

Methods of mathematical physics

Cambridge University Press - Methods of Mathematical Physics: Partial Differential Equations

METHODS OF MATHEMATICAL PHYSICS
Tutorial Sheet 3

Saddle-point Method

K: key question – explores core material
R: review question – as motivation to continue
S: standard question – provides a testing framework of the course
S: standard question – general theory testing!

3.1 Asymptotic expansion of Bessel function [6]
Given the integral representation of the Bessel function calculate the asymptotic expression of the Bessel function of the second kind given by

$$J_0^{(0)} = \frac{1}{\pi} \int_{-\infty}^{\infty} e^{ixt} \exp\left(\frac{i}{2} t^2 + \frac{1}{2}\right) dt$$
 where there is a branch cut along the negative real axis.

Ans: $J_0^{(0)} \sim \sqrt{\frac{\pi}{2}} e^{-|x|} e^{-i(x^2/4 + \pi/4)}$

3.2 Changing variable to obtain form for saddle-point method [4]
Consider

$$F(z) = \int_{-\infty}^{\infty} \frac{\cos(zt)}{t^2 + a^2} dt$$
 where there is a branch cut along the positive real axis.
 Change variable to $t = -i\sqrt{a} + \epsilon z$ then use the saddle-point method to calculate the leading behaviour of $F(z)$ as $|z| \rightarrow \infty$.
 Hint: You should find two saddle points at $\pm i/a$ but only one will contribute.
 Ans: $F(z) \sim \sqrt{\frac{\pi}{2}} e^{-iz^2/4a} e^{-i\pi/4}$

3.3 Asymptotic expansion of Legendre Polynomial [4]
The Legendre polynomial $P_n(x)$ may be defined via Gegenbauer's integral

$$P_n(x) = \frac{1}{2^n n! \Gamma(n+1)} \int_{-1}^1 (1-x^2)^{n/2} x^n dx$$
 where the contour integration is over the unit circle in the complex plane. Use the saddle point method to obtain the asymptotic expansion of $P_n(x)$ when $|x| \gg n$.

- Define $f(t) = \ln(t^2 - 1) - 3bt - cx^2$ and show that $f'(t) = 0$ at $t_0 = x^2$.
- Show that $f''(t_0) = \frac{c}{x^2(t_0 + x^2)}$.

What is in the phase of the steepest descent contour through the two saddle points? Check that the contour goes through the saddle points in the correct sense.

Description: -

- Methods of mathematical physics
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Notes: Previous ed. 1946.

This edition was published in 1950



Filesize: 41.64 MB

Tags: #Some #Mathematical #Methods #of #Physics

Mathematical Tools for Physics

Methods of Mathematical Physics: Partial Differential Equations

For some extensive subjects, which are important but need long discussion and are well treated in some standard book, we have thought it sufficient to give references. In part two, the concepts developed for discrete systems in previous chapters are extended to continuous systems.

Methods of Mathematical Physics: Volume 2, Differential Equations by Richard Courant

But aside from that it's free.

Mathematical Methods in Physics

Part I is a brief introduction to distribution theory, in which elements from the theories of ultradistributions and hyperfunctions are considered in addition to some deeper results for Schwartz distributions, thus providing a comprehensive introduction to the theory of generalized functions.

Mathematical Methods in Physics

Your final grade is determined using the following approximate formula: the homework is 40% of your grade, the midterm exam is 20%, and the final is 40%. We have also found repeatedly that the easiest way to make a statement reasonably plausible is to give a rigorous proof. The single file contains a full index also linked.

Related Books

- [Q Group summary of proceedings, 1976-1982 - summary of Q conferences](#)
- [Jinan kai bu bai nian](#)
- [Duke Ellington](#)
- [Cómo ser feliz día a día](#)
- [Essentials of numerical control](#)