12/12/23

3. FIR FILTERS

*FIR:- Impulse Response is finite.

- O/p depends only on present & past i/ps.
- FIR filters are used for linear phase response
- It is stable, can be realized in both recursive and non-recursive

→ Disadvantages:- · Complex,
· Require more filter coefficients
· Narrow Transition band

→ IMPULSE RESPONSE LINEAR PHASE FIR FILTERS:-

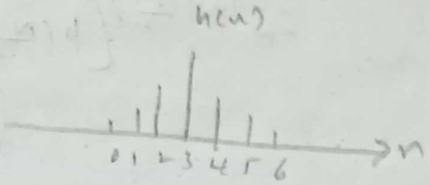
- LP FIR filters are classified into '2' types:
  1. Symmetric
  2. Anti-symmetric

① Symmetric LPFIR:-

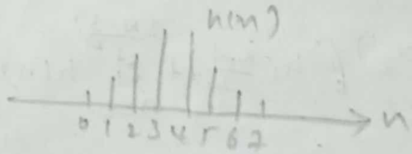
- $\boxed{h(n) = h(M-1-n)}$; $0 \le n \le M-1$

where, M = No. of samples.

* If 'M' is odd:- (7)



* If 'M' is even:- (8)

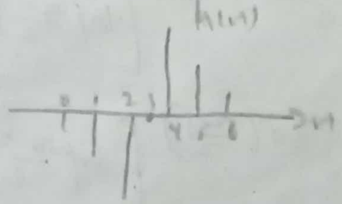


② ANTI-SYMMETRIC LPFIR:-

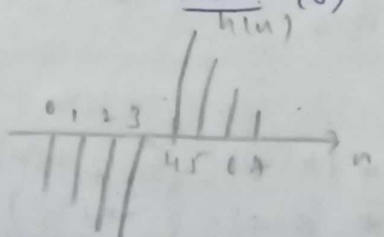
- $\boxed{h(n) = -h(M-1-n)}$; $0 \le n \le M-1$

where, M = No. of samples.

* If 'M' is Odd:- (7)



* If 'M' is even:- (8)



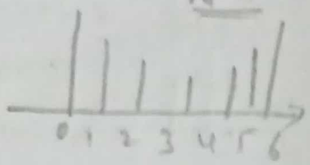
FREQUENCY RESPONSE OF LPFIR FILTERS:

CASE-1:

We know that for a symmetric FIR filter,

$$\rightarrow \boxed{h(n) = h(N-1-n)} \; ; \; 0 \le n \le N-1$$

N=7



let N=ODD

$$\Rightarrow H(e^{j\omega}) = \sum_{n=0}^{N-1} h(n) e^{-j\omega n}$$

$$= \sum_{n=0}^{\left(\frac{N-3}{2}\right)} h(n) e^{-j\omega n} + h\left(\frac{N-1}{2}\right) \cdot e^{-j\omega\left(\frac{N-1}{2}\right)} + \sum_{n=\frac{N+1}{2}}^{N-1} h(n) e^{-j\omega n}$$

put, $N-1-n=m$

$\Rightarrow n = N-1-m$

$$'' \qquad '' \qquad + \sum_{n=\frac{N+1}{2}}^{N-1} h(N-1-n) e^{-j\omega n}$$

$n = \frac{N+1}{2} \Rightarrow m = \frac{N-3}{2}$

$n = N-1 \Rightarrow m = 0$

$$= h\left(\frac{N-1}{2}\right) e^{-j\omega\left(\frac{N-1}{2}\right)} + \sum_{n=0}^{\left(\frac{N-3}{2}\right)} h(n) e^{-j\omega n} + \sum_{n=0}^{\left(\frac{N-3}{2}\right)} h(n) e^{-j\omega(N-1-n)}$$

$$= h\left(\frac{N-1}{2}\right) e^{-j\omega\left(\frac{N-1}{2}\right)} + \sum_{n=0}^{\left(\frac{N-3}{2}\right)} h(n) \left[e^{-j\omega n} + e^{-j\omega(N-1-n)}\right]$$

$$= e^{-j\omega\left(\frac{N-1}{2}\right)} \left[h\left(\frac{N-1}{2}\right) + \sum_{n=0}^{\left(\frac{N-3}{2}\right)} h(n) \left[e^{-j\omega n} \cdot e^{j\omega\left(\frac{N-1}{2}\right)}\right] + \sum_{n=0}^{\left(\frac{N-3}{2}\right)} h(n) e^{-j\omega(N-1-n)} e^{j\omega\left(\frac{N-1}{2}\right)}\right]$$

$$= e^{-j\omega\left(\frac{N-1}{2}\right)} \left[h\left(\frac{N-1}{2}\right) + \sum_{n=0}^{\left(\frac{N-3}{2}\right)} h(n) \left[e^{-j\omega\left[n-\left(\frac{N-1}{2}\right)\right]} + e^{-j\omega\left(N-1-n-\left(\frac{N-1}{2}\right)\right)}\right]\right]$$

$$N-1-n+\tfrac{N}{2}+\tfrac{1}{2}$$

$$= e^{-j\omega\left(\frac{N-1}{2}\right)} \left[h\left(\frac{N-1}{2}\right) + \sum_{n=0}^{\left(\frac{N-3}{2}\right)} h(n) \left[e^{-j\omega\left(\frac{2n-N+1}{2}\right)} + e^{-j\omega\left(\frac{3N-1-2n}{2}\right)}\right]\right]$$

$$= e^{-j\omega\left(\frac{N-1}{2}\right)} \left[h\left(\frac{N-1}{2}\right) + \sum_{n=0}^{\left(\frac{N-3}{2}\right)} h(n) \left[e^{-j\omega\left[n-\left(\frac{N-1}{2}\right)\right]} + e^{j\omega\left[n-\frac{N-1}{2}\right]}\right]\right]$$

$$\boxed{H(e^{j\omega}) = e^{-j\omega\left(\frac{N-1}{2}\right)} \left[h\left(\frac{N-1}{2}\right) + 2\sum_{n=0}^{\left(\frac{N-3}{2}\right)} h(n) \cdot \cos\left[\omega\left(n-\left(\frac{N-1}{2}\right)\right)\right]\right]}$$

$\Rightarrow H(e^{j\omega}) = |H(e^{j\omega})| . e^{j\theta(\omega)}$

$$\Rightarrow \boxed{|H(e^{j\omega})| = h\left(\frac{N-1}{2}\right) + 2\sum_{n=0}^{\left(\frac{N-3}{2}\right)} h(n)\cos\left[\omega\left(n-\left(\frac{N-1}{2}\right)\right)\right]}$$

and $e^{j\theta(\omega)} = \begin{cases} e^{-j\omega\left(\frac{N-1}{2}\right)} + 0 & ; H(e^{j\omega}) > 0 \\ e^{-j\omega\left(\frac{N-1}{2}\right)} + \pi & ; H(e^{j\omega}) < 0 \end{cases}$

$$\Rightarrow \boxed{\theta(\omega) = \begin{cases} -\omega\left(\frac{N-1}{2}\right) + 0 & ; H(e^{j\omega}) > 0 \\ -\omega\left(\frac{N-1}{2}\right) + \pi & ; H(e^{j\omega}) < 0 \end{cases}}$$

* $A > 0 \Rightarrow$
$Arg(A) = \tan^{-1}\left(\frac{IP}{RP}\right)$
$= \tan^{-1}\left(\frac{\sin 0}{\cos 0}\right)$
$= \tan^{-1}(\tan 0)$
$= 0$ //

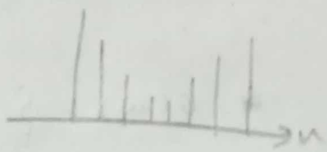
* $A < 0 \Rightarrow$
$Arg(A) = \tan^{-1}\left(\frac{IP}{RP}\right)$
$= \tan^{-1}\left(\frac{\sin\pi}{\cos\pi}\right)$
$= \pi$ //

* CASE-2:

* when $h(n)$ is symmetric & 'N' is even.

$$H(e^{j\omega}) = \sum_{n=0}^{N-1} h(n)e^{-j\omega n}$$

$$= \sum_{n=0}^{\frac{N}{2}-1} h(n)e^{-j\omega n} + \sum_{n=\frac{N}{2}}^{N-1} h(n)e^{-j\omega n}$$



$$= \sum_{n=0}^{\frac{N}{2}-1} h(n)e^{-j\omega n} + \sum_{n=\frac{N}{2}}^{N-1} h(N-1-n)e^{-j\omega n}$$

* $N-1-n = m$
$n = \frac{N}{2} \Rightarrow m = N-1-\frac{N}{2}$
$= \frac{N}{2}-1$
$n = N-1 \Rightarrow m = 0$

$$= \sum_{n=0}^{\frac{N}{2}-1} h(n)e^{-j\omega n} + \sum_{n=0}^{\frac{N}{2}-1} h(n)e^{-j\omega(N-1-n)}$$

$$= \sum_{n=0}^{\frac{N}{2}-1} h(n)\left[e^{-j\omega n}\, e^{j\omega\left(\frac{N-1}{2}\right)} + e^{-j\omega(N-1-n)} . e^{j\omega\left(\frac{N-1}{2}\right)}\right]e^{-j\omega\left(\frac{N-1}{2}\right)}$$

$$= \sum_{n=0}^{\left(\frac{N}{2}-1\right)} h[n]\left[e^{-j\omega\left(n-\left(\frac{N-1}{2}\right)\right)} + e^{+j\omega\left[N-1-n+\frac{N-1}{2}\right]}\right]e^{-j\omega\left(\frac{N-1}{2}\right)}$$

$$= \sum_{n=0}^{\left(\frac{N}{2}-1\right)} h[n]\left[e^{-j\omega\left[n-\left(\frac{N-1}{2}\right)\right]} + e^{j\omega\left[N-\frac{N}{2}-1+\frac{1}{2}-n\right]}\right]e^{-j\omega\left(\frac{N-1}{2}\right)}$$

$$= \sum_{n=0}^{\left(\frac{N}{2}-1\right)} h[n]\left[e^{-j\omega\left[n-\left(\frac{N-1}{2}\right)\right]} + e^{j\omega\left[n-\left(\frac{N-1}{2}\right)\right]}\right]e^{-j\omega\left(\frac{N-1}{2}\right)}$$

$$= e^{-j\omega\left(\frac{N-1}{2}\right)} \sum_{n=0}^{\frac{N}{2}-1} h[n]\, 2\cos\left[\omega\left[n-\frac{N-1}{2}\right]\right]$$

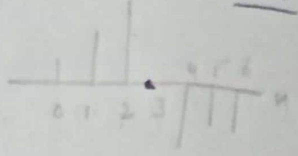
$$H(e^{j\omega}) = 2e^{-j\omega\left(\frac{N-1}{2}\right)} \sum^{\left(\frac{N}{2}-1\right)} h[n]\cos\left[\omega\left(n-\frac{N-1}{2}\right)\right]$$

CASE-3: h(n) is Anti symmetric with Odd samples:-

$$\boxed{h(n) = -h(N-1-n)} \; ; \; 0 \le n \le N-1$$

$\rightarrow H(e^{j\omega}) = \sum_{n=0}^{N-1} h(n) e^{-j\omega n}$

N=7



$= \sum_{n=0}^{(N-3)/2} h(n) e^{-j\omega n} + 0 + \sum_{n=\frac{N+1}{2}}^{N-1} h(n) e^{-j\omega n}$

$= \sum_{n=0}^{\left(\frac{N-3}{2}\right)} h(n) e^{-j\omega n} + \sum_{n=\frac{N+1}{2}}^{N-1} -h(N-1-n) e^{-j\omega n}$

$N-1-n = m$
$\rightarrow n = \frac{N+1}{2} \Rightarrow m = \frac{N-3}{2}$
$\rightarrow n = N-1 \Rightarrow m = 0$

$H(e^{j\omega}) = \sum_{n=0}^{\left(\frac{N-3}{2}\right)} h(n) e^{-j\omega n} + \sum_{n=0}^{(N-3/2)} -h(n) e^{-j\omega(N-1-n)}$

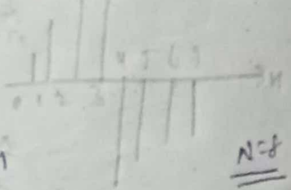
$$\Rightarrow H(e^{j\omega}) = 2e^{-j\omega\left(\frac{N-1}{2}\right)} \sum_{n=0}^{\frac{N-3}{2}} j \cdot h(n) \sin\left(\omega\left(\frac{N-1}{2} - n\right)\right)$$

* $|H(e^{j\omega})| = 2\sum_{n=0}^{\frac{N-3}{2}} h(n) \sin\left[\omega\left[\left(\frac{N-1}{2}\right) - n\right]\right]$

* $\theta(\omega) = \begin{cases} \frac{\pi}{2} - \omega\left(\frac{N-1}{2}\right) & ; |H(\omega)| > 0 \\ \frac{3\pi}{2} - \omega\left(\frac{N-1}{2}\right) & ; |H(\omega)| < 0 \end{cases}$

CASE-4: h(n) is Anti-symmetric with Even samples:-

$\rightarrow H(e^{j\omega}) = \sum_{n=0}^{N-1} h(n) e^{-j\omega n}$



N=8

$\rightarrow H(e^{j\omega}) = \sum_{n=0}^{\frac{N}{2}-1} h(n) e^{-j\omega n} + \sum_{n=\frac{N}{2}}^{N-1} -h(N-1-n) e^{-j\omega n}$

$= \sum_{n=0}^{\frac{N}{2}-1} h(n) e^{-j\omega\left(\frac{N-1}{2}\right)} \left(e^{-j\omega n} \cdot e^{j\omega\left(\frac{N-1}{2}\right)} - e^{-j\omega(N-1-n)} e^{j\omega\left(\frac{N-1}{2}\right)}\right)$

$= e^{-j\omega\left(\frac{N-1}{2}\right)} \sum_{n=0}^{\frac{N}{2}-1} h(n) \left[e^{j\omega\left(\frac{N-1}{2} - n\right)} - e^{-j\omega\left(\frac{N-1}{2} - n\right)}\right]$

$= e^{-j\omega\left(\frac{N-1}{2}\right)} \sum_{n=0}^{\frac{N}{2}-1} h(n) \cdot 2j \sin\left[\omega\left(\frac{N-1}{2} - n\right)\right]$

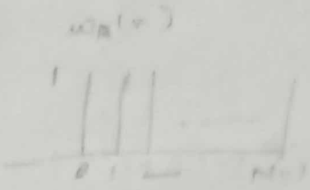
$$\boxed{H(e^{j\omega}) = 2e^{-j\omega\left(\frac{N-1}{2}\right)} \sum_{n=0}^{\frac{N}{2}-1} j h(n) \cdot \sin\left[\omega\left(\frac{N-1}{2} - n\right)\right]}$$

* $|H(e^{j\omega})| = 2\sum_{n=0}^{\frac{N}{2}-1} h(n) \sin\left[\omega\left(\frac{N-1}{2} - n\right)\right]$

* $\theta(\omega) = \begin{cases} \frac{\pi}{2} - \omega\left(\frac{N-1}{2}\right) & ; |H(e^{j\omega})| > 0 \\ \frac{3\pi}{2} - \omega\left(\frac{N-1}{2}\right) & ; |H_r(e^{j\omega})| < 0 \end{cases}$

* DESIGNING OF LP FIR FILTERS: USING WINDOW METHOD

① Rectangular Window:-



* $w_R(n) = \begin{cases} 1 & , 0 \le n \le N-1 \\ 0 & , o.w \end{cases}$

$\Rightarrow w_R(e^{j\omega}) = \sum_{n=0}^{N-1} w_R(n) e^{-j\omega n}$

$= \sum_{n=0}^{N-1} e^{-j\omega n}$ $\quad \therefore \left[\sum_{n=0}^{N-1} a^n = \frac{1-a^N}{1-a}\right]$

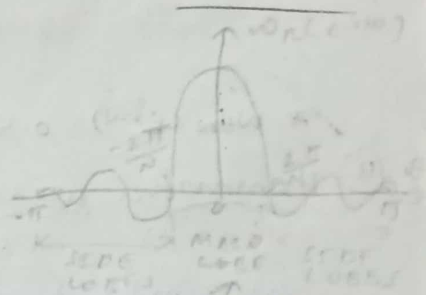
$= \frac{1-e^{-j\omega N}}{1-e^{-j\omega}}$

$= \frac{e^{-j\omega N/2}}{e^{-j\omega/2}} \left[\frac{e^{j\omega N/2} - e^{-j\frac{\omega N}{2}}}{e^{j\omega/2} - e^{-j\omega/2}}\right]$

$= e^{-j\omega\left(\frac{N-1}{2}\right)} \left[\frac{2j\sin\left(\frac{\omega N}{2}\right)}{2j\sin\left(\frac{\omega}{2}\right)}\right]$

$$\boxed{w_R(e^{j\omega}) = e^{-j\omega\left(\frac{N-1}{2}\right)} \left[\frac{\sin\left(\frac{\omega N}{2}\right)}{\sin\left(\frac{\omega}{2}\right)}\right]}$$

MAGNITUDE RESPONSE

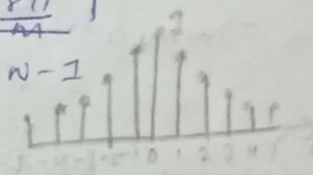


$\boxed{\text{width} = \frac{4\pi}{N}}$

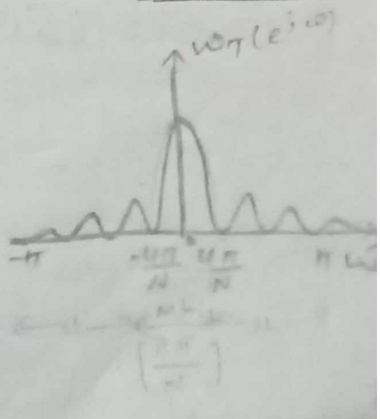
② Triangular Window / Bartlet Window:-

* $\boxed{\text{width} = \frac{8\pi}{N}}$

* $w_T(n) = \begin{cases} 1 - 2\left[\frac{n - \left(\frac{N-1}{2}\right)}{N-1}\right] & ; 0 \le n \le N-1 \\ 0 & ; o.w \end{cases}$



FREQUENCY RESPONSE



$\Rightarrow w_T(e^{j\omega}) = \sum_{n=0}^{N-1} w_T(n) e^{-j\omega n}$

$= \sum_{n=0}^{N-1} \left\{1 - 2\left[\frac{n - \left(\frac{N-1}{2}\right)}{N-1}\right]\right\} e^{-j\omega n}$

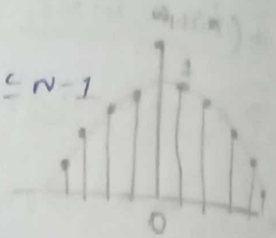
$= \sum_{n=0}^{N-1} e^{-j\omega n} - \frac{2}{N-1} \sum_{n=0}^{N-1} \left[n - \left(\frac{N-1}{2}\right)\right] e^{-j\omega n}$

$=$

$w_T(e^{j\omega}) = \frac{2}{N-1} \left[\frac{\sin\left(\frac{N-1}{4}\right)\omega}{\sin\left(\frac{\omega}{2}\right)}\right]^2$

(3) HANNING WINDOW:- $\boxed{\text{width} = \frac{8\pi}{M}}$

$$w_{HN}[n] = \begin{cases} 0.5 - 0.5\cos\left(\frac{2n\pi}{N-1}\right) & ; 0 \le n \le N-1 \\ 0 & ; ow \end{cases}$$



$$\Rightarrow W_{HN}(e^{j\omega}) = \sum_{n=0}^{N-1} \left[0.5 - 0.5\cos\left(\frac{2n\pi}{N-1}\right)\right] e^{-j\omega n}$$

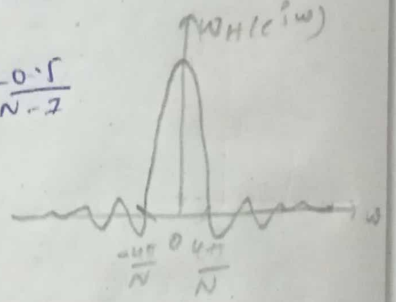
$$= 0.5 \sum_{n=0}^{N-1} (e^{-j\omega})^n - 0.5 \sum_{n=0}^{N-1} \cos\left(\frac{2n\pi}{N-1}\right) e^{-j\omega n}$$

$$= 0.5 \left(\frac{1 - e^{-j\omega N}}{1 - e^{-j\omega}}\right) - \frac{0.5}{N-1}$$

$$= 0.5 \cdot \frac{e^{-j\omega N/2}}{e^{-j\omega/2}} \left[\frac{e^{j\omega N/2} - e^{-j\omega N/2}}{e^{j\omega/2} - e^{-j\omega/2}}\right] - \frac{0.5}{N-1}$$

$$= 0.5\, e^{-j\left(\frac{N-1}{2}\right)\omega} \cdot \frac{\cancel{2}\sin\left(\frac{\omega N}{2}\right)}{\cancel{2}\sin\left(\frac{\omega}{2}\right)} - \frac{0.5}{N-1}$$

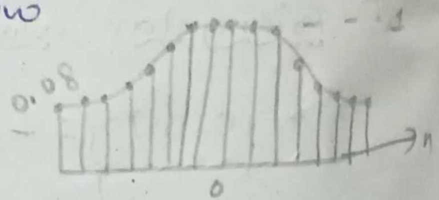
$$= 0.5\, e^{-j\left(\frac{N-1}{2}\right)\omega} \left[\frac{\sin\left(\frac{\omega N}{2}\right)}{\sin\left(\frac{\omega}{2}\right)}\right]$$



* $W_{HN}(e^{j\omega}) = 0.5\, W_R(\omega) + 0.25\left[W_R\left(\omega - \frac{2\pi}{M}\right) + W_R\left(\omega + \frac{2\pi}{M}\right)\right]$

(4) HAMMING WINDOW: $\boxed{\text{width} = \frac{8\pi}{M}}$

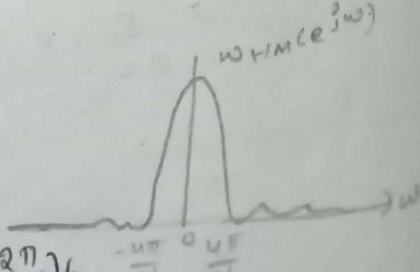
$$w_{HM}[n] = \begin{cases} 0.54 - 0.46\cos\left(\frac{2n\pi}{N-1}\right) & ; 0 \le n \le N-1 \\ 0 & ; ow \end{cases}$$



$$\Rightarrow W_{HM}(e^{j\omega}) = \sum_{n=0}^{N-1} \left[0.54 - 0.46\cos\left(\frac{2n\pi}{N-1}\right)\right] e^{-j\omega n}$$

$$= 0.54 \sum_{n=0}^{N-1} e^{-j\omega n} - 0.46 \sum_{n=0}^{N-1} \cos\left(\frac{2n\pi}{N-1}\right) e^{-j\omega n}$$

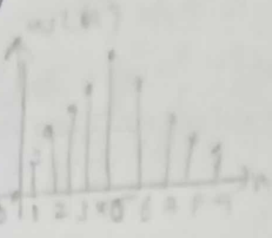
$$= 0.54\, e^{-j\left(\frac{N-1}{2}\right)\omega} \cdot \left[\frac{\sin\left(\frac{\omega N}{2}\right)}{\sin\left(\frac{\omega}{2}\right)}\right] - 0.46$$



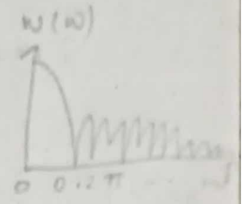
* $W_{HM}(\omega) = 0.54\, W_R(\omega) + 0.23\left[W_R\left(\omega - \frac{2\pi}{M}\right) + W_R\left(\omega + \frac{2\pi}{M}\right)\right]$

BLACKMAN WINDOW: $\boxed{\text{width} = \frac{12\pi}{M}}$

* $w_B(n) = \left[0.42 - 0.5\cos\left(\frac{2\pi n}{N-1}\right) + 0.08\cos\left(\frac{4\pi n}{N-1}\right)\right]$



$\Rightarrow w_B(e^{j\omega}) = 0.42\sum_{n=0}^{N-1} e^{-j\omega n} - 0.5\sum_{n=0}^{N-1}\cos\left(\frac{2\pi n}{N-1}\right)e^{-j\omega n}$
$+ 0.08\sum_{n=0}^{N-1}\cos\left(\frac{4\pi n}{N-1}\right)e^{-j\omega n}$



* $W_B(\omega) = 0.42\,W_R(\omega) + 0.25\left(W_R\left(\omega - \frac{2\pi}{M}\right) + W_R\left(\omega + \frac{2\pi}{M}\right)\right)$
$+ 0.04\left(W_R\left(\omega - \frac{4\pi}{M}\right) + W_R\left(\omega + \frac{4\pi}{M}\right)\right)$

* PROCEDURE TO DESIGN LPFIR USING WINDOW TECHNIQUE

- The filter characteristics are specified in frequency domain interms of the desired magnitude & phase response of the filter.

- The desired frequency response is:

* $H_d(e^{j\omega}) = \sum_{n=-\infty}^{\infty} h_d(n)e^{-j\omega n}$

* $h_d[n] = \frac{1}{2\pi}\int_{-\pi}^{\pi} H_d(e^{j\omega})e^{j\omega n}\,d\omega.$

- $h_d[n]$ is desired impulse response of infinite length, one possible way of obtaining finite length impulse response is to truncate the infinite length at $n=0$ to $N-1$ where 'N' is the length of desired sequence. But, due to abrupt truncation of the infinite impulse response, it results in oscillations in passband & stop band. This effect is known as "GIBBS PHENOMENON".

In order to reduce this effect & to obtain finite length impulse response, the infinite length impulse response is multiplied by a finite length sequence "w(n)" called a window.

Finite length impulse response, h[n] can be obtained by looking through a window and seeing these terms of $h_d[n]$. This process of obtaining h[n] from $h_d[n]$ ...

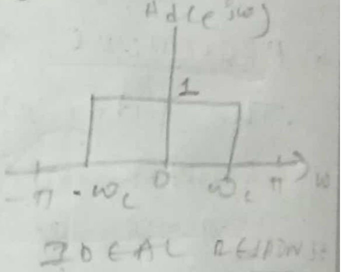
called "windowing".

After multiplying window sequence $w(n)$ with $h_d[n]$, we get a finite duration sequence $h[n]$ that satisfies desired magnitude response.

$$\boxed{h[n] = \begin{cases} h_d[n] \cdot w[n] & ; 0 \le n \le N-1 \\ 0 & ; ow \end{cases}}$$

Q: Design the symmetric FIR low pass filter, whose the desired frequency response is $H_d(e^{j\omega}) = \begin{cases} e^{-j\omega T} & ; |\omega| \le \omega_c \le \pi \\ 0 & ; ow \end{cases}$ with $\omega_c = 1$ rad/sec & $N = 7$.
odd

Sol: Given, $H_d(e^{j\omega}) = \begin{cases} e^{-j\omega T} & ; |\omega| \le \omega_c \le \pi \\ 0 & ; ow \end{cases}$



$$\Rightarrow h_d[n] = \frac{1}{2\pi} \int_{-\omega_c}^{\omega_c} e^{-j\omega T} \cdot e^{j\omega n} \, d\omega$$

$$= \frac{1}{2\pi} \int_{-\omega_c}^{\omega_c} e^{-j\omega(-n+T)} \, d\omega$$

$$= \frac{1}{2\pi} \cdot \left[\frac{e^{-j\omega(T-n)}}{j(T-n)} \right]_{-\omega_c}^{\omega_c}$$

$$= \frac{-1}{2j\pi(T-n)} \left[e^{-j\omega_c(T-n)} - e^{j\omega_c(T-n)} \right]$$

$$= \frac{1}{2\pi(T-n) \cdot j} \, 2j \sin[\omega_c(T-n)]$$

$$\rightarrow \boxed{h_d[n] = \frac{\sin[\omega_c(T-n)]}{\pi(T-n)}} \; ; \; n \neq T$$

$$\rightarrow h_d[n] = \frac{\omega_c}{\pi} \frac{\sin[\omega_c(T-n)]}{\omega_c(T-n)}$$

$$\boxed{h_d[n] = \frac{\omega_c}{\pi}} \; ; \; n = T$$

→ To obtain value of 'τ' :.

w.k.t, $h[n] = h[N-1-n]$; $h[n] = h_d[n]\,w[n]$

$\Rightarrow h_d[n]w[n] = h_d[N-1-n]\,w[n]$

$\Rightarrow \quad h_d[n] = h_d[N-1-n]$

$$\frac{\sin w_c(\tau-n)}{\pi(\tau-n)} = \frac{\sin w_c(\tau-N+1+n)}{\pi(\tau-N+1+n)}$$

$$\frac{-\sin w_c(\tau-n)}{-\pi(\tau-n)} = \frac{\sin w_c(\tau-N+1+n)}{\pi(\tau-N+1+n)}$$

$$\frac{\sin[-w_c(\tau-n)]}{-\pi(\tau-n)} = \frac{\sin w_c(\tau-N+1+n)}{\pi(\tau-N+1+n)}$$

$w_c = 1 \Rightarrow$

$$\frac{\sin[\overset{a}{-(\tau-n)}]}{-\pi\underset{a}{(\tau-n)}} = \frac{\sin(\overset{b}{\tau-N+1+n})}{\pi\underset{b}{(\tau-N+1+n)}}$$

$a = b$

$\Rightarrow -\tau + \cancel{n} = \tau - N + 1 + \cancel{n}$

$2\tau = N-1$

$$\boxed{\tau = \frac{N-1}{2}}$$

Given, $N = 7 \Rightarrow \boxed{\tau = 3}$

1) Rectangular window:-

$\boxed{h[n] = h_d[n]\,w_R[n]}$; $w_R[n] = 1$; $0 \le n \le N-1$

$\Rightarrow h[0] = h_d[0] = \frac{\sin(3)}{\pi(3)} = \frac{0.14}{3\pi} = 0.01497 \quad (n \neq \tau)$

• $h[1] = h_d[1] = \frac{\sin(2)}{2\pi} = 0.14472 \quad [n \neq \tau]$

• $h[2] = h_d[2] = \frac{\sin(1)}{\pi} = 0.26785 \quad [n \neq \tau]$

• $h[3] = h_d[3] = \frac{w_c}{\pi} = \frac{1}{\pi} = 0.3183 \quad (n = \tau)$

• $h[4] = h[2] = h_d[2] = 0.26785$

• $h[5] = h[1] = 0.14472$

• $h[6] = h[0] = 0.01497$

w.k.T.

$$|H(\omega)| = h\left(\frac{N-1}{2}\right) + 2\sum_{n=0}^{\frac{N-3}{2}} h[n]\cos\left(\omega\left(n-\frac{N-1}{2}\right)\right)$$

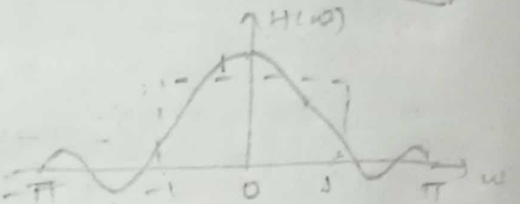
$$\Rightarrow |H(\omega)| = h(3) + 2\sum_{n=0}^{2} h[n].\cos[\omega(n-3)]$$

$$= h(3] + 2h[0]\cos(3\omega) + 2h[1]\cos(2\omega) + 2h[2]\cos\omega$$

$$= 0.3183 + 2(0.01497)\cos(3\omega) + 2(0.14472)\cos(2\omega) + 2(0.26785)\cos\omega$$

$$\boxed{|H(\omega)| = 0.3183 + 0.0298[\cos 3\omega] + 0.2894\cos(2\omega) + 0.5357\cos\omega}$$

$\Rightarrow H(0) = 1.1732$



② Using Hanning Window:-

- $w_{HN}[n] = \underline{0.5 - 0.5\cos\left(\frac{2n\pi}{N-1}\right)}$; $0 \le n \le N-1$

- $\boxed{h[n] = h_d[n].w_{HN}[n]}$

- $h[0] = h_d[0].w_{HN}[0] = h_d[0].[0.5 - 0.5] = 0$
- $h[1] = h_d[1].w_{HN}[1] = h_d[1]\left[0.5 - 0.5\cos\left(\frac{2\pi}{N-1}\right)\right] = \frac{h_d[1]}{4}$

$= \frac{0.14472}{4} = 0.03618$

- $h[2] = h_d[2].\frac{3}{4} = 0.26785\left(\frac{3}{4}\right) = \frac{0.80355}{4} = 0.2008$

- $h[3] = h_d[3].1 = 0.3813$

w_{HN}: $\frac{1}{4}$, $\frac{3}{4}$, 1

w.k.T,

$$|H(\omega)| = h\left(\frac{N-1}{2}\right) + 2\sum_{n=0}^{\frac{N-3}{2}} h[n]\cos\left[\omega\left(n-\frac{N-1}{2}\right)\right]$$

$\Rightarrow H(\omega) = h(3) + 2\sum_{n=0}^{2} h[n] \cos\left[\omega\left(n - \frac{N-1}{2}\right)\right]$

$= h(3) + 2\sum_{n=0}^{2} h[n] \cos[\omega(n-3)]$

$= h(3) + 2[h(0)\cos(3\omega) + h[1]\cos(2\omega) + h[2]\cos(\omega)]$

$= 0.3813 + 2\cdot(0) + 2\cdot(0.03618)\cdot\cos(2\omega) + 2(0.2008)\cos(\omega)$

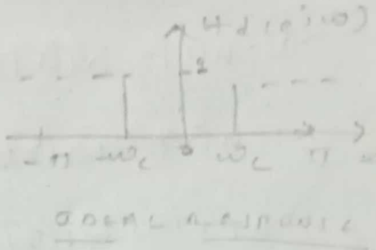
$= 0.3813 + 0 + (0.07236)\cos(2\omega) + [0.4016]\cos\omega$

$$\boxed{H(\omega) = 0.3813 + [0.07236]\cos(2\omega) + (0.4016)\cos\omega}$$

$H(0) = 0.3813 + 0.07236 + 0.4016 = 0.85526 \approx 1$

Q: Design A symmetric High Pass FIR filter which the desired frequency response is; $H_d(e^{j\omega}) = \begin{cases} e^{-j\omega T} & ; \omega_c \le |\omega| \le \pi \\ 0 & ; \text{O.W} \end{cases}$

with $\omega_c = 2$ rad/sec, $N = 7$.



Sol:- $h_d(n) = \frac{1}{2\pi}\int_{-\pi}^{\pi} H_d(e^{j\omega}) e^{j\omega n}\, d\omega$

$= \frac{1}{2\pi}\left[\int_{-\pi}^{-\omega_c} H_d(e^{j\omega}) e^{j\omega n}\, d\omega + \int_{\omega_c}^{\pi} H_d(e^{j\omega}) e^{j\omega n}\, d\omega\right]$

$= \frac{1}{2\pi}\left[\int_{-\pi}^{-\omega_c} e^{-j\omega T} e^{j\omega n}\, d\omega + \int_{\omega_c}^{\pi} e^{-j\omega T} e^{j\omega n}\, d\omega\right]$

$= \frac{1}{2\pi}\left\{\left[\frac{e^{-j\omega(T-n)}}{-j(T-n)}\right]_{-\pi}^{-\omega_c} + \left[\frac{e^{-j\omega(T-n)}}{-j(T-n)}\right]_{\omega_c}^{\pi}\right\}$

$= \frac{1}{2\pi}\left\{\frac{e^{j\omega_c(T-n)} - e^{j\pi(T-n)} + e^{-j\pi(T-n)} - e^{-j\omega_c(T-n)}}{-j(T-n)}\right\}$

$= \frac{1}{2\pi}\left\{\frac{+2j\sin\omega_c(T-n) - 2j\sin\pi(T-n)}{-j(T-n)}\right\}$

$= \frac{1}{\pi(T-n)}[\sin(\pi(T-n)) - \sin[\omega_c(T-n)]]$

$= \frac{\sin[\pi(T-n)] - \sin[2(T-n)]}{\pi(T-n)}$

$h_d[n] = \frac{\sin[\pi(n-T)] - \sin[2(n-T)]}{\pi(n-T)}$; $n \ne T$

and $\boxed{h_d[n] = 1 - \frac{2}{\pi}}$; $n = T$

and we know that $\boxed{T = \frac{N-1}{2}}$

$\Rightarrow T = \frac{7-1}{2} = 3 \Rightarrow \boxed{T = 3}$;

$$h_d[n] = \begin{cases} \frac{\sin[\pi(n-T)] - \sin[2(n-T)]}{\pi(n-T)} & ; n \neq T \\ 1 - \frac{2}{\pi} & ; n = T \end{cases}$$

* using rectangular window,

$\Rightarrow \boxed{h[n] = h_d[n] \cdot w_R[n]}$, $w_R[n] = 1$; $0 \leq n \leq N-1$

$\Rightarrow 0 \leq n \leq 6$

$\Rightarrow h[0] = h_d[0] \cdot w_R[0] = \frac{\sin[\pi(-3)] - \sin[2(-3)]}{\pi(-3)} = 0.0296$

$\Rightarrow h[1] = h_d[1] \cdot w_R[1] = \frac{\sin[\pi(-2)] - \sin[2(-2)]}{\pi(-2)} = 0.12044$

$\Rightarrow h[2] = h_d[2] \cdot w_R[2] = \frac{\sin[\pi(-1)] - \sin[2(-1)]}{\pi(-1)} = -0.289$

$\Rightarrow h[3] = h_d[3] \cdot w_R[3] = (1 - \frac{2}{\pi})(1) = 0.364$

$\Rightarrow h[4] = h_d[4] \cdot w_R[4] = \frac{\sin[\pi(1)] - \sin[2(1)]}{\pi(1)} = -0.289$

$\Rightarrow h[5] = h_d[5] \cdot w_R[5] = 0.12044$

$\Rightarrow h[6] = h_d[6] \cdot w_R[6] = 0.0296$

w.k.T,

$$|H(\omega)| = h\left(\frac{N-1}{2}\right) + 2 \sum_{n=0}^{\frac{N-3}{2}} h[n] \cos\left[\omega\left(n - \frac{N-1}{2}\right)\right]$$

$$= h(3) + 2 \sum_{n=0}^{2} h[n] \cos[\omega(n-3)]$$

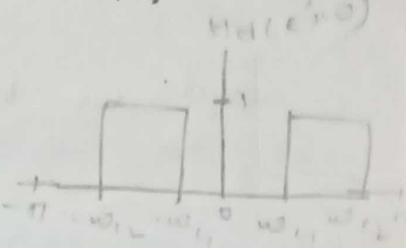
$$= h(3) + 2[h(0)\cos(3\omega) + h(1)\cos(2\omega) + h(2) \cdot \cos(\omega)]$$

$$= [0.364] + 2[0.0296]\cos(3\omega) + 2[0.12044]\cos(2\omega) + 2[-0.289]\cos(\omega)$$

$$\boxed{|H(\omega)| = 0.364 + [0.0592]\cos(3\omega) + [0.24088]\cos(2\omega) + [-0.578]\cos\omega}$$

Q: Design a symmetric BPF with desired frequency response, $H_d(e^{j\omega}) = \begin{cases} e^{-j\omega T} & ; \omega_{c_1} \le |\omega| \le \omega_{c_2} < \pi \\ 0 & ; \text{o.w} \end{cases}$

with $\omega_{c_1} = 1$ rad/sec, $\omega_{c_2} = 2$ rad/sec & $N = 7$.



Sol:

$$h_d(n) = \frac{1}{2\pi}\int_{-\pi}^{\pi} H_d(e^{j\omega}) e^{j\omega n}\, d\omega$$

$$= \frac{1}{2\pi}\left[\int_{-\omega_{c_2}}^{-\omega_{c_1}} d\omega + \int_{\omega_{c_1}}^{\omega_{c_2}} d\omega\right]$$

$$= \frac{1}{2\pi}\left\{\left[\frac{e^{-j\omega(T-n)}}{-j(T-n)}\right]_{-\omega_{c_2}}^{-\omega_{c_1}} + \left[\frac{e^{-j\omega(T-n)}}{-j(T-n)}\right]_{\omega_{c_1}}^{\omega_{c_2}}\right\}$$

$$= \frac{1}{2\pi}\left[\frac{e^{+j\omega_{c_1}(T-n)} - e^{+j\omega_{c_2}(T-n)} + e^{-j\omega_{c_2}(T-n)} - e^{-j\omega_{c_1}(T-n)}}{-j(T-n)}\right]$$

$$= \frac{1}{2\pi}\left[\frac{+2j\sin[\omega_{c_1}(T-n)] - 2j(\sin\omega_{c_2}(T-n)]}{-j(T-n)}\right]$$

$$h_d(n) = \begin{cases} \dfrac{\sin[\omega_{c_1}(T-n)] - \sin[\omega_{c_2}(T-n)]}{-\pi(T-n)} & n \ne T \end{cases}$$

$$= \frac{\sin[\omega_{c_2}(T-n)] - \sin[\omega_{c_1}(T-n)]}{\pi(T-n)}, \quad n \ne T$$

$$\boxed{h_d(n) = \frac{\sin[\omega_{c_2}(n-T)] - \sin[\omega_{c_1}(n-T)]}{\pi(n-T)}} \; ; n \ne T$$

* $n = T$

$$\Rightarrow h_d(n) = \frac{\sin 2(n-T)}{\pi(n-T)} - \frac{\sin(n-T)}{\pi(n-T)} \; ; n \ne T$$

$$= \frac{2}{\pi} - \frac{1}{\pi} \; ;$$

$$\boxed{h_d(n) = \frac{1}{\pi}} \; ; n = T$$

$$\therefore h_d(n) = \begin{cases} \dfrac{\sin[2(n-T)] - \sin(n-T)}{\pi(n-T)} & ; n \ne T \\ \dfrac{1}{\pi} & ; n = T \end{cases}$$

$\rightarrow$ we know that, $\tau = \frac{N-1}{2}$

$\Rightarrow \tau = \frac{7-1}{2}$

$\boxed{\tau = 3}$

and we have, $h_d[n] = \begin{cases} \frac{\sin[2(n-\tau)] - \sin[n-\tau]}{\pi(n-\tau)} & ; n \neq \tau \\ \frac{1}{\pi} & ; n = \tau \end{cases}$

$\therefore$ using, window technique, we have:

$\Rightarrow \boxed{h[n] = h_d[n]\, w_R[n]}$; $w_R[n] = 1$; $0 \le n \le N-1$

$0 \le n \le 6$

$\Rightarrow h[0] = h_d[0](1) = \frac{\sin[2(-3)] - \sin[(-3)]}{\pi(-3)} = -0.04462$

$h[1] = h_d[1] = \frac{\sin[2(-2)] - \sin[-2]}{\pi(-2)} = -0.26516$

$h[2] = h_d[2] = \frac{\sin[2(-1)] - \sin[-1]}{\pi(-1)} = 0.02158$

$h[3] = h_d[3] = \frac{1}{\pi} = 0.3183$

$h[4] = h_d[4] = 0.02158$

$h[5] = h_d[5] = -0.26516$

$h[6] = h_d[6] = -0.04462$

$\Rightarrow |H(\omega)| = h\left(\frac{N-1}{2}\right) + 2\sum_{n=0}^{\left(\frac{N-3}{2}\right)} h[n] \cos\left[\omega\left(n - \frac{N-1}{2}\right)\right]$

$= h(3) + 2\sum_{n=0}^{2} h[n] \cos[\omega(n-3)]$

$= h(3) + 2\,(h(0)\cos(3\omega) + h(1)\cos(2\omega) + h(2)\cos(\omega)$

$= h(3) + 2h(0)\cos(3\omega) + 2h(1)\cos(2\omega) + 2h(2)\cos\omega$

$= 0.3183 + 2[-0.04462]\cos(3\omega) + 2[-0.26516]\cos(2\omega)$

$+ 2[0.02158]\cos\omega$

$|H(\omega)| = 0.3183 - [0.08924]\cos 3\omega - [0.53032]\cos 2\omega + [0.04316]\cos \omega$

* $H(0) = 0.3183 - 0.08924 - 0.53032 + 0.04316$

$\boxed{H(0) = -0.2581}$

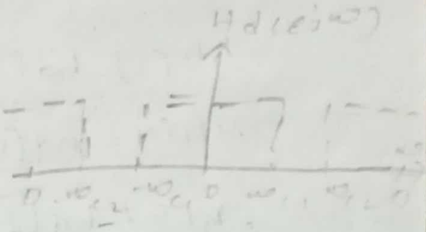
* $H(1) = 0.3183 - (0.08924)\cos(3) - (0.53032)\cos(2) + (0.04316)\cos(1)$

$= 0.3183 + 0.0883 + 0.2206 + 0.02331$

$\boxed{H(1) = 0.65051}$

Q: Design a symmetric BRF which the desired frequency response is $H_d(e^{j\omega}) = \begin{cases} e^{-j\omega T}; & 0 \le |\omega| \le \omega_{c1} \\ 0; & \omega_{c2} \le |\omega| \le \pi \end{cases}$ o.w

with $\omega_{c1} = 1$ rad/s, $\omega_{c2} = 2$ rad/sec, $N = 7$.



Sol: $h_d[n] = \frac{1}{2\pi}\int_{-\pi}^{\pi} H_d(e^{j\omega}) \cdot e^{j\omega n}\, d\omega$

$= \frac{1}{2\pi}\left[\int_{-\pi}^{-\omega_{c2}} + \int_{-\omega_{c1}}^{\omega_{c1}} + \int_{\omega_{c2}}^{\pi}\right]$

$= \frac{1}{2\pi}\left[\left[\frac{e^{j\omega(n-T)}}{j(n-T)}\right]_{-\pi}^{-\omega_{c2}} + \left[\frac{e^{j\omega(n-T)}}{j(n-T)}\right]_{-\omega_{c1}}^{+\omega_{c1}} + \left[\frac{e^{j\omega(n-T)}}{j(n-T)}\right]_{\omega_{c2}}^{\pi}\right]$

$= \frac{1}{2\pi}\left\{\frac{e^{-j\omega_{c2}(n-T)} - e^{-j\pi(n-T)} + e^{j\omega_{c1}(n-T)} - e^{-j\omega_{c1}(n-T)} + e^{j\pi(n-T)} - e^{j\omega_{c2}(n-T)}}{j(n-T)}\right]$

$= \frac{1}{2j\pi(n-T)}\left[2j\sin[\omega_{c1}(n-T)] + 2j\sin(\pi(n-T)) - 2j\sin[\omega_{c2}(n-T)]\right]$

$\boxed{h_d[n] = \frac{\sin(n-T)}{\pi(n-T)} + \frac{\sin\pi(n-T)}{\pi(n-T)} - \frac{\sin[2(n-T)]}{\pi(n-T)}}\ ; n \ne T$

If $n = T \Rightarrow \frac{1}{\pi} + \frac{\pi}{\pi} - \frac{2}{\pi} = 1 + \frac{1}{\pi} - \frac{2}{\pi}$

$\boxed{h_d[n] = 1 - \frac{1}{\pi}}\ ; n = T$

$\therefore h_d[n] = \{ \frac{1}{\pi(n-T)}[\sin(n-T) + \sin\pi(n-T) - \sin[2(n-T)]]; n \ne$

$\rightarrow \tau = \frac{N-1}{2} = \frac{7-1}{2} = 3$

and, $h_d(n) = \begin{cases} \frac{1}{\pi(n-\tau)} [\sin(n-\tau) + \sin[\pi(n-\tau)] - \sin[2(n-\tau)]] ; n \neq \tau \\ 1 - \frac{1}{\pi} ; n = \tau \end{cases}$

· using window technique, we have:

$\boxed{h[n] = h_d[n] \, w_R[n]}$; $w_R[n] = 1$; $0 \le n \le N-1$

· $h[0] = h_d[0] = \frac{1}{\pi(-3)} [\sin(-3) + \sin[\pi(-3)] - \sin[2(-3)]] = 0.044$

· $h[1] = h_d[1] = \frac{1}{\pi(-2)} [\sin(-2) + \sin[\pi(-2)] - \sin[2(-2)]] = 0.2651$

· $h[2] = h_d[2] = \frac{1}{\pi(-1)} [\sin(-1) + \sin[\pi(-1)] - \sin[2(-1)]] = -0.02$

· $h[3] = h_d[3] = 1 - \frac{1}{\pi} = 0.681$

· $h[4] = h_d[4] = -0.02$

· $h[5] = h_d[5] = 0.2651$

· $h[6] = h_d[6] = 0.044$

$\Rightarrow H(e^{j\omega}) = h\left(\frac{N-1}{2}\right) + 2\sum_{n=0}^{\frac{N-3}{2}} h[n] \cdot \cos\left[\omega\left(n - \frac{N-1}{2}\right)\right]$

$= h(3) + 2\sum_{n=0}^{2} h[n] \cos[\omega(n-3)]$

$= h[3] + 2(h(0)\cos(3\omega) + h(1)\cos 2\omega) + h(2)\cos(\omega))$

$= 0.681 + 2[0.044]\cos 3\omega + 2[0.2651]\cos 2\omega + 2[-0.02]\cos\omega$

$\boxed{H(e^{j\omega}) = 0.681 + [0.088]\cos 3\omega + [0.5302]\cos 2\omega - [0.04]\cos\omega}$

* $H(0) = 0.681 + 0.088 + 0.5302 - 0.04 = 1.2592$

* $H(1) = 0.681 + [0.088][\cos 3] + (0.5302)(\cos 2) - (0.04)(\cos 1)$

$= 0.681 + (-0.08711) + (-0.2206) - (0.0216)$

$\boxed{H(1) = 0.35169}$

→ DESIGNING OF FIR USING SAMPLING METHOD:-

① $H[k]$ can be obtained by sampling the frequency response $H_d(e^{j\omega})$ at 'N' points.

$$\boxed{H[k] = H_d(e^{j\omega})\Big|_{\omega = \frac{2k\pi}{N}}}$$

② The filter coefficients are obtained by using IDFT.

$$h(n) = IDFT\,[H(k)] = \frac{1}{N}\sum_{k=0}^{N-1} H[k]\cdot e^{j\left(\frac{2\pi}{N}\right)kn} \quad -①$$

③ $h(n)$ to be real rather than complex, to apply this constraint on ① apply certain conditions on $H[k]$.

⇒ For real valued $h[n]$:

- If $N = \text{ODD}$; $h[n] = \frac{1}{N}\left[H(0) + 2\sum_{k=1}^{\frac{N-1}{2}} Re\left[H(k)\, e^{+j\frac{2\pi}{N}kn}\right]\right]$; $n = 0 \ldots N-1$
- If $N = \text{EVEN}$; $h[n] = \frac{1}{N}\left[H(0) + 2\sum_{k=1}^{\frac{N}{2}-1} Re\left[H(k)\, e^{j\left(\frac{2\pi}{N}\right)kn}\right]\right]$; $n = 0 \ldots N-1$

where, N = No. of samples.

Q: Design a linear phase low pass FIR filter with desired frequency response $H_d(e^{j\omega}) = e^{-j\omega\left(\frac{N-1}{2}\right)}$; $0 \le |\omega| \le \frac{\pi}{2}$
0 ; $\frac{\pi}{2} \le |\omega| \le \pi$
with $N = 7$.

Sol: Given, $H_d(e^{j\omega}) = e^{-j\omega\left(\frac{N-1}{2}\right)}$; $0 \le |\omega| \le \pi/2$
0 ; $\frac{\pi}{2} \le |\omega| \le \pi$; $N = 7$

$$\Rightarrow H_d(e^{j\omega}) = \begin{cases} e^{-j(3\omega)} & ; 0 \le |\omega| \le \pi/2 \\ 0 & ; \frac{\pi}{2} \le |\omega| \le \pi \end{cases}$$

①, $H[k] = H_d(e^{j\omega})\Big|_{\omega = \frac{2k\pi}{N}}$

$$H[k] = \begin{cases} e^{-j\frac{6k\pi}{N}} & ; 0 \le k \le 7/4 \\ 0 & ; \frac{7}{4} \le k \le \frac{7}{2} \end{cases}$$

⇒ $H(0) = 1$
⇒ $H(1) = e^{-j\left(\frac{6}{7}\right)\pi}$
⇒ $H(2) = 0$
⇒ $H(3) = 0$

②, $h(n) = \frac{H(0)}{N} + \frac{2}{N}\sum_{k=1}^{N-1} Re\left[H(k)\cdot e^{j\left(\frac{2\pi}{N}\right)kn}\right]$

$$= \frac{1}{7} + \frac{2}{7}Re\left[H[1]e^{j\left(\frac{2\pi}{7}\right)n} + H[2]e^{j\ldots} + H(3)e^{\ldots}\right]$$

$$= \frac{1}{7} + \frac{2}{7}Re\left[e^{-j\left(\frac{6}{7}\right)\pi}\cdot e^{j\left(\frac{2\pi}{7}\right)n}\right] = \frac{1}{7} + \frac{2}{7}Re\left[e^{j\left(\frac{2\pi - 6}{7}\right)n}\right]$$

$$\boxed{h[n] = \frac{1}{7} + \frac{2}{7}\cos\left[\left(\frac{2\pi n - 6\pi}{7}\right)\right]}$$

$$\boxed{h[0] = \frac{1}{7} + \frac{2}{7}\cos(0) = \frac{1}{7} + \frac{2}{7} = \frac{3}{7} = 0.428}$$

* $h[0] = \frac{1}{7} + \frac{2}{7}\cos\left(-\frac{6\pi}{7}\right)$

$= \frac{1}{7} + \frac{2}{7}(-0.9)$

$= \frac{1}{7}[1 - 1.8]$

$= \frac{-0.8}{7}$

$\boxed{h[0] = -0.11}$

* $h[1] = \frac{1}{7} + \frac{2}{7}\cos\left[-\frac{4\pi}{7}\right]$

$= \frac{1}{7} + \frac{2}{7}(-0.22)$

$= \frac{1}{7}(1 - 0.44)$

$= \frac{0.56}{7}$

$\boxed{h[1] = 0.0793}$

$1.00 - 0.44 = 0.56$

* $h[2] = \frac{1}{7} + \frac{2}{7}\cos\left[-\frac{2\pi}{7}\right]$

$= \frac{1}{7} + \frac{2}{7}(0.62)$

$= \frac{1}{7}(1 + 2(0.62))$

$= \frac{1}{7}[1 + 1.24]$

$= 2.24/7$

$\boxed{h[2] = 0.321}$

* $h[3] = \frac{1}{7} + \frac{2}{7}\cos(0)$

$= \frac{1}{7} + \frac{2}{7}$

$= \frac{3}{7}$

$= 0.428571$

$\boxed{h[3] = 0.4286}$ //

$\rightarrow H(e^{j\omega})$

* REALIZATION OF FIR FILTERS :-

① DF :- W.K.T , For a FIR Filter, $H(z) = \sum_{k=0}^{M} a_k z^{-k}$

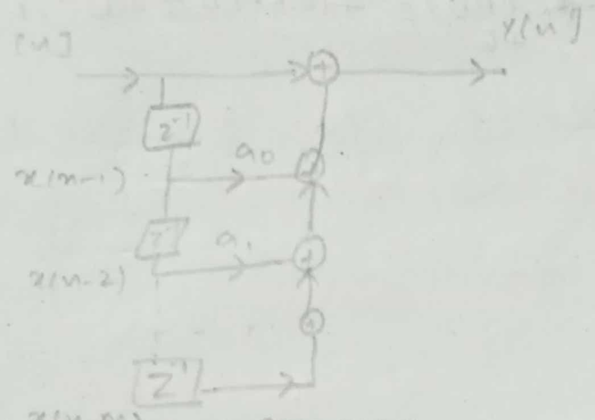
$$\Rightarrow \frac{Y(z)}{X(z)} = \sum_{k=0}^{M} a_k z^{-k}$$

$$\Rightarrow Y(z) = \sum_{k=0}^{M} a_k z^{-k} \cdot X(z)$$

$$Y(z) = a_0 X(z) + a_1 z^{-1} X(z) + \dots + a_M z^{-M} X(z)$$

Taking I.Z.T, we get :

$$y[n] = a_0 x[n] + a_1 x(n-1) + \dots + a_M x(n-M)$$



DIRECT FORM

② CASCADE FORM :-

$$H(z) = \sum_{k=0}^{M} a_k z^{-k}$$

$$Y(z) = \sum_{k=0}^{M} a_k z^{-k} \cdot X(z)$$

$$H(z) = H_1(z) \cdot H_2(z) \dots H_K(z)$$

where, $H_k(z) = a_{k0} + a_{k1} z^{-1} + a_{k2} z^{-2}$

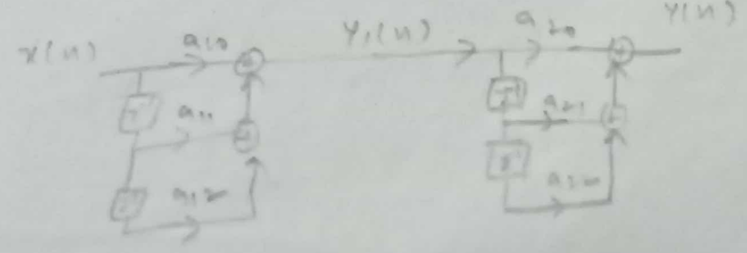
$$H(z) = H_1(z) \cdot H_2(z) = \frac{Y_1(z)}{X(z)} \cdot \frac{Y(z)}{Y_1(z)}$$

$$\Rightarrow \frac{Y_1(z)}{X(z)} = H_1(z) = a_{10} + a_{11} z^{-1} + a_{12} z^{-2}$$

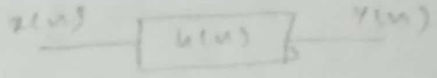
and $\frac{Y(z)}{Y_1(z)} = H_2(z) = a_{20} + a_{21} z^{-1} + a_{22} z^{-2}$

$$\Rightarrow y_1(n) = a_{10} x(n) + a_{11} x(n-1) + a_{12} x(n-2)$$

and $y(n) = a_{20} y_1(n) + a_{21} y(n-1) + a_{22} y(n-2)$



→ Tapped delay line (or) Transversal :- Tapped at discrete points, the signal input appeared in delayed versions i.e, "Transversal" in Time



- $H(z) = \sum_{k=0}^{M} a_k z^{-k}$

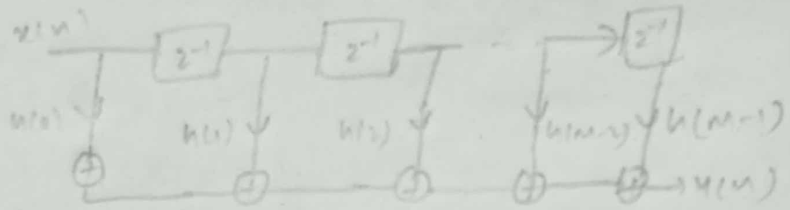
$Y(z) = \sum_{k=0}^{M} a_k z^{-k} \cdot X(z)$

$$\boxed{\Rightarrow y(n) = \sum_{k=0}^{M} a_k \cdot x(n-k)}$$

→ $y(n) = x(n) * h(n)$

$= \sum_{k=0}^{M-1} h(k)\, x(n-k)$

$$\boxed{y(n) = h(0)x(n) + h(1)x(n-1) + \dots + h(M-1)x(n-M-1)}$$



TAPPED DELAY LINE STRUCTURE

③ CANONICAL FORM:

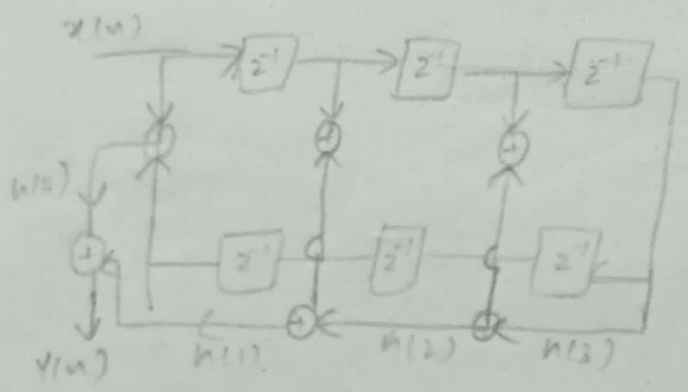
- $y(n) = \sum_{k=0}^{M-1} h(k)\, x(n-k)$

let, $M = 7$

$\Rightarrow y(n) = h(0)x(n) + h(1)x(n-1) + h(2)x(n-2) + h(3)x(n-3) + h(4)x(n-4) + h(5)x(n-5) + h(6)x(n-6)$

- $y(n) = h(0)[x(n) + x(n-6)] + h(1)[x(n-1) + x(n-5)] + h(2)[x(n-2) + x(n-4)] + h(3)x(n-3)$

$\because h(0) = h(6)$
$h(1) = h(5)$
$h(2) = h(4)$



Q: Realise the following system function using minimum number of multipliers.

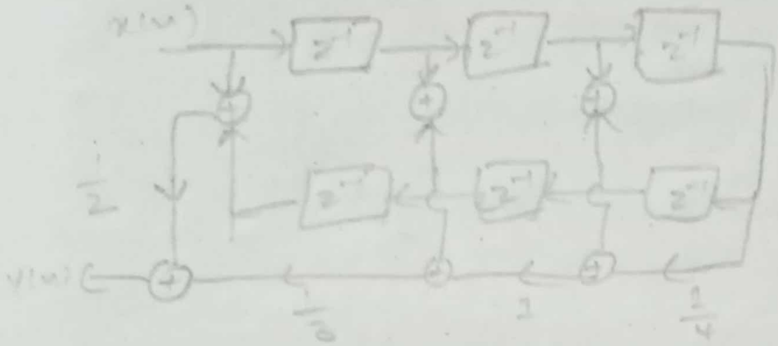
$H(z) = \frac{1}{2} + \frac{1}{3}z^{-1} + z^{-2} + \frac{1}{4}z^{-3} + z^{-4} + \frac{1}{3}z^{-5} + \frac{1}{2}z^{-6}$

sol:: Given,

$$\frac{Y(z)}{X(z)} = \frac{1}{2} + \frac{1}{3} z^{-1} + z^{-2} + \frac{1}{4} z^{-3} + z^{-4} + \frac{1}{3} z^{-5} + \frac{1}{2} z^{-6}$$

$$\rightarrow Y(z) = \frac{1}{2} x(z) + \frac{1}{3} z^{-1} x(z) + z^{-2} x(z) + \frac{1}{4} z^{-3} x(z) + z^{-4} x(z) + \frac{1}{3} z^{-5} x(z) + \frac{1}{2} z^{-6} x(z)$$

$$\overset{IZT}{\Rightarrow} y(n) = \frac{1}{2}(x(n) + x(n-6)] + \frac{1}{3}[x(n-1) + x(n-5)] + 1(x(n-2) + x(n-4)] + \frac{1}{4} x(n-3)$$ //



Q:: Obtain Cascade realization with minimum no. of multipliers for the system function

$$H(z) = \left\{ \frac{1}{2} + z^{-1} + \frac{1}{2} z^{-2} \right] \left[1 + \frac{1}{3} z^{-1} + z^{-2} \right]$$

sol::

$$H(z) = \frac{Y(z)}{X(z)}$$