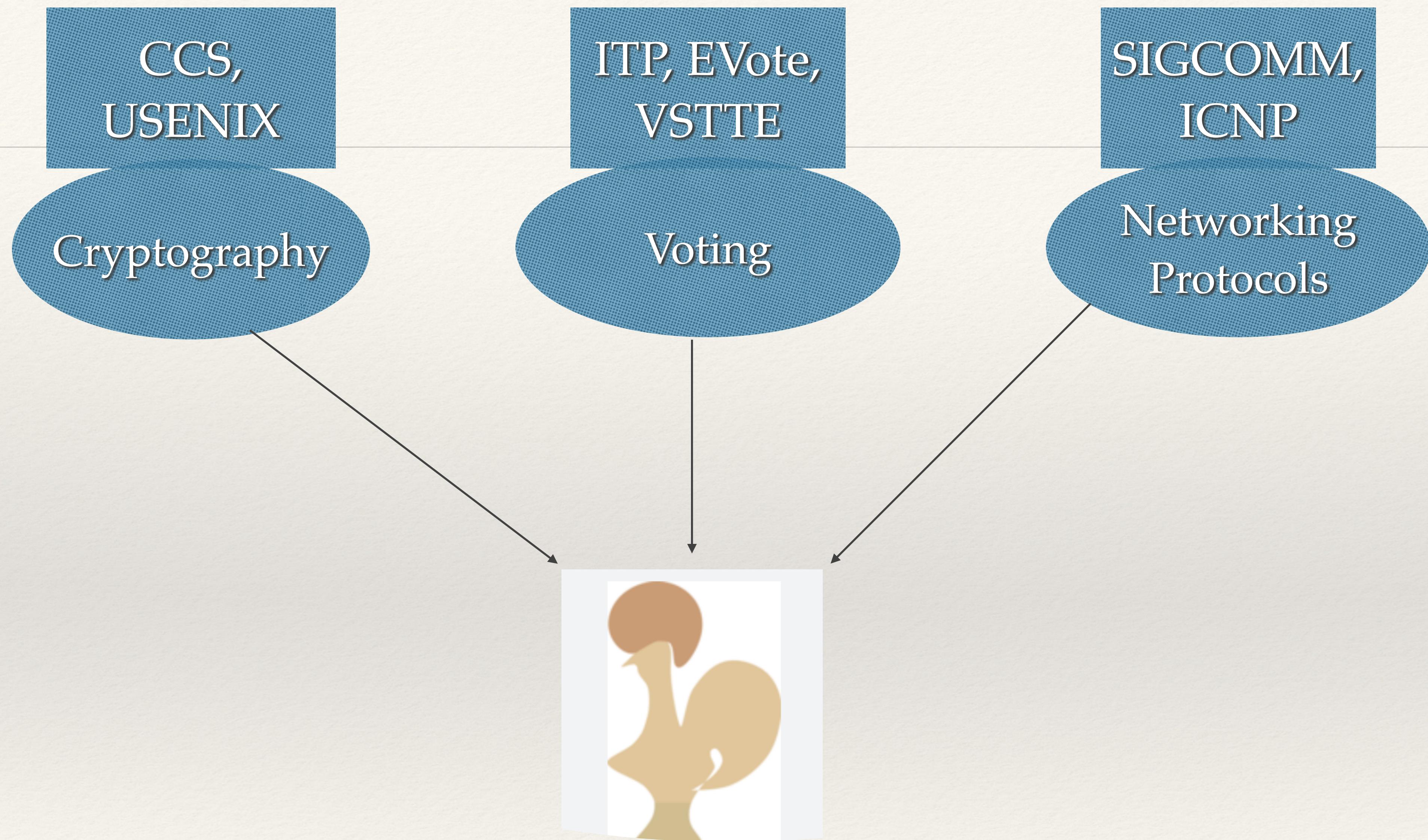


# Formal Verification for Correct and Secure Software

Mukesh Tiwari,  
University of Cambridge,  
Cambridge



UNIVERSITY OF  
CAMBRIDGE



# Moscow's blockchain voting system cracked a month before election

French researcher nets \$15,000 prize for finding bugs in Moscow's Ethereum-based voting system.

≡ **cnn politics** The Biden Presidency Facts First 2022 Midterms

## Federal review says Dominion software flaws haven't been exploited in elections



By [Sean Lyngaas](#), [Evan Perez](#) and [Whitney Wild](#), CNN

Updated 12:18 PM EDT, Fri June 3, 2022

## Experts Find Serious Problems With Switzerland's Online Voting System Before Public Penetration Test Even Begins

The public penetration test doesn't begin until next week, but experts who examined leaked code for the Swiss internet voting system say it's poorly designed and makes it difficult to audit the code for security and configure it to operate securely.



By [Kim Zetter](#)

## Flaws found in NSW iVote system yet again

Analysis of source code published at the request of the NSW Electoral Commission shows that the state's election system software was still vulnerable to attack.

# I hear you: paper ballots are great

 AEC 📝 ✅  
@AusElectoralCom

The Senate count is one of the most complex upper house counts in the world - it's so complex that we needed to write a program to distribute your preferences, as doing it by hand would mean we couldn't provide elected Senators in time to take their seat.



12:02 AM · Jun 14, 2022

13 Retweets 2 Quote Tweets 62 Likes

**Blind advocates allege NSW's removal of online voting system is a breach of human rights**

**State electoral commission accused of discrimination for suspending iVoting platform as Blind Citizens Australia takes case to watchdog**

[Home](#) > [The Ministry and its Network](#) > [News](#) > [2020](#)

A+ A-

**French citizens abroad – Approval of electronic voting for consular elections (15 January 2020)**

# Claim

Theorem Provers can help us in  
implementing correct and secure  
software

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# Machine-checking Multi-Round Proofs of Shuffle: Terelius-Wikstrom and Bayer-Groth

Authors:

Thomas Haines, *Australian National University*; Rajeev Gore, *Polish Academy of Science*; Mukesh Tiwari, *University of Cambridge*

ply says “[t]he completeness follows from the completeness of the sigma proof,” which we have just shown not to be the case since the relationship which the sigma protocol proves may be false.



**This petition was submitted during the 2010–2015  
Conservative – Liberal Democrat coalition government**

[View other petitions from this government](#)

## Petition

# Schulze method voting system

[More details](#)

Most voting systems are flawed due to their methodology (see Arrow's Impossibility Theorem for more details) but some are more 'fair' than others. The Schulze method, based on the Schwartz criterion is a voting system which is mathematically 'fairer' and will generate voting outcomes which are a better representation than most voting systems. This petition aims to start a debate into the practicalities of introducing this system into UK public voting systems.

**This petition is closed**

**This petition ran for 6 months**

**3 signatures**



10,000

# Schulze Method

$$\prod_{k=0}^n \prod_{i=0}^n \prod_{j=0}^n dist[i][j] = \max(dist[i][j], \min(dist[i][k], dist[k][j]))$$

Schulze Method

$$\prod_{k=0}^n \prod_{i=0}^n \prod_{j=0}^n dist[i][j] = \min(dist[i][j], dist[i][k] + dist[k][j])$$

Floyd-Warshall Algorithm

# Schulze Method

$C < B < A$

A

1
2

B

C

Ballot 1

$C = B < A$

1

2

2

Ballot 2

$C = B = A$

1

1

1

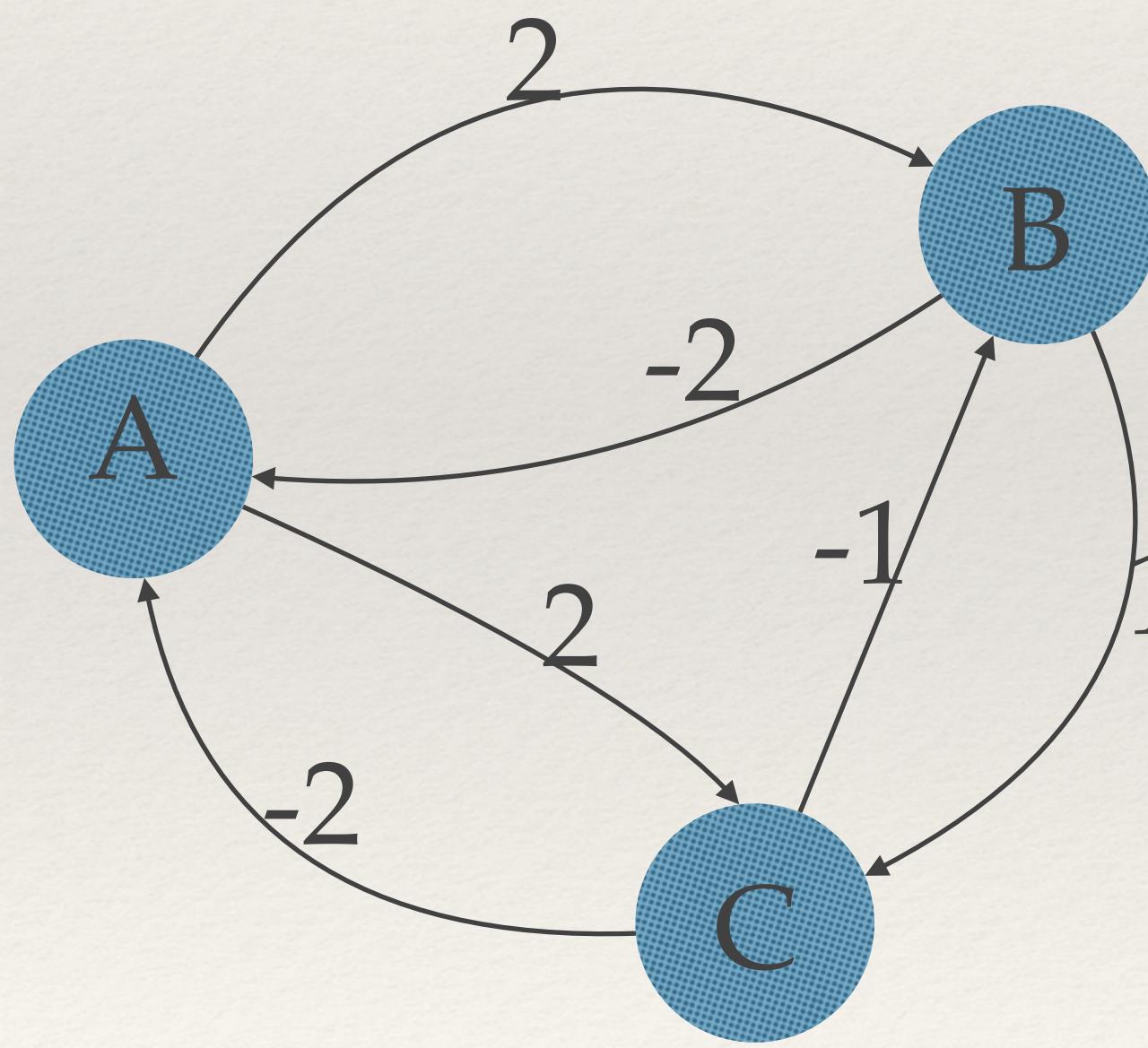
Ballot 3

# Schulze Method

		A	B	C
A	0	1	1	
B	-1	0	0	
C	-1	0	0	

$$C = B < A$$

# Schulze Method

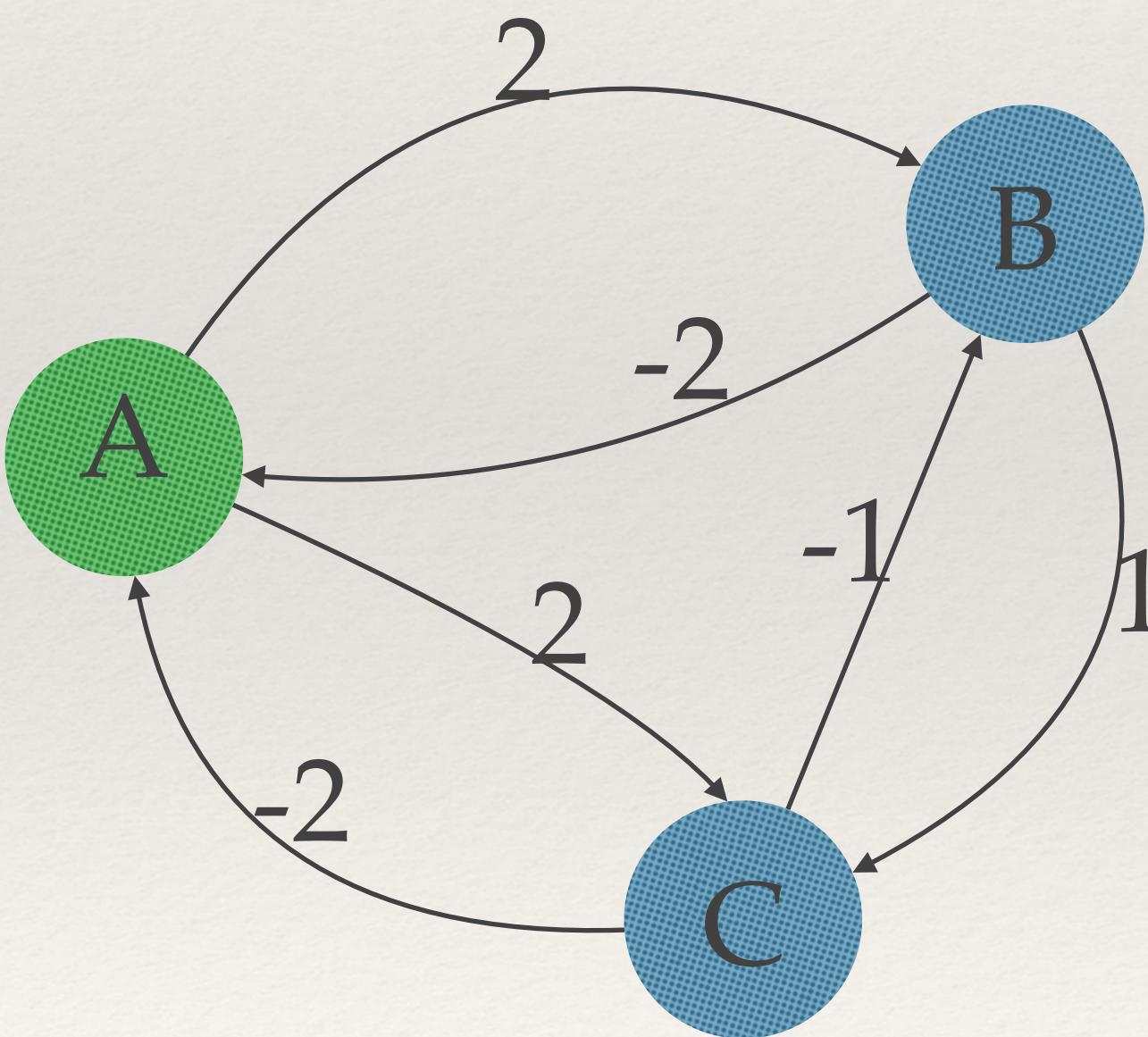


		A	B	C
A	A	0	2	2
	B	-2	0	1
C	-2	-1	0	

Margin Matrix ( $m$ )

# Schulze Method

A wins



	A	B	C
A	0	2	2
B	-2	0	1
C	-2	-1	0

Generalised Margin Matrix (M)

# Schulze Voting as Evidence Carrying Computation

[Dirk Pattinson](#)  & [Mukesh Tiwari](#) 

Conference paper

**948** Accesses | **6** [Citations](#)

Part of the [Lecture Notes in Computer Science](#) book series (LNTCS, volume 10499)

## No More Excuses: Automated Synthesis of Practical and Verifiable Vote-Counting Programs for Complex Voting Schemes

[Lyria Bennett Moses](#), [Rajeev Goré](#), [Ron Levy](#), [Dirk Pattinson](#)  & [Mukesh Tiwari](#)

Conference paper | [First Online: 06 October 2017](#)

## Modular Formalisation and Verification of STV Algorithms

[Milad K. Ghale](#) , [Rajeev Goré](#), [Dirk Pattinson](#) & [Mukesh Tiwari](#)

# The Ballot Identification Problem

A	B	C	D	E	F	G	H	I	J	K	L	M	N	O
3	2	1	4	6	7	5	11	9	10	15	14	8	13	12

## Verifiable Homomorphic Tallying for the Schulze Vote Counting Scheme

[Thomas Haines](#), [Dirk Pattinson](#) & [Mukesh Tiwari](#) 

Conference paper | [First Online: 14 March 2020](#)

418 Accesses | 1 Citations

Part of the [Lecture Notes in Computer Science](#) book series (LNCS, volume 12031)

# Encrypted Ballot

C < B < A

	A	B	C
A	E(0, r1)	E(1,r2)	E(1, r3)
B	E(-1, r4)	E(0, r5)	E(1, r6)
C	E(-1, r7)	E(-1, r8)	E(0, r9)

$$E(m, r) = (g^r, g^m \cdot h^r)$$

	A	B	C
A	E(0, r1)	E(1, r2)	E(1, r3)
B	E(-1, r4)	E(0, r5)	E(1, r6)
C	E(-1, r7)	E(-1, r8)	E(0, r9)

Ballot 1:  $C < B < A$

	A	B	C
A	E(0, r11)	E(-1, r12)	E(-1, r13)
B	E(1, r14)	E(0, r15)	E(-1, r16)
C	E(1, r17)	E(1, r18)	E(0, r19)

Ballot 2:  $A < B < C$

$$Enc(m_1, r_1) = (g^{r_1}, g^{m_1} \cdot h^{r_1})$$

$$Enc(m_2, r_2) = (g^{r_2}, g^{m_2} \cdot h^{r_2})$$

$$Enc(m_1, r_1) \cdot Enc(m_2, r_2) = (g^{r_1+r_2}, g^{m_1+m_2} \cdot h^{r_1+r_2})$$

# Challenge

What is the ordering of candidates in this ballot?

		A	B	C
A	E(0, r1)	E( <b>1</b> , r2)	E( <b>1</b> , r3)	
B	E( <b>1</b> , r4)	E(0, r5)	E(-1, r6)	
C	E( <b>1</b> , r7)	E(-1, r8)	E(0, r9)	

# Challenge

What about this one?

		A	B	C
A	E(0, r1)	E(100, r2)	E(100, r3)	
	B	E(-100, r4)	E(0, r5)	E(1, r6)
C	E(-100, r7)	E(-1, r8)	E(0, r9)	

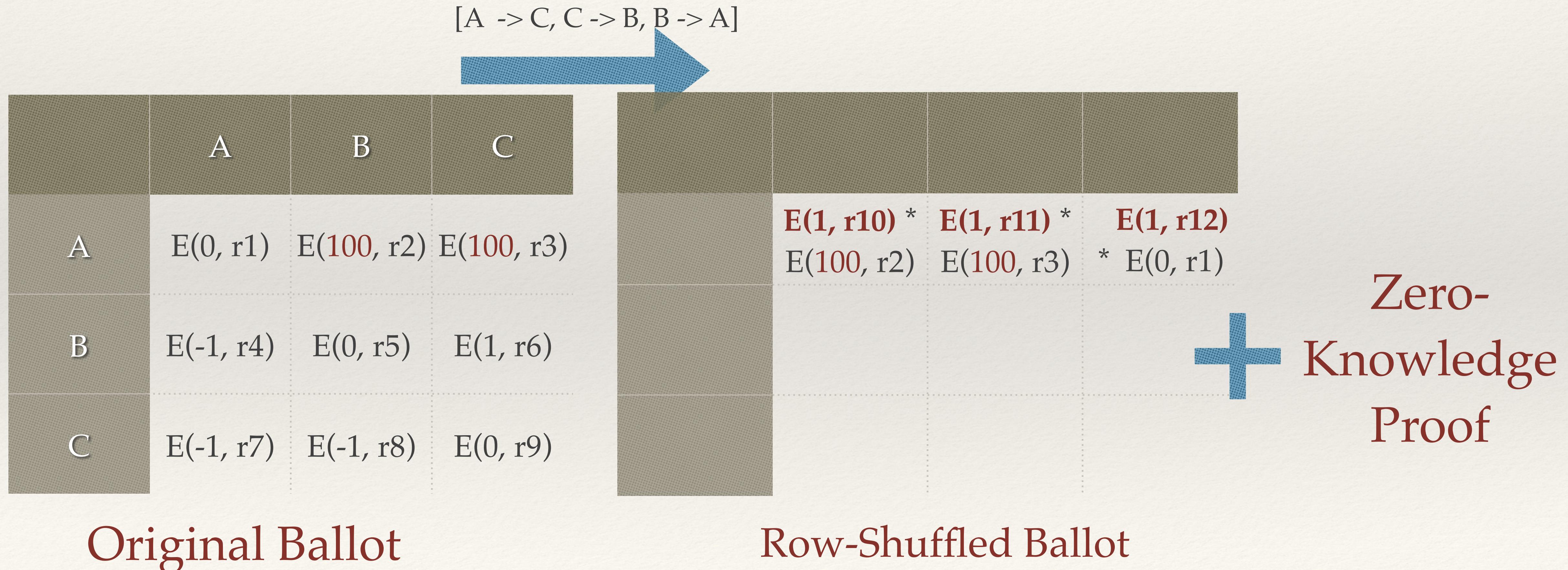
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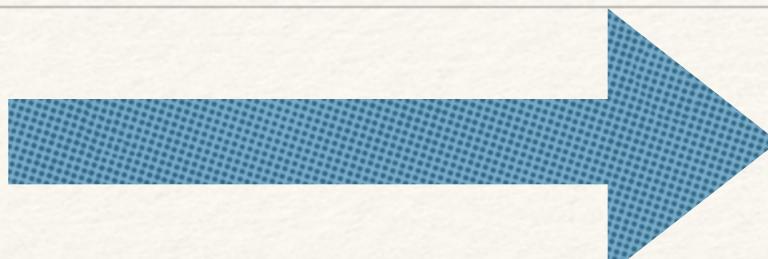
# Challenge

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Deciding if a ballot is valid or not

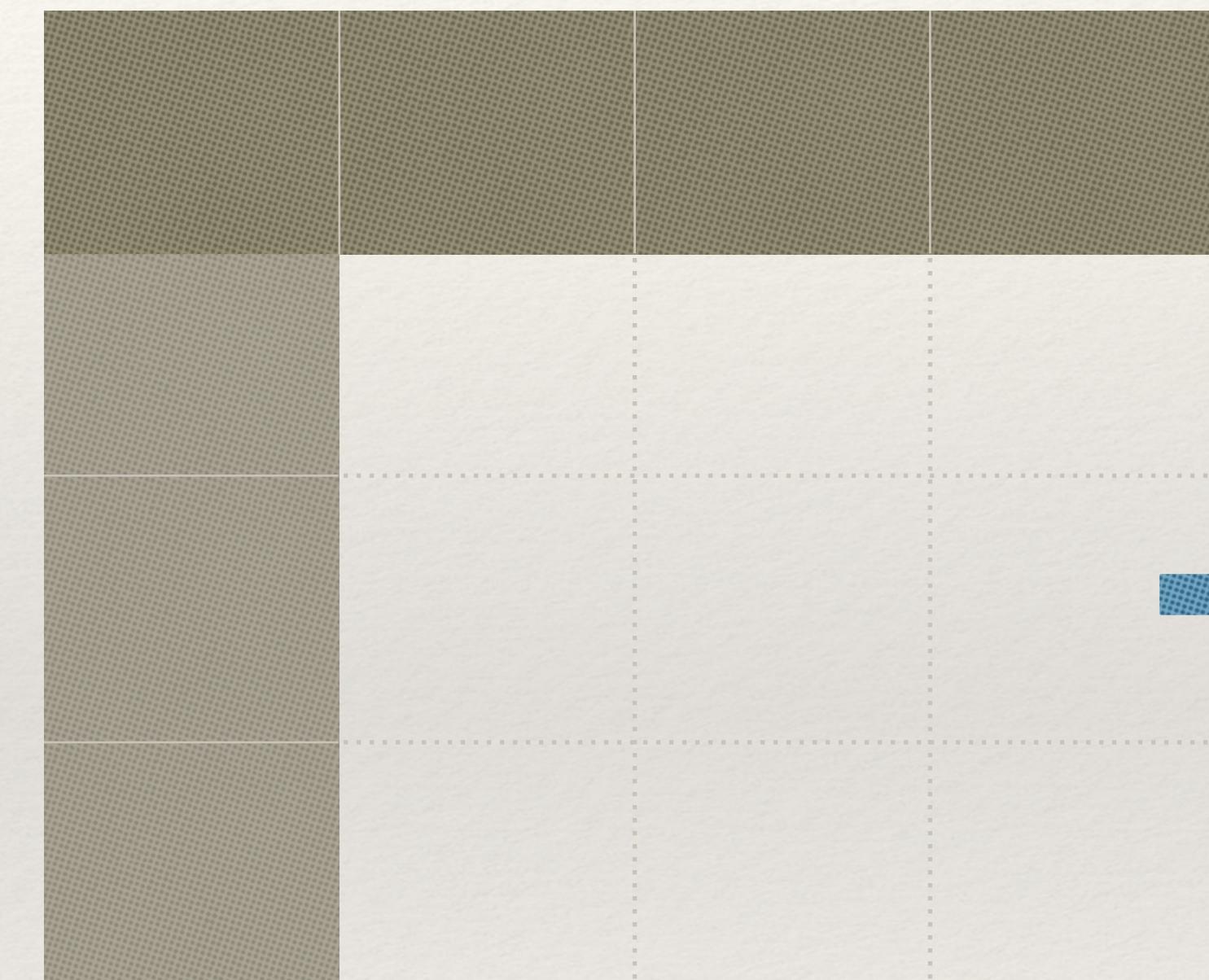
For every ballot, the counting authority generates a **secret permutation**,  
but it is **publicly verifiable** that it is a valid permutation (Zero-  
Knowledge Proof)





$E(1, r10) * E(1, r11) * E(1, r12)$ $E(100, r2) \quad E(100, r3) * E(0, r1)$			

Row-Shuffled Ballot



Row-Column-Shuffled Ballot



Zero-  
Knowledge  
Proof

## Decryption with Zero-Knowledge Proof



Row-Column-Shuffled Ballot

Decrypted Row-Column-Shuffled Ballot

# Future Projects and Possible Collaborators

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## Exploring zk-SNARK, Blockchain, and Multiparty computation for public good

- ❖ Domain Specific Language for zk-SNARK with Prof. Neil Ghani, Dr Conor McBride, Dr Robert Atkey, and Dr Fredrik Nordvall Forsberg
- ❖ Social Choice Theory with Dr Jules Hedges

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What values do I bring to Strathclyde?

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Cyber security backed by Constructive  
Logic

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# Questions?