CL 686 : Advanced Process Control
Nonlinear System 1 (Group A)

Consider a nonlinear system governed by the following set of ODEs

$$\frac{dX_1}{dt} = \frac{U_1}{100}(D - X_1) - 2k(X_2)X_1^2 \tag{1}$$

$$\frac{dX_2}{dt} = \frac{U_1}{100}(275 - X_2) + \alpha_3\ k(X_2)X_1^2 - \alpha_4(X_2 - X_3) \tag{2}$$

$$\frac{dX_3}{dt} = \frac{U_2}{10}(250 - X_3) + \alpha_5(X_2 - X_3) \tag{3}$$

$$k(X_2) = \alpha_1 \times \exp\left(\frac{-\alpha_2}{X_2}\right) \tag{4}$$

The parameters and steady state input conditions of the system are as shown in Table below

Parameter	**Nominal Value**
α_1	4.11×10^{13}
α_2	9.2055×10^{3}
α_3	284.1043
α_4	2.8571
α_5	28.5714

- **State:** X_1 (concentration of reactant A in reaction $A \rightarrow B$), X_2 (reactor temperature) and X_3 (coolant fluid temperature)
- **Manipulated Inputs:** U_1 (reactor inlet flow) and U_2 (coolant flow)

Condition for Dynamic Simulation:

- **Steady state inputs:**

$$\mathbf{U}_s = \begin{bmatrix} 120 & 30 \end{bmatrix}^T \quad ; \quad D_s = 1 \tag{5}$$

- **Equilibrium/ Steady State Operating Point**

$$\mathbf{X}_s \equiv \begin{bmatrix} 0.0192 & 384.0056 & 371.2721 \end{bmatrix}^T \tag{6}$$

- **Sapling interval (h) and simulation time**:

$$T = 0.1 \text{ min} \quad \text{and} \quad t_f = 50 \text{ min}$$

Simulate open loop plant behavior for N_s $(= t_{f/}/T) = 500$ number of samples

- **Step change in disturbance input D:**

$$D(k) = D_s + d(k) \tag{7}$$

where $d(k)$ is a step signal as follows

$$d(k) = \left\{ \begin{array}{cc} 0 & \text{for } \ k < 250 \\ -0.2 & \text{for k} \geq 250 \end{array} \right\}$$

- **Manipulated input simulation:** Random Binary Signal (RBS) Input

$$RBS\_Period = \begin{bmatrix} 25 & 30 \end{bmatrix}^T \quad \text{and} \quad RBS\_amplitude = \begin{bmatrix} 10 & 4 \end{bmatrix}^T$$

  Refer to the Quadruple Tank dynamic simulation demo program uploaded in Moodle to know about the RBS input.

- **File containing system data:** *System1_ Parameters.mat*
- **File for computing system derivatives:** *System1_ Dynamics.m*

Open Loop Plant Dynamic Simulation Algorithm

Simulate open loop plant behavior for Ns ($= t_{f/}/T$) number of samples

- **Initialization**

  Load system parameters and $(\mathbf{X}s, \mathbf{U}_s, D_s)$ from data file

  Set $\mathbf{X}(0) = \mathbf{X}s, \quad \mathbf{U}(0) = \mathbf{U}_s, \quad D(0) = D_s; \quad \mathbf{Y}(0) = \mathbf{Y}s$

  Load continuous time linear perturbation model matrices $(\mathbf{A}, \mathbf{B}, \mathbf{H}, \mathbf{C})$

  Generate discrete time linear perturbation model matrices $(\Phi, \Gamma_u, \Gamma_d)$ for the specified T

  Create matrices for storing state and input data

- **Dynamic Simulation**

  FOR k = 1 to Ns-1

  - Generate $\mathbf{u}(k)$ as *Random Binary Signal* and $d(k)$ as step change
    $\mathbf{U}(k) = \mathbf{U}_s + \mathbf{u}(k)$
    $D(k) = D_s + d(k)$
    Solve for $\mathbf{x}(k+1) = \Phi\mathbf{x}(k) + \Gamma\mathbf{u}(k) + \Gamma_d\mathbf{d}(k)$ *% Linear plant simulation*
    Find $\mathbf{X}_L(k+1) = \mathbf{X}_s + \mathbf{x}(k+1)$
    Solve for $\mathbf{X}(k+1) = F\left[\mathbf{X}(k), \mathbf{U}(k), D(k)\right]$ *% Nonlinear Plant Simulation*

  END FOR

- Display Simulation Results

  - Plot $X_i(k)$ v/s Time and $X_{L,i}(k)$ v/s Time in same figure for $i = 1, 2, 3$
    Plot $X_{L,i}(k) - X_i(k)$ v/s Time for $i = 1, 2, 3$
    Plot $U_i(k)$ v/s Time for $i = 1, 2$
    Plot $D(k)$ v/s Time