COLOR THEORY

TOPICS TO BE COVERED

The color problem

Cone response and Tristimulus Vectors

Solution to the Color problem – using CMFs

Chromaticity Diagrams

Color Spaces and Examples

Displays

Color Quantization

THE COLOR PROBLEM

How do we perceive an object?

How do we capture the image of an object?

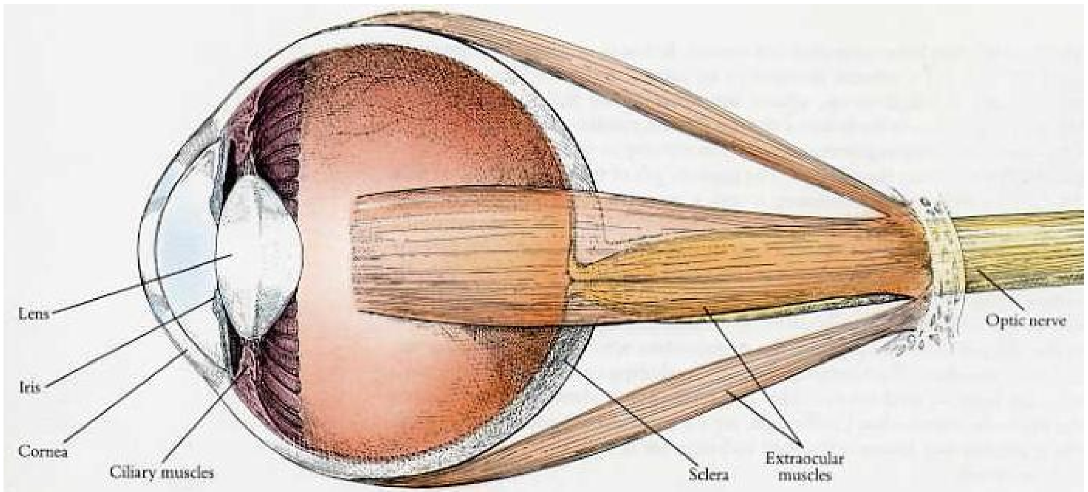
- **Digital Still Camera**
- **Camcorders**
- **Film processing/scanning**

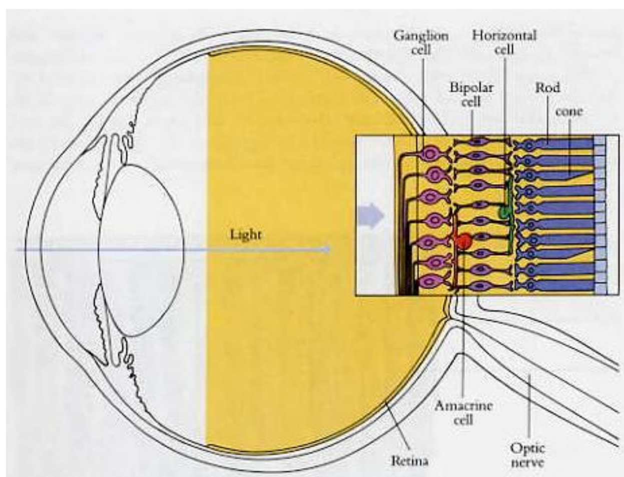
How do render an object image

- **Storage/Transfer issues**
- **Rendering on Client**
- **Printing**

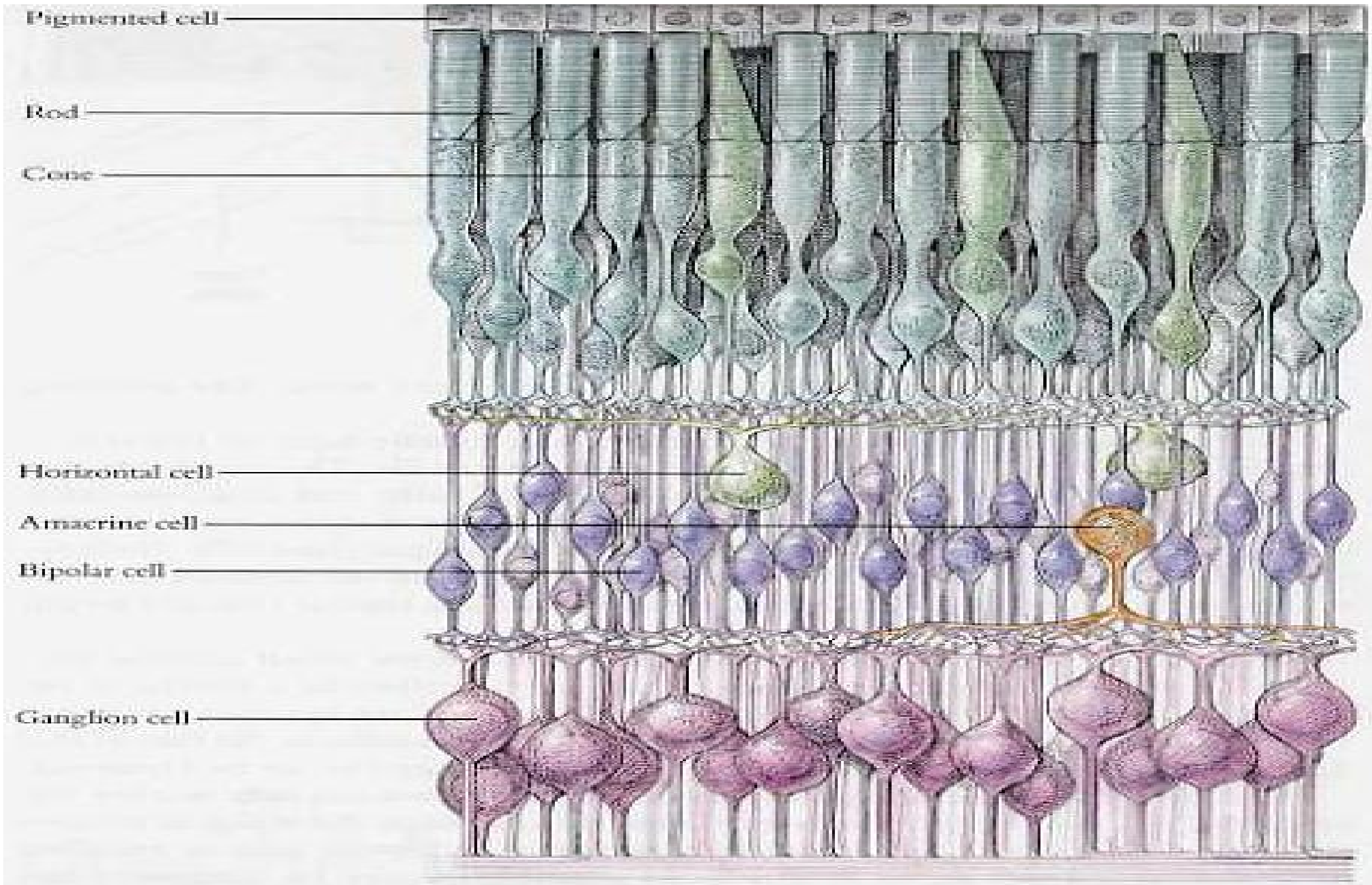
The Color Problem – How do you ensure that the colors in the image of a rendered object look the same as the original colors of that object?

EYE STRUCTURE

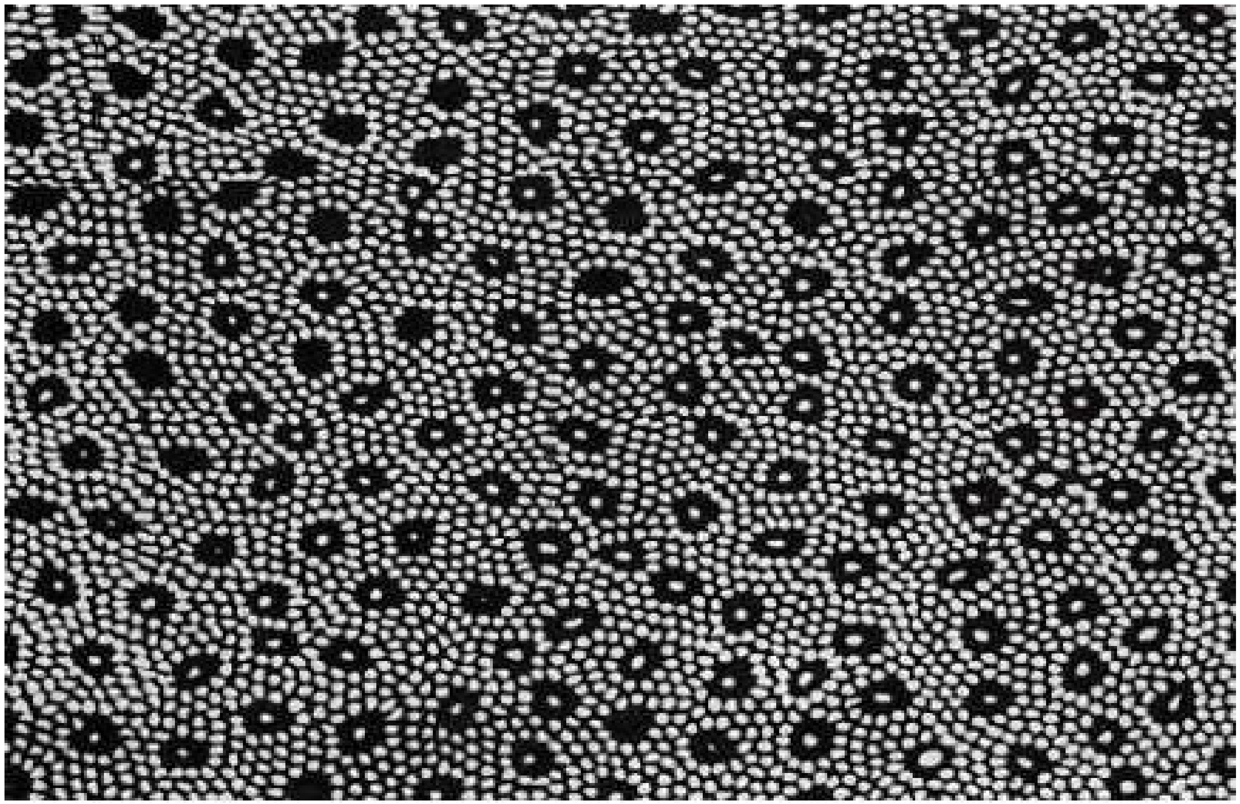




RETINA


Pigmented cell
Rod
Cone
Horizontal cell
Amacrine cell
Bipolar cell
Ganglion cell

RETINA



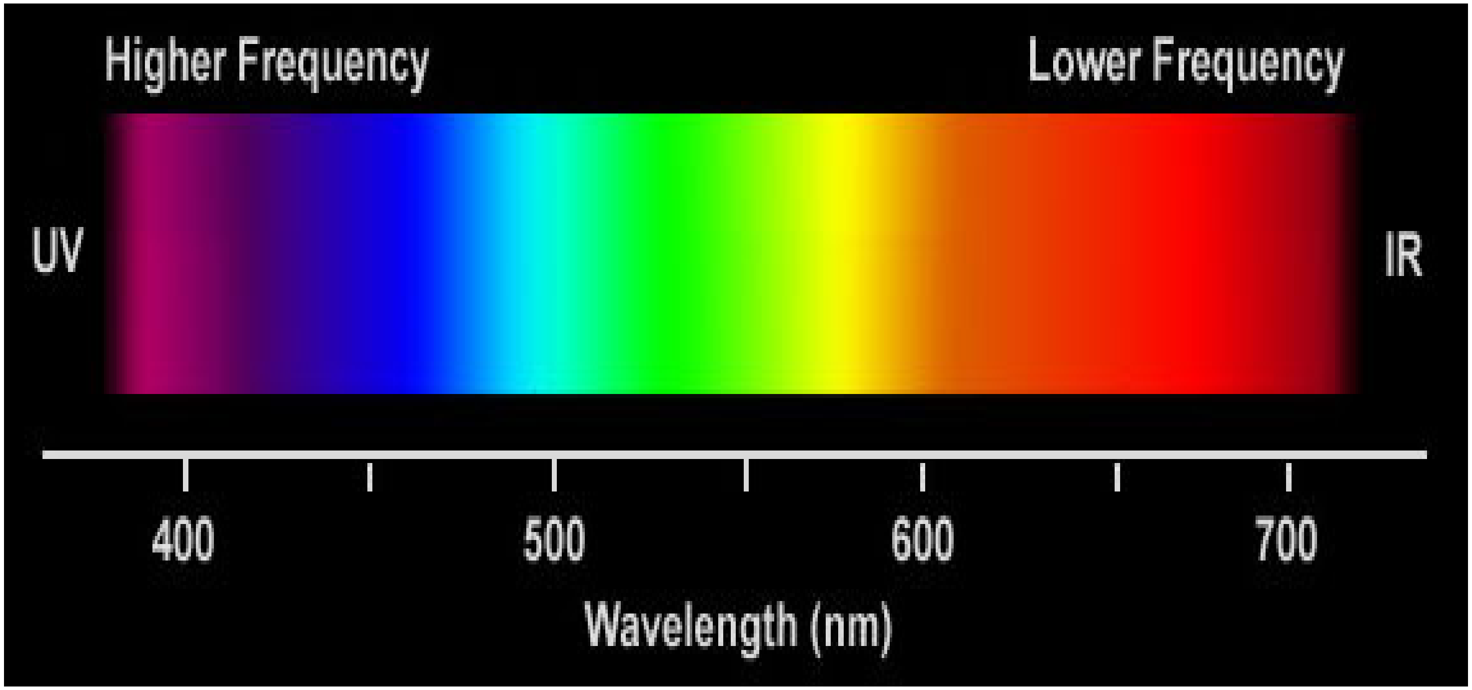
HOW DO WE PERCEIVE COLORS?

Spectrum of visible light *f*(λ): λ =360-830 nm
(λ = wavelength)

In the *retina*: 3 types of cones with different spectral absorptance (or sensitivity). The response c_i of a cone of type *i* is modeled as

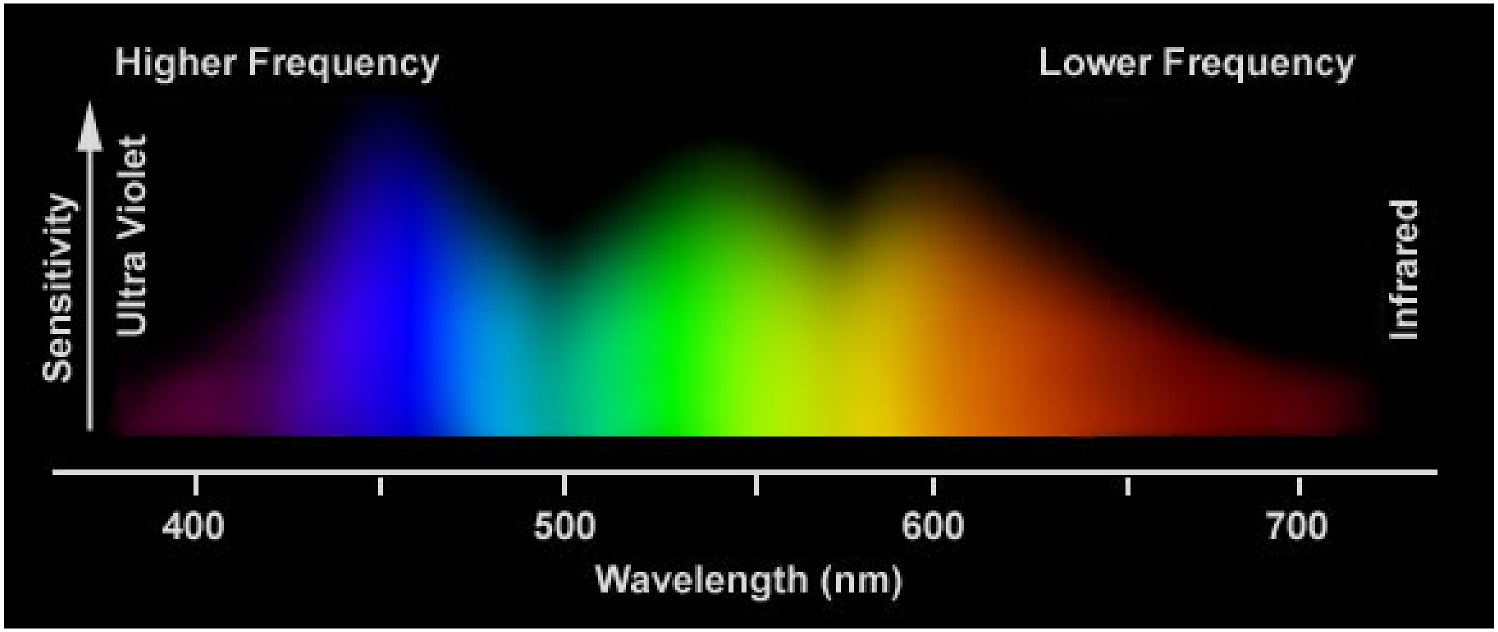
$$c_i = \int_{\lambda_{\min}}^{\lambda_{\max}} s_i(\lambda) f(\lambda) d\lambda$$

SPECTRAL DISTRIBUTION

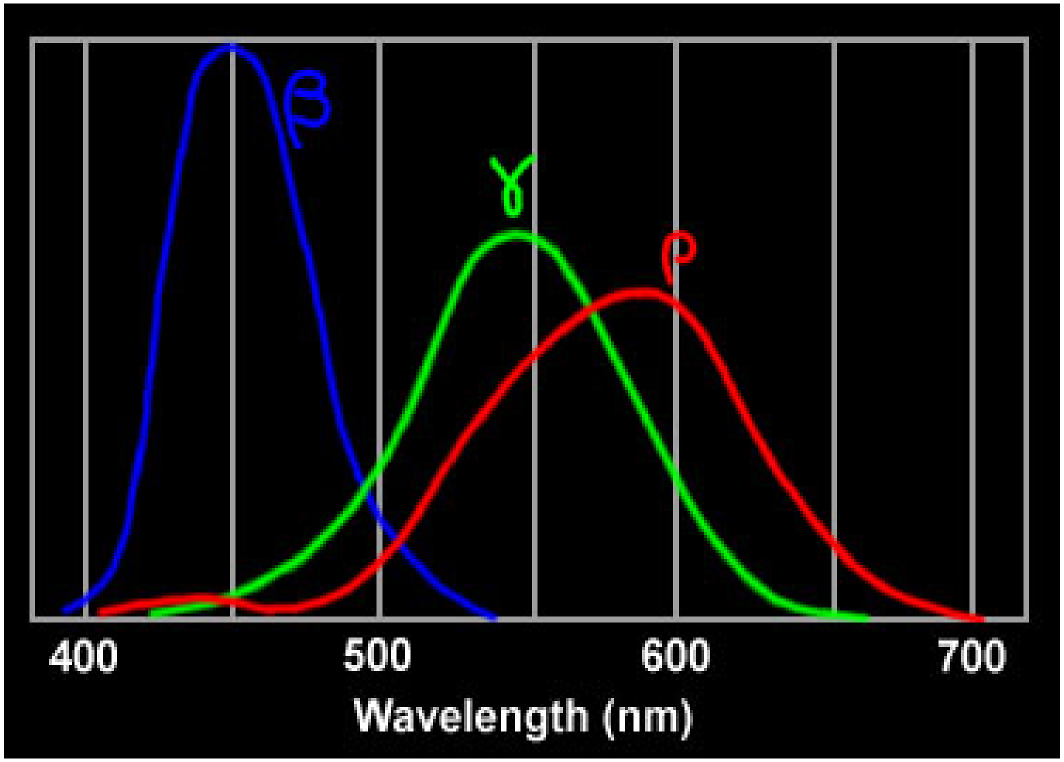


SPECTRAL SENSITIVITY

Sensitivity for Entire Spectrum



SPECTRAL SENSITIVITY OF CONES



DISCRETE FUNCTION EVALUATION OF CONE RESPONSE

Instead of evaluating the cone response as a continuous integral function, we will use discrete quantities with summations

- **Represent the light spectral stimulus $f(\lambda)$ as a vector f on *N* different wavelength (with a spectral interval of 10 nm)**
- **Represent each sensitivity $s_i(\lambda)$ as vector s_i as *N* different wavelength**

Then the cone response is represented by a vector (called tristimulus vector):

$$C = S^T f$$

TRISTIMULUS VECTORS

$C = S^T f$ is a 3-vector (called *color*) which represents how we *perceive* a light with spectrum f (which is a N-vector).

If two different spectra f_1 and f_2 produce the same color $C = S^T f_1 = S^T f_2$, then they *look the same*, that is, they are undistinguishable by the human eye.

Thus, it is *not* important that after processing, transmission and reproduction we recreate the same light spectrum f as the original, but that we reproduce a light spectrum that has the same perceived color c.

Thus, 3 values (the entries of C) are sufficient to represent the color of a pixel (instead of *N* values)

ISSUES WITH GENERATING COLOR

The tristimulus or color equation is $C = S^T f$

- We need to know S
- We need to know f

We don't actually know S; we will show soon that it doesn't matter

To understand f we need to know

- How should we capture a light spectrum? (How do we take a color picture?)
- How do we reproduce a light spectrum? (How do we render a color?)

HOW DOES A 3-CCD CAMERA WORK?

Light is refracted onto 3 CCDs (through a prism). In front of each CCD there is a *color filter* with frequency response $m_i(\lambda)$. Thus, a pixel of the i-th CCD gives an output

$$a_i = \int_{\lambda_{\min}}^{\lambda_{\max}} m_i(\lambda) f(\lambda) d\lambda$$

In discrete form: $a = M^T f$ where the i-th column of M contains the samples of $m_i(\lambda)$

Note that M can only have positive entries!

HOW DOES A COLOR PROJECTOR WORK?

Color Projectors work using Additive Synthesis: there are actually 3 light projectors focusing light onto the same spot.

Let the light spectra produced by each be: p_1, p_2, p_3 . The intensity of each projector can be controlled at each pixel via a *gain* a_i . Thus, the actual light spectrum on the screen is $g = p_1a_1 + p_2a_2 + p_3a_3 = Pa$

where $P=[\ p_1,\ p_2,\ p_3\ ]$ (size: Nx3) and

$a=[\ a_1,\ a_2,\ a_3]^T$ (size:3x1)

Are 3 projectors able to create all the *light spectra* that we want to reproduce? No! **But we just need that they create the *colors* we need to reproduce.**

COLOR PROJECTOR (2)

Let { p_1, p_2, p_3 } by three "colorimetrically independent" light sources (called *primaries*).

Then, given any spectrum f, we can always find gains a such that the light g = Pa generated by additive synthesis has the same *color* of f.

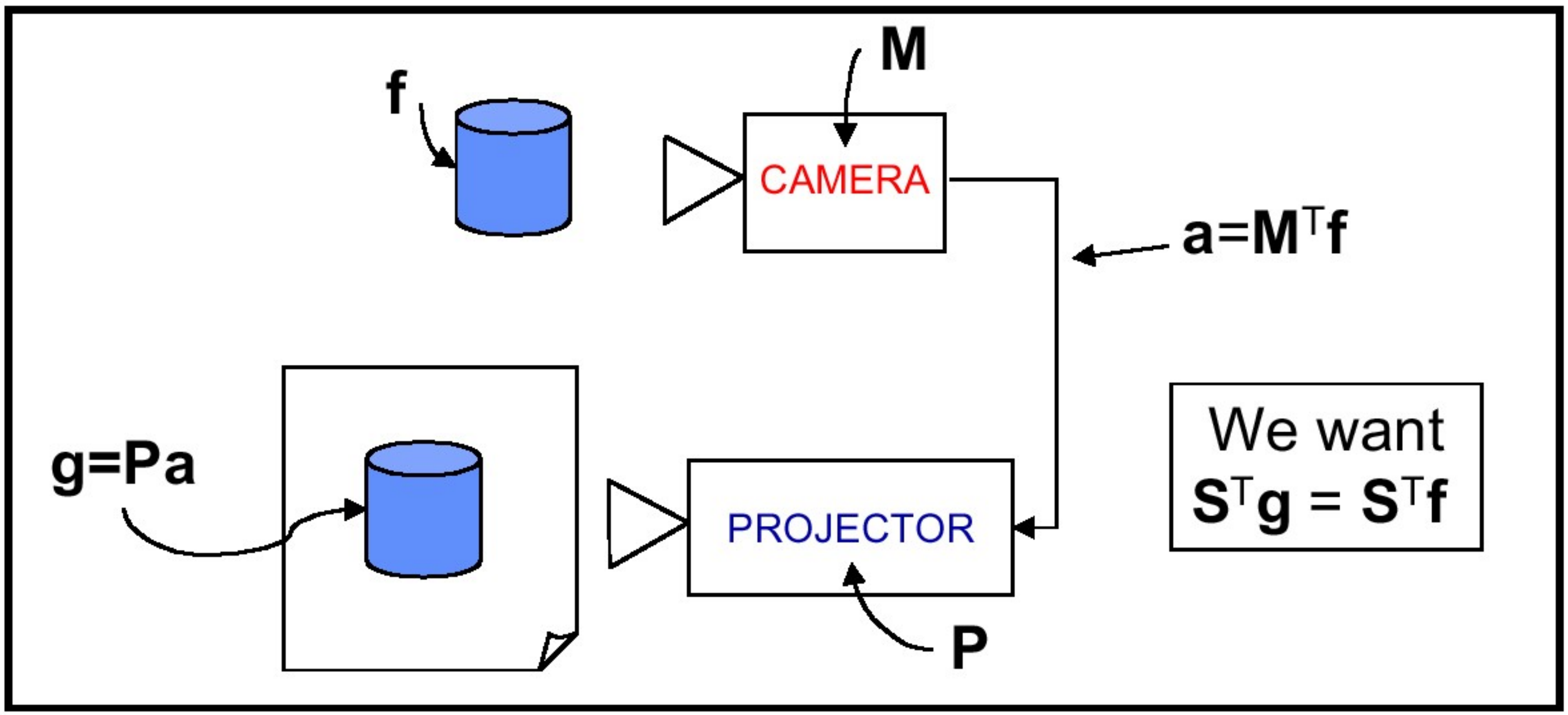
Proof: We need to find a vector a such that

$$S^T g = S^T P a = C. \text{ Just choose } a=(S^T P)^{-1} C.$$

Problem: a may have negative components, which makes no physical sense (gains and spectra can only have positive values!)

THE PROBLEM

Given a set of primaries P, how can we be sure that for any light spectrum f, the projector reproduces a light spectrum with the same color of f?



THE PROBLEM (2)

Given a set of primaries P, design the "filters" M of a color camera such that for any input spectrum f, the light g generated by the projector has the same color of f.

In other words: we want that the projectors, using as gains the output a of the camera, create a light g = Pa such that $S^Tg = S^TPa = S^Tf = C$.

And we want to do it without knowing S (which is difficult to compute)

PROBLEM SOLUTION

The solution is based on color matching experiments

The columns { m_1, m_2, m_3 } of M (size: *Nx*3) are called the color matching functions (CMF) associated with the primaries P.

Problem (again): CMF can take on negative values.

WHAT ARE COLOR MATCHING FUNCTIONS?

Like we said before: the three CMFs { m_1, m_2, m_3 } correspond to the " filters" in a color camera to generate the three color channels a = { a_1, a_2, a_3 }.

Each color signal can be used to control the intensities of the primaries (projectors) in order to generate a spectrum g = Pa that has the same color as f

$$S^T g = S^T P a$$

Note: Given a set or primaries P, there is just one set of CMFs M associated to it, and vice-versa

Remember, C is the "perceptual" color vector; a is the color vector associated to the chosen primaries

CHANGING PRIMARIES

What happens when we change primaries? (e.g., we use a different projector, or we use a CRT screen instead of a projector)

Suppose we use primaries Q instead of P. We want to find a new color vector b such that

$$S^T P a = S^T Q b$$

The solution is b=Ta with $T=(S^T Q)^{-1}(S^T P)$

(T is a 3x3 matrix which is independent of a)

Thus T is a linear operator that maps from one color space into a new one.

CIE CMFS

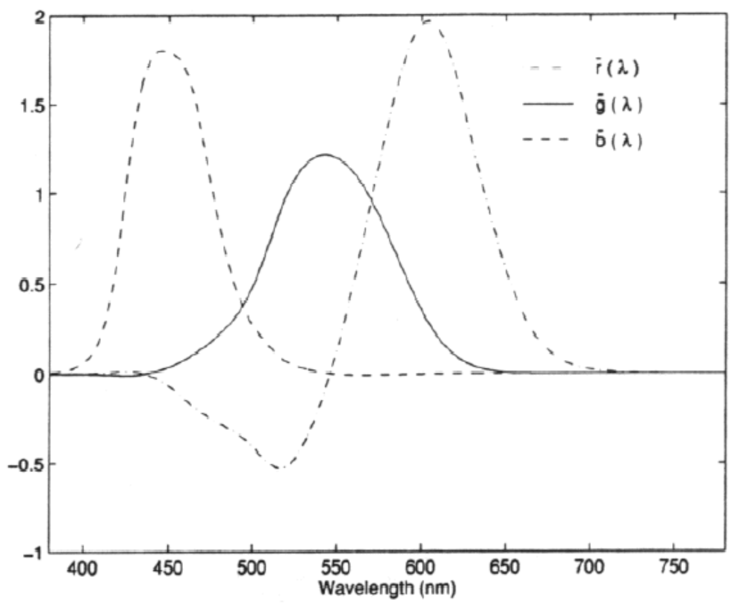
The International Commission on Illumination (CIE) has standardized two sets of CMFs: *RGB* and *XYZ*

RGB: The CMF's are associated with *monochromatic primaries* at wavelengths of 700, 546 and 435 nm.
(Note that one RGB CMF takes on -ve values.)

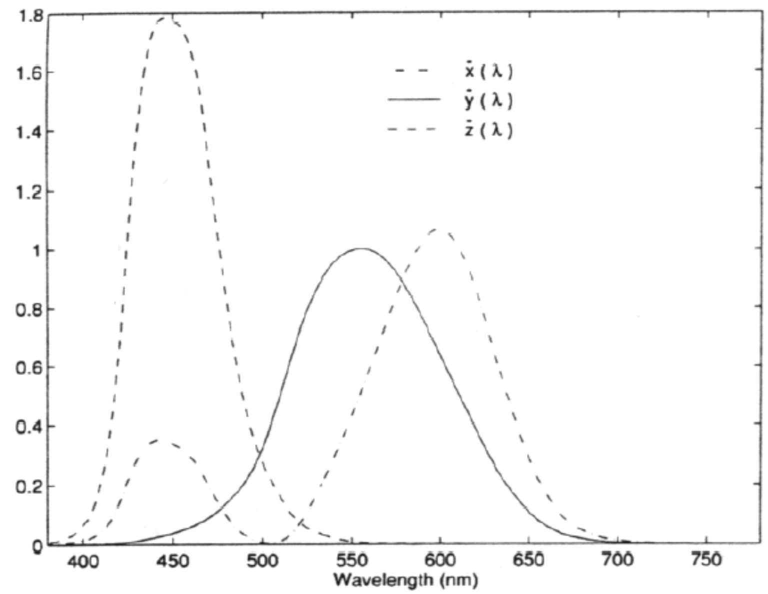
XYZ: One of the 3 CMFs, y(λ), is the *luminous efficient function*, which gives the relative sensitivity of the eye to each wavelength. *Y* is called *luminance*. (Note that all *XYZ* CMFs are positive; however, it can be shown that the corresponding primaries are not physically realizable)

Note: *{r(λ),g(λ),b(λ)}* ({*x(λ),y(λ),z(λ)*}) are the CMFs (corresponding to the columns of M); the values *{R,G,B}* {*X,Y,Z*} are the color channels (corresponding to a)

CIE CMFS

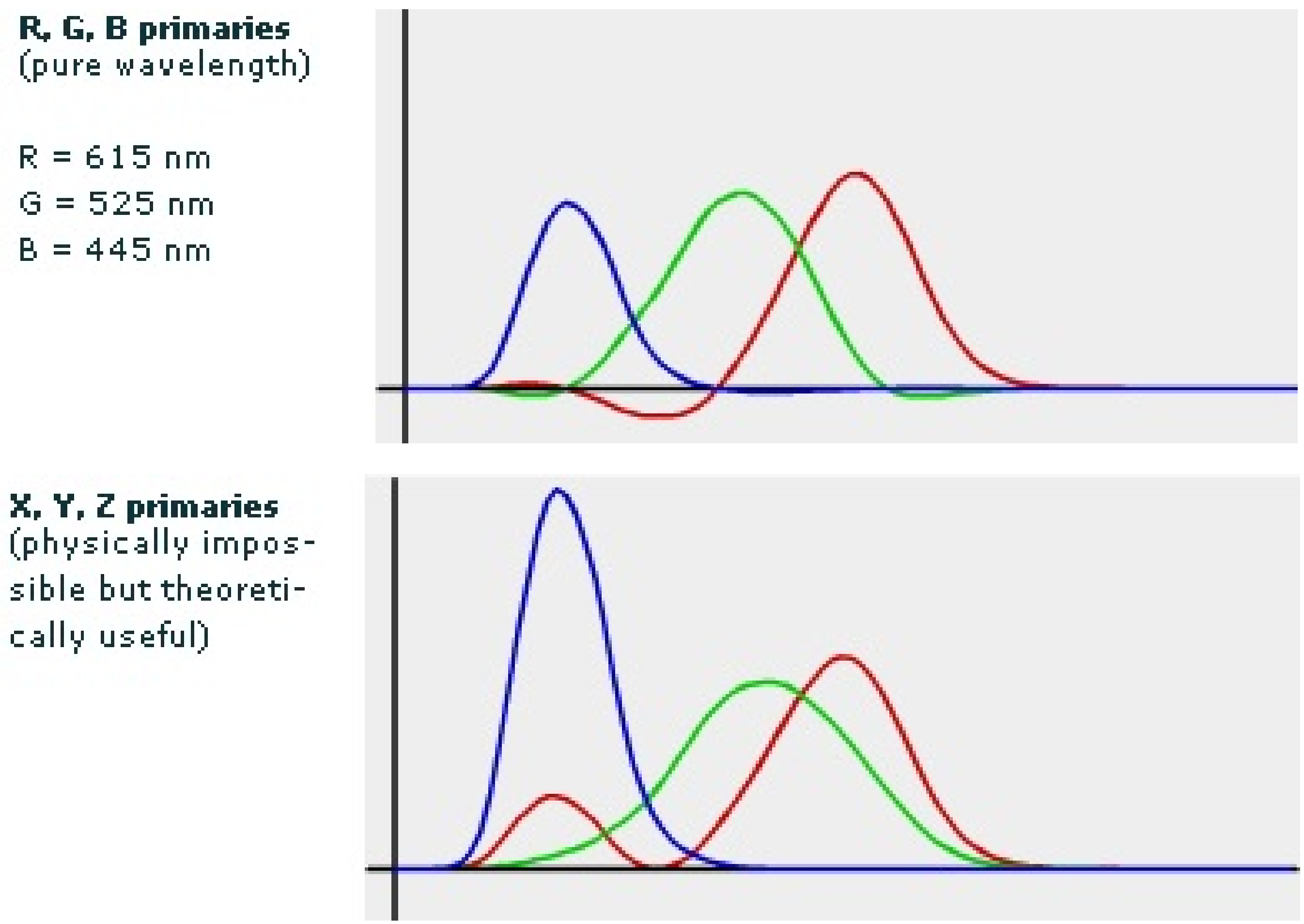


CIE RGB CMFs

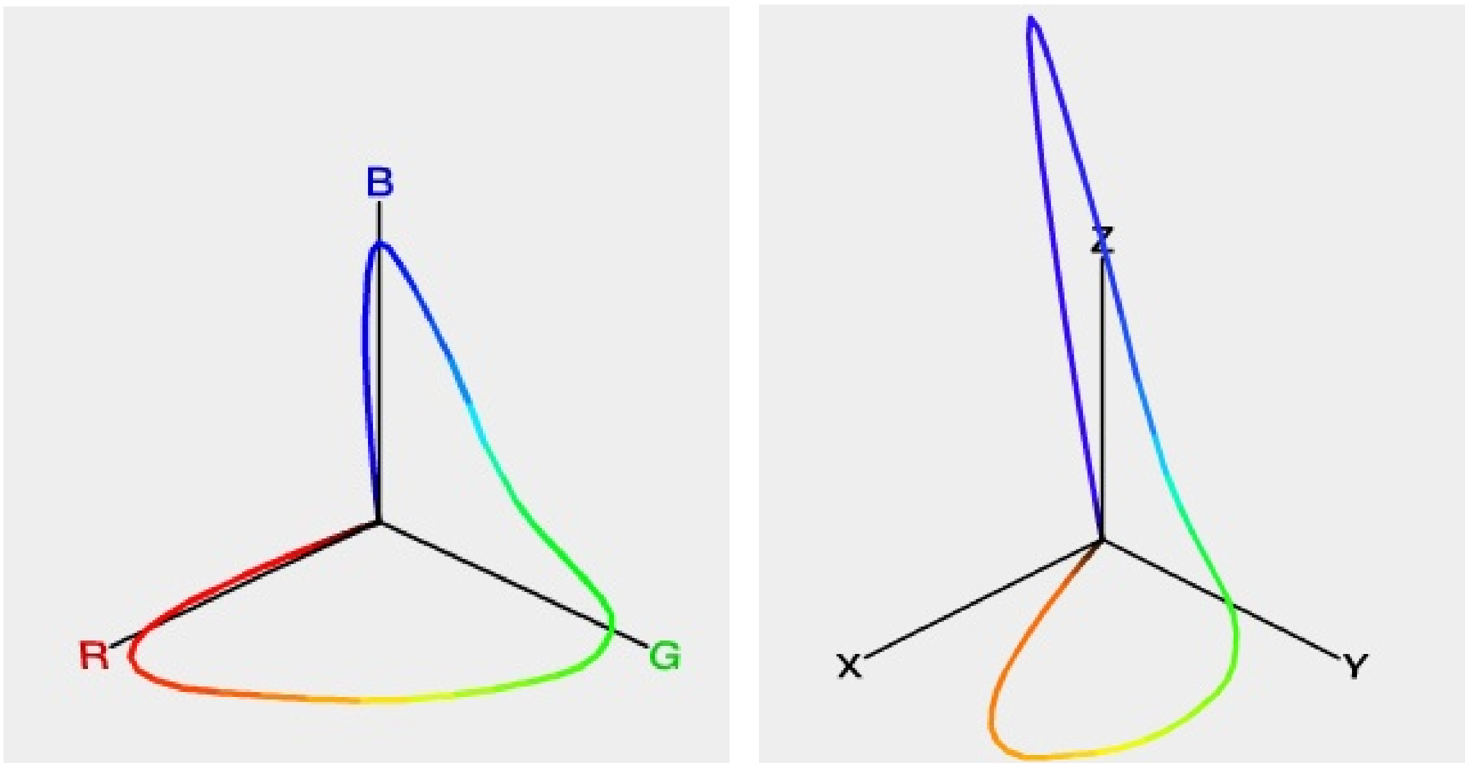


CIE XYZ CMFs

CIE CMFS


R, G, B primaries
(pure wavelength)
R = 615 nm
G = 525 nm
B = 445 nm
X, Y, Z primaries
(physically impos-
sible but theoreti-
cally useful)

RGB & XYZ COLOR SPACE



CHROMATICITY DIAGRAMS

Can we represent all colors by additive synthesis of 3 primaries? No! **That's because CMFs can assume negative values, and the frequency responses of the color filters can only be positive.**

***Problem*: represent all achievable colors on a 2-D plane (color channels** $a_1a_2a_3$ **live in a 3-D space!)**

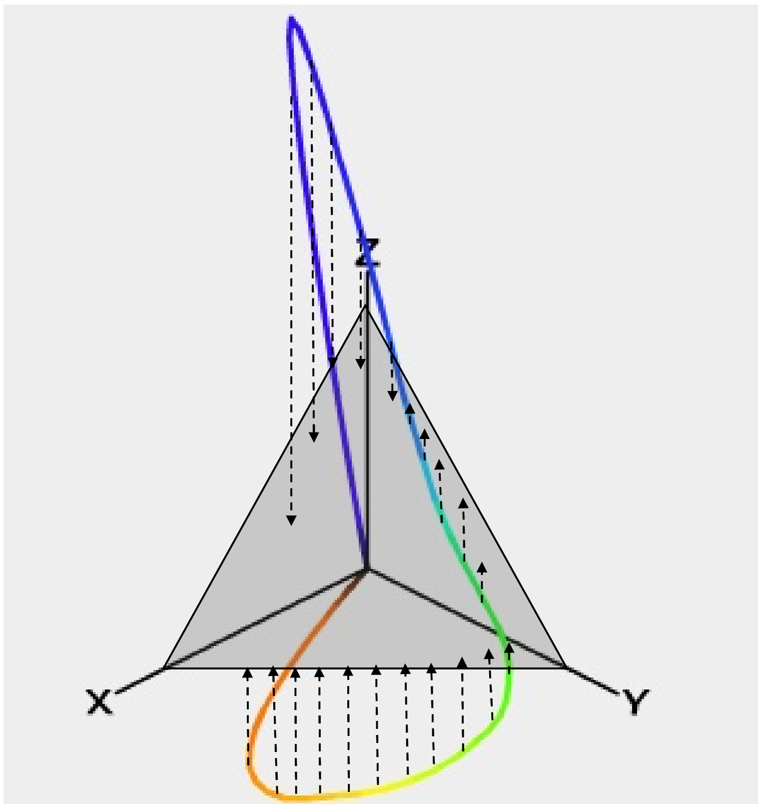
***Solution*: *normalize* colors to live in 2-D.**

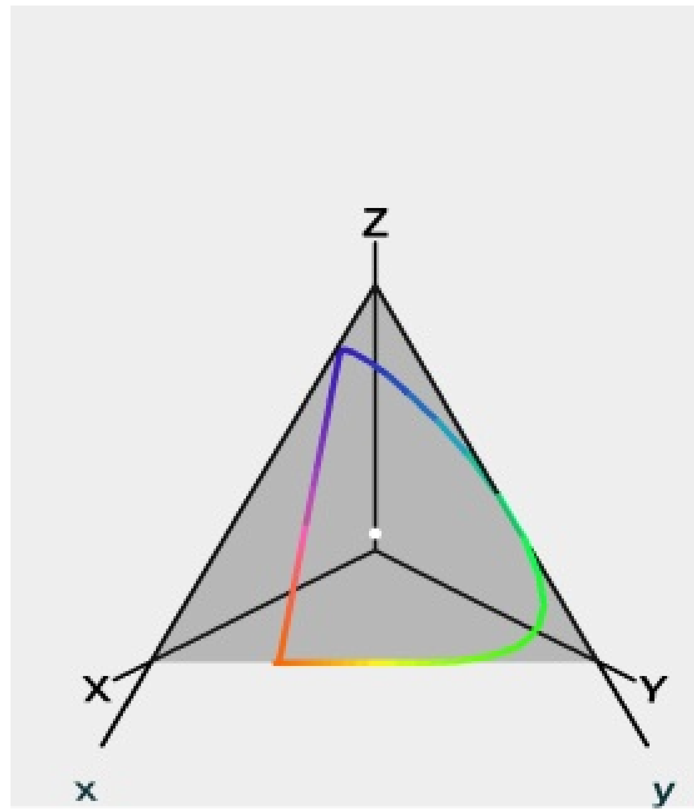
***Example*: CIE xy chromaticity diagram. Given a color value (X,Y,Z), compute the normalized values**

$$x=X/(X+Y+Z),\ y=Y/(X+Y+Z),\ z=Z/(X+Y+Z).$$

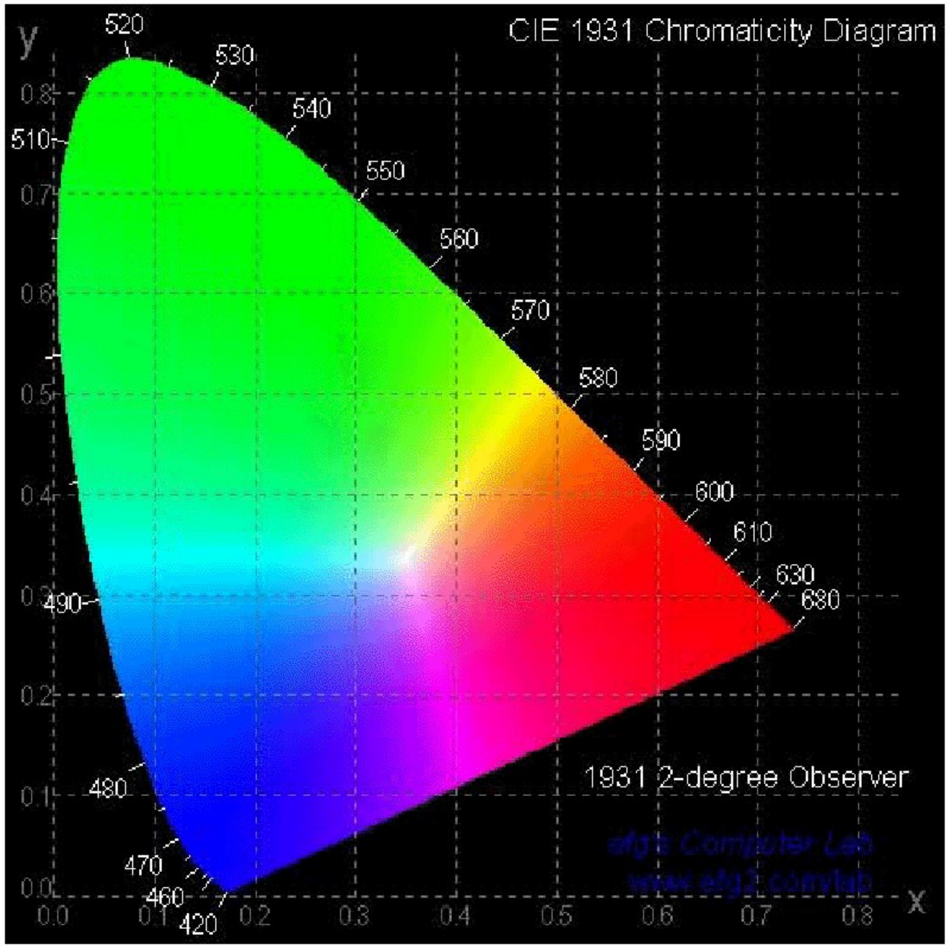
Then, plot only the colors corresponding to 2 chromaticity coordinates (x, y)

NORMALIZED XYZ SPACE



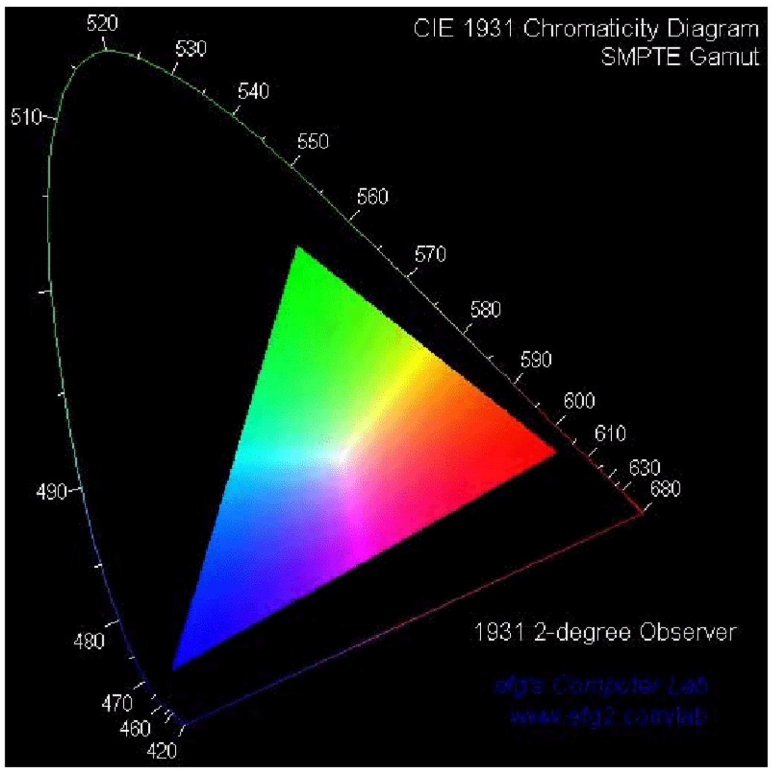


CIE *XY* CHROMATICITY DIAGRAMS



COLOR GAMUTS

Color gamut **of a color synthesis device = set of obtainable colors. Represented by a *polygonal* on the chromaticity diagram, example shown here**



VISUALIZATIONS

Color Applets

Visualization of CIE XYZ and RGB

Visualization of Color Gamuts

LINEAR TRANSFORMATIONS OF *RGB* COLOR SPACES

***RG*B: used in CRT monitors (additive synthesis)**

***YIQ* (used in NTSC TV). T=** $\begin{pmatrix} 0.299 & 0.587 & 0.114 \\ 0.596 & -0.274 & 0.322 \\ 0.211 & -0.523 & 0.312 \end{pmatrix}$

***YUV* (used in PAL, SECAM). T=** $\begin{pmatrix} 0.299 & 0.587 & 0.114 \\ -0.147 & -0.289 & 0.436 \\ 0.615 & -0.515 & -0.1 \end{pmatrix}$

***YCrCb* (for JPEG, MPEG). T=** $\begin{pmatrix} 0.299 & 0.587 & 0.114 \\ 0.500 & -0.4187 & -0.0813 \\ -0.1687 & -0.3313 & 0.5 \end{pmatrix}$

2/7/2001

WHY IS *RG*B NOT USED?

Similarly to the processing in the human visual system, transformed color spaces such as *YIQ, YUV, YCrCb* represent color with *luminance (*Y) and *chrominance* (the other 2 channels). Advantages:

- We can subsample the chrominance channels (e.g., 4:2:2, 4:2:0 subsampling schemes)
- We can quantize the chrominance channels more coarsely (with fewer bits)
- The chrominance channels are rather *uncorrelated* with the luminance channel, which yields better compression.

OTHER TRANSFORMATIONS

CMY: Cyan, magenta, yellow – complementary to red, green and blue respectively. Used for ***subtractive synthesis*** from white in color printers.

$$(C, M, Y) = (1,1,1) - (R,G,B)$$

CMY*K**: like ***CMY, uses black (K) as fourth color.

Given (C, M, Y):

$K=\min(C, M, Y)$; $C=C-K$; $M=M-K$; $Y=Y-K$

UNIFORM COLOR SPACES

CIE XYZ is not perceptually uniform because perceptual differences between colors do not correspond to equal distance in color space

CIE defined two uniform color spaces:

CIE Luv and CIE Lab:

- **L depends only on Y (luminance)**
- **Radial distance (e.g. sqrt($u^2 + v^2$)) correlates to chroma (or saturation)**
- **Angular position (e.g. $\tan^{-1}(u/v)$) correlates to hue**

HSV COLOR SPACE

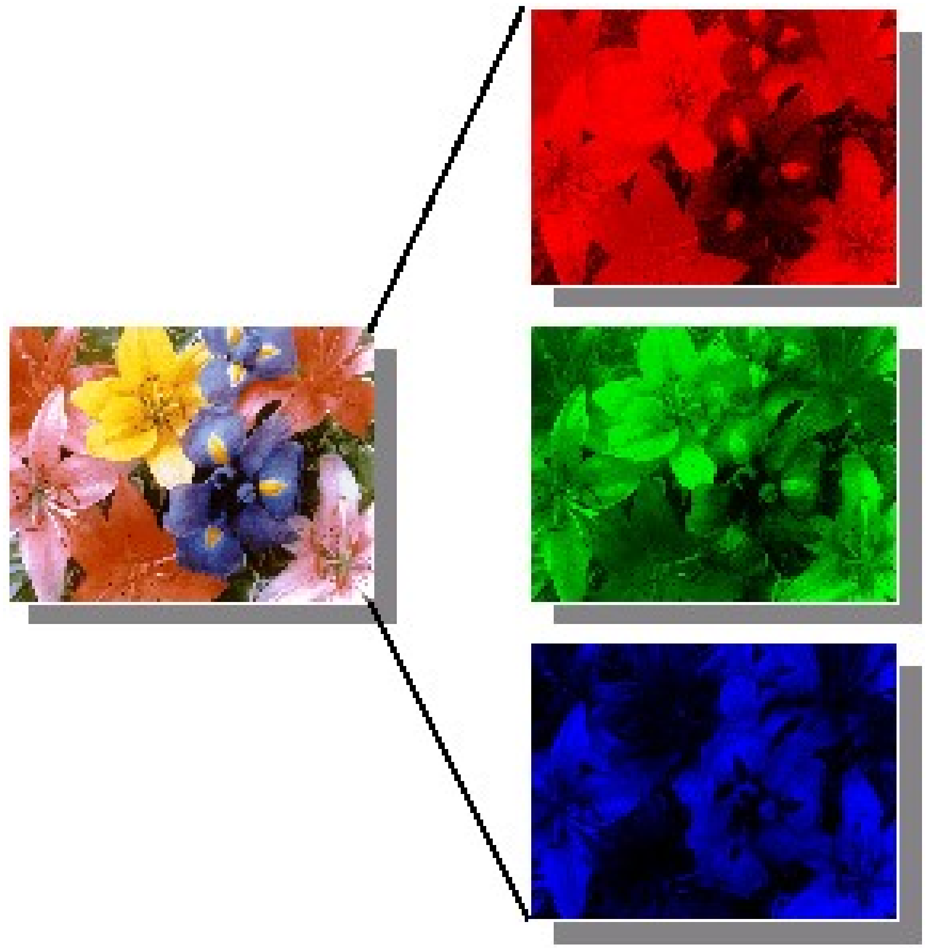
HSV is based on a cylindrical coordinate system

H (*hu*e) is measured in angles with respect to the *V* (*valu*e) axis. *S* (*saturatio*n) goes from 0 to 1.

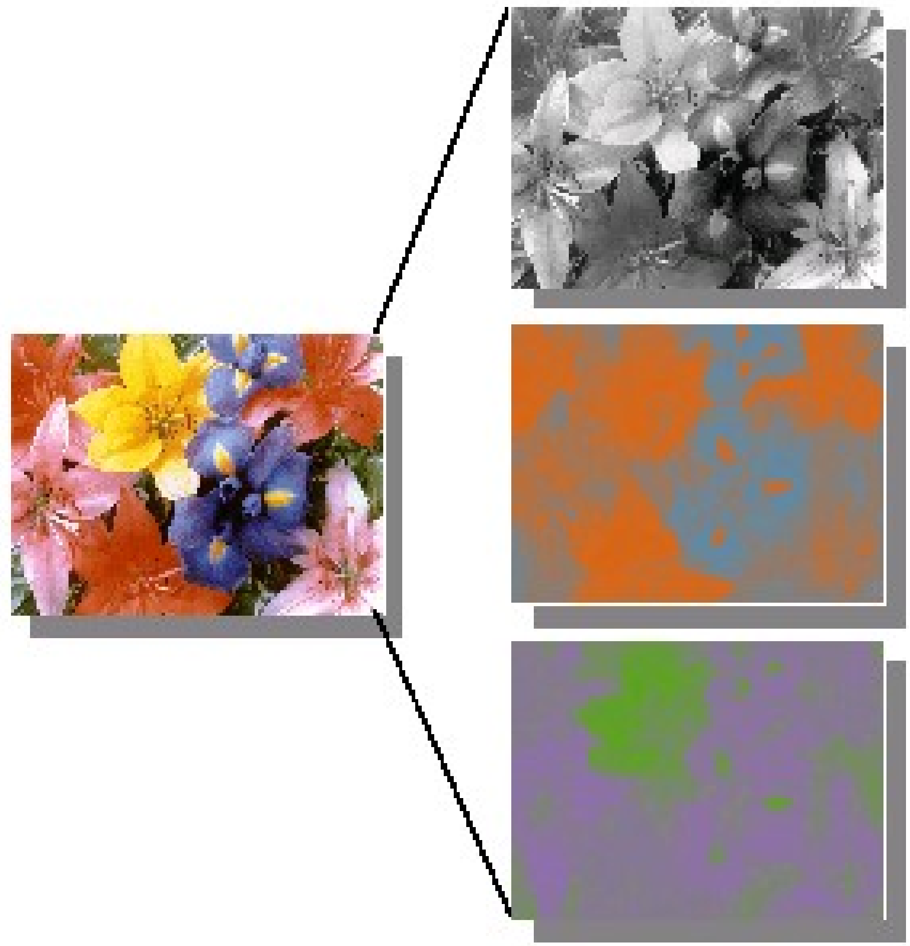
Complementary colors in HSV are 180 degrees from each other.

Hue distinguishes among colors such as red, green, purple, and yellow. *Saturation* refers to how far color is from a gray of equal intensity. *Value* represents the lightness (or brightness)

COLOR SPACE EXAMPLES – RGB



COLOR SPACE EXAMPLES – YIQ



COLOR SPACE EXAMPLES – YUV

COLOR DISPLAYS

Visible light is produce by bombardment of a thin layer of phosphor by a beam of electrons.

Shadow-mask color CRT: mosaic of *r,g,b* light emitting phosphors illuminated by 3 independent electron beams.

Color is controlled through application of different voltage to *r, g, b* guns.

How can we take into account non-linearities?

GAMMA CORRECTION

For each gun, the power of the emitted light $S_i(\lambda)$ is a function of the *control voltage* v_i. Ideally, the relationship would be linea*r:* $S_i(\lambda) = v_i\, p_i(\lambda)$ (so we would set $v_i = a_i$.) In practice, the relation is *not linear*. A better approximation is

$$S_i(\lambda) = (v_i / v)^{\gamma}\, p_i(\lambda)$$

where v is the maximum value of the voltage and γ is the exponential parameter. The monitor γ is usually approximately equal to 2.2

Thus, given a value of a_i, instead of controlling the light with voltage $v_i = a_i$, we should use $v_i = v\, a_i^{1/\gamma}$

This transformation is called *gamma correction*.

CCD CAMERAS

3 CCD

- **Light is refracted into each CCD using a prism**
- **In front of each CCD there is a color filter (corresponding to CMF functions *r ,g ,b)***
- **More expensive (need 3 CCDs per pixel) – but better resolution**

1 CCD

- **There is 1 filter *(r, g* or b) on top of each pixel. The filters are chosen according to a "mosaic" layout. E.g.**

  r g b r g b r g b
  g b r g b r g b r
  b r g b r g b r g

- **Cheaper (1 CCD) but with less color resolution (e.g.: for an *r* pixel, the other colors must be interpolated from nearby pixels.**

COLOR QUANTIZATION

What is color quantization and why is it required?

Frame buffer architecture: Image is stored in a video memory from which controllers constantly refresh the display screen. How many bits per pixel should we use? (8-12-16-24…)

The more pixels you use, the longer it takes to read and longer to refresh your monitor screen.

Palletized image: Uses a finite set of colors indexed in a table *(palette* or *color m*ap or LUT). You may choose this color mapping in two ways

- **Pre designed or fixed**
- **Adaptively - "quantized" version is similar to the original**

ADAPTIVE COLOR QUANTIZATION METHODS

The process occurs in stages

- Sampling the original image for color statistics – (Histogram)
- Choosing a color map based on those statistics
- Mapping colors to their representative colors to get new image

Different algorithms proposed and used depend on how the second step above is done

- Uniform Quantization
- Median Cut – (Heckbert)
- Octree – (Gervautz & Purgathofer)
- Peano Curve Mapping

QUESTIONS

The chromaticity diagram in (x, y) represents the normalized color matching functions X, Y and Z. Prove that

$$Z = [\ (1-x-y)/y\ ]\ Y$$

QUESTIONS

The chromaticity space is a 2D space obtained by projecting the 3D conical XYZ space onto a plane.

- **What is the equation of this plane**
- **Why is there no black color in the chromaticity space?**
- **Where do the CIE primaries lie in the chromaticity space?**