CIRCLE ASSIGNMENT

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FWC22069   IITH Future Wireless Communication (FWC)   ASSIGN-5

1   Problem Statement − The angle b/n pair of tangents drawn from the point P to the locus of circle $x^2 + y^2 + 4x - 6y + 9sin^2\alpha + 13cos^2\alpha = 0$ is 2α. The equation of locus of the point P is.
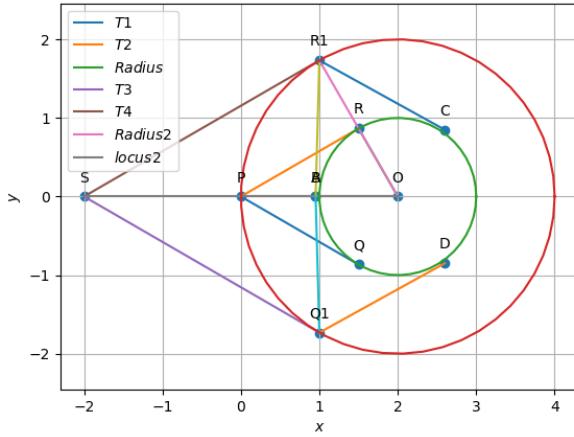

Figure 1: pair of tangents from point P

$$\mathbf{OP^2}sin^2\alpha = 4sin^2\alpha \qquad (4)$$

$$\mathbf{OP^2} = 4 \qquad (5)$$

$$(OP^2) = \|\mathbf{P} - \mathbf{O}\|^2 \qquad (6)$$

$$\mathbf{P} = \mathbf{X}, \mathbf{O} = -\mathbf{u} \qquad (7)$$

$$\|\mathbf{X} + \mathbf{u}\|^2 = 4 \qquad (8)$$

$$(\mathbf{X} + \mathbf{u})^{\mathbf{T}}.(\mathbf{X} + \mathbf{u}) = 4 \qquad (9)$$

$$\|\mathbf{X}\|^2 + \|\mathbf{u}\|^2 + 2\mathbf{X^T u} = 4 \qquad (10)$$

Hence locus of $\mathbf{OP}$ upon simplifying is

$$\mathbf{u} = (2, -3), \mathbf{f} = 4, \mathbf{f_1} = \mathbf{u^T}.\mathbf{u} - \mathbf{f} \qquad (11)$$

$$\mathbf{V} = \mathbf{I}, \mathbf{u} = -O\mathbf{u}, f_1 = 9. \qquad (12)$$

The required locus equation is

$$\mathbf{X^T}.\mathbf{X} + 2\mathbf{u^T X} + \mathbf{f_1} = 0 \qquad (13)$$

Hence the parameters of the above equation

$$\mathbf{u} = (2, -3), f_1 = 9. \qquad (14)$$

Construction

the input parameters are as follows

Symbol	Value	Description
r	2	radius
d	rcsc(θ)	radius of the circle
l	rcot(θ)	tangent(s) of the circle

solution

part 1

Hence from the given equation $x^2 + y^2 + 4x - 6y + 9sin^2\alpha + 13cos^2\alpha = 0$

$$\mathbf{O} = (-2, 3) \quad and \quad \mathbf{r} = \sqrt{\mathbf{u^T}.\mathbf{u} - \mathbf{f}} = 2\sin\alpha \qquad (1)$$

hence from Pythagoras theorem sinα=OR/OP.Here OR=2sinα.

$$\sin\alpha = 2\sin\alpha/OP. \qquad (2)$$

$$\sin^2\alpha = 4\sin^2\alpha/OP^2. \qquad (3)$$