

Topics in products of nilpotent groups

typescript - characterization of finite nilpotent groups

Corollary 1.20 Let A be a nilpotent group of class $c > 0$, and let x_1, \dots, x_d be automorphisms of A . Then, for every $q \geq 1$

$$[A^{x^q}, \langle x_1 \rangle, \dots, \langle x_d \rangle] \leq [A, \langle x_1 \rangle, \dots, \langle x_d \rangle]^q.$$

Lemma 1.21 Let A be a nilpotent group of class c , let x_1, x_2, \dots, x_d be automorphisms of A such that $[A_i, \langle x_i \rangle] = 1$ for all $i = 1, \dots, d$. Let q_1, \dots, q_d be integers ≥ 1 , and $q = q_1 \cdots q_d$. Then

$$[A^{x^{q^d}}, \langle x_1 \rangle, \dots, \langle x_d \rangle] \leq [A, \langle x_1^q \rangle, \dots, \langle x_d^q \rangle].$$

Proof. We argue by induction on $d \geq 1$. If $d = 1$, $q = q_1$, write $B = [A, \langle x^q \rangle]$. Then $B \trianglelefteq [A, x]$, and by applying Lemma 1.17 to the action of x on A/B , we have (since x^d centralizes A/B)

$$[A^{x^{q^d}}, \langle x \rangle] \leq B$$

which is what we want.

Let then $d \geq 2$. Write $s = q_1 \cdots q_{d-1}$ and $B = [A^{x^{s^{q_{d-1}}}}, \langle x_1 \rangle, \dots, \langle x_{d-1} \rangle]$. By inductive assumption

$$B \leq [A, \langle x_1^q \rangle, \dots, \langle x_{d-1}^q \rangle]. \quad (1.2)$$

Now, $q^{s^d} = s^{q^{d-1}} q^{q^{d-1}}$; thus, using Corollary 1.20,

$$[A^{x^{q^d}}, \langle x \rangle, \dots, \langle x_d \rangle] \leq [A^{x^{q^{d-1}}}, \langle x_1 \rangle, \dots, \langle x_{d-1} \rangle]^{q^{q^{d-1}}} \leq [B^{q^{q^{d-1}}}, \langle x \rangle] \leq [B^{q^d}, \langle x \rangle].$$

By the case $d = 1$ we then have

$$[A^{x^{q^d}}, \langle x_1 \rangle, \dots, \langle x_d \rangle] \leq [B^{x^{q^d}}, \langle x_d^q \rangle] \leq [B, \langle x_d^q \rangle] = [B, \langle x_d^q \rangle],$$

from which, applying (1.2), we get the desired inclusion. ■



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Tags: #locally #nilpotent #group

Nilpotent matrix

P t is isomorphic to P 1× P 2×. Furthermore, from Appendix A we recall that the radical Rad A of an associative algebra A is its unique maximal nilpotent ideal.

Nilpotent

An abelian group is precisely one for which the adjoint action is not just nilpotent but trivial a 1-Engel group. .

Nilpotent group

P t is a subgroup of G. It reads as an upper-level undergraduate text should.

characterization of finite nilpotent groups

The Theory of Nilpotent Groups

Let I k and J k be the lower central series of I and J respectively.

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