

# LU Decomposition

Monday, May 29, 2023 2:30 PM

$$A = \begin{pmatrix} -3 & 2 & -1 \\ 6 & -6 & 7 \\ 3 & -4 & 4 \end{pmatrix}$$

$$\textcircled{a} \quad \frac{I}{E_3} (E_3^{-1} E_2) E_1 A = E_3^{-1} U$$

$$\textcircled{b} \quad \frac{I}{E_2} E_2^{-1} E_2 E_1 A = E_2^{-1} E_3^{-1} U$$

$$\textcircled{c} \quad \frac{I}{E_1} E_1^{-1} E_1 A = E_2^{-1} E_3^{-1} U$$

$$E_3 E_2 E_1 A = U$$

$$U = \begin{pmatrix} -3 & 2 & -1 \\ 0 & -2 & 5 \\ 0 & 0 & -2 \end{pmatrix}$$

$$\textcircled{d} \quad \frac{I}{A} = E_3^{-1} E_2^{-1} E_1^{-1} U$$

$$\textcircled{u} E_1 = \begin{pmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad \text{Multiply 1st row by 2 and add to 2nd row}$$

$$\textcircled{u} E_2 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{pmatrix}$$

$$\textcircled{u} E_3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -1 & 1 \end{pmatrix}$$

$$A = E_3^{-1} \cdot E_2^{-1} \cdot E_1^{-1} U$$

$$= \begin{pmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ 1 & 1 & 1 \end{pmatrix}$$

$$A = L U$$

$$E_1^{-1} = \begin{pmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad E_2^{-1} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -1 & 0 & 1 \end{pmatrix} \quad E_3^{-1} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 1 \end{pmatrix}$$