Linear Independence

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Linear Independence of Vectors

" The set of vectors $\{a_1, a_2 \cdots a_n\}$ are linearly indedepent if

$$k_1 a_1 + k_2 a_2 + \cdots k_n a_n = 0$$

has only solution $k_1 = k_2 \cdots = k_n = 0$ "

Ex

$$A = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \quad B = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \quad W = \begin{pmatrix} 2 \\ 3 \\ 0 \end{pmatrix}$$

$$W = 2A + 3B$$

$$= 2\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} + 3\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$$

$$= \begin{pmatrix} 2+0 \\ 0+3 \\ 0+0 \end{pmatrix} = \begin{pmatrix} 2 \\ 3 \\ 0 \end{pmatrix}$$

A, B, W are Not linearly independent

* You can not construct another vector by linear combinati of $\{a_1 \cdots a_n\}$

✓ $A = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \quad B\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \quad w\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$ Yes linearly independent

✓ $a\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} + b\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} + c\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} a \\ b \\ c \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \therefore \begin{matrix} a=0 \\ b=0 \\ c=0 \end{matrix}$ ✓