Conversion of FA to Regular Expression :- [DFA to RE]

Arden's Theorem :-

Let A & B are 2 Regular Expressions over Σ. If A doesn't contain ε. then the following equation in x namely $x = Ax + B$ has a unique solution given by $\boxed{x = A^* B}$

$\boxed{R = PR + Q}$

Proof :- $P(P^* Q) + Q = (PP^* + \varepsilon) Q = P^* Q$

Hence the theorem is satisfied when $R = P^* Q$. This means $R = P^* Q$ is a solution $\longrightarrow$ Arden's Theorem

$\rightarrow$ Consider the above theorem Replace $\underline{R} \longrightarrow PR + Q$

$$PR + Q = P(PR + Q) + Q$$
$$= P^2 R + PQ + Q$$
$$= P^2 (PR + Q) + PQ + Q$$
$$= P^3 R + P^2 Q + PQ + Q \text{ ----- } P^{i+1} R + P^i Q + \text{ ---- } + PQ + Q$$

$$P^{i+1} R + Q \underbrace{[\varepsilon + P + P^2 + P^3 \text{ --- } P^{i-1} + P^i]}$$

Let w be a string of length i in the set R

Then w belongs to ε, $P^{i+1}R$ has no strings of length

less than $i+1$.

$$(\varepsilon + P + P^2 + \cdots P^i)Q + P^{i+1}R$$

$$\rightarrow P^*Q$$

$$\boxed{R = PR + Q \Rightarrow P^*Q}$$

$$X = AX + B \Rightarrow A^*B$$

$$Y = AY + B \Rightarrow A^*B$$

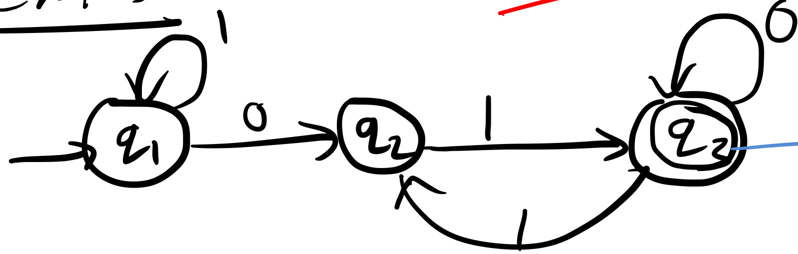
FA to RE Conversion steps:-

1. Write n equations for n states
2. If there is an edge from q to p labeled with a, then write equation as $q = ap \Rightarrow q = ap$

$$q \xrightarrow{a} p$$

3. Add epsilon (ε) if the state is a final state
4. Solve the equations by using substitution & also with arden's theorem

Ex: 1



$\to q_3 \to$ final state

① Write equations   No. of states = 3 so we will write for 3 equations

$X = AX + B \Rightarrow A^*B$ → Arden's Theorem

1) $q_1 = 0q_2 + 1q_1$

2) $\boxed{q_2 = 1q_3}$

3) $q_3 = 1q_2 + 0q_3 + \varepsilon$

$q_3 = 1q_2 + 0q_3 + \varepsilon \to$ ①

substitute q_2 in equ ①

$q_3 = 1[1q_3] + 0q_3 + \varepsilon$

$q_3 = 11q_3 + 0q_3 + \varepsilon$

$\underset{X}{q_3} = \underset{A}{(11+0)}\,\underset{X}{q_3} + \underset{B}{\varepsilon} \to A^*B$

$q_3 = (11+0)^*\varepsilon \to A^*\varepsilon = A^*$

$q_3 = (11+0)^* \to$ ②

Substitute equation ② in $q_2 = 1q_3 \Rightarrow q_2 = 1(11+0)^*$

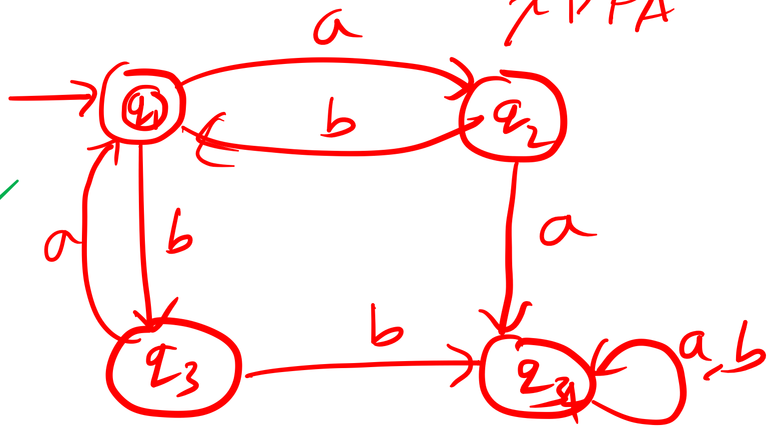
substitute q_2 in equ $q_1 = 0q_2 + 1q_1$

$\underset{X}{q_1} = \underbrace{0[1(11+0)^*]}_{B} + \underset{A}{1}\,\underset{X}{q_1} \Rightarrow$

$\boxed{q_1 = 1^*01(0+11)^*}$ ✓

Ex: 2

DFA



1) Equations

$q_1 = aq_2 + bq_3 + \varepsilon$

$q_2 = aq_4 + bq_1$

$q_3 = aq_1 + bq_4$

$q_4 = aq_4 + bq_4$

$\underset{X}{q_4} = \underset{A}{(a+b)}\underset{X}{q_4} + \phi \Rightarrow q_4 = (a+b)^* \phi$

$\boxed{q_4 = \phi}$

substitute q_4 in equation of q_3

$q_3 = aq_1 + b\phi$

$q_3 = aq_1 + \phi$

$q_2 = bq_1 + aq_4$

$q_2 = bq_1 + \phi$

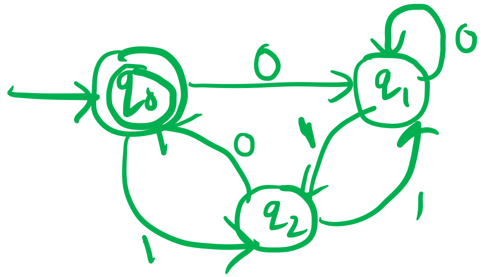
$q_1 = a[bq_1 + \phi] + b[aq_1 + \phi] + \varepsilon$

$= abq_1 + \cancel{a\phi} + baq_1 + \cancel{b\phi} + \varepsilon$

$= abq_1 + baq_1 + \varepsilon$

$\underset{X}{q_1} = \underset{A}{(ab+ba)}\underset{X}{q_1} + \underset{B}{\varepsilon} \longrightarrow A^*B$

$(ab+ba)^* \varepsilon \Rightarrow \boxed{(ab+ba)^*}$



1) Equations:

$q_0 = 0q_1 + 1q_2 + \varepsilon \rightarrow (1)$

$q_1 = 0q_1 + 1q_2 \rightarrow (2)$

$q_2 = 0q_0 + 1q_1 \rightarrow (3)$

substitute equ. (3) in equation 2

$q_1 = 0q_1 + 1[0q_0 + 1q_1]$

$= 0q_1 + 10q_0 + 11q_1$

$= (0q_1 + 11q_1) + 10q_0$

$q_1 = (0+11)q_1 + 10q_0 \Rightarrow \boxed{(0+11)^*10q_0} \rightarrow (4)$

$X = A\,X + B$

$(0+11)^*(0+11)$

a^*a

a^+

$q_2 = 0q_0 + 1q_1$

$= 0q_0 + 1[(0+11)^*10q_0]$

$= [0 + 1(0+11)^*10]q_0 \rightarrow (5)$

$q_0 = 0[(0+11)^*10q_0] + 1[0 + 1(0+11)^*10]q_0 + \varepsilon$

$q_0 = [0(0+11)^*10 + 10 + 11(0+11)^*10]q_0 + \varepsilon$

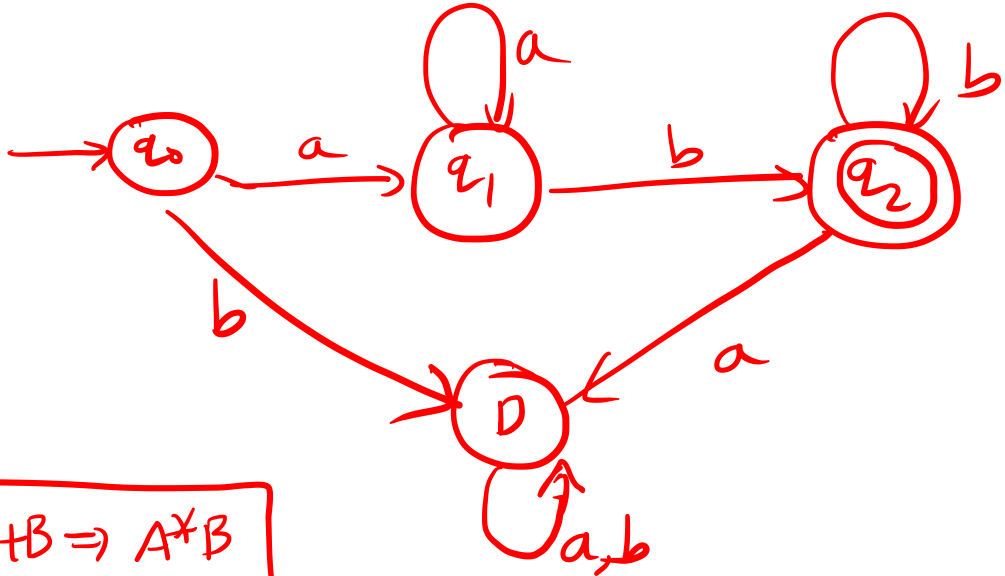
$X = \underbrace{\qquad}_{A}\; X + B$

$A^*\emptyset = \varepsilon A^* A^*$

$= \emptyset$

$\boxed{[\overbrace{0(0+11)^*10 + 10 + 11(0+11)^*}^{X}10]^*}$

$(010 + 10 + 11)(0+11)^*$



$R\emptyset = \emptyset R = \emptyset$ ✓

$R\varepsilon = \varepsilon R = R$ ✓

$X = AX + B$

$\Rightarrow \boxed{A^*B}$

$\boxed{X = AX + B \Rightarrow A^*B}$

$q_0 = aq_1 + bD \longrightarrow (1)$

$q_1 = aq_1 + bq_2 \longrightarrow (2)$

$q_2 = aD + bq_2 + \varepsilon \longrightarrow (3)$

$D = (a+b)D \longrightarrow 4$

Consider equ (4) $D = (a+b)D + \emptyset$

$X = A \; X + B \Rightarrow A^*B$

$\boxed{D = \emptyset}$

substitute (4) in all the equations

$q_0 = aq_1 \longrightarrow (5)$

$q_1 = aq_1 + bq_2 \longrightarrow (6)$

$\boxed{q_2 = bq_2 + \varepsilon} \longrightarrow$ Arden's

$q_2 = b^*\varepsilon = b^*$

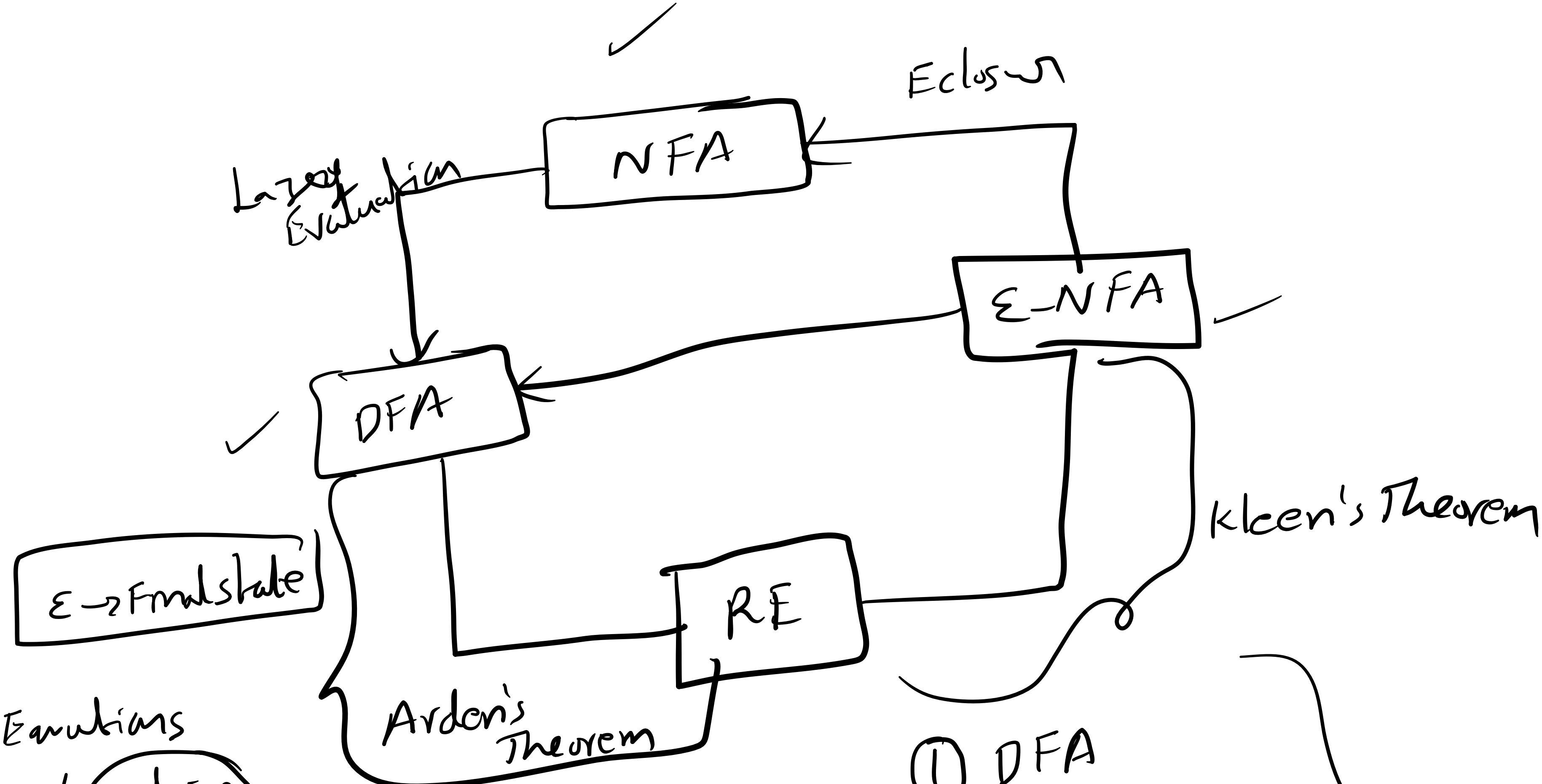
substitute in q_1 (6)

$q_1 = aq_1 + bb^*\varepsilon$

$\boxed{q_1 = aq_1 + bb^*} \longrightarrow q_1 = a^*bb^*$

substitute in equation (5)

$\boxed{q_0 = aa^*bb^*}$



Equations
↳ outgoing →
↳ Add ε to Final state q=ap

1. DFA
2. NFA
3. NFA → DFA
4. ε-NFA → NFA
5. RE → ε-NFA
6. DFA → RE