Inverse Matrix

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$$\underbrace{A A^{-1}} = I = \underbrace{A^{-1} A}$$

$$(AB)^{-1} = B^{-1} \cdot A^{-1}$$

$$(A^T)^{-1} = (A^{-1})^T$$

2×2

$$\underset{A}{\begin{pmatrix} a & b \\ c & d \end{pmatrix}} \underset{A^{-1}}{\begin{pmatrix} x_1 & x_2 \\ y_1 & y_2 \end{pmatrix}}^{?} = \underset{I}{\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}}$$

$$ax_1 + by_1 = 1$$
$$ax_2 + by_2 = 0$$
$$cx_1 + dy_1 = 0$$
$$cx_2 + dy_2 = 1$$

$$y_1 = -\frac{cx_1}{d}$$

$$y_2 = -\frac{ax_2}{b}$$

$$ax_1 + b\left(-\frac{cx_1}{d}\right) = 1$$

$$x_1\left(a - \frac{bc}{d}\right) = 1 \qquad x_1\left(\frac{ad-bc}{d}\right) = 1$$

$$x_1 = \frac{d}{ad-bc}$$

$$x_2 = \frac{-b}{ad-bc}$$

$$y_1 = -\frac{c\cancel{d}}{\cancel{d}(ad-bc)} = -\frac{c}{(ad-bc)}$$

$$y_2 = +\frac{a}{\cancel{b}} \times \frac{\cancel{b}}{(ad-bc)} = \frac{a}{(ad-bc)}$$

$$\boxed{A^{-1}} = \frac{1}{(ad-bc)} \begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$$

$\det A = (ad-bc)$

If $\det A$ is zero then there will not be any inverse matrix