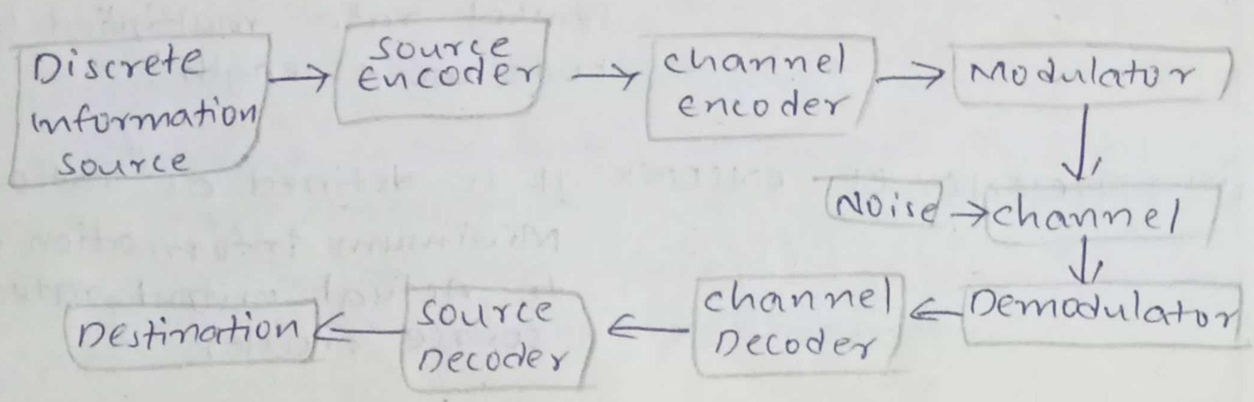
1. DIGITAL CODING OF ANALOG WAVEFORMS

. **Block Diagram of Digital Communication system:**



① INFORMATION SOURCE:– Information source may be classified based on nature of output i.e., Analog,(Digital)Discrete information source.

→ Discrete information source can be characterized by
i, Source Alphabet   ii, Symbol rate   iii, Source Alphabet probabilities
iv, Probabilistic dependence of symbols in a sequence.

→ Information rate = Symbol rate x Source Entropy
(bits/sec) (symbols/sec) (bits/symbol)

$$\boxed{R = S \times H}$$

② SOURCE ENCODER & DECODER:· source encoder assaigns code words to symbols. For each distinict symbol there is an unique code word. Code word can be of 4, 8, 16/32 bits length.

→ If it is 8-bit code word, it can generate "2^8" distinct code words.

→ Source encoders must have the following parameters.

i, BLOCK SIZE: It describes max no. of distinct code words, which can be represented by source encoder.

ii, CODE WORD LENGTH: It is No. of bits used to represent a code word.

(iii) AVERAGE DATA RATE: This is o/p bits/sec from source encoder. The data rate is equal to "product of symbol rate multiplied by code word length".

(iv) EFFICIENCY OF ENCODER: It is defined as "ratio of Minimum information rate to Actual output rate of source encoder.

(3) CHANNEL ENCODER & DECODER:- The communication channel adds noise to signals. Hence, errors are introduced in binary sequence. To overcome that channel encoding is required.

→ A channel encoder must have:

(i) CODING RATE: It depends on redundant bits added by channel encoder.

(ii) Coding Method Used

(iii) CODING EFFICIENCY: It is defined as "ratio of data rate at i/p to data rate at o/p of encoder".

(iv) Error Control Capability

(v) Feasibility of Encoder & Decoder

(4) DIGITAL MODULATORS & DEMODULATORS:

→ A DM must have following parameters:

(i) Bandwidth is needed to transmit signal.

(ii) Probability of symbol/Bit error.

(iii) Synchronous/Asynchronous method of detection

(iv) Complexity of implementation.

→ DMs are classified as:

* PCM   * Delta Modulator (DM)   * FSK   * PSK ...

⑤ COMMUNICATION CHANNEL: Communcitation channel can be characterized by following:

i, SIGNAL ATTENUATION: It occurs due to internal resistance of channel and fading of signal.

ii, Amplitude & Phase distortion: which is due to non-linear characteristics of communication channel

iii, Additive Noise interference: It is produced due to internal solid state devices & resistors.

iv, Multi Part Distortion: which occurs mostly in wireless communication.

→ Communication channel can be classified as

* GUIDED CHANNELS:
  - i, Telephone channels
  - ii, coaxial cabels
  - iii, optical fibres

* WIRELESS CHANNELS:
  - i, wireless Broadcast channels
  - ii, satelite channels
  - iii, Mobile channels

* ADVANTAGES OF DC:

- simple and cheaper than AC
- speech, video and other data may be merged and transmitted over a common channel used in multiplexing
- Using Data encryption, only permitted reciever may be allowed to detect transmitted data. This property is of importance in militay communication
- since, transmission is digital and channel encoding is used, noise doesn't accumulate from repeater to repeater in long distance communication.
- since, transmitted signal is digital in nature, large amount of noise

ICs
- Linear ICs
  - op-amps
  - Timers
  - 8038
- Digital ICs

interference may be tolerated.

- Since, channel encoding is used, errors may be detected and corrected in recievers.
- DC is adaptive to other advanced branches of Digital signal processing, Image processing, Image compression etc.

*DISADVANTAGES / LIMITATIONS OF DC:

- Due to A-D conversion, data rate becomes high. Therefore, more transmission Band width is requir for digital communication.
- DC needs synchronization incase of synchronous modulation.

* PULSE CODE MODULATION:

* Analog signal → LPF → sample → Quantize → Encoder → PCM signal

(TRANSMITTER BLOCK)

* Distorted PCM signal → Regenerative Repeater → - - → Regenerative repeater → Regenerated pcm signal

(No modulation) For long distances

(TRANSMISSIION BLOCK) (REGENATIVE REPEATER)

* i/p → Regenerative circuit → Decoder → Reconstruction filter → Destination

(RECIVER)

* FEATURES OF PCM:

i) PCM is a type of pulse modulation like PAM, PWM/PPM. But it is digital pulse modulation schem

ii) PCM o/p is in coded digital form. It is in the form of digital pulses of constant amplitude, width & position.

iii) The information is transmitted in the form of code

iv) The PCM systems consists of : PCM encoder & reciever.

v) The essential operations in PCM are sampling, quantising and encoding. All these 3 operations are performed by analog-digital convertor.

vi, In other modulation schemes, By varying analog signal; some parameter of modulation schemes will vary, But in PCM it will not happen.

* Nyquist rate must be maintained to avoid aliasing in PCM systems.

-> uniform quantization < midtread, midrise

$\boxed{\text{(Derivation)}}$ *

-> Non-uniform quantizers

-> Error signal (Q) = i/p signal - quantized signal

-> stepsize $[\Delta] = \dfrac{2m_{max}}{L \to \text{No. of levels}}$; $L = 2^R \Rightarrow R = \log_2 L$

-> quantization noise, (uniformly distributed) $f_Q(q) = \begin{cases} \dfrac{1}{\Delta} & ; -\dfrac{\Delta}{2} < q \le \dfrac{\Delta}{2} \\ 0 & ; \text{else where.} \end{cases}$

-> variance, $\sigma_Q^2 = E[Q^2]$

$$= \int_{-\Delta/2}^{\Delta/2} q^2 \cdot f_Q(q)\, dq$$

$$= \int_{-\Delta/2}^{\Delta/2} q^2 \cdot \frac{1}{\Delta}\, dq = \frac{1}{\Delta} \cdot \left[\frac{q^3}{3}\right]_{-\Delta/2}^{\Delta/2}$$

$$= \frac{1}{\Delta}\left[\frac{\Delta^3}{24} + \frac{\Delta^3}{24}\right] = \frac{\Delta^2}{12} //$$

$$\sigma_Q^2 = \left[\frac{2m_{max}}{2^R}\right]^2 \Big/ 12 = \frac{4m_{max}^2}{12 \cdot 2^{2R}} = \frac{m_{max}^2}{3 \cdot 2^{2R}}$$

* 'P' is average power of msg signal "m(t)",

* $(SNR)_{o/p} = \dfrac{P}{\sigma_Q^2} = \dfrac{P}{\frac{m_{max}^2}{3 \cdot 2^{2R}}} = \dfrac{3 \cdot P\, 2^{2R}}{m_{max}^2} = \left[\dfrac{3P}{m_{max}^2}\right] 2^{2R}$

-> From this eqn., the o/p SNR of uniform quantizer increases exponentially with increasing number of bits/sample (R)

-> For a sinusoidal modulating signal, $P = \dfrac{A_m^2}{2}$

$$\sigma_Q^2 = \frac{1}{3} A_m^2 \cdot 2^{-2R}$$

$$(SNR)_{o/p} = \frac{3}{2} \cdot 2^{2R} //$$

$$(SNR)_o = 3 \cdot 2^{(2R-1)}$$

$\boxed{SNR = 4.8 + 6R}$.

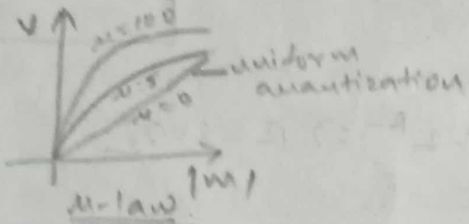
-> In "db", $(SNR)_o = 10\log 3/2 + 10 \log 2^{2R} = 1.76 + 6R$

i/p signal → Compressor → uniform quantizer → Compressed o/p

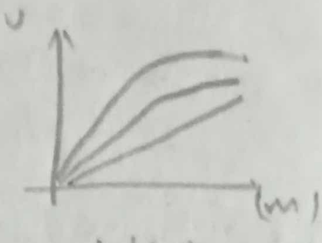
→ standard law of compression ① "μ-law" ② "A-law".

$$|v| = \frac{\ln(1+\mu|m|)}{\ln(1+\mu)} \quad ; \quad m, v = \text{i/p, o/p voltages}, \ \mu = \text{constant}.$$

→ It is assumed that m & v are scaled, so that both lie inside the interval [−1, 1]



μ-law



A-law

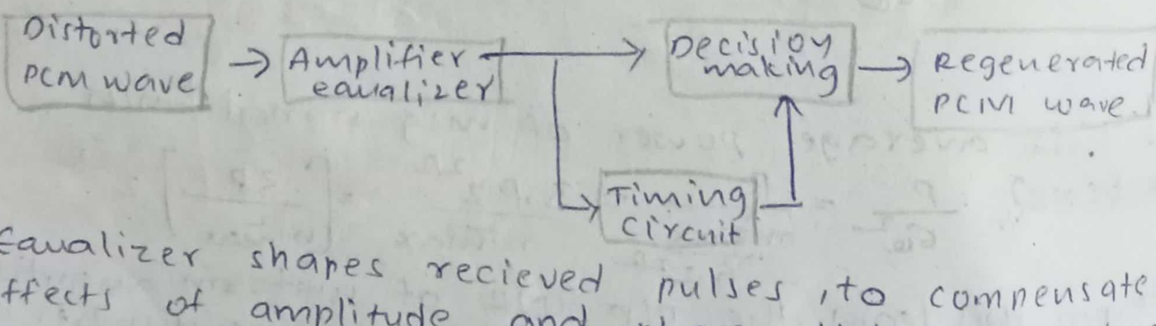
→ For a given value of μ, reciprocal slope defined by quantum steps $\frac{d|m|}{d|v|} = \frac{\ln(1+\mu)}{\mu}(1+\mu|m|)$ [μ-law]

→
$$|v| = \begin{cases} \frac{A|m|}{1+\ln A}, & 0 \le |m| \le \frac{1}{A} \\ \frac{1+\ln(A|m|)}{1+\ln A}, & \frac{1}{A} \le |m| \le 1 \end{cases} \quad ; \text{"A = constant"}$$

* PCM uses "companding" [Compression + Expansion]

→ ISI is limitation of DC. To overcome, we use equalizers.

* Analog-code pulse conversion: (REGENERATIVE REPEATER



→ Equalizer shapes recieved pulses, to compensate the effects of amplitude and phase distortions produced by non-ideal transmission characteristics of channel.

→ Timing circuit provides periodic train of pulses for sampling purpose.

→ Decision making circuit will have predefined threshold to make out a zero/one.

→ Regenerated signal ideally except for delay is same as signal which is originally transmitted. However it deviates in practice from original signal for 2 main reasons:

① "Bit errors" will appear due to channel noise and interference.

② "Jitter" will occur, where by spacing b/w recieved pulses deviates from their assaigned value

If transmitted SNR ratio is high, then regenerated PCM data is same as transmitted PCM data except for a small BER. In other words under these operating conditions, performance degradation in PCM system is essentially confined to transmission Noise i.e., Quantization Noise.

PROBABILTY OF ERROR:

→ First signal be: (symbol 1)

$$* \quad s_1(t) = \sqrt{\frac{E_{max}}{T_b}} \quad \text{—①} \quad 0 \le t \le T_b$$

peak signal energy

duration

precipitation absorption of EM wave by H_2O molecules in propagation media.

→ when symbol '0' is sent:

$$* \quad s_2(t) = 0 \quad ; \quad 0 \le t \le T_b$$

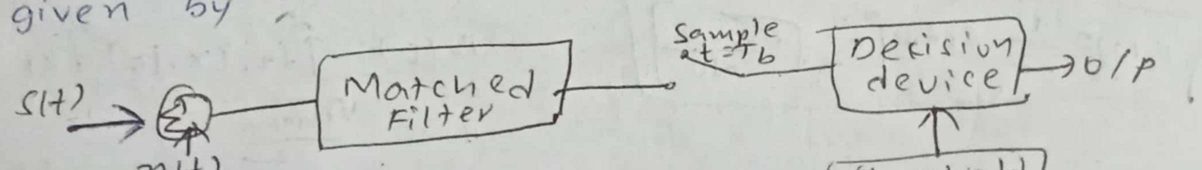
Representation of signal
→ RZ
NRZ
Manchester

• let channel be AWGN with zero mean and PSD by '$\frac{N_0}{2}$'

→ Then, recieved signal is represented by,

$$x(t) = s(t) + N(t) \quad ; \quad 0 \le t \le T_b \text{ —②}$$

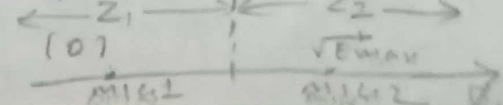
The optimum reciever uses a matched filter. The reciever structure for a binary encoded PCM is given by;



$$* \quad s_1(t) = \sqrt{E_{max}} \cdot \phi_1(t) \quad \text{—③}$$

$$\phi_1(t) = \text{Basis function} = \sqrt{\frac{1}{T_b}} \quad \text{—④}$$

An ON/OFF PCM is characterized by having a signal space i.e., one dimensional & with 2 msg points as shown in figure.



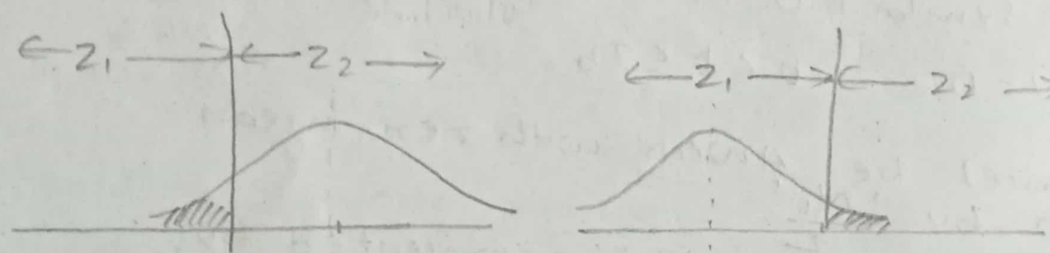
→ Two message points are represented as;

- $s_{11} = \int_0^{T_b} s_1(t).\phi_1(t)\,dt = \sqrt{E_{max}}$

- $s_{21} = \int_0^{T_b} s_2(t)\phi_1(t)\,dt = 0$ — (5)

- We assume that binary symbols '0' & '1' occur with equal probabilities and hence decision boundary is kept at Half-way point i.e., $\frac{\sqrt{E_{max}}}{2}$.

The 2 decision regions are denoted by z_1 & z_2. The decision rule is to guess symbol-1 / signal $s_1(t)$ was sent if recieved signal falls in region z_1. Similarly to guess symbol-0 / signal $s_2(t)$ was sent if recieved signal point was in 'z_2'.



→ $f_{x_1}(x_1|1)$ (2) $f_{x_1}(x_1|0)$ (1)

→ $P_e(1) = \int_{-\alpha}^{\frac{\sqrt{E_{min}}}{2}} f_x(x_1|1)\,dx_1$

$\boxed{P_e(1) = \frac{1}{2}\,\mathrm{erfc}\left[\frac{1}{2}\sqrt{\frac{E_{max}}{N_0}}\right]}$

→ $f_{x_1}(x_1|0) = \frac{1}{\sqrt{\pi N_0}} \exp\left(-\frac{x_1^2}{N_0}\right)$ — (6)

- Probability error,

$P_e(0) = \int_{\frac{\sqrt{E_{max}}}{2}}^{\alpha} f_{x_1}(x_1|0)\,dx_1$ — (7)

$= \int_{\frac{\sqrt{E_{max}}}{2}}^{\alpha} \frac{1}{\sqrt{\pi N_0}} \exp\left(\frac{-x_1^2}{N_0}\right) dx_1$

Let, $z = \frac{x_1}{\sqrt{N_0}} \Rightarrow dx_1 = \sqrt{N_0}\,dz$

$\boxed{P_e(0) = \int_{\frac{\sqrt{E_{max}}}{2\sqrt{N_0}}}^{\alpha} \frac{1}{\sqrt{\pi}} e^{-z^2}\,dz}$ — (8)

$$= \frac{1}{\sqrt{\pi}} \cdot \left[\frac{e^{-z^2}}{-2z} \right]_{\frac{\sqrt{E_{max}}}{2\sqrt{N_0}}}^{\alpha}$$

From (8),

$$P_e(0) = \frac{\text{erfc}\left(\frac{1}{2}\sqrt{\frac{E_{max}}{N_0}}\right)}{2}$$

No need

$$= \frac{1}{\sqrt{\pi}} \cdot \frac{e^{-\frac{E_{max}}{4N_0}}}{\cancel{2}\frac{\sqrt{E_{max}}}{\cancel{2}\sqrt{N_0}}}$$

$$\boxed{P_e(0) = \frac{1}{2}\text{erfc}\left(\frac{1}{2}\sqrt{\frac{E_{max}}{N_0}}\right)}$$

$$P_e(0) = \frac{\sqrt{N_0}}{\sqrt{\pi E_{max}}} \cdot e^{-\frac{E_{max}}{4N_0}}$$

$$\because \left[\text{erfc}(u) = \frac{2}{\sqrt{\pi}} \int_u^{\alpha} e^{-x^2/2}\, dx\right]$$

→ Similarly, to calculated making an error of second kind, assume symbol-1 is sent. Now, $P_e(1)$, where portion of likelihood function limits are $\frac{\sqrt{E_{max}}}{2}$.

→ The fact that "$P_e(1) = P_e(0)$" is confirmation of symmetric nature of channel. $P_e(1)$, $P_e(0)$ are conditional probabilities. We assume that A priory probability of sending '0' is "P_0". and apriory probability of sending '1' is "P_1". Hence, average probability of error is given by;

$$\boxed{P_e = P_0 P_e(0) + P_1 P_e(1)}$$

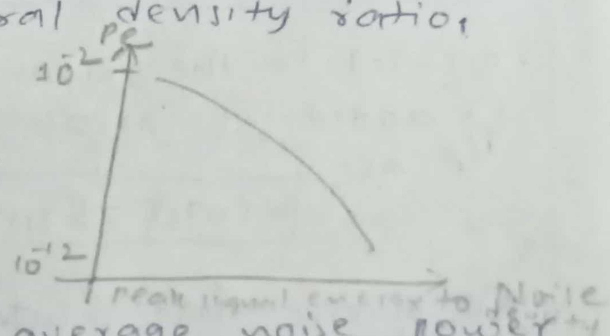
As $P_e(0) = P_e(1)$, $P_0 + P_1 = 1$ ∴ Hence, the average probability becomes Half complementary error function.

i.e, $P_e = \frac{1}{2}\text{erfc}\left(\frac{1}{2}\sqrt{\frac{E_{max}}{N_0}}\right)$

Here, $\frac{E_{max}}{N_0}$ = ratio of peak signal energy to Noise spectral density ratio.

→ $E_{max} = P_{max} \cdot T_b$

$$\Rightarrow \boxed{\frac{E_{max}}{N_0} = \frac{P_{max}}{[N_0/T_b]}}$$

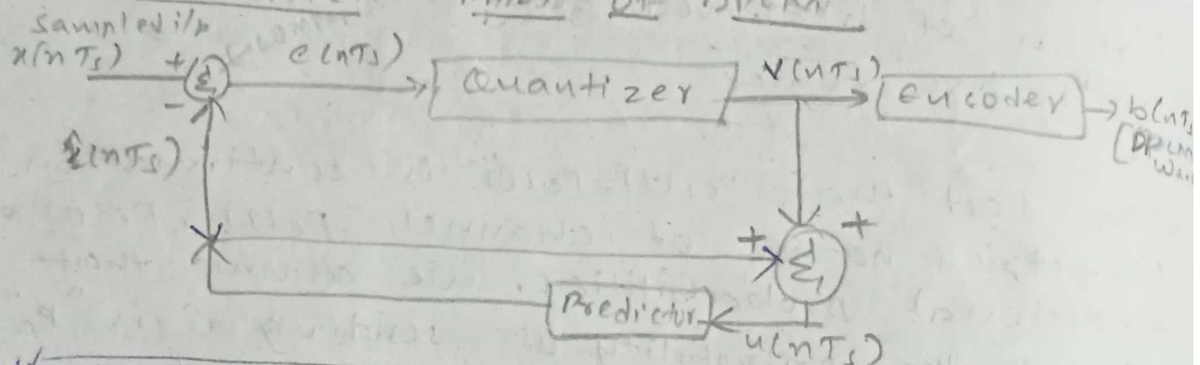


• "$\frac{N_0}{T_b}$" may be taken as average noise power contained in transmission BW $= 1/T_b$.

* DIFFERENTIAL PULSE CODE MODULATION:

In PCM, digitilization of voice/ video signal the signal is sampled at a rate slightly higher than Nyquist rate. The resulting signal doesn't change rapidly from one sample to next with result that the difference b/w adjacent as a variance i.e., smaller than the variance of signal itself. If these samples are encoded, it contains redundant information. By removing the redundancy before encoding we obtain a more efficient coded signal.

TRANSMITTER PART OF DPCM



* $\boxed{e(nT_s) = x(nT_s) - \hat{x}(nT_s)}$

i.e., $e(nT_s)$ is difference b/w unquantized i/p sample and prediction of it. The predicted value is produced by using a predictor whose i/p consists of a quantized version of i/p signal $(\hat{x}(nT_s))$

→ $e(nT_s)$ is called prediction error.

→ By encoding quantizer o/p we obtain "DPCM". The quantizer o/p is given by $[b(nT_s)]$.

- $v(nT_s) = Q[e(nT_s)]$

$$\boxed{v(nT_s) = e(nT_s) + q(nT_s)}$$

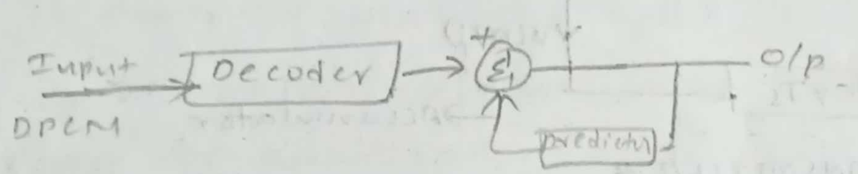
- $q(nT_s)$ is the quantization error. The quantizer o/p is added to predicted value to produce predictor i/p as:

$$\boxed{u(nT_s) = \hat{x}(nT_s) + v(nT_s)}$$

- We observe that $\hat{x}(nT_s) + v(nT_s)$ is equal to the signal $x(nT_s)$. Therefore, irrespective of properties of predictor, the quantized signal u(nTs) at predictor i/p differs from original signal by quantization error. Accordingly, your prediction is good.

→ The variance of prediction error will be smaller than the variance x(nTs), this leads to quantization error which is lesser than standard PCM. The reciever for constructing DPCM is given by;

RECIVER PART OF DPCM:



The reciever for constructing quantizer version of i/p is decoder followed by a predictor. From this analysis, we can conclude that the predictor is a transmitter-reciever operate on same sequence of samples u(nTs). With this purpose in mind the f/b path is added to quantizer in transmitter.

SNR: The o/p signal to quantization ratio of a signal coder is defined by:

$$\boxed{(SNR)_o = \frac{\sigma_x^2}{\sigma_Q^2}}$$

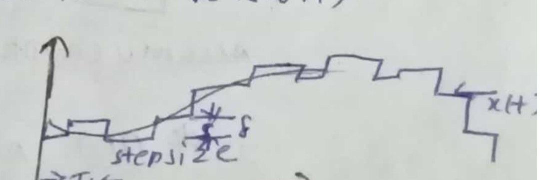
σ_x^2 = variance of i/p x(nTs)
σ_Q^2 = variance of quantization error Q(nTs)

$$(SNR)_o = \frac{\sigma_x^2}{\sigma_E^2} \times \frac{\sigma_E^2}{\sigma_Q^2}$$; σ_E^2 = variance of prediction error e(nTs)

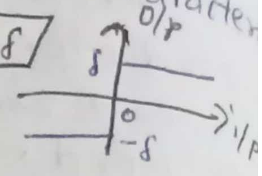
$$\boxed{(SNR)_o = (G)_p \times (SNR)_p}$$; G_p = Predictor Gain
SNR_p = Predictor error to quantization error ratio

→ To design the predictor so as to minimize σ_E^2

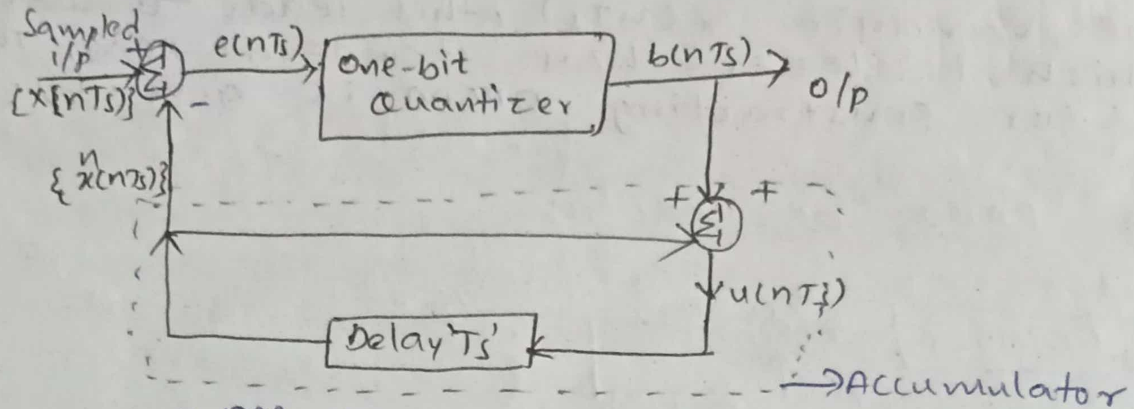
* DELTA MODULATION:-
- It is one-bit version of DPCM (Two level version)
- 'δ' can be +ve/-ve.
- 'δ' denotes absolute value of the 2 representation levels of the one bit quantizer, used in delta modulator.



. These 2 levels are indicated in the Transfer character as shown in figure and the step size, $\boxed{\Delta = 2\delta}$



• Block diagram of delta modulation is shown below:



* DM TRANSMITTER *

$* \; e(nT_s) = x(nT_s) - \hat{x}(nT_s) \quad — ①$

. $e(nT_s) = x(nT_s) - u(nT_s - T_s) \quad — ②$

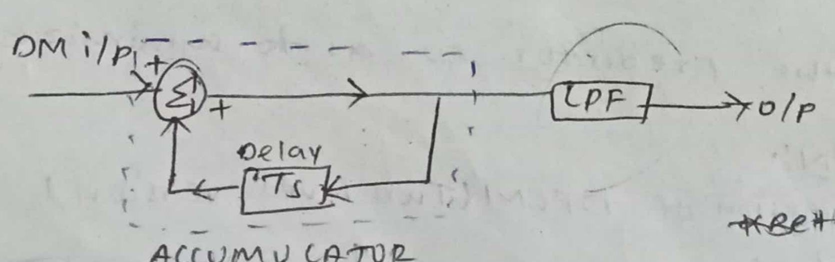
: $u(nT_s) = b(nT_s) + u(nT_s - T_s) \quad — ③$

· $b(nT_s) = \delta \, sgn\{e(nT_s)\} \quad — ④$

· T_s = Sampling period

: $e(nT_s)$ = Prediction error

- The binary quantity $b(nT_s)$ is the algebraic sign of the error $e(nT_s)$ except for scaling factor 'δ'. Infact $b(nT_s)$ is the one bit word transmitted by the DM system.
- In comparing DPCM and DM networks except for an o/p LPF, delta modulation is a special case of DPCM.
- DM offers 2 unique features: i) A one-bit code word for the o/p which eliminates the need for word framing ii) Simplicity of design for both the transmitter & Reciever



*Better than DPCM *

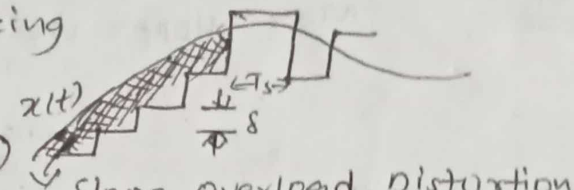
* DM RECIEVER *

1. DM suffers from 2 types of Noise:
   i) Slope overload distortion
   ii) Granular Noise

i) <u>SLOPE OVERLOAD DISTORTION</u>:-

- we denote $q(nT_s)$ as quantizing error. This $q(nT_s)$ is

$\rightarrow q(nT_s) = u(nT_s) - x(nT_s)$ — ①



Slope Overload Distortion

- To eliminate $u(nT_s - T_s)$ from ① in DM, we write

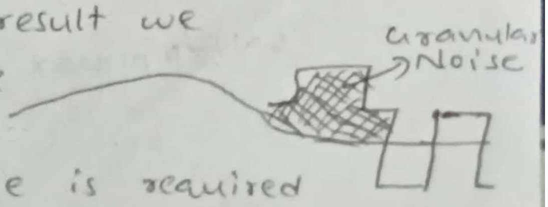
$\rightarrow e(nT_s) = x(nT_s) - x(nT_s - T_s) - q(nT_s - T_s)$ — ②

- Except for quantization error, $q(nT_s - T_s)$, the quantizer i/p is a first backward difference of i/p signals, which may be viewed as digital approximation to the derivative of i/p signal (or) equivalently as a inverse of a digital integration process.

If we consider the maximum slope of the original i/p waveform of $x(t)$, it is clear that inorder for the sequence of samples $\{u(nT_s)\}$ to increase as fast as the i/p sequence of samples $\{x(nT_s)\}$ is a region of maximum slope of, we require that the condition $\boxed{\frac{\delta}{T_s} \geq \max\left|\frac{d\,x(t)}{dt}\right|}$, otherwise slope overload will occur.

ii) <u>GRANULAR NOISE</u>:-

Granular Noise occurs when stepsize delta is too large relative to the slope characteristics of i/p waveform $x(t)$, thereby causing the staircase approximation $u(t)$ to hunt around a relatively flat segment of i/p waveform. As a result we need to have a large step size to accomodate a white dynamic range whereas a small step size is required for accurate representation of relatively low level signals.



It is therefore, clear that the choice of optimum step size that minimizes mean square value of quantizer in linear DM will be a result of a compromise b/w slope overload distortion and granular Noise.

Q: Find maximum slope of $x(t) = a_0 \cos(2\pi f_0 t)$.

sol:

$$\text{Max slope} = \max\left|\frac{dx(t)}{dt}\right|$$

$$\max\left|\frac{d}{dt}(x(t))\right| = a_0 \cdot 2\pi f_0 \cdot \sin(\ldots) = a_0 \cdot 2\pi f_0$$

$$= \max(-a_0 \sin(2\pi f_0 t) \cdot 2\pi f_0)$$

$$= \max|-2\pi f_0 a_0 \cdot \sin(2\pi f_0 t)|$$

$$* \quad \boxed{\text{max slope} = 2\pi f_0 a_0}$$

$$a_0 \, 2\pi f_0 \le \frac{\delta}{T_s}$$

$$\therefore \frac{\delta}{T_s} \ge \max\left|\frac{dx(t)}{dt}\right| \Rightarrow \frac{\delta}{T_s} \ge 2\pi f_0 a_0 \quad -③$$

$$\Rightarrow a_0 \le \frac{\delta}{2\pi f_0}$$

$$\Rightarrow a_0 \le \frac{\delta}{2\pi f_0 T_s} \quad -④$$

$$\Rightarrow a_0 \ge 2\pi f_0 T_s/\delta$$

- Maximum permissible value of the o/p signal power

$$P_{max} = \frac{a_0^2}{2} \quad -⑤ \quad \Rightarrow P_{max} = \frac{4\pi^2 f_0^2 T_s^2}{2\delta^2} = \frac{2\pi^2 f_0^2 T_s^2}{\delta^2}.$$

$$* \quad \sigma_Q^2 = \frac{\Delta^2}{12} \quad -⑥ \quad \Rightarrow \sigma_Q^2 = \frac{4\delta^2}{12} = \frac{\delta^2}{3}$$

- Average power of quantization error is uniformly distributed over a frequency interval extending from $-\frac{1}{T_s}$ to $+\frac{1}{T_s}$.

$\therefore$ Average o/p Noise power, $P_{avg} = W \cdot T_s \cdot \frac{\delta^2}{3} \quad -⑦$ (W: cut off f of LPF)

$$\therefore (SNR)_{o/p\,max} = \frac{P_{max}}{\text{avg. noise power}}$$

$$= \frac{2\pi^2 f_0^2 T_s^2}{\delta^2} \times \frac{3}{W T_s \delta^2}$$

$$= \frac{6\pi^2 f_0^2 T_s}{\delta^4}$$