Decision Properties of Regular Languages

General Discussion of "Properties"

Membership, Emptiness, Etc.

Properties of Language Classes

- A *language class* is a set of languages.
  - We have one example: the regular languages.
  - We'll see many more in this class.
- Language classes have two important kinds of properties:
  1. Decision properties.
  2. Closure properties.

Representation of Languages

- Representations can be formal or informal.
- Example (formal): represent a language by a RE or DFA defining it.
- Example: (informal): a logical or prose statement about its strings:
  - $\{0^n1^n \mid n$ is a nonnegative integer$\}$
  - "The set of strings consisting of some number of 0's followed by the same number of 1's."

Decision Properties

- A *decision property* for a class of languages is an algorithm that takes a formal description of a language (e.g., a DFA) and tells whether or not some property holds.

- Example: Is language L empty?

Subtle Point: Representation Matters

- You might imagine that the language is described informally, so if my description is "the empty language" then yes, otherwise no.

- But the representation is a DFA (or a RE that you will convert to a DFA).

- Can you tell if $L(A) = \varnothing$ for DFA A?

Closure Properties

- A *closure property* of a language class says that given languages in the class, an operator (e.g., union) produces another language in the same class.

- Example: the regular languages are obviously closed under union, concatenation, and (Kleene) closure.

  - Use the RE representation of languages.

Why Closure Properties?

1. Helps construct representations.
2. Helps show (informally described) languages not to be in the class.

Example: Use of Closure Property

- We can easily prove $L_1 = \{0^n 1^n \mid n \geq 0\}$ is not a regular language.

- $L_2 =$ the set of strings with an = number of 0's and 1's isn't either, but that fact is trickier to prove.

- Regular languages are closed under $\cap$.

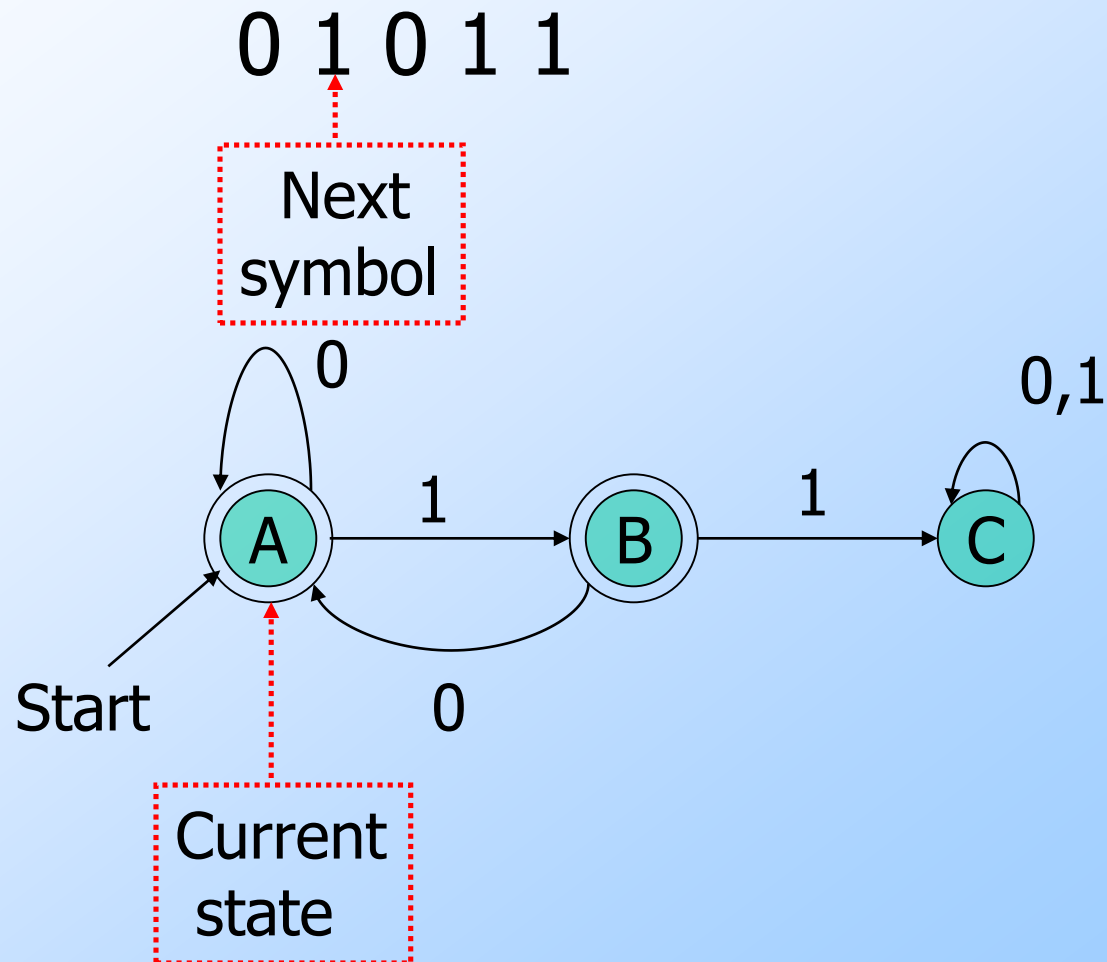- If L_2 were regular, then $L_2 \cap L(0*1*) = L_1$ would be, but it isn't.

The Membership Question

- Our first decision property is the question: "is string w in regular language L?"
- Assume L is represented by a DFA A.
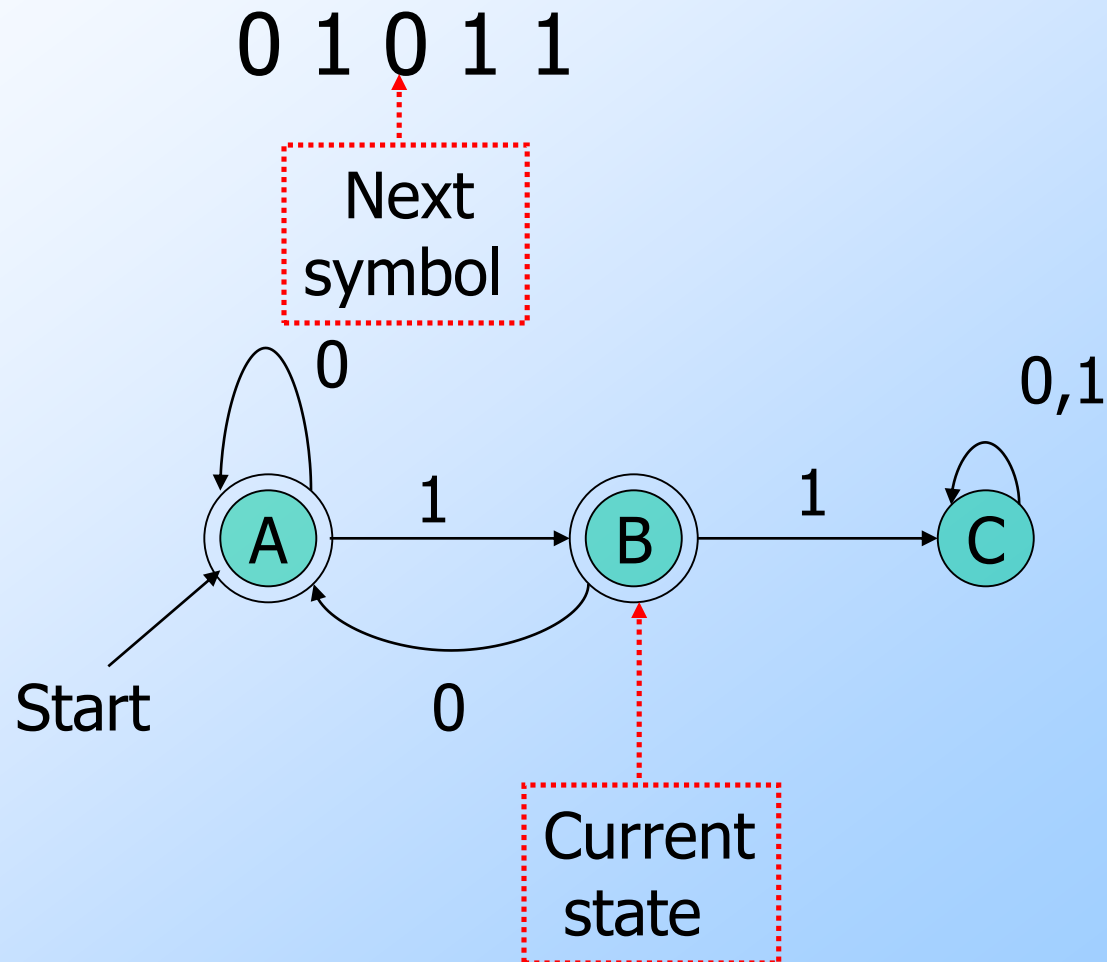- Simulate the action of A on the sequence of input symbols forming w.
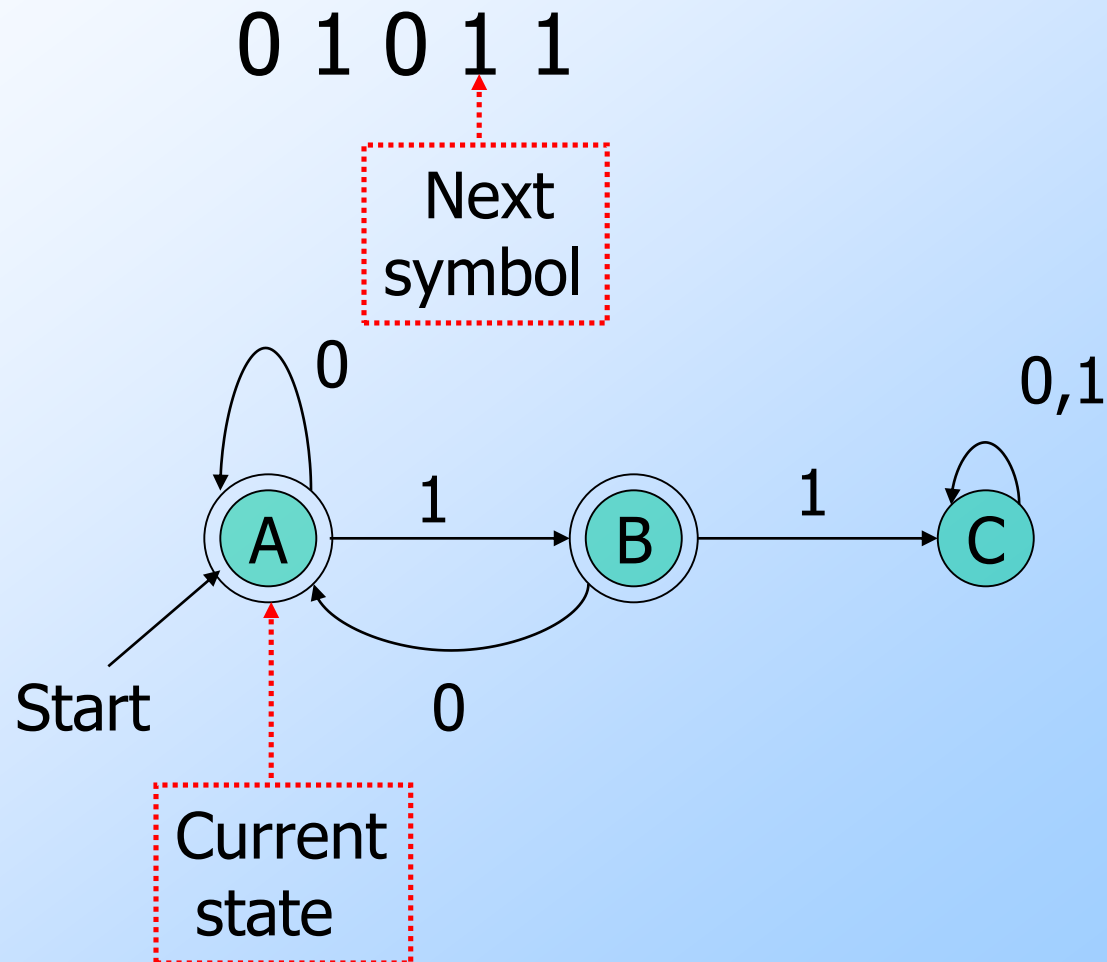
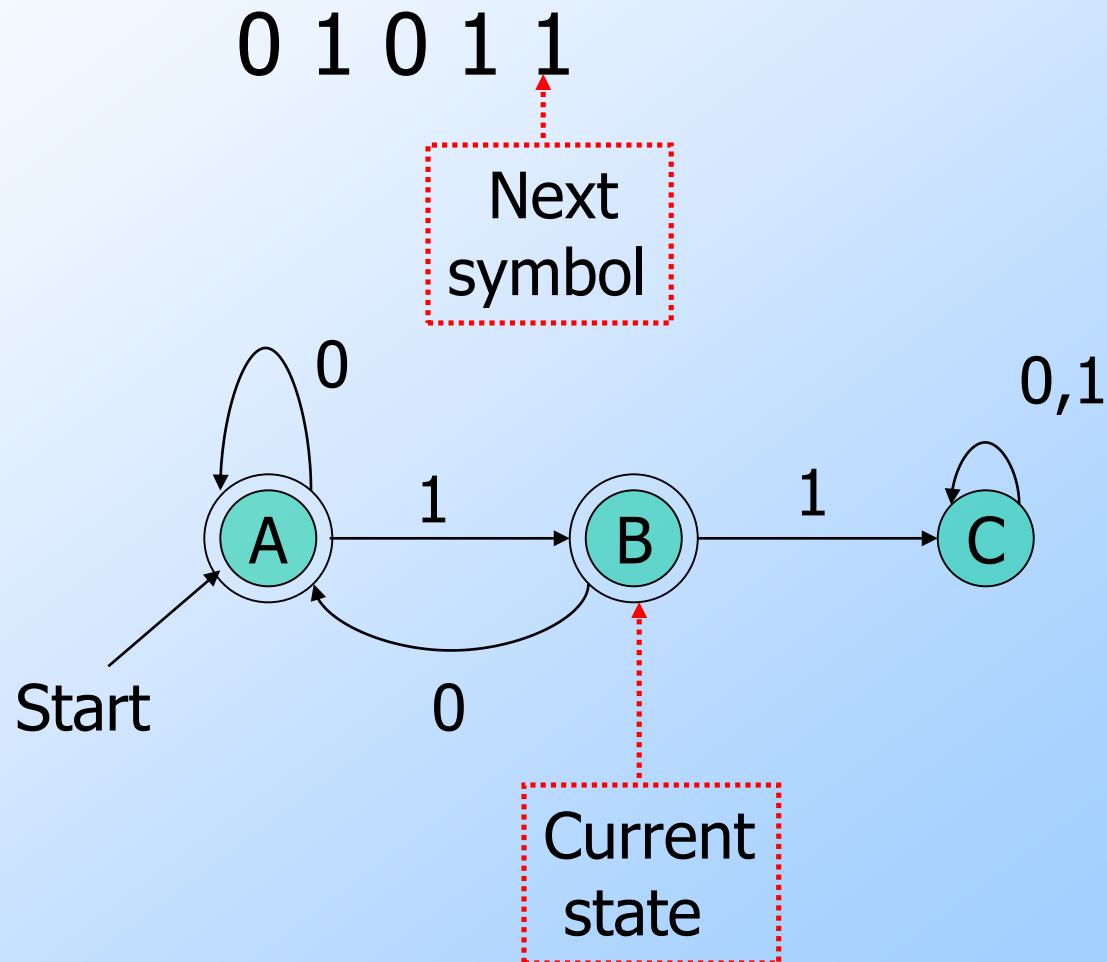# Example: Testing Membership
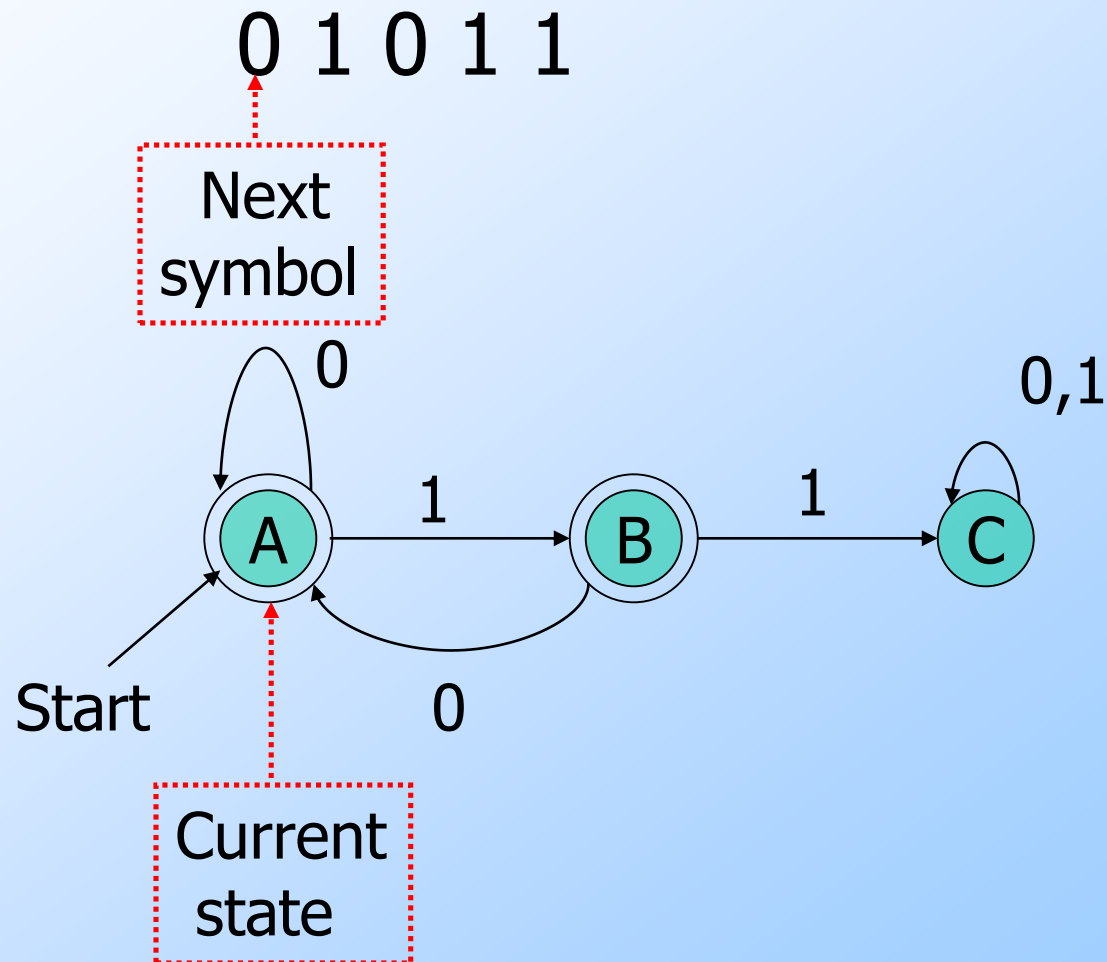
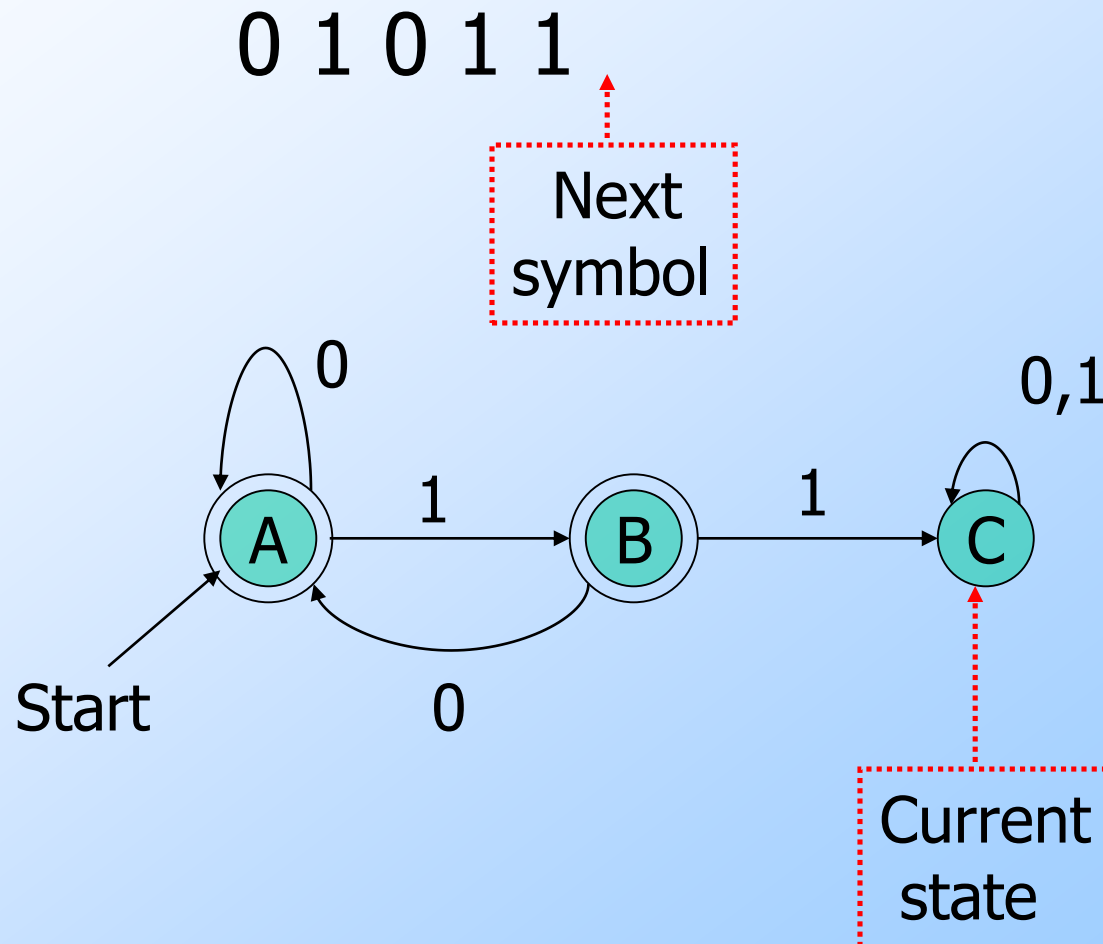# Example: Testing Membership

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0 1 0 1 1

Next symbol

Start

Current state
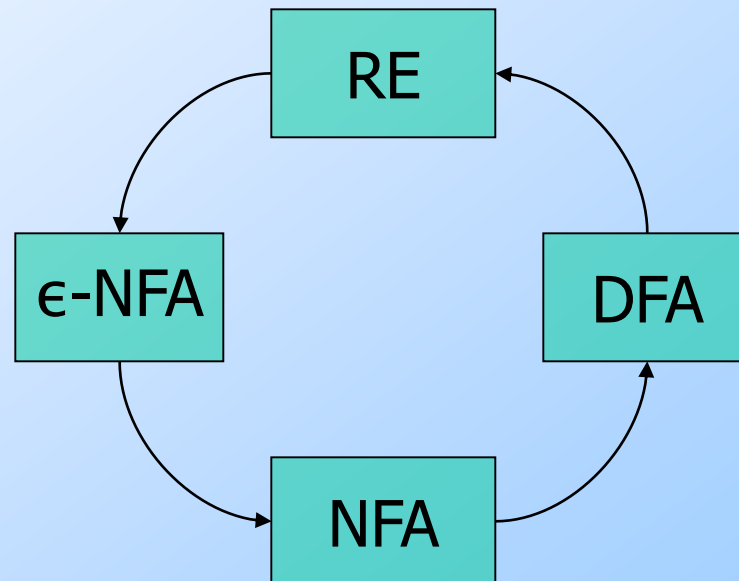
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Example: Testing Membership

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What if the Regular Language Is not Represented by a DFA?

- There is a circle of conversions from one form to another:

The Emptiness Problem: An Algorithm

- Given a regular language, does the language contain any string at all?

- Assume representation is DFA.

- Construct the transition graph.

- Compute the set of states reachable from the start state.

- If any final state is reachable, then yes, else no.