

# **Formally Verified Electronic Voting Scheme : A Case Study**

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Except where otherwise indicated, this thesis is my own original work.

Mukesh Tiwari  
22 July 2019



to my xxx, yyy (yyy is the people you want to dedicated this thesis to.)



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# Acknowledgments

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This thesis could not have been possible without the support of my supervisor, Dirk Pattinson. I really admire his ability to understand the problem, and his intuition to to make sure that I stay clear from many dead ends which I would have happily spent months. I wish I could incorporate more of his qualities, but I believe I have less optimism about my chance.





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# Abstract

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Put your abstract here.



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# Introduction

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This place is why formalizing Electronic voting scheme?

Before I start diving deep into explaining bits and pieces of this thesis (Formal verification of Schulze Method), I still need to provide persuasive argument that why did I choose to verifying Schulze algorithm in Coq theorem prover given the fact that Schulze is not used in any democratic election, and Coq is not serious business in development of mathematical theories or software artefact. Well, the honest answer is that I want to graduate (hopefully), but it's still not convincing argument because I could have chosen some other project and graduate. On the serious note, this thesis started as a quest to find the answer of question "Can we afford bug or bugs in software used for vote counting?". Given that I am computer scientist by profession, I would not try to justify my decisions by excessive use of philosophical arguments, but at this point it seems very apt to first investigate this question from philosophical point.

"People shouldn't be afraid of their government. Governments should be afraid of their people." — Alan Moore, V for Vendetta

"Those who cast the vote decides nothing. Those who count the vote decide everything." — Joseph Stalin

"The best weapon of a dictatorship is secrecy, but the best weapon of a democracy should be the weapon of openness. " — Niels Bohr

The answer depends on how you perceive democracy. For a dictator, probably bug in the vote software would be a natural choice, among many others, to rig the election. If you firmly believe in democracy and democratic values, then among many other things, transparency and bug freeness in vote counting software would also be in your agenda. There is no doubt that technology can play a critical role in maintaining the democratic values, but assuming that it is the only factor would be a gross mistake. In Azerbaijan's 2013 election, the running president Ilham Aliyev launched a iPhone app, to boost the credibility of election, which enabled the citizens of Azerbaijan to track the tallies as counting took place. There was just one problem. The app already showed that Ilham Aliyev elected before even a ballot was counted. In this particular case, technology merely helped in surfacing the problem, but it did not do any other thing. More often technology can be used to hide the transparency of system than making it evident specially in corrupt society for personal gain.

Democracy is a complex system of different actors interacting with each other in certain fashion. How to make these interaction more productive and better for

society, I leave this to political scientists and social scientist to figure out, and I stick to my job as a technology enabler. This thesis is my journey (with my supervisor) about finding a way to make vote counting software more robust and transparent.

## 1.1 Why Schulze ?

Even though Schulze method is not used in any democratic election, we settled down with it because it is interesting and at the same time, non-trivial. Schulze's method [cite Schulze] elects a single winner based on preferential votes. At the same time, Arrow's impossibility theorem [cite Arrow] states that no preferential voting scheme can have all the desired properties established by social choice theorist, the Schulze's method offers a good balance. Many of these properties are already established in his original paper. I will discuss some of its properties in next chapter.

A quantitative comparison of voting methods [cite An Optimal Single-Winner Preferential Voting System Based on Game Theory] also shows that Schulze voting is better (in a game theoretic sense) than other, more established, systems. The Schulze Method is rapidly gaining popularity in the open software community, and It is one of the most popular voting protocol over internet to elect candidates. At of 22 July 2019, Wikipedia entry on Schulze method [cite Wiki entry] shows at least 70 users, and some of notable users among them are Gentoo Foundation, Debian, GNU Privacy Guard (GnuPG), Ubuntu, and Pirate Party in Australia, Austria, Belgium, Brazil, Germany, Iceland, Italy, Netherlands, New Zealand, Sweden, Switzerland, and United States.

## 1.2 Why Coq

How many chapters you have? You may have Chapter 2, Chapter 3, Chapter 4, Chapter 5, and Chapter 6.

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# Background (Introduction of Coq, and Schulze Method)

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At the beginning of each chapter, please introduce the motivation and high-level picture of the chapter. You also have to introduce sections in the chapter.

Write about Hilbert's idea of mathematical formalism

A proof assistant is a computer program which assists users in development of mathematical proofs. The idea of developing mathematical proofs using computer goes back to Automath (automating mathematics) [cite Automath] and LCF [cite Logic for computation] project. The Automath project (1967 until the early 80's) was initiative of De Bruijn, and the aim of the project was to develop a language for expressing mathematical theories which can be verified by aid of computer. Automath was first practical project to exploit the Curry-Howard isomorphism (proofs-as-programs and formulas-as-types) [reference here]. DeBruijn was likely unaware of this correspondence, and he almost re-invented it ([Wiki entry on Curry-Howard]). Many researchers refers Curry-Howard isomorphism as Curry-Howard-DeBruijn isomorphism. Automath project can be seen as the precursor of proof assistants NuPrl [cite here] and Coq [cite coq]. Some other notable proof assistants are LCF (Logic for Computable Functions) [cite Milner?], Mizar [cite], Nqthm/ACL2 [cite], PVS [cite], HOL (a family of tools derived from LCF theorem prover), Agda [cite], and Lean [cite].

The Coq proof assistant is interactive theorem prover based on underlying theory of Calculus of Inductive Construction [Cite Pualine Mohring] which itself is a augmentation of Calculus of Construction [cite Huet and Coquand] with inductive data-type. Coq provides a highly expressive specification language Gallina for development of mathematical theories and proving the theorems about these theories. Even though Gallina is very expressive, writing proofs in Gallina is very tedious and cumbersome. In order to ease the proof development, Coq also provides tactics. The user interacting with Coq applies these tactics to build the Gallina term which otherwise would be very laborious. In this chapter, I will give a brief overview of Calculus of Construction, followed by Calculus of Inductive Construction, with a example of building proof directly using Gallina and show that how same proof can be build

easily using the tactics provided by Coq. In final section, I will try to justify my decision of using Coq for verifying Schulze method.

## 2.1 Coq in Action (Example)

Now I give a small example which defines natural number, addition of two natural numbers, and proof that addition over natural number is commutative. We can define natural number in Coq using inductive data type (listing 1.1), addition of the natural numbers (listing 1.2), and proof that addition of natural numbers is commutative written in Gallina (listing 1.3).

```
Inductive Natural : Type :=
| 0 : Natural
| Succ : Natural -> Natural
```

Listing 2.1: Inductive Data Type for Natural Numbers

More precisely, the interpretation is that Natural is a inductive type with two constructors: i) 0 representing zero, and ii) Succ representing successor which takes a Natural number and gives next Natural number.

```
Fixpoint Addition (n m : Natural) : Natural :=
  match n with
  | 0 => m
  | Succ n' => Succ (Addition n' m)
  end.
```

```
(* Notation for Addition. Now we can use + instead of
   writing Addition *)
Notation "x_+_y" := (Addition x y)
      (at level 50, left associativity).
```

Listing 2.2: Addition function for Natural Numbers

We define the addition by pattern matching on first argument **n**. When **n** is 0 (zero), then we sum is **m**, and if **n** is **Succ n'**, then sum is successor of **n' + m**.

```
Theorem Addition_by_zero : forall (n : Natural), n + 0 = n.
  refine (fix IHa (n : Natural) : n + 0 = n :=
    match n as nz return (nz + 0 = nz) with
    | 0 => eq_refl
    | Succ n' =>
      let IHn' := IHa n' in
```

Change the Addition into infix symbol + and use + in proofs. It will convey the idea more clearly

---

```

      eq_ind_r (fun m => Succ m = Succ n') eq_refl IHn'
    end).
Qed.

```

```

Lemma Successor_addition : forall (n m : Natural),
  Succ (n + m) = n + (Succ m).
refine
  (fix IHn (n : Natural) : forall m : Natural,
    Succ (n + m) = n + (Succ m) :=
    match n as nz return (forall m : Natural,
      Succ (nz + m) =
        nz + (Succ m)) with
  | 0 => fun m : Natural => eq_refl
  | Succ n' =>
    fun m : Natural =>
      eq_ind (Succ (n' + m))
        (fun t => Succ (Succ (n' + m)) = Succ t)
        eq_refl (n' + (Succ m)) (IHn n' m)
    end).
Qed.

```

```

Theorem Addition_is_commutative :
  forall (n m : Natural), n + m = m + n.
refine
  (fix IHn (n : Natural) : forall m : Natural,
    n + m = m + n :=
    match n as nz return (forall m : Natural,
      nz + m =
        m + nz) with
  | 0 => fun m : Natural => eq_ind_r (fun t => m = t)
    eq_refl
    (Addition_by_zero m)
  | Succ n' =>
    fun m =>
      eq_ind (Succ (m + n'))
        (fun t => Succ (n' + m) = t)
        (eq_ind_r (fun t => Succ t = Succ (m + n'))
          eq_refl (IHn n' m))
        (m + (Succ n'))
        (Successor_addition m n')
    end).

```

Qed.

### Listing 2.3: Addition function for Natural Numbers

We need two additional Lemma: i) *Addition\_by\_zero*, a proof of  $n + 0 = n$ , and ii) *Successor\_addition*, a proof of  $\text{Succ } (n + m) = n + (\text{Succ } m)$  to prove that addition on Natural is commutative (*Addition\_is\_commutative*),

One thing that can't escape from the reader's eyes is that the proofs written in Gallina is verbose, and they don't appear anywhere compared to a proof that would have been written by a mathematician. Well, we can lift the burden of verbosity by using tactics provided by Coq; however, there is no universally accepted solution in Coq community for second problem. There has been some research in declarative style proof writing <sup>1</sup>, but it is not widely practised in Coq community. The proof of addition on Natural is commutative re-written using tactics (Listing 1.4)

```
Lemma Addition_by_zero : forall (n : Natural), n + 0 = n.
  induction n; cbn; [auto | rewrite IHn; auto].
Qed.
```

```
Lemma Successor_addition : forall (n m : Natural),
  Succ (n + m) = n + (Succ m).
Proof.
  induction n; intros m;
  cbn; [auto | rewrite <- IHn; auto].
Qed.
```

```
Theorem Addition_is_commutative :
  forall (n m : Natural), n + m = m + n.
Proof.
  induction n; intro m;
  [rewrite Addition_by_zero |
  rewrite <- Successor_addition;
  rewrite <- IHn]; auto.
Qed.
```

### Listing 2.4: Addition function for Natural Numbers

Section 2.2 xxxx.

Section 2.3 yyyy.

---

<sup>1</sup>[www-verimag.imag.fr/~corbinea/ftp/publis/bricks-poster.pdf](http://www-verimag.imag.fr/~corbinea/ftp/publis/bricks-poster.pdf)

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## 2.2 Motivation

## 2.3 Related work

You may reference other papers. For example: Generational garbage collection [Lieberman and Hewitt, 1983; Moon, 1984; Ungar, 1984] is perhaps the single most important advance in garbage collection since the first collectors were developed in the early 1960s. (doi: "doi" should just be the doi part, not the full URL, and it will be made to link to [dx.doi.org](https://dx.doi.org) and resolve. `shortname`: gives an optional short name for a conference like PLDI '08.)

## 2.4 Summary

Summary what you discussed in this chapter, and mention the story in next chapter. Readers should roughly understand what your thesis takes about by only reading words at the beginning and the end (Summary) of each chapter.





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# Design and Implementation

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Same as the last chapter, introduce the motivation and the high-level picture to readers, and introduce the sections in this chapter.

## 3.1 Smart Design

## 3.2 Summary

Same as the last chapter, summary what you discussed in this chapter and be the bridge to next chapter.



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# Experimental Methodology

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## 4.1 Software platform

## 4.2 Hardware platform

Table 4.1 shows how to include tables and Figure 4.1 shows how to include codes.

Architecture	Pentium 4	Atom D510	i7-2600
Model	P4D 820	Atom D510	Core i7-2600
Technology	90nm	45nm	32nm
Clock	2.8GHz	1.66GHz	3.4GHz
Cores $\times$ SMT	$2 \times 2$	$2 \times 2$	$4 \times 2$
L2 Cache	1MB $\times$ 2	512KB $\times$ 2	256KB $\times$ 4
L3 Cache	none	none	8MB
Memory	1GB DDR2-400	2GB DDR2-800	4GB DDR3-1066

Table 4.1: Processors used in our evaluation.

```
1 int main(void)
2 {
3     printf("Hello_World\n");
4     return 0;
5 }
```

(a)

```
1 void main(String[] args)
2 {
3     System.out.println("Hello_World");
4 }
```

(b)

Figure 4.1: Hello world in Java and C.

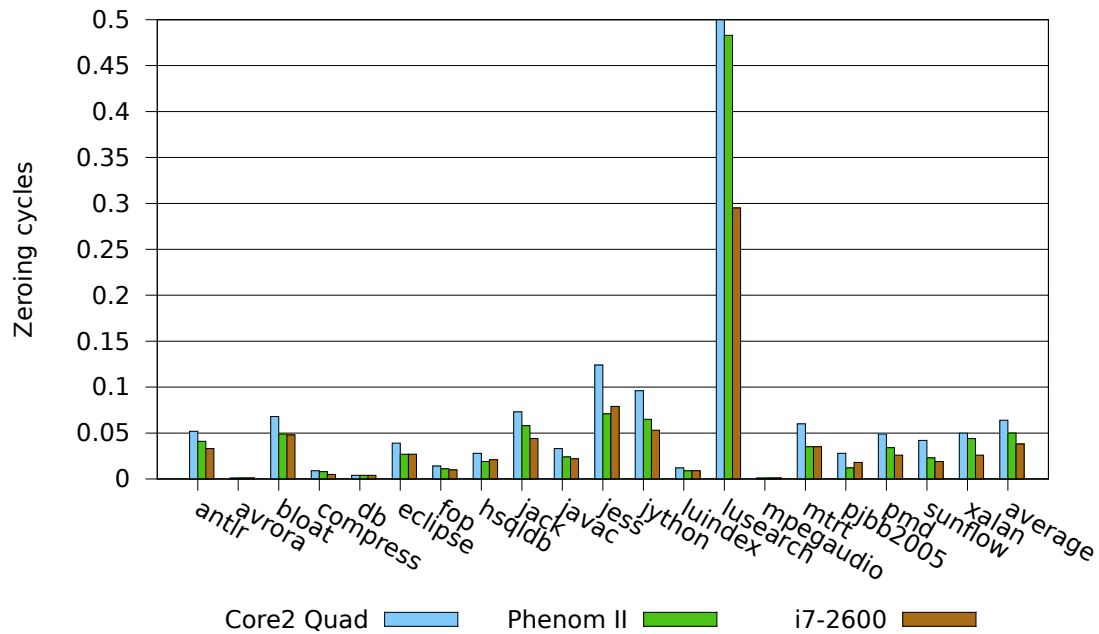
# Results

---

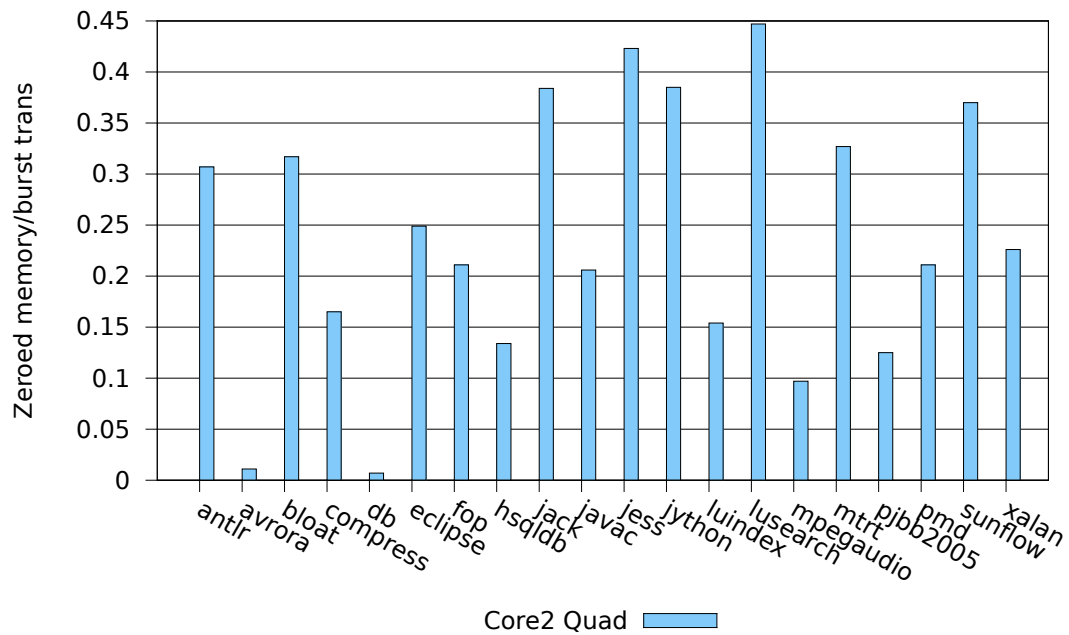
## 5.1 Direct Cost

Here is the example to show how to include a figure. Figure 5.1 includes two subfigures (Figure 5.1(a), and Figure 5.1(b));

## 5.2 Summary



(a) Fraction of cycles spent on zeroing



(b) BytesZeroed / BytesBurstTransactionsTransferred

Figure 5.1: The cost of zero initialization

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# Conclusion

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Summary your thesis and discuss what you are going to do in the future in Section 6.1.

## 6.1 Future Work

Good luck.





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# Bibliography

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- GEUVERS, H., 2009. Proof assistants: History, ideas and future. *Sadhana*, 34, 1 (Feb 2009), 3–25. doi:10.1007/s12046-009-0001-5. <https://doi.org/10.1007/s12046-009-0001-5>.
- LIEBERMAN, H. AND HEWITT, C., 1983. A real-time garbage collector based on the lifetimes of objects. *Communications of the ACM*, 26, 6 (Jun. 1983), 419–429. doi:10.1145/358141.358147. (cited on page 7)
- MOON, D. A., 1984. Garbage collection in a large LISP system. In *LFP '84: Proceedings of the 1984 ACM Symposium on LISP and Functional Programming* (Austin, Texas, USA, Aug. 1984), 235–246. ACM, New York, New York, USA. doi:10.1145/800055.802040. (cited on page 7)
- UNGAR, D., 1984. Generation scavenging: A non-disruptive high performance storage reclamation algorithm. In *SDE 1: Proceedings of the 1st ACM SIGSOFT/SIGPLAN Software Engineering Symposium on Practical Software Development Environments* (Pittsburgh, Pennsylvania, USA, Apr. 1984), 157–167. ACM, New York, New York, USA. doi:10.1145/800020.808261. (cited on page 7)