(c) **Limit.** What is the limit in (b) as $n \to \infty$? Can you confirm your result by direct integration without referring to (b)?

(d) Show path dependence with a simple example of your choice involving two paths.

13. ***ML*-Inequality, Estimation of Line Integrals.** Let $\mathbf{F}$ be a vector function defined on a curve C. Let $|\mathbf{F}|$ be bounded, say, $|\mathbf{F}| \leqq M$ on C, where M is some positive number. Show that

$$\left| \int_C \mathbf{F} \cdot d\mathbf{r} \right| \leqq ML \qquad (L = \text{Length of } C). \tag{9}$$

14. Using (9), find a bound for the absolute value of the work W done by the force $\mathbf{F} = [x^2, y]$ in the displacement from (0, 0) straight to (3, 4). Integrate exactly and compare.

15–20 INTEGRALS (8) AND (8*)

Evaluate them with $\mathbf{F}$ or f and C as follows.

15. $\mathbf{F} = [y^2, z^2, x^2]$, C: $\mathbf{r} = [3 \cos t, 3 \sin t, 2t]$, $0 \leqq t \leqq 4\pi$

16. $f = 3x + y + 5z$, C: $\mathbf{r} = [t, \cosh t, \sinh t]$, $0 \leqq t \leqq 1$. Sketch C.

17. $\mathbf{F} = [x + y, y + z, z + x]$, C: $\mathbf{r} = [4 \cos t, \sin t, 0]$, $0 \leqq t \leqq \pi$

18. $\mathbf{F} = [y^{1/3}, x^{1/3}, 0]$, C the hypocycloid $\mathbf{r} = [\cos^3 t, \sin^3 t, 0]$, $0 \leqq t \leqq \pi/4$

19. $f = xyz$, C: $\mathbf{r} = [4t, 3t^2, 12t]$, $-2 \leqq t \leqq 2$. Sketch C.

20. $\mathbf{F} = [xz, yz, x^2y^2]$, C: $\mathbf{r} = [t, t, e^t]$, $0 \leqq t \leqq 5$. Sketch C.