Alphabet:

- *An alphabet is a finite, non-empty set of symbols*
- We use the symbol ∑ (sigma) to denote an alphabet
- Examples: Click to add text
  Click to add text
  - Binary: ∑ = {0,1}
  - All lower case letters: ∑ = {a,b,c,..z}
  - Alphanumeric: ∑ = {a-z, A-Z, 0-9}
  - Engineering classes: ∑ = {1st Yr, 2nd Yr, 3rd Yr, Final Yr}

Strings:

- *A **string** or **word** is a finite sequence of symbols chosen from* $\sum$
- *Empty string is donated by* $\boldsymbol{\varepsilon}$ *(epsilon) or* $\boldsymbol{\lambda}$ (lambda).

Length of a String:

- It is the number of meaningful **symbols/alphabets** *(non- ε)* present in a string.
- Length of a string ***w***, denoted by "**|*w*|**".
  - ***E.g***., *if x = 010100 then |x| = 6*
  - *If x = 01 ε 0 ε 1 ε 00 ε* then|*x*| = *?*
- If |W|= 0, it is called an **empty string** (Denoted by **λ** or **ε**)
- *xy* = *concatenation* of two strings *x* and *y*

Powers of an alphabet:

Let ∑ be an alphabet.

- Σ^{k} = the set of all strings of length *k*
- $\Sigma^{*} = \Sigma^{0} \cup \Sigma^{1} \cup \Sigma^{2} \cup \dots$
- $\Sigma^{+} = \Sigma^{1} \cup \Sigma^{2} \cup \Sigma^{3} \cup \dots$

Kleene Closure:

- **Kleene closure** (A.K.A **Kleene operator** or **Kleene star**)
- **Definition:** The Kleene closure, ∑*, is an unary operator on a set of symbols or strings, ∑, that gives the **infinite set** of **all possible strings** of **all possible lengths** over ∑ including **ε**.
- **Representation** − $\sum^* = \sum_0 \cup \sum_1 \cup \sum_2 \cup \ldots\ldots.$ where $\sum_p$ is the set of all possible strings of length p.
- **Example** − If ∑ = {a, b},

then ∑* = {ε, a, b, aa, ab, bb, ba,……….}

Kleene Plus Closure:

- **Kleene plus closure** (A.K.A **Positive Closure**)
- **Definition:** The set $\sum^{+}$ is the infinite set of all possible strings of all possible lengths over $\sum$ excluding **ε**.
- **Representation** − $\sum^{+} = \sum_1 \cup \sum_2 \cup \ldots\ldots$

  (or) $\sum^{+} = \sum^{*} - \{\varepsilon\}$
- **Example** − If $\sum = \{a, b\}$,

  then $\sum^{+} = \{a, b, aa, ab, bb, ba, \ldots\ldots\ldots\}$

Language:

- A language is a subset of ∑* for some alphabet ∑. It can be finite or infinite.
- **Examples**:
  1. Let L be *the* language of all strings consisting of *n* 0's followed by *n* 1's:

     L = {ε,01,0011,000111,…}
  2. Let L be *the* language of all strings of with equal number of 0's and 1's:

     L = {ε,01,10,0011,1100,0101,1010,1001,…}
  3. If the language takes all possible strings of length 2 over ∑ = {a, b}, then L = { ab, bb, ba, bb}

- **Ø denotes the Empty language**

• Let L = {ε}; Is L=Ø?

The Membership Problem

Given a string $w \in \Sigma^*$ and a language L over Σ, decide whether or not $w \in L$.

Example:

Let w = 100011

Q) Is $w \in$ the language of strings with equal number of 0s and 1s?