Department of Mathematics, School of Natural Sciences, Shiv Nadar University
Monsoon Semester 2016
MAT 622 Topology – Final Exam

Marks: 40 for written work, 60 for viva.

Submission Deadline: 4 pm, Thursday, December 8.

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1. Let G be a topological group, A and B be subsets of G, and $g \in G$. Show that
   (a) A is open implies gA is open.
   (b) A is open and B is arbitrary implies AB is open.
   (c) A and B are compact implies AB is compact.
   (d) A is compact and B is closed implies AB is closed.
   (e) A and B are closed does not imply that AB is closed.
2. Let X be a locally compact space. Show that:
   (a) Every closed subspace of X is locally compact.
   (b) A continuous image of X need not be locally compact.
   (c) If $f : X \to Y$ is surjective, open, and continuous, then Y is locally compact.
   (d) If Y is Hausdorff and X is a dense subspace of Y, then X is open in Y.
3. Let X be a topological space. Prove:
   (a) X is connected iff every open cover $\{U_\alpha\}$ has the following property: Given any U_α and U_β, there is a sequence $\alpha = \alpha_1, \ldots, \alpha_n = \beta$ such that $U_{\alpha_i} \cap U_{\alpha_{i+1}} \neq \emptyset$ for every $i = 1, \ldots, n-1$.
   (b) If $A, C \subset X$ such that C is connected and $C \cap A \neq \emptyset$, $C \cap A^c \neq \emptyset$, then $C \cap \partial A \neq \emptyset$.
4. Let X be Hausdorff.
   (a) If $f : X \to X$ is continuous, prove that $\{x : f(x) = x\}$ is closed.
   (b) Suppose each point in X has a neighbourhood V such that $\bar{V}$ is regular. Prove X is regular.
