4–8 CALCULUTION OF CURL

Find curl $\mathbf{v}$ for $\mathbf{v}$ given with respect to right-handed Cartesian coordinates. Show the details of your work.

4. $\mathbf{v} = [2y^2, 5x, 0]$
5. $\mathbf{v} = xyz\,[x, y, z]$
6. $\mathbf{v} = (x^2 + y^2 + z^2)^{-3/2}\,[x, y, z]$
7. $\mathbf{v} = [0, 0, e^{-x} \sin y]$
8. $\mathbf{v} = [e^{-z^2}, e^{-x^2}, e^{-y^2}]$

9–13 FLUID FLOW

Let $\mathbf{v}$ be the velocity vector of a steady fluid flow. Is the flow irrotational? Incompressible? Find the streamlines (the paths of the particles). *Hint.* See the answers to Probs. 9 and 11 for a determination of a path.

9. $\mathbf{v} = [0, 3z^2, 0]$
10. $\mathbf{v} = [\sec x, \csc x, 0]$
11. $\mathbf{v} = [y, -2x, 0]$
12. $\mathbf{v} = [-y, x, \pi]$
13. $\mathbf{v} = [x, y, -z]$

14. **PROJECT. Useful Formulas for the Curl.** Assuming sufficient differentiability, show that
    (a) $\operatorname{curl}(\mathbf{u} + \mathbf{v}) = \operatorname{curl} \mathbf{u} + \operatorname{curl} \mathbf{v}$
    (b) $\operatorname{div}(\operatorname{curl} \mathbf{v}) = 0$
    (c) $\operatorname{curl}(f\mathbf{v}) = (\operatorname{grad} f) \times \mathbf{v} + f \operatorname{curl} \mathbf{v}$
    (d) $\operatorname{curl}(\operatorname{grad} f) = \mathbf{0}$
    (e) $\operatorname{div}(\mathbf{u} \times \mathbf{v}) = \mathbf{v} \cdot \operatorname{curl} \mathbf{u} - \mathbf{u} \cdot \operatorname{curl} \mathbf{v}$

15–20 DIV AND CURL

With respect to right-handed coordinates, let $\mathbf{u} = [y, z, x]$, $\mathbf{v} = [yz, zx, xy]$, $f = xyz$, and $g = x + y + z$. Find the given expressions. Check your result by a formula in Proj. 14 if applicable.

15. $\operatorname{curl}(\mathbf{u} + \mathbf{v})$, $\operatorname{curl} \mathbf{v}$
16. $\operatorname{curl}(g\mathbf{v})$
17. $\mathbf{v} \cdot \operatorname{curl} \mathbf{u}$, $\mathbf{u} \cdot \operatorname{curl} \mathbf{v}$, $\mathbf{u} \cdot \operatorname{curl} \mathbf{u}$
18. $\operatorname{div}(\mathbf{u} \times \mathbf{v})$
19. $\operatorname{curl}(g\mathbf{u} + \mathbf{v})$, $\operatorname{curl}(g\mathbf{u})$
20. $\operatorname{div}(\operatorname{grad}(fg))$