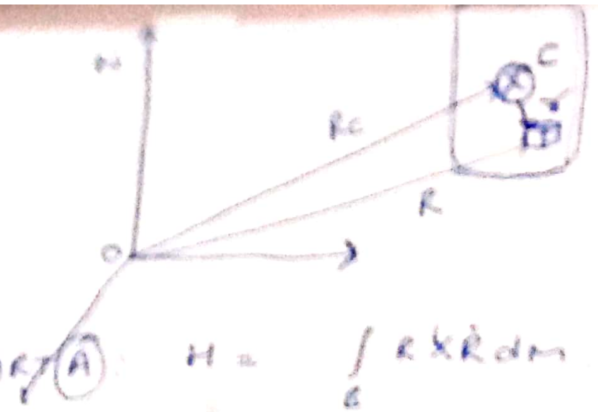

PART (A) $H = \int_B R \times \dot{R}\, dm$

$= \int_B (R_c + r) \times (\dot{R}_c + \dot{r})\, dm$

$= \int_B \underbrace{R_c \times \dot{R}_c\, dm}_{dP_c} + \cancel{\int_B (R_c \times \dot{r})\, dm}^{0} + \cancel{\int (r \times \dot{R}_c)\, dm}^{0} + \int_B (r \times \dot{r})\, dm$

com property

$H = \underbrace{R_c \times P_{com}}_{\text{Angular Momentum of COM}} + \underbrace{\int_B (r \times \dot{r})\, dm}_{H_c}$ [angular momentum about COM].

$$\boxed{H_c = \int_B (r \times \dot{r})\, dm}$$

PART (B): $H_c = \int_B (r \times \dot{r})\, dm$

$\dot{H}_c = \int_B \cancel{(\dot{r} \times \dot{r})}\, dm + \int_B (r \times \ddot{r})\, dm$.

$\dot{H}_c = \int_B r \times \underbrace{\ddot{r}\, dm}_{dF}$

$\dot{H}_c = \int_B \underbrace{r \times dF}_{d\tau}$.

$$\boxed{\dot{H}_c = \tau_c}$$

[where τ_c is the total torque about the center of mass]

PART (C):

$$KE = \frac{1}{2} {}_B\int (\dot{R}\cdot\dot{R})\,dm$$

$$R = (R_c + r)$$

$$KE = \frac{1}{2} {}_B\int (\dot{R}_c + \dot{r})\cdot(\dot{R}_c + \dot{r})\,dm$$

$$KE = \underbrace{\frac{1}{2} {}_B\int \dot{R}_c\cdot\dot{R}_c\,dm}_{K.E_{COM}} + \underbrace{\cancel{{}_B\int (\dot{R}_c\cdot\dot{r})\,dm}}_{COM} \rightarrow 0 + \underbrace{\frac{1}{2} {}_B\int (\dot{r}\cdot\dot{r})\,dm}_{KE_{ROTATIONAL}}$$

$$KE_{COM} = \frac{1}{2} M |\dot{R}_c|^2$$

$$KE_{Rot} = \frac{1}{2} {}_B\int (\dot{r}\cdot\dot{r})\,dm$$

$$\dot{r} = \cancel{\frac{{}^B d r}{dt}} \rightarrow 0 + \omega\times r \quad \text{[Transport Theorem]}$$

$$KE_{rot} = \frac{1}{2} {}_B\int \underset{a}{(\omega\times r)}\cdot\underset{(b\times c)}{(\omega\times r)}\,dm \quad (1)$$

$$a\cdot(b\times c) = b\cdot(c\times a)$$

Applying to equation (1)

$$KE_{rot} = \frac{1}{2} {}_B\int \omega\cdot(r\times\omega\times r)\,dm$$

$$KE_{rot} = \frac{1}{2} {}_B\int \omega\cdot[\tilde{r}][\tilde{r}]\,\omega\,dm$$

Since ω will be same throughout the body because it is a rigid body !!

$$KE_{rot} = \frac{1}{2}(\omega\cdot\omega) \underbrace{{}_B\int [\tilde{r}][\tilde{r}]\,dm}_{I_c}$$

$$\boxed{KE_{rot} = \frac{1}{2} I_c\, \omega\cdot\omega}$$