacle should be comparable to the wavelength of light

Types of Diffraction

	Fraunhofer Diffraction	Fresnel Diffraction
(i)	Source is at infinite distance	Source is at finite distance
ii)	Wavefront incident on the slit & the screen is plane	Wavefront incident on the slit & the screen is spherical or cylindrical
iii)	There is no path difference b/w the rays before entering the slit	There is path difference b/w the rays before entering the slit which depends upon distance b/w source & slit
iv)	Path difference depends only on angle of diffraction	Path difference b/w the rays depends on distance of slit from source as well as the screen & the angle of diffraction
(v)	Lenses are required to observe Fraunhofer diffraction.	Lenses are not required to observe Fresnel diffraction in the laboratory.

★ Fraunhofer Diffraction at Rectangular Aperture / Single Slit



Consider a parallel beam of light incident normally on a slit of width 'a' diffracted at angle 'θ'

path difference b/w extreme rays from the slit is:

$$\boxed{\Delta = a \sin\theta}$$

$\rightarrow$ phase difference, $\boxed{\phi = \frac{2\pi}{\lambda}(\Delta) = \frac{2\pi}{\lambda}(a \sin\theta)}$

The slit is now divided into 'N' parts of equal width 'da'. The width of each part is so small that it behaves like a point source.

As all parts are of equal width, the amplitude of waves transmitted by them will be same.

path difference b/w waves transmitted by 2 adjacent parts is:

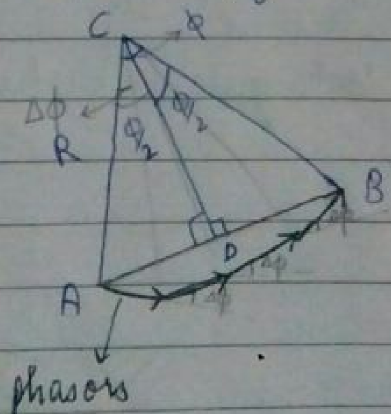
$$\boxed{\delta = da \sin\theta}$$

& hence ~~path~~ phase difference is: $\boxed{\Delta\phi = \frac{2\pi}{\lambda}(da \sin\theta)}$

$$\boxed{\begin{array}{l} a_1 = a_2 = a_3 = \dots = a_n = da = \frac{a}{N} \\ \rightarrow \sum_{n=1}^{N} a_n = a \end{array}}$$

Thus there will be 'N' amplitude phasors of the same amplitude & the same phase difference $\Delta\phi$ between adjacent phasors.

The 'N' phasors form an arc of a circle.



★ Length of arc AB represents the maximum amplitude 'E_m'

★ Length of arc AB represents the resultant amplitude 'E_θ' of the diffracted wave at angle of diffraction 'θ'

ΔACD & ΔBCD are congruent to each other

$$AD = DB = \frac{E_\theta}{2}$$

& $$\boxed{\alpha = \frac{\phi}{2} = \frac{\pi}{\lambda}(a \sin\theta)}$$

in ΔACD, $\sin\alpha = \frac{AD}{AC} = \frac{E_{\theta/2}}{R}$

$$\rightarrow \boxed{E_\theta = 2R\sin\alpha}$$

& $\theta = \frac{\text{arc}}{\text{radius}} \Rightarrow \phi = \frac{E_m}{R} = \frac{E_m}{\frac{E_\theta}{2\sin\alpha}}$

$$\rightarrow \boxed{E_\theta = E_m \frac{\sin\alpha}{\alpha}}$$

As intensity is proportional to square of amplitude,

$$I_\theta = kE_\theta^2 \quad \& \quad I_m = kE_m^2$$

$$\boxed{I_\theta = I_m\left(\frac{\sin\alpha}{\alpha}\right)^2}$$

→ PRINCIPAL MAXIMUM

$$E_\theta = \frac{E_m}{\alpha}\left[\sin\alpha\right]$$

$$E_\theta = \frac{E_m}{\alpha}\left[\alpha - \frac{\alpha^3}{3!} + \frac{\alpha^5}{5!} + \dots\right]$$

$$\rightarrow E_\theta = E_m\left[1 - \frac{\alpha^2}{3!} + \frac{\alpha^4}{5!} - \frac{\alpha^6}{7!} + \dots\right]$$

E_θ will be maximum if $\alpha = 0$

$$\rightarrow \frac{\pi}{\lambda} a\sin\theta = 0 \quad \rightarrow \boxed{\theta = 0}$$

Thus, the principal maximum is formed along the incident direction & hence is also called central maximum

MINIMA

For minimum intensity, $I_\theta = 0$

$\rightarrow I_m \left(\frac{\sin\alpha}{\alpha}\right)^2 = 0$

$\rightarrow \sin\alpha = 0$ but $\alpha \neq 0$

$\Rightarrow \boxed{\alpha = n\pi}$ $\quad n = \pm 1, \pm 2, \pm 3$

$\rightarrow \frac{\pi}{\lambda} a \sin\theta = n\pi$

★ $\boxed{a \sin\theta = n\lambda}$

n is called the order of minimum intensity

SECONDARY MAXIMA

Between any two adjacent minimum intensities, there will be a maximum intensity called secondary maximum

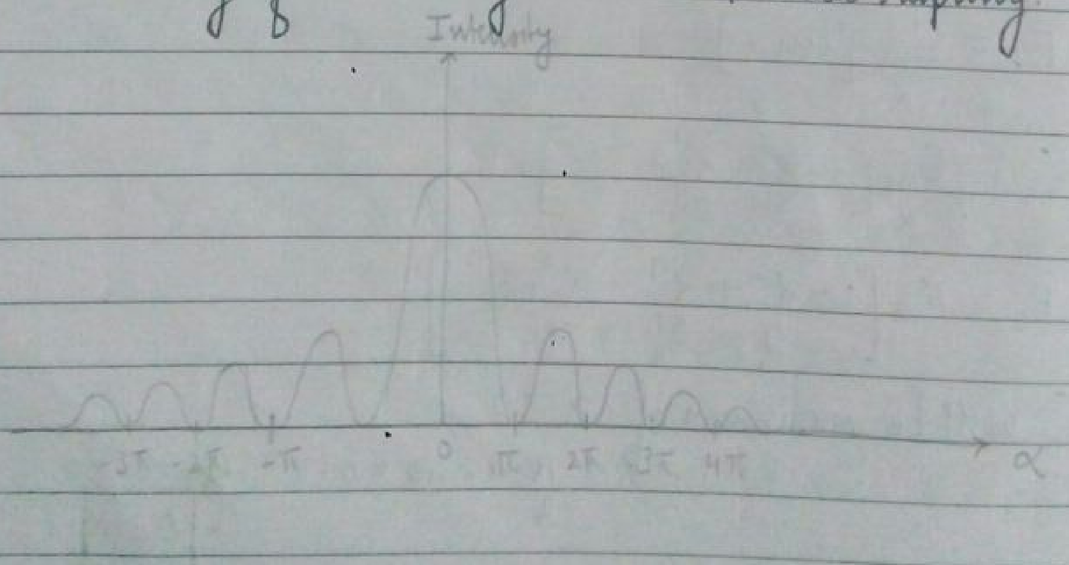
f $\sin\alpha = 1$

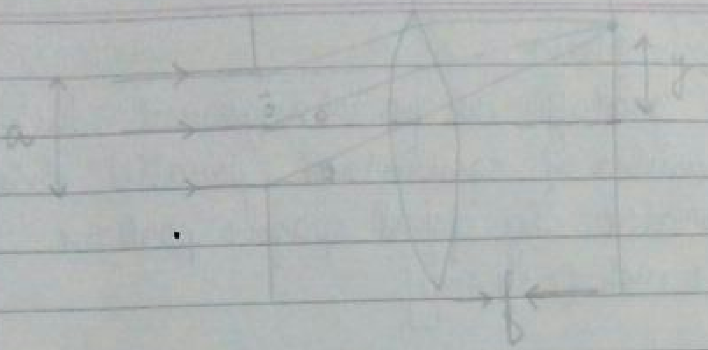
$\rightarrow \boxed{\alpha = (2n \pm 1)\frac{\pi}{2}}$

$\rightarrow \frac{\pi}{\lambda} a \sin\theta = (2n \pm 1)\frac{\pi}{2}$

$\rightarrow \boxed{a \sin\theta = (2n \pm 1)\frac{\lambda}{2}}$

★ intensity of secondary maxima decreases rapidly.





★ $f \to$ focal length of lens

Minima: $a \sin\theta = n\lambda \;\rightarrow\; \sin\theta = \frac{n\lambda}{a}$ $\because \theta$ is very small

$\rightarrow \sin\theta \simeq \theta \simeq \tan\theta$ ★

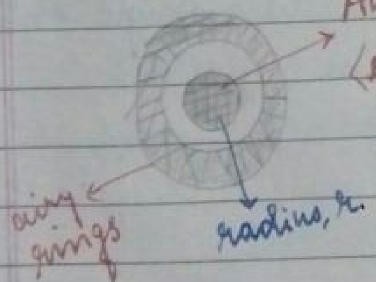
$\rightarrow \frac{n\lambda}{a} = \frac{y}{f} \;\rightarrow\; \boxed{y = \frac{nf\lambda}{a}}$

Maxima: $a \sin\theta = (2n+1)\frac{\lambda}{2}$

★

$\rightarrow \boxed{y = \frac{(2n+1)}{2}\frac{\lambda f}{a}}$

★ Circular Aperture



$\boxed{r = \frac{1.22 f\lambda}{a}}$

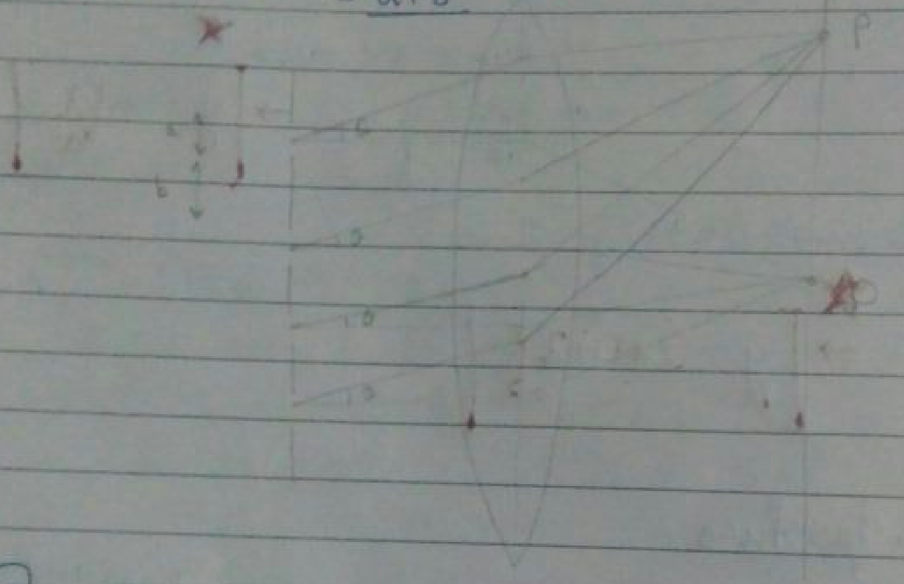
focal length

diameter of aperture

Plane Diffraction Grating

A plane diffraction grating is an arrangement which consists of a large number of equidistant parallel slits of equal width & separated by equal opaque portions
[15,000 lines in 1 inch (2.54 cm)]

Grating element: The distance b/w centres of 2 consecutive slits
$= a+b$



Resultant Amplitude at point P due to a single slit through which light is diffracted at an angle 'θ':

$$\boxed{= E_m \frac{\sin\alpha}{\alpha}}$$

$$\boxed{\alpha = \frac{\pi a \sin\theta}{\lambda}}$$

path difference b/w rays from 2 consecutive slits:

$$\Delta x = (a+b)\sin\theta$$

& corresponding phase difference, $d\phi = \frac{2\pi}{\lambda}(a+b)\sin\theta$

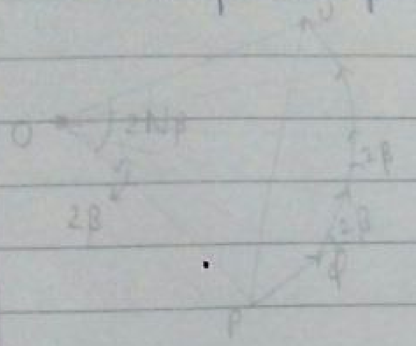
$$\boxed{\beta = \frac{\pi}{\lambda}(a+b)\sin\theta}$$

Phase difference b/w ray from 1st & n^{th} slit: $\boxed{\phi = 2N\beta}$

N= total no of slits/lines on grating

Resultant Amplitude:

N amplitude phasors → forming an arc



$$\text{angle} = \frac{\text{arc}}{\text{radius}}$$

$$\rightarrow 2\beta = \frac{P\phi}{OP}$$

$$\rightarrow OP(2\beta) = P\phi$$

$P\phi = E_m \frac{\sin\alpha}{\alpha}$

$\because \beta$ is very small,

$$2\beta \simeq \sin 2\beta$$

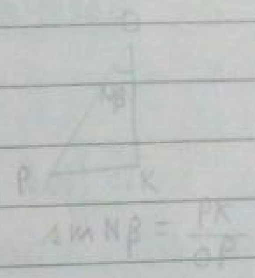
$$\rightarrow P\phi = OP(\sin 2\beta)$$

$$\rightarrow P\phi = OP(2\sin\beta \cdot \cos\beta)$$

[$\because \beta$ is very small, $\cos\beta \simeq 1$]

$$\rightarrow P\phi = 2\,OP\sin\beta \quad \text{—} (1)$$

& ~~Pφ~~ $PU = 2\,OP\sin N\beta$ — (2)



$\div$ (1) & (2)

$$\frac{PU}{P\phi} = \frac{\cancel{2}OP\sin N\beta}{\cancel{2}OP\sin\beta}$$

$$\rightarrow PU = E_\theta = P\phi\frac{\sin N\beta}{\sin\beta}$$

★

$$\rightarrow \boxed{E_\theta = E_m \frac{\sin\alpha}{\alpha}\frac{\sin N\beta}{\sin\beta}}$$

Intensity, ★ $\boxed{I_\theta = I_m \frac{\sin^2\alpha}{\alpha^2}\frac{\sin^2 N\beta}{\sin^2\beta}}$

Principal Maxima:

$$\sin\beta = 0$$

$$\rightarrow \boxed{\beta = \pm n\pi} \quad \rightarrow \frac{\pi}{\lambda}(a+b)\sin\theta = \pm n\pi$$

$$\lim_{\beta\to n\pi}\frac{\sin N\beta}{\sin\beta} = \lim_{\beta\to n\pi}\frac{N\cos N\beta}{\cos\beta} = N$$

★ Max. intensity $= \left(I_m\frac{\sin^2\alpha}{\alpha^2}\right)(N^2)$

Condition: $\boxed{(a+b)\sin\theta = \pm n\lambda}$

* $\theta = 90°$ → no of bright or dark fringe

Minima:

$$I_\theta = I_m \frac{\sin^2\alpha}{\alpha^2} \frac{\sin^2 N\beta}{\sin^2\beta}$$

$$\sin N\beta = 0$$

$$N\beta = \pm m\pi$$

$$N\left[\frac{\pi}{\lambda}(a+b)\sin\theta\right] = \pm m\pi$$

$$\boxed{N(a+b)\sin\theta = \pm m\lambda}$$

$$m = 1, 2, 3, \ldots\ldots N-1$$

★ Between ~~at~~ 2 principal maximas, (N-1) minima are obtained

$\boxed{m \neq 0}$ & $m \neq N, 2N, 3N, \ldots$

Secondary Maxima

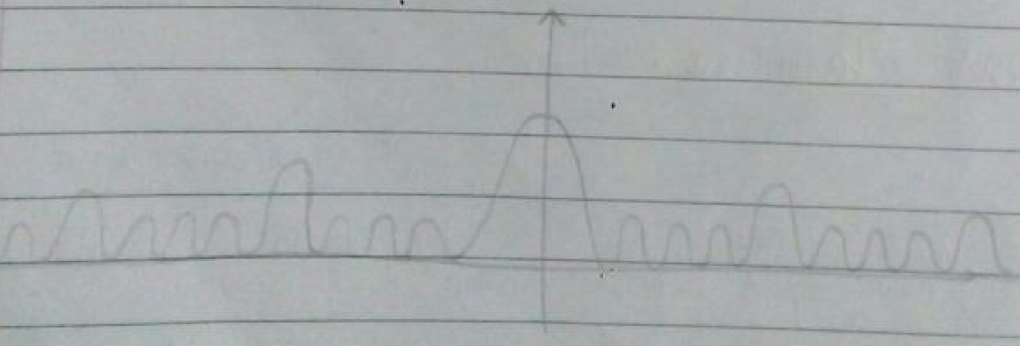
$$\cancel{E_\theta = A^2 \sin^2} \quad E_\theta = E_m \frac{\sin^2\alpha}{\alpha^2}\left(\frac{\sin N\beta}{\sin\beta}\right)^2$$

To find positions of secondary maximas; we diff. E_θ w.r.t. β

$$\frac{dE_\theta}{d\beta} = E_m \frac{\sin^2\alpha}{\alpha^2} \cdot \frac{2\sin N\beta}{\sin^3\beta}\left\{\sin\beta \cdot (\cos N\beta)N - \sin N\beta \cdot \cos\beta\right\}$$

$$\rightarrow \sin\beta \cdot (\cos N\beta)N - \sin N\beta \cdot \cos\beta = 0$$

★ (N-2) secondary maxima are formed b/w 2 principal maxima



Dispersive Power of Diffraction Grating

The dispersive power of diffraction grating is defined as change in angle of diffraction per unit change in wavelength of light

$d\theta \rightarrow$ change in angle of diffraction
$d\lambda \rightarrow$ corresponding change in wavelength of light

$$\boxed{\text{Dispersive power} = \frac{d\theta}{d\lambda}}$$

Condition for n^{th} principal maxima in diffraction grating:

$$(a+b)\sin\theta = n\lambda$$

diff. , $(a+b)\cos\theta = n\,d\lambda$

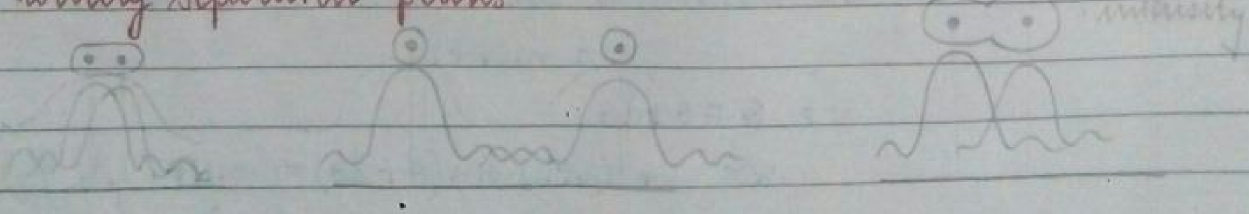
★ $$\boxed{\frac{d\theta}{d\lambda} = \frac{n}{(a+b)\cos\theta}}$$ → order of spectrum

Rayleigh Criterion for Resolution

According to Rayleigh's criterion, two closely spaced point sources are said to be <u>just resolved</u> by an optical instrument if the <u>principal maximum</u> in the diffraction pattern of one object coincides with the <u>first minimum</u> in the diffraction pattern of the other. Their resultant intensity shows a <u>small dip</u>

If the sources have very small angular separation, then central maxima in their diffraction pattern will overlap to a large extent & the resultant intensity shows uniform variation

If the sources have very large angular separation, the central maxima are widely separated & the resultant intensity shows 2 widely separated peaks



Resolving Power of Grating

The resolving power of a diffraction grating is defined as its ability to show 2 neighbouring lines in the spectrum as separate & is measured by: $\frac{\lambda}{d\lambda}$

Let m^{th} order principal maximum of wavelength λ be formed at angle 'θ'

$$\rightarrow \boxed{(a+b)\sin\theta = m\lambda} \quad \text{—①}$$

The angular separation '$d\theta$' b/w principal maxima of wavelength λ & $\lambda+d\lambda$ can be obtained by differentiating ①

$$(a+b)\cos\theta\, d\theta = m\, d\lambda$$

$$\rightarrow d\theta = \frac{m\, d\lambda}{(a+b)\cos\theta} \quad \text{—②}$$

To satisfy Rayleigh's criterion, the first minimum intensity after m^{th} order principal maximum of wavelength λ must be formed at an angle $\theta+d\theta$ so that it will coincide with principal maximum of wavelength '$\lambda+d\lambda$'

Condition for minimum intensity:

$$(a+b)\sin\theta = \frac{n}{N}\lambda$$

For $n = mN$, we get m^{th} order maximum.

For the first minimum after m^{th} order maximum:

$$n = mN+1$$

$$\rightarrow \theta = \theta + d\theta$$

$$\rightarrow (a+b)\sin(\theta+d\theta) = \left(\frac{mN+1}{N}\right)\lambda$$

★★ Grating element = ★ $\boxed{(a+b) = \frac{1}{N}}$

$$\rightarrow (a+b)\{\sin\theta \cdot \cos d\theta + \cos\theta \sin d\theta\} = \left(m + \frac{1}{N}\right)\lambda$$

For small $d\theta$, $\cos d\theta \approx 1$ & $\sin d\theta \approx d\theta$

$$(a+b)\sin\theta + (a+b)\cos\theta \cdot d\theta = m\lambda + \frac{\lambda}{N}$$

$$\rightarrow (a+b)\sin\theta = m\lambda$$

$$\rightarrow m\lambda + (a+b)\cos\theta \cdot d\theta = m\lambda + \frac{\lambda}{N}$$

$$\rightarrow (a+b)\cos\theta \cdot d\theta = \frac{\lambda}{N}$$

$$\rightarrow d\theta = \frac{\lambda}{N(a+b)\cos\theta} \quad \text{--- (3)}$$

equating (2) & (3):

$$\frac{m\,d\lambda}{(a+b)\cos\theta} = \frac{\lambda}{N(a+b)\cos\theta}$$

Resolving Power: ★ $\boxed{\frac{\lambda}{d\lambda} = mN}$

Resolving Power of a Telescope

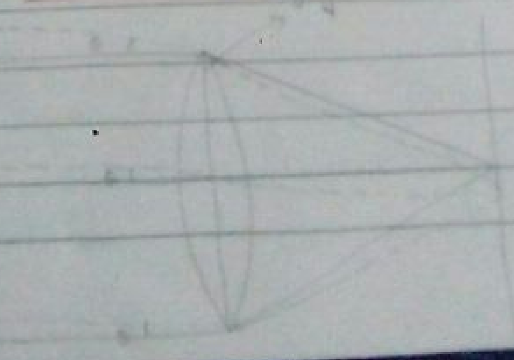
Resolving Power of a telescope is defined as the reciprocal of the smallest angle subtended at the objective lens of telescope by 2 distant objects for which the telescope produces just resolved images.

★ $\theta \rightarrow$ smallest angle subtended at the objective lens for which the 2 objects are seen to be just resolved.

★ $\boxed{R.P = \frac{1}{\theta}}$

objective lens of diameter = D

screen



As both central maxima are formed along incident direc
the angular separation b/w central maxima is 'θ'.

→ To satisfy Rayleigh's criterion, the first minimum in diffrac
pattern of first object must be formed at angle θ so as to coincide with central maximum in diffraction pattern of second object.

For first minimum intensity: $D\sin\theta = \lambda$

for small angle, $\sin\theta \approx \theta$

$\rightarrow D\theta = \lambda$

★ $\boxed{R.P = \frac{1}{\theta} = \frac{D}{\lambda}}$ D = diameter of objective lens

valid for rectangular aperture

★ For circular Aperture:

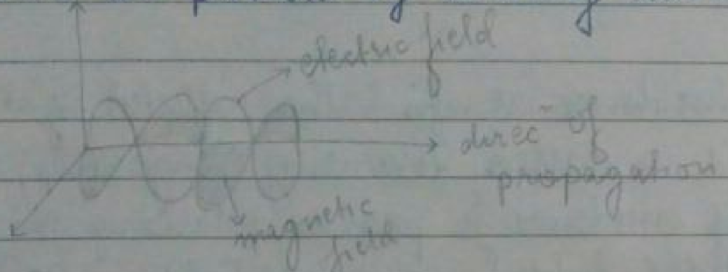
$\boxed{R.P = \frac{1}{\theta} = \frac{D}{1.22\lambda}}$ $\boxed{= \frac{r}{x}}$ → separation b/w 2 pts

→ Telescopes have $\boxed{\text{circular apertures}}$

* Polarisation of light establishes that light has <u>transverse nature</u>

Polarization

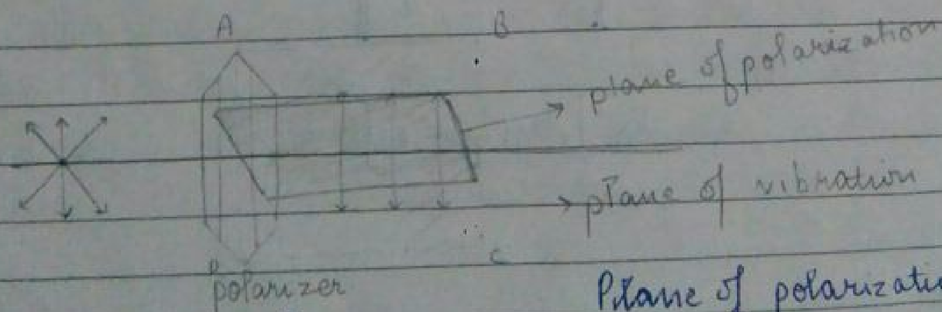
Light is an electromagnetic wave.
~~En~~ Electric field & magnetic field oscillate perpendicular to each other as well as to the direction of propagation of wave.
Light waves are produced by vibrating electric charges.



$$\boxed{c = \frac{E_0}{B_0}} \simeq 3\times10^{8}\ m/s$$

$$E_0 \ggg B_0$$

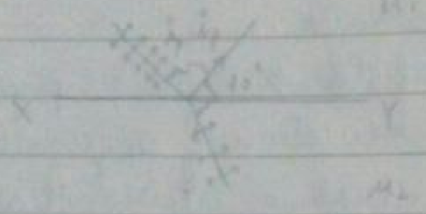
* Unpolarized Light: Light which has vibrations in all possible directions perpendicular to the direction of propagation.
* Plane Polarized Light: Light having vibrations only in one plane & in a direction perpendicular to the direction of propagation.
* Plane of Vibration: Plane containing the optic axis in which the vibrations occur.
* Plane of Polarization: Plane which is at right angles to the plane of vibration & contains the direc^n of propagation of the polarized light.



⟨calcite & Tourmaline crystal⟩

Plane of polarization is ⊥ to plane of vibration

* Polarization by Reflection



The angle of incidence at which the reflected light is completely plane polarized is called Polarising Angle (i_p) or Brewster's angle.

* When angle of incidence is equal to polarising angle, the angle b/w reflected light & refracted light is 90°

Brewster's Law:

When unpolarized light of given wavelength is incident upon the surface of a transparent substance, it experiences maximum plane polarization after reflection at the angle of incidence whose tangent is the refractive index of the substance for that wavelength.

$$\frac{\sin i_p}{\sin r} = \frac{\mu_2}{\mu_1}$$

$$r + i_p = 90^\circ \rightarrow r = 90^\circ - i_p$$

$$\rightarrow \frac{\sin i_p}{\sin(90^\circ - i_p)} = \frac{\mu_2}{\mu_1}$$

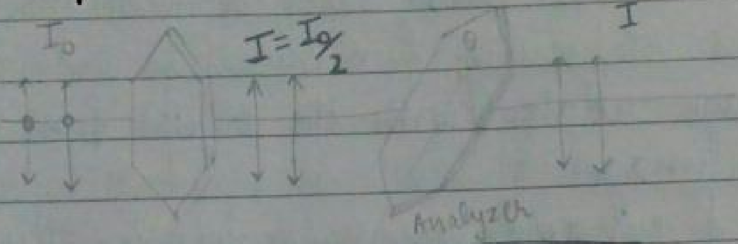
$$\rightarrow \boxed{\tan i_p = \frac{\mu_2}{\mu_1}}$$

$\because \mu_1 = 1$

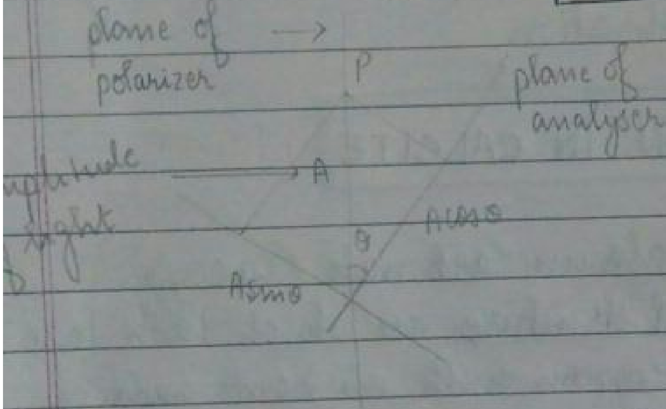
$$\rightarrow \boxed{\tan i_p = \mu}$$

★ Tourmaline crystal acts as both polarizer & analyzer

Malus Law: It states that when a beam of plane polarized light from a polarizer is incident on an analyser, the intensity of light emerging from analyser varies as the square of the cosine of angle b/w plane of transmission of the polarizer & analyzer



$$\boxed{I' \propto \cos^2\theta} \rightarrow \text{Malus law}$$



Intensity $\propto$ (Amplitude)^2

$\rightarrow I = KA^2$ — ①

& $I' = KA^2\cos^2\theta$ — ②

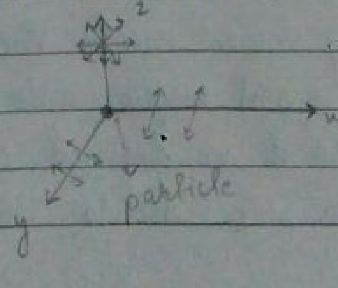
÷ ① & ②

★ $\boxed{I' = I\cos^2\theta}$

θ = angle b/w polarizer & analyzer

★ Polarisation by Scattering

When a beam of white light is passed through a medium containing particles whose size is comparable to the wavelength of light, the beam gets scattered when the scattered light is observed in a direction perpendicular to the angle of incidence, it is said to be plane polarized.



* ~~Bifurcation of a beam of light occurs during refraction in an optically anisotropic medium~~

★ **Double Refraction or Birefringence** is an optical property in which a single ray of unpolarized light entering an anisotropic medium [splits] into [two] rays.
When objects are seen through crystals like calcite, quartz or tourmaline, two images of an object are observed for certain orientations of these crystals.

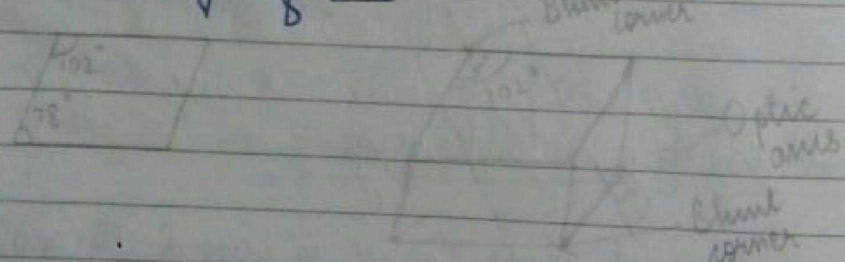
→ On rotating the crystal it is observed that one of the images remains stationary & is known as Ordinary image.

→ The other image rotates with the crystal & is known as extraordinary image.

The ordinary & extraordinary waves forming these images are observed to be plane polarized with mutually perpendicular planes of vibration.

DOUBLE REFRACTION IN CALCITE

Calcite is a crystal of calcium carbonate ($CaCO_3$)
It has a rhombohedral structure in which each face of the crystal is a parallelogram with an acute angle of 78° & obtuse angle of 102°



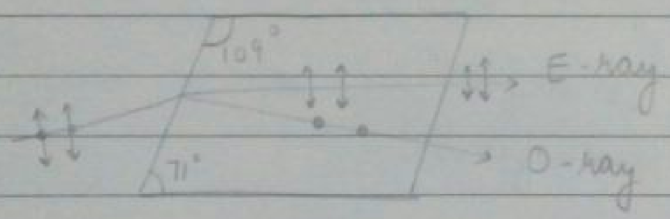
★ There are 2 diagonally opposite corners in the crystal where all the 3 angles are obtuse called BLUNT CORNERS.
At all other corners, there are 2 acute angles & 1 obtuse angle.

OPTIC AXIS: An imaginary line inside the crystal from one of the blunt corners making equal angles with all the 3 edges or any other line parallel to it is defined as the optic axis.

* If light is incident parallel or perpendicular to optic axis,

double refraction is not observed

PRINCIPAL PLANE: A plane perpendicular to that face of the crystal on which light is incident & which contains the optic axis. The principal plane does not intersect the optic axis.
All principal planes are parallelograms with acute angle of $\boxed{71^\circ}$ & obtuse angle of $\boxed{109^\circ}$
Vibrations of the ordinary ray (O-ray) are perpendicular to principal plane while those of the extraordinary ray (E-ray) are parallel to principal plane



Huygen's Wave Theory of Double Refraction

Huygen explained the phenomenon of double refraction based on the following assumptions:
Every point in a double refracting medium is a source of 2 types of wavefronts:

(a) Ordinary Wavefront: which is $\boxed{\text{spherical}}$ as ordinary ~~waves~~ rays travel with the same velocity in all directions. The refractive index of the crystal is same for these ~~waves~~ rays in all directions.

$\boxed{\mu_o = \text{constant}}$ $\boxed{V_o = \text{constant}}$

Ordinary rays obey laws of refraction.

(b) Extraordinary Wavefront: which is $\boxed{\text{ellipsoidal}}$ as extraordinary waves travel with different velocity in different directions. The refractive index of the crystal for these waves is different in different directions. $\boxed{\mu_e \neq \text{constant}}$ $\boxed{V_e \neq \text{constant}}$
Extraordinary rays do not obey laws of refraction.

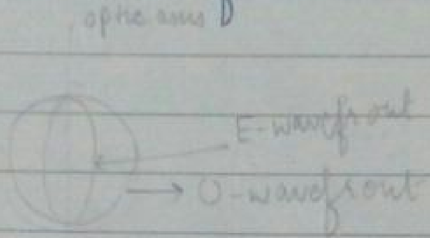
) The O-waves & E-waves travel with the same velocity along the optic axis hence the 2 wavefronts meet at the optic axis $\boxed{v_o = v_e}$

(iii) In some crystals, the velocity (v_o) of the ordinary waves is greater than the velocity (v_e) of the extraordinary waves in all directions except along the optic axis. Such crystals are known as POSITIVE CRYSTALS

As $\boxed{v_o > v_e}$ & $\mu = \frac{c}{v}$

$\rightarrow \boxed{\mu_o < \mu_e}$ (in all directions except optic axis)

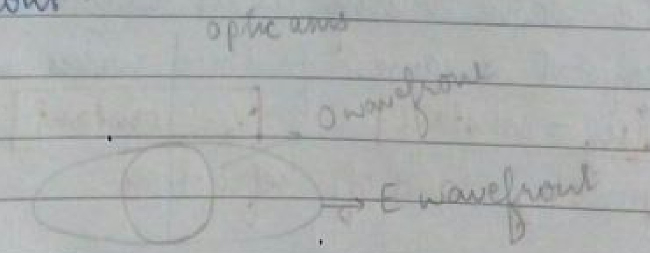
★ The spherical O-wavefront lies outside the ellipsoidal E-wavefront



Quartz & ice are positive crystals

(iv) In some crystals the velocity $\boxed{v_e > v_o}$ in all directions except along the optic axis. For these crystals $\boxed{\mu_o > \mu_e}$. These crystals are known as NEGATIVE CRYSTALS

★ The O-wavefront which is spherical lies inside the ellipsoidal E-wavefront.



Calcite is a negative crystal

→ Propagation of Light:

→ LCD