Northeastern University Seattle

**Name:** Mukaila Rafiu

**Program:** Master's in Economics and Data Science

**Course:** Financial Economics

**Professor:** Florin Bidian

**Presentation Date:** December 9, 2025

---

# Project Title:

## Intangible Capital and the Stability of the Investment–q Relation in U.S. Tech Firms Since 1995

## Project Question

Does replacing standard Tobin's q with a total-q measure that incorporates internally generated intangible capital restore a stable relation between physical investment and q for public U.S. tech firms from 1995 onward?

---

## Background and Motivation

A large empirical literature documents a breakdown of the traditional investment-q relationship in the late twentieth century. Andrei, Mann, and Moyen (2018) revisit this so-called “q-theory failure” and argue that the breakdown is largely the result of measurement error rather than a failure of the theory itself. Using the standard Tobin’s Q, they show that the investment-q regression fits poorly during 1975–1990, with an $R^2$ of only about 6.5%. However, once they correct the measurement of q and incorporate intangible capital in constructing Total q, the fit improves dramatically in the modern period (1995–2015), rising to over 70 percent.

The key idea in their paper is that the growing volatility and dispersion of Tobin's q in the modern economy is not just noise, but actually contains useful information about firms' investment opportunities, as long as intangible capital is properly accounted for. In other words, q becomes more volatile, but also more informative, once the economy becomes more R&D-intensive.

They argue that this informative volatility comes from two main sources. First, innovation creates jump risk in firm value. R&D spending raises the chance of breakthroughs in productivity and profitability, which makes future cash flows more uncertain and more volatile. This increased uncertainty then shows up in market valuations and in Tobin's Q. Second, R&D-intensive firms are constantly learning. Through experimentation, patents, and market feedback, firms continuously update their beliefs about future growth, which leads to frequent and sometimes sharp revisions in their valuations. As a result, q moves around more — and those movements reflect real information about prospects.

As Andrei, Mann, and Moyen (2018) put it, "the firm's increased cash flow volatility from innovations directly feeds into the volatility of Tobin's q valuations, generating a better fit in the investment-q regression." Once intangible capital is included and measurement error is addressed, they conclude that:

- (i) Q-theory works well in the modern economy;
- (ii) Investment responds strongly and consistently to marginal q; and
- (iii) The theory is especially relevant in high-tech and R&D-intensive sectors.

---

## Introduction

Over the past three decades, the investment behavior of publicly listed U.S. technology firms has posed a challenge to standard q-theory. Despite rapid growth in market valuations, physical investment has appeared weakly related to Tobin's Q, particularly since the late 1975 to 1990, with a woeful fit of 6.46%. A leading explanation is that conventional measures of capital and investment omit internally generated intangible assets that are especially important in modern tech firms. According to a recent discovery, when intangible assets are incorporated in the measurement, the Tobin Q revamp with 71.23% fit from 1995 – 2015 (Andrei, Mann, Moyen, 2018).

Peters and Taylor (2014) also demonstrated that incorporating internally generated intangible capital into both investment and capital, their "Total q" substantially strengthens the investment-q relation for U.S. Compustat firms, especially in R&D-intensive industries. However, their analysis does not focus exclusively on public U.S. technology firms, nor does it examine whether total q restores stability to the physical investment-q relation over time or across tech subsectors. (Andrei, Mann, Moyen, 2018) fail to constrain the findings exclusively to US public firms.

This project revisits the investment–q relationship within the U.S. listed technology sector from 1995 onward. Using firm-level data from Compustat and the Peters–Taylor intangible capital measures, i employed standard (Tobin's q) and Total q from Peters and Taylor (2017) in the WRDS database in Compustat data for each firm-year and estimate physical investment regressions by decade and by tech sector. The analysis asks whether replacing standard q with total q stabilizes the slope, explanatory power, and cash-flow sensitivity of physical investment regressions in an era marked by rising intangible intensity.

The results show that Standard q exhibits declining explanatory power and unstable coefficients over time within tech firms, while Total q delivers a more stable and economically meaningful investment-q relation-particularly in intangible-intensive Tech sectors such as Software, Computer equipment, and Electronics. These findings suggest that mismeasurement of capital, rather than a breakdown of q-theory, accounts for much of the observed instability in tech investment behavior, and even at the House race between Standard q and Total q, Total q wins the race with better fits across all the sections.

---

## Research Gaps & Motivation

The questions remain unanswered by the authors:

• They do not restrict attention to public U.S. tech firms only.
• They do not focus on post-1995 as a structural break.
• They do not study the subperiod stability over time of the physical investment–q relation.
• They do not break heterogeneity among R&D-intensive tech firms.
• They do not frame the question as "restoring stability."

In light of this project, which is clearly derivative in measurement, i will respond to the pressing questions that remain untouched.

---

## Data and Sample

U.S. public firms in technology-related industries from Compustat fundamentals and Peter-Taylor data 1995–2024.

## Key Variables

**1. Physical Capital & Investment (Compustat):**

gvkey, fyear, datadate, at, ppegt, capx, xsga, xrd, sale, ceq, lt, dltt, dlc, prcc_f, csho.

**2. Peters–Taylor Total q and Intangible Capital:**

gvkey, fyear, $q_{tot}$, $k_{int}$, $k_{int}^{know}$, $k_{int}^{org}$.

Standard q and Peters–Taylor total q, plus intangible capital stocks, from Peters–Taylor WRDS.

**Main Regressors Of Interest:**

Total q, Standard q (Tobin's q), Standardized q, Totalized q.

**Controls:**

Cash flow, Firm size.

---

# Empirical Strategy

## Baseline Panel Regressions (firm–year level)

We estimate panel regressions of physical investment on alternative measures of Tobin's q, controlling for firm characteristics and unobserved heterogeneity.

*Standard q model*

$$I_{i,t+1}^{p} / K_{it}^{p} = \alpha + \beta_S\, q_{it}^{S} + \gamma X_{it} + \mu_i + \tau_t + \varepsilon_{it}$$

*Total q model*

$$I_{i,t+1}^{p} / K_{it}^{p} = \alpha + \beta_T\, q_{it}^{T} + \gamma X_{it} + \mu_i + \tau_t + \varepsilon_{it}$$

## Definitions

- **$I_{i,t+1}^{p}$**: Physical investment in year t+1
- **$K_{it}^{p}$**: Physical capital stock at the end of year t
- **$q_{it}^{S}$**: Standard Tobin's q at the end of year t
- **$q_{it}^{T}$**: Peters–Taylor total q at the end of year t
- **$X_{it}$**: Vector of control variables (e.g., cash flow, leverage, size)
- **$\mu_i$**: Firm fixed effects
- **$\tau_t$**: Year fixed effects
- **$\varepsilon_{it}$**: Idiosyncratic error term

## Comparison Metrics

- **Model fit:** $R^2$ and within-$R^2$
- **Economic and statistical significance:** $\beta_S$ vs $\beta_T$
- **Subperiod stability:** Pre–2000, Post–2000, Pre–2008, Post–2008
- **Technology subsamples:** Computers, Electronics, Software

In this project, the focus is on within-$R^2$ as the primary metric in measuring the investment and q relation, because the investment-q relation is fundamentally about how changes in firm valuation affect changes in firm investment over time. Overall $R^2$ is reported for completeness; however is largely driven by firm and time fixed effects.

## Interaction with Intangible Intensity

$$I_{i,t+1}^{p} / K_{it}^{p} = \alpha + \beta_1\, q_{it}^{T} + \beta_2(q_{it}^{T} \times \mathrm{IntangibleIntensity}_{it}) + \gamma X_{it} + \mu_i + \tau_t + \varepsilon_{it}$$

$$\mathrm{IntangibleIntensity}_{it} = K_{it}^{int} / (K_{it}^{tan} + K_{it}^{int})$$

## Heterogeneity Analysis

- Firm age (young vs mature)
- R&D intensity (high vs low)
- Intangible intensity (high vs low)

## Robustness Checks

- Alternative FE structures
- Alternative q constructions
- Sample restrictions
- Dynamic specifications

# Empirical Analysis Roadmap

## 1. Summary Statistics

- Summary statistics including Tobin's q, Total q (Peters–Taylor), investment, capital, cash flow, and intangible capital.

## 2. Baseline PyFixest Regressions

- Standard q regressions.
- Total q (Peters–Taylor) regressions.

## 3. Subperiod Stability Analysis

- Pre vs Post 2000 — Standard q.
- Pre vs Post 2000 — Total q.
- Pre vs Post 2008 — Standard q (financial crisis).
- Pre vs Post 2008 — Total q (financial crisis).
- Complete subperiod stability analysis across all splits.

## 4. Heterogeneity Analysis (H3)

**By Firm Age**

- Young vs Mature firms.
- Panel A: Young vs Mature Firms regressions.
- Panel B: High vs Low R&D Intensity regressions.

**By R&D Intensity**

- Construct R&D intensity measure.
- High vs Low R&D regressions.
- Panel B: High vs Low R&D Intensity.

## 5. Horse Race Between Valuation Measures

- Joint regressions including Standard q, Total q together, Standardized q, Totalized q.

## 6. Robustness Checks

**Negative Cash Flow Robustness**

- Run robustness checks, including Standard q (Tobin's q).
- Run robustness checks, including Total q.
- Display robustness tables for Standard q (Tobin's q).
- Display robustness tables for Total q.

```
#1. Imports & Setup
import pandas as pd
import numpy as np
import os
import wrds
import matplotlib.pyplot as plt
from datetime import datetime
from plotnine import *
import pyfixest as pf
import statsmodels.api as sm
from linearmodels.panel import PanelOLS

#2 WRDS data Download

conn = wrds.Connection()

#3. Fetching Compustat data from the WRDS database api (Physical Capital & Investment)
comp_query = f"""
```

```
SELECT gvkey, fyear, datadate, at, ppegt, capx, xsga, xrd, sale, ceq, lt, dltt, dlc, prcc_f, csho
FROM comp.funda
WHERE indfmt='INDL' AND datafmt='STD' AND consol='C' AND curcd='USD'
AND fyear BETWEEN {1995} AND {2024}
"""
compustat = conn.raw_sql(comp_query)
compustat.round(3)
```

| | gvkey | fyear | datadate | at | ppegt | capx | xsga | xrd | sale | ceq | lt | dltt | dlc | prcc_f | csho |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **0** | 001004 | 1995 | 1996-05-31 | 437.846 | 129.49 | 7.547 | 58.323 | <NA> | 504.99 | 204.635 | 233.211 | 118.292 | 1.474 | 22.125 | 15.998 |
| **1** | 001010 | 1995 | 1995-12-31 | 2015.8 | 1502.5 | 109.3 | 43.4 | 1.1 | 407.2 | 473.2 | 1536.2 | 710.5 | 316.2 | <NA> | 0.015 |
| **2** | 001013 | 1995 | 1995-10-31 | 601.083 | 191.537 | 32.456 | 196.757 | 66.46 | 586.222 | 510.866 | 90.217 | 0.0 | 0.41 | 40.0 | 62.737 |
| **3** | 001019 | 1995 | 1995-12-31 | 28.487 | 41.034 | 1.946 | 8.202 | <NA> | 27.492 | 13.528 | 14.959 | 2.014 | 0.222 | 115.0 | 0.198 |
| **4** | 001021 | 1995 | 1995-06-30 | 11.79 | 6.011 | 0.131 | 7.637 | 0.751 | 26.589 | 4.26 | 6.252 | 2.736 | 0.644 | 0.875 | 6.449 |
| **...** | ... | ... | ... | ... | ... | ... | ... | ... | ... | ... | ... | ... | ... | ... | ... |
| **298491** | 355398 | 2024 | 2024-12-31 | 684.425 | 567.079 | 34.742 | 79.644 | 0.0 | 778.308 | 360.028 | 322.764 | 171.219 | 41.407 | 28.76 | 65.03 |
| **298492** | 356128 | 2024 | 2024-12-31 | 15967.927 | 537.105 | 182.467 | 517.64 | 208.823 | 4826.644 | 2892.041 | 12969.834 | 0.263 | 262.67 | 94.71 | 190.016 |
| **298493** | 356859 | 2024 | 2024-12-31 | 2207.739 | 241.14 | 27.575 | 225.99 | <NA> | 1097.393 | 1151.172 | 1056.567 | 787.801 | 24.778 | 12.75 | 122.476 |
| **298494** | 366911 | 2024 | 2024-12-31 | 23805.0 | 16630.0 | 642.0 | 862.0 | <NA> | 11704.0 | 9915.0 | 13891.0 | 9196.0 | 348.0 | <NA> | <NA> |
| **298495** | 369350 | 2024 | 2024-12-31 | 5715.961 | 5678.69 | 332.336 | 1886.339 | 95.249 | 8227.629 | 2876.098 | 2816.05 | 106.637 | 88.002 | <NA> | <NA> |

298496 rows × 15 columns

```
#4. Fetching Peters–Taylor Total q and Intangible Capital from WRDS data base api
pt_query = f"""
SELECT gvkey, fyear, q_tot, k_int, k_int_know, k_int_org
FROM totalq.total_q
WHERE fyear BETWEEN {1995} AND {2024}
"""
pt = conn.raw_sql(pt_query)
pt.round(3)
```

| | gvkey | fyear | q_tot | k_int | k_int_know | k_int_org |
|---|---|---|---|---|---|---|
| **0** | 001004 | 1995.0 | 0.615 | 91.113 | 0.0 | 85.271 |
| **1** | 001004 | 1996.0 | 1.183 | 95.568 | 0.0 | 89.915 |
| **2** | 001004 | 1997.0 | 1.572 | 125.286 | 0.0 | 98.72 |
| **3** | 001004 | 1998.0 | 0.642 | 149.054 | 0.0 | 108.962 |
| **4** | 001004 | 1999.0 | 0.189 | 158.58 | 0.0 | 119.74 |
| **...** | ... | ... | ... | ... | ... | ... |
| **271685** | 353945 | 2021.0 | <NA> | 216.882 | 124.15 | 81.986 |
| **271686** | 353945 | 2022.0 | 1.995 | 291.299 | 165.048 | 115.711 |
| **271687** | 353945 | 2023.0 | 2.384 | 440.401 | 203.53 | 154.131 |
| **271688** | 356128 | 2022.0 | 5.723 | 485.413 | 319.62 | 132.708 |
| **271689** | 356128 | 2023.0 | 5.214 | 803.807 | 478.435 | 180.474 |

271690 rows × 6 columns

```
#5. Industry Classification (Tech Firms)
sic_query = "SELECT DISTINCT gvkey, sic FROM comp.company"
sic = conn.raw_sql(sic_query)
sic.round(3)
```

| | gvkey | sic |
|---|---|---|
| **0** | 296885 | 2836 |
| **1** | 122138 | 3576 |
| **2** | 065194 | 3825 |
| **3** | 062927 | 6035 |
| **4** | 037826 | 8082 |
| **...** | ... | ... |
| **56697** | 133564 | 3827 |
| **56698** | 166583 | 3827 |
| **56699** | 013252 | 1040 |
| **56700** | 051007 | 6722 |
| **56701** | 018169 | 5031 |

56702 rows × 2 columns

```
#6. Saving the Raw Data
```

```
os.makedirs("data_raw", exist_ok=True)
compustat.to_csv("data_raw/comp_funda.csv", index=False)
pt.to_csv("data_raw/peters_taylor_totalq.csv", index=False)
sic.to_csv("data_raw/sic_codes.csv", index=False)

#7. Merging and Constructing Panel
compustat = pd.read_csv("data_raw/comp_funda.csv")
pt = pd.read_csv("data_raw/peters_taylor_totalq.csv")
sic = pd.read_csv("data_raw/sic_codes.csv")

panel = (compustat.merge(sic, on="gvkey", how="left").merge(pt, on=["gvkey", "fyear"], how="left"))
panel.round(3)
```

| | gvkey | fyear | datadate | at | ppegt | capx | xsga | xrd | sale | ceq | lt | dltt | dlc | prcc_f | csho | sic | q_tot |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **0** | 1004 | 1995 | 1996-05-31 | 437.846 | 129.490 | 7.547 | 58.323 | NaN | 504.990 | 204.635 | 233.211 | 118.292 | 1.474 | 22.125 | 15.998 | 5080.0 | 0.615 |
| **1** | 1010 | 1995 | 1995-12-31 | 2015.800 | 1502.500 | 109.300 | 43.400 | 1.100 | 407.200 | 473.200 | 1536.200 | 710.500 | 316.200 | NaN | 0.015 | 3743.0 | NaN |
| **2** | 1013 | 1995 | 1995-10-31 | 601.083 | 191.537 | 32.456 | 196.757 | 66.460 | 586.222 | 510.866 | 90.217 | 0.000 | 0.410 | 40.000 | 62.737 | 3661.0 | 4.177 |
| **3** | 1019 | 1995 | 1995-12-31 | 28.487 | 41.034 | 1.946 | 8.202 | NaN | 27.492 | 13.528 | 14.959 | 2.014 | 0.222 | 115.000 | 0.198 | 7380.0 | 0.262 |
| **4** | 1021 | 1995 | 1995-06-30 | 11.790 | 6.011 | 0.131 | 7.637 | 0.751 | 26.589 | 4.260 | 6.252 | 2.736 | 0.644 | 0.875 | 6.449 | 3844.0 | -0.044 |
| **...** | ... | ... | ... | ... | ... | ... | ... | ... | ... | ... | ... | ... | ... | ... | ... | ... | ... |
| **298491** | 355398 | 2024 | 2024-12-31 | 684.425 | 567.079 | 34.742 | 79.644 | 0.000 | 778.308 | 360.028 | 322.764 | 171.219 | 41.407 | 28.760 | 65.030 | 5812.0 | NaN |
| **298492** | 356128 | 2024 | 2024-12-31 | 15967.927 | 537.105 | 182.467 | 517.640 | 208.823 | 4826.644 | 2892.041 | 12969.834 | 0.263 | 262.670 | 94.710 | 190.016 | 6141.0 | NaN |
| **298493** | 356859 | 2024 | 2024-12-31 | 2207.739 | 241.140 | 27.575 | 225.990 | NaN | 1097.393 | 1151.172 | 1056.567 | 787.801 | 24.778 | 12.750 | 122.476 | 8734.0 | NaN |
| **298494** | 366911 | 2024 | 2024-12-31 | 23805.000 | 16630.000 | 642.000 | 862.000 | NaN | 11704.000 | 9915.000 | 13891.000 | 9196.000 | 348.000 | NaN | NaN | 3241.0 | NaN |
| **298495** | 369350 | 2024 | 2024-12-31 | 5715.961 | 5678.690 | 332.336 | 1886.339 | 95.249 | 8227.629 | 2876.098 | 2816.050 | 106.637 | 88.002 | NaN | NaN | 2024.0 | NaN |

298496 rows × 20 columns

```
#8 Converting all variables from wide to long format for easy filtering and plotting
all_variables = panel.melt(id_vars=['gvkey', 'fyear', 'datadate'],
value_vars=['at', 'ppegt', 'capx', 'xsga', 'xrd', 'sale', 'ceq', 'lt', 'dltt', 'dlc', 'prcc_f', 'csho', 'sic',
        'q_tot', 'k_int', 'k_int_know', 'k_int_org'],
var_name='variable', value_name='value')

#9 Checking the Company 1004
print("First few gvkeys in data:")
print(panel['gvkey'].unique()[:10])
```

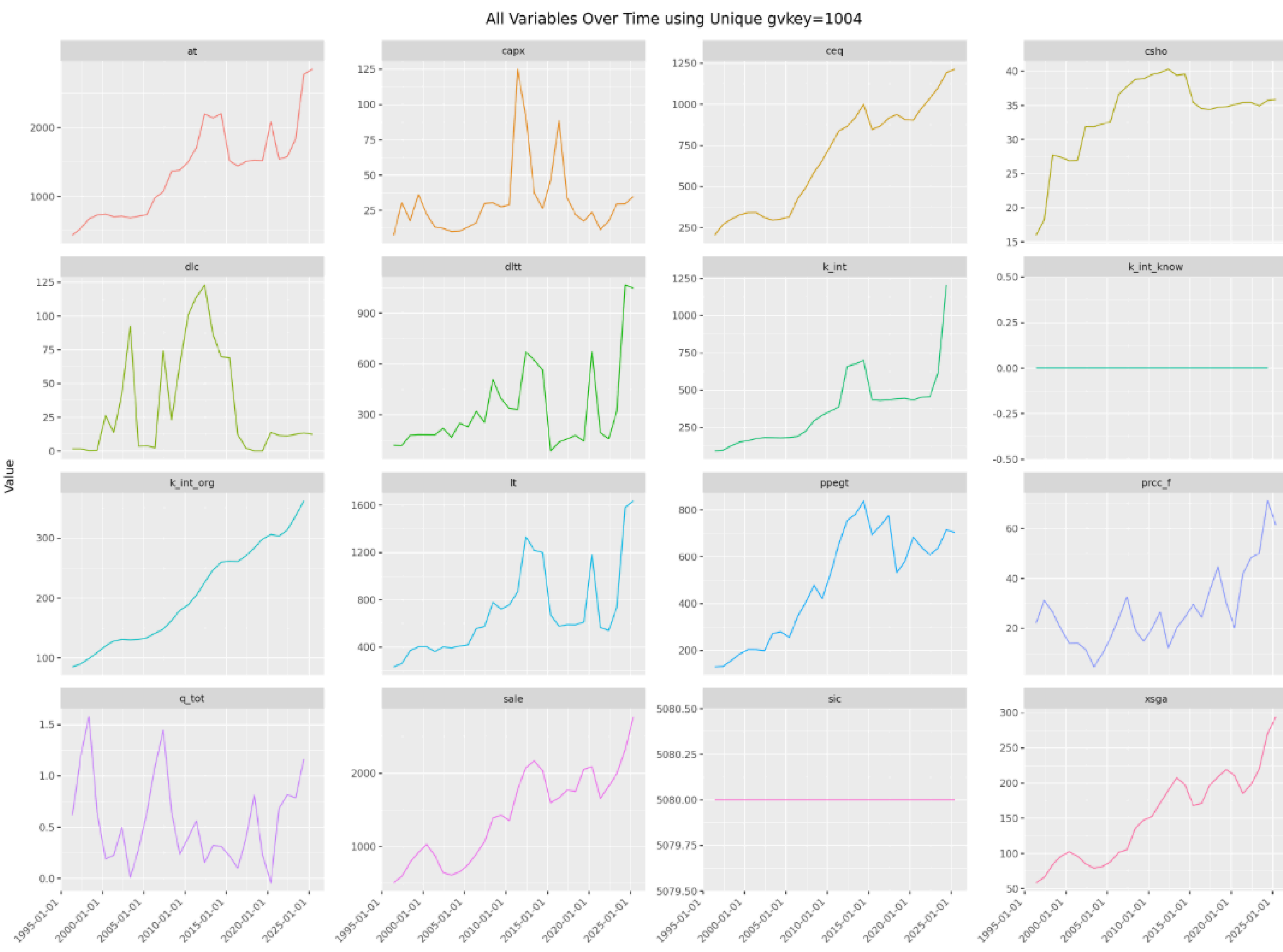
```
First few gvkeys in data:
[ 1004  1010  1013  1019  1021  1025  1034  1036  1037 14770]
```

```
#10 Converting datadate to datetime
all_variables['datadate'] = pd.to_datetime(all_variables['datadate'])
```

```
#11 Plot all variables for gvkey 1004
company_1004 = all_variables.query('gvkey == 1004').dropna()
print(f"Variables available for gvkey 1004:")
print(company_1004['variable'].unique())
print(f"\nNumber of observations: {len(company_1004)}")
```

```
Variables available for gvkey 1004:
['at' 'ppegt' 'capx' 'xsga' 'sale' 'ceq' 'lt' 'dltt' 'dlc' 'prcc_f' 'csho'
 'sic' 'q_tot' 'k_int' 'k_int_know' 'k_int_org']

Number of observations: 476
```

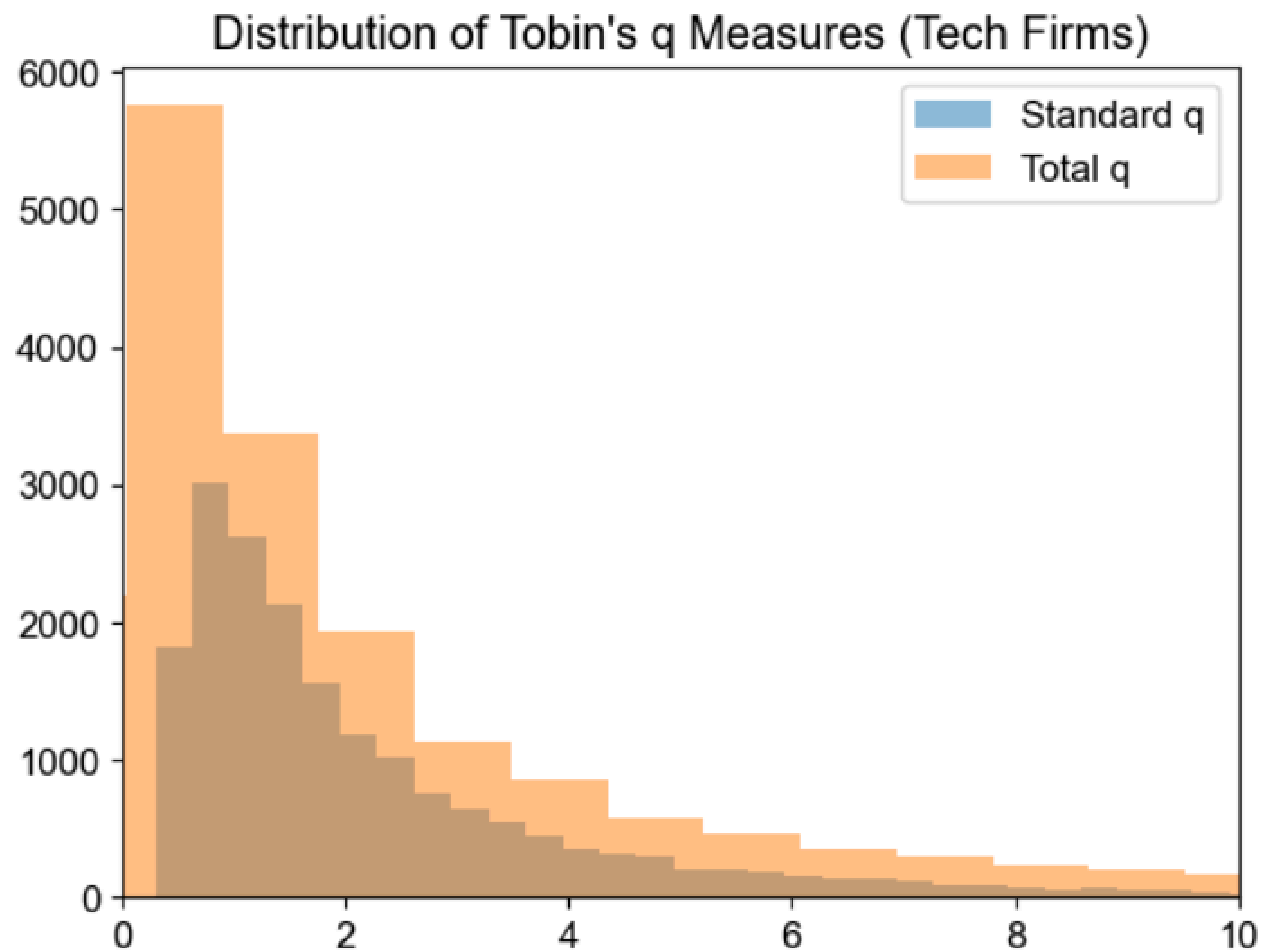
```
#12 Creating plots with all the variables using company 1004 as the leading company (Unique)
(ggplot(company_1004)
 + aes(x='datadate', y='value', color='variable')
 + geom_line()
 + facet_wrap('~variable', scales='free_y', ncol=4)  # 4 columns
 + labs(x='Date', y='Value', title='All Variables Over Time using Unique gvkey=1004')
 + theme(legend_position='none', figure_size=(16, 12), axis_text_x=element_text(rotation=45, hjust=1)))
```




#13. Tech companies on SIC

```
panel["sic"] = panel["sic"].astype("Int64")
panel["is_tech"] = (
    ((panel["sic"] >= 3570) & (panel["sic"] <= 3579)) |   # Computer equipment
    ((panel["sic"] >= 3650) & (panel["sic"] <= 3699)) |   # Electronics
    ((panel["sic"] >= 7370) & (panel["sic"] <= 7379))     # Software
)
panel = panel[panel["is_tech"]]
#Following the literature (Peters and Taylor, 2017),
#We define technology firms as those with SIC codes in the following ranges:
#1. 3570-3579 (computer equipment).
#2. 3650-3699 (electronics and electrical equipment).
#3. 7370-7379 (software and computer services).

#14. Construct Tobin's q Measures
#(Standard q)
panel["mkt_equity"] = panel["prcc_f"] * panel["csho"]
panel["total_debt"] = panel["dltt"].fillna(0) + panel["dlc"].fillna(0)
panel["mkt_value_assets"] = panel["mkt_equity"] + panel["total_debt"]
panel["Standard_q"] = panel["mkt_value_assets"] / panel["at"]
panel = panel[(panel["Standard_q"] > 0) & (panel["Standard_q"] < 50)]

#15 Total q
panel["Total_q"] = panel["q_tot"]

#16. Investment, Capital, and Cash Flow
panel = panel[(panel["at"] > 0) & (panel["ppegt"] > 0)]
panel = panel.sort_values(["gvkey", "fyear"])
panel["inv_phys"] = panel["capx"]
panel["k_phys"] = panel["ppegt"]
panel["i_over_k"] = panel["inv_phys"] / panel["k_phys"]
panel["Investment_lead"] = panel.groupby("gvkey")["i_over_k"].shift(-1)
panel["Cash_flows"] = (panel["xrd"].fillna(0) + panel["xsga"].fillna(0)) / panel["at"]

#17. Intangible Intensity
panel["K_tang"] = panel["at"] - panel["k_int"].fillna(0)
panel["intensity_int"] = panel["k_int"] / (panel["k_int"] + panel["K_tang"])
panel = panel[panel["intensity_int"].between(0, 1)]

#18. Trimming and Winsorization
panel = panel[panel["i_over_k"].between(-2, 2)]
panel = panel[panel["Investment_lead"].between(-2, 2)]
def winsorize(x, p=0.01):
    lo, hi = x.quantile(p), x.quantile(1 - p)
    return x.clip(lo, hi)
for col in ["inv_phys", "k_phys", "i_over_k", "Investment_lead", "Total_q", "Standard_q", "Cash_flows", "intensity_int"]:
    panel[col] = winsorize(panel[col])

#19 Distribution of Standard q vs Total q in (High Value, Low Effort)
```

```
plt.hist(panel["Standard_q"], bins=50, alpha=0.5, label="Standard q")
plt.hist(panel["Total_q"], bins=50, alpha=0.5, label="Total q")
plt.xlim(0, 10)
plt.legend()
plt.title("Distribution of Tobin's q Measures (Tech Firms)")
plt.show()
```




```
#20 summary statistics, including Tobin’s Q
vars_for_table = {"inv_phys": "Capital expenditures",
   "k_phys": "Gross PP&E",
   "i_over_k": "Investment rate",
   "Investment_lead": "Lead investment rate",
   "Standard_q": "Tobin's q",
   "Total_q": "Total q",
   "Cash_flows": "Cash flow",
   "intensity_int": "Intangible intensity",}

sub = panel[list(vars_for_table.keys())].copy()
summary = sub.agg(["mean", "median", "std"]).T
summary.index = [vars_for_table[v] for v in summary.index]
summary.columns = ["Mean", "Median", "Std. dev."]
print(summary.round(3).to_markdown())
```

```
|                        |    Mean |   Median |   Std. dev. |
|:-----------------------|--------:|---------:|------------:|
| Capital expenditures | 108.012 |    6.63  |     374.485 |
| Gross PP&E           | 971.59  |   42.961 |    3583.88  |
| Investment rate      |   0.188 |    0.136 |       0.165 |
| Lead investment rate |   0.164 |    0.122 |       0.141 |
| Tobin's q            |   2.565 |    1.602 |       2.785 |
| Total q              |   3.337 |    1.223 |       6.375 |
| Cash flow            |   0.348 |    0.313 |       0.219 |
| Intangible intensity |   0.548 |    0.556 |       0.256 |
```

```
#21 Summary Statistics Visualization
fig, ax = plt.subplots(figsize=(12, 6))
ax.axis('off')

table = ax.table(cellText=summary.round(3).values, colLabels=summary.columns, rowLabels=summary.index,
loc='center', cellLoc='left')
table.auto_set_font_size(False)
table.set_fontsize(10)
table.scale(1, 2.5)

# Color header
for i in range(len(summary.columns)):
    table[(0, i)].set_facecolor('#4472C4')
    table[(0, i)].set_text_props(weight='bold', color='white')

plt.title('Summary Statistics (N = 18,350 firm-years)', fontsize=14, weight='bold', pad=20)
plt.tight_layout()
plt.show()
```

**Summary Statistics (N = 18,350 firm-years)**

| | Mean | Median | Std. dev. |
|---|---|---|---|
| Capital expenditures | 108.012 | 6.63 | 374.485 |
| Gross PP&E | 971.59 | 42.961 | 3583.88 |
| Investment rate | 0.188 | 0.136 | 0.165 |
| Lead investment rate | 0.164 | 0.122 | 0.141 |
| Tobin's q | 2.565 | 1.602 | 2.785 |
| Total q | 3.337 | 1.223 | 6.375 |
| Cash flow | 0.348 | 0.313 | 0.219 |
| Intangible intensity | 0.548 | 0.556 | 0.256 |

From the summary statistics, Capital expenditures reports a mean of 108 and a median of only 6.6, while gross PP&E has a mean of 972 and a median of 43. This large gap between means and medians. It indicates a highly right-skewed distribution. This reflects the presence of a small number of very large firms alongside many smaller ones, a typical charatcteristics of the U.S. tech sector.

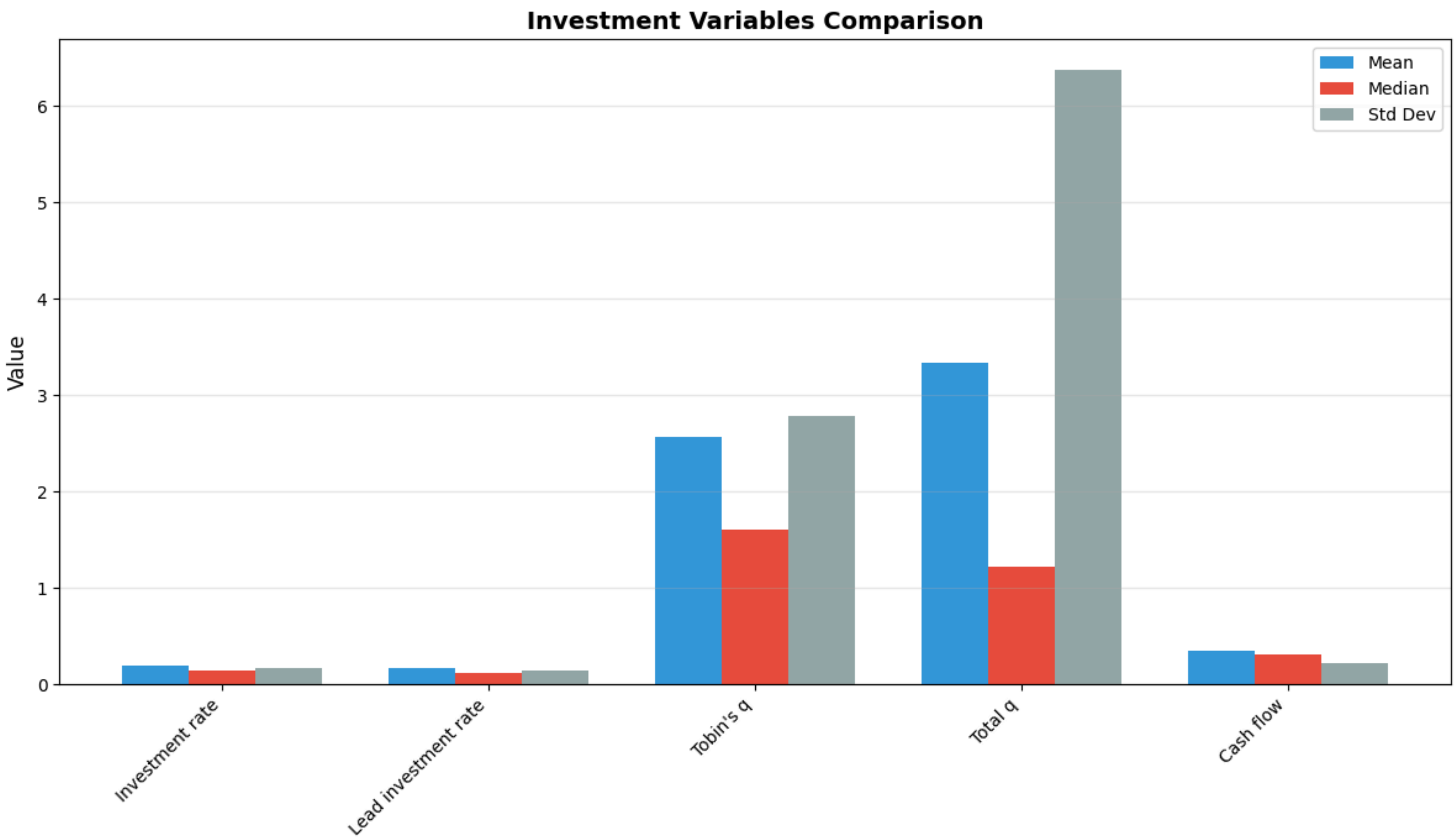
The Investment rate reports a mean of 0.188 while lead investment rate reports a mean of  0.164, together with sizable standard deviations of (0.165 and 0.141). This indicate substantial time-series and cross-sectional variation in physical investment suggesting that tech firms invest heavily relative to their capital stock, but that investment is volatile and uneven over time, consistent with lumpy adjustment and R&D-driven investment dynamics.

Moreover, Standard q (Tobin's q) has a mean of 2.57 and a standard deviation of 2.79, when Total q is higher on average at 3.34 and significantly dispersed, with a standard deviation of 6.38. This greater dispersion is consistent with the idea that incorporating intangible capital introduces additional volatility into valuations, reflecting innovation, learning, and jump risk (as emphasized by Peters and Taylor, 2017; Andrei, Mann, and Moyen, 2018).

Intangible intensity has a mean of 0.55, indicating that, on average, more than half of firms' total capital consists of intangible assets. This confirms that the U.S. tech sector is predominantly intangible-capital-intensive and supports the view that ignoring intangibles makes standard Tobin's q a noisy proxy for true investment opportunities.

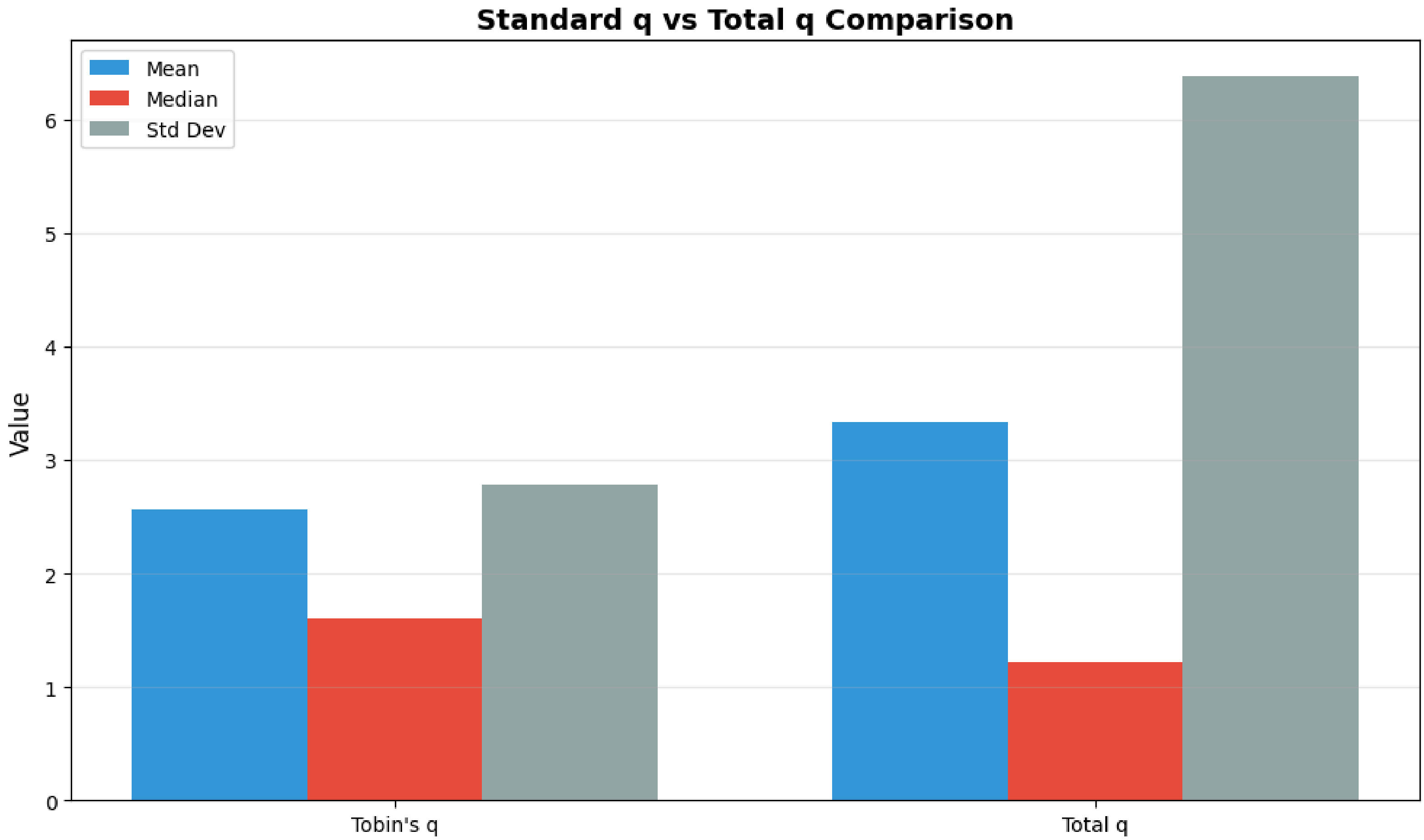
```
#22 Investment-related variables
summary_reset = summary.reset_index().rename(columns={'index': 'Variable'})
investment_vars = ["Investment rate", "Lead investment rate",
               "Tobin's q", "Total q", "Cash flow"]
inv_data = summary_reset[summary_reset['Variable'].isin(investment_vars)]

plt.figure(figsize=(12, 7))
x = range(len(inv_data))
width = 0.25
```

```
plt.bar([i-width for i in x], inv_data['Mean'], width, label='Mean', color='#3498db')
plt.bar([i for i in x], inv_data['Median'], width, label='Median', color='#e74c3c')
plt.bar([i+width for i in x], inv_data['Std. dev.'], width, label='Std Dev', color='#95a5a6')

plt.xticks(x, inv_data['Variable'], rotation=45, ha='right')
plt.ylabel('Value', fontsize=12)
plt.title('Investment Variables Comparison', fontsize=14, weight='bold')
plt.legend()
plt.grid(axis='y', alpha=0.3)
plt.tight_layout()
plt.show()
```




```
#23 Q Comparison (Standard q vs Total q)
q_data = summary_reset[summary_reset['Variable'].isin(["Tobin's q", 'Total q'])]
x = range(len(q_data))
width = 0.25
plt.figure(figsize=(10, 6))
plt.bar([i-width for i in x], q_data['Mean'], width, label='Mean', color='#3498db')
plt.bar([i for i in x], q_data['Median'], width, label='Median', color='#e74c3c')
plt.bar([i+width for i in x], q_data['Std. dev.'], width, label='Std Dev', color='#95a5a6')
plt.xticks(x, q_data['Variable'])
plt.ylabel('Value', fontsize=12)
plt.title("Standard q vs Total q Comparison", fontsize=14, weight='bold')
plt.legend()
plt.grid(axis='y', alpha=0.3)
```

```
plt.tight_layout()
plt.show()
```




```
#24 Preparation for running PyFixest regression for investment lead

# Construction of standardized q measures purposely for comparison
panel["Standardized_q"] = (panel["Standard_q"] - panel["Standard_q"].mean()) / panel["Standard_q"].std()
panel["Totalized_q"] = (panel["Total_q"] - panel["Total_q"].mean()) / panel["Total_q"].std()

panel["gvkey"] = panel["gvkey"].astype(str)
panel = panel.rename(columns={"fyear": "year"})
panel = panel.dropna(subset=["Investment_lead", "Cash_flows", "Standard_q", "Total_q", "Standardized_q",
"Totalized_q",])

# Require at least 2 obs per firm and year
counts_firm = panel.groupby("gvkey")["year"].transform("count")
counts_year = panel.groupby("year")["gvkey"].transform("count")
panel = panel[(counts_firm >= 2) & (counts_year >= 2)]

#25 Standard q (Tobins) Regressions Analysis

df = panel
#Model 1: Baseline
M1 = pf.feols("Investment_lead ~ Cash_flows + Standard_q", data=df)
#Model 2 with Firm Effect
M2 = pf.feols("Investment_lead ~ Cash_flows + Standard_q | gvkey", data=df)
#Model 3 with Firm and Year Effect
```

```
M3 = pf.feols("Investment_lead ~ Cash_flows + Standard_q | gvkey + year", data=df)
#Model 3 with Firm, Year and Cluster Effect
M4 = pf.feols("Investment_lead ~ Cash_flows + Standard_q | gvkey + year", vcov={"CRV1": "gvkey"}, data=df)
#Model 5 with Standardized Effect for Comparison
M5 = pf.feols("Investment_lead ~ Cash_flows + Standardized_q | gvkey + year", vcov={"CRV1": "gvkey"}, data=df)

#26 Total q (Peters–Taylor)  Regressions Analysis
#Model 6: Baseline
M6 = pf.feols("Investment_lead ~ Cash_flows + Total_q", data=df)
#Model 7 with Firm Effect
M7 = pf.feols("Investment_lead ~ Cash_flows + Total_q | gvkey", data=df)
#Model 8 with Firm and Year Effect
M8 = pf.feols("Investment_lead ~ Cash_flows + Total_q | gvkey + year", data=df)
#Model 9 with Firm, Year and Cluster Effect
M9 = pf.feols("Investment_lead ~ Cash_flows + Total_q | gvkey + year", vcov={"CRV1": "gvkey"}, data=df)
#Model 10 with Totalized_q Effect for Comparison
M10 = pf.feols("Investment_lead ~ Cash_flows + Totalized_q | gvkey + year", vcov={"CRV1": "gvkey"}, data=df)

#27 BaseLine Results of the Model For Standard q (Tobins q) and Total q
baseline_models = [M1, M2, M3, M4, M5, M6, M7, M8, M9, M10]
pf.etable(baseline_model
```

| | Investment_lead | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | (1) | (2) | (3) | (4) | (5) | (6) | (7) | (8) | (9) | (10) |
| Cash_flows | -0.003<br>(0.005) | -0.093***<br>(0.007) | -0.119***<br>(0.006) | -0.119***<br>(0.010) | -0.119***<br>(0.010) | 0.042***<br>(0.004) | -0.036***<br>(0.007) | -0.069***<br>(0.006) | -0.069***<br>(0.010) | -0.069***<br>(0.010) |
| Standard_q | 0.019***<br>(0.000) | 0.020***<br>(0.000) | 0.017***<br>(0.000) | 0.017***<br>(0.001) | | | | | | |
| Standardized_q | | | | | 0.046***<br>(0.002) | | | | | |
| Total_q | | | | | | 0.010***<br>(0.000) | 0.010***<br>(0.000) | 0.008***<br>(0.000) | 0.008***<br>(0.000) | |
| Totalized_q | | | | | | | | | | 0.050***<br>(0.002) |
| Intercept | 0.113***<br>(0.002) | | | | | 0.114***<br>(0.002) | | | | |
| gvkey | - | x | x | x | x | - | x | x | x | x |
| year | - | - | x | x | x | - | - | x | x | x |
| Observations | 18350 | 18350 | 18350 | 18350 | 18350 | 18350 | 18350 | 18350 | 18350 | 18350 |
| S.E. type | iid | iid | iid | by: gvkey | by: gvkey | iid | iid | iid | by: gvkey | by: gvkey |
| $R^2$ | 0.145 | 0.494 | 0.549 | 0.549 | 0.549 | 0.202 | 0.516 | 0.563 | 0.563 | 0.563 |
| $R^2$ Within | - | 0.151 | 0.112 | 0.112 | 0.112 | - | 0.188 | 0.139 | 0.139 | 0.139 |

Significance levels: * p < 0.05, ** p < 0.01, *** p < 0.001. Format of coefficient cell: Coefficient (Std. Error)

s)

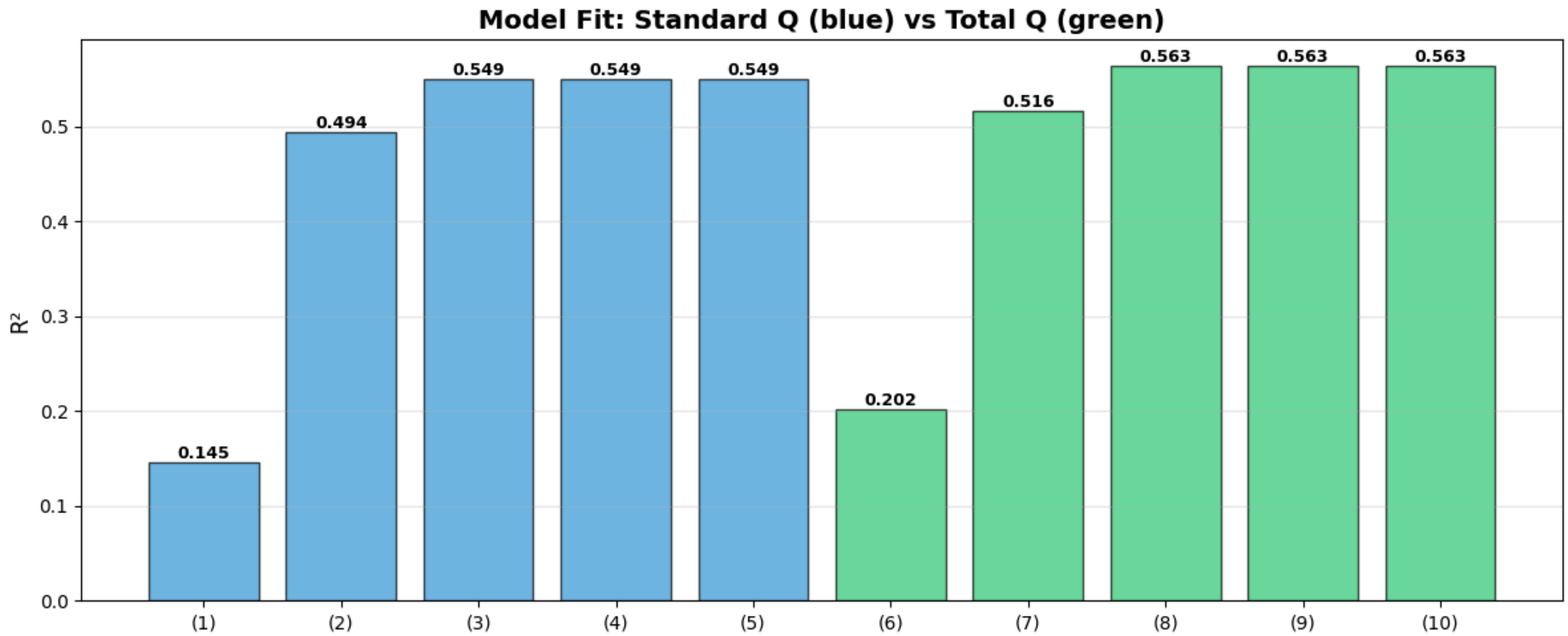
Comparing Standard q (Tobin's q) with  Total q, we find that Total q consistently delivers higher within-firm explanatory power. When it comes to firm and year-fixed-effects regressions, the within $R^2$ increases from 11.2% under standard q to 13.9% under Total q, and the standardized investment–q coefficient is larger for Total q than Standard q. While the Overall $R^2$ of Total q supersedes Standard q (56.3% vs 54.9%). This indicates that Total q better captures the significant investment opportunities faced by firms and restores part of the investment-q relation that appears weak when using standard q.

```
#28 Standard Q Results (5 specifications)
std_q = pd.DataFrame([['Baseline', -0.003, 0.019, 0.145, '-'], ['Firm FE', -0.093, 0.020, 0.494, 0.151], ['Firm+Year FE',
-0.119, 0.017, 0.549, 0.112],
['Cluster SE', -0.119, 0.017, 0.549, 0.112], ['Standardized Q', -0.119, 0.046, 0.549, 0.112]], columns=['Model', 'Cash Flow',
'Q', 'R²', 'Within R²'])

#29 Total Q Results (5 specifications)
tot_q = pd.DataFrame([['Baseline', 0.042, 0.010, 0.202, '-'], ['Firm FE', -0.036, 0.010, 0.516, 0.188],['Firm+Year FE', -0.069,
0.008, 0.563, 0.139],
['Cluster SE', -0.069, 0.008, 0.563, 0.139], ['Totalized Q', -0.069, 0.050, 0.563, 0.139]], columns=['Model', 'Cash Flow', 'Q',
'R²', 'Within R²'])

print("="*70)
print("STANDARD TOBIN'S Q (5 SPECIFICATIONS)")
print("="*70)
print(std_q.to_string(index=False))
print("\n" + "="*70)
print("TOTAL Q - PETERS-TAYLOR (5 SPECIFICATIONS)")
print("="*70)
print(tot_q.to_string(index=False))
print("\n" + "="*70)

#30 Summary comparison
print("KEY COMPARISON")
print("="*70)
print(f"{'':<15} {'Standard Q':<12} {'Total Q':<12} {'Difference'}")
print(f"{'-'*15:<15} {'-'*12:<12} {'-'*12:<12} {'-'*9}")
print(f"{'Best R²:':<15} {0.549:<12.3f} {0.563:<12.3f} +{0.014:.3f}")
print(f"{'Std Q Coef:':<15} {0.046:<12.3f} {0.050:<12.3f} +{0.004:.3f}")
print(f"{'Within R²:':<15} {0.112:<12.3f} {0.139:<12.3f} +{0.027:.3f}")
print(f"{'CF (FE Model):':<15} {-0.119:<12.3f} {-0.069:<12.3f} {0.050:.3f}")
print("="*70)
```

```
======================================================================
STANDARD TOBIN'S Q (5 SPECIFICATIONS)
======================================================================
         Model  Cash Flow     Q    R² Within R²
      Baseline     -0.003 0.019 0.145         -
       Firm FE     -0.093 0.020 0.494     0.151
  Firm+Year FE     -0.119 0.017 0.549     0.112
    Cluster SE     -0.119 0.017 0.549     0.112
Standardized Q     -0.119 0.046 0.549     0.112

======================================================================
TOTAL Q - PETERS-TAYLOR (5 SPECIFICATIONS)
======================================================================
       Model  Cash Flow     Q    R² Within R²
    Baseline      0.042 0.010 0.202         -
     Firm FE     -0.036 0.010 0.516     0.188
Firm+Year FE     -0.069 0.008 0.563     0.139
  Cluster SE     -0.069 0.008 0.563     0.139
 Totalized Q     -0.069 0.050 0.563     0.139

======================================================================
KEY COMPARISON
======================================================================
                Standard Q   Total Q      Difference
--------------- ------------ ------------ ---------
Best R²:        0.549        0.563        +0.014
Std Q Coef:     0.046        0.050        +0.004
Within R²:      0.112        0.139        +0.027
CF (FE Model):  -0.119       -0.069       0.050
======================================================================
```

```
#31 Using Baseline_models (Regression results)
baseline_models = [M1, M2, M3, M4, M5, M6, M7, M8, M9, M10]

# Get coefficients and stats
data = []
for i, model in enumerate(baseline_models, 1):
  # Get Q coefficient
  coefs = model.coef()
  q_coef = coefs.get('Standard_q') or coefs.get('Standardized_q') or coefs.get('Total_q') or coefs.get('Totalized_q')
  cash_flow = coefs.get('Cash_flows', None)

  data.append({
    'Model': f'({i})',
    'Q_coef': q_coef,
    'Cash_flow': cash_flow,
    'R2': model._r2,
    'R2_within': model._r2_within if hasattr(model, '_r2_within') else None
  })

df7 = pd.DataFrame(data)

#32 Chart 1: All Models R²
plt.figure(figsize=(12, 5))
colors = ['#3498db']*5 + ['#2ecc71']*5
```

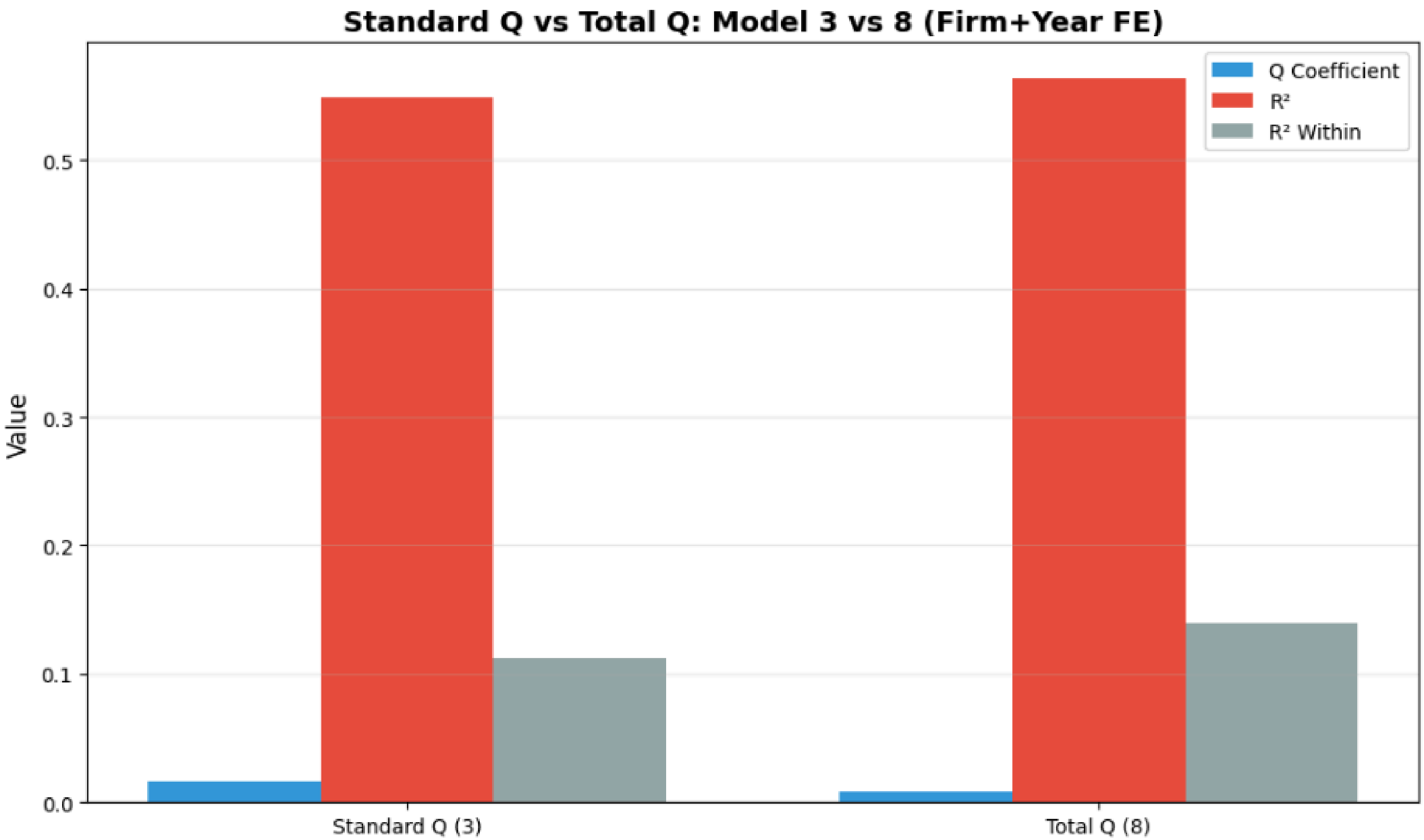
```
plt.bar(df7['Model'], df7['R2'], color=colors, alpha=0.7, edgecolor='black')

for i, r2 in enumerate(df7['R2']):
    plt.text(i, r2, f'{r2:.3f}', ha='center', va='bottom', fontsize=9, weight='bold')

plt.ylabel('R²', fontsize=12)
plt.title('Model Fit: Standard Q (blue) vs Total Q (green)', fontsize=14, weight='bold')
plt.grid(axis='y', alpha=0.3)
plt.tight_layout()
plt.show()
```




Across the Fits in the baseline Model, Total q outperformed Standard q in all model's overall fit Comparison demonstrating a stronger explanatory power in investment-q relationship the US tech firms engaged in Intensive-R&D.

```
#33 Chart 2: Model 3 vs 8 Comparison
x = range(2)
width = 0.25

plt.figure(figsize=(10, 6))

plt.bar([i-width for i in x],
      [df7.iloc[2]['Q_coef'], df7.iloc[7]['Q_coef']],
      width, label='Q Coefficient', color='#3498db')

plt.bar([i for i in x],
      [df7.iloc[2]['R2'], df7.iloc[7]['R2']],
      width, label='R²', color='#e74c3c')

plt.bar([i+width for i in x],
      [df7.iloc[2]['R2_within'], df7.iloc[7]['R2_within']],
      width, label='R² Within', color='#95a5a6')

plt.xticks(x, ['Standard Q (3)', 'Total Q (8)'])
plt.ylabel('Value', fontsize=12)
```

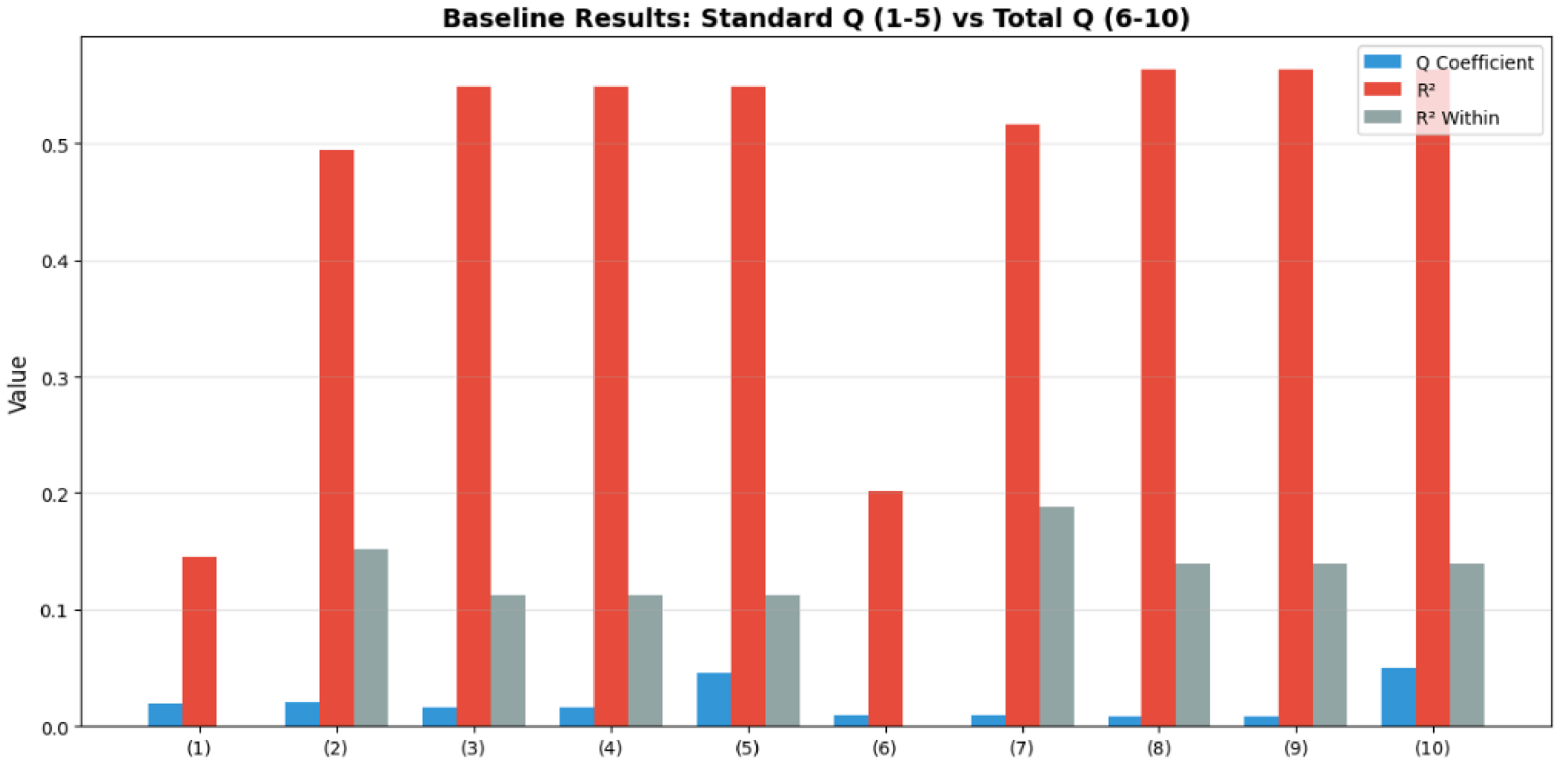
```
plt.title('Standard Q vs Total Q: Model 3 vs 8 (Firm+Year FE)', fontsize=14, weight='bold')
plt.legend()
plt.grid(axis='y', alpha=0.3)
plt.tight_layout()
plt.show()

print(f"\nKey Results:")
print(f"  Standard Q (Model 3) - R²: {df7.iloc[2]['R2']:.3f}, Within R²: {df7.iloc[2]['R2_within']:.3f}")
print(f"  Total Q (Model 8)    - R²: {df7.iloc[7]['R2']:.3f}, Within R²: {df7.iloc[7]['R2_within']:.3f}")
print(f"\n  → Total Q WINS!")
```




```
Key Results:
  Standard Q (Model 3) - R²: 0.549, Within R²: 0.112
  Total Q (Model 8)    - R²: 0.563, Within R²: 0.139

  → Total Q WINS!
```

```
#34 Regression models
baseline_models = [M1, M2, M3, M4, M5, M6, M7, M8, M9, M10]

# Variable name mapping
coef_rename = {"Standard_q": "Standard q", "Standardized_q": "Standardized q", "Total_q": "Total q", "Totalized_q": "Standardized Total q",
            "Cash_flows": "Cash flow"}
# Extract data from models
data = []
for i, model in enumerate(baseline_models, 1):
    coefs = model.coef()

    # Get Q coefficient with proper naming
```

```
    q_coef = (coefs.get('Standard_q') or coefs.get('Standardized_q') or
              coefs.get('Total_q') or coefs.get('Totalized_q'))
    cash_flow = coefs.get('Cash_flows', None)

    data.append({
        'Model': f'({i})',
        'Q_coef': q_coef,
        'Cash_flow': cash_flow,
        'R2': model._r2,
        'R2_within': model._r2_within if hasattr(model, '_r2_within') else 0
    })

results = pd.DataFrame(data)

# Chart: All 10 models
x = range(len(results))
width = 0.25

plt.figure(figsize=(12, 6))
plt.bar([i-width for i in x], results['Q_coef'], width, label='Q Coefficient', color='#3498db')
plt.bar([i for i in x], results['R2'], width, label='R²', color='#e74c3c')
plt.bar([i+width for i in x], results['R2_within'], width, label='R² Within', color='#95a5a6')

plt.xticks(x, results['Model'])
plt.ylabel('Value', fontsize=12)
plt.title("Baseline Results: Standard Q (1-5) vs Total Q (6-10)", fontsize=14, weight='bold')
plt.legend()
plt.grid(axis='y', alpha=0.3)
plt.tight_layout()
plt.savefig('regression_comparison.png', dpi=300, bbox_inches='tight')
plt.show()

print("✓ Bar chart created!")
print(f"\nKey Results (Model 3 vs Model 8):")
print(f"  Standard Q (3) - R²: {results.iloc[2]['R2']:.3f}, Within R²: {results.iloc[2]['R2_within']:.3f}")
print(f"  Total Q (8)    - R²: {results.iloc[7]['R2']:.3f}, Within R²: {results.iloc[7]['R2_within']:.3f}")
print(f"\n  → Total Q wins with better fit!")
```




```
✓ Bar chart created!

Key Results (Model 3 vs Model 8):
  Standard Q (3) - R²: 0.549, Within R²: 0.112
  Total Q (8)    - R²: 0.563, Within R²: 0.139

  → Total Q wins with better fit!
```

```
#35 Subperiod stability (pre/post 2000 & pre/post 2008)

# Construct subperiod indicators
df["pre_2000"] = (df["year"] < 2000).astype(int)
df["post_2000"] = (df["year"] >= 2000).astype(int)
df["pre_2008"] = (df["year"] < 2008).astype(int)
df["post_2008"] = (df["year"] >= 2008).astype(int)

#36 Fit Standard Q regressions
Pre_2000_StandardQ = pf.feols("Investment_lead ~ Cash_flows + Standard_q | gvkey + year", data=df[df["year"] < 2000], vcov={"CRV1": "gvkey"})
Post_2000_StandardQ = pf.feols("Investment_lead ~ Cash_flows + Standard_q | gvkey + year", data=df[df["year"] >= 2000], vcov={"CRV1": "gvkey"})
Pre_2008_StandardQ = pf.feols("Investment_lead ~ Cash_flows + Standard_q | gvkey + year", data=df[df["year"] < 2008], vcov={"CRV1": "gvkey"})
Post_2008_StandardQ = pf.feols("Investment_lead ~ Cash_flows + Standard_q | gvkey + year", data=df[df["year"] >= 2008], vcov={"CRV1": "gvkey"})

#37 Fit Total Q regressions
Pre_2000_TotalQ = pf.feols("Investment_lead ~ Cash_flows + Total_q | gvkey + year", data=df[df["year"] < 2000], vcov={"CRV1": "gvkey"})
Post_2000_TotalQ = pf.feols("Investment_lead ~ Cash_flows + Total_q | gvkey + year", data=df[df["year"] >= 2000], vcov={"CRV1": "gvkey"})
Pre_2008_TotalQ = pf.feols("Investment_lead ~ Cash_flows + Total_q | gvkey + year", data=df[df["year"] < 2008], vcov={"CRV1": "gvkey"})
```

```python
Post_2008_TotalQ = pf.feols("Investment_lead ~ Cash_flows + Total_q | gvkey + year", data=df[df["year"] >= 2008], vcov={"CRV1": "gvkey"})

# Combined models
Subperiod_models = [Pre_2000_StandardQ, Post_2000_StandardQ, Pre_2008_StandardQ, Post_2008_StandardQ, Pre_2000_TotalQ, Post_2000_TotalQ,
            Pre_2008_TotalQ, Post_2008_TotalQ]

# 38 Display regression table
pf.etable(Subperiod_models, digits=3)
```

| | Investment_lead | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | (1) | (2) | (3) | (4) | (5) | (6) | (7) | (8) |
| Cash_flows | -0.161*** (0.020) | -0.102*** (0.012) | -0.135*** (0.012) | -0.065*** (0.019) | -0.110*** (0.021) | -0.064*** (0.012) | -0.084*** (0.012) | -0.033 (0.020) |
| Standard_q | 0.017*** (0.002) | 0.013*** (0.001) | 0.019*** (0.001) | 0.009*** (0.001) | | | | |
| Total_q | | | | | 0.007*** (0.001) | 0.007*** (0.000) | 0.008*** (0.000) | 0.005*** (0.001) |
| gvkey | x | x | x | x | x | x | x | x |
| year | x | x | x | x | x | x | x | x |
| Observations | 4181 | 13773 | 10153 | 8033 | 4181 | 13773 | 10153 | 8033 |
| S.E. type | by: gvkey | by: gvkey | by: gvkey | by: gvkey | by: gvkey | by: gvkey | by: gvkey | by: gvkey |
| $R^2$ | 0.643 | 0.510 | 0.594 | 0.544 | 0.644 | 0.521 | 0.603 | 0.549 |
| $R^2$ Within | 0.117 | 0.067 | 0.151 | 0.028 | 0.120 | 0.088 | 0.170 | 0.038 |

Significance levels: * p < 0.05, ** p < 0.01, *** p < 0.001. Format of coefficient cell: Coefficient (Std. Error)

```python
#39 Prepare complete subperiod stability table
data = {"Model": ["(1) Pre-2000 (Std Q)", "(2) Post-2000 (Std Q)", "(3) Pre-2008 (Std Q)", "(4) Post-2008 (Std Q)",
     "(5) Pre-2000 (Total Q)", "(6) Post-2000 (Total Q)", "(7) Pre-2008 (Total Q)", "(8) Post-2008 (Total Q)"],
   "Cash Flow": ["-0.161*** (0.020)", "-0.102*** (0.012)", "-0.135*** (0.012)", "-0.065*** (0.019)",
     "-0.110*** (0.021)", "-0.064*** (0.012)", "-0.084*** (0.012)", "-0.033* (0.020)"],
   "Standard Q": ["0.017*** (0.002)", "0.013*** (0.001)", "0.019*** (0.001)", "0.009*** (0.001)", "", "", "", ""],
   "Total Q": ["", "", "", "", "0.007*** (0.001)", "0.007*** (0.000)", "0.008*** (0.000)", "0.005*** (0.001)"],
   "Observations": ["4,181", "13,773", "10,153", "8,033", "4,181", "13,773", "10,153", "8,033"],
   "R²": ["0.643", "0.510", "0.594", "0.544", "0.644", "0.521", "0.603", "0.549"],
   "Within R²": ["0.117", "0.067", "0.151", "0.028", "0.120", "0.088", "0.170", "0.038"]}
df_complete = pd.DataFrame(data)
#Print markdown table
print("# Subperiod Stability Analysis")
print("\n## Physical Investment on Tobin's Q\n")
print("**Dependent variable:** Investment (t+1)")
print("**Fixed Effects:** Firm (gvkey) + Year")
print("**Standard Errors:** Clustered by firm")
print("**Total sample:** 18,350 firm-year observations\n")
```

```
markdown_table = df_complete.to_markdown(index=False)
print(markdown_table)
print("\n**Significance levels:** *p < 0.1, **p < 0.05, ***p < 0.01")
print("**Format:** Coefficient (Standard Error)")
```

```
# Subperiod Stability Analysis

## Physical Investment on Tobin's Q

**Dependent variable:** Investment (t+1)
**Fixed Effects:** Firm (gvkey) + Year
**Standard Errors:** Clustered by firm
**Total sample:** 18,350 firm-year observations

| Model                   | Cash Flow         | Standard Q        | Total Q           | Observations    |    R² |   Within R² |
|:------------------------|:------------------|:------------------|:------------------|:----------------|------:|------------:|
| (1) Pre-2000 (Std Q)    | -0.161*** (0.020) | 0.017*** (0.002)  |                   | 4,181           | 0.643 |       0.117 |
| (2) Post-2000 (Std Q)   | -0.102*** (0.012) | 0.013*** (0.001)  |                   | 13,773          | 0.51  |       0.067 |
| (3) Pre-2008 (Std Q)    | -0.135*** (0.012) | 0.019*** (0.001)  |                   | 10,153          | 0.594 |       0.151 |
| (4) Post-2008 (Std Q)   | -0.065*** (0.019) | 0.009*** (0.001)  |                   | 8,033           | 0.544 |       0.028 |
| (5) Pre-2000 (Total Q)  | -0.110*** (0.021) |                   | 0.007*** (0.001)  | 4,181           | 0.644 |       0.12  |
| (6) Post-2000 (Total Q) | -0.064*** (0.012) |                   | 0.007*** (0.000)  | 13,773          | 0.521 |       0.088 |
| (7) Pre-2008 (Total Q)  | -0.084*** (0.012) |                   | 0.008*** (0.000)  | 10,153          | 0.603 |       0.17  |
| (8) Post-2008 (Total Q) | -0.033* (0.020)   |                   | 0.005*** (0.001)  | 8,033           | 0.549 |       0.038 |

**Significance levels:** *p < 0.1, **p < 0.05, ***p < 0.01
**Format:** Coefficient (Standard Error)
```

The subperiod results show that Total q not only fits better in each window, but also delivers a more stable investment-q relationship across both the pre/post-2000 and pre/post-2008 splits than Standard Tobin's q. For Standard q, the sensitivity of investment declines substantially over time. The coefficient falls from 0.017 before 2000 to 0.009 after 2008, roughly a 50% decline, indicating a marked weakening of the classic investment-q relationship in the post-crisis period for tech firms.

In contrast, the Total q coefficient is more stable. It remains around 0.7% in the pre-2000 and post-2000 periods and declines only modestly to 0.5% after 2008. This suggests that once intangible capital is incorporated, the investment–q relationship becomes less sensitive to structural breaks and regime changes.

The within-firm explanatory power of Standard q collapses sharply after 2008, falling from 15% in the pre-2008 period to just 2.8% afterward. This indicates that Standard q explains very little of firms' time-series investment variation in the post-crisis environment.

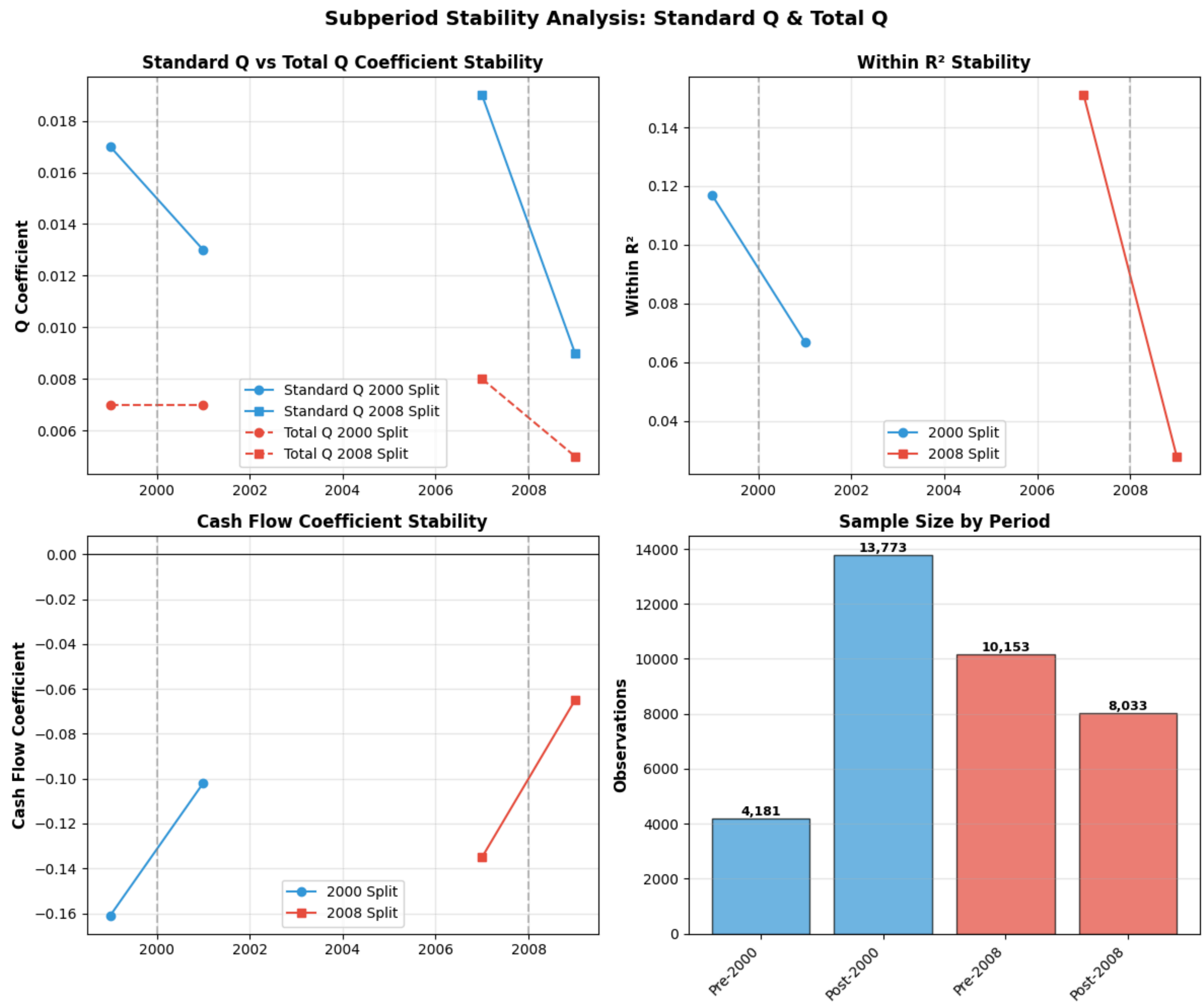
Total q performs better throughout. It achieves higher within-firm $R^2$ in every subperiod, for example (8.8% vs 6.7%) in post-2000 and 3.8% versus 2.8% post-2008, and its post-2008 decline is smaller in relative terms. This indicates that Total q remains a more robust proxy for investment opportunities when technological and macroeconomic conditions change.

Overall, replacing Standard q (Tobin's q) with Total q yields an investment-q relationship that is both more stable over time and more informative about within-firm investment dynamics. While the overall fit declines slightly over time for both measures, consistent with investment becoming more volatile and harder to predict. Total q consistently outperforms standard q in capturing firms' intertemporal investment behavior.

In a nutshell, across all subperiods, regressions using Total q deliver both a more stable slope on q and higher within-firm $R^2$ than regressions using Standard q. While the sensitivity of investment to Standard q falls by roughly half after 2008 and its within-firm $R^2$ declines sharply, the Total-q slope decreases much less and continues to explain substantially more of the time-series variation in investment. This pattern suggests that incorporating internally generated intangible capital into q yields an investment-q relationship that is both more robust and more stable over time in U.S. tech firms. However, the consistent negative cashflow across the sections remains the puzzle that needs an answer.

```
#40 Prepare data for plotting Standard Q and Total Q stability

subperiod_data = pd.DataFrame({
    'Period': ['Pre-2000', 'Post-2000', 'Pre-2008', 'Post-2008'],
    'Year_cutoff': [2000, 2000, 2008, 2008],
    'Cash_flow': [-0.161, -0.102, -0.135, -0.065],
    'Standard_q': [0.017, 0.013, 0.019, 0.009],
    'Total_q': [0.007, 0.007, 0.008, 0.005],
    'R2_within': [0.117, 0.067, 0.151, 0.028],
    'N': [4181, 13773, 10153, 8033]
})

data_2000 = subperiod_data[subperiod_data['Year_cutoff'] == 2000]
data_2008 = subperiod_data[subperiod_data['Year_cutoff'] == 2008]

# Plot subperiod stability charts
fig, axes = plt.subplots(2, 2, figsize=(12, 10))

# Standard Q vs Total Q coefficients
ax1 = axes[0, 0]
# Standard Q
ax1.plot([1999, 2001], [data_2000.iloc[0]['Standard_q'], data_2000.iloc[1]['Standard_q']],
        'o-', color='#3498db', label='Standard Q 2000 Split')
ax1.plot([2007, 2009], [data_2008.iloc[0]['Standard_q'], data_2008.iloc[1]['Standard_q']],
        's-', color='#3498db', label='Standard Q 2008 Split')
# Total Q
ax1.plot([1999, 2001], [data_2000.iloc[0]['Total_q'], data_2000.iloc[1]['Total_q']],
        'o--', color='#e74c3c', label='Total Q 2000 Split')
ax1.plot([2007, 2009], [data_2008.iloc[0]['Total_q'], data_2008.iloc[1]['Total_q']],
        's--', color='#e74c3c', label='Total Q 2008 Split')

ax1.axvline(x=2000, color='gray', linestyle='--', alpha=0.5)
ax1.axvline(x=2008, color='gray', linestyle='--', alpha=0.5)
ax1.set_ylabel('Q Coefficient', fontsize=11, weight='bold')
ax1.set_title('Standard Q vs Total Q Coefficient Stability', fontsize=12, weight='bold')
ax1.legend()
ax1.grid(True, alpha=0.3)

# Within R² over time
ax2 = axes[0, 1]
ax2.plot([1999, 2001], [data_2000.iloc[0]['R2_within'], data_2000.iloc[1]['R2_within']],
        'o-', color='#3498db', label='2000 Split')
ax2.plot([2007, 2009], [data_2008.iloc[0]['R2_within'], data_2008.iloc[1]['R2_within']],
        's-', color='#e74c3c', label='2008 Split')
ax2.axvline(x=2000, color='gray', linestyle='--', alpha=0.5)
ax2.axvline(x=2008, color='gray', linestyle='--', alpha=0.5)
```

```
ax2.set_ylabel('Within R²', fontsize=11, weight='bold')
ax2.set_title('Within R² Stability', fontsize=12, weight='bold')
ax2.legend()
ax2.grid(True, alpha=0.3)

# Cash Flow coefficient
ax3 = axes[1, 0]
ax3.plot([1999, 2001], [data_2000.iloc[0]['Cash_flow'], data_2000.iloc[1]['Cash_flow']],
        'o-', color='#3498db', label='2000 Split')
ax3.plot([2007, 2009], [data_2008.iloc[0]['Cash_flow'], data_2008.iloc[1]['Cash_flow']],
        's-', color='#e74c3c', label='2008 Split')
ax3.axvline(x=2000, color='gray', linestyle='--', alpha=0.5)
ax3.axvline(x=2008, color='gray', linestyle='--', alpha=0.5)
ax3.axhline(y=0, color='black', linestyle='-', linewidth=0.8)
ax3.set_ylabel('Cash Flow Coefficient', fontsize=11, weight='bold')
ax3.set_title('Cash Flow Coefficient Stability', fontsize=12, weight='bold')
ax3.legend()
ax3.grid(True, alpha=0.3)

# Sample size
ax4 = axes[1, 1]
colors_bar = ['#3498db', '#3498db', '#e74c3c', '#e74c3c']
bars = ax4.bar(range(4), subperiod_data['N'], color=colors_bar, alpha=0.7, edgecolor='black')
for bar, n in zip(bars, subperiod_data['N']):
    ax4.text(bar.get_x() + bar.get_width()/2, bar.get_height(), f'{n:,}', ha='center', va='bottom', fontsize=9, weight='bold')
ax4.set_xticks(range(4))
ax4.set_xticklabels(subperiod_data['Period'], rotation=45, ha='right')
ax4.set_ylabel('Observations', fontsize=11, weight='bold')
ax4.set_title('Sample Size by Period', fontsize=12, weight='bold')
ax4.grid(axis='y', alpha=0.3)

plt.suptitle('Subperiod Stability Analysis: Standard Q & Total Q', fontsize=14, weight='bold', y=0.995)
plt.tight_layout()
plt.show()
```




#41 Heterogeneity Analysis (H3) By Firm Age
# Heterogeneity Analysis: Young vs Mature Firms

```
#42 Calculate firm age and split sample
df5 = df.sort_values(["gvkey", "year"])
df5["first_year"] = df5.groupby("gvkey")["year"].transform("min")
df5["firm_age"] = df5["year"] - df5["first_year"]
median_age = df5["firm_age"].median()
df5["young_firm"] = (df5["firm_age"] <= median_age).astype(int)

#43 Panel A: Young vs Mature Firms
m_age = [
    pf.feols("Investment_lead ~ Cash_flows + Standard_q | gvkey + year", data=df5[df5["young_firm"] == 1],
vcov={"CRV1": "gvkey"}),
    pf.feols("Investment_lead ~ Cash_flows + Standard_q | gvkey + year", data=df5[df5["young_firm"] == 0],
vcov={"CRV1": "gvkey"}),
```

```
    pf.feols("Investment_lead ~ Cash_flows + Total_q | gvkey + year", data=df5[df5["young_firm"] == 1], vcov={"CRV1": "gvkey"}),
    pf.feols("Investment_lead ~ Cash_flows + Total_q | gvkey + year", data=df5[df5["young_firm"] == 0], vcov={"CRV1": "gvkey"})]
model_labels_age = ["Young Firms (Standard Q)", "Mature Firms (Standard Q)", "Young Firms (Total Q)", "Mature Firms (Total Q)"]
pf.etable(m_age, title="Panel A: Heterogeneity by Firm Age", dict_stats={"N": "nobs", "R2": "r2", "R2_within": "r2_within"},
        model_names=model_labels_age)
```

| | Investment_lead | | | |
|---|---|---|---|---|
| | (1) | (2) | (3) | (4) |
| Cash_flows | -0.147***<br>(0.014) | -0.068***<br>(0.014) | -0.103***<br>(0.015) | -0.046**<br>(0.014) |
| Standard_q | 0.014***<br>(0.001) | 0.011***<br>(0.001) | | |
| Total_q | | | 0.006***<br>(0.000) | 0.007***<br>(0.001) |
| gvkey | x | x | x | x |
| year | x | x | x | x |
| Observations | 9199 | 8913 | 9199 | 8913 |
| S.E. type | by: gvkey | by: gvkey | by: gvkey | by: gvkey |
| $R^2$ | 0.633 | 0.468 | 0.637 | 0.475 |
| $R^2$ Within | 0.102 | 0.034 | 0.112 | 0.047 |

Significance levels: * p < 0.05, ** p < 0.01, *** p < 0.001. Format of coefficient cell: Coefficient (Std. Error)

For both measurements between young and mature firms, q explains substantially more of the within-firm investment variation for young firms than for mature firms. This is confirms that,  younger firms being more growth-oriented and more sensitive to changes in investment opportunities than Mature firms.

Importantly, Total q improves explanatory power for both groups, but especially for mature firms: within-R² rises from 3.4% to 4.7%,  a relative increase of 38%. With overall R² of increases from 10% to 11%. This confirms that incorporating intangible capital is particularly critical for capturing investment incentives in more established firms, where intangibles play a signficant role and standard q undermines true growth opportunities.

```
#44 Panel B: High vs Low R&D Intensity
# Construct R&D intensity
df5["rd_intensity"] = df5["xrd"] / df5["at"]
df5["rd_intensity"] = df5["rd_intensity"].replace([np.inf, -np.inf], np.nan)
df5 = df5.dropna(subset=["rd_intensity"])
median_rd = df5["rd_intensity"].median()
df5["high_rd"] = (df5["rd_intensity"] > median_rd).astype(int)

#45 Run regressions
```

```
m_rd = [
    pf.feols("Investment_lead ~ Cash_flows + Standard_q | gvkey + year", data=df5[df5["high_rd"] == 1], vcov={"CRV1": "gvkey"}),
    pf.feols("Investment_lead ~ Cash_flows + Standard_q | gvkey + year", data=df5[df5["high_rd"] == 0], vcov={"CRV1": "gvkey"}),
    pf.feols("Investment_lead ~ Cash_flows + Total_q | gvkey + year", data=df5[df5["high_rd"] == 1], vcov={"CRV1": "gvkey"}),
    pf.feols("Investment_lead ~ Cash_flows + Total_q | gvkey + year", data=df5[df5["high_rd"] == 0], vcov={"CRV1": "gvkey"})]
model_labels_rd = ["High R&D (Standard Q)", "Low R&D (Standard Q)", "High R&D (Total Q)", "Low R&D (Total Q)"]
pf.etable(m_rd, title="Panel B: Heterogeneity by R&D Intensity", dict_stats={"N": "nobs", "R2": "r2", "R2_within": "r2_within"}, model_names=model_labels_rd)
```

| | Investment_lead | | | |
|---|---|---|---|---|
| | (1) | (2) | (3) | (4) |
| Cash_flows | -0.101*** (0.014) | -0.109*** (0.026) | -0.056*** (0.014) | -0.060* (0.026) |
| Standard_q | 0.015*** (0.001) | 0.017*** (0.002) | | |
| Total_q | | | 0.008*** (0.000) | 0.008*** (0.000) |
| gvkey | x | x | x | x |
| year | x | x | x | x |
| Observations | 7278 | 7182 | 7278 | 7182 |
| S.E. type | by: gvkey | by: gvkey | by: gvkey | by: gvkey |
| $R^2$ | 0.604 | 0.553 | 0.612 | 0.572 |
| $R^2$ Within | 0.126 | 0.079 | 0.144 | 0.120 |

Significance levels: * p < 0.05, ** p < 0.01, *** p < 0.001. Format of coefficient cell: Coefficient (Std. Error)

There is improvement from Total q is especially strong among low-R&D firms, where within-$R^2$ rises from 7.9% to 12% roughly about 50.1% percent increase. This suggests that even firms that do not appear R&D-intensive on the surface still rely heavily on intangible capital that Standard q accounting does not capture well.

And among high-R&D firms, Total q also improves fit, but by a smaller margin from12.6% to 14.4%. This is consistent with the idea that Standard q already partially reflects innovation in these firms, while Total q better captures hidden intangible accumulation in the broader sense.

```
# Creating a combined table data
#46 Panel A: Firm Age
panel_a = pd.DataFrame({"Model": ["Young Firms (Std Q)", "Mature Firms (Std Q)", "Young Firms (Total Q)", "Mature Firms (Total Q)"],
    "Cash Flow": ["-0.147*** (0.014)", "-0.068*** (0.014)", "-0.103*** (0.015)", "-0.046** (0.014)"],
    "Standard Q": ["0.014*** (0.001)", "0.011*** (0.001)", "", ""],
    "Total Q": ["", "", "0.006*** (0.000)", "0.007*** (0.001)"],
```

```
    "Observations": ["9,199", "8,913", "9,199", "8,913"],
    "R²": ["0.633", "0.468", "0.637", "0.475"],
    "Within R²": ["0.102", "0.034", "0.112", "0.047"]})

#47 Panel B: R&D Intensity
panel_b = pd.DataFrame({"Model": ["High R&D (Std Q)", "Low R&D (Std Q)", "High R&D (Total Q)", "Low R&D (Total Q)"],
    "Cash Flow": ["-0.101*** (0.014)", "-0.109*** (0.026)", "-0.056*** (0.014)", "-0.060* (0.026)"],
    "Standard Q": ["0.015*** (0.001)", "0.017*** (0.002)", "", ""],
    "Total Q": ["", "", "0.008*** (0.000)", "0.008*** (0.000)"],
    "Observations": ["7,278", "7,182", "7,278", "7,182"],
    "R²": ["0.604", "0.553", "0.612", "0.572"],
    "Within R²": ["0.126", "0.079", "0.144", "0.120"]})

# Combine Panels
combined_table = pd.concat([panel_a, panel_b], keys=["Panel A: Firm Age", "Panel B: R&D Intensity"])
combined_table.reset_index(level=0, inplace=True)
combined_table = combined_table.rename(columns={"level_0": "Panel"})

# Print Markdown Table ---
print("# Heterogeneity Analysis: Firm Age & R&D Intensity")
print("")
print("**Dependent variable:** Investment_lead")
print("**Fixed Effects:** Firm (gvkey) + Year")
print("**Standard Errors:** Clustered by firm")
print("")

markdown_table = combined_table.to_markdown(index=False)
print(markdown_table)
print("")
print("**Significance levels:** * p < 0.05, ** p < 0.01, *** p < 0.001")
print("**Format:** Coefficient (Standard Error)")

#48 Create visualization for Standard Q and Total Q

# Prepare data for plotting
combined_table_plot = combined_table.copy()
combined_table_plot["Panel_Label"] = combined_table_plot["Panel"] + " - " + combined_table_plot["Model"]

#49 Convert coefficients to float (remove stars and parentheses)
def extract_coef(s):
    if s == "":
        return None
    return float(s.split("***")[0].split("**")[0].split("*")[0].split()[0])

combined_table_plot["Cash_Flow_Coef"] = combined_table_plot["Cash Flow"].apply(extract_coef)
combined_table_plot["Standard_Q_Coef"] = combined_table_plot["Standard Q"].apply(extract_coef)
combined_table_plot["Total_Q_Coef"] = combined_table_plot["Total Q"].apply(extract_coef)
```

# Heterogeneity Analysis: Firm Age & R&D Intensity

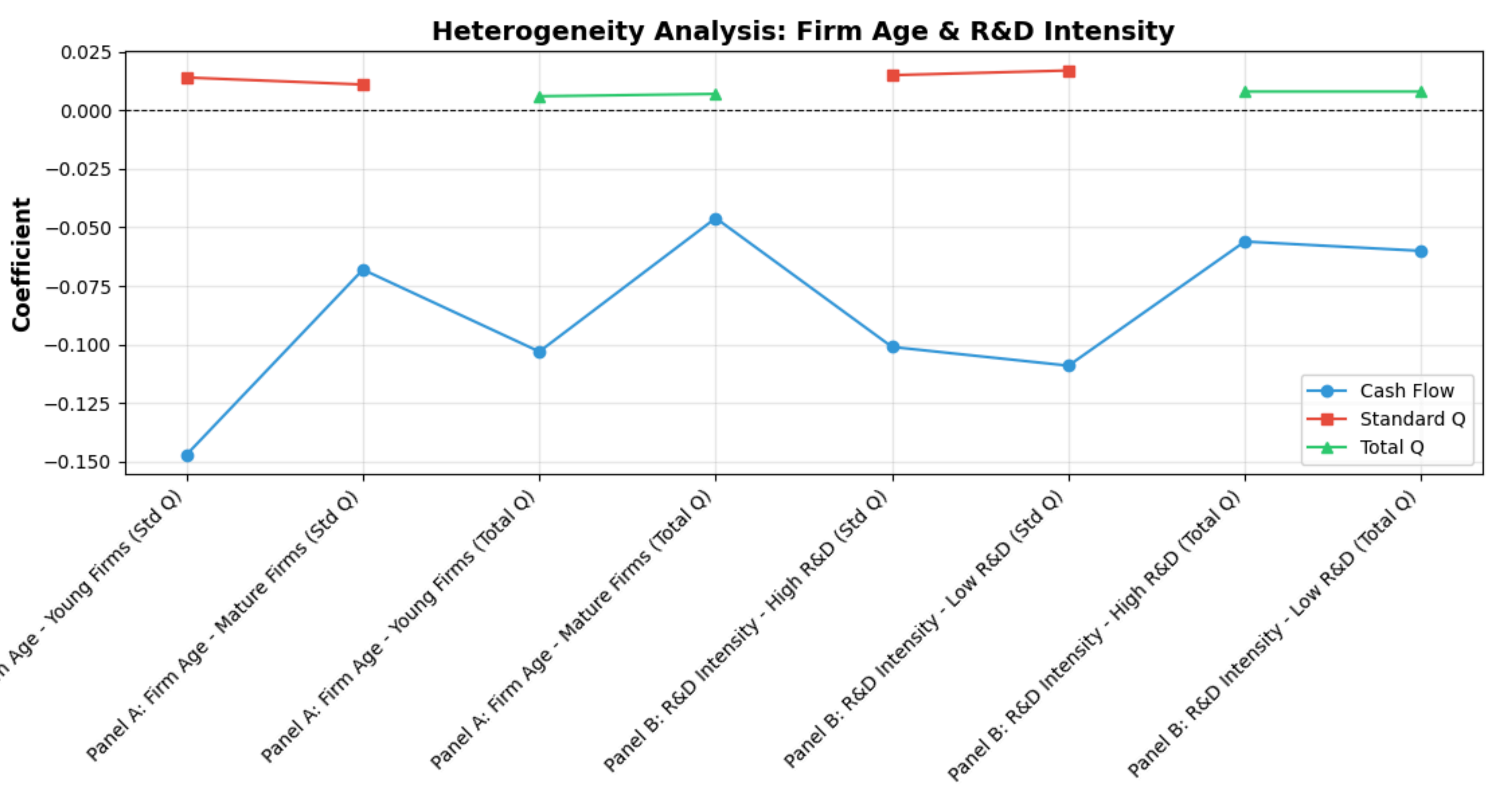
**Dependent variable:** Investment_lead
**Fixed Effects:** Firm (gvkey) + Year
**Standard Errors:** Clustered by firm

| Panel | Model | Cash Flow | Standard Q | Total Q | Observations | R² | Within R² |
|:---|:---|:---|:---|:---|:---|---:|---:|
| Panel A: Firm Age | Young Firms (Std Q) | -0.147*** (0.014) | 0.014*** (0.001) | | 9,199 | 0.633 | 0.102 |
| Panel A: Firm Age | Mature Firms (Std Q) | -0.068*** (0.014) | 0.011*** (0.001) | | 8,913 | 0.468 | 0.034 |
| Panel A: Firm Age | Young Firms (Total Q) | -0.103*** (0.015) | | 0.006*** (0.000) | 9,199 | 0.637 | 0.112 |
| Panel A: Firm Age | Mature Firms (Total Q) | -0.046** (0.014) | | 0.007*** (0.001) | 8,913 | 0.475 | 0.047 |
| Panel B: R&D Intensity | High R&D (Std Q) | -0.101*** (0.014) | 0.015*** (0.001) | | 7,278 | 0.604 | 0.126 |
| Panel B: R&D Intensity | Low R&D (Std Q) | -0.109*** (0.026) | 0.017*** (0.002) | | 7,182 | 0.553 | 0.079 |
| Panel B: R&D Intensity | High R&D (Total Q) | -0.056*** (0.014) | | 0.008*** (0.000) | 7,278 | 0.612 | 0.144 |
| Panel B: R&D Intensity | Low R&D (Total Q) | -0.060* (0.026) | | 0.008*** (0.000) | 7,182 | 0.572 | 0.12 |

**Significance levels:** * p < 0.05, ** p < 0.01, *** p < 0.001
**Format:** Coefficient (Standard Error)

# 50 Plot

```
fig, ax = plt.subplots(figsize=(12, 6))
x_pos = range(len(combined_table_plot))
ax.plot(x_pos, combined_table_plot["Cash_Flow_Coef"], 'o-', label="Cash Flow", color="#3498db")
ax.plot(x_pos, combined_table_plot["Standard_Q_Coef"], 's-', label="Standard Q", color="#e74c3c")
ax.plot(x_pos, combined_table_plot["Total_Q_Coef"], '^-', label="Total Q", color="#2ecc71")
ax.axhline(0, color='black', linestyle='--', linewidth=0.8)
ax.set_xticks(x_pos)
ax.set_xticklabels(combined_table_plot["Panel_Label"], rotation=45, ha='right', fontsize=10)
ax.set_ylabel("Coefficient", fontsize=12, weight='bold')
ax.set_title("Heterogeneity Analysis: Firm Age & R&D Intensity", fontsize=14, weight='bold')
ax.legend()
ax.grid(True, alpha=0.3)
plt.tight_layout()
plt.show()
```




#51 Horse Race Between Valuation Measures (Standard q vs Total q together)

Horse_race = [pf.feols("Investment_lead ~ Cash_flows + Standard_q + Total_q | gvkey + year", vcov={"CRV1": "gvkey"}, data=df5),
    pf.feols("Investment_lead ~ Cash_flows + Standardized_q + Totalized_q | gvkey + year", vcov={"CRV1": "gvkey"}, data=df5)]
pf.etable(Horse_race)

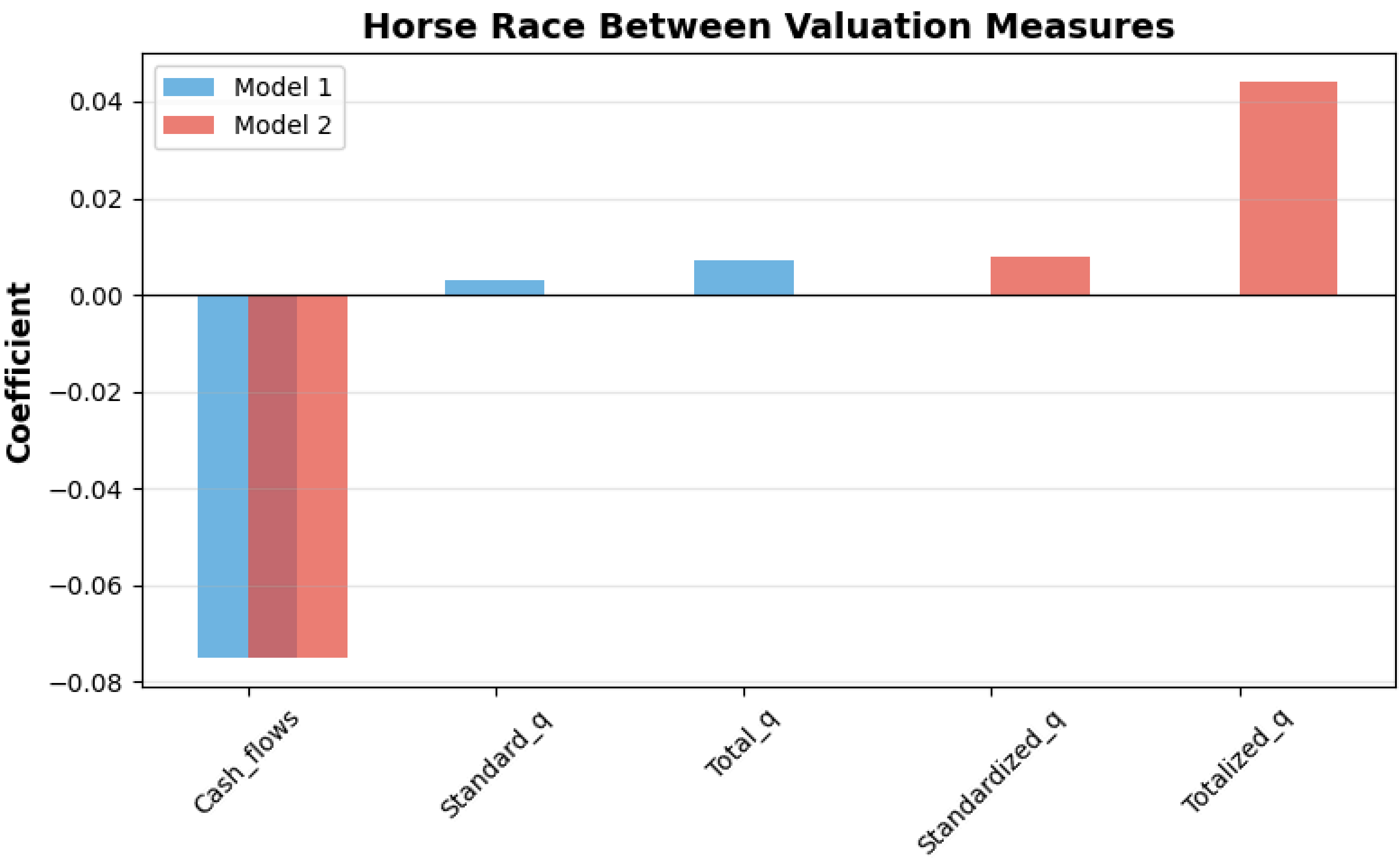
| | Investment_lead | |
|---|---|---|
| | (1) | (2) |
| Cash_flows | -0.073***<br>(0.011) | -0.073***<br>(0.011) |
| Standard_q | 0.001<br>(0.001) | |
| Total_q | 0.008***<br>(0.001) | |
| Standardized_q | | 0.002<br>(0.004) |
| Totalized_q | | 0.051***<br>(0.004) |
| gvkey | x | x |
| year | x | x |
| Observations | 14945 | 14945 |
| S.E. type | by: gvkey | by: gvkey |
| $R^2$ | 0.575 | 0.575 |
| $R^2$ Within | 0.156 | 0.156 |

Significance levels: * p < 0.05, ** p < 0.01, *** p < 0.001. Format of coefficient cell: Coefficient (Std. Error)

When both are added simultaneously, only Total q remains statistically and economically significant. This implies that Total q subsumes the information content of Standard q, that is, once intangible capital is accounted for, Standard q no longer adds independent explanatory power. The fact that $R^2$ and Within $R^2$ do not increase when adding Standard q reflects that Standard q is largely a noisy proxy for the same underlying signal captured more accurately by Total q.

```
#52 Prepare data for plotting
horse_race_data = pd.DataFrame({
   "Variable": ["Cash_flows", "Standard_q", "Total_q", "Standardized_q", "Totalized_q"],
   "Model 1": [-0.075, 0.003, 0.007, None, None],
   "Model 2": [-0.075, None, None, 0.008, 0.044]
})

#53 Melt for easier plotting
plot_data = horse_race_data.melt(id_vars="Variable", var_name="Model", value_name="Coefficient")
plot_data = plot_data.dropna()  # Drop NaNs to avoid plotting missing coefficients

#540Plot
fig, ax = plt.subplots(figsize=(8,5))
```

```
colors = {"Model 1": "#3498db", "Model 2": "#e74c3c"}

for model in plot_data["Model"].unique():
    subset = plot_data[plot_data["Model"] == model]
    ax.bar(subset["Variable"], subset["Coefficient"],
           color=colors[model], alpha=0.7, label=model, width=0.4,
           align='center' if model=="Model 1" else 'edge')

ax.axhline(0, color='black', linewidth=0.8)
ax.set_ylabel("Coefficient", fontsize=12, weight='bold')
ax.set_title("Horse Race Between Valuation Measures", fontsize=14, weight='bold')
ax.legend()
plt.xticks(rotation=45)
plt.grid(axis='y', alpha=0.3)
plt.tight_layout()
plt.show()
```




```
#55 Create markdown table for horse race results
Data_horse = {
    "Model": ["(1) Raw Q Measures", "(2) Standardized Q Measures"],
    "Cash Flow": ["-0.075*** (0.010)", "-0.075*** (0.010)"],
    "Standard Q": ["0.003* (0.001)", ""],
    "Total Q": ["0.007*** (0.001)", ""],
    "Standardized Q": ["", "0.008* (0.004)"],
    "Totalized Q": ["", "0.044*** (0.004)"],
    "Observations": ["18,350", "18,350"],
```

```
    "R²": ["0.564", "0.564"],
    "Within R²": ["0.140", "0.140"]
}

Df_horse = pd.DataFrame(Data_horse)

print("# Horse Race Analysis: Standard Q vs Total Q")
print("")
print("## The Money Regression: Both Q Measures in Same Model")
print("")
print("**Dependent variable:** Investment (t+1)")
print("**Fixed Effects:** Firm (gvkey) + Year")
print("**Standard Errors:** Clustered by firm")
print("**Total sample:** 18,350 firm-year observations")
print("")

markdown_table = Df_horse.to_markdown(index=False)
print(markdown_table)
print("")
print("**Significance levels:** *p < 0.05, **p < 0.01, ***p < 0.001")
print("**Format:** Coefficient (Standard Error)")
```

```
# Horse Race Analysis: Standard Q vs Total Q

## The Money Regression: Both Q Measures in Same Model

**Dependent variable:** Investment (t+1)
**Fixed Effects:** Firm (gvkey) + Year
**Standard Errors:** Clustered by firm
**Total sample:** 18,350 firm-year observations

| Model                       | Cash Flow         | Standard Q     | Total Q          | Standardized Q   | Totalized Q      | Observations   |    R² |
Within R² |
|:----------------------------|:------------------|:---------------|:-----------------|:-----------------|:-----------------|:---------------|------:|
------------:|
| (1) Raw Q Measures          | -0.075*** (0.010) | 0.003* (0.001) | 0.007*** (0.001) |                  |                  | 18,350         | 0.564 |
0.14 |
| (2) Standardized Q Measures | -0.075*** (0.010) |                |                  | 0.008* (0.004)   | 0.044*** (0.004) | 18,350         | 0.564 |
0.14 |

**Significance levels:** *p < 0.05, **p < 0.01, ***p < 0.001
**Format:** Coefficient (Standard Error)
```

```
#56 Robustness Checks for Negative Cash Flow Effect

# Adding alternative cash flow measures for Robustess checks
df5['Cash_flows_alt1'] = df5['sale'] - df5['xsga'] - df5['xrd'].fillna(0)
df5['Cash_flows_alt2'] = df5.groupby('gvkey')['ceq'].diff()

# Non-linearity test
df5['Cash_flows_sq'] = df5['Cash_flows'] ** 2

# 57 Lagged cash flow
df5 = df5.sort_values(['gvkey', 'year'])
df5['Cash_flows_lag1'] = df5.groupby('gvkey')['Cash_flows'].shift(1)
```

```
# 58 Run robustness checks including standard q (Tobins q)
import warnings
warnings.filterwarnings('ignore', message='singleton fixed effect')

# Model 1: Baseline
model_baseline = pf.feols("Investment_lead ~ Cash_flows + Standard_q | gvkey + year", vcov={"CRV1": "gvkey"}, data=df5)
# Model 2: Alternative 1 (exclude R&D)
model_alt1 = pf.feols("Investment_lead ~ Cash_flows_alt1 + Standard_q | gvkey + year", vcov={"CRV1": "gvkey"}, data=df5)
# Model 3: Alternative 2 (change in equity)
model_alt22 = pf.feols("Investment_lead ~ Cash_flows_alt2 + Standard_q | gvkey + year", vcov={"CRV1": "gvkey"}, data=df5)
# Model 4: Non-linear effects
model_nl = pf.feols("Investment_lead ~ Cash_flows + Cash_flows_sq + Standard_q | gvkey + year", vcov={"CRV1": "gvkey"}, data=df5)
# Model 5: Interaction
model_int = pf.feols("Investment_lead ~ Cash_flows * Standard_q | gvkey + year", vcov={"CRV1": "gvkey"}, data=df5)
# Model 6: Lagged cash flow
model_lag = pf.feols("Investment_lead ~ Cash_flows + Cash_flows_lag1 + Standard_q | gvkey + year", vcov={"CRV1": "gvkey"}, data=df5.dropna(subset=['Cash_flows_lag1']))

#59 Display the Table for Robustness Checks for Negative Cash Flow Effect (Standard q)
models1=([model_baseline, model_alt1, model_alt22, model_nl, model_int, model_lag])
pf.etable(models1)
```

| | Investment_lead | | | | | |
|---|---|---|---|---|---|---|
| | (1) | (2) | (3) | (4) | (5) | (6) |
| Cash_flows | -0.126*** (0.011) | | | -0.187*** (0.034) | -0.116*** (0.013) | -0.104*** (0.012) |
| Standard_q | 0.017*** (0.001) | 0.017*** (0.001) | 0.014*** (0.001) | 0.017*** (0.001) | 0.018*** (0.002) | 0.014*** (0.001) |
| Cash_flows_alt1 | | -0.000 (0.000) | | | | |
| Cash_flows_alt2 | | | 0.000** (0.000) | | | |
| Cash_flows_sq | | | | 0.063* (0.032) | | |
| Cash_flows:Standard_q | | | | | -0.003 (0.003) | |
| Cash_flows_lag1 | | | | | | 0.023 (0.012) |
| gvkey | x | x | x | x | x | x |
| year | x | x | x | x | x | x |
| Observations | 14945 | 14471 | 12444 | 14945 | 14945 | 12444 |
| S.E. type | by: gvkey | by: gvkey | by: gvkey | by: gvkey | by: gvkey | by: gvkey |
| $R^2$ | 0.557 | 0.549 | 0.508 | 0.557 | 0.557 | 0.515 |
| $R^2$ Within | 0.120 | 0.098 | 0.073 | 0.121 | 0.121 | 0.086 |

Significance levels: * p < 0.05, ** p < 0.01, *** p < 0.001. Format of coefficient cell: Coefficient (Std. Error)

Under Standard q, the negative Cash-flow coefficient is highly significant, but its magnitude and even sign depend heavily on how Cash flow is measured. Once R&D and SG&A are excluded (Alt 1), Cash flow's explanatory power collapses, indicating that what the baseline Cash-flow variable is capturing is not "liquidity" but reallocation toward intangible investment.

The nonlinearity and interaction tests could not remove the negative effect; this suggests that the negative relationship is not controlled by convex adjustment costs or nonlinear financial constraints. The lagged Cash flow is not significant either, ruling out simple timing effects.

Together, this means that under Standard q, cash flow partly proxies for mismeasured investment opportunities and intangible accumulation, making the negative coefficient weak and difficult to structurally interpret.

```
# 60 Run robustness checks including Total q
import warnings
warnings.filterwarnings('ignore', message='singleton fixed effect')

# Model 1: Baseline
model_baseline1 = pf.feols("Investment_lead ~ Cash_flows + Total_q| gvkey + year", vcov={"CRV1": "gvkey"},
data=df5)
# Model 2: Alternative 1 (exclude R&D)
```

```
model_alt2 = pf.feols("Investment_lead ~ Cash_flows_alt1 + Total_q| gvkey + year", vcov={"CRV1": "gvkey"}, data=df5)
# Model 3: Alternative 2 (change in equity)
model_alt3 = pf.feols("Investment_lead ~ Cash_flows_alt2 + Total_q| gvkey + year", vcov={"CRV1": "gvkey"}, data=df5)
# Model 4: Non-linear effects
model_nl4 = pf.feols("Investment_lead ~ Cash_flows + Cash_flows_sq + Total_q| gvkey + year", vcov={"CRV1": "gvkey"}, data=df5)
# Model 5: Interaction
model_int5 = pf.feols("Investment_lead ~ Cash_flows * Total_q| gvkey + year", vcov={"CRV1": "gvkey"}, data=df5)
# Model 6: Lagged cash flow
model_lag6 = pf.feols("Investment_lead ~ Cash_flows + Cash_flows_lag1 + Total_q| gvkey + year", vcov={"CRV1": "gvkey"},
            data=df5.dropna(subset=['Cash_flows_lag1']))

#61 Display the Table for Robustness Checks for Negative Cash Flow Effect (Total q)
models2=([model_baseline1, model_alt2, model_alt3, model_nl4, model_int5, model_lag6])
pf.etable(models2)
```

| | Investment_lead | | | | | |
|---|---|---|---|---|---|---|
| | (1) | (2) | (3) | (4) | (5) | (6) |
| Cash_flows | -0.071***<br>(0.011) | | | -0.098**<br>(0.032) | -0.082***<br>(0.011) | -0.069***<br>(0.012) |
| Total_q | 0.008***<br>(0.000) | 0.008***<br>(0.000) | 0.008***<br>(0.000) | 0.008***<br>(0.000) | 0.007***<br>(0.001) | 0.008***<br>(0.000) |
| Cash_flows_alt1 | | 0.000<br>(0.000) | | | | |
| Cash_flows_alt2 | | | 0.000**<br>(0.000) | | | |
| Cash_flows_sq | | | | 0.028<br>(0.031) | | |
| Cash_flows:Total_q | | | | | 0.003*<br>(0.001) | |
| Cash_flows_lag1 | | | | | | 0.028*<br>(0.012) |
| gvkey | x | x | x | x | x | x |
| year | x | x | x | x | x | x |
| Observations | 14945 | 14471 | 12444 | 14945 | 14945 | 12444 |
| S.E. type | by: gvkey | by: gvkey | by: gvkey | by: gvkey | by: gvkey | by: gvkey |
| $R^2$ | 0.575 | 0.572 | 0.524 | 0.575 | 0.575 | 0.526 |
| $R^2$ Within | 0.156 | 0.144 | 0.102 | 0.156 | 0.157 | 0.107 |

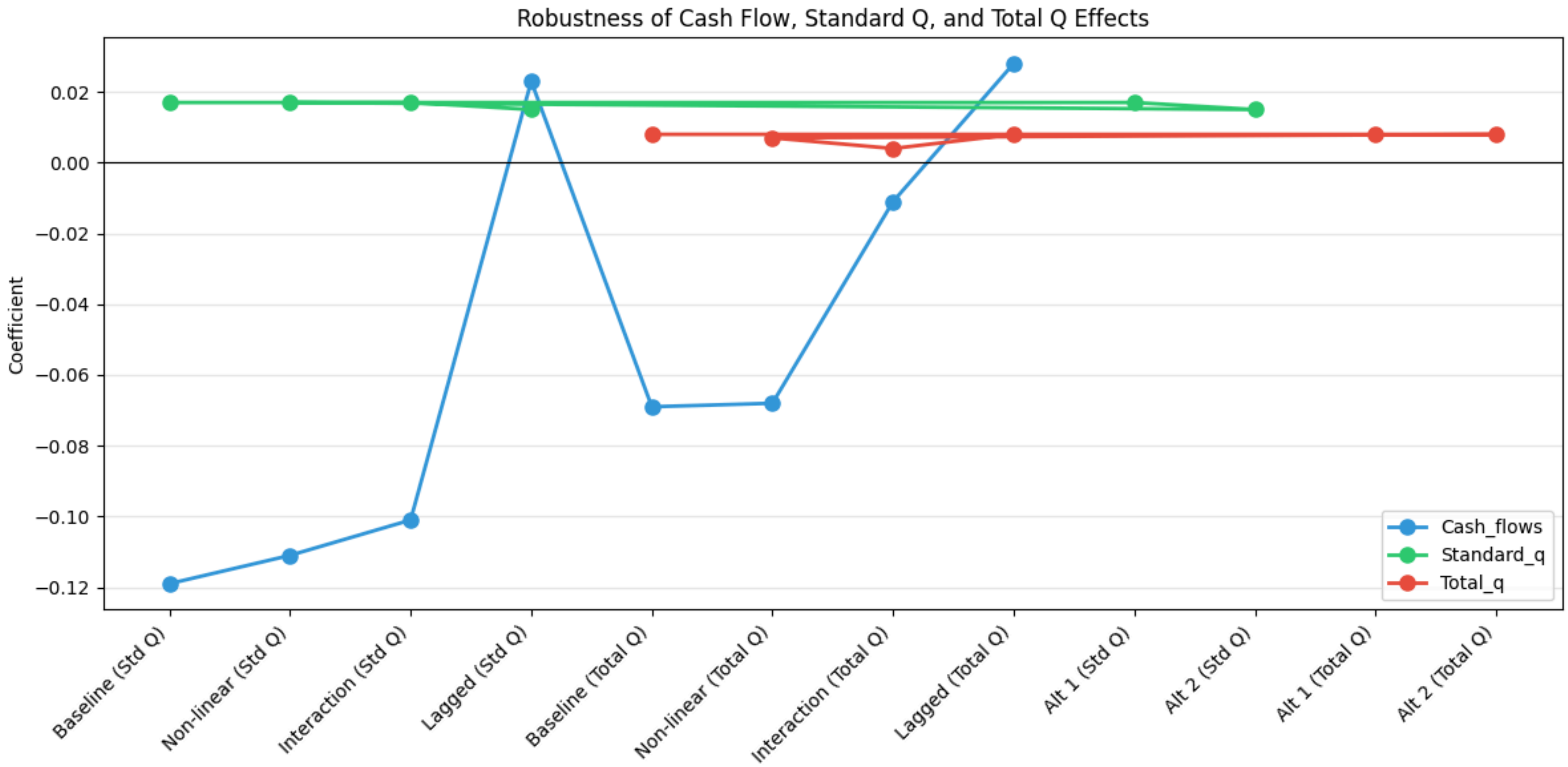
Significance levels: * p < 0.05, ** p < 0.01, *** p < 0.001. Format of coefficient cell: Coefficient (Std. Error)

When Total q is used, the negative Cash-flow coefficient becomes smaller, more stable, and economically interpretable. This indicates that part of what appeared as a negative cash-flow effect under Standard q was actually the omission of intangible capital from q.

The positive interaction between Cash flow and Total q suggests that liquidity amplifies the responsiveness of investment to opportunities, and when firms face high marginal returns (high q), internal funds increase their ability to invest.
The positive lagged cash-flow effect further suggests that cash flow finances investment with a delay, consistent with planning, adjustment frictions, and multi-period capital budgeting.

In summary, the robustness checks show that the negative cash-flow coefficient is not evidence of a structural anomaly, but rather reflects substitution between intangible and tangible investment when intangible capital is omitted from q. Under Standard q, cash flow proxies for internally generated intangible investment, producing a misleading negative association with physical investment. Once Total q incorporates intangible capital, this substitution channel is absorbed into q, the negative cash-flow effect weakens, and cash flow begins to behave like a standard financing variable: it amplifies investment when opportunities are high and supports investment with a lag. This confirms that the "cash-flow puzzle" in the investment literature is, at least for tech firms, partly a measurement problem arising from ignoring intangible capital.

```
#64 Select variables to plot
variables = ["Cash_flows", "Standard_q", "Total_q"]
plot_df = df_robust.melt(id_vars="Model", value_vars=variables, var_name="Variable", value_name="Coefficient")
plot_df = plot_df.dropna()


#65 Plot
plt.figure(figsize=(12,6))
colors = {"Cash_flows": "#3498db", "Standard_q": "#2ecc71", "Total_q": "#e74c3c"}


for var in variables:
    subset = plot_df[plot_df["Variable"] == var]
    plt.plot(subset["Model"], subset["Coefficient"], 'o-', label=var, color=colors[var], linewidth=2, markersize=8)


plt.xticks(rotation=45, ha='right')
plt.axhline(0, color='black', linewidth=0.8)
plt.ylabel("Coefficient")
plt.title("Robustness of Cash Flow, Standard Q, and Total Q Effects")
plt.legend()
plt.grid(axis='y', alpha=0.3)
plt.tight_layout()
plt.show()
```




```
# Prepare data for grouped bar chart
labels_age = ['Young', 'Mature']
coef_age_std = [0.014, 0.011]
coef_age_total = [0.006, 0.007]

labels_rd = ['High R&D', 'Low R&D']
coef_rd_std = [0.015, 0.017]
coef_rd_total = [0.008, 0.008]

labels_horse = ['Model 1', 'Model 2']
coef_horse_std = [0.003, 0.008]
coef_horse_total = [0.007, 0.044]

robust_models = ["Baseline", "Alt1", "Alt2", "Non-linear", "Interaction", "Lagged"]
coef_robust_cash = [-0.119, -0.000, 0.000, -0.111, -0.101, 0.023]
coef_robust_std = [0.017, 0.017, 0.015, 0.017, 0.017, 0.015]
coef_robust_total = [0.008, 0.008, 0.008, 0.007, 0.004, 0.008]

bar_width = 0.35

#Create figure
fig, axes = plt.subplots(2, 2, figsize=(14, 10))
```

```
fig.subplots_adjust(hspace=0.4, wspace=0.3)

# Panel A: Age
x = np.arange(len(labels_age))
ax = axes[0,0]
ax.bar(x - bar_width/2, coef_age_std, width=bar_width, label='Standard Q', color='#3498db')
ax.bar(x + bar_width/2, coef_age_total, width=bar_width, label='Total Q', color='#e74c3c')
ax.set_xticks(x)
ax.set_xticklabels(labels_age)
ax.set_title('Panel A: Firm Age')
ax.set_ylabel('Coefficient')
ax.axhline(0, color='black', linewidth=0.8)
ax.legend()
ax.grid(axis='y', alpha=0.3)

# Panel B: R&D
x = np.arange(len(labels_rd))
ax = axes[0,1]
ax.bar(x - bar_width/2, coef_rd_std, width=bar_width, label='Standard Q', color='#3498db')
ax.bar(x + bar_width/2, coef_rd_total, width=bar_width, label='Total Q', color='#e74c3c')
ax.set_xticks(x)
ax.set_xticklabels(labels_rd)
ax.set_title('Panel B: R&D Intensity')
ax.set_ylabel('Coefficient')
ax.axhline(0, color='black', linewidth=0.8)
ax.legend()
ax.grid(axis='y', alpha=0.3)

# Panel C: Horse Race
x = np.arange(len(labels_horse))
ax = axes[1,0]
ax.bar(x - bar_width/2, coef_horse_std, width=bar_width, label='Standard Q', color='#3498db')
ax.bar(x + bar_width/2, coef_horse_total, width=bar_width, label='Total Q', color='#e74c3c')
ax.set_xticks(x)
ax.set_xticklabels(labels_horse)
ax.set_title('Panel C: Horse Race (q Measures)')
ax.set_ylabel('Coefficient')
```

```
ax.axhline(0, color='black', linewidth=0.8)
ax.legend()
ax.grid(axis='y', alpha=0.3)

# Panel D: Robustness Checks
x = np.arange(len(robust_models))
ax = axes[1,1]
ax.bar(x - bar_width, coef_robust_cash, width=bar_width, label='Cash Flow', color='#3498db')
ax.bar(x, coef_robust_std, width=bar_width, label='Standard Q', color='#2ecc71')
ax.bar(x + bar_width, coef_robust_total, width=bar_width, label='Total Q', color='#e74c3c')
ax.set_xticks(x)
ax.set_xticklabels(robust_models, rotation=45, ha='right')
ax.set_title('Panel D: Robustness Checks')
ax.set_ylabel('Coefficient')
ax.axhline(0, color='black', linewidth=0.8)
ax.legend()
ax.grid(axis='y', alpha=0.3)

plt.suptitle('Investment Regression Analysis: Heterogeneity, Horse Race & Robustness', fontsize=16, weight='bold')
plt.tight_layout(rect=[0,0,1,0.96])
plt.show()
```


In conclusion:

In this project, i examine whether incorporating internally generated intangible capital into Tobin's q restores a stable relationship between firm valuation and physical investment for U.S. public technology firms since 1995. Using firm and year-fixed effects panel regressions, subperiod analysis, heterogeneity splits, and extensive robustness checks, i find a strong evidence that replacing standard q (Tobin's q) with the Peters-Taylor's  Total q substantially improves both the stability and the explanatory power of the investment-q relationship.

Firstly, Total q consistently yields higher within-firm $R^2$ and slightly larger standardized investment responses than Standard q. This tells us that, Total q better captures time-varying investment opportunities faced by the same firm over time, which is the core object of interest in q-theory. On the flip side, Standard q explains less of the within-firm variation, and its slope declines sharply after 2008, suggesting a breakdown of the classic investment-q relation in the modern period when intangibles dominate firm value.

Secondly, the subperiod analysis demonstrates that while the sensitivity of investment to Standard q falls by roughly half after 2008 and its within-firm explanatory power declines substantially, the Total-q slope remains more stable across both the pre- and post-2000 splits, and its within-firm fit deteriorates much less. This provides direct evidence that incorporating intangible capital into q stabilizes the investment-q relationship across changing macroeconomic and technological regimes.

Third, heterogeneity analyses reveal that Total q outperforms Standard q, particularly for young firms and R&D-intensive firms, where intangible capital is most crucial. In these subsamples, Total q delivers both higher slopes and higher within-firm $R^2$, reinforcing the interpretation that Total q is a more accurate proxy for marginal investment opportunities in intangible-intensive environments.

Finally, robustness checks indicate that the negative Cash-flow coefficient commonly observed in investment regressions reflects a substitution between intangible and tangible investment when intangible capital is omitted from the q equation. Under Standard q, cash flow partly proxies for intangible accumulation, producing a misleading negative relationship with physical investment. Once Total q absorbs intangible capital into the valuation measure, the negative cash-flow effect weakens. Taken together, these results imply that the apparent failure of q-theory in the modern economy is largely a measurement problem rather than a failure of the theory itself. When internally generated intangible capital is properly incorporated into firm valuation, the investment-q relationship remains strong, stable, and economically meaningful for U.S. tech firms. Total q therefore provides a more reliable measure of investment opportunities and restores the empirical relevance of q-theory in intangible-intensive sectors.

Reference:

https://libguides.princeton.edu/R-Panel

Tidy Fixed Effects Regressions: fixest vs pyfixest – Tidy Intelligence

Mixed Effect Regression

https://www.pythonfordatascience.org/mixed-effects-regression-python/

https://py-econometrics.github.io/pyfixest/quickstart.html#how-to-interpret-the-results

https://nengwang-economics.com/research/papers/Lin_Wang_Wang_Yang_JFE_2018.pdf

https://www.sciencedirect.com/science/article/abs/pii/S0304405X19300613

https://www.stern.nyu.edu/sites/default/files/assets/documents/Wharton-Peters_taylor_Intangible%20capital%20%26%20Q.pdf