Model III : (Multiservice Model) (M/M/S): (∞/FCFS)

Numerical Problem - 02

Q: Four counters are being run on the frontier of a country to check the passports and necessary papers of the tourists. The tourists choose any counter at random. If the arrival at the frontier is poisson at the rate λ and the service time is exponential with parameter $\frac{\lambda}{2}$, What is the steady state average queue at each counter?

Solution:

$S = 4$

$\lambda = \lambda$

$\mu = \frac{\lambda}{2}$

$\therefore \rho = \frac{\lambda}{\mu S}$

$\rho = \frac{\lambda}{\frac{\lambda}{2} \times 4^2} = \frac{1}{2}$

$\rho = \frac{1}{2}$

@start Practicing

Expected queue length, $L_q = P_s \dfrac{\rho}{(1-\rho)^2}$

$$= \frac{\left(\frac{\lambda}{\mu}\right)^s P_0}{s!} \cdot \frac{\rho}{(1-\rho)^2}$$

Thus,

$$P_0 = \left[\sum_{n=0}^{s-1} \frac{(s\rho)^n}{n!} + \frac{(s\rho)^s}{s!\,(1-\rho)}\right]^{-1}$$

$$= \left[\sum_{n=0}^{3} \frac{(\cancel{4}^{2} \cdot \frac{1}{\cancel{2}})^n}{n!} + \frac{(\cancel{4}^{2} \cdot \frac{1}{\cancel{2}})^4}{4!\,(1-\frac{1}{2})}\right]^{-1}$$

$4! = 4 \times 3 \times 2 \times 1$

$2^0 = 1$

$0! = 1$

$$= \left[\sum_{n=0}^{3} \frac{2^n}{n!} + \frac{2^4}{\cancel{24}^{12} \times \frac{1}{\cancel{2}}}\right]^{-1}$$

$$= \left[\left(1 + 2 + \frac{2^2}{2!} + \frac{2^3}{3!}\right) + \frac{\cancel{16}^{4}}{\cancel{12}^{3}}\right]^{-1}$$

$$= \left[\left(1 + 2 + \frac{\cancel{4}^{2}}{\cancel{2}} + \frac{\cancel{8}^{4}}{\cancel{6}_{3}}\right) + \frac{4}{3}\right]^{-1}$$

$$= \left[5 + \frac{4}{3} + \frac{4}{3}\right]^{-1}$$

$$= \left[\frac{15 + 4 + 4}{3}\right]^{-1} = \left[\frac{23}{3}\right]^{-1} = \frac{3}{23}$$

$$\therefore P_0 = \frac{3}{23}$$

So, Expected queue length $= \dfrac{\left(\frac{\lambda}{\mu}\right)^{s} P_0}{s!} \cdot \dfrac{\rho}{(1-\rho)^2}$

$$= \frac{\left(\frac{\lambda}{\lambda/2}\right)^{4} \cdot \frac{3}{23}}{4!} \cdot \frac{\frac{1}{2}}{\left(1 - \frac{1}{2}\right)^2}$$

$$= \frac{2^4 \cdot \frac{3}{23}}{24} \cdot \frac{\frac{1}{2}}{\frac{1}{4}}$$

@Start Practicing

$$= \overset{4}{\cancel{16}} \times \frac{\cancel{3}}{23} \times \frac{1}{\underset{\underset{4}{\cancel{12}}}{\cancel{24}}} \times \frac{1}{\cancel{2}} \times \overset{2}{\cancel{4}}$$

$$= \frac{4}{23}$$

$$\therefore \; L_q = \frac{4}{23}.$$