Matrix Representation Of Linear Regression Model

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$y = mx + c$

$X_1 \quad X_2 \qquad \qquad x_p \quad Y$

CRIM	ZN	INDUS	CHAS	NOX	RM	AGE	DIS	RAD	TAX	PTRATIO	B	LSTAT	Price
0.00632	18.0	2.31	0.0	0.538	6.575	65.2	4.0900	1.0	296.0	15.3	396.90	4.98	24.0
0.02731	0.0	7.07	0.0	0.469	6.421	78.9	4.9671	2.0	242.0	17.8	396.90	9.14	21.6
0.02729	0.0	7.07	0.0	0.469	7.185	61.1	4.9671	2.0	242.0	17.8	392.83	4.03	34.7
0.03237	0.0	2.18	0.0	0.458	6.998	45.8	6.0622	3.0	222.0	18.7	394.63	2.94	33.4
0.06905	0.0	2.18	0.0	0.458	7.147	54.2	6.0622	3.0	222.0	18.7	396.90	5.33	36.2

$1 \; \vdots \; n$

$\underline{\underline{Y}} = \begin{pmatrix} \cancel{y_1} \\ \cancel{\vdots} \\ \cancel{y_n} \end{pmatrix} = (y_1 \cdots y_n)^T \sim$ Response / Target

Vector

$\underline{\underline{x_1}} = \begin{pmatrix} x_{11} \\ \vdots \\ x_{n1} \end{pmatrix} \quad \underline{\underline{x_2}} = \begin{pmatrix} x_{12} \\ \vdots \\ x_{n2} \end{pmatrix} \quad \cdots \quad \underline{\underline{x_p}} = \begin{pmatrix} x_{1p} \\ \vdots \\ x_{np} \end{pmatrix}$

$\underline{\underline{x_1}}, \underline{\underline{x_2}} \rightsquigarrow$ Predictors

$X = (1, \underline{\underline{x_1}}, \underline{\underline{x_2}} \cdots x_p)$ where

$1 = (1, \cdots 1)^T$

$\beta = (\beta_0, \beta_1 \cdots \beta_p)^T$ Vector of Parameters

$\varepsilon = (\varepsilon_1 \cdots \varepsilon_n)^T \sim$ Vector of Error Term

$y_i = \beta_0 + \beta_1 x_{i1} + \cdots x_{ip} + \varepsilon_i$ (for all $i = 1 \cdots n$)

$y_1 = \beta_0 + \beta_1 x_{11} + \cdots x_{1p} + \varepsilon_1$

$$\begin{pmatrix} y_1 \\ \vdots \\ y_n \end{pmatrix} = \begin{pmatrix} 1 & x_{11} & \cdots & x_{1p} \\ 1 & & & \vdots \\ 1 & & & \vdots \\ 1 & x_{1n} & & x_{np} \end{pmatrix} \begin{pmatrix} \beta_0 \\ \vdots \\ \beta_p \end{pmatrix} + \begin{pmatrix} \varepsilon_1 \\ \vdots \\ \varepsilon_n \end{pmatrix}$$

$$\boxed{\underline{\underline{Y}} = X \underline{\underline{\beta}} + \underline{\underline{\varepsilon}}}$$

$$\boxed{\underbrace{\underline{\underline{Y}}}_{(n \times 1)} = \underbrace{X}_{n \times (p+1)} \overbrace{\underline{\underline{\beta}}}^{(p+1)\times 1} + \underbrace{\underline{\underline{\varepsilon}}}_{n \times 1}}$$

$\underline{\underline{n \times 1}} + \underline{\underline{n \times 1}}$