Assignment 13b-

Problem 19

conditions. State the difference between Fig. 291 in Sec. 12.3 and Fig. 295.

5–7 LATERALLY INSULATED BAR

Find the temperature $u(x, t)$ in a bar of silver of length 10 cm and constant cross section of area 1 cm^2 (density 10.6 g/cm^3, thermal conductivity 1.04 cal/(cm sec °C), specific heat 0.056 cal/(g °C) that is perfectly insulated laterally, with ends kept at temperature 0°C and initial temperature $f(x)$ °C, where

5. $f(x) = \sin 0.1\pi x$

6. $f(x) = 4 - 0.8|x - 5|$

7. $f(x) = x(10 - x)$

8. Arbitrary temperatures at ends. If the ends $x = 0$ and $x = L$ of the bar in the text are kept at constant temperatures U_1 and U_2, respectively, what is the temperature $u_1(x)$ in the bar after a long time (theoretically, as $t \to \infty$)? First guess, then calculate.

9. In Prob. 8 find the temperature at any time.

10. Change of end temperatures. Assume that the ends of the bar in Probs. 5–7 have been kept at 100°C for a long time. Then at some instant, call it $t = 0$, the temperature at $x = L$ is suddenly changed to 0°C and kept at 0°C, whereas the temperature at $x = 0$ is kept at 100°C. Find the temperature in the middle of the bar at $t = 1, 2, 3, 10, 50$ sec. First guess, then calculate.

18–25 TWO-DIMENSIONAL PROBLEMS

18. Laplace equation. Find the potential in the rectangle $0 \leqq x \leqq 20, 0 \leqq y \leqq 40$ whose upper side is kept at potential 110 V and whose other sides are grounded.

19. Find the potential in the square $0 \leqq x \leqq 2, 0 \leqq y \leqq 2$ if the upper side is kept at the potential 1000 $\sin \frac{1}{2}\pi x$ and the other sides are grounded.

20. CAS PROJECT. Isotherms. Find the steady-state solutions (temperatures) in the square plate in Fig. 297 with $a = 2$ satisfying the following boundary conditions. Graph isotherms.

(a) $u = 80 \sin \pi x$ on the upper side, 0 on the others.

(b) $u = 0$ on the vertical sides, assuming that the other sides are perfectly insulated.

(c) Boundary conditions of your choice (such that the solution is not identically zero).

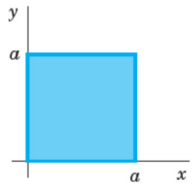

Fig. 297. Square plate