Inner And Outer Products

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$$A = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix} \quad B = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$$

3×1 $\quad$ 3×1

$m \times n$ $\quad$ $n \times p$

A is normalized if $\|A\| = 1$

Orthogonal + normalized

"Orthonormal"

$$A^T B = (a_1 \; a_2 \; a_3) \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$$

$$= a_1 b_1 + a_2 b_2 + a_3 b_3$$

$A^T B = 0 \Rightarrow A, B$ Orthogonal

norm $\|A\| = (A^T A)^{\frac{1}{2}} = \sqrt{a_1^2 + a_2^2 + a_3^2}$

Outer Product

$$AB^T = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix} (b_1 \; b_2 \; b_3)$$

3×1 $\quad$ 1×3 $\Rightarrow$ 3×3

$$= \begin{pmatrix} a_1 b_1 & a_1 b_2 & a_1 b_3 \\ a_2 b_1 & a_2 b_2 & a_2 b_3 \\ a_3 b_1 & a_3 b_2 & a_3 b_3 \end{pmatrix}$$