Coefficients of weight function as its derivatives are called secondary variables (SV).
In our case, a$\frac{du}{dx}$ is secondary variable.
If SV is specified in the boundary, then the conditions are called natural boundary condition (NBC).
Now we convert our differential equation and boundary condition to weak form.

- Step 1
  We write the weighted integral statement, i.e;
  $\int_0^L$ w[-$\frac{d}{dx}$(a$\frac{du}{dx}$) - f]dx = 0
  It is equivalent to differential equations and does not include any boundary conditions and thus it must be differentiable as order of differential equation.

- Step 2
  Weakening Differentiable of ϕ_i
  0= $\int_0^L$ [w[-$\frac{d}{dx}$(a$\frac{du}{dx}$)]- wf] dx
  Integrating by parts, we get,
  0= $\int_0^L$[(a$\frac{dw}{dx}\frac{du}{dx}$) - wf] dx - [wa$\frac{du}{dx}$]

- Step 3
  Least Square Method,
  $\frac{d}{dC_j}$ $\int_0^L$ R^2 dx=0 $\forall$ j = 1,2,...,N
  This also gives N equations for N unknown coefficients.

- Step 4
  We desire that,
  $\int_0^L$ $w_i(x)$ Rdx = 0 $\forall$ i=1,2,...,N
  where w_i's are N linearly independent functions called weight functions.

  Substituting U_N(x) in our Differential Equation,
  -$\frac{d}{dx}$ [a(x)$\frac{dU_N}{dx}$] = f(x) for 0< x <L.
  If this equally holds for all x $\in$[0,L] the solution is exact. Since, we assume it only as an approximation.
  Define Residual Function as R(x,c_1, c_2, c_3,...,c_N)as
  R = -$\frac{d}{dx}$[a(x)$\frac{dU_N}{dx}$] - f(x)
  R $\neq$0 $\forall$x $\in$ [0, L] as a U_N approximation.
  Now we have various ways to minimize R in some senses over the domain.

  1. Collocation Method
     It forces that R is zero at selected N points of the domain.
     i.e; R(x, ,c_1, c_2, c_3,...,c_N) = 0 $\forall$ x=x_i, i $\in$1,2,...,N

Consider
-$\frac{d}{dx}$[a(x)$\frac{du}{dx}$] = f(x) for 0< x <L.
For u(x) subject to the boundary conditions

u(0)= $u_0 a(x)$du$_{\overline{dx}}|_{x=L}$ = Q_L

where a(x) and f(x) are known functions.
u_0 and Q_L are known values
In case of bar (only axial load and structure)
u = displacement
a(x)= EA (stiffness)
f = distributed axial force
Q_L= axial load
Now, we want an approximation of u(x) in the form
u(x)≈u_N(x) = $\sum_{j=1}^{N} c_j\phi_j$ + (x) +ϕ_0(x)