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Model III (M/M/S) : (∞/FCFS) (Multiservice Model)

1) $$P_n = \begin{cases} \dfrac{1}{n!}\left(\dfrac{\lambda}{\mu}\right)^n P_0 & , n = 0, 1, 2 \ldots S-1 \\ \dfrac{1}{S!}\cdot\dfrac{1}{S^{n-S}}\left(\dfrac{\lambda}{\mu}\right)^n P_0 & , n = S, S+1 \ldots \end{cases}$$

2) $$P_0 = \left[\sum_{n=0}^{S-1} \frac{(S\rho)^n}{n!} + \frac{(S\rho)^S}{S!\,(1-\rho)}\right]^{-1}$$

3) Traffic intensity, $\rho = \dfrac{\lambda}{\mu S}$

4) Length of queue, $L_q = P_S \dfrac{\rho}{(1-\rho)^2}$, where $P_S = \dfrac{\left(\frac{\lambda}{\mu}\right)^S P_0}{S!}$

5) Length of the system, $L_s = L_q + \dfrac{\lambda}{\mu}$

6) Waiting time in the system, $W_s = \dfrac{L_s}{\lambda}$ or $W_q + \dfrac{1}{\mu}$

7) Waiting time in the queue, $W_q = \dfrac{L_q}{\lambda}$ or $P_S \cdot \dfrac{1}{S\mu(1-\rho)^2}$

8) The mean no. of individuals who actually wait is given by, $L(L>0) = \frac{1}{1-\rho}$

9) The mean waiting time in the queue for those who actually wait is given by $W(W>0)$,

$$= \frac{1}{s\mu-\lambda}$$

10) Probability $P(W>0) = \frac{P_s}{1-\rho}$

11) Probability that there will be someone waiting $= \frac{P_s \rho}{1-\rho}$

12) Average number of idle servers $= S - $ (Avg. no. of customer served)

13) Efficiency of M/M/S model $= \frac{\text{Avg. no. of customers served}}{\text{Total no. of customers served}}$.

Model III : (Multiservice Model) $(M/M/S):(\infty/FCFS)$

Numerical Problem - 01

Q: A supermarket has two girls ringing up sales at the counters. If the service time for each customer is exponential with mean four minutes and if people arrive in a Poisson fashion at the counter, at the rate of 10 per hour, then calculate,

(i) The probability of ~~having~~ customer to wait for service.

(ii) The expected percentage of idle time for each girl.

(iii) If a customer has to wait, find the expected length of his waiting time.

Solution: Probability of waiting time for service is,

$$P(W>0) = \frac{P_s}{1-\rho} = \frac{\left(\frac{\lambda}{\mu}\right)^s P_0}{s!\,(1-\rho)}$$

Given,

$$\lambda = \frac{10}{60} \text{ per min.} = \frac{1}{6} \text{ per min.}$$

$$\mu = \frac{1}{4} \text{ per min.}$$

$$S = 2$$

$$\rho = \frac{\lambda}{\mu S} = \frac{\frac{1}{\cancel{6}_{3}}}{\frac{1}{\cancel{4}_{2}} \times \cancel{2}} = \frac{1}{3}$$

We know,

$$\therefore P_0 = \left[\sum_{n=0}^{s-1} \frac{(s\rho)^n}{n!} + \frac{(s\rho)^s}{s!\,(1-\rho)}\right]^{-1}$$

$$P_0 = \left[\sum_{n=0}^{1} \frac{\left(2\cdot\frac{1}{3}\right)^n}{n!} + \frac{\left(2\cdot\frac{1}{3}\right)^2}{2!\left(1-\frac{1}{3}\right)}\right]^{-1}$$

$0! = 1$

$a^0 = 1$

$$= \left[\left(1+\frac{2}{3}\right) + \frac{\frac{\cancel{2}\,4}{\cancel{9}\,3}}{\cancel{2}\times\frac{2}{\cancel{3}}}\right]^{-1}$$

$$= \left[\frac{5}{3}+\frac{1}{3}\right]^{-1} = \left(\frac{\cancel{6}\,2}{\cancel{3}}\right)^{-1} = 2^{-1} = \frac{1}{2}$$

Thus, probability $(W>0) = \dfrac{\left(\frac{\lambda}{\mu}\right)^s P_0}{s!\,(1-\rho)} = \dfrac{\left(\frac{\frac{1}{\cancel{6}\,3}}{\frac{1}{\cancel{4}\,2}}\right)^2 \times \frac{1}{2}}{2!\left(1-\frac{1}{3}\right)}$

$$= \frac{\left(\frac{2}{3}\right)^2 \times \frac{1}{2}}{2\times\frac{2}{3}} = \frac{\frac{\cancel{4}}{\cancel{9}\,3}\times\frac{1}{2}}{\cancel{2}\times\frac{2}{\cancel{3}}} = \frac{1}{6}$$

(ii) The fraction of the time the service remain busy, (i.e traffic intensity) is given by,

$$\rho = \frac{\lambda}{\mu S} = \frac{1}{3}$$

fraction of the time the service remains idle is,

$$= \left(1 - \frac{1}{3}\right) = \frac{2}{3}.$$

percentage of idle time of each girl $= \left(\frac{2}{3} \times 100\right)\%$

$$= 67\%.$$

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(iii) Expected length of customer who has to wait in queue,

$$(W/W>0) = \frac{1}{s\mu-\lambda}$$

$$= \frac{1}{\cancel{2}\times\frac{1}{\cancel{4}2}-\frac{1}{6}}$$

$$= \frac{1}{\frac{1}{2}-\frac{1}{6}} = \frac{1}{\frac{3-1}{6}}$$

$$= \frac{1}{\frac{\cancel{2}}{\cancel{6}3}} = \frac{1}{\frac{1}{3}} = 3 \text{ min}.$$