Model I : (M/M/1) (∞/FCFS) or Birth and Death Model

λ = Mean arrival rate (no. of arrival per unit time)

μ = Mean service rate (no. of customer served per unit time)

(i) Traffic intensity, $\boxed{\rho = \frac{\lambda}{\mu}}$

(ii) Expected (average) no. of units in the system L_s

$$\boxed{L_s = \frac{\frac{\lambda}{\mu}}{1 - \frac{\lambda}{\mu}} = \frac{\lambda}{\mu - \lambda}}$$

(iii) Expected (average) queue length L_q.

$$\boxed{L_q = L_s - \frac{\lambda}{\mu} = \frac{\lambda^2}{\mu(\mu - \lambda)}}$$

(iv) Expected Waiting time in the queue, W_q

$$\boxed{W_q = \frac{\lambda}{\mu(\mu - \lambda)}}$$

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(v) Expected Waiting line in the system, W_s

$$\boxed{W_s = W_q + \frac{1}{\mu} = \frac{1}{\mu - \lambda}}$$

(vi) Expected waiting time of a customer who has to wait $(W/W>0)$

$$\boxed{(W/W>0) = \frac{1}{\mu-\lambda} = \frac{1}{\mu(1-\rho)}}$$

(vii) Expected length of non-empty queue,

$$\boxed{(L/L>0) = \frac{\mu}{\mu-\lambda} = \frac{1}{1-\rho}}$$

(viii) Probability of Waiting time in the queue $\geq t$

$$= \int_t^\infty \lambda(1-\lambda)\, e^{-(\mu-\lambda)W}\, dW$$

(ix) Probability of queue size $\geq N = \rho^N$

$$= \int_t^\infty \rho(\mu-\lambda)\, e^{-(\mu-\lambda)w}\, dW$$

(x) Probability (Waiting time in the system $\geq t$)

$$= \int_t^{\infty} \rho(\mu-\lambda)\, e^{-(\mu-\lambda)w}\, dw$$

Littles formulas

$$\boxed{\begin{aligned} L_s &= \lambda W_s \\ W_q &= W_s - \frac{1}{\mu} \\ L_q &= L_s - \frac{\lambda}{\mu} \end{aligned}}$$

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Q: A T.V. mechanic finds that the time spent on his Jobs has an exponential distribution with mean 30 minutes, if he repairs sets in the order in which they come in. If the arrival of sets is approximately poisson with an average rate of 10 per eight-hour day, what is the mechanics expected idle time each day? How many Jobs are ahead of the average sets just brought in?

Solution: $\mu = \frac{1}{30}$, $\lambda = \frac{10}{8 \times 60} = \frac{1}{48}$. $\Rightarrow \frac{\lambda}{\mu} = \frac{30}{48}$

Expected no. of Jobs are,

$$L_s = \frac{\frac{\lambda}{\mu}}{1-\frac{\lambda}{\mu}} = \frac{\lambda}{\mu-\lambda} = \frac{\frac{1}{48}}{\frac{1}{30}-\frac{1}{48}}$$

$$= \frac{\frac{1}{48}}{\frac{48-30}{30 \times 48}} = \frac{1}{\cancel{48}} \times \frac{\overset{5}{\cancel{30}} \times \cancel{48}}{\underset{3}{\cancel{18}}}$$

$L_s = \frac{5}{3}$

$L_s = 1\frac{2}{3}$ Jobs.

Since the fraction of the time the mechanic is busy $= \frac{\lambda}{\mu}$

$\therefore$ The no. of hours for which the repairman remains busy in an eight-hour day.

$$= 8\left(\frac{\lambda}{\mu}\right) = 8 \times \frac{30}{48} = 5 \text{ hours.}$$

Therefore, the time for which the mechanic remain idle in an eight-hour day $= (8-5)$ hours.

$$= 3 \text{ hours.}$$