MODEL II (M/M/1) : (N/FCFS)

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This model differs from model I in the sense that, the maximum number of customers in the system is limited to N. Arrivals will not exceed N in any case.

The various measures of this model are,

1. $P_0 = \frac{1-\rho}{1-\rho^{N+1}}$ where, $\rho = \frac{\lambda}{\mu}$ ($\frac{\lambda}{\mu} > 1$ is allowed)

2. $P_N = \frac{1-\rho}{1-\rho^{N+1}} \rho^n$, for $n = 0, 1, 2, \ldots N$

3. $L_q = L_s - \frac{\lambda}{\mu}$

4. $L_s = P_0 \sum_{n=0}^{N} n \rho^n$

5. $L_q = L_s - \frac{\lambda}{\mu}$

6. $W_s = \frac{L_s}{\lambda}$

7. $W_q = W_s - \frac{1}{\mu}$

Q: In a railway marshalling yard, goods train arrive at the rate of 30 trains per day. Assume that the inter-arrival time follows an exponential distribution and the service time is also to be assumed as exponential with mean of 36 minutes. Calculate,

(i) the probability that the yard is empty.

(ii) the average queue length, assuming that the line capacity of the yard is nine trains.

(Considered $N = 9$).

Solution :

$$\lambda = \frac{30}{24 \times 60} = \frac{1}{48} \text{ train per min.}$$

$$\therefore \rho = \frac{\lambda}{\mu} = \frac{36}{48} = 0.75$$

$$\mu = \frac{1}{36} \text{ per min.}$$

(i) The probability that the queue is empty is given by,

$$P_0 = \frac{1-\rho}{1-\rho^{N+1}} \quad (\text{Where, } N = 9)$$

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$$= \frac{1 - 0.75}{1-(0.75)^{10}}$$

$$= \frac{0.25}{0.943} = 0.265$$

(ii) Average no. of train in the system,

$$L_s = P_0 \sum_{n=0}^{N} n\rho^n$$

$$= 0.265 \left[0 + \rho + 2\rho^2 + 3\rho^3 + 4\rho^4 + 5\rho^5 + 6\rho^6 + 7\rho^7 + 8\rho^8 + 9\rho^9 \right]$$

$$= (0.265)\left[0.75 + 1.125 + 1.265 + 1.2656 + 1.1865 + 1.0678 + 0.9343 + 0.80090 + 0.6757 \right]$$

$$= (0.265)(9.0702) = 2.5529 .$$

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Average queue length is given by,

$$L_q = L_s - \frac{\lambda}{\mu}$$

$$= (2.5529 - 0.75)$$

$$= 1.8029$$

$$= 2 \text{ (approx)}$$