1–10 PARAMETRIC REPRESENTATIONS

What curves are represented by the following? Sketch them.

1. $[3 + 2 \cos t, 2 \sin t, 0]$
2. $[a + t, b + 3t, c - 5t]$
3. $[0, t, t^3]$
4. $[-2, 2 + 5 \cos t, -1 + 5 \sin t]$
5. $[2 + 4 \cos t, 1 + \sin t, 0]$
6. $[a + 3 \cos \pi t, b - 2 \sin \pi t, 0]$
7. $[4 \cos t, 4 \sin t, 3t]$
8. $[\cosh t, \sinh t, 2]$
9. $[\cos t, \sin 2t, 0]$
10. $[t, 2, 1/t]$

11–20 FIND A PARAMETRIC REPRESENTATION

11. Circle in the plane $z = 1$ with center $(3, 2)$ and passing through the origin.
12. Circle in the yz-plane with center $(4, 0)$ and passing through $(0, 3)$. Sketch it.
13. Straight line through $(2, 1, 3)$ in the direction of $\mathbf{i} + 2\mathbf{j}$.
14. Straight line through $(1, 1, 1)$ and $(4, 0, 2)$. Sketch it.
15. Straight line $y = 4x - 1$, $z = 5x$.
16. The intersection of the circular cylinder of radius 1 about the z-axis and the plane $z = y$.
17. Circle $\frac{1}{2}x^2 + y^2 = 1$, $z = y$.
18. Helix $x^2 + y^2 = 25$, $z = 2 \arctan (y/x)$.
19. Hyperbola $4x^2 - 3y^2 = 4$, $z = -2$.
20. Intersection of $2x - y + 3z = 2$ and $x + 2y - z = 3$.
21. **Orientation.** Explain why setting $t = -t^*$ reverses the orientation of $[a \cos t, a \sin t, 0]$.
22. **CAS PROJECT. Curves.** Graph the following more complicated curves:

    (a) $\mathbf{r}(t) = [2 \cos t + \cos 2t, 2 \sin t - \sin 2t]$ (*Steiner's hypocycloid*).

    (b) $\mathbf{r}(t) = [\cos t + k \cos 2t, \sin t - k \sin 2t]$ with $k = 10, 2, 1, \frac{1}{2}, 0, -\frac{1}{2}, -1$.

    (c) $\mathbf{r}(t) = [\cos t, \quad \sin 5t]$ (a *Lissajous curve*).

    (d) $\mathbf{r}(t) = [\cos t, \quad \sin kt]$. For what k's will it be closed?

    (e) $\mathbf{r}(t) = [R \sin \omega t + \omega R t, \quad R \cos \omega t + R]$ (*cycloid*).

24–28 TANGENT

Given a curve C: $\mathbf{r}(t)$, find a tangent vector $\mathbf{r}'(t)$, a unit tangent vector $\mathbf{u}'(t)$, and the tangent of C at P. Sketch curve and tangent.

24. $\mathbf{r}(t) = [t, \frac{1}{2}t^2, 1]$, $\quad P$: $(2, 2, 1)$
25. $\mathbf{r}(t) = [10 \cos t, 1, 10 \sin t]$, $\quad P$: $(6, 1, 8)$
26. $\mathbf{r}(t) = [\cos t, \sin t, 9t]$, $\quad P$: $(1, 0, 18\pi)$
27. $\mathbf{r}(t) = [t, 1/t, 0]$, $\quad P$: $(2, \frac{1}{2}, 0)$
28. $\mathbf{r}(t) = [t, t^2, t^3]$, $\quad P$: $(1, 1, 1)$

29–32 LENGTH

Find the length and sketch the curve.

29. Catenary $\mathbf{r}(t) = [t, \cosh t]$ from $t = 0$ to $t = 1$.
30. **Circular helix** $\mathbf{r}(t) = [4 \cos t, 4 \sin t, 5t]$ from $(4, 0, 0)$ to $(4, 0, 10\pi)$.

31. **Circle** $\mathbf{r}(t) = [a \cos t, a \sin t]$ from $(a, 0)$ to $(0, a)$.

32. **Hypocycloid** $\mathbf{r}(t) = [a \cos^3 t, a \sin^3 t]$, total length.

33. **Plane curve.** Show that Eq. (10) implies $\ell = \int_a^b \sqrt{1 + y'^2}\, dx$ for the length of a plane curve $C: y = f(x), z = 0$, and $a = x = b$.

34. **Polar coordinates** $\rho = \sqrt{x^2 + y^2}, \theta = \arctan(y/x)$ give

$$\ell = \int_\alpha^\beta \sqrt{\rho^2 + \rho'^2}\, d\theta,$$

where $\rho' = d\rho/d\theta$. Derive this. Use it to find the total length of the **cardioid** $\rho = a(1 - \cos\theta)$. Sketch this curve. *Hint.* Use (10) in App. 3.1.

35–46 CURVES IN MECHANICS

Forces acting on moving objects (cars, airplanes, ships, etc.) require the engineer to know corresponding ***tangential*** and ***normal accelerations***. In Probs. 35–38 find them, along with the ***velocity*** and ***speed***. Sketch the path.

35. **Parabola** $\mathbf{r}(t) = [t, t^2, 0]$. Find $\mathbf{v}$ and $\mathbf{a}$.

36. **Straight line** $\mathbf{r}(t) = [8t, 6t, 0]$. Find $\mathbf{v}$ and $\mathbf{a}$.

37. **Cycloid** $\mathbf{r}(t) = (R \sin \omega t + Rt)\mathbf{i} + (R \cos \omega t + R)\mathbf{j}$. This is the path of a point on the rim of a wheel of radius R that rolls without slipping along the x-axis. Find $\mathbf{v}$ and $\mathbf{a}$ at the maximum y-values of the curve.

38. **Ellipse** $\mathbf{r} = [\cos t, 2 \sin t, 0]$.