PROBLEM SET 10.4

1–10 LINE INTEGRALS: EVALUATION BY GREEN'S THEOREM

Evaluate $\int_C \mathbf{F}(\mathbf{r}) \cdot d\mathbf{r}$ counterclockwise around the boundary C of the region R by Green's theorem, where

1. $\mathbf{F} = [y, -x]$, C the circle $x^2 + y^2 = 1/4$
2. $\mathbf{F} = [6y^2, 2x - 2y^4]$, R the square with vertices $\pm(2, 2)$, $\pm(2, -2)$
3. $\mathbf{F} = [x^2 e^y, y^2 e^x]$, R the rectangle with vertices $(0, 0)$, $(2, 0)$, $(2, 3)$, $(0, 3)$
4. $\mathbf{F} = [x \cosh 2y, 2x^2 \sinh 2y]$, R: $x^2 \leqq y \leqq x$
5. $\mathbf{F} = [x^2 + y^2, x^2 - y^2]$, R: $1 \leqq y \leqq 2 - x^2$
6. $\mathbf{F} = [\cosh y, -\sinh x]$, R: $1 \leqq x \leqq 3$, $x \leqq y \leqq 3x$
7. $\mathbf{F} = \operatorname{grad}(x^3 \cos^2(xy))$, R as in Prob. 5
8. $\mathbf{F} = [-e^{-x} \cos y, -e^{-x} \sin y]$, R the semidisk $x^2 + y^2 \leqq 16$, $x \geqq 0$

where $\mathbf{k}$ is a unit vector perpendicular to the xy-plane. **Verify** (10) and (11) for $\mathbf{F} = [7x, -3y]$ and C the circle $x^2 + y^2 = 4$ as well as for an example of your own choice.

13–17 INTEGRAL OF THE NORMAL DERIVATIVE

Using (9), find the value of $\int_C \frac{\partial w}{\partial n}\, ds$ taken counterclockwise over the boundary C of the region R.

13. $w = \cosh x$, R the triangle with vertices $(0, 0)$, $(4, 2)$, $(0, 2)$.
14. $w = x^2 y + xy^2$, R: $x^2 + y^2 \leqq 1$, $x \geqq 0$, $y \geqq 0$
15. $w = e^x \cos y + xy^3$, R: $1 \leqq y \leqq 10 - x^2$, $x \geqq 0$
16. $W = x^2 + y^2$, C: $x^2 + y^2 = 4$. Confirm the answer by direct integration.