PROBLEM SET 10.9

1–10 DIRECT INTEGRATION OF SURFACE INTEGRALS

Evaluate the surface integral $\iint_S (\text{curl } \mathbf{F}) \bullet \mathbf{n} \, dA$ directly for the given $\mathbf{F}$ and S.

1. $\mathbf{F} = [z^2, -x^2, 0]$, S the rectangle with vertices $(0, 0, 0)$, $(1, 0, 0)$, $(0, 4, 4)$, $(1, 4, 4)$
2. $\mathbf{F} = [-13 \sin y, 3 \sinh z, x]$, S the rectangle with vertices $(0, 0, 2)$, $(4, 0, 2)$, $(4, \pi/2, 2)$, $(0, \pi/2, 2)$
3. $\mathbf{F} = [e^{-z}, e^{-z} \cos y, e^{-z} \sin y]$, S: $z = y^2/2$, $-1 \leqq x \leqq 1$, $0 \leqq y \leqq 1$
4. $\mathbf{F}$ as in Prob. 1, $z = xy$ $(0 \leqq x \leqq 1,\ 0 \leqq y \leqq 4)$. Compare with Prob. 1.
5. $\mathbf{F} = [z^2, \frac{3}{2}x, 0]$, S: $0 \leqq x \leqq a$, $0 \leqq y \leqq a$, $z = 1$
6. $\mathbf{F} = [y^3, -x^3, 0]$, S: $x^2 + y^2 \leqq 1$, $z = 0$
7. $\mathbf{F} = [e^y, e^z, e^x]$, S: $z = x^2$ $(0 \leqq x \leqq 2,\ 0 \leqq y \leqq 1)$
8. $\mathbf{F} = [z^2, x^2, y^2]$, S: $z = \sqrt{x^2 + y^2}$, $y \geqq 0$, $0 \leqq z \leqq h$
9. Verify Stokes's theorem for $\mathbf{F}$ and S in Prob. 5.
10. Verify Stokes's theorem for $\mathbf{F}$ and S in Prob. 6.
11. **Stokes's theorem not applicable.** Evaluate $\oint_C \mathbf{F} \bullet \mathbf{r}' \, ds$, $\mathbf{F} = (x^2 + y^2)^{-1}[-y, x]$, C: $x^2 + y^2 = 1$, $z = 0$, oriented clockwise. Why can Stokes's theorem not be applied? What (false) result would it give?
12. **WRITING PROJECT. Grad, Div, Curl in Connection with Integrals.** Make a list of ideas and results on this topic in this chapter. See whether you can rearrange or combine parts of your material. Then subdivide the material into 3–5 portions and work out the details of each portion. Include no proofs but simple typical examples of your own that lead to a better understanding of the material.

13–20 EVALUATION OF $\oint_C \mathbf{F} \bullet \mathbf{r}' \, ds$

Calculate this line integral by Stokes's theorem for the given $\mathbf{F}$ and C. Assume the Cartesian coordinates to be right-handed and the z-component of the surface normal to be nonnegative.

13. $\mathbf{F} = [-5y, 4x, z]$, C the circle $x^2 + y^2 = 16$, $z = 4$