21/12/23

UNIT-4:- DIGITAL MODULATION TECHNIQUES

Selection of digital Modulation scheme:-

-> Required parameters are:-

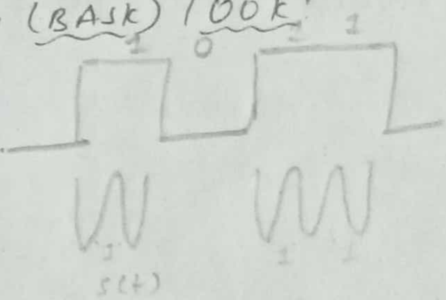
- Maximum data rate
- Minimum probability of symbol error
- Minimum Transmitted Power
- Minimum Channel Band width
- maximum Resistance to interfering symbol
- Minimum Circuit Complexity.

ASK (OOK)
PSK -> BPSK, QPSK
M-ary -> 8PSK, 16PSK
FSK -> BFSK
MSK
GMSK
-> OFDM
QAM

-> Some of these Goals pose conflicting requirements.

EX: Goals 1&2 are in conflict with Goals 3&4.
The best way is to satisfy as many of these requirements as possible.

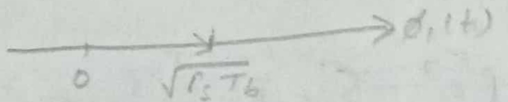
① Binary Amplitude shift keying (BASK) / OOK:



- In ASK, there is only one unit energy carrier and it is switched ON/OFF, depending on the i/p binary sequence.

$$\boxed{s(t) = \sqrt{2P_S}\ \cos(2\pi f_c t)} \quad —①$$

-> Signal space diagram:-



- $\phi_1(t)$ = Basis Function
- '0' : No signal } Message Points
- '1' = Amplitude
- T_b = Bit duration

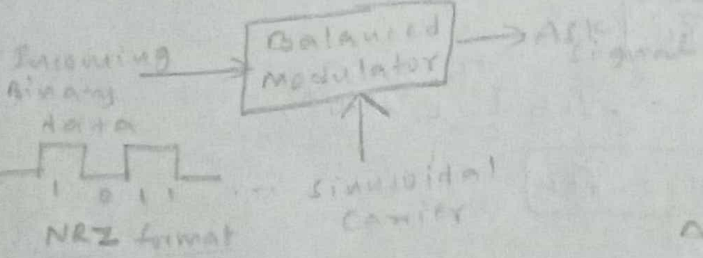
$$① \Rightarrow s(t) = \sqrt{P_S T_b} \cdot \underbrace{\sqrt{2/T_b}\ \cos(2\pi f_c t)}_{\phi_1(t)}$$

$$s(t) = \sqrt{P_S T_b} \cdot \phi_1(t)$$

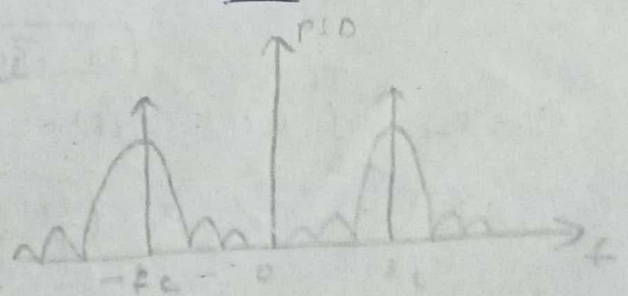
- The distance b/w two message points is, $d = \sqrt{P_S T_b}$

$$\boxed{d = \sqrt{E_b}}$$

Generation of ASK:-



* PSD *



- ~~Applications~~ LIMITATIONS:-
- Used in limited applications bcz of signal representation in Amplitude.
- More prone to Noise.

② COHERENT BINARY PSK:-

Coherent $\Downarrow$ phase of Recieved carrier freq = Modulating carrier freq

NRZ

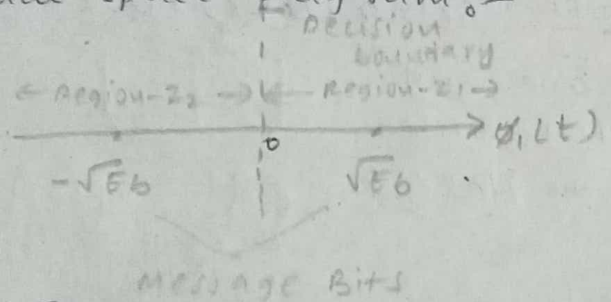
(1). $s_1(t) = \sqrt{\dfrac{2E_b}{T_b}} \cos(2\pi f_c t)$

(0). $s_2(t) = \sqrt{\dfrac{2E_b}{T_b}} \cos(2\pi f_c t + \pi) = -\sqrt{\dfrac{2E_b}{T_b}} \cos(2\pi f_c t)$ $\Big\}\ 0 \le t < T_b$

- Basis function, $\phi_1(t) = \sqrt{\dfrac{2}{T_b}} \cdot \cos(2\pi f_c t)$

$\Rightarrow s_1(t) = \sqrt{E_b} \cdot \phi_1(t)$; $s_2(t) = -\sqrt{E_b} \cdot \phi_1(t)$

→ signal space Diagram:-



- If msgbit is in region-Z₁ $\Rightarrow$ msg bit = 1.
- If msg bit is in region-Z₂ $\Rightarrow$ msg bit = 0.

- $s_{11} = \int_0^{T_b} s_1(t) \cdot \phi_1(t)\, dt = \int_0^{T_b} +\sqrt{E_b} \cdot \phi_1(t)\, \phi_1(t) dt$

$= \sqrt{E_b} \cdot \int_0^{T_b} \phi_1^2(t)\, dt$

$= \cancel{\sqrt{E_b} \int_0^{T_b} \phi_1^2(t)} = \sqrt{E_b} \int_0^{T_b} \dfrac{2}{T_b} \cdot \cos^2(2\pi f_c t) dt$

$= \dfrac{2\sqrt{E_b}}{2 \cdot T_b} \int_0^{T_b} [1 + \cos(4\pi f_c t)]\, dt$

$= \dfrac{\sqrt{E_b}}{T_b} \left[T_b + \dfrac{\sin(4\pi f_c T_b)}{4\pi f_c}\right]$

* $s_{11} = \sqrt{E_b}[1 + \text{sinc}(4 f_c T_b)]$ $\quad \dfrac{\sin \pi a}{\pi a}$ sinc a

$\boxed{s_{11} = \sqrt{E_b}}$

lly, $s_{21} = \int_0^{T_b} s_2(t) \cdot \phi_1(t)\, dt = -\sqrt{E_b} \int_0^{T_b} \phi_1^2(t) dt$

$\boxed{s_{21} = -\sqrt{E_b}}$

- Likelihood Function:-
  - If '$x(t)$' is recieved signal and 'x_1' is observation scalar: $x_1 = \int_0^{T_b} x(t) \cdot \phi_1(t)\, dt$

→ If symbol '0' is transmitted i.e., $s_2(t)$ is transmitted, then likelihood function is defined as:

$$f_{x_1}(x_1|0) = \frac{1}{\sqrt{\pi N_0}} \exp\left[\frac{-1}{N_0}(x_1 - s_{21})^2\right]$$

which is a conditional probability.

$$\Rightarrow \boxed{f_{x_1}(x_1|0) = \frac{1}{\sqrt{\pi N_0}} \exp\left[\frac{-1}{N_0}(x_1 + \sqrt{E_b})^2\right]}$$

→ probability error:-

→ The conditional probability of reciever deciding in favour of the symbol-1, given that symbol was transmitted. It can be labelled as "$P_e(0)$" and given by:

$$P_e(0) = \int_0^{\alpha} f_{x_1}(x|0)\, dx_1$$

$$P_e(0) = \int_0^{\alpha} \frac{1}{\sqrt{\pi N_0}} \exp\left[\frac{-1}{N_0}(x_1 + \sqrt{E_b})^2\right] dx_1$$

$$= \cancel{\frac{1}{\sqrt{\pi N_0}} \cdot \frac{\exp\left[\frac{-1}{N_0}(x_1 + \sqrt{E_b})^2\right]}{\frac{-1}{N_0} \cdot 2(x_1 + \sqrt{E_b})}}$$

(or)

let $z = \frac{1}{\sqrt{N_0}}(x_1 + \sqrt{E_b})$; $x_1 = 0 \Rightarrow z = \sqrt{\frac{E_b}{N_0}}$; $x_1 = \alpha \Rightarrow z = \alpha$

$dz = \frac{1}{\sqrt{N_0}} dx$

$$\Rightarrow P_e(0) = \int_{\sqrt{E_b}/\sqrt{N_0}}^{\alpha} \frac{1}{\sqrt{\pi N_0}} \exp(-z^2)\, dz \cdot \sqrt{N_0}$$

$$P_e(0) = \frac{1}{\sqrt{\pi}} \int_{\frac{\sqrt{E_b}}{\sqrt{N_0}}}^{\alpha} \exp(-z^2)\, dz = \frac{1}{2} \operatorname{erfc}\left(\sqrt{\frac{E_b}{N_0}}\right) \quad -①$$

$$\cancel{= \frac{1}{\sqrt{\pi}} \left[\frac{\exp(-z^2)}{-2z}\right]_{\frac{E_b}{\sqrt{N_0}}}^{\alpha}}$$

$$\cancel{= \frac{-1}{2\sqrt{\pi}} \left\{-\exp\left(-\frac{E_b^2}{N_0}\right)\right\}}$$

$$\cancel{P_e(0) = \frac{1}{2\sqrt{\pi}} \cdot \exp\left(-\frac{E_b^2}{N_0}\right)}$$

$$\boxed{Q(v) = \frac{1}{\sqrt{2\pi}} \int_v^{\alpha} \exp\left(-\frac{x^2}{2}\right) dx}$$

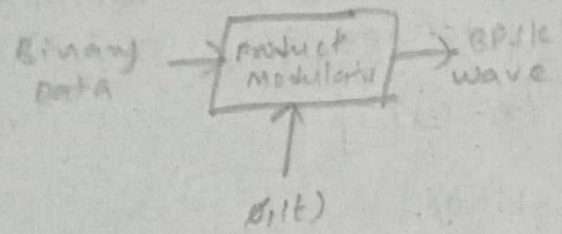
→ similarly, the probability error of '1' is obtained as:

$$\boxed{P_e(1) = \frac{1}{2} \operatorname{erfc}\left(\sqrt{\frac{E_b}{N_0}}\right)} \quad -②$$

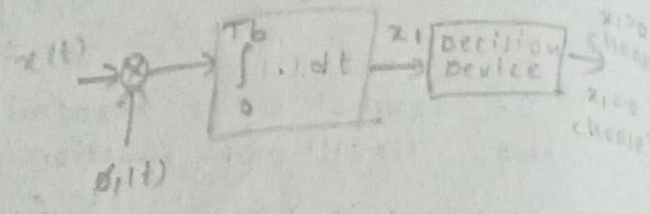
∴ From ① & ②

Any error is given as: $\boxed{P_e = \frac{1}{2} \operatorname{erfc}\left(\sqrt{\frac{E_b}{N_0}}\right)}$

→ Generation of BPSK:-



→ Detection of BPSK:



③ COHERENT BINARY FSK:- / FSK:

- $s_i(t) = \begin{cases} \sqrt{\dfrac{2E_b}{T_b}} \cos(2\pi f_i t) & ; 0 \le t \le T_b \\ 0 & ; \text{elsewhere} \end{cases}$

where, $\boxed{f_i = \dfrac{n_c + i}{T_b}}$, n_c is a fixed integer.

- $s_1(t)$ & $s_2(t)$ are orthogonal but not normalised to have unit energy.

$\Rightarrow \quad \phi_i(t) = \begin{cases} \sqrt{\dfrac{2}{T_b}} \cos 2\pi f_i t & ; 0 \le t < T_b \\ 0 & ; \text{else where} \end{cases}$

→ coefficients, $s_{ij} = \int_0^{T_b} s_i(t)\phi_j(t)\,dt$

$$s_{ij} = \sqrt{\frac{2E_b}{T_b}} \cdot \sqrt{\frac{2}{T_b}} \int_0^{T_b} \cos(2\pi f_i t) \cdot \cos(2\pi f_j t)\,dt$$

* If $i = j$;

$$= \frac{2\sqrt{E_b}}{2 \cdot T_b} \int_0^{T_b} [1 + \cos(4\pi f_i t)]\,dt$$

$$= \frac{\sqrt{E_b}}{T_b} \left[T_b + \frac{\sin(4\pi f_i T_b)}{4\pi f_i} \right]$$

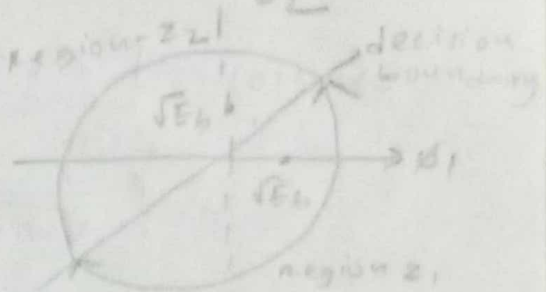
$$= \sqrt{E_b}\,[1 + \text{sinc}(4 f_i T_b)].$$

$$\boxed{s_{ij} = \sqrt{E_b}}$$

* If $i \ne j$, $\boxed{s_{ij} = 0}$

$\therefore \quad s_{ij} = \begin{cases} \sqrt{E_b} & ; i = j \\ 0 & ; i \ne j \end{cases}$

→ signal space diagram:-



* $s_1 = \begin{bmatrix} \sqrt{E_b} \\ 0 \end{bmatrix}$; $s_2 = \begin{bmatrix} 0 \\ \sqrt{E_b} \end{bmatrix}$

→ from message points, s_1, s_2, we write observation vector of 'x' as:

two elements
- $x_1 = \int_0^{T_b} x(t)\,\phi_1(t)\,dt$
- $x_2 = \int_0^{T_b} x(t)\cdot\phi_2(t)\,dt$

→ If $x(t)$ is recieved signal, then
- If '1' is transmitted ⇒ $x(t) = s_1(t) + w(t)$
- If '0' is transmitted ⇒ $x(t) = s_2(t) + w(t)$

where, w = white noise with PSD $\frac{N_0}{2}$ and mean '0'.

→ let us define a guassian random variable as:- (L)
- sample value, $\boxed{l = x_1 - x_2}$

- The conditional mean of random variable 'L' given that symbol '1' is transmitted is given as:-

* $\boxed{E[L|1] = E[x_1|1] - E[x_2|1] = \sqrt{E_b}}$

- conditional mean of random variable 'L' given that symbol '0' is transmitted is equal to:

* $\boxed{E[L|0] = E[x_1|0] - E[x_2|0] = -\sqrt{E_b}}$

- The variance of the random variable 'L' is independent of which binary symbol was transmitted. since random variables X_1 & X_2 are statistically independent, each with variance $\frac{N_0}{2}$, we can write variance of 'L' as

* $\boxed{var[l] = var[x_1] + var[x_2] = N_0}$

→ suppose that, the symbol '0' was transmitted, then the corresponding value of the conditional probability density function of random variable 'L' equals:

* $f(l|0) = \frac{1}{\sqrt{2\pi N_0}} \cdot \exp\left[-\frac{(l+\sqrt{E_b})^2}{2N_0}\right]$

- Since, the condition $x_1 > x_2$, $l > 0$, reciever treats the output as '1'. we deduce that the conditional probability of error * $\boxed{P_e(0) = P(l > 0 \mid \text{symbol '0' was sent})}$

$\Rightarrow P_e(0) = \int_0^{\alpha} f(l|0)\,dl$

$$= \int_0^{\alpha} \frac{1}{\sqrt{2\pi N_0}} \exp\left[-\frac{(l+\sqrt{E_b})^2}{2N_0}\right] dl$$

$$= \frac{1}{\sqrt{2\pi N_0}} \int_0^{\alpha} \exp\left[-\frac{(l+\sqrt{E_b})^2}{2N_0}\right] dl$$

$$= \frac{1}{\sqrt{2\pi N_0}} \int_{\sqrt{\frac{E_b}{2N_0}}}^{\alpha} \exp[-(z^2)]\, dz \sqrt{2N_0}$$

$z = \frac{l+\sqrt{E_b}}{\sqrt{2N_0}}$

$l = 0 \Rightarrow z = \sqrt{\frac{E_b}{2N_0}}$

$l = \alpha \Rightarrow z = \alpha$

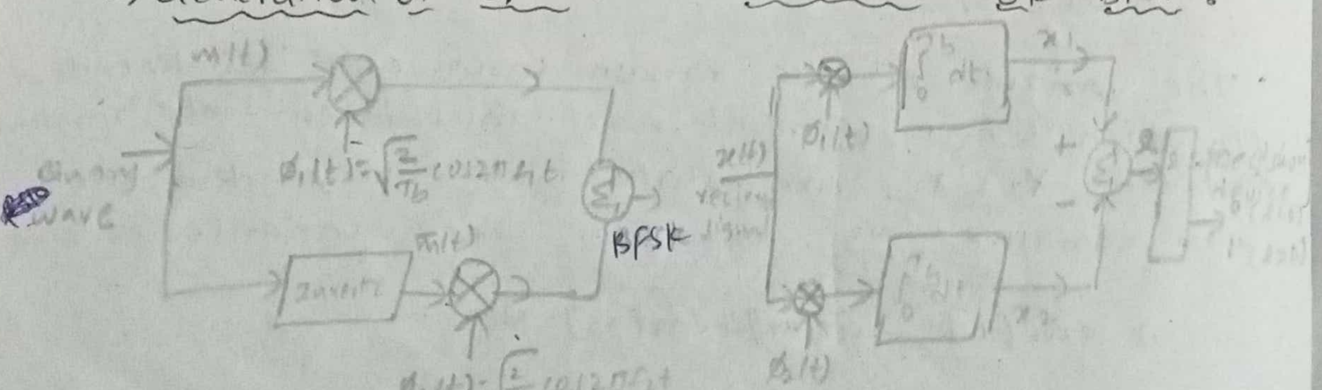
$$= \frac{1}{\sqrt{\pi}} \int_{\sqrt{\frac{E_b}{2N_0}}}^{\alpha} \exp(-z^2)\, dz$$

$$\boxed{P_e(0) = \frac{1}{2}\,\mathrm{erfc}\left(\sqrt{\frac{E_b}{2N_0}}\right)}$$

lly $P_e(1) = \frac{1}{2}\,\mathrm{erfc}\left(\sqrt{\frac{E_b}{2N_0}}\right)$

$\therefore$ Average probability of error $= \frac{1}{2}\,\mathrm{erfc}\left(\sqrt{\frac{E_b}{2N_0}}\right)$ //

→ Generation of BFSK & DETECTION OF BFSK :-



M-2

④ Quadrature Phase shift keying (QPSK) :-

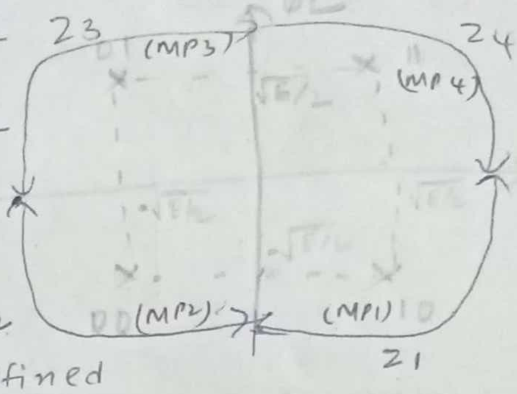
The QPSK represents the output in one of four equally spaced values $\frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}$ & $\frac{7\pi}{4}$.

i.e., $s_i(t) = \begin{cases} \sqrt{\frac{2E}{T}} \cos\left[2\pi f_c t + (2i-1)\frac{\pi}{4}\right] & ; 0 \le t \le T \\ 0 & ; \text{elsewhere} \end{cases}$

where,

- E = Transmitted signal energy / symbol
- T = symbol Duration
- $f_c = \frac{n_c}{T}$; n_c = fixed integral
- $i = 1, 2, 3, 4$
- Each phase is represented by a "dibit". 10, 00, 01, 11 ($0 \le t \le T$) — Gray encoded dibits

Input dibits	phase of QPSK signal	coordinates of message points s_{i1}	s_{i2}
1 0	$\pi/4$	$\sqrt{E/2}$	$-\sqrt{E/2}$
0 0	$3\pi/4$	$-\sqrt{E/2}$	$-\sqrt{E/2}$
0 1	$5\pi/4$	$-\sqrt{E/2}$	$\sqrt{E/2}$
1 1	$7\pi/4$	$\sqrt{E/2}$	$\sqrt{E/2}$



→ signal space diagram:
(or) Constellation diagram

→ **Probability of error:**

- The recieved signal '$x(t)$' is defined by:

$$\boxed{x(t) = s_i(t) + w(t)} \; ; \; 0 \le t \le T \; , i = 1, 2, 3, 4$$

- $$s_i(t) = \begin{cases} \sqrt{\frac{2E}{T}} \left[\cos(2\pi f_c t) \cdot \cos(2i-1)\frac{\pi}{4} - \sin(2\pi f_c t) \cdot \sin(2i-1)\frac{\pi}{4}\right] & ; 0 \le t \le T \\ 0 & ; \text{elsewhere} \end{cases}$$

- $\phi_1(t) = \sqrt{\frac{2}{T}} \cos 2\pi f_c t$
- $\phi_2(t) = \sqrt{\frac{2}{T}} \sin 2\pi f_c t$

$$\Rightarrow s_i(t) = \left\{ \sqrt{E}\left[\cos(2i-1)\frac{\pi}{4}\right]\phi_1(t) - \left[\sin(2i-1)\frac{\pi}{4}\right]\sqrt{E} \cdot \phi_2(t) \right.$$

- The observation vector 'x' of a coherent QPSK reciever has got 2 elements x_1, x_2 which are represented as:

$$* \; x_1 = \int_0^T x(t)\phi_1(t)\,dt = \sqrt{E} \cdot \cos(2i-1)\frac{\pi}{4} + w_1$$

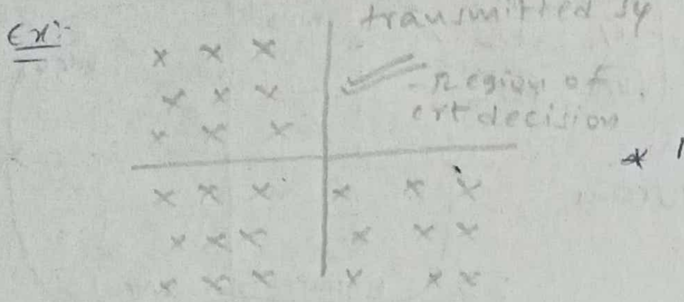
$$* \; x_2 = \int_0^T x(t)\phi_2(t)\,dt = -\sqrt{E} \sin(2i-1)\frac{\pi}{4} + w_2$$

- The x_1, x_2 are sample values of independent guassian random variables with mean values equal to $\sqrt{E}\cos(2i-1)\frac{\pi}{4}$ and $-\sqrt{E}\sin(2i-1)\frac{\pi}{4}$, with a common variance equal to $\frac{N_0}{2}$.

→ The decision rule is to guess '$s_1(t)$' was transmitted if the recieved signal point associated with the observation vector 'x' falls inside the region $-Z_1$, guess $s_2(t)$ was transmitted if recieved signal point falls inside the region $-Z_2$ and so on.

An erroneous decision will be made if, for example signal s_4 is transmitted but noise $w(t)$ is such that the recieved signal point falls outside the region Z-4.

Ex:-



→ we write the probability of correct decision as:-

$$* \; P_c = \int_0^{\alpha} \frac{1}{\sqrt{\pi N_0}} \exp\left[-\frac{(x_1-\sqrt{E/2})^2}{N_0}\right] dx_1$$

$$* \int_0^{\alpha} \frac{1}{\sqrt{\pi N_0}} \exp\left(-\frac{(x_2-\sqrt{E/2})^2}{N_0}\right)$$

• select region $x_1 \geq 0$ & $x_2 \geq 0$

$$* \text{ let } \frac{x_1-\sqrt{E/2}}{\sqrt{N_0}} = \frac{x_2-\sqrt{E/2}}{\sqrt{N_0}} = z$$

$$\Rightarrow P_c = \int_{-\sqrt{\frac{E}{2N_0}}}^{\alpha} \frac{1}{\sqrt{\pi N_0}} \exp[-z^2] . \sqrt{N_0}\, dz \; * \int_{-\sqrt{\frac{E/2}{N_0}}}^{\alpha} \frac{1}{\sqrt{\pi N_0}} \exp(-z^2)\, dz . \sqrt{N_0}$$

$$= \frac{1}{(\sqrt{\pi})^2} \int_{-\sqrt{\frac{E}{2N_0}}}^{\alpha} [\exp[-z^2]]^2 dz$$

$$* \; \frac{1}{\sqrt{\pi}} \int_{-\sqrt{\frac{E}{2N_0}}}^{\alpha} \exp(-z^2)\, dz$$

$$= 1 - \frac{1}{2} \operatorname{erfc}\left(\sqrt{\frac{E}{2N_0}}\right)$$

$$= \frac{1}{\pi} \int_{-\sqrt{\frac{E}{2N_0}}}^{\alpha} [\exp(-z^2)]^2 dz$$

$$= \left\{1 - \frac{1}{2} \operatorname{erfc}\left(\sqrt{\frac{E}{2N_0}}\right)\right\}^2$$

$$= 1 + \frac{1}{4} \operatorname{erfc}^2\left(\sqrt{\frac{E}{2N_0}}\right) - \operatorname{erfc}\left(\sqrt{\frac{E}{2N_0}}\right)$$

$$\boxed{P_c = 1 - \operatorname{erfc}\left(\sqrt{\frac{E}{2N_0}}\right) + \frac{1}{4} \operatorname{erfc}^2\left(\sqrt{\frac{E}{2N_0}}\right)}$$

But $\frac{E}{2N_0} >> 1$

$$\Rightarrow \boxed{P_c = 1 - \text{erfc}\sqrt{\frac{E}{2N_0}}}$$

$$\Rightarrow \boxed{P_e = \text{erfc}\sqrt{\frac{E}{2N_0}}}$$

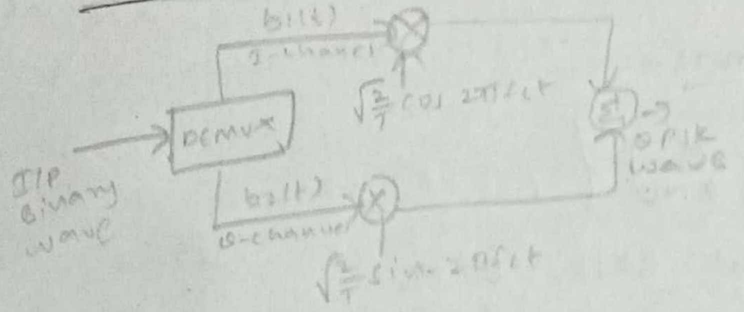
* $P_e + P_c = 1$

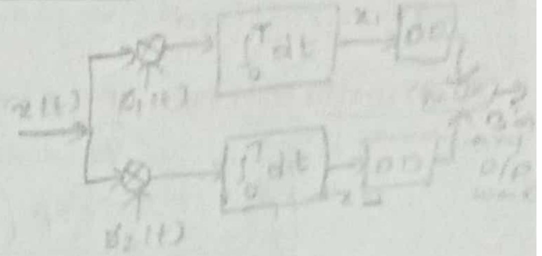
$P_e = 1 - P_c$

- $E_{symbol} = 2E_b$

$$\Rightarrow \boxed{P_e \simeq \text{erfc}\sqrt{\frac{E_b}{N_0}}}$$

→ Generation of QPSK ; Detection of QPSK :





⊛ Non-Coherent Binary Modulation Technique :-

Coherent detection exploits knowledge of the carrier waves phase difference there by providing the optimum error performance attainable with a digital modulation format. But, it is impracticable to have knowledge of the carrier phase at the reciever. Hence, we use the Non-coherent detection.

Consider a binary signalling scheme that involve the use of 2-orthogonal signals $s_1(t)$ & $s_2(t)$ which have equal energy. During the time period $0 \le t \le T$ one of these signals is sent over an imperfect channel that shifts carrier phase by unknown amount.

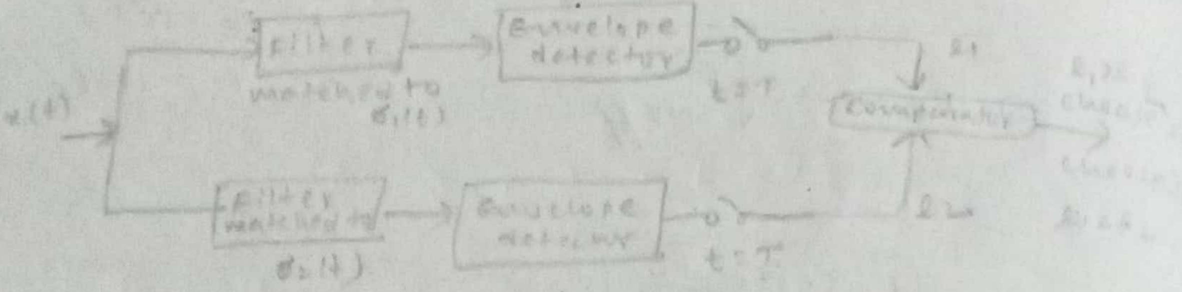
→ $g_1(t)$ & $g_2(t)$ represents phase shifted versions of $s_1(t)$ & $s_2(t)$. $g_1(t)$, $g_2(t)$ remain orthogonal and of equal energy regardless of the unknown carrier phase. we refer to such a signalling scheme as "Non-coherent orthogonal Modulation", Depending on how we define $s_1(t)$ & $s_2(t)$ Non-coherent binary fsk and non-coherent BPSK may be treated as special cases of this modulation schemes.

→ The recieved signal $x(t) = g_1(t) + w(t)$

$x(t) = g_2(t) + w(t)$; $0 \le t \le T$

where, $w(t)$ = Gaussian Noise with PSD $\frac{N_0}{2}$.

→ Generation:-



Reciever structure for Generalised Binary reciever

→ Generalised reciever for noncoherent orthogonal

→ **Probability of error:-**

$$\boxed{P_e = \frac{1}{2}\exp\left(\frac{-E}{2N_0}\right)} \quad (PSK + DPSK)$$

→ For Non-coherent BFSK, $E = E_b$
$T = T_b$

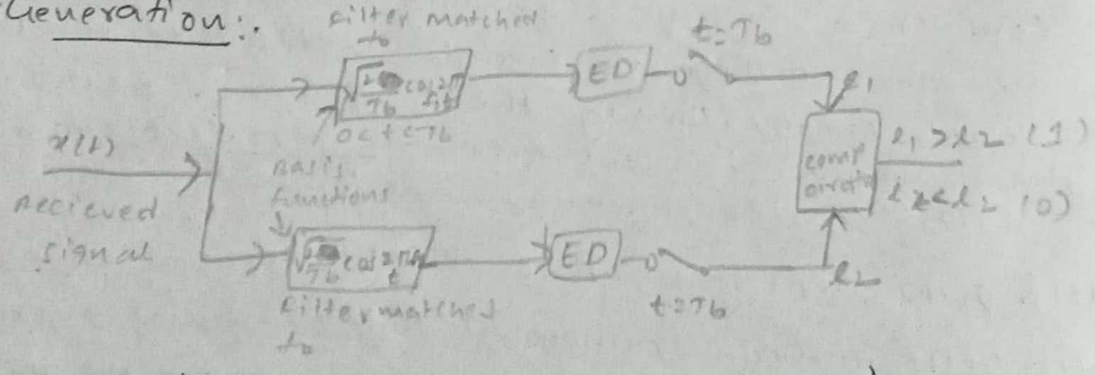
$$\Rightarrow P_e = \frac{1}{2}\exp\left(\frac{-E_b}{2N_0}\right)$$

→ properties of error function

* Non-Coherent BFSK system:-

→ $s_i(t) = \begin{cases} \sqrt{\frac{2E_b}{T_b}}\cos 2\pi f_i t & ; 0 \le t \le T_b \\ 0 & ; ow \end{cases}$

• Generation:-



• probability of error, $P_e = \frac{1}{2}\exp\left(\frac{-E_b}{2N_0}\right)$

* Non-coherent DPSK scheme:-

The DPSK eliminates the need for a coherent reference signal at the reciever by combining two basic operations at the transmitter.

1. Differential Encoding of i/p binary wave
2. Phase shift keying

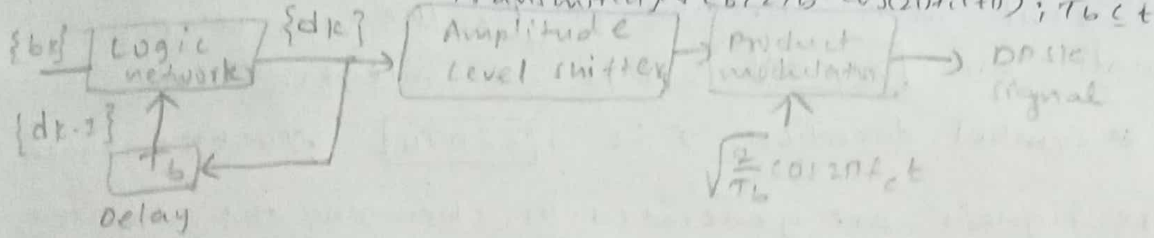
Therefore, it is called as Differential phase shift keying. In effect, to send symbol '0', we phase advance the current signal wave form by 180° and for send symbol 1, we leave the phase of current signal wave form unchanged.

The reciever is a equipped wave, a storage capability. So that, it can measure the relative phase difference b/w the wave forms recieved during 2 sucessive bit intervals.

• Generation:-

$*\ s_1(t) = \begin{cases} \sqrt{\frac{E_b}{2T_b}} \cdot \cos 2\pi f_c t & ; 0 \le t \le T_b \\ \sqrt{E_b/2T_b} \cdot \cos 2\pi f_c t & ; T_b \le t \le 2T_b \end{cases}$ [symbol-1 is transmitted]

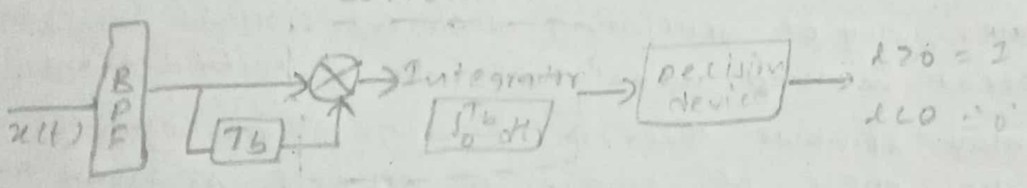
$*\ s_2(t) = \begin{cases} \sqrt{E_b/2T_b} \cdot \cos 2\pi f_c t & ; 0 \le t \le T_b \\ \sqrt{E_b/2T_b} \cdot \cos(2\pi f_c t + \pi) & ; T_b \le t \le 2T_b \end{cases}$ [symbol-0 is transmitted]



DPSK transmitter

• DPSK Reciever:-

correlator = product device followed by '∫'



* Total time duration, $T = 2T_b$

* Total Energy, $E = 2E_b$

∴ Probability of error, $P_e = \frac{1}{2} \exp\left(\frac{-2E_b}{2N_0}\right)$

$$\boxed{P_e = \frac{1}{2} \exp\left(\frac{-E_b}{N_0}\right)}$$

* $\boxed{d_k = d_{k-1} b_k + \overline{d_{k-1}}\, \overline{b_k}}$. modulo-2

Let, input be {bk} 1 0 0 1 0 0 1 1

⇒ $\{\overline{b_k}\}$ {0 1 1 0 1 1 0 0}

· If change = '0'
- If no change = '1'.

and {dk}* = 1 0 1 1 0 1 1 1

{dk-1} = 1 1 0 1 1 0 1 1

$\{\overline{d_{k-1}}\}$ = 0 0 1 1 0 1 0 0

$* \{b_k \cdot d_{k-1}\} = \{1\ 0\ 0\ 1\ 0\ 0\ 1\ 1\}$

$* \{\bar{b}_k\ \bar{d}_{k-1}\} = \{0\ 0\ 1\ 0\ 0\ 1\ 0\ 0\}$

$\therefore d_k = \{1\ 0\ 1\ 1\ 0\ 1\ 1\ 1\}$ //

* M-ARRAY MODULATION SCHEMES:-

M-2

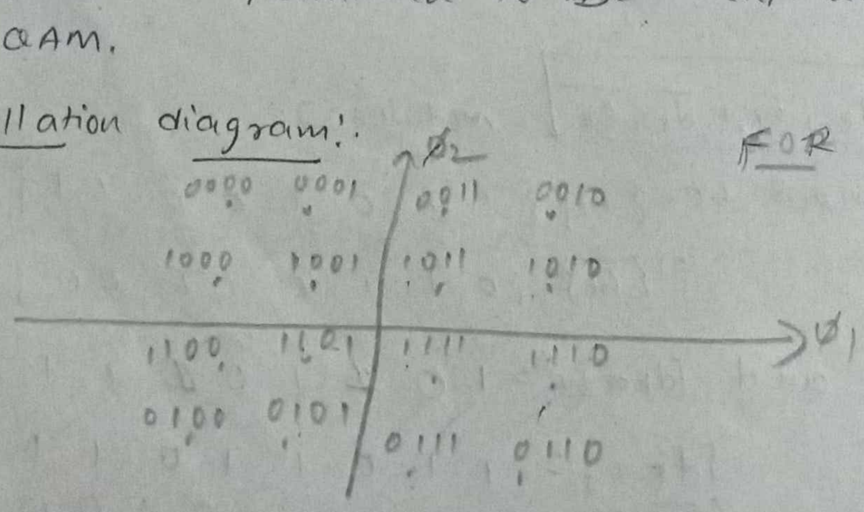
- M-Array scheme: M- Possible signals $s_1(t), s_2(t)$ an - - - $s_M(t)$ are sent in a duration of T. The no. of possible signals are $\boxed{M = 2^n}$; n is an integer.
  - * symbol duration 'T' is $\boxed{T = nT_b}$; T_b = bit duration
- -> These signals are generated by changing the amplitude, phase/ frequency of a carrier wave in M discrete steps. which are called as: M-array ASK, M-array PSK, M-array FSK.
- -> Another way of generating M-array signals is combine different method modulations into a hybrid form.

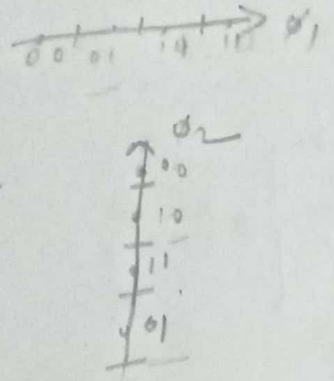
(1) we may combine discrete changes in both the amplitude and the phase of carrier to produce M-array Amplitude phase keying i.e., M-array APK.

- QAM:-
  - -> A special form of this modulation called M-array QAM has some attractive properties.
- M-array systems are used to conserve BW at the expense of increased power.
- In M-array PSK, the in-phase and quadrature phase components are permitted to be independent, then we get QAM.

- Constellation diagram:

FOR 16 QAM





. M-array constellation diagram consists of a square lattice of message points. Here, $M=16$.

-> In general, M-array QAM enables transmission of "$M = L^2$" independent symbols. For this type of QAM, we write:

$$\boxed{s_i(t) = \sqrt{\frac{2E_0}{T}}\, a_i \cos(2\pi f_c t) + \sqrt{\frac{2E_0}{T}}\, b_i \sin(2\pi f_c t)}$$

. where,

E_0 = Energy of signal with lowest amplitude

and a_i, b_i = A pair of independent integers choosen in accordance with the location of the pertinent message point.

-> Basis functions are :-

. $\phi_1(t) = \sqrt{\frac{2}{T}} \cos(2\pi f_c t)$; $0 < t < T$

. $\phi_2(t) = \sqrt{\frac{2}{T}} \sin(2\pi f_c t)$ "

. The signal '$s_i(t)$' consists of 2 phase quadrature carriers each of which is modulated by a set of discrete amplitudes. Hence the name quadrature amplitude modulation.

The coordinates of i^{th} message point are "$a_i\sqrt{E_0}$, $b_i\sqrt{E_0}$" where a_i, b_i are integers.

. For 16 QAM, $(a_i, b_i) = \left[\quad\right]_{L\times L}$; $L = \sqrt{M}$

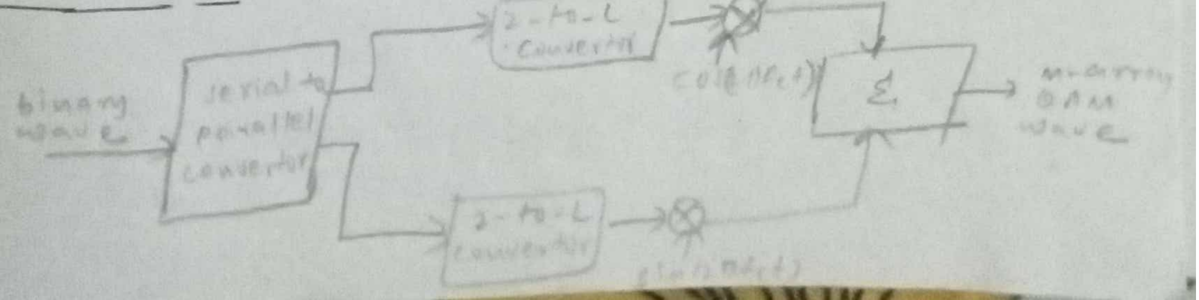
-> probability error for M-array QAM is:-

$$\boxed{P_e = 2\left(1 - \frac{1}{\sqrt{M}}\right) \operatorname{erfc}\left[\sqrt{\frac{3 \cdot E_{avg}}{2(M-1)N_0}}\right]}$$

where,

$E_{avg} = \frac{2(M-1)E_0}{3}$

-> Generation of M-array QAM:-



→ Detection of QAM (M-array):