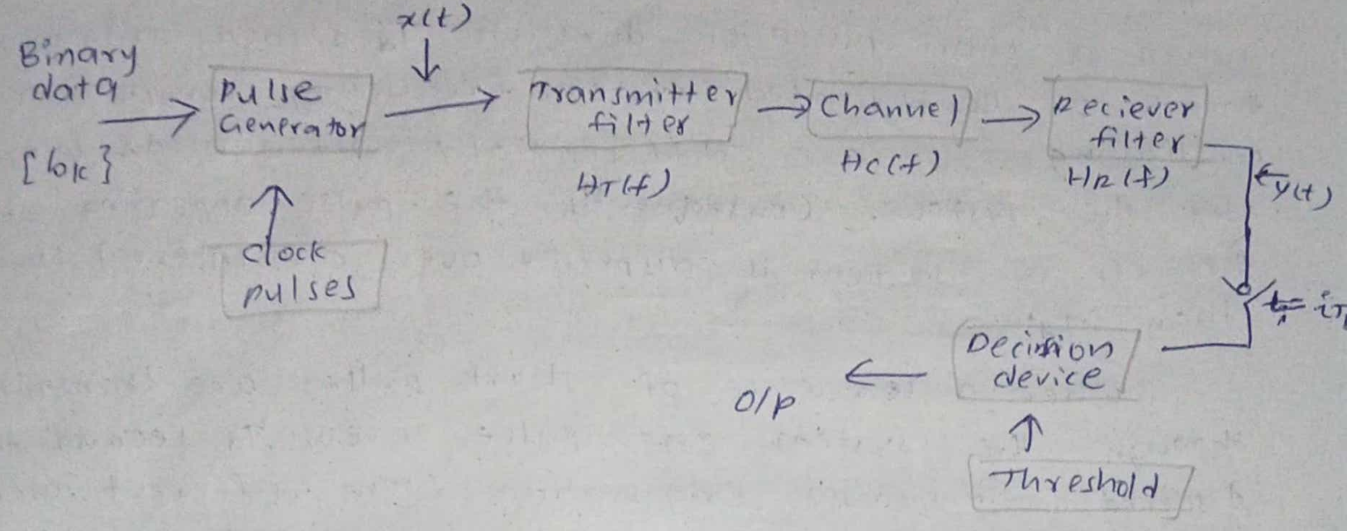
14/11/23

UNIT-2

* ISI: Inter Symbol Interfarence:-



* Baseband Binary data Transmission system.

- $x(t) = \sum_{-\alpha}^{\alpha} a_k \cdot v(t - kT_b)$ —① ; $v(t)$ = Basic pulse

  $v(0) = 1$

  a_k = Data type & format

  b_k = i/p binary data sequence

  $x(t)$ = PAM waves (discrete)

- $y(t) = \mu \sum_{k=-\alpha}^{\alpha} a_k p(t - kT_b)$ —②

  SF

* $\mu p(t)$ = Response of cascade connection of transmitting filter, the channel and the recieving filter, which is produced by the pulse $v(t)$ applied to i/p of cascaded connection.

- $p(0) = 1$

- $\mu p(f) = V(f) \cdot H_T(f) \cdot H_C(f) \cdot H_R(f)$ —③

Recieving filter o/p is sampled at $t_i = iT_b$ (i an integer)

$\Rightarrow y(t_i) = \mu \sum_{k=-\alpha}^{\alpha} a_k\, p(iT_b - kT_b)$ —④

$= \underbrace{\mu a_i}_{k=i} + \mu \sum_{k=-\alpha}^{\alpha} a_k (iT_b - kT_b) \}\ k \neq i$

- (μa_i) is produced by i^{th} transmitted bit.
- 2^{nd} term is due to the residual effect of all other transmitting bits and decoding of i^{th} bit, which represents ISI.

· ISI arises because of imperfection in overall frequency response of the system.

· when a short pulse of duration 'T_d' seconds is transmitted through a Band limited system, frequency components constituting are differentially attenuated & delayed by the system. Consequently, the pulse appearing at the o/p of system is dispersed over an interval longer than 'T_d'.

when a sequence of short pulses are transmitted, one pulse every 'T_d' second that through the system, one pulse every T_d second the dispersed responses originating from different intervals will interfere thereby resulting ISI.

The presence of ISI, introduces errors in decision device at the reciever o/p.

*NYQUIST CRITERION FOR DISTORTIONLESS BASEBAND TRANSMISSION:

· The reciever extracts and decodes corresponding sequence of bytes, a_k from the output $y(t)$. Extraction requires sampling o/p $y(t)$ at $t = iT_b$. Decoding requires weighted pulse contribution, $a_k \cdot p(iT_b - kT_b)$ for $k = i$, is free from ISI should be written as $p(iT_b - kT_b) = 1, i = k$; $0, i \neq k$

Normalisation leads to $p(0) = 1$; If $p(t)$ satisfies the above condition and we write o/p "$y(t_i) = \mu \cdot t_i$" representing zero ISI (perfect reception in absence of noise).

$$* \quad P_\delta(f) = R_b \cdot \sum_{n=-\alpha}^{\alpha} P(f - nR_b) \quad ; \text{ Frequency domain represent}$$

where,

$R_b = 1/T_b$ and

$P_\delta(f)$ = Fourier Transform of an infinite periodic sequence of delta function of period T_b.

$$\Rightarrow P_\delta(f) = \int_{-\alpha}^{\alpha} \sum_{m=-\alpha}^{\alpha} \left[p(mT_b) \cdot \delta(t - mT_b) \right] \exp(-j2\pi f t)\, dt$$

- $m = i - k$
  $i = k \Rightarrow m = 0$
  $i \neq k \Rightarrow m \neq 0$

$$\Rightarrow P_f(t) = \int_{-\alpha}^{\alpha} p(0) \cdot \delta(t) \cdot \exp(-i 2\pi f t)\, dt = p(0) \quad ; m = 0$$

- For zero ISI,

$$\Rightarrow \quad R_b \cdot \sum_{n=-\alpha}^{\alpha} P(f - n R_b) = 1$$

$$\boxed{\sum_{n=-\alpha}^{\alpha} P(f - n R_b) = 1/R_b = T_b}$$ (only for PAM)

which is required condition for distortion less Transmission.

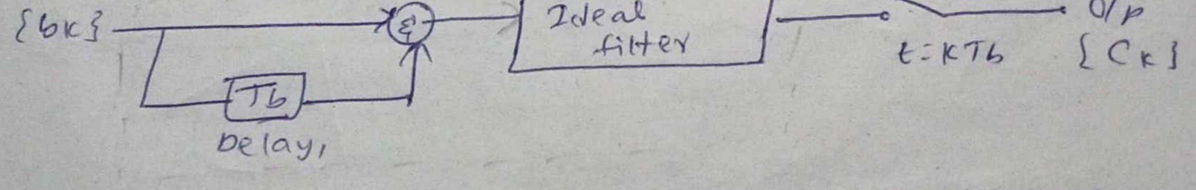
It provides a measure for reconstructing BL function to overcome the effects of ISI.

* CORRELATIVE CODING:-

Here, ISI is used in a controlled manner to get better performance of sy system. ISI is added to transmitted signal.

- We can achieve a bitrate of "2 B0" bits/sec. where 'B0' is channel BW.
- The schemes that follow the principle are called "Correlative coding schemes / partial response signalling schemes".
- There are 2 types of Correlative. Coding:
  1. Duo Binary correlative coding
  2. Modified DuoBinary Correlative coding

① Duo Binary Correlative Coding:-



- Consider a binary i/p sequence {bk} consisting of uncorrelated binary digits is having duration Tb secs.

'1' represented by +1V and
'0' represented by −1V.

→ These have been converted into a 3 level o/p −2V, 0, +2V
To produce this we use the diagram where, for every unit impulse applied to this duobinary conversion filter 2 unit impulses face T_b seconds appear at o/p represented by :

$$\boxed{C_k = b_k + b_{k-1}} \quad \text{—} \; (1)$$

- Input sequence $\{b_k\}$ of uncorrelated binary digits is transformed into a sequence of $\{C_k\}$ of correlated digits
- The correlation b/w adjacent transmitted levels may be viewed as introducing ISI, into the transmitted signal in an artificial manner. But this ISI is under designer control.

Ideal design element producing delay of 'T_b' as a TF "$\exp(-j2\pi f T_b)$" of a simple filter is $1 + \exp(-j2\pi f T_b)$ — (2)
↓
one delay

- Overall TF $\boxed{H(f) = H_c(f) \cdot [1 + \exp(-j2\pi f T_b)]}$ — (2)

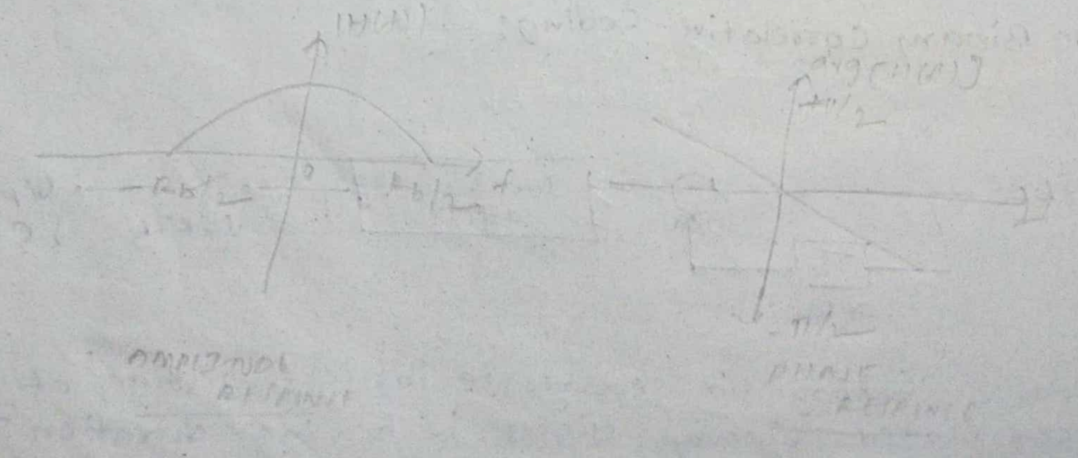
$$= H_c(f) \cdot [\exp(j\pi f T_b) + \exp(-j\pi f T_b)] \cdot \exp(-j\pi f T_b)$$

$$H(f) = H_c(f)\,[2\cos(\pi f T_b)] \cdot \exp(-j\pi f T_b)$$

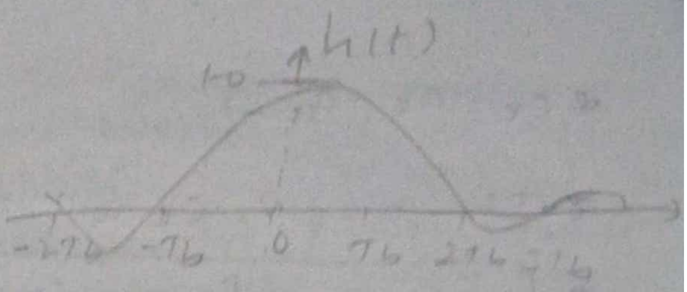
→ For an ideal Filter, $BW = R_b/2 = B_0$

$$H_c(f) = \begin{cases} 1; & |f| \le R_b/2 \\ 0; & \text{otherwise} \end{cases}$$

$$\Rightarrow H(f) = \begin{cases} 2\cos(\pi f T_b) \cdot \exp(-j\pi f T_b) & ; |f| \le R_b/2 \\ 0 & ; \text{o.w} \end{cases} \quad \text{—} \; (3)$$



$$\boxed{h(t) = \frac{T_b^2}{\pi t}\,\frac{\sin(\pi t/T_b)}{(T_b - t)}}$$

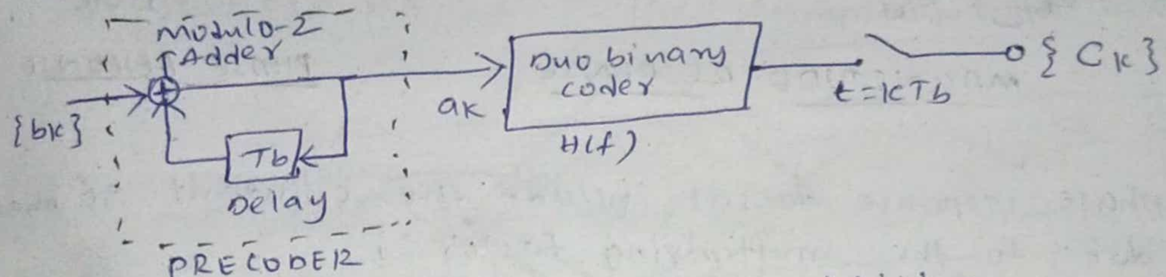


IMPULSE RESPONSE OF DUOBINARY CONVERSION FILTER

Previous symbol is stored and used to get present symbol and giving to f/b. In this discussion, we assume that 'c_k' is recieved without error and '$\hat{b}_{k-1}$' at time '$t=(k-1)T_b$' corresponds to a correct decision.

→ The drawback of this coder is that, once errors are made they tend to propogate along.

② Modified DuoBinary Correlative Coding :—

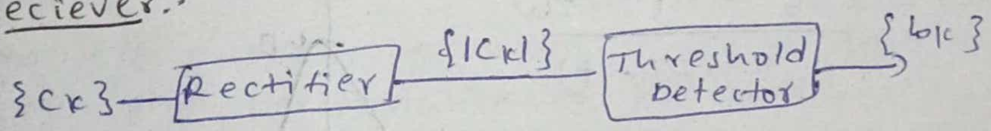


$a_k = b_k + a_{k-1}$ → modulo-2 addition

$c_k = a_k + a_{k-1}$

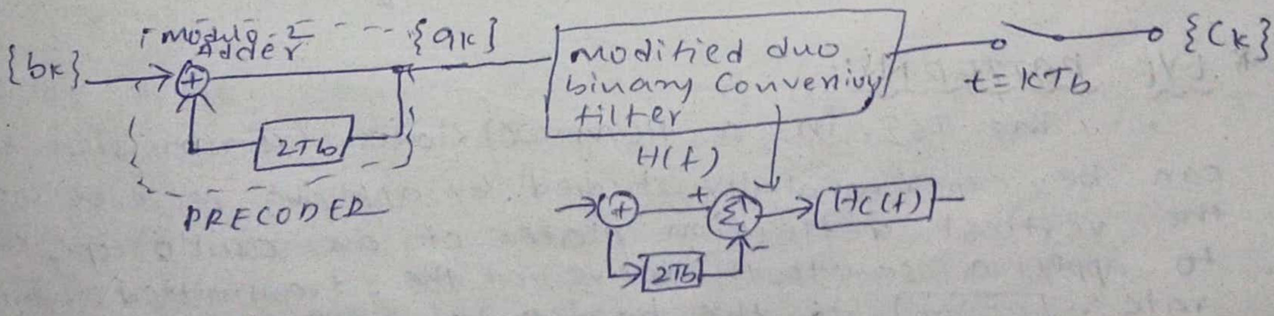
→ $c_k = \pm 2v$ if b_k = '0'
$\quad\;\; = 0$ if b_k = '1' } Decision Rule

→ **Reciever**:-



→ $\{b_k\}$ = '0' if $|c_k| > 1$
$\quad\quad\quad$ '1' if $|c_k| < 1$ //

→ Modified Duobinary Signalling :—
This involves a correlation span of 2 binary digits. This is achied by subtracting input binary digits faced 2 Tb seconds apart.



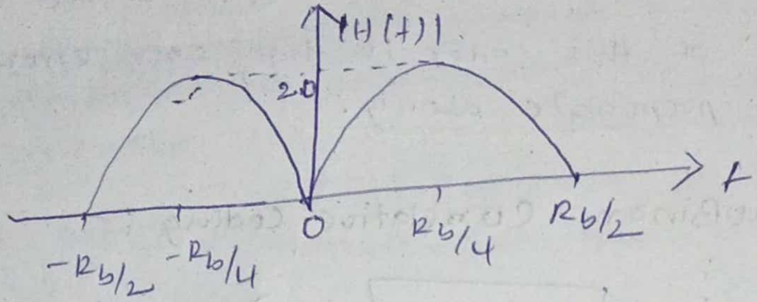
* $C_k = a_k - a_{(k-2)}$

* If $a_k = \pm 1V$, C_k will take three values: $+2V, 0, -2V$

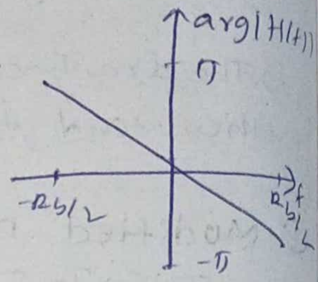
* $\boxed{H(f) = H_c(f)\left[1 - \exp(j 4\pi f T_b)\right]}$

After modification, we get (go through)

$$H(f) = \begin{cases} 2j\sin(2\pi f T_b) \cdot \exp(-j2\pi f T_b) & ; |f| \le R_b/2 \\ 0 & ; \text{ow} \end{cases}$$



MAGNITUDE RESPONSE



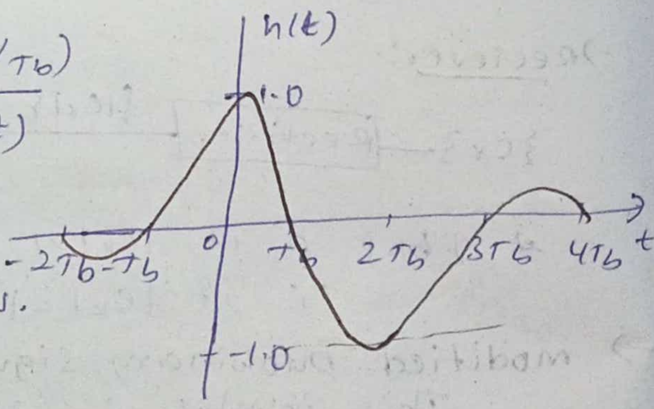
PHASE RESPONSE

-> The phase response doesn't include the constant 90° phase shift due to the multiplying factor 'j'.

-> O/p has no DC component, in practise many communication channels can't transmit a DC component.

-> After applying inverse Fourier Transform, we get

$$h(t) = \frac{2T_b^2 \sin(\pi t/T_b)}{\pi t(2T_b - t)}$$



-> we get 3 distinguishable levels at sampling intervals.

Coming to the detection, similar procedure is followed and the detection decision is, if $|C_k| < 1V \Rightarrow$ '0'

$|C_k| > 1V \Rightarrow$ '1'

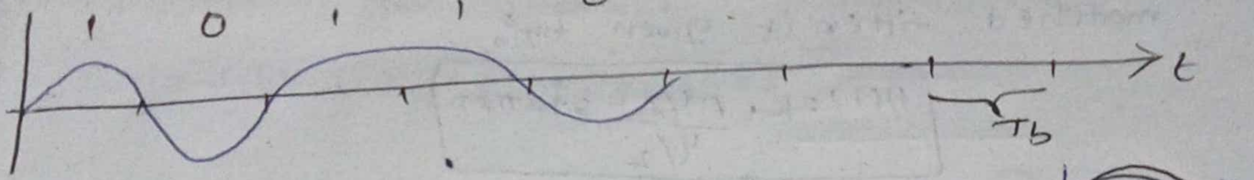
* EYE PATTERNS:-

The ISI in a PCM (or) data transmission system can be experimentally studied by applying recieved wave to the vertical deflection plates of an oscilloscope and to apply a sawtooth wave at the transmitted symbol rate $\boxed{R = 1/T}$ to the horizontal reflection plates.

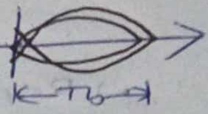
The waveforms in sucessive symbol intervals are there by translated into one interval on the oscillosco display for the case of a binary wave, for which $t = T_b$

The resultant display is called an eye pattern bcz of it's resembleuce to human eye for the binary waves.

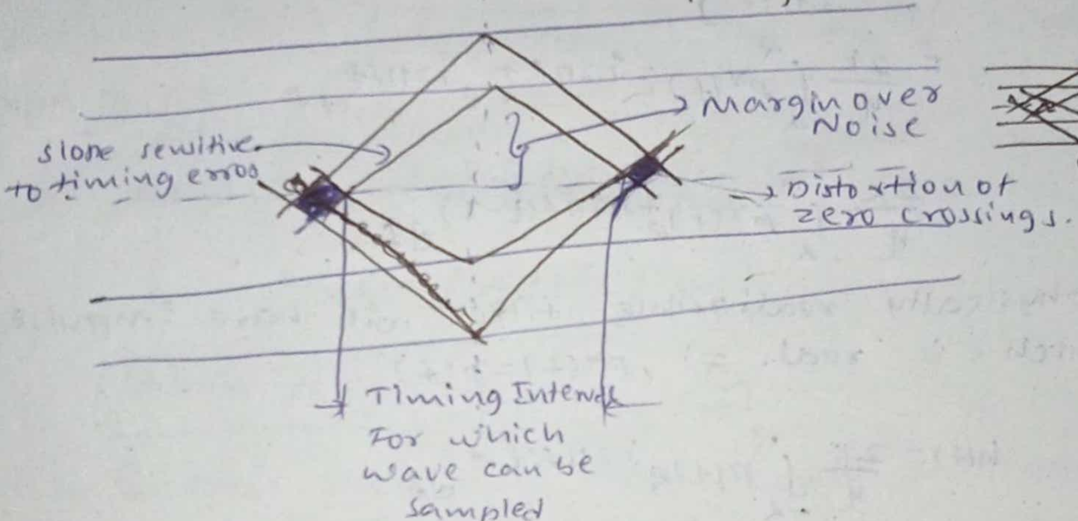
The interior region of the eye pattern is called the eye opening.

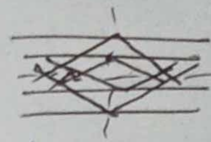


->If superimposed in one period T_b, we get:



which is a eye pattern. On expanding this eye pattern, we get:





-> An EYE pattern provides a great deal of information about the performance of the pertinent system.

1. The width of eye opening defines the time interval over which the recieved wave can be sampled without error from ISI. The preffered time for sampling is instant of time at which the eye is opened widest.

2. The sensitivity of the system to timing error is determined by the rate of closure of eye as the sampling time is varied.

3. The hieght of the eye opening at a specified sampling time defines the margin over noise.

4. When the effect of ISI is severe, traces from the upper portion of the eye pattern crosses traces from the lower portion. As a result, the eye is completely closed.

In such a situation, it is impossible to avoid errors due to the combined presence of ISI and noise in the system and a solution has to be found to correct from the

-> **MATCHED FILTER:** An optimum filter which yields a maximum ratio "$P_0^2(T)/\sigma_0^2$" is called a "Matched Filter" when input noise is white. The transfer function of matched filter is given by:

$$\boxed{H(f) = K \cdot \frac{P^*(f)}{\eta/2} e^{-j2\pi fT}}$$

-> Response of this filter to unit strength impulse applied at $t=0$, is

$$h(t) = F^{-1}\{H(f)\}$$

$$= \frac{2K}{\eta}\int_{-\infty}^{\infty} P^*(f) e^{-j2\pi fT} \cdot e^{j2\pi ft} df$$

$$= \frac{2K}{\eta}\int_{-\infty}^{\infty} P^*(f) e^{+j2\pi f(t-T)} df$$

-> physically realizable filter will have impulse response, which is real. => $P^*(f) = P(f)$.

$$h(t) = \frac{2K}{\eta}\int_{-\infty}^{\infty} P(f) e^{j2\pi f(t-T)} df$$

$$= \frac{2K}{\eta} P(T-t)$$

$\because [P(t) = s_1(t) - s_2(t)]$

$$\boxed{h(t) = \frac{2K}{\eta}[s_1(T-t) - s_2(T-t)]}$$

The impulsive response of a matched filter consists of $P(t)$ rotated about $t=0$ and then delayed long enough to make filter realizable.

-> **probability error of matched filter:-**

probability error $= \left[\frac{P_0^2(T)}{\sigma_0^2}\right]_{max}$

$$= \frac{2}{\eta}\int_{-\infty}^{\infty} |P(f)|^2 df$$

-> From parseval's theorem, => $\int_{-\infty}^{\infty} |P(f)|^2 df = \int_0^T P^2(t) dt$

$$\Rightarrow \left[\frac{P_0^2(T)}{\sigma_0^2}\right]_{max} = \frac{2}{\eta}\int_0^T [s_1(t) - s_2(t)]^2 \, dt$$

$$= \frac{2}{\eta}\left[\int_0^T [s_1^2(t) + s_2^2(t) - 2s_1(t).s_2(t)]dt\right]$$

$$= \frac{2}{\eta}[E_{s1} + E_{s2} - 2E_{s12}]$$

Energies of s_1, s_2 — Energy due to correlation b/w $s_1(t)$ & $s_2(t)$.

→ If $s_2(t) = -s_1(t) \Rightarrow E_{s1} = E_{s2} = -E_{s12} = E_s$

$$\Rightarrow \left[\frac{P_0^2(T)}{\sigma_0^2}\right]_{max} = \frac{2}{\eta}[4E_s] = \frac{8E_s}{\eta} \quad \text{— (1)}$$

→ using $P_0(T) = s_{01}(T) - s_{02}(T)$, we get:

$$P_e = \frac{1}{2}\operatorname{erfc}\left[\frac{P_0(T)}{2\sqrt{2}\sigma_0}\right] = \frac{1}{2}\operatorname{erfc}\left[\frac{P_0^2(T)}{8\sigma_0^2}\right]^{1/2} \quad \text{— (2)}$$

From (1) & (2)

$$(P_e)_{min} = \frac{1}{2}\operatorname{erfc}\left\{\frac{1}{8}\left[\frac{P_0^2(T)}{\sigma_0^2}\right]_{max}\right\}^{1/2}$$

$$= \frac{1}{2}\operatorname{erfc}\left[\frac{1}{8}\cdot\frac{8E_s}{\eta}\right]^{1/2}$$

$$\boxed{(P_e)_{min} = \frac{1}{2}\operatorname{erfc}\left[\frac{E_s}{\eta}\right]^{1/2}} \quad \text{— (3)}$$

It indicates error probability depends only on signal energy and not on the signal waveshape.

→ If, $s_1(t) = V \quad ; 0 \le t \le T$

$s_2(t) = -V \quad ; 0 \le t \le T$

$$\Rightarrow h(t) = \frac{2K}{\eta}[s_1(T-t) - s_2(T-t)] \text{ → pulse}$$

$$= \frac{2K}{\eta}(2V)\cdot[u(t) - u(t-T)]$$

$$h(t) = \frac{4KV}{\eta}[u(t) - u(t-T)]$$

↓ no effect

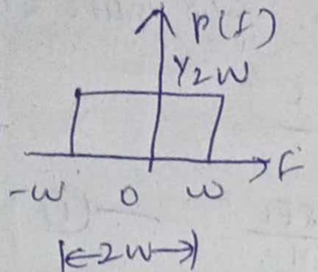
→ we can select K in above eqn such that $\frac{4KV}{\eta} = 1$.

$$\Rightarrow \boxed{H(s) = \frac{1}{s} - \frac{e^{-sT}}{s}}$$

$\frac{1}{s}$: integration from $t=0$; $\frac{e^{-sT}}{s}$: integration from $t=T$.

=) The overall response of matched filter is an integral from $t=0$ to $t=T$ and zero thereafter. so that so as the determination of one bit is concerned we ignore the response after $t=T$.

* Ideal Nyquist channel:-



The simplest way of satisfying

$$\sum_{n=-\alpha}^{\alpha} P(f - nf_b) = T_b \quad (or)$$

$$\sum_{n=-\alpha}^{\alpha} P(f - nR_b) = T_b \quad -①$$

* GR of P(f) *

- For $n=0$, LHS corresponds to $P(f)$ and it represents a frequency function with the narrowest band which satisfies -①

- The range of frequencies for $P(f)$ will extend will extend from $-W$ to W ($-B_0$ to B_0) where W/B_0 corresponds to half the bit rate.

- Hence, $W = \frac{f_b}{2} / \frac{R_b}{2}$.

- This equation is to specify the frequency function $P(f)$ to be in the form of a rectangular function, is shown as:

$$P(f) = \begin{cases} \frac{1}{2W} ; & -W<f<W \\ 0 ; & |f| > W \end{cases}$$

$$P(f) = \frac{1}{2W} \,\mathrm{rect}\left(\frac{f}{2W}\right) \longrightarrow ①$$

- The overall system BW 'W' is defined by: $W = \frac{R_b}{2} = \frac{1}{2T_b}$ ②

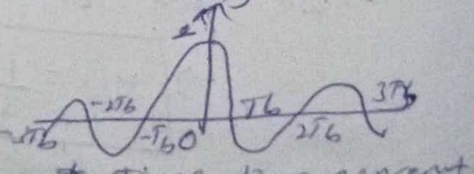
- The signal that produces zero ISI can be obtained by taking the IFT of $P(f)$.

This means that we have,

$$p(t) = F^{-1}[P(f)] \Rightarrow p(t) = F^{-1}\left[\frac{1}{2W}\mathrm{rect}\left(\frac{f}{2W}\right)\right]$$

$$\Rightarrow p(t) = \mathrm{sinc}(2Wt)$$

$$= \frac{\sin(2Wt)}{2Wt} \quad -③$$



The frequency domain representation & Time domain representation is called as "Ideal Nyquist channel".

* **Advantages of sinc pulse:- (INC)**

1. BW requirement is reduced.
2. ISI is reduced to zero.

* **Disadvantages of INC:**

- There are 2 Disadvantages:-

1. The $p(f)$ is plot from $-W$ to W & zero eleswhere This is physically unrealizable because of abrupt transitions at the edges $\pm W$.

2. The function $p(t)$ decreases as $1/t$ for large 't', resulting in slow rate of decay.

Q1. A delta modulator system is designed to operate at five times Nyquist rate for signal having a bandwidth equal to 3 KHz bandwidth. Calculate maximum amplitude of a 2KHz input sinusoid for which the delta modulator does not have slope overload. Give quantizing step size 250mv. Derive formula that you use.

$$\ast \; A_m < \frac{\delta}{2\pi f_m T_s}$$

- To avoide slope overload,

$$\boxed{\Delta f_s \geq \left| \frac{d\,m(t)}{dt} \right|_{max}}$$

- $f_s = 5 \times 2 f_m = 5 \times 2 \times 3k = 30KHz$

$$\therefore \Delta f_s \geq 2\pi A_m f_m. \qquad \because A_m \sin(2\pi f_m t)$$

$$\Rightarrow \boxed{A_m \geq \delta / 2\pi f_m T_s}$$

$$250 \times 10^{-3} \times 30 \times 10^{3} \geq (2\pi) A_m \times$$

$$A_m \geq \frac{250 \times 10^{-3} \times f_s}{2(\pi) \cdot f_m} \geq \frac{\cancel{250}^{125} \times 10^{-3} \times \cancel{30}^{15} \times \cancel{10^3}}{\cancel{2} \times \pi \times \cancel{2} \times \cancel{10^3}} \geq 597.13mV$$

$$\Rightarrow \boxed{A_m \leq 0.597 V}$$