PROBLEM SET 9.4

1–8 SCALAR FIELDS IN THE PLANE

Let the temperature T in a body be independent of z so that it is given by a scalar function $T = T(x, t)$. Identify the isotherms $T(x, y) = \text{const}$. Sketch some of them.

1. $T = x^2 - y^2$

2. $T = xy$

3. $T = 3x - 4y$

4. $T = \arctan(y/x)$

5. $T = y/(x^2 + y^2)$

6. $T = x/(x^2 + y^2)$

7. $T = 9x^2 + 4y^2$

8. CAS PROJECT. Scalar Fields in the Plane. Sketch or graph isotherms of the following fields and describe what they look like.

(a) $x^2 - 4x - y^2$ **(b)** $x^2y - y^3/3$

(c) $\cos x \sinh y$ **(d)** $\sin x \sinh y$

(e) $e^x \sin y$ **(f)** $e^{2x} \cos 2y$

(g) $x^4 - 6x^2y^2 + y^4$ **(h)** $x^2 - 2x - y^2$

9–14 SCALAR FIELDS IN SPACE

What kind of surfaces are the **level surfaces** $f(x, y, z) = \text{const}$?

9. $f = 4x - 3y + 2z$

10. $f = 9(x^2 + y^2) + z^2$

11. $f = 5x^2 + 2y^2$

12. $f = z - \sqrt{x^2 + y^2}$

13. $f = z - (x^2 + y^2)$

14. $f = x - y^2$

15–20 VECTOR FIELDS

Sketch figures similar to Fig. 198. Try to interpet the field of $\mathbf{v}$ as a velocity field.

15. $\mathbf{v} = \mathbf{i} + \mathbf{j}$

16. $\mathbf{v} = -y\mathbf{i} + x\mathbf{j}$

17. $\mathbf{v} = x\mathbf{j}$

18. $\mathbf{v} = x\mathbf{i} + y\mathbf{j}$

19. $\mathbf{v} = x\mathbf{i} - y\mathbf{j}$

20. $\mathbf{v} = y\mathbf{i} - x\mathbf{j}$

21. CAS PROJECT. Vector Fields. Plot by arrows:

(a) $\mathbf{v} = [x, x^2]$

(b) $\mathbf{v} = [1/y, 1/x]$

(c) $\mathbf{v} = [\cos x, \sin x]$

(d) $\mathbf{v} = e^{-(x^2+y^2)}[x, -y]$

22–25 DIFFERENTIATION

22. Find the first and second derivatives of $\mathbf{r} = [3 \cos 2t, 3 \sin 2t, 4t]$.

23. Prove (11)–(13). Give two typical examples for each formula.

24. Find the first partial derivatives of $\mathbf{v}_1 = [e^x \cos y, e^x \sin y]$ and $\mathbf{v}_2 = [\cos x \cosh y, -\sin x \sinh y]$.

25. WRITING PROJECT. Differentiation of Vector Functions. Summarize the essential ideas and facts and give examples of your own.