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Model-I (M/M/1) : (∞/FCFS) Numerical Problem-2

Q: Arrivals at a telephone booth are considered to Poisson, with an average time of 10 minutes between one arrival and the next. The length of a phone call is assumed to be distributed exponentially with mean three minutes.

(i) What is the average length of the queue that forms from time to time?

(ii) The telephone department will install a second booth when convinced that an arrival would have to wait at least three minutes for the phone. By how much time must the flow of arrivals be increased in order to justify a second booth?

(iii) Estimate the fraction of a day that the phone will be in use.

(iv) Find the average number of units in the system.

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Solution :

(i) $\lambda = \frac{1}{10}$, $\mu = \frac{1}{3}$

Average length of non-empty queue,

$$(L/L>0) = \frac{\mu}{\mu - \lambda} = \frac{\frac{1}{3}}{\frac{1}{3} - \frac{1}{10}} = \frac{\frac{1}{\cancel{3}}}{\frac{7}{\cancel{3}\times 10}} = \frac{10}{7} = 1.43$$

person

(ii)

$W_q = 3$, $\mu = \frac{1}{3}$, $\lambda = \lambda'$

$$W_q = \frac{\lambda}{\mu(\mu - \lambda)}$$

$$\Rightarrow 3 = \frac{\lambda'}{\frac{1}{3}\left(\frac{1}{3} - \lambda'\right)} \Rightarrow \frac{1}{3} - \lambda' = \lambda'$$

$$\Rightarrow \frac{1}{3} = 2\lambda'$$

$$\Rightarrow \lambda' = \frac{1}{6} = 0.16$$

Hence, increase in the arrival rate $= 0.16 - 0.10$

$= 0.06$ arrival per min.

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(iii) The fraction of a day that the phone will be in busy = Traffic Intensity.

$$\rho = \frac{\lambda}{\mu} = \frac{\frac{1}{10}}{\frac{1}{3}} = 0.3.$$

(iv) Average no. of unit in the system,

$$L_s = \frac{\lambda}{\mu - \lambda} = \frac{\frac{1}{10}}{\frac{1}{3} - \frac{1}{10}} = \frac{1}{\cancel{10}} \times \frac{\cancel{10} \times 3}{7} = 0.43 \text{ person.}$$