## Wavelets 9735

1 pixel image 9 → 9 ×

2 pixel image 9,7 → 9 ×

7 ×

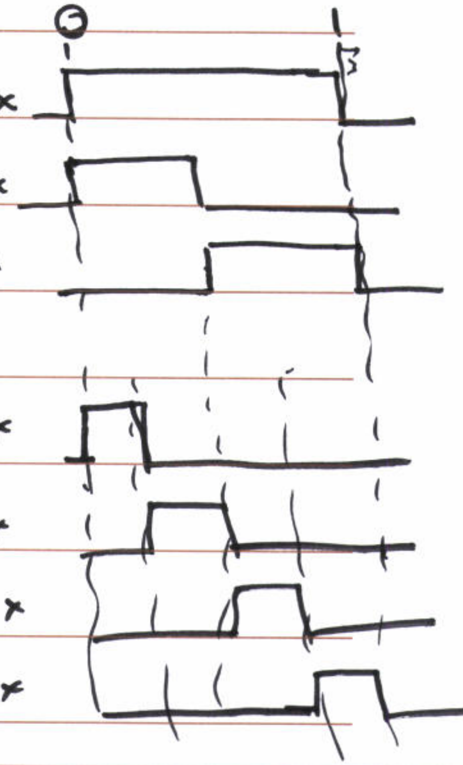
4 pixel image 9735 →

9 ×

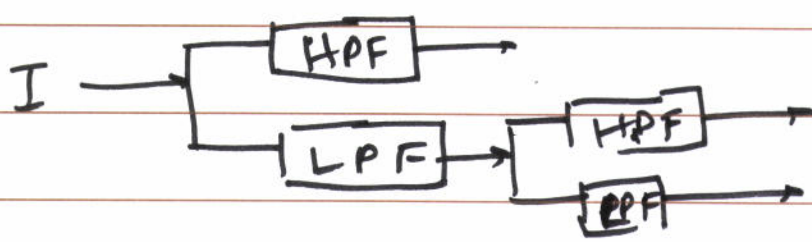
7 ×

3 ×

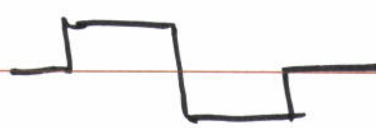
5 ×



In wavelet theory → express with LPF & HPF




LPF →  box

HPF →  harr

9 7 3 5 → 8 4 1 -1

→ 6 2 1 -1

box basis $\phi_i^j(x) = \phi(2^j x - i)$

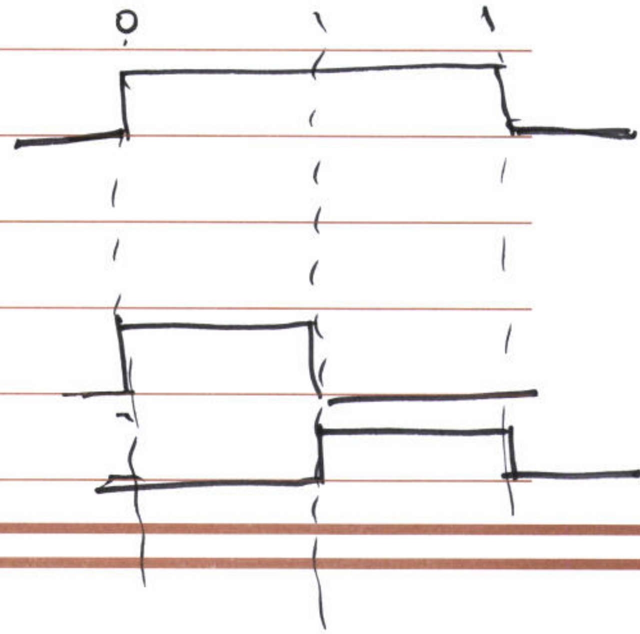
$i = 0, 1, 2 \ldots 2^j - 1$

$\phi(x) = 1 \quad \forall \; 0 \leq x < 1$

$\phantom{\phi(x)} = 0 \quad$ otherwise

$j = 0$

$\phi_0^0(x) = \phi(x)$




$j = 1$

$\phi_0^1(x) = \phi(2x)$

$\phi_1^1(x) = \phi(2x - 1)$

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$2x - 1 = y$

$0 \leq y < 1$

$0 \leq 2x - 1 < 1$

$1 \leq 2x < 2$

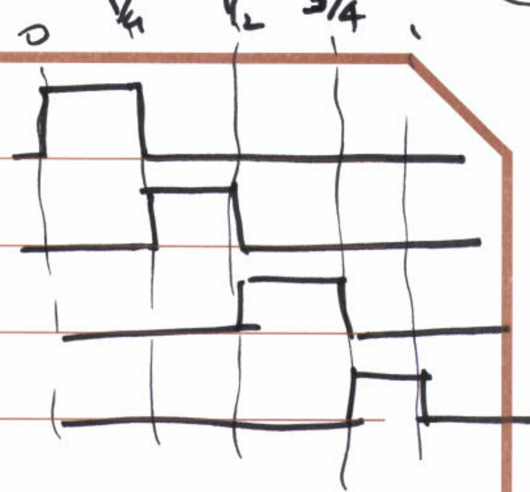
$1/2 \leq x < 1$

$\phi_0^2(x) = \phi(4x)$

$\phi_1^2(x) = \phi(4x-1)$

$\phi_2^2(x) = \phi(4x-2)$

$\phi_3^2(x) = \phi(4x-3)$



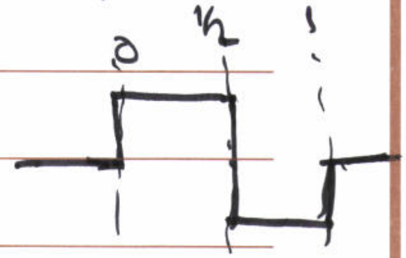

9 7 3 5

$$I = 9 \times \phi_0^2(x) + 7\,\phi_1^2(x) + 3\,\phi_2^2(x) + 5\,\phi_3^2(x)$$

Harr function

$\psi_i^j(x) = \psi(2^j x - i) \quad i = 0, 1, 2 \ldots 2^j - 1$

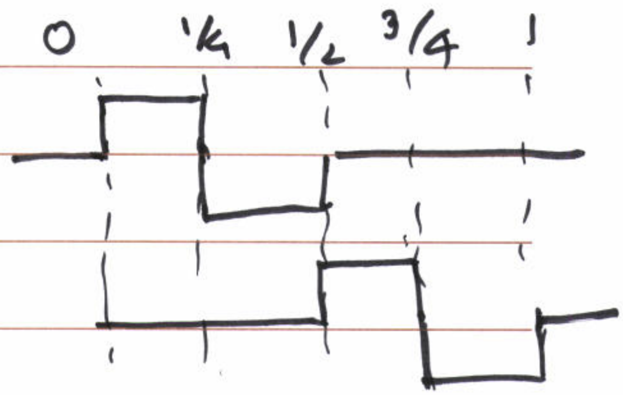
$$\psi(x) = \begin{cases} 1 & 0 \le x < 1/2 \\ -1 & 1/2 \le x < 1 \\ 0 & \text{otherwise} \end{cases}$$




$\psi_0^0(x) = \psi(x)$

$\psi_0^1(x) = \psi(2x)$

$\psi_1^1(x) = \psi(2x-1)$




$\cancel{2x-1 = 1}$

$\cancel{0 \le y < 1}$

$\cancel{0 \le 2x-1 < 1}$

$\cancel{1 \le 2x < 2}$

$\frac{1}{2} \le x < 1$

$$2x-1 = y$$

$0 \le y < 1/2$ $\qquad$ $1/2 \le y < 1$

$0 \le 2x-1 < 1/2$ $\qquad$ $1/2 \le 2x-1 < 1$

$1 \le 2x < 3/2$ $\qquad$ $3/2 \le 2x < 2$

$1/2 \le x < 3/4$ $\qquad$ $3/4 \le x < 1$

$$I = 9 \quad 7 \quad 3 \quad 5$$

$$= \underset{\substack{\parallel \\ 9}}{c_0^2}\,\phi_0^2(x) + \underset{\substack{\parallel \\ 7}}{c_1^2}\,\phi_1^2(x) + \underset{\substack{\parallel \\ 3}}{c_2^2}\,\phi_2^2(x) + \underset{\substack{\parallel \\ 5}}{c_3^2}\,\phi_3^2(x)$$

$$I = 8 \quad 4 \quad 1 \quad -1$$

$$= \underset{\substack{\parallel \\ 8}}{c_0^1}\,\phi_0^1(x) + \underset{\substack{\parallel \\ 4}}{c_1^1}\,\phi_1^1(x) + \underset{\substack{\parallel \\ 1}}{d_0^1}\,\psi_0^1(x) + \underset{\substack{\parallel \\ -1}}{d_1^1}\,\psi_1^1(x)$$

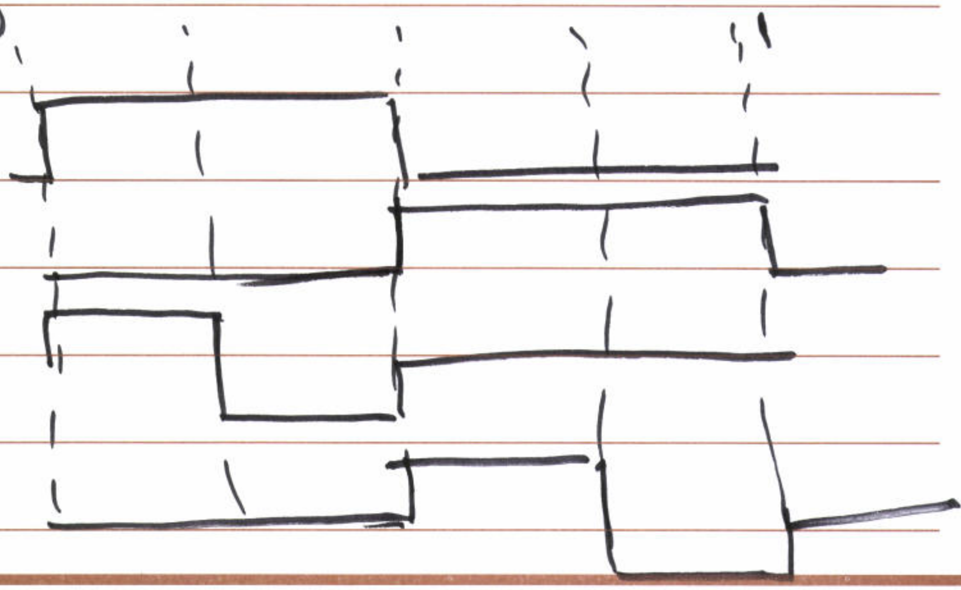
0 $\qquad$ 1

$c_0^1 = 8 \times$

$c_1^1 = 4 \times$

$d_0^1 = 1 \times$

$d_1^1 = -1 \times$



9 $\quad$ 7 $\quad$ 3 $\quad$ 5

⑤

$I = 6 \quad 2 \quad 1 \quad -1$

$= c_0^0 \phi_0^0(x) + d_0^0 \psi_0^0(x) + d_0^1 \psi_0^1(x) + d_1^1 \psi_1^1(x)$

$c_0^0 = 6 \times$

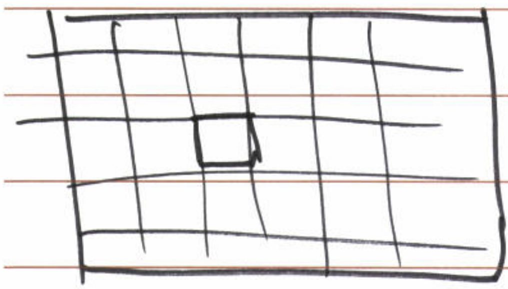
$d_0^0 = 2 \times$

$d_0^1 = 1 \times$

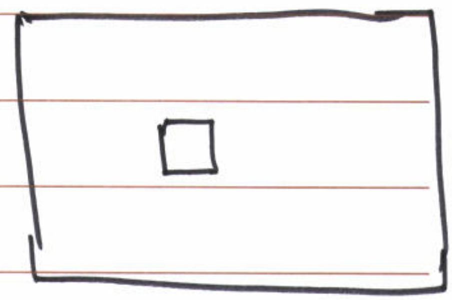
$d_1^1 = -1 \times$

9 7 3 5

# Dithering



A



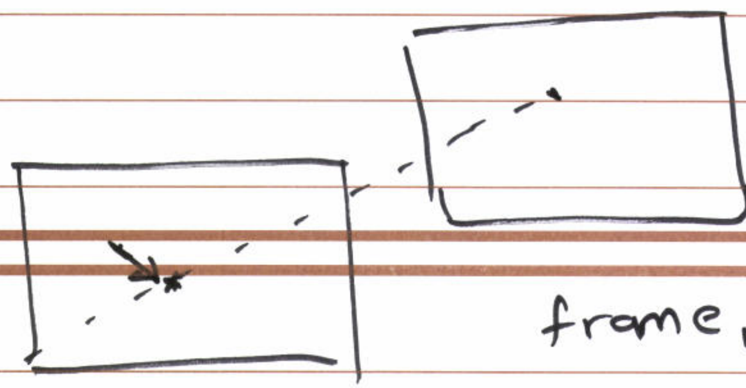
B

avg intensity

## Video compression

From frame to frame → pixels change

- noise
- objects move
- camera moves
- lighting changes



$\text{frame}_{n+1}$

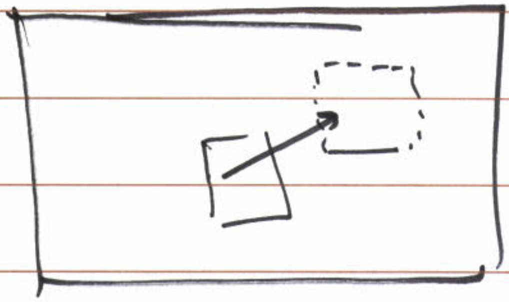
$\text{frame}_n$

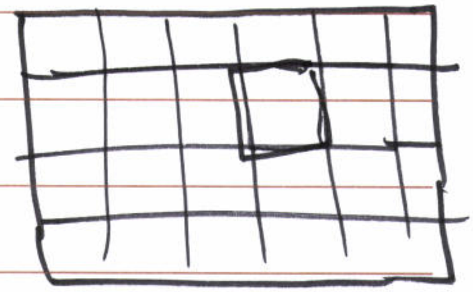
$$C_n[i\ j] \neq C_{n+1}[i\ j]$$

$$C_n[i+dx, j+dy] = C_{n+1}[i,j]$$

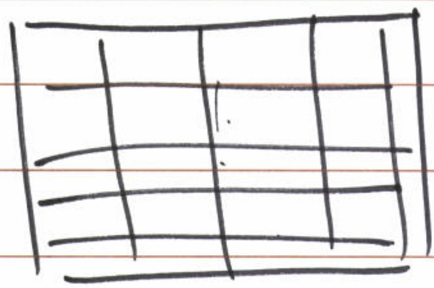
$dx\ dy$ = motion vector

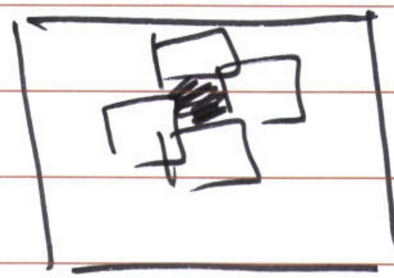
$dr\ dg\ db$





$\text{frame}_n$ $\text{frame}_{n+1}$





error metrics

$$MAD = \frac{\sum_{p=1}^{m} \sum_{q=1}^{n} \left| C_{n+1}(p,q) - C_n(p-dx, q-dy) \right|}{mn}$$

$$MSD = \left( \qquad \right)^2$$

SAD