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Model IV $(M/M/S):(N/FCFS)$

This model is essentially the same as model III, except that the maximum number of customers in the system is limited to N, where $N > S$ (S = No. of channels).

$$\therefore \quad \lambda_n = \begin{cases} \lambda, & 0 \le n \le N \\ 0, & n \ge N \end{cases}$$

$$\mu_n = \begin{cases} n\mu, & 0 \le n \le S \\ S\mu, & S \le n \le N \end{cases}$$

Formula:

1) $$P_n = \begin{cases} \dfrac{1}{n!}\left(\dfrac{\lambda}{\mu}\right)^n P_0, & 0 \le n < S \\ \dfrac{1}{S^{n-S}\, S!}\left(\dfrac{\lambda}{\mu}\right)^n P_0, & S \le n \le N \end{cases}$$

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2) $$P_0 = \left[\sum_{n=0}^{s-1} \frac{1}{n!} \left(\frac{\lambda}{\mu}\right)^n + \sum_{n=s}^{N} \frac{1}{s^{n-s}\, s!} \left(\frac{\lambda}{\mu}\right)^n \right]^{-1}$$

3) $$L_q = \sum_{n=s}^{N} (n-s) P_n$$

4) $$L_s = L_q + S - P_0 \sum_{n=0}^{s-1} \frac{(s-n)(s\rho)^n}{n!}$$

5) $$W_q = W_s - \frac{1}{\mu}$$

6) $$W_s = \frac{L_s}{\lambda(1-P_N)}.$$

7) Traffic intensity, $\rho = \frac{\lambda}{\mu S}$ or, $S\rho = \frac{\lambda}{\mu}$

Numerical Problems on (M/M/S) : (N/FCFS)

Q: A barber shop has two barber and three chairs for customers. Assume that the customers arrive in a poisson fashion at a rate of five per hour and each barber services customers according to an exponential distribution with 15 minutes. Further, if a customer arrives and there are no empty chairs in the shop, he will leave. What is the expected no. of customers in the shop?

solution :

$S = 2$

$N = 3$

$\lambda = \frac{5}{60} = \frac{1}{12}$ per min

$\mu = \frac{1}{15}$ per min,

$\frac{\lambda}{\mu} = \frac{1}{12} \times 15 = \frac{5}{4}$

$\rho = \frac{\lambda}{\mu S}$

$S\rho = \frac{\lambda}{\mu} = \frac{5}{4}$

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$$P_0 = \left[\sum_{n=0}^{s-1} \frac{1}{n!} \left(\frac{\lambda}{\mu}\right)^n + \sum_{n=s}^{N} \frac{1}{s^{n-s} \; s!} \left(\frac{\lambda}{\mu}\right)^n \right]^{-1}$$

$$= \left[\sum_{n=0}^{1} \frac{1}{n!} \left(\frac{5}{4}\right)^n + \sum_{n=2}^{3} \frac{1}{2^{n-2} \; 2!} \left(\frac{5}{4}\right)^n \right]^{-1}$$

$0! = 1$

$a^0 = 1$

$$= \left[\left(1 + \frac{5}{4}\right) + \left(\frac{1}{2} \times \frac{25}{16} + \frac{1}{2 \times 2} \left(\frac{125}{64}\right)\right) \right]^{-1}$$

$$= \left[1 + \frac{5}{4} + \frac{25}{32} + \frac{125}{256} \right]^{-1}$$

$$= \left[\frac{256 + 320 + 200 + 125}{256} \right]^{-1}$$

$$= \left[\frac{901}{256} \right]^{-1} = \frac{256}{901} = 0.28$$

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$$P_n = \begin{cases} \frac{1}{n!}\left(\frac{\lambda}{\mu}\right)^n P_0 & , \quad 0 \leq n < s \\ \frac{1}{s^{n-s} * s!}\left(\frac{\lambda}{\mu}\right)^n P_0 & , \quad s \leq n \leq N \end{cases}$$

if $n=3$,

$0 \leq 3 < 2$ ✗

$2 \leq 3 \leq 3$ ✓

$$= \begin{cases} \frac{1}{n!}\left(\frac{5}{4}\right)^n * 0.28 & , \quad 0 \leq n < s \\ \frac{1}{2^{n-2} * 2!}\left(\frac{5}{4}\right)^n * 0.28 & , \quad s \leq n \leq N \end{cases}$$

Thus, $L_s = L_q + s - P_0 \sum_{n=0}^{s-1} \frac{(s-n)(s\rho)^n}{n!}$

$$= \sum_{n=s}^{N} (n-s) P_n + s - P_0 \sum_{n=0}^{s-1} \frac{(s-n)(s\rho)^n}{n!}$$

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$$= \sum_{n=2}^{3} (n-2) P_n + 2 - 0.28 \sum_{n=0}^{1} \frac{(2-n)\left(\frac{5}{4}\right)^n}{n!}$$

$$= 0 + P_3 + 2 - 0.28\left(2 + 1.25\right)$$

$$= P_3 + 2 - 0.28 \times 3.25$$

$$= P_3 + 1.09$$

$$= \left[\frac{1}{2^{3-2} \times 2!}\left(\frac{5}{4}\right)^3 \times 0.28\right] + 1.09$$

$$= \left[\frac{1}{4} \times (1.25)^3 \times 0.28\right] + 1.09$$

$$= \frac{1.9531 \times 0.28}{4} + 1.09 \qquad = \frac{0.5468}{4} + 1.09$$

$$= 1.226$$