

$$A = \begin{pmatrix} -3 & 2 & -1 \\ 6 & -6 & 7 \\ 3 & -4 & 4 \end{pmatrix} \quad \underline{A = LU} \quad L = \begin{pmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ -1 & 1 & 1 \end{pmatrix} \quad U = \begin{pmatrix} -3 & 2 & -1 \\ 0 & -2 & 5 \\ 0 & 0 & -2 \end{pmatrix}$$

$$Ly = \begin{pmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ -1 & 1 & 1 \end{pmatrix} \begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix} = b = \begin{pmatrix} -1 \\ -7 \\ -6 \end{pmatrix}$$

$$Ax = b$$

$$A \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} -1 \\ -7 \\ -6 \end{pmatrix}$$

A ($m \times n$)
 x ($n \times 1$) Vector
 b ($m \times 1$) Vector
 y ($n \times 1$) Vector
 $-$ col Vector

Forward
Sub

$$y_1 = -1$$

$$-2y_1 + y_2 = -7 \quad y_1 = -1 \quad y_2 = -9$$

$$-y_1 + y_2 + y_3 = -6 \quad y_3 = 2$$

$$Ax = b$$

$$\rightarrow LUx = b$$

$$\rightarrow L(Ux) = b$$

$(m \times n) \quad (n \times 1)$
 $m \times 1$

$det \quad y = Ux$
 $Ly = b \sim \text{Solve } y \quad \textcircled{I}$
 $Ux = y \sim \text{Solve } x \quad \textcircled{II}$

$$Ux = y$$

$$\begin{pmatrix} -3 & 2 & -1 \\ 0 & -2 & 5 \\ 0 & 0 & -2 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} -1 \\ -9 \\ 2 \end{pmatrix}$$

$$-3x_1 + 2x_2 - x_3 = -1$$

$$-2x_2 + 5x_3 = -9$$

$$-2x_3 = 2$$

Backward
Sub

$$x_3 = -1$$

$$x_2 = 2$$

$$x_1 = 2$$

$$x \begin{pmatrix} 2 \\ 2 \\ -1 \end{pmatrix}$$