

PROBLEM SET 10.6

1-10 FLUX INTEGRALS (3) $\int_S \mathbf{F} \cdot \mathbf{n} \, dA$

Evaluate the integral for the given data. Describe the kind of surface. Show the details of your work.

- $\mathbf{F} = [-x^2, y^2, 0]$, $S: \mathbf{r} = [u, v, 3u - 2v]$,
 $0 \leq u \leq 1.5$, $-2 \leq v \leq 2$
- $\mathbf{F} = [e^y, e^x, 1]$, $S: x + y + z = 1$, $x \geq 0$, $y \geq 0$,
 $z \geq 0$
- $\mathbf{F} = [0, x, 0]$, $S: x^2 + y^2 + z^2 = 1$, $x \geq 0$,
 $y \geq 0$, $z \geq 0$
- $\mathbf{F} = [e^y, -e^z, e^x]$, $S: x^2 + y^2 = 25$, $x \geq 0$,
 $y \geq 0$, $0 \leq z \leq 2$
- $\mathbf{F} = [x, y, z]$, $S: \mathbf{r} = [u \cos v, u \sin v, u^2]$,
 $0 \leq u \leq 4$, $-\pi \leq v \leq \pi$
- $\mathbf{F} = [\cosh y, 0, \sinh x]$, $S: z = x + y^2$, $0 \leq y \leq x$,
 $0 \leq x \leq 1$
- $\mathbf{F} = [0, \sin y, \cos z]$, S the cylinder $x = y^2$, where
 $0 \leq y \leq \pi/4$ and $0 \leq z \leq y$
- $\mathbf{F} = [\tan xy, x, y]$, $S: y^2 + z^2 = 1$, $2 \leq x \leq 5$,
 $y \geq 0$, $z \geq 0$
- $\mathbf{F} = [0, \sinh z, \cosh x]$, $S: x^2 + z^2 = 4$,
 $0 \leq x \leq 1/\sqrt{2}$, $0 \leq y \leq 5$, $z \geq 0$
- $\mathbf{F} = [y^2, x^2, z^4]$, $S: z = 4\sqrt{x^2 + y^2}$, $0 \leq z \leq 8$,
 $y \geq 0$
- CAS EXPERIMENT. Flux Integral.** Write a program for evaluating surface integrals (3) that prints intermediate results (\mathbf{F} , $\mathbf{F} \cdot \mathbf{N}$, the integral over one of

the two variables). Can you obtain experimentally some rules on functions and surfaces giving integrals that can be evaluated by the usual methods of calculus? Make a list of positive and negative results.

12-16 SURFACE INTEGRALS (6) $\iint_S G(\mathbf{r}) \, dA$

Evaluate these integrals for the following data. Indicate the kind of surface. Show the details.

- $G = \cos x + \sin x$, S the portion of $x + y + z = 1$ in the first octant
- $G = x + y + z$, $z = x + 2y$, $0 \leq x \leq \pi$,
 $0 \leq y \leq x$
- $G = ax + by + cz$, $S: x^2 + y^2 + z^2 = 1$, $y = 0$,
 $z = 0$
- $G = (1 + 9xz)^{3/2}$, $S: \mathbf{r} = [u, v, u^3]$, $0 \leq u \leq 1$,
 $-2 \leq v \leq 2$
- $G = \arctan(y/x)$, $S: z = x^2 + y^2$, $1 \leq z \leq 9$,
 $x \geq 0$, $y \geq 0$
- Fun with Möbius.** Make Möbius strips from long slim rectangles R of grid paper (graph paper) by pasting the short sides together after giving the paper a half-twist. In each case count the number of parts obtained by cutting along lines parallel to the edge. (a) Make R three squares wide and cut until you reach the beginning. (b) Make R four squares wide. Begin cutting one square away from the edge until you reach the beginning. Then cut the portion that is still two squares wide. (c) Make