PROBLEM SET 10.7

1–8 APPLICATION: MASS DISTRIBUTION

Find the total mass of a mass distribution of density σ in a region T in space.

1. $\sigma = x^2 + y^2 + z^2$, T the box $|x| \leqq 4$, $|y| \leqq 1$, $0 \leqq z \leqq 2$
2. $\sigma = xyz$, T the box $0 \leqq x \leqq a$, $0 \leqq y \leqq b$, $0 \leqq z \leqq c$
3. $\sigma = e^{-x-y-z}$, T: $0 \leqq x \leqq 1 - y$, $0 \leqq y \leqq 1$, $0 \leqq z \leqq 2$
4. σ as in Prob. 3, T the tetrahedron with vertices (0, 0, 0), (3, 0, 0), (0, 3, 0), (0, 0, 3)
5. $\sigma = \sin 2x \cos 2y$, T: $0 \leqq x \leqq \frac{1}{4}\pi$, $\frac{1}{4}\pi - x \leqq y \leqq \frac{1}{4}\pi$, $0 \leqq z \leqq 6$
6. $\sigma = x^2y^2z^2$, T the cylindrical region $x^2 + z^2 \leqq 16$, $|y| \leqq 4$
7. $\sigma = \arctan(y/x)$, T: $x^2 + y^2 + z^2 \leqq a^2$, $z \geqq 0$
8. $\sigma = x^2 + y^2$, T as in Prob. 7

9–18 APPLICATION OF THE DIVERGENCE THEOREM

Evaluate the surface integral $\iint_S \mathbf{F} \cdot \mathbf{n}\, dA$ by the divergence theorem. Show the details.

9. $\mathbf{F} = [x^2, 0, z^2]$, S the surface of the box $|x| \leqq 1$, $|y| \leqq 3$, $0 \leqq z \leqq 2$
10. Solve Prob. 9 by direct integration.
11. $\mathbf{F} = [e^x, e^y, e^z]$, S the surface of the cube $|x| \leqq 1$, $|y| \leqq 1$, $|z| \leqq 1$
12. $\mathbf{F} = [x^3 - y^3, y^3 - z^3, z^3 - x^3]$, S the surface of $x^2 + y^2 + z^2 \leqq 25$, $z \geqq 0$
13. $\mathbf{F} = [\sin y, \cos x, \cos z]$, S, the surface of $x^2 + y^2 \leqq 4$, $|z| \leqq 2$ (a cylinder and two disks!)
14. $\mathbf{F}$ as in Prob. 13, S the surface of $x^2 + y^2 \leqq 9$, $0 \leqq z \leqq 2$

15. $\mathbf{F} = [2x^2, \frac{1}{2}y^2, \sin \pi z]$, S the surface of the tetrahedron with vertices (0, 0, 0), (1, 0, 0), (0, 1, 0), (0, 0, 1)
16. $\mathbf{F} = [\cosh x, z, y]$, S as in Prob. 15
17. $\mathbf{F} = [x^2, y^2, z^2]$, S the surface of the cone $x^2 + y^2 \leqq z^2$, $0 \leqq z \leqq h$
18. $\mathbf{F} = [xy, yz, zx]$, S the surface of the cone $x^2 + y^2 \leqq 4z^2$, $0 \leqq z \leqq 2$

19–23 APPLICATION: MOMENT OF INERTIA

Given a mass of density 1 in a region T of space, find the moment of intertia about the x-axis

$$I_x = \iiint_T (y^2 + z^2)\, dx\, dy\, dz.$$

19. The box $-a \leqq x \leqq a$, $-b \leqq y \leqq b$, $-c \leqq z \leqq c$
20. The ball $x^2 + y^2 + z^2 \leqq a^2$
21. The cylinder $y^2 + z^2 \leqq a^2$, $0 \leqq x \leqq h$
22. The paraboloid $y^2 + z^2 \leqq x$, $0 \leqq x \leqq h$
23. The cone $y^2 + z^2 \leqq x^2$, $0 \leqq x \leqq h$
24. Why is I_x in Prob. 23 for large h larger than I_x in Prob. 22 (and the same h)? Why is it smaller for $h = 1$? Give physical reason.
25. Show that for a solid of revolution, $I_x = \dfrac{\pi}{2}\displaystyle\int_0^h r^4(x)\, dx.$

    Solve Probs. 20–23 by this formula.